Universitat Autònoma de Barcelona

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ESSAYS ON TIME-CONSISTENCY OF OPTIMAL FISCAL POLICY

Begoña Domínguez Manzano

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Preface

Saving The Odyssey

Sirens were beautiful creatures whose seductive singing lured hearers to abandon ship and swim toward them. Dangerous rocks surrounded the Isle of the Sirens, and those who jumped overboard would inevitably be killed. To avoid this, Ulysses ordered his crew to fill their ears with wax and to tie him up to the mast.

"you are to tie me up,
tight as a splint,
erect along the mast,
lashed to the mast,
and if I shout and beg to be untied,
take more turns of the rope to muffle me."

Homer, The Odyssey

The crew did not hear the Sirens and rowed on until the danger was past. Ulysses is then released, thereby setting the paradigm for overcoming temptation or, in other words, solving the problem of time-inconsistency.

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Chapter 1

Introduction

The primary goal of a benevolent government is to provide maximum welfare to the individuals populating the economy. Fiscal and monetary policies are the main macroeconomic instruments in the hands of the government and, therefore, constitute a central concern in macroeconomic theory.

The usual framework to study optimal policy was provided by Ramsey (1927). The Ramsey problem was defined for a static economy as follows. In order to finance an exogenous stream of public spending, the government selects optimal excise taxes so as to maximize the welfare of the representative individual. In the dynamic extension of the Ramsey problem, the government makes decisions once and for all at the initial date. This one-time choice could be seen as the result of a full-commitment that enables the current government to tie the actions of future governments. The resulting policy is such that once the government at the initial date selects a policy plan for all future dates, the successive governments are bound to set the policy that is the continuation of the plan that was chosen at the initial date. However, this scheme does not fit how actual governments take their policy decisions. The actual policy design is better described as a policy plan that is selected sequentially through time by a sequence of governments. In this case, the resulting policy does not coincide in general with the announced Ramsey policy.[1] Therefore, as Kydland and Prescott (1977) showed, the Ramsey policy is time-inconsistent.

An optimal policy selected by a government at a given date is said to be time-inconsistent when it is no longer optimal when reconsidered at some later date, even though no relevant information has been revealed. The time-inconsistency problem of optimal policy arises under very general conditions; it appears in dynamic economies populated by individuals with rational expectations. In particular, in a representative agent model with a benevolent government, the optimal policy is time-inconsistent when the government has no lump-sum taxes at its disposal.[2] The source of time-inconsistency lies in the possibility of re-optimizing taking into account the up-to-date history. As Faig (1994) made clear, different endowments call for different policy plans and, once these endowments have changed, the government has incentives to change the policy plan in order to set a less distorsionary taxation. Capital taxes illustrate very well this problem. As Chamley (1986) showed, a government should promise low future capital taxes in order to encourage investment. However, once this investment has taken place, the current capital income is a pure rent

[1]The dynamic version of the Ramsey problem could be viewed as a static game with a government and a continuum of private agents as players, whereas without commitment the problem resembles a sequential game with an infinity of governments and private agents as players. In the former game, the Ramsey policy and allocation correspond to a subgame perfect Nash equilibrium. However, in the latter, the Ramsey policy and allocation are not subgame perfect Nash equilibrium and, hence, they are not the solution of the game.

[2]The time-inconsistency of optimal policy also appears in economies where lump-sum taxation is available. This happens under certain assumptions over preferences; for instance, when preferences change over time, if government and individual preferences differ or when heterogeneity among individuals is assumed (for a review, see Chari, Kehoe and Prescott (1989)).

and should be taxed heavily. This is known as the capital levy problem. Moreover, the optimal labor tax rate for future dates must take into account how this policy affects the capital accumulation in the time interval. However, once the capital investment is bygone, these effects are not taken into account. Thus, the optimal labor tax rate is different from the announced policy. Under rational expectations, the individuals can anticipate those incentives to deviate from the previously selected policy. As a result, the government faces a credibility problem. Hence, without commitment, the optimal policy cannot be implemented and this time-inconsistency implies a welfare loss.[3]

The concept of time-inconsistency was first introduced by Strotz (1955). In that paper the time-inconsistency problem follows from changes in preferences over time. However, the reasons above described make an optimal policy time-inconsistent even in contexts where preferences are time-consistent in Strotz's sense. Due to this fact, Prescott (1977) and Kydland and Prescott (1977) argued that control theory is not an appropriate tool for dynamic economic planning. Later on, Calvo (1978) and Fischer (1980) showed how this time-inconsistency affects the design of monetary policy and capital taxation, respectively. As a consequence, from then on, economists are urged upon finding a solution to the time-inconsistency problem.

A solution to the design problem without commitment must require policies to be sequentially rational. This means that the policy must maximize welfare at each date taking into account that private agents have rational expectations and would react optimally. Among the different methods that have been developed to solve the time-inconsistency

[3]On the basis of numerical examples, Judd (1987) concluded that actual U.S. tax rates on labor and capital income are not the rates consistent with minimizing the excess burden of taxation. The author linked this result to the inability to commit that the policy-maker faces.

problem, we will focus on two methods, namely, debt restructuring and reputation.[4] The first method is that of Lucas and Stokey (1983), who showed how the careful selection of the maturity of debt can provide the right incentives to future governments so as to continue with the announced policy. The second method was developed by Barro and Gordon (1983). In this paper they showed the role of reputation as a possible solution to the time-inconsistency problem. Since the problem resides mainly in the expectations that individuals make about the future policy plan, the policy-maker may be able to build a reputation so as to make the announced policy credible.

In the next lines we aim to describe how the literature on debt restructuring and reputation has evolved and which are, from our point of view, their main limitations. The debt restructuring method was developed for a barter economy without capital by Lucas and Stokey (1983). They showed that the optimal fiscal policy could be made time-consistent through the optimal management of the structure of debt when governments commit to honoring debt and are allowed to issue this debt with a sufficiently rich maturity structure. This analysis was extended to an open economy by Persson and Svensson (1986) and by Faig (1991) and to a monetary economy by Alvarez, Kehoe and Neumeyer (2002). In a model with endogenous government consumption and public capital, Faig (1994) made the optimal fiscal policy time-consistent through restructuring debt indexed to consumption and debt indexed to leisure maturing at any moment in the future. All in all, the general principle of this literature is that the time-inconsistency problem can be solved through debt

[4] There are also other methods to solve the time-inconsistency problem. Among those, we find the backward solution for finite-horizon models. For infinite-horizon economies, we obtain the Markov perfect equilibria, which include the limit of the backward solution, if that limit exists. Another method appears when the policy-maker and the policy-making institution are independent; in this case decisions may be made time-consistent through delegation.

restructuring for economies without private capital as long as the government has enough variety of debt at its disposal. This method suffers of two main limitations. First, for economies with a private stock of capital, this method cannot solve the time-inconsistency problem because of the capital levy problem. Second, under a limited debt structure, the government lacks of enough instruments to solve completely the time-inconsistency problem.

The reputation method has been widely used in monetary policy. However, as Persson and Tabellini (1994) have argued, the literature on fiscal policy is not so considerable mainly because of technical limitations. The analysis of fiscal policy requires a dynamic framework, and the solution to dynamic models with incentive constraints is very complex. For an infinite-horizon version of Fischer's capital taxation model, Chari and Kehoe (1990) solved those technical problems by assuming that capital accumulates in subperiods but is non-storable between periods. They characterized the set of sustainable equilibrium for this repeated environment, and they obtained that with low enough discount rate the Ramsey solution is time-consistent. Later on, Chari and Kehoe (1993) analyzed these trigger mechanisms in a dynamic environment. For an economy without capital, governments choose their policy plan under a sufficiently rich debt structure, but they may default on their debt payments. Hence, they avoided the time-inconsistency problem present on Lucas and Stokey (1983), that is, the incentives to change the policy plan, and focused on the time-inconsistency caused by the default on debt payments. They characterized the set of sustainable equilibria and obtained that such mechanism cannot support equilibria with positive debt. More recently, Benhabib and Rustichini (1997) studied the properties of the optimal taxes under a reputation mechanism. In order to simplify the analysis, governments are not allowed to issue debt. They obtained that the optimal time-consistent capital tax rate is different from zero at the steady state. As it becomes clear, the main limitation of the literature on fiscal policy under a reputation lies in the need of simplifying the dynamic framework. This limitation has left many interesting questions unanswered. Among them, we point out the following questions. Is there a role for debt restructuring under a reputation? If so, how should governments issue debt under a reputation? Which are the properties of the optimal time-consistent taxes for economies with a market for public debt?

This dissertation has three chapters exploring the role of debt in the time-inconsistency problem of optimal policy. Chapter 2 analyzes the optimal management of the composition of debt under a reputation mechanism. Chapter 3 studies the properties of the optimal time-consistent taxes for an economy with public debt. Finally, Chapter 4 studies the role of debt restructuring in the time-inconsistency problem of optimal fiscal policy for an economy with private capital and endogenous growth achieved via public capital.

The debt restructuring method cannot solve the time-inconsistency problem if the available debt structure is not sufficiently rich. Under a limited debt structure, Chapter 2 considers reputational mechanisms to overcome the time-inconsistency problem. In particular, we explore the effects of an enrichment of the debt structure, that is, an increase in the maturity of debt for an economy with a reputation mechanism. We find that an enrichment of the debt structure increases the costs of maintaining the reputation and, thus, it can make the optimal policy time-inconsistent. This result applies for economies where a reputation can sustain the full-commitment policy. In addition, we use numerical solution methods to show that, even when the reputation mechanism cannot sustain the full-commitment, an increase in the maturity may weaken the reputation and reduce welfare. In other words, governments that face credibility problems would find optimal to shorten the maturity of debt. This theoretical result captures the empirical evidence that governments tend to issue a larger proportion of short-term debt, the less credible is their policy program.

The previous literature on optimal taxation without commitment has focused only on economies without public debt and the main result was that the optimal time-consistent capital taxes are different from zero at the steady state. In particular, Benhabib and Rustichini (1997) obtained that the optimal time-consistent policy is characterized by subsidies to capital at the steady state. The intuition for this result is that capital subsidies encourage the accumulation of capital, which could become high enough to act as a commitment device against deviation from the announced policy. Chapter 3 explores whether the previous result holds in the presence of a market for public debt. To this end, we allow governments to issue debt. The implications of this new environment are twofold. On the one hand, the time-inconsistency problem might be worsened by the possibility of defaulting on debt payments. On the other hand, governments have a new instrument, debt, to affect the benefits and costs of deviating from the announced policy. Given that the government is benevolent and that the choice of not issuing bonds is still a sustainable outcome, the optimal management of debt alleviates the consequences of the time-inconsistency problem. In the presence of debt, we extend the Chamley-Judd result to economies without commitment showing that the optimal time-consistent capital tax rate is zero at the steady state. Thus, once governments have the possibility of issuing debt, the central commitment device against deviation is public debt.

The debt restructuring method cannot make the optimal policy time-consistent in the presence of the capital levy problem. This problem has been widely recognized by the literature, and it has limited the debt restructuring method to models with no private capital and no growth. Chapter 4 develops a model with private capital and endogenous growth achieved via public capital. Given the relevant role that the government plays in the growth process, providing a solution to the time-inconsistency problem of fiscal policy is crucial. Since the capital levy problem makes debt restructuring unable to solve the time-inconsistency problem, we consider a zero tax rule on capital income. More precisely, the current and future governments choose the optimal policy subject to a zero capital tax constraint at all dates. We show that the careful management of debt can make the optimal policy subject to this restriction on capital taxes time-consistent. We characterize the distinctive properties of debt restructuring for economies with private capital and endogenous growth. In addition, we use numerical methods in order to compare the time-consistent policy with the full-commitment policy without the restriction on capital taxes. We obtain that the time-consistent policy is quite close to the full-commitment policy without the restriction of zero capital taxes both in growth and in welfare terms.

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Chapter 2

Reputation in a Model with a Limited Debt Structure

2.1 Introduction

This paper studies the time-inconsistency problem of optimal fiscal policy for an economy with a limited debt structure and a reputation mechanism. In the absence of full-commitment, debt restructuring cannot solve the time-inconsistency problem if the available debt structure is not sufficiently rich. Under the realistic assumption of a limited debt structure, reputational mechanisms are introduced to overcome the time-inconsistency problem. We find that an enrichment of the debt structure, that is, an increase in the maturity of debt, can make the optimal policy time-inconsistent. In other words, governments that face credibility problems would find optimal to shorten the maturity of debt. This theoretical result captures the empirical evidence that governments tend to issue a larger proportion of short-term debt, the less credible is their policy program.

In a seminal paper Lucas and Stokey (1983) presented a pathbreaking analysis of the time-inconsistency problem of optimal fiscal policy. Their main result was that an optimal policy could be made time-consistent if governments commit to honoring debt and this debt is issued with a sufficiently rich maturity structure. For a barter economy with exogenous public spending and no capital, they showed how the careful selection of the maturity of debt indexed to consumption could provide the right incentives to future governments so as to continue with the announced policy. This method has been called debt restructuring. Later on, Rogers (1989) proved that this variety of debt could not enforce time-consistency for an economy with endogenous government spending. Since she considered a finite-horizon economy, the time-consistent policy was given by the backward solution of the model. She found that increasing the maturity of debt leads to time-consistent policies that provide more welfare. In this sense, an enrichment of the debt structure could limit the costs of time-inconsistency and shorter maturities would magnify these costs. More recently, Faig (1994) showed that, when government spending is endogenous, the time-inconsistency problem could be solved if governments issue bonds indexed to consumption and to leisure maturing at any moment in the future. All in all, the general principle of this literature is that the time-inconsistency problem can be solved through debt restructuring as long as the government has enough variety of debt at its disposal. During the last decades, it has been argued that such rich debt structures have no clear counterpart in actual economies. For instance, Lucas (1986) argued that "it is hard to imagine an economic constitution that spells out its provision in terms of infinite sequence of contingent claims"; Rogers (1989) recognized that the government's menu of assets is quite limited. Turning to actual assets markets, we can find debt issued in real terms that resembles the empirical counterpart of debt indexed to consumption. For instance, Treasury Inflation-Protected Securities are issued with a 5-, 10-, and 30-year maturity by the U.S. Treasury since 1997. These securities vary with the consumer price index. However, the explicit provision of debt indexed to leisure or debt maturing at any future date is quite questionable. Then, it must be acknowledged that governments have access to a limited composition of debt. Thus, in the absence of any other mechanism, debt restructuring can limit but not solve completely the time-inconsistency problem.

Under a limited debt structure, a government may try to build a reputation so as to make the announced policy credible.[1] We concentrate on this type of mechanisms and, in particular, on the one developed by Chari and Kehoe (1993). In their paper governments choose the policy plan under a sufficiently rich debt structure, but they may default on their debt payments. Hence, they avoided the time-inconsistency problem present in Lucas and Stokey (1983), namely, the incentives to change the policy plan, and focused on the time-inconsistency caused by the default on debt. In this framework they defined a trigger mechanism, which specifies infinite reversion to a Markov equilibrium. Given that governments have incentives to default on positive debt and to honor negative debt payments, this Markov equilibrium is such that the government debt is never positive. In the present paper governments commit to honoring debt and the time-inconsistency problem is that of Lucas and Stokey (1983). In this environment, as Faig (1994) made clear, different endowments call for different policy plans and, once these endowments have changed, the government has

[1] See Barro and Gordon (1983).

incentives to change the policy plan. In our framework these endowments are the debt obligations, which have a particular maturity structure.[2] Notice that, once the current savings decisions have been taken, the next government faces different debt obligations than those that the previous government was facing for the same time profile. The endowments have changed and, thus, continuing with the announced policy is not longer optimal. Following this reasoning, our Markov equilibrium is such that the new issues of debt make the next government have the same debt obligations as those of the previous government. Hence, no incentives to deviate from the policy plan appear. This Markov equilibrium allows us to define a set of sustainable equilibria.

In this paper we focus on the best sustainable equilibrium. Following Benhabib and Rustichini (1997), we introduce explicitly the constraint that continuing with the announced policy must yield a discounted utility that is at least as high as the one that the individual would obtain if the government deviates from the announced policy. The policy after a deviation is defined as the reversion to the Markov equilibrium. We formulate the allocation selection and policy design as an optimization problem subject to this period-by-period incentive compatibility constraint. The resulting policy is time-consistent and, furthermore, the best sustainable policy that the government can attain.

We explore the role of the debt structure for the best sustainable policy. Our main result is that, if the reputation can sustain the full-commitment policy for a given maturity of debt, then a sufficient increase in the maturity can make the full-commitment

[2] In both Chari and Kehoe (1993) and in our model, debt is a state variable, which gives rise to a dynamic rather than a repeated game. This fact has two consequences. First, our trigger mechanism is typically different from those that appear in static models. Second, the Folk theorem of Fundenberg and Maskin (1986) cannot be applied directly in our setup because this theorem applies in repeated games with several large agents and our model is a dynamic game with one large agent (the government) and a continuum of competitive private agents.

policy time-inconsistent. The intuition for this result is as follows. If continuing with the full-commitment provides more welfare than deviating from it for a given maturity of debt, the full-commitment is time-consistent. The reputation mechanism can sustain the full-commitment for that debt structure. Suppose now a richer debt structure in terms of effective maturity. On the one hand, the policy under full-commitment depends on the present value of the inherited debt, but it is independent of its composition. On the other hand, the policy after a deviation is endogenously determined and depends not only on the value of the inherited debt but on its composition. We show that the welfare value of deviating increases in the effective maturity of debt. Moreover, we find that there exists a maturity of debt above which deviating can provide more welfare than continuing with the full-commitment. As a result, the full-commitment policy is not longer time-consistent, which in turn implies a welfare loss. Therefore, the best sustainable policy involves shorter maturities of the existing debt. This analytical result is complemented with a numerical exercise. Our analytical findings apply to economies in which, for a given maturity, a reputation can sustain the full-commitment. We use numerical methods to compute the welfare gains or losses from an increase in the effective maturity of debt both when the full-commitment can be sustained and when it cannot be sustained. We obtain that, even when the reputation mechanism cannot sustain the full-commitment, an increase in the maturity of debt reduces welfare.

Summing up, we find that there is a maximum maturity consistent with a credible policy plan. Is this theoretical finding confirmed by the observed debt issuance patterns? Are credibility problems affecting the actual composition of public debt? If it is so, how?

We find several references arguing that credibility affects the actual maturity structure of debt. For instance, Calvo (1997) and Barro (1999) claimed that credibility problems could seriously interfere with the placement of long-term bonds. In this sense, Campbell (1995) also argued that a committed government could reduce the cost of debt servicing by issuing short-term debt. However, most empirical studies on the maturity of debt do not focus on its relationship with credibility but on the causal relation between short-term debt and liquidity crisis. As Figure 1 shows for different episodes in the 90s, Mexico, Korea, Thailand, Russia, and Brazil had levels of short-term debt that exceeded international reserves. These high levels of short-term debt have been identified as one of the main determinants of the respective liquidity crisis that these countries suffered.[3] However, what led these countries to these high levels of short-term debt? A possible explanation could be the lack of confidence of the investors in the government policy.[4] Clearly, that lack of confidence would reflect not only the time-inconsistency problem that the present paper studies but also the possibility of default on debt payments.

[Insert Figure 1 about here.]

The empirical relationship between credibility and the maturity of debt has been examined in different papers. Alesina, Prati and Tabellini (1990) studied the Italian policy in the 80s and found that the government paid a premium on long-term debt due to lack of confidence. Later on, Missale and Blanchard (1994) showed that debt and maturity have

[3] For instance, see Rodrik and Velasco (1999).

[4] Rodrik and Velasco (1999) identified per-capita incomes, financial sophistication, debt to GDP ratios, corruption, and openness as possible determinants of short-term debt. They found that per capita incomes and M2 to GDP ratios are robust and positively related to short maturities. Corruption is positive but not significant. More open economies (which tend to be creditworthy) have statistically significant higher ratios of long-term debt to GDP. One could argue that creditworthiness and corruption are measures of credibility.

moved in opposite directions in Ireland, Italy and Belgium over the last 30 years. The intuition for this result is that governments tried to keep their non-inflation pledge credible by decreasing maturity as debt increases.

The main contribution on the empirical evidence on public debt management and credibility is found in Missale, Giavazzi and Benigno (1997), which examined 62 episodes of fiscal stabilization in OECD countries over the last two decades. They showed that the credibility of the program, measured as the change in long-term interest rates at the start of a stabilization, is an important determinant on the choice of public debt. They found that, at the start of a stabilization, governments tend to issue a larger share of short maturity debt the less credible is the program. The present paper develops a model that captures this empirical evidence by arguing that short-term debt may be the optimal response to credibility problems.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 and 4 study the policy under full-commitment and under reputation, respectively. Section 5 reports the numerical results. Section 6 concludes. Finally, the Appendices include proofs, figures, and explain the numerical solution method.

2.2 The Model

Our economy is a version of that of Lucas and Stokey (1983). This version departs from the original model in that government spending is endogenous. This government spending can be financed through time-variant tax rates on labor income and through debt.[5]

[5] The first-best allocation would be attainable if governments could levy taxes on consumption and on labor income. In that case, the time-inconsistency problem would disappear. As usual in the literature, consumption taxes are excluded to assure distorsionary taxation (see Zhu (1995)).

This public debt consists of bonds that are indexed to consumption and that mature at different future dates, with a maximum maturity of N dates. More precisely, the government at date t can issue sequences $\{_{t+1}b_s\}_{s=t+1}^{t+N}$, that enter in the economy at the end of date t, of claims by the individual to consumption goods at date $s = t+1$, ..., $t+N$. Through the issue of these bonds, governments promise debt payments, interest and principal, that can be viewed as consumption units that the individual receives at some future date. We do not consider the possibility of issuing claims to leisure.[6] Therefore, public debt is issued with a limited structure in terms of both maturity calendar and debt-type variety.

We consider an economy populated by identical infinitely-lived individuals. Each individual is endowed with an initial debt and with $d > 0$ units of time per period that can be either devoted to leisure $x_t \in (0, d)$ or to output production $y_t = d - x_t$. This output is produced by a representative competitive firm, which maximizes profits given the factor price. As a result, labor is paid its marginal product, that is, a unitary wage rate. Given that output is non-storable and can be used for both private consumption c_t and public spending g_t, the resource constraint is

$$c_t + x_t + g_t \leq d, \text{ for } t = 0, 1, 2, ... \tag{2.1}$$

The representative individual derives utility from private consumption, public spending, and leisure so that his objective is to maximize the sum of discounted utilities

$$\sum_{t=0}^{\infty} \beta^t \left[U\left(c_t, x_t\right) + Z\left(g_t\right) \right], \tag{2.2}$$

[6] Faig (1994) showed that the time-inconsistency problem could be solved in this environment through the careful issue of claims to consumption and to leisure with no maturity restrictions.

with $\beta \in (0,1)$. The utility functions $U(\cdot,\cdot)$ and $Z(\cdot)$ take the following forms:

$$U(c_t, x_t) = \theta \ln c_t + (1-\theta) \ln x_t, \tag{2.3}$$

$$Z(g_t) = \gamma \ln g_t, \tag{2.4}$$

where $\theta \in (0,1)$ and $\gamma > 0$ measure the importance of private consumption relative to leisure and the importance of public consumption, respectively. Taking the government policy as given, the representative individual chooses bonds and non-negative amounts of private consumption and leisure so as to maximize his welfare (2.2) subject to the budget constraint

$$c_t + \sum_{s=t+1}^{t+N} q_t^s \left({}_{t+1}b_s - {}_t b_s \right) \leq q_t^t \, {}_t b_t + (1-\tau_t) y_t, \text{ for } t = 0, 1, 2, ..., \tag{2.5}$$

and the no-Ponzi-game condition

$$\lim_{t \to \infty} \sum_{s=t+1}^{t+N} \eta_t q_t^s \, {}_{t+1}b_s = 0. \tag{2.6}$$

Here η_t is the Lagrange multiplier for constraint (2.5) at date t, τ_t is the labor income tax rate at date t, and $q_t^s = 1/(1 + {}_t r_s)$, where q_t^s and ${}_t r_s$ are respectively the price and interest rate at date t of debt maturing at date s. We normalize q_t^t to be equal to 1 for all dates t. The first-order conditions for this optimization problem are

$$\frac{U_x(c_t, x_t)}{U_c(c_t, x_t)} = (1-\tau_t), \text{ for } t = 0, 1, 2, ..., \tag{2.7}$$

$$\frac{\beta^{s-t} U_c(c_s, x_s)}{U_c(c_t, x_t)} = \frac{q_t^s}{q_s^s}, \text{ for } s = t+1, ..., t+N, \text{ and } t = 0, 1, 2, ..., \tag{2.8}$$

where U_c and U_x denote the partial derivatives of the instantaneous utility function (2.3) with respect to consumption and leisure, respectively. Other derivatives of the utility function will follow a similar notation. An allocation for consumers is a sequence $\{a_t\}_{t=0}^{\infty}$, where $a_t = \left(c_t, x_t, \{{}_{t+1}b_s\}_{s=t+1}^{t+N} \right)$.

The government budget constraint is

$$g_t + q_t^t \ {}_tb_t \leq \tau_t y_t + \sum_{s=t+1}^{t+N} q_t^s \left({}_{t+1}b_s - {}_tb_s \right), \text{ for } t = 0, 1, 2, ..., \tag{2.9}$$

where $\{{}_0b_s\}_{s=0}^{N-1}$ is the initial debt. For the purposes of comparison between different maturities of debt, the initial debt is assumed to mature at date 0, that is, ${}_0b_s = 0$ for all dates $s \geq 1$. The government policy at date t is $\pi_t = \left(\tau_t, g_t, \{q_t^s\}_{s=t+1}^{t+N}\right)$.

A competitive equilibrium for this economy is defined as follows:

Definition 1 *Given the policy* $\{\pi_t\}_{t=0}^{\infty}$ *and the initial debt* ${}_0b_0$, a *competitive equilibrium allocation is a sequence* $\{a_t\}_{t=0}^{\infty}$ *such that: (i) the representative individual maximizes his welfare* (2.2) *subject to the budget constraint* (2.5) *and the No-Ponzi-game condition* (2.6)*; (ii) labor is paid its marginal product; and (iii) all markets clear (equation* (2.1) *is satisfied with equality).*

We will next turn to the policy selection. We will first consider the economy under full-commitment and we will model the economy without commitment afterwards.

2.3 The Full-Commitment Policy

In this section we will assume that future governments commit to the optimal policy chosen by the current government. This assumption can be viewed as a full-commitment among the successive governments that makes the optimal policy planned at date 0 sustainable. Formally, under full-commitment, the government at date 0 selects a policy plan $\pi = \{\pi_t\}_{t=0}^{\infty}$. Then, taking into account π, consumers choose their allocation rule at each date t. An allocation rule is a sequence of functions $f = \{f_t\}_{t=0}^{\infty}$ that map policies π_t into allocations a_t.

This policy plan and allocation rule are obtained by solving a simple programming problem. First, an allocation is chosen so as to maximize the welfare of the representative individual subject to the competitive equilibrium conditions. Then, taking into account these conditions, the allocation determines the optimal taxes. In order to define the government optimization problem, we plug the first-order conditions (2.7) and (2.8) into the budget constraint (2.5) to obtain

$$U_{c_t}\left(c_t - {}_tb_t\right) - U_{x_t}\left(d - x_t\right) + \sum_{s=t+1}^{t+N} \beta^{s-t} U_{c_s}\left({}_{t+1}b_s - {}_tb_s\right) = 0, \text{ for } t = 0, 1, 2, ... \quad (2.10)$$

The government chooses the sequences $\left\{c_t, x_t, g_t, \{{}_{t+1}b_s\}_{s=t+1}^{t+N}\right\}_{t=0}^{\infty}$ that maximize the welfare of the representative individual (2.2) subject to the resource constraint (2.1), the budget constraint (2.10), and the transversality condition

$$\lim_{t\to\infty} \sum_{s=t+1}^{t+N} \beta^s U_{c_s}\ {}_{t+1}b_s = 0, \quad (2.11)$$

given the initial debt ${}_0b_0$.

The solution of this problem is characterized by constraints (2.1), (2.10), and (2.11) and the following first-order conditions for consumption, leisure, public spending, and debt, respectively:

$$\mu_t = U_{c_t} + \lambda_t\left[U_{c_t} + U_{c_tc_t}\left(c_t - {}_tb_t\right)\right] + \sum_{k=\max[0,t-N]}^{t-1} \lambda_k U_{c_tc_t}\left({}_{k+1}b_t - {}_kb_t\right), \quad (2.12)$$

$$\mu_t = U_{x_t} + \lambda_t\left[U_{x_t} - U_{x_tx_t}\left(d - x_t\right)\right], \quad (2.13)$$

$$\mu_t = Z_{g_t}, \quad (2.14)$$

$$\lambda_t = \lambda_{t+1} \text{ for } {}_{t+1}b_s, \text{ where } s = t+1, ..., t+N, \quad (2.15)$$

for all dates $t \geq 0$, where μ_t and λ_t are the Lagrange multipliers associated with constraints (2.1) and (2.10), respectively.[7] We obtain the optimal labor tax rates and interest factors from the equilibrium conditions (2.7) and (2.8). The optimal management of debt results from equation (2.15). This equation clearly implies the standard result of the irrelevance of the debt structure. Therefore, the government cares about the amount of debt to issue, but it is indifferent about its composition.[8]

We now describe the dynamics of the policy and allocation. Given that the initial debt matures at date 0, we can state the following proposition:

Proposition 1 *If* ${}_0b_0 \underset{(\leq)}{\geq} 0$, *then the optimal allocation-policy pair under full-commitment* $\{a_t, \pi_t\}_{t=0}^{\infty}$ *exhibits a one-period transition, where* $c_0 \underset{(\leq)}{\geq} c_{ss}$, $x_0 \underset{(\geq)}{\leq} x_{ss}$, $g_0 \underset{(\geq)}{\leq} g_{ss}$ *and* $\tau_0 \underset{(\geq)}{\leq} \tau_{ss}$.

Proof. See Appendix 2.7. ■

Proposition 1 characterizes the steady state and the dynamics of the economy under full-commitment. Combining equation (2.15) with conditions (2.12) and (2.13) becomes clear that the only source of transition is the initial inherited debt structure. As a result, the transition towards the steady state takes as many periods as effective maturity of the initial inherited debt.

Under full-commitment the optimal policy is sustainable independently of the debt structure. However, in the absence of such a commitment, future governments will reconsider the optimal plan. In fact, once the current savings decisions have been taken, the

[7] As pointed out by Lucas and Stokey (1983), second-order conditions are not clearly satisfied because they involve third and second derivatives of the utility function. Therefore, we assume that an optimal interior solution exists.

[8] Multiple solutions for the composition of debt arise because one condition, equation (2.15), is obtained to solve for N variables, ${}_{t+1}b_s$ for all maturities $s = t+1, ..., t+N$.

amount and composition of debt that the government at date $t+1$ must reimburse are different from those that the government at date t was required to from date $t+1$ onwards. Since the debt obligations have changed, it is not longer optimal to continue with the announced policy. Moreover, given that governments have access to a limited debt structure, the optimal fiscal policy cannot be made time-consistent through debt restructuring. In this environment a government may try to build a reputation so as to make the announced policy credible.

2.4 The Policy under a Reputation

From this section on we will assume that future governments commit to honoring debt, but they can reconsider both taxation and spending plans subject to a reputation mechanism. In this framework governments must weigh the benefits of deviating from the announced policy against the loss of reputation that leads to a reversion to a Markov equilibrium. In this section we first describe how decisions are taken for an economy without commitment. Next, we define a sustainable equilibrium. We find a Markov equilibrium, which allows us to provide the set of sustainable equilibria. Finally, we select the best sustainable equilibria.

Without commitment, both government and consumers take their decisions sequentially. At the beginning of date t the government chooses a current policy as a function of the history $h_{t-1} = (\pi_s | \, s = 0, ..., t-1)$, denoted $\sigma_t(h_{t-1})$, and a plan for future policies under all possible future histories. Given a history h_{t-1}, the policy plan σ induces future histories by $h_t = (h_{t-1}, \sigma_t(h_{t-1}))$ and so on. A continuation policy of σ

is $(\sigma_t(h_{t-1}), \sigma_{t+1}(h_{t-1}, \sigma_t(h_{t-1})), ...)$. After the government sets the current policy, the representative agent chooses consumption, leisure, and new debt holdings for date t as a function of history h_t, denoted $f_t(h_t)$, and a plan for future allocations. Given a history h_t and a policy plan σ_t, a continuation allocation of f is $(f_t(h_t), f_{t+1}(h_t, \sigma_{t+1}(h_t)), ...)$. We will next frame a continuation policy and a continuation allocation into an optimization problem.

Consider first the government at date t. Given some history h_{t-1} and given that future allocations evolve according to f, the government chooses a continuation policy that maximizes the welfare of the representative individual

$$\sum_{s=t}^{\infty} \beta^{s-t} \left[U(c_s(h_s), x_s(h_s)) + Z(g_s(h_{s-1}))\right], \tag{2.16}$$

subject to

$$g_s(h_{s-1}) + {}_s b_s(h_{s-1}) \leq \tau_s(h_{s-1}) y_s(h_s) + \sum_{r=s+1}^{s+N} q_t^s(h_{s-1})\left({}_{s+1}b_r(h_s) - {}_s b_r(h_{s-1})\right), \tag{2.17}$$

for all dates $s \geq t$, where the future histories are induced by σ from h_{t-1}. The solution of this problem provides a value of welfare that is denoted $V(h_{t-1}; \sigma, f)$.

Consider now the representative individual at date t. Given some history h_t and given that future policies evolve according to σ, the representative individual chooses a continuation allocation to maximize

$$\sum_{s=t}^{\infty} \beta^{s-t} \left[U(c_s(h_s), x_s(h_s)) + Z(g_s(h_{s-1}))\right], \tag{2.18}$$

subject to

$$c_t(h_t) + \sum_{r=t+1}^{t+N} q_t^r\left({}_{t+1}b_r(h_s) - {}_t b_r(h_{t-1})\right) \leq {}_t b_t(h_{t-1}) + (1-\tau_t) y_t(h_t), \tag{2.19}$$

for date t, and

$$c_s(h_s) + \sum_{r=s+1}^{s+N} q_t^s(h_{s-1})\left({}_{s+1}b_r(h_s) - {}_{s}b_r(h_{s-1})\right) \leq {}_{s}b_s(h_{s-1}) + (1-\tau_s(h_{s-1}))y_s(h_s), \tag{2.20}$$

for all dates $s > t$, where future histories are induced by σ from h_t. We denote the welfare resulting from this optimization problem by $W(h_t;\sigma,f)$. From these programs follows a definition for a sustainable equilibrium:

Definition 2 *A sustainable equilibrium is a pair (σ, f) that satisfies the following conditions: (i) Given the allocation rule f, the continuation policy of σ solves the government's problem for every history h_{t-1}; (ii) given a policy plan σ, the continuation allocation of f solves the consumer's problem for every history h_t.*

2.4.1 A Markov Equilibrium

A sustainable equilibrium is utility-Markov if the past history influences payoffs only to the extent that it changes the inherited debt structure. A formal definition is as follows:

Definition 3 *A sustainable equilibrium is said to be utility-Markov if for any pair of histories h_{t-1} and h'_{t-1} such that $\{{}_tb_s(h_{t-1})\}_{s=t}^{t+N-1} = \{{}_tb_s(h'_{t-1})\}_{s=t}^{t+N-1}$, then it results that (i) $V(h_{t-1};\sigma,f) = V(h'_{t-1};\sigma,f)$, and (ii) $W(h_{t-1},\pi_t;\sigma,f) = W(h'_{t-1},\pi_t;\sigma,f)$, where V and W are defined in equations (2.16) and (2.18), respectively.*

Following Chari and Kehoe (1993), we will find a Markov equilibrium by means of solving two different programs. The first program defines the following value function:

$$V\left(\{{}_tb_s\}_{s=t}^{t+N-1}\right) = \max \sum_{s=t}^{\infty} \beta^{s-t}\left[U\left(c_s, x_s\right) + Z\left(g_s\right)\right], \tag{2.21}$$

subject to the resource constraint (2.1) at date $s \geq t$, the government budget constraint (2.9) at date $s \geq t$, the first-order conditions (2.7) and (2.8) at date $s \geq t$, the incentive constraint

$$\sum_{s=r}^{\infty} \beta^{s-r}\left[U\left(c_s, x_s\right) + Z\left(g_s\right)\right] \geq V\left(\{{}_rb_s\}_{s=r}^{r+N-1}\right), \tag{2.22}$$

at date $s \geq t$, and the transversality condition (2.11), given the initial debt $\{{}_tb_s\}_{s=t}^{t+N-1}$. Equation (2.22) ensures that the government will have no incentives to deviate from the announced plan. By choosing the sequence $\{a_s, \pi_s\}_{s=t}^{\infty}$ that solves problem (2.21), we obtain a policy plan σ^m for the government.

The second program defines the value function

$$W\left(\{{}_tb_s\}_{s=t}^{t+N-1}, \pi_t\right) \equiv \max \sum_{s=t}^{\infty} \beta^{s-t}\left[U\left(c_s, x_s\right) + Z\left(g_s\right)\right], \tag{2.23}$$

subject to the incentive constraint (2.22) at date $s \geq t$, the budget constraint for the individual (2.5) at date $s \geq t$, the first-order conditions (2.7) and (2.8) at date $s \geq t$, the government budget constraint (2.9) at date $s \geq t+1$, and the transversality condition (2.11), given π_t and the initial debt $\{{}_tb_s\}_{s=t}^{t+N-1}$. We choose the sequence $\{a_s, \pi_{s+1}\}_{s=t}^{\infty}$ that solves the dynamic optimization problem (2.23), which yields the consumer allocation rule f^m. Notice that this allocation rule must be defined for all possible histories, including those in which the government deviates. Thus, the government budget constraint is not required to hold at date t.

These two programs are recursive by construction. Therefore, as the next lemma shows, the solution of these problems constitutes a sustainable equilibrium:

Lemma 1 *The pair* (σ^m, f^m) *is a sustainable equilibrium.*

Proof. See Appendix 2.7. ■

A policy plan and an allocation rule that satisfy their respective programs form a sustainable equilibrium given that, as follows from constraint (2.22), no deviation would improve welfare. Moreover, that sustainable equilibrium is utility-Markov by construction. On the one hand, if a unique solution exists, this is uniquely determined by the inherited debt structure. On the other hand, if there is more than one solution, they should provide the same welfare. This Markov equilibrium is characterized as follows:

Lemma 2 *The pair* (σ^m, f^m) *satisfies* ${}_{t+1}b_s = {}_tb_s$ *for* $s = t+1, ..., t+N$.

Proof. See Appendix 2.7. ■

Lemma 2 says that the utility-Markov equilibrium (σ^m, f^m) satisfies that the new issues of debt are such that the government at date $t+1$ has the same debt obligations as those of the government at date t from date $t+1$ on. This equilibrium is sustainable and Markov. Notice that the existence of the time-inconsistency problem is intimately associated with the state variables. In the absence of state variables linking periods, the model becomes static and no time-inconsistency problems appear. In a model with state variables, decisions on current and future variables take into account how these decisions affect the dynamics of current and future state variables but do not consider how they did affect their evolution in the past. Thus, once the state variables evolve, decisions become time-inconsistent. Conversely, given that the state variables do not change in our

Markov equilibrium, this equilibrium is obviously sustainable. Moreover, this sustainable equilibrium is Markov since past history influences welfare only through the inherited debt structure.

Lemma 2 characterizes an equilibrium that is sustainable. However, there are typically other possible sustainable equilibria. In order to find the set of sustainable equilibria, we define the revert-to-Markov equilibria as follows:

Definition 4 *Let us consider a sequence of policies and allocations* (π, a). *A revert-to-Markov allocation and a revert-to-Markov policy are defined as follows:*

(i) For any history h_t, *a revert-to-Markov allocation is the allocation* a_t *given by* a *if the policy* $(\pi_0, ..., \pi_t)$ *has been chosen according to* π. *If a first deviation from* π *occurs at date* t, that is, $\hat{\pi}_t \neq \pi_t$, *then the equilibrium allocation at date* $t+j$ *for all* $j \geq 0$ *is given by the utility-Markov equilibrium in Lemma 2.*

(ii) For any history h_{t-1}, *a revert-to-Markov policy plan specifies continuation with* (π, a) *as long as past policies* $(\pi_0, ..., \pi_{t-1})$ *have been chosen according to* π. *If there was ever a deviation from* π, *the policy reverts to that of the utility-Markov equilibrium in Lemma 2.*

Once the revert-to-Markov equilibria are defined, we characterize the set of sustainable equilibria as follows:

Proposition 2 *A pair* (π, a) *is a sustainable equilibrium if and only if*

(i) (π, a) *is a competitive equilibrium at date* 0.

(ii) For all dates t *the following inequality holds:*

$$\sum_{s=t}^{\infty} \beta^{s-t} \left[U\left(c_s, x_s\right) + Z\left(g_s\right)\right] \geq V^D \left(\left\{{}_t b_s\right\}_{s=t}^{t+N-1}\right),$$

where V_t^D *denotes the welfare value provided by the Markov outcome,*[9] *which is*

$$V^D = \sum_{s=t}^{\infty} \beta^{s-t} \left[U \left(c_s^D, x_s^D \right) + Z \left(g_s^D \right) \right]. \tag{2.24}$$

Proof. See Appendix 2.7. ■

Proposition 2 defines the set of sustainable equilibria as the revert-to-Markov equilibria. Once the composition of the inherited debt changes, the government has incentives to deviate. It will do so if the benefits outweigh the costs of deviating. The rewards come from the possibility of re-optimizing taking into account the new endowments. Lemma 2 and Proposition 2 specify the consequences of deviating. If the government deviates from the previously announced plan, the continuation policy and allocation will be specified by the Markov equilibrium from then on. The dynamics after a deviation are characterized in the following proposition:

Proposition 3 *If* ${}_s b_j \underset{(\leq)}{\geq} 0$ *for maturity* $s = j, ..., j + N - 1$*, then the allocation-policy pair after a deviation at date* t $\left\{ a_j^D, \pi_j^D \right\}_{j=t}^{\infty}$ *exhibits a transition of* N *periods, where* $\left(c_j^D \middle/ g_j^D \right) \underset{(\leq)}{\geq} \left(c_{ss}^D / g_{ss}^D \right)$, $x_j^D \underset{(\leq)}{\geq} x_{ss}^D$, $g_j^D \underset{(\geq)}{\leq} g_{ss}^D$.

Proof. See Appendix 2.7. ■

Proposition 3 describes the dynamics of the economy after a deviation. If the government inherits bonds with a maturity of 1, 2, ... and N periods, then the economy displays a transition of N periods. The pattern that the variables follow after a deviation depends on the sign of this inherited debt.

[9] The variables c_s^D, x_s^D, and g_s^D denote consumption, leisure, and government spendings after a deviation, respectively. Other variables after a deviation will be denoted analogously.

The welfare value after deviation, that is the deviation value V_t^D, plays an important role in determining the best sustainable policy that can be attained. As we have shown, this deviation value depends on the inherited debt structure. In particular, for a given maturity N, there is a composition of this debt that makes the deviation value maximum, denoted $V^{*D}(N)$. The next proposition shows how the inherited debt and the maturity affect the deviation value and the maximum deviation value:

Proposition 4 *For all maturities $j = t, ..., t + N - 1$, $\frac{\partial V_t^D}{\partial_t b_j} = -\beta^{j-t} \frac{\theta \lambda_j^D}{c_j^D} \leq 0$. Moreover, if $M \geq N$, then $V^{*D}(M) \geq V^{*D}(N)$.*

Proof. See Appendix 2.7. ■

The policy after a deviation of Lemma 2 describes a government whose choices are subject to a given management of debt. In this context the government uses debt to enforce time-consistency, but it cannot use debt to smooth consumption across periods. Proposition 4 characterizes the welfare value after a deviation. This proposition says that the deviation value is greater the smaller the inherited debt is, and that the maximum deviation value increases with the maturity of this debt. The former result is obvious since the more indebted an economy is, the less welfare can provide to their agents. An intuition for the latter result is that the losses from the inability to smooth consumption are smaller under a higher maturity of debt.

2.4.2 The Best Sustainable Equilibrium

In the previous section we have provided a Markov equilibrium, which allowed us to define the set of sustainable equilibria. In this section we characterize the best sustainable equilibrium. The government chooses the best sustainable equilibrium by selecting an allocation among the different sustainable equilibria so as to maximize the welfare of the representative individual. In order to do so, the government must take into account the period-by-period constraint that the welfare value of continuing with the announced policy must be higher than the welfare value of deviating from it. These conditions can be viewed as incentive compatibility constraints linking the current and future governments as follows:

$$\sum_{t=i}^{\infty} \beta^{t-i} \left[U\left(c_t, x_t\right) + Z\left(g_t\right) \right] \geq V^D \left(\left\{ {}_i b_s \right\}_{s=i}^{i+N-1} \right), \text{ for date } i = 1, 2, ... \tag{2.25}$$

Let us define the government optimization problem. The government chooses the sequences $\left\{ c_t, x_t, g_t, \left\{ {}_{t+1} b_s \right\}_{s=t+1}^{t+N} \right\}_{t=0}^{\infty}$ that maximize the welfare of the representative individual (2.2) subject to the resource constraint (2.1), the budget constraint (2.10), the incentive compatibility constraint (2.25), and the transversality condition (2.11), given the initial debt ${}_0 b_0$.

Under the incentive compatibility constraints (2.25), the solution of this optimization problem is clearly time-consistent since it takes into account the future governments' trade-off between continuing with or deviating from the announced policy. However, these incentive constraints (2.25) make our optimization problem very non-recursive. We overcome this difficulty by considering the approach suggested by Marcet and Marimón (1999). According to these authors, a problem with incentive constraints can be analyzed by an equivalent saddle point problem. The corresponding saddle point expression for our problem is the following:

$$L \equiv \sum_{t=0}^{\infty} \beta^t \left[\Gamma_t l_0 + \Lambda_t l_1 \right], \tag{2.26}$$

subject to the resource constraint (2.1), where

$$\begin{aligned} \Gamma_{t+1} &= \left(1, \Gamma_t^1 + \xi_t, \lambda_t, \lambda_{t-1}, ..., \lambda_{t-N+1}\right), \text{ with } \Gamma_0 = (1, 0, ..., 0), && (2.27) \\ \Lambda_t &= (\xi_t, \lambda_t), \text{ with } \xi_0 = 0, \text{ and } \lambda_t = 0 \text{ for dates } t < 0, && (2.28) \end{aligned}$$

and

$$l_0 = \begin{bmatrix} U(c_t, x_t) + Z(g_t) \\ U(c_t, x_t) + Z(g_t) \\ U_{c_t}\left({}_t b_t - {}_{t-1} b_t \right) \\ \vdots \\ U_{c_t}\left({}_{t-N+2} b_t - {}_{t-N+1} b_t \right) \\ U_{c_t}\ {}_{t-N+1} b_t \end{bmatrix} \text{ and } l_1 = \begin{bmatrix} U(c_t, x_t) + Z(g_t) - V_i^D\left(\{ {}_i b_s \}_{s=i}^{i+N-1} \right) \\ U_{c_t}(c_t - {}_t b_t) - U_{x_t}(d - x_t) \end{bmatrix}. \tag{2.29}$$

Our original optimization problem is equivalent to this saddle point problem. Solving the system $(2.26) - (2.29)$, we obtain precisely a Lagrangian for the original optimization problem, where λ_t and ξ_t are the Lagrange multipliers for constraints (2.10) and (2.25), respectively. Thus, Lagrangian (2.26) provides a new formulation for our original optimization problem. Under general conditions, Marcet and Marimón (1999) extended dynamic programming theory to show that this formulation is recursive in the sense that it satisfies a saddle point functional equation, which is analogous to a Bellman equation. Given this result, we can guarantee that the variables of the model are time-invariant functions of the state $\{ {}_t b_s \}_{s=t}^{t+N-1}$ and the co-state variables, $\Gamma_t^1, \lambda_{t-1}, ..., \lambda_{t-N}$. In this Lagrangian we minimize with respect to Λ_t and maximize with respect to the control and state variables. The solution of this problem is characterized by constraints (2.1), (2.10), (2.11), and (2.25)

and the following first-order conditions for consumption, leisure, public spending, and debt, respectively:

$$\begin{aligned}
\mu_t &= \left(1+\Gamma_t^1+\xi_t\right) U_{c_t} + \lambda_t \left[U_{c_t} + U_{c_t c_t}\left(c_t - {}_t b_t\right)\right] && (2.30)\\
&\quad + \sum_{k=\max[0,t-N]}^{t-1} \lambda_k U_{c_t c_t}\left({}_{k+1}b_t - {}_k b_t\right), \\
\mu_t &= \left(1+\Gamma_t^1+\xi_t\right) U_{x_t} + \lambda_t \left[U_{x_t} - U_{x_t x_t}\left(d - x_t\right)\right], && (2.31)\\
\mu_t &= \left(1+\Gamma_t^1+\xi_t\right) Z_{g_t}, && (2.32)\\
0 &= U_{c_s}\left(\lambda_t - \lambda_{t+1}\right) - \xi_{t+1}\frac{\partial V_{t+1}^D}{\partial\ {}_{t+1}b_s}, && (2.33)
\end{aligned}$$

for all dates $t \geq 0$, where μ_t, λ_t and ξ_t are the Lagrange multipliers for constraints (2.1), (2.10) and (2.25), respectively.[10] We obtain the optimal taxes and interest factors from the equilibrium conditions (2.7) and (2.8). The optimal structure of debt is chosen according to condition (2.33). In contrast to the full-commitment, this condition says that as long as the incentive constraints (2.25) are binding, the composition of debt is not longer irrelevant. Thus, the government under a reputation cares not only about the amount of debt to issue but also about the maturity of public debt.

We next study the dynamics of the best sustainable policy. The transition is driven by the initial debt structure and also by the effort that the government makes to outweigh future incentives to deviate from the announced policy. This effort is measured by the multipliers $\{\xi_t\}_{t\geq 1}$ for the incentive compatibility constraints (2.25). The properties of these dynamics are summarized as follows:

Proposition 5 *For $N = 1$, the allocation-policy pair in the best sustainable equilibrium*

[10] In the absence of concavity, our characterization is necessary but may not be sufficient for a solution. As usual in this literature, we assume that an interior solution exists.

$\{a_j, \pi_j\}_{j=t}^{\infty}$ *exhibits a transition towards the steady state, where* $\xi_j \geq \xi_{ss} = 0$, *and* $\lambda_j \leq \lambda_{ss}$. *Moreover, if* ${}_j b_j \underset{(\leq)}{\geq} 0$, *then* $(c_j/g_j) \underset{(\leq)}{\geq} (c_{ss}/g_{ss})$.

Proof. See Appendix 2.7. ■

Proposition 5 characterizes the transition towards the steady state for the best sustainable equilibrium with one-period debt. The properties of this transition depend on the sign of the inherited debt. An interesting feature is that the incentive compatibility constraints (2.25) are not binding at the steady state. Therefore, the steady state is incentive-unconstrained.

The best sustainable policy lies between the full-commitment and the policy after deviation in the following sense. The best sustainable policy yields a welfare that is at most the welfare under full-commitment, but it is greater than the deviation value, both evaluated at the initial date 0. Notice, moreover, that the value of continuing with the full-commitment at some particular date $t > 0$ is different from the welfare of the full-commitment computed at that date t as initial date, due to time-inconsistency problems. In particular, the deviation could yield a value of welfare that is greater than the value of continuing with the full-commitment. In that case, the full-commitment would not be sustainable. Hence, the best sustainable policy would provide less welfare than the full-commitment, both evaluated at the initial date 0. The analysis of the continuation value under full-commitment, denoted V_{FC}^{C}, and the maximum deviation value V^{*D}, allows us to draw some conclusions on the best sustainable policy. Those results are summarized in the next proposition:

Proposition 6 *If* $V_{FC}^{C} \geq V^{*D}(N)$ *for some finite maturity* N *and* ${}_0b_0 \neq 0$*, then the best sustainable equilibrium is characterized by the following properties:*

(i) For all maturities $P \in \{1, 2, ...N\}$*,* $V_{FC}^{C} < V^{*D}(P)$ *holds. Therefore, the full-commitment policy is time-consistent for all debt structures.*

(ii) There exits a finite maturity $\widehat{N}$*, that is strictly greater than* N*, such that for all maturities* $M \in \left\{\widehat{N}, \widehat{N}+1, ..., \infty\right\}$*,* $V_{FC}^{C} < V^{*D}(M)$ *holds. Therefore, there exists at least a debt structure that makes the full-commitment policy time-inconsistent.*

Proof. See Appendix 2.7. ■

Proposition 6 refers to economies where the reputation can sustain the full-commitment. For these economies, a sufficient increase in the maturity of debt can make the full-commitment solution not longer time-consistent and, hence, reduce welfare. The intuition for this result is as follows. If continuing with the full-commitment provides more welfare than the maximum deviation value for a given maturity N, that is, $V_{FC}^{C} \geq V^{*D}(N)$, the full-commitment is time-consistent. The welfare value of continuing with the full-commitment is independent of the maturity. However, Proposition 4 shows that the benefits from deviating are increasing in the effective maturity of debt. Moreover, the proof of Proposition 6 shows that, when the maturity N tends to infinity, the maximum deviation value is strictly larger than the welfare value of the continuation with the full-commitment. Thus, there exists a finite maturity $\widehat{N}$ above which the deviation can provide more welfare than the full-commitment. Therefore, a sufficient enrichment of the debt structure can make the full-commitment policy time-inconsistent. The best sustainable policy is characterized by shortening the maturity of debt, which allows us to sustain the full-commitment. Figure

2 illustrates this result.[11] This figure shows how an increase in the maturity shifts the maximum deviation value above the continuation value.

[Insert Figure 2 about here.]

Proposition 6 considers economies that have an initial deficit or an initial surplus, that is, economies with some initial debt either positive or negative. When the initial debt of the economy is zero, the economy satisfies the following corollary:

Corollary 1 *If* ${}_0b_0 = 0$, *then the full-commitment policy is time-consistent for all debt structures.*

Proof. See Appendix 2.7. ■

Corollary 1 shows that the best sustainable policy coincide with the full-commitment for economies with initial zero debt. Moreover, the debt structure is irrelevant. Thus, the reputation mechanism can sustain the full-commitment policy for all debt structures. This result holds because, when the initial debt is zero, the continuation with the full-commitment policy provides more welfare than the deviation policy.

We have shown in Proposition 6 that, under some conditions, a limited debt structure may be more preferable. We should argue how restrictive these conditions are. Table 1 shows several examples where $V_{FC}^{C} \geq V^{*D}(N)$ holds for some finite N under different parameter and initial values. This exercise also shows that larger maturities shift the deviation value above the continuation value and, hence, reduce welfare. In the next section we assess numerically how large these welfare losses could be.

[11]For Figure 2 and Table 1, we have considered the following parameter and initial values: $\beta = 0.95$, $\theta = 1/3$, $\gamma = 1$, $d = 100$, and ${}_0b_0 = 10$.

[Insert Table 1 about here.]

2.5 Numerical Solution

In order to find a numerical solution, we apply the parametrized expectations algorithm (PEA) developed by Den Haan and Marcet (1990).[12] This method replaces agents' expectations about future variables with an approximating function of the state and co-state variables with some coefficients on these variables.[13] This algorithm consists of the following steps. First, we choose some reasonable parameter and initial values.[14] Second, we propose a parametrized form of the current and past state variables to substitute expectations on future variables. Next, we obtain time-series of the variables, with which we compute the conditional expectation. Then, we perform a non-linear regression of these expressions so as to estimate the coefficients. Finally, we iterate this process until a fix point is found.

We consider two economies, A and B. These economies share the same parameter values but differ in their initial debt holdings, $b_0^A = 8$ and $b_0^B = 10$. The time-series are simulated for 1000 periods. The endowment per period d equals 100. The discount rate β is 0.95. The parameter θ is 1/3 so as to obtain reasonable values for labor. The value of γ is 2 in order to consider short maturities. Notice that the higher maturities are considered, the greater number of first-order conditions for debt (2.33) and, hence, the greater number

[12] A numerical solution for a dynamic non-linear economy requires computational methods in both a stochastic or in a non-stochastic environment. Given the complexity of constraint (2.25), we find PEA a suitable computational method for our deterministic setup. Other papers have applied this method to a non-stochastic economy (see, for example, Domeij (1999)).

[13] This can be carried out thanks to Marcet and Marimón (1999) that guarantee that the model variables are a time-invariant function in state and co-state variables.

[14] We use the NLSYS library of GAUSS 2.2.

of equations must be parametrized. This may make a numerical solution impossible to obtain. The speed of adjustment ω for PEA, defined in Appendix 2.8, is 0.5. For this set of parameter and initial values, we consider an increase in the effective maturity of debt from one to two dates. Initially, governments only issue debt that matures in one date. Afterwards, governments issue non-zero bonds that mature both in one and in two dates.

Applying PEA, we obtain time-series for the two economies.[15] The main difference between these two economies is that with one-period debt, the full-commitment policy is only sustainable for economy B. When one and two-period debt are issued, the full-commitment cannot be sustained for any of these economies. Let us see how different the best sustainable and the full-commitment policy are. In Figures 3 and 4 we observe that both private and public consumption are smaller in the short and in the long-run for the best sustainable policy. In the medium term the best sustainable policy may provide larger consumption than under full-commitment. The opposite result applies concerning leisure and taxes. Let us now consider how an increase in the maturity of debt affects these variables. For economy A, an increase in the maturity broadens the previously described differences between the full-commitment and the best sustainable policy. For economy B, this increase in the maturity of debt moves our economy out of the full-commitment solution.

[Insert Figures 3 and 4 about here.]

We describe now how an increase in the effective maturity of debt affects welfare, as Tables 2 and 3 show. Economy A is an interesting case because the reputation cannot make the full-commitment sustainable, that is, $V_{FC}^{C} \geq V^{*D}(N)$ does not hold for all N.

[15]In this numerical exercise we check the second-order conditions of the optimization problems.

Therefore, this economy does not satisfy the requirements of Proposition 6. The welfare under full-commitment takes a value of 199.361. For the best sustainable policy, the increase in the maturity reduces welfare from 199.360 (-0.05%) to 199.354 (-0.35%).[16] Therefore, we find that, even when the reputation mechanism cannot sustain the full-commitment, larger maturities lead to policies that provide less welfare. An intuition for this result can be drawn from Figure 3. At the bottom of this figure, we see the dynamics of the Lagrange multiplier for the incentive compatibility constraint (2.25). With one-period debt, this multiplier decreases rapidly towards zero. When governments issue one and two-period debt, the multiplier follows an interesting sinuous pattern towards zero described as follows: at odd dates, it is higher and decreases more slowly towards zero than with one-period debt; at even dates, it is almost zero. Thus, increasing the maturity from one to two-periods reduces the effort to satisfy the incentive constraint (2.25) in the next period, but it weakens the reputation mechanism over all the life-horizon. Therefore, even when the reputation mechanism cannot sustain the full-commitment, an increase in the maturity of debt may weaken the reputation and reduce welfare.

[Insert Tables 2 and 3 about here.]

The economy B illustrates the results of Proposition 6. With one-period debt, the full-commitment policy is sustainable. Once governments also issue two-period bonds, the full-commitment cannot be sustained and the economy suffers a welfare loss. Numerically, the increase in the maturity of debt moves welfare from 199.164 to 199.149 (-0.75%).

[16] These percentages show the difference in welfare between the best sustainable and the full-commitment policy.

2.6 Conclusions

This paper has studied the time-inconsistency problem of optimal fiscal policy for an economy with a limited debt structure and a reputation mechanism. In this framework we have explored the role of the debt structure in relation to the time-inconsistency problem. Under a limited debt structure, a reputation may sustain the full-commitment policy. However, a sufficiently large increase in the maturity of debt weakens this reputation mechanism and can make the optimal policy time-inconsistent. By means of numerical assessment, we have also concluded that, even when the reputation mechanism cannot sustain the full-commitment, increasing the maturity of debt can lead to time-consistent policies that provide less welfare.

A number of key issues arise from this result. First, we have shown that the management of the composition of debt affects the credibility of the fiscal policy. In particular, we have argued that shorter maturities can make the policy announcements credible. These findings have some policy implications. First, we have identified an instrument for the policy-makers to reduce their credibility problems. Second, our results urge upon the need of coordination between the policy-making institutions. We have shown that debt management affects the final policy outcomes. Therefore, this interrelation should be embedded in the policy selection, which is particularly relevant when policy decisions are taken by different institutions.

Second, to study time-inconsistency issues, the specification of the time-horizon, either finite or infinite, does matter. In a finite-period economy, the time-consistent policy comes from the backward solution of the model. In that setup Rogers (1989) showed that shorter maturities increase the costs of the time-inconsistency problem. However, in an infinite-horizon economy, reputational forces come into play, which were absent in a finite-period setup. Thus, there is typically a large set of sustainable equilibria. From this set, we have focused on the best sustainable equilibrium. In contrast to Rogers (1989), we have found that shorter maturities reduce the costs of the time-inconsistency problem.

As a possible extension, it would be interesting to consider the possibility of liquidity crisis. As we have already commented, short maturities have been highlighted as one of the main determinants of liquidity crises. It is obviously more likely that a government falls short of liquidity when the debt obligations need to be met in a short-term rather than spread along a larger horizon. There the debt structure would play a different role than the one in this paper. In that framework the credibility gains associated with short-term debt would be undermined by the chances of suffering a liquidity crisis.

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2.7 Proofs of the Propositions and Lemmas

Proof of Proposition 1

Using equations (2.3) , (2.4) , (2.14) and (2.15), conditions (2.12) and (2.13) become

$$\frac{\theta}{c_t}\left[1+\lambda_0\left(\frac{{}_0b_t}{c_t}\right)\right]=\frac{\gamma}{g_t}, \tag{2.34}$$

$$\frac{(1-\theta)}{x_t}\left[1+\lambda_0\left(\frac{d}{x_t}\right)\right]=\frac{\gamma}{g_t}, \tag{2.35}$$

for all dates $t \geq 0$. Given that ${}_0b_s = 0$ for all maturities $s \geq 1$, the steady state is reached at date 1. Next, combining condition (2.35) with (2.1) , we get

$$\frac{c_t}{g_t}=\frac{d}{g_t}-\frac{x_t}{g_t}-1=\frac{\gamma}{\lambda_0\left(1-\theta\right)}\left(\frac{x_t}{g_t}\right)^2-\left(\frac{1}{\lambda_0}+1\right)\frac{x_t}{g_t}-1>0,$$

which implies

$$\frac{\partial\left(\frac{c_t}{g_t}\right)}{\partial\left(\frac{x_t}{g_t}\right)}>0. \tag{2.36}$$

Let us assume that ${}_0b_0 \geq 0$. Then, conditions (2.34) and (2.36) imply $(c_0/g_0) \geq (c_{ss}/g_{ss})$ and $(x_0/g_0) \geq (x_{ss}/g_{ss})$, respectively. Next, taking into account equations (2.1) and (2.35), we obtain $g_0 \leq g_{ss}$, $x_0 \leq x_{ss}$ and $c_0 \geq c_{ss}$. Using equation (2.7), we get $\tau_0 \leq \tau_{ss}$. Finally, if ${}_0b_0 \leq 0$, the previous inequalities are reversed. ■

Proof of Lemma 1

First, we must show that given a policy plan σ^m, the continuation allocation of f^m solves the consumer's problem (2.18) for every history h_t. Note that the solution of problem

(2.23) for $\left(\{{}_tb_s\}_{s=t}^{t+N-1}, \pi_t\right)$ at date $t+j$ for $j \geq 1$ coincides with the solution of (2.21) for $\{{}_{t+1}b_s\}_{s=t+1}^{t+N}$ from date $t+1$ on. Therefore, the policies that solve problem (2.23) are π_t and, by the recursivity of (2.21), those generated by σ^m for date $s > t$. Then, the policies that solve problem (2.23) are exactly the policies that the consumer faces when solving (2.18). Moreover, given that constraint (2.5) holds for all dates $s \geq t$ for problem (2.23), equations (2.19) and (2.20) are satisfied. The allocations generated from f^m are the optimal response to these policies. Thus, they solve problem (2.18).

Second, we must prove that given the allocation rule f^m, the continuation policy of σ^m solves the government problem (2.16) for all histories h_{t-1}. It suffices to show that no deviation improves welfare. Thus, if consumers follow the allocation rule f^m and the government policies from date $t+1$ on are generated from σ^m, then there is no policy π_t at date t which satisfies the budget constraint (2.17) and improves welfare. This is the case since σ^m and f^m satisfy constraint (2.22). ■

Proof of Lemma 2

To characterize the Markov equilibrium (σ^m, f^m), we will focus on the government problem (2.21) yielding σ^m. We first set the initial period at date 0 and, then, at date 1. At the initial date 0 we obtain the following first-order conditions for consumption, leisure, public consumption and bond holdings, respectively:

$$\left(1+\xi_{1,\,0}+...+\xi_{t,\,0}\right)U_{c_t}+\lambda_{t,\,0}\left[U_{c_t}+U_{c_tc_t}\left(c_t-{}_tb_t\right)\right] \qquad (2.37)$$

$$+\sum_{k=\max[0,t-N]}^{t-1}\lambda_{k,\,0}U_{c_tc_t}\left({}_{k+1}b_t-{}_kb_t\right)=\mu_{t,\,0},$$

$$\left(1+\xi_{1,\,0}+...+\xi_{t,\,0}\right)U_{x_t}+\lambda_{t,\,0}\left[U_{x_t}-U_{x_tx_t}\left(d-x_t\right)\right]=\mu_{t,\,0}, \qquad (2.38)$$

$$\left(1+\xi_{1,\,0}+...+\xi_{t,\,0}\right)Z_{g_t}=\mu_{t,\,0}, \qquad (2.39)$$

$$U_{c_s}\left(\lambda_{t,\,0}-\lambda_{t+1,\,0}\right)=\xi_{t+1,\,0}\frac{\partial V^D_{t+1}}{\partial\ {}_{t+1}b_s}, \qquad (2.40)$$

for all dates $t \geq 0$, where $\mu_{t,\,0}$, $\lambda_{t,\,0}$ and $\xi_{t,\,0}$ are the Lagrange multipliers for constraints (2.1), (2.10) and (2.22), respectively, from the government problem at date 0.[17] At the initial date 1 the government problem yields the following first-order conditions for consumption, leisure, public consumption and bonds, respectively:

$$\left(1+\xi_{2,\,1}+...+\xi_{t,\,1}\right)U_{c_t}+\lambda_{t,\,1}\left[U_{c_t}+U_{c_tc_t}\left(c_t-{}_tb_t\right)\right] \qquad (2.41)$$

$$+\sum_{k=\max[1,t-N]}^{t-1}\lambda_{k,\,1}U_{c_tc_t}\left({}_{k+1}b_t-{}_kb_t\right)=\mu_{t,\,1},$$

$$\left(1+\xi_{2,\,1}+...+\xi_{t,\,1}\right)U_{x_t}+\lambda_{t,\,1}\left[U_{x_t}-U_{x_tx_t}\left(d-x_t\right)\right]=\mu_{t,\,1}, \qquad (2.42)$$

$$\left(1+\xi_{2,\,1}+...+\xi_{t,\,1}\right)Z_{g_t}=\mu_{t,\,1}, \qquad (2.43)$$

$$U_{c_s}\left(\lambda_{t,\,1}-\lambda_{t+1,\,1}\right)=\xi_{t+1,\,1}\frac{\partial V^D_{t+1}}{\partial\ {}_{t+1}b_s}, \qquad (2.44)$$

for all dates $t \geq 1$, where $\mu_{t,\,1}$, $\lambda_{t,\,1}$ and $\xi_{t,\,1}$ are the Lagrange multipliers for constraints (2.1), (2.10) and (2.22) from the government problem at date 1.

As proved in Lemma 1, the pair (σ^m, f^m) is a sustainable equilibrium. Then, the sequence $\{a_s, \pi_s\}_{s=1}^{\infty}$ that solves the government problem at date 0 must solve the

[17]Equation (2.10) summarizes the budget constraint (2.5) and the first-order conditions (2.7) and (2.8).

government problem at date 1, and at any arbitrary date $t \geq 1$. Is there a debt structure different from ${}_1b_s = {}_0b_s$ for all maturities $s = 1, ..., N$, such that decisions at date 0 and at date 1 coincide?

Using equation (2.39), the first-order conditions (2.37) and (2.38) from the government problem at date 0 are

$$\left(1+\xi_{1,\,0}\right) U_{c_1} + \lambda_{1,\,0}\left[U_{c_1} + U_{c_1c_1}\left(c_1 - {}_1b_1\right)\right] + \lambda_{0,\,0}U_{c_1c_1}\left({}_1b_1 - {}_0b_1\right) = \tag{2.45}$$

$$\left(1+\xi_{1,\,0}\right) Z_{g_1},$$

$$\left(1+\xi_{1,\,0}\right) U_{x_1} + \lambda_{1,\,0}\left[U_{x_1} - U_{x_1x_1}\left(d - x_1\right)\right] = \left(1+\xi_{1,\,0}\right) Z_{g_1}, \tag{2.46}$$

for date 1. Similarly, from the government problem at date 1 we obtain

$$U_{c_1} + \lambda_{1,\,1}\left[U_{c_1} + U_{c_1c_1}\left(c_1 - {}_1b_1\right)\right] = Z_{g_1}, \tag{2.47}$$

$$U_{x_1} + \lambda_{1,\,1}\left[U_{x_1} - U_{x_1x_1}\left(d - x_1\right)\right] = Z_{g_1}. \tag{2.48}$$

Solving equations (2.46) and (2.48) for the same allocation, we get

$$\lambda_{1,\,1} = \left[\frac{\lambda_{1,\,0}}{1+\xi_{1,\,0}}\right]. \tag{2.49}$$

Taking into account equation (2.49) and that the same allocation solves conditions (2.45) and (2.47), ${}_1b_1 = {}_0b_1$ must hold. Next, let us consider the government problem at date t and at date $t+1$. Since the same allocation and policy solve the first-order conditions for date $t+1$, then ${}_{t+1}b_{t+1} = {}_tb_{t+1}$. In particular, we obtain that ${}_2b_2 = {}_1b_2$.

Conditions (2.45) and (2.46) from the government problem at date 0 are

$$\left(1+\xi_{1,\,0}+\xi_{2,\,0}\right)U_{c_2}+\lambda_{2,\,0}\left[U_{c_2}+U_{c_2c_2}\left(c_2-{}_2b_2\right)\right]+\lambda_{1,\,0}U_{c_2c_2}\left({}_2b_2-{}_1b_2\right) \tag{2.50}$$

$$+\lambda_{0,\,0}U_{c_2c_2}\left({}_1b_2-{}_0b_2\right)=\left(1+\xi_{1,\,0}+\xi_{2,\,0}\right)Z_{g_2},$$

$$\left(1+\xi_{1,\,0}+\xi_{2,\,0}\right)U_{x_2}+\lambda_{2,\,0}\left[U_{x_2}-U_{x_2x_2}\left(d-x_2\right)\right]=\left(1+\xi_{1,\,0}+\xi_{2,\,0}\right)Z_{g_2}, \tag{2.51}$$

for date 2. Similarly, from the government problem at date 1 we get

$$\left(1+\xi_{2,\,1}\right)U_{c_2}+\lambda_{2,\,1}\left[U_{c_2}+U_{c_2c_2}\left(c_2-{}_2b_2\right)\right]+\lambda_{1,\,1}U_{c_2c_2}\left({}_2b_2-{}_1b_2\right) \tag{2.52}$$

$$=\left(1+\xi_{2,\,1}\right)Z_{g_2},$$

$$\left(1+\xi_{2,\,1}\right)U_{x_2}+\lambda_{2,\,1}\left[U_{x_2}-U_{x_2x_2}\left(d-x_2\right)\right]=\left(1+\xi_{2,\,1}\right)Z_{g_2}. \tag{2.53}$$

To solve conditions (2.51) and (2.53) for the same allocation, it must hold that

$$\lambda_{2,\,1}=\left[\frac{1+\xi_{2,\,1}}{1+\xi_{1,\,0}+\xi_{2,\,0}}\right]\lambda_{2,\,0}. \tag{2.54}$$

Taking into account equation (2.54) and ${}_2b_2={}_1b_2$, if the same allocation solves conditions (2.50) and (2.52), then ${}_1b_2={}_0b_2$ holds. Moreover, considering the government problems at the initial date t and at the initial date $t+1$, the corresponding first-order conditions from these problems for date $t+2$ imply ${}_{t+1}b_{t+2}={}_tb_{t+2}$. Iterating this process, it is easy to see that the solution of the government problem (2.21) satisfies ${}_1b_s={}_0b_s$ for all maturities $s=1,2,...,N$. Hence, for an arbitrary date t, the pair (σ^m,f^m) satisfies ${}_{t+1}b_s={}_tb_s$ for $s=t+1,...,t+N$. ■

Proof of Proposition 2

Suppose first that a given policy plan and an allocation rule pair is sustainable. Then, it should form a competitive equilibrium at date 0. Moreover, a reversion to the Markov equilibrium is always feasible. Hence, the welfare value of this pair must be at least as large as the welfare value after deviation.

Next, suppose that a policy plan and an allocation rule satisfy statement (i) and (ii) of Proposition 2. We must show that this pair is a sustainable equilibrium. First, this pair is optimal since it is a competitive equilibrium at date 0 by condition (i). Second, no deviation will be desirable since, by condition (ii), it provides a greater welfare than any deviation. Thus, they form a sustainable equilibrium. ■

Proof of Proposition 3

Lemma 2 characterizes the economy after a deviation. Introducing ${}_{t+1}b_s = {}_tb_s$ for maturities $s = t+1, ..., t+N$ into equation (2.10), we obtain

$$U_{c_s^D}\left(c_s^D - {}_sb_s\right) - U_{x_s^D}\left(d - x_s^D\right) = 0, \tag{2.55}$$

where ${}_sb_s = {}_tb_s$ for all maturities $s = t, ..., t+N-1$, and zero otherwise. After a deviation at date t, the government chooses the sequence $\left\{c_s^D, x_s^D, g_s^D\right\}_{s=t}^{\infty}$ that maximizes welfare (2.24) subject to constraints (2.1) and (2.55) given the initial debt ${}_rb_s = {}_tb_s$ for all maturities $s = t, ..., t+N-1$ and dates $r \geq t$. The first-order conditions can be summarized as follows:

$$\frac{\theta}{c_t^D}\left[1 + \lambda_t^D\left(\frac{{}_tb_t}{c_t^D}\right)\right] = \frac{\gamma}{g_t^D}, \tag{2.56}$$

$$\frac{(1-\theta)}{x_t^D}\left[1 + \lambda_t^D\left(\frac{d}{x_t^D}\right)\right] = \frac{\gamma}{g_t^D}, \tag{2.57}$$

where ${}_tb_s = {}_rb_s$ for dates $r \geq t$ with maturity $s = t, ..., t+N-1$, which clearly implies a transition of N periods. Let us assume that ${}_tb_t \geq 0$ for some date t. Considering the budget constraint (2.55), it can be easily shown that $x_t^D \geq x_{ss}^D$. From equation (2.56), we get $\left(c_t^D / g_t^D\right) \geq \left(c_{ss}^D / g_{ss}^D\right)$. The resource constraint (2.1) implies $g_t^D \leq g_{ss}^D$. Finally, if ${}_tb_t \leq 0$ for some date t, the previous inequalities are reversed. ■

Proof of Proposition 4

Taking the derivative of the deviation value (2.24) with respect to ${}_tb_s$, which is exogenous for the deviation policy, we get

$$\frac{\partial V_t^D}{\partial\ {}_tb_s} = \beta^{s-t}\left[\frac{\theta}{c_s^D}\left(\frac{\partial c_s^D}{\partial\ {}_tb_s}\right) + \frac{(1-\theta)}{x_s^D}\left(\frac{\partial x_s^D}{\partial\ {}_tb_s}\right) + \frac{\gamma}{g_s^D}\left(\frac{\partial g_s^D}{\partial\ {}_tb_s}\right)\right]. \tag{2.58}$$

Equations (2.56), (2.57), (2.1) and (2.55) describe the deviation policy and, by implicit derivation, we obtain

$$-\frac{\theta}{(c_s^D)^2}\left[1 + 2\lambda_s^D \frac{{}_tb_s}{c_s^D}\right]\left(\frac{\partial c_s^D}{\partial\ {}_tb_s}\right) + \frac{\gamma}{(g_s^D)^2}\left(\frac{\partial g_s^D}{\partial\ {}_tb_s}\right) + \frac{\theta\ {}_tb_s}{(c_s^D)^2}\left(\frac{\partial \lambda_s^D}{\partial\ {}_tb_s}\right) + \frac{\theta\lambda_s^D}{(c_s^D)^2} = 0, \tag{2.59}$$

$$-\frac{(1-\theta)}{(x_s^D)^2}\left[1 + 2\lambda_s^D \frac{d}{x_s^D}\right]\left(\frac{\partial x_s^D}{\partial\ {}_tb_s}\right) + \frac{\gamma}{(g_s^D)^2}\left(\frac{\partial g_s^D}{\partial\ {}_tb_s}\right) + \frac{(1-\theta)\, d}{(x_s^D)^2}\left(\frac{\partial \lambda_s^D}{\partial\ {}_tb_s}\right) = 0, \tag{2.60}$$

$$\frac{\partial c_s^D}{\partial\ {}_tb_s} + \frac{\partial x_s^D}{\partial\ {}_tb_s} + \frac{\partial g_s^D}{\partial\ {}_tb_s} = 0, \tag{2.61}$$

$$\frac{\theta_t b_s}{(c_s^D)^2}\left(\frac{\partial c_s^D}{\partial\ {}_tb_s}\right) + \frac{(1-\theta)\, d}{(x_s^D)^2}\left(\frac{\partial x_s^D}{\partial\ {}_tb_s^c}\right) - \frac{\theta}{c_s^D} = 0. \tag{2.62}$$

Solving for $\dfrac{\partial g_s^D}{\partial\ {}_tb_s}$ and $\dfrac{\partial c_s^D}{\partial\ {}_tb_s}$ in equations (2.61) and (2.62), the derivative (2.58) can be written as

$$\frac{\partial V_t^D}{\partial\ {}_tb_s} = \beta^{s-t}\left[\left[\frac{\theta}{c_s^D} - \frac{\gamma}{g_s^D}\right]\frac{c_s^D}{{}_tb_s} + \left[\frac{(1-\theta)}{x_s^D} - \frac{\gamma}{g_s^D} - \left(\frac{(1-\theta)}{(x_s^D)^2}\right)\left(\frac{\left(c_s^D\right)^2}{\theta\ {}_tb_s}\right)\left[\frac{\theta}{c_s^D} - \frac{\gamma}{g_s^D}\right]\right]\left(\frac{\partial x_s^D}{\partial\ {}_tb_s}\right)\right],$$

which, using conditions (2.56) and (2.57), becomes

$$\frac{\partial V_t^D}{\partial \ _t b_s} = -\beta^{s-t}\left(\frac{\theta \lambda_s^D}{c_s^D}\right), \tag{2.63}$$

which is clearly negative.

We will next show that the maximum deviation value increases in the maturity of debt. Let us consider the optimization problem once the government deviates at an arbitrary date t and allow for the selection of the debt structure that maximizes the deviation value.[18] Observe that this inherited debt structure satisfies the government budget constraint for the policy before deviation at date $t-1$.[19] Then, we find the sequences $\{c_s^D, x_s^D, g_s^D\}_{s=t}^{\infty}$ and $\{_t b_s\}_{s=t}^{t+M-1}$ that maximize the welfare (2.24) subject to the resource constraint (2.1) at date $s \geq t$, the budget constraint (2.55) at date $s \geq t$, and the constraint on the initial composition of debt,

$$-K + \sum_{s=t}^{t+M-1} \beta^{s-t+1} A_s \ _t b_s = 0, \tag{2.64}$$

with

$$K = U_{c_{t-1}}\left(c_{t-1} - \ _{t-1}b_{t-1}\right) - U_{x_{t-1}}\left(d - \ x_{t-1}\right) - \sum_{s=t}^{t+M-1} \beta^{s-t+1} A_s \ _{t-1}b_s,$$

where $A_s = U_{c_s}$. Equation (2.64) is just the budget constraint (2.10) at date $t-1$ for the policy before deviation. The first-order conditions for this problem are

$$\frac{\theta}{c_t^D}\left[1 + \lambda_t^D\left(\frac{_t b_t}{c_t^D}\right)\right] = \frac{\gamma}{g_t^D}, \tag{2.65}$$

[18] Here we analyze the policy after deviation. Hence, the policy announcements of the policy before deviation are exogenous. For the purpose of finding the composition of debt that maximizes the deviation value, we select the inherited debt structure.

[19] Note also that, for the policy after a deviation at date t, the government budget constraint at date $t-1$ is not longer satisfied.

$$\frac{(1-\theta)}{x_t^D}\left[1+\lambda_t^D\left(\frac{d}{x_t^D}\right)\right]=\frac{\gamma}{g_t^D}, \tag{2.66}$$

$$\lambda_t^D\frac{\theta}{c_t^D}=\vartheta^D, \tag{2.67}$$

where ϑ^D is the Lagrange multiplier for constraint (2.64). Constraints (2.1), (2.55), (2.64) and conditions (2.65) – (2.67) yield the policy, the allocation, and the initial composition of debt that maximize the welfare after deviation. Given that conditions (2.65) – (2.67) specify an identical pattern for each date, the debt structure that maximizes the deviation value (2.24) at date t is ${}_tb_s=\bar{b}$ for all maturities $s=t,...,t+M-1$. Thus, a composition of debt ${}_tb_s=\bar{b}'$ for $s=t,...,t+N-1$ provides a smaller deviation value. Hence, if $M\geq N$, then $V^{*D}(M)\geq V^{*D}(N)$. ■

Proof of Proposition 5

We proceed by contradiction. Let us assume $\xi_t\neq 0$. After a deviation, constraints (2.1) and (2.10) become

$$c^D+x^D+g^D=d, \tag{2.68}$$

$$1-\theta\left(\frac{b}{c^D}\right)-(1-\theta)\left(\frac{d}{x^D}\right)=0, \tag{2.69}$$

respectively. Combining conditions (2.65) and (2.66), we obtain

$$\left(\frac{d}{x^D}\right)\left(\frac{1-\theta}{x^D}\right)\left[\left(\frac{\gamma}{g^D}\right)-\left(\frac{\theta}{c^D}\right)\right]=\left(\frac{b}{c^D}\right)\left(\frac{\theta}{c^D}\right)\left[\left(\frac{\gamma}{g^D}\right)-\left(\frac{1-\theta}{x^D}\right)\right]. \tag{2.70}$$

For the best sustainable policy, the equations equivalent to (2.68) – (2.70) are

$$c + x + g = d, \tag{2.71}$$

$$1 - \theta\left(\frac{b}{c}\right) - (1-\theta)\left(\frac{d}{x}\right) + \beta\theta\left(\frac{b}{c}\right) = 0, \tag{2.72}$$

$$\left(\frac{d}{x}\right)\left(\frac{1-\theta}{x}\right)\left[\left(\frac{\gamma}{g}\right) - \left(\frac{\theta}{c}\right)\right] = \left(\frac{b}{c}\right)\left(\frac{\theta}{c}\right)\left[\left(\frac{\gamma}{g}\right) - \left(\frac{1-\theta}{x}\right)\right]. \tag{2.73}$$

Moreover, combining conditions (2.30) and (2.32) , we get

$$\frac{\theta}{c_t}\left[1 + \Gamma_t^1 + \xi_t + \lambda_t\left(\frac{{}_t b_t}{c_t}\right)\right] = \frac{\gamma}{g_t}\left[1 + \Gamma_t^1 + \xi_t\right]. \tag{2.74}$$

Solving for ξ_t, we obtain

$$\xi_t = (\lambda_t - \lambda_{t-1})\left(\frac{\theta}{c}\right)\left[\frac{\left(\frac{b}{c}\right)}{\left(\frac{\gamma}{g}\right) - \left(\frac{\theta}{c}\right)}\right], \tag{2.75}$$

at the steady state. Next, introducing equation (2.63) into (2.33), we can write

$$\xi_t = (\lambda_t - \lambda_{t-1})\left(\frac{\theta}{c}\right)\left[\frac{\left(\frac{b}{c^D}\right)}{\left(\frac{\gamma}{g^D}\right) - \left(\frac{\theta}{c^D}\right)}\right]. \tag{2.76}$$

Using equations (2.75) and (2.76), $(c/g) = (c^D/g^D)$ must hold. Moreover, conditions (2.70) and (2.73) imply $(x/g) = (x^D/g^D)$. Combining now constraints (2.68) and (2.71), we get $c = c^D$, $x = x^D$ and $g = g^D$. However, in that case, equations (2.69) and (2.72) cannot be satisfied. Hence, $\xi_t = 0$ at the steady state.

Since $\xi_{ss} = 0$ and $\dfrac{\partial V_{t+1}^D}{\partial\ {}_{t+1}b_{t+1}} < 0$, condition (2.33) implies $\lambda_j \leq \lambda_{ss}$. From equation (2.30), if ${}_j b_j \geq 0$, then $(c_j/g_j) \geq (c_{ss}/g_{ss})$. Finally, if ${}_s b_j \leq 0$, then this inequality is reversed. ■

Proof of Proposition 6

For some available maturity of debt N, the welfare value of continuing with the full-commitment is greater than the maximum deviation value. Proposition 4 shows that the maximum deviation value increases in the maturity of debt. Therefore, for all maturities $P \in \{1, 2, ...N\}$, the full-commitment policy provides a continuation value that is also larger than the maximum deviation value. Thus, the incentive compatibility constraints (2.25) are not binding. Hence, the full-commitment is time-consistent for all possible debt structures.

We will next consider a debt structure with an unlimited maturity and show that $V_{FC}^{C} < V^{*D}(\infty)$. Notice that, for the full-commitment to be sustainable under all possible debt structures, the full-commitment must provide a larger continuation value than the maximum deviation value for $N = \infty$. Let us characterize the value of continuation with the full-commitment. The government problem at the initial date 0 specifies a plan for date 1 on, which is characterized by the following conditions:

$$\frac{\theta}{c_t} = \frac{\gamma}{g_t}, \tag{2.77}$$

$$\frac{(1-\theta)}{x_t}\left[1 + \lambda_0\left(\frac{d}{x_t}\right)\right] = \frac{\gamma}{g_t}. \tag{2.78}$$

Equations (2.77) and (2.78) determine the value of continuing with the full-commitment policy V_{FC}^{C}. Consider now the optimal choice at the initial date 1 if reputation costs were absent. If that were the case, the optimal response would be

$$\frac{\theta}{c_t}\left[1 + \lambda_1\left(\frac{{}_1b_t}{c_t}\right)\right] = \frac{\gamma}{g_t}, \tag{2.79}$$

$$\frac{(1-\theta)}{x_t}\left[1 + \lambda_1\left(\frac{d}{x_t}\right)\right] = \frac{\gamma}{g_t}. \tag{2.80}$$

Notice that, since reputation threats are absent, the system $(2.79)-(2.80)$ must provide a greater welfare than the one implied by equations $(2.77)-(2.78)$. Next, the maximum deviation value $V^{*D}(\infty)$ at date 1 is described by

$$\frac{\theta}{c_t^D}\left[1+\lambda_t^D\left(\frac{{}_1\bar{b}}{c_t^D}\right)\right]=\frac{\gamma}{g_t^D}, \tag{2.81}$$

$$\frac{(1-\theta)}{x_t^D}\left[1+\lambda_t^D\left(\frac{d}{x_t^D}\right)\right]=\frac{\gamma}{g_t^D}. \tag{2.82}$$

Given that the deviation policy follows the same pattern at each date, all variables are constant and, in particular, $\lambda_t^D=\bar{\lambda}$ for all dates t. Observe now that the systems $(2.79)-(2.80)$ and $(2.81)-(2.82)$ just describe the same allocation for ${}_1b_t={}_1\bar{b}$. Hence, $V_{FC}^C<V^{*D}(\infty)$. Thus, we have shown that $V^{*D}(N)$ is strictly increasing in N and that, under $N=\infty$, the maximum deviation value exceeds the value of continuing with the full-commitment policy. Then, there exits a finite maturity $\widehat{N}$ strictly greater than N such that for all maturities $M\in\left\{\widehat{N},\widehat{N}+1,...,\infty\right\}$, we obtain $V_{FC}^C<V^{*D}(M)$. Therefore, for all maturities M, there is at least a debt structure that makes the full-commitment policy time-inconsistent. ■

Proof of Corollary 1

We know from Proposition 1 that, if ${}_0b_0=0$, the economy under full-commitment is always at the steady state. Moreover, the government runs a balanced budget. From the proof of Proposition 4, the composition of debt that maximizes the deviation value under a balanced budget is clearly ${}_tb_s=\bar{b}=0$ for all maturities s, which gives rise to the same allocation and policy than under full-commitment. Therefore, $V_{FC}^C\geq V^D(N)$ under all possible debt structures. Hence, the full-commitment policy is always time-consistent. ■

2.8 Numerical Solution Method

In this appendix we describe the numerical solution method. Under full-commitment, a numerical solution can be obtained without the help of any sophisticated method. Notice that, since all initial debt matures at zero, the model displays one-period transition. Then, the solution is determined by equations (2.1), (2.34), and (2.35) evaluated both at date 0 and at the steady state, together with the implementability constraint

$$U_{c_0}(c_0 - \ _0b_0) - U_{x_0}(d - \ x_0) + \frac{\beta}{1-\beta}\left[U_{c_{ss}}(c_{ss}) - U_{x_{ss}}(d - \ x_{ss})\right] = 0,$$

which results from adding constraint (2.10) over time t and imposing the transversality condition (2.11). The model has 7 equations and the same number of unknowns and, hence, any non-linear library or optimization algorithm can solve that system without any difficulty. Without commitment, a numerical solution becomes rather complicated when the incentive compatibility constraints (2.25) are binding at some date t.[20] The solution is characterized by constraints (2.1), (2.10), (2.25), conditions (2.30) – (2.33), and the transversality condition (2.11). Observe that equations (2.10), (2.11), (2.25), and (2.33) depend on future variables. For the purpose of solving the system, the conditions that depend on future variables must be replaced by functions that only depend on variables at the past and the current date.

We use the parametrized expectations algorithm by Den Haan and Marcet (1990), which replaces equations of future variables by a function of the state and co-state variables. This method is suitable for finding the steady state of the model. As suggested by Marshall (1992), we introduce time in the function so as to approximate the transition. Let us

[20]If the incentive compatibility constraint (2.25) never binds, then the best sustainable and the full-commitment policy coincide.

consider the economy with one-period debt. In this case, the model can be written so that the only equation to approximate is given by the sum to infinity included in the incentive compatibility constraint (2.25),

$$\sum_{s=t+1}^{\infty} \beta^{s-t} \left[U\left(c_s, x_s\right) + Z\left(g_s\right) \right]. \tag{2.83}$$

The following steps describe how the model is solved:

(i) We choose some reasonable parameter and initial values.

(ii) We propose a parametrized form to substitute the term (2.83).[21] In particular, we choose

$$\Psi\left({}_t b_t, \xi_{t-1}, \lambda_{t-1}, \ln\left(\frac{h}{t}\right); \delta\right) = \exp\left(\delta_1 + \delta_{2t} b_t + \delta_3 \xi_{t-1} + \delta_4 \lambda_{t-1} + \delta_5 \ln\left(\frac{h}{t}\right)\right),$$

where h is taken to be 100 and t is a time index that goes from 1 to 100. Thus, after period 100, $\ln\left(\frac{h}{t}\right)$ becomes negligible. The form Ψ suits our purpose since it gives a positive image and a transition. Logarithmic functions are not considered because ξ_{t-1} may take values that are very close to zero.

(iii) We find the vector δ that makes Ψ the best predictor of (2.83). To this end, we solve

$$S(\delta) = \arg\min_{\delta} \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \left[U\left(c_s(\delta), x_s(\delta)\right) + Z\left(g_s(\delta)\right) \right] - \Psi\left({}_t b_t, \xi_{t-1}, \lambda_{t-1}, \ln\left(\frac{h}{t}\right); \delta\right) \right].$$

First, we substitute the term (2.83) by the function Ψ. Then, an initial value for coefficient δ is needed.[22] We obtain series for all the variables.[23] For this series, we compute the

[21] We introduce ξ_{t-1} as a co-state variable instead of its aggregation on time Γ_t^1 because the former provides a more accurate solution.

[22] An accurate performance of this method needs good initial values. To this end, we first solve the model but restricted to a finite transition. Then, we estimate initial coefficients with the time-series that we have obtained.

[23] In order to satisfy the transversality condition (2.11), time-series are computed as follows. First, we

term (2.83) and perform a non-linear regression of (2.83) on Ψ. This regression yields an estimated coefficient $S(\delta)$ with which a new coefficient δ^1 is obtained by following the next scheme:

$$\delta^1 = (1-\omega)\,\delta^0 + \omega S(\delta)\,,$$

where $\omega \in (0,1]$ is the speed of convergence. With this coefficient δ^1, we compute a new series and run another regression until we find a fix point of S.

For the economy with two-period debt, we need to approximate two expectations (2.83) and c_{t+1} in (2.33). In this case, we choose

$$\Psi_1 = \exp\left(\delta_{11} + \delta_{12t}b_t + \delta_{13t}b_{t+1} + \delta_{14}\xi_{t-1} + \delta_{15}\lambda_{t-1} + \delta_{16}\lambda_{t-2} + \delta_{17}\ln\left(\frac{h}{t}\right)\right),$$

and

$$\Psi_2 = \exp\left(\delta_{21} + \delta_{22t}b_t + \delta_{23}\xi_{t-1}\right),$$

respectively. Then, we iterate the process as explained above.

guess the multiplier λ_0 for the budget constraint (2.10) at date 0. Next, we obtain the time-series. Then, we check whether λ_0 satisfies the implementability constraint that results from adding the constraint (2.10) over time and imposing the transversality condition (2.11). Finally, we search for the value of λ_0 that satisfies the implementability constraint.

2.9 Figures and Tables

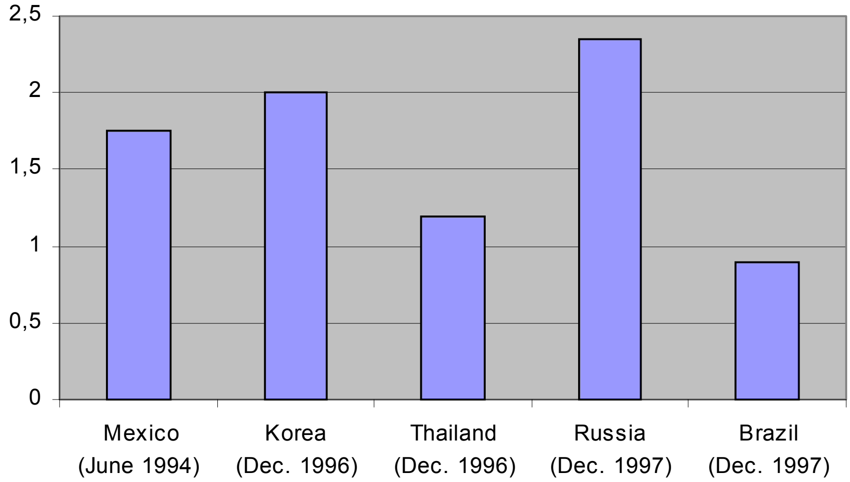

FIGURE 1. Ratio of Short-Term Debt to Reserves

Sources: Bank for International Settlements; and IMF, International Financial Statistics, various issues.

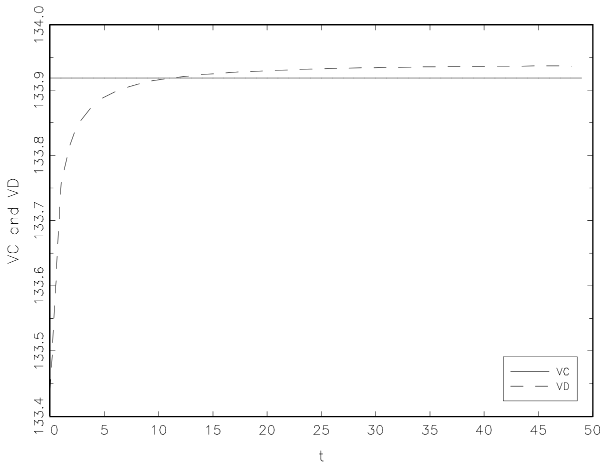

FIGURE 2. Continuation and Deviation Value

TABLE 1. Changes in Parameter and Initial Values

Parameter Change	$\mathbf{V}^{C}_{FC}-\mathbf{V}^{*D}$	$\hat{\mathbf{N}}$	$\left(\frac{g_{ss}}{y_{ss}}\right)$	$\left(\frac{l_{ss}}{y_{ss}}\right)$
$-$	-0.00158209	13	0.750	0.323
$\boldsymbol{\beta} = \mathbf{0.975}$	-0.00045825	27	0.750	0.328
$\boldsymbol{\beta} = \mathbf{0.925}$	-0.00171404	8	0.750	0.318
$\boldsymbol{\theta} = \mathbf{0.35}$	-0.00020039	14	0.740	0.340
$\boldsymbol{\theta} = \mathbf{0.31}$	-0.00019815	10	0.763	0.300
$\boldsymbol{\gamma} = \mathbf{2}$	-0.10025427	2	0.857	0.322
$\boldsymbol{\gamma} = \mathbf{0.25}$	$-1.5499E-06$	55	0.428	0.326
$\mathbf{d} = \mathbf{105}$	-0.00139905	13	0.750	0.324
$\mathbf{d} = \mathbf{95}$	$-3.7690E-06$	55	0.428	0.326
${}_0\mathbf{b}_0 = \mathbf{25}$	-0.00566113	12	0.750	0.315
${}_0\mathbf{b}_0 = \mathbf{5}$	-0.00021513	13	0.750	0.327

$\hat{N}$ stands for the critical maturity length below which $V^{C}_{FC} \geq V^{*D}$.

Benchmark parameter and initial values are the following: $\beta = 0.95$, $\theta = 1/3$, $\gamma = 1$, $d = 100$, and ${}_0b_0 = 10$.

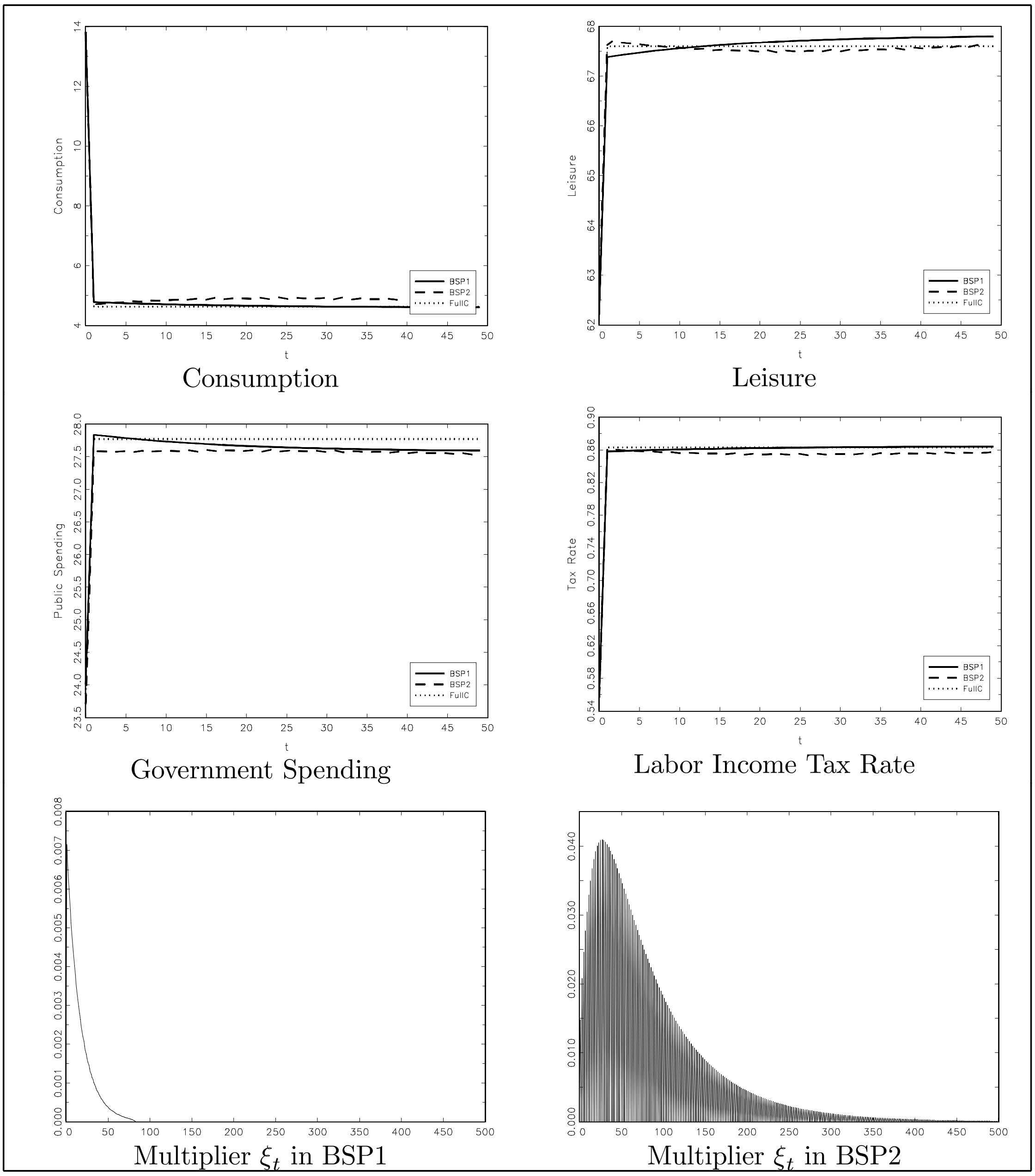

FIGURE 3. For Economy A, the Dynamics of Consumption, Leisure, Public Spending, Labor Income Tax Rate, and the Lagrange Multiplier of the Incentive Compatibility Constraint (2.25) under Full-Commitment (FullC), under the Best Sustainable Policy with One-Period Debt (BSP1) and with One and Two-Period Debt (BSP2)

TABLE 2. Results of the Numerical Solution of the Economy A

	FULLC	**BSP1**	**BSP2**
$\mathbf{c}_{ss}$	4.628	4.596	4.534
$\mathbf{l}_{ss}$	67.601	67.821	68.258
$\mathbf{g}_{ss}$	27.770	27.581	27.207
$\boldsymbol{\tau}_{ss}$	0.863	0.864	0.867
$\mathbf{b}_{ss}$	3.840	4.696	3.253 3.253
Welfare	199.361	199.360	199.354
MaxE	–	0.03297	0.602 0.176
MeanE	–	0.032	0.476 0.051
MSE	–	0.001	0.248 0.009
DA	–	3	2 (*but* 3 *in* # 1, 2 *and* 7; *but* 1 *in* # 4) 1 (*but* 0 *in* # 3)

FULLC stands for the policy under full-commitment.

BSP1 stands for the best sustainable policy with one-period debt.

BSP2 stands for the best sustainable policy with one and two-period debt.

MaxE stands for the maximum error or difference between the equation to parametrize and the parametrized form at any t. Analogously, MeanE and MSE stand for the mean error and the mean square error, respectively.

DA stands for digits of accuracy. Thus, the number of digits for which the final coefficient vector δ and $S(\delta)$ coincide. For instance, $2\,(\textit{but } 1 \textit{ in } \# \, 4)$ says that there are 2 DA in all estimated coefficients but 1 DA in coefficient number 4.

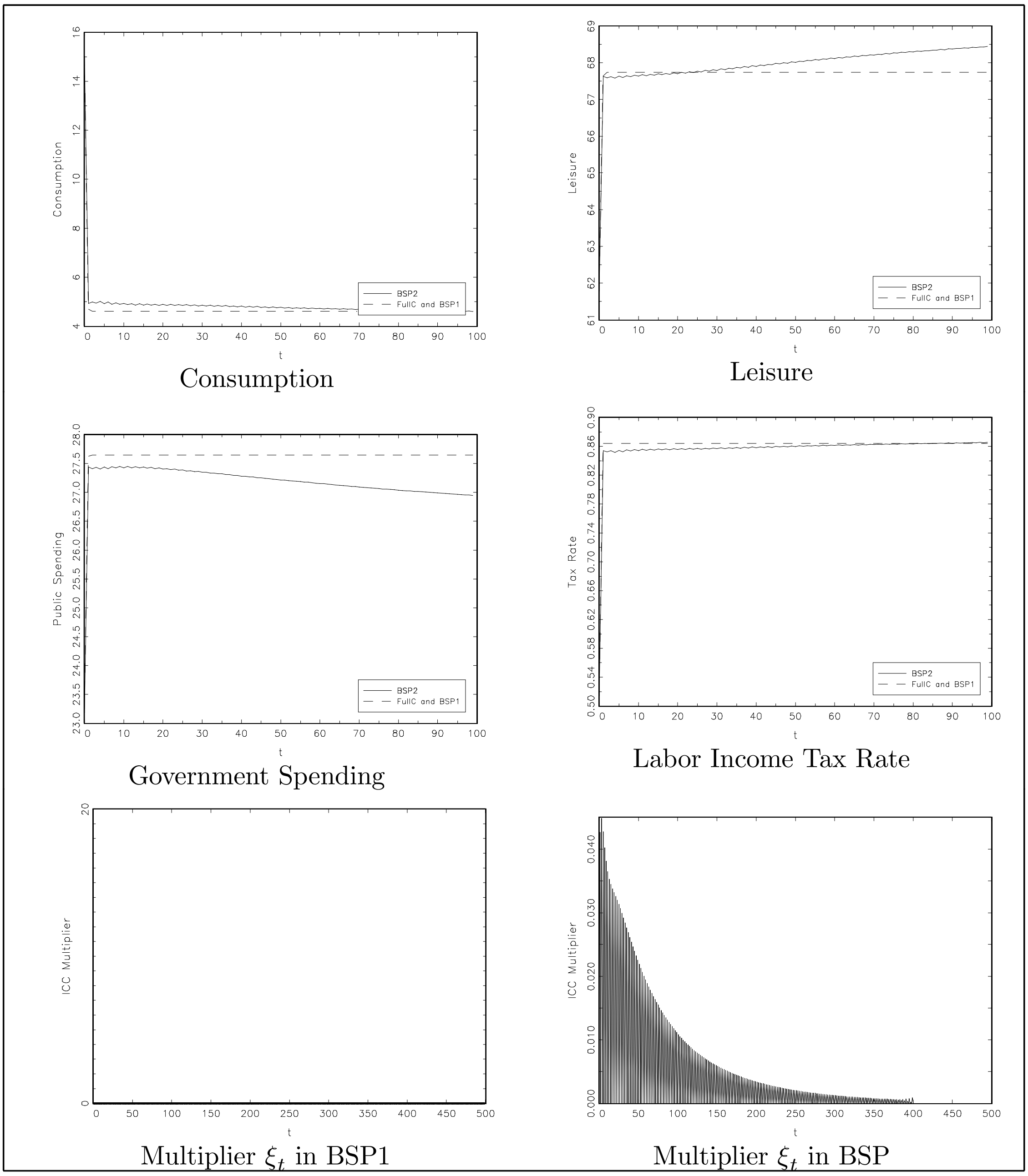

FIGURE 4. For Economy B, the Dynamics of Consumption, Leisure, Public Spending, Labor Income Tax Rate, and the Lagrange Multiplier of the Incentive Compatibility Constraint (2.25) under Full-Commitment (FullC), under the Best Sustainable Policy with One-Period Debt (BSP1) and with One and Two-Period Debt (BSP2)

TABLE 3. Results for the Numerical Solution of the Economy B

	FULLC	**BSP1**	**BSP2**
$\mathbf{c}_{ss}$	4.608	4.608	4.436
$\mathbf{l}_{ss}$	67.743	67.743	68.944
$\mathbf{g}_{ss}$	27.648	27.648	26.618
$\boldsymbol{\tau}_{ss}$	0.863	0.863	0.871
$\mathbf{b}_{ss}$	4.393	4.393	4.510 4.510
Welfare	199.164	199.164	199.149
MaxE	–	–	0.449 0.122
MeanE	–	–	0.271 −0.007
MSE	–	–	0.089 0.003
DA	–	–	2 (*but* 3 *in* # 1 *and* 6; *but* 1 *in* # 4) 1 (*but* 0 *in* # 3)

FULLC stands for the policy under full-commitment.

BSP1 stands for the best sustainable policy with one-period debt.

BSP2 stands for the best sustainable policy with one and two-period debt.

MaxE stands for the maximum error or difference between the equations to parametrize and the parametrized form at any t. Analogously, MeanE and MSE stand for the mean error and the mean square error, respectively.

DA stands for digits of accuracy. Thus, the number of digits for which the final coefficient vector δ and $S(\delta)$ coincide. For instance, $2\,(but\ 1\ in\ \#\ 4)$ says that there are 2 DA in all estimated coefficients but 1 DA in coefficient number 4.