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**DISSIPATIVITY AND PASSIVITY-RELATED PROPERTIES  
IN NONLINEAR DISCRETE-TIME SYSTEMS**

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Instituto de Organización y Control de Sistemas Industriales

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**A mis padres**

Por todo



# El apoyo, la confianza, la ayuda, el amor

Vicenta López del Olmo, Francisco Navarro Alcantud, Domingo de Jesús Cortés Rodríguez.

Enric Fossas i Colet, Dorinda Fernández Marquina, Juanan Pérez Ortiz, Araceli González García, Federica Boiani, Jorge Martín Espada, Eni Pradas Caulín, Joaquín Álvarez Gallegos, Hebertt Sira Ramírez, Jaime Álvarez Gallegos, Salvador Ros Torrecillas, Paquito Navarro López, Araceli Cortés Rodríguez, José Yépez, Leira Márquez, Josep Maria Mirats i Tur, Isa Martínez Moya, Georgina Domínguez Machaen, Montse Bruguera, Josefa Ruíz Candelas, Mercedes Vía, Isidoro Hernán Losada, José Antonio Gálvez.

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# The support, the trust, the help, the love

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# Resumen

El propósito de la presente tesis es el estudio de la disipatividad en sistemas no lineales discretos. Dicho trabajo de investigación presenta nuevas contribuciones en la teoría de control no lineal discreto basado en disipatividad y en el estudio de las propiedades de sistemas disipativos no lineales. Los resultados conseguidos se dividen en tres objetivos principales:

1. **La caracterización de sistemas disipativos múltiple entrada múltiple salida no lineales discretos de estructura general**, lo que también se conoce como condiciones de Kalman-Yakubovich-Popov (KYP). Las condiciones de KYP ya existentes se extienden a una clase de sistemas disipativos discretos no lineales múltiple entrada múltiple salida que son no afines en el control. La clase de sistemas disipativos estudiada se denomina disipatividad QSS, es decir, sistemas disipativos cuyas funciones de almacenamiento y suministro de energía,  $V$  y  $s$ , satisfacen que  $V(f(x,u))$  y  $s(h(x,u),u)$  son cuadráticas en  $u$ . También se proporcionan condiciones necesarias y suficientes para la caracterización de sistemas conservativos QSS discretos no afines en el control.
2. **El problema de disipatividad por realimentación en sistemas no lineales discretos**. Se proponen dos formas de abordar dicho problema:
  - (a) **El problema de la disipatividad por realimentación a través de la relación fundamental de la disipatividad**. Se da solución al problema de la disipatividad por realimentación para sistemas única entrada única salida discretos no lineales no afines en el control, mediante cuatro metodologías basadas en la igualdad fundamental de la disipatividad. Se proponen condiciones suficientes bajo las cuales la disipatividad por realimentación es posible.
  - (b) **El problema de pasivización mediante las propiedades del grado relativo y la dinámica cero del sistema no pasivo original**. El problema de transformación de un sistema no pasivo a uno que lo es se resuelve mediante realimentación de estado para una clase de sistemas múltiple entrada múltiple salida no lineales discretos afines en el control, usando las propiedades del grado relativo y la dinámica cero del sistema no pasivo original. Se puede considerar como una extensión al caso pasivo de los resultados ya existentes, referentes al problema de transformar un sistema que no es conservativo a uno que lo es mediante realimentación de estado.
3. **El problema de estabilización basado en disipatividad en sistemas no lineales discretos**. El método de *Moldeo de Energía e Inyección de Amortiguamiento* se extiende a sistemas generales no lineales discretos única entrada única salida, además de analizar algunas de las propiedades de estabilidad de una clase de sistemas disipativos y de sistemas que se pueden transformar a disipativos por realimentación. También, se establecen condiciones suficientes bajo las cuales dichos sistemas son estabilizables.

Otros objetivos secundarios han sido alcanzados, como son: el estudio del grado relativo y la dinámica cero de sistemas pasivos no lineales discretos, algunas conclusiones acerca de la conservación de la pasividad bajo la interconexión por retroalimentación negativa y la interconexión paralela, algunas notas acerca de la conservación y pérdida de la disipatividad y pasividad con el muestreo, además, las propiedades en el dominio de la frecuencia de los sistemas disipativos se usan y se relacionan con algunos de los criterios de estabilidad más importantes basados en la respuesta en frecuencia. También, los métodos de control basados en disipatividad diseñados se aplican al problema de regulación de un modelo discreto con interpretación física: un convertidor buck, para el que se mejora la respuesta en lazo abierto.

El hecho de haber tratado sistemas discretos generales nos ha permitido dar una serie de resultados para sistemas no lineales continuos no afines en el control. Dos problemas se han propuesto, principalmente: el estudio de la disipatividad por realimentación para sistemas no lineales no afines única entrada única salida y el uso de los resultados de disipatividad por realimentación. Estos resultados se han utilizado para extender al caso no lineal no afín única entrada única salida el método de estabilización de *Moldeo de Energía e Inyección de Amortiguamiento*.

**Palabras clave:** Disipatividad, Pasividad, Sistemas no Lineales Discretos, Disipatividad por Realimentación, Pasivización, Control Basado en Disipatividad, Respuestas Frecuenciales, Estabilización por Realimentación

# Summary

This dissertation is devoted to dissipativity-related concepts in the nonlinear discrete-time setting, and presents several new contributions which are not covered by the existing nonlinear discrete-time dissipativity-based control theory and the study of the properties of nonlinear discrete-time dissipative systems.

The study of dissipativity given in this dissertation is concentrated in the state-space or internal description representation of systems. The results achieved are classified into three main goals or problems to solve, such as:

1. **The characterization of dissipative multiple-input multiple-output nonlinear discrete-time systems of general form**, what is regarded as Kalman-Yakubovich-Popov (KYP) conditions. The KYP conditions existing in the literature are extended to a class of nonlinear multiple-input multiple-output dissipative discrete-time systems which are non-affine in the control input. The class of dissipativity characterized is regarded as *QSS*-dissipativity, that is, dissipative systems whose storage and supply functions,  $V$  and  $s$ , satisfy that  $V(f(x, u))$  and  $s(h(x, u), u)$  are quadratic in  $u$ . Necessary and sufficient conditions for the characterization of *QSS*-lossless discrete-time systems which are non-affine in the control input are also given.
2. **The feedback dissipativity problem in the nonlinear discrete-time setting**. Two approaches are proposed to deal with this topic:
  - (a) **The feedback dissipativity problem through the fundamental dissipativity inequality**. The feedback dissipativity problem is solved for single-input single-output nonlinear discrete-time non-affine-in-the-control-input systems by means of four methodologies based on the fundamental dissipativity equality. Sufficient conditions under which feedback dissipativity is possible are proposed. The first approach proposes the control achieving feedback dissipativity as the explicit solution of the dissipativity equality, the second one uses the speed-gradient algorithm in its discrete-time version. The third and fourth methodologies are of approximate type.
  - (b) **The feedback passivity problem through the properties of the relative degree and zero dynamics of the non-passive system**. The problem of rendering a system passive via state feedback is solved for a class of multiple-input multiple-output nonlinear discrete-time systems which are affine in the control input using the properties of the relative degree and the zero dynamics of the non-passive system. It is an extension to the passivity case of the results reported in the literature for the losslessness feedback problem.
3. **The dissipativity-based stabilization problem in nonlinear discrete-time systems**. The dissipativity-based controller design methodology of the *Energy Shaping and Damping Injection* is extended to general nonlinear single-input single-output discrete-time systems, in addition to, the analysis of some stability properties of a class of dissipative and feedback dissipative single-input single-output

nonlinear discrete-time systems. Furthermore, sufficient conditions under which a class of feedback dissipative systems is stabilizable are proposed.

Other secondary goals in the dissipativity properties exploration in discrete-time systems are achieved, mainly: the study of the relative degree and zero dynamics of passive nonlinear discrete-time systems, some conclusions about passivity preservation under feedback and parallel interconnections, some notes on the non-preservation and preservation of dissipativity, and its special case of passivity, under sampling, in addition, dissipativity frequency-domain properties have been used and related to some of the most important frequency-based feedback stability criteria. Furthermore, the feedback dissipativity and dissipativity-based control results are applied to solve the regulation problem in a discrete-time model with physical interpretation: the DC-to-DC buck converter, whose open-loop response is improved by means of the use of some of the stabilization methods proposed.

The fact of treating general discrete-time systems has allowed us to extend some dissipativity-related definitions to the case of continuous-time nonlinear non-affine-in-the-input systems. Two main problems are presented, namely: the study of the feedback dissipativity problem for nonlinear non-affine single-input single-output systems based upon the fundamental dissipativity equality, and the use of the feedback dissipativity results in order to extend the *Energy Shaping and Damping Injection* controller design method to the case of non-affine single-input single-output nonlinear systems.

**Key Words:** Dissipativity, Passivity, Nonlinear Discrete-time Systems, Feedback Dissipativity, Passivation, Dissipativity-based Control, Frequency Responses, Feedback Stabilization

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# List of abbreviations

DBC	Dissipativity-based control .....	9
PBC	Passivity-based control .....	9
KYP	Kalman-Yakubovich-Popov conditions .....	13
SPRD0	Strictly positive-real discrete functions with relative degree zero .....	16
ISP	Input strict passivity .....	17
OSP	Output strict passivity .....	17
VSP	Very strict passivity .....	17
FGS	Finite gain stable .....	17
IDA	Interconnection and damping assignment .....	28
CMAC	Cerebellar model articulation computer neural network .....	30
ESDI	Energy shaping plus damping injection .....	31
QSS	Quadratic storage supply .....	52
ODE	Ordinary differential equation .....	63
PWM	Pulse-width modulation .....	64
DC	Direct current .....	63
SG	Speed-gradient algorithm .....	72
ZIO	Zero-input-output property of supply functions .....	102
LTI	Linear time-invariant .....	129
QS	Quadratic storage .....	132
SISO	Single-input single-output .....	143
MIMO	Multiple-input multiple-output .....	143

