Universitat Autònoma de Barcelona

Departament de Física

Grup de Física Teòrica

Symmetry breaking in particle physics from extra dimensions.

Ruptura de simetrías en física de partículas
mediante dimensiones extra.

Leandro Da Rold

Universitat Autònoma de Barcelona
Grup de Física Teòrica
Institut de Física d'Altes Energies

2005/2006

Memoria presentada para optar al grado de
Doctor en Física

Director:
Dr. Àlex Pomarol Clotet

a Natalia

Contents

Chapter 1

Introduction

Symmetry is at the basis of our knowledge of nature. It has been one of the most powerful tools to build our present understanding in theoretical physics. Let us start by giving a few examples. The theoretical framework that provides the basis of the fundamental high energy description of nature is quantum field theory. The standard model (SM) is the quantum field theory that describes, with an incredible precision, the non-gravitational interactions between the fundamental particles: the strong and electroweak (EW) gauge interactions. The basic guide to derive the SM from the experimental results has been symmetry principles. Moreover, the basic ingredients that lead to a consistent quantum field theory are: symmetry principles, quantum mechanics and the cluster decomposition principle [1]. The classical theory of gravity, described by General Relativity, is one of the most beautiful examples of how far we can go by applying symmetry principles. Also the Hamiltonian formulation of classical mechanics and the classical unification of electricity and magnetism are based on symmetries.

Thus symmetry has played a crucial role as a guide to build our present knowledge on the fundamental laws of nature. However, there are many symmetries that are only partially observed in nature. As a typical example we can mention the Heisenberg ferromagnet, an array of interacting spin 1/2 magnetic dipoles. Although the Hamiltonian describing this system is rotationally invariant, the ground state is aligned in some direction breaking the original symmetry. Concerning particle physics, the usual example is chiral symmetry that, although spontaneously broken by quark and gluon condensates and explicitly by the quark masses, provides one of the first successful phenomenological models of hadrons. Other examples are the EW symmetry of the SM, that is spontaneously broken and the scale invariance of quantum chromodynamics, broken by quantum effects. The list is very long, but we want to stress here that breaking symmetries is as important as symmetries they-self. In general, symmetries imply relations between physical quantities, thus breaking symmetries is not an easy task if one wants to preserve some properties arising from these relations. One of the problems of the breakdown of symmetries is that it is not natural to impose different symmetries to different sectors of a theory. Quantum effects will in general induce divergent breaking terms, spoiling the properties inherited from the original invariance. Much of the current research is directly related with the study and comprehension of this subject. For these reasons this thesis is devoted to the study of symmetry breaking in particle physics.

In this thesis we will consider the possibility of breaking symmetries in theories with extra dimensions. The main motivation is that physics of extra dimensions provides new mechanisms of symmetry breaking. Some of them are based on the possibility of having different compactifications of the extra space. Compactifying the extra dimensions in a smooth manifold the Lorentz invariance is spoiled. The fields propagating in the compact extra dimensions have to satisfy a periodicity condition, thus if the lagrangian is invariant under some symmetry group, it is possible to impose non-trivial boundary conditions for the bulk fields, that can be used to break 4D symmetries. This is the Scherk-Schwarz mechanism [2]. If the manifold has singular points translation invariance along the extra dimensions is also broken, the simplest example is given by one extra dimension compactified on a segment. In this case there are fields that can propagate only along the boundaries of the space, thus it is also possible to break symmetries by boundary conditions (assigning different boundary conditions to the components of a given multiplet). Therefore the symmetry is spoiled on the boundaries, and from the 4D point of view the theory is no longer symmetric. We want to stress here the following property of symmetry breaking in extra dimensions: physics of extra dimensions offer the possibility of breaking symmetries by 4D non-local mechanisms. One example is given by the Scherk-Schwarz mechanism described above, where the symmetry breaking is a global effect due to the compactification of the space. As a second example let us assume that two sectors of a given theory are localized on different boundaries and respect different symmetries. By exchange of bulk fields the boundary sectors can be communicated, thus breaking the different symmetries. Since the breakdown is a nonlocal effect in the extra dimension, it leads to finite corrections of the symmetric sector. An example is given by the Hosotani mechanism for symmetry breaking [3]. See Ref. [4] for a complete list of references and a careful description of the Hosotani mechanism.

In this thesis we will consider the breakdown of the chiral and the electroweak symmetries in the SM. The chiral symmetry of QCD plays a very important role in the low energy description of hadrons. The QCD lagrangian with massless u and d quarks has an exact $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ chiral symmetry. This symmetry, if exact and unbroken, would require any hadron state to exhibit a degeneracy with another hadron state of opposite parity and equal spin, baryon number and strangeness. But this doubling of states is not observed in the hadron spectrum. Thus the chiral symmetry $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ must be broken spontaneously to the vector subgroup $\mathrm{SU}(2)_{L+R}$. The spectrum of hadrons does exhibit an approximate SU(2) isospin symmetry, under which we can group the mesons and baryons in irreducible representations. This symmetry is not exact because the quark masses break it explicitly. Although we know the fundamental lagrangian of QCD, it is not possible with present techniques to show that $SU(2)_L \otimes SU(2)_R$ is spontaneously broken down to $SU(2)_{L+R}$. It is also impossible to give analytic predictions of the scale of the condensates that break these symmetries, the spectrum of hadrons, their couplings or decay constants from the fundamental theory [1]. The reason is that at low energies QCD is a strongly coupled theory. The $SU(2)_{L+R}$ symmetry can be extended to an even less exact $SU(3)_{L+R}$ by including the s quark. Thus, as the breaking of the chiral symmetry determines the low energy phenomenology of hadrons, it is very important to improve our understanding on this subject. A few years ago, Maldacena stated a conjecture [5] relating a supersymmetric strongly coupled theory with a string theory in higher dimensions. The conjecture stated that there is a string theory in a ten dimensional space ($AdS_5 \times S^5$) that is dual to a 4D strongly coupled conformal field theory (CFT) with $N = 4$ supersymmetries and a large number of colors N. Although there is not a mathematical proof of the holographic AdS/CFT conjecture it has passed many non-trivial tests. By deforming the string theory one can expect to obtain a dual description of certain deformed 4D theory (for example a non-supersymmetric non-CFT 4D theory similar to QCD) [6]. Since a higher dimensional string theory can be described at low energies by an effective field theory in higher dimensions, in some range of energies it should be possible to describe a strongly coupled 4D theory in terms of a weakly interacting field theory with extra dimensions [7]. This suggests a more phenomenological approach to 4D strongly interacting theories by using extra dimensional field theories. The higher dimensional description can not give us information about the fundamental 4D degrees of freedom. Moreover, a theory with extra dimensions is not even renormalizable. However, in some special regimes, it could be possible to capture some essential features of the more involved string theory. Thus, there are several motivations to study physics of extra dimensions in this context. First, it might be that we can learn something about QCD in particular, and about strongly coupled theories in general, studying extra dimensional theories. Second, with QCD we have experimental results of a strong theory that guide us to know what is essential to establish a predictive correspondence. Finally, we are testing the holographic map. Therefore, the first part of this thesis will be devoted to the study of QCD through extra dimensional theories. We will propose an effective 5D theory able to describe the spontaneous breakdown of chiral symmetry for mesons. We will describe the vector, the axial-vector, the scalar and the pseudo-scalar sectors of mesons with a very simple model in warped 5D space. Since the 5D model is weakly coupled we are able to do explicit calculations. The model predicts the masses, the decay constants and the interactions in terms of the 5D parameters. We calculate the different contributions to the low energy chiral lagrangian for pions of QCD. Almost all the predictions are in good agreement with the experimental results. We show that our predictions are robust under modifications of the 5D metric in the IR, and that some of the relations arise as a consequence of the 5D gauge

[1] Nevertheless by lattice simulations it is possible to obtain numerical results concerning confinement in QCD.

symmetry.

In the second part of this thesis, we will consider the EW sector of the SM. The SM with a fundamental scalar Higgs suffers a serious problem of instability that signals our ignorance over the mechanism of EW symmetry breaking. In this context we can consider the Higgs only as a parametrization of the EW symmetry breakdown. Therefore we expect new physics at a scale of a few TeV. The most elegant and economical alternative to explain the EW symmetry breaking are technicolor theories, where the EW symmetry is spontaneously broken by a strongly interacting sector [8]. This is inspired in the chiral breaking of QCD. Nevertheless these theories suffer from large deviations to the EW precision tests. A more sophisticated and realistic scenario is to have a composite Higgs boson, where the fundamental constituents are fields of a strongly coupled theory [9]. In this theories, however, due to the EW precision tests, it is necessary for the Higgs mass to be smaller than the composite scale. This can be achieved by an approximate global symmetry protecting the mass of the composite Higgs. Again this idea is inspired in the pion of QCD, a composite scalar particle. The pion mass is protected because this field is the Goldstone boson corresponding to the spontaneous breaking of the chiral symmetry. This is a very attractive scenario to explain the origin of the fundamental theory of EW symmetry breaking. But to really test these proposals, we must be able to calculate physical quantities with enough precision. The actual non-perturbative techniques are far away to reach the experimental accuracy. However, inspired by the AdS/CFT conjecture, it is possible to build extra dimensional theories that resembles strongly interacting 4D theories with a large number of colors. The extra dimensional theories are weakly coupled, and it is possible to calculate physical quantities. Thus we will propose a 5D model that can accomplish a realistic theory of EW symmetry breaking. We will a composite Higgs model along the lines of Ref.[10]. In this model the Higgs arises from the fifth component of a 5D gauge theory. The higher dimensional gauge symmetry protects the Higgs from acquiring a mass at tree level. However, if one breaks the symmetry in one of the boundaries, by quantum corrections one can generate a potential for the Higgs, that is localized towards the other boundary. As this is a finite volume effect, the potential is finite. Since the 5D model is weakly interacting we can compute many physical quantities, as the Higgs potential, the contributions of the strong sector to the electroweak precision observables and the spectrum of new particles. We will show that contributions from the top quark can trigger EW symmetry breaking, and that for a large region of the parameter space, the electroweak precision observables are below their experimental bounds. The model of Ref.[10] suffers from large deviations to the vertex $Zb_L\bar{b}_L$. In this thesis we will show that there is a subgroup of the custodial symmetry that can protect the interaction $Zb\bar{b}$. Therefore, embedding the SM into an appropriate multiplet of the higher dimensional gauge theory, the extra contributions to $Zb_L\bar{b}_L$ can be canceled. The model predicts a light fermionic resonance with a mass of $0.5-1.5$TeV that should be seen at LHC. The top quark is mostly a composite state and we expect deviations from the SM in this sector.

The thesis is organized in the following way. We start in chapter 2 with a brief review of symmetry principles and mechanisms of symmetry breaking in 4D. We consider the effective low energy description of theories with broken symmetries and its possible high energy completions. In particular we consider as an example a non-linear σ model.

In chapter 3 we give an introduction to physics of extra dimension. We show how to describe 5D theories using the holographic prescription. This procedure allows to interpret an extra dimensional theory in a 4D language. We elaborate on the dictionary relating 5D and 4D theories, with special attention to the phenomenological applications, in particular to the different mechanisms of symmetry breaking. We consider scalar, fermionic and gauge fields. We compare the holographic description with the traditional Kaluza-Klein approach. We show how to localize zero modes in different points of the extra dimension. For the case of AdS_5 space we derive the relation between 4D operators of a given scaling dimension and the mass of the 5D field.

In chapter 4 we propose a 5D model to study the chiral symmetry breaking of QCD in the meson sector, in particular the vector, axial-vector, scalar and pseudoscalar sectors [11], [12] (the same model was proposed also in Ref. [13]). We calculate the correlators for the different sectors of the model, and obtain from them the spectrum and decay constants. We show how to expand the correlators at large Euclidean momentum and match them with the operator product expansion of the corresponding correlators in QCD. We also obtain the correlators at low momentum, and extract from them the low energy constants. The 5D gauge symmetry leads to several interesting sum rules, and also leads to vector meson dominance for the pion interactions. We compute the predictions for the constants of the low-energy chiral lagrangian of QCD, the quark masses and other physical quantities. We show that, within the range of validity of our model, all the predictions are in good agreement with the experimental results.

Chapter 5 is devoted to study the breakdown of the EW symmetry [14]. First we describe a model in 4D arising from a strongly coupled theory. The Higgs is a pseudo Goldstone boson of the strong sector. Integrating out all the resonances leads to an effective 4D lagrangian for the external sector, that corresponds to the fermions and gauge fields of the SM. The interesting physical quantities can be expressed in terms of a few correlators that encode the effects of the strong dynamics. We calculate the Higgs potential and the EW precision parameters in terms of these correlators. Next we present a 5D model that leads to the same effective lagrangian after integrating out the bulk degrees of freedom. By matching the correlators with the 5D predictions we are able to compute the Higgs potential and the EW precision observables. We show that there there is a symmetry protecting the interaction $Zb_L\bar{b}_L$ from extra contributions [15]. We also give several predictions for the spectrum of new particles and show that there is a correlation between the Higgs mass and the lightest fermionic masses.

As most of the calculations have been made at tree level, in chapter 6 we develop a formalism to compute radiative corrections in theories with extra dimensions [16]. We consider the special case of flat space and expand the propagators in terms of winding modes. Thus instead of summing over Kaluza-Klein modes, as in the usual approach to extra dimensions, to obtain the loop corrections we have to sum over winding modes. The method is very useful to separate finite from divergent contributions. We show a few examples where there are no divergent corrections and the finite contributions are predictions of the theory. In particular we consider a mechanism able to stabilize the size of the extra dimension by computing a two loop potential in a toy model. We also suggest some applications of this method to calculate radiative corrections in the model of chiral symmetry breaking of chapter 4 and in the model of EW symmetry breaking of chapter 5.

In chapter 7 we summarize and give a brief discussion of the prospects for the future.

Chapter 2

Symmetry breaking in 4D and its 5D completion

A symmetry implies a relation between the different parameters of the theory that in many cases determines the behavior of the physical states. In general, symmetries lead to interesting properties at high energies, for example they can protect the masses of the particles against divergent corrections. This is the case of internal local symmetries that protect the masses of the gauge bosons. Another example is given by the chiral symmetry that can protect the masses of the chiral fermions. Thus breaking symmetries is non-trivial if one wants to preserve some of the properties inherited form the invariance of the theory.

We are interested in the low energy description of theories with spontaneously broken symmetries. If the theory we want to study involves different scales of energy, it is possible to describe the low energy dynamics independently of the details of the high energy physics. The appropriate framework to describe the low energy physics is effective field theory, where low is defined with respect to some scale of energy Λ, characterizing the underlying theory. The relevant degrees of freedom of the effective field theory are only those corresponding to the states with masses $m < \Lambda$, the heavier degrees of freedom are integrated out. Thus one obtains a non-renormalizable theory describing the interactions between the light degrees of freedom, this interactions can be organized in powers of E/Λ. The effective lagrangian contains all the terms compatible with the cluster decomposition principle, perturbative unitarity and the symmetries of the physics that one wants to describe. Although there is an infinite number of terms with increasing powers of fields, at a given order in the energy expansion, the low energy theory is specified by a finite number of couplings. Therefore the theory is renormalizable order by order and one obtains a consistent and predictive theory at low energies. In effective field theories the symmetries are non-linearly realized, this realization of the symmetries gives extraordinary constrains in the interactions.

The most beautiful example of an effective field theory describing broken symmetries is the chiral lagrangian of QCD, the low energy description of the strong interactions describing the physics of pions. The building block of this description is the pattern of symmetry breaking. Given this pattern the lightest degrees of freedom are automatically determined and the interactions are enormously constrained. We will elaborate more on the chiral symmetry breaking described by the chiral lagrangian in chapter 4.

Another important example of a non-linear realization of a symmetry is given by technicolor. In this case a strongly interacting sector accounts for the breakdown of the EW symmetry. We will consider the EW symmetry breakdown by a strong sector in chapter 5.

Both of these examples can be described by an effective non-linear σ model. We will consider the pattern of symmetry breaking SU(2)$_L\otimes$SU(2)$_R$ $\to$SU(2)$_V$. Under the conserved (broken) symmetry transformation with infinitesimal parameter $\vec{\theta}_V = \sigma_a \theta^a_V$ ($\vec{\theta}_A$) the pion field $\vec{\pi}$ of the non-linear σ model rotates as

$$\delta_V \vec{\pi} = \vec{\theta}_V \times \vec{\pi}\,, \qquad \delta_A \vec{\pi} = f_\pi \vec{\theta}_A \left(1 - \frac{\pi^2}{f_\pi^2}\right) + \frac{2\vec{\pi}}{f_\pi}(\vec{\theta}_A \cdot \vec{\pi})\,. \tag{2.1}$$

The lagrangian describing the pion interactions is given by

$$\mathcal{L}_{eff} = \frac{1}{2}\frac{(\partial_\mu \vec{\pi})^2}{(1+\pi^2/f_\pi^2)^2} + \dots\,, \tag{2.2}$$

where the dots stand for higher order terms. This lagrangian is invariant under the broken symmetry because the "covariant" derivative $\vec{\mathcal{D}}_\mu = \partial_\mu \vec{\pi}/(1+\pi^2/f_\pi^2)$ rotates as

$$\delta_A \vec{\mathcal{D}}_\mu = 2(\vec{\pi} \times \vec{\theta}_A)\vec{\mathcal{D}}_\mu\,. \tag{2.3}$$

Thus $\vec{\mathcal{D}}_\mu$ transforms linearly (although with a field-dependent parameter) and Eq. (2.2) is invariant. By including higher order terms we can calculate pion interactions to any desired order in the pion energy, provided that $E < 4\pi f_\pi$. It is enough to build this higher order terms by using $\vec{\mathcal{D}}_\mu$. In this way the lagrangian $\mathcal{L}_{eff}$ is invariant under the full chiral symmetry, but the symmetry is realized non-linearly.

A non-linear model, being an effective low energy description, has an UV cut-off Λ ($\Lambda \sim 4\pi f_\pi$ for the non-linear σ model). This means that at energies larger than Λ one is not allowed to make an expansion in powers of E/Λ, perturbative unitarity is lost and the theory is not predictive anymore. Thus one has to replace the effective theory by a new theory including the degrees of freedom of the heavy particles. We will call this procedure a completion of the effective theory. To complete an effective theory one has to know which are the relevant variables at higher energies and also their interactions. For the case of the non-linear σ model one can consider different possibilities to extend this model at energies higher than the cut-off scale. One possibility is to include a radial excitation ρ linearizing the σ model. This is what happens with the Higgs sector, where the radial excitation corresponds to the physical scalar field and the angular variables are eaten by the massive gauge bosons. The radial field in this case is the variable that unitarizes the theory at high energies. The lagrangian of the linear sigma model is given by

$$\mathcal{L}_\phi \;=\; \frac{1}{2}\left[(\partial_\mu\phi)^2+(\partial_\mu\sigma)^2\right]-\frac{1}{2}m^2(\phi^2+\sigma^2)-\frac{\lambda}{4}(\phi^2+\sigma^2)^2\,, \tag{2.4}$$

where ϕ is a triplet to be associated with the pion field and the field σ has been introduced to obtain a chiral invariant lagrangian, $\sigma+i2T^a\phi_a$ transforms as a $(\mathbf{2},\mathbf{2})$ of $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$, with $T^a=\sigma^a/2$ the SU(2) generators.

The minimum of the scalar potential depends on the sign of m^2/λ. Assuming $\lambda>0$, for $m^2>0$ the minimum is at $\sigma=\phi_a=0$, whereas for $m^2<0$ there is a minimum at $(\phi^2+\sigma^2)=|m^2/\lambda|$. In this second case there is a spontaneous breakdown of the chiral symmetry. As we want to preserve unbroken the $\mathrm{SU}(2)_{L+R}$ subgroup, the physical vacuum is left invariant by the generators $Q^{aV}=Q^{aL}+Q^{aR}$ and is broken by the generators $Q^{aA}=Q^{aL}-Q^{aR}$, where $Q^{aL,R}$ are the generators of $\mathrm{SU}(2)_{L,R}$. This implies that $\langle 0|\phi_a|0\rangle=0$ and $\langle 0|\sigma|0\rangle=-|m^2/\lambda|^{1/2}$. Thus we can define a physical field σ' by $\sigma=\langle 0|\sigma|0\rangle+\sigma'$. Rewriting the lagrangian of Eq. (2.4) in terms of the new variable we obtain a massive scalar $m_{\sigma'}=\sqrt{2}|\mu|$ and a triplet of massless pions ϕ_a, the Goldstone bosons of the spontaneously broken chiral symmetry.

One can compute the matrix element of the axial current between the vacuum and the massless ϕ_a and one obtains $\langle 0|\sigma|0\rangle=-f_\pi$. Therefore f_π is the only scale of the theory.

In place of the variables ϕ_a and σ, one can define new variables

$$\rho=\sqrt{\phi_a\phi_a}\,,\qquad \pi_a=f_\pi\frac{\phi_a}{\phi_a+\rho}\,. \tag{2.5}$$

Integrating out the massive field ρ we obtain the effective non-linear σ model of Eq. (2.2).

The σ model can be extended to account for an explicit breaking of the chiral symmetry, preserving the $\mathrm{SU}(2)_{L+R}$ subgroup. A very simple example is obtained by adding to the lagrangian a term $-\nu\sigma$. Minimizing the new potential we obtain the following condition for $\langle 0|\sigma|0\rangle$

$$-\nu-\lambda\langle 0|\sigma|0\rangle^3-\mu^2\langle 0|\sigma|0\rangle=0\,. \tag{2.6}$$

By shifting the variable $\sigma=\langle 0|\sigma|0\rangle+\sigma'$ we obtain a mass term for the pion fields $m_\pi^2=\mu^2+\lambda\langle 0|\sigma|0\rangle=\nu/f_\pi$. The axial current A_μ is not conserved anymore and we obtain

$$\partial_\mu A^a_\mu=m_\pi^2 f_\pi\phi_a\,. \tag{2.7}$$

This is the PCAC relation. Therefore, due to the explicit breaking the pions are no longer massless, they become pseudo Goldstone bosons (PGB).

There is another important property that we want to stress. Introducing an explicit breaking of the symmetry, in general has the effect of forcing the direction of the vacuum into an alignment with the symmetry breaking term. This is known as the vacuum alignment condition [17].

Another possibility to extend the non-linear effective model is a drastic change of the degrees of freedom at high energies, like in QCD, where the variables of the non-linear σ model describe composite states, the pions. In this case the basic constituents of the fundamental theory are the quarks and gluons. Thus, instead of introducing a radial excitation to recover unitarity, one has to change the variables at high energies. By describing the theory in terms of quarks and gluons there are no problems with unitarity. The quark masses give an explicit breaking of the chiral symmetry inducing a finite mass for the pions. Alternatively 't Hooft showed that in the limit of large number of colors, QCD can be described in terms of an infinite number of stable resonances [18]. With this description there is an appropriate infinite tower of states that unitarizes the pion interactions at high energies.

We will consider in this thesis the possibility to give a completion of the non-linear σ model describing the spontaneously broken symmetry by working with extra dimensions. This is possible because in extra dimensional theories one can break a symmetry group G to a subgroup H by boundary conditions. Using naive dimensional analysis (NDA) [19, 20, 21], one can estimate that 5D gauge theories will become strongly interacting at a scale $\Lambda \sim 24\pi^3/g_5^2$, where g_5 is the 5D gauge coupling. [1] This cut-off is larger than the 4D non-linear σ model cut-off $4\pi f_\pi$. To see this we write the 5D cut-off in terms of 4D quantities as $\Lambda \sim 12\pi^2 f_\pi/g_4$ (where the matching conditions are $1/g_4^2 = 2\pi R/g_5^2$ and $g_4 f_\pi \sim 1/R$, with $2\pi R$ the length of the extra dimension). Comparing both results, the cut-off of the 5D theory is parametrically larger by a factor $3\pi/g_4$. Physically what happens is the following, every 5D field contains an infinite tower of 4D fields, the KK modes, thus there is a tower of KK states that ensures unitarity up to an energy scale Λ which is not arbitrarily large but can be well above the 4D cut-off $4\pi f_\pi$.

We have described three different scenarios to complete the effective non-linear σ model. The different completions of the same low energy physics give different signatures at high energies. In Fig. 2.1 we show a picture of the amplitude of the elastic $W - W$ scattering (a similar picture is valid for the elastic pion scattering). The different curves correspond to a technicolor-like model where unitarization is achieved by the strong dynamics, the SM where unitarization is accomplished by the Higgs, and finally a model where unitarization is achieved by a tower of resonances. Therefore we should be able to distinguish between this models in the future high energy experiments.

[1] This result is valid whether the gauge group G is broken by boundary conditions or not, provided that the theory does not contain small parameters.

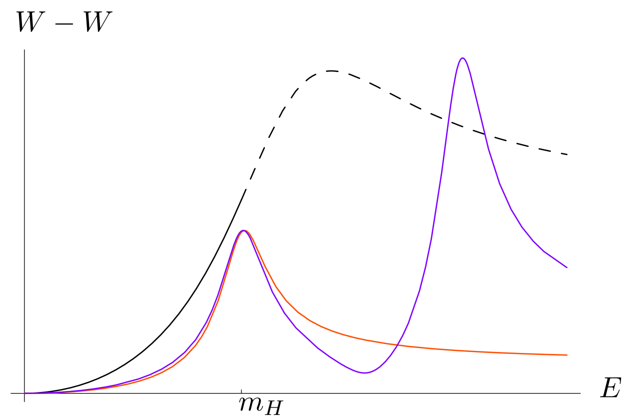


Figure 2.1: *Picture of the amplitude of the elastic $W-W$ scattering in the three different examples. The red line corresponds to the linear completion of the σ model, where the Higgs unitarizes the amplitude. The black line corresponds to a technicolor-like model, where the amplitude increases as E^2 at low energies (continuous line) and unitarization is accomplished by the unknown underlying fundamental physics (dashed-line). The blue line corresponds to a model where unitarization is accomplished by a tower of resonances. As can be seen from this picture the first resonance gives only a partial unitarization. A similar picture is valid for the $\pi-\pi$ elastic scattering.*

Chapter 3

Introduction to 5D spaces and holography

In this chapter we will consider the KK and the holographic approach for scalar Φ, fermionic ψ and gauge A_M fields in 5D. Both descriptions give the same predictions. As an example we will show explicitly the spectrum matching. We will discuss how to localize zero modes in the fifth dimension and also some properties of warped spaces important for phenomenology. For the particular case of AdS space we will compute the KK wave functions and show, in the holographic context, how to associate the bulk field masses with the dimension of 4D operators.

We consider a general not factorizable metric, where the 4D Minkowski metric is multiplied by a warp factor that depends on the extra dimension. In conformal coordinates the 5D metric is defined by

$$ds^2 = a^2(z)(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \equiv g_{MN}dx^M dx^N \, , \tag{3.1}$$

where a is the warp factor and z is the fifth coordinate. $(M = \mu, 5)$ is the 5D spacetime index, where $\mu = 0, 1, 2, 3$ and $\eta_{\mu\nu} = diag(+ - --)$ is the 4D Minkowski metric. In the case of flat space $a = 1$ and in the case of AdS$_5$ a is given by

$$a(z) = \frac{L}{z} \, , \tag{3.2}$$

where L is the AdS curvature radius. We will compactify the extra dimension by putting two boundaries, one at $z = L_0$ and another at $z = L_1$. Then the action is defined on the line segment $L_0 \leq z \leq L_1$. The boundaries at $z = L_0$ and $z = L_1$ will be called UV and IR boundary respectively [22].

The 5D action is given by

$$\begin{aligned} S_5 = \frac{1}{g_5^2}\int d^4x \int dz \, \sqrt{g}\Big[\; - \; & \frac{1}{4}A_{MN}A^{MN} + \frac{1}{2}|D_M\Phi|^2 - \frac{1}{2}m_\Phi^2|\Phi|^2 \\ + \; & \frac{i}{2}\,\bar{\Psi}e_A^M\Gamma^A D_M\Psi - \frac{i}{2}\,(D_M\Psi)^\dagger\Gamma^0 e_A^M\Gamma^A\Psi - m_\Psi\bar{\Psi}\Psi\Big] \, , \end{aligned} \tag{3.3}$$

where g is the determinant of the metric, e_A^M is the inverse vielbein $e_A^M = e_N^B g^{MN} \eta_{AB}$. The vielbein is defined by $g_{MN} = e_M^A e_N^B \eta_{AB}$. The gamma matrices are $\Gamma^A = \{\gamma^\mu, -i\gamma^5\}$ and in terms of the warped factor $e_A^M = \delta_A^M / a(z)$. D_M is the gauge covariant derivative and $\nabla_M = D_M + \omega_{M\,AB}[\Gamma_A, \Gamma_B]$ is the curved space covariant derivative for fermion fields, with $\omega_{M\,AB}[\Gamma_A, \Gamma_B]$ the spin connection. We have factored out a coefficient $1/g_5^2$, then g_5 is the 5D expansion parameter. We will work with $\mathrm{Dim}[A] = 1$, $\mathrm{Dim}[\Phi] = 1$ and $\mathrm{Dim}[\Psi] = 3/2$, then $\mathrm{Dim}[g_5^2] = -1$. The action (3.3) includes all the 5D quadratic terms consistent with gauge and coordinate invariance for fields of spin not bigger than one.

On the boundaries of the 5D spacetime, at $z = L_0, L_1$, translation symmetry is broken. This boundaries extend over the x^μ directions and we will locate there two 3-branes. These 3-branes can support 4D field theories that can couple to the bulk fields. The 4D action on the boundaries is

$$S_{bound} = \int d^4x (\sqrt{-g_0}\mathcal{L}_0 + \sqrt{-g_1}\mathcal{L}_1)\,, \tag{3.4}$$

where $g_{0,1}$ is the determinant of the metric induced on the branes and $\mathcal{L}_{0,1}$ is the 4D Lagrangian of the boundary fields.

To obtain the equations of motion of the 5D fields we require an extremum of the action $\delta S = 0$. In theories with boundaries we have to take into account the localized terms also. For this reason we have to compute the variation of the full action, the bulk terms and the boundary terms. In general, the variation of the 5D action can be written as [23]

$$\delta S = \delta(S_5 + S_{bound}) = \int d^5x\, \delta\phi(\mathcal{D}\phi) + \int d^4x\, \delta\phi(\mathcal{B}\phi)\Big|_{L_0}^{L_1}\,, \tag{3.5}$$

where ϕ is any bulk field. $\mathcal{B}\phi$ stands for the boundary terms obtained from integration by parts over the extra coordinate, but also from the variation of the original boundary terms. The boundary terms arising from integration by parts on the x^μ directions are automatically zero if we assume that $\phi = 0$ at the 4D boundary $x^\mu = \infty$ (we are not interested on 4D non-trivial configurations).

Requiring the first term of Eq. (3.5) to vanish we obtain the bulk equation of motion $\mathcal{D}\phi = 0$. To cancel the second term of Eq. (3.5) we have to impose the appropriate boundary conditions, that with our notation correspond to $\delta\phi|_{L_{0,1}} = 0$ or $\mathcal{B}\phi|_{L_{0,1}} = 0$.

The usual approach to theories with compact extra dimensions consists in decomposing the fields in an infinite series of 4D fields, the KK modes, and compute with them the physical quantities. As these 4D fields are mass-eigenstates, the particle content in a KK description is very intuitive. However, the KK description is not always the most appropriate approach to understand the physical results. There is an alternative approach to describe theories with extra dimensions, called the holographic or boundary description, that in some cases is much more useful. It consists in separating the bulk fields from their boundary values and treat them as different degrees of freedom. In theories with two boundaries, it is possible to consider a one boundary description, separating just the variables on one boundary (for example at $z = L_0$) from the other variables, or a two boundary description, separating the variables on both boundaries from the bulk. Some of the advantages of this approach are [24]

- Theories with a compact extra dimension offer the possibility of breaking symmetries, like gauge symmetry or supersymmetry, by boundary conditions. This means that the bulk and the boundaries respect different symmetries. The KK modes are a mixing of boundary and bulk degrees of freedom and for this reason they do not have well defined symmetry transformations. On the holographic approach the treatment of the symmetries is much more transparent, because we separate degrees of freedom supported on the bulk from degrees of freedom supported on the boundaries.

- If bulk and boundary fields are weakly coupled, it is possible to treat the bulk as a small perturbation of the boundary. This happens for spaces that can be approximated by AdS_5 for small z, and also if the boundary fields have large kinetic terms. In this case it is possible to make an expansion in the boundary-bulk coupling that simplifies the computations.

- Theories with compact extra dimensions have certain properties that resemble 4D strongly coupled theories with a large number of colors. These theories have an infinite tower of 4D states with the same quantum numbers, that can be associated with the infinite tower of resonances of a large N theory. Thus it is possible to establish a correspondence where the bulk is matched to the strong sector of the 4D theory and the boundary fields match with external sources. This qualitative correspondence has its roots in the Maldacena conjecture [5], as we will explain later.

3.1 The holographic description

To obtain the holographic description of a 5D theory with boundaries we proceed in the following way. We consider the partition function $\mathcal{Z} = \int d\phi\, e^{iS(\phi)}$ of the 5D theory. We integrate the bulk fields fixing their value on the UV boundary $\phi(x, z = L_0) = \phi^0(x)$ (one source description) and obtain a partition function depending on ϕ^0

$$\mathcal{Z}[\phi^0] = \int_{\phi\big|_{L_0}=\phi^0} d\phi\; e^{iS(\phi)} = e^{iS_{\mathrm{eff}}(\phi^0)}\,. \tag{3.6}$$

To integrate over the bulk fields we have to solve the 5D equations of motion and substitute the solution back into the action. In general, for an interacting theory it is not possible to solve the equations of motion. Instead we can solve the bulk equations for a free field, this is equivalent to work at tree level. On the IR boundary we have to choose boundary conditions that cancel the IR terms $\mathcal{B}\phi|_{L_1} = 0$. For a scalar or a gauge field without boundary terms on the IR we can either choose Neumann or Dirichlet boundary conditions. The effective action $S_{\rm eff}$ is a 4D action for the UV field ϕ^0, and in general it is non-local. There can be also extra terms localized on the UV boundary, Eq. (3.4). If the UV field ϕ^0 is dynamical we have to integrate over all its possible configurations, obtaining the partition function

$$\mathcal{Z} = \int d\phi^0 \, e^{i\, S_{UV}[\phi^0] + i\, S_{\rm eff}[\phi^0]} \,, \tag{3.7}$$

where $S_{UV} = \int d^4x \sqrt{-g_0}\mathcal{L}_0$ are the extra local terms on the UV boundary. If the terms on S_{UV} dominate over the terms of $S_{\rm eff}$, the effective theory is essentially given by S_{UV} and the bulk terms give a small correction. In this case the boundary field ϕ^0 has a small mixing with the bulk resonances and it is an approximate mass-eigenstate. This happens for example for AdS$_5$ spaces, but is possible to mimic this situation in flat 5D spaces by adding large kinetic terms for the UV boundary fields.

Inspired on the AdS/CFT conjecture, we can establish an holographic correspondence using the 4D boundary action defined above: the 4D theory on the UV boundary is dual to a 4D strongly coupled field theory (SCFT) in the limit of large number of colors N. This statement can be quantified in the following way

$$\mathcal{Z}[\phi^0] = \int d\phi_{SCFT} \; e^{iS_{SCFT}[\phi_{SCFT}] + i\int d^4x \, \phi^0 \mathcal{O}} \,, \tag{3.8}$$

where S_{SCFT} is the SCFT action with ϕ_{SCFT} the general SCFT fields, and $\mathcal{O}$ is an SCFT operator made of ϕ_{SCFT}. This means that the fields ϕ^0 are the sources for the correlators of the SCFT operators $\mathcal{O}$. Then at the classical level the 5D theory is equivalent to a 4D SCFT in the large-N limit.

In SCFT we can calculate n-point functions as

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle = \frac{\delta^n \ln \mathcal{Z}[\phi^0]}{\delta\phi^0 \cdots \delta\phi^0}\bigg|_{\phi^0=0} . \tag{3.9}$$

At the classical level Eq. (3.9) simplifies to

$$\langle \mathcal{O} \cdots \mathcal{O} \rangle = \frac{\delta^n S_{\rm eff}[\phi^0]}{\delta\phi^0 \cdots \delta\phi^0}\bigg|_{\phi^0=0} , \tag{3.10}$$

and we obtain the connected Green functions of the SCFT from the on-shell bulk action. In theories with a large number of colors N, the n-point functions can be written as an infinite sum over narrow resonances. This is a consequence of the large-N limit [18]. On the 5D side, the n-point functions are calculated in terms of 5D propagators that can be decomposed as infinite sums over the 4D propagators of the KK modes. In this way the infinite tower of resonances of the SCFT match with the infinite tower of KK modes. Let us consider as an example the two-point function that in the lowest order in $1/N$ can be written as [25]

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle = \sum_n \frac{F_n^2}{p^2+m_n^2}\,, \tag{3.11}$$

where m_n and $F_n = \langle 0|\mathcal{O}|n\rangle$ are respectively the masses and decay constants of the resonances created by $\mathcal{O}$. The matrix element of an operator between the vacuum and a resonance is of order $\sqrt{N}$, i.e. $F_n \sim \sqrt{N}$ and in the 5D theory $F_n \sim 1/g_5$, where g_5 is the expansion parameter of the 5D theory, Eq. (3.3). These results can be generalized to n-point functions, with $n > 2$, and the correspondence between the 4D and the 5D theories can be established in a similar way.

The above approach is inspired in the AdS/CFT conjecture. This correspondence was first stated by Maldacena [5]. It related a string theory in some specific geometry (type IIB string theory on $\mathrm{AdS}_5\times\ \mathrm{S}^5$) with a 4D supersymmetric theory ($\mathcal{N} = 4$ SU(N) 4D gauge theory, with $\mathcal{N}$ the number of supersymmetry generators). Although there is no rigorous proof of this correspondence it has passed many nontrivial tests, and it is conjectured to be an exact duality. However in this work we are interested in a more phenomenological version of this correspondence, although still able to capture some essential features of the duality. This is the correspondence we described in the previous paragraphs. This correspondence assumes the existence of a 4D SCFT with a set of operators $\mathcal{O}$ that couple to external sources ϕ^0. Moreover, this SCFT should have certain properties related to properties of the 5D weakly coupled theory, according to Eq. (3.8). For example, given that the 5D theory has a set of symmetries, after integrating out the bulk fields the resulting generating function $\mathcal{Z}$ will have some properties related to these symmetries. This means that the SCFT should reflect these symmetries also. But in general we are not able to calculate in an SCFT, we ignore the nature of the operators, the spectrum, the couplings. We can not even know if an SCFT with the same $\mathcal{Z}$ as that of the 5D theory can exist. Therefore, we should consider the holographic interpretation as a 4D description of the 5D effective theory, very useful in some particular cases. However we can still learn many things about SCFT from the holographic interpretation. The holographic approach that we have described can also help to determine what is essential to establish a more sophisticated version of the correspondence. Then it is important to obtain a dictionary relating the different theories.

We will discuss briefly some entries of the dictionary:

- The internal local symmetries of the 5D bulk correspond to global symmetries of the 4D SCFT. This can be understood because the bulk fields have well defined transformations properties, then integrating out this fields we end up with an effective action S_{eff} that contains information about the bulk symmetries. As the n-point functions are derived from S_{eff}, they will satisfy the Ward identities corresponding to the symmetries of the bulk. On the other hand, the boundary action S_{UV} can respect different symmetries, that will correspond to symmetries of the external sector of the SCFT, the sources ϕ^0.

- The symmetries of the 5D spacetime correspond to symmetries of the spacetime of the SCFT. The most popular example corresponds to the 5D spacetime being AdS_5, with no boundaries, i.e.: $L_0 \to 0$ and $L_1 \to \infty$. The AdS_5 space has a local SO(4,2) symmetry, that corresponds in the 4D theory to a conformal symmetry of the quantum field theory (CFT). In a conformal theory it is not possible to define a space of mass eigenstates, and the natural objects are the operators $\mathcal{O}$ made up of elementary fields. These operators can be organized by their dimensions under scale transformations, thus the momentum dependence of the n-point functions $\langle \mathcal{O} \cdots \mathcal{O} \rangle$ is dictated by the dimensions of the operators.

- Another important entry is the one related to broken symmetries. Let us consider an SCFT with a symmetry broken in the IR. This happens for example in QCD where the scale invariance is broken at low energies. Introducing an IR boundary on a 5D theory we sharply end the space breaking the translation symmetry. Thus we introduce a mass scale $m \sim 1/(L_1 - L_0)$, as can be seen by computing the spectrum. This breaking of the symmetry is reflected on the n-point functions at energies below the mass scale $E \leq m$. For AdS_5 space, the dual 4D theory has conformal symmetry. In a conformal theory it is not possible to define particle states or a S matrix. Adding an IR brane corresponds to a deformation on the 4D theory leading to a breakdown of the conformal invariance in the IR. Therefore the theory has a spectrum of particles and one can define a S matrix.

- The energy scales in the 5D theory are scaled by the warp factor $a(z)$. If the 5D spacetime can be approximated by AdS_5 in the UV boundary, the presence of an UV brane can be associated to an UV scale $\Lambda_{UV} \sim 1/L_0$. This corresponds to putting an UV cutoff on the SCFT. Thus at energies below Λ_{UV} the 4D theory is conformal but at $E \sim \Lambda_{UV}$ this symmetry is explicitly broken. Taking the limit $\Lambda_{UV} \to \infty$ the SCFT remains conformal at high energies. However, we can also work with a finite Λ_{UV} that means that the sources ϕ^0 become dynamical fields. In this case the SCFT will induce kinetic terms for the sources that will be contained in S_{eff} defined in Eq. (3.6). It is also possible to add explicit kinetic terms in the UV brane S_{UV}.

- It is well known that in extra dimensional theories with boundaries one can break symmetries by boundary conditions. An example is given by a 5D theory in AdS_5 with a gauge symmetry G in the bulk. One can break this symmetry to a smaller group $H \subset G$

by assigning Dirichlet boundary conditions to the generators we want to break. Since an internal gauge symmetry in the bulk corresponds to a global symmetry in the SCFT, breaking the local symmetry in the IR by boundary conditions corresponds to a global symmetry in the SCFT broken at low energies $E \sim 1/L_1$.

3.2 Scalar fields

We consider now the case of a free scalar field in warped space. We will work in momentum space along the four non-compact dimensions. From Eq. (3.3) we can obtain the 5D equation of motion for a free scalar field (leading order in g_5) in conformal coordinates

$$\left[\partial^2 - a^{-3}\partial_5 a^3 \partial_5 + a^2 m_\phi^2\right] \Phi = 0\,, \tag{3.12}$$

where $\partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$.

After integration by parts we obtain the following boundary term

$$\mathcal{L}_{\rm bound} = -\frac{1}{2g_5^2}\left[a^3\Phi\partial_5\Phi\right]\Big|_{L_0}^{L_1} + a^4 m_{UV}^2\Phi^2\Big|_{L_0} - a^4 m_{IR}^2\Phi^2\Big|_{L_1}\,, \tag{3.13}$$

where we have included masses m_{UV} and m_{IR} on the UV and IR boundaries. We have not consider UV or IR kinetic terms, but they can be included straightforward.

We will compare now the KK and the holographic description of the scalar theory.

3.2.1 Kaluza-Klein description of a 5D scalar field

To obtain the KK description we perform a separation of variables and express the 5D field Φ as an infinite sum over 4D fields

$$\Phi(x,z) = \sum_n \phi^{(n)}(x) f_n^\phi(z)\,. \tag{3.14}$$

$\phi^{(n)}$ are mass eigenstates satisfying $\partial^2\phi^{(n)}(x) = -m_{\phi_n}^2\phi^{(n)}(x)$ and $f_n^\phi(z)$ are the KK wave functions that describe the bulk profile of the KK modes. Introducing Eq. (3.14) into the bulk equation of motion we obtain

$$(-a^{-3}\partial_5 a^3\partial_5 + a^2 m_\phi^2) f_n^\phi(z) = m_{\phi_n}^2 f_n^\phi(z)\,, \tag{3.15}$$

with $f_n^\phi(z)$ normalized as

$$\int dz\, a^3 f_m^\phi(z) f_n^\phi(z) = \delta_{mn} \tag{3.16}$$

to obtain canonical kinetic terms for the KK modes. The KK Eq. (3.15) is a linear equation of second order and it has two solutions $h_1^\phi(m_n,z)$ and $h_1^\phi(m_n,z)$ that depend on the particular profile of the metric. To obtain the explicit form of $h^\phi_{1,2}$ we have to specify the z dependence of the warp factor. We can write the KK wave function in terms of generic functions $h^\phi_{1,2}(m_n, z)$ as

$$f^\phi_n(z) = \frac{1}{N_n}\left[h^\phi_1(m_{\phi_n}, z) + b(m_{\phi_n})\, h^\phi_2(m_{\phi_n}, z)\right] , \tag{3.17}$$

where N_n is a normalization constant determined by Eq. (3.16) and b depends on the boundary conditions.

To fully determine the KK wave functions we have to specify the boundary conditions that fix the coefficient b and the masses m_n. Imposing boundary conditions to cancel the UV and IR boundary terms we obtain two equations

$$\begin{aligned} b(m_{\phi_n}) &= -\left.\frac{\partial_5 h^\phi_1(m_{\phi_n}, z) + 2a g_5^2 m_{IR}^2 h^\phi_1(m_{\phi_n}, z)}{\partial_5 h^\phi_2(m_{\phi_n}, z) + 2a g_5^2 m_{UV}^2 h^\phi_2(m_{\phi_n}, z)}\right|_{L_0} , \\ b(m_{\phi_n}) &= -\left.\frac{\partial_5 h^\phi_1(m_{\phi_n}, z) + 2a g_5^2 m_{IR}^2 h^\phi_1(m_{\phi_n}, z)}{\partial_5 h^\phi_2(m_{\phi_n}, z) + 2a g_5^2 m_{IR}^2 h^\phi_2(m_{\phi_n}, z)}\right|_{L_1} . \end{aligned} \tag{3.18}$$

The UV and IR boundary conditions interpolate between Neumann and Dirichlet for $m_{UV,IR} = 0, \infty$. Eq. (3.18) determines the mass spectrum.

When the spacetime is a slice of AdS$_5$, the functions $h^\phi_{1,2}$ are given in terms of Bessel functions [26]

$$h^\phi_1(m_{\phi_n}, z) = \frac{z^2}{L_1^2} J_\beta(m_{\phi_n} z) , \qquad h^\phi_2(m_{\phi_n}, z) = \frac{z^2}{L_1^2} Y_\beta(m_{\phi_n} z) , \tag{3.19}$$

where

$$\beta = \sqrt{4 + m_\phi^2 L^2}\, . \tag{3.20}$$

If the space is asymptotically AdS on the UV, the functions $h^\phi_{1,2}$ can be approximated by Bessel functions for $z \to 0$. If on the IR the metric has deviations of order one with respect to AdS we expect $h^\phi_{1,2}$ to deviate order one with respect to the Bessel functions deep in the IR.

Let us consider the zero mode of a scalar field in AdS$_5$ space. The zero mode wave function is obtained from Eq. (3.15) imposing $m_{\phi_0} = 0$ and the general solution is

$$f^\phi_0(z) = c_1 z^{2-\beta} + c_2 z^{2+\beta} \tag{3.21}$$

where c_1 and c_2 are integration constants to be determined imposing the boundary conditions. Imposing Neumann or Dirichlet boundary conditions leads to $c_1 = c_2 = 0$ and there is no zero mode. This means that to obtain a zero mode we have to include boundary terms, as the UV and IR mass terms in Eq. (3.13). In this case the boundary conditions lead to the following equations for $c_{1,2}$:

$$c_1 z_0^{1-\beta}(2 - \beta + 2g_5^2 L m_{UV}^2) + c_2 z_0^{1+\beta}(2 + \beta + 2g_5^2 L m_{UV}^2) = 0\, , \tag{3.22}$$

$$c_1 z_1^{1-\beta}(2 - \beta + 2g_5^2 L m_{IR}^2) + c_2 z_1^{1+\beta}(2 + \beta + 2g_5^2 L m_{IR}^2) = 0\, . \tag{3.23}$$

with β defined in Eq. (3.20).

For general values of m_{UV} and m_{IR} these equations lead to $c_1 = c_2 = 0$ again. But for special values of the boundary masses there is a zero mode. Of special interest is the situation when either c_1 or c_2 vanishes, that corresponds to

$$(2+\beta+2g_5^2 L m_{UV}^2) = (2+\beta+2g_5^2 L m_{IR}^2) = 0 \qquad \Rightarrow c_1 = 0\,, \tag{3.24}$$

$$(2-\beta+2g_5^2 L m_{UV}^2) = (2-\beta+2g_5^2 L m_{IR}^2) = 0 \qquad \Rightarrow c_2 = 0\,. \tag{3.25}$$

Thus the zero mode is given by

$$f_0^\phi = \frac{z^\gamma}{N_0}\,, \tag{3.26}$$

where N_0 is a normalization constant and

$$\gamma \equiv \gamma_\pm = 2 \pm \beta\,. \tag{3.27}$$

For real values of β (which implies $m_\phi^2 > -4/L^2$ that is the bound for the stability of AdS space [27]) the exponent takes values over $-\infty < 2-\beta < \infty$, leading to the possibility of having a localized zero mode. By comparing the kinetic term of the zero mode with the case of flat space, we obtain that the zero mode is flat for $\gamma = 1$, it is localized towards the UV for $\gamma < 1$ and it is localized in the IR for $\gamma > 1$. Since $\gamma_+ = 2+\beta$, the zero mode in this case is always localized in the IR. For $\gamma_- = 2-\beta$ the zero mode can be localized in the UV ($1 < \beta$), in the IR ($0 \leq \beta < 1$) or it can be delocalized ($\beta = 1$). Therefore the localization of the massless mode can be determined choosing the value of the boundary and bulk masses.

3.2.2 Holographic description of a 5D scalar field

Let us study now the holographic approach for a theory with scalar fields (we follow reference [23]). We will consider the holographic description with sources in the UV. We work in momentum space on the 4D direction and in coordinate space in the extra dimension. The solution of the 5D equation of motion (3.12) can be expressed in terms of the functions $h_{1,2}^\phi$ as

$$\Phi(p,z) = h_1^\phi(p,z) + b(p)\, h_2^\phi(p,z)\,, \tag{3.28}$$

where p is the 4D momentum. The factor $b(p)$ is determined imposing the IR boundary conditions

$$b(p) = -\frac{\partial_5 h_1^\phi(p,L_1) + 2ag_5^2 m_{IR}^2 h_1^\phi(p,L_1)}{\partial_5 h_2^\phi(p,L_1) + 2ag_5^2 m_{IR}^2 h_2^\phi(p,L_1)}\,. \tag{3.29}$$

Substituting the solution back into the action we obtain an effective lagrangian that corresponds, according to the holographic prescription, to the generating functional of the two-point function

$$\mathcal{L}_{\rm eff} = \frac{1}{2g_5^2} a^3 \Phi\big(\partial_5\Phi + 2g_5^2 a m_{UV}^2 \Phi\big)\big|_{L_0}\,. \tag{3.30}$$

As we said before, Eq. (3.8), the boundary field play the role of the external source coupled to the SCFT. In general the matching of the 5D field on the boundary $\Phi\big|_{L_0} = \Phi^0$ with the 4D source $\tilde{\phi}$ is given by

$$\Phi\big|_{L_0} \equiv \Phi^0 = \alpha\tilde{\phi}\,, \tag{3.31}$$

where α is a normalization constant to be computed in every specific model. After this normalization of the boundary fields we obtain

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\Pi_\phi(p^2)\tilde{\phi}^2\,, \tag{3.32}$$

The correlator Π_ϕ is obtained by substitution of the solution of the bulk equation of motion (3.28) into the effective lagrangian (3.30)

$$\Pi_\phi(p^2) = \frac{\alpha^2 a^3}{g_5^2}\frac{\partial_5(h_1^\phi + b\,h_2^\phi) + 2g_5^2 a m_{UV}^2(h_1^\phi + b\,h_2^\phi)}{h_1^\phi + b\,h_2^\phi}\Big|_{L_0}\,. \tag{3.33}$$

In some cases the matching of the boundary field with the SCFT source involves a fifth derivative. We will see an explicit example in chapter 4, where the holographic approach is used to describe a 5D model of mesons. In that model the pion field is related to the fifth derivative of a 5D field. Therefore the relation between the sources is given by

$$\partial_5\Phi\big|_{L_0} = \alpha\tilde{\phi}\,. \tag{3.34}$$

Introducing Eq. (3.34) into the effective lagrangian of Eq. (3.30) we obtain the following correlator

$$\Pi_\phi(p^2) = \frac{\alpha^2 a^3}{g_5^2}\frac{h_1^\phi + b\,h_2^\phi}{\partial_5(h_1^\phi + b\,h_2^\phi) + 2g_5^2 a m_{UV}^2(h_1^\phi + b\,h_2^\phi)}\Big|_{L_0}\,. \tag{3.35}$$

According to Eq. (3.9), for a nondynamical source field the poles of Π_ϕ determine the pure SCFT mass spectrum. It is straightforward to see that poles of the correlator Π_ϕ correspond to the KK spectrum of the bulk scalar field defined by Eq. (3.18) with the appropriate boundary conditions. The spectrum determined by Eq. (3.33) match with the KK spectrum for a field with UV Dirichlet boundary conditions, and the poles of Eq. (3.35) match with the KK spectrum for a field with Neumann boundary conditions.

To avoid UV boundary effects, that in the 4D theory corresponds to the effects of the external source, we have to take the limit $L_0 \to 0$. In this process we usually find divergent terms that have to be cancelled by adding the appropriate counterterms. Then the correlator Π_ϕ and the two-point function $\langle\mathcal{O}\mathcal{O}\rangle$ differ by local divergent terms

$$\langle\mathcal{O}\mathcal{O}\rangle = \lim_{L_0\to 0}(\Pi_\phi + \text{counterterms})\,. \tag{3.36}$$

However, as these divergences are local, the poles of Π_ϕ and $\langle\mathcal{O}\mathcal{O}\rangle$ coincide. We will elaborate more on this for AdS space.

We will discuss the scaling properties and the effects of the UV cutoff for spaces that are AdS on the UV. We will consider the case in which the boundary field Φ^0 matches with the SCFT source $\tilde{\phi}$ according to Eq. (3.31). For AdS space the functions $h^{\phi}_{1,2}$ are explicitly given in terms of Bessel functions

$$h_1^{\phi}(p,z) = z^2 J_{\beta}(pz)\,, \qquad h_2^{\phi}(p,z) = z^2 Y_{\beta}(pz)\,, \tag{3.37}$$

where β is defined in Eq. (3.20).

To obtain massless resonances, we will include boundary masses satisfying either Eq. (3.24) or (3.25). The different boundary conditions will give rise to different holographic branches, corresponding to $\gamma_{\pm}$ in Eq. (3.27).

γ_- branch

We start with the γ_- branch. As a first step we study the properties of the 5D theory integrating the bulk and after that we present a 4D SCFT lagrangian with the same properties. Taking the limit $L_0 \to 0$ in Eq. (3.33), the correlator can be approximated by

$$\Pi_{\phi}(p^2) \simeq \alpha^2 \frac{L}{g_5^2}\Big[\frac{p^2}{2\beta-2} - p^{2\beta} L_0^{2\beta-2} \frac{2^{1-2\beta}\pi}{\Gamma(\beta)^2}\Big(\frac{1}{b(p)} + cotg(\pi\beta)\Big) + \dots\Big]\,, \tag{3.38}$$

where the dots stand for higher order terms, we have absorbed a warp factor $a(L_0) = L/L_0$ in the coefficient α of Eq. (3.31). In Eq. (3.38) we have included only the first analytic term and the first non-analytic term. This expression is valid for non-integer β, for integer β the term $cotg(\pi\beta)$ is absent and there is a logarithm instead. The non-analytic term corresponds to the SCFT contribution to the two-point function. The poles of the correlator are given by the zeroes of $b(p)$, defined in Eq. (3.29). They depend on the details of the metric in the IR and coincide with the poles of $\langle \mathcal{O}\mathcal{O}\rangle$.

To analyze the scaling dimensions of the operator $\mathcal{O}$ we take the limit of large Euclidean momentum. If the space is AdS also in the IR, the factor $1/b(p)$ in Eq. (3.38) is exponentially suppressed $\sim e^{-pL_1}$. Therefore, for large momentum the leading non-analytic contribution to the correlator scales with momentum as $p^{2\beta}$. To extract the leading non-analytic term we rescale the field $\tilde{\phi}$ by an amount $z_0^{\beta-1}$ Thus from Eqs. (3.36)and (3.38) we get

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle = \lim_{L_0\to 0}(\Pi_{\phi} + \text{counterterms}) = -\alpha^2 \mathrm{p}^{2\beta}\frac{\mathrm{L}}{\mathrm{g}_5^2}\frac{2^{1-2\beta}\pi\,\mathrm{cotg}(\pi\beta)}{\Gamma(\beta)^2}\,. \tag{3.39}$$

To obtain the dimensions of the operator $\mathcal{O}$ we Fourier transform the correlator $\langle \mathcal{O}\mathcal{O}\rangle$ to coordinate space

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ipx} \langle \mathcal{O}(p)\mathcal{O}(-p)\rangle\,. \tag{3.40}$$

Therefore the operator $\mathcal{O}$ has dimension scale

$$\dim \mathcal{O} = 2 + \beta = 2 + \sqrt{4 + m_\phi^2}\,. \tag{3.41}$$

This is a very important property of 5D theories in AdS$_5$, it allows us to relate the masses of 5D fields with the dimension of the operators in the SCFT.

Extra UV degrees of freedom can be introduced by keeping L_0 finite. In this case the boundary field can mix with the bulk fields. Then to obtain the mass spectrum one has to invert the whole quadratic term in $S_{UV} + S_{\text{eff}}$. If the UV action S_{UV} is absent the spectrum is given by the zeroes of Π_ϕ, that matches with the KK mass spectrum of Eq. (3.18) for Neumann boundary conditions on the UV.

If the space is AdS also in the IR, $b(p)$ can be obtained in terms of Bessel functions by using Eq. (3.37). Expanding the two-point function for small momentum $pL_1 \to 0$ we obtain

$$\Pi_\phi(p^2) \simeq \alpha^2 \frac{L}{g_5^2}\Big[\frac{p^2}{2\beta - 2}\Big(1 - (\frac{z_0}{z_1})^{2\beta-2}\Big) + \dots\Big]\,, \tag{3.42}$$

where the dots stand for higher order analytic terms. In this limit the non-analytic term of Eq. (3.38) vanishes and the kinetic term receives an extra contribution from the massive bulk states.

It is possible to write a dual 4D lagrangian describing an SCFT coupled to an external source, that agrees with the holographic description of the 5D theory given above. Below the UV scale this lagrangian is given by [23]

$$\mathcal{L}_{\text{4D}} = \frac{Z_\phi}{2}(\partial_\mu \tilde{\phi})^2 + \frac{\omega_\phi}{\Lambda_{UV}^{\beta-1}}\tilde{\phi}\mathcal{O} + \mathcal{L}_{\text{UV}}(\tilde{\phi}) + \mathcal{L}_{\text{SCFT}} + \dots\,, \tag{3.43}$$

where Z_ϕ and ω_ϕ are dimensionless running couplings. We have written only the dominant higher dimensional term. This lagrangian describes a massless dynamical source $\tilde{\phi}$ linearly coupled to the SCFT. The bare kinetic term Z_ϕ is matched to the first analytic term in Eq. (3.38) at scale Λ_{UV}: $Z_\phi(\Lambda_{UV}) = L/g_5^2(2\beta - 2)$. Taking $\Lambda_{UV} = 1/L_0 \to \infty$ the source becomes non-normalizable and decouples from the SCFT. Keeping Λ_{UV} finite the source $\tilde{\phi}$ can propagate. Since it is coupled to the SCFT operators, there is a mixing between the source and the SCFT bound states. For $\beta - 1 > 0$ the coupling of the source to the SCFT is irrelevant, it corresponds in the 5D description to a zero mode localized on the UV brane, i.e.: $\gamma_- > 1$. The mixing with the SCFT is negligible at low energies and the source is an approximate massless state. On the other hand, for $\beta - 1 = 0$ ($-1 < \beta - 1 < 0$) the coupling to the SCFT is marginal (irrelevant). In these cases the massless state is a mixture of the source and the SCFT resonances. In the 5D side it corresponds to a flat zero mode (localized in the IR brane).

Running coupling between the scalar source and the SCFT

The coupling between the source and the SCFT can be understood in terms of a renormalization group equation. Following [28] and [23] we define a dimensionless coupling at low energy $\mu \sim 1/L_1$ by $\lambda(\mu) = \omega(\mu)(\mu/\Lambda_{UV})^{\beta-1}/\sqrt{Z(\mu)}$ that satisfies the renormalization group equation

$$\mu\frac{d\lambda}{d\mu} = (\dim[\mathcal{O}] - 3)\,\lambda + c\frac{N}{16\pi^2}\lambda^3 + \ldots\,, \tag{3.44}$$

where c is a constant, $\dim[\mathcal{O}] = 2 + \beta$ as defined in Eq. (3.41) and we have defined the large number colors N by

$$\frac{L}{g_5^2} = \frac{N}{16\pi^2}\,. \tag{3.45}$$

The dots in Eq. (3.44) stand for higher order terms. The second term of Eq. (3.44) arises from the SCFT contribution to the wave function renormalization. The low energy value of λ depends on the dimension of $\mathcal{O}$. For $\beta - 1 \geq 0$ the coupling decreases with the energy scale μ and we obtain

$$\lambda(\mu) \sim (\frac{\mu}{\Lambda_{UV}})^{\beta-1}\,, \tag{3.46}$$

therefore the mixing between the source and the SCFT is very small at low energies. For $-1 \leq \beta - 1 \leq 0$ the coupling flows towards a fixed point

$$\lambda(\mu) = 4\pi\sqrt{\frac{1-\beta}{cN}}\,. \tag{3.47}$$

In this case the mixing between the source and the SCFT states is large.

γ_+ branch

We consider now the description corresponding to the branch γ_+. We describe first the properties of the 5D theory in the holographic approach and then we present the dual 4D SCFT.

Taking the limit $L_0 \to 0$ in Eq. (3.33), the correlator can be approximated by

$$\Pi_\phi(p^2) \simeq \alpha^2\frac{L}{g_5^2}\Big[-\frac{2\beta}{L_0^2} + \frac{p^2}{2\beta-2} - p^{2\beta}L_0^{2\beta-2}\frac{2^{1-2\beta}\pi}{\Gamma(\beta)^2}\Big(\frac{1}{b(p)} + cotg(\pi\beta)\Big) + \ldots\Big]\,, \tag{3.48}$$

where the dots stand for higher order terms. To obtain Eq. (3.48) we have absorbed a warp factor $a(L_0) = L/L_0$ in the coefficient α (defined in Eq. (3.31)). This correlator agrees with the result for the γ_- case, except for the constant term. Since the scaling properties are the same, $\dim\mathcal{O} = 2 + \sqrt{4 + m_\phi^2L^2}$.

At low energy the behavior of the theory is different is different than in the γ_- case. To understand this branch we consider that the space is AdS also in the IR and take the low momentum limit $pL_1 \to 0$, thus

$$\Pi_\phi(p^2) \simeq \frac{L}{g_5^2}\Big[-\frac{2\beta}{L_0^2} + \frac{p^2}{2\beta-2} - \frac{1}{(pL_1)^2}(\frac{L_0}{L_1})^{2\beta-1}\frac{8\beta^2(1+\beta)}{L_1} + \ldots\Big]\,, \tag{3.49}$$

where the dots stand for higher order terms. At low energies the correlator has a pole at zero momentum, corresponding to a massless composite state. In the 5D theory this pole corresponds to a zero KK mode localized in the IR boundary. The source receives a mass of order $1/L_0$ and becomes very heavy. The dual lagrangian below the UV scale is given by [23]

$$\mathcal{L}_{4\mathrm{D}} = \frac{Z_\phi}{2}(\partial_\mu \tilde{\phi})^2 - \frac{m_0^2}{2}\tilde{\phi}^2 + \frac{\omega_\phi}{\Lambda_{UV}^{\beta-1}}\tilde{\phi}\mathcal{O} + \mathcal{L}_{\mathrm{UV}}(\tilde{\phi}) + \mathcal{L}_{\mathrm{SCFT}} + \dots \,, \tag{3.50}$$

where m_0 and Z_ϕ are the source mass and kinetic term. By matching the dual theory with the 5D description at scale $\Lambda_{UV} = 1/L_0$ we obtain: $m_0^2(\Lambda_{UV}) = 2L\beta/(g_5 L_0)^2$ and $Z_\phi(\Lambda_{UV}) = L/g_5^2/(2\beta - 2)$. For infinite Λ_{UV} the source decouples from the CFT. For Λ_{UV} finite the source receives a mass of order Λ_{UV}^2. The SCFT has a massless composite state. For $\beta \geq 1$ the coupling between the source and the SCFT sector is irrelevant, and the mixing can be neglected. Therefore the massless eigenstate is mostly a composite SCFT state. For $0 \leq \beta \leq 1$ the coupling is marginal or relevant and the massless state is a mixing between the source and the SCFT states.

3.3 Fermionic fields

We will study the KK and the holographic description for the fermion field of Eq. (3.3) (we will follow reference [28] for the holographic description).

The Clifford algebra in five dimensions includes the four Dirac matrices γ^μ plus an additional γ^5 matrix. This γ^5 is the parity transformation matrix for 4D spinors. Therefore a 5D spinor contains left- and right-handed 4D components, and the simplest 5D spinor is a Dirac spinor instead of a Weyl or a Majorana spinor. This means that a bulk fermion is not chiral but vector like. However the SM contains chiral fermions. To obtain chiral fermions in 5D theories (and in general in higher dimensional theories) we will consider extra spaces with boundaries (orbifolds). Therefore imposing appropriate boundary conditions over the fermion fields it is possible to obtain chiral fermions from higher dimensions.

A 5D fermion is a Dirac fermion $\Psi = \Psi_L + \Psi_R$, where Ψ_L and Ψ_R are the left- and right-handed components respectively, defined by $\gamma_5 \Psi_{L,R} = \mp \Psi_{L,R}$. To obtain the equation of motion for the fermion field we have to calculate the variation of the action with respect to Ψ and $\bar{\Psi}$. As in this process one generates boundary terms we will explain it in detail. To obtain the variation with respect to $\bar{\Psi}$ we have to integrate by parts the term of Eq. (3.3) containing $(D_M \Psi)^\dagger$ and we obtain

$$\frac{1}{g_5^2}\int d^4x \int_{L_0}^{L_1} dz \, \sqrt{g}\left[i\,\bar{\Psi} \mathcal{D} \Psi\right] + \frac{1}{2g_5^2}\int d^4x \, a^4 (\bar{\Psi}_R \Psi_L - \bar{\Psi}_L \Psi_R)\Big|_{L_0}^{L_1}, \tag{3.51}$$

where $\mathcal{D} = e^M_A \Gamma^A D_M - M$ is the 5D Dirac operator. To obtain the variation with respect to Ψ we integrate by parts the term of Eq. (3.3) containing $(D_M\Psi)$ and get

$$\frac{1}{g_5^2}\int d^4x \int_{L_0}^{L_1} dz\,\sqrt{g}\,\left[i\,\overline{\mathcal{D}\Psi}\Psi\right] + \frac{1}{2g_5^2}\int d^4x\,a^4(\bar{\Psi}_L\Psi_R - \bar{\Psi}_R\Psi_L)\Big|_{L_0}^{L_1}. \tag{3.52}$$

We vary Eq. (3.51) with respect to $\bar{\Psi}$ and Eq. (3.52) with respect to Ψ. The variation of the bulk terms leads to the 5D Dirac equation, therefore the bulk action is minimized if the bulk fields satisfy $\mathcal{D}\Psi = 0$. If we do not include extra terms on the branes the variation of the boundary terms leads to

$$0 = \frac{1}{2g_5^2}\int d^4x\,\sqrt{-g_{\rm ind}}\,\left(\bar{\Psi}_L\,\delta\Psi_R + \delta\bar{\Psi}_R\,\Psi_L - \bar{\Psi}_R\,\delta\Psi_L - \delta\bar{\Psi}_L\,\Psi_R\right)\Big|_{L_0}^{L_1}. \tag{3.53}$$

The IR boundary term vanishes if we impose a Dirichlet condition $\Psi_L\big|_{L_1} = 0$ (or $\Psi_R\big|_{L_1} = 0$) and we are left with just the UV boundary term.

3.3.1 Kaluza-Klein description of a 5D fermionic field

In the KK description we cancel the UV boundary term by imposing a Dirichlet boundary condition $\Psi_L\big|_{L_0} = 0$ (or $\Psi_R\big|_{L_0} = 0$) (see ref. [29] and references therein for a general review of the KK decomposition of fermions in 5D spaces). The 5D equation of motion relates Ψ_L and Ψ_R and is given by

$$\not{p}\Psi_{L,R} \mp (2\frac{\partial_5 a}{a} + \partial_5 \pm a m_\psi)\Psi_{R,L} = 0\,, \tag{3.54}$$

where $\not{p} = p_\mu\gamma^\mu$. We apply separation of variables and decompose the 5D fermion field as an infinite series of 4D fields

$$\Psi(x,z)_{L,R} = \sum_n \psi^{(n)}_{L,R}(x) f_n^{\psi_{L,R}}(z)\,. \tag{3.55}$$

where $\psi^{(n)}_{L,R}$ are mass eigenstates satisfying $\not{p}\,\psi^{(n)}_{L,R}(x) = m_{L,R\,n}\psi^{(n)}_{L,R}(x)$ and $f_n^{\psi_{L,R}}(z)$ are the KK wave functions that describe the bulk profile of the KK modes. We insert Eq. (3.55) into Eqs. (3.54) and after some manipulation we obtain two separate equations of second order for the wave functions $f_n^{\psi_{L,R}}$

$$\Big[m^2_{L,R\,n} + (2a^{-1}\partial_5 a + \partial_5)^2 - a^2 m_\psi^2 \pm m_\psi\partial_5 a\Big] f^\psi_{L,R\,n}(z) = 0\,. \tag{3.56}$$

The wave functions $f_n^{\psi_{L,R}}(z)$ are normalized as

$$\int dz\,a^4 f_m^{\psi_{L,R}}(z) f_n^{\psi_{L,R}}(z) = \delta_{mn} \tag{3.57}$$

to obtain canonic kinetic terms for the 4D fields $\psi^{(n)}_{L,R}$. Similar to the scalar field, Eq. (3.56) has two independent solutions $h_{1,2}^{\psi_{L,R}}$ that depend on the specific profile of the warp factor $a(z)$.

We write $f_n^{\psi_{L,R}}$ in terms of $h_{1,2}^{\psi_{L,R}}$ as

$$f_n^{\psi_{L,R}}(z) = \frac{1}{N_n}\left[h_1^{\psi_{L,R}}(m_{L,Rn}, z) + b_{L,R}(m_{L,Rn})\, h_2^{\psi_{L,R}}(m_{L,Rn}, z)\right] , \tag{3.58}$$

where N_n is a normalization constant determined by Eq. (3.57) and b depends on the boundary conditions. As Eq. (3.54) relates the left- and the right-handed fermions, once we fix one component on each boundary the other component is automatically fixed. For example, imposing Dirichlet boundary conditions on the UV or IR boundaries for one of the components: $f_0^{\psi_{R,L}}\big|_{L_{0,1}} = 0$, Eq. (3.54) leads to the UV or IR boundary conditions for the other component: $[2a^{-1}\partial_5 a + \partial_5 - am_\psi]f_0^{\psi_{L,R}}\big|_{L_{0,1}} = 0$. Therefore b is given by

$$\begin{aligned} b_{L,R}(m_{L,Rn}) &= -\left.\frac{\tilde{h}_1^{\psi_{L,R}}(m_{L,Rn}, z)}{\tilde{h}_2^{\psi_{L,R}}(m_{L,Rn}, z)}\right|_{L_0} , \\ b_{L,R}(m_{L,Rn}) &= -\left.\frac{\tilde{h}_1^{\psi_{L,R}}(m_{L,Rn}, z)}{\tilde{h}_2^{\psi_{L,R}}(m_{L,Rn}, z)}\right|_{L_1} . \end{aligned} \tag{3.59}$$

Imposing Dirichlet boundary conditions for the right-handed component we obtain $\tilde{h}_{1,2}^{\psi_R} = h_{1,2}^{\psi_R}$ and $\tilde{h}_{1,2}^{\psi_L} = [2a^{-1}\partial_5 a + \partial_5 - am_\psi]h_{1,2}^{\psi_L}$. On the other hand, imposing Dirichlet boundary conditions for the lefft-handed component $\tilde{h}_{1,2}^{\psi_L} = h_{1,2}^{\psi_L}$ and $\tilde{h}_{1,2}^{\psi_R} = [2a^{-1}\partial_5 a + \partial_5 - am_\psi]h_{1,2}^{\psi_R}$. Eq. (3.59) determines the mass spectrum $m_{L,Rn}$. For AdS space the functions $h_{1,2}^{\psi_{L,R}}$ are given in terms of Bessel functions

$$h_1^{\psi_{L,R}}(m_{L,Rn}, z) = \frac{z^{5/2}}{L_1^{5/2}} J_{\beta_{L,R}}(m_{L,Rn} z) , \qquad h_2^{\psi_{L,R}}(m_{L,Rn}, z) = \frac{z^{5/2}}{L_1^{5/2}} Y_{\beta_{L,R}}(m_{L,Rn} z) , \tag{3.60}$$

where the index β is strictly positive and it is given by

$$\beta = \beta_{L,R} = |m_\psi\, L \pm 1/2| . \tag{3.61}$$

If the 5D space is AdS only in the UV, the functions $h_{1,2}^{\psi_{L,R}}$ can be approximated by Bessel functions only for $z \to 0$.

We are interested in the zero mode of a fermion field because it will correspond to the SM fermions. We consider AdS$_5$ space and solve the KK Eq. (3.56) for $m_{L,R_0} = 0$. The solution is given by

$$f_0^{\psi_{L,R}}(z) = \frac{z^{2\mp m_\psi L}}{N_{L,R0}} \tag{3.62}$$

where N_0 is the normalization constant determined by

$$1 = \int_{L_0}^{L_1} dz\, a(z)^4 \frac{z^{4\mp 2m_\psi L}}{N_{L,R0}^2} = \int_{L_0}^{L_1} dz \frac{z^{\mp 2m_\psi L}}{N_{L,R0}^2} . \tag{3.63}$$

The discussion on the boundary conditions implies that just one of the zero modes survive, either the left- or the right-handed component. In this way 5D theories with boundaries generate a chiral spectrum. On the other hand the massive modes are vector-like. For different values of m_ψ it is possible to localize the zero mode in the boundaries. To understand this localization it is useful to consider Eq. (3.63) in the limit of $L_1 \to \infty$ [29]. For a Left (Right) zero mode the integral is convergent if $m_\psi L > -1/2$ ($m_\psi L < 1/2$), and thus the Left (Right) fermion is localized on the UV brane. Sending $L_0 \to 0$ the integral is convergent for $m_\psi L < -1/2$ ($m_\psi L > 1/2$), and thus the Left (Right) fermion is localized on the IR brane. For $m_\psi L = -1/2$ ($m_\psi L = 1/2$) the Left (Right) zero mode is delocalized.

3.3.2 Holographic description of a 5D fermionic field

We turn to the holographic description of a fermionic field in terms of an UV source. As the 5D action contains first-order derivatives only, it is not possible to fix on the UV Ψ_L and Ψ_R independently. We are allowed to fix just one of the fields, obtaining a left- or right-handed description, but not both of them. We will take as the fixed variable the left-handed component $\Psi_L(x, z = L_0) = \Psi_L^0$ (to obtain the right-handed description one has to fix $\Psi_R(x, z = L_0) = \Psi_R^0$, see ref. [28]). This means that $\delta\Psi_L|_{L_0} = 0$ and Ψ_R is free to vary on the UV. Therefore we must add an extra term to the action, adjusted to cancel the variation of the UV term

$$\tilde{S}_{UV} = \frac{1}{2g_5^2} \int d^4x\, a^4\, \left(\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L\right)\big|_{L_0} . \tag{3.64}$$

We can also add extra UV terms S_{UV}, but only for the source field Ψ_L

$$S_{UV}[\Psi_L^0] = \int d^4x\, \sqrt{-g_0}\mathcal{L}_0(\Psi_L^0) = \int d^4x\, a^4(L_0)\big[- \bar{\Psi}_L^0\, \not{p} \Psi_L^0 + \ldots\big] . \tag{3.65}$$

To obtain the left-handed description we have to solve the 5D Dirac equation and insert it back into the action. The bulk terms cancel out but the boundary term $\tilde{S}_{UV}$, defined in Eq. (3.64), gives a contribution S_{eff}. Hence the whole boundary action is given by $S_{UV} + S_{\text{eff}}$. To solve the 5D Dirac equation we write the 5D field $\Psi_{L,R}$ as a function of its boundary value $\Psi_{L,R}^0$

$$\Psi_{L,R}(p, z) = \frac{f_{L,R}(p, z)}{f_{L,R}(p, L_0)} \Psi_{L,R}^0(p) , \tag{3.66}$$

where $p = \sqrt{p^2}$. Inserting Eq. (3.66) into Eqs. (3.54), we obtain two first-order coupled equations for $f_{L,R}$. We transform them in two separate equations of second order

$$\left[p^2 + (2a^{-1}\partial_5 a + \partial_5)^2 - a^2 m_\psi^2 \pm m_\psi \partial_5 a\right] f_{L,R}(p, z) = 0 , \tag{3.67}$$

with two independent solutions each. Proceeding similar to the KK decomposition we write $f_{L,R}$ in terms of the independent solutions $h_{1,2}^{\psi_{L,R}}$

$$f_{L,R}(p, z) = h_1^{\psi_{L,R}}(p, z) + b_{L,R}(p)\, h_2^{\psi_{L,R}}(p, z) , \tag{3.68}$$

where b depends on the boundary conditions. If there are not extra boundary terms b is given by

$$b_{L,R}(p) = \; - \; \frac{\tilde{h}_1^{\psi_{L,R}}(p, L_1)}{\tilde{h}_2^{\psi_{L,R}}(p, L_1)} \,, \tag{3.69}$$

where $\tilde{h}_{1,2}^{\psi_R} = h_{1,2}^{\psi_R}$ and $\tilde{h}_{1,2}^{\psi_L} = [2a^{-1}\partial_5 a + \partial_5 - am_\psi]h_{1,2}^{\psi_L}$ for a right-handed component with Dirichlet conditions on the IR, and $\tilde{h}_{1,2}^{\psi_L} = h_{1,2}^{\psi_L}$ and $\tilde{h}_{1,2}^{\psi_R} = [2a^{-1}\partial_5 a + \partial_5 - am_\psi]h_{1,2}^{\psi_R}$ for a left-handed component with Dirichlet conditions on the IR. Eqs. (3.66) and (3.54) lead to a relation between the boundary fields $\Psi^0_{L,R}$

$$\not{p} \frac{\Psi^0_{L,R}}{f_{L,R}(p, L_0)} = p \frac{\Psi^0_{R,L}}{f_{R,L}(p, L_0)} \,. \tag{3.70}$$

From this equation we can obtain Ψ^0_R as a function of Ψ^0_L and inserting it into the boundary action $\tilde{S}_{UV}$ we obtain the boundary action for the left-source.

$$\int d^4x \, \bar{\Psi}^0_L \frac{a^4(L_0) p f_R(p, L_0)}{g_5^2 \not{p} f_L(p, L_0)} \Psi^0_L \,. \tag{3.71}$$

To canonically normalize the kinetic term of the source in Eq. (3.65) we define

$$\tilde{\Psi}^0_L = \frac{\Psi^0_L}{a(L_0)^{3/2}} \,. \tag{3.72}$$

Therefore the whole boundary action is $S_{UV}[\tilde{\Psi}^0_L] + S_{\text{eff}}[\tilde{\Psi}^0_L]$, with the effective lagrangian given by

$$\mathcal{L}_{\text{eff}} = \bar{\tilde{\Psi}}{}^0_L \Pi_\psi(p) \tilde{\Psi}^0_L \,, \tag{3.73}$$

where

$$\Pi_\psi(p) = \frac{a(L_0) p f_R(p, L_0)}{g_5^2 \not{p} f_L(p, L_0)} = \frac{a(L_0) p}{g_5^2 \not{p}} \frac{h_1^{\psi_R} + b_R(p) h_2^{\psi_R}}{h_1^{\psi_L} + b_L(p) h_2^{\psi_L}}\Big|_{L_0} \,. \tag{3.74}$$

The holographic procedure says that the correlator Π_ψ corresponds to the two-point function $\langle \mathcal{O}_R \mathcal{O}_R \rangle$ of a 4D theory with a source $\mathcal{L} = \tilde{\Psi}^0_L \mathcal{O}_{\mathcal{R}}$. For a non-dynamical source field the poles of Π_ψ determine the pure SCFT mass spectrum. If we don't introduce UV boundary terms ($S_{UV} = 0$) to cancel $\tilde{S}_{UV}$ defined in Eq. (3.64) we have to impose $\Psi^0_R = 0$. This condition can be obtained computing the variation of $\tilde{S}_{UV}$ with respect to Ψ^0_L. In the KK description the condition $\Psi^0_R = 0$ corresponds to an even bulk fermion Ψ_L, as can be checked by comparing with the KK spectrum calculated above. To obtain a bulk fermion Ψ_L with odd UV boundary conditions, in the KK description we have to impose $\Psi^0_L = 0$. In the holographic description we have to add and extra fermion Ψ'_R on the UV boundary that acts as a Lagrange multiplier. By adding the UV term

$$\mathcal{L}_{UV} = \frac{a(L_0)}{g_5^2} \bar{\Psi}'_R \Psi^0_L + h.c. \,, \tag{3.75}$$

the equation of motion for Ψ'_R gives $\Psi^0_L = 0$. Thus in this case Ψ^0_L acts as a non-dynamical source coupled to the SCFT.

We will discuss the scaling properties and the effects of the UV cutoff for the fermion fields when the space is AdS_5 (we will follow Ref. [28]). In this case the functions $h_{1,2}^{\psi_{L,R}}$ are given by

$$h_1^{\psi_{L,R}}(p,z) = \frac{z^{5/2}}{L_1^{5/2}} J_{\beta_{L,R}}(pz)\,, \qquad h_2^{\psi_{L,R}}(p,z) = \frac{z^{5/2}}{L_1^{5/2}} Y_{\beta_{L,R}}(pz)\,, \tag{3.76}$$

where the index β is defined in Eq. (3.61). Thus the correlator is

$$\Pi_\psi(p) = \frac{Lp}{g_5^2 L_0\, \not{p}} \frac{J_{\beta_R}(pL_0) + b_R(p) Y_{\beta_R}(pL_0)}{J_{\beta_L}(pL_0) + b_L(p) J_{\beta_L}(pL_0)}\,, \tag{3.77}$$

where b depends on the IR boundary conditions. For $\Psi_R|_{L_1} = 0$ we obtain

$$b_L(p) = -\frac{J_{\beta_L-1}(pL_1)}{Y_{\beta_L-1}(pL_1)}\,, \qquad b_R(p) = -\frac{J_{\beta_R}(pL_1)}{Y_{\beta_R}(pL_1)} \tag{3.78}$$

and for $\Psi_L|_{L_1} = 0$ b is given by

$$b_L(p) = -\frac{J_{\beta_L}(pL_1)}{Y_{\beta_L}(pL_1)}\,, \qquad b_R(p) = -\frac{J_{\beta_R+1}(pL_1)}{Y_{\beta_R+1}(pL_1)} \tag{3.79}$$

For different values of m_ψ we obtain different holographic theories. This behaviour can be expected from the localization of the zero modes. Therefore we consider three different cases, depending on whether $m_\psi L$ is bigger, equal or lower than 1/2.

Holographic theory for $m_\psi L > 1/2$

We consider first the case $m_\psi L > 1/2$. We change to Euclidean momentum and take the limits $L_0 \to 0$ and $L_1 \to \infty$ in Eq. (3.77), and get

$$\Pi_\psi(p) \simeq \frac{iL\,\not{p}}{g_5^2}\Big[\frac{1}{2\beta_L - 2} - \frac{2^{1-2\beta_L}\Gamma(1-\beta_L)\cos(\pi\beta_L)}{\Gamma(\beta_L)}(pL_0)^{2\beta_L-2} + \dots\Big]\,, \tag{3.80}$$

where we have included just the first analytic and non-analytic terms, the dots stand for higher order terms. The analytic term gives a kinetic term for the source. The non-analytic term corresponds to the contribution of the SCFT states to the two point function. To extract the scaling dimensions of the operator $\mathcal{O}_R$ coupled to the source ψ^0_L we re-scale the field ψ^0_L by an amount $L_0^{\beta-1}$ and get

$$\langle \mathcal{O}_R(p)\mathcal{O}_R(-p)\rangle = \lim_{L_0\to 0}(\Pi_\psi + \text{counterterms}) = \not{p}\mathrm{p}^{2\beta-2}\frac{\mathrm{L}}{\mathrm{g}_5^2}\frac{2^{1-2\beta_\mathrm{L}}\Gamma(1-\beta_\mathrm{L})\cos(\pi\beta_\mathrm{L})}{\Gamma(\beta_\mathrm{L})}\,. \tag{3.81}$$

To obtain the dimensions of the operator $\mathcal{O}_R$ we Fourier transform the correlator $\langle\mathcal{O}_R\mathcal{O}_R\rangle$ to coordinate space and get

$$\dim \mathcal{O}_R = 3/2 + \beta_L = 3/2 + |m_\psi L + 1/2|\,. \tag{3.82}$$

Thus for fermion fields we can relate the 5D masses with the dimension of the operators in the SCFT through Eq. (3.82).

The spectrum of the pure SCFT is given by the poles of Eq. (3.77) in the limit $L_0 \to 0$. Keeping the UV brane at finite L_0, the divergent terms are interpreted as terms for the source field that becomes dynamical, as can be seen from the kinetic term in Eq. (3.80). This means that the boundary field becomes dynamical and can mix with the bulk. To obtain the mass spectrum we have to invert the whole quadratic term in $S_{UV} + S_{\text{eff}}$. If S_{UV} is absent the spectrum corresponds to the zeroes of Π_ψ, that matches with the KK spectrum of a fermion with even boundary conditions in the UV.

To obtain information about the massless states we take the limit of low momentum $pL_1 \to 0$. From the KK decomposition we know that if the left-handed component satisfies Dirichlet conditions on both boundaries, the right-handed component has a zero mode. Moreover, for $m_\psi L > 1/2$ this mode is localized on the IR boundary. Therefore we should find a massless pole in the correlator in this case. Expanding Eq. (3.77) with the specified boundary conditions we obtain

$$\Pi_\psi(p) \simeq \frac{L}{g_5^2}\frac{1}{\not{p}}\frac{\beta_L L_0^{2\beta_L-2}2}{L_1^{2\beta_L}} + \dots\,, \tag{3.83}$$

where the dots stand for higher orders in powers of pL_1. The pole in the correlator signals the massless right-handed mode. Imposing Dirichlet boundary conditions in the IR for the right-handed component the pole is absent. This is consistent with the KK picture.

It is possible to write a dual 4D lagrangian with the same properties as the ones described above. It consists in a 4D SCFT coupled to an external left-handed source Ψ_L^0 [28]

$$\mathcal{L} = Z_\psi\, \bar{\Psi}_L^0 \not{p}\Psi_L^0 + \frac{\omega}{\Lambda_{UV}^{\beta_L-1}}\left(\bar{\Psi}_L^0\mathcal{O}_R + h.c.\right) - i\xi\,\frac{\bar{\mathcal{O}}_R \not{p}\mathcal{O}_R}{\Lambda_{UV}^{2\beta_L}} + \mathcal{L}_{\text{UV}}(\Psi_L^0) + \mathcal{L}_{\text{SCFT}} + \dots\,, \tag{3.84}$$

where Z_0, ω and ξ are dimensionless running couplings. The dots stand for higher dimensional operators, suppressed by higher powers of Λ_{UV}. This lagrangian describes a source Ψ_L^0 linearly coupled to the SCFT. The scaling dimension of the operator $\mathcal{O}$ agrees with Eq. (3.82). We match the bare kinetic term Z_ψ with the first analytic term in Eq. (3.80) at the scale Λ_{UV}: $Z_\psi(\Lambda_{UV}) = L/g_5^2(2\beta_L - 2)$. As $m_\psi L > 1/2$, the coupling of the source with the SCFT is always irrelevant, and the mixing of the source with the SCFT is negligible. Taking the limit $\Lambda_{UV} \to \infty$ the source decouples from the SCFT. The non-analytic factor $(L_0/L_1)^{2\beta_L-2}$ in Eq. (3.83) indicates that the massless resonance is excited by the source through the coupling of Eq. (3.84).

Holographic theory for $-1/2 \leq m_\psi L \leq 1/2$

In this case the 4D dual lagrangian is the same as Eq. (3.84). Most of the properties of this theory are the same as the one described above. In particular the scaling dimensions of the operator $\mathcal{O}$ are given by the same equation. However there is an important difference with that case: for these values of the 5D fermion mass the coupling of the source and the SCFT is always relevant. Thus the mixing between the source and the resonances can not be neglected. This picture is consistent with the KK approach because for these range of masses the zero mode becomes delocalized.

Holographic theory for $m_\psi L < -1/2$

To study the holographic theory in this range of 5D fermionic masses we start by taking the limits $L_0 \to 0$ and $L_1 \to \infty$ in Eq. (3.77). By working in Euclidean momentum we obtain

$$\Pi_\psi(p) \simeq \frac{iL\not{p}}{g_5^2}\Big[\frac{2\beta_L}{(pL_0)^2} + \frac{2^{1+2\beta_L}\Gamma(1+\beta_L)^2 cotan(\pi\beta_L)}{\pi}(pL_0)^{2\beta_L-2} + \dots\Big], \tag{3.85}$$

where we show only the first analytic and non-analytic terms. There is an important difference with the first case: the pole at zero momentum. It means that the source at the boundary excites a massless state from the bulk. The non-analytic term corresponds to the SCFT contribution to the two-point function. The dimension of the operator $\mathcal{O}_R$ that couples to the source is given by Eq. (3.82).

The massive spectrum of the theory is given by the poles of Π_ψ in the limit of $L_0 \to 0$. In this limit the source decouples and we are left with just the bulk spectrum that coincides with the KK massive spectrum, as can be checked from Eq. (3.77). To obtain information about the massless states we consider first the case $\Psi_L|_{L_1} = 0$. From the analysis of the KK spectrum we know that if Ψ_L has Dirichlet boundary conditions in the UV also, then there is a right-handed zero mode. Therefore the correlator Π_ψ must reproduce this result. Taking the limit $pL_1 \to 0$ in Eq. (3.77) with the specified IR boundary condition we obtain

$$\Pi_\psi(p) \simeq \frac{L}{g_5^2}\Big(\frac{2\beta_L}{\not{p}L_0^2} + \dots\Big), \tag{3.86}$$

where the dots stand for convergent terms. As we expected there is massless right-handed state in this case. We turn to the case $\Psi_R|_{L_1} = 0$. Since $\Psi_L|_{L_0} = 0$, non of the components have Neumann boundary conditions on both branes, and from the KK analysis we know that there is no chiral spectrum. By expanding Eq. (3.77) for $pL_1 \to 0$ with L_0 small but finite we get

$$\Pi_\psi(p) \simeq \frac{2\beta_L\not{p}L}{g_5^2}\frac{1-(L_0/L_1)^{2\beta_L-2}}{4\beta_L(\beta_L-1)-(pL_1)^2(L_0/L_1)^{2\beta_L}}. \tag{3.87}$$

Taking the limit $p \to 0$ there is no massless pole in the correlator. However, if we take first the limit $L_0 \to 0$, we obtain the correlator of Eq. (3.86).

The dual 4D SCFT is more involved in this case. The pole in Eq. (3.85) can not be assigned to an SCFT resonance, because in the limit $L_1 \to \infty$ the conformal symmetry is restored. It can not be cancelled by a counterterm because it is not a local term. To explain this term we have to introduce a new degree of freedom, an elementary field χ_R, that couples to the source. Therefore the source excites this new field and gives the pole in Eq. (3.85). The 4D lagrangian is

$$
\begin{aligned}
\mathcal{L} =& \mathcal{L}_{\rm CFT} + \mathcal{L}_{\rm UV}(\Psi_L^0) + Z_0\, \bar{\Psi}_L^0 \,\not{p}\Psi_L^0 + \tilde{Z}_0\, \bar{\chi}_R \,\not{p}\chi_R + \eta\Lambda_{UV}\left(\bar{\chi}_R\Psi_L^0 + \bar{\Psi}_L^0\chi_R\right) \\
&+ \left[\frac{\omega}{\Lambda_{UV}^{\beta_L-1}}\,\bar{\Psi}_L^0\mathcal{O}_R - i\frac{\tilde{\omega}}{\Lambda_{UV}^{\beta_L}}\,\bar{\chi}_R\,\not{p}\mathcal{O}_R + h.c.\right] - i\xi\,\frac{\bar{\mathcal{O}}_R\,\not{p}\mathcal{O}_R}{\Lambda_{UV}^{2\beta_L}} + \ldots\,,
\end{aligned}
\tag{3.88}
$$

where the field χ_R couples to the source through a mass term of order Λ_{UV}, and it couples to the SCFT through the same operator $\mathcal{O}_R$. The field χ_R and the source Ψ_L marry each other and become massives, with a large mass $\sim \Lambda_{UV}$. In the case $\Psi_L|_{L_1} = 0$, we obtained a massless pole in the correlator that corresponds to an exchange of the field χ_R. This means that the SCFT is not chiral in this case, since we have associated the massless right-handed KK mode to χ_R. On the other hand, for $\Psi_R|_{L_1} = 0$, there is no massless pole in the correlator, Eq. (3.87). As in the 4D dual lagrangian χ_R is a chiral field that can lead to a massless pole, the SFCT must have a massless resonance which marries χ_R. Therefore the SCFT is chiral, it has a massless left-handed resonance which acquires a mass by marrying χ_R.

Running coupling between the fermionic source and the SCFT

We can interpret the coupling of the source with the SCFT in terms of the renormalization equation group, similar to the case of the scalar field. Following ref. [10] we define a dimensionless coupling at low energy $\mu \sim 1/L_1$ by $\lambda(\mu) = \omega(\mu)(\mu/\Lambda_{UV})^{\beta_L-1}/\sqrt{Z(\mu)}$ that satisfies the renormalization group equation

$$
\mu\frac{d\lambda}{d\mu} = (\dim[\mathcal{O}_R] - 5/2)\,\lambda + c\frac{N}{16\pi^2}\lambda^3 + \ldots\,,
\tag{3.89}
$$

where c is a constant and N is defined in Eq. (3.45). The dots stand for higher order terms. The first term corresponds to the anomalous dimension of the operator $\mathcal{O}_R$. The second term arises from the SCFT contribution to the wave function renormalization. The low energy value of λ depends on the dimension of $\mathcal{O}_R$. For $\dim[\mathcal{O}_R] > 5/2$ the coupling decreases with the energy scale μ. At energies below the IR cutoff $1/L_1$ we obtain

$$
\lambda(\mu) \sim (\frac{\mu_{\rm IR}}{\Lambda_{UV}})^{\dim[\mathcal{O}_R]-5/2}\,,
\tag{3.90}
$$

therefore the mixing between the source and the SCFT is very small at low energies. For $\dim[\mathcal{O}_R] < 5/2$ the coupling flows towards a fixed point

$$
\lambda(\mu) = 4\pi\sqrt{\frac{5/2 - \dim[\mathcal{O}_R]}{cN}}\,.
\tag{3.91}
$$

In this case the mixing between the source and the SCFT states is large.

3.4 Gauge fields

The KK and the holographic description for gauge fields will be studied in detail in next chapters. However in this section we will derive the equation of motion and will briefly elaborate on the KK and the holographic description for a general warped space.

To study the gauge fields we have to fix the gauge. We choose to work in the unitary gauge where only physical configurations survive. Thus we add to Eq. (3.3) the following gauge fixing term

$$\mathcal{L}_{GF} = -\frac{a}{2\xi g_5^2}\left[\partial_\mu A_\mu - \frac{\xi}{a}\partial_5(aA_5)\right]^2 . \tag{3.92}$$

By taking the limit $\xi \to \infty$ we obtain an equation for A_5

$$\partial_5(aA_5) = 0 . \tag{3.93}$$

After integration by parts, the 5D equation of motion for the vector A_μ is

$$\left[(\partial^2 - a^{-1}\partial_5 a\partial_5)\eta_{\mu\nu} - \partial_\mu\partial_\nu\right] A_\nu = 0 . \tag{3.94}$$

The are also boundary terms due to the integration by parts

$$\mathcal{L}_{\rm bound} = \frac{a}{2g_5^2}\left(A_\mu\partial_5 A_\mu - 2A_\mu\partial_\mu A_5\right)\Big|_{L_0}^{L_1} . \tag{3.95}$$

We have not included extra terms on the boundaries.

3.4.1 Kaluza-Klein description of a 5D gauge field

To obtain the KK description we expand the vector field as $A_\mu(x,z) = \sum_n A_\mu^{(n)}(x) f_n^A(z)$. The KK equation is given by

$$(-a^{-1}\partial_5 a\partial_5) f_n^A = m_{A_n}^2 f_n^A . \tag{3.96}$$

The wave functions f_n^A are normalized according to $\int dz\, a\, f_m^A f_n^A = \delta_{mn}$. Formally we can express the KK wave functions in terms of two independent solutions, as we did for the scalar and fermion fields

$$f_n^A(z) = \frac{1}{N_n}\left[h_1^A(m_{A_n},z) + b(m_{A_n})\, h_2^A(m_{A_n},z)\right] , \tag{3.97}$$

where b and m_{A_n} are determined by the boundary conditions. The solutions $\tilde{h}_{1,2}^A$ depend on the specific profile of the warp factor. To cancel the boundary terms in Eq. (3.95) we can either choose Dirichlet or Neumann boundary conditions, therefore b is given by

$$b(m_{A_n}) = -\frac{\tilde{h}_1^A(m_{A_n},L_0)}{\tilde{h}_2^A(m_{A_n},L_0)} , \tag{3.98}$$

$$b(m_{A_n}) = -\frac{\tilde{h}_1^A(m_{A_n},L_1)}{\tilde{h}_2^A(m_{A_n},L_1)} , \tag{3.99}$$

where $\tilde{h}^A_{1,2} = h^A_{1,2}, \partial_5 h^A_{1,2}$ respectively for Dirichlet and Neumann boundary conditions. Eq. (3.98) determines the KK massive spectrum of the vector fields.

For AdS$_5$ space the functions $h^A_{1,2}$ are Bessel functions

$$h^A_1(m_{A_n}, z) = \frac{z}{L_1} J_1(m_{A_n} z)\,, \qquad h^A_2(m_{A_n}, z) = \frac{z}{L_1} Y_1(m_{A_n} z)\,. \tag{3.100}$$

A 5D gauge field has a zero mode only for Neumann boundary conditions in both boundaries. Since the KK equation of the zero mode is $\partial_5 f^A_0 = 0$, the zero mode wave function is flat. Imposing Neumann boundary conditions on A_μ, the field A_5 must satisfy Dirichlet boundary conditions to cancel the boundary terms. Therefore Eq. (3.93) implies that $A_5 = 0$. On the other hand, if some of the components A^a_μ have Dirichlet boundary conditions on one of the branes at least, there are no zero modes for these components. The symmetry along the generators T^a is broken. Breaking gauge symmetries by boundary conditions is a very efficient method that will be used in chapter 5 to build a model of EW symmetry breaking.

On the other hand, if $A_\mu|_{L_{0,1}} = 0$, we can impose Neumann boundary conditions for A_5: $\partial_5 a A_5|_{L_{0,1}} = 0$. Thus there is a zero mode A^0_5 with a wave function proportional to the inverse warp factor $f^{A_5}_0 = a^{-1}/N_0$. In chapter 5 this zero mode will be associated with the Higgs field. By the 5D gauge invariance a potential for A_5 is forbidden, but as we will see a finite potential can be generated by quantum effects.

3.4.2 Holographic description of a 5D gauge field

In chapter 4 we will give a detailed holographic description of a 5D gauge field. We will consider the holographic description with sources on the UV brane only. The solution of the 5D equation of motion for A_μ is a linear combination of $h^A_{1,2}$:

$$A_\mu(p, z) = A^0_\mu(p) \frac{h^A_1(p, z) + b(p)\, h^A_2(p, z)}{h^A_1(p, L_0) + b(p)\, h^A_2(p, L_0)}\,, \tag{3.101}$$

where $A^0_\mu(p)$ is the value of the gauge field in the UV: $A_\mu(p, L_0) = A^0_\mu(p)$, and $b(p)$ is determined by the IR boundary conditions. If we do not include extra terms in the IR, we obtain

$$b(p) = -\frac{\tilde{h}^A_1(p, L_1)}{\tilde{h}^A_2(p, L_1)}\,, \tag{3.102}$$

with $\tilde{h}^A_{1,2} = h^A_{1,2}, \partial_5 h^A_{1,2}$ respectively for Dirichlet and Neumann boundary conditions. Substituting the solution back into the action we obtain the effective lagrangian that corresponds to the generating functional of the two-point function

$$\mathcal{L}_{\text{eff}} = -\frac{P_{\mu\nu}}{2g_5^2} a A_\mu \partial_5 A_\nu \big|_{L_0} = \frac{P_{\mu\nu}}{2} A^0_\mu \Pi_A(p) A^0_\nu\,. \tag{3.103}$$

where $P_{\mu\nu} = \eta_{\mu\nu} - p_\mu p_\nu / p^2$ and Π_A is given by

$$\Pi_A(p) = -\frac{a(L_0)}{g_5^2} \frac{\partial_5 h_1^A(p, L_0) + b(p)\, \partial_5 h_2^A(p, L_0)}{h_1^A(p, L_0) + b(p)\, h_2^A(p, L_0)} \,. \tag{3.104}$$

The boundary field plays the role of the external source coupled to the SCFT. For a non-dynamical source the poles of the correlator give the spectrum of the pure CFT, they correspond to the KK spectrum of a gauge field with Dirichlet boundary conditions on the UV. To decouple the sources from the CFT we have to send the UV cutoff to infinity, i.e. $L_0 \to 0$. Keeping L_0 small but finite the CFT generates a kinetic term for the source, thus the source mixes with the resonances. To obtain the mass spectrum one has to invert the whole kinetic term. If there are not extra UV kinetic terms the spectrum is determined by the zeroes of Π_A, that matches with the KK spectrum for Neumann boundary conditions on the UV.

As discussed in chapter 2, global symmetries lead to conserved currents $\partial^\mu J_\mu = 0$. Let as check that the effective theory described by Eq. (3.103) also leads to a conserved current. The current J_μ is obtained by the variation of the effective lagrangian with respect to the source. Therefore in momentum space $\partial^\mu J_\mu$ is given by

$$-p^\mu \frac{P_{\mu\nu}}{2} \Pi_A(p) A_\nu^0 = 0 \,. \tag{3.105}$$

where the equality follows from the contraction $P_{\mu\nu} p_\mu$. Thus a gauge symmetry in the bulk leads to a global conserved current. A gauge symmetry can be spontaneously broken by the Higgs mechanism, leading to massive vector bosons for the broken generators. This mechanism of symmetry breaking can also be implemented in a 5D theory, leading to massive 5D vector fields. Since 5D gauge symmetries correspond in the holographic description to global 4D symmetries, a spontaneous breaking of a local 5D symmetry corresponds to a spontaneous breaking of a 4D global symmetry, thus leading to massless Goldstone boson. In chapter 4 we will use this entry of the dictionary to obtain pions as Goldstone bosons.

If the space is AdS, the functions $h_{1,2}^A$ are given in terms of Bessel functions

$$h_1^A(p, z) = z J_1(pz) \,, \qquad h_2^A(p, z) = z Y_1(pz) \,. \tag{3.106}$$

where $b(p) = -J_\beta(pL_1)/Y_\beta(pL_1)$, with $\beta = 1, 0$ respectively for Dirichlet and Neumann boundary conditions on the IR. Therefore the correlator Π_A is given by

$$\Pi_A(p^2) = -\frac{L}{g_5^2} \frac{p}{L_0} \frac{J_0(pL_0) + b(p) Y_0(pL_0)}{J_1(pL_0) + b(p) J_1(pL_0)} \,. \tag{3.107}$$

Chapter 4

Chiral symmetry breaking from five dimensional spaces

4.1 Introduction

The string/gauge duality [5] has allowed us in the last years to gain new insights into the problem of strongly coupled gauge theories. Although a string description of real QCD has not yet been formulated, different string constructions have been able to describe gauge theories with certain similarities to QCD. In Refs. [30] several authors computed the spectrum of glueballs and the glueball condensates from higher dimensional models. Recently, the incorporation of D7-branes in the $\mathrm{AdS}_5\times \mathrm{S}^5$ background [31] has allowed to address flavor issues [32].

We will consider a more phenomenological approach to QCD by using 5D field theories in Anti-de-Sitter (AdS) (some examples of related approaches can be found in [33]-[47], the list is not exhaustive). This approach is based on the AdS/CFT correspondence [7] that relates strongly coupled conformal field theories (CFT) to weakly coupled 5D theories in AdS. This is a more modest attempt but, in certain regimes, it grasps the generic features of the more involved string constructions.

This approach can be useful to study chiral symmetry breaking in the sector of mesons of QCD. It is known from the OPE that the vector-vector current correlator for large Euclidean momentum, $p \gg \Lambda_{\mathrm{QCD}}$, is given in the chiral limit by [48]

$$\Pi_V(p^2) = p^2 \left[\beta \ln \frac{\mu^2}{p^2} + \frac{\gamma}{p^4} + \frac{\delta}{p^6} + \cdots \right] , \tag{4.1}$$

where $\beta \simeq N_c/(12\pi^2)$, $\gamma \simeq \alpha_s \langle G^2_{\mu\nu}\rangle/12\pi$ and $\delta \simeq -28\pi\alpha_s\langle\bar{q}q\rangle^2/9$ are almost momentum-independent coefficients. Similar expression holds for the axial-axial, scalar and pseudoscalar correlators. Therefore QCD behaves in Eq. (4.1) as a near-conformal theory in the ultraviolet (UV) in which the breaking of the conformal symmetry is given by the condensates. The correlator Π_V, on the other hand, must have, according to the large-N_c expansion, single poles in the imaginary axis of p corresponding to colorless vector resonances. These properties of QCD can be implemented in a 5D theory in AdS. The condensates $\langle\mathcal{O}\rangle$ are described, in the AdS side, by vacuum expectation values (VEV) of scalars Φ whose masses are related to the dimension d of $\mathcal{O}$ by Eq. (3.41) [7]. Confinement and the mass gap in QCD can be obtained in the AdS$_5$ by compactifying the fifth dimension. Alike large-N_c QCD, the 5D theory is also described as a function of weakly coupled states corresponding to the mesons.

In this chapter we will present a simple 5D model to study chiral symmetry breaking in QCD [11, 12]. We will calculate the vector, axial, scalar and pseudoscalar correlators, $\Pi_V, \Pi_A \Pi_S$ and Π_P, and derive from them the masses and decay constants of the vector, axial-vector, scalar and pseudo-Goldstone (PGB) mesons. We will also calculate their interactions and show some generic properties of 5D models. As an example, we will study the electromagnetic form factor of the pions and show how vector-meson dominance (VMD) appears. We will also compute the scalar contribution to the PGB interactions. Finally, we will derive the PGB chiral lagrangian arising from this 5D model and we will give the predictions for the L_i coefficients as well as for the PGB masses. We will compare all these predictions with the experimental data.

4.2 A 5D model for chiral symmetry breaking

The 5D analog of QCD with 3 flavors consists in a theory with a SU(3)$_L\otimes$SU(3)$_R$ gauge symmetry in the 5D bulk and a parity defined as the interchange $L \leftrightarrow R$. We will not consider the extra U(1)$_{L,R}$ that involves the anomaly. We will work in conformal coordinates with the metric defined in Eq. (3.1). We will compactify the space by putting two boundaries, one at $z = L_0$ (UV-boundary) and another at $z = L_1$ (IR-boundary). The limit $L_0 \to 0$ should be taken after divergences are canceled by adding counterterms on the UV boundary [7].

The only fields in the bulk that we will consider are the gauge boson fields, L_M and R_M, and a scalar field Φ transforming as a $(\mathbf{3_L},\mathbf{\bar{3}_R})$ whose VEV will be responsible for the breaking of the chiral symmetry. The action is given by

$$S_5 = \int d^4x \int dz\, \mathcal{L}_5 \,, \tag{4.2}$$

where

$$\mathcal{L}_5 = \sqrt{g}\, M_5 \,\mathrm{Tr}\left[-\frac{1}{4}L_{MN}L^{MN} - \frac{1}{4}R_{MN}R^{MN} + \frac{1}{2}|D_M\Phi|^2 - \frac{1}{2}M_\Phi^2|\Phi|^2\right] , \tag{4.3}$$

the covariant derivative is defined as

$$D_M\Phi = \partial_M\Phi + iL_M\Phi - i\Phi R_M \,, \tag{4.4}$$

and g is the determinant of the metric. We have defined $L_M = L_M^a T^a$ where $M = (\mu, 5)$, $\Phi = \mathbb{1}/\sqrt{3}\,\Phi_s + \Phi_a T_a$ and $\mathrm{Tr}[T^aT^b] = \delta_{ab}$. The coefficient $M_5 = 1/g_5^2$ (see chapter 3) has been factored out in front of the lagrangian so that $1/\sqrt{M_5}$ is the 5D expansion parameter playing the role of $1/\sqrt{N_c}$ in QCD. We will parametrize the scalar field as $\Phi = (v+S)\, e^{iP/v(z)}$ where $v(z) \equiv \langle\Phi\rangle$ and S corresponds to a real scalar and P to a real pseudoscalar ($S \to S$ and $P \to -P$ under $L \leftrightarrow R$). They transform as $\mathbf{1}+\mathbf{8}$ under $\mathrm{SU}(3)_V$.

Let us study $v(z)$ in the case of AdS_5. We assume $M_\Phi^2 = -3/L^2$ that corresponds in the CFT to an operator of dimension 3 such as $\bar{q}q$, as we can see from Eq. (3.41). Solving the bulk equation of motion for $\langle\Phi\rangle$ we get

$$v(z) = c_1\, z + c_2\, z^3\,, \tag{4.5}$$

where c_1 and c_2 are two integration constants. They can be determined as a function of the value of $v(z)$ at the boundaries:

$$c_1 = \frac{\widetilde{M}_q L_1^3 - \xi\, L_0^2}{L L_1 (L_1^2 - L_0^2)}\,, \qquad c_2 = \frac{\xi - \widetilde{M}_q L_1}{L L_1 (L_1^2 - L_0^2)}\,, \tag{4.6}$$

where we have defined

$$\widetilde{M}_q \equiv \frac{L}{L_0} v\big|_{L_0}\,, \qquad \xi \equiv L\, v\big|_{L_1}\,. \tag{4.7}$$

By the AdS/CFT correspondence, a nonzero $\widetilde{M}_q$ is equivalent to put an explicit breaking of the chiral symmetry in the CFT (such as adding quark masses). On the other hand, a nonzero value of c_2 corresponds to an spontaneous breaking of the chiral symmetry in the IR, playing the role of the condensate $\langle\bar{q}q\rangle$ in QCD. Therefore the value of c_2 is determined dynamically by minimizing the action. In order to get a nonzero value for c_2 in the chiral limit ($\widetilde{M}_q = 0$) we add a potential for Φ on the IR-boundary:

$$\mathcal{L}_{\mathrm{IR}} = -a^4 V(\Phi)\big|_{L_1}, \qquad V(\Phi) = -\frac{1}{2} m_b^2\, \mathrm{Tr}\,|\Phi|^2 + \lambda\, \mathrm{Tr}\,|\Phi|^4\,. \tag{4.8}$$

An origin for this type of potentials can be found in string constructions [32, 38]. To determine the value of c_2, or equivalently the value of ξ, we must minimize the effective 4D action obtained after substituting Eq. (4.5) into the 5D action. For $L_0 \to 0$, this is given by

$$S_{\mathrm{eff}} \simeq -\int d^4x\, \mathrm{Tr}\left\{ M_5 L \left[\frac{-\widetilde{M}_q^2}{2L_0^2} + \frac{\widetilde{M}_q^2}{L_1^2} - 2\frac{\xi \widetilde{M}_q}{L_1^3} + \frac{3}{2}\frac{\xi^2}{L_1^4} \right] + V(\xi)\frac{L^4}{L_1^4} \right\}\,, \tag{4.9}$$

that is minimized for

$$\xi^2 = \frac{\mathbb{1}}{4\lambda}\left(m_b^2 L^2 - 3 M_5 L\right) + \mathcal{O}(\widetilde{M}_q)\,. \tag{4.10}$$

This 5D model depends on 5 parameters: [1] $\widetilde{M}_q$, M_5, L_1, ξ and λ. The value of $\widetilde{M}_q$ is related to the quark masses as we will see below. The value of M_5 is related to N_c and $1/L_1$

[1] We trade m_b^2 for ξ by means of Eq. (4.10). In the following we will take $\xi \to \xi\mathbb{1} + \mathcal{O}(\widetilde{M}_q)$ and treat ξ as a parameter.

corresponds to the mass gap to be related to Λ_{QCD}. The model has then two extra parameters with respect to QCD, ξ and λ. However, as we will see, the vector, axial and pseudoscalar sectors of the model does not depend on the value of λ. Therefore we can fix the value of the other four parameters by matching this sector of the model with the corresponding sector of QCD. The scalar sector will fix the value of λ. An estimate of its value can be obtained using naive dimensional analysis (NDA) that gives $\lambda \sim 1/(16\pi^2) \sim 10^{-2} - 10^{-3}$.

Few comments are in order. Using naive dimensional analysis one can estimate that this 5D theory becomes strongly coupled at a scale $\sim 24\pi^3 M_5$. This implies that extra (stringy) physics must appear at this scale or, equivalently, that this is the scale that suppresses higher dimensional operators in Eq. (4.3). We estimate this scale to be around few GeV. Second, we are neglecting the backreaction on the metric due to the presence of the scalar VEV. Although a nonzero energy-momentum tensor of Φ will affect the geometry of the space producing a departure from AdS, this effect will only be relevant at z very close to the IR-boundary, and therefore it will not substantially change our results. We will comment on this in the last section. Notice that neglecting the backreaction corresponds to freeze other possible condensates that turn on in the presence of the quark condensate.

4.3 Vector, axial-vector, scalar and PGB sectors

We are interested in studying the different sectors of the model. It is convenient to define the vector and axial gauge bosons:

$$\begin{aligned} V_M &= \frac{1}{\sqrt{2}}\left(L_M + R_M\right), \\ A_M &= \frac{1}{\sqrt{2}}\left(L_M - R_M\right). \end{aligned} \tag{4.11}$$

Let us first consider the chiral limit $\widetilde{M}_q = 0$. In this case we have $v \propto \mathbb{1}$ and the symmetry breaking pattern U(3)$_L \otimes$ U(3)$_R \to$ U(3)$_V$. By adding the gauge fixing terms

$$\begin{aligned} \mathcal{L}^V_{GF} &= -\frac{M_5 a}{2\xi_V}\,\mathrm{Tr}\left[\partial_\mu V_\mu - \frac{\xi_V}{a}\partial_5(aV_5)\right]^2, \\ \mathcal{L}^A_{GF} &= -\frac{M_5 a}{2\xi_A}\,\mathrm{Tr}\left[\partial_\mu A_\mu - \frac{\xi_A}{a}\partial_5(aA_5) - \xi_A\sqrt{2}a^2 vP\right]^2, \end{aligned} \tag{4.12}$$

the gauge bosons V_μ and A_μ do not mix with the scalars A_5 and P. We will take the limit $\xi_{V,A} \to \infty$, i.e.

$$\partial_5(aV_5) = 0\,, \qquad P = -\frac{1}{\sqrt{2}a^3 v}\partial_5(aA_5)\,. \tag{4.13}$$

After integration by parts the 5D quadratic terms for the gauge bosons, the scalar and the pseudoscalar A_5 are

$$\begin{aligned}
\mathcal{L}_V &= \frac{aM_5}{2}\,\mathrm{Tr}\left\{V_\mu\left[(\partial^2 - a^{-1}\partial_5 a\partial_5)\eta_{\mu\nu} - \partial_\mu\partial_\nu\right]V_\nu\right\}\,, \\
\mathcal{L}_A &= \frac{aM_5}{2}\,\mathrm{Tr}\left\{A_\mu\left[(\partial^2 - a^{-1}\partial_5 a\partial_5 + 2v^2a^2)\eta_{\mu\nu} - \partial_\mu\partial_\nu\right]A_\nu\right\}\,, \\
\mathcal{L}_S &= -\frac{a^3M_5}{2}\,\mathrm{Tr}\left\{S[\partial^2 - a^{-3}\partial_5 a^3\partial_5 + a^2M_\Phi^2]S\right\}, \\
\mathcal{L}_{A_5} &= -\frac{aM_5}{2}\,\mathrm{Tr}\left\{A_5\big[\partial^2\mathcal{D} + \mathcal{D}(2v^2a^2\mathcal{D})\big]A_5\right\},
\end{aligned} \tag{4.14}$$

where $\mathcal{D}$ is a differential operator defined by

$$\mathcal{D} = 1 - \partial_5\left(\frac{1}{2v^2a^3}\partial_5 a\right)\,. \tag{4.15}$$

There are also boundary terms that, after using the 5D equation of motion for A_5 (*i.e.*, $\mathcal{D}A_5 = -\partial^2A_5/(2v^2a^2)$) and Eq. (4.10), can be written as [2]

$$\begin{aligned}
\mathcal{L}_{\mathrm{bound}} &= \frac{M_5 a}{2}\,\mathrm{Tr}\left[V_\mu\partial_5V_\mu - 2V_\mu\partial_\mu V_5 + A_\mu\partial_5A_\mu - 2A_\mu\partial_\mu A_5 - A_5\frac{\partial^2}{2v^2a^3}\partial_5(aA_5) - a^2S\partial_5S\right]\Big|_{L_0}^{L_1} \\
&- a^4V(S)\big|_{L_1} + M_5a^3\,\mathrm{Tr}[S]\partial_5 v\big|_{L_0}\,,
\end{aligned} \tag{4.16}$$

where

$$V(S)\big|_{L_1} = m_S^2\,\mathrm{Tr}[S^2]\big|_{L_1} + \mathcal{O}(S^3), \qquad m_S^2 = \frac{4\lambda\xi^2}{L^2} - \frac{3M_5}{2L} + \mathcal{O}(\widetilde{M}_q)\,. \tag{4.17}$$

The IR-boundary terms can be cancelled by imposing the following boundary conditions:

$$\partial_5V_\mu\big|_{L_1} = V_5\big|_{L_1} = \partial_5A_\mu\big|_{L_1} = A_5\big|_{L_1} = \left[M_5\partial_5 + 2am_S^2\right]S\big|_{L_1} = 0\,. \tag{4.18}$$

The UV-boundary conditions will be discussed later. The interactions between the fields that we will be considering are

$$\mathcal{L}_{VAA_5} = i\sqrt{2}aM_5\,\mathrm{Tr}\left[A_\mu[\partial_5V_\mu,\pi] + \frac{1}{2}A_\mu[V_\mu,A_5]\delta(z-L_0)\right]\,, \tag{4.19}$$

$$\mathcal{L}_{VA_5A_5} = \frac{iaM_5}{\sqrt{2}}\,\mathrm{Tr}\left(\partial_\mu A_5[V_\mu,A_5]\right) + \frac{iM_5}{2\sqrt{2}a^3v^2}\,\mathrm{Tr}\left(\partial_\mu\partial_5(aA_5)[V_\mu,\partial_5(aA_5)]\right)\,, \tag{4.20}$$

$$\mathcal{L}_{SA_5A_5} = \frac{a^3M_5}{2}\,\mathrm{Tr}\left[\frac{S}{v^3a^6}\Big(\partial_\mu\partial_5(aA_5)\Big)^2 - 4vS(\mathcal{D}A_5)^2\right]\,, \tag{4.21}$$

$$\mathcal{L}_{A_5^4} = \frac{M_5}{96a^9v^6}\,\mathrm{Tr}\left[\Big(\partial_5(aA_5)\overleftrightarrow{\partial_\mu}\partial_5(aA_5)\Big)^2\right]\,. \tag{4.22}$$

The SVV interaction is absent. This is a consequence of the $\mathrm{U}(3)_V$ invariance and the fact that only dimension-four operators are considered in Eq. (4.3). This interaction, however, could be induced by higher-dimensional operators or loop effects.

[2] One obtains the same result if, instead of the equation of motion, one uses the mass eigenfunction equation, $\mathcal{D}A_5 = m^2A_5/(2v^2a^2)$, as we will do later to perform a KK decomposition of the sector.

With the above lagrangian for the scalar and pseudoscalar sector we can calculate any relevant physical quantity. We will be considering two approximations. First, we will be working at the tree-level. According to the discussions in chapter 3 this corresponds to work in the large-N_c limit. Since loop effects are expected to be of order $1/N_c$, our predictions for QCD quantities will have a 30% uncertainty. Second, we will take the chiral limit $\widetilde{M}_q \to 0$. For the pseudoscalar sector this limit will be taken in the following way. We will first perform the calculations with $c_1 \to 0$ and fixed L_0 (this is equivalent to $\widetilde{M}_q \to \xi L_0^2/L_1^3$ and $c_2 \to \xi/(LL_1^3)$). Next we will take the limit $L_0 \to 0$. This procedure simplifies the calculations and avoids singularities at $z = L_0$.

4.3.1 The current-current correlators $\Pi_{V,A}$

The vector and axial vector correlators of QCD are defined by

$$\Pi^{\mu\nu}_{V,A}(p^2) = \int d^4x e^{ipx} \langle J^\mu_{V,A}(x) J^\nu_{V,A}(0) \rangle \,, \tag{4.23}$$

where $J^\mu_V = \bar{q}\gamma^\mu q$ and $J^\mu_A = \bar{q}\gamma^\mu\gamma_5 q$.

In QCD the generating functional $\mathcal{S}$ of the current-current correlators is calculated by integrating out the quarks and gluons as a function of the external sources. This must be equivalent in the large-N_c limit to integrate all the colorless resonances at tree-level. The AdS/CFT correspondence tells us that this generating functional is the result of integrating out, at tree-level, the 5D gauge fields restricted to a given UV-boundary value:

$$V_\mu\big|_{L_0} = v_\mu \,, \qquad A_\mu\big|_{L_0} = a_\mu \,. \tag{4.24}$$

The boundary fields v_μ and a_μ play the role of external sources coupled respectively to the vector and axial-vector QCD currents. The effective lagrangian that gives the generating functional of the two-point correlators $\Pi_{V,A}$ is given by:

$$\mathcal{L}_{\text{eff}} = \frac{P_{\mu\nu}}{2} \operatorname{Tr}\left[v_\mu \Pi_V(p^2) v_\nu + a_\mu \Pi_A(p^2) a_\nu \right] \,. \tag{4.25}$$

For the AdS$_5$ space the Π_V can be calculated analytically (see Eq. (3.106) and refs. [49, 10]):

$$\Pi_V(p^2) = -M_5 L \frac{ip}{L_0} \frac{J_0(ipL_1)Y_0(ipL_0) - J_0(ipL_0)Y_0(ipL_1)}{J_0(ipL_1)Y_1(ipL_0) - J_1(ipL_0)Y_0(ipL_1)} \,, \tag{4.26}$$

where J_n, Y_n are Bessel functions of order n and p is the Euclidean momentum. For large momentum, $pL_1 \gg 1$, the dependence on p of the correlators is dictated by the conformal symmetry and we find

$$\Pi_V(p^2) \simeq -\frac{M_5 L}{2} p^2 \ln(p^2 L_0^2) \,. \tag{4.27}$$

$1/L_0$ plays the role of a UV-cutoff that can be absorbed in the bare kinetic term of v_μ. The coefficient of Eq. (4.27) must be matched to the QCD β-function of Eq. (4.1). We get

$$M_5 L = \frac{N_c}{12\pi^2} \equiv \tilde{N}_c\,, \tag{4.28}$$

that fixes the value of the 5D coupling. The next to leading terms in the large momentum expansion of Eq. (4.27) appear suppressed exponentially with the momentum $\sim e^{-pL_1}$, contrary to the QCD Π_V correlator of Eq. (4.1). This is because in our 5D model the vector V_M does not couple to $\langle\Phi\rangle^2$, and therefore it does not feel the breaking of the conformal symmetry coming from $\langle\Phi\rangle^2$. In fact the only breaking of the conformal symmetry that V_M feels arises from the IR-boundary that sharply cuts the AdS$_5$ space, but these effects decouple exponentially at large momentum. To reproduce the extra terms of Eq. (4.1), we would have to consider either higher-dimensional operators mixing V_M with $\langle\Phi\rangle^2$ or IR deviations from the AdS$_5$ space.

In large-N_c QCD the correlators $\Pi_{V,A}$ can be rewritten as a sum over narrow resonances:

$$\Pi_V = p^2 \sum_n \frac{F^2_{V_n}}{p^2 + M^2_{V_n}}\,, \quad \Pi_A = p^2 \sum_n \frac{F^2_{A_n}}{p^2 + M^2_{A_n}} + F^2_\pi\,. \tag{4.29}$$

F_{V_n} and F_{A_n} are the vector and axial-vector decay constants and the poles of $\Pi_{V,A}$ give the mass spectrum. For the AdS$_5$ space the masses M_{V_n} are determined by the poles of Eq. (4.26):

$$J_0(M_{V_n}L_1) \simeq 0 \ \longrightarrow \ M_{V_n} \simeq \left(n - \frac{1}{4}\right)\frac{\pi}{L_1}\,. \tag{4.30}$$

For the $n = 1$ resonance, the rho meson, we have $M_\rho \simeq 2.4/L_1$ that we will use to determine the value of L_1

$$M_\rho \simeq 770 \text{ MeV} \ \rightarrow \ \frac{1}{L_1} \simeq 320 \text{ MeV}\,. \tag{4.31}$$

The vector decay constants are given by the residues of the poles of Π_V/p^2. We obtain

$$F^2_{V_n} = \tilde{N}_c \frac{\pi M_{V_n}}{L_1}\frac{Y_0(M_{V_n}L_1)}{J_1(M_{V_n}L_1)}\,. \tag{4.32}$$

Using Eqs. (4.28), (4.31) and (4.32) we obtain $F_{V_1} \simeq 140$ MeV to be compared with the experimental value $F_\rho = 153$ MeV. For the higher resonances we obtain $F_{V_{2,3}} \simeq 210, 270$ MeV.

The correlator Π_A depends on the z-dependent mass of A_μ and cannot be calculated analytically. Numerical analysis is therefore needed to obtain the masses and decay constants of the axial-vector mesons. Analytical formulas, however, can be obtained if we approximate the 5D mass of A_μ as a IR-boundary 4D mass, $M_{\rm IR}$. This is expected to be a good approximation since the scalar VEV $v(z)$ that gives a mass to A_μ grows towards the IR-boundary as $v(z) \simeq (z/L_1)^3\xi/L$ and is only relevant for values of z close to the IR-boundary. The value of $M_{\rm IR}$ is determined by

$$\int_{L_0}^{L_1} dz\, a^3(z) M^2_{\rm IR} A_\mu\, \delta(z - L_1) = M_5 \int_{L_0}^{L_1} dz\, 2a^3(z) v^2(z) A_\mu\,. \tag{4.33}$$

The effect of a IR-boundary mass is simply to change the IR-boundary condition from Eq. (4.18) to $\left[M_5\partial_5+a^2M_{\rm IR}^2\right]A_\mu\big|_{L_1}=0$ [26], and therefore the equation that determines the mass spectrum changes from Eq. (4.30) to

$$J_0(M_{A_n}L_1)\simeq-\int_{L_0}^{L_1}dz\,\frac{2a^3(z)v^2(z)}{M_{A_n}L}zJ_1(M_{A_n}z)\,. \tag{4.34}$$

In Fig. 4.1 we show the value of the mass of the lowest state as a function of ξ. We compare the exact numerical value of M_{A_1} and the approximate value coming from Eq. (4.34). We see that the difference is below the 10%. For $\xi\simeq 4$ we find that M_{A_1} coincides with the experimental mass of the a_1, $M_{a_1}\simeq 1230$ MeV. We then see that the experimental data favor values of ξ around 4. For this value $\xi\simeq 4$ we also find that $F_{A_1}\simeq 160$ MeV. For the second resonance we find, for $\xi=4$, $M_{A_2}\simeq 2$ GeV and $F_{A_2}\simeq 200$ MeV. For heavier axial-vector resonances the right-hand side of Eq. (4.34) can be neglected and then their masses approach to the values of the vector masses Eq. (4.30) (and similarly for the decay constants).

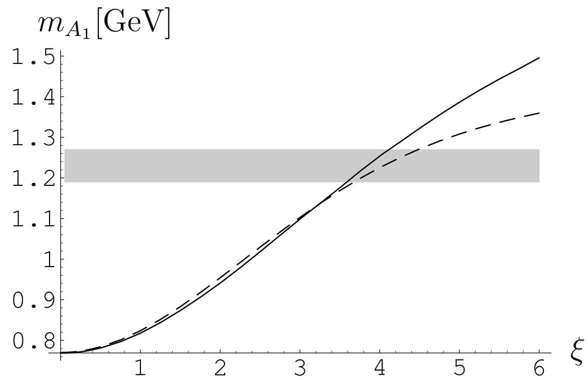


Figure 4.1: *Mass of the first axial-vector resonance as a function of ξ. The solid line is the exact result, while the dashed line corresponds to the approximate value coming from Eq. (4.34). The shadow band shows the experimental value $M_{a_1}=1230\pm 40$ MeV.*

In the large and small momentum limits the correlator Π_A can also be calculated analytically without the need of the above approximation. Furthermore these analytical expressions simplify enormously if $\xi\gg 1$. In this limit we find that the dependence of Π_A on ξ is simply dictated by the conformal symmetry. For small p, we have

$$\Pi_A(p^2)=\Pi_A(0)+p^2\Pi_A'(0)+\mathcal{O}(p^4)\,, \tag{4.35}$$

where for $\xi\gg 1$

$$\Pi_A(0)\;=\;F_\pi^2\simeq\frac{2^{5/3}\pi}{3^{1/6}\Gamma(\frac{1}{3})^2}\frac{\tilde N_c\,\xi^{2/3}}{L_1^2}\,, \tag{4.36}$$

$$\Pi_A'(0)\;\simeq\;-\tilde N_c\left[\ln\frac{L_0}{L_1}+\ln\xi^{1/3}+\frac{4\gamma+\pi\sqrt{3}-\ln 12}{12}\right]. \tag{4.37}$$

From Eq. (4.36), making use of Eqs. (4.28) and (4.31), we get

$$F_\pi \simeq 87 \left(\frac{\xi}{4}\right)^{\frac{1}{3}} \text{ MeV}\,, \tag{4.38}$$

in excellent agreement with the experimental value for $\xi \simeq 4$. We have checked that the approximate value of F_π Eq. (4.38) differs from the exact value by less than 10% if $\xi \gtrsim 3$. Adding an explicit breaking of the chiral symmetry, $\widetilde{M}_q \neq 0$, gives an extra contribution to $\Pi_A(0)$. By expanding around $\widetilde{M}_q = 0$, we obtain $\Pi_A(0) = \Pi_A^{(0)}(0) + \widetilde{M}_q L_1 \Pi_A^{(1)}(0) + \cdots$, where, in the limit $\xi \gg 1$, $\Pi_A^{(0)}$ is given by Eq. (4.36) and

$$\Pi_A^{(1)}(0) \simeq \frac{2^{8/3} 3^{2/3} \pi}{\Gamma(\frac{1}{6})^2} \frac{\tilde{N}_c\, \xi^{1/3}}{L_1^2} \left(1 - \frac{2\Gamma(\frac{1}{6})}{6^{4/3}\sqrt{\pi}} \frac{1}{\xi^{2/3}}\right) . \tag{4.39}$$

Eq. (4.39) gives a contribution to F_π proportional to the quark masses $\widetilde{M}_q$.

In the large momentum limit Π_A is given in the chiral limit by

$$\Pi_A(p^2) = -p^2 \left[\frac{\tilde{N}_c}{2} \ln(p^2 L_0^2) + \frac{c_6}{p^6} + \mathcal{O}(\frac{1}{p^{12}})\right] , \qquad \text{where} \quad c_6 = -\frac{16}{5} \frac{\tilde{N}_c\, \xi^2}{L_1^6} . \tag{4.40}$$

As we said before, corrections to Eq. (4.40) are expected if the 5D metric departs in the IR from AdS. Nevertheless, these corrections cancel out in the left-right correlator $\Pi_{LR} = \Pi_V - \Pi_A$. Therefore, at large momentum we have

$$\Pi_{LR}(p^2) \simeq \frac{c_6}{p^4} + \cdots\,, \tag{4.41}$$

independently of variations in the AdS$_5$ metric. Eq. (4.40) gives

$$c_6 \simeq -1.4 \times 10^{-3} \left(\frac{\xi}{4}\right)^2 \text{GeV}^6\,, \tag{4.42}$$

to be compared with the QCD value $c_6 = -4\pi\alpha_s \langle \bar{q}q\rangle^2 \simeq -1.3 \times 10^{-3}$ GeV^6 extracted from the evaluation of the condensate of Ref. [50]. We must notice, however, that Eq. (4.41) will be affected by higher-dimensional operators such as $\mathrm{Tr}[\Phi^\dagger L_{MN} \Phi R^{MN}]$.

From the analysis of the vector and axial-vector sectors we have determined the values of $M_5 = N_c/12\pi^2$, $L_1 \simeq 1/320$ Mev^{-1} and $\xi \simeq 4$. They can be fixed by using, for example, the QCD values for N_c, M_ρ and M_{a_1}. Our predictions for the scalar and pseudoscalar sectors and for the interactions will be given using the above values (although in certain cases we will study the dependence of the predictions on ξ). This leaves the scalar sector of the theory depending only on one parameter, λ.

4.3.2 The scalar and pseudoscalar correlators $\Pi_{S,P}$

In this section we will calculate the scalar and pseudoscalar two-point correlator. In QCD these are defined as

$$\Pi_{S,P}(p^2) = -\int d^4x e^{ipx} \langle J_{S,P}(x) J_{S,P}(0)\rangle \,, \tag{4.43}$$

where $J_S = \bar{q}q$ and $J_P = i\bar{q}\gamma_5 q$. The correlators $\Pi_{S,P}$ can be obtained from the generating functional $\mathcal{S}$ according to

$$\Pi_S = \frac{\delta^2 \mathcal{S}}{\delta s^2} \,, \qquad \Pi_P = \frac{\delta^2 \mathcal{S}}{\delta p_s^2} \,, \tag{4.44}$$

where s and p_s are the scalar and pseudoscalar external sources coupled to QCD:

$$\mathcal{L} = -\,\mathrm{Tr}[\bar{q}_L\, \phi\, q_R] + h.c., \quad \phi = M_q + s + ip_s \,. \tag{4.45}$$

The UV boundary condition for the 5D scalar field is

$$\Phi\big|_{L_0} = \alpha \frac{L_0}{L}\phi \,, \tag{4.46}$$

where the constant α will be determined by matching with the QCD correlators in the UV as we will see later. Up to the quadratic order in the fields, Eq. (4.46) leads to

$$S\big|_{L_0} = \alpha\frac{L_0}{L}\left(s + \alpha\frac{p_s^2}{2\widetilde{M}_q}\right), \qquad P\big|_{L_0} = -\frac{\partial_5(aA_5)}{\sqrt{2}a^3 v}\Big|_{L_0} = \alpha\frac{L_0}{L}p_s \,. \tag{4.47}$$

Let us calculate $\mathcal{S} = \int d^4x\, \mathcal{L}_{\rm eff}$ at the quadratic level for S and A_5. By solving the equations of motion from Eq. (4.14) with the boundary conditions of Eqs. (4.18) and (4.47), and substituting the solution back into the action, we get (in momentum space) [3]

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\Pi_S(p^2)\,\mathrm{Tr}[s^2] + \frac{1}{2}\Pi_P(p^2)\,\mathrm{Tr}[p_s^2] + \Gamma_S\,\mathrm{Tr}[s] \,. \tag{4.48}$$

For a AdS$_5$ space Π_S can be given analytically at the tree-level. We obtain

$$\Pi_S(p^2) = \alpha^2 M_5 L\left[\frac{1}{L_0^2} + \frac{ip}{L_0}\frac{J_0(ipL_0) + b(p)Y_0(ipL_0)}{J_1(ipL_0) + b(p)Y_1(ipL_0)}\right] \,, \tag{4.49}$$

where J_n, Y_n are Bessel functions, p is the Euclidean momentum and $b(p)$ is determined by the IR-boundary condition of Eq. (4.18):

$$b(p) = -\frac{ipL_1 J_2(ipL_1) - \frac{8\lambda\xi^2}{M_5 L}J_1(ipL_1)}{ipL_1 Y_2(ipL_1) - \frac{8\lambda\xi^2}{M_5 L}Y_1(ipL_1)} \,. \tag{4.50}$$

Taking the limit $L_0 \to 0$ we find

$$\Pi_S(p^2) \simeq \alpha^2 M_5 L\left[\frac{1}{L_0^2} + \frac{1}{2}p^2\ln(p^2 L_0^2) + \frac{\pi p^2}{2b(p)}\right] \,. \tag{4.51}$$

[3] There is also a mixing term between p_s and the longitudinal part of $A_\mu|_{L_0}$ that we are not writing.

The divergent terms for $L_0 \to 0$ can be absorbed in a bare mass and a bare kinetic term for s. After this renormalization the correlator is finite. For large momentum $pL_1 \gg 1$, we find

$$\Pi_S(p^2) \simeq \frac{\alpha^2 M_5 L}{2} p^2 \ln p^2 \,, \tag{4.52}$$

as expected from the conformal symmetry. Matching with QCD in which at large momentum we have

$$\Pi_S^{\rm QCD}(p^2) \simeq \frac{N_c}{8\pi^2} p^2 \ln p^2 \,, \tag{4.53}$$

we obtain, using Eq. (4.28),

$$\alpha = \sqrt{3} \,. \tag{4.54}$$

The next to leading terms in the large momentum expansion in Eq. (4.52) are suppressed exponentially, contrary to QCD where one finds that the scalar correlator has power corrections. This is because we assumed, for simplicity, that the scalar had a potential only on the IR-boundary. In more realistic models such as those arising from string theories the scalar potential is present in the 5D bulk (although peaked towards the IR). In these cases the scalar correlator has power corrections. Also, if the 5D metric deviate in the IR from AdS or if higher-dimensional operators are included in Eq.(4.3), then power corrections can be present in Π_S.

For small momentum $\Pi_S(p^2)$ can be approximated by

$$\Pi_S(p^2) \simeq 3\tilde{N}_c \left[-\frac{2}{L_1^2} + \frac{\tilde{N}_c}{2\lambda\xi^2 L_1^2} \right] + \mathcal{O}(p^2) \,. \tag{4.55}$$

The scalar correlator Eq. (4.49) can also be written as a sum over infinitely narrow resonances, similarly as in large-N_c QCD:

$$\Pi_S(p^2) = \sum_n \frac{F_{S_n}^2 M_{S_n}^2}{p^2 + M_{S_n}^2} \,. \tag{4.56}$$

Therefore the masses of the scalar resonances can be determined by finding the poles of Eq. (4.51), *i.e.*, by the equation $b(p) = 0$. In Fig. 4.2 we plot the value of the mass of the first and second scalar resonance as a function of λ for $\xi = 4$, that is the preferred value of ξ for the axial sector. The first resonance mass ranges from $M_{S_1} = 0$ MeV ($\lambda \to 0$) to $M_{S_1} = 1226$ MeV ($\lambda \to \infty$). We compare this value with the masses of the a_0 states (since these are the QCD scalars whose masses are not very sensitive to M_q). We see that for a value of λ close to its NDA estimate, $\lambda \sim 10^{-2} - 10^{-3}$, the mass of the first scalar resonance is closer to that of $a_0(980)$ than to that of $a_0(1450)$. Nevertheless we must recall that we are working in the large-N_c limit and then corrections can be as large as 30%. Consequently we cannot discard to associate $S^{(1)}$ with $a_0(1450)$. The scalar decay constants F_{S_n} are determined by the residues of Π_S. We obtain

$$F_{S_n}^2 = \frac{3\tilde{N}_c \pi M_{S_n}^2 \left(\frac{8\lambda\xi^2}{M_5 L} Y_1(M_{S_n} L_1) - M_{S_n} L_1 Y_2(M_{S_n} L_1) \right)}{M_{S_n} L_1 \left(1 - \frac{8\lambda\xi^2}{M_5 L}\right) J_0(M_{S_n} L_1) + \left(\frac{8\lambda\xi^2}{M_5 L} + M_{S_n}^2 L_1^2 - 2 \right) J_1(M_{S_n} L_1)} \,. \tag{4.57}$$

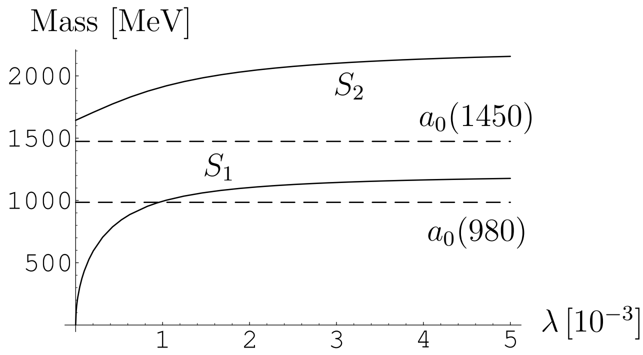


Figure 4.2: *Mass of the first and second scalar resonances as a function of λ for $\xi = 4$. The dashed lines show the experimental values for the masses of the scalar resonances $a_0(980)$ and $a_0(1450)$.*

For $\lambda \simeq 10^{-3}$ we obtain $M_{S_1} \simeq 1$ GeV and $F_{S_1} \simeq 260$ MeV, while for the second resonance we get $M_{S_2} \simeq 1900$ MeV and $F_{S_2} \simeq 370$ MeV. Using this result we can calculate the value of the coupling c_m defined in Ref. [52]. We obtain $c_m = F_{S_1} M_{S_1}/(4B_0) \simeq 41$ MeV (taking the value of B_0 from Eq. (4.100)) very close to the value used in Ref. [52]: $c_m \simeq 42$ MeV.

To calculate the pseudoscalar correlator Π_P we must rely on numerical analysis. Only for small and large momentum we are able to give analytical results. For large momentum $pL_1 \gg 1$ we have

$$\Pi_P(p^2) = \frac{3\tilde{N}_c}{L_0^2} + p^2 \left[\frac{3\tilde{N}_c}{2} \ln(p^2 L_0^2) - \frac{c_6^P}{p^6} + \mathcal{O}(\frac{1}{p^{12}}) \right] , \qquad \text{where} \quad c_6^P = -\frac{64}{5} \frac{3\tilde{N}_c\, \xi^2}{L_1^6} . \quad (4.58)$$

Again the divergences can be cancelled by adding a proper mass and a kinetic term for the pseudoscalar p_s on the UV-boundary. From Eqs. (4.52) and (4.58) we can obtain the correlator $\Pi_{SP} = \Pi_S - \Pi_P$ at large momentum. It drops as $\Pi_{SP} \sim c_6^P/p^4$. Comparing with $\Pi_{LR} = \Pi_V - \Pi_A \sim c_6/p^4$, we find $c_6^P = 12\, c_6$ in strong disagreement with QCD in which one has $c_6^P = 3\, c_6$. This can be improved if, as we said, we consider more realistic theories where the scalar potential is present in the 5D bulk and therefore Π_S has power corrections.

At low momentum [4] and for $\xi \gg 1$ we find

$$\Pi_P(p^2) \simeq \frac{2\widetilde{B}_0^2 F_\pi^2}{p^2} - \tilde{N}_c \widetilde{B}_0^2 + \mathcal{O}(p^2)\,, \quad (4.59)$$

where

$$\widetilde{B}_0 = \frac{2\sqrt{3}\tilde{N}_c \xi}{F_\pi^2 L_1^3}\,. \quad (4.60)$$

[4] In order to obtain the correct result it is important to take the limit $L_0 \to 0$ before taking $p^2 \to 0$ [28].

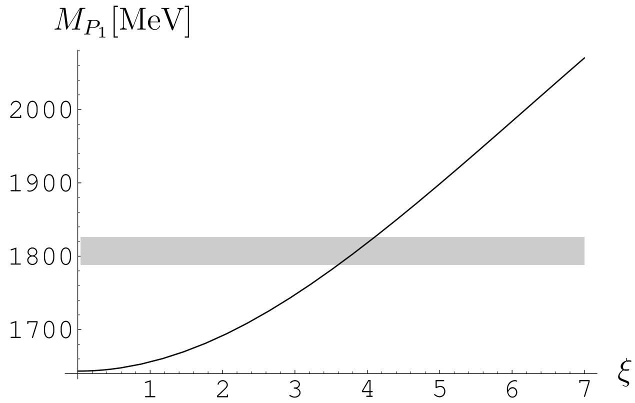


Figure 4.3: *Mass of the first massive pseudoscalar resonance as a function of ξ. The shadow band shows the experimental value for $\pi(1800)$.*

The first term of Eq. (4.59) shows a pole at $p^2 = 0$ as expected due to the presence of the massless PGB.

By looking at the poles of Π_P we can find the pseudoscalar masses. The lowest mode is the massless PGB of the spontaneous chiral symmetry breaking. There is a nonet of PGBs but we must recall that the inclusion of the $\mathrm{U}(1)_A$-anomaly will give mass to the singlet [38]. The mass of the first massive resonance is shown in Fig. 4.3 as a function of ξ. We see that its value is far from the mass of the $\pi(1300)$ state. Nevertheless, we find that, for $\xi \simeq 4$, M_{P_1} is close to the mass of $\pi(1800)$ suggesting that this could be the state to be associated with our first massive pseudoscalar resonance. For this resonance we find a decay constant $F_{P_1} \simeq 374$ MeV.

Finally, we calculate the linear term in Eq. (4.48) to be associated in QCD with the $\bar{q}q$ condensate: $\Gamma_S = -\langle J_S \rangle$. We find

$$\Gamma_S = \sqrt{3}\tilde{N}_c \frac{\widetilde{M}_q L_1^2 + 2\xi L_0^2/L_1 - 3\widetilde{M}_q L_0^2}{L_0^2(L_1^2 - L_0^2)} \quad \xrightarrow{\widetilde{M}_q \to 0} \quad \frac{2\sqrt{3}\tilde{N}_c\,\xi}{L_1(L_1^2 - L_0^2)} \quad \xrightarrow{L_0 \to 0} \quad \frac{2\sqrt{3}\tilde{N}_c\,\xi}{L_1^3}\,. \tag{4.61}$$

4.4 Interactions

To calculate the couplings between the resonances it is convenient to perform a Kaluza-Klein (KK) decomposition of the 5D fields. We expand the fields in a tower of 4D mass-eigenstates, $V_\mu(x,z) = \frac{1}{\sqrt{M_5 L}} \sum_n f_n^V(z) V_\mu^{(n)}(x)$, and equivalently for the other fields (see chapter 3 for the details). To cancel the UV-boundary terms of Eq. (4.16) we impose

$$V_\mu\big|_{L_0} = A_\mu\big|_{L_0} = S\big|_{L_0} = 0\,, \qquad P\big|_{L_0} \propto \partial_5(aA_5)\big|_{L_0} = 0\,. \tag{4.62}$$

For the electromagnetic subgroup of $\mathrm{SU}(3)_V$, however, we must consider the boundary condition $\partial_5 V_\mu\big|_{L_0} = 0$ in order to have a massless mode in the spectrum, the photon, whose wave-function satisfies

$$\partial_5 f_0^V = 0\,. \tag{4.63}$$

In the limit $L_0 \to 0$ this state becomes non-normalizable since its kinetic term diverges as $\tilde{N}_c \ln(L_1/L_0)$. To keep it as a dynamical field, we can fix $1/L_0$ to a large but finite value. In the absence of UV-boundary kinetic terms, this value of $1/L_0$ defines the scale of the Landau pole [49]: [5]

$$\frac{1}{e^2(\mu)} = -\tilde{N}_c \ln(L_0\mu)\,. \tag{4.64}$$

The wave-functions of the KK modes $V_\mu^{(n)}$ ($n \neq 0$) are given for the AdS$_5$ case by Eq. (3.100)[26]:

$$f_n^V(z) = \frac{z}{N_{V_n} L_1}\left[J_1(M_{V_n} z) + b(M_{V_n}) Y_1(M_{V_n} z)\right] \quad \xrightarrow{L_0\to 0} \quad \frac{z}{N_{V_n} L_1} J_1(M_{V_n} z)\,, \tag{4.65}$$

where $b(M_{V_n}) = -\frac{J_1(M_{V_n}L_0)}{Y_1(M_{V_n}L_0)}$ and N_{V_n} is a constant fixed by canonically normalizing the field. The masses M_{V_n} are determined by the condition $\partial_5 f_n^V(z)\big|_{L_1} = 0$ that coincides with the poles of Eq. (4.26). For the vector KK modes associated to the electromagnetic subgroup, we have $b(M_{V_n}) = -\frac{J_0(M_{V_n}L_0)}{Y_0(M_{V_n}L_0)}$. In this case the KK masses are different by factors of order e^2 from the values of Eq. (4.30). This is expected since the KK modes are mass-eigenstates and their masses incorporate corrections due to the mixing of the resonances in Eq. (4.29) with the photon. In Fig. 4.4 we plot the wave-function of the first two KK modes.

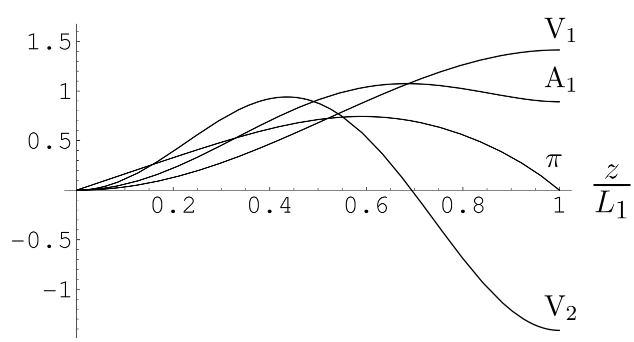


Figure 4.4: *Wave-function of the $n = 1, 2$ vector resonance, the $n = 1$ axial-vector resonance and the PGB.*

For the axial-vector A_μ there is no massless mode. The KK wave-functions cannot be obtained analytically and one must rely in numerical analysis. In Fig. 4.4 we plot the wave-

[5] This is equivalent to add a kinetic term on the UV-boundary with coupling $1/e_0^2 = \tilde{N}_c \ln(L_0\mu) + 1/e^2(\mu)$ that cancels, in the limit $L_0 \to 0$, the divergence in the kinetic term of the massless mode and normalizes this state.

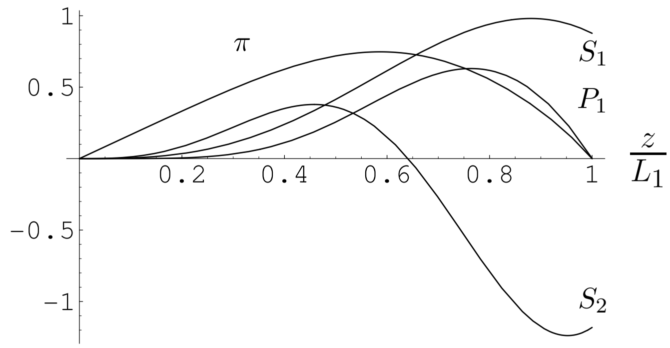


Figure 4.5: *Wave-functions of the $n = 1, 2$ scalar resonance, the PGB and the first massive pseudoscalar for $\xi = 4$ and $\lambda = 10^{-3}$.*

function of the first KK mode, the a_1, for the AdS$_5$ case. Throughout this section we will work in the chiral limit.

The wave-functions of the KK-modes $S^{(n)}$ are given by

$$f_n^S(z) = \frac{z^2}{N_{S_n} L_1^2}\left[J_1(M_{S_n}z) - \frac{J_1(M_{S_n}L_0)}{Y_1(M_{S_n}L_0)}Y_1(M_{S_n}z)\right] \quad \xrightarrow{L_0\to 0} \quad \frac{z^2}{N_{S_n}L_1^2}J_1(M_{S_n}z)\,, \tag{4.66}$$

where N_{S_n} is a constant fixed by canonically normalizing the fields, $\int a^3 (f_n^S)^2 dz/L = 1$. In Fig. 4.5 we plot the wave-functions of the first two KK-modes.

Finally, the equation that determines the wave-functions of the pseudoscalars can be obtained from Eq. (4.14). This is given by

$$\mathcal{D} f_n^P = \frac{M_{P_n}^2}{2v^2a^2} f_n^P\,. \tag{4.67}$$

The lowest state, $P^{(0)} \equiv \pi$, is the PGB arising from the chiral symmetry breaking, that in the limit $L_0 \to 0$ is massless. Its wave-function is given by

$$f^\pi(z) \quad \xrightarrow{L_0\to 0} \quad \frac{z^3}{L_1^3 N_0}\left[I_{2/3}\left(\frac{\sqrt{2}\xi}{3}\frac{z^3}{L_1^3}\right) - \frac{I_{2/3}\left(\sqrt{2}\xi/3\right)}{K_{2/3}\left(\sqrt{2}\xi/3\right)}K_{2/3}\left(\frac{\sqrt{2}\xi}{3}\frac{z^3}{L_1^3}\right)\right]\,, \tag{4.68}$$

where N_0 is determined by the condition $-\frac{1}{2a^2v^2L} f^\pi \partial_5 (a f^\pi)|_{L_0} = 1$. The wave-function of the massive modes must be obtained numerically from Eq. (4.67) with the normalization condition $\int dz\, (f_n^P M_{P_n})^2/(2v^2aL) = 1$. The wave-functions of π and $P^{(1)}$ are shown in Fig. 4.5 (the π wave-function is also shown in Fig. 4.4).

4.4.1 Vector-PGB meson interactions

The interactions between the different resonances are easily obtained from Eqs. (4.19)-(4.22) and integrating over z with the corresponding wave-functions. Here we present some phenomenologically relevant examples. The first one is the coupling of the photon to $A_\mu^{(n)}\pi$. Using Eqs. (4.19), (4.62) and (4.63), we obtain that this coupling is zero:

$$g_{A_n\gamma\pi} = 0\,. \tag{4.69}$$

Eq. (4.69) is a consequence of electromagnetic gauge invariance which implies that $p^\nu \mathcal{M}_{\mu\nu} = 0$ where p^ν is the momentum of the photon and $\mathcal{M}_{\mu\nu}$ is the vertex $A_\mu^{(n)}\gamma_\nu\pi$. In the 5D model of Eq. (4.3), in which only dimension 4 operators are considered, we have at tree level that $\mathcal{M}_{\mu\nu}$ can only be proportional to $g_{\mu\nu}$ and then Eq. (4.69) follows from the condition of gauge invariance. Eq. (4.69) has the interesting consequence that, at the leading order in large-N_c, the branching ratio of $a_1 \to \gamma\pi$ vanishes. This coupling, however, could be induced from 5D higher-dimensional operators or quantum loop effects.

Another example is the vector coupling to two PGB. From Eq. (4.20) we get

$$\mathcal{L}_{V_n\pi\pi} = i\frac{g_{n\pi\pi}}{\sqrt{2}}\,\mathrm{Tr}\left(\partial_\mu\pi[V_\mu^{(n)},\pi]\right), \tag{4.70}$$

where $g_{n\pi\pi}$ is given by

$$g_{n\pi\pi} = \frac{1}{\sqrt{M_5 L^3}}\int dz\, a f_n^V\left[(f_0^\pi)^2 + \frac{(\partial_5(af_0^\pi))^2}{2a^4v^2}\right]. \tag{4.71}$$

In Fig. 4.6 we show the coupling of the first three KK modes as a function of ξ for the AdS$_5$ case. One can see that the heavier is the KK mode (larger n), the smaller is its coupling to PGB. This can be understood as a consequence of the increase in the oscillations of the KK wave-function as n increases (see Fig. 4.4), that implies a smaller contribution to the integral Eq. (4.71) for larger n.

Finally, we consider the four-pion interaction. It receives contributions coming from the exchange of vector resonances, scalar resonances, and the four-interaction of Eq. (4.22). At the order p^2, the chiral symmetry tells us that this coupling must be $(1/12F_\pi^2)\,\mathrm{Tr}[(\pi\overleftrightarrow{\partial_\mu}\pi)^2]$. This implies the following sum rule:

$$g_{\pi^4} + \sum_n \frac{g_{n\pi\pi}^2}{M_{V_n}^2} = \frac{1}{3F_\pi^2}\,, \tag{4.72}$$

where g_{π^4} denotes the four-interaction

$$g_{\pi^4} = \frac{1}{24M_5L^2}\int dz\,\frac{\left[\partial_5(af^\pi)\right]^4}{a^9v^6}\,. \tag{4.73}$$

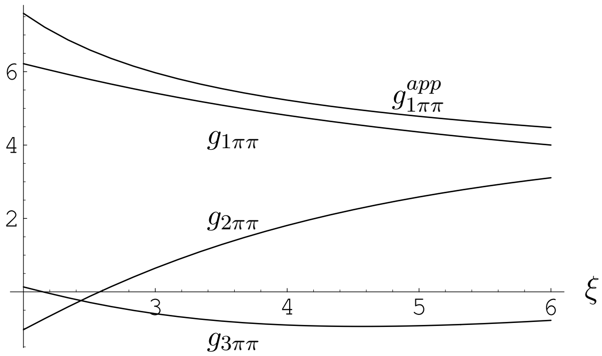


Figure 4.6: *Coupling of the $n=1,2,3$ vector resonance to two PGB. We also show the approximate value for $n=1$ given by $g^{app}_{1\pi\pi}=M_{V_1}/(\sqrt{3}\,F_\pi)$ -see Eq. (4.74).*

We find that for values $\xi \gtrsim 3$ the contribution g_{π^4} is small and only the vector contribution dominates. This is saturated (at the 90% level) by the first resonance, the rho meson, leading to the following approximate relation

$$M_\rho^2 \simeq 3F_\pi^2 g^2_{\rho\pi\pi}\,. \tag{4.74}$$

In Fig. 4.6 we plot the approximate value of $g_{1\pi\pi}$ that arises from Eq. (4.74), and it is shown that the difference from its exact value is only $\sim 10\%$. Eq. (4.74) differs by a factor 2/3 from the KSRF relation [53], $M_\rho^2 \simeq 2F_\pi^2 g^2_{\rho\pi\pi}$, that is known to be experimentally very successful. The approximate relation Eq. (4.74) had been found previously in a specific extra dimensional model [36]. We have shown here, however, that it is a general prediction of 5D models independent of the space geometry. It only relies on the 5D gauge symmetry that forbids terms with four A_5.

4.4.2 The electromagnetic form factor of the pion

The electromagnetic form factor of the pion, $\mathcal{F}_\pi(p)$ where p is the momentum transfer, corresponds to the coupling of the pion to the external vector field v_μ. In the 5D picture the pion does not couple directly to v_μ but only through the interchange of the vector resonances. This is because the pion wave-function is zero at the UV-boundary and therefore the pion can only couple to the UV-boundary fields through the KK states. This implies that the form factor of the pion can be written as

$$\mathcal{F}_\pi(p) = \sum_n g_{n\pi\pi}\frac{M_{V_n}F_{V_n}}{p^2+M^2_{V_n}}\,. \tag{4.75}$$

The quantization of the electric charge of the pion implies $\mathcal{F}_\pi(0)=1$ from which one can derive the sum rule $\sum_n g_{n\pi\pi}F_{V_n}/M_{V_n}=1$. Above we saw that the coupling $g_{n\pi\pi}$ and the inverse of the mass decrease as n increases implying that this sum rule is mostly dominated by the first resonance and therefore

$$g_{\rho\pi\pi} F_\rho \simeq M_\rho \, . \tag{4.76}$$

For $\xi \simeq 4$, this relation is fulfilled at the 88% level. For larger values of ξ, however, Eq. (4.76) is not so well satisfied since the contribution of the second resonance becomes important. Eq. (4.76) together with Eq. (4.74) allows us to write a relation between the ρ and π decay constants

$$F_\rho \simeq \sqrt{3} F_\pi \, . \tag{4.77}$$

At large momentum the contribution of each $n > 1$ resonance to $\mathcal{F}_\pi(p)$ becomes sizable since the small value of $g_{n\pi\pi}$ for $n > 1$ is compensated by the large value of $M_{V_n} F_{V_n}$. Nevertheless, the total contribution coming from summing over all the modes with $n > 1$ approximately cancels out, implying that the rho meson dominates in Eq. (4.75) even at large momentum. This can be seen in Fig. 4.7 where we compare the exact result for $\mathcal{F}_\pi(p)$ to the result in which only one resonance is considered $\mathcal{F}_\pi(p) = M_\rho^2/(p^2 + M_\rho^2)$. The cancellation of the contribution of the heavy modes to $\mathcal{F}_\pi(p)$ is a consequence of the conformal symmetry. At large momentum transfer the conformal symmetry tells us that the electromagnetic form factor of a scalar hadron drops as $1/p^{(2\tau_h - 2)}$ where $\tau_h =$Dim$[\mathcal{O}_h] - s$ being $\mathcal{O}_h$ the local operator that creates the hadron from the vacuum and s the spin of the operator [33]. For the case of the pion we have that $\tau_h = 2$ (where $\mathcal{O}_h$ is the axial-vector current operator) and then $\mathcal{F}_\pi(p)$ must drop as $1/p^2$. This large momentum behaviour coincides with that of the rho contribution to $\mathcal{F}_\pi(p)$.

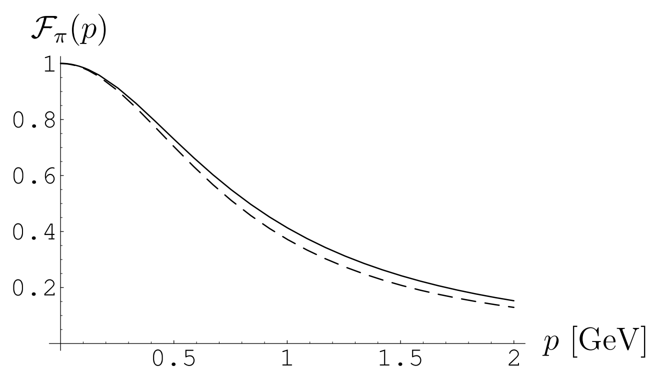


Figure 4.7: *Electromagnetic pion form factor as a function of the transfer momentum p for $\xi = 4$. The solid line is the exact result, while the dashed line is obtained by considering only the rho meson (VMD).*

The hypothesis that the electromagnetic form factor of the pion is dominated by the rho meson, that goes under name of VMD, was proposed long ago. It did not have any theoretical motivation, but it led to a good agreement with experiments tough. We have seen, however,

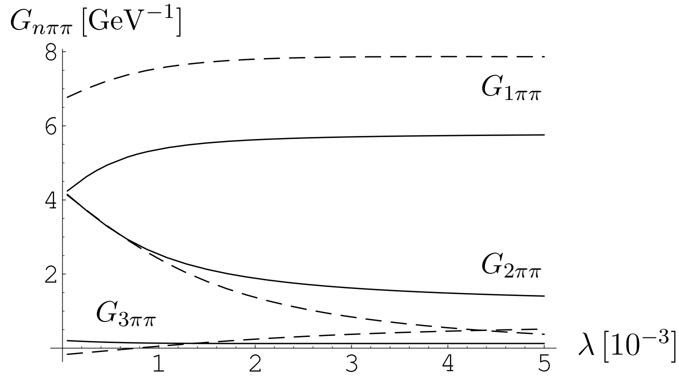


Figure 4.8: *Coupling of the* $n = 1, 2, 3$ *scalar resonance to two PGBs as a function of* λ *for* $\xi = 4$ *(solid line) and* $\xi = 3$ *(dashed line).*

that VMD in $\mathcal{F}_\pi(p)$ appears as an unavoidable consequence of this 5D model for $\xi \sim 4$ (see also Ref. [36]).

4.4.3 Scalar-PGB meson interactions

We consider the coupling between scalar and pesudoscalar fields. The coupling of a scalar to two PGBs comes from Eq. (4.21). We obtain

$$\mathcal{L}_{S_n\pi\pi} = G_{n\pi\pi}\,\mathrm{Tr}[S^{(n)}(\partial_\mu\pi)^2]\,, \tag{4.78}$$

where $G_{n\pi\pi}$ is given by

$$G_{n\pi\pi} = \frac{1}{\sqrt{M_5 L^3}} \int dz\, f_n^S \frac{[\partial_5(af^\pi)]^2}{2a^3v^3}\,. \tag{4.79}$$

In Fig. 4.8 we show the coupling of the first modes as a function of λ for $\xi = 3, 4$. We find that $G_{n\pi\pi}$ becomes smaller as n increases. This property is also present in the coupling between a vector resonance and two PGBs, and it is due to the oscillatory behaviour of the KK wave-functions. Associating $S^{(1)}$ with $a_0(980)$, we find that $M_{S_1} \simeq 980$ MeV for $\lambda \simeq 10^{-3}$, and the prediction of the 5D model for the $a_0\pi\eta$ coupling is $G_{1\pi\pi} \simeq 5.4$ GeV^{-1} for $\xi = 4$. In the notation of Ref. [52] we find $c_d = F_\pi^2 G_{1\pi\pi}/2 \simeq 20$ MeV to be compared to the value $|c_d| \simeq 32$ MeV given there. If the width of $a_0(980)$ is dominated by the decay to $\eta\pi$ we find

$$\Gamma(a_0 \to \eta\pi) \simeq 27 - 56 \text{ MeV}\ , \quad \text{for } \xi = 4 - 3\,. \tag{4.80}$$

Unfortunately, the experimental value of the width of $a_0(980)$ has a large uncertainty $\Gamma(a_0) = 50 - 100$ MeV [54].

4.4.4 (Pseudo)Scalar contributions to PGB interactions

By integrating the heavy scalar resonances we obtain the following four-PGB interaction

$$\mathcal{L}^{(8)}_{\pi^4} = \frac{1}{2}\left\{\mathrm{Tr}[(\partial_\mu\pi)^2(\partial_\nu\pi)^2] - \frac{1}{3}\mathrm{Tr}^2[(\partial_\mu\pi)^2]\right\}\sum_n \frac{G^2_{n\pi\pi}}{p^2+M^2_{S_n}}\,, \tag{4.81}$$

from the scalar octet and

$$\mathcal{L}^{(1)}_{\pi^4} = \frac{1}{6}\mathrm{Tr}^2[(\partial_\mu\pi)^2]\sum_n \frac{G^2_{n\pi\pi}}{p^2+M^2_{S_n}}\,, \tag{4.82}$$

from the scalar singlet. The sum over the KK-modes in Eqs. (4.81) and (4.82) is dominated by the first resonance. At large momentum we find that the first resonance gives 82% of the total contribution and this percentage rises to 94% at zero momentum (for $\lambda \simeq 10^{-3}$). Therefore, as in the vector case, we find that the scalar mediation of the four-PGB interaction is dominated by the exchange of the first resonance.

Four-PGB interactions can also arise from Eq. (4.22). We find

$$\mathcal{L}_{\pi^4} = \frac{g_{\pi^4}}{4}\,\mathrm{Tr}[(\pi\overleftrightarrow{\partial_\mu}\pi)^2]\,, \tag{4.83}$$

with g_{π^4} defined in Eq. (4.73). At high energies the four-PGB amplitude arising from Eq. (4.83) grows as $\sim E^2$. Nevertheless, this bad energy behaviour of the four-PGB amplitude is cured by the contribution arising from Eqs. (4.81) and (4.82) that cancels the E^2 terms. This occurs thanks to the sum rule

$$\sum_n G^2_{n\pi\pi} = 6\,g_{\pi^4}\,. \tag{4.84}$$

Eq. (4.84) is a property of any 5D model in which the breaking of the chiral symmetry is realized by the Higgs mechanism.

We can also calculate the coupling of the PGB to the source s that defines the scalar form factor of the PGB. Apart from a contact piece given by

$$\mathcal{L}_{\pi^2 s} = -\widetilde{B}_0\,\mathrm{Tr}[\pi^2 s]\,, \tag{4.85}$$

this coupling is mediated by the octet and singlet scalar resonances that gives respectively

$$\begin{aligned}
\mathcal{L}^{(8)}_{\pi^2 s} &= \left\{\mathrm{Tr}[(\partial_\mu\pi)^2 s] - \frac{1}{3}\mathrm{Tr}[(\partial_\mu\pi)^2]\,\mathrm{Tr}[s]\right\}\sum_n \frac{G_{n\pi\pi}F_{S_n}M_{S_n}}{p^2+M^2_{S_n}}\,,\\
\mathcal{L}^{(1)}_{\pi^2 s} &= \frac{1}{3}\mathrm{Tr}[(\partial_\mu\pi)^2]\,\mathrm{Tr}[s]\sum_n \frac{G_{n\pi\pi}F_{S_n}M_{S_n}}{p^2+M^2_{S_n}}\,.
\end{aligned} \tag{4.86}$$

The scalar form factor of the PGB is then given by (normalized to unity at zero momentum)

$$\mathcal{F}^S_\pi(p) = 1 - \frac{p^2}{2\widetilde{B}_0}\sum_n \frac{G_{n\pi\pi}F_{S_n}M_{S_n}}{p^2+M^2_{S_n}}\,. \tag{4.87}$$

At low momentum the sum in Eq. (4.87) is dominated by the first resonance that gives 75% of the total contribution (for $\lambda \simeq 10^{-3}$). At large momentum we find that the form factor goes as $1/p^2$, as expected from the conformal symmetry [33]. The cancellation of the constant term in $\mathcal{F}_\pi^S(p)$ occurs due to the sum rule

$$\sum_n G_{n\pi\pi} F_{S_n} M_{S_n} = 2\widetilde{B}_0 \,. \tag{4.88}$$

This sum rule is fulfilled in any 5D model whose metric approaches to AdS$_5$ for $z \to 0$ (conformal theories in the UV). In Eq. (4.88) we find that the first two resonances give a similar contribution, while the contributions of the heavier resonances tend to cancel out. Therefore we see that $\mathcal{F}_\pi^S(p)$ is very well approximated by the exchange of only the first two resonances.

4.4.5 The effective lagrangian for the ρ meson

We have seen that the rho meson gives the largest contribution to the pion couplings. Therefore in order to obtain the chiral lagrangian for the PGB, it is convenient to write first the effective lagrangian for the rho meson.

In order to make contact with the literature [51], we will write the effective lagrangian not in the mass-eigenstate basis but in the basis defined by v_μ as in Eq. (4.24) and the rho field V_μ transforming under the SU(3)$_V$ symmetry as $V_\mu \to hV_\mu h^\dagger + i/g\, h\partial_\mu h^\dagger$. From now on we will follow the notation and definitions of Ref. [51]. The effective lagrangian for V_μ invariant under the chiral symmetry can be written as

$$\mathcal{L}_V = -\frac{1}{4}\,\mathrm{Tr}[V_{\mu\nu}V^{\mu\nu}] + \frac{1}{2}M_\rho^2\,\mathrm{Tr}\left[V_\mu - \frac{i}{g}\Gamma_\mu\right]^2 - \frac{\widetilde{F}_\rho}{2\sqrt{2}M_\rho}\,\mathrm{Tr}[V_{\mu\nu}f_+^{\mu\nu}] + \cdots \,, \tag{4.89}$$

where

$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger(\partial_\mu - iR_\mu)u + u(\partial_\mu - iL_\mu)u^\dagger\right\} \,, \qquad f_+^{\mu\nu} = uF_L^{\mu\nu}u^\dagger + u^\dagger F_R^{\mu\nu}u \,, \tag{4.90}$$

and $u^2 = U$ being U a parametrization of the PGB:

$$U = e^{i\sqrt{2}\,\pi/F_\pi} \,. \tag{4.91}$$

The lagrangian Eq. (4.89) does not contain all possible chiral terms of $\mathcal{O}(p^4)$. We have neglected couplings between π and V_μ involving more than one derivative since these couplings do not arise from a 5D lagrangian. (We have also neglected trilinear couplings between vectors since they do not play any role in our analysis).

Matching the above lagrangian with the 5D AdS theory we obtain

$$\begin{aligned} \frac{1}{g} &= 2\sqrt{2} g_{\rho\pi\pi}\frac{F_\pi^2}{M_\rho^2} \,, \\ \widetilde{F}_\rho &= F_\rho - \frac{M_\rho}{\sqrt{2}g} \,. \end{aligned} \tag{4.92}$$

Notice that our 5D model predicts a nonzero value for $\widetilde{F}_\rho$ (we find $\widetilde{F}_\rho \simeq 40$ MeV for $\xi = 4$) differently from Ref. [51] or models where the rho is considered a Yang-Mills field [55] in which one has $\widetilde{F}_\rho = 0$.

4.5 The chiral lagrangian for the PGB

By integrating all the heavy resonances we can obtain the effective lagrangian for the PGB. This lagrangian is fixed by the chiral symmetry up to some unknown coefficients. In these section we will give the prediction of the AdS_5 model for these coefficients.

At low energies it is almost impossible to describe QCD in terms of the fundamental degrees of freedom, quarks and gluons. Instead, it is natural to describe the physics in terms of the asymptotic hadronic states. Due to the big number of such states, this description is also very difficult. However, at very low energies there is a great simplification. Below the scale of the massive resonances $\Lambda \sim m_\rho$, the spectrum contains only an octet of light particles, the PGB, whose interactions are constrained by the pattern of symmetry breaking. The basic ingredients to build the low energy effective lagrangian are the chiral symmetry and a systematic expansion in terms of increasing powers of momentum. Assuming the spontaneous symmetry breaking $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \to \mathrm{SU}(3)_V$, the Goldstone theorem says that there is an octet of pseudoscalar massless bosons. If the chiral symmetry is explicitly broken by an external source, the pseudoscalars will become massives (PGB). The PGB fields π can be written as a function of a unitary matrix $U(\pi)$, transforming under the chiral group as

$$U(\pi) \to g_R\, U(\pi)\, g_L^\dagger\,, \tag{4.93}$$

where $g_{L,R}$ are elements of $\mathrm{SU}(3)_{L,R}$. The 3×3 matrix π is given by

$$\pi \equiv T^a \pi^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix} \tag{4.94}$$

We organize the chiral lagrangian for the PGB π in increasing powers of momentum. Due to the unitarity of the matrix U at least two derivatives are needed to obtain a non trivial lagrangian. Up to $\mathcal{O}(p^2)$, this is given by

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\,\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]. \tag{4.95}$$

This effective description becomes more powerful if we introduce external sources. The external fields can be used to incorporate electroweak interactions and explicit breaking of the chiral symmetry. Moreover the external sources allow us to obtain the effective realization of the n-point functions of the fundamental theory. We introduce external sources L_μ and R_μ associated to the left and right currents of the chiral symmetry, and a complex scalar source χ associated to the scalar and pseudoscalar currents. Therefor the lowest order lagrangian consistent with the chiral symmetry is

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\,\mathrm{Tr}\left[D_\mu U^\dagger D^\mu U \,+\, U^\dagger\chi \,+\, \chi^\dagger U\right], \tag{4.96}$$

where

$$D_\mu U = \partial_\mu U - iR_\mu U + iUL_\mu\,, \tag{4.97}$$

and

$$\chi = \,2B_0\,(M_q + ip)\;, \qquad M_q = \mathrm{Diag}(m_u, m_d, m_s)\,. \tag{4.98}$$

The constant B_0 is related to the quark condensate by $\langle\bar{q}q\rangle = -B_0F_\pi^2$. In the AdS$_5$ model F_π is given by Eq. (4.36).

For the prediction of B_0 we can use Eq. (4.61):

$$\langle\bar{q}q\rangle = -F_\pi^2 B_0 = -2\sqrt{3}\tilde{N}_c\frac{\xi}{L_1^3} \simeq -(226\ \mathrm{MeV})^3\left(\frac{\xi}{4}\right)\,, \tag{4.99}$$

that leads to

$$B_0 = \frac{2\sqrt{3}\tilde{N}_c\xi}{F_\pi^2 L_1^3} \simeq 1520\left(\frac{\xi}{4}\right)^{\frac{1}{3}}\ \mathrm{MeV}\,. \tag{4.100}$$

Notice that $B_0 = \widetilde{B}_0$ as it should be, since the first term of Eq. (4.59) can also be deduced by integrating out the PGB at tree-level in the chiral lagrangian. The relation $B_0 = \widetilde{B}_0$ also leads to the right matching of Eq. (4.85) with the chiral lagrangian. The value of the quark masses M_q is related to the VEV of Φ on the UV-boundary. Using Eqs. (4.7), (4.46) and (4.54) we obtain [6]

$$M_q = \frac{1}{\sqrt{3}}\widetilde{M_q}\,. \tag{4.101}$$

From the chiral lagrangian we have

$$(m_\pi^2)_{ab} = 2B_0\,\mathrm{Tr}\left[M_q T_a T_b\right]\,, \tag{4.102}$$

that for $m_{\pi^0} \simeq 135$ MeV and $m_{K^0} \simeq 498$ MeV gives

$$m_u + m_d = 11.5\ \mathrm{MeV}\;, \qquad m_s = 150\ \mathrm{MeV}\,. \tag{4.103}$$

The value of the quark masses in Eq. (4.103) are scale independent. This is because we took $M_\Phi^2 = -3/L^2$ that corresponds, by the AdS/CFT dictionary, to fix the dimension of M_q to be

[6] In Refs. [13, 11] the quark masses did not have the correct normalization since the value of α was not calculated.

exactly one. In QCD however the quark masses evolve with the energy scale μ. To minimize this discrepancy we must compare our predictions with the experimental values of the quark masses taken at the lowest energy scale (~ 1 GeV). From Ref. [54] we have $m_u + m_d = 7 - 16$ MeV and $m_s = 108 - 175$ MeV at $\mu \sim 1$ GeV in good agreement with Eq. (4.103).

At the $\mathcal{O}(p^4)$ the chiral lagrangian has extra terms given by [56]

$$\begin{aligned}
\mathcal{L}_4 &= L_1 \,\mathrm{Tr}^2\big[D_\mu U^\dagger D^\mu U\big] + L_2 \,\mathrm{Tr}\,\big[D_\mu U^\dagger D_\nu U\big]\,\mathrm{Tr}\,\big[D^\mu U^\dagger D^\nu U\big] + L_3 \,\mathrm{Tr}\,\big[D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\big] \\
&+ L_4 \,\mathrm{Tr}\,\big[D_\mu U^\dagger D^\mu U\big]\,\mathrm{Tr}\,\big[U^\dagger \chi + \chi^\dagger U\big] + L_5 \,\mathrm{Tr}\,\big[D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U\right)\big] \\
&+ L_6 \,\mathrm{Tr}^2\big[U^\dagger \chi + \chi^\dagger U\big] + L_7 \,\mathrm{Tr}^2\big[U^\dagger \chi - \chi^\dagger U\big] + L_8 \,\mathrm{Tr}\,\big[\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi\big] \\
&- iL_9 \,\mathrm{Tr}\,\big[F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U\big] + L_{10} \,\mathrm{Tr}\,\big[U^\dagger F_R^{\mu\nu} U F_{L\mu\nu}\big] \,. \qquad (4.104)
\end{aligned}$$

The coefficients $L_{1,2,3}$ are responsible for four-pion interactions at $\mathcal{O}(p^4)$, while L_9 gives a contribution to the electromagnetic form factor of the pion at $\mathcal{O}(p^2)$. From the discussion of the previous section we know that the dominant contribution to these processes arises from the rho meson exchange. Therefore the main contribution to $L_{1,2,3,9}$ will arise by integrating out this particle. Thus we will give first the vector contributions to the coefficients L_i. Using the effective lagrangian Eq. (4.89) with Eqs. (4.92), we obtain [7]

$$L_1 = \frac{g_{\rho\pi\pi}^2 F_\pi^4}{8 M_\rho^4} \,, \qquad (4.105)$$

$$L_2 = 2L_1 \,, \qquad L_3 = -6L_1 \,, \qquad (4.106)$$

$$L_9 = \frac{g_{\rho\pi\pi} F_\rho F_\pi^2}{2 M_\rho^3} \,. \qquad (4.107)$$

Using Eqs. (4.74) and (4.76), we get

$$L_1 \simeq \frac{F_\pi^2}{24 M_\rho^2} \,, \qquad L_9 \simeq \frac{F_\pi^2}{2 M_\rho^2} \,. \qquad (4.108)$$

The coefficients $L_{4,6}$ are zero at the tree-level (leading order in the large-N_c expansion). L_7 will not be studied here since it arises from integrating out the singlet PGB that becomes massive when the $\mathrm{U}(1)_A$ anomaly is considered. L_8 only receives contributions from the scalar sector. L_5 and L_{10} can be calculated from the correlators $\Pi_{V,A}$:

$$L_5 = \frac{1}{16 B_0} \frac{d\Pi_A}{dM_q}\bigg|_{M_q=0} \,, \qquad (4.109)$$

$$L_{10} = \frac{1}{4}\big[\Pi'_A(0) - \Pi'_V(0)\big] \,, \qquad (4.110)$$

[7] These coefficients are induced after performing the redefinition $V_\mu \to V_\mu + i\Gamma_\mu/g$ in Eq. (4.89). After this redefinition the rho meson couples to the pion only at $\mathcal{O}(p^3)$ and then it does not induce a contribution to Eq. (4.104) when it is integrated out.

From Eqs. (4.26), (4.36)-(4.39) and (4.99) we obtain for $\xi \gg 1$ with $\lambda\xi^2$ fixed

$$L_5 \simeq \frac{\tilde{N}_c \pi^3}{\sqrt{3}\Gamma(\frac{1}{3})^6}\left[1 - \frac{2\tilde{N}_c}{3F_\pi^2 L_1^2}\right] + \frac{F_\pi^4 L_1^4}{192\lambda\xi^4} \simeq 1.2\cdot 10^{-3}\left[1 - 0.23\left(\frac{4}{\xi}\right)^{\frac{2}{3}} + 0.09\left(\frac{10^{-3}}{\lambda}\right)\left(\frac{4}{\xi}\right)^{\frac{8}{3}}\right], \tag{4.111}$$

$$L_{10} \simeq -\frac{\tilde{N}_c}{4}\left[\ln \xi^{1/3} + \frac{4\gamma + \pi\sqrt{3} - \ln 12}{12}\right] \simeq -5.7\cdot 10^{-3}\left[\ln\left(\frac{\xi}{4}\right)^{\frac{1}{3}} + 1\right]. \tag{4.112}$$

The coefficient L_5 can also be calculated from the scalar sector (see below).

At tree-level, the (pseudo)scalar resonances only contribute to $L_{1,3,4,5,6,8}$. The contributions to the coefficients L_1 and L_3 coming from the octet and singlet scalar can be read from Eqs. (4.81) and (4.82). We obtain

$$L_1^{(8)} = -\frac{1}{3}L_3^{(8)}, \qquad L_1^{(1)} = -L_1^{(8)}, \tag{4.113}$$

$$L_3^{(8)} = \sum_n \frac{G_{n\pi\pi}^2 F_\pi^4}{8M_{S_n}^2}, \qquad L_3^{(1)} = 0. \tag{4.114}$$

The octet and singlet contribution to the coefficient L_1 cancels out, as expected from large-N_c [56], and only L_3 gets a nonzero scalar contribution. For $\lambda \simeq 10^{-3}$ and $\xi = 4$ (3) we obtain $L_3^{(8)} \simeq 0.2\cdot 10^{-3}$ ($0.3\cdot 10^{-3}$). Adding the vector contribution to L_3 we get $L_3 \simeq -2.4\cdot 10^{-3}$ ($-1.7\cdot 10^{-3}$) to be compared with the experimental value [57] $L_3^{\rm exp} \simeq -3.5 \pm 1.1$. The scalar contribution to L_4 and L_5 can be obtained from Eq. (4.86):

$$L_4^{(8)} = -\frac{1}{3}L_5^{(8)}, \qquad L_4^{(1)} = -L_4^{(8)}, \tag{4.115}$$

$$L_5^{(8)} = \frac{F_\pi^2}{8B_0}\sum_n \frac{G_{n\pi\pi}F_{S_n}}{M_{S_n}}, \qquad L_5^{(1)} = 0. \tag{4.116}$$

As expected from large-N_c, the total contribution to L_4 is zero. The value of L_5 is shown in Fig. 4.9 as a function of M_{S_1} for $\xi = 3, 4$. For $M_{S_1} \sim 1$ GeV we obtain $L_5 \simeq 1.1\cdot 10^{-3}$ in good agreement with experiments.

Finally, the coefficient $L_{6,8}$ can be computed from the correlators $\Pi_{S,P}$. We have

$$L_6^{(8)} = -\frac{1}{3}L_8^{(8)}, \qquad L_6^{(1)} = -L_6^{(8)}, \tag{4.117}$$

$$L_8^{(8)} = \frac{1}{32B_0^2}\frac{d}{dp^2}\Big[p^2\big(\Pi_S(p^2) - \Pi_P(p^2)\big)\Big]\Big|_{p^2=0}, \qquad L_8^{(1)} = 0. \tag{4.118}$$

Then $L_6 = L_6^{(8)} + L_6^{(1)} = 0$, as expected from large-N_c. Using Eqs. (4.55), (4.59) and (4.100) in the above equation, we obtain

$$L_8 \simeq \frac{\tilde{N}_c}{32}\left[1 - \frac{6}{B_0^2 L_1^2} + \frac{3\tilde{N}_c}{2\lambda\xi^2 B_0^2 L_1^2}\right] \simeq 8\cdot 10^{-4}\left[1 - 0.27\left(\frac{4}{\xi}\right)^{\frac{2}{3}} + 0.11\left(\frac{10^{-3}}{\lambda}\right)\left(\frac{4}{\xi}\right)^{\frac{8}{3}}\right]. \tag{4.119}$$

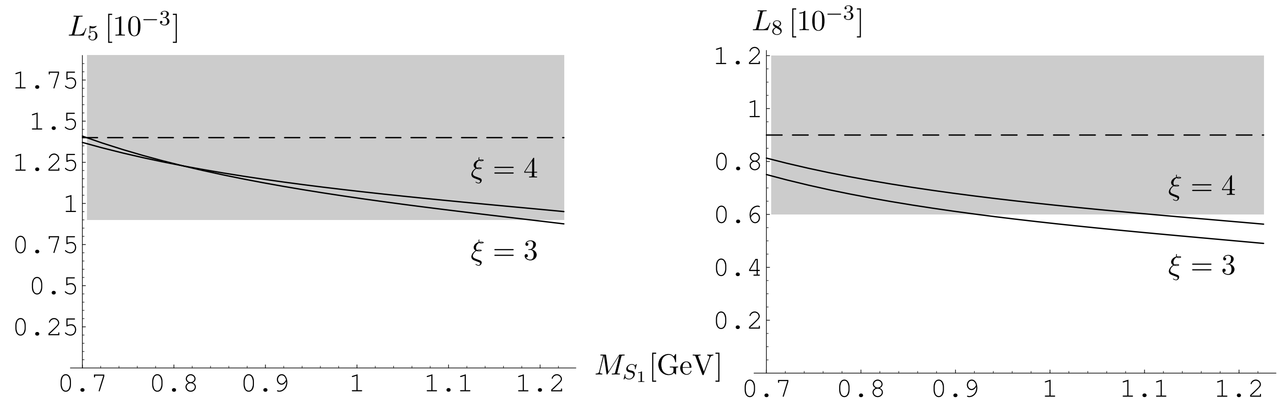


Figure 4.9: *Prediction for L_5 and L_8 as a function of M_{S_1}. The horizontal line corresponds to the experimental value with the error bands [57].*

Notice that this expression is only valid for $\xi \gg 1$ with $\lambda\xi^2$ fixed. In Fig. 4.9 we show the exact value of L_8 as a function of M_{S_1}. For $M_{S_1} \simeq 1$ GeV and $\xi = 4$ we obtain $L_8 \simeq 0.6 \cdot 10^{-3}$ again in good agreement with the experimental value. From Fig. 4.9 one can see that small values of M_{S_1} are preferred. The coefficient L_8 can also be written as

$$L_8 = \frac{1}{32B_0^2}\left[F_{S_1}^2 + \sum_{n=1}^{\infty}\left(F_{S_{n+1}}^2 - F_{P_n}^2\right)\right] , \tag{4.120}$$

that shows that in the limit where the chiral symmetry is restored, $\xi \to 0$ and $F_{S_{n+1}} \to F_{P_n}$, only the first term remains. For $\xi \simeq 4$ we find that the first term still dominates (it gives 70% of the total contribution for $\lambda \simeq 10^{-3}$) since the other resonances, being so heavy, are not very sensitive to chiral symmetry breaking.

To summarize, in Table 1 we compare the experimental values of L_i with the predictions of our AdS$_5$ model for the value $\xi = 4$. We give the exact values of our predictions although we find that the predictions in the limit $\xi \gg 1$ differ by less than a 10% from the exact results. Comparing the predictions with the experimental values we find that the discrepancy is always below the 30%.

Finally, we also calculate the coefficient of the operator $\mathrm{Tr}[Q_R U Q_L U^\dagger]$ responsible for the electromagnetic pion mass difference ($Q_{L,R}$ are the left- and right-handed charges) [52]. This coefficient is given by $e^2 C = (m_{\pi^+}^2 - m_{\pi^0}^2)F_\pi^2/2$ where

$$m_{\pi^+} - m_{\pi^0} \simeq \frac{3\alpha}{8\pi m_\pi F_\pi^2}\int_0^\infty dp^2\left(\Pi_A - \Pi_V\right) . \tag{4.121}$$

Taking Π_V from Eq. (4.26) and calculating Π_A numerically in the chiral limit for $\xi = 4\,(5)$ we find $\Delta m_\pi \simeq 3.6\,(4)$ MeV to be compared with the experimental value $\Delta m_\pi \simeq 4.6$ MeV.

	Experiment	AdS$_5$
L_1	0.4 ± 0.3	0.4
L_2	1.4 ± 0.3	0.9
L_3	-3.5 ± 1.1	-2.4
L_4	-0.3 ± 0.5	0.0
L_5	1.4 ± 0.5	0.9
L_6	-0.2 ± 0.3	0.0
L_8	0.9 ± 0.3	0.6
L_9	6.9 ± 0.7	5.4
L_{10}	-5.5 ± 0.7	-5.5

Table 4.1: *Experimental values of the L_i at the scale M_ρ in units of* 10^{-3} *[57] and the predictions of the AdS$_5$ model for the value $\xi = 4$.*

The coefficients L_i and C have been previously calculated using different approximations. For example, in Refs. [52, 58] these coefficients were calculated from an effective theory of resonances, showing a good agreement with the experimental data. It would be interesting to study the relation between the approach presented here with those of Refs. [52, 58].

4.6 Conclusions

We have presented a 5D model that describes some of the properties of QCD related to chiral symmetry breaking. Alike large-N_c QCD, this model is defined by a set of infinite weakly coupled resonances. The vector, axial vector and PGB sectors depend only on one parameter, ξ, related to the quark condensate (apart from the other 3 parameters of the model that are fixed by the 3 parameters that define QCD: the mass gap $\Lambda_{\rm QCD}$, M_q, and N_c). The scalar sector depends also on the parameter λ. We have obtained predictions for the masses and decay constants of the vector, axial-vector, PGB and scalar mesons. These predictions are in good agreement with the experimental data. A summary of some of the results is given in Table 1 and Figs. 4.9 and 4.10 that shows that, within a 30%, they agree with the data.

The 5D gauge invariance of the model leads to interesting sum rules among the couplings and masses of the resonances from which we obtain $M_\rho^2 \simeq 3\, g_{\rho\pi\pi}^2 F_\pi^2$, $F_\rho \simeq \sqrt{3}\, F_\pi$, and the vanishing of the BR of a_1 into $\pi\gamma$ at the tree-level. Another prediction of the model is the realization of VMD in the electromagnetic form factor of the pion.

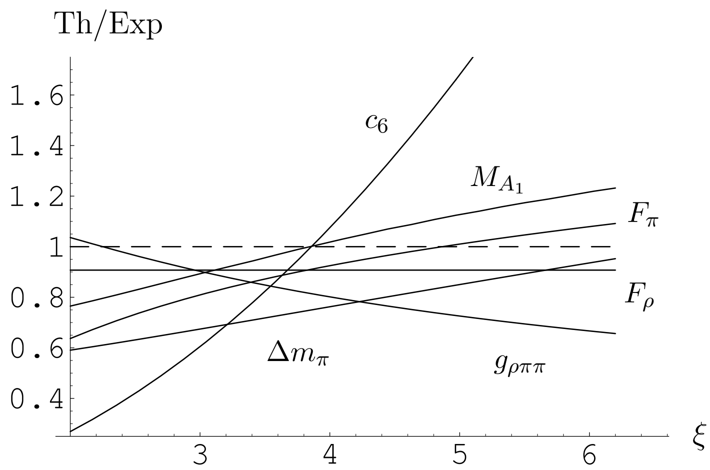


Figure 4.10: *Predictions of the model for some physical quantities as a function of* ξ *divided by their experimental value. We have taken* $M_q = 0$.

We have found a good agreement for the the quark masses. For the first massive pseudoscalar resonance we have obtained a mass around 1800 MeV, quite different from the mass of the lowest QCD pseudoscalar resonance $\pi(1300)$. This has suggested us to associate this state to $\pi(1800)$. We have also given predictions for the scalar couplings and decay constants but the absence of clean experimental data has not allowed us to compare them with QCD.

Previous approaches to calculate the scalar and pseudoscalar spectrum and/or determine their contribution to L_i can be found in Refs. [58]-[61]. In particular, the analysis of Refs. [59, 60] has certain similarity with ours. Refs. [59, 60] work in the large-N_c limit where QCD is described as a theory of infinite hadron resonances. These sets of infinite hadrons, however, are approximated in Refs. [59, 60] by taking only the lowest modes, and their masses and couplings are determined by demanding a good high-energy behaviour of the correlators and form factors. In our approach we have shown that the correlators and form factors have the correct high-energy behaviour since this is dictated by the conformal symmetry. We have also found that, in certain cases, it can be a good approximation to take only the lowest resonance. Therefore in these cases our approach and that of Refs. [59, 60] give similar results. Nevertheless, we have showed that the single-resonance approximation is not always justified (for example in Eq. (4.88)) and this approximation can lead to large errors in the determination of the scalar parameters.

Since the results presented here depend on the AdS_5 metric Eq. (3.2), one can wonder whether the results are robust under possible deviations from AdS. For example, if the backreaction on the metric due to $\langle\Phi\rangle^2$ or other possible condensates are included in the model, we expect the warp factor $a(z)$ to depart from AdS in the IR. Nevertheless if we want the theory to be almost conformal in the UV, the warp factor for $z \ll L_1$ (where $1/L_1$ gives the mass gap) must behave as

$$a(z) \simeq \frac{L}{z}\left[1+\sum_i c_i\left(\frac{z}{L_1}\right)^{d_i}\right], \tag{4.122}$$

where c_i are numerical constants related to the singlet condensates $\langle\mathcal{O}_i\rangle$ and $d_i = \mathrm{Dim}[\mathcal{O}_i]$. In QCD $d_i \geq 4$. Eq. (4.122) implies that only for values of z quite close to L_1 the metric will deviate from AdS. Therefore, unless the coefficients c_i are very large, we do not expect large deviations from our results. The c_i, however, are restricted by the curvature of the space $\mathcal{R}$. We have checked that for $\mathcal{R} \sim 1/L_1^2$, our results are not substantially modified by deformations of the AdS metric in the IR. As an example, we have compared some of our results to those with the metric of Ref. [31], [8] $a(z) = \frac{\pi L}{2L_1 \sin[\pi z/2L_1]}$, and we have found that the differences are smaller than 10%. We can then conclude that more realistic string constructions of QCD, such as those of Refs. [32], must lead to quantitatively similar results. In Ref. [45] a systematic expansion of the metric has been done, matching the coefficients c_i of Eq. (4.122) with the OPE of the vector and axial-vector correlators.

[8] This is the induced metric on the D7-brane on which the gauge bosons propagate in Ref. [31].

Chapter 5

Electroweak symmetry breaking from five dimensions: a composite Higgs model

5.1 Introduction

The standard model (SM) has had an incredible phenomenological success describing the physics at energies below the electroweak (EW) scale. However the SM is not the fundamental theory of EW symmetry breaking. The main reason is that the Higgs potential of the SM is not stable under quantum corrections. This problem suggests that the SM should be replaced by another theory, like a supersymmetric theory, a string theory, a quantum theory of gravity or a strongly coupled theory.

The aim of models beyond the SM can be summarize in the following three minimal requirements:

- Provide a solution to the hierarchy problem, i.e.: they provide a natural mechanism to explain why the Planck scale $M_{Pl} \sim 10^{18}$ GeV is much bigger than the EW scale $M_{EW} \sim 10^2$ GeV. Let us give a very brief description of the hierarchy problem. The quantum corrections to the Higss mass are quadratically divergent with the cut-off of the theory. Thus to obtain a light Higgs with parameters of $\mathcal{O}(1)$ the scale of new physics must be $\Lambda_{NP} \leq 1$ TeV. On the oder hand, if the new physics scale is so low, we should see any indirect effect in the precision tests. For example, we expect new physics to induce higher dimensional operators, like four fermion interaction suppressed by this scale. The precision tests constrain Λ_{NP} to be larger than a few TeV, thus we are led to the LEP paradox [62]. Therefore the ambitious solution to the hierarchy problem is to give a picture for physics up to the Planck scale, stabilizing this hierarchy and explaining the

EW symmetry breaking.

- Explain the flavor structure of the SM. A complete model would have to predict the fermionic spectrum of the SM, the masses and mixing angles. Although explaining the flavor structure can be too ambitious, there is a minimal requirement: operators inducing flavor changing neutral currents must be suppressed. Thus the models beyond the SM must provide a GIM-like mechanism. One also needs to suppress operators inducing proton decay.

- Pass the precision EW tests. Any theory beyond the SM is constrained by high precision experiments at LEP and SLAC, and the deviations from the SM at low energies must be very small. Models beyond the SM are called universal if the deviations from the SM reside only in the self-energies of the vector bosons. In this case, and if the scale of new physics is sensibly higher than M_{EW}, the deviations from the SM are described by four parameters that encode the low energy behaviour of the vector self-energies [63, 64]. Since high precision experiments constrain this parameters, the universal theories beyond the SM must be compatible with the experimental bounds to be realistic alternatives. The models we will be interested in are universals up to the interaction $Zb\bar{b}$. Thus we have to check that they satisfy the constraints for deviations in this interaction.

In chapter 4 we considered the AdS/CFT duality to study different properties of strongly interacting theories. We introduced a simple model with fundamental fields in AdS$_5$ to investigate the chiral symmetry breaking of QCD in the sector of mesons. Thus, in this chapter, we will consider the breaking of the EW symmetry by a strongly interacting sector in the context of the AdS/CFT conjecture. Moreover, we also aim the new sector to explain the flavor structure of the SM and to pass the EW precision tests.

One of the most economical alternatives to explain the breaking of the EW symmetry are technicolor theories. In these theories the EW symmetry is broken by the interactions with a strongly coupled sector. This idea is inspired in QCD, where the strong interactions spontaneously break the chiral symmetry SU(2)$_L$⊗SU(2)$_R$ →SU(2)$_{L+R}$, as explained in the previous chapter. However, in technicolor theories, due to the strong interactions, it is not possible to calculate the precision parameters. In some cases it is possible to estimate them, and their contribution to the Peskin-Takeuchi S parameter is larger than the experimental bound. From 5D higgsless models we can obtain analytical predictions for the S parameter. In terms of L_{10} computed in the previous chapter S is given by $S = -16\pi L_{10}$, with $F_\pi = 246$ GeV. Electroweak precision tests tell us that $S \lesssim 0.3$, a constraint difficult to be satisfied in the present Higssless models [65, 66]. From Eq. (A.7) we can derive the dependence of S on d

$$S \simeq -4\pi N_c \left[\ln \xi^{1/d} + \frac{\gamma + \psi\left(\frac{2+d}{2d}\right) - \psi\left(\frac{2}{d}\right) - \psi\left(\frac{1}{d}\right) - \ln \frac{d^2}{2}}{2d} \right] . \tag{5.1}$$

For a fixed value of F_π we find that the dependence on d is very weak, and S changes only a few per cent when varying d. This implies that S in these type of models is always sizable.

In this chapter we will consider the alternative of having a Higgs as a composite particle arising from a strongly coupled field theory. In these theories there is a scale Λ_{NP} at which new particles appears, like m_ρ in QCD. This scale must be larger than the scale of the Higgs mass to avoid constraints from searches of new particles states $\Lambda_{NP} > M_{EW}$. Therefore the Higgs mass must be protected by some approximate symmetry even below the scale of new particles. The pion of QCD provides a good example, it is a Goldstone boson coming from the spontaneous breaking of the chiral symmetry. There is an explicit breaking of the chiral symmetry due to the gauging of the EW group and the quark masses, and the pion becomes a pseudo Goldstone boson (PGB) with a small mass. This suggests that the Higgs could be a PGB of a strongly interacting sector, with a global symmetry explicitly broken by the SM external sector. The Higgs mass is generated by quantum effects between the SM and the strong sector. The analog of the pion decay constant f_π sets the scale of EW symmetry breaking, and as in QCD it is related to the scale of new physics by $f_\pi \sim \Lambda_{NP}/(4\pi)$. There are additional conditions over this composite Higgs. In order to obtain a large enough Higgs mass, the quartic self-coupling must be $\mathcal{O}(1)$. Also the Yukawa couplings to the quark and lepton fields must have the appropriate values. In this kind of models the precision parameters will receive contributions from the strong sector and one has to calculate their values to know if the model passes the EW precision tests. As we will see the most constraining observable is the Peskin-Takeuchi S parameter. In this chapter we will describe a model with the properties mentioned above and will calculate some physical quantities as the Higgs mass, the precision parameters and the masses of the lightest new particles.

The most important obstacle to test whether the theory depicted above can be a serious model of EW symmetry breaking is calculability. It is very difficult to obtain quantitative predictions in a strongly coupled theory because of the non perturbative effects. To avoid this problem we will consider the holographic approach from 5D theories in a slice of AdS$_5$. As we argue in section 3.1, models in AdS$_5$ can mimic strongly coupled conformal field theories (CFT) with a large number of colors N.

In this chapter we present a model along the lines of Ref. [10]. In that paper the authors considered a 4D model with an SO(5)$\otimes$U(1)$_X$ global symmetry. This is the minimal group with the following properties: it contains the EW gauge group, after spontaneous symmetry breaking it generates a Goldstone boson corresponding to the Higgs field and it has an unbroken SO(3) custodial symmetry. The Higgs is the Goldstone boson corresponding to the spontaneous breaking SO(5)$\rightarrow$SO(4) by the strong dynamics, thus it is a **4** of SO(4). Since SO(4)$\sim$SU(2)$_L\otimes$SU(2)$_R$ (with a parity L $\leftrightarrow$ R), the Goldstone boson has the appropriate quantum numbers to be associated with the Higgs field. The SM fermions are assumed to couple linearly to the fermionic operators of the CFT that form complete multiplets of SO(5). The authors considered these operators to be a **4** of SO(5). The SM sector explicitly breaks down the global symmetry to the EW subgroup and by loop effects it generates a mass for the PGB. In this model one can calculate the Higgs potential at one loop in terms of the two-point functions of the SM vector and fermion fields. These functions encode all the effects of the strong dynamics, thus they are computed in a 5D model that mimics the strong sector. In a large range of the parameter space EW symmetry breaking is triggered by the top quark contributions to the Higgs potential. The Higgs acquires a vacuum expectation value (VEV) that is related to the EW symmetry breaking scale f_π by $v = \epsilon f_\pi$. The value of the parameter ϵ is model dependent, and for small ϵ the S parameter is small enough to pass the EW precision tests. The model predicts a very light Higgs, $m_H \leq 140$GeV. A complete analysis of the EW precision tests is performed in Ref. [68]. In particular the authors perform a detailed analysis of the decay $Z \rightarrow b_L\bar{b}_L$ and conclude that to satisfy the experimental bounds on this decay one needs a significant tuning of a few percent. A priori it is not easy to avoid large contributions to $Zb\bar{b}$ because the top, being heavy, couples strongly to the strong sector, and since b_L is in the same doublet as t_L, one expects large modifications to $Zb\bar{b}$. We will show that there is a subgroup of the custodial symmetry that can protect the interaction $Zb\bar{b}$. This is the same subgroup that protects the T parameter of the SM from radiative corrections. This symmetry cannot protect simultaneously $Zt\bar{t}$ and $Wt\bar{b}$. Therefore, by an appropriate election of the representation of the fermionic operators $Zb\bar{b}$ is protected and the theory is realistic in a large region of the parameter space. We also expect large modifications of the interactions $Zt\bar{t}$ and $Wt\bar{b}$ that could be observed in the future experiments. This symmetry can be used for any model beyond the SM with a new sector that is invariant under the global custodial symmetry. In this chapter we will show the general argument to protect these interactions and will consider its application to our model. Thus, by embedding the fermionic fields in appropriate multiplets, the model can pass these EW precision tests. In particular, for a **10** of SO(5), q_L is included in a bidoublet $(\mathbf{2},\mathbf{2})$ of SU(2)$_L\otimes$SU(2)$_R$, and $Zb\bar{b}$ is protected. Therefore we present an extension of the minimal model [10], with the fermions embedded in a **10** of SO(5). We will compute the Higgs potential and other physical quantities. We will show that the S parameter constrain excludes 75% of the parameter space. We obtain a correlation between the Higgs mass and the lightest fermionic resonance $m_H \sim m_1$. Thus a small Higgs mass implies a small $m_1 \sim 0.5-1.5$TeV, and we obtain a little hierarchy between m_H and the scale of new fermionic resonances. The vector resonances are similar to the original model and in general are heavier than the fermionic resonances $\sim 1-3$TeV.

This chapter is organized in the following way. In section 5.2 we describe the 4D CFT model. We will integrate out all the CFT and obtain an effective low energy lagrangian that, at the quadratic level, can be parametrized in terms of a few form factors. We will derive the one loop Higgs potential in section 5.3. In section 5.4 we will present the 5D model that leads to same effective lagrangian as the 4D theory. We will compute the tree level two point functions and match the 4D form factors to the 5D ones. In section 5.5 we will show our results for the precision EW parameters. We will also give predictions for the spectrum of the model. In section 5.6 we show symmetry protecting $Zb\bar{b}$ and calculate the extra contributions to the vertex $Zb_L\bar{b}_L$ in our model. In section 5.7 we give our conclusions.

5.2 4D model

We will consider a 4D theory with a strongly interacting sector with a large number of colors N. We assume that the theory is conformal at high energies and that the conformal symmetry is spontaneously broken at a IR scale of order TeV. Thus there is a mass gap and the theory has a discrete spectrum of particles with the lightest masses of order TeV. The theory has a global symmetry $\mathrm{SU}(3)_c\otimes\mathrm{SO}(5)\otimes\mathrm{U}(1)_X$, and hence the operators and states of this sector are in complete multiplets of this group. We assume that this global symmetry is spontaneously broken by the strong dynamics to $\mathrm{SU}(3)_c\otimes\mathrm{SO}(4)\otimes\mathrm{U}(1)_X$ at a scale f_π. We will assume that the operator breaking the global symmetry has a large scaling dimension. Due to the spontaneous breaking there is a Goldstone boson that is a $\mathbf{4}$ of SO(4), with the right quantum numbers to be the Higgs boson. It is a real bidoublet $(\mathbf{2},\mathbf{2})$ of $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$. The SM fields are elementary and are external to the CFT. The $\mathrm{SU}(2)\otimes\mathrm{U}(1)_Y$ symmetry included in the global group of the CFT is gauged by the external fields, that couple to the conserved currents of the CFT. Hypercharge is given by $Y=T^{3R}+X$. As the $\mathrm{SU}(3)_c$ does not play any role in our analysis we will neglect it in the following. The SM fermions also couple to the strong sector, we assume that they couple linearly through CFT operators $\mathcal{L}=\lambda\bar{\Psi}\mathcal{O}$, with $\mathcal{O}$ transforming as a $\mathbf{10}$ of SO(5). The lagrangian of the model is given by

$$\mathcal{L}=\mathcal{L}_{\rm CFT}+\mathcal{L}_{\rm SM}+J^{a_L\,\mu}L^a_\mu+J^\mu_Y B_\mu+\sum_r\lambda_r\,\bar{\psi}_r\mathcal{O}_r+\text{h.c.}\,, \tag{5.2}$$

where $\psi_r=\{q_L,u_R,d_R,l_L,e_R\}$ are the SM fermions and L^a_μ $(a=1,2,3)$, B_μ stand for the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge bosons. A family index is understood.

From chapter 3 we know that the running coupling λ satisfies a renormalization group equation. By choosing the scaling dimension of the fermionic operator $\mathcal{O}$, it is possible to determine the value of the running coupling λ. Therefore depending on $\mathrm{d}im[\mathcal{O}]$, the fermions will have a small (large) mixing with the CFT, and thus a small (large) Yukawa coupling. To obtain a large enough top mass, we will need a large Yukawa coupling, and thus the top will be mostly composite.

At tree level, due to the SO(5)→SO(4) breaking, the theory has a large space of degenerate vacua. A subspace of vacua preserve the SM SU(2)⊗U(1)$_Y$ symmetry, and the orthogonal subspace does not preserve this symmetry. Therefore the EW symmetry can be broken or not by the vacuum depending on the strong dynamics. The SM fields do no respect the full SO(5) global symmetry of the CFT. Thus, due to the couplings with the SM fields, the global symmetry is explicitly broken, and the Higgs acquires a mass. This means that the large space of vacua is lifted by loop effects and the Higgs becomes a PGB. The gauge contributions to the potential tend to align the vacuum along the EW preserving subspace of vacua, and do not trigger EW symmetry breaking [9]. The fermion contribution to the potential can misalign the vacuum, being the top contribution the biggest one.

The Higgs is the Goldstone boson and is parametrized in terms of a unitary matrix $\Sigma(\Pi)$, where Π is given in terms of the broken generators $T^{\hat{a}}$, $\hat{a}=1,2,3,4$ as

$$\Sigma = \Sigma_0 e^{\Pi/f_\pi} \ , \qquad \Sigma_0 = (0,0,0,0,1) \ , \qquad \Pi = -iT^{\hat{a}}h^{\hat{a}}\sqrt{2} \ . \tag{5.3}$$

The tree level vacuum is characterized by Σ_0. Σ can be simplified by using the SO(5) generators

$$\Sigma = \frac{\sin h/f_\pi}{h}\left(h^1, h^2, h^3, h^4, h\cot h/f_\pi\right) \ , \qquad h = \sqrt{(h^{\hat{a}})^2} \, . \tag{5.4}$$

Quantum effects will generate a potential for h. The VEV of h is determined by minimizing this potential. Defining $\epsilon = \sin\langle h\rangle/f_\pi$, the true vacuum is characterized by

$$\langle\Sigma\rangle = \left(0,0,\epsilon,0,\sqrt{1-\epsilon^2}\right) . \tag{5.5}$$

ϵ takes values between 0 and 1, for $\epsilon = 0$ there is no EW symmetry breaking and for $\epsilon = 1$ there is maximal EW symmetry breaking.

We can obtain a low energy effective lagrangian of the above theory by integrating out all the heavy resonances of the CFT, thus obtaining a non-local lagrangian for the external fields. As this effective lagrangian will respect the symmetries of the strong sector, it is convenient to write it in an SO(5) invariant form. Since the SM fields do not fill complete multiplets of SO(5), we will promote them to obtain complete multiplets. The elementary fermions are promoted to fill complete adjoint representations of SO(5). An adjoint representation of SO(5) has ten components, a **10** of SO(5) is decomposed under SO(4) as $\mathbf{10} \sim \mathbf{4}+\mathbf{6}$. As SO(4)∼SU(2)$_L$⊗SU(2)$_R$ (with a parity L ↔ R), it is useful to decompose this multiplet under the SU(2)$_L$⊗SU(2)$_R$ group. A **10** of SO(5) contains a bidoublet, a right triplet and a left triplet of SU(2)$_L$×SU(2)$_R$: $\mathbf{10} \sim (\mathbf{3},\mathbf{1})+(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{3})$. The SM model fermions q_L, u_R, d_R are embedded into the adjoint representation as

$$\Psi_q = \begin{bmatrix} (\mathbf{3},\mathbf{1})^{\mathbf{q}}_{\mathbf{L}} \\ (\mathbf{2},\mathbf{2})^{\mathbf{q}}_{\mathbf{L}} = \begin{bmatrix} q'_L \\ q_L \end{bmatrix} \\ (\mathbf{1},\mathbf{3})^{\mathbf{q}}_{\mathbf{L}} \end{bmatrix}, \qquad \Psi_u = \begin{bmatrix} (\mathbf{3},\mathbf{1})^{\mathbf{u}}_{\mathbf{R}} \\ (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{R}} \\ (\mathbf{1},\mathbf{3})^{\mathbf{u}}_{\mathbf{R}} = \begin{bmatrix} \chi^u_R \\ u_R \\ d'_R \end{bmatrix} \end{bmatrix}, \qquad \Psi_d = \begin{bmatrix} (\mathbf{3},\mathbf{1})^{\mathbf{d}}_{\mathbf{R}} \\ (\mathbf{2},\mathbf{2})^{\mathbf{d}}_{\mathbf{R}} \\ (\mathbf{1},\mathbf{3})^{\mathbf{d}}_{\mathbf{R}} = \begin{bmatrix} \chi^d_R \\ u'_R \\ d_R \end{bmatrix} \end{bmatrix}. \tag{5.6}$$

The extra components are non-dynamical external sources of the CFT sector. They are not physical degrees of freedom and they are introduced to write an SO(5) invariant effective lagrangian. The leptons are embedded into adjoint representations in a similar way. As we want the SM fermions to have the right hypercharge, the X charge of the fermions can be 2/3 or $-1/3$. Eq. (5.6) corresponds to X= 2/3, that as we will explain later, suppress the extra contributions to the interaction $Zb\bar{b}$.

We also introduce additional non-dynamical components in the gauge sector to fill complete adjoint representations A_μ and B'_μ of SO(5)$\times$U(1)$_X$, but only the gauge fields of SU(2)$_L\times$U(1)$_Y$ are dynamical.

We can now write the non-local lagrangian for the sources in a SO(5)$\times$U(1)$_X$ invariant way. It is given by the most general lagrangian compatible with the symmetries. After integrating out all the CFT states at tree level, including fluctuations of the Higgs field around a constant classical background Σ, the most general effective Lagrangian for the external fields is, in momentum space and at the quadratic level,

$$\begin{aligned} \mathcal{L}_{\text{eff}} =& \frac{1}{2} P_{\mu\nu} \Big[\Pi^B_0(p)\, B'^{\mu} B'^{\nu} + \Pi_0(p)\, \mathrm{Tr}\big[A^\mu A^\nu\big] + \Pi_1(p)\, \Sigma A^\mu A^\nu \Sigma^T \Big] \\ &+ \sum_{r=q,u,d} \Big[\mathrm{Tr}\big[\bar{\Psi}_r \not{p}\, \Pi^r_0(p) \Psi_r\big] + \Sigma \Psi_r \not{p}\, \Pi^r_1(p) \Psi_r \Sigma^T \Big] \\ &+ \sum_{r=u,d} \Big[\mathrm{Tr}\big[\bar{\Psi}_q M^r_0(p) \Psi_r\big] + \Sigma \Psi_q M^r_1(p) \Psi_r \Sigma^T \Big], \end{aligned} \tag{5.7}$$

where $P_{\mu\nu} = \eta_{\mu\nu} - p_\mu p_\nu / p^2$. In Eq. (5.7) we have not written terms for the dynamical external fields not induced by the strong dynamics, as bare kinetic terms and gauge fixing terms. But they can be included straightforward, and we will consider the effect of bare kinetic terms in our analysis. The effects of the strong dynamics are contained on the form factors $\Pi(p), M(p)$ and they can not be computed perturbatively in the 4D theory. If the sources are non-dynamical the poles of the two-point functions give the spectrum of the theory. On the other hand, if the sources are dynamical we have to invert the whole quadratic action. If there are not extra boundary terms, the spectrum is given by the zeroes of the two-point functions. As we will see below, to calculate the S parameter and the Higgs potential we only need these two-point functions.

From Eq. (5.7) one can derive the low-energy effective theory. This is the theory of the light states, the SM fields and the Higgs. This procedure is equivalent to the chiral lagrangian in QCD. It is obtained by performing an expansion in derivatives and light fields over m_ρ:

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm yuk} - V(\Sigma) + \Delta\mathcal{L}\,. \tag{5.8}$$

The term $\mathcal{L}_{\rm kin}$ contains the kinetic terms of the dynamical fields

$$\mathcal{L}_{\rm kin} = \frac{f_\pi^2}{2}\left(D_\mu\Sigma\right)\left(D^\mu\Sigma\right)^T + \sum_{r=q,u,d} Z_r\,\bar\psi_r\,\not{D}\psi_r - \frac{1}{4g^2}W^{a_L}_{\mu\nu}W^{a_L\,\mu\nu} - \frac{1}{4g'^2}B_{\mu\nu}B^{\mu\nu}\,, \tag{5.9}$$

where $Z_q = \Pi_0^q(0)+\Pi_1^q(0)/2$, $Z_{u,d} = \Pi_0^{u,d}(0)$, $f_\pi^2 = \Pi_1(0)$, $1/g^2 = -\Pi_0'(0)$, $1/g'^2 = -[\Pi_0^B{}'(0) + \Pi_0'(0)]$. The kinetic term for Σ includes the gauge field mass term, we obtain $M_W^2 = g^2v^2/4$, where we have defined the EW symmetry breaking scale

$$v \equiv \epsilon f_\pi = f_\pi \sin\frac{\langle h\rangle}{f_\pi} = 246~\mathrm{GeV}\,. \tag{5.10}$$

The term $\mathcal{L}_{\rm yuk}$ contains the Yukawa couplings between the Higgs and the elementary fermions and comes from the expansion of the last term of Eq. (5.7):

$$\mathcal{L}_{\rm yuk} = \frac{\sin(h/f_\pi)\ \cos(h/f_\pi)}{4}\left[M_1^u(0)\,\bar q_L\begin{pmatrix}u_R\\0\end{pmatrix} + \sqrt{2}M_1^d(0)\,\bar q_L\begin{pmatrix}0\\d_R\end{pmatrix} + \mathrm{h.c.}\right]. \tag{5.11}$$

When the Higgs acquires a VEV the fermions get a mass that is given by the zeroes of the corresponding two-point function. However, for $\epsilon = 1$, although there is EW symmetry breaking the Yukawa coupling vanishes, and the SM fermions are massless. This is because the remaining SO(4) is aligned in a direction such that it protects the SM fermions from acquiring a mass. For $\epsilon \ll 1$ the fermion mass terms derived from Eq. (5.11) can be approximated by

$$m_u \simeq \frac{M_1^u(0)}{4\sqrt{Z_qZ_u}}\,\frac{v}{f_\pi} \equiv y_u\,v\,, \qquad m_d \simeq \frac{\sqrt{2}M_1^d(0)}{4\sqrt{Z_qZ_d}}\,\frac{v}{f_\pi} \equiv y_d\,v\,. \tag{5.12}$$

By NDA $y_{u,d} \sim \lambda_{u,d}\lambda_q\sqrt{N}/4\pi$. According to chapter 3 the coupling of the fermions to the CFT depends on the value of $\gamma_{q,u,d} = \mathrm{dim}\mathcal{O}_{\rm q,u,d} - 5/2$. Then by choosing them to be positive $\lambda_{q,u,d}$ are strongly suppressed at low energies. This mechanism can explain the hierarchical structure of the fermion masses in a natural way [26]:

$$m_{u,d} \sim \frac{\sqrt{N}}{4\pi}\left(\frac{\mu_{\rm IR}}{\Lambda}\right)^{\gamma_q+\gamma_{u,d}} v\,. \tag{5.13}$$

To obtain a large top mass we require $\gamma_u < 0$ and $\gamma_q \simeq 0$ for the third quark generation. A negative γ_u implies that the physical right-handed top quark is mostly composite. Flavour changing neutral current (FCNC) effects are also suppressed by the small couplings $\lambda_{u,d,q}$, the theory has GIM-like mechanism (see, for example [26, 70, 71, 72]).

The Higgs potential is generated at one loop. The gauge contributions tend to align the vacuum along an $SU(2)_L \times U(1)_Y$-preserving direction, but the fermion contributions can trigger EW symmetry breaking, playing the top quark the most important role. The Higgs acquires a VEV and breaks the SO(4) symmetry to the custodial SO(3).

The term $\Delta\mathcal{L}$ contains the higher order operators of the expansion. Among them there are the operators contributing to the precision parameters constrained by LEP. We will consider the case of $\Lambda_{NP} > M_{EW}$, then the universal effects of the new sector can be parametrized by a set of coefficients that describe the low energy behaviour of the SM vector self-energies. Based on symmetry principles and absence of fine-tuning, the most relevant effects of new physics can be encoded in the following form factors [64]

		Adimensional form factors	custodial	$SU(2)_L$
$\widehat{S}$	$=$	$g^2\Pi'_{W_3B}(0)$	$+$	$-$
$\widehat{T}$	$=$	$\frac{g^2}{M_W^2}[\Pi_{W_3W_3}(0) - \Pi_{W^+W^-}(0)]$	$-$	$-$
Y	$=$	$\frac{g^2 M_W^2}{2}\Pi''_{BB}(0)$	$+$	$+$
W	$=$	$\frac{g^2 M_W^2}{2}\Pi''_{W_3W_3}(0)$	$+$	$+$

In this table we also show the symmetries that these form factors preserve. There are higher derivative terms contributing to the two-point functions, but if Λ_{NP} is big enough compared with M_{EW}, as is the case in our model, they can be neglected. Experimental data constrain the new physics contributions to these form factors to be very small. In universal models, regardless the value of the Higgs mass, $\widehat{S}, \widehat{T}, W$ and Y must be of order 10^{-3}. The authors of [64] performed a global fit with a light and a heavy Higgs, obtaining

m_H	$10^3\widehat{S}$	$10^3\widehat{T}$	$10^3 Y$	$10^3 W$
115GeV	0.0 ± 1.3	0.1 ± 0.9	0.1 ± 1.2	-0.4 ± 0.8
800GeV	-0.9 ± 1.3	2.0 ± 1.0	0.0 ± 1.2	-0.2 ± 0.8

In the 4D model described above, the $\widehat{T}$ parameter does not receive any contribution at tree level from the CFT due to the custodial symmetry. Nevertheless, we expect quantum contributions to this parameter, being the most important due to top interactions. The authors of ref. [68] calculated the one-loop contribution to $\widehat{T}$ in the minimal composite Higgs model. The leading effect comes from t_R fermions running in the loop. The authors conclude that the corrections to $\widehat{T}$ are compatible with current data and they don not imply a significant amount of tuning.

The $\widehat{S}$ parameter receives contributions from the third term of Eq. (5.7):

$$\Delta\mathcal{L} \supset \frac{1}{2}\Pi'_1(0)\, W^{a_L}_{\mu\nu} B^{\mu\nu}\, \Sigma T^{a_L} Y \Sigma^T \,, \tag{5.14}$$

where T^{a_L}, Y are respectively the generators of $SU(2)_L$ and hypercharge. By defining $S =$

$g^2/(16\pi)\widehat{S}$, Eq. (5.14) gives

$$S = 4\pi\Pi_1'(0)\epsilon^2\,. \tag{5.15}$$

The parameters W and Y are small in the present model, since they arise from dimension-six operators and are thus suppressed by a factor $(g^2 f_\pi^2/m_\rho^2)$ compared to S and T.

5.3 Higgs potential

The elementary sector explicitly breaks the SO(5) symmetry, then loops of elementary fields can transmit this breaking to the CFT sector and generate a Higgs potential. The main contributions come from the top quark, that couples strongly to the CFT sector, and from the gauge and bottom fields. At one loop the Coleman-Weinberg potential is

$$V(h) = \frac{9}{2}\int\frac{d^4p}{(2\pi)^4}\,\log\Pi_W - (2N_c)\int\frac{d^4p}{(2\pi)^4}\left[\log\Pi_{b_L} + \log\left(p^2\Pi_{t_L}\Pi_{t_R} - \Pi^2_{t_Lt_R}\right)\right], \tag{5.16}$$

where $\Pi_i(p)$ are the self-energies of the corresponding SM fields in the background of h. These can can be written as functions of the form factors of Eq. (5.7)

$$\begin{aligned}
\Pi_W &= \Pi_0 + \frac{\Pi_1}{4}\sin^2\frac{h}{f_\pi}\,, &(5.17)\\
\Pi_{t_Lt_R} &= \frac{M_1^u}{4}\sin(\frac{h}{f_\pi})\cos(\frac{h}{f_\pi})\,, &(5.18)\\
\Pi_{b_L} &= \Pi_0^q + \frac{\Pi_1^q}{2}[1-\sin^2(\frac{h}{f_\pi})]\,, &(5.19)\\
\Pi_{t_L} &= \Pi_0^q + \frac{\Pi_1^q}{2}[1-\frac{1}{2}\sin^2(\frac{h}{f_\pi})]\,, &(5.20)\\
\Pi_{t_R} &= \Pi_0^u + \frac{\Pi_1^u}{4}\sin^2(\frac{h}{f_\pi})\,. &(5.21)
\end{aligned}$$

The potential of Eq. (5.16) has a constant divergent term. Up to this piece the potential is finite, because the form factors Π_1, M_1 decay for large momentum as $|\langle\Phi\rangle|^2/p^{2d}$ (with Euclidean momentum), where Φ is the CFT operator of dimension $d \gg 1$ responsible for the SO(5) breaking. Then we can expand the logarithms in Eq. (5.16) and write an approximate formula

$$V(h) \simeq \alpha + \beta\sin^2(\frac{h}{f_\pi}) + \gamma\sin^4(\frac{h}{f_\pi})\,, \tag{5.22}$$

where α is a divergent constant and

$$\beta = \int\frac{d^4p}{(2\pi)^4}\left[\frac{9\Pi_1}{8\Pi_0} + 2N_c\Big(\frac{3\Pi_1^q}{4\Pi_0^q+2\Pi_1^q} - \frac{\Pi_1^u}{4\Pi_0^u} - \frac{(M_1^u)^2}{-p^2\Pi_0^u(16\Pi_0^q+8\Pi_1^q)}\Big)\right], \tag{5.23}$$

$$\begin{aligned}
\gamma = \int\frac{d^4p}{(2\pi)^4}\Bigg\{\frac{-9\Pi_1^2}{64\Pi_0^2} + 2N_c \quad &\Bigg[\quad \frac{(\Pi_1^q)^2}{2(2\Pi_0^q+\Pi_1^q)^2} + \frac{\Pi_1^q\Pi_1^u - (M_1^u)^2/p^2}{8\Pi_0^u(2\Pi_0^q+\Pi_1^q)}\\
&+ \quad \frac{1}{8}\Big(\frac{\Pi_1^q}{2\Pi_0^q+\Pi_1^q} - \frac{\Pi_1^u}{2\Pi_0^u} - \frac{(M_1^u)^2}{-4p^2\Pi_0^u(2\Pi_0^q+\Pi_1^q)}\Big)^2\Bigg]\Bigg\}\,. \qquad (5.24)
\end{aligned}$$

Minimizing this potential we obtain

$$\epsilon = \sqrt{-\frac{\beta}{2\gamma}}\,. \tag{5.25}$$

For $\beta < 0$ there is dynamical EW symmetry breaking. The gauge fields contribution to β is positive and thus it tends to align the VEV in the $\mathrm{SU}(2)_L$ preserving direction. The fermions q_L and u_R contribute with different signs and tend to cancel, but the q_L term has a coefficient 3 compared to the u_R term. If the last term of Eq. (5.23) is large enough we obtain a misalignment of the vacuum.

We can approximate the physical Higgs mass for $\epsilon \ll 1$ by the following equation

$$m_{\rm Higgs}^2 \simeq \frac{8\gamma\,\epsilon^2}{f_\pi^2} \sim 4\frac{N_c}{N}\,y_t^2\,v^2\,. \tag{5.26}$$

The Higgs mass results from the interaction of the external sources with the CFT, and it is proportional to $1/N$. This is easy to understand from the 5D point of view, the Higgs mass arises from the gauge interaction whose expansion parameter is g_5^2, and as we saw in section 3.1 $N \sim 1/g_5^2$. Remarkably, the quartic coupling is $\mathcal{O}(1)$.

To verify if this theory can be a real alternative of EW symmetry breaking we have to compute β, γ and the electroweak precision observables as the S parameter. As the 4D theory is strongly coupled we will calculate these quantities in the 5D theory.

5.4 5d model

In this section we describe a 5D model that can mimic the 4D strongly coupled theory presented before. The holographic model is weakly coupled and we show how to compute the relevant form factors. We will see that at low energies this 5D model can be described by the effective lagrangian of Eq. (5.7). We consider a 5D spacetime that is a slice of AdS_5, with the metric given by Eqs. (3.1) and (3.2).

5.4.1 Gauge sector

We will consider a 5D gauge symmetry $\mathrm{SU}(3)_c\times\mathrm{SO}(5)\times\mathrm{U}(1)_X$ reduced to $\mathrm{SU}(3)_c\times\,\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ on the UV-brane and to $\mathrm{SU}(3)_c\times\mathrm{SO}(4)\times\mathrm{U}(1)_X$ on the IR-brane. This breaking is accomplished by imposing Dirichlet boundary conditions for the gauge fields corresponding to the broken generators. As the $\mathrm{SU}(3)_c$ does not play any role in our analysis we will neglect it.

The 5D action for the gauge sector is

$$S_5 = \int d^4x \int dz\,\sqrt{g}\,\left[-\frac{1}{4g_5^2}\mathrm{Tr}(A_{MN}A^{MN}) - \frac{1}{4g_5'^2}B'_{MN}B'^{MN})\right]\,, \tag{5.27}$$

where g is the determinant of the metric, g_5 and g'_5 are respectively the 5D gauge couplings of the SO(5) and U(1)$_X$ groups. The covariant derivative is defined by

$$D_M = \partial_M + i(g_5 A_M + g'_5 B'_M)\,, \tag{5.28}$$

where $A_M = A^b_M T^b$, $M = (\mu, 5)$ and $\mathrm{Tr}[T^a T^b] = \delta_{ab}$.

We split the SO(5) gauge fields into the SO(5)/SO(4) coset components and the SO(4) components. As SO(4)$\sim$SU(2)$_L\times$SU(2)$_R$ we write A_M as

$$A_M = L^b_M T^{L\,b} + R^b_M T^{R\,b} + X^{\hat{b}}_M T^{\hat{b}}\,, \tag{5.29}$$

where $L^b_M \equiv W^{L\,b}_M$, $b = 1,2,3$ and $\hat{b} = 1,2,3,4$. The unbroken generators correspond to the following linear combinations

$$L^a_\mu\,, \qquad B_\mu = \frac{g'_5 R^3_\mu + g_5 B'_\mu}{\sqrt{g_5^2 + g_5'^2}}\,. \tag{5.30}$$

We will analyze the 5D quadratic terms for A_M (it is straightforward to include the U(1)$_X$ gauge field). We work in the unitary gauge by adding the gauge fixing term

$$\mathcal{L}_{GF} = -\frac{1}{2g_5^2 \xi\, kz} Tr\left[\partial_\mu A_\mu - \xi\, z\partial_5(A_5/z)\right]^2\,. \tag{5.31}$$

In this gauge there is no mixing between the A_μ and A_5 fields in the bulk. Taking the limit $\xi \to \infty$ (see refs. [69],[11])

$$\partial_z(A_5/z) = 0 \tag{5.32}$$

The 5D quadratic terms for the gauge fields are

$$\mathcal{L}_5 = \frac{1}{2g_5^2 kz}\,\mathrm{Tr}\left\{A_\mu\left[(\partial^2 - kz\partial_5\frac{1}{kz}\partial_5)\eta_{\mu\nu} - \partial_\mu\partial_\nu\right]A_\nu + A_5\partial^2 A_5\right\}\,. \tag{5.33}$$

There are also boundary terms

$$\mathcal{L}_{\rm bound} = \frac{1}{2g_5^2 kz}\,\mathrm{Tr}\left(A_\mu\partial_5 A_\mu - 2A_\mu\partial_\mu A_5\right)\Big|_{L_0}^{L_1}\,. \tag{5.34}$$

The boundary conditions for A_5 are Dirichlet for the fields of the unbroken generators (to cancel the mixing boundary terms) and Neumann for the fields whose generators we want to break. This implies that A_5 is non-vanishing only in its SO(5)/SO(4) components (see section 3.4). Solving Eq. (5.32) with the specified boundary conditions we get

$$\begin{aligned} L^a_5(x,z) &= R^a_5(x,z) = 0\,,\\ X^{\hat{a}}_5(x,z) &= \zeta(z)h^{\hat{a}}(x)\,, \qquad \zeta(z) = z\sqrt{2/(L_1^2 - L_0^2)}\,, \end{aligned} \tag{5.35}$$

here and after a and $\hat{a}$ run respectively over the SO(4) generators (unbroken on the IR-brane)and the SO(5)/SO(4) generators (broken on the IR-brane). In Eq. (5.35) $\zeta(z)$ is normalized to obtain a canonical kinetic term for $h(x)$. Physical fluctuations of X_5 correspond to a 4D scalar field $h(x)$ transforming as a **4** of SO(4), the Higgs. From the point of view of the 5D theory, a potential for X_5 is forbidden at tree level by gauge invariance, but it is generated radiatively as a finite-volume effect from non-local operators. This is the Hosotani mechanism for symmetry breaking [3]. This means that the Higgs is massless at tree level, but it can acquire a finite mass by quantum effects. The fields in the boundary break the symmetry protecting the Higgs mass, and transmit this breaking through the exchange of virtual particles.

5.4.2 Fermion sector

The SM fermions are embedded into 5D Dirac spinors ξ_i that live in the bulk and belong to the $\mathbf{10_{2/3}}$ representation of SO(5)×U(1)$_X$. For each quark family we define

$$\xi_q = \begin{bmatrix} (\mathbf{3},\mathbf{1})^{\mathbf{q}}_{\mathbf{L}}(--) & (\mathbf{3},\mathbf{1})^{\mathbf{q}}_{\mathbf{R}}(++) \\ (\mathbf{2},\mathbf{2})^{\mathbf{q}}_{\mathbf{L}} = \begin{bmatrix} q'_L(-+) \\ q_L(++) \end{bmatrix} & (\mathbf{2},\mathbf{2})^{\mathbf{q}}_{\mathbf{R}} = \begin{bmatrix} q'_R(+-) \\ q_R(--) \end{bmatrix} \\ (\mathbf{1},\mathbf{3})^{\mathbf{q}}_{\mathbf{L}}(--) & (\mathbf{1},\mathbf{3})^{\mathbf{q}}_{\mathbf{R}}(++) \end{bmatrix}$$

$$\xi_u = \begin{bmatrix} (\mathbf{3},\mathbf{1})^{\mathbf{u}}_{\mathbf{L}}(++) & (\mathbf{3},\mathbf{1})^{\mathbf{u}}_{\mathbf{R}}(--) \\ (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{L}}(+-) & (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{R}}(-+) \\ (\mathbf{1},\mathbf{3})^{\mathbf{u}}_{\mathbf{L}} = \begin{bmatrix} \chi^u_L(++) \\ u^{c\prime}_L(-+) \\ d^{c\prime}_L(++) \end{bmatrix} & (\mathbf{1},\mathbf{3})^{\mathbf{u}}_{\mathbf{R}} = \begin{bmatrix} \chi^u_R(--) \\ u_R(+-) \\ d'_R(--) \end{bmatrix} \end{bmatrix} \tag{5.36}$$

$$\xi_d = \begin{bmatrix} (\mathbf{3},\mathbf{1})^{\mathbf{d}}_{\mathbf{L}}(++) & (\mathbf{3},\mathbf{1})^{\mathbf{d}}_{\mathbf{R}}(--) \\ (\mathbf{2},\mathbf{2})^{\mathbf{d}}_{\mathbf{L}}(+-) & (\mathbf{2},\mathbf{2})^{\mathbf{d}}_{\mathbf{R}}(-+) \\ (\mathbf{1},\mathbf{3})^{\mathbf{d}}_{\mathbf{L}} = \begin{bmatrix} \chi^d_L(++) \\ u^{c\prime\prime}_L(++) \\ d^{c\prime\prime}_L(-+) \end{bmatrix} & (\mathbf{1},\mathbf{3})^{\mathbf{d}}_{\mathbf{R}} = \begin{bmatrix} \chi^d_R(--) \\ u'_R(--) \\ d_R(+-) \end{bmatrix} \end{bmatrix}$$

where leptons are realized in a similar way. Here $(\pm,\pm)$ is a shorthand notation to denote a Neumann $(+)$ or Dirichlet $(-)$ boundary condition on each brane. Chiralities under the 4D Lorentz group have been denoted with L, R. We have split the $\mathbf{10_{2/3}}$ of SO(5) in multiplets of SU(2)$_L$ × SU(2)$_R$. As explained in section 3.3, massless modes in Eq. (5.36) arise from $(+,+)$

fields, these are q_L, $(\mathbf{3},\mathbf{1})^{\mathbf{q}}_{\mathbf{R}}$,$(\mathbf{1},\mathbf{3})^{\mathbf{q}}_{\mathbf{R}}$, $(\mathbf{3},\mathbf{1})^{\mathbf{u,d}}_{\mathbf{L}}$, $u^{c\prime\prime}_L$, $d^{c\prime}_L$ and $\chi^{u,d}_L$.

It is possible to write mass and kinetic mixing terms between the different fermions ξ_q, ξ_u, ξ_d respecting the gauge symmetry. The bulk kinetic and mass terms can be simultaneously diagonalized. We will work in this basis. In the IR boundary one can include SO(4)-invariant masses.

We will associate zero modes with the SM model fermions. To get rid of the extra massless states we add extra fields on the IR-brane, $\widetilde{(\mathbf{3},\mathbf{1})}_R$ and $\widetilde{(\mathbf{1},\mathbf{3})}_R$ and the following SO(4)-invariant IR boundary terms

$$\overline{(\mathbf{3},\mathbf{1})}^{u,d}_L \widetilde{(\mathbf{3},\mathbf{1})}_R + \overline{(\mathbf{1},\mathbf{3})}^{u,d}_L \widetilde{(\mathbf{1},\mathbf{3})}_R \tag{5.37}$$

and

$$\tilde{M}_{u,d}\big[\overline{(\mathbf{3},\mathbf{1})}^{u,d}_L (\mathbf{3},\mathbf{1})^q_R + \overline{(\mathbf{1},\mathbf{3})}^{u,d}_L (\mathbf{1},\mathbf{3})^q_R\big] + \tilde{m}_{u,d}\overline{(\mathbf{2},\mathbf{2})}^q_L (\mathbf{2},\mathbf{2})^{u,d}_R \,. \tag{5.38}$$

The extra fields on the IR will marry the extra zero modes. Therefore, the massless states become a mixture of different fields

- q_L mixes with q^u_L and q^d_L, these are the SU$(2)_L$ doublets with $T^3_R = -1/2$ in ξ_u and ξ_d respectively;

- u_R mixes with u^q_R and $\tilde{u}_R$, these are the $T^3_R = 0$ component of $(\mathbf{1},\mathbf{3})^{\mathbf{q}}_{\mathbf{R}}$ and $\widetilde{(\mathbf{1},\mathbf{3})}_R$ respectively;

- d_R mixes with d^q_R and $\tilde{d}_R$, these are the $T^3_R = -1$ component of $(\mathbf{1},\mathbf{3})^{\mathbf{q}}_{\mathbf{R}}$ and $\widetilde{(\mathbf{1},\mathbf{3})}_R$ respectively.

The mixing angles depend on the 5D bulk masses $M^i_5 = c_i k$ and on the IR masses $\tilde{m}_{u,d}, \tilde{\mathrm{M}}_{u,d}$.

The Yukawa couplings are gauge couplings with X_5, included in the 5D covariant derivative of the bulk fermions. As the fermions belong to a **10** of SO(5), X_5 only connects the bidoublets of SU$(2)_L\times$SU$(2)_R$ with the triplets of SU$(2)_L$ or with the triplets of SU$(2)_R$. Moreover X_5 connects fermions of opposite Lorentz chirality inside the same multiplet ξ_i, for example

$$(\mathbf{2},\mathbf{2})^i_L \leftarrow X_5 \rightarrow (\mathbf{3},\mathbf{1})^i_R\,, \qquad (\mathbf{2},\mathbf{2})^i_L \leftarrow X_5 \rightarrow (\mathbf{1},\mathbf{3})^i_R\,, \tag{5.39}$$

for $i = q, u, d$. The physical masses arise from the Yukawa couplings between the massless states, and can be suppressed depending on the mixing angles.

5.4.3 Matching to the 4D theory through the holographic approach

To obtain the 5D prediction for the form factors of Eq. (5.7) we match the two theories on the SO(4)-invariant vacuum: $\Sigma = \Sigma_0$ (i.e. $h = 0$). We compute first the gauge sector. We solve the 5D equations of motion for the gauge fields at the quadratic level, restricted to the proper UV-boundary conditions, and get

$$\mathcal{L}_{\rm eff} = \frac{1}{2} P_{\mu\nu} \big[\Pi_a(p) A^{a\,\mu} A^{a\,\nu} + \Pi_{\hat{a}}(p) A^{\hat{a}\,\mu} A^{\hat{a}\,\nu} \big] \,. \tag{5.40}$$

As before, indexes a ($\hat{a}$) run over the SO(4) (SO(5)/SO(4)) generators, and

$$\Pi_{a,\hat{a}}(p) = -\frac{1}{g_5^2 k} \frac{p}{L_0} \frac{Y_0(pL_0)\tilde{J}_{0,1}(pL_1) - \tilde{Y}_{0,1}(pL_1) J_0(pL_0)}{Y_1(pL_0)\tilde{J}_{0,1}(pL_1) - \tilde{Y}_{0,1}(pL_1) J_1(pL_0)} \,, \tag{5.41}$$

where the functions $\tilde{J}_{0,1}$ are given by

$$\tilde{J}_1(pL_1) = J_1(pL_1) \,, \qquad \tilde{J}_0(pL_1) = J_0(pL_1) - \frac{g_5^2 k}{g_{\rm IR}^2} \, pL_1 \, J_1(pL_1) \tag{5.42}$$

and similar equations hold for $\tilde{Y}_{0,1}$. The factor $1/g_{\rm IR}^2$ is the coefficient of the SO(4) boundary kinetic term on the IR-brane. For simplicity we have not included the $U(1)_X$ gauge boson in Eq. (5.40), but it is straightforward to include it. Its correlator is given by similar formulas. We have not written down possible boundary kinetic terms on the UV-brane though they can be included directly, we will consider the effects of these terms in our analysis. Eq. (5.40) must be matched to Eq. (5.7) after setting $\Sigma \to \Sigma_0$. Identifying the fields with Neumann (Dirichlet) boundary conditions on the UV-brane with the dynamical (non-dynamical) external fields of the 4D theory we get

$$\Pi_a(p) = \Pi_0(p) \,, \qquad \Pi_{\hat{a}}(p) = \Pi_0(p) + \frac{1}{2}\Pi_1(p) \,. \tag{5.43}$$

From this equation we obtain the gauge form factors:

$$\Pi_0(p) = \Pi_a(p) \;, \qquad \Pi_1(p) = 2\big[\Pi_{\hat{a}}(p) - \Pi_a(p)\big] \,. \tag{5.44}$$

To make contact with the 4D strongly coupled theory described in section 5.2 we will define the number of colors N in the CFT in terms of the 5D parameters. We will match the perturbative expansion parameter $1/N$ in the 4D theory with perturbative expansion parameter in 5D according to Eq. (3.45). For the large N description of the 4D theory to be reliable the condition $N \gg 1$ has to be satisfied, implying a maximum value for the 5D coupling in units of the curvature. As we want the 5D theory to be weakly coupled, from NDA we obtain:

$$\frac{\pi}{\Lambda_S L} = \frac{g_5^2}{24\pi^2 L} \ll 1 \,, \tag{5.45}$$

where Λ_S is the strong cutoff scale of the 5D theory. By demanding the value of the effective $SU(2)_L$ coupling to match with its electroweak value $g \simeq 0.65$, we can derive a lower bound for $g_5^2 k$. We express g in terms of g_5 and the boundary gauge couplings as:

$$\frac{1}{g^2} = \frac{\ln(L_1/L_0)}{g_5^2 k} + \frac{1}{g_{\rm UV}^2} + \frac{1}{g_{\rm IR}^2} \,, \tag{5.46}$$

where $1/g_{\rm UV}^2$ and $1/g_{\rm IR}^2$ are the SU(2)$_L$ UV-brane and the SO(4) IR-brane kinetic terms. From Eq. (5.46) we obtain

$$\frac{g_5^2}{L} > g^2 \ln \frac{L_1}{L_0} \sim 16\,. \tag{5.47}$$

Using Eq. (3.45) to express g_5 in terms of N we get an upper bound for N. Therefore N is restricted in the interval $1 \ll N \lesssim 10$.

The fermionic form factors can be obtained integrating out the bulk fermionic fields with fixed values on the UV-boundary. It is possible to fix on the boundary either the left- or the right-handed component of each fermion (but not both of them) obtaining the left- or right-handed source description [28] (see section 3.3). As ξ_q contains the SM fermion q_L we will fix the left-handed component of this field on the UV-brane, $\xi_{q\,L} = \big((\mathbf{3},\mathbf{1})^{\mathbf{q}}_{\mathbf{L}}, (\mathbf{2},\mathbf{2})^{\mathbf{q}}_{\mathbf{L}}, (\mathbf{1},\mathbf{3})^{\mathbf{q}}_{\mathbf{L}}\big)^{\mathbf{T}}$. For ξ_u, that contains the SM fermion u_R, we will fix the right-handed component on the UV-brane, $\xi_{u\,R} = \big((\mathbf{3},\mathbf{1})^{\mathbf{u}}_{\mathbf{R}}, (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{R}}, (\mathbf{1},\mathbf{3})^{\mathbf{u}}_{\mathbf{R}}\big)^{\mathbf{T}}$. For simplicity we omit ξ_d. Integrating out the bulk fields at tree level, we obtain the following quadratic terms for the fermionic boundary fields

$$\begin{aligned}\mathcal{L}_{\rm eff} =&\Pi_{(2,2)^q_L}(p)\,\overline{(2,2)}^q_L\,\not p(2,2)^q_L + \Pi_{(3,1)^q_L}(p)\,\overline{(3,1)}^q_L\,\not p(3,1)^q_L + \Pi_{(1,3)^q_L}(p)\,\overline{(1,3)}^q_L\,\not p(1,3)^q_L\\ &+ \Pi_{(2,2)^u_R}(p)\,\overline{(2,2)}^u_R\,\not p(2,2)^u_R + \Pi_{(3,1)^u_R}(p)\,\overline{(3,1)}^u_R\,\not p(3,1)^u_R + \Pi_{(1,3)^u_R}(p)\,\overline{(1,3)}^u_R\,\not p(1,3)^u_R\\ &+ M_{(2,2)^u}\overline{(2,2)}^q_L(2,2)^u_R + M_{(3,1)^u}\overline{(3,1)}^q_L(3,1)^u_R + M_{(1,3)^u}\overline{(1,3)}^q_L(1,3)^u_R\,.\end{aligned} \tag{5.48}$$

The fermionic form factors Π_i, M_i depend on the IR boundary masses $\tilde m_{u,d}, \tilde M_{u,d}$ that mix the different 5D multiplets. In the appendix B we compute the form factors in terms of the 5D propagators. By matching Eq. (5.48) with Eq. (5.7) in the vacuum Σ_0 we obtain

$$\begin{aligned}\Pi_0^r(p) &= \Pi_{(3,1)^r}(p) = \Pi_{(1,3)^r}(p)\,,\\ \Pi_1^r(p) &= 2\big[\Pi_{(2,2)^r}(p) - \Pi_{(3,1)^r}(p)\big]\,,\\ M_0^s(p) &= M_{(3,1)^s}(p) = M_{(1,3)^s}(p)\,,\\ M_1^s(p) &= 2\big[M_{(2,2)^s}(p) - M_{(3,1)^s}(p)\big]\,,\end{aligned} \tag{5.49}$$

where $r = q, u, d$ and $s = u, d$.

As usual in the AdS/CFT correspondence, the 5D fermion masses m_i are related to the anomalous dimensions of the CFT operators. The precise relation depends on the chirality of the source on the UV-brane, and is given by [28]

$$\gamma_q = \left|m_q L + \frac{1}{2}\right| - 1\,, \qquad \gamma_{u,d} = \left|m_{u,d} L - \frac{1}{2}\right| - 1\,. \tag{5.50}$$

For the light fermions we can satisfy Eq. (5.13) by fixing $\gamma_{q,u,d} > 0$. This implies $m_q L > 1/2$ and $m_{u,d}L < -1/2$. For the top we will require $\gamma_q \simeq 0$ and $\gamma_u < 0$, this implies $m_q L \simeq 1/2$ and $|m_u L| < 1/2$.

Having matched the 4D form factors with their 5D predictions, we can compute the relevant physical quantities.

5.5 Predictions from 5 dimensions

The 5D model presented in the last section has the same effective lagrangian as the 4D strongly coupled CFT containing the Higgs boson. Once we match the correlators of both theories, we can make predictions on the physical observables using the 5D correlators. As we argue in section 3.1, the n-point functions of the strongly coupled CFT in the large N limit can be written as an infinite sum over narrow resonances. We start with the sector of vector resonances, that is the same as in the minimal composite model. We write the correlators $\Pi_{a,\hat{a}}$ as

$$\Pi_a(p) = p^2 \sum_n \frac{F_{a_n}^2}{p^2 + m_{a_n}^2}\,, \qquad \Pi_{\hat{a}}(p) = p^2 \sum_n \frac{F_{\hat{a}_n}^2}{p^2 + m_{\hat{a}_n}^2} + \frac{1}{2} f_\pi^2\,. \tag{5.51}$$

To obtain f_π^2 we evaluate the correlator $\Pi_{\hat{a}}$ of Eq. (5.41) at zero momentum and get

$$f_\pi^2 = \frac{4L}{g_5^2} \frac{1}{L_1^2}\,. \tag{5.52}$$

We can also calculate the masses and decay constants from Eq. (5.41). For the first massive resonances the masses can be approximated by

$$m_\rho \equiv m_{a_1} \simeq \frac{3\pi/4}{\sqrt{1 + 9\pi^2/32\, z_{\rm IR}}} \frac{1}{L_1}\,, \qquad m_{\hat{a}_1} \simeq \frac{5\pi}{4} \frac{1}{L_1}\,, \tag{5.53}$$

where $z_{\rm IR} = g_5^2/g_{\rm IR}^2 L$.

5.5.1 Electroweak precision tests

One of the most constraining physical quantities is the S parameter, given in Eq. (5.15) in terms of the vector correlators. Using Eqs. (5.41) and (5.44) we obtain the prediction of the 5D model for S

$$S = \frac{3}{8} \frac{N}{\pi} \epsilon^2 \left[1 + \frac{4}{3} z_{\rm IR} \right]\,. \tag{5.54}$$

As discussed in section 5.2, LEP experiments give a very stringent bound on the S parameter: $S \lesssim 0.3$ [64], that can be translated into a bound over the parameter ϵ of the model

$$\epsilon \lesssim 0.5 \sqrt{\left(\frac{10}{N}\right) \frac{1}{1 + 4/3\, z_{\rm IR}}}\,. \tag{5.55}$$

Thus the maximum value of ϵ depends on the large number of colors N and also on the IR kinetic term. Low values of N lead to less stringent bounds on ϵ, however for low N the perturbative expansion is not trustable. By fixing $f_\pi \epsilon = v$, and using the value for f_π and L_1 from Eqs. (5.52) and (5.53), we can translate the bound over ϵ as a condition over the first vector state m_ρ [10]. Therefore imposing $S \lesssim 0.3$ the lowest vector resonance depends on the ratio z_{IR}. For $z_{IR} = 0$ ($z_{IR} = \infty$) m_ρ must be heavier than ~ 2.3TeV (~ 1.6TeV).

To obtain the value of ϵ we have to calculate the Higgs potential of Eq. (5.16). By using the correlators obtained from the 5D model we can calculate ϵ and check whether there is EW symmetry breaking. We also have to check that the S parameter is below the upper bound of Eq. (5.55) and that the top quark mass acquires the correct value after EW symmetry breaking. As the potential cannot be computed analytically we relay on numerical analysis. The 5D correlators depend on the following parameters: the 5D gauge coupling of the SO(5) group g_5, the UV kinetic terms for the $SU(2)_L$ group $1/g_{UV}^2$, the IR kinetic term for the SO(4) group $1/g_{IR}^2$, the bulk fermion masses m_q and m_u, and the IR masses $\tilde{m}_u$ and $\tilde{M}_u$. For simplicity we have not considered the effect of fermion kinetic terms on the boundaries, as well as the contribution to the potential due to the bottom quark an the hypercharge field. We fix the value of the parameters in the following way. The value of the UV gauge coupling g_{UV} is fixed by requiring that the effective $SU(2)_L$ coupling has its experimental value, Eq. (5.46). The value of the IR gauge coupling g_{IR} is set to be of loop order $1/g_{IR}^2 = 1/(4\pi)^2$. We express the value of the bulk coupling g_5 in terms of the large number of colors N, that is constrained to $1 \ll N \leq 10$. Typically we take $N = 4, 6, 8, 10$. The value of the bulk fermion masses lie in the interval $-1/2 < m_{q,u}L < 1/2$. For masses out of this range the top mass becomes too small. The IR fermion masses take values $\mathcal{O}(1)$, we have used $-2.2 \leq \tilde{m}_u L \leq 2.2$ and $-2.2 \leq \tilde{M}_u L \leq 2.2$. L_1 is fixed by Eq. (5.10).

We perform a detailed scanning over the parameter space in the following way. We fix the IR boundary masses $\tilde{m}_u$ and $\tilde{M}_u$, and the number of colors N, and vary m_q and m_u over the specified values. Therefore for every $(\tilde{m}_u, \tilde{M}_u, N)$ we obtain a plot in the plane (m_u, m_q). We define four regions in this plane:

1. A region were there is no electroweak symmetry breaking: $\epsilon = 0$,

2. A region where there is electroweak symmetry breaking and the S parameter satisfies the experimental bound: $0 \leq \epsilon \leq 0.5[10/(N(1 + 4/3z_{IR}))]^{1/2}$,

3. A region where there is electroweak symmetry breaking but the S parameter is bigger than the experimental bound: $0.5[10/(N(1 + 4/3z_{IR}))]^{1/2} \leq \epsilon \leq 1$,

4. A region where there is electroweak symmetry breaking but the SM fermions are massless: $\epsilon = 1$.

Given that there is EW symmetry breaking and the SM fermions are massive, comparing the second and third regions we obtain the constrains in the parameter space for $(\tilde{m}_u, \tilde{M}_u, N)$ fixed.

In Fig. 5.1 we show the predictions of the model for $\tilde{m}_u L = 1, \tilde{M}_u L = -2$ and $N = 4, 6, 8$. Due to the N dependence of the upper bound for ϵ, Eq. (5.55), the allowed region is bigger for

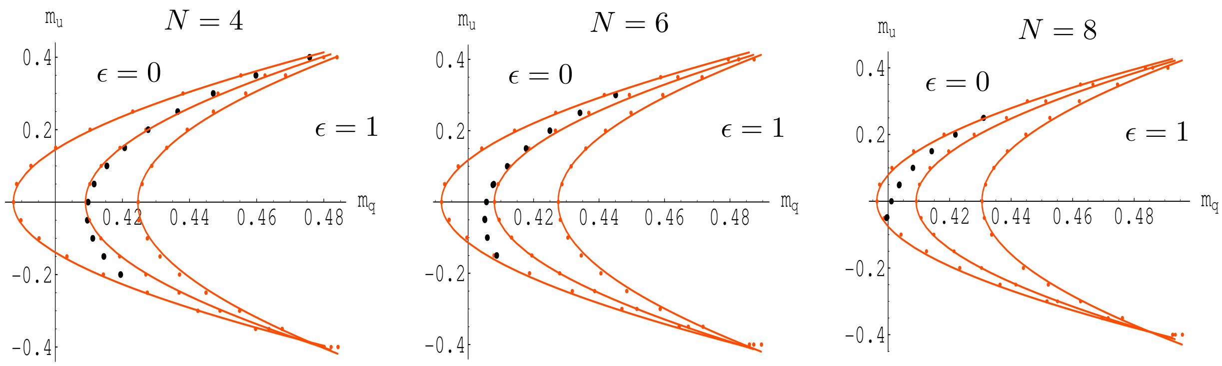


Figure 5.1: *Values of ϵ in the plane (m_u, m_q) for IR boundary masses $\tilde{m}_u L = 1, \tilde{M}_u L = -2$ and $N = 4, 6, 8$. The red dots correspond to $\epsilon = 0, 0.5[10/(N(1 + 4/3 z_{IR}))]^{1/2}, 1$, from left to right. The black dots correspond to $m_t = 150$ GeV. The red lines interpolates between the red points, and are given by $m_u = a\sqrt{m_q^2 - b^2}$, with $a \sim 1.5$ and $b \sim 0.4/L$.*

smaller N. Changing the values of the IR masses the different regions in the plane (m_q, m_u) change, but the tuning does not change much. Scanning over the parameter space we obtain that the region allowed by the S parameter constrain is of order 25%.

The top mass is not fixed in the plots of Fig. 5.1, thus m_t varies over the plane. The dashed lines in the plot correspond to $m_t = 150$ GeV. For $N \geq 10$ the top mass is in general smaller than its experimental value, only for very special values of the IR masses the top becomes heavy enough.

5.5.2 Spectrum of resonances

The spectrum of vector resonances is the same as in the minimal composite Higgs model (see Ref. [68] for a detailed analysis of the vector spectrum). Before EW symmetry breaking the L and B vector resonances, Eq. (5.30), are CFT composite states with a small a mixing with the external sources proportional to the UV boundary gauge coupling. Their spectra are given by the zeroes of $(\Pi_0 - p^2/g_{UV}^2)$ and $(\Pi_0 + \Pi_0^B - p^2/g_{UV}'^2)$ respectively, where g'_{UV} is the kinetic term of the $\mathrm{U}(1)_Y$ in the UV and Π_0^B is similar to Π_0 changing $\{g_5, g_{IR}\} \to \{g'_5, g'_{IR}\}$, with g'_{IR} the IR kinetic term of the $\mathrm{U}(1)_X$.

The other vector resonances are pure CFT composite states. There is a tower of fields transforming as $\mathbf{2}_{\pm 1/2}$ under $\mathrm{SU}(2)\otimes\mathrm{U}(1)_Y$, whose masses are given by the poles of $(\Pi_0 + \Pi_1/2)$. There is also a tower of fields transforming as $\mathbf{1}_{\pm 1}$, whose masses correspond to the poles of Π_0, and a tower of singlets whose masses are given by the poles of $(\Pi_0 - \Pi_0^B)$.

After EW symmetry breaking the different towers are mixed by the Higgs VEV. Therefore the spectrum of charged L vectors is given by

$$\text{zeroes}\big[\Pi_0 + \frac{\epsilon^2}{4}\Pi_1 - \frac{p^2}{g_{UV}^2}\big]\,, \tag{5.56}$$

and the tower of neutral vectors (Z) is given by

$$\text{zeroes}\big[(\Pi_0 - \frac{p^2}{g_{UV}^2})(\Pi_0 + \Pi_0^B - \frac{p^2}{g_{UV}'^2}) + \frac{\epsilon^2}{4}\Pi_1(\Pi_0^B + 2\Pi_0 - \frac{p^2}{g_{UV}^2} - \frac{p^2}{g_{UV}'^2})\big]\,. \tag{5.57}$$

For moderate values of the parameters the Higgs mass is bigger than 100 GeV. Remarkably it is heavier than in Ref. [10]. Adjusting the parameters to obtain $m_t = 150$ GeV, the Higgs mass is almost constant for constant N. For $N = 4, 6, 8$ we obtain $m_H \sim 160, 180, 220$ GeV respectively.

The spectrum of fermion resonances is more complicate due to the mixing of the different towers accomplished by the IR masses and the Higgs VEV. Following Ref. [68], we will consider as an example the case of two sources $\Psi_{1,2}$ coupled to two mixed towers of resonances. Thus the two-point function obtained after integrating-out all the resonances is 2×2 matrix $\Pi(p)_{ij}$, $i, j = 1, 2$. If both sources are dynamical the spectrum is given by the zeroes of the determinant of $\Pi(p)$. This is because diagonalizing $\Pi(p)$ each tower of resonances couple to one independent combination of the sources, thus the zeroes of the eigenvalues of $\Pi(p)$ correspond to the zeroes of its determinant. If both sources are non-dynamical, the spectrum is given by the poles of any $\Pi(p)_{ij}$. The poles of the eigenvalues of $\Pi(p)$ correspond to the poles of any of the elements $\Pi(p)_{ij}$. If Ψ_1 is dynamical and Ψ_2 is not, the physical spectrum is given by the zeroes of $\Pi(p)_{11}$. Although Ψ_1 is directly coupled to one tower of resonances only, it can probe all the spectrum of resonances due to the mixings.

Before EW symmetry breaking there are towers of q_L, t_R and b_R whose spectrum are given respectively by the zeroes of $\not{p}(\Pi_0^q + \Pi_1^q/2)$, $\not{p}\Pi_0^u$ and $\not{p}\Pi_0^d$. There are also towers of q_L' ($\mathbf{2}_{7/6}$), χ_R ($\mathbf{1}_{5/3}$) and a triplet ($\mathbf{3}_{2/3}$), whose spectrum are given respectively by the poles of $\not{p}(\Pi_0^q + \Pi_1^q/2)$, $\not{p}\Pi_0^u$ and $\not{p}\Pi_0^d$.

After EW symmetry breaking there is tower of t's (with hypercharge $+2/3$) with masses

$$\text{zeroes}\Big[p^2(\Pi_0^q + \frac{\Pi_1^q}{2})\Pi_0^u + p^2\frac{\epsilon^2}{4}(\Pi_0^q\Pi_1^u - \Pi_0^u\Pi_1^q) - p^2\frac{\epsilon^4}{16}\Pi_1^q\Pi_1^u - \frac{\epsilon^2 - \epsilon^4}{16}M_1^{u\,2}\Big]\,, \tag{5.58}$$

and a tower of b's (with hypercharge $-1/3$) whose masses are given by

$$\text{zeroes}\Big[\Pi_0^q + \Pi_1^q\frac{1-\epsilon^2}{2}\Big]\,. \tag{5.59}$$

The lightest resonance is given by the first fermionic KK state $q_{L\,1}$. In Fig. 5.2 we show the mass of this resonance as a function of the 5D mass m_u. The different colors indicate that

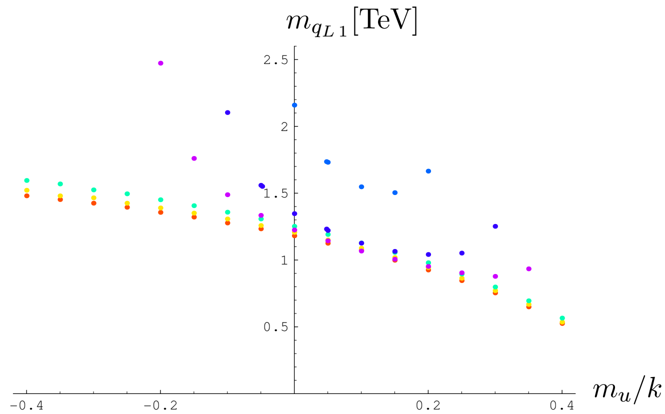


Figure 5.2: *Mass of the lightest resonance, $m_{q_{L\,1}}$, as a function of m_u. The red, yellow and green points correspond to constant ϵ and $N = 4, 6, 8$. The violet, blue and light blue points correspond to $m_t = 150$ GeV and $N = 4, 6, 8$.*

we have given different values to the 5D parameters. We have fixed either $m_t = 150$ GeV or ϵ such that it saturates its upper bound, and $N = 4, 6, 8$. The points of constant ϵ correspond to the lower values of $m_{q_{L\,1}}$. The points giving large masses for $m_u \to \pm 1/2L$ correspond to $m_t = 150$ GeV. In this case $m_{q_{L\,1}}$ becomes very large because ϵ is too small.

The mass of the lightest fermionic resonance has a correlation with the Higgs mass. To obtain an approximate equation relating both masses we can approximate the coefficient γ of Eq. (5.24) by

$$\gamma \simeq -N_c \int \frac{d^4p}{(2\pi)^4} \frac{(M_1^u)^2}{8p^2 \Pi_0^u (\Pi_0^q + \Pi_1^q/2)} , \tag{5.60}$$

where we only keep the term of Eq. (5.24) that gives the most important contribution. By defining the mass scale Λ

$$\Lambda^2 = -2 \int dp\, p \frac{F_M(p)}{F_M(0)} , \qquad \text{where} \qquad F_M(p) = \frac{M_1^u(p)^2}{\Pi_0^u (\Pi_0^q + \Pi_1^q/2)]} \tag{5.61}$$

we can approximate the Higgs mass by

$$m_h^2 \simeq \frac{N_c}{\pi^2} \frac{m_t^2}{v^2} \epsilon^2 \Lambda^2 , \tag{5.62}$$

with m_t given by Eq. (5.12). As the form factor $F_M(p)$ is dominated by the first resonance, the scale Λ satisfies $\Lambda \sim m_{q_{L\,1}}$. Scanning over the parameter space this relation is fulfilled with an error that oscillates between $20 - 40\%$. Therefore the Higgs mass is proportional to the lightest fermionic resonance $m_H \sim m_{q_{L\,1}}$. In Fig. 5.3 we show the correlation between them for $N = 8$ and $m_t = 150$ GeV. The parameter ϵ is such that it saturates the upper bound of Eq. (5.55). By fixing these parameters the resulting Higgs mass is light 116 GeV $\leq m_H \leq$ 161 GeV.

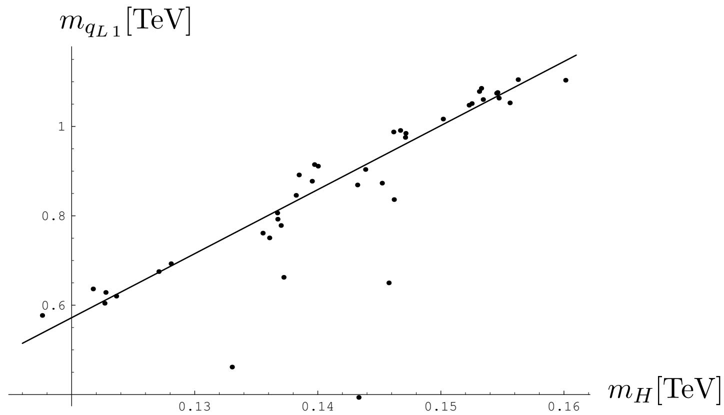


Figure 5.3: *Correlation between the Higgs mass and the lightest resonance mass. The points are obtained scanning over the parameter space with $N = 8$, $m_t = 150GeV$ and $\epsilon \simeq 0.52$ defined by the upper bound of Eq. (5.55).*

5.6 A symmetry for $Zb\bar{b}$

To obtain a successful model of EW symmetry breaking it is necessary to pass the EW precision tests. In models where the top has a strong coupling with the new sector, we expect large modifications in the interactions $Zt\bar{t}$. Since b_L is in the same multiplet as t_L, we also expect large corrections in the coupling $Zb\bar{b}$. As this coupling is in agreement with its SM value at the 0.25%, it is difficult to success in this sector. In fact, in the minimal Higgs model [10], the contribution of the KK modes to the decay $Z \to b_L\bar{b}_L$ is too large. To make this corrections small enough one needs an amount of tuning of order a few percent in that model.

We will show in this section that there is a subgroup of the custodial symmetry that can protect the interaction $Zb\bar{b}$ from these large corrections. Moreover, the interaction $Zb_L\bar{b}_L$ is safe even in the case of large contributions to the vertices $Zt_L\bar{t}_L$ and $Wt_L\bar{b}_L$, that are obtained from the first one by SU(2)$_L$ transformations. The custodial symmetry can protect also the coupling $Zb_R\bar{b}_R$. However experimental data suggest that this coupling deviates from its SM prediction. As we will see, the symmetries protecting these interactions can be implemented in a large class of models.

Let us consider a new sector with the pattern of global symmetry breaking

$$SU(2)_L \otimes SU(2)_R \to SU(2)_V\,, \tag{5.63}$$

and a parity under the interchange $L \leftrightarrow R$ (P_{LR}). We can classify the operators $\mathcal{O}$ of this new sector by their transformation properties under this global symmetry, in particular by their isospin number $T_{L,R}$ and its third component $T^3_{L,R}$. Let us assume that each SM field is coupled to a single operator $\mathcal{O}$, such that we can assign a $\{T_{L,R}, T^3_{L,R}\}$ to each SM field, although they are not in complete representations of SU(2)$_L\otimes$SU(2)$_R$. We consider then the corrections to the coupling $Z\Psi\bar{\Psi}$ at zero momentum, with Ψ a SM fermion. This interaction is given by

$$\frac{ig}{\cos\theta_W}\left[Q^3_L - Q\sin^2\theta_W\right] Z^\mu \bar{\psi}\gamma_\mu\psi\,, \tag{5.64}$$

where Q^3_L and Q are respectively the third component of the left charge and the electric charge. As the electric charge is conserved, it can not receive corrections. On the other hand, there are two subgroups of the custodial group that can protect Q^3_L. The first one is U(1)$_L\otimes$U(1)$_R\otimes P_{LR}$ that is broken to U(1)$_V\otimes P_{LR}$. P_{LR} is a symmetry of the new sector, but in general it is not a symmetry of the SM sector. Thus in general the coupling $\mathcal{L}_{int} = \bar{\Psi}\mathcal{O}_\Psi + h.c.$ between the new sector and the field Ψ of the SM breaks P_{LR}. Nevertheless, if Ψ is an eigenstate of P_{LR}, P_{LR} is a good symmetry of the interaction. In this case the quantum numbers of Ψ satisfy

$$T_L = T_R\ , \qquad T^3_R = T^3_L\,. \tag{5.65}$$

Therefore the correction to the charge Q^3_L of Ψ vanishes. The proof is the following. The charge $Q^3_V = Q^3_L + Q^3_R$ is conserved due to the U(1)$_V$ invariance, thus it is not modified by the interactions and we obtain

$$\delta Q_V = \delta Q^3_L + \delta Q^3_R = 0\,. \tag{5.66}$$

The P_{LR} symmetry impose the corrections to Q^3_L to be equal to the corrections to Q^3_R for an eigenstate Ψ:

$$\delta Q^3_L = \delta Q^3_R\,. \tag{5.67}$$

From Eqs. (5.66) and (5.67) we obtain $\delta Q^3_L = 0$. Thus the interaction $Z\Psi\bar{\Psi}$, with Ψ a SM fermion that satisfies Eq. (5.65), is protected by the U(1)$_V\otimes P_{LR}$ symmetry.

The other subgroup of the custodial symmetry that can protect Q^3_L is given by the following discrete transformation $T^3_{L,R}\to -T^3_{L,R}$. We will denote this symmetry by P_C. Under P_C: $L^3\to -L^3$, and Ψ is an eigenstate of P_C if

$$T^3_L = T^3_R = 0\,. \tag{5.68}$$

In this case we obtain $\delta Q^3_L = 0$. The proof is the following. The current $\bar{\Psi}\gamma_\mu\Psi$ is even under P_C if Ψ is an eigenstate of this symmetry. Since L^3_μ is odd under P_C, the interaction is not present. Therefore, the interaction $Z\Psi\bar{\Psi}$ is protected by P_C invariance if Ψ satisfies Eq. (5.68).

Since this symmetries protect the interactions with the Z only at zero momentum, we expect corrections of order p^2/Λ^2_{NP}.

According to the previous discussion, if the new sector respects the custodial symmetry, it is possible to embed the SM fermions in multiplets such that the interaction $Zb\bar{b}$ is protected. Assigning to b_L the quantum numbers

$$b_L: \qquad T_L = 1/2 = T_R\ , \qquad T^3_L = -1/2 = T^3_R\,, \tag{5.69}$$

the coupling $Zb_L\bar{b}_L$ does not receive corrections from the new sector. As t_L is in the same $\mathrm{SU}(2)_L$ doublet as b_L, it has the following quantum numbers:

$$t_L: \qquad T_L = 1/2 = T_R\ , \qquad T_L^3 = 1/2 = -T_R^3\,. \tag{5.70}$$

Therefore $Zt_L\bar{t}_L$ is not protected and we expect large modifications. The same happens for $Wt_L\bar{b}_L$, thus as the couplings of the top with the gauge bosons are poorly known, future experiments will be able to test this scenario.

5.6.1 Operator analysis

We consider an operator analysis for the coupling of $q_L = (t_L, b_L)$ and t_R to the gauge bosons W and Z and analyze the consequences of the custodial symmetry. To obtain the assignments of Eqs. (5.69) and (5.70) we have to embed q_L in a $\mathbf{4_{2/3}}$ multiplet of SO(4)$\otimes$U$(1)_X$, or, equivalently,

$$q_L \ \in\ (\mathbf{2},\mathbf{2})_{\mathbf{2/3}} \equiv Q_L\,, \tag{5.71}$$

multiplet of $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\otimes\ \mathrm{U}(1)_X$. In the low energy effective theory there are two single-trace dimension 4 operators that can contribute to the Z couplings:

$$\mathcal{L} = ic_1\,\mathrm{Tr}\left[\bar{Q}_L\gamma^\mu Q_L\hat{V}_\mu\right] - ic_2\,\mathrm{Tr}\left[\bar{Q}_L\gamma^\mu V_\mu Q_L\right], \tag{5.72}$$

where $Q_L = \sigma_\mu Q_L^\mu$ is a 2×2 matrix field [1], $V_\mu = (D_\mu U)U^\dagger$, $\hat{V}_\mu = (D_\mu U)^\dagger U$, and the covariant derivative is defined as $D_\mu U = \partial_\mu U + ig\sigma_a W_\mu^a U/2 - ig' B_\mu U\sigma_3/2$. U is the unitary matrix that contains the Goldstone boson that parametrize the symmetry breaking $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R \to\mathrm{SU}(2)_V$. By imposing P_{LR}, under which $U\to U^\dagger$, $V_\mu \leftrightarrow \hat{V}_\mu$ and $Q_L \to \sigma_\mu^\dagger Q_L^\mu$, we obtain $c_1 = c_2$. There is also a double-trace operator that can contribute to the Z coupling to q_L:

$$\mathcal{L} = ic_3\,\mathrm{Tr}\left[\bar{Q}_L\gamma^\mu D_\mu U\right]\mathrm{Tr}[U^\dagger Q_L] + h.c.\,. \tag{5.73}$$

This operator is however subleading with respect to those of Eq. (5.72) in a $1/N$ expansion of the sector beyond the SM. In the 5D models this means that it can only be induced at the one-loop level.

To obtain the contributions to $Zb_L\bar{b}_L$, $Zt_L\bar{t}_L$ and $Wt_L\bar{b}_L$ we plug in Eqs. (5.72) and (5.73)

$$Q_L = \sigma_- b_L + \sigma_0 t_L + \dots\ , \quad U = \mathbb{1}\ , \quad D_\mu U = \frac{ig\sigma_3}{2\cos\theta_W}Z_\mu + \frac{ig\sigma_+}{\sqrt{2}}W_\mu^+ + \dots\,, \tag{5.74}$$

where $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/2$ and $\sigma_0 = (\mathbb{1}+\sigma_3)/2$. This gives

$$\frac{g}{\cos\theta_W}\left[\frac{c_1-c_2}{2}\,\bar{b}_L\gamma^\mu b_L + \frac{c_1+c_2-c_3}{2}\,\bar{t}_L\gamma^\mu t_L\right]Z_\mu + \frac{g}{\sqrt{2}}\,(c_2-c_3)\,\bar{t}_L\gamma^\mu b_L W_\mu^+ + h.c.\,. \tag{5.75}$$

[1] We use the basis $\sigma_\mu = (\mathbb{1}, \mathrm{i}\sigma_1, \mathrm{i}\sigma_2, \mathrm{i}\sigma_3)$ where σ_a are the Pauli matrices.

As expected from the symmetry argument, the contributions to $Zb_L\bar{b}_L$ cancels if invariance under P_{LR} is imposed ($c_1 = c_2$), while the contributions to the top couplings are different from zero.

The embedding of t_R in a multiplet of the custodial group is determined by the top mass operator $\bar{q}_L U t_R$. These operator can arise from two SU(2)$_L\otimes$SU(2)$_R\otimes$ U(1)$_X$ invariant operators:

$$a)\ \ \overline{(\mathbf{2},\mathbf{2})}_{\mathbf{2/3}}(\mathbf{2},\mathbf{2})_{\mathbf{0}}(\mathbf{1},\mathbf{1})_{\mathbf{2/3}}\ ,\quad \text{or}\quad b)\ \ \overline{(\mathbf{2},\mathbf{2})}_{\mathbf{2/3}}(\mathbf{2},\mathbf{2})_{\mathbf{0}}(\mathbf{1},\mathbf{3})_{\mathbf{2/3}}\,. \tag{5.76}$$

This implies respectively the two following embeddings for t_R:

$$a)\ t_R\ \in (\mathbf{1},\mathbf{1})_{\mathbf{2/3}}\ ,\quad \text{or}\quad b)\ t_R\ \in\ (\mathbf{1},\mathbf{3})_{\mathbf{2/3}} + (\mathbf{3},\mathbf{1})_{\mathbf{2/3}}\,. \tag{5.77}$$

They corresponds respectively to a $\mathbf{1_{2/3}}$ and a $\mathbf{6_{2/3}}$ multiplet of SO(4)$\otimes$U(1)$_X$. In both cases t_R has $T^3_L = T^3_R = 0$ that corresponds to the condition Eq. (5.68). Therefore its coupling to the Z is protected by the P_C symmetry. [2] We can also perform an operator analysis for the Z coupling to t_R. For the case (a), no invariant operator can be written since $\mathrm{Tr}[V_\mu] = \mathrm{Tr}[\hat{V}_\mu] = 0$. For the case (b), we have that t_R corresponds to the $T^3_L = T^3_R = 0$ state in $(\mathbf{1},\mathbf{3})_{\mathbf{2/3}} \equiv U_R$. We have two dimension 4 operators that can contribute to the Z coupling of t_R:

$$\mathcal{L} = ic_4\,\mathrm{Tr}\left[\bar{U}_R\gamma^\mu U_R\hat{V}_\mu\right] + ic_5\,\mathrm{Tr}\left[\bar{U}_R\gamma^\mu\hat{V}_\mu U_R\right]. \tag{5.78}$$

Using $U_R = \sigma_3 t_R + ...$ we find that, as expected, the contribution to $Zt_R\bar{t}_R$ vanishes.

In theories in which the Higgs arises as a PGB from the symmetry breaking O(5)$\rightarrow$ O(4), we must embed the fermion multiplets into SO(5) representations. We find two very simple options. For the case (a) we can use the $\mathbf{5_{2/3}}$ multiplet that decomposes as

$$\mathbf{5_{2/3}} = (\mathbf{2},\mathbf{2})_{\mathbf{2/3}} + (\mathbf{1},\mathbf{1})_{\mathbf{2/3}}\,, \tag{5.79}$$

and contains the multiplets of Eqs. (5.71) and (5.77). For the case (b) we can embed the top in the $\mathbf{10_{2/3}}$ multiplet:

$$\mathbf{10_{2/3}} = (\mathbf{2},\mathbf{2})_{\mathbf{2/3}} + (\mathbf{3},\mathbf{1})_{\mathbf{2/3}} + (\mathbf{1},\mathbf{3})_{\mathbf{2/3}}\,. \tag{5.80}$$

In Ref. [10] the SM fermions were embedded in the **4** of SO(5). We see that just by changing their embedding from the **4** to the **5** (or the **10**) of SO(5) we are automatically driven to the right charge assignment for b_L. [3] This leads to composite Higgs models with the same properties as those of Ref. [10], with the exception of the large corrections to $Z\bar{b}_L b_L$.

[2] For the case (a) it is interesting to remark that t_R is a singlet of the custodial symmetry and therefore loop effects involving this field will not generate corrections to the T parameter.

[3] It would be possible, for example, to embed the SM fermions into a $\mathbf{10}_{-1/3}$ of SO(5) also. In this case b_L has non zero axial charge, with $T^{3L} = -T^{3R} = -1/2$, and the vertex $Zb_L\bar{b}_L$ is not protected.

5.6.2 $Zb\bar{b}$ in the 5D model

To compute the correction to the interaction $Z\Psi\bar{\Psi}$ it is convenient to use the KK description. A tree level there are two types of contributions: mediated by vector resonances and mediated by fermionic resonances. Concerning the vector fields, the most important piece comes from the exchange of the first KK resonances L_μ and R_μ at zero momentum. Therefore, for 5D fermion masses satisfying $|mL| < 1/2$ (as is the case for the top and bottom), it can be approximated by

$$\delta g \simeq (T^{3R} - T^{3L})\epsilon^2 \frac{\sqrt{2}}{L_1^2 m_\rho^2} \frac{1/2 - mL}{1 - 2mL/3} \,, \tag{5.81}$$

where m_ρ is the mass of the first KK vector resonance. Thus assigning to b_L the charges of Eq. (5.69) the gauge contribution to $Zb_L\bar{b}_L$ vanishes.

The contributions from the first fermion KK resonance are the dominant one, and are of the form

$$\delta g \simeq (T^{3L}_{KK} - T^{3L}) \sin^2\theta_{KK} \,, \tag{5.82}$$

where θ_{KK} is the mixing angle between the KK mode and the SM fermion after EW symmetry breaking.

For q_L embedded in $(\mathbf{2},\mathbf{2})_{\mathbf{2/3}}$ of $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_X$, only fermionic KKs in the representations $(\mathbf{1},\mathbf{1})_{\mathbf{2/3}}$, $(\mathbf{1},\mathbf{3})_{\mathbf{2/3}} \oplus (\mathbf{3},\mathbf{1})_{\mathbf{2/3}}$ and $(\mathbf{3},\mathbf{3})_{\mathbf{2/3}}$ can mix with b_L or t_L at order ϵ. The coefficients of the operators in Eqs. (5.72) and (5.73) then read:

$$c_1 = c_2 \simeq \frac{1 - 2m_q L}{2\sqrt{2}(3 - 2m_q L)}\,\epsilon^2 + \frac{1}{2}\sin^2\theta_{KK}^{(1,1)} + \frac{1}{2}\sin^2\theta_{KK}^{(3,1)} - \frac{3}{4}\sin^2\theta_{KK}^{(3,3)} \,, \quad c_3 = 0 \,. \tag{5.83}$$

Here $\theta_{KK}^{(1,1)}$ is the mixing angle between t_L and the KK in the $(\mathbf{1},\mathbf{1})_{\mathbf{2/3}}$ representation, and $\theta_{KK}^{(3,1)}$ ($\theta_{KK}^{(3,3)}$) is the mixing angle between b_L and the KK in the $(\mathbf{3},\mathbf{1})_{\mathbf{2/3}}$ ($(\mathbf{3},\mathbf{3})_{\mathbf{2/3}}$) representation. In the case of a composite Higgs model where q_L is embedded in a $\mathbf{10}_{\mathbf{2/3}}$ of SO(5), the result is that of Eq. (5.83) with only the gauge and $(\mathbf{3},\mathbf{1})_{\mathbf{2/3}}$ fermionic contributions turned on. Eq. (5.75) together with Eq. (5.83) gives us the tree-level correction to the couplings of the Z and the W to the SM fermions. Corrections of order $\sim 10\%$ or even larger are thus possible if q_L is strongly coupled to the 5D bulk dynamics (i.e.: for $-1/2 < m_q L \lesssim 0$), and they could be observed in future experiments that probe the couplings of the top quark.

5.7 Conclusions

The theory of EW symmetry breaking is still unknown. The most popular model beyond the SM, supersymmetry, is already highly constrained. For this reason it is very important to look for new alternatives. The idea of a Higgs arising as a PGB can explain the hierarchy between the EW scale and the scale of new physics. In particular, a Higgs as composite of a strong sector is one of the most economical scenarios, but until recently it has evade a detailed quantitative calculation.

Inspired in the AdS/CFT conjecture, models with extra dimensions can be interpreted as strongly coupled field theories. In Refs. [69, 10], two examples of holographic models were given. The last model provides a realistic description of EW symmetry breaking, but the interaction $Zb\bar{b}$ receives large corrections. In this chapter we have presented an extension of this minimal model by embedding the fermionic fields in a higher representation of the symmetry group, a **10** of SO(5). We showed that the top quark can trigger EW symmetry breaking, and that the S parameter constrains are satisfied in a large region of the parameter space. Moreover, constrains from the decay $Z \to \bar{b}_L b_L$, that were a serious problem for the minimal model, is not an issue in the extended model. We have shown that the custodial symmetry $\mathrm{SU}(2)_V \otimes P_{LR}$ can protect the interaction $Zb\bar{b}$ from corrections. However, the interactions $Zt\bar{t}$ and $Wt\bar{b}$ can not be protected at the same time and can receive large corrections. The model predicts a light Higgs boson with mass 116 GeV$\lesssim m_H \lesssim$ 161 GeV, that has a correlation with the lightest fermionic resonance 600 GeV$\lesssim m_{q_{L\,1}} \lesssim$ 1100 GeV. The lightest vector resonances are heavier than the fermionic resonances $\sim 1-3$ TeV. Thus the model gives definite predictions that can be tested in the future high energy experiments like LHC.

Chapter 6

Radiative corrections in 5D theories expanding in winding modes

6.1 Introduction

In this chapter we will develop a formalism that allows to compute loop corrections in field theories with extra dimensions, separating UV divergences from finite contributions. In order to do so we will decompose the propagators in winding modes. In a 5D theory these modes are obtained by propagation around the circle of the extra dimension. Two paths with different windings are topologically different because one can not deform one into the other, thus they give different contributions to physical quantities. We will show that UV divergences in loop integrals are associated to the zero winding mode and that non zero modes correspond to long distances and therefore they give finite contributions [73].

We will use the winding decomposition of the 5D propagators to compute radiative corrections on 5D theories in flat space [16]. The results corresponding to the divergent corrections are valid for curved spaces also. We will consider theories compactified in a smooth space and we will also consider singular spaces (orbifolds) where translation symmetry is broken. We will show that the boundaries can support 4D fields by calculating the one loop corrections that lead to localized terms. As an interesting application we will apply this formalism to compute a two loop effective potential. We will show that this potential can stabilize large extra dimensions when there are terms localized on the boundaries. The method can be extended to compute radiative corrections in realistic models. In particular it would be very interesting to calculate radiative corrections in the holographic duals of QCD, where we expect corrections to be of order 30%.

This chapter is organized in the following way. In section 6.2 we define winding modes working on a mixed momentum-coordinate representation. We apply this idea to a 5D toy model in section 6.3 and show the simplicity of this method in some particular cases. In section 6.4 we compute a two loop effective potential and section 6.5 is for conclusions.

6.2 Winding modes

Let us consider a 5D space that is a direct product of a 4D Minkowski space $\mathcal{M}^4$ an a compact manifold $\mathcal{C}^1$, thus the 5D space is given by $\mathcal{M}^4 \times \mathcal{C}^1$. We assume that the 1D compact space can be obtained by identifying the real line with a discrete infinite group G acting freely on $\mathcal{R}^1$, thus we can express the compact space as $\mathcal{C}^1 = \mathcal{R}^1/G$. In this case one can associate a winding mode to every element of the group G. The simplest example of a compact space obtained in this way is the circle $\mathcal{C}^1 = S^1$, where $G = \mathcal{Z}$, the set of integer numbers, with the sum defined as the group operation. In this example we obtain the compact space identifying $y \sim y + n2\pi R$, where R is the radius of the circle. Due to the identification $0 \leq y < 2\pi R$. The index n labels the winding modes. It is immediate to generalize this method to higher dimensional spaces (see Ref. [16] for a generalization to 6D spaces).

The procedure we described above to compactify an infinite space suggests the following algorithm to obtain the 5D propagators: we calculate the propagators of the 5D fields in the infinite space and identifying $y \sim y + n2\pi R$ we obtain the corresponding ones on the compact space.

As an example we will calculate the massless scalar propagator on Euclidean 5D spacetime. We will work on a mixed representation (p_μ, y), where p_μ is the 4D momentum and y is the coordinate in the extra dimension. The propagator is determined by the following equation

$$(p^2 - \partial_y^2)\widetilde{G}(p; y - y') = \delta(y - y'), \tag{6.1}$$

where we have Fourier transform to momentum space only on the 4D space, and $p^2 = p^\mu p_\mu$. Solving this equation we obtain

$$\widetilde{G}(p; y - y') = \frac{e^{-p|y-y'|}}{2p}. \tag{6.2}$$

As translation invariance is not broken the propagator only depends on $|y - y'|$. If we consider a massive scalar field we just have to replace $p^2 \to p^2 + m^2$. To obtain the propagator in the compact space we have to identify $|y - y'| \sim |y - y' + 2n\pi R|$ (see Fig. 6.1), thus we restrict $y, y' \in [0, 2\pi R)$ and summing over windings we obtain

$$\widetilde{G}^{cir}(p; y, y') = \sum_{n=-\infty}^{n=\infty} \widetilde{G}(p; y - y' + 2n\pi R) = \sum_n \frac{e^{-p|y-y'+2n\pi R|}}{2p}. \tag{6.3}$$

The series can be resumed but we want to consider the contribution of each mode. In the last equation we can see that for $n \neq 0$ the propagator is exponentially damped at high energies, therefore loop integrals involving the propagator with non zero winding modes will lead to finite results. On the other hand, for $n = 0$, the propagator decays as p^{-1} for $y = y'$ and there is no factor that can force the momentum integrals to be finite. For these reason, if we compactify the extra dimension on a circle, we will obtain divergent contributions from the winding zero mode, and finite contributions from the other modes. Therefore, using the winding formalism, it is very easy to separate divergent from finite contributions. The divergent terms are associated to short distances, thus divergences arising from zero modes are the same as the divergences of the uncompactified theory.

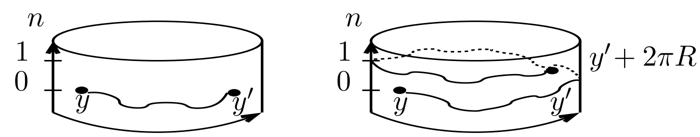


Figure 6.1: *Two equivalent contributions for the propagator between y and y'. In the vertical axis we represent the number of windings.*

Let us compare the winding mode expansion with the KK approach with a specific example. The radiative corrections to the mass terms are dominated by high energy effects. Thus, as high energies implies small distances, the small winding contributions dominate over the large winding ones, and in general it is enough to consider the first terms in the series to obtain the most important part. This is not the case when we do KK decomposition, where we have to sum over all the tower of KK resonances to obtain the final result. Therefore in this case the winding method is much simpler.

Orbifold compactification

Orbifolds are used to obtain 4D chiral fermions from a higher dimensional theory. In general we can obtain an orbifold with a discrete group F acting non-freely on the compact space $\mathcal{C}$. The points of $\mathcal{C}$ left invariant by F are the fixed points of the extra space, and therefore $\mathcal{C}/F$ is singular. The simplest example is the orbifold S^1/Z_2, where Z_2 is the parity transformation in the extra dimension, $Z_2 : y \to -y$. Due to this identification the extra coordinate is restricted to a smaller interval, $0 \le y \le \pi R$. The fixed points are located at $y = 0, \pi R$. As the fields also transform under a parity transformation, we have to specify the field parities.

we will consider a scalar field on S^1/Z_2 with parity $Z_2\phi(x^\mu, y) = \phi(x^\mu, -y) = \pm\phi(x^\mu, y)$. Due to the identification $y \sim -y$, the propagation from y to y' and the propagation from y to $-y'$ describe the same process, thus the propagator is given by [74]

$$\widetilde{G}^{orb}_{\pm}(p; y, y') = \sum_n \left(\frac{e^{-p|y-y'+2n\pi R|}}{2p} \pm \frac{e^{-p|y+y'+2n\pi R|}}{2p}\right), \tag{6.4}$$

where $y, y' \in [0, \pi R]$, and $\pm$ stands for the field parity. This propagator depends on $(y + y')$ due to the breaking of translation invariance. In this equation we can see that the propagator decays as p^{-1} for $(y \to 0, y' \to 0, n = 0)$ and $(y \to \pi R, y' \to \pi R, n = -1)$, thus we expect divergences localized on the fixed points of the orbifold [75]. These localized terms are not forbidden because they do not break any symmetry of the theory.

6.3 5D radiative corrections in the model $\lambda\phi^4$

We will consider a toy model with a scalar self-interacting field and we will calculate the one-loop radiative corrections to the mass and the coupling. The scalar action is defined by

$$S = \int d^4x\, dy\, [\frac{1}{2}(\partial_M \phi)^2 - \frac{\lambda}{4!}\phi^4]. \tag{6.5}$$

By naive dimensional analysis the theory has a cutoff $\Lambda \sim 24\pi^3\lambda^{-1}$. Therefore in performing quantum corrections we will cutoff the 4D momentum integral at the scale Λ. The loop contributions that depend on Λ will signal the divergences of the 5D theory.

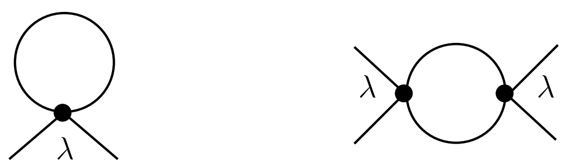


Figure 6.2: *Feynman diagrams for one-loop mass and vertex in the scalar interacting theory.*

In Fig. 6.2 we show the Feynman diagrams that renormalize the two point function and the coupling at one loop. Thus the effective action with one-loop quantum corrections can be written as

$$S_{eff} = S_{cl} + S_2 + S_4 + ... = S_{cl} - \int dy\; m^2(y)\phi^2(y) - \int dydy'\phi^2(y)\lambda(y,y')\phi^2(y') + ... \tag{6.6}$$

where S_{cl} corresponds to the tree level action and S_n correspond the one-loop term with n-fields. We will consider first the case of an extra dimension compactified on a circle. In this case the one loop contribution to the two-point function with zero external momentum is given by

$$m^2_{cir} = \frac{\lambda}{2}\int \frac{d^4p}{(2\pi)^4}(\frac{1}{2p} + \sum_{n\neq 0}\frac{e^{-p2|n|\pi R}}{2p}) = \frac{\lambda}{16\pi^2}(\frac{\Lambda^3}{6} + \frac{1}{(2\pi R)^3}\sum_{n\neq 0}\frac{1}{|n|^3}), \tag{6.7}$$

where we have thrown away terms that cancel when $\Lambda R \to \infty$. This result can be compared with the result in terms of KK modes [76]. As we argued in the previous section, divergences are associated to the zero winding modes and finite terms to $n \neq 0$. If there is a symmetry (like supersymmetry or a gauge symmetry) protecting the mass term from divergent contributions, the finite term is a prediction of the theory.

Next we calculate the term S_4 at one loop. For simplicity, we will take $\phi(y) = \phi_c$ =constant. We expand the propagators in powers of the external momentum and keep the zeroth order terms. Thus S_4 is given by

$$S_4 = -\frac{\lambda^2\phi_c^4}{2}\int dy \int dy' \sum_{n,n'} \int \frac{d^4p}{(2\pi)^4}\widetilde{G}(p, y - y' + 2n\pi R)\widetilde{G}(p, y' - y + 2n'\pi R). \tag{6.8}$$

As there are two propagators involved, we have to sum over two winding indexes. When the topology of the extra space is more complicated (for example when there is more than one extra dimension) it is useful to express this equation in terms of just one propagator as

$$S_4 = -\mathcal{I}\lambda^2\phi_c^4/2, \tag{6.9}$$

where $\mathcal{I}$ is given by [1]

$$\mathcal{I} = -\int dy \sum_n \int \frac{d^4p}{(2\pi)^4}\frac{d}{dp^2}\widetilde{G}(p; y, y + 2n\pi R). \tag{6.10}$$

Integrating over the coordinates and momentum the one-loop contribution to $\lambda(y, y')$ is given by

$$\lambda_{cir} = \frac{\lambda^2}{64\pi^2}(\Lambda - \sum_{n\neq 0}\frac{1}{|n|\pi R}), \qquad \phi = \phi_c. \tag{6.11}$$

As the sum over windings is logarithmically divergent in the IR, we have to introduce an IR cut-off. This means that we sum until a maximum winding number $n_{max} = (2\pi R\mu_{ir})^{-1}$ regulating the long distance behavior. If the field is massive the mass is the natural cut-off and there are no IR divergences. The IR logarithm is similar to the 4D case, this can be easily understood in terms of the KK modes. The zero K-K mode is massless, therefore this mode can propagate long distances inducing the IR divergence.

6.3.1 Radiative corrections on orbifolds

We consider the same scalar theory of Eq. 6.5 with the extra dimension compactified on an orbifold. The one-loop contributions to S_2 on S^1/Z_2 are generated from the following equation (in Ref. [77] the same calculation was performed using the KK description)

$$S_2 = -\frac{\lambda}{2}\int_0^{\pi R} dy\, \phi^2(y)\int \frac{d^4p}{(2\pi)^4}\sum_n(\frac{e^{-p2|n|\pi R}}{2p} \pm \frac{e^{-p2|y+n\pi R|}}{2p}) \equiv S_S \pm S_Z, \tag{6.12}$$

where S_S stands for the same contribution to S_2 as in the circle case, except that $y \in (0, \pi R)$. The second term depends on y and it gives a new contribution. It has divergences for $n = 0, -1$.

[1] To see that Eq. (6.8) and Eq. (6.9) are the same we can write Eq. (6.8) in K-K modes without external momenta as $\sum_{p_5}\int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+p_5^2)^2}$, with p_5 the momentum in the extra dimension. The integrand can be written as $\frac{1}{(2\pi)^4}\frac{-d}{dp^2}(p^2 + p_5^2)^{-1}$, and by Fourier transformation we obtain Eq. (6.9).

To separate the localized divergent contributions from the constant ones we expand $\phi^2(y)$ in powers of y around the fixed points y_{fp}. Expanding ϕ^2 to second order, S_Z is given by

$$S_Z \simeq -\frac{\lambda}{16\pi^2}\sum_{fp}\{\phi^2(y_{fp})[\sum_{n\neq 0,-1}\frac{(1+2n)sgn(n)}{n^2(1+n)^2 32\pi^2 R^2}+\frac{\Lambda^2}{8}-\frac{1}{16\pi^2R^2}]+\partial_y^2\phi^2(y_{fp})\frac{\log(\Lambda R)}{16}\}. \tag{6.13}$$

The quadratic divergences are associated to the zero winding mode when $y\to 0$ and to the winding mode $n=-1$ when $y\to\pi R$, thus these divergences are localized on the fixed points of the orbifold. There is also a logarithmically divergent kinetic term along the direction of the extra dimension.

From Eq. (6.12) we can see that $m^2(y)$ has two contributions. The first one is the same as in the circle, Eq. (6.7), and the second contribution depends on the position in the extra dimension. If we split $m^2(y)$ in a divergent and a finite contributions as $m^2(y) = m^2_{div}(y) + m^2_f(y)$, the divergent contribution is given by

$$m^2_{div\pm}(y) = \frac{\lambda}{16\pi^2}\{\frac{\Lambda^3}{6}\pm\sum_{fp}\delta_{fp}[\frac{\Lambda^2}{8}+\frac{\log(\Lambda R)}{16}\partial_y^2]\}. \tag{6.14}$$

The finite term can be computed for a constant field, and is given by

$$m^2_{f\pm} = \frac{\lambda}{128\pi^5R^3}[\sum_{n\neq 0}\frac{1}{|n|^3}\pm\sum_{n\neq 0,-1}\frac{(1+2n)sgn(n)}{2n^2(1+n)^2}\mp 1], \qquad \phi=\phi_c. \tag{6.15}$$

To obtain the radiative correction to S_4 we expand the fields in powers of y around the fixed points y_{fp}. We just take the zeroth order term in the power series and use Eqs. (6.9) and (6.10) with the propagator of the orbifold space. In this case S_4 is given by

$$S_4 \simeq -\frac{\lambda^2}{16\pi^2}\sum_{fp}\phi^4(y_{fp})[\frac{\Lambda\pi R}{8}+\sum_{n>1}(\frac{1}{4n}\pm\frac{1}{8}\log\frac{n+1}{n-1})\pm\frac{\log(\Lambda R)}{4}]. \tag{6.16}$$

The linear UV divergence is due to the zero winding mode. The logarithmic divergence is localized on the fixed points and is associated to $n=0,-1$, thus it is a 4D divergence. Therefore we can write S_4 as

$$S_4 = -\int dy\, \lambda^{\pm}(y)\,\phi^4(y), \tag{6.17}$$

$$\lambda^{\pm}(y) = \lambda^{\pm}_f(y) + \frac{\lambda^2}{64\pi^2}\sum_{fp}\delta_{fp}[\frac{\Lambda\pi R}{2}\pm\log(\Lambda R)], \tag{6.18}$$

where $\lambda^{\pm}_f(y)$ is a finite coupling. For a constant field $\lambda^{\pm}_f$ is given by

$$\lambda^{\pm}_f(y) = \frac{\lambda^2}{16\pi^2}[\frac{1}{\pi R}\sum_{n>1}(\frac{1}{2n}\pm\frac{1}{4}\log\frac{n+1}{n-1})], \qquad \phi=\phi_c. \tag{6.19}$$

If the field is even there are logarithmic IR divergences, as in the circle, on the other hand if the field is odd there are not IR divergences. This is easier to understand in terms of KK modes: only the even field has a massless mode that can propagate long distances and induce the IR divergence.

In dimensional regularization scheme, the effective scale Λ in the 4D logarithm of the coupling, Eq. (6.18), is replaced by an arbitrary scale μ. Further discussions in this scheme can be found in Ref. [75].

6.3.2 One-loop contribution to the gauge coupling

We consider as an application of the previous formalism, a 5D theory with gauge fields and a scalar charged field, transforming with a representation S of the gauge group. We want to obtain the radiative contributions to gauge coupling due to the interactions with the scalar fields in a orbifold space. Thus we consider the one loop scalar contribution to the vacuum polarization Π_{MN}. This have been computed with KK modes in Ref. [78], here we will obtain the same result with winding modes.

As we want to obtain the effective 4D theory, we consider constant fields in the bulk (zero KK modes) and integrate over the extra dimension. We define g_0 as the effective 4D coupling of an abelian theory at the scale Λ. Thus, after some manipulations, we can write the main contribution to the one-loop vacuum polarization as

$$\Pi(k^2=0) = \frac{g_0^2}{3}\sum_{q_5}\int \frac{d^4q}{(2\pi)^4}\frac{1}{(q_4^2+q_5^2)^2} = -\frac{g_0^2}{3}\int dy \int \frac{d^4p}{(2\pi)^4}\frac{d}{dp^2}\widetilde{G}^{orb}_{\pm}(p;y,y). \tag{6.20}$$

As there two scalar propagators with vanishing external momentum, the momentum integral is the same as in the one loop contribution to the self-coupling λ . On the right hand side of Eq. (6.20) we have written the vacuum polarization in terms of one propagator, as we did for the scalar coupling. Thus, using the results obtained in the previous section, the gauge coupling of the effective 4D theory is given by

$$g^{-2} = g_0^{-2}[1 + g_0^2\beta_0\log(\mu_{ir}R) - g_0^2\beta_1\log(\Lambda R)], \tag{6.21}$$

where we only show the logarithmic contribution and

$$\frac{\beta_0}{2} = \beta_1 = \frac{1}{48\pi^2}. \tag{6.22}$$

In Eq. (6.21), as in Eq. (6.18) for the one-loop scalar coupling, the $\log(\Lambda R)$ is due to localized contributions.

If the gauge group is non-abelian, we only have to modify the charges and multiply by $t(S)$, where $tr[T_a(S)T_b(S)] = t(S)\delta_{ab}$. Eq. (6.21) gives the scalar contribution to the 4D effective coupling at one loop, for a theory with a cut-off scale Λ. We can consider a theory with a different cut-off Λ', and one-loop coupling g'. Then the relation between the couplings g and g' is given by

$$g' = g[1 + g_0^2\beta_1 \log\frac{\Lambda'}{\Lambda}]. \tag{6.23}$$

6.4 Radion stabilization by two loop effects

As an application of the winding formalism, we compute in this section the leading two loop contributions to the effective potential for the radion in a product space $\mathcal{R}^4 \times S^1/Z_2$. We will see that under certain symmetry assumptions we can obtain a Coleman-Weinberg potential that can stabilize a large extra dimension

6.4.1 Scalar potential

Let us consider a scalar 5D theory as the one described in section 6.3.1, with the extra dimension compactified in an orbifold (in Refs. [79] the authors considered more realistic set-ups). Since the effective potential for the radion vanishes at tree level, we have to compute radiative corrections to obtain a sensible effective potential (see Fig. 6.3) [2]. To calculate the quantum corrections we will use the winding formalism. Let us start with the one-loop term correction to the radion potential. In terms of KK modes the one loop effective potential is given by

$$V^{(1)} = \sum_{k_5}\int\frac{d^4k}{(2\pi)^4}\log(k^2+k_5^2)\,. \tag{6.24}$$

To transform this potential to the winding representation we can write the last equation as

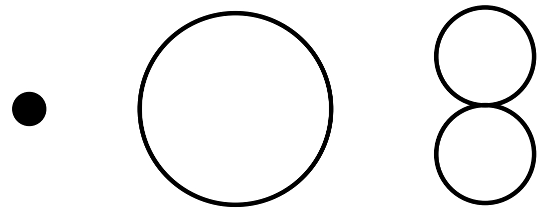
$$V^{(1)} = \sum_{k_5}\int\frac{d^4k}{(2\pi)^4}\int dk^2\frac{1}{(k^2+k_5^2)}. \tag{6.25}$$

The last factor is the scalar propagator expanded in KK modes, thus we can replace it by the one with winding modes and integrate over momentum and coordinate space:

$$\begin{aligned} V^{(1)} &= \sum_n\int\frac{d^4k}{(2\pi)^4}\int dk^2\int_0^{\pi R}dy\,[\widetilde{G}(k,2n\pi R)\pm\widetilde{G}(k,2y+2n\pi R)] \\ &= \frac{1}{8\pi^2}[\frac{\Lambda^5}{5}\pi R - \frac{3\zeta(5)}{8\pi^4R^4}\mp\frac{\Lambda^4}{16}], \end{aligned} \tag{6.26}$$

where $\widetilde{G}$ is defined in Eq. (6.2) and ζ is the Riemann zeta function ($\zeta(\alpha)=\sum_{n>0}1/n^\alpha$). The finite term corresponds to non zero windings and the terms $\mathcal{O}(\Lambda^4)$ and $\mathcal{O}(\Lambda^5)$ correspond to localized and bulk divergences respectively.

[2]If the scalar vev $\langle\phi\rangle \neq 0$, we also have to include a two loop diagram, with two three-point vertices, each of them proportional to the vev [80].



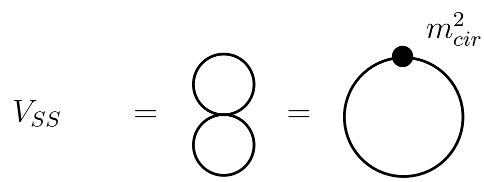
(a) (b) (c)

Figure 6.3: *Perturbative expansion of the effective potential for the radion. The first diagram (a) is the tree level contribution, it cancels out in the present toy model.*

Next we calculate the two loop term $V^{(2)}$, shown in (c) of Fig. 6.3. It is given by

$$V^{(2)} = \frac{\lambda}{2}\sum_{n,n'}\int_0^{\pi R} dy \int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}[\widetilde{G}(k,2n\pi R) \pm \widetilde{G}(k,2y+2n\pi R)] \\ \times[\widetilde{G}(q,2n'\pi R) \pm \widetilde{G}(q,2y+2n'\pi R)]. \tag{6.27}$$

Once more zero winding modes give divergent contributions and non zero modes give the finite corrections. If we call $\widetilde{G}_S$ the first term of the orbifold propagator (equal to the circle propagator) and $\widetilde{G}_Z$ the second term of the orbifold propagator (obtained with Z_2 identification), that is $\widetilde{G}^{orb}_{\pm} = \widetilde{G}_S \pm \widetilde{G}_Z$, then we can write equation (6.27) as $V^{(2)} = V_{SS} \pm 2V_{SZ} + V_{ZZ}$. We consider first the term V_{SS}, every loop is similar to the one loop contribution to the mass. Thus V_{SS} of Eq. (6.27) can be interpreted as one loop with a massless propagator $\widetilde{G}_S$ and a mass insertion $m^2_{cir} \propto \lambda[\Lambda^3/6 + \zeta(3)/(8\pi^3R^3)]$, as is shown in the following Feynman diagram



However, since the mass m_{cir} is of order Λ, it can not be considered a perturbation. Therefore we have to consider terms with arbitrary number of mass insertions, as is shown in Fig. 6.4. If we sum the series of Fig. 6.4, we obtain a loop with a massive propagator and a mass vertex. As we discussed in section 6.2, a massive propagator is obtained replacing $p \rightarrow \sqrt{p^2 + m^2_{cir}}$. If the mass is large, $m_{cir} \gg R^{-1}$ (as is the case because $m_{cir} \sim \Lambda \gg R^{-1}$), we can approximate the propagator by $e^{-m_{cir}|y+2n\pi R|}/m_{cir}$. In this case the propagator is exponentially damped and cancels before making windings. Then the only relevant contributions are divergent, and $V_{SS} \simeq 2\lambda\Lambda^6\pi R/(96\pi^2)^2$. If there is a symmetry that protects the masses form divergent contributions, as a local gauge symmetry, then the finite contributions are given by $V_{SS} \simeq 2\lambda\zeta(3)^2/(128^2\pi^9R^5)$.

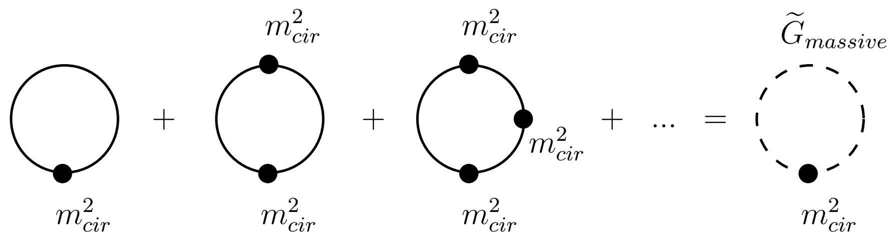


Figure 6.4: *Feynman diagrams giving the one loop effective potential with mass insertions. The mass m_{cir} itself is one loop, it is given by the one loop contribution to the mass. The continuous lines are for massless propagators and the dashed line is for the massive one.*

We can make a similar analysis for the other topologies obtaining similar results. Summing over topologies we get

$$V^{(2)} = \frac{\lambda}{64\pi^4}[A\ (\pi R)\Lambda^6 + B\Lambda^5 + C(\pi R)^{-5}], \tag{6.28}$$

where $A \sim 10^{-1}, B \sim 10^{-1}, C \sim 10^{-2}$, and the sign of B depends on field parity under Z_2. If there is no symmetry protecting V from divergences, the divergent terms are dominant, in the other case we only get the R^{-5} finite term.

6.4.2 Effect of brane kinetic terms

We want to obtain a potential able to stabilize a large extra dimension. Therefore we consider new interactions localized on the boundaries: we add to the previous set-up fermion fields localized on the fixed points of the extra dimension. If these 4D fields couple to the bulk ones, there are new contributions to the two loop effective potential. At two loops there is a new term with a fermionic-loop localized on the branes and a scalar-loop on the bulk, as is shown in Fig. 6.5.

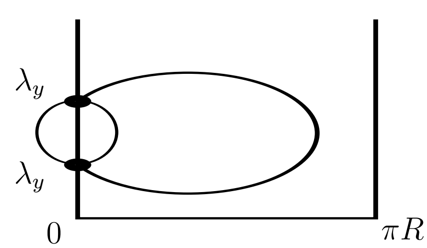


Figure 6.5: *Feynman diagram with one localized fermionic loop and one bulk scalar loop, contributing to the effective potential.*

We will consider the following interaction between the bulk and boundary fields: $\mathcal{L}_{int} = \delta(y)\lambda_y\phi\bar{\psi}\psi$ and we will calculate the two loop contribution due to this new vertex. The 4D fermionic loop has several terms given by

$$\frac{\lambda_y^2}{8\pi^2}[-2\Lambda^2 + \frac{5}{3}q^2 - q^2 \log \frac{q^2}{\Lambda^2}]. \tag{6.29}$$

We are interested in the logarithmic kinetic contributions. The first and second terms are local, but the third is not local and will lead to new interesting effects. The term of the scalar effective action that couples to the brane loop is

$$\frac{\lambda_y^2}{8\pi^2}\int_0^{\pi R} dy \int_0^{\pi R} dy' \int \frac{d^4p}{(2\pi)^4}\delta(y)\delta(y')\phi(y)p^2 \log(\frac{p^2}{\Lambda^2})\phi(y'). \tag{6.30}$$

Thus the two loop contribution to the radion potential with a loop localized on one of the branes is given by

$$\begin{aligned} V_b^{(2)} &= \frac{\lambda_y^2}{8\pi^2}\sum_n \int \frac{d^4p}{(2\pi)^4}\widetilde{G}(p; 2n\pi R)p^2 \log(\frac{p^2}{\Lambda^2}) \\ &= \frac{\lambda_y^2}{64\pi^4}[\frac{\Lambda^5}{25} + \sum_{n\neq 0}\frac{50 - 24\gamma - 24\log(2n\pi R\Lambda)}{(2n\pi R)^5}], \end{aligned} \tag{6.31}$$

where as usual the divergent term is due to the zero winding mode contribution.

To obtain the potential at two loops we have to add $V^{(1)} + V^{(2)} + V_b^{(2)}$. Let us suppose that for each boson there is a fermion with equal boundary conditions. Then the finite part of $V^{(1)}$ cancels out because the fermionic loops have a minus sign compared with the bosonic ones. Furthermore, if there is a symmetry protecting the effective potential from divergent terms, then only the finite terms in $V^{(2)}$ and $V_b^{(2)}$ remain, and the two-loop effective potential is given by

$$V \sim \frac{1}{R^5}[D - \log(\Lambda R)], \qquad \text{where} D \sim 1. \tag{6.32}$$

This is a Coleman-Weinberg potential [81] that can stabilize a large extra dimension, $R \gg \Lambda^{-1}$, with 5D parameters of the same order. When we calculated the radiative corrections in the previous sections, we assumed the extra dimension to be large compared with the inverse cut-off. Thus stabilizing a large extra dimension we obtain a consistent scenario.

6.5 Conclusions and discussions

We have used the winding formalism to compute radiative corrections on a theory with extra dimensions. It allows us to separate, in a very clear and intuitive way, cut-off dependent from finite corrections. Radiative corrections also generate masses, kinetic terms and couplings localized on the boundaries. We also obtained the logarithmic contribution to the 4D effective gauge coupling.

We analyzed the possibility of getting a potential stabilizing the size of the extra dimension with a toy model. We saw that it is very simple to compute the two loop effective potential with winding modes. We also showed a scenario with brane terms and bulk fields where the extra volume can be stabilized with a Coleman-Weinberg potential. This mechanism can be extended to more realistic models.

In the previous chapters we argue that higher dimensional theories can mimic strongly coupled 4D theories with a large number of colors. In chapter 4 we proposed a 5D model in warped space to study the chiral breaking of QCD. All the calculations were given at tree level, that corresponds in the 4D theory to the leading order in $1/N$. Since in QCD $N = 3$ we expect corrections of order 30% in the expansion in powers of $1/N$. In the 5D model this corrections will correspond to the expansion in powers of the 5D coupling g_5. There are interesting cases where one can compute the loop modifications to the tree level predictions. In particular we will consider the holographic model of chiral symmetry breaking. First, we consider the decay $a_1 \to \pi\gamma$. In our model with only dimension 4 operators this decay vanishes. We know from experimental results, that this decay ratio, although being small, is different from zero. Therefore it would be very interesting to compute this interaction at one loop level. Another tree level result is the relation $m_\rho^2 = 3g_{\rho\pi\pi}^2 F_\pi^2$, that is a consequence of the 5D gauge invariance, and is independent of the details of the metric. Thus this prediction is robust, but it differs by 2/3 from the KSRF relation $m_\rho^2 = 2g_{\rho\pi\pi}^2 F_\pi^2$, that is consistent with the experimental results. It would be important to calculate the one loop correction to this relation, to see whether it decreases the factor 3 or not.

There are also interesting radiative calculations concerning the 5D model of EW symmetry breaking that we presented in chapter 5. In particular, one of the most constraining observables in this kind of models is the S parameter. Thus it would be interesting to compute the one loop correction to the operator responsible for S parameter. It is important to know if these contributions make the S parameter larger or not.

Chapter 7

Conclusions

In this thesis we have used extra dimensions to study symmetry breaking in particle physics. In particular we have presented two different models with extra dimensions to study symmetry breaking in the SM. We have shown that there is a symmetry able to protect the $Zb\bar{b}$ interaction that is very useful to build models of EW symmetry breaking. We have also shown a method to calculate radiative corrections in theories with extra dimensions.

First, we have investigated the chiral symmetry breaking of QCD. We presented a 5D model describing different properties of QCD related to the chiral symmetry breaking. In particular it can describe the vector, the axial-vector, the scalar and pseudoscalar sectors of mesons. The model has 5 parameters, three are related to the parameters of QCD (the number of colors N_c, the mass gap Λ_{QCD} and the quark mass M_q) and the other two parameters are free. One of them is related to the quark condensate that spontaneously breaks the chiral symmetry, and the other only affects the scalar sector of the theory, and is thus fixed by the mass of the first scalar resonance. Similar to large N_c QCD, the model has infinite towers of resonances that are weakly coupled, therefore we are able to do perturbative calculations and make predictions. The masses, decay constants and couplings of the resonances are in very good agreement with the experimental values for QCD. By computing the correlators in the Euclidean we can also make predictions for the condensates and for the low energy constants, that in general are in good agreement with their experimental values.

The 5D gauge invariance of the model determines some very interesting sum rules involving the couplings, masses and decay constants. In particular the model predicts vector meson dominance for the pion interactions. It also predicts a vanishing branching ratio $a_1 \to \pi\gamma$ and a modified KSFR relation $m_\rho^2 = 3g_{\rho\pi\pi}^2 F_\pi^2$. As we discussed in the previous chapter, it would very interesting to calculate the radiative corrections to this relations.

We have also calculated the predictions for the low energy effective lagrangian of QCD, the chiral lagrangian for the pions, as well as the quark masses. All the results are in good agreement with the experimental data.

We have checked that the predictions are robust under modifications of the 5D metric in the IR. By including power corrections to the AdS metric it is possible to account for for other condensates of QCD. We expect this corrections to appear if we include the backreaction of the metric due to the vacuum expectation value of the scalar fields. Another approach would be to consider higher dimensional operators. We have checked that these higher dimensional operators play the role of z dependent masses or can be absorbed as corrections to the metric. We point out that in this process we end up with different effective metrics, depending on the fields and on the operators that we include. Thus higher dimensional operators will give contributions to the operator product expansion (OPE) of the different correlators. One has to check that the coefficients of the OPE match with the QCD prediction and that the spectrum reproduce the experimental results. As there are many operators that one can include, it would be nice to obtain a relation between them from the DBI action. We are working on this subject.

This analysis can be extended to include the effects of m_s, to compute three- and four-point functions, and to study other sectors of QCD. We are also working on this subject.

In the second part of this thesis we have investigated the EW symmetry breaking. We have presented a 5D model that describes the dynamical breakdown of the EW symmetry by a strong sector. This kind of models open up the possibility of studying the dynamical breaking of the EW symmetry at a quantitative level and they should be taken as a real alternative to go beyond the SM, at the same level as supersymmetry. Our model consists in the SM fermions and gauge bosons plus extra dimensions that play the role of a strong sector. The sector beyond the SM contains a massless composite Higgs arising from a spontaneous breakdown of a global symmetry. The remaining global symmetry is broken by the gauge sector of the SM and the Higgs acquires a finite mass by radiative corrections. The fermions are coupled linearly to the strong sector and can trigger EW symmetry breaking. The model is fully realistic and pass the EW precision tests in a large region of the parameter space. It predicts a Higgs mass of order $\sim 116-250$ GeV, and the lightest resonance corresponding to a fermionic particle is of order $\sim 0.6-1.5$ TeV. The lightest vector resonances of the model are heavier than the fermionic ones, and in order to have a large top mass the top sector must be almost composite. This are predictions of the model that can be tested at LHC in the next years.

We have also shown that there is a symmetry that can protect the extra contributions to the vertex $Zb_L\bar{b}_L$. This symmetry is a subgroup of the custodial group that protects the T parameter from radiative corrections, plus a discrete parity defined as the interchange $L \leftrightarrow R$. This allows to protect the decay $Z \to b\bar{b}$, that are strongly constrained by experiments. This symmetry can also protect $Z\psi\bar{\psi}$, with ψ any fermion of the SM. However it is not possible to protect several fermions at the same time. As the top couplings are poorly known, it is enough obtaining a cancellation of the contributions to the bottom vertex. Thus we expect corrections in the interactions $Zt\bar{t}$ and $Wt\bar{b}$.

Finally we have shown a method to compute radiative corrections in theories with extra dimensions in terms of winding modes. We considered several toy models and calculated the one-loop contributions to the masses and couplings. We also found a two loop potential able to stabilize the size of the extra dimension. This mechanism can be generalized to more realistic theories.

The methods of the last chapter can be used to calculate the radiative corrections of either the higher dimensional model of QCD or the higher dimensional model of EW symmetry breaking. We have suggested several sectors where it would be interesting to compute the radiative corrections.

Appendix A

Chiral symmetry breaking induced by an operator of dimension d

In this appendix we give the expression for Π_A in the different limits studied in the text for the case in which the breaking of the chiral symmetry arises from a VEV of a scalar Φ with an arbitrary 5D mass M_Φ. This corresponds in the CFT to turning on an operator of dimension $d = \sqrt{4 + M_\Phi^2 L^2} + 2$.

For small momentum we have $\Pi_A(p) = \Pi_A^{(0)}(0) + M_q L_1 \Pi_A^{(1)}(0) + p^2 \Pi_A'(0) + \cdots$ where in the limit $\xi \gg 1$:

$$
\begin{aligned}
\Pi_A^{(0)}(0) &\simeq \frac{2^{(1-1/d)} d^{(1-2/d)} \pi}{\sin(\pi/d)\,\Gamma(\frac{1}{d})^2} \frac{\tilde{N}_c \, \xi^{2/d}}{L_1^2} \,, && \text{(A.1)}\\
\Pi_A^{(1)}(0) &\simeq \frac{2^{(1-1/d)} \Gamma(\frac{2+d}{2d}) \Gamma(\frac{3}{d})}{d^{(1-2/d)} \Gamma(\frac{4+d}{2d})} \frac{\tilde{N}_c \, \xi^{1-2/d}}{L_1^2} \left(1 - \frac{d^{2/d} \Gamma(\frac{4+d}{2d}) \Gamma(\frac{4}{d})}{2^{1/d} \Gamma(\frac{6+d}{2d}) \Gamma(\frac{1}{d})} \frac{1}{\xi^{2/d}} \right) , && \text{(A.2)}\\
\Pi_A'(0) &\simeq -\tilde{N}_c \ln \frac{L_0}{L_1} - \tilde{N}_c \left[\ln \xi^{1/d} + \frac{\gamma + \psi\left(\frac{2+d}{2d}\right) - \psi\left(\frac{2}{d}\right) - \psi\left(\frac{1}{d}\right) - \ln \frac{d^2}{2}}{2d} \right] , && \text{(A.3)}
\end{aligned}
$$

where $\psi(x) = \Gamma'(x)/\Gamma(x)$.

In the large momentum limit we have

$$
\Pi_A(p^2) = -p^2 \left[\frac{\tilde{N}_c}{2} \ln(p^2 L_0^2) + \frac{c_{2d}}{p^{2d}} + \cdots \right] , \tag{A.4}
$$

where

$$
c_{2d} = -\frac{d\,\sqrt{\pi}}{2(d-1)} \frac{\Gamma(d)^3}{\Gamma(d+\frac{1}{2})} \frac{\tilde{N}_c \xi^2}{L_1^{2d}} \,. \tag{A.5}
$$

From the above expressions we can derive L_5 and L_{10}:

$$
L_5 \simeq \tilde{N}_c \frac{2^{(-2-2/d)} \pi \Gamma(\frac{2+d}{2d}) \Gamma(\frac{3}{d})}{\sin(\pi/d)\,\Gamma(\frac{4+d}{2d}) \Gamma(\frac{1}{d})^2} \left(1 - \frac{d^{2/d} \Gamma(\frac{4+d}{2d}) \Gamma(\frac{4}{d})}{2^{1/d} \Gamma(\frac{6+d}{2d}) \Gamma(\frac{1}{d})} \frac{1}{\xi^{2/d}} \right) , \tag{A.6}
$$

$$L_{10} \simeq -\frac{\tilde{N}_c}{4}\left[\ln \xi^{1/d} + \frac{\gamma + \psi\left(\frac{2+d}{2d}\right) - \psi\left(\frac{2}{d}\right) - \psi\left(\frac{1}{d}\right) - \ln \frac{d^2}{2}}{2d}\right]. \tag{A.7}$$

Appendix B

Fermionic propagator and self-energies in AdS$_5$

In this appendix we will derive the 5D propagator for a fermionic field. We consider first the 5D scalar propagator G^ϕ in AdS$_5$ space with boundary masses. From Eq. (3.3) we can obtain the equation for the propagator

$$\left[z^2\partial_z^2 + z\partial_z - \left(-p^2z^2 + \beta^2\right)\right]\hat{G}^\phi(z,z';p) = -\frac{z}{k}\delta(z-z'). \tag{B.1}$$

where $\beta = \sqrt{4 + m_\phi^2/k^2}$, and $\hat{G}^\phi$ is the propagator for the re-scaled field $\hat{\Phi} = \Phi/(kz)^2$. The relation between $\hat{G}^\phi$ and G^ϕ is $G^\phi = (kz)^2(kz')^2\hat{G}^\phi$.

We Fourier transform to Euclidean momentum space in the x^μ directions, and imposing modified Neumann boundary conditions, the scalar propagator is

$$\begin{aligned}\hat{G}(z,z';p) =& \frac{-L_0}{(X_I/X_K - Z_I/Z_K)} \\ &\times \left(I_\beta(|p|z_<) - \frac{Z_I}{Z_K}K_\beta(|p|z_<)\right)\left(I_\beta(|p|z_>) - \frac{X_I}{X_K}K_\beta(|p|z_>)\right),\end{aligned} \tag{B.2}$$

where $z_<$ ($z_>$) is the smaller (grater) of z and z', and I_β, K_β are modified Bessel functions. The coefficients X_I, X_K, Z_I and Z_K are obtained by imposing the boundary conditions

$$\begin{aligned}X_I &= |p|L_1 I_{\alpha-1}(|p|L_1) - \left(\alpha - s/2 - z_1|p|^2L_1^2L_0^{-1} - m_1L_0\right)I_\alpha(|p|L_1), \\ X_K &= -|p|L_1K_{\alpha-1}(|p|L_1) - \left(\alpha - s/2 - z_1|p|^2L_1^2L_0^{-1} - m_1L_0\right)K_\alpha(|p|L_1),\end{aligned} \tag{B.3}$$

$$\begin{aligned}Z_I &= |p|L_0I_{\alpha-1}(|p|L_0) - \left(\alpha - s/2 + z_0|p|^2L_0 + m_0L_0\right)I_\alpha(|p|L_0), \\ Z_K &= -|p|L_0K_{\alpha-1}(|p|L_0) - \left(\alpha - s/2 + z_0|p|^2L_0 + m_0L_0\right)K_\alpha(|p|L_0),\end{aligned} \tag{B.4}$$

The 5D fermion propagator between the points z, z' in the extra dimension is given by (see for example [69]):

$$S(p,z,z') = (k^2 z z')^{5/2}\left[\not{p} + \gamma^5\left(\partial_z + \frac{1}{2z}\right) + \frac{m_\psi}{kz}\right]\left[P_R\, G_R(p,z,z') + P_L\, G_L(p,z,z')\right] , \qquad \text{(B.5)}$$

where $P_{R,L} = (1 \pm \gamma^5)/2$ and G_R is given by the scalar propagator with $\beta = |m_\psi/k + 1/2|$, $m_{UV} = -m_{IR} = -m_\psi$ and $L_0 = L_1 = 0$. G_L is obtained from G_R by changing $m_\psi \to m_\psi$.

We calculate the self-energies $\Pi_{q,u,d}(p)$ and $M_{u,d}(p)$ in terms of the 5D propagators. These correlators are given by the inverse of the propagator with $z = z' = L_0$. Adding IR masses $\tilde{m}_u, \tilde{M}_u$, the different fermionic fields can mix. Resumming the perturbative series of IR mass insertions one obtains [10]

$$\begin{aligned}\Pi_{(2,2)^q_L}(p) =& \frac{k}{p^2}\,\frac{1}{G^{(++)}_{Rq}(L_0,L_0)}\left\{1 - \frac{G^{(++)}_{Rq}(L_0,L_1)G^{(++)}_{Rq}(L_1,L_0)}{G^{(++)}_{Rq}(L_0,L_0)}\times\right.\\ &\left.\times\,\frac{\tilde{m}_u^2\, p^2\,(kL_1)^2\, G^{(-+)}_{L\,q^u}(L_1,L_1)}{1-\tilde{m}_u^2\, p^2\,(kL_1)^2\, G^{(-+)}_{Rq}(L_1,L_1)G^{(-+)}_{L\,q^u}(L_1,L_1)}\right\},\end{aligned} \qquad \text{(B.6)}$$

$$\begin{aligned}\Pi_{(3,1)^q_L}(p) =& \frac{k}{p^2}\,\frac{1}{G^{(+-)}_{RQ}(L_0,L_0)}\left\{1 - \frac{\tilde{G}^{(-+)}_{LQ}(L_0,L_1)\tilde{G}^{(+-)}_{RQ}(L_1,L_0)}{G^{(+-)}_{RQ}(L_0,L_0)}\times\right.\\ &\left.\times\,\frac{\widetilde{M}_u^2\,(kL_1)^2\, G^{(++)}_{RQ^u}(L_1,L_1)}{1-\widetilde{M}_u^2\, p^2\,(kL_1)^2\, G^{(++)}_{LQ}(L_1,L_1)G^{(++)}_{RQ^u}(L_1,L_1)}\right\},\end{aligned} \qquad \text{(B.7)}$$

$$\begin{aligned}\Pi_{(2,2)^u_R}(p) =& \frac{k}{p^2}\,\frac{1}{G^{(++)}_{L\,q^u}(L_0,L_0)}\left\{1 - \frac{G^{(++)}_{L\,q^u}(L_0,L_1)G^{(++)}_{L\,q^u}(L_1,L_0)}{G^{(++)}_{L\,q^u}(L_0,L_0)}\times\right.\\ &\left.\times\,\frac{\tilde{m}_u^2\, p^2\,(kL_1)^2\, G^{(-+)}_{Rq}(L_1,L_1)}{1-\tilde{m}_u^2\, p^2\,(kL_1)^2\, G^{(-+)}_{L\,q^u}(L_1,L_1)G^{(-+)}_{Rq}(L_1,L_1)}\right\},\end{aligned} \qquad \text{(B.8)}$$

$$
\begin{aligned}
\Pi_{(3,1)^u_R}(p) =& \frac{k}{p^2}\,\frac{1}{G^{(+-)}_{L\,Q^u}(L_0,L_0)}\Bigg\{1-\frac{\tilde G^{(-+)}_{R\,Q^u}(L_0,L_1)\tilde G^{(+-)}_{L\,Q^u}(L_1,L_0)}{G^{(+-)}_{L\,Q^u}(L_0,L_0)}\times \\
&\qquad\times\;\frac{\widetilde{M}^2_u\,(kL_1)^2\,G^{(++)}_{L\,Q}(L_1,L_1)}{1-\widetilde{M}^2_u\,p^2\,(kL_1)^2\,G^{(++)}_{R\,Q^u}(L_1,L_1)G^{(++)}_{L\,Q}(L_1,L_1)}\Bigg\}\,,
\end{aligned} \tag{B.9}
$$

$$
\begin{aligned}
M_{(2,2)^u}(p) =& -\tilde m_u k^2 L_1\,\frac{G^{(++)}_{L\,q^u}(L_0,L_1)G^{(++)}_{R\,q}(L_1,L_0)}{G^{(++)}_{L\,q^u}(L_0,L_0)G^{(++)}_{R\,q}(L_0,L_0)}\times \\
&\qquad\times\;\frac{1}{1-\tilde m^2_u\,p^2\,(kL_1)^2\,G^{(-+)}_{R\,q}(L_1,L_1)G^{(-+)}_{L\,q^u}(L_1,L_1)}\,,
\end{aligned} \tag{B.10}
$$

$$
\begin{aligned}
M_{(3,1)^u}(p) =& -\frac{\widetilde{M}_u k^2 L_1}{p^2}\,\frac{\tilde G^{(-+)}_{R\,Q^u}(L_0,L_1)\tilde G^{(+-)}_{R\,Q}(L_1,L_0)}{G^{(+-)}_{L\,Q^u}(L_0,L_0)G^{(+-)}_{R\,Q}(L_0,L_0)}\times \\
&\qquad\times\;\frac{1}{1-\widetilde{M}^2_u\,p^2\,(kL_1)^2\,G^{(++)}_{L\,Q}(L_1,L_1)G^{(++)}_{R\,Q^u}(L_1,L_1)}\,.
\end{aligned} \tag{B.11}
$$

where the functions $\tilde G_{R,L}(z,z')$ are defined by

$$
\tilde G_{R,L}(z,z') = \left[\pm\partial_z + \frac{(c\pm 1/2)}{z}\right] G_{R,L}(z,z')\,. \tag{B.12}
$$

Bibliography

[1] S. Weinberg, "The Quantum theory of fields. Vol. 1: Foundations, Vol. 2: Modern applications,"

[2] J. Scherk and J. H. Schwarz, Nucl. Phys. B **153** (1979) 61.

[3] Y. Hosotani, Phys. Lett. B **126** (1983) 309.

[4] M. Quiros, arXiv:hep-ph/0302189.

[5] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231;

[6] A. Zaffaroni, PoS **RTN2005** (2005) 005.

[7] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428** (1998) 105;
E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253.

[8] S. Weinberg, Phys. Rev. D **13** (1976) 974; Phys. Rev. D **19** (1979) 1277;
L. Susskind, Phys. Rev. D **20** (1979) 2619.

[9] D. B. Kaplan and H. Georgi, Phys. Lett. B **136**, 183 (1984); B **136**, 187 (1984);
H. Georgi, D. B. Kaplan and P. Galison, Phys. Lett. B **143**, 152 (1984);
H. Georgi and D. B. Kaplan, Phys. Lett. B **145**, 216 (1984);
M. J. Dugan, H. Georgi and D. B. Kaplan, Nucl. Phys. B **254**, 299 (1985).

[10] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B **719** (2005) 165 [arXiv:hep-ph/0412089].

[11] L. Da Rold and A. Pomarol, Nucl. Phys. B **721** (2005) 79;

[12] L. Da Rold and A. Pomarol, JHEP **0601** (2006) 157 [arXiv:hep-ph/0510268].

[13] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **95** (2005) 261602.

[14] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, in preparation.

[15] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, arXiv:hep-ph/0605341.

[16] L. Da Rold, Phys. Rev. D **69** (2004) 105015 [arXiv:hep-th/0311063].

[17] R. F. Dashen, Phys. Rev. **183** (1969) 1245.

[18] G. 't Hooft, Nucl. Phys. B **72** (1974) 461; Nucl. Phys. B **75** (1974) 461.

[19] A. Manohar and H. Georgi, Nucl. Phys. B **234** (1984) 189.

[20] H. Georgi and L. Randall, Nucl. Phys. B **276** (1986) 241.

[21] Z. Chacko, M. A. Luty and E. Ponton, JHEP **0007** (2000) 036 [arXiv:hep-ph/9909248].

[22] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [arXiv:hep-ph/9905221]; Phys. Rev. Lett. **83** (1999) 4690 [arXiv:hep-th/9906064].

[23] T. Gherghetta, arXiv:hep-ph/0601213.

[24] A. Pomarol, *Prepared for 28th Johns Hopkins Workshop on Current Problems in Particle Theory: Hyperspace, Superspace, Theory Space and Outer Space, Baltimore, Maryland, 5-8 Jun 2004*

[25] E. Witten, Nucl. Phys. B **160** (1979) 57.

[26] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586** (2000) 141 [arXiv:hep-ph/0003129].

[27] P. Breitenlohner and D. Z. Freedman, Phys. Lett. B **115** (1982) 197.

[28] R. Contino and A. Pomarol, JHEP **0411** (2004) 058 [arXiv:hep-th/0406257].

[29] C. Csaki, J. Hubisz and P. Meade, arXiv:hep-ph/0510275.

[30] C. Csaki, H. Ooguri, Y. Oz and J. Terning, JHEP **9901** (1999) 017 [arXiv:hep-th/9806021];
J. G. Russo, Nucl. Phys. B **543** (1999) 183 [arXiv:hep-th/9808117];
C. Csaki, Y. Oz, J. Russo and J. Terning, Phys. Rev. D **59** (1999) 065012 [arXiv:hep-th/9810186];
C. Csaki, J. Russo, K. Sfetsos and J. Terning, Phys. Rev. D **60** (1999) 044001 [arXiv:hep-th/9902067].

[31] A. Karch and E. Katz, JHEP **0206** (2002) 043.

[32] For certain realizations of string theories describing flavor see:
M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, JHEP **0307**, 049 (2003);
T. Sakai and J. Sonnenschein, JHEP **0309**, 047 (2003);
C. Nunez, A. Paredes and A. V. Ramallo, JHEP **0312** (2003) 024;
J. Babington, J. Erdmenger, N. J. Evans, Z. Guralnik and I. Kirsch, Phys. Rev. D **69** (2004) 066007;
N. J. Evans and J. P. Shock, Phys. Rev. D **70** (2004) 046002;
K. Ghoroku and M. Yahiro, Phys. Lett. B **604**, 235 (2004);
K. Suzuki, arXiv:hep-th/0411076;
S. Kuperstein, JHEP **0503** (2005) 014;
D. Bak and H. U. Yee, Phys. Rev. D **71**, 046003 (2005);
A. Paredes and P. Talavera, Nucl. Phys. B **713**, 438 (2005);
I. Kirsch and D. Vaman, Phys. Rev. D **72** (2005) 026007;
R. Apreda, J. Erdmenger and N. Evans, arXiv:hep-th/0509219.

[33] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. **88** (2002) 031601; JHEP **0305**, 012 (2003);
H. Boschi-Filho and N. R. F. Braga, Phys. Lett. B **560** (2003) 232.

[34] A. Karch, E. Katz and N. Weiner, Phys. Rev. Lett. **90**, 091601 (2003);

[35] H. Boschi-Filho and N. R. F. Braga, JHEP **0305** (2003) 009

[36] D. T. Son and M. A. Stephanov, Phys. Rev. D **69** (2004) 065020.

[37] S. Hong, S. Yoon and M. J. Strassler, JHEP **0404** (2004) 046, arXiv:hep-th/0409118 and arXiv:hep-ph/0501197;
G. F. de Teramond, C. R. U. and S. J. Brodsky, arXiv:hep-th/0501022.

[38] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113** (2005) 843; arXiv:hep-th/0507073.

[39] D. T. Son and M. A. Stephanov, Phys. Rev. D **69** (2004) 065020.

[40] S. Hong, S. Yoon and M. J. Strassler, JHEP **0404** (2004) 046; arXiv:hep-th/0409118; arXiv:hep-ph/0501197.

[41] S. J. Brodsky and G. F. de Teramond, Phys. Lett. B **582** (2004) 211; Phys. Rev. Lett. **94**, 201601 (2005); arXiv:hep-ph/0509269.

[42] J. Hirn and V. Sanz, arXiv:hep-ph/0507049.

[43] N. Evans, J. P. Shock and T. Waterson, Phys. Lett. B **622** (2005) 165.

[44] E. Katz, A. Lewandowski and M. D. Schwartz, arXiv:hep-ph/0510388.

[45] J. Hirn, N. Rius and V. Sanz, Phys. Rev. D **73**, 085005 (2006) [arXiv:hep-ph/0512240].

[46] J. Erlich, G. D. Kribs and I. Low, Phys. Rev. D **73** (2006) 096001 [arXiv:hep-th/0602110].

[47] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, arXiv:hep-ph/0602229.

[48] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147** (1979) 385; Nucl. Phys. B **147** (1979) 448.

[49] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP **0108** (2001) 017.

[50] M. Jamin, Phys. Lett. B **538** (2002) 71.

[51] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. B **223** (1989) 425.

[52] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321** (1989) 311.

[53] K. Kawarabayashi and M. Suzuki, Phys. Rev. Lett. **16** (1966) 255;
Riazuddin and Fayyazuddin, Phys. Rev. **147** (1966) 1071.

[54] S. Eidelman *et al.* [Particle Data Group Collaboration], Phys. Lett. B **592** (2004) 1.

[55] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54** (1985) 1215.

[56] J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465.

[57] A. Pich, arXiv:hep-ph/9806303.

[58] S. Peris, M. Perrottet and E. de Rafael, JHEP **9805** (1998) 011.

[59] M. F. L. Golterman and S. Peris, Phys. Rev. D **61** (2000) 034018.

[60] M. Jamin, J. A. Oller and A. Pich, Nucl. Phys. B **622** (2002) 279.

[61] A. Bramon, R. Escribano and J. L. L. Martinez, Phys. Rev. D **69** (2004) 074008.

[62] R. Barbieri and A. Strumia, arXiv:hep-ph/0007265.

[63] M. E. Peskin and T. Takeuchi, Phys. Rev. D **46** (1992) 381.

[64] R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, Nucl. Phys. B **703** (2004) 127 [arXiv:hep-ph/0405040].

[65] C. Csaki, C. Grojean, L. Pilo and J. Terning, Phys. Rev. Lett. **92** (2004) 101802.

[66] R. Barbieri, A. Pomarol and R. Rattazzi, Phys. Lett. B **591** (2004) 141;

[67] M. A. Luty and T. Okui, arXiv:hep-ph/0409274.

[68] K. Agashe and R. Contino, arXiv:hep-ph/0510164.

[69] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B **671** (2003) 148 [arXiv:hep-ph/0306259].

[70] S. J. Huber and Q. Shafi, Phys. Lett. B **498** (2001) 256 [arXiv:hep-ph/0010195].

[71] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP **0308**, 050 (2003) [arXiv:hep-ph/0308036].

[72] K. Agashe, G. Perez and A. Soni, Phys. Rev. D **71**, 016002 (2005) [arXiv:hep-ph/0408134].

[73] N. Arkani-Hamed, L. Hall, Y. Nomura, Smith, Weiner, Nucl. Phys. B605(2001)81, [arXiv:hep-ph/0102090].

[74] R. Barbieri, R. Contino, P. Creminelli, R. Rattazzi, C.A. Scrucca, Phys. Rev. D66(2002)024025, [arXiv:hep-th/0203039];
D. Marti, A. Pomarol, Phys. Rev. D66(2002)125005, [arXiv:hep-ph/0205034].

[75] H. Georgi, A. K. Grant, G. Hailu, Phys. Lett. B506(2001)207, [arXiv:hep-ph/0012379].

[76] I. Antoniadis, S. Dimopoulos, A. Pomarol, M. Quiros, Nucl. Phys. B544(1999)503, [arXiv:hep-ph/9810410];
A. Delgado, A. Pomarol, M. Quiros, Phys. Rev. D60(1999)095008, [arXiv:hep-ph/9812489].

[77] H. Cheng, K. T. Matchev, M. Schmaltz, Phys. Rev. D66(2002)036005, [arXive: hep-ph/0204342].

[78] See for example L. J. Hall, Y. Nomura, Phys. Rev. D64(2001)055003, [arXive: hep-ph/0103125];
R. Contino, L. Pilo, R. Rattazzi, E. Trincherini, Nucl. Phys. B622(2002)227, [arXiv: hep-ph/0108102];
J.F. Oliver, J. Papavassiliou, A. Santamaria, Phys. Rev. D67(2003)125004, [arXive: hep-ph/0302083].

[79] G. von Gersdorff, M. Quiros, A. Riotto, Nucl. Phys. B **689**(2004)76;
G. von Gersdorff, A. Hebecker, Nucl. Phys. B **720**(2005)211.

[80] R. Jackiw, Phys. Rev. D 9(1974)1686.

[81] S. Coleman, E. Weinberg, Phys. Rev. D 7(1973)1888.

Agradecimientos

Este proyecto comenzó hace unos cuantos años a muchos quilómetros de Barcelona. El camino recorrido ha sido largo y han habido muchas escollos, momentos díficiles y de inseguridad. Pero también ha estado lleno de buenos momentos y de alegrías. Ha sido una experiencia sumamente enriquecedora en muchos aspectos, en la cual hemos caminado mucho. Digo hemos porque no lo he hecho sólo, sino que me han acompañado muchas personas, más de las que aquí podría nombrar, para ellas va mis más profundo agradecimiento.

Primero que nada quiero agradecer a dos personas que han sido fundamentales en la realización de este proyecto. A mi director de tesis, Àlex, que me ha tenido una paciencia infinita para discutir conmigo innumerables fenómenos del mundo de la física. Siempre ha tenido una gran generosidad y entusiasmo para transmitir sus conocimientos y ha sido para mí una fuente inagotable de ideas, inspiración y creatividad.

La otra persona que ha sido fundamental en la realización de esta tesis es Natalia. Con ella comenzamos a soñar este proyecto en Argentina, y juntos hemos alcanzado este objetivo. Su comprensión y afecto en este camino han sido imprescindibles. En particular quiero agradecerle su paciencia y constancia para acompañarme, aún en los momentos más difíciles.

En el ifae he pasado la mayor cantidad de mi tiempo. Mis compañeros de "habitáculo", el Nino, el Ariel y el Juli, qué aguante han tenido. La he pasado muy bien y hemos hecho grandes amistades que han superado las fronteras del despacho. Agradezco también a todos los compañeros con los que he compartido charlas, comidas, discusiones, salidas: Germano, David, Carla, Antoni, Gabi Z., Oriol P., Santi, Alex, Sergio, Oscar, Javi R., Andrea, Carles, Pere, Javi V., Caro, Olga, Gabriel. Al Juli y a Jaume, que me han ayudado con mi parálisis informática.

También quiero agradecer al resto de la gente del ifae, con las que he compartido estos años y de quienes he aprendido mucha física. A la gente que ha hecho críticas constructivas de mi trabajo: Santi, Rafa, Eduard de Rafael, también al resto del grupo con quienes he compartido muy buenas charlas a la hora de la comida: Ramón, Emili, Eduard, y en los seminarios y pasillos del ifae: Albert, Toni, Mariano, Pedro, John, Quim, Ana, Ramón.

Agradezco a Roberto Contino, que me ha respondido muchas preguntas y con quien he mantenido una correspondencia muy provechosa.

A los miembros del tribunal: Santi Peris, Mariano Quirós, Jorge Russo, Nuria Rius, Arcadi Santamaría, Rafel Escribano y Roberto Emparán, les agradezco su buena predisposición para aceptar evaluar esta tesis.

Quiero agradecer a las secretarias del departamento: Celia y Mercé, y del ifae: Montse, que me han ayudado a no perderme en el laberinto de la burocracia. También quiero a agradecer a Celia por todo el apoyo que me dio en mis primeros días en Barcelona.

Agradezco a mi familia catalana, el Joan, la Susana, el David, el Adri, la Luci y el Bruno, la Nuria, el César, el Raimón, la Mireia y el Iván, que casi sin conocerme confiaron en mí y me abrieron las puertas de sus casas.

A la tía Kelli, la Irene, la Julia y el Claudio, les agradezco habernos apoyado desde el principio en todos los aspectos. Su afecto, proximidad y soporte han sido inmensos.

Al Goyo, al Danilo y a la María, que en los días más difíciles no se movieron de mi lado y fueron un soporte esencial para afrontar tantos problemas.

A mis padres y a toda mi familia, que siempre han confiado en mí y que me han apoyado en todo lo que hizo falta para que realizara este proyecto.

Mis amigos de Barcelona han hecho que este proyecto sea mucho más que lo que yo pensaba. A ellos les agradezco haber creado esta atmósfera tan estimulante: Flaca, Seba, Max, Melanie, René, Noelia, Emi, Huevo, Belén, Raquel, Diego, Ana D. P., Martín, Irene, Teo. A todos los amigos que han pasado de visita: el Cuchu en cuya compañia emprendería cualquier proyecto, el Sequi que es una fuente de inspiración y estímulo para pensar los desafíos de la física, Sparky, el Demonio, el Rolo, el Bocha, la Ceci, Martín y Eli.

La construcción de un proyecto colectivo supera todas mis espectativas. Por eso quiero agradecer a todos los que forman el Ateneu Rosa de Foc. En especial al Francesc, que ha compartido conmigo todos los espacios y que me ha apoyado en todo momento. Su amistad es algo sumamente valioso. A la Galia entera, que resiste ahora y siempre al opresor. La Galia ha sido para mí un espacio vital de crecimiento y formación. Al Mía, con quien hemos tejido una gran amistad enhebrada en nuestra cesta agroecológica. Al Àlex, con quien empecé compartiendo la hora de la comida en la UAB, y que es un ejemplo de constancia y dedicación.

La montaña ha sido y es para mí un espacio vital, donde se ponen en juego muchas cosas y se estrechan los lazos y la confianza. Agradezo a los amigos con los que he compartido la roca de Catalunya y de los Pirineos: Pepe, Magda, José Ángel, Germano, Xavi.

Mis amigos de argentina, la juventus: Hueso, Javi, Migue, Mariano, Juanma, Fede, y el equipo de las estrellas (Gabi te considero una estrella más), que siempre han estado a mi lado, más allá de los kilómetros. A ellos les agradezco todo y les digo que no se relajen, cuando menos se lo esperen me van a tener de nuevo por allá.

Quiero agradecer a la gente con la que di los primeros pasos que me llevaron a este proyecto, en especial a César y Rafael que siempren me han apoyado. También al IB y a la universidad pública argentina, que pese a todo siguen siendo de gran calidad.

Por último quiero dedicar unas palabras a Victoria, que ha sido quien me ha propuesto, seguramente sin quererlo, los retos más difíciles y que me han dado las satisfacciones más grandes. A ella le agradezco su vitalidad sin fin y su alegría a lo largo de esta etapa.