Essays on Macroeconomic Dynamics

A thesis presented

by

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A mis padres

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Creating a new theory is not like destroying an old barn and erecting a skyscraper in its place. It is rather like climbing a mountain, gaining new and wider views, discovering new connections between our starting point and its rich environment. But the point from which we started still exists and can be seen, although it appears smaller and forms a tiny part of our broad view gained by the mastery of obstacles on our adventurous way up.

(Attributed to Albert Einstein)

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Foreword

The thrust of this Thesis is to explore the behavior of economic agents in a changing environment, developing models which are genuine dynamic structures built on solid macroeconomic foundations. Macroeconomics is a discipline that studies the structure, performance and behavior of the economy as a whole. The main concern of macroeconomists is to analyze and attempt to understand the underlying determinants of aggregate trends in the economy with respect to the total output of goods and services, unemployment, inflation and the balance of payments. The primary aim of the present research is to develop, as comprehensively and rigorously as possible, an understanding about the way the economy functions and how it is likely to react to specific policies, as well as to the wide variety of demand and supply shocks that may cause instability. Macroeconomic theory, consisting of a set of views about the way the economy operates and organized within a logical framework (or model), forms the basis upon which economic policy is designed and implemented. Hence, the subject matter analyzed in this research is important, not only for its contribution to academic research, but also because in a way or another macroeconomic events influence the lives and welfare of all of us. In this sense, an understanding by private agents and policy makers of the factors which determine macroeconomic performance is essential in order to design and implement economic policies and take decisions which have the potential to vastly improve economic welfare.

Most questions in macroeconomics involve the consideration of events at more than one date. In this Thesis, I follow the tradition that attempts to study, simultane-

ously and explicitly, the time horizon that is relevant for the question at hand. This view was brought into prominence by Irving Fisher at the turn of the past century and has become a common feature of modern macroeconomic theories. Notwithstanding, the methods used by researchers to analyze dynamic macroeconomic relations and to address issues in macroeconomic policy have evolved considerable over the years. Traditionally, macroeconomic models were backward looking in nature, meaning that agents only looked at the past. With the advent of rational expectations in the mid-1970s the methods of macroeconomic dynamics changed in a fundamental way. The key methodological innovation was based on the observation that although certain economic variables were backward looking, others embodied expectations about the future and were therefore forward looking. Consequently, dynamic macroeconomic models, as those developed in this Thesis, should be based on a combination of backward and forward-looking dynamics, reflecting the fact that some economic variables are tied to the past, whereas others are looking to the future.

A formal macroeconomic inquiry has to be built entirely from microeconomic, general equilibrium fundamentals, being explicit about the tastes or preferences of individuals, of the resources available to them and of the technologies and information agents can use. In this respect, the analysis must involve deriving the behavioral relationships of the macro model from the intertemporal optimization of micro agents. The environment under which economic agents must take decisions is characterized by uncertainty. Thus, the researcher has to incorporate, in the model to be developed, those sources of uncertainty of interest in the particular environment under consideration. These premises are undertaken by the dynamic stochastic general equilibrium (DSGE) paradigm, which have become the most

important approach in modern macroeconomic analysis.

The Thesis is organized in two separate parts. The first one covers the topics of monetary macroeconomics and is divided in two Chapters. Chapter 1 is concerned with the study of the role of capacity constraints in determining the magnitude of the outcomes of unexpected monetary policy actions. Chapter 2 performs an empirical evaluation of the relationship between inflation and utilization, two key elements of monetary policy analysis. The second Part of the Thesis is about international macroeconomics, analyzing the dynamics of capital flows in a small open economy. A more detailed description of the contents of each of the three Chapters, as well the motivation underlying them and contribution, is given in the sequel.

The first Part of the Thesis is motivated by the claim made by the Nobel laureate Robert E. Lucas (1996) in which it is stated that the understanding that people have over the interaction of money with the economy remains, to a large extent, unresolved and that this an issue that still creates controversy among academicians and policy makers. The objective of this research effort is to contribute into the development of a theoretical framework for quantifying monetary policy actions, taking into consideration some relevant aspects that characterize actual economies and to provide evidence in this regard. I believe that once there is a good theory for evaluating monetary policy, the welfare and economic benefits of alternative policy rules can be deduced and consequently, a more rational choice of them could be done. It is a matter of fact that the interaction of money with the real economy is complicated. The reason is that monetary policy does not act through a single channel. Hence, to be successful in conducting monetary policy, it is necessary to determine

and compare the likely economic effects of all the different alternatives is hands of policy makers. Obviously, the only practical option is to experiment with artificial economies or models.

In the first Chapter of this Thesis, I develop an analytical framework that is aimed at deepening the understanding of the complex manner in which money affects the economy. An important characteristic for a good model to have is the ability to reproduce real world's response to simple monetary policy experiments. In this sense, the model that I propose seeks to determine how different is such response depending on the extent to which economic resources are being used in the economy. Thus, the asymmetric effects of monetary policy are investigated. The focus of the present inquiry is on the role of variable capacity utilization as a relevant factor influencing the dynamic relationship between monetary policy and economic activity. The source of the asymmetry considered is directly linked to the bottlenecks and stock-outs that emerge from the existence of capacity constraints in the real side of the economy. In the context proposed, money has real effects due to the presence of rigidities in households' portfolio decisions in the form of a Lucas-Fuerst 'limited participation' constraint. The model features variable capacity utilization rates across firms due to demand uncertainty. A monopolistic competitive structure provides additional effects through optimal mark-up changes. The overall message of the Thesis for monetary policy is that the same actions may have different effects depending on the capacity utilization rate of the economy. This line of research is expected to be extended in two ways. One involves the design and evaluation of monetary policy rules that take into consideration the asymmetries existing in the economy. Another line of research involves the analysis of firm

heterogeneity and the propagation of monetary policy shocks, taking as basis the model economy I have developed.

The characteristically close connection between macroeconomic theory and empirical work drove me to analyze, in Chapter 2, the joint dynamics of two key variables for the conduct of monetary policy: inflation and the aggregate capacity utilization rate. In this regard, an econometric procedure useful for estimating dynamic rational expectation models with unobserved components has been developed and applied in this context. The method combines the flexibility of the unobserved components approach, based on the Kalman recursion, with the power of the general method of moments estimation procedure. A 'hybrid' Phillips curve relating inflation to the capacity utilization gap and incorporating forward and backward looking components is estimated. The results that I obtained show that such a relationship in non-linear: the slope of the Phillips curve depends significantly on the magnitude of the capacity gap. These are important findings that provide support for studying the implications of asymmetric monetary policy rules.

As mentioned above, the second Part of the Thesis, Chapter 3, is about international macroeconomics and, more specifically, about the behavior of the current account in a small open economy. Movements in the current account are deeply intertwined with, and convey information about, the actions and expectations of all market participants in an open economy. For this reason, researchers and policymakers must regard capital flows dynamics as an important macroeconomic issue. In this regard, trying to explain its movements, seeking to uncover the factors that induce those changes and assessing its sustainable level is an endeavor of crucial academic and political relevance. The focus of my research

is on the macroeconomic consequences resulting from the fact that people must take their decisions in a more integrated world. The processes of increased globalization experienced in recent years have been reflected in a notably increase in global trade of financial assets, with the consequent exposure of an economy to foreign instabilities. In this context, policy makers face two main challenges; first, they must know how to limit the potentially adverse effects of this higher interdependence; and second, they should be able to capture the possible benefits of this particular scenario. With this thesis, I aim at providing an adequate tool to those agents involved in the decision processes, for helping them to choose the best alternatives in their hands.

The Chapter concerning the study of the macroeconomic performance of monetary policy was based on a DSGE model. However, an important drawback of stochastic intertemporal optimization models is that they are often very formal and difficult to solve. This means that the tractability only can be achieved under some restrictive conditions. In some cases, considering a deterministic model might yield much more transparent intuitions and results. This is the avenue followed in the Chapter analyzing the dynamics of capital flows in a small open economy. Specifically, I examine the dynamic response of capital flows when the economy faces a disturbance on world interest rates. Making use of a fully optimizing macrodynamic model, I characterize the precise path that follows the current account in the transition to the new equilibrium. I show that such dynamics depend on the net foreign asset position of the country and on the relative speed of adjustment between investment and the intertemporal substitution of consumption. I study how the degree of international capital mobility affects the adjustment path of the current account. A mone-

tary extension of the model is performed. In this context, I analyze the interaction of the degree of domestic financial development with the external behavior of the economy. The framework that I have developed is suitable to analyze policy issues such as those related to the optimal degree of capital mobility in open economies.

My main goal in writing this Thesis has been to provide scientific arguments concerning some open questions in macroeconomics. Along the way, I have learnt how to deal with unknown elements and I have discovered, at the same time, new avenues of research that are worthwhile pursuing, most of which ought to be left for the near future. In this vein, and as Robert Solow (2000) points out, the heterogeneity of agents with respect to their underlying beliefs, expectations, market power, access to capital and so on, constitute some of the frictions and sources of imperfections that seems to characterize actual macroeconomic behavior. Exploring these issues constitute an exciting task for future work. The conceptual and analytical difficulties involved are not trivial, but the payoffs are likely to be worthwhile.

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Chapter 1

Endogenous Capacity Utilization and the Asymmetric Effects of Monetary Policy

1.1   Introduction

What are the effects of central bank policy? Do they depend on the state of the economy? How should monetary policy be conducted in the short run? For many years, macroeconomists have grappled with these questions, but have not yet reached a consensus. Achieving a thorough notion of the mechanics that constitute the monetary transmission mechanism requires a deep exploration of the nontrivial structure of the complete economy. This is not a straightforward task for either theorists and applied economists. From a theoretical point of view, the main difficulty has been to develop models that can generate

the salient features of aggregate time series, which is the first step towards reliable policy analysis. Models of the transmission mechanism should generate a response of economic variables to a monetary policy shock consistent with those found in the data in, at least, three dimensions: sign, timing and magnitude.

The literature has provided us with models that are able to replicate reasonably well the sign and timing of the transmission mechanism. However, models that can adequately account for the magnitude of the responses to monetary policy remain to be developed. A relevant aspect in this regard refers to asymmetries: depending on the state of the economy, similar policy actions will generate quantitatively different effects on the main economic variables. In this study, I consider the hypothesis of capacity utilization constraints in the real side of the economy and portfolio rigidities in the financial sector, as the basis for developing an analytical framework consistent with the aforementioned features of the monetary transmission mechanism. Such a framework consists of a dynamic stochastic general equilibrium model which displays the non-neutralities of money needed to perform policy analysis in the short run, as well as the production inflexibilities that are able to generate the asymmetric dynamics of key macroeconomic variables documented in empirical research.

1.1.1 Capacity Utilization and Monetary Policy Performance

In the literature, there are several explanations for the asymmetric response that monetary policy generates on the main macroeconomic variables. One of these arguments is known as the capacity constraint hypothesis.[1] The idea is that some firms find it difficult

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[1]Other arguments are based on "menu costs" and nominal wage rigidities. Dupasquier and Ricketts (1997) briefly survey some of the different sources of asymmetries in this regard. Another strand in the

to increase their capacity to produce in the short run, giving rise to supply shortages and production bottlenecks. This is going to have important implications on one particular relation which is at the heart of the science of monetary policy, the Phillips curve. In this regard, when the economy experiences strong aggregate demand, the impact on inflation will be greater when more firms are restricted in their ability to raise output in the short run. Consequently, the short-run aggregate supply equation or Phillips curve will display a convex shape, which has relevant consequences for the performance of a monetary policy aimed at controlling inflation. Certainly, if the economy is initially weak, easing monetary conditions will primarily affect output, but if the economy is initially strong, a monetary expansion will mainly affect prices.

Recently, a great deal of research has been devoted to test empirically the asymmetric effects of monetary policy from the existence of a convex Phillips curve. In this vein, Cover (1992), Karras (1996) and Alvarez-Lois (2000) provide evidence of asymmetries between positive and negative monetary shocks on output and prices. Weise (1999) making use of an econometric methodology that allows to test for the different types of asymmetries finds that monetary shocks have dramatically different effects depending on the state of the economy. But prices and output are not the only macroeconomic variables studied in this context, there is also evidence of asymmetries in the behavior of nominal interest rates.[2]

Despite the empirical evidence and the strong theoretical arguments put forward, there is certainly a lack of a general equilibrium approximation to the issue of asymmetries within the monetary macroeconomic literature. This Thesis aims at filling this gap, de-

literature emphasizes the role credit market imperfections in the monetary transmission mechanism. See, for instance, Bernanke, Gertler and Gilchrist (1998).

[2]See, for instance, Enders and Granger (1998) for evidence in this regard.

veloping a quantitative model of the monetary transmission mechanism and analyzing its implications for the conduct of monetary policy.

1.1.2 Modeling Capacity Within a Monetary Framework

The model developed here has two basic ingredients: (i) it incorporates a real side with production inflexibilities that result in variable rates of utilization across firms and (ii) it considers portfolio constraints that create a short run non-neutrality of monetary policy. Regarding the first component, the model presented here follows the formulation of Fagnart, Licandro and Portier (1999) in modelling the issue of capacity utilization.[3] These authors introduce idiosyncratic demand uncertainty and a rich modeling of the production sector (firms heterogeneity and absence of an aggregate production function) within a monopolistic competitive business cycle model. The bulk of their model relies on three basic aspects: first, the limited possibilities of a short run substitutability between production factors; second, the presence of uncertainty at the time of capacity choices, which explains the presence of underutilized equipments; and third, the existence of idiosyncratic uncertainty which results in a nondegenerate distribution of utilization rates across firms. In equilibrium, a proportion of firms face demand shortages and have idle capacities, while others are at full capacity and are unable to serve any extra demand. Moreover, the monopolistic competitive environment provides an additional source of dynamics through optimal mark-up changes.

Regarding the second ingredient of the model, namely the monetary side, this study considers the existence of participation constraints in the financial market, which create

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[3]Probably, the first attempt to rationalize explicitly equipment idleness in a real business cycle model is due to Cooley, Hansen and Prescott (1995).

non-neutralities of monetary policy. Specifically, the effects of an unexpected monetary policy action are firstly felt through the demand for money and the short term interest rate -the *liquidity effect*- which subsequently affects investment and output, known this as *output effect.* The magnitude and persistence of such effects are clearly an important issue, as they capture a key nonneutral effect of monetary policy. Explaining the strong relationship between money and real activity in a general equilibrium theory involves facing two challenges. The first is to provide a theory in which money is valued in equilibrium. This is done assuming a cash-in-advance constraint. Secondly, and more difficult, it is to show how monetary policy has real effects in a world where economic agents are behaving rationally, without simply assuming some *ad hoc* form of money illusion. The limited participation paradigm provides a rationale for this issue.[4] The basic idea is that money plays a role in the economy due to its asymmetric distribution to economic agents: money is firstly distributed to financial intermediaries an then to firms before it finally reaches consumers' hands.[5]

Two features describe the mechanism working in these models (i) changes in the money supply initially involve the monetary authority and financial sector only and (ii) the representative household's supply of funds, through bank deposits, is predetermined relative to monetary shocks. Under these circumstances, an unanticipated money injection increases the share of liquid assets held by financial intermediaries. Thus, firms are forced to absorb the excess of liquidity in the economy. The market clearing interest rate falls as a result. The liquidity effect can generate a strong real response to monetary policy by changing the

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[4] A second strand of the literature, known as the new neoclassical synthesis -see Goodfriend and King (1997)- highlights the role of nominal frictions in shaping key features of monetary economies.

[5] Basic references in the literature on limited participation models include Lucas (1990), Fuerst (1992) and Christiano (1991).

financial costs of hiring factors of production. The existence of production inflexibilities that arise due to the existence of capacity utilization constraints will condition the intensity of the liquidity and output effects.[6] Depending on the magnitude of these inflexibilities, the response of the economy to a monetary shock will differ notably.[7] These asymmetries are quantified in the model presented here.

The outline of the Chapter is as follows. Section 1.2 presents a formal description of the model's behavioral aspects. Section 1.3 offers a characterization of the general equilibrium of the economy and its qualitative properties. The implications for short-run dynamics are analyzed at this stage. Section 1.4 studies the quantitative dimension of the model, what involves the computation of impulse responses of the main variables in the model to a monetary policy shock and other numerical simulation exercises. Section 1.5 offers some concluding remarks and possible lines for further research.

1.2 The Model Economy

The basic structure of the model is taken from Christiano, Eichenbaum and Evans (1998) and Fagnart, Licandro and Portier (1999). I consider an economy consisting of households, financial intermediaries, a central bank in charge of the conduct of monetary policy and two productive sectors: a competitive sector producing a final good and a monopolistic sector providing intermediate goods. These intermediate goods are the only inputs necessary for the production of the final good. The final good can be used either for consumption

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[6]Christiano and Eichenbaum (1995) also analyses these margins.

[7]Finn (1996) and Cook (1999) analyze the role of capital underutilization in a monetary quantitative framework. The description of the underutilization phenomenon , which follows Burnside and Eichenbaum (1996) depreciation in use models, is highly stylized, however.

or for investment purposes. Capital and labor are used in the production of intermediate goods by means of a putty-clay technology.[8] This specification of the production function allows for the introduction of a simple, but realistic, concept of capacity. Each input firm makes its investment, pricing and employment decisions under idiosyncratic demand uncertainty, that is, before knowing the exact demand for its production. This structure implies that intermediate goods firms can be either sales or capacity constrained; it also allows different firms to face different capacity constraints. Consequently, this source of uncertainty is what explains the presence of heterogeneity between firms at equilibrium regarding the degree of utilization of their productive capacities.

These production side particularities are embedded into an otherwise standard limited participation model. Before proceeding to describe in more detail the different aspects that constitute the basis of the model economy, it is convenient to define the information sets that appear in the model.

$$\Omega_{0,t} = \text{economy-wide variables dated at time } t-1 \text{ and earlier}$$

$$\Omega_{1,t} = \text{includes } \Omega_{0,t} \text{ and period } t \text{ aggregate monetary shock}$$

At this point, it is also useful to describe the elements that represent the state of the economy in the model I am developing. These are the aggregate stock of capital K, the capital-labor ratio X and the realization of the monetary policy shock, μ.

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[8]Capital and labor are substitutes *ex ante*, i.e., before investing, but complement *ex post*, i.e., when equipment is installed. This implies that each firm makes a capacity choice when investing.

1.2.1 Final Good Firms

At time t, a single final good, denoted by $\mathcal{Y}$, is produced by a representative firm which sells it in a perfectly competitive market. Such commodity can either be used for consumption or for investment. There is no fixed input, which implies that the optimization program of these firms remain purely static. The production activities are carried out by combining a continuum of intermediate goods, indexed by $j \in [0,1]$. The production technology is represented by a constant return-to-scale CES function defined as follows

$$\mathcal{Y}_t = \left[\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} v_{j,t}^{\frac{1}{\epsilon}} \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{1.1}$$

with $\epsilon > 1$ being the elasticity of substitution of inputs and where $Y_{j,t}$ is the quantity of input j used in production at date t. Here, $v_{j,t} \geq 0$ is a productivity parameter corresponding to input j. It is assumed to be drawn from a stochastic process i.i.d. distributed across time and input firms,[9] with a log normal distribution function $\mathrm{F}(v)$ that has unit mean and is defined over the support $[\underline{v},\bar{v}]$ with $0 < \underline{v} < 1 < \bar{v}$. The representative firm purchases inputs to intermediate good firms taking into account that the supply of each input j is limited to an amount $\bar{Y}_{j,t}$. Assuming a uniform non-stochastic rationing scheme, the optimization program of the final firm can be written as follows

$$\max_{\{\mathcal{Y}_t,Y_{j,t}\}} \mathbf{P}_t \mathcal{Y}_t - \int_0^1 P_{j,t} Y_{j,t} \mathrm{d}j , \tag{P1}$$

subject to

$$Y_{j,t} \leq \bar{Y}_{j,t} \ \forall j \in [0,1] ,$$

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[9] In order to keep the model tractable, it is assumed that the idiosyncratic shock is not serially correlated. Thus, its realization influences exclusively contemporary production and employment decisions, but not investment decisions.

where $\mathbf{P}_t$ is the price of the final good which is taken as given by the firm. When maximizing profits, the final firm faces no uncertainty: it knows the input prices $\{P_{j,t}\}$, the supply constraints $\{\bar{Y}_{j,t}\}$ and the productivity parameters $\{v_{j,t}\}$. It is important to notice that the inclusion of supply constraints in the problem above is due to the particular structure of the model, where input producing firms set their prices before the idiosyncratic shock is realized.

The solution to (P.1) determines the quantity that the final good firm is going to make for the goods produced by each intermediate firm. Under deterministic quantity constraints and a uniform rationing scheme, effective demands are not well defined. Realized transactions can be derived, however.[10] The quantity of inputs used will be determined by the corresponding idiosyncratic productivity level of each intermediate firm as described in the next result:

Lemma 1 (Realized Transactions) *The optimal allocation of inputs across intermediate good firms is given by the following system of equations*

$$Y_{j,t} = \begin{cases} \mathcal{Y}_t v_{j,t} \left(\frac{P_{j,t}}{\mathbf{P}_t} \right)^{-\epsilon} & if & \underline{v}_{j,t} \leq v_{j,t} \leq \bar{v}_{j,t} \\ \bar{Y}_{j,t} & otherwise \end{cases} \qquad (1.2)$$

with

$$\tilde{v}_{j,t} = \frac{\bar{Y}_{j,t}}{\mathcal{Y}_t \left(\frac{P_{j,t}}{\mathbf{P}_t} \right)^{-\epsilon}} \; . \qquad (1.3)$$

The variable $\tilde{v}_{j,t}$ determines the critical value of the productivity parameter $v_{j,t}$ for which the unconstrained demand equals the supply constraint $\bar{Y}_{j,t}$. The term $(P_{j,t}/\mathbf{P}_t)^{-\epsilon}$ appearing in the demand function of a firm with excess capacities represents, at given $\mathcal{Y}_t$,

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[10]For a detailed discussion on the theory of effective demands see Green (1980) or Svensson (1980).

the positive spillover effects a firm with idle resources benefits from. As mentioned above, for tractability purposes I shall assume that all intermediate firms are *ex-ante* equal. This symmetry means that input prices and capacities are the same across firms. Assuming that a law of large numbers applies in the present context, the final output supply can be expressed as follows

$$\mathcal{Y}_t = \left[\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} v_{j,t}^{\frac{1}{\epsilon}} \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.4}$$

or taking into account equation (1.2),

$$\mathcal{Y}_t = \left\{ \left[\left(\frac{P_{j,t}}{\mathbf{P}_t} \right)^{-\epsilon} \mathcal{Y}_t \right]^{\frac{\epsilon-1}{\epsilon}} \int_{\underline{v}}^{\tilde{v}_t} v \mathrm{dF}\left(v \right) + \bar{Y}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^{\bar{v}} v^{\frac{1}{\epsilon}} \mathrm{dF}\left(v \right) \right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{1.5}$$

Recall that $\mathrm{F}(v)$ is the distribution function of idiosyncratic shocks; thus, for a proportion $\mathrm{F}(\tilde{v})$ of intermediate firms, the realized value of the productivity parameter is below $\tilde{v}$. Some manipulation of the previous expression allows one to write relative prices as a function of $\tilde{v}_t$, the proportion of firms with excess capacities

$$\frac{P_{j,t}}{\mathbf{P}_t} = \left\{ \int_{\underline{v}}^{\tilde{v}_t} v \mathrm{dF}\left(v \right) + \tilde{v}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^{\bar{v}} v^{\frac{1}{\epsilon}} \mathrm{dF}\left(v \right) \right\}^{\frac{1}{\epsilon-1}}. \tag{1.6}$$

The right hand side of this expression is increasing in $\tilde{v}$ and bounded above by 1. To see this, first notice that the marginal productivity of a supply-constrained input, $\partial \mathcal{Y}_t / \partial Y_{j,t}$, remains larger that its real marginal cost, $P_{j,t}/\mathbf{P}_t$, while they are equal for unconstrained inputs.[11] Thus, in the case that some input is supply-constrained, one obtains that

$$\mathcal{Y}_t = \int_0^1 \frac{\partial \mathcal{Y}_t}{\partial Y_{j,t}} Y_{j,t} \mathrm{d}j > \int_0^1 \left(P_{j,t}/\mathbf{P}_t \right) Y_{j,t} \mathrm{d}j \tag{1.7}$$

where the first equality is achieved by applying the Euler Theorem. The (normalized to 1) real price of the final good is equal to the shadow price index for intermediate inputs,

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[11]With constant returns to scale, this implies positive profits despite the perfect competitive assumption.

which is computed by using the marginal productivities of inputs in the production of final output, that is,

$$1 = \left[\int_0^1 \left(\frac{\partial \mathcal{Y}_t}{\partial Y_{j,t}} \right)^{1-\epsilon} v_{j,t} \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.8}$$

where the price-index expression is obtained from the maximization problem of the final-good firm. Notice that when no supply constraints are binding, $\tilde{v} \rightarrow \bar{v}$, the model shrinks to the standard case and $\partial \mathcal{Y}_t / \partial Y_{j,t} = (P_{j,t}/\mathbf{P}_t)$. In such a case, the symmetric equilibrium relative price of an intermediate good with respect to the final good, $P_{j,t}/\mathbf{P}_t$, is equal to one. Moreover, under these circumstances, the optimal production level of the final firm $\mathcal{Y}_t$ is indeterminate. However, when some input is supply-constrained, the final good price is larger than the input price.

$$\mathbf{P}_t = \left[\int_0^1 \left(\frac{\partial \mathcal{Y}_t}{\partial Y_{j,t}} \right)^{1-\epsilon} v_{j,t} \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}} > \left[\int_0^1 P_{j,t}^{1-\epsilon} v_{j,t} \mathrm{d}j \right]^{\frac{\epsilon}{\epsilon-1}} = P_{j,t}. \tag{1.9}$$

where the last equality is due to the fact that all input firms charge the same price $P_{j,t} = P_t$ for all j. Consequently, the spillover term $(P_{j,t}/\mathbf{P}_t)^{-\epsilon}$ is larger than one. This term is going to play a significant role in the model's behavior, as will be stressed later.

1.2.2 Intermediate Good Firms

In this sector, each intermediate good is produced by a monopolistically competitive firm making use of capital and labor, which are combined for production through a *putty-clay* technology. Intermediate firms start period t with a predetermined level of capacity. Such a production plan cannot be adapted to the needs of the firm within the period. Hence, investment achieved during period $t-1$ becomes productive at date t. Investment consists of the design of a production plan by simultaneously choosing a quantity

of capital goods $K_{j,t}$ and employment capacity $N_{j,t}$ according to the following Cobb-Douglas technology:

$$\bar{Y}_{j,t} = K_{j,t}^{\alpha} N_{j,t}^{1-\alpha} \tag{1.10}$$

where $0 < \alpha < 1$. The variable $N_{j,t}$ represents the maximum number of available work-stations in the firm. Hence, the firm is at full capacity when all these work-stations are operating full-time. As it is common in models featuring a *putty-clay* technology, it is convenient to express investment decision as the choice of both $K_{j,t}$ and a capital-labor ratio $X_{j,t} \equiv K_{j,t}/N_{j,t}$. Consequently, the expression in (1.10) can be rewritten as

$$\bar{Y}_{j,t} = X_{j,t}^{\alpha-1} K_{j,t}, \tag{1.11}$$

from where the technical productivity of the installed equipments can be deduced. For the case of capital, it is given by $X_{j,t}^{\alpha-1}$, whereas $X_{j,t}^{\alpha}$ represents that of labor, so that this production function displays constant returns-to-scale in the within-period labor. In particular, if the firm uses a quantity of labor $L_{j,t}^{d}$ smaller than $N_{j,t}$, it then produces $X_{j,t}^{\alpha} L_{j,t}^{d}$ units of intermediate good. Once the idiosyncratic (demand) shock $v_{j,t}$ is revealed, the firm instantaneously adjusts its labor demand $L_{j,t}^{d}$ to cover the needs of its production plan, $Y_{j,t}$, that is,

$$L_{j,t}^{d} = \frac{Y_{j,t}}{X_{j,t}^{\alpha}} = \frac{1}{X_{j,t}^{\alpha}} \min \left\{ \mathcal{Y}_t v_{j,t} \left(\frac{P_{j,t}}{\mathbf{P}_t} \right)^{-\epsilon}, \bar{Y}_{j,t} \right\} . \tag{1.12}$$

In order to finance such productive activities, intermediate good firms must borrow the necessary amount of money from a financial intermediary since cash earnings do not arrive in time to finance the period wage bill. Specifically, firms rent labor at a wage W_t which is

paid with cash obtained from the financial intermediary at an interest rate $R_t^L > 0$. At the end of the period, the firm pays back the loan and the interests: $W_t L_{j,t}^d (1 + R_t^L)$.

After observing the aggregate shocks, but before knowing the idiosyncratic one, input producing firms take their price decisions. Input prices are announced on the basis of (rational) expectations, before the exact value of the demand for their production is realized. This price-setting assumption has the advantage of giving a symmetric equilibrium in prices, avoiding in this manner price aggregation difficulties. It is worthwhile to point out that this assumption on the price behavior of input firms should not have important implications on the manner in which the economy responds to aggregate shocks: since prices are announced at the time shocks are known, they are perfectly flexible in this sense.

The price decision is static and the same rule will be followed by all firms given that, *ex-ante*, all of them are identical; that is, $P_t = P_{j,t}$. Consequently, each firm chooses a price in order to maximize current period expected profits,

$$P_t \equiv \arg\max E_v \left[P_t Y_{j,t} - \left(1 + R_t^L\right) W_t L_{j,t}^d \right] \tag{1.13}$$

which by (1.12) is

$$P_t \equiv \arg\max E_v \left[\left(P_t - \frac{\left(1 + R_t^L\right) W_t}{X_{j,t}^\alpha} \right) Y_{j,t} \right] \tag{1.14}$$

where $Y_{j,t}$ is the level of input-goods produced by firm j. The particular amount of those input-goods produced will depend on the demand shock faced by each firm. Such a demand is derived from expression (1.2) or more specifically

$$E_v \left(Y_{j,t}\right) = \left(\frac{P_t}{\mathbf{P}_t}\right)^{-\epsilon} \mathcal{Y}_t \int_{\underline{v}}^{\tilde{v}_t} v \mathrm{dF}\left(v\right) + \bar{Y}_t \int_{\tilde{v}_t}^{\bar{v}} \mathrm{dF}\left(v\right). \tag{1.15}$$

Taking into account these considerations, the optimal price decision can be characterized by the following result:

Lemma 2 (Intermediate-Goods Pricing) *The price decision of any input firm j at date t adopts the following expression:*

$$P_t = \left(1 - \frac{1}{\epsilon\pi\left(\tilde{v}_t\right)}\right)^{-1} \frac{\left(1 + R_t^L\right)W_t}{X_t^{\alpha}} \tag{1.16}$$

where $\pi\left(\tilde{v}_t\right)$ represents the probability of excess capacity in the economy, that is, $\pi\left(\tilde{v}_t\right)$ is a weight measure of the proportion of firms for which demand is smaller than their productive capacity,

$$\pi\left(\tilde{v}_t\right) = \frac{\left(\frac{P_t}{\mathbf{P}_t}\right)^{-\epsilon}\mathcal{Y}_t}{E_v\left(Y_{j,t}\right)} \int_{\underline{v}}^{\tilde{v}_t} v dF\left(v\right). \tag{1.17}$$

Notice that $\pi\left(\tilde{v}\right)$ depends only on $\tilde{v}$, as becomes clear from the combination of equations (1.15) and (1.3) above,

$$\pi\left(\tilde{v}_t\right) = \frac{\int_{\underline{v}}^{\tilde{v}_t} v d\mathrm{F}\left(v\right)}{\int_{\underline{v}}^{\tilde{v}_t} v d\mathrm{F}\left(v\right) + \tilde{v}_t \int_{\tilde{v}_t}^{\bar{v}} d\mathrm{F}\left(v\right)} \tag{1.18}$$

The pricing mechanism resulting from (1.16) implies that intermediate firms set their price as a mark-up over the marginal cost.[12] The mark-up rate depends negatively on the (absolute) value of the price elasticity of expected sales, which is defined as the elasticity of expected sales to expected demand, $\pi\left(\tilde{v}\right)$, times the price elasticity of expected demand, ϵ. This means that when $\pi\left(\tilde{v}\right)$, the probability of a sales constraint, is large, that is, when more input firms are likely to produce under their full capacity level, firm's actual market

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[12]The derivation of this condition supposes that each monopolistic firm only considers the direct effect of its price decision on demand and neglects all indirect effects (e.g. the effects through $\mathcal{Y}_t$). This approximation is reasonable in a context where there is a continuum of firms.

power is reduced, implying a smaller mark-up rate. Notice that when no firm is producing at full capacity, that is, when $\pi(\tilde{v}) = 1$, the pricing rule implies a constant mark-up over the marginal cost as in the standard case.[13] It is assumed that $\pi(\tilde{v}) > 1/\epsilon$ in order to prevent the input price being zero.

The decision concerning the installment of the productive capacity of each input firm has a dynamic nature. The objective of each firm is to maximize its dividend, $\Pi^f_{j,t}$, which is the amount of cash that remains after investment and fixed costs expenditures are made, $\mathbf{P}_t(I_t + \Psi)$, business loans (including wage payments), $W_t L^d_{j,t}(1 + R^L_t)$, are repaid to financial intermediaries and input goods, $P_t Y_{j,t}$, are delivered for cash. More compactly,

$$\Pi^f_{j,t} = P_t Y_{j,t} - W_t L^d_{j,t}(1 + R^L_t) - \mathbf{P}_t(I_t + \Psi). \tag{1.19}$$

Investment in new capital goods, I_t, is used to augment the future capital stock in the intermediate business sector, according to the following law of motion, where δ is the corresponding rate of depreciation,

$$I_{t+1} = K_{t+1} - (1 - \delta)K_t. \tag{1.20}$$

Firms choose a contingency plan $\{K_{t+1}, X_{t+1}\}^\infty_{t=o}$ to maximize the expected discounted value of the dividend flow

$$E_{\Omega_{1,0}}\left[\sum_{t=0}^\infty \Delta_{t+1}\Pi^f_t\right] \tag{1.21}$$

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[13]In the standard case, $\pi(\tilde{v}_t) = 1$, the pricing rule reduces to

$$P_t = \left(\frac{\epsilon - 1}{\epsilon}\right)^{-1}\frac{(1 + R_t)W_t}{X^\alpha_t},$$

which is similar to that in the paper of Christiano et al. (1997). In the sticky-price version of their model, firms set their price equal to a constant mark-up over a weighted expectation of the marginal cost. In the present model, firms can perfectly foresee R and W so that prices are flexible in this respect. Notice that both models are not directly comparable since the production function is *Cobb-Douglas* in Christiano et al. (1997) but *Putty-Clay* here.

subject to (1.12), (1.15), (1.16), given the stochastic process for $\left\{R_t^L, W_t, \Delta_t\right\}_{t=0}^{\infty}$ and given K_0 and X_0, with expectations formed rationally under the assumed information structure. For firms to act in the best interests of their shareholders, the stochastic discount factor Δ_{t+1} should correspond to the representative household's relative valuation of cash across time, which requires

$$\Delta_{t+1} = \frac{\beta^{t+1} U_c\left(C_{t+1}, L_{t+1}\right)}{\mathbf{P}_{t+1}} \tag{1.22}$$

where β is the discount factor and U_c is the marginal utility for the household of consumption, as will be explained later. Thus, the value of the firm for the shareholder derives from the flow of dividends that are paid at the end of each period with cash. The reason the subscript $t + 1$ appears is because the shareholder has to wait until next period to use this cash to buy consumption goods. Regarding the optimal production plan of an intermediate-good firm, the next result summarizes these decisions:

Lemma 3 (Capacity Choice) *The optimal decision of investment in capital* K_{t+1} *and capital-labor ratio* X_{t+1} *is given, respectively, by the following Euler equations*

$$E_{\Omega_{1,0}}\left\{\Delta_{t+1}\mathbf{P}_t - \Delta_{t+2}\mathbf{P}_{t+1}\left(1-\delta\right)\right\} = E_{\Omega_{1,0}}\left\{\Delta_{t+2}\left(1 - F\left(\tilde{v}_{t+1}\right)\right)\Phi_{t+1}\left(\frac{\bar{Y}_{t+1}}{K_{t+1}}\right)\right\} \tag{1.23}$$

and

$$E_{\Omega_{1,0}}\left\{\Delta_{t+2}\Phi_{t+1}\left(\frac{\bar{Y}_{t+1}}{X_{t+1}}\right)\left[\left(\frac{\alpha\left(\epsilon-1\right)}{\tilde{v}_{t+1}}\right)\int_{\underline{v}}^{\tilde{v}_{t+1}} v\,dF\left(v\right) - \int_{\tilde{v}_{t+1}}^{\bar{v}} dF\left(v\right)\right]\right\} = 0 \tag{1.24}$$

where

$$\Phi_{t+1} \equiv P_{t+1} - \frac{\left(1 + R_{t+1}^L\right)W_{t+1}}{X_{j,t+1}^{\alpha}}. \tag{1.25}$$

The first equation states that the optimal capital stock is such that the expected user cost of capital is equal to its expected revenue, which is given by the discounted increase in profits generated by an additional unit of capital corrected by the probability of operating such unit. From the second equation one can observe the trade-off faced by the intermediate firm when choosing the optimal capital-labor ratio. When increasing the capital-labor ratio, the firm increases its labor productivity, which is given by X_t^α, something that has a favorable effect on its competitive position in case of excess capacities. However, increasing X_t means that the maximum level of employment available in period t will be lower, and likewise the maximum volume of sales of the firm. The optimal capital-labor ratio will be such that the two opposite effects on expected profits are equal in the margin.

1.2.3 Money Supply and Financial Intermediation

In this model, banks' main task is the provision of liquidity to their customers, the input producing firms. Banks begin each period with assets and liabilities that consist solely of the funds deposited with them by the households, D_t. Competition among banks for these deposits determines the market-clearing gross interest rate, $\left(1 + R_t^D\right)$, which is payable at the end of the period. Banks finance their lending activities with household deposits, as well as with funds obtained from cash injections, $\mathcal{M}_t$, made by the monetary authority every period. The asset side of banks' balance sheet is composed by loans, B_t^S, that are supplied to intermediate firms. The bank charges a gross lending rate equal to $\left(1 + R_t^L\right)$. Financial intermediation is assumed to be a costless activity. With no barriers to entry, competitive forces will ensure that the equilibrium interest rate on loans equals the rate paid on deposits, that is, $R_t^L = R_t^D$. Moreover, in equilibrium, the financial intermediaries

will supply inelastically the total amount of loanable funds at their disposal:

$$B_t^S = D_t + \mathcal{M}_t. \tag{1.26}$$

At the end of the period, banks remit $\Pi_t^b = (1 + R_t) \mu_t M_t$ as dividends to households, where μ_t is the growth rate of money,

$$\mu_t \equiv \frac{M_{t+1} - M_t}{M_t} = \frac{\mathcal{M}_t}{M_t} \tag{1.27}$$

which is assumed to follow an AR(1) stochastic process[14]

$$\mu_t = (1 - \rho_\mu) \mu + \rho_\mu \mu_{t-1} + \varepsilon_{\mu_t}, \tag{1.28}$$

with $0 < \rho_\mu < 1$ and ε_{μ_t} is an i.i.d. shock to μ_t with zero mean and standard deviation σ_μ. The random variable ε_{μ_t} is assumed to be orthogonal to all other variables in the model.

1.2.4 Households

The economy is populated by a continuum of homogeneous households of unit measure. These agents value alternative stochastic streams of a (composite) consumption good C_t and labor L_t^s, according to the following lifetime expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U (C_t, 1 - L_t^s) \tag{1.29}$$

where $\beta > 0$ represents households' intertemporal discount factor. Here, E_t denotes the expectation operator conditional on the information at date t. Throughout the Chapter, it

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[14]Christiano, Eichenbaum and Evans (1998) show that this is a good approximation when money is measured by broad monetary aggregates such as M2, but when the concept of money refers to M1 or even the monetary base, the monetary policy shock is better represented by a second order MA process.

is assumed that the function $U(\cdot)$ is given by

$$U\left(C_t, L_t^s\right) = \begin{cases} \frac{\left[C_t^\gamma (1-L_t^s)^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma} & \text{for } \sigma \neq 1 \\ \\ \gamma \log\left(C_t\right) + (1-\gamma) \log\left(1 - L_t^s\right) & \text{for } \sigma = 1 \end{cases} \tag{1.30a}$$

Here $1 - L_t$ denotes the quantity of leisure time, and the total time for work -the time endowment- is set at 1. The curvature parameter σ measures the relative risk aversion. The parameter γ is a scalar between 0 and 1 and it represents the consumption expenditure share in the utility function.

The representative household begins period t holding an amount M_t of liquid assets that represent the economy's stock of money. At this point in time, it decides how much money is going to be deposited in a saving account, D_t. The remaining currency $M_t - D_t$, together with labor income, will be used to finance purchases of a consumption good. Therefore, the household faces the following cash constraint in the final goods market:

$$\mathbf{P}_t C_t \leq M_t - D_t + W_t L_t^s \tag{1.31}$$

where L_t^s represents the fraction of time actually devoted to work and W_t is the wage paid in the competitive labor market for each unit of time supplied. Importantly, portfolio decisions take place before the realization of the monetary shock. As a result, the equilibrium rate of interest falls, and output and employment rises. Income for the household is derived from several sources: labor income, $W_t L_t^s$, which are the only source of income available to finance current period transactions; profits from financial intermediaries Π^b and from input firms Π^f, and rents from bank deposits. Thus, the stock of money, M_{t+1}, in the hands of the household at the end of period t is given by

$$M_{t+1} \leq M_t - D_t + W_t L_t^s - \mathbf{P}_t C_t + \left(1 + R_t^d\right) D_t + \Pi^f + \Pi^b. \tag{1.32}$$

In summary, the household's problem is to maximize (1.29) subject to (1.31)-(2.10) by choice of contingency plans for $\{C_t, D_t, L_t^s\}_{t=0}^{\infty}$ given the stochastic process for

$$\left\{ P_t, \mathbf{P}_t, W_t, R_t^L, R_t^d, \Pi_t^f, \Pi_t^b \right\}_{t=0}^{\infty}$$

with expectations formed rationally under the assumed information structure. Moreover, the household must respect the constraint $0 \leq M_t - D_t \leq M_{t-1}$. The first order conditions to the previous problem are represented by a set of Euler equations together with some appropriate boundary conditions. It is assumed that the conditions for an interior solution are satisfied, and thus the cash in advance constraint (1.31) and the money stock equation (2.10) are binding. Next, I proceed to summarize these conditions.

Lemma 4 *The optimal behavior of the household is characterized as follows: the optimal consumption and labor decisions are given by*

$$-\frac{U_{L_t}}{U_{C_t}} = \frac{W_t}{\mathbf{P}_t} \tag{1.33}$$

and the optimal portfolio choice

$$E_{\Omega_{0,t}} \left\{ \frac{U_{C_t}}{\mathbf{P}_t} - \frac{\beta \left(1 + R_t^d\right) U_{c,t+1}}{\mathbf{P}_{t+1}} \right\} = 0, \tag{1.34}$$

where U_c and U_L denote the partial derivatives of U with respect to C and L respectively; from the Cash-in-Advance constraint, consumption is derived

$$C_t = \frac{M_t - D_t + W_t L_t^s}{\mathbf{P}_t}. \tag{1.35}$$

The formulation and results in this section are rather standard within the literature of limited participation models. Equation (1.33) governs the household's consumption and

labour decision. Equation (1.34) is associated with the household's portfolio decision. Note that the decision on deposits is made conditional on $\Omega_{0,t}$ which excludes the time period t shocks from the time t information set. Since households cannot immediately adjust their nominal savings, a monetary shock disproportionately affects banks reserves and, hence, the supply of loanable funds. This creates the liquidity effect. Formally, we can proceed as in Fuerst (1992) and write condition (1.34) as follows

$$\Lambda_t \equiv \left(1 + R_t^d\right) E_{\Omega_{1,t}} \left\{ \beta \frac{U_{c,t+1}}{\mathbf{P}_{t+1}} \right\} - \frac{U_{C_t}}{\mathbf{P}_t}$$

and

$$E_{\Omega_{0,t}} \left\{ \Lambda_t \right\} = 0. \tag{1.36}$$

Fuerst (1992) refers to Λ_t as the *liquidity effect*. One can think of it is as the difference, at date t, between the value of money in the goods market and its value in the loan market. When $\Lambda_t < 0$, money is more valuable in the goods market since households will be willing to borrow at a higher rate than if they had the opportunity to do so. In this case, the loan market is relatively liquid. The variable Λ_t would be zero if the households could choose the portfolios contemporaneously, as in the standard Cash-in-Advance model.[15] However, here it is zero only in expected value. Notice that it is possible to write the gross nominal interest rate as

$$\left(1 + R_t^d\right) = \frac{\frac{U_{C_t}}{\mathbf{P}_t} + \Lambda_t}{E_{\Omega_{1,t}} \left\{ \beta \frac{U_{c,t+1}}{\mathbf{P}_{t+1}} \right\}}, \tag{1.37}$$

so that a positive money shock (injected through the loan market) reduces the value of money in the loan market. As a result, Λ_t is negative and would reduce the nominal interest

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[15]This condition is $(1 + R_t) = \beta E_t \left\{ \frac{\mathbf{P}_{t+1}}{\mathbf{P}_t} \frac{U_{C_t}}{U_{c,t+1}} \right\}$ from where the Fisher Effect can be deduced.

rate. This effect is compensated by the anticipated inflation effect. Fisherian fundamentals hold only on average, not period by period.

1.3 Qualitative Properties

In this section I explore some of the insights and qualitative implications, with the corresponding intuition, that can be derived from the model economy presented above. To that end, I first describe the equilibrium that characterizes the economy. Next, I proceed with the analysis of the long run properties of the model, which are derived from stationary equilibrium. This latter concept of equilibrium will be the basis for the dynamic analysis that will be performed in the next section. I also study the influences of the parameters on the stationary equilibrium. This section ends with the implications of capacity utilization for the shape of the short-run dynamics.

1.3.1 The Competitive Equilibrium

A competitive equilibrium for this model can be defined in the usual way. Given the initial productive equipments K_0 and X_0, the initial monetary growth rate μ_0 with its corresponding stochastic process (1.28), a competitive equilibrium for the model economy described above can be stated as follows,

Definition 5 (Competitive Equilibrium) *The general equilibrium of the economy during any period $t \geq 0$ is determined by a stochastic process for prices $\left\{\mathbf{P}_t, P_t, R_t^L, R_t^d, W_t, \Delta_t\right\}_{t=0}^{\infty}$ a quantity vector $\left\{K_t, X_t, C_t, D_t, L_t, \mathcal{Y}_t\right\}_{t=0}^{\infty}$ and a proportion of firms $\left\{F\left(\tilde{v}_t\right)\right\}_{t=0}^{\infty}$ that result from the optimal choices (consistent with the available information) of the central bank, the*

households and the firms. In a competitive equilibrium these choices are required to be made

under rational expectations and consistent with the following market-clearing conditions:

$$\mathcal{Y}_t = C_t + K_{t+1} - (1 - \delta)K_t + \Phi$$

$$L_t^s = L_t^d$$

$$W_t L_t^d = D_t + \mathcal{M}_t$$

$$M_t^d = M_t^s$$

which represent the goods, labor, loans and money markets, respectively.

In the previous definition, the aggregate allocation and pricing functions depend on the relevant state. In particular, deposits, D_t, are a function of the information set $\Omega_{0,t}$ whereas all other price and allocation rules are elements of $\Omega_{1,t}$, where $\Omega_{0,t}$ and $\Omega_{1,t}$ are defined as above. Recall that financial intermediation is a costless activity and, hence, $R_t^L = R_t^d$. Moreover, at equilibrium, $\mathrm{F}(\tilde{v}_t)$ represents the proportion of firms that underuse their productive capacities, i.e., those for which $v_{j,t} \in [\underline{v}, \tilde{v}_t]$. The variable $\pi(\tilde{v}_t)$ weights this proportion of firms by the relative importance of their production in total output. An important feature of this equilibrium is its symmetry: all input firms j choose the same capacity level and take the same pricing decisions. With all prices identical, aggregate employment, denoted by L_t, is equal to individual expected employment levels (up to a scaling factor):

$$L_t = \frac{\left(\frac{P_t}{\mathbf{P}_t}\right)^{-\epsilon} \mathcal{Y}_t}{X_t^\alpha} \int_{\underline{v}}^{\tilde{v}_t} v_t \mathrm{dF}\left(v_t\right) + \frac{K_t}{X_t} \int_{\tilde{v}_t}^{\overline{v}} \mathrm{dF}\left(v_t\right) \tag{1.38}$$

where K_t and X_t stand for aggregate capital and capital/labor respectively at time $t-1$ and available at time t, and

$$\tilde{v}_t = \frac{\bar{Y}_t}{\mathcal{Y}_t \left(\frac{P_t}{\mathbf{P}_t}\right)^{-\epsilon}} \tag{1.39}$$

represents the ratio of productive capacity to expected demand for intermediate inputs. Notice that, as $v_{j,t} < \tilde{v}_t$, the aggregate productive capacity is underutilized at equilibrium. The individual capacity utilization rates are given by:

$$\mathcal{C}_{j,t} = \begin{cases} \left(\frac{P_t}{\mathbf{P}_t}\right)^{-\epsilon} \mathcal{Y}_t v_{j,t}/\bar{Y}_t & \text{if} \quad v_{j,t} \le \tilde{v}_t \\ 1 & \text{if} \quad v_{j,t} > \tilde{v}_t \end{cases} \tag{1.40}$$

which introduced into (1.7) yields the aggregate capacity utilization rate,

$$\mathcal{C}_t \equiv \frac{\mathcal{Y}_t}{\bar{Y}_t}. \tag{1.41}$$

For a given distribution $F(v_t)$ and thus given σ_v^2, there is a decreasing relationship between the capacity utilization rate, $\mathcal{C}_t$, and the weighted proportion of firms with idle resources, $\pi(\tilde{v}_t)$, which subsequently determines the mark-up rate. The aggregate capacity utilization rate is directly linked to the proportion of firms that produce at full capacity, $(1 - \pi(\tilde{v}_t))$. At given price elasticity of demand, ϵ, this implies a positive relationship between the capacity utilization and mark-up rates

$$\text{Mark-Up} \equiv \left(1 - \frac{1}{\epsilon \pi(\tilde{v}_t)}\right)^{-1}.$$

1.3.2 Implications for Short Run Dynamics

Next, a diagrammatic representation of the labor market equilibrium at given capacity level is presented. This will prove useful for understanding the short-run implications

of the model. Specifically, the diagrammatic apparatus will provide some intuition on the interactions between capacity utilization and markup variations in the short run. As a result, it will be very useful to understand why the short run effects of a same shock are expected to depend crucially on the value of the capacity utilization rate at the time the particular shock takes place.

In Figure 1.1, the upward sloping curve represents the aggregate labor supply schedule, as given in equation (1.33). The other curve, concave and sloping downwards, represents the macroeconomic labor demand curve given in equation (1.38). In the very short run, at given capacity, the labor demand curve intersects both axes. The intersection with the horizontal axis is due to the fact that even at zero real wage rates, the short-run demand for labor is bounded above by the maximum number of work stations corresponding to the full employment of installed capacities.

Notice that when $\tilde{v} \to \underline{v}$, equation (1.38) reduces to the following expression:

$$L_t^d = \frac{K_t}{X_t}. \tag{1.42}$$

In the opposite case, when all firms have idle resources, and thus underutilize their productive capacities, the proportion of firms $\pi(\tilde{v}_t) = 1$ and the real wage rate given in (1.16) becomes,

$$\frac{W_t}{P_t} = \left(1 - \frac{1}{\epsilon}\right) \frac{X_t^\alpha}{\left(1 + R_t^L\right)}. \tag{1.43}$$

Along the short-run labor demand curve there is a negative relationship between the demand elasticity of sales, $\pi(\tilde{v}_t)$, and employment, L_t. Also, a downward shift along the short-run labor demand curve increases the mark-up, since the proportion of firms at full

capacity is larger and so is the spill-over effect from constrained to unconstrained firms. The implications of a monetary policy shock on the response of the labor market are shown in Figure 1.2. An unanticipated expansionary monetary policy shock leads to a reduction in the short-term nominal interest rate through the *liquidity effect*. This implies that the maximum feasible real wage rate increases. The short-run labor demand curve intersects now the vertical axis at a higher value. The labor supply curve is not affected by the monetary shock, since wages enter into the cash-in-advance constraint. As a result, the equilibrium in the labor market implies a rise in employment. The number of firms producing at full capacity also increases. This fact produces a positive spillover into the remaining firms that have idle resources. The market power of these firms naturally rises and hence does the mark-up in the economy.

The capacity utilization rate also moves in the same direction. It is important to notice that the effects of the monetary disturbance are going to depend crucially on the state of the economy at the time of the shock, with the state determined by the capacity utilization rate. Hence, further reductions in the nominal interest rate achieved through expansionary policies will have less impact on employment and, as will be shown later, a higher effect on prices.

An important shortcoming of the previous intuition about the short-run effects of a monetary shock is that it is based on an exogenous movement in the interest rate. However, the equilibrium rate of interest is determined jointly with other variables in the model such as employment and output. The results below aim at providing a general equilibrium insight into this issue.

Proposition 6 *The impact effect of an unanticipated monetary policy shock on employment*

is positive,

$$L_{\mu,t} \equiv \frac{d \log L_t}{d \log \mu_t} \equiv \frac{dL_t}{d\mu_t} \frac{\mu_t}{L_t} > 0$$

as is the instantaneous correlation with output

$$\mathcal{Y}_{\mu,t} \equiv \frac{d \log \mathcal{Y}_t}{d \log \mu_t} \equiv \frac{d\mathcal{Y}_t}{d\mu_t} \frac{\mu_t}{\mathcal{Y}_t} > 0$$

Proof. Taking the ratio of the loan market-clearing condition, $W_t L_t = D_t + \mathcal{M}_t$

to the cash equation, equation (1.31), one obtains

$$\Gamma_t = \frac{W_t}{P_t} \frac{L_t}{C_t} = \frac{D_t + \mathcal{M}_t}{M_t + \mathcal{M}_t}. \tag{1.44}$$

Notice that since $D_t < M_t$ and both variables are predetermined relative to $\mathcal{M}_t$, the response

of Γ_t to an innovation in the rate of growth of money $\mu_t \equiv \mathcal{M}_t / M_t$ is positive, that is,

$$\Gamma_{\mu,t} \equiv \frac{d\Gamma_t}{d\mu_t} = \frac{M_t - D_t}{\left[M_t \left(1 + \mu_t \right) \right]^2} > 0 \tag{1.45}$$

which establishes, for example, that a monetary contraction creates a relative shortage of

liquidity in the financial market.

Now, introducing (1.33) into (1.44) one gets

$$\Gamma_t = -\frac{U_{L_t}}{U_{C_t}} \frac{L_t}{C_t}; \tag{1.46}$$

but from (1.30a)

$$-\frac{U_{L_t}}{U_{C_t}} = \left(\frac{\gamma}{1 - \gamma} \right) \frac{C_t}{(1 - L_t)} \tag{1.47}$$

so that

$$\Gamma_t = \left(\frac{\gamma}{1 - \gamma} \right) \frac{L_t}{(1 - L_t)}. \tag{1.48}$$

Differentiating implicitly the previous equation yields

$$L_{\mu,t} \equiv \frac{d \log L_t}{d \log \mu_t} = \left(\frac{1-\gamma}{\gamma} \right) \frac{\mu_t \Gamma_{\mu,t}}{L_t / \left(1 - L_t \right)^2} > 0 \qquad (1.49)$$

From (1.4) and (1.12), final output is a function that depends positively on employment

$$\mathcal{Y}_t = \left[\int_{\underline{v}}^{\bar{v}} \left(X^\alpha L_t \right)^{\frac{\epsilon-1}{\epsilon}} v_t^{\frac{1}{\epsilon}} \, \mathrm{dF}\left(v \right) \right]^{\frac{\epsilon}{\epsilon-1}} . \qquad (1.50)$$

Hence, a positive (negative) monetary shock increases (reduces) output in the short run, that is,

$$sign \left(\frac{d \log \mathcal{Y}_t}{d \log \mu_t} \right) = sign \left(\frac{d \log L_t}{d \log \mu_t} \right) > 0 \qquad (1.51)$$

■

The previous result and the fact that, in the short-run, the level of installed equipment is fixed, imply a change in the capacity utilization rate in the economy. At the same time, the market power of those firms with idle resources is increased. This can be expressed, more formally, as follows:

Corollary 7 *In the short-run, an increase (decrease) in the equilibrium level of employment, due to an unanticipated change in the rate of growth of the money supply, rises (reduces) the mark-up, as well as the capacity utilization rate, whereas it decreases (increases) the price relation between intermediate and final goods.*

Proof. The capacity utilization rate was defined in equation (1.41) as

$$\mathcal{C}_t \equiv \frac{\mathcal{Y}_t}{\overline{Y}_t}.$$

Given that in the short-run $\bar{Y}_t$, the capacity level in the economy, is fixed and since from the previous proposition final output increases, it follows immediately that the capacity utilization rate increases.

From (1.3), the capacity utilization rate can be alternatively expressed as

$$\mathcal{C}_t = \frac{(P_t/\mathbf{P}_t)^\epsilon}{\tilde{v}_t}. \tag{1.52}$$

Substituting the price relation for its value given in (1.6), it follows that the capacity utilization rate depends only on $\tilde{v}_t$, the cut-off value of the idiosyncratic shock

$$\mathcal{C}_t \equiv \mathcal{C}\left(\tilde{v}_t\right) = \frac{1}{\tilde{v}_t} \left\{ \int_{\underline{v}}^{\tilde{v}_t} v \mathrm{dF}\left(v\right) + \tilde{v}_t^{\frac{\epsilon-1}{\epsilon}} \int_{\tilde{v}_t}^{\bar{v}} v^{\frac{1}{\epsilon}} \mathrm{dF}\left(v\right) \right\}^{\frac{\epsilon}{\epsilon-1}}. \tag{1.53}$$

Since, in the short-run, the capacity utilization rate increases after a positive monetary policy shock, the cut-off value $\tilde{v}_t$ must decrease (recall that the price relation $(P_t/\mathbf{P}_t)$, is a decreasing function of $\tilde{v}_t$).

Notice that the weighted proportion of firms with idle resources, $\pi\left(\tilde{v}_t\right)$, depends positively on $\tilde{v}_t$, as becomes clear after rewriting (1.18) as

$$\pi\left(\tilde{v}_t\right) = \left[1 + \frac{\int_{\tilde{v}_t}^{\bar{v}} \mathrm{dF}\left(v\right)}{\int_{\underline{v}}^{\tilde{v}_t} v \mathrm{dF}\left(v\right)} \right]^{-1} \tag{1.54}$$

Thus, the unanticipated monetary policy shock increases the mark-up since this variable, defined as,

$$\text{Mark-Up} \equiv \left(1 - \frac{1}{\epsilon \pi\left(\tilde{v}_t\right)} \right)^{-1} \tag{1.55}$$

depends negatively on the proportion $\pi\left(\tilde{v}_t\right)$. ∎

It is worthwhile stressing the highly non-linear relationship that exists between the mark-up and the capacity utilization rate. This means that in a high capacity economy, the

effect on the mark-up of an extra increase in the capacity utilization rate due, for instance, to a monetary policy shock will be higher than the effect of the same policy in a low capacity economy. Next, I analyze the response of the nominal interest rate and the real wage rate to an unanticipated monetary policy shock.

Proposition 8 *For a fixed level of investment and the utility function in (1.30a), the impact effect of an unanticipated monetary policy shock on the nominal interest rate is negative, while the real wage rate responds positively to the same shock. That is,*

$$R_{\mu,t} \equiv \frac{d \log \left(1 + R_t \right)}{d \log \mu_t} < 0$$

and

$$\left(W / \mathbf{P} \right)_{\mu,t} \equiv \frac{d \log \left(W_t / \mathbf{P}_t \right)}{d \log \mu_t} > 0.$$

Proof. From (1.33) and (1.47), the equilibrium real wage rate is given by

$$\frac{W_t}{\mathbf{P}_t} = \frac{\gamma}{1 - \gamma} \frac{C_t}{\left(1 - L_t \right)}. \tag{1.56}$$

Taking into account the final-goods market clearing condition, this can be expressed as

$$\frac{W_t}{\mathbf{P}_t} = \frac{\gamma}{1 - \gamma} \frac{\mathcal{Y}_t - K_{t+1} + (1 - \delta) K_t + \Phi}{\left(1 - L_t \right)} \tag{1.57}$$

from where

$$\frac{d \log \left(W_t / \mathbf{P}_t \right)}{d \log \mu_t} = \frac{\gamma}{1 - \gamma} \left(\frac{d \log \left(\mathcal{Y}_t \right)}{d \log \mu_t} - \frac{d \log \left(1 - L_t \right)}{d \log \mu_t} \right) > 0 \tag{1.58}$$

which is positive since in the previous proposition it was proven that employment and output are positively related to an unanticipated monetary policy shock.

Now, from (1.16) one can solve the gross nominal interest rate as a function of the mark-up, the ratio of intermediate to final-good prices and the wage rate as follows,

$$(1 + R_t) = \left(1 - \frac{1}{\epsilon \pi \left(\tilde{v}_t\right)}\right) \left(\frac{P_t}{\mathbf{P}_t}\right) \frac{X_t^\alpha}{W_t/\mathbf{P}_t}. \tag{1.59}$$

Consequently, the effect on the nominal interest rate due to the unanticipated monetary shock acts through three channels, namely the mark-up, the price relation and the real wage rate:

$$\frac{d \log \left(1 + R_t\right)}{d \log \mu_t} = \frac{d \log \left(1 - 1/\epsilon \pi \left(\tilde{v}_t\right)\right)}{d \log \mu_t} + \frac{d \log \left(P_t/\mathbf{P}_t\right)}{d \log \mu_t} - X_t^\alpha \frac{d \log \left(W_t/\mathbf{P}_t\right)}{d \log \mu_t} \tag{1.60}$$

The monetary shock rises employment and final output implying a reduction in the real wage rate. As discussed above, higher employment levels imply a higher capacity utilization rate and other related variables such as the mark-up and the proportion of firms producing at full capacity, $1 - \pi \left(\tilde{v}_t\right)$, so that,

$$sign \left(\frac{d \log \left(1 - 1/\epsilon \pi \left(\tilde{v}_t\right)\right)}{d \log \mu_t}\right) < 0$$

Moreover, the price ratio decreases in response to an unanticipated monetary shock. This is so because from (1.6) this ratio depends only on the cut-off value $\tilde{v}_t$, which is negatively related with the equilibrium level of employment

$$sign \left(\frac{d \log \left(P_t/\mathbf{P}_t\right)}{d \log \mu_t}\right) < 0$$

From the discussion above, it follows that the derivative in (1.60) is negative. ∎

Altogether, this version of the model displays the *liquidity effect* of a money supply shock, as well as the other features that characterize monetary economies. As noted above,

the presence of capacity constraints as well as the monopolistic competitive environment provide a rich source of dynamics. In particular, the liquidity effect in (1.60) can be decomposed into three elements, the one corresponding to the change in the real wage, which also appears in standard limited participation models; the one corresponding to the change in the mark-up and the one corresponding to the change in the input-price relation. These last two elements are particular of the model presented here and provide the key determinant of the asymmetric effects of the monetary policy shocks. The implications of this feature of the model is discussed in more detail below.

Proposition 9 *The magnitude of the response of employment (output) and the real wage rate to an unanticipated change in the growth rate of money is negatively related with the capacity utilization rate at the time of the shock, whereas the opposite is true for the nominal rate of interest.*

Proof. The strategy of the proof is the following: first it is shown that the magnitude of the response of employment to the monetary shock depends negatively on the level of employment at the time of the shock; next it is proven that in a high (low) capacity economy employment will be higher (lower).

Recall that the ratio of funds passing through the loan market to funds passing through the goods market, defined in (1.44), can be expressed as a function of the of labor to leisure ratio,

$$\Gamma_t = \left(\frac{\gamma}{1-\gamma}\right) \frac{L_t}{(1-L_t)}.$$

Thus, the larger is the employment level, the larger is the corresponding ratio of funds Γ_t. This implies that for a given rate of growth of money, $\mu_t \equiv M_t/\mathcal{M}_t$ the ratio D_t/M_t is also

high since,

$$\Gamma_t = \frac{D_t + \mathcal{M}_t}{M_t + \mathcal{M}_t} = \frac{(D_t/M_t) + \mu}{1 + \mu}.$$

But a high value of Γ_t implies a low value of $(M_t - D_t)$ so that the change in the pool of funds passing through the financial intermediary that are lent to firms

$$\frac{d\Gamma_t}{d\mu_t} = \frac{M_t - D_t}{[M_t (1 + \mu_t)]^2} \tag{1.61}$$

will be low.

Next, it remains to be shown that in a high capacity utilization rate economy, employment is higher that in a low capacity utilization rate. To do this, notice that the capacity utilization rate, $\mathcal{C}_t$, was defined in equation (1.41) as the ratio of current output, $\mathcal{Y}_t$, to maximum output, $\bar{Y}_t$, that is,

$$\mathcal{C}_t \equiv \frac{\mathcal{Y}_t}{\bar{Y}_t}.$$

Since in the short-run the maximum level of output is fixed, a high level of capacity is obtained with a more intensive use of existing resources. This implies that current output will be higher. But this can be achieved only with a higher level of employment, since current output is given by

$$\mathcal{Y}_t = \left[\int_{\underline{v}}^{\bar{v}} (X^\alpha L_t)^{\frac{\epsilon-1}{\epsilon}} v_t^{\frac{1}{\epsilon}} \, \mathrm{dF}(v) \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{1.62}$$

Altogether, in the short run, a high level of capacity utilization rate implies a high level of employment, and thus the effect of the monetary shock on employment will be lower. The impact on the other macro variables follows from applying the results in the previous proposition. ∎

Next, it is shown how final prices and output respond asymmetrically to the unanticipated monetary policy shock. The result is illustrated in Figure 1.3 and the argument is as follows: assume first that the economy is in the equilibrium point (Y,P) and an unanticipated monetary policy shock takes place. The monetary shock shifts the short-run supply curve to the right from SS to $S'S'$, since it reduces production costs by driving down the equilibrium rate of interest. Notice that the supply curve is vertical at the maximum output level, Y^*. The money injection increases, at the same time, aggregate demand of the final good through the household's cash-in-advance constraint, from DD to $D'D'$. The new equilibrium is reached at (Y',P'). If a new monetary injection occurs, the supply moves to $S''S''$ and demand to $D''D''$. The intersection of both curves determines the equilibrium value of output and price level at (Y'',P''). The increase in prices will be higher than the corresponding increase in production when comparing the two equilibrium allocations and prices. This intuition is proved more formally in the following result.

Proposition 10 *The short-run response of the price level to an unanticipated monetary policy shock depends positively on the capacity utilization rate at the time of the shock.*

Proof. Combining the cash-in-advance constraint in equation (1.31), evaluated at equality, and the loan market-clearing condition, $W_t L_t = D_t + \mathcal{M}_t$, one obtains

$$P_t C_t = M_t + \mathcal{M}_t.$$

Making use of the goods market-clearing condition, consumption can be substituted out, which yields

$$P_t \left(\mathcal{Y}_t - K_{t+1} + (1 - \delta)K_t - \Phi \right) = M_t + \mathcal{M}_t$$

assuming that capital is kept constant, since the focus is on the intra-temporal response of the variables and noting that $\mu_t \equiv \mathcal{M}_t/M_t$, the previous equation becomes

$$P_t \mathcal{Y}_t = 1 + \mu_t. \tag{1.63}$$

Taking logarithms in the previous equation and differentiating it with respect to the gross rate of monetary growth, it follows that

$$\frac{d\log(\mathbf{P}_t)}{d\log(1 + \mu_t)} + \frac{d\log(\mathcal{Y}_t)}{d\log(1 + \mu_t)} = 1. \tag{1.64}$$

Now, from the previous results the response of output was shown to be positive and negatively related to the capacity utilization rate of the economy. Hence, the response of prices is larger the smaller is the effect on output. $\blacksquare$

Up until now, the short-run or impact effects of a monetary shock have been explored. However, in order to explore the dynamics of the model, it is necessary to determine the equilibrium laws of motion of the theoretical economy by means of a numerical approximation algorithm. This is precisely the objective of the next section, where the quantitative properties of the model are evaluated and simulation exercises are performed as well.

1.4 Quantitative Analysis

In this section, I describe the quantitative properties of the model economy. The objective is to illustrate the interactions between capacity utilization and mark-up rate changes by analyzing numerically the dynamic behavior of some key macroeconomic variables in response to a monetary shock. One of the results I pursue is to show how the same shock can have significantly different short run effects depending on the characteristics of

the economy at the time the shock occurs. The variable of reference is the level of the capacity utilization rate. In order to compute the impulse response functions, the model has to be solved numerically. The solution method adopted is based on a linear approximation of the equilibrium policy rules about the non-stochastic steady state.

1.4.1 Parameter Values

The model is calibrated to match the long-run properties of the post-war US time-series with the non-stochastic steady state values of the model. The parameterization follows, in some extent, Christiano, Eichenbaum and Evans (1998) and Fagnart, Licandro and Portier (1999). The time period is one quarter. The parameter for preferences and technology are assigned values that are standard in the business cycle literature. Table 1.1 summarizes the values of the calibrated parameters which are described in the sequel. The discount factor is set at $(\beta) = (1.03)^{-0.25}$; the utility parameter is chosen so that one third of the time endowment in the steady state corresponds to labor, hence, the consumption expenditure share in the utility function $(\gamma) = 0.35$; the relative risk aversion $(\sigma) = 2$. Model calibration requires that capital's share on aggregate income $(\alpha) = 0.3485$; the annual depreciation rate of 10% implies a value $(\delta) = 0.018$; the elasticity of intermediate goods is chosen to obtain a markup ratio of 1.7 and thus, $(\epsilon) = 8.7364$; the fixed cost that assures zero monopolistic profits is $\Phi = 0.1057$. I deal, in what follows, with the calibration of the aggregate uncertainty components. As stated above, I will follow the common practice in the related research by assuming an AR(1) process for the mean growth rate of money. In particular, the mean growth rate of money $(\mu) = 0.016$, a value that corresponds to the mean quarterly growth rate of the monetary base in the U.S. as obtained in Cook (1999)

for the period 1970:1-1995:1. The persistence of the monetary shock $(\rho_\mu) = 0.32$ with the standard deviation of $(\sigma_\mu) = 0.0038$.

The structural parameters that determine the aggregate capacity utilization rate are two: the variance of the idiosyncratic shock and the degree of substitutability among intermediate goods. These parameters are chosen in order to reproduce two different situations, each featuring a different long-run capacity utilization rate. In this manner, it will be possible to study how different the dynamic properties of the model under these two different scenarios are. Specifically, the high capacity economy is characterized by a low variability of the idiosyncratic shock, $\sigma_v^2 = 0.25$, and a high value of input substitutability, $\epsilon = 15$. The opposite is true for the low capacity economy, that is $\sigma_v^2 = 1.75$ and $\epsilon = 4.85$. The steady state properties of the fully parameterized model under different scenarios are summarized in Table 1.2.

1.4.2   Dynamic Properties

Recall that the main objective of this Chapter is to provide a formal theoretical background to the recently documented asymmetric responses of key macroeconomic variables to unanticipated monetary policy shocks. In this sense, it is studied whether the level of utilization of the productive capacity of the economy alters the dynamic properties of the model. To achieve this target, the equilibrium laws of motion of prices and quantities are approximated using the undetermined coefficients method described in Christiano (1998). Specifically, the model is linearized about the non-stochastic steady state and the impulse responses computed next. The impulse response functions represent the response, over time, of the elements of the endogenous variables to a pulse in one of the elements of the

vector of stochastic innovations. An important characteristic for a good model to have is its ability to reproduce real world's response to simple monetary policy experiments. This section reports the dynamic responses of selected variables in the model to a one percent increase in the gross rate of monetary growth in period 3, from the process

$$\mu_t = (1 - \rho_\mu) \, \mu + \rho_\mu \mu_{t-1} + \varepsilon_{\mu_t} \text{ with } \sigma_\mu = 0.0032. \tag{1.65}$$

Despite the one-time nature of the shock, the growth rate of money will stay above trend for several quarters given that the autocorrelation coefficient is $\rho_\mu = 0.32$. The impulse responses of the two parameterized models described in the previous section are compared. In this manner, it is possible to analyze the quantitative importance of the capacity utilization constraints as a source of asymmetry in the dynamics of the economy.

A number of results are worth noting here. First, both versions of the model are able to reproduce the stylized facts of monetary policy. According to many studies in the identified VAR literature, an expansive money supply shock leads to an increase of employment, aggregate output and real wages and to a decrease of nominal interest rates. A liquidity effect is found in the model in that the monetary shock leads to a decrease in nominal interest rates and an increase in capital and labor. The capital/labor ratio also increases after the shock, thus, the maximum level of employment available in the period after the shock decreases, and likewise the maximum volume of sales of input firms. This negative effect is compensated by an increase in firms' labor productivity, something that has a favorable effect on their competitive position in case of excess capacities. The liquidity effect also causes output to rise immediately. Employment and investment respond to the policy shock much like output. Another important feature of the result is that real

wages rise after a positive money shock. The real wage rate exerts upward pressure on the marginal cost of hiring labor, which had declined in the impact period because of the lower interest rates. Lower production costs push input prices downwards leading to an increase in input demands and inducing to a more extensive use of productive capacities in all the firms with idle productive resources. Reflecting the dynamics of output, prices initially rise and later decrease to slowly return to its non-stochastic steady state value.[16] The mark-up and the capacity utilization rates increase, whereas the weighted proportion of firms underusing their resources falls. The dynamics of these variables, in particular the increase in mark-ups induced by the higher capacity utilization, will partially offset the reduction in input prices. Consequently, the magnitude of the response of variables such as prices, output an employment, will depend crucially on the magnitude of the response of the mark-up. Recall that the response of this variable is closely related to the proportion of firms producing at full capacity. When capacity is high, the spill-over effect described above is high and thus is the market power and mark-up. Consequently, in situations of high capacity, and implied high mark-up's, the *liquidity effect* is to some extent augmented by a *capacity effect*. This is the source of asymmetry that is found in the responses of the main macroeconomic variables of this model.

The important result of this exercise is that the responses of the endogenous variables to the monetary shock depend crucially on the extent to which real resources are used. Panel (a) of Figures 4 to 13 show the impulse responses to an unanticipated shock happening in the third quarter, whereas Panel (b) shows the impact effect of the same shock

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[16]In this version of the model, the dynamic response functions of the endogenous variables lack persistence. For instance, output and employment do not display the delayed hump shape response that the estimated response functions exhibit as reported in Chirstiano et al. (1998).

for a level of capacity utilization rate ranging, at the time of the monetary policy action, from 65% to 95%.[17] Notice that the response of output, labor, real wages and investment is stronger when the capacity utilization rate is low. Intuitively, when the economy experiences a low level of capacity utilization, an expansive monetary policy shock will lead to a strong increase in output since less firms are producing at full capacity. Thus, in the low capacity case, the constraints that are associated to the predetermined level of equipment are less restrictive for a large set of input producing firms. The resulting expansion in output is achieved with the subsequent increase in employment. Under this same environment, the response of the nominal interest rate is higher due to a strong liquidity effect. The equilibrium interest rate at which firms will accept the new currency is much lower when more firms cannot increase their production due to the existence of capacity utilization constraints. Notice also the highly non-linear path that the impact response of this variable traces when the capacity utilization rate, at the time of the shock, increases. This result is, somehow, related to that of Cook (1999) who develops a model in which firms cannot transfer capital across sectors.

As expected, output prices are more sensible in a high capacity economy provided that investment adjusts sluggishly. The monetary shock also produces an impact change on some other important endogenous variables. The response of the capital-labor ratio is significantly different depending on the capacity utilization rate of the economy. The capacity utilization rate increases more and the price relation decreases more, the lower is the capacity at the time of the shock. The mark-up increases more in a high capacity economy,

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[17]The simulations are obtained by varying the parameters ϵ and σ_v in order to achieve a given capacity utilization rate. It must be pointed out that the results are independent of the specific combination chosen of those parameters.

and the weighted proportion of firms producing while having idle resources also decreases more in a high capacity economy. The dynamics of these variables also display a remarkably non-linear shape. It must be pointed out that the monetary shock does not produce a dynamic response in these capacity-related variables. This reflects the intraperiod nature of the real frictions embedded in the model. As a result, the qualitative characteristics of the contemporaneous impact do not extend beyond the period of the monetary policy shock. The asymmetric dynamics are not kept along time. Hence, a rather interesting extension of the model presented here is to achieve more persistency in the response of the endogenous variables to the monetary shock, for instance, by extending the intratemporal nature of the idiosyncratic shock towards an intertemporal dimension.

Finally, Figure 1.14 represents a Pseudo Phillips curve. Each point in this figure corresponds to a cumulated increase in the capacity and inflation gap due to a series of 1% unanticipated monetary policy shocks. It is relevant to see the non-linearity in such a relationship. For low levels of capacity utilization rate, the monetary policy shock exerts more pressure on real economic activity than on prices. As the economy moves toward a situation with a higher rates of aggregate capacity utilization, the effects of subsequent monetary policy shocks are comparatively more intense on prices. This result illustrates the direct link between the empirical findings described, for instance, in Chapter 2 of this Thesis with those coming from the theoretical model of this Chapter.

1.5 Concluding Remarks and Extensions

Despite the empirical evidence and the strong theoretical arguments, there is a lack of a general equilibrium approximation to the issue of asymmetries in the monetary macroeconomic literature. This Thesis aims at filling this gap, developing a quantitative model of the monetary transmission mechanism in this regard and analyzing its implications for the conduct of monetary policy. The overall message of this thesis for monetary policy is that the same central bank actions may have quantitatively different macroeconomic effects depending on the extent to which productive resources are being used, that is, depending on the capacity utilization rate in the economy. A dynamic stochastic general equilibrium model consistent with these facts is developed. Specifically, it has been considered the interaction of endogenous capacity utilization (derived from productive constraints and firm heterogeneity) and market power within a quantitative macroeconomic model of the monetary transmission mechanism. The monetary structure of the model assumes a Lucas-Fuerst 'limited participation' constraint. In the real side, the fact that firms face a positive probability of being producing at variable capacity provides credible microfoundations to the idea of *ex post* inflexibilities in production sector that have recently been the object of study in the related literature.

The source of the asymmetry is directly linked to the bottlenecks and stock-outs that emerge from the existence of capacity constraints in the real side of the economy. Hence, these constraints act as a source of amplification of monetary shocks and generates asymmetries in the response of key macroeconomic variables. These effects interact additionally with those emerging from the imperfectly-competitive environment that characterizes the

intermediate-good sector through optimal mark-up changes. Within the structure of the model, a non-walrasian pricing behavior in line with intertemporal 'sticky' price models could easily be incorporated and thus, follow the results of recent empirical evaluation exercises of dynamic stochastic general equilibrium models, such as those of Christiano, et al. (1997), where it is claimed that a combination of limited participation with sticky-price behavior could successfully account for the basic stylized facts observed in the data. Hence, developing a model equipped with both types of frictions will be of notably interest.

The quantitative analysis presented here focuses on the asymmetric effects of monetary policy, but there is an important issue that has to be considered: the timing of the response of the macroeconomic variables to the shock. In the model above, such response is immediate and there is a lack of propagation. Andolfatto et al. (2000) generate persistent liquidity effects assuming that individuals are not able to perfectly observe the current monetary policy shock. It will be interesting to incorporate this latter feature into the model presented here and see how the resulting outcome is.

The empirically plausible asymmetry of the Phillips curve, due to the fact that some firms find it difficult to increase their capacity to produce in the short run, is going to have important implications for the conduct of monetary policy. In this sense, Nobay and Peel (2000) have shown that the analysis of optimal discretionary monetary policy under a non-linear Phillips Curve yields results that are in marked contrast with those obtained under the conventional linear paradigm. All these particularities, that are likely to offer interesting insights into the monetary transmission mechanism, are worthwhile exploring using the analytical framework developed here.

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Table 1.1: Calibrated Parameters

|		
Consumption/leisure share	γ	0.35
Risk Aversion	σ	2
Intertemporal discount rate	β	$(1.03)^{-0.25}$
Capital share	α	0.3485
Depreciation rate	δ	0.018
Fixed costs	Φ	0.1057
Autocorrelation monetary shock	ρ_μ	0.32
Standard deviation monetary shock	σ_μ	0.0038
Mean monetary shock	μ	0.016

Table 1.2: Stationarity Properties

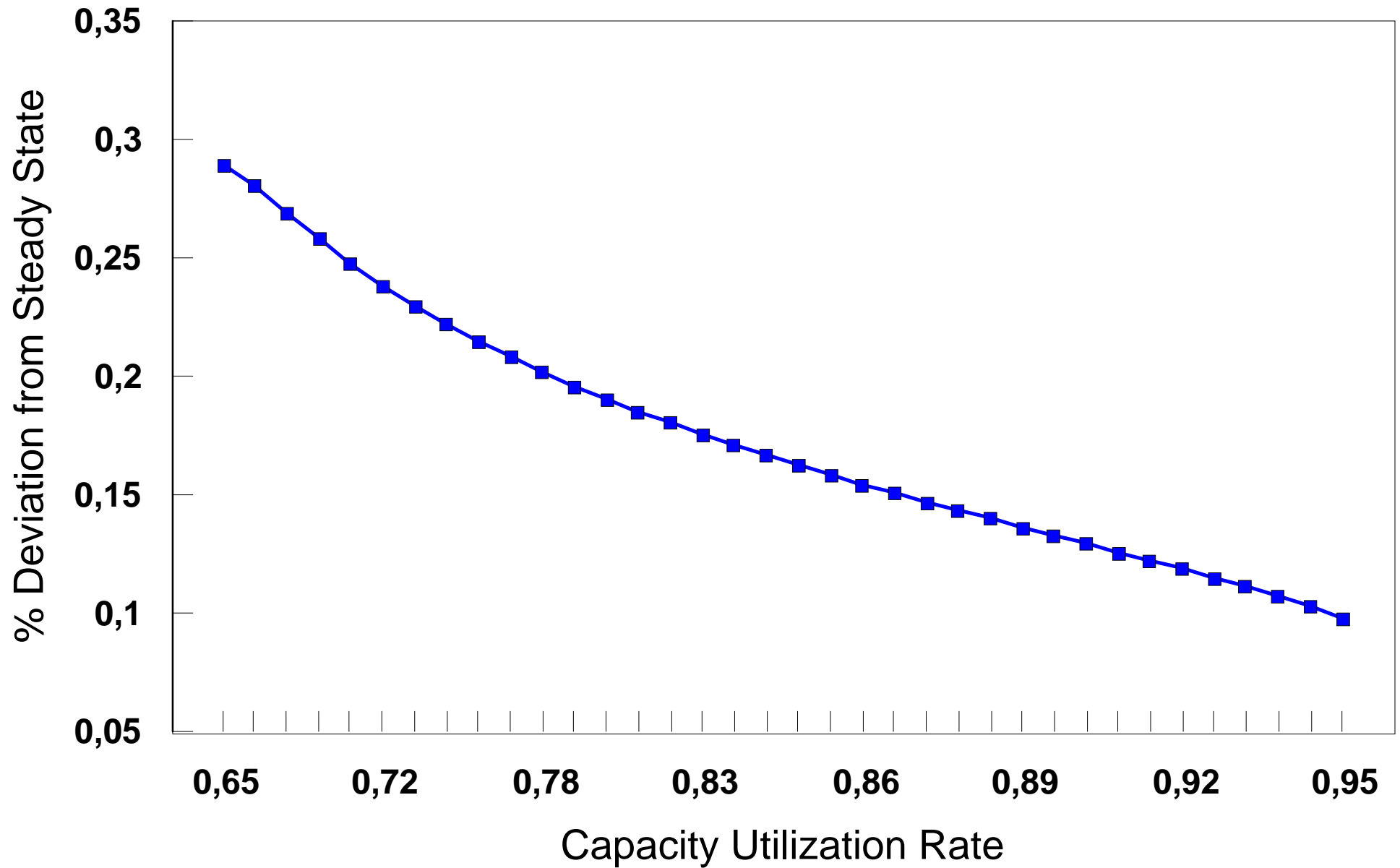|		Low Capacity: $\sigma_v^2 = 1.75$, $\epsilon = 4.85$	High Capacity: $\sigma_v^2 = 0.25$, $\epsilon = 15$
Output	$\mathcal{Y}$	0.87	1.23
Investment	$\delta K/\mathcal{Y}$	0.17	0.30
Consumption	C	0.59	0.83
Capital	K	9.48	16.70
Capital-Labor	X	20.24	50.49
Labor	L	0.32	0.32
Capital/output	$K/\mathcal{Y}$	10.88	13.49
Consumption/Output	$C/\mathcal{Y}$	0.69	0.67
Leisure-Labor Ratio	$(1-L)/L$	2.11	2.16
Mark-up	$(1 - 1/\epsilon\pi)^{-1}$	1.93	1.65
Monopolistic profits	$M.P./\mathcal{Y}$	0.48	0.13
Deposits	D/M	0.67	0.85
Price Relation	$\mathbf{P}/P$	0.87	0.96
Firms Full Capacity	$\mathrm{F}(v)$	0.39	0.60
Capacity utilization	$\mathcal{C}$	0.65	0.95
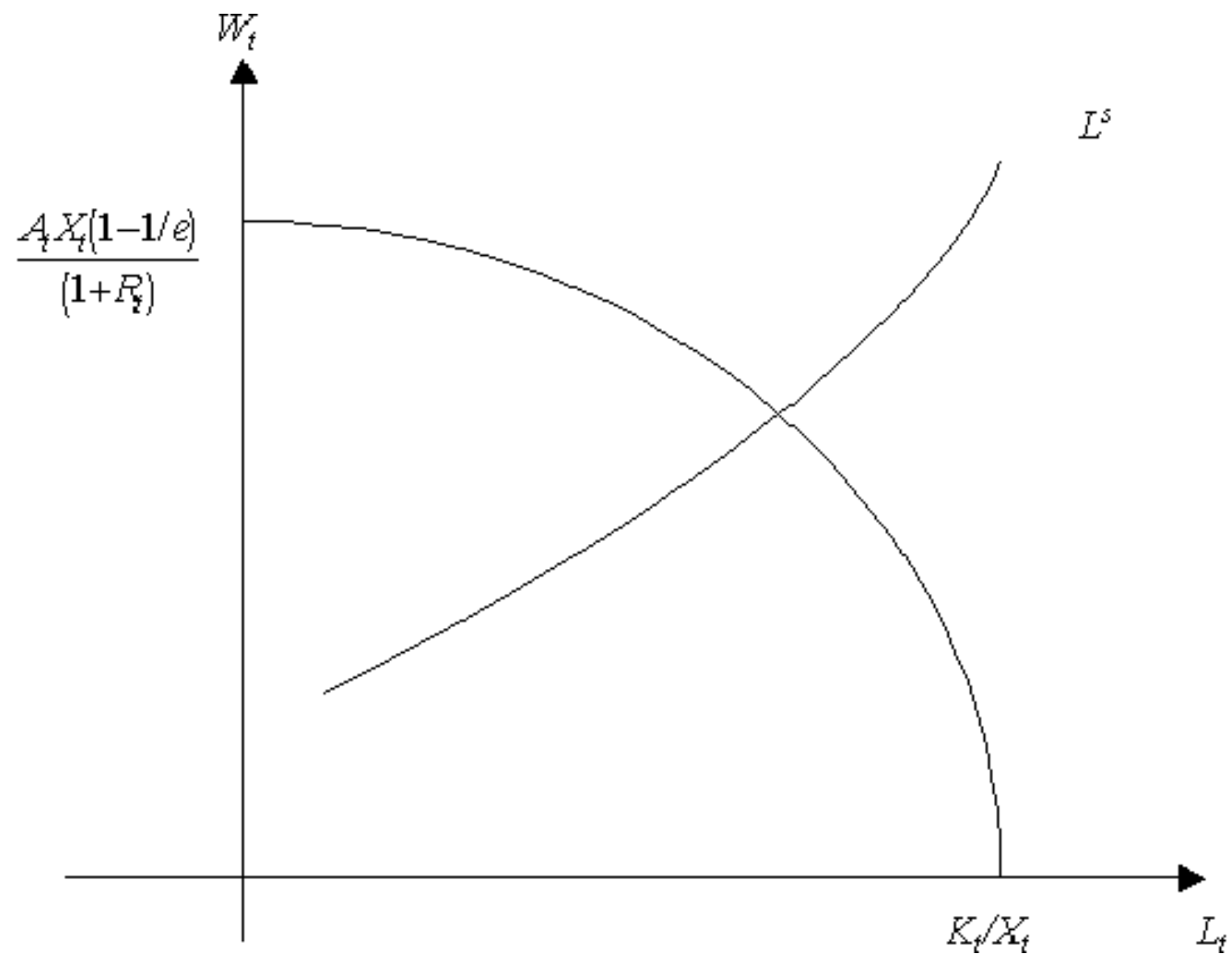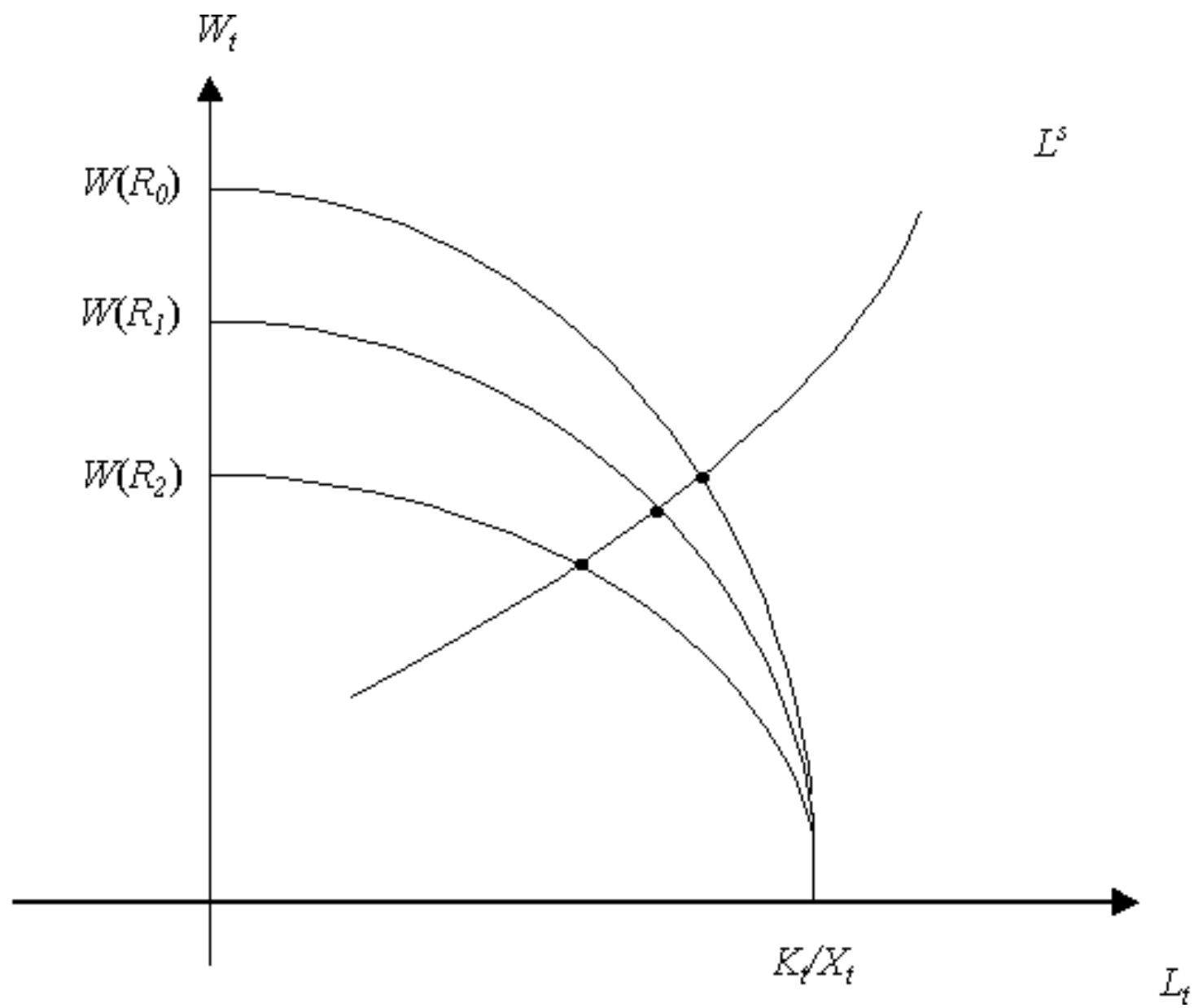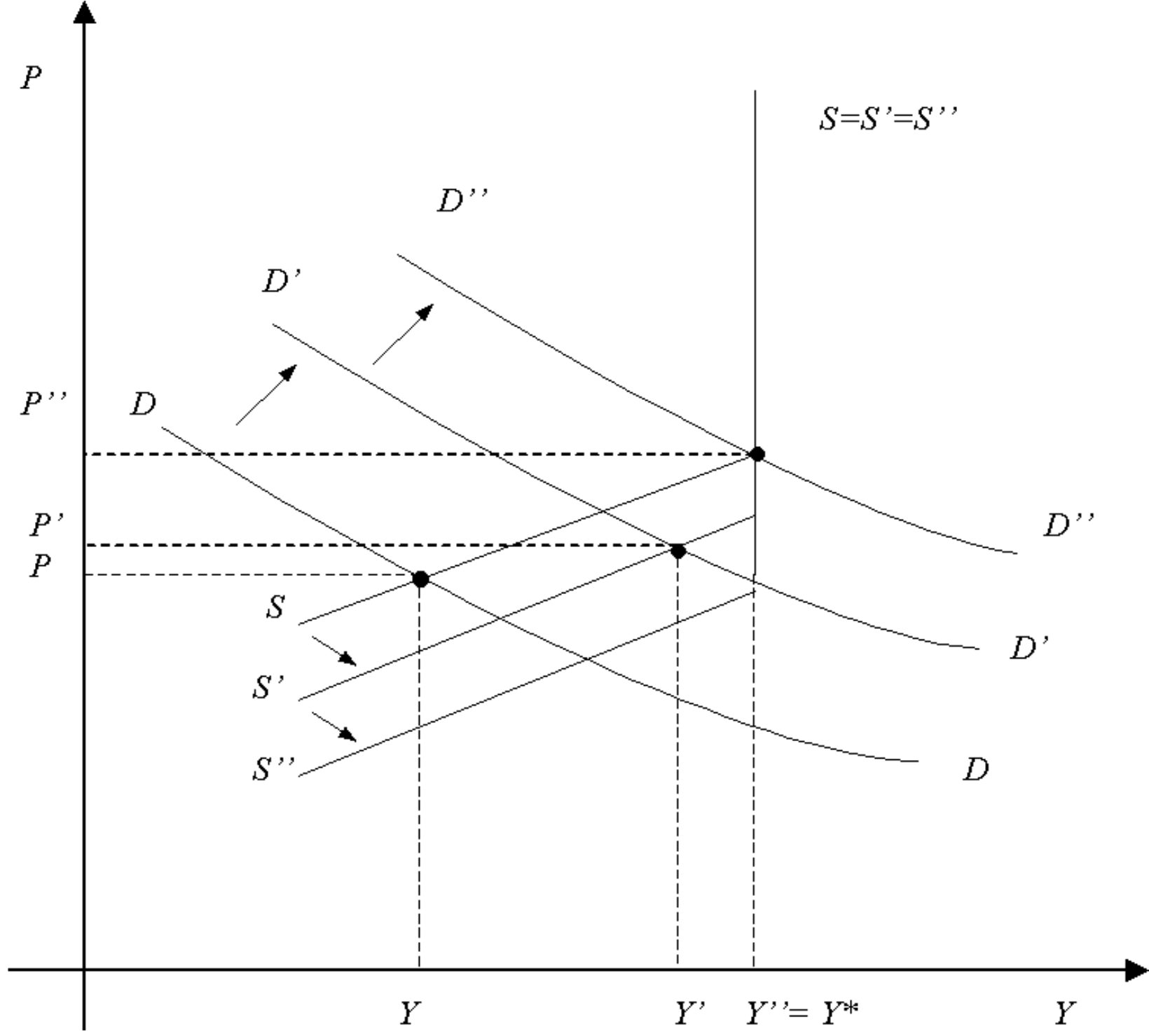
Impulse Responses to a 1% Shock in Money Growth Rate
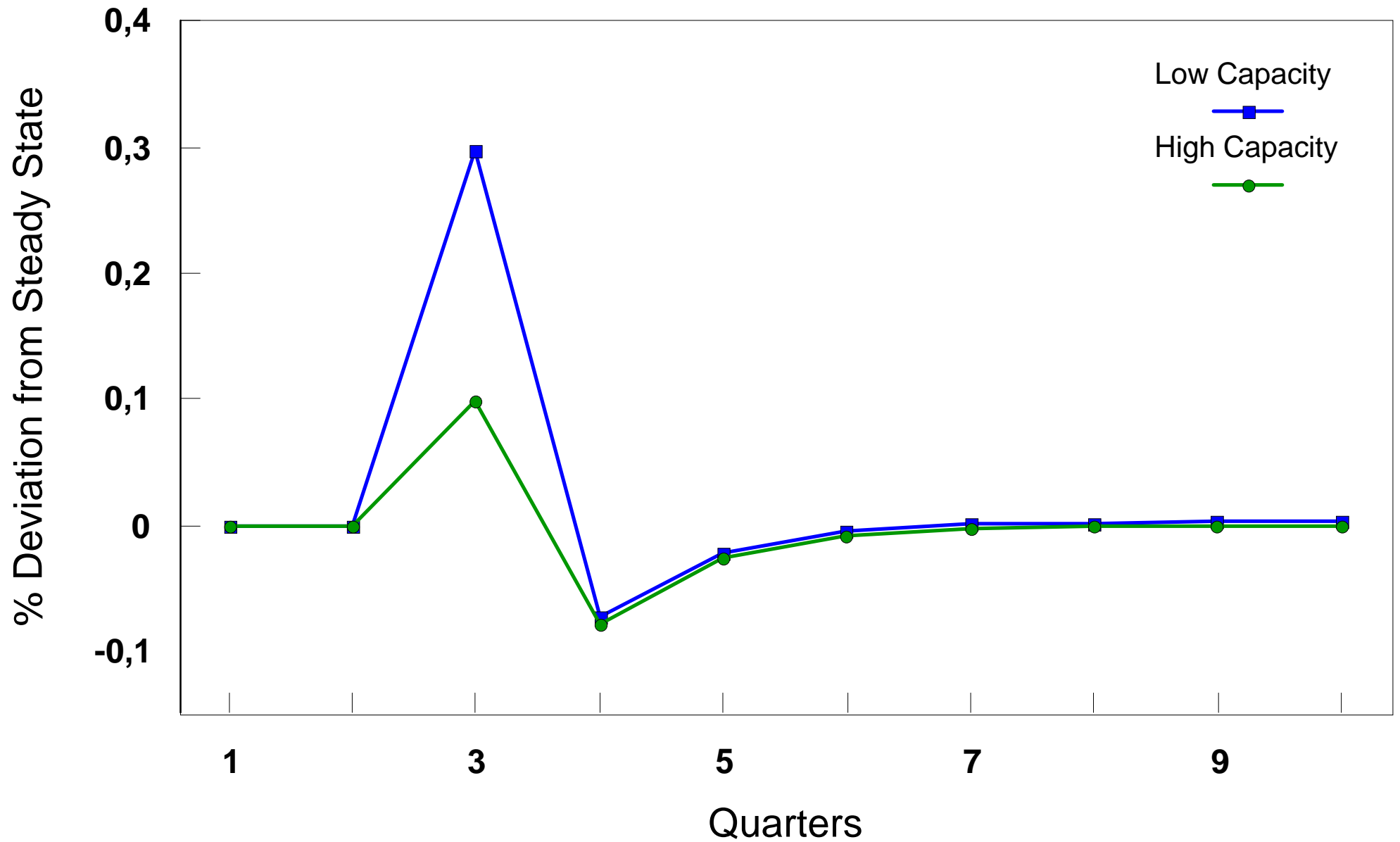
Impact Effect due to a 1% Shock in Money Growth Rate

Y-axis: % Deviation from Steady State
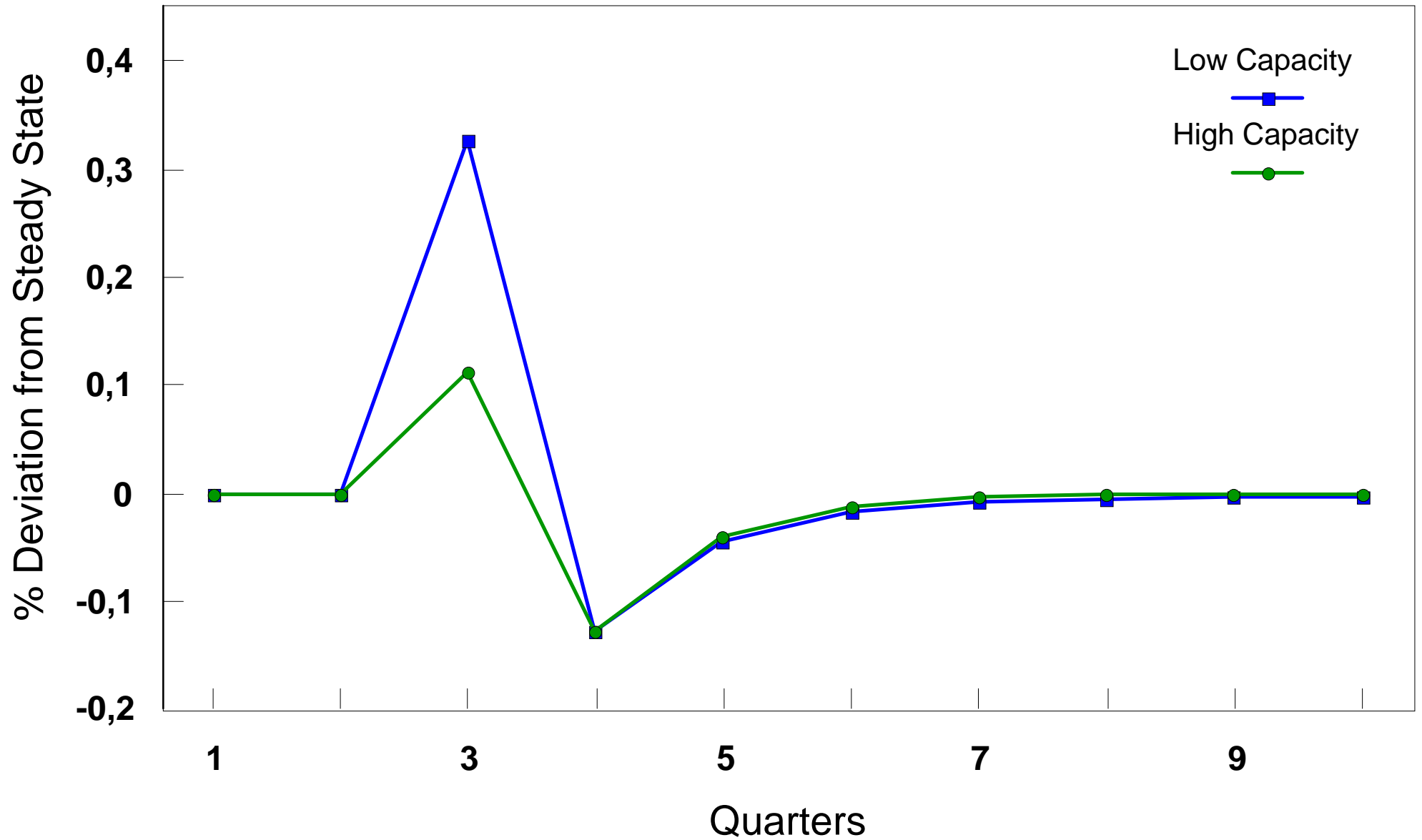
X-axis: Capacity Utilization Rate

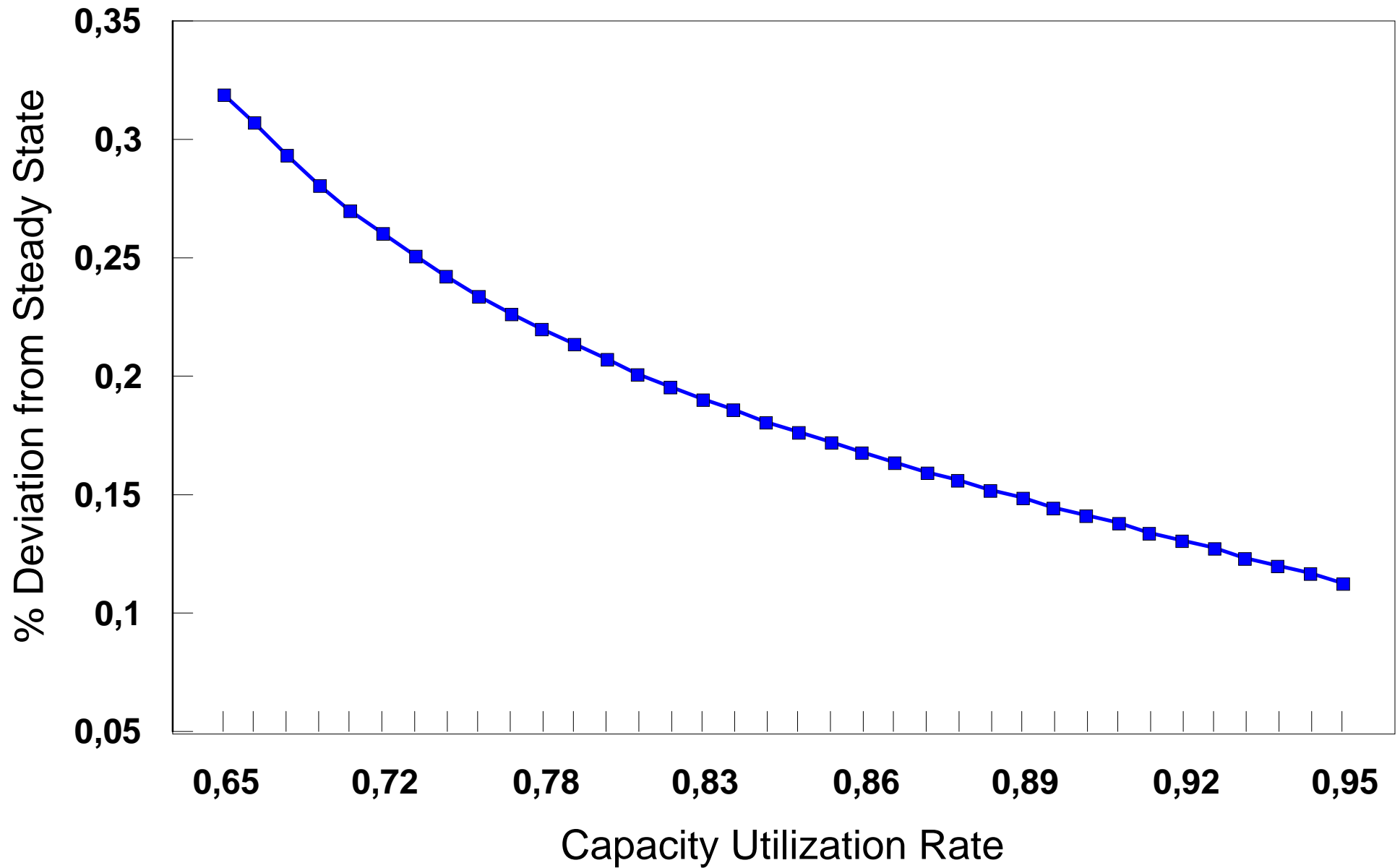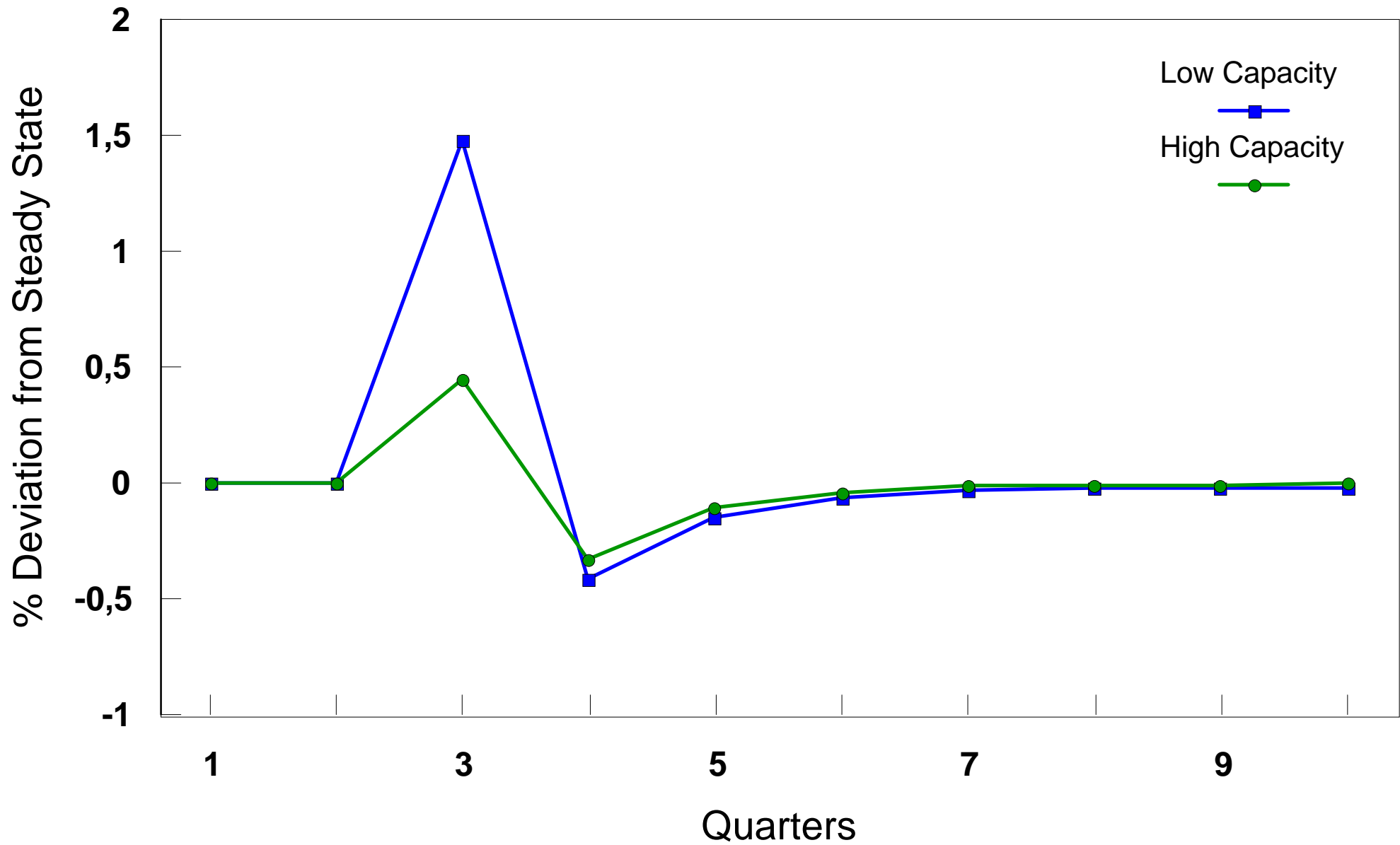**Impulse Responses to a 1% Shock in Money Growth Rate**

Impact Effect due to a 1% Shock in Money Growth Rate

Impulse Responses to a 1% Shock in Money Growth Rate

Impact Effect due to a 1% Shock in Money Growth Rate

% Deviation from Steady State

Capacity Utilization Rate

Impulse Responses to a 1% Shock in Money Growth Rate

Impact Effect due to a 1% Shock in Money Growth Rate

% Deviation from Steady State (y-axis): 0,11 – 0,19

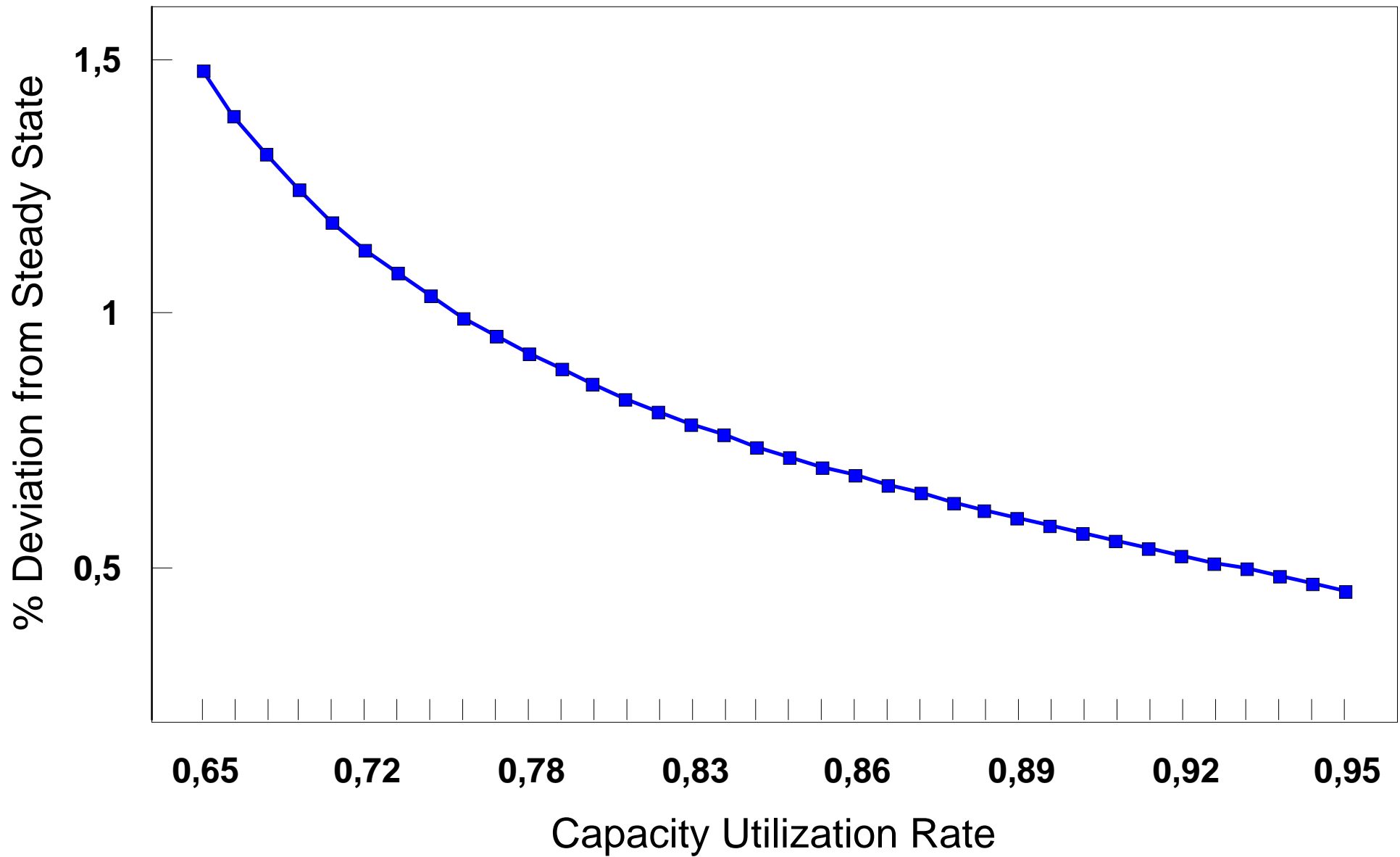Capacity Utilization Rate (x-axis): 0,65 – 0,95

Impulse Responses to a 1% Shock in Money Growth Rate

Impact Effect due to a 1% Shock in Money Growth Rate

Deviation from Steady State

Capacity Utilization Rate

Impulse Responses to a 1% Shock in Money Growth Rate

One-period-ahead Effect due to a 1% Shock in Money Growth Rate

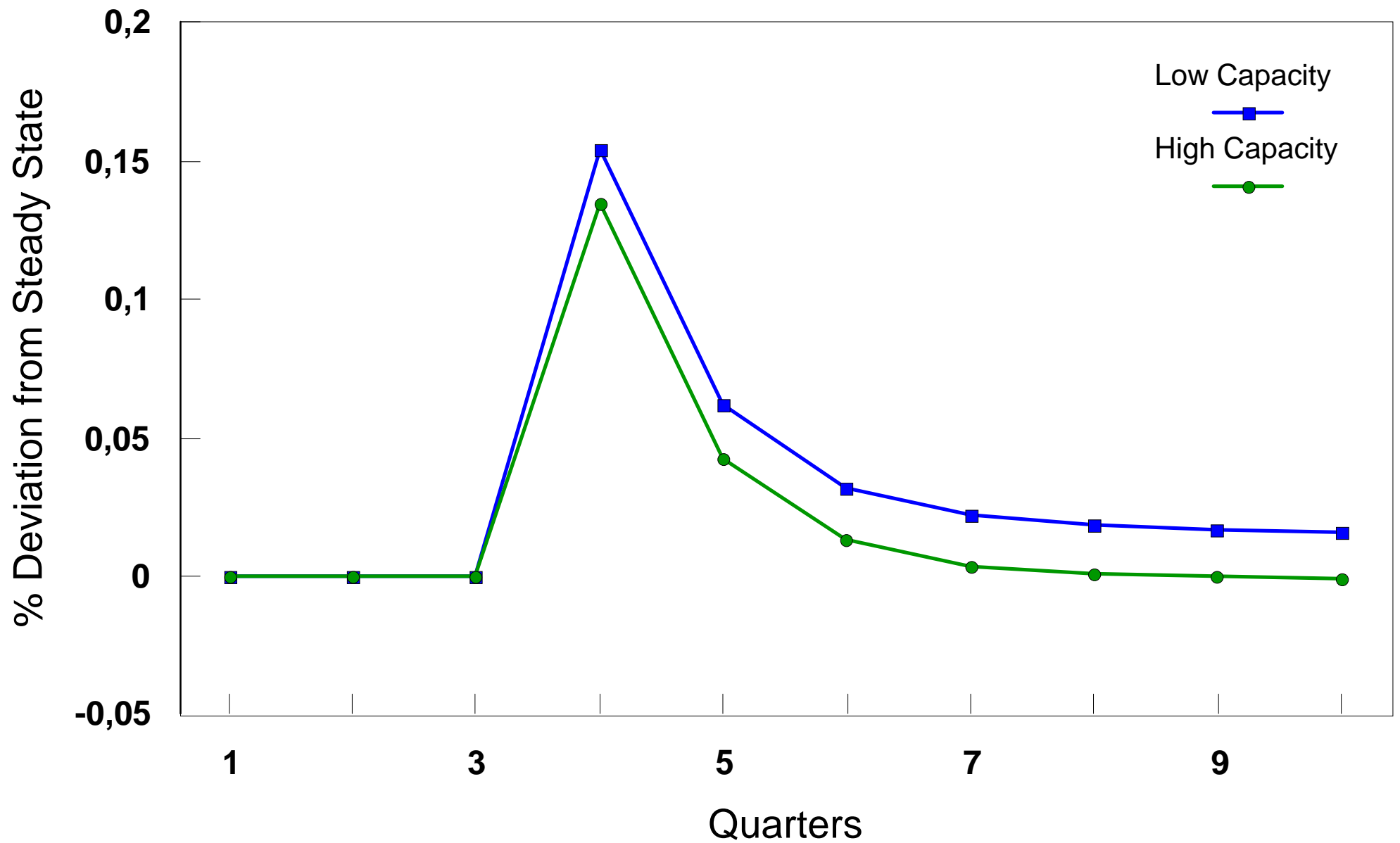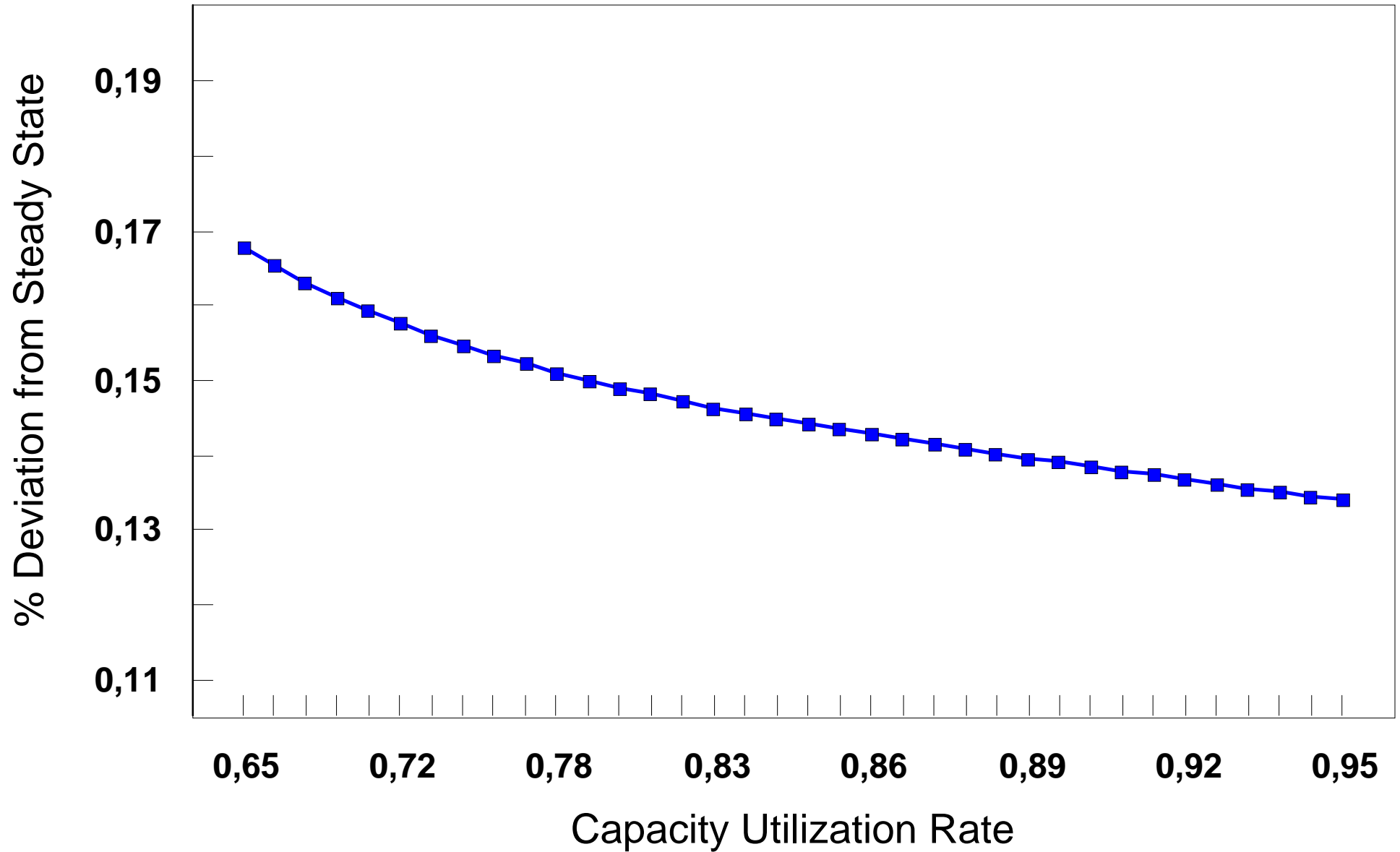**Impulse Responses to a 1% Shock in Money Growth Rate**

Impact Effect due to a 1% Shock in Money Growth Rate

Impulse Responses to a 1% Shock in Money Growth Rate

Impact Effect due to a 1% Shock in Money Growth Rate

% Deviation from Steady State
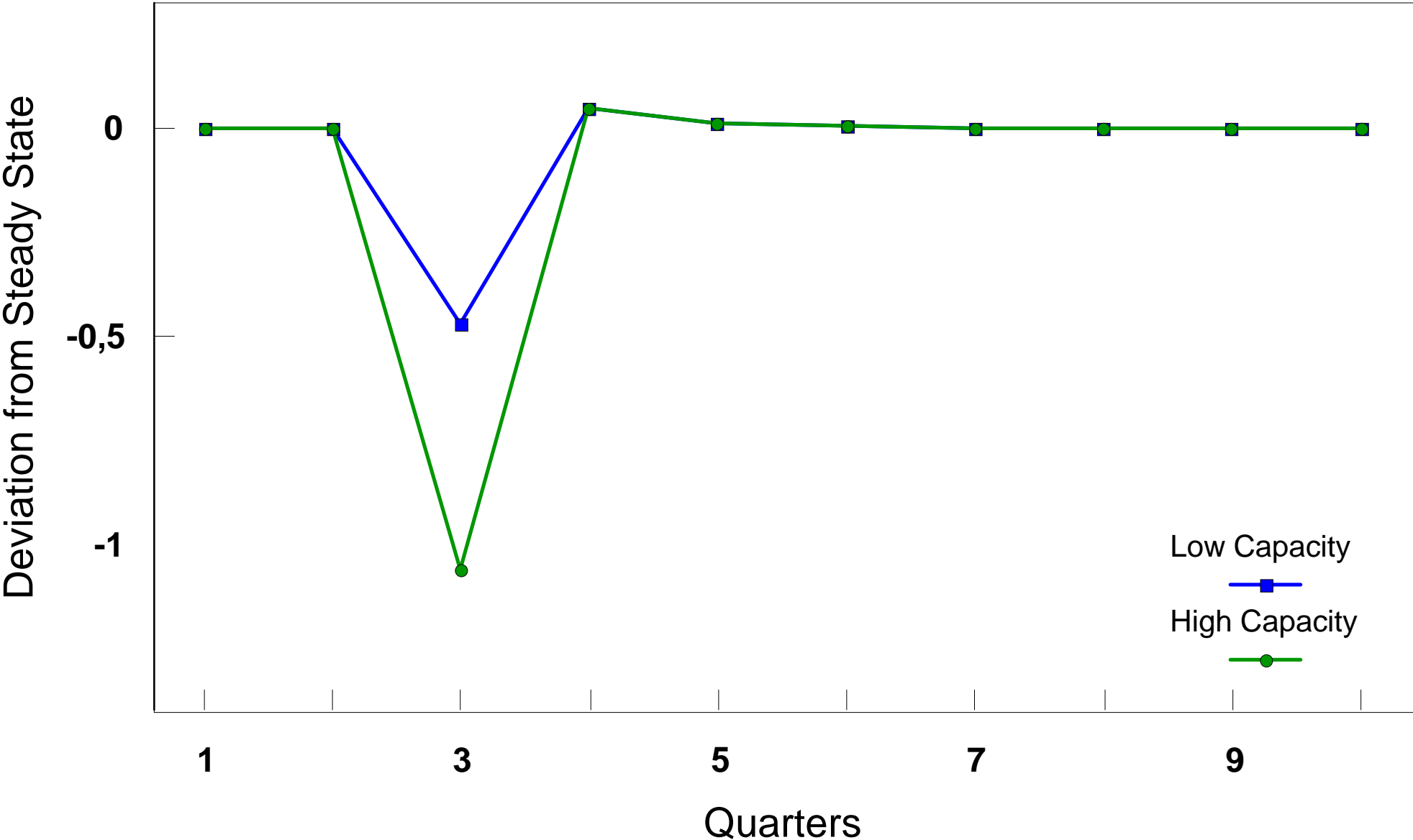
Capacity Utilization Rate

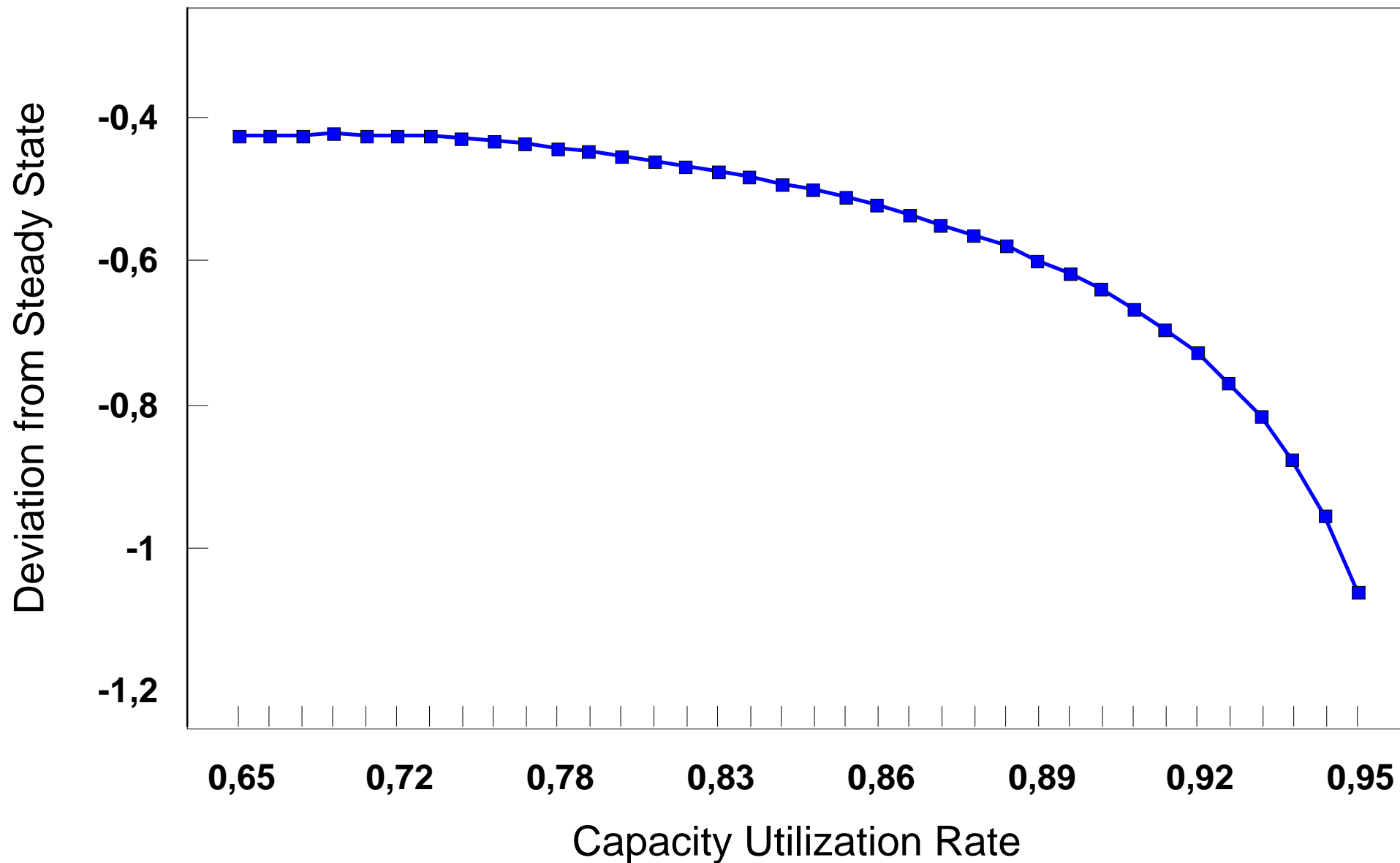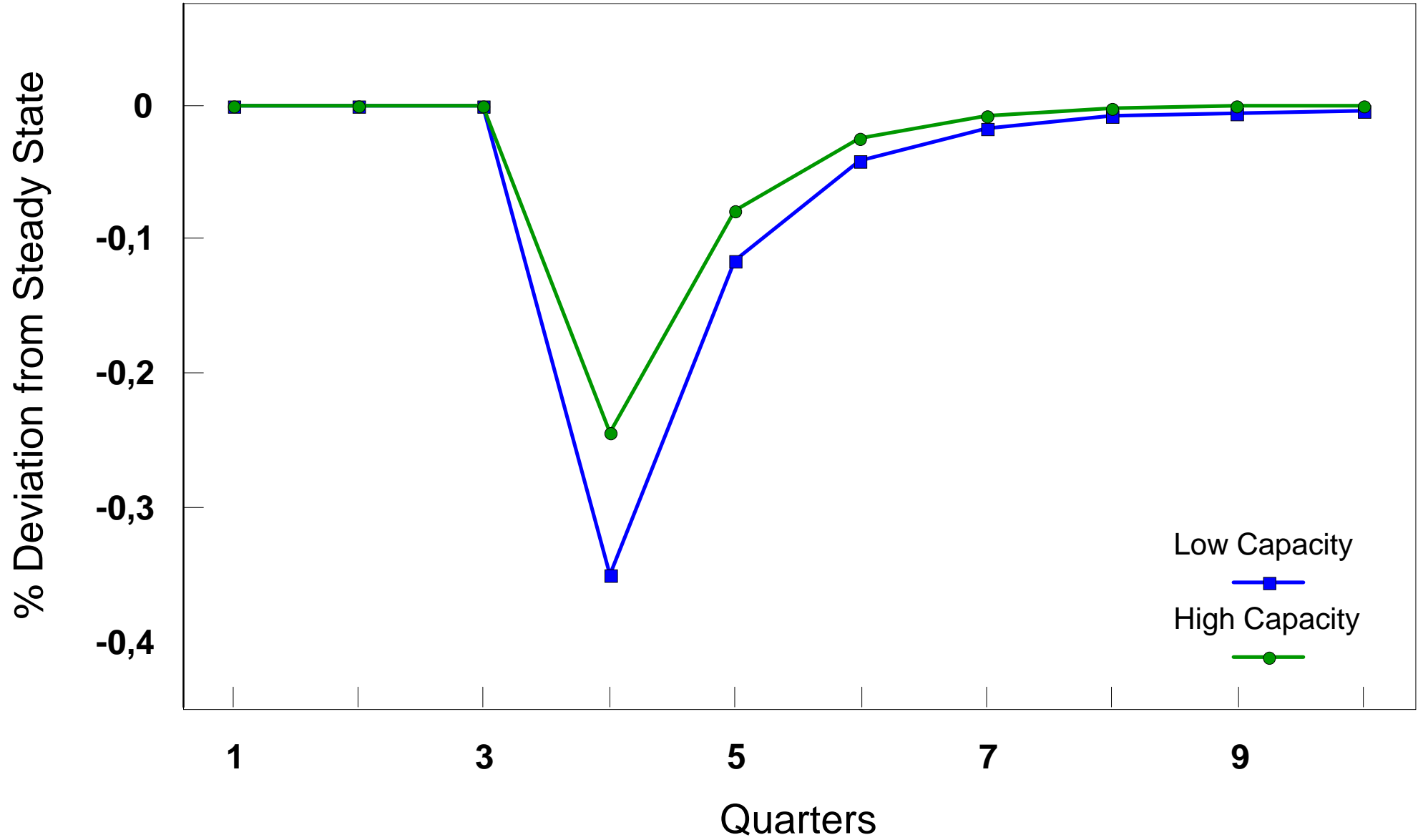**Impulse Responses to a 1% Shock in Money Growth Rate**

Impact Effect due to a 1% Shock in Money Growth Rate

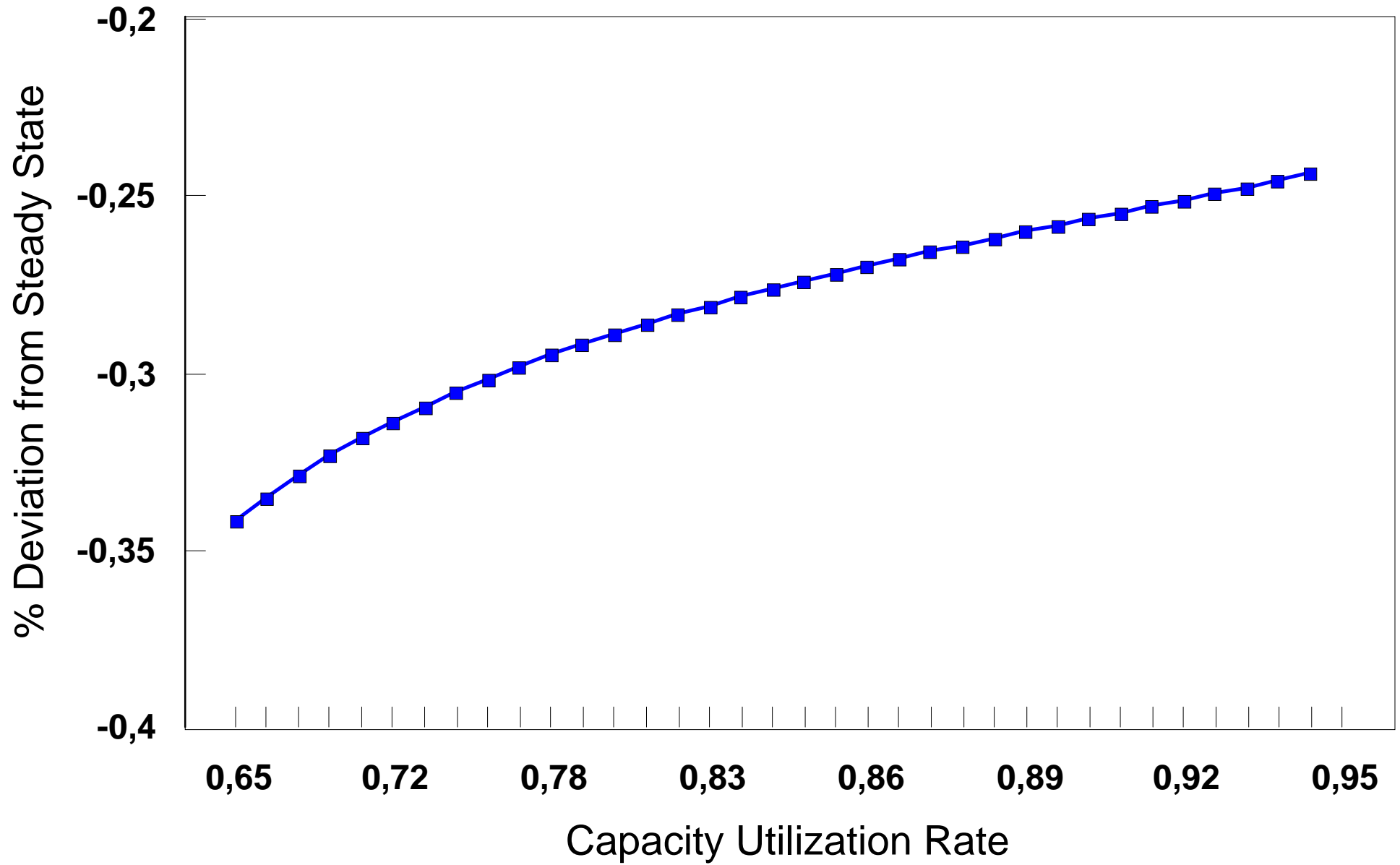**Impulse Responses to a 1% Shock in Money Growth Rate**
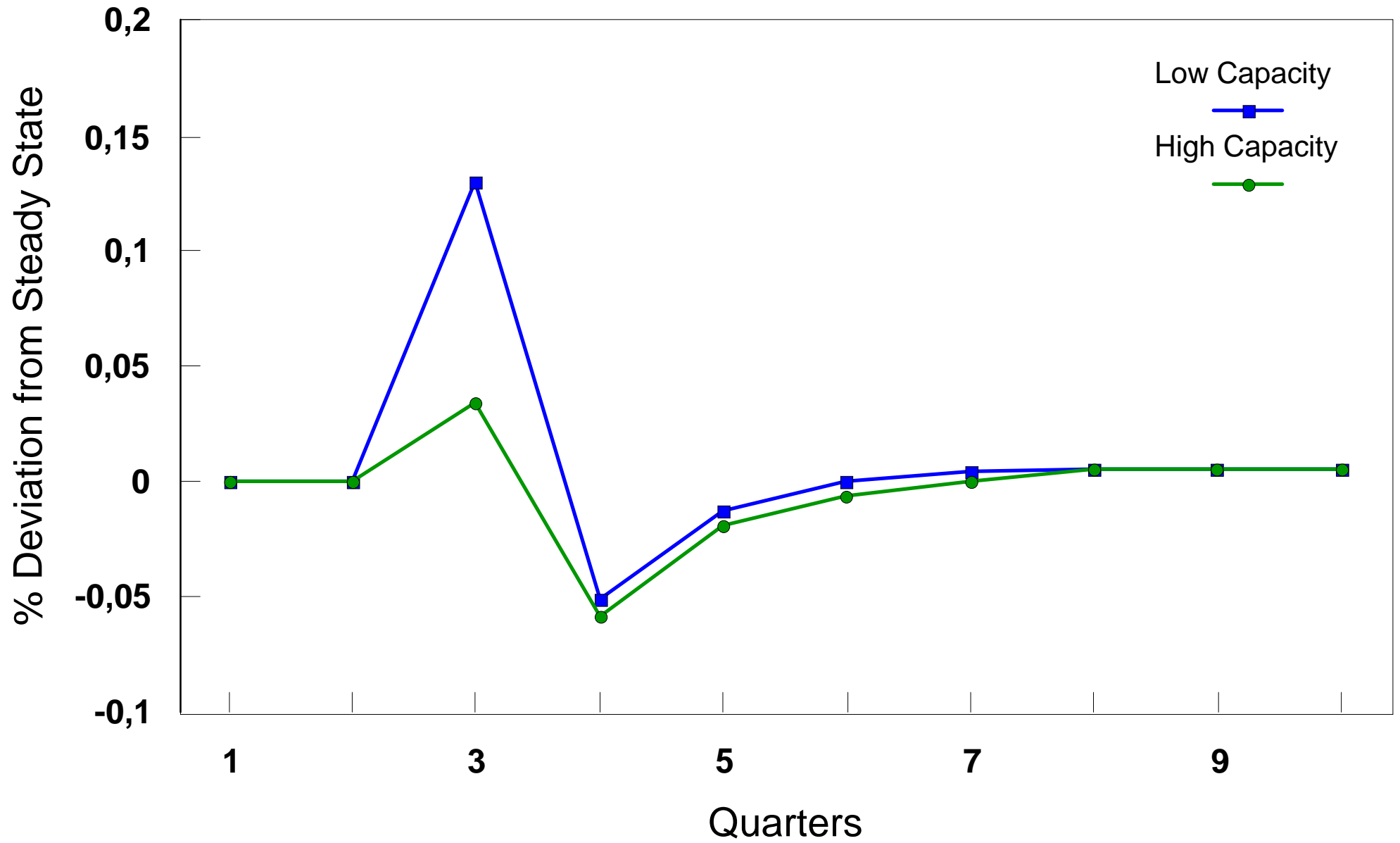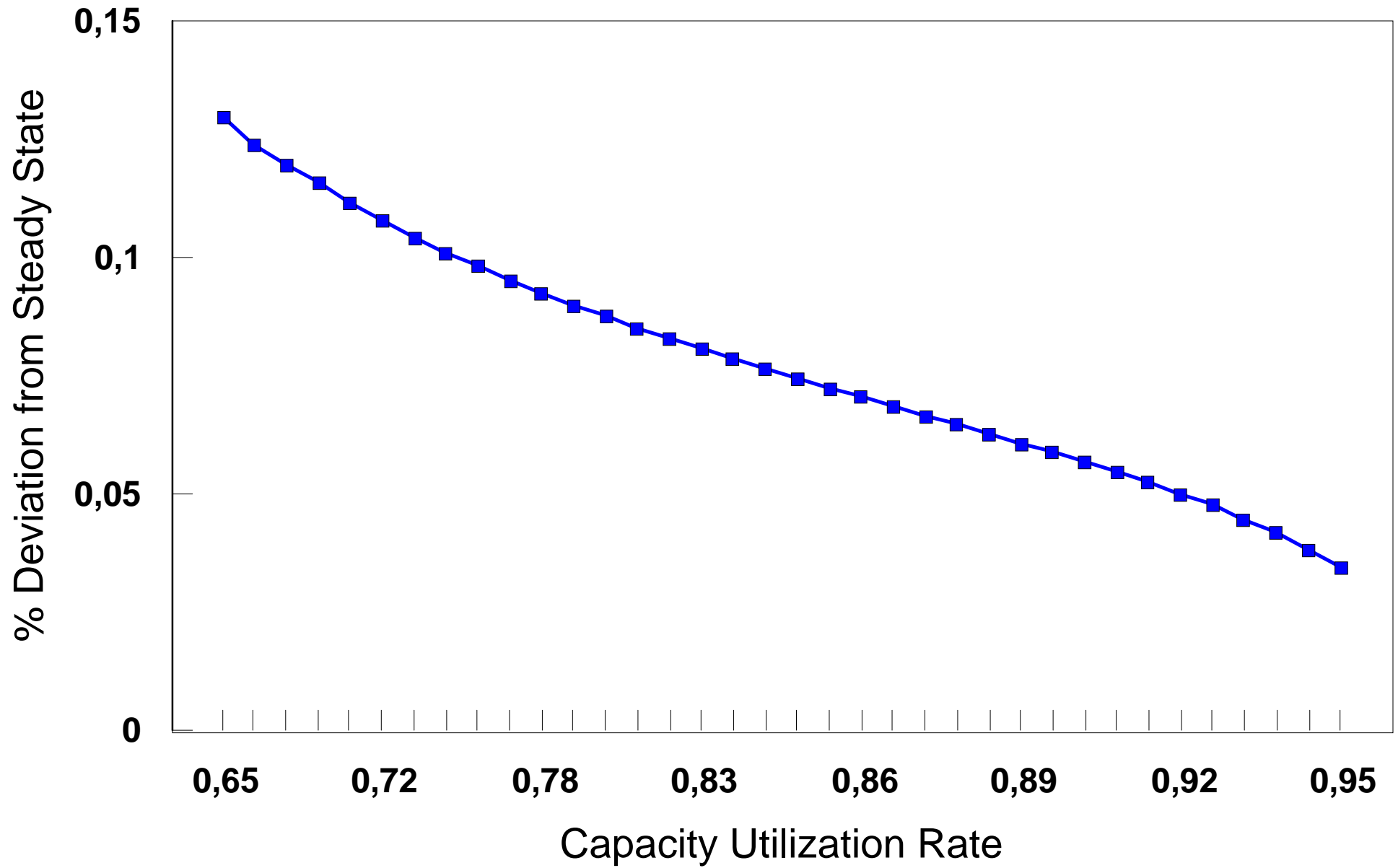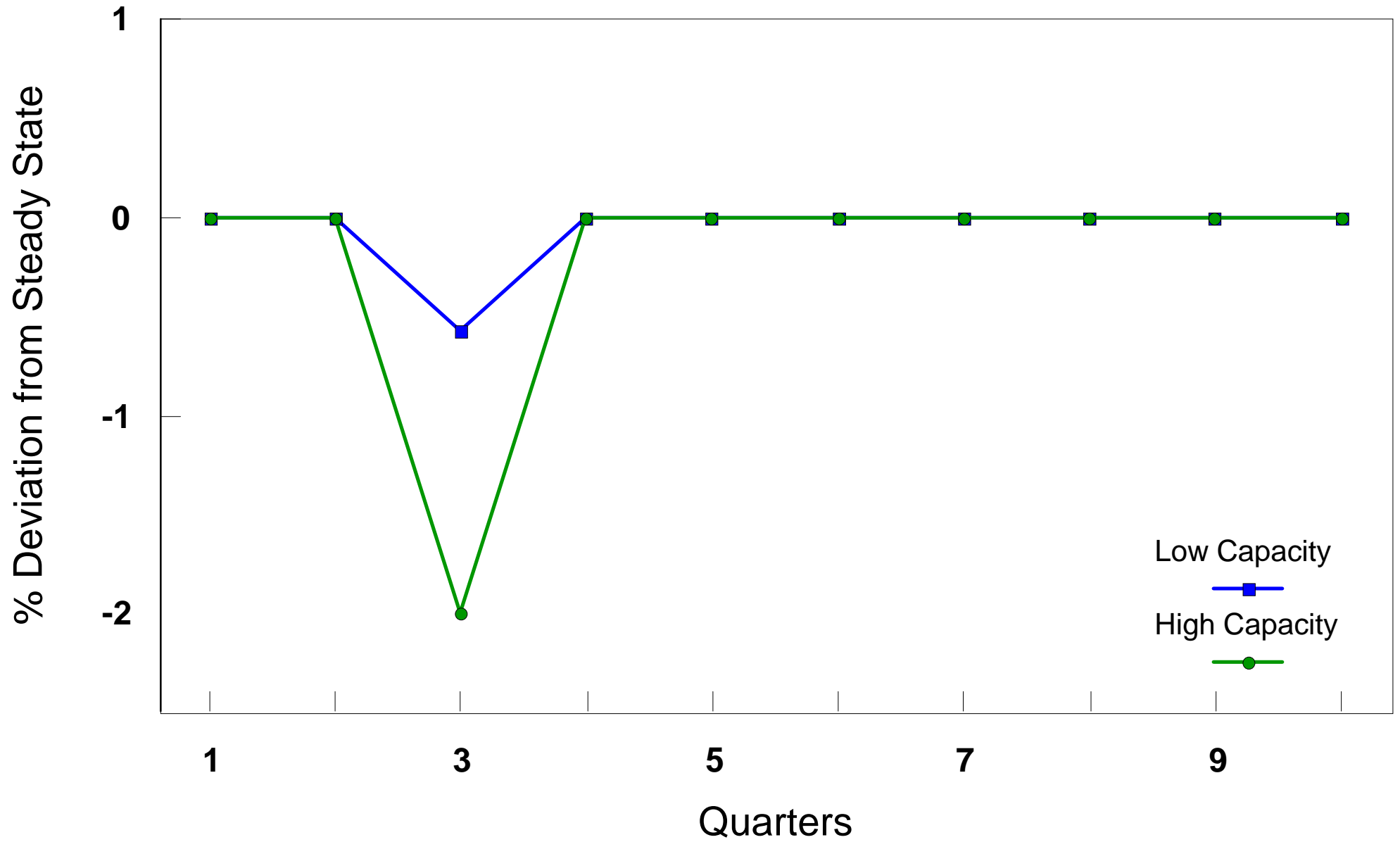
Impact Effect due to a 1% Shock in Money Growth Rate

% Deviation from Steady State
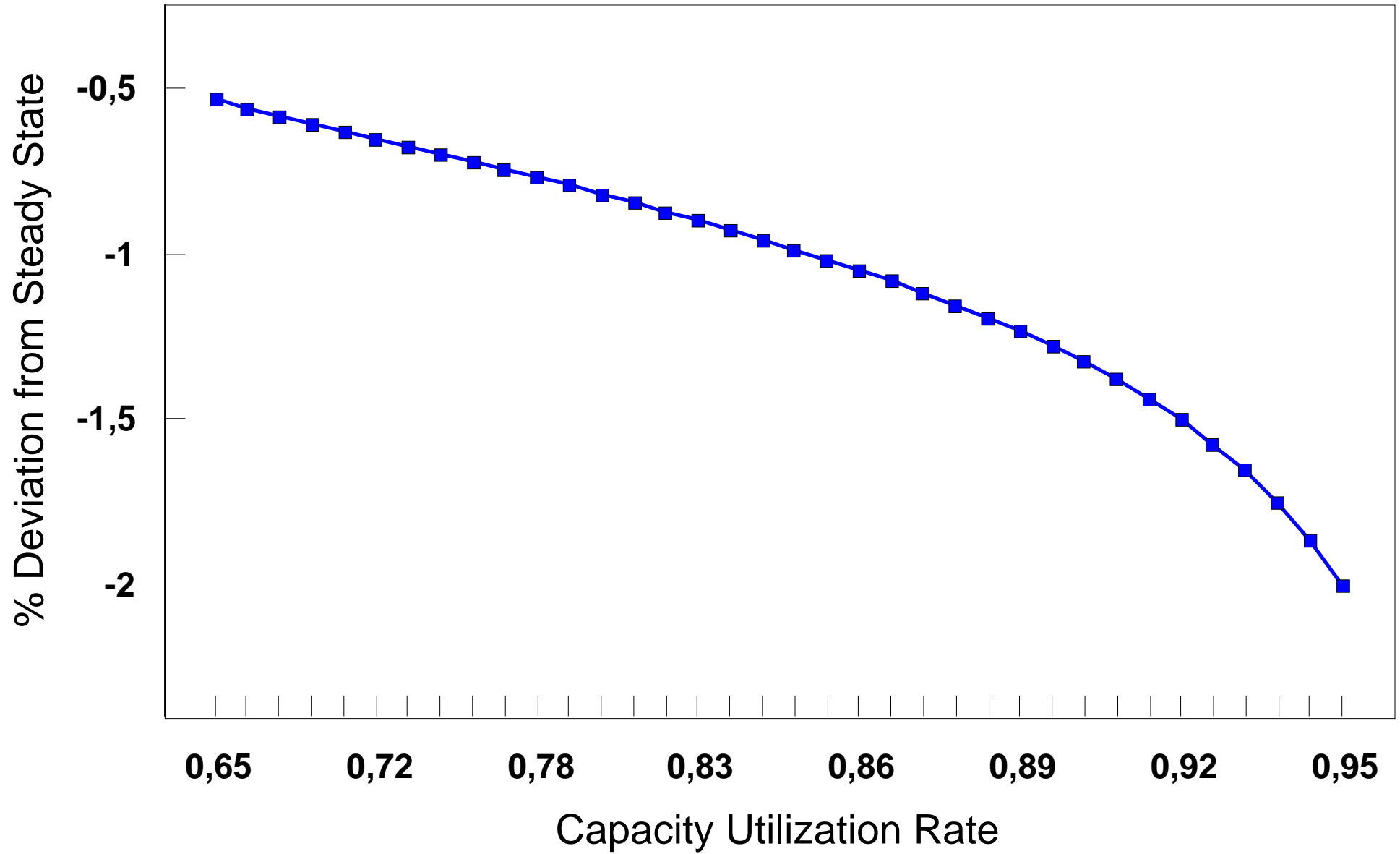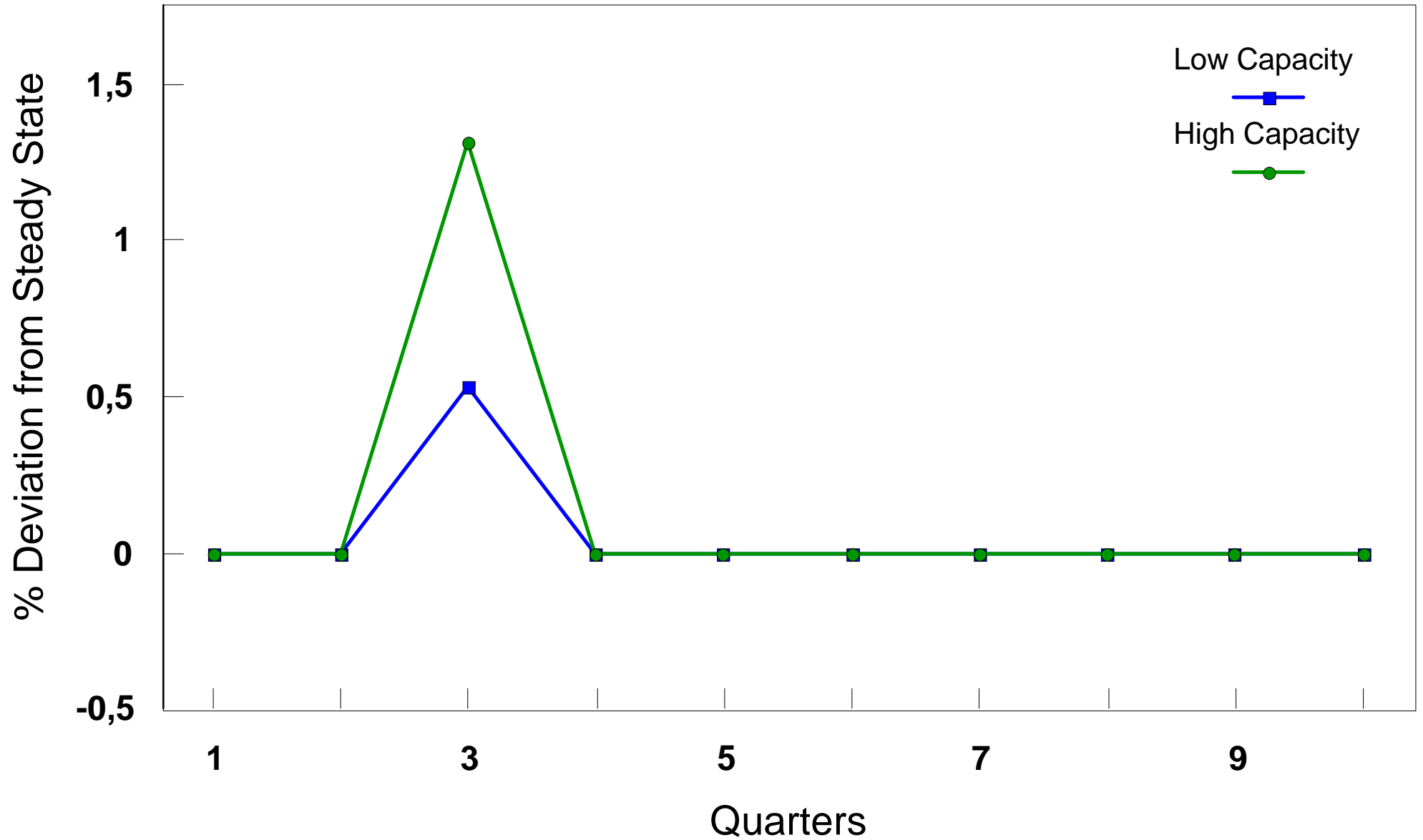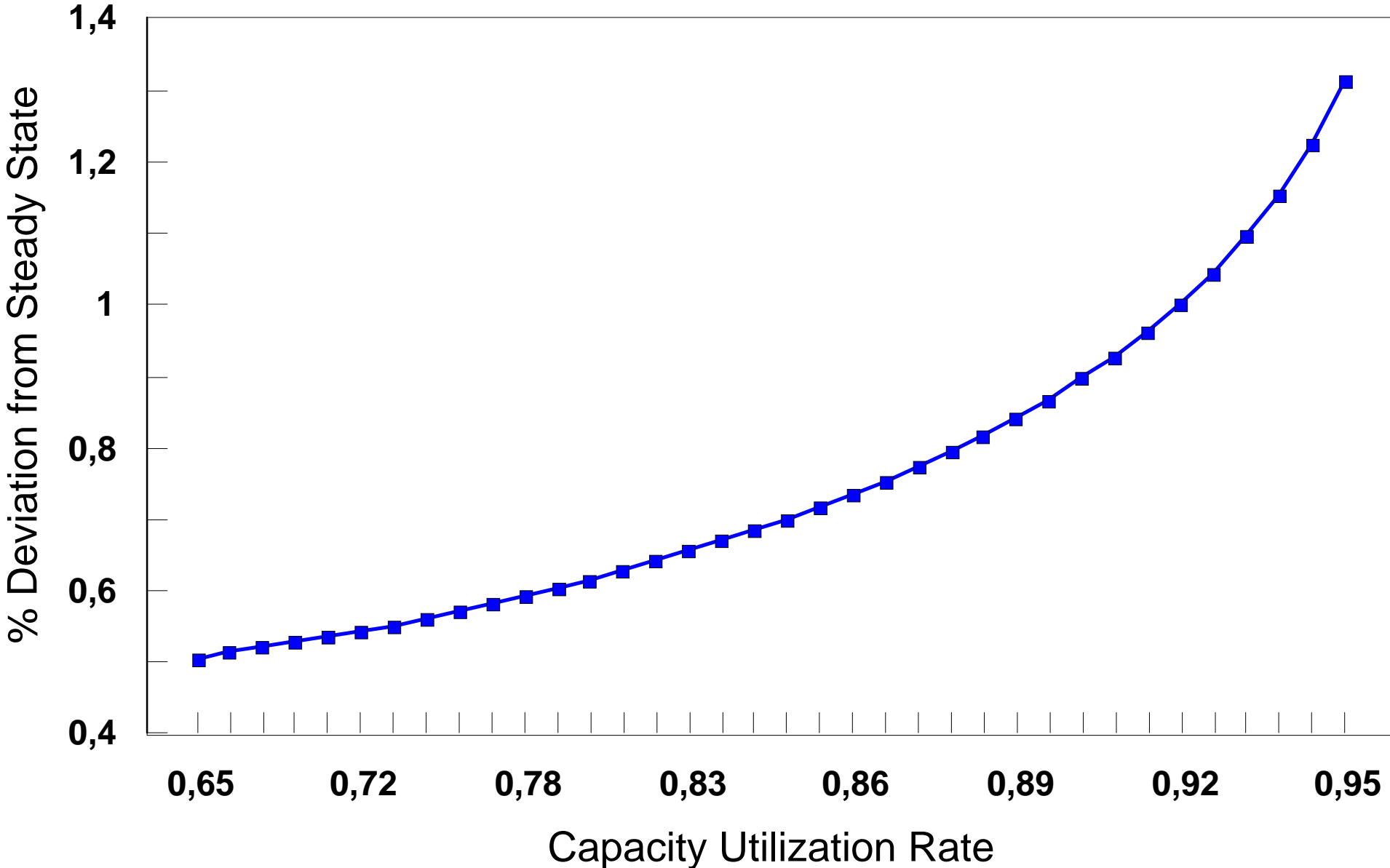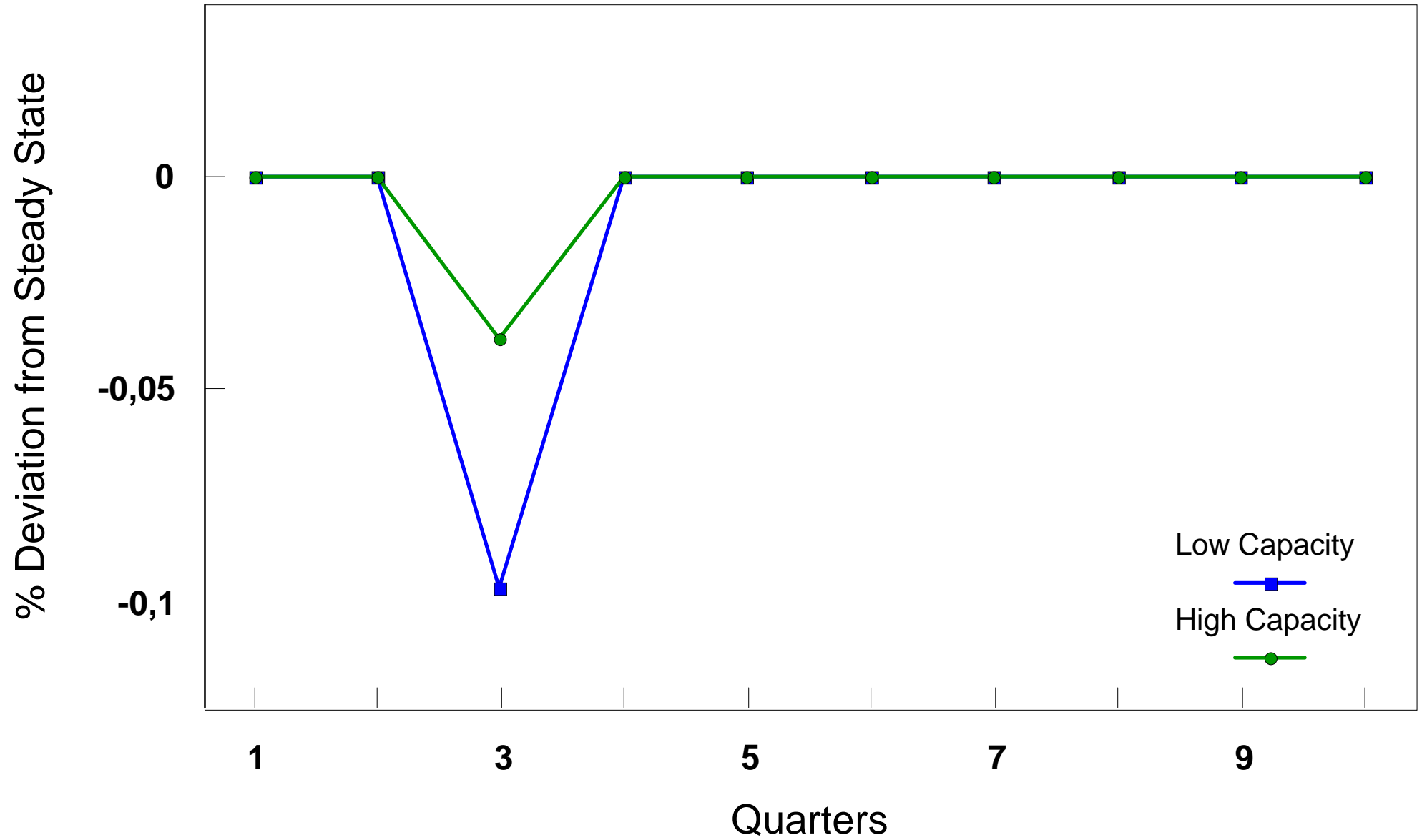
Capacity Utilization Rate
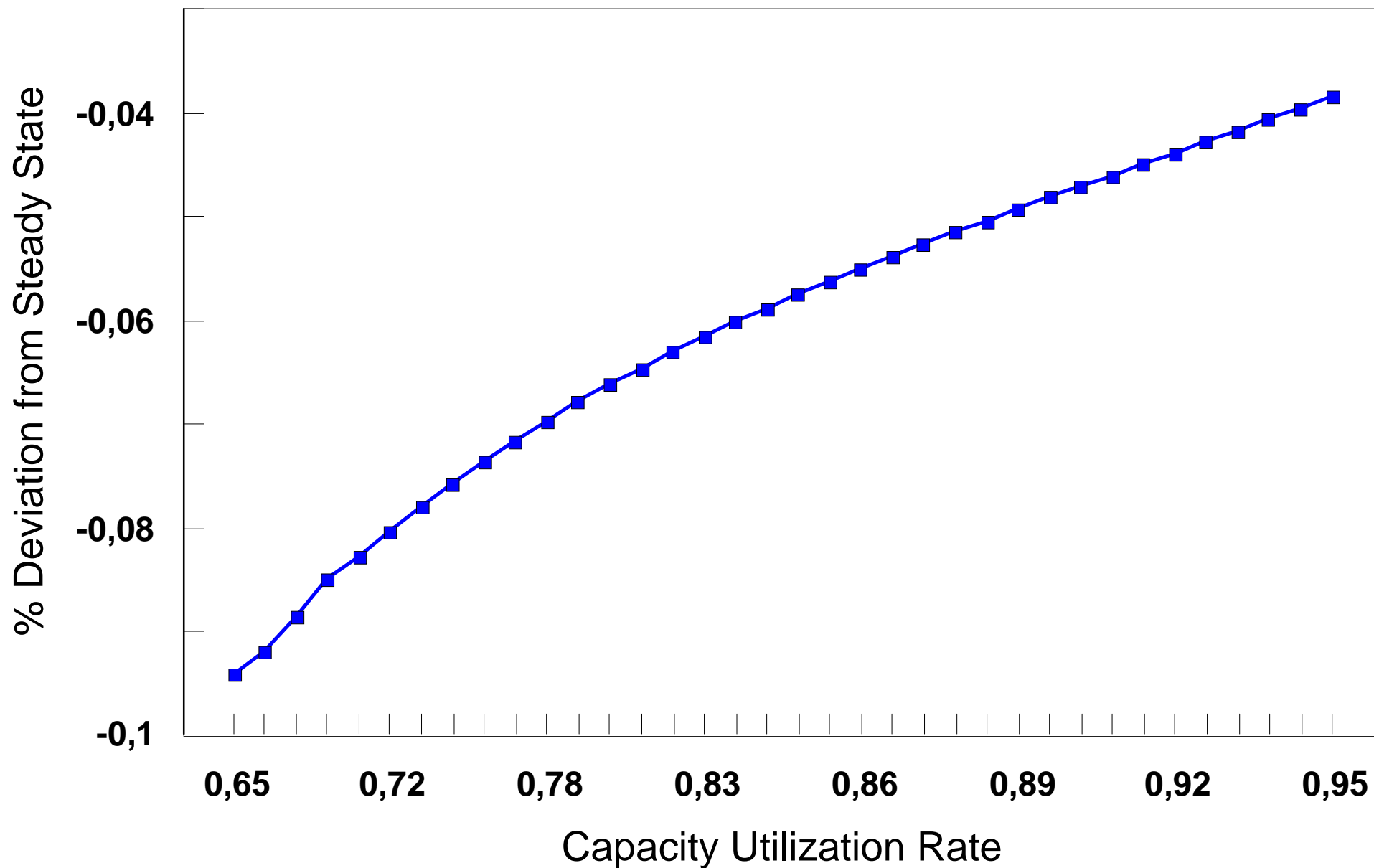
Chapter 2

Asymmetries and the

Capacity-Inflation Trade-Off

2.1 Introduction

As a first step towards a truly understanding of the effects stemming from monetary policy, one needs to explore the precise nature of the relationship between prices and real economic activity, namely the Phillips curve. This is so because the appropriate course of monetary policy depends crucially on the short-run inflation dynamics, as a recent strand of research has stressed.[1] This Chapter explores empirically such a relationship, providing evidence on its asymmetric shape. The argument behind this finding involves the existence of capacity utilization constraints in the real side of the economy. When analyzing the activity-inflation trade off, the standard approach in the literature is to consider some sort of linear (or linearized) Phillips curve. Specifically, the change in inflation relative to

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[1]See, *inter alia*, the work of Clarida, Galí and Gertler (1999) and Goodfriend and King (1997).

expected inflation is assumed to be proportional to some measure of overall real activity, with such relationship being constant over time. However, there are strong arguments, both theoretical and empirical, that can be put forward in support of an asymmetric activity-inflation trade-off. One of these arguments is known as the capacity constraint hypothesis.[2] The idea is that some firms find it difficult to increase their capacity to produce in the short run, creating production bottlenecks and supply shortages. Thus, when the economy experiences strong aggregate demand, the impact on inflation will be greater when more firms are restricted in their ability to raise output in the short run. In this framework, the short-run aggregate supply equation or Phillips curve has a convex shape, which has relevant consequences for the performance of a monetary policy aimed at controlling inflation. If the economy is initially weak, easing monetary conditions will primarily affect output, but if the economy is initially strong, a monetary expansion will mainly affect prices. In this context, Friedmans's *pushing a string* argument applies. Another implication of a convex Phillips curve is that the more stable output is, the higher the level of output will be in the economy, on average.[3] Again, the issue of asymmetry has important implications for the design and conduct of monetary policy [4]

To carry out the analysis, I make us of an econometric procedure useful for es-

[2]Other arguments are based on "menu costs", nominal wage rigidities and credit market mperfections. Dupasquier and Ricketts (1997) briefly survey some of the different sources of asymmetries, performing an empirical investigation in this regard. See also Weise (1999).

[3]The thinking behind this result is that given the lags in the effects of monetary policy, there is an incentive for pre-emptive tighten responses to inflationary pressure. If central bankers act in this way, it will prevent the economy from moving too far up the level where inflation begins to rise more rapidly, thereby avoiding the need for a larger negative output gap in the future to reverse this large rise in inflation. See MacKlem (1997) for a full discussion in this respect.

[4]In this sense, Nobay and Peel (2000) have shown that the analysis of optimal discretionary monetary policy under a non-linear Phillips Curve yields results that are in marked contrast with those obtained under a conventional linear paradigm. Shaling, (1999) extends Svensson's inflation targeting model with a convex Phillips curve and derives optimal monetary policy rules.

timating dynamic rational expectation models with unobserved components. The method combines the flexibility of the unobserved components approach based on the Kalman recursion, with the power of the general method of moments (GMM) estimation procedure.[5] The method is applied to the estimation of a 'hybrid' Phillips curve that relates inflation to the capacity utilization gap. The term 'hybrid' means that both forward and backward looking components are considered in the specification of the Phillips curve. The empirical results that are obtained with the application of this method provide clear evidence on the asymmetric relationship existing in the joint dynamics of prices and economic activity, the source of such an asymmetry being the existence of capacity constraints in the real side of the economy. The Phillips curve considered here is based on the capacity gap, which is the difference between the non accelerating inflationary capacity utilization rate (NAICU) and the current aggregate capacity utilization rate.[6] As a by- product of the econometric procedure, time-varying estimates of the NAICU are obtained.

An additional relevant implication over the role of capacity utilization in the conduct of monetary policy is the possibility of developing asymmetric monetary policy rules. In light of the arguments and results given here, such kinds of rules can be easily justified. For example, to the extent that the effect on inflation becomes disproportionately larger as the capacity gap increases, the policy response should become more aggressive with each incremental increase in the gap. From either a theoretical and an empirical point of view, it would be desirable to explore these issues in depth but, unfortunately, this is beyond the

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[5]In a different context, Bacchetta and Gerlach (1997) apply a procedure that combines instrumental variables estimation with the Kalman filter.

[6]Conventionally, Phillips curves are based on the employment gap or on the output gap. The latter is defined as the difference between potential output and current output, where the former is typically viewed as the trend component of the latter, with the trend estimated in various ways. See Gerlach ans Smets (1999) ans Kichian (1999) with the references there in.

scope of the present analysis and has to be left for future research.

The outline of the Chapter is as follows. Section 2.2 introduces the Phillips curve relationship. Section 2.3 presents the econometric methodology used in the estimation of the empirical model, with the results being reported in Section 2.4. Some conclusions and possible lines for further research are offered in Section 2.5.

2.2   An Empirical 'Hybrid' Short-Run Phillips Curve

There is evidence that capacity is a leading indicator of future inflationary pressures, despite the apparently spurious dynamics existing between the joint behavior of these two magnitudes.[7] Figure 2.1 shows the dynamics of the capacity utilization and the level of the inflation rate for the U.S. economy over the last three decades. The associated cross correlogram is shown in Figure 2.2. The simple correlation between the contemporaneous level of these two variables is negative and very low (-0.2) a fact that could suggest that the specification of the Phillips curve below could be at odds with the data. However, this correlation increases (0.18) when the variables under consideration are inflation and lagged, at least four periods, capacity utilization rate. This result reinforces, but not sufficiently, the idea of capacity as a leading indicator of inflation pressures. Indeed, from Figures 2.3 and 2.4, where it is shown the historical series and the corresponding scattered graph, it is not clear the existence of a positive relationship in the dynamics of prices and capacity. Consequently, these dynamics are not well captured simply by taking into account the level of the inflation rate and some lag in the capacity utilization rate. Better results are found,

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[7]de Kock and Nada-Vicencs (1996) revealed that capacity pressures provide a signal about future inflation at the 5 per cent level for Canada, the U.S., Japan and Germany. See also Staiger, Stock and Watson (1997).

however, when inflationary influences are subsumed in last period's inflation rate. A measure of acceleration in the inflation rate is considered in this case. The cross correlations are shown in Figure 2.5. It is noticeably the positive correlation (0.7) reached when acceleration and the two-quarter lagged capacity are considered.[8] In this case, the historical series, shown in Figure 2.6, seem to follow a close pattern over time and the corresponding scattered graph, shown in Figure 2.7, points toward a positive relationship. Hence, given that one of the overriding objectives of most central banks around the world is to achieve and maintain a low and stable rate of inflation, the information provided in capacity utilization rate series is expected to be an important and useful input in the design of monetary policy rules, a fact that policy makers cannot disregard.

In light of the discussion above, in this section I analyze the short run trade-off between inflation and economic activity, this latter variable being measured by the capacity utilization rate of the economy. More specifically, I specify and estimate a capacity-based Phillips curve, stressing the (non-linearity) asymmetry in such a relationship. As a by-product of the estimation procedure, time-varying NAICU rate estimates are obtained. This is of special interest given the ongoing debate on this concept and the scepticism on the reliability of existing NAICU estimates. I start by specifying an expectations-augmented Phillips curve as follows

$$\pi_t = \pi_t^e + \gamma_t \left(\mathcal{C}_t - \mathcal{C}_t^* \right) + v_t \; , \tag{2.1}$$

where π_t, π_t^e, $\mathcal{C}_t^*$ and $\mathcal{C}$ are, respectively, the inflation rate, the expected inflation, the NAICU

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[8]Stock and Watson (1999) using univariate time series models conclude that Phillips curves based on capacity utilization outperform, in a forecasting dimension, those based on unemployment. In multivariate time series models, capacity utilization also tends to be among the most important indicators of inflation as Cecchetti (1995) shows. See Corrado and Mattey (1997) for additional arguments.

and the observed capacity utilization rate. The error term, v_t is assumed to be white noise. Notice that the NAICU is implicitly defined as the capacity rate that makes inflation expectations consistent with observed inflation. In equation (2.1) there are two unobserved variables, π_t^e and $\mathcal{C}_t^*$. The manner in which these variables are dealt with is now analyzed.

Regarding expected inflation, several approaches have been followed in the literature. In the 'traditional' formulation of the Phillips curve, expectations are assumed to be rational and backward-looking, that is, $\pi_t^e \equiv E_{t-1}\pi_t$. A simple forecasting structure is considered which implies that

$$\pi_t^e = \pi_{t-1}. \tag{2.2}$$

However, it must be pointed out that expectations formation is sensitive to monetary policy inflation and expectations based only on past inflation rates might thus be inappropriate. This argument is at the heart of the New Keynesian paradigm, where the use of forward looking expectations is advocated. Reconciling the new Phillips curve with the data has not been always successful, however. One of the possible explanations is the loss of inertia in inflation when only future inflation is considered. Hence, a plausible correct specification for (2.2) is to use a combination of backward- and forward-looking components, that is,

$$\pi_t^e = \lambda E_t \pi_{t+1} + (1 - \lambda) E_{t-1} \pi_t . \tag{2.3}$$

where the parameter $\lambda \in (0, 1)$ measures the importance of future and lag inflation in the component of inflation expectations.[9] Taking into account (2.2), the previous equation can

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[9]This specification of expected inflation is the basis for what Galí and Gertler (1999) and some others authors call a 'hybrid New Phillips' curve. The motivation for this approach is largely empirical. Some plausible justification for it can be achieved making reference to the existence of adaptive expectations on part of a subset of price setters.

be rewritten as

$$\pi_t^e = \lambda E_t \pi_{t+1} + (1 - \lambda) \pi_{t-1}. \tag{2.4}$$

The expression for the 'hybrid' Phillips curve is obtained after introducing (2.4) into (2.1),

$$\pi_t = \lambda E_t \pi_{t+1} + (1 - \lambda) \pi_{t-1} + \gamma_t (\mathcal{C}_t - \mathcal{C}_t^*) + v_t. \tag{2.5}$$

Notice that this formulation nests the 'traditional' specification of the Phillips curve as a special case, when $\lambda = 0$. Notice also that both the trade-off parameter γ_t and the NAICU are potentially time varying. A fairly general formulation for this non-observable variable is specified as a random walk[10]

$$\mathcal{C}_t^* = \mathcal{C}_{t-1}^* + \eta_t, \tag{2.6}$$

while the slope coefficient γ_t is assumed to be a linear function of the NAICU gap

$$\gamma_t = k_0 + 100 k_1 \left(\mathcal{C}_{t-1} - \mathcal{C}_{t-1}^* \right). \tag{2.7}$$

Under the capacity constraint hypothesis, therefore, the value of k_1 is expected to be positive and significant: when the gap is positive and the economy is booming, prices should go up very rapidly. This argument is illustrated in Figure 2.8, which represents a (non-linear) asymmetric Phillips curve. When the economy experiences an excess of supply, that is, when the capacity gap is negative, the response in prices will be moderate. The opposite is likely to happen in the case of excess demand.

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[10]This assumption is also considered, among others, in Gordon (1997) and Stock (1999).

2.2.1   Econometric Methodology

Next, I proceed to describe the general econometric methodology that will be used in the estimation of the model consisting of equations (2.5), (2.6) and (2.7). Such a methodology uses the general method of moments (GMM) to estimate the parameters of the model and the Kalman recursion to obtain optimal forecasts of the unobserved components, in this case the NAICU.[11] The Kalman-GMM algorithm is a recursive procedure where in each iteration, given a set of structural parameters, a series of the unobserved variables is generated using the Kalman filter. With this data, a set of orthogonality conditions is constructed and a new set of parameter estimates is obtained applying an appropriate GMM technique which consists of the minimization of a loss function. In what follows, a more detailed description of the econometric approach is presented.

State-Space Representation

The model to be estimated consists of the 'hybrid' Phillips curve

$$\pi_t = \lambda E_t \pi_{t+1} + (1 - \lambda) \pi_{t-1} + \gamma_t \left(\mathcal{C}_t - \mathcal{C}_t^* \right) + v_t,$$

with the unobservable NAICU given by the equation

$$\mathcal{C}_t^* = \mathcal{C}_{t-1}^* + \eta_t$$

and where the slope or trade-off parameter is assumed to evolve according to the following equation

$$\gamma_t = k_0 + 100 k_1 \left(\mathcal{C}_{t-1} - \mathcal{C}_{t-1}^* \right).$$

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[11] For a good exposition of state space models and the Kalman algorithm see Chapter 3 of Harvey (1989) and Chapter 14 in Hamilton (1994). This latter reference also covers the GMM methodology.

In order to make the model estimable, a few manipulations have to be made with respect to the forward looking component in the Phillips curve. In particular, the unobserved forecast variable $E_t \pi_{t+1}$ is replaced by its realized value plus a forecast error term, u_t. This yields an equation of the form

$$\pi_t = \lambda \pi_{t+1} + (1 - \lambda) \pi_{t-1} + \gamma_t \left(\mathcal{C}_t - \mathcal{C}_t^* \right) + \varepsilon_t, \tag{2.8}$$

where the error term, ε_t, is a linear combination of the error in the forecast of π_{t+1} and the exogenous disturbance term, v_t, that is,

$$\varepsilon_t \equiv v_t + u_t = v_t + \lambda \left(E_t \pi_{t+1} - \pi_{t+1} \right). \tag{2.9}$$

Notice that the term $\left(E_t \pi_{t+1} - \pi_{t+1} \right)$ can be though of as an inflation surprise. Within the context of rational expectations, the prediction error, u_t, is free of autocorrelation, hence, since v_t is white noise, the composite error term, ε_t, is also free of autocorrelation, having the following distribution[12]

$$\varepsilon_t \sim N \left(v_t, \sigma_\varepsilon^2 \right). \tag{2.10}$$

The transformed model consists of equations (2.6), (2.7) and (2.8). Since it contains unobserved components, as well as time-varying parameters, it is useful to adopt a state-space representation for it. In particular, the 'hybrid' Phillips curve in (2.5) is taken as the *measurement* equation which can be written, in general form, as

$$\mathbf{y}_t = \mathbf{z}_t \alpha_t + \mathbf{d}_t + \varepsilon_t, \qquad t = 1, ..., T \tag{2.11}$$

---

[12]Note that, despite its temporal subindex, the variable ε_t is realized in period $t + 1$.

where $\mathbf{y}_t$ is a time series of endogenous variables that, in the specific case at hand, is $\mathbf{y}_t = \pi_t$, the $N \times m$ matrix $\mathbf{z}_t$ contains explanatory variables[13], but in the specific context at hand it is a $m-$dimensional vector:

$$\mathbf{z}_t = \left[\pi_{t+1}, \pi_{t-1}, -\gamma_t\right],$$

$\mathbf{d}_t$ is, in general, an $N \times 1$ vector of observed variables, ε_t is an $N \times 1$ vector of serially uncorrelated disturbances with mean zero with $\mathbf{H}$ being its covariance matrix. In the present model, these elements are scalars and have the following expression

$$\mathbf{d}_t = \gamma_t \mathcal{C}_t \quad \text{and} \quad \mathbf{H} = \sigma_\varepsilon^2.$$

The $m \times 1$ vector α_t contains elements that are not observable, but are known to be generated by a first-order Markov process,

$$\alpha_t = \mathbf{T}_t \alpha_{t-1} + \eta_t, \qquad t = 1, \ldots, T \tag{2.12}$$

where $\mathbf{T}_t$ is an $m \times m$ matrix and η_t is a $m \times 1$ vector of serially uncorrelated disturbances with means zero and covariance matrix $\mathbf{Q}_t$. The covariance matrix $\mathbf{Q}_t$ and the transition matrix $\mathbf{T}_t$ are assumed to be time invariant, so that $\mathbf{Q}_t = \mathbf{Q}$ and $\mathbf{T}_t = \mathbf{T}$ for all t. The error term η_t is assumed to be normally distributed and uncorrelated, in all time periods, with the error in the measurement equation, ε_t that is,

$$E\left(\varepsilon_t \eta_\tau\right) = 0 \quad \text{for all } t \text{ and } \tau. \tag{2.13}$$

Equation (2.12) is the *transition* equation of the state-space system. For the particular model presented above,

$$\alpha_t = [a_{1t}, a_{2t}, \mathcal{C}_t^*]',$$

---

[13]In the present environment, this vector might contain endogenous variables, that is, $\mathrm{E}(z_{i,t}\varepsilon_t) \neq 0$ for some i.

in which, $a_{1t} = \lambda$ and $a_{2t} = 1 - \lambda$ implying that $\mathbf{Q}$ has all elements zero except the last one in the diagonal, that will be denoted as σ_η^2. Hence, the only parameter that really varies with time is the NAICU, $\mathcal{C}_t^*$. The transition matrix is assumed to have the following form,

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

from where it follows that the NAICU is modeled, in a parsimonious manner, as a random walk

$$\mathcal{C}_t^* = \mathcal{C}_{t-1}^* + \eta_t. \tag{2.14}$$

The specification of the state-space model is completed with two further assumptions. First, the initial state vector α_0 has a mean of $\mathbf{a}_0$ and a covariance matrix $\mathbf{P}_0$; and second, the disturbances ε_t and η_t are uncorrelated with the initial state. Notice that the system matrices $\mathbf{z}_t$, $\mathbf{H}$ and $\mathbf{Q}$ depend on a set of unknown parameters that have to be estimated. These elements are usually known as hyperparameters and will be denoted by an $n \times 1$ vector ψ_0. In the present context, the vector of hyperparameters is

$$\psi = \left\{ \lambda, \gamma\left(k_0, k_1\right); \sigma_\varepsilon^2, \sigma_\eta^2 \right\}' \tag{2.15}$$

and will be estimated by GMM. Once the model has been put in state space form, it is possible to apply the Kalman algorithm, which is a recursive procedure that can be used for making inference about the state of the nature, α.

The Kalman-GMM Estimation Procedure

The model's hyperparameters are estimated by means of the general method of moments (GMM). The reason for using this approach, rather than the more efficient maximum likelihood, relies on the fact that the empirical model presented above contains explanatory variables that are correlated with the composite disturbance term. In the model at hand, future inflation, π_{t+1}, and the composite error term, ε_t, are correlated.[14]

The existence endogenous explanatory variables is a common feature of dynamic rational expectation models and, hence, the estimation by instrumental variables is the usual approach followed in this context. What is particular in the case studied in the present study is the consideration of unobserved components as explanatory variables. Thus, the Kalman algorithm, which sequentially produces linear projections of the unobserved state

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[14]To see this, first notice that future inflation, being a covariance-stationary process can be written using Wold's decomposition as

$$\pi_{t+1} = \sum_{s=0}^{\infty} \omega_s a_{t+1-s},$$

where a_{t+1-s} is white noise, with constant variance σ_a^2, and $\sum_{s=0}^{\infty} |a_{t+1-s}| < \infty$, with $E_t a_{t+1-s} = 0$ for all $s > 0$. Hence, the covariance between future inflation and the composite disturbance is

$$\mathrm{Cov}\left(\varepsilon_t, \pi_{t+1}\right) = \mathrm{Cov}\left(v_t, \pi_{t+1}\right) + \lambda \mathrm{Cov}\left(\left(E_t \pi_{t+1} - \pi_{t+1}\right), \pi_{t+1}\right),$$

which taking into account $E\left(v_t a_s\right) = 0$ for all t and s yields

$$\mathrm{Cov}\left(\varepsilon_t, \pi_{t+1}\right) = 0 + \lambda \mathrm{Cov}\left(\omega_0 a_{t+1}, \sum_{s=0}^{\infty} \omega_s a_{t+1-s}\right) = \lambda \omega_0^2 \sigma_a^2$$

and, under these same assumptions, the variance of the composite disturbance is

$$\mathrm{Var}\left(\varepsilon_t\right) = \sigma_v^2 + \lambda^2 \omega_0^2 \sigma_a^2$$

and the correlation, normalizing $\omega_0 = 1$, is

$$\mathrm{Corr}\left(\varepsilon_t, \pi_{t+1}\right) = \frac{\lambda}{\sqrt{\sigma_v^2/\sigma_a^2 + \lambda^2} + \sqrt{\sum_{s=0}^{\infty} \omega_s^2}}.$$

variables, is *inserted* within the optimization routine of the GMM estimation procedure. In this sense, the Kalman recursion takes the hyperparameters, ψ, as given and produces time-series estimates of the state variables, α, and the error term in the measurement equation, ε. The set of orthogonality conditions can be constructed next and the loss function evaluated. The process iterates until an optimum is reached.

As mentioned above, the estimation procedure involves two parts. First, I will present a procedure for computing the value of the unobserved variables in the model. The vector of hyperparameters is given at this stage; the way in which these elements are determined is explained later. Hence, let $\mathbf{a}_{t-1|t-1}$ denote the *optimal*[15] estimator of α_{t-1}, based on the observations up to and including $\mathbf{y}_{t-1}$, that is,

$$\mathbf{a}_{t-1|t-1} = \mathbf{E}_t\left(\alpha_{t-1}\right) = \mathbf{E}\left(\alpha_{t-1} \mid \mathbf{Y}_{t-1}\right).$$

where $\mathbf{Y}_{t-1} \equiv \left(\mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, ..., \mathbf{y}'_1, \mathbf{z}'_{t-1}, \mathbf{z}'_{t-2}, ..., \mathbf{z}'_1\right)$. Let $\mathbf{P}_{t-1|t-1}$ denote the covariance matrix of the estimation error,

$$\mathbf{P}_{t-1|t-1} = \mathbf{E}\left\{\left[\alpha_{t-1} - \mathbf{E}_t\left(\alpha_{t-1}\right)\right]\left[\alpha_{t-1} - \mathbf{E}_t\left(\alpha_{t-1}\right)\right]'\right\}.$$

Now, given $\mathbf{a}_{t-1|t-1}$ and $\mathbf{P}_{t-1|t-1}$, the optimal estimator of α_t is given by the transition equation (2.12),

$$\mathbf{a}_{t|t-1} = \mathbf{E}\left(\alpha_t \mid \mathbf{z}_t, \mathbf{Y}_{t-1}\right) = \mathbf{T}_t \mathbf{a}_{t-1|t-1},$$

where it has been assumed that $\mathbf{z}_t$ is predetermined with respect to the state α_{t-s} for $s = 0, 1, ...$ what means that $\mathbf{z}_t$ provides no information about α_{t-s}. Under the same assumptions,

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[15]The estimates are optimal in the particular case of gaussian disturbances and predetermined or exogenous variables. Since in the present context, this is not the case, the statistical properties of the estimators remain uncertain.

the covariance matrix of the estimated error is

$$\mathbf{P}_{t|t-1} = \mathbf{T}_t \mathbf{P}_{t-1|t-1} \mathbf{T}_t' + \mathbf{Q}_t, \qquad t = 1, \ldots\ldots T.$$

These last two equations are known as the *prediction* equations.

Once the new observation, $\mathbf{y}_t$, becomes available, the estimator of α_t, namely $\mathbf{a}_{t|t-1}$, can be updated using the formulae for updating linear projections[16]

$$\mathbf{a}_{t|t-1} + \mathbf{P}_{t|t-1} \mathbf{Z}_t' \mathbf{F}_t^{-1} \varepsilon_{t|t-1}$$

where $\varepsilon_{t|t-1}$ can be interpreted as a vector of prediction errors, that is,

$$\varepsilon_{t|t-1} = \mathbf{y}_t - \mathbf{z}_t \mathbf{a}_{t|t-1} - \mathbf{d}_t.$$

The MSE associated with the updated projection of the state is

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1} \mathbf{z}_t' \mathbf{F}_t^{-1} \mathbf{z}_t \mathbf{P}_{t|t-1},$$

which has a conditional variance of the following form

$$\mathbf{F}_t = \mathbf{z}_t \mathbf{P}_{t|t-1} \mathbf{z}_t' + \mathbf{H}_t, \qquad t = 1, \ldots\ldots, T.$$

Next, giving starting values for the conditional mean of the state vector $\mathbf{a}_0$ and its conditional variance $\mathbf{P}_0$, the Kalman filter proceeds iteratively for $t = 1$ to $t = T$, delivering optimal estimators of the state vector as new observations become available. An important aspect of this procedure is the initialization of the algorithm. In this respect, if prior information is available on all the elements of the state vector, α_0, then it has a proper prior distribution with known mean, $\mathbf{a}_0$, and bounded covariance matrix, $\mathbf{P}_0$. Once

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[16] See Chapter 4 of Hamilton (1994) for a detailed derivation of these expressions.

the filter estimates of the state vector α_t have been computed and the hyperparameters been estimated, it is possible to improve their *quality* by the procedure of smoothing.[17] The idea of smoothing is to estimate α_t taking into account the information available after time t. The smoothed estimator, denoted by $\mathbf{a}_{\tau|T}$, can be obtained by applying the *fixed-interval* algorithm. This procedure consists of a set of recursions which start with the final quantities $\mathbf{a}_T$, and $\mathbf{P}_T$, given by the Kalman filter, and works backwards. The equations are

$$\mathbf{a}_{t|T} = \mathbf{a}_t + \mathbf{P}_t^* \left(\mathbf{a}_{t+1|T} - \mathbf{T}_{t+1} \mathbf{a}_t \right)$$

and

$$\mathbf{P}_{t|T} = \mathbf{P}_t + \mathbf{P}_t^* \left(\mathbf{P}_{t+1|T} - \mathbf{P}_{t+1|t} \right) \mathbf{P}_t^{*\prime},$$

where

$$\mathbf{P}_t^* = \mathbf{P}_t \mathbf{T}_{t+1}' \mathbf{P}_{t+1|t}^{-1}, \ \ t = T - 1, ..., 1$$

with $\mathbf{a}_{T|T} = \mathbf{a}_T$ and $\mathbf{P}_{T|T} = \mathbf{P}_T$.

The Kalman algorithm procedure above takes as given the vector of hyperparameters, but these elements have to be estimated. Given the particular nature of the model, in particular the endogeneity of some explanatory variables, makes the Generalized Method of Moments (GMM) an appropriate estimation procedure. This method assumes that the statistical model implies a set of r orthogonality conditions of the form

$$E\left\{ \mathbf{h}\left(\psi_0, \mathbf{w}_{t|t-1} \right) \right\} = \mathbf{0}, \tag{2.16}$$

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[17]The term *quality* is this context refers to obtaining estimates of the state variables with a lower mean squared error.

where $\mathbf{w}_{t|t-1}$ is an $(h \times 1)$ vector of variables observed and predicted at date t, the vector of true parameters is ψ_0, and $\mathbf{h}(\cdot)$ is a differentiable $r-$ dimensional vector value function. In the specific case at hand, the orthogonality condition is given by the combination of the disturbance term in (2.8),

$$\varepsilon_{t|t-1} = \pi_t - \lambda \pi_{t+1} - (1-\lambda)\pi_{t-1} - \gamma_t \left(\mathcal{C}_t - \mathcal{C}_{t|t-1}^* \right), \qquad (2.17)$$

with an $r-$ dimensional vector $\mathbf{u}_t$ of variables (instruments) dated at time t or earlier that are orthogonal to $\varepsilon_{t|t-1}$, the vector of prediction errors and where $\mathcal{C}_{t|t-1}^*$ is the Kalman-estimate of the NAICU. The orthogonality condition has the following expression

$$E\left(\varepsilon_{t|t-1} \mid \mathbf{u}_t\right) = 0,$$

so that the function $\mathbf{h}\left(\psi_0, \mathbf{w}_{t|t-1}\right) = \varepsilon_{t|t-1}\mathbf{u}_t$, which by equation (2.17) and (2.16) results in

$$E\left\{ \left(\pi_t - \lambda \pi_{t+1} - (1-\lambda)\pi_{t-1} - \gamma_t \left(\mathcal{C}_t - \mathcal{C}_{t|t-1}^* \right) \right) \mathbf{u}_t \right\} = 0. \qquad (2.18)$$

The next step involves the choice of the instrument set $\mathbf{u}_t$. In this case, one could include those variables that help predict changes in the inflation rate. Bivariate Granger-causality tests are performed as a manner to check the forecasting capability of each instrument. Given that the set of instruments will possibly exceed the number of parameters, the model is said to be overidentified. In such a case, the GMM method consists in a two-step nonlinear two-stage least squares procedure that yields consistent estimates of the true vector of parameters $\psi_0 = \left\{ \lambda, \gamma(k_0, k_1); \sigma_\varepsilon^2, \sigma_\eta^2 \right\}'$. Thus, the GMM estimate $\hat{\psi}$ is the value of the vector ψ that minimizes

$$Q\left(\psi, \mathcal{Y}_{T|T-1}\right) \equiv \left[\mathbf{g}\left(\psi, \mathcal{Y}_{T|T-1}\right)\right]' \hat{S}_T^{-1} \left[\mathbf{g}\left(\psi, \mathcal{Y}_{T|T-1}\right)\right],$$

where $\mathcal{Y}_{T|T-1} \equiv \left(\mathbf{w}_{T|T-1}, \mathbf{w}_{T-1|T-2}, ...\mathbf{w}_{1|0} \right)'$ is a $(Th \times 1)$ vector and $\mathbf{g}(\cdot)$ is the sample mean of $\mathbf{h}(\cdot)$, that is,

$$\mathbf{g}\left(\psi, \mathcal{Y}_{T|T-1}\right) = \frac{1}{T} \sum_{t=1}^{T} \mathbf{h}\left(\psi, \mathbf{w}_{t|t-1}\right)$$

and $\hat{S}$ is the asymptotic covariance matrix of $\sqrt{T}\mathbf{g}(\cdot)$. In the first step, $\mathbf{S}_T$ is set equal to the identity matrix $\mathbf{I}_N$. The resulting estimate $\hat{\psi}$ is used to construct a better estimate of $\mathbf{S}_T$. For the case that is heteroskedastic and serially correlated, one could use the following White's estimate of $\mathbf{S}_T$

$$\hat{S} = \hat{\Gamma}_{0,T} + \sum_{v=1}^{q} \left\{ 1 - \left[v/\left(q+1\right) \right] \right\} \left(\hat{\Gamma}_{v,T} + \hat{\Gamma}'_{v,T} \right)$$

with

$$\hat{\Gamma}_{v,T} = (1/T) \sum_{t=v+1}^{T} \left[\mathbf{h}\left(\psi, \mathbf{w}_{t|t-1}\right) \mathbf{h}\left(\psi, \mathbf{w}_{t-v|t-1-v}\right) \right]'.$$

In order to analyze the overall specification of the model, the use of a simple procedure has been suggested in the GMM literature which consists of checking that the orthogonality conditions are effectively close to zero when evaluated at the estimated parameters $\hat{\psi}$. This test is known as test for overidentifying restrictions and the statistic associated with it has asymptotically a χ^2 distribution with $(r-a)$ degrees of freedom. The test statistic has the following functional form

$$\left[\sqrt{T}\mathbf{g}\left(\hat{\psi}, \mathcal{Y}_{T|T-1}\right) \right]' \hat{S}_T^{-1} \left[\sqrt{T}\mathbf{g}\left(\hat{\psi}, \mathcal{Y}_{T|T-1}\right) \right] \overset{L}{\rightarrow} \chi^2\left(r-a\right).$$

The Kalman-GMM is a sequential procedure that starts by making an initial guess as to the numerical values of the unknown parameters, $\psi_0^{(1)}$. For these initial numerical values for the population parameters, the matrices $\mathbf{T}$, $\mathbf{Q}$ and $\mathbf{H}$ can be constructed from the

expressions just given and iterate on the Kalman recursion. The sequences $\left\{\mathbf{a}_{t|t-1}\right\}_{t=1}^{T}$ and $\left\{\mathbf{P}_{t|t-1}\right\}_{t=1}^{T}$ resulting from these iterations could then be used to compute the error terms $\left\{\varepsilon_{t|t-1}\right\}_{t=1}^{T}$ which together with the set of instruments $\mathbf{u}_t$ allows to define the orthogonality conditions $\mathbf{h}\left(\psi_0^{(1)}, \mathbf{w}_{t|t-1}\right)$ and then to obtain the loss function $Q\left(\psi_0^{(1)}, \mathcal{Y}_{T|T-1}\right)$. The procedure continues until this function $Q\left(\psi_0^{(j)}, \mathcal{Y}_{T|T-1}\right)$ is minimized and estimates of the hyperparameters of the model, $\psi_0^{(j)}$, are thereby found. This completes the description of econometric methodology used in the estimation of the empirical model.

Recall that the disturbance term ε_t was assumed to be normally distributed. The basic derivation of the Kalman algorithm works under this condition, as well as the exogeneity of the explanatory variables. As shown , for instance, in Hansen and Hodrick (1980), the disturbance term in the class of models considered here has a moving average representation of order $\kappa - 1$, κ being the forecasting horizon. In this particular case, $\kappa = 1$, so that the model is effectively gaussian.[18] If, additionally, the right hand side variables, $\mathbf{z}_t$, were assumed to be exogenous, then the Kalman filter would yield optimal predictions of the state variables within the set of forecasts with respect to any function of $(\mathbf{z}_t, \mathbf{Y}_{t-1})$. It remains to prove the properties of the Kalman filter carry over with endogenous explanatory variables.

In summary, the model consisting of equations (2.5), (2.6) and (2.7) is estimated by means of a procedure that uses the general method of moments (GMM) to estimate the parameters of the model and the Kalman recursion to obtain optimal forecasts of the unobserved components, in this case the NAICU.

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[18] A more general specification of the error term in this context is considered, for instance, in Kichian (1999). This author analyses the possibility of having ARCH errors in a state-space framework for measuring potential output.

2.3   Estimation Results and Discussion

The estimation uses quarterly data for the economy of the United States. The sample period extends from 1960:Q1 to 2000:Q1. Quarterly inflation at an annual rate was computed from data on the U.S. GDP Implicit Price Deflator according to the following formula: $400 \cdot \log(p_t/p_{t-1})$, where p_t is the price deflator with 1996 as base year, seasonally adjusted and provided by the U.S. Department of Commerce, Bureau of Economic Analysis. Seasonally adjusted data on the capacity utilization rate, measured it as a percentage of total capacity in the manufacturing sector, was obtained from the series provided by the Federal Reserve Board.

The analysis has several steps. Firstly, a preliminary analysis of the data is performed. The series for inflation and capacity utilization rate are pictured in Figure 2.1. At first glance, capacity seems to be stationary, whereas inflation displays a more erratic behavior. To confirm this intuition, a stationary analysis is performed for each of the series. Results are presented in the first two rows of Table 2.1. The null hypothesis of non-stationary is more clearly rejected for capacity utilization than for inflation, which seems to be weakly stationary. In light of these results, the Phillips curve in equation (2.5) is a relationship between two stationary variables. Nevertheless, the characterization of inflation as a stationary process must be regarded with caution.

Before moving into the estimation of the Phillips Curve, it is worthwhile investigating the behavior of the capacity utilization rate alone since it contents valuable information that could the be used in the estimation of the Phillips curve. In particular, the presence of non-linearities in the capacity utilization rate is explored using a two-regime threshold

autoregressive (TAR) model.[19] Specifically, capacity is assumed to follow a process of the form

$$C_t = (\alpha_0 + \alpha_1 C_{t-1} + \cdots + \alpha_p C_{t-p}) \, 1 \, (q_t \leq \gamma) +$$

$$(\beta_0 + \beta_1 C_{t-1} + \cdots + \beta_p C_{t-p}) \, 1 \, (q_t > \gamma) + \varepsilon_t,$$

where $1(\cdot)$ denotes the indicator function and q_t is a known function of the data. The error ε_t is a martingale difference sequence with respect to the past history of C_t. The autoregressive order is $p \geq 1$ and γ is the threshold parameter. The parameters of interest are estimated by least squares. Importantly, the threshold function that better fits the data is the average between the capacity utilization rate of the current period, C_t, and that of the previous period, C_{t-1}, that is, $q_t = (C_t + C_{t-1}) / 2$. The results of the estimation show that the data generating process follows, at the 5% level, a first order autoregressive threshold process, with the value of the threshold being $\hat{\gamma} \simeq 0.83$. This estimate is quite precise as it can be deduced after looking at Figure 2.9, where it is shown the values of a likelihood ratio test for different values of the parameter γ. The values of this parameter γ that are beneath the dotted line yield the confidence region for the estimate. In the present case, this region is very small what implies a high precision of the results. The TAR model is tested against a linear autoregressive model for the data generating process. Figure 2.10 shows the outcomes of an F-test, for the null of linearity, with respect to different values of the parameter γ. The results show that the TAR process is statistically significant for values of the threshold parameter around 0.83.

Additionally, the estimated value of the threshold $\hat{\gamma}$ implies that the TAR model

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[19]See Hansen (1997) for a detailed description of TAR models.

splits the regression function into two regimes. Hence, depending on the past value of the capacity utilization rate, the economy moves from a low capacity regime to a high capacity one. This is illustrated in Figure 2.11, where it is plotted the capacity utilization rate of the U.S. economy over the period 1960-2000, coded wether the observation falls in regime 1 (low utilization), or regime 2 (high utilization). The observations for which $(C_t + C_{t-1})/2$ falls in the confidence region of $\hat{\gamma}$ are coded as uncertain. There are few observations in this category what reflects the high precision of the estimate $\hat{\gamma} \simeq 0.83$. It is important to notice that the threshold value is close to the average capacity utilization rate of 82% which is the value the Board of Governors assign to the non inflationary rate of capacity utilization.

Once the basic structure of the empirical model has been vindicated, the strategy of the rest of this section is the following. I first estimate a benchmark model in which, both, the NAICU and the trade-off parameter, γ_t, are constant. These are constraints that will be relaxed in due course. In the benchmark set-up, therefore, any kind of time variation in the parameters is allowed. The equation under consideration is

$$E\left\{ \left(\pi_t - \lambda \pi_{t+1} - (1 - \lambda) \pi_{t-1} - \gamma \mathcal{C}_t + \gamma \mathcal{C}^* \right) \mathbf{u}_t \right\} = 0, \tag{2.19}$$

where the vector of hyperparameters to be estimated is $\psi_0 = \{\lambda, \gamma, \mathcal{C}^*\}'$. The instruments will include all variables that are potentially useful for forecasting inflation. Among these variables are lagged values of inflation, capacity, the long-short term interest rate spread, wage inflation and a measure of productivity. The interest rate spread is computed as the difference between the 10-Year Treasury Rate, at constant maturity rates, and the 3-Month Treasury Bill Rate, at auction averages, both series provided by the Federal Reserve Board of Governors. Wage inflation is computed as the annualized rate of growth of quarterly unit

labor costs in the nonfarm business sector. The series is seasonally adjusted and provided by the U.S. Department of Labor, Bureau of Labor Statistics. The same source provides a series of seasonally adjusted output per hour in the nonfarm business sector that is used to construct a measure for overall productivity. In order to check the forecasting performance of each instrument, bivariate Granger-causality tests are performed. The results are displayed in Table 2.2. The capacity utilization rate and wage inflation contain significant information for forecasting inflation. The interest rate spread and the productivity measure also have this capability but to a lesser extent than the other two variables. On the other hand, inflation has a poor forecasting capability with respect to these variables, the exception being wage inflation so that a problem of endogeneity could arise. However, this is not expected to be the case since the order of the magnitude of the causality of inflation with respect to wages is not very high. Equation (2.19) was estimated using a two step non-linear two stage procedure with four lags of the instruments described above. The results are shown in Table 2.3. The overall specification of the model is well supported by the results of the test for overidentifying restrictions. The χ^2 statistic is 15.6 and the 99% critical value with 18 degrees of freedom is 34.8. It is remarkable, furthermore, that estimates for the equilibrium capacity rate and the weighting parameter λ are very significant. Recall that the estimate of the NAICU is approximately equal to the 82% used by the Board of Governors as the non accelerating rate of capacity utilization. The value of the weighting parameter λ is slightly smaller than the widely used value of 0.5 which is the basis of the 'sticky' inflation model of Fuhrer and Moore (1995).

The important point is that the slope coefficient is not very significant, probably

because the model does not capture the dynamics of the data in a right way. Hence, the next step is to relax the assumption of constancy in the slope of the 'hybrid' Phillips curve (2.19). To that end, the parameter γ is now considered to be a time-invariant linear function of the lagged NAICU gap.

$$\gamma_t = k_0 + 100k_1 \left(\mathcal{C}_{t-1} - \mathcal{C}^*\right).$$

At this stage, the non accelerating inflationary capacity utilization rate is kept constant.

$$E\left\{\left(\pi_t - \lambda\pi_{t+1} - (1 - \lambda)\pi_{t-1} - \gamma_t \left(\mathcal{C}_t - \mathcal{C}^*\right)\right)\mathbf{u}_t\right\} = 0,$$

where the vector of hyperparameters to estimate is $\psi_0 = \{\lambda, \mathcal{C}^*, \gamma\left(k_0, k_1\right)\}'$. The set of instruments include the second to the fifth lagged value of inflation, capacity, the long-short term interest rate spread, wage inflation and a measure of productivity. The results of the GMM estimation yield a value for the χ^2 statistic in the overidentifying test of 11.9 which is smaller than 26.2, the 99% critical value with 12 degrees of freedom. Thus, the model is well specified. The estimated coefficients are shown in Table 2.4 from where it can be seen that the weighting parameter λ and the NAICU are slightly higher than in the previous model, but they are again very significative. The important finding refers to the estimated value of the coefficient k_1 which is positive and significant at the 5 percent level. This suggest that the capacity constraint hypothesis cannot be disregarded as a source of (nonlinearity) asymmetry in the short-run Phillips curve.

The final step is to estimate the model with both the slope and the NAICU being time variant, which is the model given by equations (2.5)-(2.7). Results are shown in Table 2.5. The Kalman-GMM estimates of the parameter vector $\psi_0 = \left\{\lambda, \gamma\left(k_0, k_1\right); \sigma_\varepsilon^2, \sigma_\eta^2\right\}'$ are

significant with a χ^2 statistic in the overidentification test of 12.9 and since the 99% critical value with 13 degrees of freedom is equal to 27.7, the model seems to be well specified. It must be pointed out that the variances of the transition and measurement equation, σ_ε^2 and σ_η^2 respectively, are not estimated directly, but with an iterative procedure. In Figure 2.12, the capacity utilization rate and the time-varying NAICU, obtained from the estimation of the Phillips curve, are displayed. It can be inferred that both the NAICU and the observed capacity utilization rate have exhibited a low variability in the last two decades, with values ranging from 82 to 83 per cent. This result could complement those obtained by McConnell and Pérez-Quirós (2000), who document a decline in the volatility of real GDP growth since the first quarter of 1984. Related studies in this context document similar patterns for other key macroeconomic variables.

The results in this section have two main contributions: first, it responds to the need for a reliable measure of the NAICU put forward by some authors, since the only available measure is the *ad hoc* Federal Reserve Board estimate of 82%; and second, it provides empirical support for the consideration of asymmetries in the design of monetary policy rules by central bankers.

2.4 Concluding Remarks and Extensions

The goal of this Chapter has been to shed light into the nature of the monetary transmission mechanism. To that end, the joint dynamics of two key macroeconomic variables, prices and the capacity utilization rate, the latter being a measure of real economic activity, have been studied. To carry out the analysis, an econometric procedure

useful for estimating dynamic rational expectation models with unobserved components is used. The method combines the flexibility of the unobserved components approach based on the Kalman recursion, with the power of the GMM estimation procedure. The method is applied to the estimation of a 'hybrid' Phillips curve that relates inflation to the capacity utilization gap, where the term 'hybrid' means that in the specification of the Phillips curve, both forward and backward looking components are considered. The results have shown that such a relationship is non-linear: the slope of the Phillips curve depends significantly on the magnitude of the capacity gap. This finding has important implications for the conduct of monetary policy and particularly for the design of monetary policy rules. These rules could be evaluated within an appropriate framework and its performance compared to simple feedback rules, such as Taylor-type rules which have been the object of increased attention among monetary policy practitioners and monetary theorists. These rules are characterized by an aggressive response of the interest rate to high inflation and a high output gap. According to the results of this Chapter, such a response should be made dependent on the degree of utilization of economic resources, considering, for instance, state dependent coefficients. Moreover, as an alternative to the output gap, one could use the NAICU gap. Indeed, recent studies that analyze the implications of measurement errors for the design of monetary policy, such as Orphanides et al. (1999), show that the results using the capacity utilization rate are more encouraging that those based on the output gap, despite the high correlation between both series. Hence, an interesting extension will be the consideration of a reduced form model in the spirit of Rudebusch and Svensson (1999), where the particularities described here will be considered.

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Table 2.1: Stationarity Tests

Series	Model	Lags	ADF (t - statistic)
Inflation, π_t	Constant - No Trend	0	-3.00*
Capacity, $\mathcal{C}_t$	Constant - No Trend	3	-4.07***
Wage Inflation	Constant - No Trend	1	-4.72***
Spread	Constant - No Trend	2	-3.51*
Productivity	Constant - No Trend	1	-8.08***

Note: Asterisks denote the rejection of the null hypothesis at the (*) 10%, (**) 5% and (***) 1% significance levels. Critical values are taken from Hamilton (1994) Table B.6 page 763, these are -2.57 at 10%, -2.88 at the 5% and -3.46 at the 1% level. The model Constant-No Trend is $y_t = \xi_1 \Delta y_{t-1} + \xi_2 \Delta y_{t-2} + \cdots + \cdots + \xi_p \Delta y_{t-p} + \alpha + \rho y_{t-1} + u_t$ refers to the Case 2 in Hamilton (1994).

Table 2.2: Granger Causality Tests

Causality Sense	F-Statistic	Causality Sense	F-Statistic
Capacity-Inflation	7.21***	Inflation-Capacity	1.95
Wages-Inflation	8.23***	Inflation-Wages	2.89**
Productivity-Inflation	3.26**	Inflation-Productivity	1.12
Spread-Inflation	3.75***	Inflation-Spread	1.11

Note: Asterisks denote rejection of the null hypothesis at the (**) 5% and (***) 1% significance levels. Critical values are taken from Hamilton (1994) Table B.4 page 760, these are 2.44 at the 5% and 3.47 at the 1% level. The F statistic has p degrees of freedom in the nominator and (t-2p-1) in the denominator, where p (=4) is the number of lags and t (=156) is the sample size.

Table 2.3: *Hybrid* Phillips Curve: Constant Slope and NAICU

$$E\left\{\left(\pi_t - \lambda\pi_{t+1} - (1-\lambda)\pi_{t-1} - \gamma\mathcal{C}_t + \gamma\mathcal{C}^*\right)\mathbf{u}_t\right\} = 0$$

Variable	Coefficient	Standar Error	T-statistic	P-value
λ	0.43295	0.0644	6.7238	0.00
γ	1.4155	1.0515	1.3462	0.08
$\mathcal{C}^*$	0.8283	0.0248	33.333	0.00

Table 2.4: *Hybrid* Phillips Curve: Time Varying Slope and Constant NAICU

$$E\left\{\left(\pi_t - \lambda\pi_{t+1} - (1-\lambda)\pi_{t-1} - \gamma_t\left(\mathcal{C}_t - \mathcal{C}^*\right)\right)\mathbf{u}_t\right\} = 0$$

$$\gamma_t = k_0 + 100k_1\left(\mathcal{C}_{t-1} - \mathcal{C}^*\right)$$

| Variable	Coefficient	Standar Error	T-statistic	P-value
λ	0.44003	0.007791	5.63294	0.00
k_0	8.22689	2.769037	2.99514	0.05
k_1	0.92696	0.349778	2.65446	0.05
$\mathcal{C}^*$	0.84971	0.006993	121.8313	0.00

Table 2.5: *Hybrid* Phillips Curve: Time Varying Slope and NAICU

$$E\left\{\left(\pi_t - \lambda\pi_{t+1} - (1-\lambda)\,\pi_{t-1} - \gamma_t\left(\mathcal{C}_t - \mathcal{C}^*_{t|T}\right)\right)\mathbf{u}_t\right\} = 0$$

$$\gamma_t = k_0 + 100k_1\left(\mathcal{C}_{t-1} - \mathcal{C}^*_{t-1|T}\right)$$

| Variable	Coefficient	Standar Error	T-statistic	P-value
λ	0.43443	0.761940	5.70172	0.00
k_0	7.05163	0.905789	7.78507	0.00
k_1	1.16649	0.490279	2.37924	0.00
σ^2_ε	0.93472	-	-	-
σ^2_η	0.00025	-	-	-
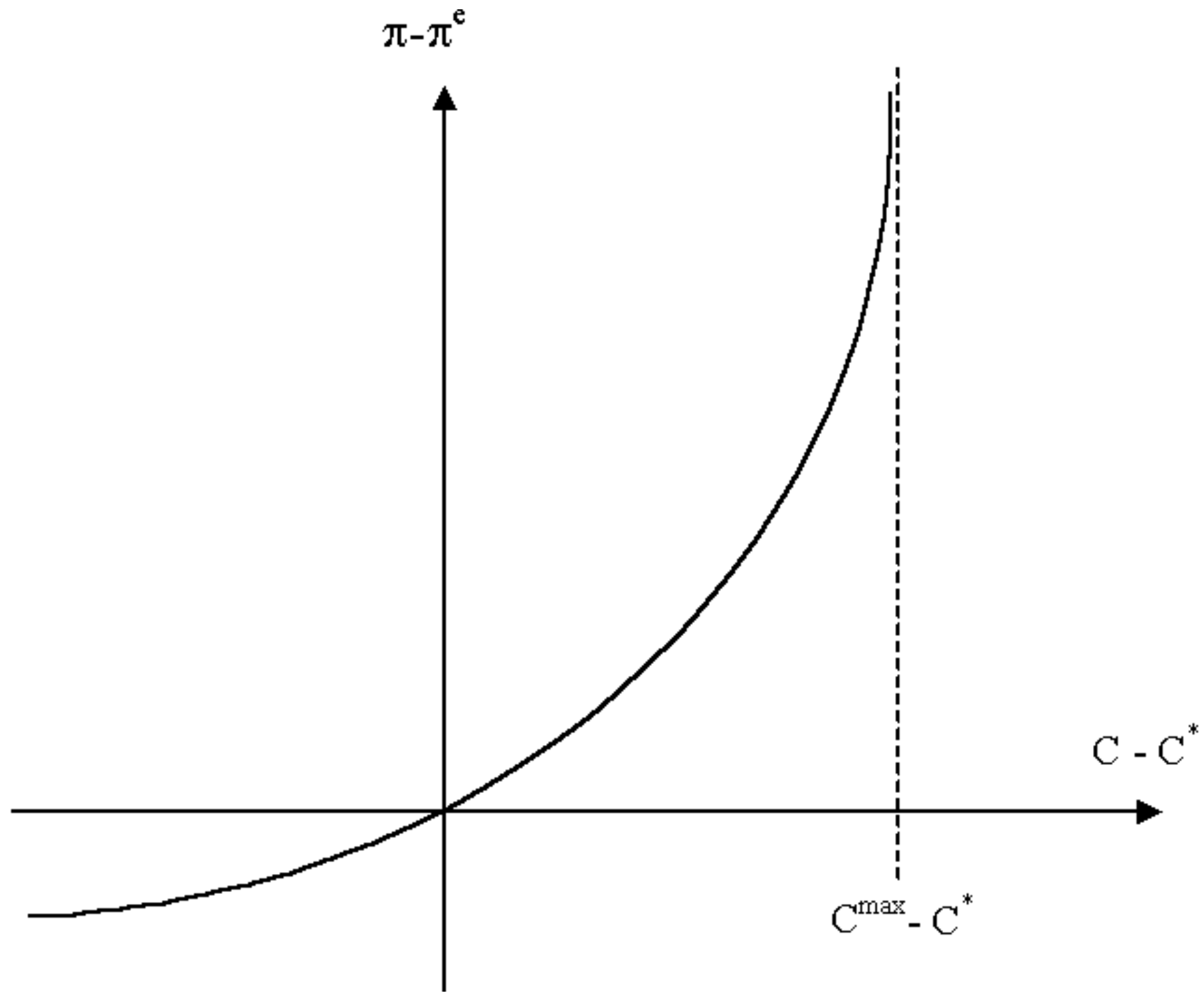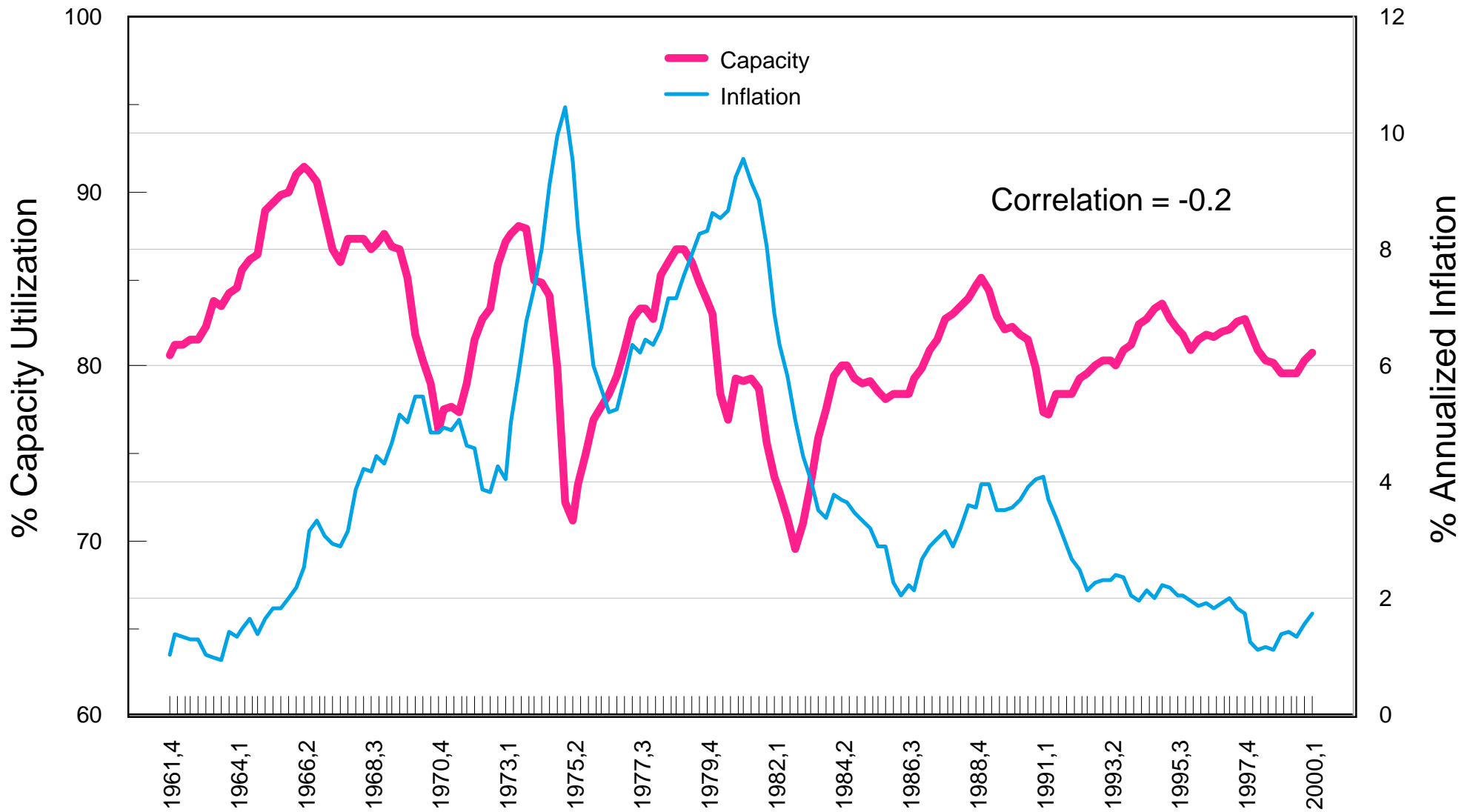
Note: Inflation is the four-quarter moving average of the annualized quarterly GDP-Deflator
Capacity Utilization is the quarter average percent of capacity in manufacturing sector
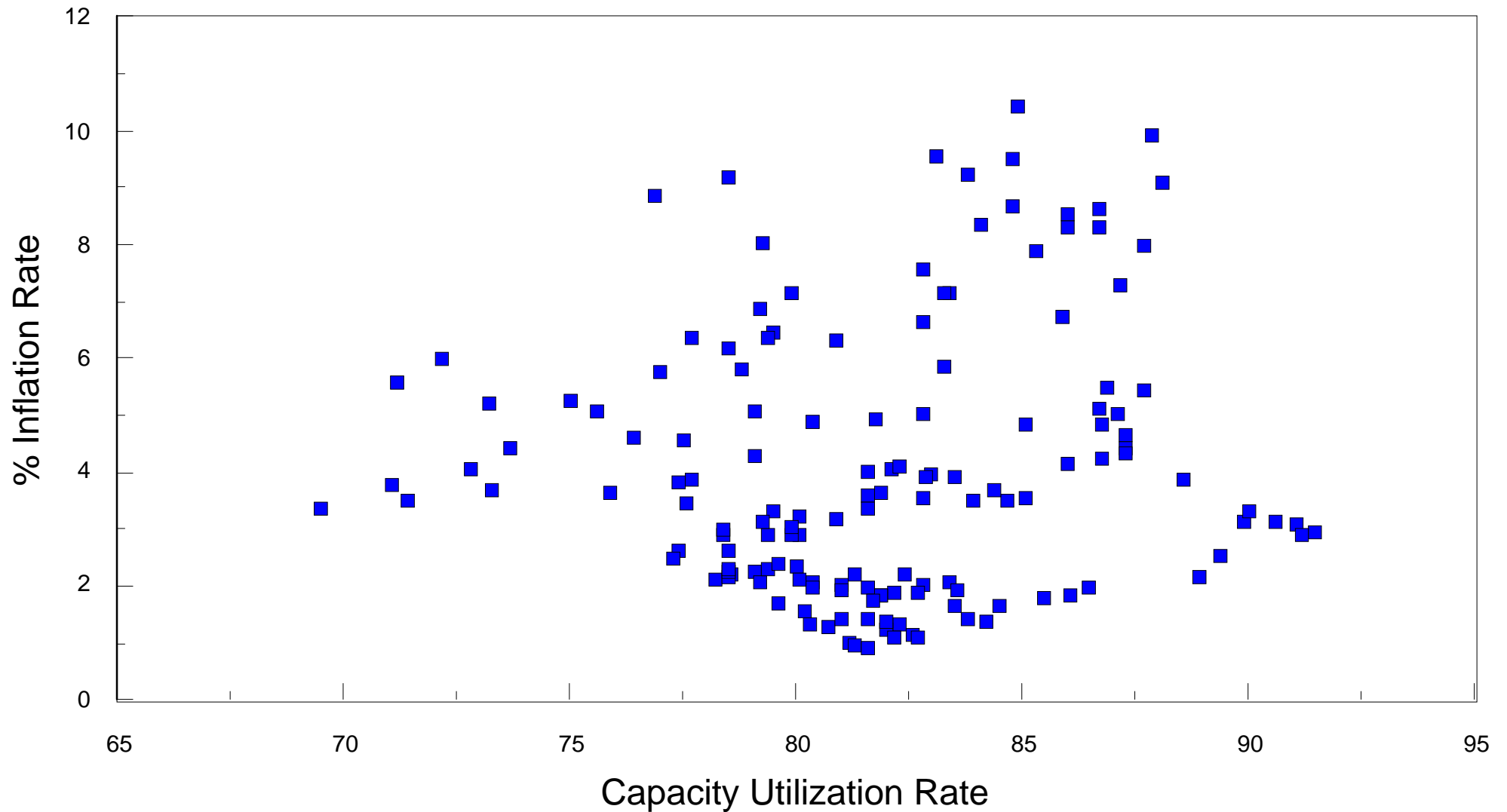**Source:** Board of Governos Federal Reserve System and the U.S. Department of Commerce
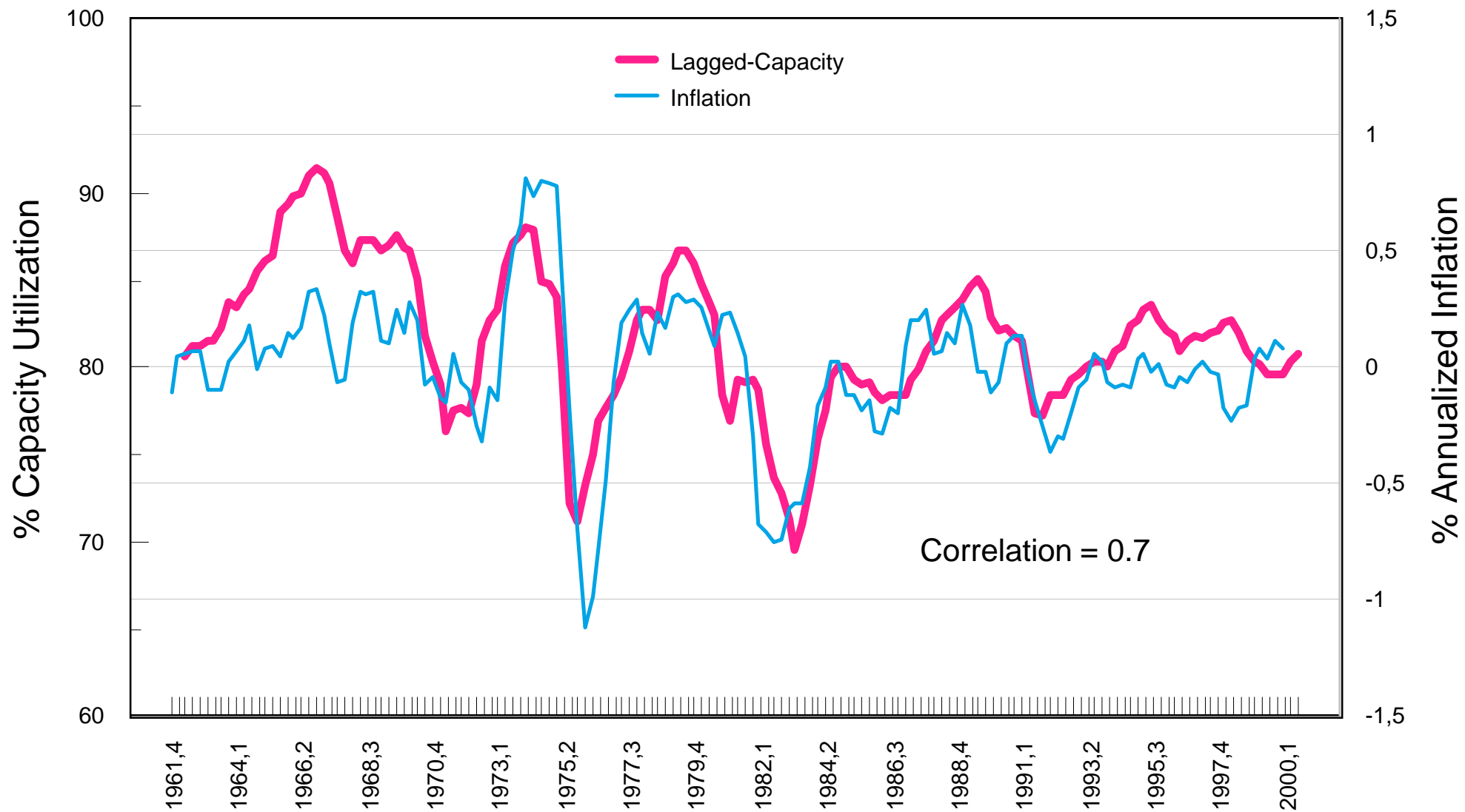
Capacity (t) and Inflation (t+i)
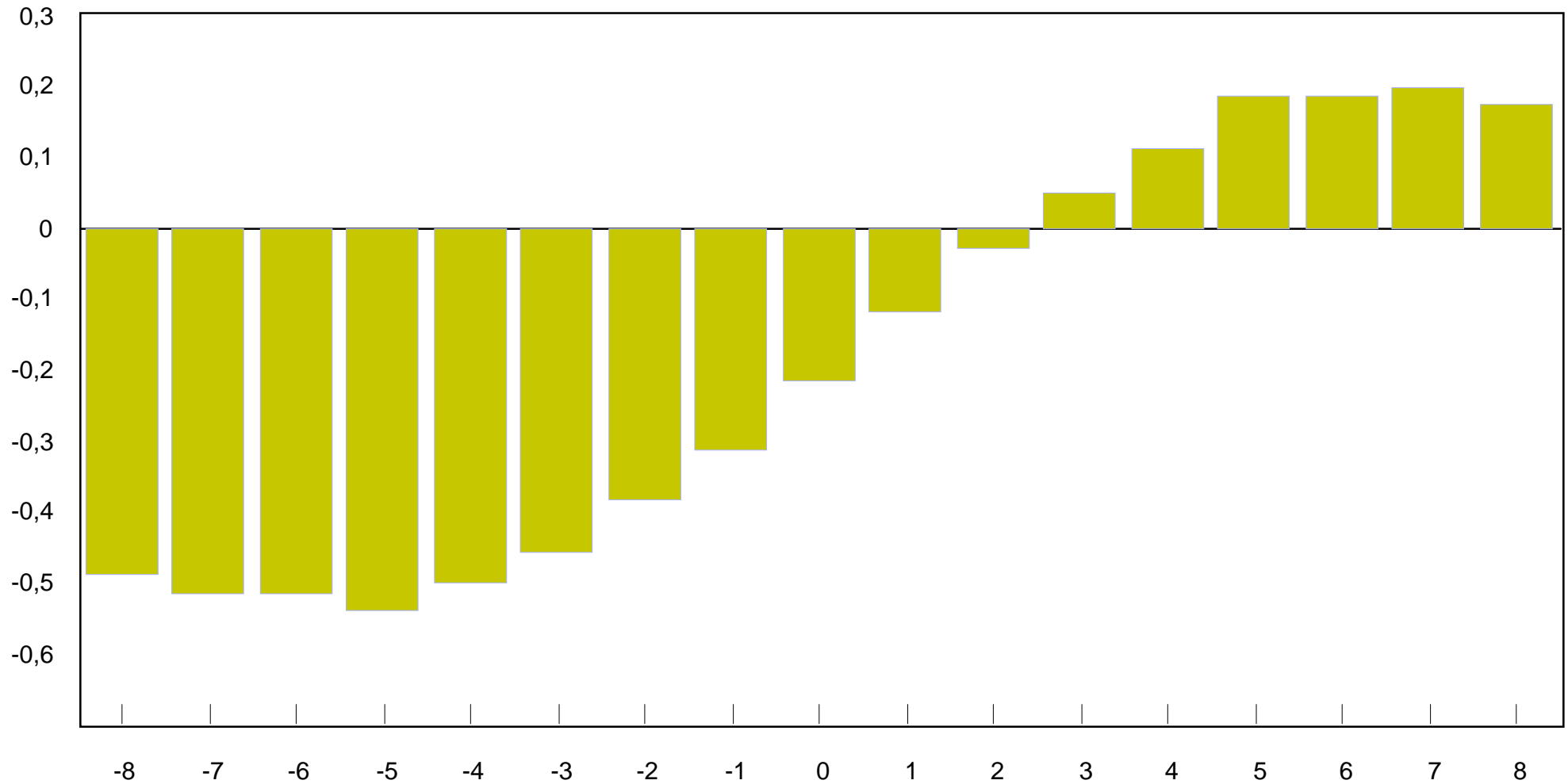
Note: Inflation is the four-quarter moving average of the annualized quarterly GDP-Deflator
Capacity Utilization is the quarter average percent of capacity in manufacturing sector
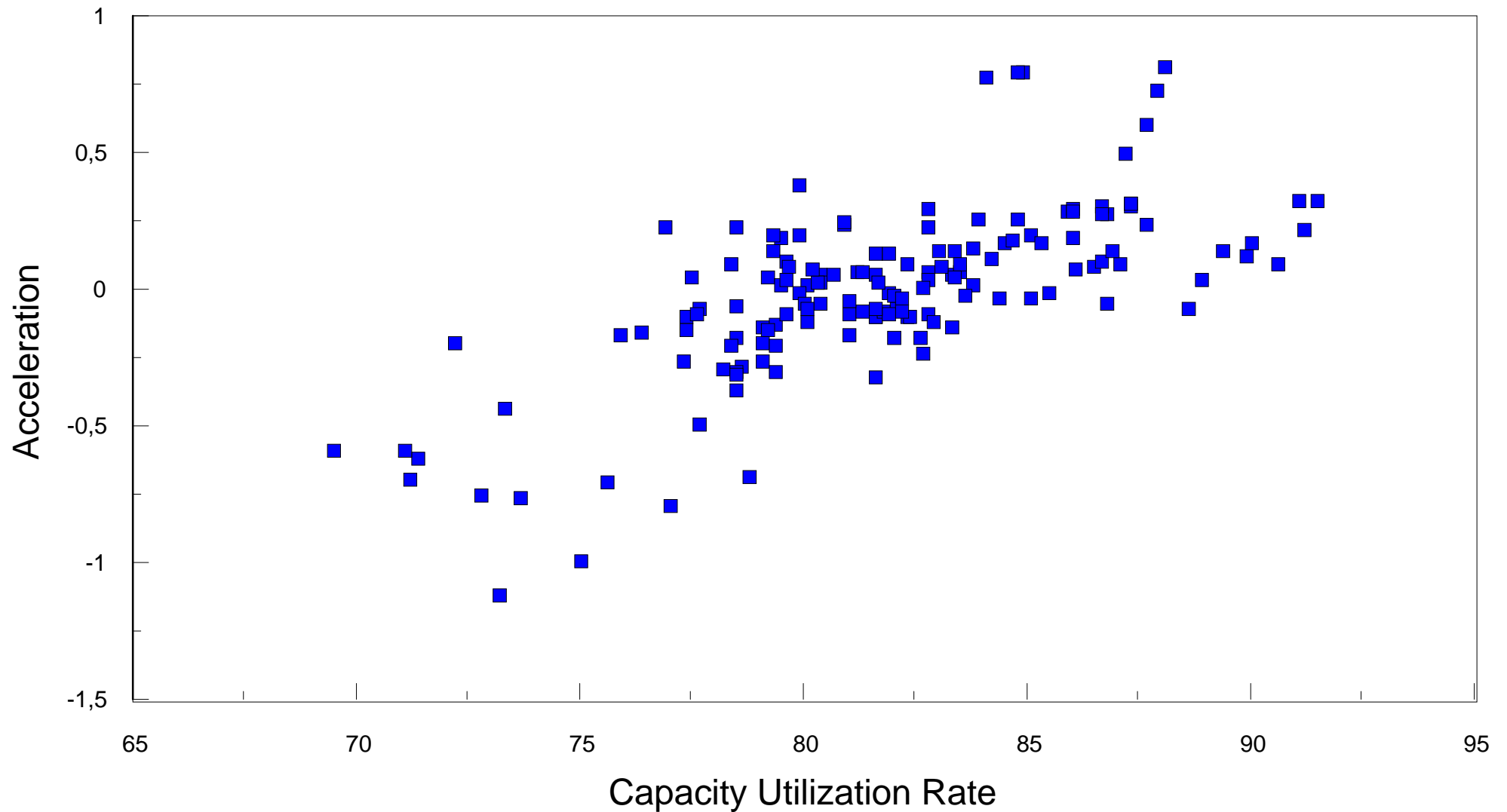**Source**: Board of Governors Federal Reserve System and the U.S. Department of Commerce

Note: Inflation is the four-quarter moving average rrate of change in the GDP-deflator
Capacity Utilization is the quarter average percent of capacity in manufacturing sector
Sourc : Board of Governors Federal Reserve System and the U.S. Department of Commerce
Sample Period: 1961:Q1 - 2000:Q1

Note: Acceleration is the first difference of the annualized GDP-Deflator rate of inflation
Lagged capacity is the two lagged value of the capacity utilization rate.
Source: Board of Governors Federal Reserve System and the U.S. Department of Commerce

Note: Acceleration is the first difference of the GDP-deflator annualized inflation rate
Capacity Utilization is the quarter average percent of capacity in manufacturing sector
Sourc : Board of Governors Federal Reserve System and the U.S. Department of Commerce
Sample Period: 1961:Q1 - 2000:Q1

Capacity (t) and Acceleration (t+i)

Note: Acceleration is the first difference of the annualized GDP-Deflator rate of inflation
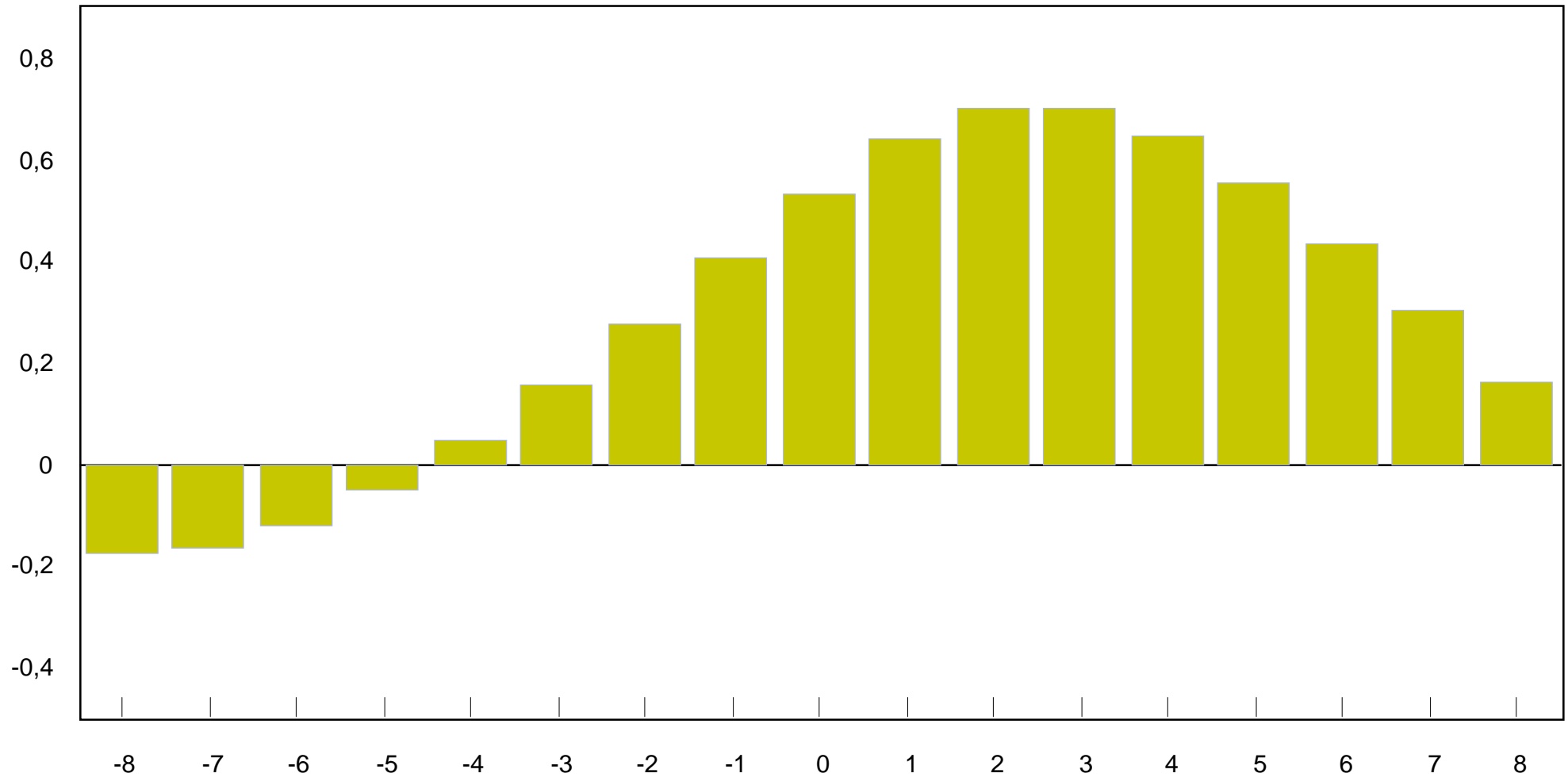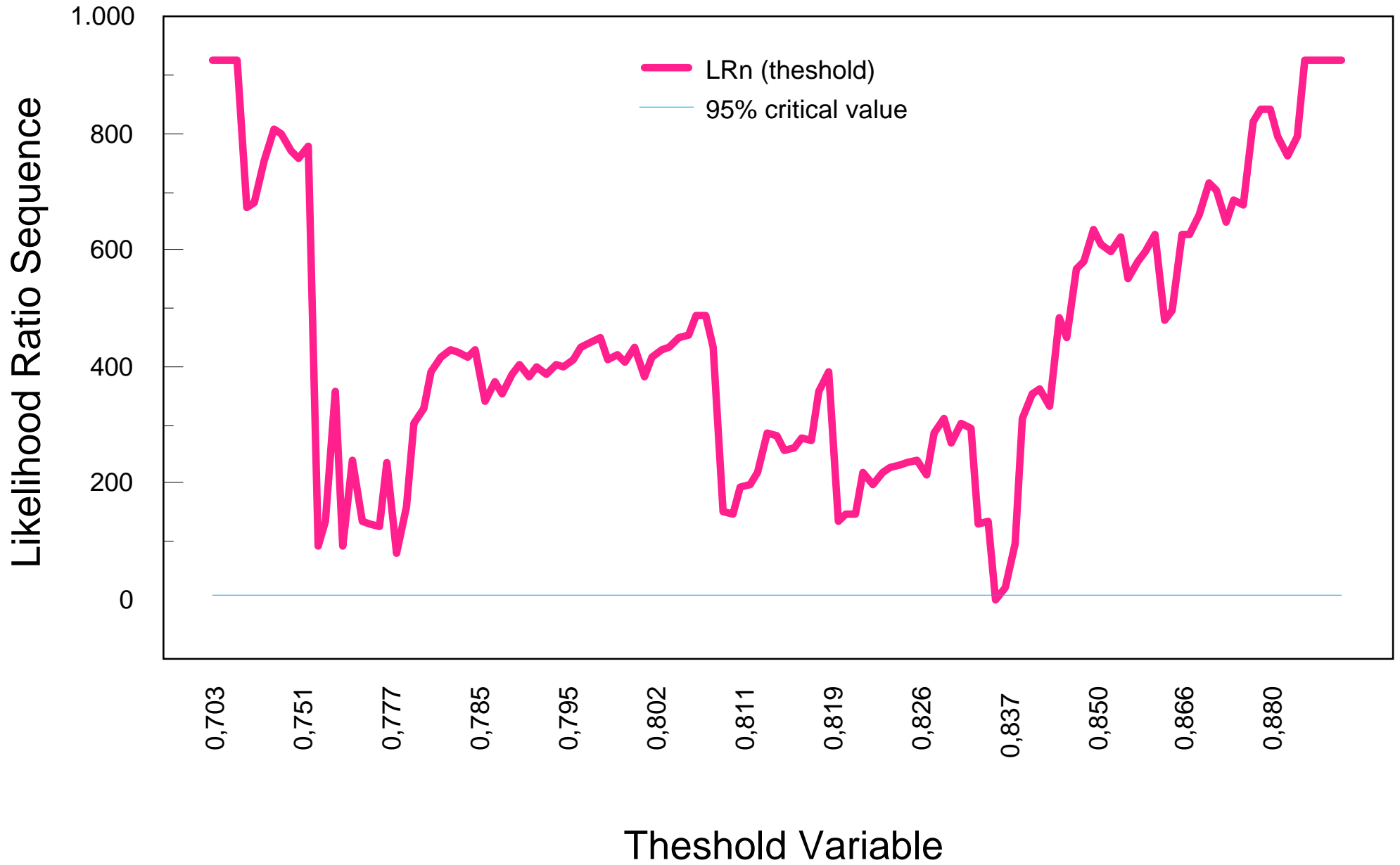Lagged capacity is the two lagged value of the capacity utilization rate.
Source: Board of Governors Federal Reserve System and the U.S. Department of Commerce

Confidence Interval Construction for Theshold

- LRn (theshold)
- 95% critical value

Likelihood Ratio Sequence

Theshold Variable

X-axis labels: 0,703 0,751 0,777 0,785 0,795 0,802 0,811 0,819 0,826 0,837 0,850 0,866 0,880

Y-axis labels: 0 200 400 600 800 1.000

F test for Theshold

Fn (theshold) — vertical axis

Theshold Variable — horizontal axis

— LRn (theshold)
— 95% Critical value
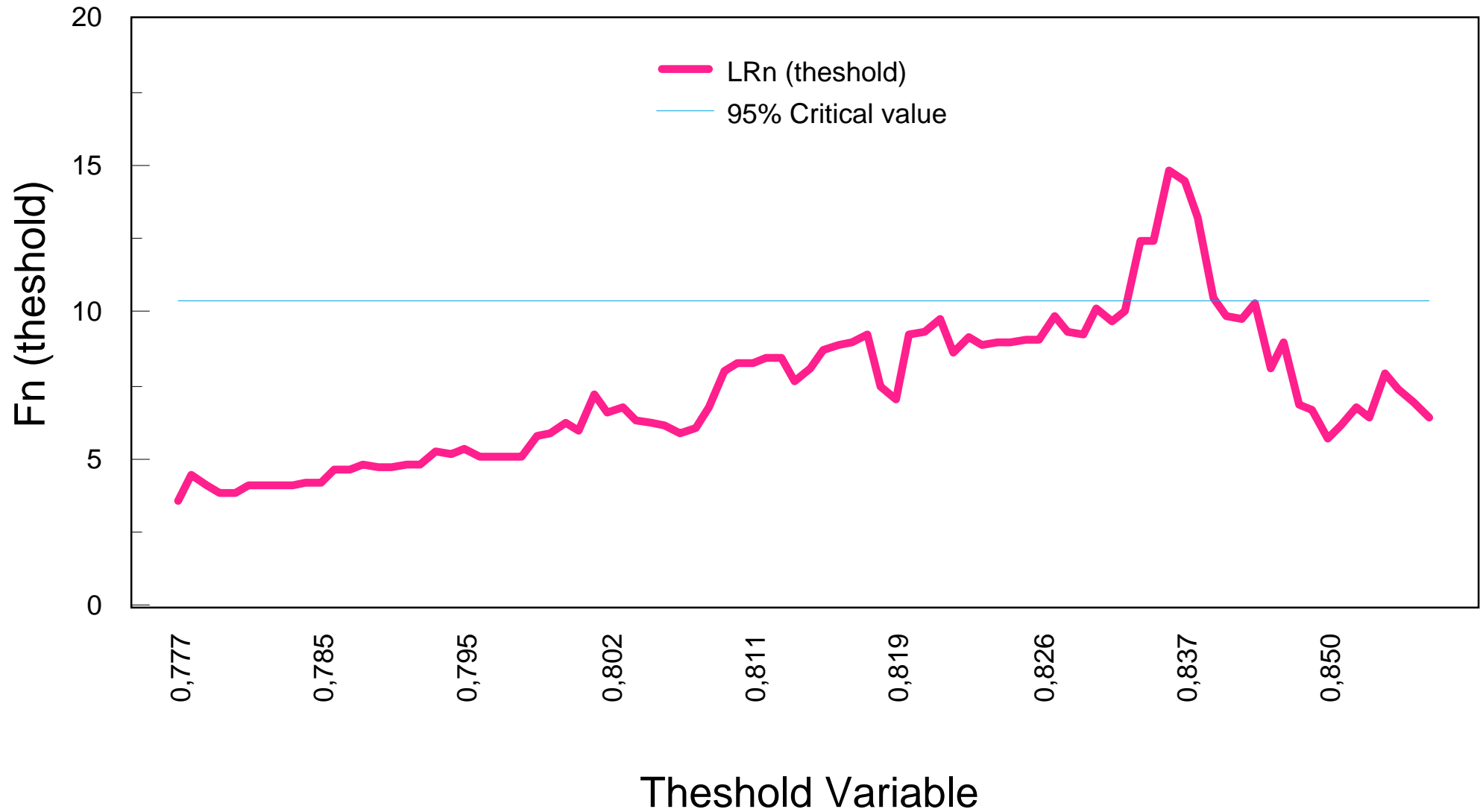
Note: Reject the null H0: Linearity if F sequence exceeds Critical Value

Classification by Regimes

Capacity utilization rate vs. Date

Regime 1
Uncertain
Regime 2
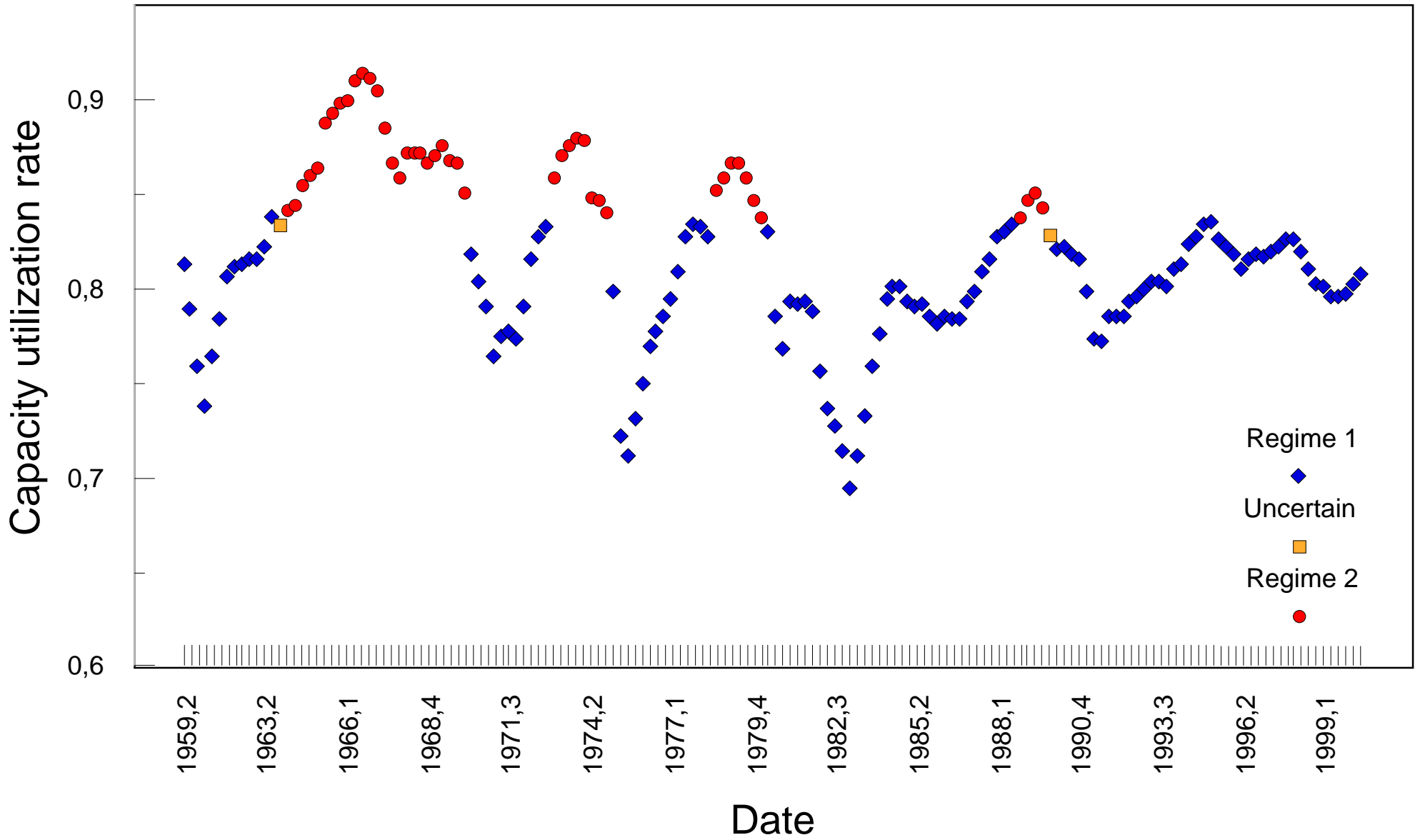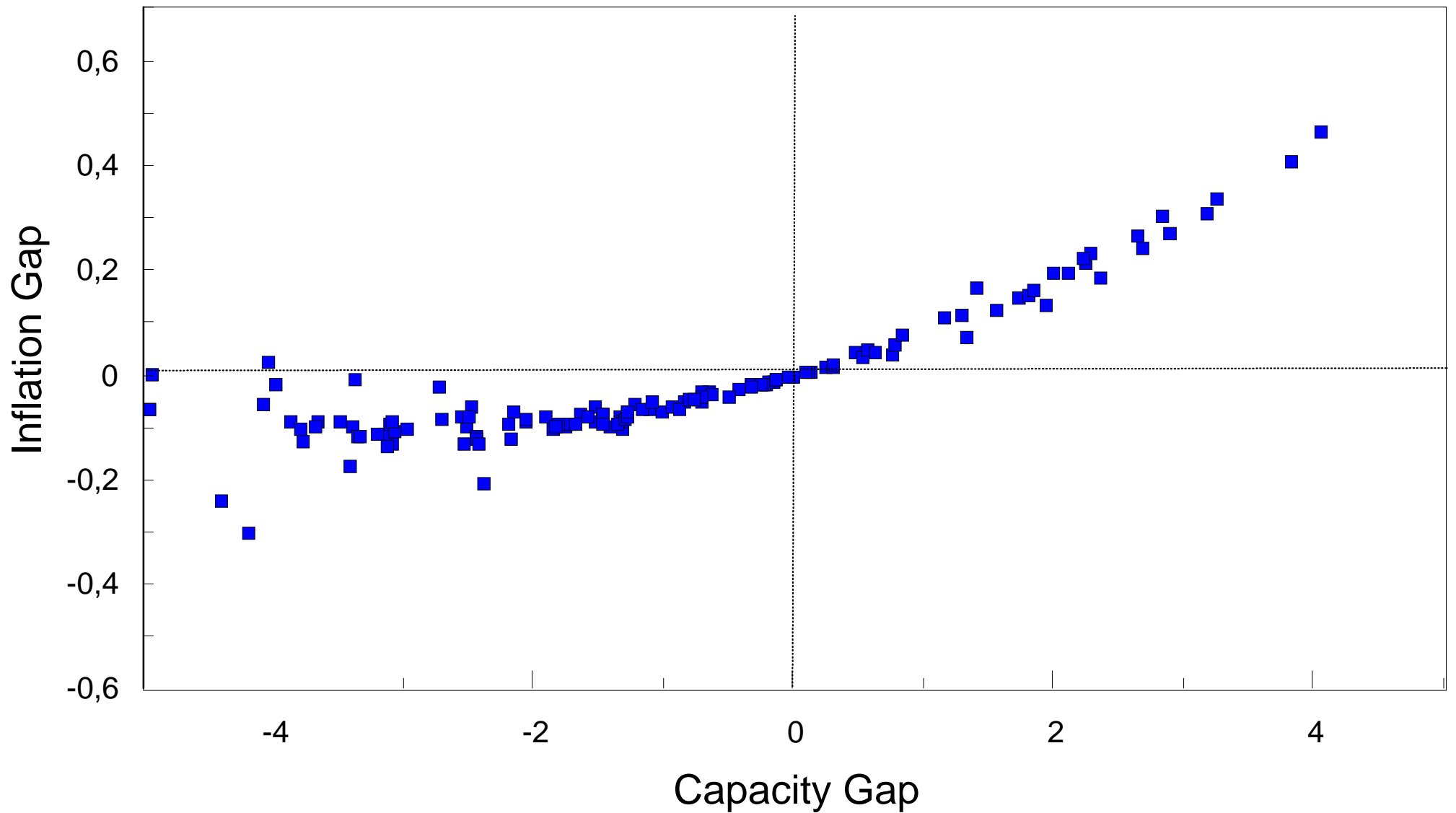
Note: Representation of the estimated Phillips Curve with time-varying slope and NAICU according to the equation $E\left\{\left(\pi_t - \lambda\pi_{t+1} - (1-\lambda)\pi_{t-1} - \gamma_t\left(C_t - C^*_{t|T}\right)\right)u_t\right\} = 0$ and $\gamma_t = k_0 + 100k_1\left(C_{t-1} - C^*_{t-1|T}\right)$
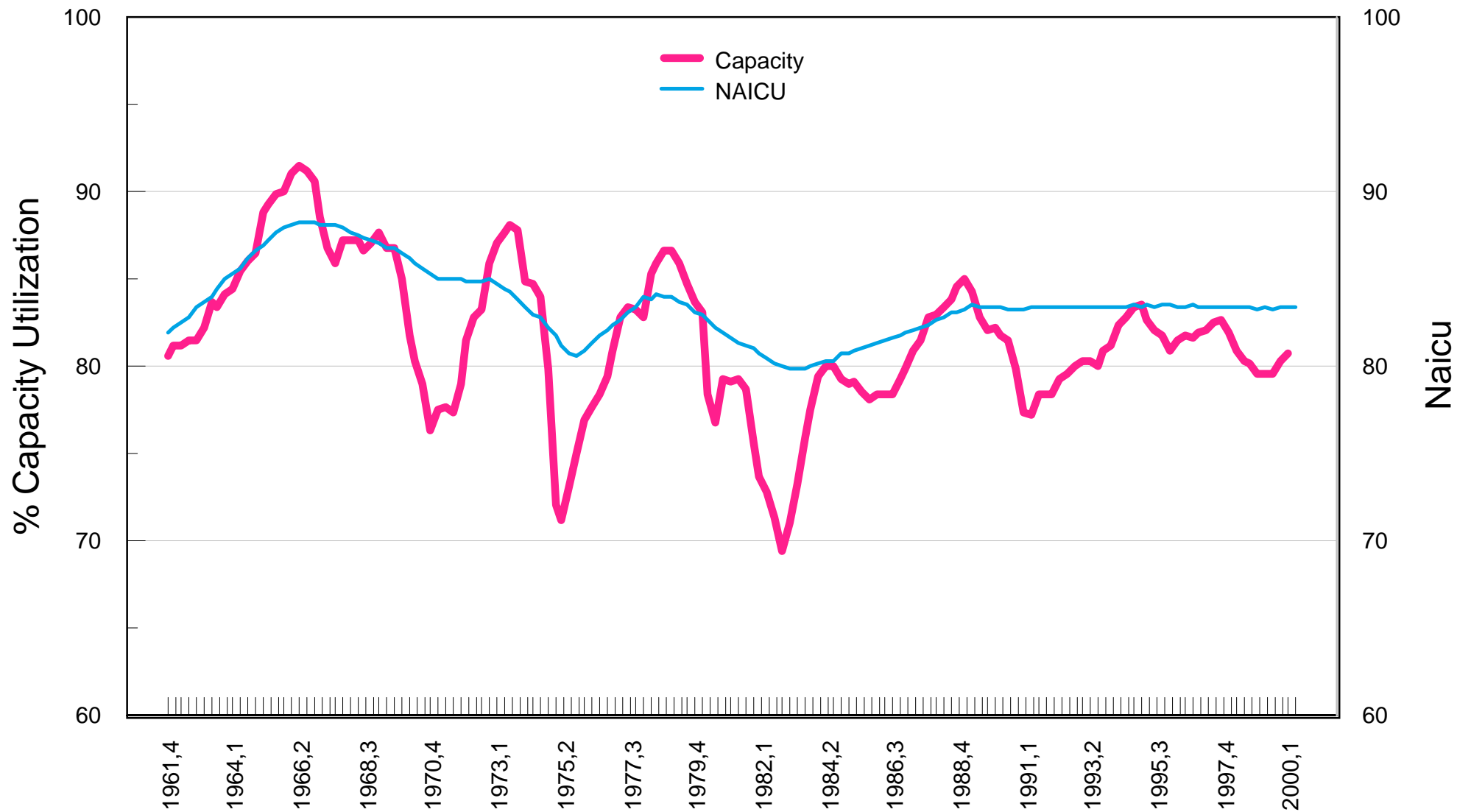
Note: Naicu is the non-accelerating rate of inflation estimated using the Kalman-Gmm method
         Capacity Utilization is the quarter average percentage of capacity in manufacturing sector
Source: Board of Governors of Federal Reserve System and the U.S. Department of Commerce

Chapter 3

Capital Flows and Foreign Interest Rate Disturbances in a Small Open Economy

3.1   Introduction

As world financial markets become more integrated and economic agents more internationally diversified, the exposure of an economy to foreign shocks increases notably. Moreover, events in a given country affect the performance of neighboring economies to a much more significant extent. As a consequence, the study of the impact that the process of increasing globalization experienced in recent years has on key macroeconomic variables becomes of particular interest. One of the main challenges faced by policy-makers around the world has been the design of procedures that limit the potentially adverse effects that

this higher interdependence might have on the economy. At the same time, however, policy-makers should be able to capture the possible benefits that may be generated under this scenario. Given that capital movements have turned out to be a major contributor to interdependence among economies, it would clearly be of great interest to have a better understanding of their interaction with the main macroeconomic variables of a country. With this in mind, the present Chapter of this Thesis attempts to provide a theoretical framework that allows for the study of the dynamic behavior of capital flows in a small open economy which is integrated into the international financial scene. Specifically, the aim of this study is to analyze the main influences on these dynamics as reflected in the behavior of the current account. To this end, a dynamic optimization macroeconomic model in the spirit of Matsuyama (1987) is used. The model to be considered is that of a small open economy based on intertemporal optimization by individuals with a finite life-span. Economic agents' decisions are modelled within a framework of perfect foresight, which in the absence of uncertainty is equivalent to a world of rational expectations. Other possible theoretical approaches to the analysis of capital flows exist, an example being the representative agent models of Uctum (1991), Turnovsky (1995) or Agénor (1998). Nevertheless, finite-horizon models, such as the one considered here, provide a very useful approach to the study of current account dynamics since their structure allows one to combine investment and savings dynamics in a convenient way. Moreover, the existence of a stable steady-state for the small open economy is guaranteed even in the case of a constant rate of time preference that is different from the world interest rate, whereas in infinite horizon models the aforementioned rates must be equal in order to rule out

degenerate dynamics.

The present theoretical inquiry extends the original work of Matsuyama (1987) through the analysis of the dynamics of the current account when the economy faces a disturbance in world interest rates. It is proven that the effects of this sort of shock depend mainly on the net foreign asset position of the domestic economy, as well as on the speed of adjustment of both investment in productive capital and the intertemporal substitution of consumption, that is, saving. For instance, if the country is initially a net creditor on the international financial stage, an increase in foreign interest rates will induce a net outflow of capital. The opposite is likely to occur for the case of a net debtor country. The particular path that the current account follows when facing this particular shock will depend on the relative magnitude of saving and investment dynamics. A detailed discussion of these issues is provided, illustrating some of the cases with simple numerical examples.

Imperfect Capital Mobility and Money

An important issue that is considered in the present analysis is the treatment of the degree of international capital mobility. Most of the theoretical literature on current account dynamics builds upon the assumption of perfect capital mobility. Nevertheless, empirical studies do not always support this assumption, leaving the question as an open debate.[1] Even though the issue of the mobility of capital has been treated from different perspectives, the overall conclusion reached is that capital is certainly mobile across countries but not as much as would be expected. Given this, it is natural to think that a high

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[1] Three main approaches can be highlighted: the Consumption Insurance Approach -see Obstfeld (1994), the so called Law of One Price -see Frenkel (1993) and Popper (1993), and the International Allocation of Investment Course -see Feldstein and Baccheta (1991).

degree of capital mobility is an assumption that can only be applied to a limited number of economies. Several reasons can be put forward in this regard, including transaction costs, government intervention and regulation policies, costs of gathering and processing information, financial constraints and other such capital market imperfections.[2] Nevertheless, it is also obvious that global trade in financial assets has experienced a substantial increase in recent years and that there is a widespread trend towards deregulation of domestic and international capital market activities. In theory, the potential benefits of international capital mobility are clear. Individuals gain the opportunity to smooth consumption by borrowing or diversifying abroad, and savings are directed towards the world's most productive investment opportunities. However, the size and extent to which these gains have been attained in practice remains uncertain, and it would be interesting to analyze in detail the economic and welfare implications arising from a more economically-integrated world. This would certainly help policy makers to deal with the consequences of the irreversible process upon which society is nowadays embarked. While this issue is out of the scope of the present work, it constitutes an exciting line for future research.

The benchmark model, upon which the bulk of the analytical discussion is based, takes into account the presence of exogenously-given imperfections on the mobility of capital. This feature is modeled by means of a country-specific interest cost that domestic investors must pay on foreign asset holdings. In one of the two extensions of the benchmark model, this characteristic is further studied. In particular, following Agénor (1998), Senhadji (1997) and Fisher (1995), the existence of an upward-sloping supply of foreign

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[2]Hermalin and Rosen (1999) provide a framework for understanding the sources of risk in international borrowing that are absent in a purely domestic context: the risk that sovereign borrowers will default and the risk of macroeconomic instability that stems from the impact of net capital flows.

loans is considered. Hence, the interest costs arising from imperfections or rigidities on the markets for international financial investment are endogenized. As a result, it is possible to study disturbances in world interest rates differently from those which are particular to the interest cost faced by the domestic economy. Additionally, within the present framework it is feasible to analyze the consequences of exogenous shocks, under different degrees of capital mobility, on the dynamics of capital flows. In particular, it is proven that the speed of adjustment of the intertemporal substitution of consumption, and thus the dynamics of capital flows, depends on the net asset position of the economy and on the strength of international capital market imperfections. Importantly, the present study places a special emphasis on the effects that differing degrees of capital mobility, measured through the cost of interests, have on the dynamic behavior of capital flows.

In recent times, a considerable amount of research has been devoted to the importance that the level of development of the domestic financial system has in the propagation of macroeconomics disturbances.[3] As a first approach to this, the benchmark model is extended through the incorporation of a monetary side as in Park (1994). The approach adopted here lies on the Sidrausky tradition of including money in the utility function.[4] The introduction of money has interesting consequences on the dynamics of the current account. In particular, the parameter relating to the marginal utility of money holdings, which can be associated with the level of domestic financial development, plays an important role. It is shown that the higher the weight of money in the utility function, which

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[3]Aghion, Bacchetta and Banerjee (1998) develop a model to study the effects of capital account liberalizations in economies at an intermediate level of financial development. Edwards and Vegh (1997) analyze how the banking sector affects the propagation of shocks in a small open economy.

[4]By facilitating transactions, money is assumed to yield direct utility to the consumer that is not associated with other assets such as bonds, which yield only an indirect utility through the income they generate and the consumption goods they enable the agent to purchase.

one could identify with a poor financial system, the slower are the transitional dynamics of intertemporal substitution of consumption, and hence the dynamic adjustment of capital flows in the face of a given shock.

The Chapter is organized as follows. Section 3.2 presents the economic environment with exogenous imperfect capital mobility across countries. The long run effects of a world interest rate disturbance are analyzed in this section, as well as the dynamics followed by the economy toward the new steady state. Two extensions of the baseline model are presented in Section 3.3 and Section 3.4. In the former, the imperfection on international capital flows are endogenized, while in the latter a monetary side is introduced into the model. Finally, Section 3.5 contains some concluding remarks and presents possible lines for future research.

3.2   The Model

3.2.1   Exogenous Imperfect Capital Mobility

The model deals with a small open economy that produces a single good that can be either consumed, traded or used for domestic capital formation. Output is produced according to a well-behaved production function using physical capital and labor. Inputs are assumed to be immobile across frontiers. Residents in the domestic economy can trade in financial assets. Hence, under perfect capital mobility, and assuming domestic assets and foreign bonds are perfectly substitutes, asset market equilibrium requires that the real interest rate in the domestic economy, r_t, must be equal to the world interest rate r_t^*. Nevertheless, one of the main aspects that distinguishes the present model from those

previously employed in the literature relates to the assumption of perfect capital mobility.[5] In the framework proposed here, capital is not completely mobile across countries. This consideration can be justified because of the existence of legal restrictions and liquidity and risk constraints that apply to capital account transactions. In this way, some sort of sluggishness in capital mobility that is in closer accordance with reality, especially in developing countries, is introduced. Analytically, the idea of imperfect international capital mobility is modelled with the introduction of a country-specific interest cost on foreign bonds held by domestic residents. This, in turn, would give rise to a divergence or wedge between the domestic and world interest rates. As a first step of our analysis, it is considered that such an interest cost, denoted by μ, is an exogenously given parameter.[6] Under this scenario, the uncovered interest parity condition becomes

$$r_t = r^* + \mu_t \tag{3.1}$$

which in the sequel is referred to as the international *effective* security arbitrage condition.

Households

Following Blanchard (1985), the economy is inhabited by individuals with a finite life-span who possess a constant probability p of death. A crucial assumption is the existence of perfect annuity markets. One can assume that because of the uncertain lifetimes, all loans require the purchase of life insurance in addition to regular interest payments. In case of

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[5]Following Obstfeld (1995), capital is said to be freely mobile within a multicountry region when its residents face no official obstacles to the negotiation and execution of financial trade anywhere and with anyone within the region and face transaction costs that are no greater for parties residing in the same country.

[6]One can justify such an assumption by considering, for instance, the existence of a tax that is levied on foreign traded bonds. Another possibility would be the existence of an international financial agency that, on the basis of studies of different parameters and variables, issues an assessment of the risk associated with the domestic economy.

death, the estate is transferred to the life insurance company which, in turn, guarantees to cover outstanding debts. It is assumed that there is a large number of individuals in each cohort so that the frequency of those who survive equals the survival probability, $1 - p$. That is, at each instant of time a cohort of new consumers of size p is born without inheritance. By assuming that there are a large number of consumers, such contracts contingent on death can be offered risklessly if the risk of death is purely individualistic and there is no aggregate uncertainty. Furthermore, it is assumed that there is competition among insurance companies. Under such circumstances, the zero-profit condition ensures that the percentage insurance premium equals the probability of death p.[7]

The economy has a steady population normalized to one, with pe^{-ps} being the number of the consumers aged s. The real wealth of the representative generation-s agent can be decomposed into human and non-human (portfolio) wealth. The former comes from labor supplied inelastically, earning a competitive real wage of w_t. Asset holdings constitute the other source of real income -non-human wealth- which will be denoted by a_t^s. Consumers' portfolios consist of foreign bonds, b_t, and domestic equities, v_t. The flow budget constraint of the consumer born at time s can, therefore, be expressed in real terms as

$$\dot{a}_t^s = (r_t + p)\, a_t^s + w_t - c_t^s \text{ for all } t > s \qquad (3.2)$$

where $a_t^s \equiv v_t^s + b_t^s$. Notice that the effective return on the financial asset is $(r_t + p)$. This is explained as follows: consumers with positive wealth may contract to receive pa, and to pay a contingent on their death. In the absence of a bequest motive, consumers will have such a contract; consumers with negative wealth are required to pay an insurance premium

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[7]See Frenkel and Razin (1987) for more details.

pa because they have a default risk with probability p. It is assumed that consumers are born without financial inheritance, that is, $a_s^s = 0$. Consumers also need to satisfy the following intertemporal solvency condition which prevents them from continuing to borrow infinitely:

$$\lim_{v \to +\infty} a_v^s \exp\left(-\int_t^v (r_z + p)\, dz\right) \geqq 0 \tag{3.3}$$

Consequently, the representative generation-s agent has to choose the level of real consumption, c_t , and real financial asset holdings, a_t , which solves her intertemporal utility optimization problem. Thus, at any point of time t, the individual born at time s selects a time path $\{c_v^s, a_v^s\}_{v=t}^\infty$ that maximizes her expected lifetime utility function

$$E_t\left[\int_t^{+\infty} U\left(c_v^s\right) e^{\theta(t-v)} dv\right] = \int_t^{+\infty} U\left(c_v^s\right) e^{(\theta+p)(t-v)} dv \tag{3.4}$$

where θ is the pure rate of time preference. Notice that since p is the probability of death it follows that $\theta + p$ is the constant effective rate of time preference.[8] Next, in order to get closed-form expressions it has been chosen a logarithmic functional form for the instantaneous utility function: $U\left(c_v^s\right) = \log c_v^s$. The optimal consumption at time t of a consumer born at time s is obtained, as shown in the appendix, by forming the Hamiltonian associated with the optimization program and combining the first order necessary (and sufficient) conditions with the consumer's present value budget constraint. As a result, the following expression is obtained:

$$c_t^s = [p + \theta]\left(a_t^s + h_t^s\right) \tag{3.5}$$

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[8]To see the importance of the existence of perfect annuity markets, consider the case that such a market does not exist and that the government distributes the non-human wealth left by those who die to those who have just been born in a lump-sum fashion. In this case, the Blanchard economy behaves as if it were an economy with infinitely lived consumers with a pure rate of time preference equal to $\theta + p$. As a result, Ricardian equivalence holds and there is no steady state with positive consumption unless $r = \theta + p$.

where h_t^s denotes the expected human wealth of the consumer at time t. The next step is to aggregate the main macroeconomic variables across generations. This has been done, as in Blanchard (1985), according to the formula:

$$X_t = \int_{-\infty}^{t} x_t^s p e^{p(s-t)} ds, \text{ for } x_t = c_t^s, a_t^s \text{ and } h_t^s. \tag{3.6}$$

The dynamics of aggregate consumption C_t, portfolio wealth A_t and human wealth H_t, therefore adopt the following forms as shown in the appendix[9]

$$C_t = [p + \theta] (A_t + H_t)$$

$$\dot{A}_t = r_t A_t + w_t - C_t \tag{3.7}$$

$$\dot{H}_t = (r_t + p) H_t - w_t$$

The Euler equation is also derived and is given by:

$$\dot{C} = (r_t - \theta) C_t - [p (p + \theta)] A_t \tag{3.8}$$

If the evolution of w_t is characterized, then (3.7.b) and (3.8) represent the dynamics of savings behavior in the model which, together with investment, describes the evolution of the current account.

Firms' Behavior and Dynamic Investment Decisions

On the production side, a representative firm operating in the home country produces the single traded good according to a neo-classical production function. The good is sold on a competitive market. Two inputs -capital and labor- are needed for the production

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[9]Notice that the finite-horizon structure of the consumer side of the model causes aggregate human wealth to be discounted at a higher rate $(r_t + p)$ than aggregate non-human wealth.

of the good. It is assumed that there is a competitive labor market where workers inelastically supply their effort, obtaining at each period of time a real wage rate w_t. In order to model investment decisions in a dynamic manner, it is introduced costs to the installation of productive capital. Such costs are described by the non linear function $J\left(\dot{K}_v/K_v\right)$ where K is the stock of productive capital in the domestic economy at date t. Hence, at every point of time, the production decision is reduced to the choice of a path of labor demand and capital investment $\left\{L_v, \dot{K}_v\right\}_{v=t}^{\infty}$ which maximizes the present value of future dividends, given the production and the capital-stock adjustment technology available. Therefore, the firm's optimal behavior can be described by following expression:

$$V_t = \max_{\left(\dot{K}_v, L_v\right)} \int_t^{+\infty} \left\{F\left(\cdot\right) - w_v L_v - \dot{K}_v - J\left(\frac{\dot{K}_v}{K_v}\right) K_v\right\} \exp\left(-\int_t^v r_z dz\right) dv$$

where $F\left(K_t, L_t\right)$ is a linear homogeneous production function, net of capital stock depreciation, satisfying the usual regularity properties. In order to focus the analysis on investment dynamics, the labor decision is not explicitly studied, thus the optimizing behavior of the firm reduces to a Tobin's q theory of investment. To be specific, the optimality conditions of this problem, together with the transversality condition requiring (q, K) to converge to the steady state value $(1, K^*)$, will determine the unique dynamics of aggregate capital stock, as well as the relative value of the firm. The equations describing the optimal behavior of investment in productive capital are[10]

$$\dot{K}_v = \phi\left[q_v - 1\right] K_v \tag{3.9}$$

$$\dot{q}_v = q_v r_v - F_K + \left(q_v - 1\right)\phi\left(q_v - 1; \chi\right) + J\left[\phi\left(q_v - 1\right)\right] \tag{3.10}$$

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[10]See Hayashi (1982) for details about the derivation of these equations.

where $F_K \equiv F_K (K_t, 1)$ is the marginal product of capital, equal to r at equilibrium. The shadow price of investment, q_t , shows how the firm's present value of future cash flows is affected when the capital constraint is relaxed. Furthermore, the linear homogeneity of the production function and investment technology equalizes marginal q and average q, implying that $qK_t = V_t$. The function ϕ is defined as an inverse function of J', and satisfies $\phi(0) = 0$ and $\phi' = \frac{1}{J''} > 0$.

Dynamic Macroeconomic Relationships and Steady State Analysis

Combining consumers' and firms' efficiency conditions, it is possible to obtain the following system of differential equations which describes the dynamic relationship among the relevant variables of the economy. Such a system of equations is useful to characterize the evolution of saving and investment:

$$\dot{C}_t = (r^* + \mu_t - \theta) C_t - (p(\theta + p)) (B_t + q_t K_t)$$

$$\dot{B}_t = r_t B_t + F(K_t, 1) - C_t - G_t - K_t \phi(q_t - 1; \chi) - J(\phi(q_t - 1)) K_t \qquad (3.11)$$

$$\dot{K}_t = \phi(q_t - 1) K_t$$

$$\dot{q}_t = r_t q_t - F_K - (q_t - 1) \phi(q_t - 1) + J(\phi(q_t - 1))$$

The first equation in the preceding system shows that the rate of change of consumption depends both on non-human wealth and the level of consumption. This feature of the model allows for the comparison of different steady states associated with different equilibrium interest rates. The second equation in (3.11) corresponds to the balance of payments identity, that is, to a change in foreign asset holdings $\dot{B}_t$. This equation shows that the current account is equal to GNP ($F(K,1)$ + capital depreciation + rB) minus absorp-

tion ($C + G + I$, where G denotes government spending and I denotes gross investment: $I = \dot{K} + J\left(\dot{K}/K\right)K+$ capital depreciation). Notice that the *Fisherian Separation Theorem* holds in this model. This means that investment decisions can be made separately from saving decisions for a given interest rate.

The steady state is computed by setting the differential equations of the system in (3.7) as well as in (3.11), to zero. Simple algebra leads to the following set of equations

$$C^* = \frac{-p\left(p + \theta\right)\left\{F\left(K^*, 1\right) - \left[\left(r^* + \mu\right)\right]K^*\right\}}{\left(r^* + \mu + p\right)\left[r^* + \mu - p - \theta\right]}$$

$$r = F_{K^*}\left(K^*, 1\right)$$

$$w^* = F\left(K^*, 1\right) - \left[r^* + \mu\right]K^* \qquad (3.12)$$

$$0 = \left(r^* + \mu\right)B^* + F\left(K^*, 1\right) - C^*$$

Notice that a positive level of consumption is guaranteed whenever $r < p + \theta$. In the next section it is shown that this condition also guarantees saddle path stability. Finally, combining the first and last equation of the above system (3.12), it can be obtained a relationship between the steady state values of human and non-human wealth:

$$A^* = \left(\frac{r - \theta}{\left(p + \theta\right) - r}\right)H^* \qquad (3.13)$$

Given that human wealth is positive, one has that if $r < \theta$ the economy is a net dissaver, disaccumulating non-human wealth over time. If $r = \theta$, it has a constant consumption, and given that labor income is constant, non-human wealth is zero along the steady state. Recall that the capital account is determined residually by the current account, which has two components: the trade account and the services account. The steady state condition on the balance of payments is that capital account surplus must be zero, i.e. $\dot{B} = 0$. This,

in turn, implies that if the country is a net debtor, then the economy must run a trade surplus in order to be able to pay the interest on foreigners' holdings of domestic assets.

Long Run Comparative Statics

In this section, it is studied the long run effects on the current account that are motivated by unanticipated and once-and-for-all shocks in foreign interest rates. The next result characterizes such long run effects:

Proposition 11 *If initially the country is a net creditor on the international financial scene, $B^* > 0$, an increase in foreign interest rates will result in a higher long run level of foreign assets. However, the opposite is likely to happen for the case of a net debtor country, $B^* < 0$, with a low level of fixed productive capital K^*.*

Formally, this result can be derived as follows. First, combining the steady state equations in (3.12), it is possible to express the steady state level of net foreign bonds as

$$B^* = \frac{1}{\Phi} \left[(r - \theta) F(K^*, 1) - p(p + \theta) K^* \right] \tag{3.14}$$

where $\Phi = -(r + p)[r - (p + \theta)]$. Notice that Φ is positive since $r < (p + \theta)$ as has been discussed above. Next, differentiating (3.14) with respect to r^* and noting that $r = r^* + \mu$, the following expression results

$$\frac{dB^*}{dr^*} = \frac{1}{\Phi} \left\{ F(K^*, 1) + (r + p)[r - (p + \theta)] \frac{dK^*}{dr^*} + (2r - \theta) B^* \right\} \tag{3.15}$$

where the sign of the term $2r - \theta$ is assumed to be positive, as otherwise the economy would always be net dissaver ($r < \theta$). The change in the long run level of productive capital

resulting from a shock on interest rates, dK^*/dr^*, can be obtained by implicit differentiation of $r = F_{K^*}(K^*, 1)$. Doing this, one finds that

$$\frac{dK^*}{dr^*} = \frac{1}{F_{KK}} < 0$$

thus, if $B^* > 0$, the sign of the derivative in (3.15) will be positive, whereas it will depend on the particular long run level of net foreign investment in the domestic economy in the case where $B^* < 0$. These results are consistent with the findings of other researches such as Uctum (1991), Agénor and Hoffmaister (1996) and Agénor (1998). Razin (1995) also analyses the response of the current account to changes in interest rates arising from productivity-enhancing shocks, finding, as it was done here, that the current account has to be positively correlated with such shocks.

Current Account Dynamics

The procedure for analyzing the dynamics of capital flows when an external disturbance hits the economy has the following steps: (i) the system of differential equations in (3.11) that describes the equilibrium dynamics of the economy is linearized around the unique steady state; (ii) the resulting linear system is solved for the convergent path allowing in this manner, to describe the behavior of the current account as a function of the dynamics of investment and intertemporal consumption, that is, of saving; and (iii) the effects that the disturbances have on both the long run level of foreign debt and domestic fixed capital are compared.

According to the procedure described above, the system of differential equations in

(3.11) is linearized around its steady state values, leading to the following set of equations:

$$
\begin{bmatrix} \dot{C} \\ \dot{B} \\ \dot{K} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} r - \theta & -p\left(p + \theta\right) & -p\left(p + \theta\right) & -p\left(p + \theta\right)K^* \\ -1 & r^* + \mu & r^* + \mu & -\phi'\left(0\right)K^* \\ 0 & 0 & 0 & \phi'\left(0\right)K^* \\ 0 & 0 & -F_{KK} & r^* + \mu \end{bmatrix} \begin{bmatrix} C - C^* \\ B - B^* \\ K - K^* \\ q - 1 \end{bmatrix} \tag{3.16}
$$

The analysis of the stability properties of the linearized system is carried out by calculating the eigenvalues of the coefficient matrix and studying their sign[11]

$$
\begin{array}{ll}
\lambda_1 = \frac{r}{2} + \frac{1}{2}\left(r^2 - 4F_{KK}\phi'\left(0\right)K\right)^{1/2} & \lambda_3 = r + p \\
\lambda_2 = \frac{r}{2} - \frac{1}{2}\left(r^2 - 4F_{KK}\phi'\left(0\right)K\right)^{1/2} & \lambda_4 = r - p - \theta
\end{array}
$$

Solving the system (3.16) will allow for the study of the dynamic responses of the model to different disturbances. In particular, it will be possible to analyze the evolution of foreign asset holdings after a given shocks hits the economy. The general solution of the linear non-homogeneous system of differential equations given in (3.16) can be written as

$$
\begin{bmatrix} C\left(t\right) - C^* \\ B\left(t\right) - B^* \\ K\left(t\right) - K^* \\ q\left(t\right) - 1 \end{bmatrix} = c_2 \begin{bmatrix} \left(r - \lambda_2\right)\left(1 + x_2\right) \\ x_2 \\ 1 \\ \lambda_2/\phi'\left(0\right)K^* \end{bmatrix} \exp\left(\lambda_2 t\right) + c_4 \begin{bmatrix} r - \lambda_4 \\ 1 \\ 0 \\ 0 \end{bmatrix} \exp\left(\lambda_4 t\right)
$$

where λ_2 and λ_4 are the stable eigenvalues.[12] The former is associated with the dynamics of investment whereas the latter corresponds to those of the intertemporal substitution of

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[11]Clearly, λ_3 is positive and λ_4 is negative since it has been proven that a positive level of consumption at the steady state requires $r < p + \theta$. Moreover, the other two roots, λ_1 and λ_2, have different signs since they are the solution to the second order equation $\lambda^2 - r\lambda + F_{KK}\phi'\left(0\right)K^* = 0$, which has a negative independent term. Hence, it will be considered λ_2 as the negative root and it will be assumed that $\lambda_2 \neq \lambda_4$.

[12]A unique convergent path to the steady state exists since the number of predetermined variables (B and K) equals the number of negative eigenvalues, and the number of jump variables (q and C) equals the number of positive eigenvalues.

consumption, that is, to saving. Moreover, the parameter x_2 consists of

$$x_2 = -1 - \frac{p(p+\theta)\lambda_2}{(\lambda_2 - \lambda_4)(2r - \theta - \lambda_2 - \lambda_4)\phi'(0)} \tag{3.17}$$

The sign of x_2 could be positive or negative: it approaches infinity as λ_2 approaches λ_4 from above and negative infinity when approaching from below. On the other hand, c_2 and c_4 are undetermined coefficients which can be calculated by setting $t=0$ in the previous system. In particular, these values are $c_2 = K(t) - K^*$ and $c_4 = (B(t) - B^*) - x_2(K(t) - K^*)$. In order to analyze the evolution of foreign asset holdings in the domestic economy, which is the main focus of the present study, the second equation in the previous system is differentiated with respect to time

$$\dot{B} = x_2[\lambda_2 \exp(\lambda_2 t) - \lambda_4 \exp(\lambda_4 t)](K_0 - K^*) + \lambda_4 \exp(\lambda_4 t)[B_0 - B^*] \tag{3.18}$$

Equation (3.18) shows that the behavior of the current account is regulated by shifts in the steady state levels of K and B, the magnitude of the stable roots and the sign of x_2. Characterizing the dynamic path is rather complicated, so in order to facilitate the analysis equation (3.18) is reformulated as follows

$$\dot{B} = A_2 \exp(\lambda_2 t) + A_4 \exp(\lambda_4 t) \tag{3.19}$$

where $A_2 \equiv \lambda_2(K(0) - K^*)x_2$ and $A_4 \equiv \lambda_4[(B(0) - B^*) - x_2(K(0) - K^*)]$. These derivations lead to the following result:

Proposition 12 *For a Blanchard economy that is in a net creditor position, the dynamic path described by its current account after a rise in world interest rates is characterized as follows:*

- Case i: $\lambda_2 < \lambda_4$ and $x_2 < 0$. In this case, one has that A_2 is positive but the sign of A_4 is not clearly determined. The initial impact on the current account is given by the expression $\dot{B} = x_2 \left[K\left(0\right) - K^* \right] \left(\lambda_2 - \lambda_4 \right) + \lambda_4 \left[B\left(0\right) - B^* \right]$ and is positive. The remaining path towards the new steady state will depend on the sign of A_4. It is straightforward to prove that when A_4 is positive, the current account is in surplus throughout the adjustment process, whereas a deficit may occur when A_4 is negative. A simple graphical illustration is shown in Figures 1 and 2.

- Case ii: $\lambda_2 > \lambda_4$ and $x_2 < 0$. As before, A_2 is positive but the sign of A_4 is ambiguous, as is the sign of the initial effect on the current account, $\dot{B}\left(0\right)$. In the case that $\dot{B}\left(0\right)$ adopted a positive sign, one would have a situation like the previous case when A_4 was greater than zero (see Figure 3.1). Otherwise, the path followed by the current account would be as illustrated in Figure 3.3.

- Case iii: $\lambda_2 > \lambda_4$ and $x_2 > 0$. The first inequality means that the adjustment of investment is slow relative to the intertemporal substitution of consumption. This scenario implies that A_2 is negative whereas A_4 is positive. The immediate effect on the current account is also positive. However, the current account may run into a deficit while converging to the new steady state, as depicted in Figure 3.4.

Notice that a case like $\lambda_2 < \lambda_4$ and $x_2 > 0$ is not possible since the fractional expression on the right hand side of (3.17) will always be positive and thus x_2 could not be greater than zero.

3.3   Endogenous Imperfect Capital Mobility

Until now, it has been assumed that the interest cost associated with imperfections in or restrictions on the markets for international capital were an exogenously-given parameter, μ_t. This implied that it could not be distinguished between shocks arising from disturbances in world interest rates, r_t^*, and those due to specific financial shocks affecting the domestic economy. In this section, this assumption is relaxed by endogenizing μ_t, the interest rate differential.[13] Specifically, it will be made dependent on the level of net foreign assets held in the domestic economy. The intuition behind this assumption is the existence of an upward-sloping supply schedule for foreign debt, embodying the idea of the existence of restrictions on these markets. Analytically, it will be considered, as in Fisher (1995), Senhadji (1997) and Agénor (1998), that the interest cost is a function both of the outstanding level of net foreign debt, B_t, and a parameter η that measures the degree of international capital mobility of a particular country.[14] This parameter will play an important role in the dynamics of the current account, since the shape of such dynamics will depend on the level of capital mobility, that is, the particular value of η. Hence, the specific interest cost can be expressed as $\mu_t \equiv \mu\left(B_t^*, \eta\right)$ and the interest rate parity condition now becomes

$$r_t = r_t^* + \mu\left(B_t^*, \eta\right) \tag{2'}$$

It must be pointed out that since the cost of interest depends on the aggregate level of net foreign debt, individuals cannot internalize this cost when making their saving decisions.

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[13]More formally, one could explicitly introduce into the model financial intermediaries operating internationally. These firms exist due to their higher ability to mobilize substantial pooling of funds from diverse sources. Moreover, they are better informed about the functioning of the international financial market. International financial intermediation must be a costly activity for these firms to play a nontrivial macroeconomic role.

[14]It is assumed that $\eta = 0$ implies no specific restrictions on such mobility.

That is, $\mu\left(B_t^*, \eta\right)$ is taken as given by the consumers.[15]

Taking into account these arguments, the next step is to analyze the dynamic response of the current account to shocks to the interest rate differential. The following result shows that the larger the imperfections or restrictions associated to the access to international debt markets, the more dramatic the reaction of the current account to a financial country-specific shock:

Proposition 13 *The magnitude of the response of B^* to the interest rate disturbance will depend on the sensitivity of the interest rate differential $\mu\left(B_t^*, \eta\right)$ to the parameter η, which measures the degree of external imperfections. The sensitivity is given by the derivative of μ w.r.t. η, that is, by μ_η.*

In order to show this result, one has to proceed, as in the previous section, by studying the effects of such disturbances on the long run values of the stock of productive capital and on the volume of net foreign debt. Notice that the level of foreign debt cannot now be explicitly solved for, as was possible in the case where there was an exogenous interest cost. The *Implicit Function Theorem* is used to obtain the change in B^* due to the disturbance above mentioned. The steady state level of foreign debt is implicitly defined by equation (3.14):

$$B^* - \frac{1}{\Phi}\left[\left(r^* + \mu\left(B^*, \eta\right) - \theta\right) F\left(K^*, 1\right) - p\left(p + \theta\right) K^*\right] = 0 \qquad (3.20)$$

Recall that the long run level of productive capital depends on the real interest rate r and hence, by (2), on $r^* + \mu\left(B^*, \eta\right),$ the effective interest rate.[16] In order to analyze the impact

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[15]This could be done, however, in representative agent models with infinite horizon -see Agénor and Hoffmainster (1996).

[16]As mentioned previously, endogenizing the interest cost η permits to study shocks due to foreign interest

of a shock to the interest rate differential μ, expression (3.20) is implicitly differentiated w.r.t. η, which gives the following result:

$$\frac{dB^*}{d\eta} = [1 - \Theta\mu_B]^{-1}\Theta\mu_\eta \tag{3.21}$$

where

$$\Theta \equiv \left(\frac{da}{d\mu} - \frac{a}{\Phi}\frac{d\Phi}{d\mu}\right)\frac{1}{\Phi} \tag{3.22}$$

and μ_η and μ_B represent the partial derivative of $\mu(B^*, \eta)$ with respect to its first and second argument, with $a \equiv [r^* + \mu(B^*, \eta) - \theta]F(K^*, 1) - p(p + \theta)K^*$. Clearly, the absolute value of the magnitude of expression (3.21) depends, ceteris paribus, positively on μ_η.

The dynamics of the current account can be obtained, as before, by linearizing the model around the steady state. In this case, the only modification with respect to the previous situation can be found in the equation of the balance of payments identity. Because of the structure of the macroeconomic equilibrium, the speed of adjustment of productive capital is not affected by this change, that is, the eigenvalue λ_2 is the same as before. Nevertheless, the transitional dynamics of intertemporal substitution of consumption are modified in this new scenario.[17] Recall that the interest cost μ depends also on the level of capital market imperfections, measured by the parameter η, as does its derivative, hence $\mu_B \equiv \mu_B[\eta]$. At this point, it will be assumed that $\mu_{B\eta}[\eta]$ is positive, that is, as the level of imperfect capital mobility η increases, domestic residents have to pay (receive) proportionally more (less) for their asset holdings. Altogether, the corresponding eigenvalue

---

rate disturbances and those due to changes in the country specific interest cost separately. As becomes clear from (2), the sign of the effect on the current account of both types of disturbances will be the same.

[17]The *stable* eigenvalue associated to saving is

$\lambda_4 = (B^*\mu_B[B^*] + r) - \frac{\theta}{2} - \frac{1}{2}\left[B^*\mu_B[B^*](B^*\mu_B[B^*] + 2\theta) + \theta^2 + 4p(p + \theta)\right]^{1/2}$

is a function of η, that is, $\lambda_4 \equiv \lambda_4[\eta]$. Therefore, it is possible to analyze how changes in international financial market conditions affect the dynamics of savings and, thus, of capital flows. This is the focus of the next result:

Proposition 14 *A high degree of capital market imperfections, i.e., high value of η, will give rise to a slow adjustment of the current account to a given shock when the economy is in a net creditor position. The opposite is likely to occur when the domestic economy is in a net debtor position in foreign markets.*

The proof of the previous result could be given analytically by simply differentiating λ_4 with respect to the parameter η. Nevertheless, a graphical exposition would be more intuitive. To this end, the parameters in λ_4 have assigned values that are usual in the literature: $p = 0.03$,$\theta = 0.035$ and $r = 0.033$. Moreover, the level of foreign assets was set equal to unity, meaning that $B^* = 1$ when the country is a net creditor, whereas for the case of a net foreign indebtedness $B^* = -1$. Figures 5 and 6 show how the speed of adjustment associated with saving (*stable* eigenvalue λ_4) changes as the parameter η is modified. It is important to notice the opposing effects obtained depending on the net foreign asset position of the country.

3.4   Capital Flows in a Monetary Economy

Throughout the previous analysis, the monetary side was kept in the background and the exchange rate regime was implicitly assumed to be a flexible one. However, the introduction of money would certainly be of great interest since changes in money holdings would give another source of capital movements to be taken into account. Thus, in this

section the original Matsuyama (1987) model is extended by introducing a monetary side. It will be considered the case of perfect capital mobility with a flexible exchange rate regime satisfying Purchasing Power Parity (PPP). Under perfect capital mobility, and assuming domestic assets and foreign bonds are perfect substitutes, asset market equilibrium requires uncovered interest rate parity (UIP) to hold at all times. Money is introduced into the model on a money-in-the-utility function basis. Consumers' portfolios consist of foreign bonds b_t, domestic equities v_t and real money balances, m_t. The instantaneous utility function is additively separable in consumption and real money balances.[18] The parameter γ is a positive constant that represents the weight of money in the utility function and it will play an important role in the dynamics of the current account as it is shown later.

The central bank is assumed to follow a simple money growth rate rule. Letting M_t denote real balances at time t, with σ being the constant rate of growth of nominal money supply and π representing the domestic rate of inflation, the money growth rate rule adopted by the central banks can be written as

$$\dot{M}_t = (\sigma - \pi_t) M_t \tag{3.23}$$

The basic structure of the model coincides with that in Section 2, thus the system of differential equations describing the dynamic relationship among the relevant macroeconomic variables of the economy is obtained in a similar manner. In particular, it is obtained

---

[18]Logarithmic preferences $U\left(c_v^s, m_v^s\right) = \gamma \log m_v^s + \log c_v^s$ were chosen in order to simplify the exposition; the same results would hold for any separable utility function. Non-separable preferences would introduce effects unrelated to the main issues at hand.

the following set of equations,

$$\dot{C}_t = (r_t^* - \theta)\, C_t - \left(\frac{p\,(\theta + p)}{1 + \gamma}\right)(M_t + B_t^c + q_t K_t)$$

$$\dot{B}_t = r^* B_t + F(K_t, 1) - C_t - g_t - K_t \phi(q_t - 1) - J\ (\phi(q_t - 1))\, K_t$$

$$\dot{K}_t = \phi(q_t - 1)\, K_t \tag{3.24}$$

$$\dot{q}_t = r^* q_t - F_K - (q_t - 1)\, \phi(q_t - 1) + J\ (\phi(q_t - 1))$$

The dynamics of the current account can be derived in the same way as in the model described in the previous Section. The eigenvalues associated with investment dynamics do not change with respect to the version of the model without money. Nevertheless, the stable root that drives the dynamics of saving now becomes

$$\lambda_4' = r - \frac{\theta}{2} - \frac{1}{2}\left(\theta^2 + 4\frac{p\,(p + \theta)}{1 + \gamma}\right)^{1/2}$$

An interesting exercise involves the study of the consequences on the dynamics of saving as a result of changes to the parameter γ which can be associated with the marginal utility of money. For instance, an improvement in the financial system which affects the costs of holding cash may have consequences on the overall economy through the adjustment process that consumers carry out on their portfolios.

Proposition 15 *The higher the weight of money in the utility function, the lower the absolute value of the stable root λ_4', meaning that the dynamics of adjustment of savings, and thus the current account, after a shock will be slower.*

A simple numerical exercise using the usual parameter values serves to illustrate the effect that a higher weight of money in the utility function (a rise in the parameter γ)

has on the dynamics of intertemporal substitution of consumption, and thus on the current account. Figure 3.7 shows the parameter γ on the horizontal axis, whereas λ'_4 is measured along the vertical axis.

The discussion of the dynamics of the current account is basically analogous to that presented in the previous sections. The assumption of separability in the utility function implies that the study of the real side of the economy can be carried out separately from the monetary side. Money is superneutral in this model and thus has no effects on the real side of the economy, meaning that real variables such as the capital stock or consumption are not dependent on the rate of monetary growth. Under the weaker condition that the utility function is multiplicatively separable in consumption and money balances, it can be proved that the superneutrality property of money still holds in the long run but not along the transition, a feature that certainly would enrich the analysis. It could also be of interest to introduce financial intermediaries such as banks, a feature that would allow to analyze how shocks to the banking system affect the dynamics of capital flows. Nevertheless, the role of money in the decision process of portfolio selection cannot satisfactory be dealt with until uncertainty is introduced into the model.

3.5   Concluding Remarks and Extensions

This study has analyzed the dynamic behavior of capital flows within a fully optimizing framework. Based on the seminal work of Matsuyama (1987), the effects on the current account of foreign interest rate disturbances have been considered, proving that the effects of these shocks depend mainly on the net foreign asset position of the domestic econ-

omy, as well as on the speed of adjustment of investment and intertemporal substitution of consumption.

An endogenously determined country-specific interest cost on foreign assets was introduced with the objective of analyzing the role played by the degree of capital mobility on the dynamics of capital flows. The interest cost was assumed to be a function of the volume of foreign asset trading in the economy and a parameter measuring shocks to this country-specific interest cost. This extension allows to distinguish disturbances in world interest rates from those particular to the interest cost faced by the domestic economy. In addition, it has been possible to analyze the consequences on the dynamics of capital flows of exogenous shocks under different degrees of capital mobility, proving that the speed of adjustment of intertemporal substitution of consumption, that is, saving, will depend on the net asset position of the economy and the extent of international capital market imperfections. It would be of great interest, however, to introduce microeconomic fundamentals into the model. As an example, explicitly incorporating financial companies that provide services to investors seeking foreign funds would be a very suggestive extension of the analysis.

Regarding the role played by money, the results obtained should, naturally, be a focus of further research. The first extension involves the functional form of the utility function. In the model considered here, money enters in the utility function separately from consumption. This consideration leads to the strong results of superneutrality of money, meaning that real variables such as the capital stock or consumption are not dependent upon the rate of monetary growth. Turnovsky (1995) points out that under the weaker condition

where the utility function is multiplicatively separable in consumption and money balances, money would still be superneutral in the long run but not during the transition. Thus, it would be useful to analyze how the results in this Chapter are enriched when a wider class of monetary specifications are considered.

An exciting extension of the model could be achieved by introducing explicitly a banking system. In this sense, it could be possible to analyze how this sector of the economy affects the propagation of shocks. The role of money in the decision process cannot really be dealt with adequately until uncertainty is introduced into the model. Hence, future research should point in this direction. The model considered here assumes complete absence of uncertainty and risk. An especially interesting line for further research would be the development of general equilibrium intertemporal small open economy models within a stochastic framework. For instance, Grinols and Turnovsky (1994) develop a stochastic model of exchange rate determination and asset prices along these lines. These kind of set-ups are interesting since they allow to address a number of issues that are object of attention in international macroeconomics. For instance, it would be interesting to analyze the response of capital flows not only to changes in the levels of the macroeconomic variables but also to changes in their associated risk.

Another promising line for further research is constituted by the analysis of the links between the behavior of the real exchange rate and that of capital flows: Kollman (1997) presents a model dealing with this sort of issue. Nevertheless his is a model without capital accumulation; It is fair to think that the introduction of good-producing firms taking investment decisions, as it has been done in the present study, will notably improve

our understanding of the dynamic behavior of the main macroeconomic aggregates of the small open economy.

Appendix Chapter 3

Solving the Consumer's Optimization Problem

Let $\mathcal{H}_v \equiv \log c_v^s + \lambda_v \left[(r_v + p) a_v^s + w_v - c_v^s - \tau_v \right]$ denote the present value Hamiltonian associated with the problem. The first order necessary (and sufficient) conditions are

$$\frac{\partial \mathcal{H}_v}{\partial c_v^s} \equiv 0 \implies \frac{1}{c_v^s} = \lambda_v$$

$$-\frac{\partial \mathcal{H}_v}{\partial a_v^s} \equiv \dot{\lambda}_v - \lambda_v \left(p + \theta \right) \tag{A.1}$$

$$0 = \lim_{v \longrightarrow +\infty} \lambda_v c_v^s \exp\left[(p + \theta)(t - v) \right]$$

Taking logs on both sides of (A.1.a) and differentiating w.r.t. time, one obtains

$$\frac{\dot{\lambda}_v}{\lambda_v} = -\frac{\dot{c}_v^s}{c_v^s} \tag{A.2}$$

From (A.1.b) one gets $\dfrac{\dot{\lambda}_v}{\lambda_v} = -\left(r_v - \theta \right)$ which when introduced in (A.2) gives a first order differential equation for consumption whose solution, for a given value c_t^s, takes the following form:

$$c_v^s = c_t^s \exp\left(\int_t^v (r_z - \theta)\, dz \right) \ \text{ for } v \in [t, \infty) \tag{A.3}$$

Introducing this equation into the consumer's present value budget constraint

$$\int_t^{+\infty} c_v^s \exp\left(-\int_t^v (r_z + p)\, dz \right) dv \leqq a_t^s + h_t^s \tag{A.4}$$

where

$$h_t \equiv \int_t^{+\infty} (w_v - \tau_v) \exp\left(-\int_t^v (r_z + p)\, dz \right) dv \tag{A.5}$$

allows one to solve for the optimal consumption at time t, of a consumer born at time s:

$$c_t^s = [p + \theta] \left(a_t^s + h_t^s \right)$$

Aggregating the Consumers' Equations

First, net human wealth is aggregated across individuals by applying the aggregation formula (3.6) to the individual value obtained from equation (A.5). This gives rise to the following expression

$$H_t = \int_{-\infty}^{t} \left\{ \int_t^{+\infty} (w_v - \tau_v) \exp\left(-\int_t^v (r_z + p)\, dz \right) dv \right\} p e^{p(s-t)}\, ds$$

Next, changing the order of integration one gets

$$H_t = \int_t^{+\infty} \left\{ \int_{-\infty}^{t} (w_v - \tau_v)\, p e^{p(s-t)}\, ds \right\} \exp\left(-\int_t^v (r_z + p)\, dz \right) dv$$

$$= \int_t^{+\infty} Y_t^s \exp\left(-\int_t^v (r_z + p)\, dz \right) dv$$

where $Y_t^s = \int_{-\infty}^{t} (w_t - \tau_t)\, p e^{p(s-t)}\, ds$ is aggregate labor income net of taxes. Following Blanchard (1985), it is assumed that all agents work and have the same productivity, i.e., labor income is equally distributed. This allows one to write $Y_t^s = w_t - \tau_t$. Therefore, aggregate net human wealth is given by

$$H_t = \int_t^{+\infty} (w_t - \tau_t) \exp\left(-\int_t^v (r_z + p)\, dz \right) dv \qquad \text{(A.6)}$$

Using the Leibniz rule, the derivative w.r.t. time of the previous equation gives the aggregate level of human wealth,

$$\dot{H}_t = (r_t + p) H_t - w_t + \tau_t$$

Aggregate consumption is obtained by applying the aggregation formula (3.6) to equation (3.5) and using equation (A.6), which yields

$$C_t = [p + \theta] (A_t + H_t)$$

Finally, in order to obtain the aggregate level of assets held by the consumers, simply apply the formula given in (3.6) to a_s, and differentiate it w.r.t. time, leading to

$$\dot{A}_t = a_s^s - pA_t + \int_{-\infty}^{t} \dot{a}_t^s p e^{p(s-t)} ds$$

Next, using the flow budget constraint given in equation (3.2) it is possible to write

$$\dot{A}_t = -pA_t + \int_{-\infty}^{t} [(r_t + p) a_t^s + w_t - \tau_t - c_t^s] p e^{p(s-t)} ds$$

$$= r_t A_t + w_t - \tau_t - C_t$$

where the last equality comes from applying the aggregation formula.

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Change Current Account: $B(t)$

Periods After Shock
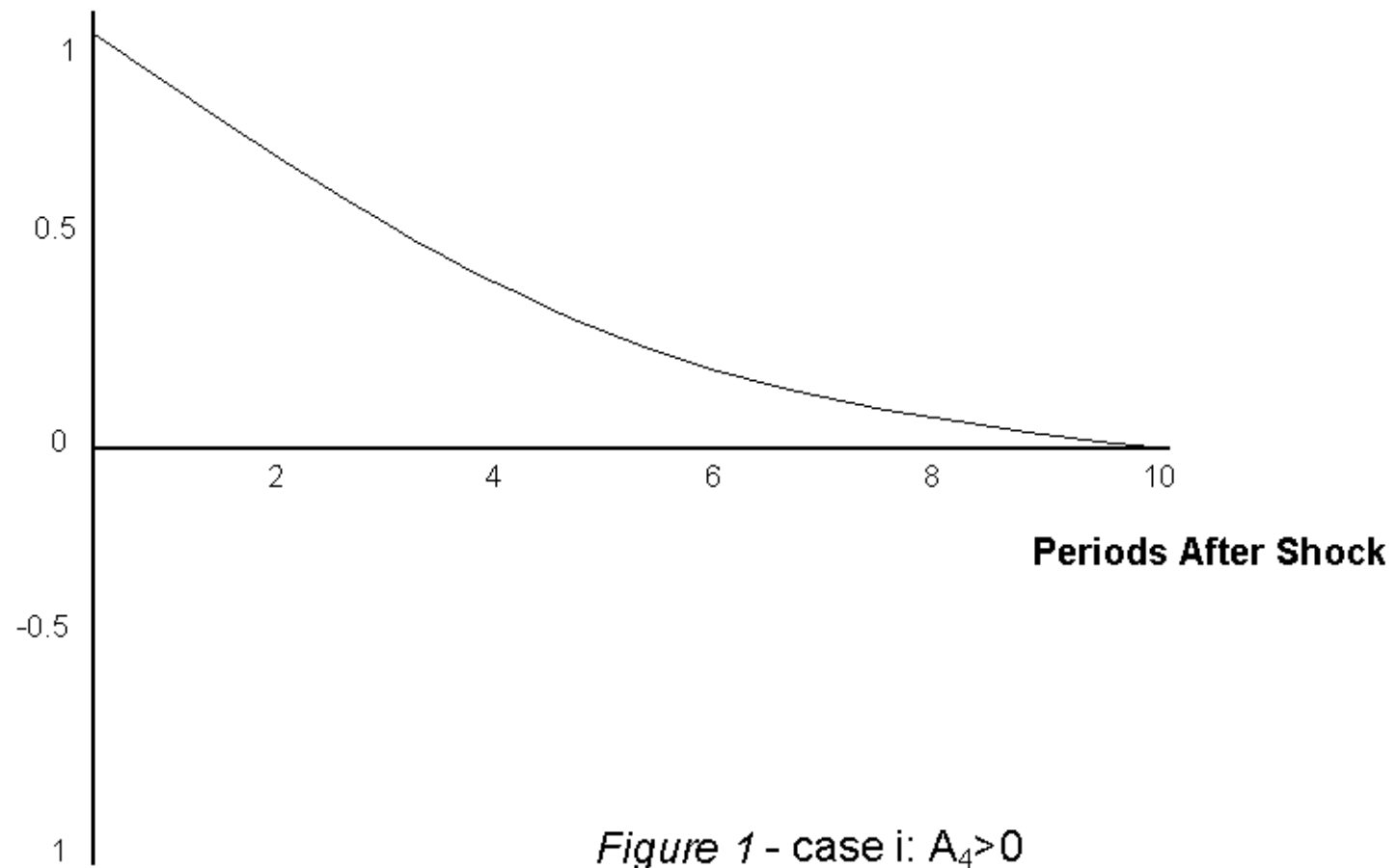
Figure 1 - case i: $A_4 > 0$

Change Current Account: $\dot{B}(t)$

Periods After Shock
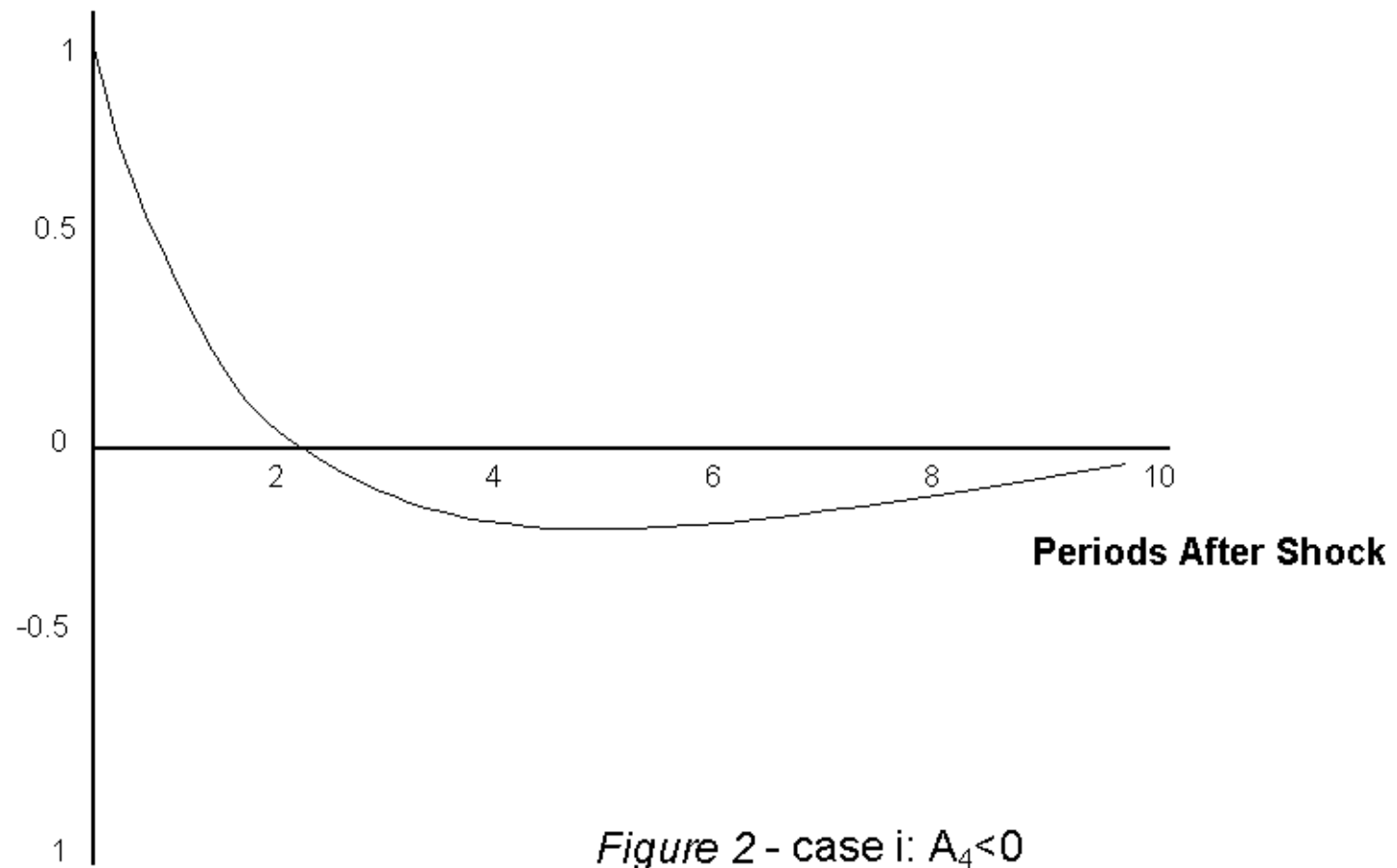
Figure 2 - case i: $A_4 < 0$

Change Current Account: $\dot{B}(t)$
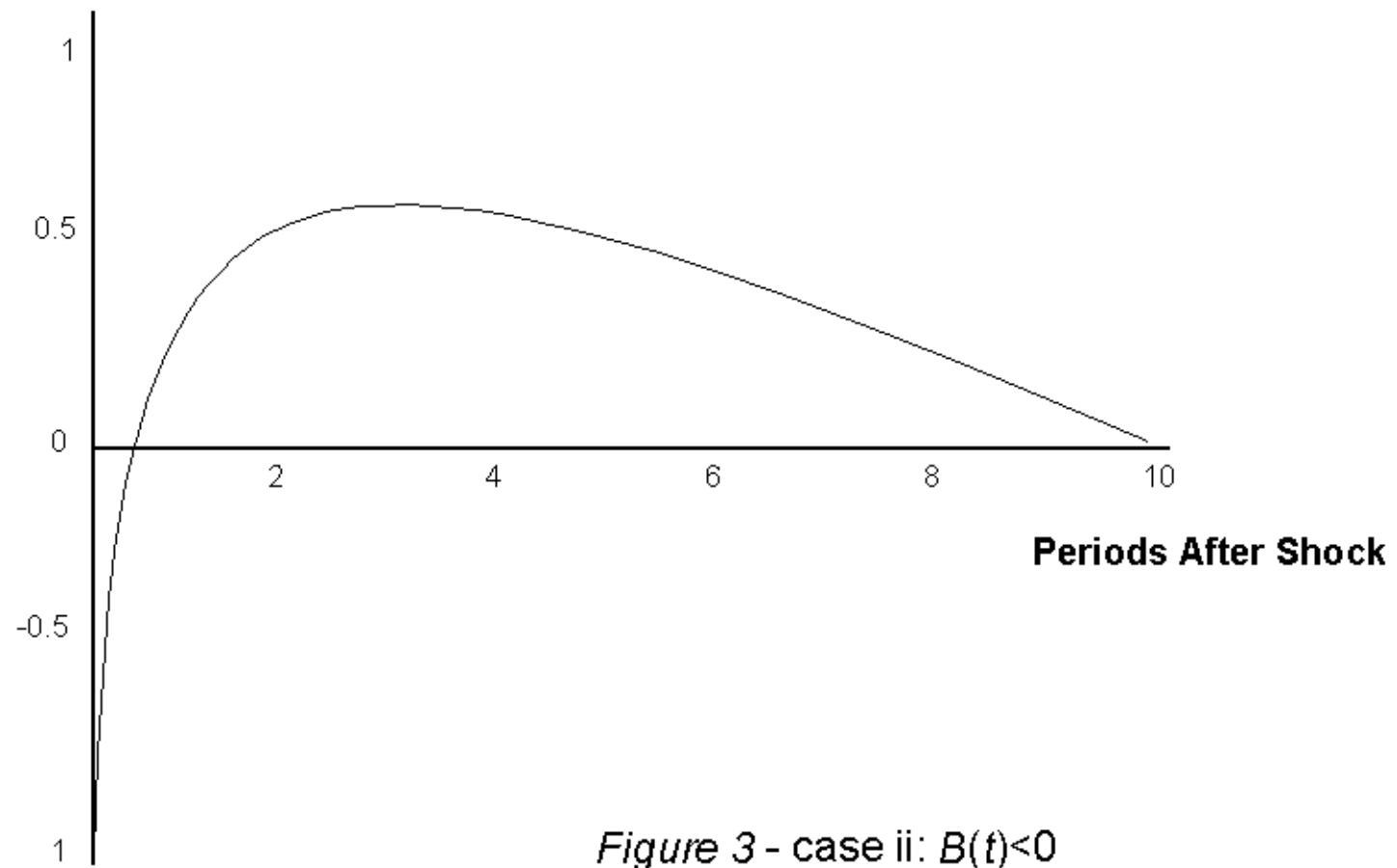
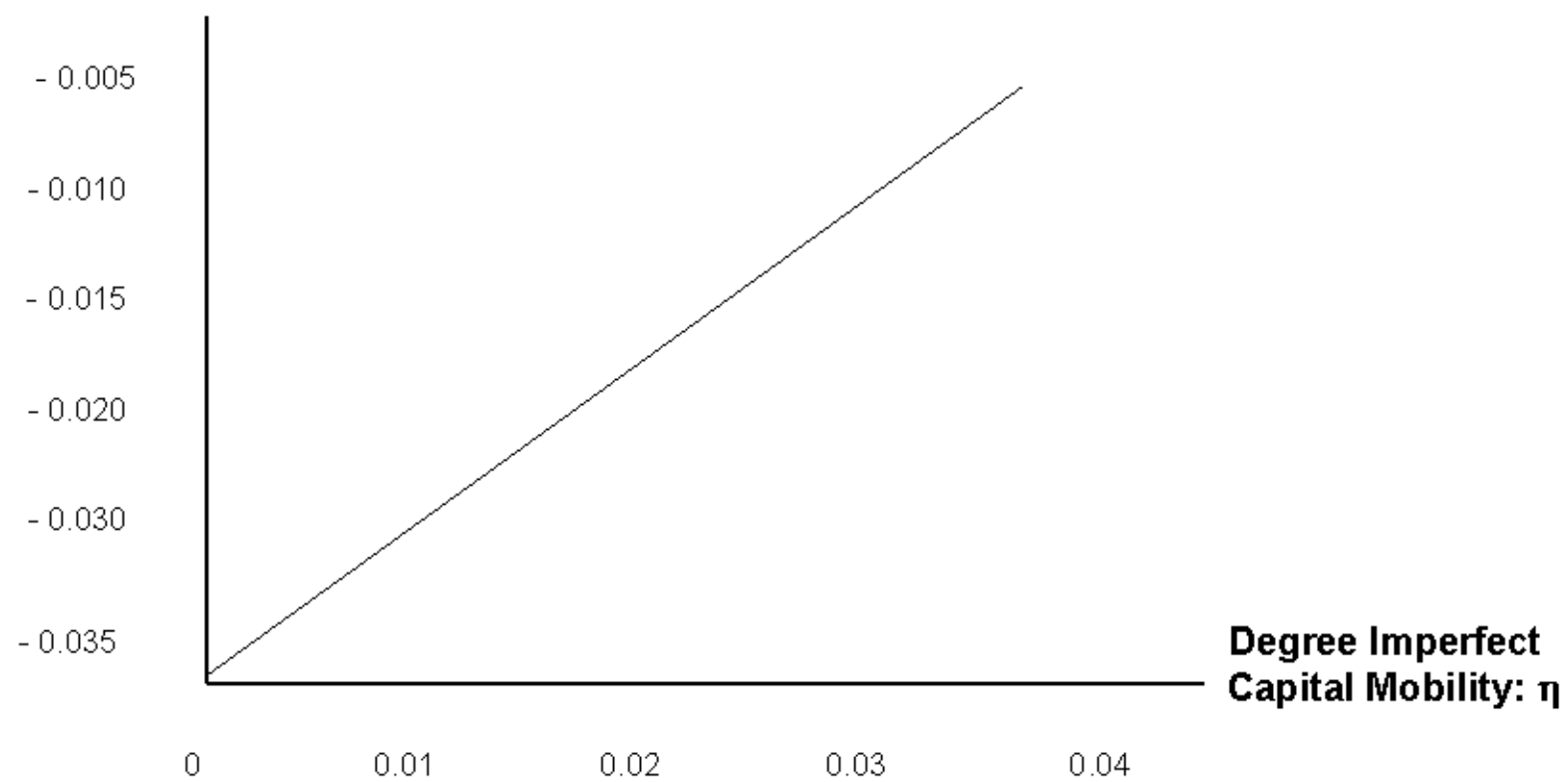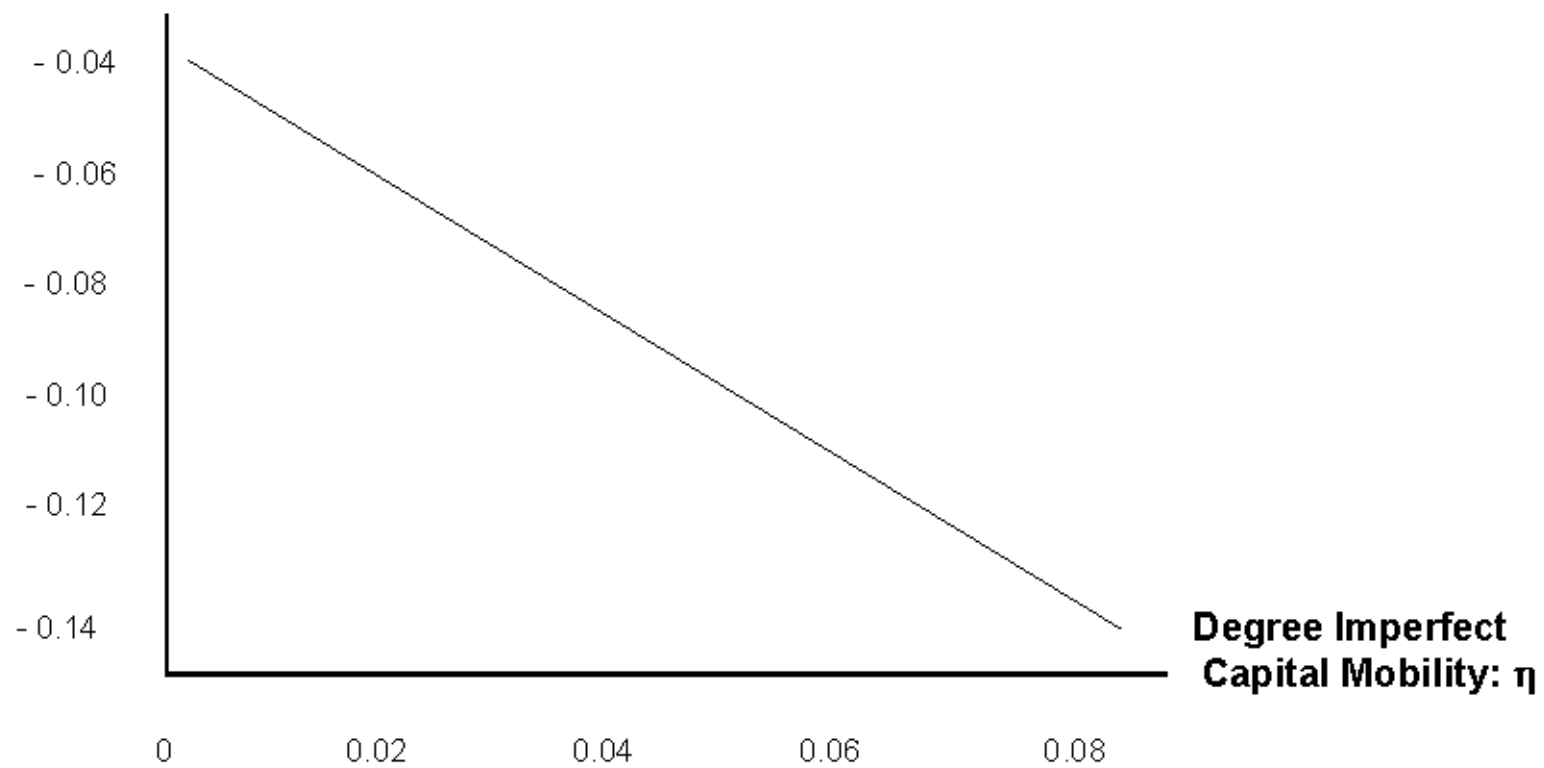*Figure 3* - case ii: $B(t)<0$

Change Current Account: $B(t)$

Periods After Shock
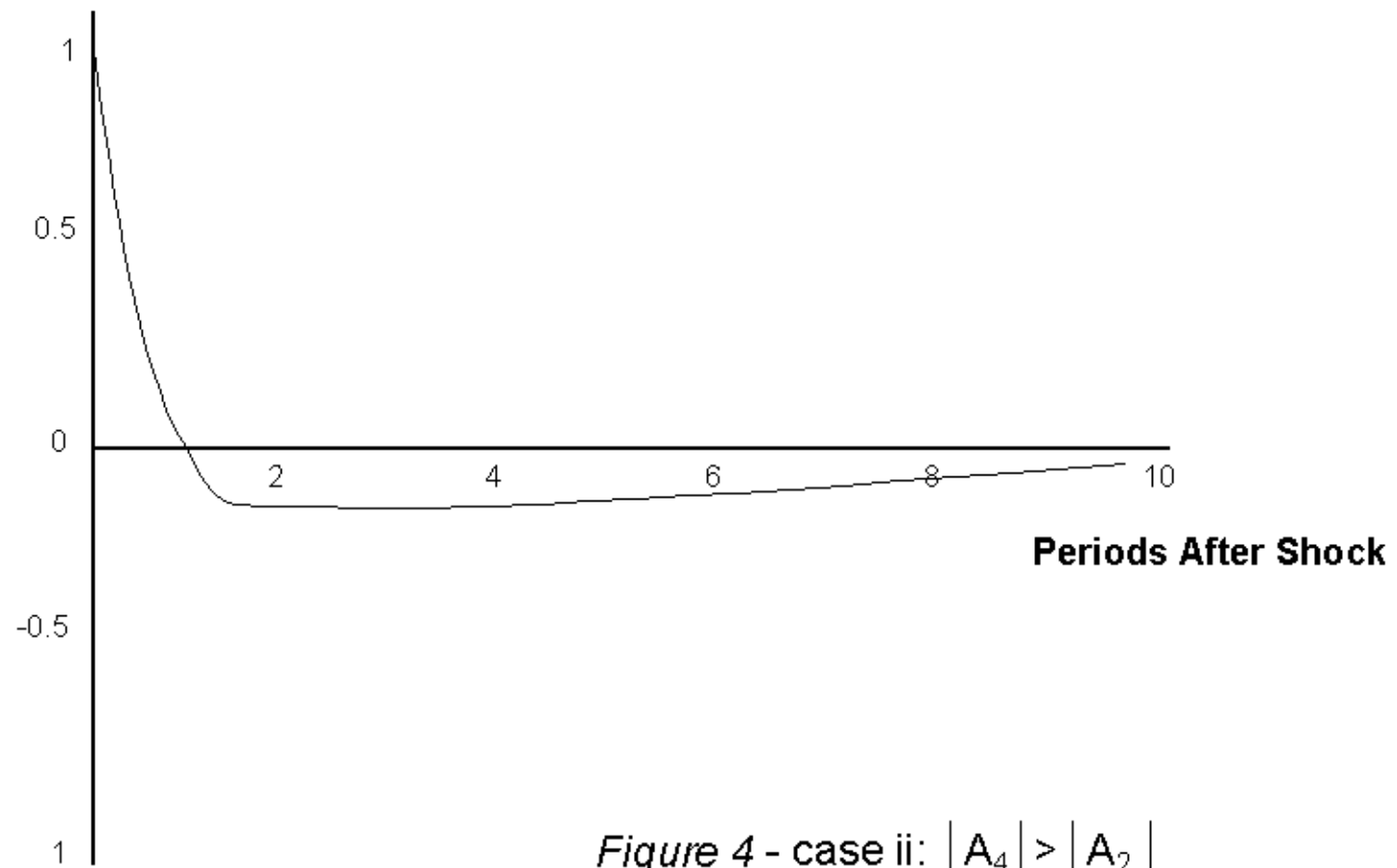
Figure 4 - case ii: $|A_4| > |A_2|$

Figure 5 - Net Creditor Economy

Figure 6 - Net Debtor Economy

Figure 7