Chapter 1

Introduction

In this chapter, the reasons for the dissipativity and passivity-related properties to be studied in nonlinear discrete-time systems will be described. The new contributions and main results of this dissertation will be briefly presented. Finally, the contents of this work will be outlined.

1.1 Motivation

We are devoted to advance in the study of the behaviour of nonlinear discrete-time systems by means of its energy properties.

If we want to modify the behaviour of some system dynamics, what is usually called "control", it must be approximated by means of a model. The richest approximation of reality is a **nonlinear** model. The representations we are interested in will be time-depending in order to define the evolution of the system as time goes forward. These time-depending representations may be of two kinds: the continuous-time or the discrete-time ones. The latter appears when time is divided in intervals and the system dynamics is only evaluated at the extremes of these intervals.

A great deal of phenomena appears when **discrete-time systems** are treated, some of these phenomena are not present in the continuous-time counterpart. Although the interest of discrete-time dynamics is important by itself, we can also bring up for consideration the discrete-time dynamics obtained from continuous-time ones, in other words, the study of discretized systems.

Most discrete-time systems are sampled-data systems obtained from continuous-time systems by means of a sample-and-hold element. In practice, discrete-time systems appear continually in the control systems field. Computer-controlled systems are sampled-data systems, since computers work in discrete time and therefore, they are discrete-time systems by nature. Thus, the calculations carried out on a digital computer in order to control a system by implementing a control algorithm result in a sampled-data system. For such a system, the input to the system is kept constant over a certain period of time, regarded as the sampling period. This procedure will give rise to transform the properties of the original system. The motivation of treating this kind of systems is of paramount clearness.

Besides, we can claim the possibility of associating a discrete-time system to a continuous-time system by means of the Poincaré map. The field of discrete-time systems offers fascinating ways in the dynamical systems theory progress.

There are a lot of problems broadly treated in the continuous-time case which have not attracted as significant attention in the discrete-time domain, this is the case of the dissipativity and passivity-related properties. The works dedicated to this field in the discrete-time case are considerably less than the ones appearing for the continous-time one.

There are three main problems in the study of nonlinear discrete-time systems:

1. **The discretization of a nonlinear continuous-time system, searching for the best representation**. The characteristics of a discretized system will depend in large measure on the kind of discretization made. The best discrete-time representation of a continuous-time system is the exact discretization, which is not always possible, since the system flow is needed to be obtained. The approximated solutions to the discretization problem are presented as more usual alternatives:
   - Discrete system as the approximation of the system time-response.
   - Discretization methods based upon the approximation of the derivative (multistep methods or single-step methods, as Euler, Heun, Taylor and Runge-Kutta ones).
   - The use of the algebraic approach:
     - The use of series expansion of exponential type *(Monaco and Normand-Cyrot, 1986)* [102], *(Monaco and Normand-Cyrot, 1990)* [108].
     - Consideration of input-output models in addition to Volterra series *(Monaco and Normand-Cyrot, 1986)* [103, 102]), *(Monaco and Normand-Cyrot, 1987)* [104]).

   An interesting solution to the discretization problem is given by the multirate sampling technique *(Grizzle and Kokotovic, 1988)* [47], *(Monaco and Normand-Cyrot, 1988)* [107], *(Monaco and Normand-Cyrot, 1990)* [108], *(Monaco and Normand-Cyrot, 1991)* [109], with its applications *(Chelouah, 1994)* [20], *(Chelouah and Petitot, 1995)* [21], *(Monaco and Normand-Cyrot, 1992)* [110], *(Georgiou et al., 1992)* [43], where the input is sampled in a faster way than the states and the outputs. In addition, it is presented as a solution to the loss of the linearization property under the discretization of the system.

2. **The effect of the discretization on the system properties**. Intuitively, it can be understood that the fact of sampling a continuous-time system will change its initial properties, such as: stability, controllability, observability, relative-degree property, zero dynamics characteristics, dissipativity features, and so on. We could think about what happens in relation to dissipativity and feedback dissipativity properties of a system when this one is discretized. This will not be the goal of our work, on the contrary, discrete models will be studied. Anyway, we may conjecture that if the relative degree and the zero dynamics play an important role on

the characterization of the passivity and the passivation properties for continuous-time systems, then the study of the preservation under sampling of passivity and passivation properties seems to be based upon the study of the changes in the relative degree and the stability properties of the zero dynamics, which, in general, are not preserved under sampling. For a recent study of the relative degree and zero dynamics under sampling see *(Monaco and Normand-Cyrot, 2001)* [114].

3. **The study of intrinsically discrete-time systems**. There are two main points of view in the study of discrete-time systems, mainly:
   - Using an algebraic basis (standing out *(Aranda-Bricaire et al., 1996)* [8], *(Grizzle, 1993)* [48], *(Fliess, 1990)* [33]).
   - Via a geometric approach (for instance, *(Grizzle, 1985)* [46], *(Monaco and Normand-Cyrot, 1984)* [101], *(Jakubczyk and Sontag, 1990)* [60]).

In this dissertation, dissipativity and passivity will be treated from the point of view of the third approach pointed out before. Above these approaches, two ways in treating the study of discrete-time systems can be put forward, that is:

1. Using the existing approaches for the continuous-time case and adapting them for the discrete-time case. This line will be followed in most of this dissertation.
2. Using mathematical tools defined in the discrete domain, in order to take the most of the properties of discrete-time dynamics. Discrete-time systems, and mainly nonlinear ones, have their own characteristics different from continuous-time ones, that is the reason why different analysis tools and approaches from the ones used for the continuous-time case are compelled to use.

We are devoted to treat **discrete-time systems** of the form:

$$x(k+1) = f(x(k),u(k)) \tag{1.1}$$
$$y(k) = h(x(k),u(k)) \tag{1.2}$$

where $f : \mathscr{X} \times \mathscr{U} \to \mathscr{X}$, $h : \mathscr{X} \times \mathscr{U} \to \mathscr{Y}$ are smooth maps, with $\mathscr{X}$ an open subset of $\Re^n$ and $\mathscr{U}$, $\mathscr{Y}$ open subsets of $\Re^m$. This is a general representation for discrete-time systems. Using an affine structure will not necessarily simplify the problem, so it is worthy to consider more general models, in addition, considering the case of a discrete-time system coming from a continuous-time one, we can notice that an exact discretization, generally, does not yield an affine model in the input.

This research merges the worth of deeping in the nonlinear discrete-time control field to the worth of studying systems dynamics in terms of their **energy** characteristics. The study of the behaviour of a system in terms of the energy it can store or dissipate has an extraordinary value, since it gives a rather physical and intuitive interpretation of problems, such as, the system stability properties or the transformation of the system to another one, in such a way that the new system behaves as we want. The study of systems by means of their associated energy is very appropriate in the analysis of mechanical, electromechanical or electrical systems, among others.

This work will be centered on the study of the energy of systems in terms of their **dissipativity** property and its particular case of **passivity**. A dissipative system is such a system which cannot store all the energy that has been given, that is, it dissipates energy in some way. We are interested in the definition of a dissipative system based on the existence of a *storage function* (representing the energy stored by the system), a *supply function* (external energy received by the system) and a *dissipation function* (representing the total energy dissipated by the system in some time interval). The idea of stored energy can be used connected to the system stability, considering the stored energy function as a Lyapunov-like function. In fact, dissipative systems benefit from stability properties. A passive system with an equilibrium point at the origin, and having a differentiable positive definite storage function, which is zero at the origin, is stable in the sense of Lyapunov considering a zero-input to the system. Furthermore, passive systems are *weakly minimum phase* systems *(Byrnes et al., 1991)* [12], since if the system output is rendered zero by means of an adequate feedback, the remaining dynamics or *zero dynamics* is Lyapunov stable.

Besides, dissipativity concepts will make possible to distinguish different parts or components of the systems dynamics. Depending the stored energy of the system along the trajectories of the system decreases, increases or remains, we will speak of the dissipative, non-dissipative or invariant-energy (also called lossless) part of the system, respectively.

Dissipative and passive systems are concluded to enjoy highly desirable properties, namely, the ones referring to stability and representation properties which may simplify the system analysis and control design. Hence, we are motivated to handle with dissipative systems, and if they are not dissipative trying to convert them into dissipative ones. A way which comes to us in order to make this transformation is a state feedback. For control purposes, we are accustomed to making transformations in the system structure, for example, the linearization by state feedback or coordinate transformations, a solution giving rise to a new system representation which can hide a lot of system structural properties. The fact of making a system dissipative or passive, known as *feedback dissipativity* or *passivation*, respectively, appears to be a more natural system transformation. The result of the feedback dissipativity process is a system which keeps on being nonlinear and has interesting characteristics for controlling some of its variables. There are a lot of examples of taking advantage of the dissipativity approach in the study of mechanical and electro-mechanical systems. In Chapter 2, some of the most important applications of the dissipativity-based approach will be mentioned, and along this dissertation, its beneficial characteristics will be reviewed.

1.2 Main contributions

The contributions of this dissertation are classified into three main goals or problems to solve, such as:

1. **The characterization of dissipative systems of general form** represented by the system (1.1)-(1.2), what is regarded as Kalman-Yakubovich-Popov conditions. In the literature, the Kalman-Yakubovich-Popov conditions have been established for the nonlinear discrete-time case for systems affine in the control input, that is: see *(Byrnes and Lin, 1993)* [13] and *(Byrnes and Lin, 1994)* [14] for the passivity and losslessness case, respectively, and *(Sengör, 1995)* [151], *(Göknar and Sengör, 1998)* [44] for the dissipativity and losslessness cases. The existing conditions for the dissipativity and losslessness cases are extended to a class of nonlinear

multiple-input multiple-output dissipative and lossless discrete-time systems of the form (1.1)-(1.2) in Chapter 4. The class of dissipativity and losslessness characterized is regarded as *QSS*-dissipativity and *QSS*-losslessness, respectively.

2. **The feedback dissipativity problem**, and its special case of feedback passivity or passivation problem, i.e., the transformation of a nonlinear discrete-time system of the form (1.1)-(1.2) into a dissipative (or passive) one by means of a static-state feedback law. This problem has not been studied in the literature in the nonlinear discrete-time setting. Indeed, for the linear case, the problems of feedback passivity and feedback (Q,S,R)-dissipativity are treated in the framework of the positive real control problem and the (Q,S,R)-dissipative control problem, respectively, in connection with H_∞ design, see *(de Souza and Xie, 1992)* [164], *(Souza et al., 1993)* [165] for the positive control problem and *(Tan et al., 1999)* [170], *(Tan et al., 2000)* [171] for the (Q,S,R)-dissipative control problem. Concerning the nonlinear case, only the feedback losslessness problem has been solved for affine-in-the-input systems *(Byrnes and Lin, 1994)* [14]. Therefore, this is the main contribution of this dissertation. Two approaches are proposed to deal with this problem:

   (a) **Solving the feedback dissipativity problem through the fundamental dissipativity inequality**. Chapter 5 is devoted to this goal. In this chapter, sufficient conditions under which a class of single-input single-output non-affine discrete-time systems are feedback dissipative are given, as well as, the proposal of four methodologies in order to solve the feedback dissipativity problem in such systems. The feedback losslessness problem for non-affine-in-the-input systems is also treated, this can be also considered as a new contribution.

   (b) **Solving the feedback passivity problem of a class of multiple-input multiple-output affine-in-the-input discrete-time systems using the properties of the relative degree and the zero dynamics**. These results are presented in Chapter 7 and are an extension to the passivity case of the ones given in *(Byrnes and Lin, 1994)* [14] where the feedback losslessness problem is reported. The fact of concluding the special properties of the relative degree of passive nonlinear discrete-time systems which are affine in the control input is also a new contribution.

3. **The dissipativity-based stabilization problem**. In other words, the use of dissipativity and feedback dissipativity properties for control purposes. Chapter 6 deals with this problem. In this chapter, the feedback dissipativity results achieved in Chapter 5 are used. The consequences of feedback dissipativity properties in systems stability are shown. The main contribution is the extension of the *Energy Shaping and Damping Injection* methodology, proposed and used in the continuous-time setting, to the case of single-input single-output systems of the form (1.1)-(1.2), in addition to the application of the new feedback dissipativity techniques proposed in Chapter 5 for stabilization purposes. The passivity-based stabilization approaches existing in the nonlinear discrete-time domain given by two main approaches, that is, Byrnes and Lin's work *(Lin and Byrnes, 1995)* [87, 89], *(Lin, 1996)* [91] and Sengör's work *(Sengör, 1995)* [151] will not be used, since the *Energy Shaping and Damping Injection* approach appears to be more appropriate in order to be combined with our feedback dissipativity results.

In all the mentioned results, the state-space or internal description representation of systems is used. Besides, passivity and feedback passivity have been treated as special cases of dissipativity and feedback dissipativity, respectively, with the exclusion of the results given in Chapter 7. Another kind of approach and spirit is given in Chapter 8; the frequency-domain properties of passivity and dissipativity are used to give light to some problems not broadly treated in the literature, such as: passivity preservation under feedback and parallel interconnections, some notes on dissipativity and passivity under sampling, and the study of the relations between dissipativity and some of the most important frequency-based feedback stability criteria in the discrete-time domain, namely: Tsypkin's, Popov's and the circle criteria.

The results given for the feedback dissipativity and the dissipativity-based stabilization problems are applied to two examples. One of these examples is the discrete-time model of the DC-to-DC buck converter. The stabilization of the DC-to-DC buck converter by means of the dissipativity-based discrete-time techniques proposed can be also considered as a contribution of this dissertation.

On the other hand, the fact of treating general discrete-time systems has allowed us to extend some dissipativity-related definitions for the case of general single-input single-output continuous-time systems of the form

$$\begin{aligned} \dot{x}(t) &= f(x(t),u(t)) \\ y(t) &= h(x(t),u(t)) \end{aligned}$$

Chapter 3 collects these results. The main contributions of this chapter are the following ones:

1. The proposal of the feedback dissipativity problem as an extension to the nonlinear non-affine case of the feedback passivity problem given in *(Sira-Ramírez, 1998)* [159] for nonlinear systems which are affine in the control input.
2. The use of the feedback dissipativity results in order to extend the *Energy Shaping and Damping Injection* controller design method to the case of non-affine nonlinear systems.

1.3 Outline

The present dissertation is divided into the following chapters.

To begin with, a revision of the work preceding ours will be made in Chapter 2, not only the nonlinear discrete-time case, but also the linear and the continuous-time cases. Three main parts will be considered in this chapter, corresponding to the three main goals of this dissertation, that is:

1. Characterization of dissipative systems.
2. The feedback dissipativity problem.
3. Dissipativity-based stabilization and control.

Some dissipativity-related results achieved in the continuous-time case for non-affine-in-the-input nonlinear systems are given in Chapter 3. A set of necessary and sufficient conditions fulfilled for a class of multiple-input multiple-output dissipative systems are proposed. The feedback dissipativity problem is treated for a class of single-input single-output dissipative systems. Besides, some stability implications are derived for dissipative systems with special supply functions. These stability conclusions, in addition to the feedback dissipativity results, are used in order to extend the *Energy Shaping and Damping Injection* controller design methodology to the case of non-affine nonlinear systems. Finally, a Generalized Hamiltonian-type form for nonlinear systems is derived; and feedback dissipative systems, i.e., systems which can be rendered dissipative, are shown to exhibit special features and stabilization properties if they are written in the form proposed.

Chapter 4 is devoted to the proposal of the *Kalman-Yakubovich-Popov* conditions for a class of multiple-input multiple-output discrete-time systems of the form (1.1)-(1.2). Either the dissipativity or the losslessness case is treated. A new class of dissipative systems is introduced, what is regarded as *QSS*-dissipative systems, i.e., dissipative systems whose storage (V) and supply (s) functions satisfy $V(f(x,u))$ and $s(h(x,u),u)$ are quadratic in u. *QSS*-dissipativity characterization will be used to solve the passivation problem in nonlinear discrete-time systems affine in the control input in Chapter 7.

The feedback dissipativity problem for single-input single-output systems of the form (1.1)-(1.2) is treated in Chapter 5. In this chapter, the definitions and formalization of the feedback dissipativity problem are given. First of all, sufficient conditions under which a class of non-affine discrete-time systems can be rendered dissipative are derived. Then, four feedback dissipativity methodologies are proposed. They are based on the establishment of the input u which satisfies the fundamental dissipativity inequality. The first method proposes an implicit solution for this problem. The second one uses the speed-gradient algorithm in its discrete-time version. The last two methods are of approximate type. In these last two cases, dissipativity is conceived as a "perturbation" of the storage energy invariance or the system losslessness situations. For these feedback dissipativity methodologies, the errors of the approximation are bounded, and sufficient conditions under which the approximation made is valid are given. The four feedback dissipativity methods are illustrated by means of two examples: a discrete-time model of the DC-to-DC buck converter proposed in this chapter, and an academic nonlinear discrete-time system. For the examples, the feedback passivity problem is treated. The validity of the passifying control and the admissible values for the constants appearing in the passifying scheme are analyzed for each example.

The problem of using dissipativity and feedback dissipativity properties proposed in Chapter 5 for stabilization purposes is dealt with in Chapter 6. In this chapter, some dissipativity stability-related results will be formalized. Sufficient conditions under which a class of single-input single-output feedback dissipative systems of the form (1.1)-(1.2) can be stabilizable are given, in addition to, the extension of the *Energy Shaping and Damping Injection* methodology to the case of a class of dissipative single-input single-output nonlinear discrete-time systems of the form (1.1)-(1.2). The four feedback dissipativity methods proposed in Chapter 5 are used in the *Energy Shaping and Damping Injection* scheme in order to stabilize two systems orbits around a desired fixed point. The systems treated are the two examples presented in Chapter 5.

An alternative to the passivation problem given in Chapter 5 is proposed in Chapter 7. Here, the feedback passivity problem for a class of multiple-input multiple-output nonlinear discrete-time systems affine in the control input is solved using the properties of the relative degree and zero dynamics of the non-passive system. For this aim, the special properties that the relative degree and zero dynamics of passive discrete-time systems, either linear or nonlinear, exhibit are shown. The passivation methodology proposed in this chapter is used for stabilization purposes; it is applied to the stabilization around the system fixed point of the nonlinear example proposed in Chapter 5. The feedback passivity scheme is used to passify the DC-to-DC buck converter example, the frequency-domain properties of the passified system are analyzed.

A different chapter in spirit to the rest of the chapters of this dissertation is Chapter 8. This chapter is conceived as a results collecting one in which some kind of formalism is lost. Chapter 8 differs from Chapter 2 due to the fact that it gives some small contributions. Some implications of dissipativity and passivity in the discrete-time setting are collected, and the frequency-domain properties of dissipativity and passivity are used in order to illustrate such special characteristics that dissipative and passive systems exhibit. The properties studied are, mainly: the preservation of passivity under parallel and feedback interconnections, the study of the preservation of passivity and dissipativity under sampling, and the use of the frequency-domain implications of dissipative systems in order to study nonlinear feedback systems absolute stability in the discrete-time domain. All the implications of dissipativity and passivity presented in this chapter can be considered as approaches to explore and study in a deeper way in the future.

The conclusions and suggestions for further research are emplaced in Chapter 9. In this chapter, the conclusions and comments for future work given in each chapter are collected, in addition to give some research lines initiated which have not been closed yet.

After Chapter 9, the Appendix and the Bibliography are presented. In the Appendix, some of the most important dissipativity and passivity characterizations appeared in the literature for the continuous-time and discrete-time cases are given.