Chapter 2

Background and literature review

2.1 Introduction

The control of a system in terms of the energy it can store or dissipate is known as *Dissipativity-Based Control* (DBC) or *Passivity-Based Control* (PBC), depending upon the class of dissipativity used, and gives us a rather intuitive approach exploiting the system physical properties and the interconnection of the system with its environment.

This chapter is concerned with the DBC problem, and it consists of three different parts corresponding to the main goals of this dissertation, that is:

1. Characterization of dissipativity and its special case of passivity.
2. Study of feedback dissipativity and passivation properties.
3. Use of dissipativity, passivity, feedback dissipativity and passivation properties for control purposes.

In each of these parts, a review of the main existing results in the literature is given. The basic definitions implied in each of these points will be presented, either for the continuous-time or the discrete-time case.

The first part of this chapter devoted to the characterization of dissipativity is divided into two sections, Section 2.2 and Section 2.3. Section 2.2 deals with the basic definitions of dissipativity using the state-space system representation and dissipativity formalization by means of the storage and supply functions. Section 2.3 is concerned with the Kalman-Yakubovich-Popov (KYP) lemma, in the linear case, and what is regarded as KYP conditions for dissipativity and passivity in the nonlinear case. It collects the most important existing results referring the dissipativity and passivity characterization through the KYP conditions. The second part of this chapter, given in Section 2.4, presents the basic definitions and research lines concerning the problems of feedback dissipativity and feedback passivity. Section 2.5 is the last part of the chapter, it gives a note on the stability implications for passive and dissipative systems, as well as the applications of the dissipativity and feedback dissipativity properties for control purposes. Finally, a preview of the main results obtained in this dissertation are outlined, together with a brief discussion of the most important open problems in the dissipativity-related research. Along the chapter, the results which will be presented in the rest of the chapters composing this dissertation are briefly pointed out.

2.2 Definition of a dissipative system

Dissipativity and its particular case of passivity were born from the observation of physical systems' behaviour. Such concepts are the formalization of physical energy processes. Dissipativity ideas emerged in the circuit theory field, from the phenomenon of dissipation of energy across resistors. The abstraction of the connections between input-output behaviour, internal system description and properties of energy functions is the basis for dissipative systems. Precisely, due to the fact that dissipativity merges all these concepts, it acts as a powerful tool for analyzing systems.

The energy concept is very useful in the analysis of physical systems. Many systems can be studied from its **sources** and **losses of energy**. Having the idea of the gain and the loss of energy, intuitively, a dissipative system is such a system which cannot store all the energy that is given. A dissipative system dissipates energy and does not produce it, that is, any increase of stored energy is only due to external sources. This definition implies the existence of three energy-like functions: the *storage function* (representing the energy stored by the system), the *supply function* (the energy injected to the system from an external source, which restricts the manner in which the system absorbs energy) and the *dissipation function*. The supply function is interpretated as an input power, denomination inherited from the circuit theory. Depending upon the form of the supply function, different kinds of dissipativity are obtained; passivity is the one which has attracted more attention. The one who defined dissipativity and passivity concepts by means of the notion of the storage, the supply rate and the dissipation rate functions was *Willems* in the early 70's *(Willems, 1972)* [179, 180].

2.2.1 The continuous-time case

To begin with continuous-time systems, the following definition for dissipativity is presented. Let the system,

$$\dot{x} = f(x,u), \quad x \in \mathscr{X}, \quad u \in \mathscr{U} \tag{2.1}$$

$$y = h(x,u), \quad y \in \mathscr{Y} \tag{2.2}$$

where $f : \mathscr{X} \times \mathscr{U} \to \mathscr{X}$, is a smooth mapping of its arguments with $\mathscr{X}$ an open subset of $\Re^n$, and $\mathscr{U}$ an open subset of $\Re^m$ and a function $h : \mathscr{X} \times \mathscr{U} \to \mathscr{Y}$, with $\mathscr{Y}$ an open subset of $\Re^m$. Associated with system (2.1)-(2.2) we consider a $\mathscr{C}^1$ function called the *supply function*, denoted by $s(y,u)$, with $s : \mathscr{Y} \times \mathscr{U} \to \Re$, satisfying

$$\int_0^t |s(y(\sigma),u(\sigma))| d\sigma < \infty, \quad \forall t \geq 0$$

Definition 2.1 (Willems, 1972) *[179] System (2.1) with a properly chosen output (2.2) is said to be dissipative with respect to the supply function $s(y,u)$ if there exists a positive* *definite, $\mathscr{C}^1$ function, addressed as the* storage function, $V : \mathscr{X} \to \Re$, $V(0) = 0$, *such that for any t_0 and any $t_f > t_0$, the following inequality is satisfied, irrespectively of the initial value of the state $x(t_0)$,*

$$V(x(t_f)) - V(x(t_0)) \leq \int_{t_0}^{t_f} s(y(\sigma), u(\sigma))\, d\sigma, \quad \forall (x,u) \in \mathscr{X} \times \mathscr{U} \tag{2.3}$$

If the above inequality is a strict inequality the system is said to be *strictly dissipative*.

It is easy to see that for C^1 storage energy functions, the dissipativity inequality adopts the following *infinitesimal* form,

$$\dot{V}(x) \leq s(y,u) \tag{2.4}$$

Inequalitiy (2.4) expresses that the rate of increase of the stored energy is not higher than the energy supplied to the system.

From (2.3) the three main elements for the characterization of dissipative systems can be distinguished, that is: the output, the energy stored by the system, and the injected energy from an external source. The output plays an important role in the dissipativity definition, since, dissipativity is an input-output property. Therefore, the problem of the election of the output for which we want the system to be dissipative has to be faced.

Definition 2.2 (Khalil, 1996) *[76] System (2.1)-(2.2) is* passive *if it is dissipative with respect to the supply rate $s(y,u) = y^T u$. The system is* strictly input passive *if there exists a constant $\varepsilon > 0$ such that the system is dissipative with respect to $s(y,u) = y^T u - \varepsilon u^T u$. The system is* strictly output passive *if there exists a constant $\delta > 0$ such that the system is dissipative with respect to $s(y,u) = y^T u - \delta y^T y$. The system is* strictly state passive *if there exists a constant ρ and $\psi(x)$ a positive semi-definite function of x called the state dissipation rate, such that the system is dissipative with respect to $s(y,u) = y^T u - \rho\,\psi(x)$.*

Definition 2.3 *System (2.1)-(2.2) is said to be lossless with respect to the supply function $s(y,u)$ if there exists a storage function $V(x)$ such that for any t_0 and any $t_f > t_0$, the following equality is satisfied, irrespectively of the initial value of the state $x(t_0)$,*

$$V(x(t_f)) - V(x(t_0)) = \int_{t_0}^{t_f} s(y(\sigma)u(\sigma))\, d\sigma, \quad \forall (x,u) \in \mathscr{X} \times \mathscr{U} \tag{2.5}$$

The definition of a dissipative system can also be established in terms of an equality for the energy balance, instead of an inequality of the form (2.3). Therefore, as in *(Hill and Moylan, 1980)* [55] Definition 2.1 is rewritten in the following way.

Definition 2.4 *System (2.1) with a properly chosen output function (2.2) is said to be dissipative (respectively, strictly dissipative) with respect to the supply function $s(y,u)$ if there exist a positive definite function V, $V : \mathscr{X} \to \Re$, with $V(0) = 0$, regarded as the storage function, and a continuous function $\phi : \mathscr{X} \times \mathscr{U} \to R$ with $\phi(\cdot,u)$ positive (respectively, strictly positive) for each $u \in \mathscr{U}$, such that, for any t_0 and any $t_f > t_0$, the following equality is satisfied, irrespectively of the initial value of the state $x(t_0)$,*

$$V(x(t_f)) - V(x(t_0)) = \int_{t_0}^{t_f} [s(y(\sigma), u(\sigma)) - \phi(x(\sigma), u(\sigma))]\, d\sigma, \ \forall (x,u) \in \mathscr{X} \times \mathscr{U} \tag{2.6}$$

The function ϕ will be regarded as the *dissipation rate* function in the sense proposed in *(Hill and Moylan, 1980)* [55].

For $\mathscr{C}^1$ storage energy functions, the dissipativity equality (2.6) adopts the following *infinitesimal* form,

$$\dot{V}(x) = s(y,u) - \phi(x,u) \tag{2.7}$$

2.2.2 The discrete-time case

The concept of dissipativity can also be formalized for the case of discrete-time systems. The supply function s also appears to be associated with the system, now a discrete-time dynamics. Besides, it is assumed that for any $u \in \mathscr{U}$ and for any initial condition $x(0)$, $s(y(k),u(k)) \in \Re$ for all $k \geq 0$.

Consider discrete-time systems of the form

$$x(k+1) = f(x(k),u(k)), \;\; x \in \mathscr{X}, \; u \in \mathscr{U} \tag{2.8}$$

$$y(k) = h(x(k),u(k)), \;\; y \in \mathscr{Y} \tag{2.9}$$

where $f : \mathscr{X} \times \mathscr{U} \to \mathscr{X}$, and $h : \mathscr{X} \times \mathscr{U} \to \mathscr{Y}$ are smooth maps with $\mathscr{X} \subset \Re^n$, $\mathscr{U}$, $\mathscr{Y} \subset \Re^m$.

Definition 2.5 (Lin, 1993) *[82] System (2.8) with a properly chosen output (2.9) is said to be dissipative with respect to the supply rate s if there exists a positive definite function V, $V : \mathscr{X} \to \Re$, with $V(0) = 0$, regarded as the storage function such that the following inequality is satisfied for all $x(0)$, for all $k \in \mathscr{Z}_+ := \{0,1,2,\dots\}$,*

$$V(x(k+1)) - V(x(k)) \leq s(y(k),u(k)), \;\; \forall (x(k),u(k)) \in \mathscr{X} \times \mathscr{U}, \forall k \tag{2.10}$$

Inequality (2.10) can be rewritten as

$$V(x(k+1)) - V(x(0)) \leq \sum_{i=0}^{k} s(y(i),u(i)), \; \forall k, \forall u(k), \; \forall x(0) \tag{2.11}$$

Either expression (2.10) or (2.11) is regarded as the dissipation inequality in the discrete-time domain.

Another definition for dissipative systems can be obtained, changing the dissipation inequality by an equality. Then, Definition 2.5 is rewritten as the following one.

Definition 2.6 *A system of the form (2.8) with a properly chosen output (2.9) is said to be dissipative (respectively, strictly dissipative) with respect to the supply function $s(y,u)$ if there exist a positive definite function V, $V : \mathscr{X} \to \Re$, with $V(0) = 0$, regarded as the storage function, and a continuous function $\phi : \mathscr{X} \times \mathscr{U} \to \Re$, with $\phi(\cdot,u)$ positive (respectively, strictly positive) for each $u \in \mathscr{U}$ such that, the following equality is satisfied, irrespectively of the initial value of the state x,*

$$V(x(k+1)) - V(x(k)) = s(y(k),u(k)) - \phi(x(k),u(k)), \; \forall (x(k),u(k)) \in \mathscr{X} \times \mathscr{U}, \forall k \tag{2.12}$$

In this case, as in the continuous-time domain, function ϕ will be also regarded as the *dissipation rate* function.

2.3 Dissipativity characterization: the KYP lemma

If we look up the historical evolution of dissipativity formalization, we can say that the first ideas of dissipativity emerged in the circuit theory field, from the phenomenon of dissipation of energy across resistors, the works of *(Zames, 1966)* [190] or *(Vidyasagar, 1977)* [175] are the most representative ones. Besides, *(Willems, 1972)* [179, 180] studied dissipative systems inspired by the circuit theory, as well as thermodynamics and mechanics, and connected them with feedback control theory. A different research line was initiated by *Wu and Desoer*'s work *(Wu and Desoer, 1970)* [182], cast in terms of the system input-output properties and from a general operator theoretic viewpoint. An advance in the concept of passivity is made by *(Popov, 1973)* [141] who established passivity as an important feedback property by means of the concepts of hyperstability and absolute stability. The extension of dissipativity results to the case of nonlinear systems which are affine in the control input was given by *Hill and Moylan*, *(Hill and Moylan, 1976)* [53], *(Hill and Moylan, 1977)* [54], *(Hill and Moylan, 1980)* [55], *(Moylan, 1974)* [116]. In conclusion, the dissipativity and the passivity properties in physical systems are formalized in different ways.

The most important formalization of the characteristics of a passive system is the *Kalman-Yakubovich-Popov* (KYP) property or the KYP lemma, which is a set of necessary and sufficient conditions for a system to meet in order to be passive.

It must be pointed out that the KYP lemma, originally, established the connection between passivity conditions (i.e., a set of properties that any passive sytem fulfils) and positive real transfer functions *(Kalman, 1963)* [68], *(Yakubovich, 1963)* [186], *(Yakubo vich, 1973)* [187], *(Popov, 1964)* [140]. Thus, the KYP was first proposed for continuous-time linear systems, founding the relation between passivity and frequency stability properties. However, the denomination of KYP property has been adopted to name the set of necessary and sufficient conditions that a passive or a dissipative system satisfies, whether it is linear or nonlinear.

2.3.1 Dissipativity and passivity implications in the linear case

In the linear case, an equivalence between passivity and positive transfer functions is produced, as well as, the connection of passivity and frequency-domain stability concepts. The most important results concerning this point will be described in this section.

2.3.1.1 The continuous-time case

The concept of positive real functions is originated in network theory as the frequency-domain formulation of the fact that the time integral of the energy input to a passive network must be positive. Then, a positive real system is defined in the following way.

Definition 2.7 (Byrnes et al., 1991) *[12] A system Σ is said to be positive real if for all $u \in \mathscr{U}$, $t \geq 0$*

$$\int_0^t y^T(\sigma)u(\sigma)d\sigma \geq 0$$

whenever $x(0) = 0$, where the integral is considered along the system trajectories.

Passive systems are related to positive real systems. The relation between passive and positive real systems is based on the reachability property from the equilibrium point $x = 0$, see *(Byrnes et al., 1991)* [12].

The study of positive realness translates into the study of positive real transfer functions for linear systems. The implications that the positive realness property has in the frequency domain has been analyzed for linear systems. The relation between passivity and positive transfer functions is given by the KYP lemma in the linear case, which can be regarded as a frequency stability property. It is known that a linear positive real system as defined in Definition 2.7 is equivalent to positive realness of its associated transfer function *(Byrnes et al., 1991)* [12].

The KYP lemma was first proposed in *(Kalman, 1963)* [68], *(Popov, 1964)* [140], *(Yakubovich, 1963)* [186], *(Yakubovich, 1973)* [187], and then it was extended to multivariable systems by *(Anderson, 1967)* [3] and *(Anderson and Vongpanitlerd, 1973)* [5] for continuous-time linear systems of the following form:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.13}$$
$$y(t) = Cx(t) + Du(t) \tag{2.14}$$

where $x \in \Re^n$, $u, y \in \Re^m$. From now and so on, the notations of ≥ 0, > 0, ≤ 0 and > 0 are used to denote positive semi-definiteness, positive definiteness, negative semi-definiteness and negative definiteness of matrices, respectively.

In the linear case, the KYP lemma is also regarded as the *Positive Real Lemma*.

Theorem 2.1 (KYP lemma) (Khalil, 1996) *[76] Let the system (2.13)-(2.14), where A is Hurwitz, (A, B) controllable, and (A, C) observable. If there exist a real symmetric positive definite matrix P and real matrices W and L satisfying*

$$PA + A^T P = -L^T L \tag{2.15}$$
$$PB = C^T - L^T W \tag{2.16}$$
$$W^T W = D + D^T \tag{2.17}$$

regarded as the passivity conditions*, then the transfer function $G(s) = C(sI - A)^{-1}B + D$ is positive real, that is, it satisfies the following conditions*

(i) *all elements of $G(s)$ are analytic for $Re(s) > 0$,*

(ii) $G(j\omega) + G^T(-j\omega) \geq 0, \forall \omega \in \Re$, *for which $s = j\omega$ is not a pole of any element of $G(s)$, and*

(iii) *the eigenvalues of A on the imaginary axis are simple and the corresponding residues*

$$\lim_{s \to s_0} (s - s_0) G(s)$$

(s_0 a pole of $G(s)$) are Hermitian and nonnegative definite matrices.

Conversely, if $G(s)$ is positive real, then for any minimal realization of $G(s)$, there exist $P > 0$, W and L which satisfy the passivity conditions (2.15)-(2.17).

■

There are different definitions for positive real and strictly positive real transfer functions in the literature, we could point out a recently appeared one which is less restrictive, and is known as *wide positive realness (Xiao and Hill, 1998)* [183]. For more references on this topic, see Chapter 8.

2.3.1.2 The discrete-time case

The discrete-time counterpart of the KYP lemma is proposed in *(Hitz and Anderson, 1969)* [58], it will be presented below and is also known as the *Discrete Positive Real Lemma*.

Consider the linear time-invariant multiple-input multiple-output discrete-time system:

$$x(k+1) = Ax(k) + Bu(k) \tag{2.18}$$

$$y(k) = Cx(k) + Du(k) \tag{2.19}$$

The associated transfer function of this system is,

$$G(z) = C(zI - A)^{-1}B + D$$

Assume that system (2.18)-(2.19) is controllable.

Theorem 2.2 (Hitz and Anderson, 1969) *[58] Let $G(z)$ be a square matrix of real rational functions of z with no poles in $|z| > 1$ and simple poles only on $|z| = 1$, and let (A,B,C,D) be a minimal realization of $G(z)$. If for (A,B,C,D) there exist a real symmetric positive definite matrix P and real matrices L and W such that*

$$A^T PA - P = -L^T L \tag{2.20}$$

$$A^T PB = C^T - LW \tag{2.21}$$

$$W^T W = (D + D^T) - B^T PB \tag{2.22}$$

then, the transfer function $G(z)$ is discrete positive real, that is, satisfies the conditions

(i) *$G(z)$ has elements analytic in $|z| > 1$, and*

(ii) *$G^*(z) + G(z) \geq 0$ in $|z| > 1$,*

where the superscript asterisk denotes the operation of complex conjugation.

Conversely, if $G(z)$ is discrete positive real, then for any minimal realization of $G(z)$, there exist $P > 0$, W, L which satisfy the passivity conditions (2.20)-(2.22).

■

Condition **(ii)** in Theorem 2.2 implies that

$$G^T(e^{-j\omega}) + G(e^{j\omega}) \geq 0,$$

for all real ω such that no element of $G(z)$ has a pole at $z = e^{j\omega}$ *(Hitz and Anderson, 1969)* [58].

Remark 2.1 *Either for the continuous-time case or the discrete-time one, passivity conditions (2.15)-(2.17) or (2.20)-(2.22), respectively, establish that the system is passive with respect to a storage function of the form* $V(x) = \frac{1}{2}x^T Px$.

As occurring in the continuous-time case, conditions for a transfer function to be positive real are rewritten in different ways, for example in *(Fernández and Ortega, 1987)* [31] it is given an algebraic formulation of the positive-real conditions in terms of a polynomial inequality whose solution is numerically stable.

The works appearing in the literature studying the passivity properties or positive realness in the discrete-time linear case are much more less than those appearing for the continuous-time setting. Referring the KYP property, we can stand out *(Premaratne and Jury, 1994)* [142] and *(Halanay and Ionescu, 1993)* [52] where the KYP "positivity conditions" are replaced with more general ones in order to achieve less restrictive conditions in order to stabilize the system. A very interesting and recent work is *(Tan et al., 2000)* [171] where the properties of a class of dissipative discrete-time systems are extended for the case of uncertain discrete-time systems. Strictly positive real discrete-time functions are also treated in *(Mosquera and Pérez, 2001)* [115]. Another recent work in relation to discrete positive-real functions is *(Fernández-Anaya and Vallin- Hernández, 2000)* [32] where strictly positive-real discrete functions with relative degree zero (SPRD0) are characterized. The discrete-time bounded real lemma is also revisited in *(Vaidyanathan, 1985)* [174], for the lossles case, and in *(Wimmer, 2000)* [181].

The generalized version of the KYP lemma, for the dissipativity linear discrete-time case is given in *(Goodwin and Sin, 1984)* [45] for supply functions of the form:

$$s(y,u) = y^T Qy + 2y^T Su + u^T Ru, \tag{2.23}$$

where Q, S, R are appropriately dimensioned matrices, with Q and R symmetric.

Definition 2.8 *Dissipative systems with supply functions of the form (2.23) are regarded as* (Q,S,R)–dissipative systems.

Lemma 2.1 (Goodwin and Sin, 1984) *[45] Let* $G(z)$ *a transfer function description, and* $M(z) = R + G^H(z)S + S^T G(z) + G^H(z)QG(z)$, *with* $G^H(z)$ *denoting the hermitian transpose of* $G(z)$. *Let the system (2.18)-(2.19) be a minimal realization of* $G(z)$. *Consider a supply function of the form (2.23). Then* $\forall z$ *s.t.* $|z| \geq 1$, $M(z)$ *is positive definite if and only if there exist a real symmetric positive definite matrix P and real matrices L and W such that*

$$A^T PA - P = \hat{Q} - L^T L \tag{2.24}$$

$$A^T PB = \hat{S} - LW \tag{2.25}$$

$$B^T PB = \hat{R} - W^T W \tag{2.26}$$

with

$$\begin{aligned} \hat{Q} &= C^T QC \\ \hat{S} &= C^T QD + C^T S \\ \hat{R} &= R + D^T S + S^T D + D^T QD \end{aligned}$$

■

Conditions (2.24)-(2.26) can be considered as the characterization of dissipativeness for storage functions of the form $V = \frac{1}{2}x^T Px$, with P a positive definite symmetric matrix, and supply functions given in (2.23). Special cases of dissipativeness can be derived choosing different values for Q, S and R *(Goodwin and Sin, 1984)* [45]:

1. Passivity: $Q = R = 0$, $S = \frac{1}{2}I$
2. Input strict passivity (ISP) : $Q = 0$, $S = \frac{1}{2}I$, $R = -\varepsilon I$
3. Output strict passivity (OSP) : $Q = -\delta I$, $S = \frac{1}{2}I$, $R = 0$
4. Very strict passivity (VSP) : $Q = -\delta I$, $S = \frac{1}{2}I$, $R = -\varepsilon I$
5. Finite gain stable (FGS) : $Q = -I$, $S = 0$, $R = k^2 I$,

where ε and δ are small positive scalars, I the identity matrix, and k an arbitrary constant.

The frequency-domain characteristics of passive and dissipative systems and their equivalence to positive real transfer functions, in addition to their stability implications will be revisited in Chapter 8.

2.3.2 The nonlinear case

2.3.2.1 The continuous-time case

The conditions that a nonlinear continuous-time dissipative system fulfils are established by *(Hill and Moylan, 1976)* in [53] for (Q,S,R)–dissipative systems.

The generalization of passivity conditions (2.15)-(2.17) for nonlinear continuous-time systems is given by *(Moylan, 1974)* [116], in the line of Willem's results, using the notions of storage and supply functions in order to define passivity. For the nonlinear case, what is called KYP lemma only establishes necessary and sufficient conditions for a system to be passive, no relation with the positive realness characteristic is made. However, for the nonlinear case, positive systems are also defined (see Definition 2.7) and related to passive systems, as it is established in *(Byrnes et al., 1991)* [12].

Either dissipativity conditions or passivity conditions for nonlinear continuous-time systems will be important to bear in mind, due to the fact that the nonlinear discrete-time results existing in the literature are based on them.

Let a system described by

$$\dot{x} = f(x) + g(x)u \tag{2.27}$$
$$y = h(x) + J(x)u \tag{2.28}$$

where f and h are real vector functions of the state vector x, and g and J are real matrix functions of x. It is supposed that f, g, h, and J have continuous derivatives of all orders. The input u and the output y have the same dimensions, therefore, J is a square matrix.

Conditions for systems of the form (2.27)-(2.28) to be (Q,S,R)–dissipative and passive are established in *(Hill and Moylan, 1976)* [53] and *(Moylan, 1974)* [116], respectively. See Theorems A.1 and A.2 in Appendix A at the end of this dissertation. Necessary and sufficient conditions for continuous-time (Q,S,R)-dissipative and (Q,S,R)-lossless systems which are affine in the control input are revisited in *(Sengör, 1995)* [151] in the framework of abstract dynamical energy systems.

Necessary and sufficient conditions for passivity given in *(Moylan, 1974)* [116] are rewritten in *(Byrnes et al., 1991)* [12], for systems (2.27)-(2.28) with $J=0$. Let the system,

$$\dot{x} = f(x) + g(x)u \quad (2.29)$$

$$y = h(x) \quad (2.30)$$

Definition 2.9 (Byrnes et al., 1991) *[12] System (2.29)-(2.30) has the KYP property if there exists a $\mathscr{C}^1$ nonnegative function $V:\mathscr{X}\to\Re$, $\mathscr{X}\subset\Re^n$, with $V(0)=0$, such that*

$$L_{f(x)}V(x) \leq 0, \ \forall x\in\mathscr{X} \quad (2.31)$$

$$L_{g(x)}V(x) = h^T(x), \ \forall x\in\mathscr{X} \quad (2.32)$$

As it is shown in *(Byrnes et al., 1991)* [12], system (2.29)-(2.30) which has the KYP property is passive, with storage function V, and, conversely, a passive system of the form (2.29)-(2.30) having a $\mathscr{C}^1$ storage function has the KYP property. The robust version of the KYP property for nonlinear continuous-time systems which are affine in the control input is given by *(Lin and Shen, 1999)* [94].

From passivity conditions appearing either in the linear case or in the nonlinear case, two kinds of conditions can be identified: a stability-kind condition and input-output relation conditions, both written in different ways depending on the structure of the system considered. On the one hand, conditions (2.15), (2.20), (A.6), (2.31) are the ones regarded as stability-kind conditions. These relations endow the storage function V the property of being a Lyapunov-like function for the system dynamics obtained by considering $u=0$. On the other hand, relations (2.16)-(2.17), (2.21)-(2.22), (A.7)-(A.8), (2.32) are the ones referred as input-output conditions. The distinction of these two kinds of conditions for passivity it will be of utmost importance in order to propose and understand the KYP conditions for nonlinear discrete-time systems.

We must pay attention to the fact that no KYP lemma or necessary and sufficient conditions for passivity or dissipativity have been defined for the case of having a non-affine structure in the control input, i.e., for nonlinear general systems of the form (2.1)-(2.2). The results appearing in the literature which treat the non-affine case are the ones presented by *Lin* in *(Lin, 1995)* [86] and *(Lin, 1996)* [93], who gives necessary conditions for a non-affine nonlinear continuous-time system to be passive (see Proposition A.1 in Appendix A), and furthermore, uses the passivity properties of a system in order to achieve its asymptotic stabilization. The characterization of a class of dissipative non-affine nonlinear systems of the form (2.1)-(2.2) will be presented in Chapter 3.

2.3.2.2 The discrete-time case

Referring the discrete-time case, there are a lot of dissipativity-related problems remaining unsolved, standing out the establishment of the KYP lemma for general nonlinear discrete-time systems, that is, necessary and sufficient conditions for a nonlinear discrete-time system of the form (2.8)-(2.9) to be dissipative or passive.

Four main approaches are distinguished in the literature concerning the extension of dissipativity and passivity concepts for the discrete-time case:

- Wu and Desoer's work. Dissipativity-related concepts.
- Lin and Byrnes' work. Passivity and losslessness characterizations.
- Sengör and Göknar's work. Dissipativity and losslessness characterizations.
- Monaco and Normand-Cyrot's work. Passivity and losslessness characterizations.

The initial results are given by *(Wu and Desoer, 1970)* [182] in terms of the system input-output properties, connecting passivity with feedback stabilization from a general operator theoretic viewpoint. This approach will not be of our interest, we will focus on the state-space representation.

Most of the studies existing in the literature referring to passivity in the discrete-time domain are given in *(Byrnes and Lin, 1993)* [13], *(Byrnes and Lin, 1994)* [14], *(Lin and Byrnes, 1995)* [87, 89] and can be considered as the extension to discrete-time affine-in-control nonlinear systems of the philosophy underlying *(Byrnes et al., 1991)* [12]. The systems under study in Byrnes and Lin's work have an affine structure in the control input:

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k))u(k) && (2.33) \\ y(k) &= h(x(k)) + J(x(k))u(k) && (2.34) \end{aligned}$$

where $x \in \Re^n$, $u \in \Re^m$, and $y \in \Re^m$, f, g, h and J are smooth maps, all of appropriate dimensions.

The great amount of their results are concerning lossless discrete-time systems, although some results are given for the case of passive discrete-time systems.

Necessary and sufficient conditions for systems of the form (2.33)-(2.34) to be lossless are given in *(Byrnes and Lin, 1993)* [13], *(Byrnes and Lin, 1994)* [14]. These conditions are shown as follows and are regarded as the KYP lemma for lossless discrete-time systems.

Theorem 2.3 (Byrnes and Lin, 1994) *[14] A system of the form (2.33)-(2.34) with a $\mathscr{C}^2$ storage function V is lossless, if and only if*

A)

$$\begin{aligned} V(f(x)) &= V(x) && (2.35) \\ \left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)} g(x) &= h^T(x) && (2.36) \\ g^T(x)\left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)} g(x) &= J^T(x) + J(x) && (2.37) \end{aligned}$$

B) *$V(f(x)+g(x)u)$ is quadratic in u.*

■

An extension of these conditions for discrete-time systems of the form (2.8)-(2.9) will be given in Chapter 4 of the present dissertation.

Concerning passivity characterization, necessary and sufficient conditions are found for affine discrete-time systems of the form (2.33)-(2.34) to be passive *(Byrnes and Lin, 1993)* [13], for storage functions such that $V(f(x)+g(x)u)$ is quadratic in u. Basically, this result can be considered as an extension of the KYP lemma presented in *(Moylan, 1974)* [116], see (A.6)-(A.8) in Appendix A.

Theorem 2.4 (Byrnes and Lin, 1993) *[13] Suppose there exists a positive definite function V, $V:\Re^n \to \Re$, $V(0)=0$, such that $V(f(x)+g(x)u)$ is quadratic in u. A discrete-time nonlinear system of the form (2.33)-(2.34) is passive with storage function V, if and only if there are real functions $l(x)$ and $k(x)$, all of appropriate dimensions, such that*

$$V(f(x))-V(x) = -\frac{1}{2}l^T(x)l(x) \tag{2.38}$$

$$\left.\frac{\partial V(\alpha)}{\partial \alpha}\right|_{\alpha=f(x)} g(x)+l^T(x)k(x) = h^T(x) \tag{2.39}$$

$$J^T(x)+J(x)-g^T(x)\left.\frac{\partial^2 V(\alpha)}{\partial \alpha^2}\right|_{\alpha=f(x)} g(x) = k^T(x)k(x) \tag{2.40}$$

■

Conditions (2.38)-(2.40) will be extended for discrete-time systems of the form (2.8)-(2.9) for the dissipativity case in Chapter 4. In addition, conditions (2.38)-(2.40) will be used to solve the passivation problem for systems of the form (2.33)-(2.34) in Chapter 7.

Necessary conditions for a system to be passive are also proposed in *(Lin and Byrnes, 1995)* [87], which are an adaptation for the passive case of the necessary and sufficient conditions for a system (2.33)-(2.34) to be lossless, see conditions (2.35)-(2.37). These necessary conditions for passivity are also generalized for the non-affine case *(Lin, 1995)* [86]. These results are presented in Propositions A.2 and A.3 in Appendix A.

There is another interesting way of treating dissipativity definitions in the literature, Sengör and Göknar's work *(Sengör, 1995)* [151], *(Göknar and Sengör, 1998)* [44], where a generalized KYP Lemma for dissipativity and losslessness is given for discrete-time affine-in-control nonlinear systems of the form (2.33)-(2.34). In addition, the results given are used to solve the stabilization problem. In Sengör's work, the definitions for lossless and dissipative systems are given in the framework of abstract dynamical energy systems by means the axiomatic definition of dynamical systems, see *(Desoer, 1970)* [24]. A general model is used which embodies discrete-time and continuous-time processes. Lossless and dissipative systems are characterized using the definition of gradient-like functions as conservative potential functions or as internal energy functions, respectively. For the non-affine-in-the-input discrete-time case, they do not give necessary and sufficient conditions on f and h for systems (2.8)-(2.9) to be dissipative. On the contrary, they give an alternative definition for dissipativity (and losslessness) to the one given by the fundamental dissipativity inequality given in Definition 2.5 or Definition 2.6 by means of the use of special storage functions which take the form of what is addressed as gradient-like functions. For the affine-in-the-input discrete-time case, they propose an adaptation of the necessary and sufficient conditions for dissipativity proposed in *(Hill and Moylan, 1976)* [53]. Sengör and Göknar give the definition of a dissipative system by means of the definition of internal energy functions. The kind of systems treated are dynamical energy systems, see Definitions A.1 and A.2 in Appendix A. A system is dissipative if an internal energy function can be associated with it. Then, the dissipativity conditions for nonlinear affine-in-the-input and discrete-time systems are proposed, see Theorems A.3 and A.4 in Appendix A. The ellegance of this work lies on the fact that since the definition of dissipativity is very general, it is easy to show the relation between all the energy-related concepts.

The fourth existing approach for the discrete-time case in connection with passivity concepts is the work of *(Monaco and Normand-Cyrot, 1997)* [111], *(Monaco and Normand-Cyrot, 1999)* [113]. They obtain necessary and sufficient conditions for general non-affine discrete-time nonlinear systems of the form $x(k+1) = f(x(k), u(k))$, $y(k) = h(x(k), u(k))$, with $u \in \Re$ and $y \in \Re^n$ to be lossless and passive. The authors phocus on systems which can be expanded by exponential Lie series. The use of the proposed exponential representation of discrete-time dynamics allows the use of differential geometric concepts in the discrete-time domain *(Monaco and Normand-Cyrot, 1997)* [112]. Indeed, they represent discrete-time dynamics as coupled difference and differential equations: the difference equation models the state evolution, and the control effect is modeled by a differential equation. The equivalence of these concepts with respect to the continuous-time affine case is proved by studying classical structural properties as invariance *(Grizzle, 1985)* [46], *(Monaco and Normand-Cyrot, 1984)* [101] and accessibility *(Jakubczyk and Sontag, 1990)* [60].

2.4 The feedback dissipativity and passivation problems

This section establishes what means the fact of rendering a system dissipative or passive by means of a feedback control law. The main results given in the literature are examined, not only for the continuous-time case, but also for the discrete-time case.

2.4.1 Basic definitions

As dissipative systems are attractive for our control purposes, we are motivated to transform a system which is not dissipative into one that is dissipative, this action will be pointed out as *feedback dissipativity*. The action of converting a non-passive system into a passive system will be regarded as *passivation* or *feedback passivity*. The passivation problem will be treated as a particular case of the feedback dissipativity one.

In this dissertation, feedback dissipativity and passivation studies are restricted to the action of rendering a system dissipative (respectively, passive) by means of a static state feedback. The idea of feedback dissipativity can be easily understood from the fundamental passivity inequality (2.3), or from its infinitesimal version (2.4).

Consider a general nonlinear continuous-time system of the form (2.1)-(2.2). Let $\gamma : \mathscr{X} \times \mathscr{U} \to \mathscr{U}$ be a C^1 function of its arguments, with $\mathscr{X} \subset \Re^n$ and $\mathscr{U} \subset \Re^m$. A nonlinear static state feedback control law is denoted by the expression $u = \gamma(x, v)$ with $v \in \mathscr{U}$.

Definition 2.10 *A feedback control law $u = \gamma(x, v)$ is locally* regular *if for all $(x, v) \in \mathscr{X} \times \mathscr{U}$ it follows that $\partial\gamma/\partial v \neq 0$.*

Regularity is regarded as a highly convenient property of static state feedback control laws since it leads to locally invertible input coordinate transformations. By the *feedback transformed* system we mean the system $\dot{x} = f(x, \gamma(x, v))$, $y = h(x, \gamma(x, v))$.

Definition 2.11 *Consider a system of the form (2.1)-(2.2) and two scalar functions $V(x)$, $s(y, u)$ as a storage function and a supply function, respectively. The system is said to be* feedback dissipative *with the functions V and s if there exists a* regular *static state feedback control law of the form, $u = \gamma(x, v)$, such that the feedback transformed system is dissipative.*

In other words, the following relation is satisfied

$$\dot{V} \leq s(y, v) \tag{2.41}$$

or equivalently,

$$\frac{\partial V}{\partial x} f(x, \gamma(x, v)) \leq s(y, v) \tag{2.42}$$

with $y = h(x, \gamma(x, v))$.

Similarly, if relations (2.41), (2.42) are strict inequalities, then the system is said to be feedback strictly dissipative.

The existence of a feedback control law, of the form $u = \gamma(x, v)$, for which the system is rendered dissipative must be assessed from the existence of solutions, for the control input u, of the following inequality,

$$\frac{\partial V}{\partial x} f(x, u) \leq s(h(x, u), v) \tag{2.43}$$

The solution of this problem will be revisited in Chapter 3.

For the passivation problem, passivation conditions can be established in terms of the KYP conditions. Since we do not have the KYP lemma for general systems of the form (2.1)-(2.2), this idea will be presented for nonlinear systems which are affine in the control input (2.29)-(2.30). A feedback control law which is affine in the control input will be used, i.e., $u = \alpha(x) + \beta(x)v$. Therefore, a regular control law of the form $u = \alpha(x) + \beta(x)v$ is regular if for all $x \in \mathscr{X}$ it follows that $\beta(x) \neq 0$.

Definition 2.12 *A system of the form (2.29)-(2.30) is said to be feedback passive with the storage function V and the supply function s if there exists a static state regular control law $u = \alpha(x) + \beta(x)v$ such that for the closed-loop system*

$$\dot{x} = [f(x) + g(x)\alpha(x)] + g(x)\beta(x)v$$

there exists a positive $\mathscr{C}^1$ function $V(x)$ satisfying $\forall x \in \mathscr{X}$,

$$L_{[f(x)+g(x)\alpha(x)]}V(x) \leq 0 \quad (2.44)$$

$$L_{g(x)\beta(x)}V(x) = h^T(x) \quad (2.45)$$

For the discrete-time case, the definition for a system that can be rendered dissipative by means of a static state feedback control law is also established from the fundamental dissipativity inequality in the discrete-time setting (2.10).

Definition 2.13 *Consider a system of the form (2.8)-(2.9) and two scalar functions $V(x)$, $s(y,u)$ considered as a storage function and a supply rate function, respectively. The system is said to be* feedback dissipative *with the functions V and s if there exists a* regular *static state feedback control law of the form, $u = \alpha(x,v)$, with v the new input, such that the feedback transformed system $x(k+1) = f(x(k), \alpha(x(k), v(k)))$ is dissipative.*

In other words, the following relation is satisfied

$$V(f(x,\alpha(x,v))) - V(x) \leq s(y,v) \quad (2.46)$$

with $y = h(x, \alpha(x,v))$.

Similarly, if relation (2.46) is a strict inequality, then the system is said to be feedback strictly dissipative.

The existence of a feedback control law, of the form $u = \alpha(x,v)$, for which the system is rendered dissipative must be assessed from the existence of solutions, for the control input u, of the following relation,

$$V(f(x,u)) - V(x) \leq s(h(x,u),v) \quad (2.47)$$

Definition 2.14 *Systems of the form (2.1)-(2.2) or (2.8)-(2.9) are said to be feedback passive with the functions V and s if they are feedback dissipative with $s = y^T u$.*

Definition for feedback dissipative systems will be rewritten by means of using the fundamental dissipativity equality (which introduces the dissipation rate function) instead of the dissipativity inequality, and by means of adapting Definition 2.4 for the continuous-time case, or Definition 2.6 for the discrete-time case for what it will be regarded as (V,s)-dissipativity, see Chapters 3 and 4. The definitions for feedback dissipativity given (Definitions 2.11 and 2.13) imply a definition of dissipativity slightly different from the ones given in Definition 2.1 for the continuous-time case and Definition 2.5 for the discrete-time case. In Definitions 2.1 and 2.5, the system is dissipative with an associated supply function s, the storage function V is not associated a priori with the system, while for the proposed feedback dissipativity problem function V is associated a priori with the system which is rendered dissipative. (V,s)-dissipative systems will be the class of dissipative systems treated in this dissertation.

Feedback dissipativity for nonlinear continuous-time systems will be revisited in Chapter 3. The property of feedback dissipativity for nonlinear discrete-time system will be treated in Chapter 5 from different points of view, basically making use of the fundamental dissipativity equality. The feedback passivity property will be treated in Chapter 7 making use of the properties of the relative degree and zero dynamics of passive discrete-time systems.

2.4.2 Research lines

The problem of passivation has attracted more attention that the feedback dissipativity problem in the literature. The problem of passivation is solved for the continuous-time setting from different viewpoints. As far as we know, the solution of the problems of passivation and feedback dissipativity in the nonlinear discrete-time setting has not been established yet.

2.4.2.1 Continuous-time case

The idea of rendering a system passive by means of a static state feedback was first proposed and solved by *(Kokotović and Sussmann, 1989)* [77] introducing the concept of *feedback positive systems*. A general solution to feedback passivity was given by the work of *(Byrnes et al., 1991)* [12], where necessary and sufficient conditions for passivation of nonlinear affine-in-control continuous-time systems are established in terms of geometric properties of the system, such as, the relative degree and the stability of its zero dynamics. These results were later extended by *(Santosuosso, 1997)* [147] to systems having an affine feedthrough term between the input and the output.

Referring the linear case, the results of *(Saberi et al., 1990)* [145] appeared, where it is presented that any controllable linear system of the form $\dot{x} = Ax + Bu, y = Cx$ is feedback equivalent to a positive real system, if and only if, CB is nonsingular and the system is weakly minimum phase. The problem of passivation oriented to solve the stabilization problem is proposed in *(Kelkar et al., 2000)* [72] and *(Xie et al., 1998)* [185]. In some works the problem of passivation is treated as what is regarded as the positive real control problem (see Chapter 8) consisting in designing a controller which renders the closed-loop transfer function positive real, see *(Bao et al., 1996)* [10], *(Kelkar and Joshi, 1998)* [71] in which not only passivity is ensured, but also an H_∞ performance. The linear feedback dissipativity has also been addressed in the literature in the works of *(Picci and Pinzoni, 1992)* [137] and *(Yuliar et al., 1998)* [189].

An alternative approach to the one presented in *(Byrnes et al., 1991)* [12] for solving the passivation problem in nonlinear systems is presented by *Sira-Ramírez* in *(Sira-Ramírez and Angulo-Núñez, 1997)* [157], *(Sira-Ramírez, 1998)* [159] which will be briefly explained afterwards.

Another approach for solving the passivation problem is of iterative type. In this line, the speed-gradient algorithm is used *(Fradkov, 1991)* [35], *(Fradkov et al., 1995)* [36, 37], *(Fradkov and Pogromsky, 1998)* [40], *(Seron et al., 1995)* [153], as well as iterative learning algorithms *(Arimoto and Naniwa, 2000)* [9].

The problem of output feedback passivity has been also proposed for nonlinear affine-in-the-input systems, either for the static case *(Jiang and Hill, 1998)* [62], *(Fradkov and Hill, 1998)* [39], *(Chun and Seo, 2001)* [22], or for the dynamic case *(Chun and Seo, 2001)* [22]. The problem of output feedback passivity consists of achieving passivity by feedback using the output and the new control input. In the recent work of *(Chun and Seo, 2001)* [22], sufficient conditions for static and dynamic output passivation are proposed. The static output feedback passivity is an extension of the one given in *(Jiang and Hill, 1998)* [62]. They achieve dynamic output feedback passivity by means of observer-based control and a supplementary input.

Feedback passivity by means of a state feedback for systems containing uncertain elements is also studied, see *(Seron et al., 1995)* [153], *(Su and Xie, 1996)* [166] and *(Lin and Shen, 1999)* [94]. A recent work which treats this problem is *(Duarte-Mermoud et al., 2002)* [26] where multiple-input multiple-output nonlinear systems with linear parametric uncertainties are rendered passive by means of an adaptive static state controller.

All these passivation-related results are for the affine continuous-time case. The problem of passivation for non-affine continuous-time systems is proposed in Fradkov's work *(Fradkov et al., 1995)* [37] where the passivation problem is seen as a goal-oriented problem by means of the *speed-gradient* algorithm. Another recent approach to the problem of feedback dissipativity of general continuous-time systems is the one given by *(Sira-Ramírez and Navarro-López, 2000)* [161]. These two approaches *(Fradkov et al., 1995)* [37] and *(Sira-Ramírez and Navarro-López, 2000)* [161] will be extended for the discrete-time case in Chapter 5.

An interesting survey of passivity and passivation properties and their implications in dynamical systems is given in *(Sepulchre et al., 1997)* [152], where definite connections between passivity, Lyapunov-based control and inverse optimal feedback can be found. Other works referring feedback passivity are *(Byun et al., 2000)* [15], *(Jadbabaie and Abdallah, 1997)* [59] for the nonlinear case, and *(Pogromsky et al., 1998)* [138] for nonlinear and linear switching systems.

We are interested in studying the feedback dissipativity problem and its particular case of passivation problem for nonlinear dynamics. Works referring the passivation problem are more numerous in the literature than the ones treating the feedback dissipativity problem, indeed, they can give ideas to treat the feedback dissipativity problem.

Two passivation research lines will be highlighted at this point: on the one hand, the approach presented in *(Byrnes et al., 1991)* [12] which establishes necessary and sufficient conditions for a system to be feedback passive via the properties of the relative degree and the zero dynamics. This result will be the basis for most of the passivation-concerned results existing in the discrete-time setting. On the other hand, the work appeared in *(Sira-Ramírez and Angulo-Núñez, 1997)* [157], *(Sira-Ramírez, 1998)* [159] is pointed out, in which the passivation conditions are the conditions for which there exists a control fulfilling the fundamental passivity inequality. Our feedback dissipativity studies in the nonlinear discrete-time setting will use the ideas underlying these two approaches.

Geometric solution

The solution of the passivation problem of nonlinear affine-in-control continuous-time systems is given in *(Byrnes et al., 1991)* [12]. They take benefit of the characteristics that the relative degree and the zero dynamics have for a passive system. It is proven that any passive system of the form (2.29)-(2.30) with a positive storage function, necessarily has relative degree one at $x = 0$ and is weakly minimum phase, i.e., the system zero dynamics is Lyapunov stable. These conditions are used to characterize the feedback passivity.

Therefore, the passivation is established as a structural property of the system, an intrinsic geometrical system property, a fact that can be considered as an advantage.

The main problem of this approach is that for feedback passivity one must search for an output with respect to which not only will the system have relative degree one, but also have a Lyapunov stable zero dynamics. On this line, the non-minimum phase systems are left out for passivation considerations.

This approach will be extended for the discrete-time case in Chapter 7.

Using the passivity inequality

The second approach for solving the passivation problem which is interesting to explain since it will be used in the sequel is the one proposed in *(Sira-Ramírez, 1998)* [159], where the passivation condition is established as a geometric *transversality condition*. The basis of this approach is that given a system of the form $\dot{x} = f(x) + g(x)u$ and a $\mathscr{C}^1$ positive definite function associated with the system, V, such that $V : \mathscr{X} \to \Re$, known as the storage function, the vector field f is decomposed into three parts: the dissipative part, the non-dissipative part and the invariant part, depending the stored energy of the system along the trajectories of the system decreases, increases or remains, respectively.

The feedback law proposed in order to render the system passive is of the type $u = \alpha(x) + \beta(x)v$. The underlying idea of the passivation method is the proposal of a control u so that the closed-loop system can fulfil the passivity inequality with v the new input of the system. This approach associates the function V with the system, i.e., the open-loop storage function with respect to which the closed-loop system will be passive is established. The passivation condition guarantees the existence of the control u which makes the closed-loop system fulfil the passivity inequality.

A geometric interpretation of the proposed passivation process is given. The storage function V induces a foliation on the system state-space defined by the different energy levels $V(x) = constant$. The passivation process aims to project the system non-dissipative part on the V-tangent distribution along the direction of the field g.

The ideas underlying this approach will be used in order to solve the feedback dissipativity problem in the discrete-time domain in Chapter 5.

2.4.2.2 Discrete-time case

As far as we know, either the problem of feedback dissipativity or its particular case of passivation of nonlinear discrete-time systems has not been solved. The origin of this difficulty is that the structural properties of a passive system have not been established for the discrete-time setting yet. It must be also pointed out that general KYP conditions have not been established yet for the discrete-time domain. Then, there is a lot to do in this field.

The nonlinear passivation-related results reported in the literature are the ones proposed in Byrnes and Lin's work, see *(Byrnes and Lin, 1993)* [13], *(Byrnes and Lin, 1994)* [14], *(Lin, 1993)* [82]. They give solution to the problem of *feedback losslessness*, the problem of rendering a discrete-time affine system of the form (2.33)-(2.34) lossless by means of a static state feedback.

The feedback losslessness characterization is proposed in the line of *(Byrnes et al., 1991)* [12] by means of the properties of the relative degree and zero dynamics of the system. It will be interesting to explore which are the difficulties for achieving feedback passivity, and why there only exist results for the feedback losslessness case. The main problem to solve is to find the structural geometric properties of a discrete-time passive system. The passivation problem will be solved by means of the study of the properties of the relative degree and zero dynamics for a class of nonlinear discrete-time systems in Chapter 7.

Referring the linear case, the problem of feedback passivity and feedback (Q,S,R)-dissipativity is treated in the framework of what is regarded as positive real control problem or quadratic dissipative control problem, respectively, in connection with H_∞ design. In *(de Souza and Xie, 1992)* [164], *(Souza et al., 1993)* [165] a feedback controller is designed for the closed-loop system to be robust asymptotic stable and positive real. Concerning the feedback dissipativity problem the works of *(Tan et al., 1999)* [170], *(Tan et al., 2000)* [171] are pointed out. They are concerned with what is regarded as the robust dissipative control problem, i.e., the design of a feedback control law which achieves robust asymptotic stability and strictly (Q,S,R)-dissipativity. Necessary and sufficient conditions for the solvability of the dissipative control problem and the robust dissipative control problem are given. Their solution is based on the solution of a linear matrix inequality. Both the linear static state feedback and the dynamic output feedback controllers are considered. This approach will not be considered in this dissertation.

2.5 The DBC problem. A blend of dissipativity properties and control purposes

Dissipative and passive systems have interesting stability properties, then their feedback stabilization can be proposed. The stabilization is another interesting problem to solve from the dissipativity point of view.

In this section, the stability properties of passive and dissipative systems are treated, in addition to the use of these characteristics for control purposes. Not only will system stability be treated, but also some notes on the stability of interconnected passive systems.

2.5.1 Dissipativity and stability

The results appearing in the literature concerning the study of stability and stabilization properties of passive systems are more numerous than the ones related to the passivation problem, either for the continuous-time case or the discrete-time case.

The relation between passivity and Lyapunov stability can be established by considering the storage function $V(x)$ as a Lyapunov function. This fact can be easily followed from the KYP conditions, actually, from the first-type ones previously regarded as stability-kind condition (see Section 2.3). In addition, it cannot be forgotten that, as Section 2.3 presented, for the case of linear systems, the characterization of passive systems through the KYP lemma gives a frequency stability property.

Dissipativity and passivity concepts in connection to the stabilization problem have been extensively used in many different works in the literature, some of the most recent works being: *(Byrnes et al., 1991)* [12], *(Sepulchre et al., 1997)* [152], *(van der Schaft, 1996)* [148], *(Ortega et al., 1998)* [132], *(Fradkov and Pogromsky, 1998)* [40], *(Fradkov et al., 1999)* [41], *(Lozano et al., 2000)* [97].

An advance in the formalization of the PBC in the nonlinear continuous-time setting is given by the recent and interesting practical-oriented work of Ortega, van der Schaft and coworkers, see *(Ortega et al, 2001)* [134] and *(van der Schaft and Ortega, 2001)* [149] and references therein, either theory or applications. These works contribute in the PBC theory, giving a more general framework of application than the one given in *(Ortega et al., 1999)* [133]. They view the plant and controller system as energy-transformation devices interconnected in order to achieve a desired behaviour. The structure interconnecting the plant and the controller is power-conserving, and its geometric formalization is made by means of Dirac structures. Furthermore, an alternative stabilization method to the standard PBC is proposed. It is the *Interconnection and Damping Assignment* (IDA) methodology which overcomes the dissipation obstacle of the standard PBC methodology *(Ortega and Spong, 1989)* [127], that is, the standard PBC is only applicable when the energy dissipated by the system is bounded. In the IDA controller design, the storage function is obtained as a result of the chosen desired subsystems interconnection and damping.

Although the problem of passivation of nonlinear discrete-time systems is not solved, the passivity approach is used to solve the stabilization problem, in absence of a KYP lemma. Lin's work follows this line. We, then, find the solution of the problem of stabilization in bilinear systems *(Lin and Byrnes, 1994)* [83] or for more general discrete-time systems *(Lin and Byrnes, 1995)* [87, 89], *(Lin, 1996)* [91]. The works *(Lin, 1995)* [90], *(Lin and Byrnes, 1994)* [84, 85] can be also pointed out.

The same is true for the non-affine nonlinear continuous-time case. General necessary and sufficient conditions for feedback passivity have not been established for this case; however, the only existing necessary KYP conditions for non-affine systems *(Lin, 1995)* [86], *(Lin, 1996)* [93] allow the stabilization problem to be solved for this kind of general continuous-time systems. The problem of the stabilization of non-affine passive systems is also treated in *(Shiriaev, 2000)* [155] by means of the use of the V-detectability concept, a generalization of the zero-state detectability presented in *(Byrnes et al., 1991)* [12]. Another recent work uses the *Energy Shaping plus Damping Injection* (ESDI) method for the stabilization of non-affine continuous-time systems *(Sira-Ramírez and Navarro-López, 2000)* [161].

In this dissertation, we are more interested in the problem of studying dissipativity, passivity, feedback dissipativy, and passivation problems of nonlinear discrete-time systems, however, we will also treat, as a secundary goal, the use of dissipativity and passivity for stabilization purposes. We will not analyze the structural properties that a dissipative or passive system has to fulfil in order to be stabilizable. On the contrary, dissipativity and feedback dissipativity definitions will be used to propose a DBC design method for nonlinear discrete-time systems, see Chapter 6. In addition, frequency-domain characteristics of dissipativity will be proposed as a way for analyzing frequency-based feedback stability criteria in the discrete-time setting, see Chapter 8. The fascinating feature of dissipativity is that dissipative systems can exhibit a specially intimate relationship between input-output stability (whose basis is the boundness of input-output mappings) and state-space stability (commonly referring to the zero-input response to a non-zero initial state). For linear systems, asymptotic and input-output stability are equivalent, see *(Willems, 1970)* [178]; the partial generalization of this result to nonlinear systems is made in *(Hill and Moylan, 1980)* [55] through dissipativity concepts, this problem has not been treated for discrete-time systems. Although there are a lot of stability studies of feedback systems in terms of input-output properties, the advantage of dissipativity is that it has a frequency-domain interpretation, this fact is stated in the KYP lemma. See Chapter 8 for more details.

Since stabilization of passive systems is more usually treated than passivation, a question arises immediately: is the passivation property of a system stronger than its stabilization property? A passive system is Lyapunov stable, but not every stable system is passive. Anyway, passive systems are not exactly a subset of stable systems, the reason for this fact is basically the condition of detectability-type that is needed for a passive system to be stabilizable. In linear systems, a passive system is stabilizable if it is detectable, in other words, the unobservable parts of the system must be asymptotically stable *(Sepulchre et al., 1997)* [152]. The analogous concept to detectability translated for the nonlinear case gives rise to the *zero-state detectability* in *(Byrnes et al., 1991)* [12]. These basic results are given for affine nonlinear continuous-time systems of the form (2.29)-(2.30). The zero-state detectability concept is extended for non-affine systems by means of the V-detectability concept *(Shiriaev, 2000)* [155].

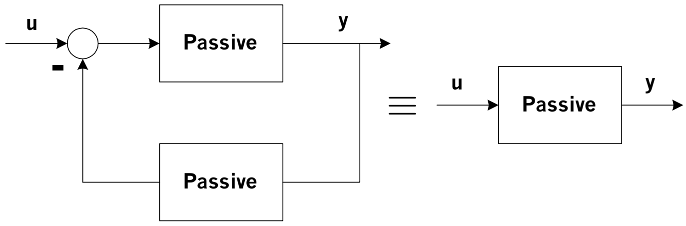

Figure 2.1: Passive block interconnection

Another remarkable stability property of passive systems, indeed, one of the most important passivity results, and besides one of the significant results of feedback interconnections is that a negative feedback loop consisting of two passive systems is passive *(Sepulchre et al., 1997)* [152], see Figure 2.1. In addition, under an additional detectability condition, this feedback is also stable.

In case a controllable linear system is passive, then, its transfer function is positive real, that is, it has a Nyquist plot on the right-half plane, which means that an infinite gain proportional control can be introduced without destabilizing the system. These frequency stability properties are also exploited for interconnected systems, they appear to be as a complement of the gain property characterized by the small gain theorems. The connection between passivity and the small gain theorems are presented in *(Desoer and Vidyasagar, 1975)* [25] (also treating the discrete-time case), *(van der Schaft, 1996)* [148], or *(Khalil, 1996)* [76]. The study of the preservation of passivity and other kinds of dissipativity under block interconnection is revisited in Chapter 8 for the discrete-time setting.

If we speak of stability, passivity and block interconnection we are compelled to mention the relations between passivity and the concept of *hyperstability* or *absolute stability*. A linear system is said to be absolutely stable if its feedback interconnection to any static nonlinearity in a sector (α, β) is globally asymptotically stable. This concept have strong relations with the concept of passivity and passivity of interconnected systems. These broader implications of passivity are established in the seminal *Popov*'s work *(Popov, 1973)* [141], either for the continuous or the discrete-time case.

In conclusion, passivity is an interesting tool for studying the systems stability. It is presented to be an alternative to the Lyapunov-like methods for stabilization purposes. The passivity approach gives, as its main advantages: passivity preservation under block interconnection and the valuable physical orientation of the stabilization problem, having energy functions with a physical meaning.

2.5.2 Dissipativity: a tool for stabilization

A fundamental control problem is that of stabilization of the system trajectories around desired equilibria. This problem can be solved exploiting the interesting properties referring stability that dissipative and passive systems present.

The application of the passivity approach to systems control is found in a great variety of works written in the last decades. There are mainly devoted to mechanical (such as robots), electro-mechanical (such as induction motors) and purely electrical systems (such as DC-to-DC power converters), for a survey see *(Ortega et al., 1997)* [131]. Some examples of application to robotics and Lagragian systems are the works of *(Ortega and Spong, 1989)* [127], *(Ortega et al., 1995)* [130], *(Ortega et al., 1999)* [133] and the ones presented by Kelly and coworkers *(Kelly et al., 1998)* [73], *(Kelly and Santibáñez, 1998)* [74], *(Kelly, 1999)* [75]. Electro-mechanical systems were addressed in *(Ortega, 1991)* [128], *(Ortega and Espinosa, 1991)* [129], *(Nicklasson et al., 1997)* [126] or *(Espinosa et al., 1997)* [30]. Passivity-based controllers for power electronics were given, for instance, in *(Sira-Ramírez et al., 1997)* [158] and *(Escobar and Sira-Ramírez*, 1998) [28]. Passivity-based control design has also been applied to other kind of nonlinear systems, see *(Sira-Ramírez and Angulo-Núñez, 1997)* [157] and the works of *(Egeland and Godhavn, 1994)* [27], *(Johanssen and Egeland, 1993)* [63], and *(Fossen and Strand, 1999)* [34].

Passivity has also appeared related to other kinds of control approaches, giving a simplification of them since it takes advantage of the physical system properties. We can stand out: the nonlinear observer design, in which by means of passivity characteristics a reduction of the observer parameters to choose is achieved *(Fossen and Strand, 1999)* [34]; the adaptive control *(Ortega et al., 1997)* [131], *(Andrievsky et al., 1996)* [7], *(Jiang et al., 1996)* [61], *(Panteley et al., 1998)* [135] or *(Seron et al., 1995)* [153]; the robust control *(Guillard et al., 1996)* [49] or *(Lin and Byrnes, 1996)* [92], as well as the use of passivity as a previous step in the application of other types of controllers (as is the case of the passivation together with the sliding-mode control, *(Escobar, 1999)* [29]). The passivity-based control is combined with the property of differential flatness in *(Sira-Ramírez, 2000)* [160]. We can also stand out the use of the property of passivity in neural networks, see *(Yu and Li, 2000)* [188], and *(Commuri and Lewis, 1997)* [19] where it is shown that selecting the weight-update law to ensure state-strict passivity of the system is sufficient to prove stability for the closed-loop system with the Cerebellar Model Articulation Computer (CMAC) neural network. Passivity concepts has also been applied to fuzzy control systems, see *(Calcev et al., 1998)* [16] or to the problem of synchronization, see *(Fradkov and Markov, 1997)* [38] or *(Sira-Ramírez and Cruz-Hernández, 2000)* [162]. The passivity approach is also used in order to study the stability of hybrid systems, denoting by an hybrid system a system which switches between different operation modes or continuous behaviours. In this line, two recent works can be pointed out: *(Žefran et al., 2001)* [192] and *(Pogromsky et al., 1998)* [138]. The former proposed an extension of the notion of passivity to hybrid systems in such a way that passivity guarantees stability; multiple storage functions, one for each operating mode, are used, and it is applied to haptics devices and to teleoperation. The latter solves the problem of finding an appropriate switching rule such that the closed-loop system is globally asymptotically stable with zero input and passive, here, the passivation is achieved via controlled switching and is oriented to stabilization purposes.

The control design methodology attracting our attention is the *Energy Shaping plus Damping Injection* (ESDI) one, which consists of modifying the stored energy of the system for the desired equilibrium to be a minimum of a new energy function for the closed-loop system and the addition, through state or output feedback, of the required dissipation in order to enhance the dissipation structure of the underlying stabilization error system. Generalities and details of this technique can be found in *(Ortega, 1991)* [128], *(Ortega et al., 1998)* [132] and the work of *(Sira-Ramírez et al., 1997)* [158]. In the literature, the ESDI idea has been applied in two different ways, the main differences between them are related to the way the *Energy Shaping* is made. On the one hand, we have the approach of *(Ortega et al., 1999)* [133] and *(Maschke et al., 1998)* [99], for which the expression for the closed-loop system Lyapunov-like energy function will not be needed to be known in order to ensure the stability of the system equilibrium point. An application of this dissipativity-based control is given by the author of this dissertation, see *(Navarro-López and Fossas-Colet, 2001)* [117]. On the other hand, the work of *(Sira-Ramírez, 1998)* [159] and *(Sira-Ramírez and Navarro-López, 2000)* [161] is based on the explicit knowledge of the expression for the system energy. Indeed, the energy shaping is made in a different way in the two approaches. In the first one, the new energy function is conceived as a variation of the initial one ($V+\Delta V$, considering V the initial energy function), on the contrary, in the second one, the energy shaping is represented by the energy associated with the error system, which definition is based on the proposal of an auxiliary dynamics. A difference between the two mentioned approaches can be also detected through the purpose of the dissipation injected to the system.

The ESDI method proposed in *(Sira-Ramírez, 1998)* [159] will be extended for nonlinear non-affine-in-control systems in Chapter 3, for the continuous-time case, and in Chapter 6 for the discrete-time case.

As it can be seen, the dissipativity and passivity stabilization-related results in the discrete-time setting are much less numerous than the ones given in the continuous-time case.

2.6 Conclusions

The study of dissipativity and passivity properties in a system is divided into three main problems:

- The characterization of passivity and dissipativity by means of the KYP conditions.
- The study of the feedback dissipativity and the passivation properties of a system.
- The use of dissipativity, passivity, feedback dissipativity and passivation properties for control purposes, what is known as DBC or PBC.

This chapter has reviewed dissipativity-related results appeared in the literature. Most of the non-existing results and open problems are for nonlinear discrete-time systems, as well as nonlinear non-affine-in-control continuous-time systems.

Among the results which have not been established yet, the following ones can be stood out:

- **The establishment of necessary and sufficient conditions for the characterization of either dissipative or passive general nonlinear discrete-time systems, what is regarded as the KYP properties**. The characterization of passive discrete-time systems is given in the literature for linear systems *(Hitz and Anderson, 1969)* [58], and for nonlinear systems affine in the control input in *(Byrnes and Lin, 1993)* [13]. Necessary conditions for a system to be passive are also proposed in a different line in *(Lin and Byrnes, 1995)* [87], which are generalized for the non-affine case in *(Lin, 1995)* [86]. Lossless systems affine in the control input are also characterized *(Byrnes and Lin, 1994)* [14], *(Sengör, 1995)* [151]. Necessary and sufficient conditions for dissipative systems affine in the control input are presented in *(Sengör, 1995)* [151] and *(Göknar and Sengör, 1998)* [44]; the linear case is treated in *(Goodwin and Sin, 1984)* [45]. In both cases, the supply function takes the form $s(y,u) = y^T Qy + 2y^T Su + u^T Ru$. For single-input multiple-output nonlinear non-affine-in-the-input systems, the KYP conditions are proposed in *(Monaco and Normand-Cyrot, 1997)* [111], *(Monaco and Normand-Cyrot, 1999)* [113], in this approach, the authors phocus on systems which can be expanded by exponential Lie series.
- **The KYP lemma for the non-affine-in control continuous-time case**. There only exist necessary conditions for the characterization of passive general systems in *(Lin, 1995)* [86], *(Lin, 1996)* [93]. Dissipativity of non-affine continuous-time systems will be briefly examined in Chapter 3.
- **The establishment of the conditions that a nonlinear discrete-time system has to fulfil in order to be feedback dissipative or feedback passive**. Only the feedback losslessness property has been treated *(Byrnes and Lin, 1994)* [14]. The problems of feedback dissipativity and feedback passivity will be treated in Chapters 5 and 7, respectively.
- **Implications of dissipativity and passivity in feedback systems stability in the discrete-time setting**. Dissipativity can be related to the most important frequency-based feedback stability criteria. Some comments on this topic will be given in Chapter 8.
- **The implications of passivity and dissipativity in the relative degree and the zero dynamics of nonlinear discrete-time systems** have not been established yet, they have only been studied for the losslessness case, see *(Byrnes and Lin, 1994)* [14]. These properties for the passivity case will be examined in Chapter 7 in addition to use them for the characterization of the feedback passivity property for a class of nonlinear discrete-time systems. Some comments on the relative degree properties of (Q,S,R)–dissipative systems are also given in Chapter 7, as well as, a word on the zero dynamics of a class of dissipative systems.
- **The concepts of dissipativity and passivity have not been exploited in the nonlinear discrete-time setting in an adequate way**. The existing results are

adaptations of continuous-time results, maybe that is the reason why these results are limited and non-general. They have not taken advantage of the discrete-time dynamics properties. It is necessary to use or define discrete mathematical tools to treat dissipativity and passivity in the discrete-time domain. In this dissertation, we have followed the already existing approaches in order to treat dissipativity-related concepts.

- **Characterization of dissipativity, passivity, feedback dissipativity and passivation of multiple-input multiple-output nonlinear discrete-time systems**. In this dissertation, the study of dissipativity and losslessness for a class of multiple-input multiple-output nonlinear discrete-time systems is treated in Chapter 4. The feedback dissipativity problem is only treated for single-input single-output systems in Chapter 5, whereas Chapter 7 deals with the passivation of a class of multiple-input multiple-output nonlinear discrete-time systems.

- **The study of the preservation of dissipativity and passivity under block interconnection in the discrete-time setting**. This point will be treated in Chapter 8.

- **The study of the relation between input-output stability and state-space stability through dissipativity concepts in the discrete-time setting**. This will not be studied in this dissertation.

In conclusion, there is much to do in the field of discrete-time dissipativity, standing out in the nonlinear discrete-time setting.