Chapter 6

Dissipativity-based stabilization in the nonlinear discrete-time setting

6.1 Introduction

In this chapter, some dissipativity stability-related results in the nonlinear discrete-time setting will be formalized. The most important and immediate consequences of dissipativity and feedback dissipativity properties in systems stability will be shown for a class of nonlinear discrete-time systems. The feedback dissipativity results achieved in Chapter 5 will be used to tackle the stabilization problem. Sufficient conditions under which a system which can be rendered (V,s)-dissipative can be locally stabilizable are given. The extension of the ESDI dissipativity-based stabilization procedure to a class of dissipative nonlinear single-input single-output discrete-time systems will be proposed. It will be an extension of the ESDI methodology existing for the continuous-time nonlinear case, see *(Ortega and Spong, 1989)* [127] and *(Sira-Ramírez, 1998)* [159], and it is based on the results presented in Chapter 3. The proposed dissipativity-based control scheme is illustrated by means of the two examples used in Chapter 5. The feedback dissipativity methodologies presented in Chapter 5 will be used in the application of the ESDI control scheme to the examples so as to solve the stabilization of the system orbits around a desired equilibrium. Simulations are presented. The interest in the application of the ESDI methodology in the examples is that, on the one hand, the performance of the open-loop buck response is improved and, on the other hand, in the nonlinear example, a desired fixed point (which is unstable in open loop) is stabilized.

The chapter is organized as follows. Section 6.2 presents some preliminary notes on the stability properties of (V,s)-dissipative systems. Section 6.3 deals with stability properties of a class of feedback dissipative systems. Here, sufficient conditions for a class of feedback dissipative systems to be stabilizable are posed. Section 6.4 presents the extension of the ESDI design method to nonlinear discrete-time systems. This dissipativity-based stabilization method is applied to two examples: the discrete-time model for the DC-to-DC buck converter (5.13) and an academic nonlinear example

(5.23), both presented in Chapter 5. Sections 6.5 and 6.6 give, respectively, the ESDI methodology applied to the buck example model using the feedback dissipativity methods presented in Section 5.2 and Section 5.3. Sections 6.7 and 6.8 present, respectively, the ESDI methodology applied to the nonlinear model (5.23) using the feedback dissipativity methods presented in Section 5.4.1 and Section 5.4.2. Conclusions and suggestions for further research are given in the last section.

6.2 Preliminary notes

Let nonlinear single-input single-output discrete-time systems of the form (5.1)-(5.2) having $\overline{x}$ as an isolated fixed point of $f(x,\overline{u})$, with $\overline{u}$ a constant, i.e., $f(\overline{x},\overline{u}) = \overline{x}$. Consider a positive definite C^2 function $V : \mathscr{X} \to \Re$ associated with the system (5.1)-(5.2) and addressed as the *storage function*. A second C^2 function is also considered, called the *supply function*, denoted by $s(y,u)$, with $s : \mathscr{Y} \times \mathscr{U} \to \Re$. Let a $\mathscr{C}^1$ regular nonlinear static state feedback control law $u = \alpha(x,v)$, $\alpha : \mathscr{X} \times \mathscr{U} \to \mathscr{U}$ with $\alpha(\overline{x},0) = \overline{u}$ and $(0,0)$, $(\overline{x},\overline{u})$ contained in $\mathscr{X} \times \mathscr{U}$. Let a dissipation rate function ϕ as given in Definition 4.1, now, with $\phi(\overline{x},\overline{u}) = 0$.

Definition 6.1 *The supply function $s(y,u)$ is said to satisfy the zero-input-output (ZIO) property if*

$$\begin{aligned} s(0,u) &= 0, \quad \forall\, u \in \mathscr{U} \\ s(y,0) &= 0, \quad \forall\, y \in \mathscr{Y} \end{aligned} \tag{6.1}$$

All the results relating stability are given for a class of dissipative systems with supply function s satisfying the ZIO property.

(V,s)-dissipativity and V-passivity have immediate consequences referring to the stability of the system when no control is applied and the stability of the zero dynamics, as it will be illustrated in Theorem 6.2. *Lyapunov's Stability Theorem* for discrete-time systems is presented, since it will be used in the sequel.

Theorem 6.1 LYAPUNOV'S STABILITY THEOREM (LaSalle, 1986) *[80] Consider a discrete-time system of the form $x(k+1) = f(x(k))$. Let $\overline{x}$ be a fixed point of the system. The system dynamics will be restricted to a neighbourhood S of $\overline{x} \in \Re^n$, $f : S \to S$. Suppose that there exists a continuous, positive definite function $V : S \to \Re^+$, with $V(\overline{x}) = 0$. If $V(x(k+1)) - V(x(k)) \leq 0$, $\forall x \in S$, then $\overline{x}$ is a stable equilibrium (if $V(x(k+1)) - V(x(k)) < 0$, then $\overline{x}$ is asymptotically stable).*

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Theorem 6.2 *For (V,s)-dissipative systems of the form (5.1)-(5.2) with positive definite storage functions and supply functions satisfying the ZIO property, the fixed point, $\overline{x}$, of the zero-input dynamics $x(k+1) = f(x(k),0)$ is Lyapunov stable (resp., asymptotically stable if the system is strictly (V,s)-dissipative). Similarly, if the output y of this class of dissipative systems is held to be zero in an indefinite fashion by means of an appropriate control input, then the* zero dynamics *is Lyapunov stable (resp., asymptotically stable if the system is strictly (V,s)-dissipative).*

Proof. These statements can be proved restricting (4.3) for $u = 0$ in the first case, and for $y = 0$ in the second one, and considering conditions (6.1) in addition to Lyapunov's Stability Theorem 6.1.

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6.3 Dissipativity, passivity and stability

A nonlinear regular static state feedback control law of the form $u = \alpha(x, v)$, which achieves either (V, s)-*dissipativity* or *strict* (V, s)-*dissipativity*, induces an *implicit damping injection* which makes the system locally stable (resp., locally asymptotically stable if strict (V, s)-dissipativity is achieved) for certain particular values of the transformed control input. The following theorem clarifies this assertion.

Theorem 6.3 (Navarro-López et al., 2002) *[119] Consider the system (5.1)-(5.2), and two scalar functions $V(x)$ and $s(y, v)$ as a storage function and a supply function satisfying the ZIO property, respectively. Suppose $\bar{x}$ an isolated fixed point for $f(x, \bar{u})$, with $\bar{u}$ a constant. Let $\phi(x, u)$ be a dissipation rate (resp., strict dissipation rate) function. Suppose there exists a feedback control law, $u = \alpha(x, v)$, defined in an open neighbourhood $\mathscr{W} = \widetilde{\mathscr{X}} \times \widetilde{\mathscr{U}}$ with $\widetilde{\mathscr{X}} \subset \mathscr{X}$, $\widetilde{\mathscr{U}} \subset \mathscr{U}$ which renders the system (V, s)-dissipative (resp., strictly (V, s)-dissipative). Consider $x = \bar{x}$ the unique x for which $V(x) = 0$ and $\phi(x, \alpha(x, 0)) = 0$. Let $\mathscr{W}$ invariant with respect to $x(k+1) = f(x(k), \alpha(x(k), 0))$ and $(\bar{x}, \bar{u}) \in \mathscr{W}$. Then, for all $x \in \widetilde{\mathscr{X}}$, the control law $u = \alpha(x, 0)$ locally stabilizes (resp., locally asymptotically stabilizes) the system to $\bar{x}$.*

Proof. Since $u = \alpha(x, v)$ achieves strict (V, s)-dissipativity in an open neighbourhood $\mathscr{W} = \widetilde{\mathscr{X}} \times \widetilde{\mathscr{U}} \subset \mathscr{X} \times \mathscr{U}$ (the argument is the same for (V, s)-dissipativity), relation (4.3) can be considered with $u = \alpha(x, v)$ and $s(y, v)$, then

$$V(f(x, \alpha(x, v))) - V(x) = s(h(x, \alpha(x, v)), v) - \phi(x, \alpha(x, v)), \;\; \forall (x, v) \in \mathscr{W} \tag{6.2}$$

In particular, for $v = 0$ and considering (6.1), (6.2) yields to,

$$\begin{aligned} V(f(x, \alpha(x, 0))) - V(x) &= s(h(x, \alpha(x, 0)), 0) - \phi(x, \alpha(x, 0)) = \\ &= -\phi(x, \alpha(x, 0)) < 0, \;\; \forall x \in \widetilde{\mathscr{X}} \end{aligned} \tag{6.3}$$

Taking into account (6.3) and the fact that $x = \bar{x}$ is the unique x for which $V(\bar{x}) = 0$ and $\phi(\bar{x}, \alpha(\bar{x}, 0)) = 0$, the result of the theorem follows from fundamental Lyapunov's Stability Theorem 6.1, and consequently, for all $x \in \widetilde{\mathscr{X}}$, $f(x, \alpha(x, 0))$ converges to $\bar{x}$.

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Proposition 6.1 (Navarro-López et al., 2002) *[119] Consider the system (5.1)-(5.2) and two scalar functions $V(x)$ and $s(y, v)$ as a storage function and a supply function satisfying the ZIO property, respectively. Suppose $\bar{x}$ an isolated fixed point for $f(x, \bar{u})$. Let*

$\phi(x,u)$ be a dissipation rate (resp., strict dissipation rate) function. Let $V(\bar{x}) = 0$ and $\phi(\bar{x},\bar{u}) = 0$. The system (5.1)-(5.2) is locally stabilizable (resp., asymptotically stabilizable) to $\bar{x}$ by the control $u = \alpha(x,0)$ with $\alpha(x,v)$ a control law which renders the system (V,s)-dissipative (resp., strictly (V,s)-dissipative) if there exist $(x_0,u_0) \in \mathscr{X} \times \mathscr{U}$ and $\mathscr{W}$ a neighbourhood of (x_0,u_0) containing $(\bar{x},\bar{u})$ for which the following relations are valid

1.
$$V(f(x_0,u_0)) - V(x_0) = -\phi(x_0,u_0), \tag{6.4}$$

2.
$$\left.\frac{\partial V(z)}{\partial z}\right|_{z=f(x,u)} \frac{\partial}{\partial u} f(x,u) + \frac{\partial}{\partial u}\phi(x,u) \neq 0, \qquad \forall (x,u) \in \mathscr{W}, \tag{6.5}$$

3. *$\mathscr{W}$ invariant with respect to $x(k+1) = f(x(k),\alpha(x(k),0))$.*

Proof. Let $u = \alpha(x,v)$ achieve strict (V,s)-dissipativity (the argument is the same for (V,s)-dissipativity). Let $\mathscr{X} \times \mathscr{U}$ an open neighbourhood. Consider the following $\mathscr{C}^1$ function $F : \mathscr{X} \times \mathscr{U} \to \mathscr{R}$ defined by:

$$F(x,u) = V(f(x,u)) - V(x) + \phi(x,u)$$

From (6.4) and (6.5), by the implicit function theorem, there exist open neighbourhoods $\widetilde{\mathscr{X}} \subset \mathscr{X}$ of x_0, $\widetilde{\mathscr{U}} \subset \mathscr{U}$ of u_0, with $\mathscr{W} = \widetilde{\mathscr{X}} \times \widetilde{\mathscr{U}}$, and a unique map $\beta : \widetilde{\mathscr{X}} \to \widetilde{\mathscr{U}}$ such that

$$V(f(x,\beta(x))) - V(x) = -\phi(x,\beta(x)), \;\; \forall x \in \widetilde{\mathscr{X}} \tag{6.6}$$

Since s satisfies the ZIO property, for $(x,u,v=0)$, conditions (6.4) and (6.5) are equivalent to:

1.
$$V(f(x_0,u_0)) - V(x_0) = s(h(x_0,u_0),0) - \phi(x_0,u_0), \tag{6.7}$$

2.
$$\left.\frac{\partial V(z)}{\partial z}\right|_{z=f(x,u)} \frac{\partial}{\partial u} f(x,u) - \left.\frac{\partial s}{\partial z_1}\frac{\partial}{\partial u} h(x,u)\right|_{(z_1,v)=(h(x,u),0)} + \frac{\partial}{\partial u}\phi(x,u) \neq 0,$$
$$\forall (x,u) \in \mathscr{W} \tag{6.8}$$

From these conditions, the implicit function theorem assures that there exists an open neighbourhood of (x_0,u_0), $\mathscr{W} = \widetilde{\mathscr{X}} \times \widetilde{\mathscr{U}}$, and an unique $u = \left.\alpha(x,v)\right|_{v=0}$ such that,

$$V(f(x,\alpha(x,0))) - V(x) = s(h(x,\alpha(x,0)),0) - \phi(x,\alpha(x,0)), \qquad \forall x \in \widetilde{\mathscr{X}}$$

By the unicity of $\beta(x)$ and $\alpha(x,0)$, then $\beta(x) = \alpha(x,0)$.

Supposing $(\bar{x},\bar{u}) \in \mathscr{W}$ and $\mathscr{W}$ invariant with respect to $x(k+1) = f(x(k),\alpha(x(k),0))$, a neighbourhood $\mathscr{S}$ of $\bar{x}$ can be found, with $\mathscr{S} \subset \widetilde{\mathscr{X}}$ such that

$$V(f(x,\alpha(x,0))) - V(x) < 0, \;\; \forall x \in \mathscr{S} \tag{6.9}$$

Then, the result of the proposition follows.

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6.4 The energy shaping plus damping injection method

6.4.1 Extension of the ESDI method to the nonlinear discrete-time case

In this section, the passivity-based stabilization methodology of the ESDI is extended to the nonlinear discrete-time case. The ESDI method consists in modifying the closed-loop system stored energy and in adding the required dissipation. In the literature, the ESDI idea has been applied in two main different ways: the main difference between them is related to the way the *energy shaping* is performed. The approach of *(Ortega et al., 1997)* [131] shapes the stored energy of the system for the desired equilibrium to be the minimum of the new energy function for the closed-loop system. On the other hand, in the classic passivity-based control approach *(Ortega and Spong, 1989)* [127], the definition of the controller is derived from a copy of the system with additional damping; here, the energy shaping is represented by the energy associated to an error dynamics, the definition of which is based on the proposal of an auxiliary dynamics, see for example *(Sira-Ramírez, 1998)* [159] and *(Sira-Ramírez and Navarro-López, 2000)* [161]. We will follow the second mentioned approach.

This section will adapt the ESDI approach proposed in Section 3.6 and *(Sira-Ramírez and Navarro-López, 2000)* [161] to the discrete-time setting.

The main idea of the method in the discrete-time domain is the following one. Consider the system (5.1)-(5.2). Let suppose there exists a regular state feedback control law $u = \alpha(x,v)$ that renders the system strictly (V,s)-dissipative with $V(x)$ and $s(y,v)$ satisfying (6.1) the storage and the supply functions, respectively. Then, a strict dissipation rate function $\phi(x,u)$ must exist such that the following relation holds in a neighbourhood which contains the origin,

$$V(f(x,\alpha(x,0))) - V(x) = -\phi(x,\alpha(x,0)) \qquad (6.10)$$

The control law $\alpha(x,0)$ defined by (6.10) asymptotically stabilizes the origin of the original system, see Theorem 6.3. If a different point is desired to be stabilized, an error system can be generated by proposing an auxiliary system based on the structure of the original system dynamics. The damping is implicitly introduced by means of function ϕ, whereas in the continuous-time case it is explicitly injected in the auxiliary dynamics. These ideas are formally established in the following result.

Theorem 6.4 (Navarro-López et al., 2002) *[119] Consider the system (5.1)-(5.2) and two scalar functions $V(x)$ and $s(y,v)$ as a storage function and a supply function satisfying the ZIO property, respectively. Let $\phi(x,u)$ be a strict dissipation rate function. Suppose there exists a regular nonlinear static state feedback control law $u = \alpha(x,v)$ defined in an open neighbourhood $\mathscr{W} = \widetilde{\mathscr{X}} \times \widetilde{\mathscr{U}}$ containing $(x = 0, v = 0)$ with $\widetilde{\mathscr{X}} \subset \mathscr{X}$, $\widetilde{\mathscr{U}} \subset \mathscr{U}$ which renders the system (5.1)-(5.2) strictly (V,s)-dissipative. Consider $x = 0$ the unique x for which $V(x) = 0$, $\phi(x,\alpha(x,0)) = 0$. Then, for $v = 0$ the following relation is satisfied*

$$V(f(x,\alpha(x,0))) - V(x) = -\phi(x,\alpha(x,0)), \;\; \forall x \in \widetilde{\mathscr{X}} \qquad (6.11)$$

and the control $u = \alpha(x-\xi,0)$ locally asymptotically stabilizes the origin of the error dynamics $e = x - \xi$. ξ is the desired dynamics of the system and is defined as

$$\xi(k+1) = f(x(k),u(k)) - f(x(k)-\xi(k),\alpha(x(k)-\xi(k),0)) \qquad (6.12)$$

Proof. Consider the modified stored energy function $V(x(k)-\xi(k))$. Then,

$$V(x(k+1)-\xi(k+1))-V(x(k)-\xi(k)) = V(f(x(k),u(k))-\xi(k+1))-V(x(k)-\xi(k)) \tag{6.13}$$

If the system is feedback strictly dissipative (Definition 5.1) by means of a regular static feedback $u=\alpha(x,v)$, and considering $v=0$, then the following relation is satisfied, taking into account (6.1):

$$V(f(x(k),u(k))-\xi(k+1))-V(x(k)-\xi(k)) = -\phi(x(k)-\xi(k),\alpha(x(k)-\xi(k),0)) < 0 \tag{6.14}$$

Substituting (6.12) in (6.14) and considering $e=x-\xi$, one yields to

$$V(f(e(k),\alpha(e(k),0)))-V(e(k)) < 0. \tag{6.15}$$

Therefore, $V(e(k+1))-V(e(k))<0$, taking into account $e(k+1)=f(e(k),\alpha(e(k),0))$, and from fundamental results of Lyapunov's discrete stability theory, it can be said that the origin of the error dynamics is locally asymptotically stable.

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The control $\alpha(x,0)$ obtained from (6.11) will be regarded as the feedback dissipativity control or passifying control, whereas the control to be applied to the original system will be obtained from (6.12), being called stabilizing control and written as u^*. The ESDI control design proposed is depicted in Figure 6.1.

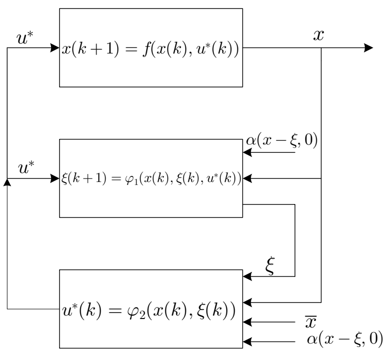

Figure 6.1: ESDI control scheme.

Four main steps can be distinguished in the ESDI control scheme, they are presented as follows:

Step 1 Computation of the control $\alpha(e,0)$, $\forall e \in \widetilde{\mathscr{X}_e}$, with $\widetilde{\mathscr{X}_e} \subset \mathscr{X}_e$ the neighbourhood where $\alpha(e,0)$ is defined, and $\mathscr{X}_e \subset \Re^n$ may not in general coincides with $\mathscr{X}$. Control $\alpha(e,0)$ is the control rendering the error dynamics strictly (V,s)-dissipative, it is obtained from relation

$$V(f(e,\alpha(e,0)))-V(e) = s(h(e,\alpha(e,0)),0)-\phi(e,\alpha(e,0)) \tag{6.16}$$

supposing s to satisfy the ZIO property, i.e., $s(h(e,\alpha(e,0)),0)=0$. Sufficient conditions for feedback dissipativity given in Theorem 5.1 will be used with $x_0=e_0$, $v_0=0$ and $u_0=\alpha(e_0,0)$. Note that relation (6.16) with s satisfying the ZIO property assures that $f(e,\alpha(e,0))$ converges to $e=0$ for all $e\in\widetilde{\mathscr{X}_e}$, see Theorem 6.3.

Step 2 Computation of the stabilizing control $u=u^*(x,e)$ from the proposed auxiliary dynamics.

Step 3 Computation of the auxiliary dynamics (6.12) using $u^*(x,e)$.

Step 4 Computation of the system state. The stabilizing control $u^*(x,e)$ is applied to (5.1)-(5.2).

In **Step 1**, different feedback dissipativity methodologies can be used in order to obtain the control $\alpha(e,0)$. In this chapter, the four feedback dissipativity schemes proposed in Chapter 5 will be applied to the stabilization to a desired point of the orbits of the discrete-time model of the DC-to-DC buck converter and of a nonlinear academic example, both posed in Chapter 5.

Remark 6.1 *Note that in this methodology the existence of $V(x)$ and $\phi(x,u)$ is supposed a priori. We consider that for systems with a physical energy interpretation this fact is not a drawback. Indeed, this method has been successfully applied in continuous-time systems.*

Remark 6.2 *The presented ESDI methodology is an alternative stabilization method to the passivity-based stabilization approaches existing for the discrete-time case proposed by* (Lin, 1995) *[86] and* (Lin and Byrnes, 1995) *[87], who instead of using the proposal of an auxiliary dynamics and shaping the energy associated to an error dynamics, they propose a stabilizing state feedback law taking advantage of the necessary conditions to be fulfilled by a passive system. Moreover, they assure asymptotic stability by presuming a detectability-type condition. Our proposal is more practical oriented.*

In the next two sections, the ESDI stabilization method will be applied to two examples, posed in Chapter 5: the discrete-time model of the DC-to-DC buck converter (5.13) and a nonlinear discrete-time model given by (5.23). The dissipation rate function in each example will be the same as the one proposed in Chapter 5 for feedback dissipativity purposes. The ESDI scheme will be presented, no feedback dissipativity technique in order to obtain $\alpha(e,0)$ is considered, i.e., **Step 1** is only indicated, so far. In Sections 6.5, 6.6, 6.7 and 6.8 the feedback dissipativity techniques proposed in Chapter 5 will be used in order to propose $\alpha(e,0)$.

6.4.2 Example 1. Dissipativity-based stabilization of the buck converter

In this section, the ESDI control scheme is applied to the discrete-time model for the DC-to-DC buck converter given by (5.13). Although the model is linear, it is appropriate to illustrate the proposed dissipativity-based control methodology. It is an example for which the energy concepts introduced have a physical interpretation.

We aim to stabilize the output voltage x_2 to a constant value $\bar{x}_2\in(0,1]$. The energy associated to the system (5.15) is considered as the storage function V, and $s(k)=$

$y(k)v(k)$ as the supply function, with $y(k) = x_2(k) + \hat{u}(k)$. Let $\hat{u}$, $v \in [0,1]$, $x_1 \in [0, \gamma\rho]$, $x_2 \in [0, \rho]$, with $\rho > 1$ and γ the normalized load. Let the error dynamics $e = (e_1, e_2)^T = (x_1 - \xi_1, x_2 - \bar{x}_2)^T$ with $e_1, e_2 \in [-\rho_x, \rho_x]$, ξ_1 the auxiliary dynamics for x_1, and the control $\alpha(e_1, e_2, v) \in [-\rho_u, \rho_u]$ defined in $[-\rho_x, \rho_x] \times [-\rho_x, \rho_x] \times [-\rho_u, \rho_u]$ with ρ_x and ρ_u positive constants.

First of all, a function ϕ must be proposed. This function will be chosen as in (5.16) where μ is a positive constant. The constant μ represents the damping injection to the system; the smaller μ is, the slower the convergence to the fixed point is. The stabilization of the system strongly depends on the form of ϕ. Function ϕ will be used in the passivation of the error dynamics e, so it will be considered as $\phi(e, \alpha(e, 0))$, a function which is always positive and it is zero for the equilibrium state $e = (0, 0)^T$, then Theorem 6.4 can be applied.

The ESDI control scheme is applied and four main steps can be distinguished in it, namely:

Step 1 Computation of control $\alpha(e, 0)$. Control $\alpha(e, 0)$ is the control which renders the error dynamics strictly V-passive with the new input v considered to be zero. Two different feedback dissipativity techniques will be used in this example: the first feedback dissipativity method proposed in Chapter 5 based on the explicit solution of the control α obtained from the fundamental dissipativity equality which will be presented in Section 6.5, and the use of the SG algorithm in order to obtain the control which achieves strict feedback passivity, given in Section 6.6.

Step 2 Computation of the stabilizing control $\hat{u}^*(x, e)$ from the proposed auxiliary dynamics,

$$\xi_1(k+1) = ax_1(k) - bx_2(k) + [\gamma(-a+1) + b]\hat{u}(k) - f_1(e(k), \alpha(e(k), 0)) \tag{6.17}$$

$$\xi_2(k+1) = bx_1(k) + cx_2(k) + (-\gamma b - c + 1)\hat{u}(k) - f_2(e(k), \alpha(e(k), 0)) \tag{6.18}$$

with

$$\begin{aligned} f_1(e(k), \alpha(e(k), 0)) &= ae_1(k) - be_2(k) + [\gamma(-a+1) + b]\alpha(e(k), 0) \\ f_2(e(k), \alpha(e(k), 0)) &= be_1(k) + ce_2(k) + (-\gamma b - c + 1)\alpha(e(k), 0) \end{aligned}$$

Due to the fact that we want to stabilize ξ_2 to a constant value $\bar{x}_2 \in (0, 1]$; then, $\xi_2(k) = \overline{\xi}_2 = \bar{x}_2$, $\forall k$, and $\hat{u}^*$ is obtained from (6.18),

$$\hat{u}^*(k) = \frac{\bar{x}_2 - [bx_1(k) + cx_2(k)] + f_2(e(k), \alpha(e(k), 0))}{(-\gamma b - c + 1)} \tag{6.19}$$

Step 3 Computation of the auxiliary dynamics (6.17) using $\hat{u}^*(k)$,

$$\xi_1(k+1) = ax_1(k) - bx_2(k) + [\gamma(-a+1) + b]\hat{u}^*(k) - f_1(e(k), \alpha(e(k), 0)) \tag{6.20}$$

Step 4 Computation of the system state. Control $\hat{u}^*(k)$ is applied to (5.13),

$$\begin{aligned} x_1(k+1) &= ax_1(k) - bx_2(k) + [\gamma(-a+1) + b]\hat{u}^*(k) \\ x_2(k+1) &= bx_1(k) + cx_2(k) + (-\gamma b - c + 1)\hat{u}^*(k) \end{aligned}$$

The control scheme designed for the buck converter will use two different feedback dissipativity techniques in Sections 6.5 and 6.6.

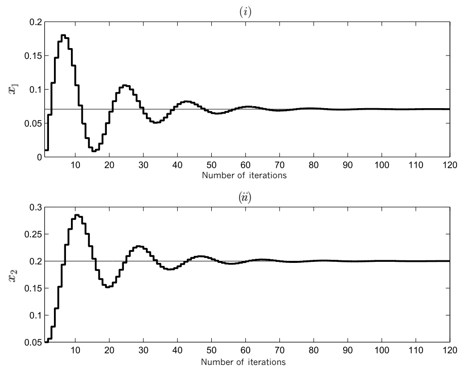

Figure 6.2: Buck converter open-loop response for $\hat{u} = \hat{\overline{u}} = \overline{x}_2 = 0.2$ (i) current x_1 (ii) voltage x_2.

Remark 6.3 *It must be pointed out that the system (5.13) is stable in open-loop considering $\hat{u} = \overline{x}_2$. Then, the responses which will be obtained for the stabilized buck by means of the ESDI control scheme are pretended to have a better performance that the one presented in Figure 6.2.*

6.4.3 Example 2. Dissipativity-based stabilization of a nonlinear example

Let the system (5.23) and $x_1 \in [-\varepsilon_{x_1}, \varepsilon_{x_1}]$, $x_2 \in [-\varepsilon_{x_2}, \varepsilon_{x_2}]$, $u, v \in [-\varepsilon_u, \varepsilon_u]$, with ε_{x_1}, ε_{x_2}, ε_u positive constants. We aim to stabilize the variable x_1 to a constant value $\overline{x}_1 = \overline{\xi}_1 \neq 0$, $|\overline{x}_1| < \varepsilon_{x_1}$. Let the error dynamics $e = (e_1, e_2)^T = (x_1 - \overline{x}_1, x_2 - \xi_2)^T$ with $e_1 \in [-\varepsilon_{x_1}, \varepsilon_{x_1}]$, $e_2 \in [-\varepsilon_{x_2}, \varepsilon_{x_2}]$ and the control $\alpha(e_1, e_2, v) \in [-\varepsilon_u, \varepsilon_u]$ defined in $[-\varepsilon_{x_1}, \varepsilon_{x_1}] \times [-\varepsilon_{x_2}, \varepsilon_{x_2}] \times [-\varepsilon_u, \varepsilon_u]$.

Consider as the storage function the energy associated to the system $V = x_1(k)^2 + x_2(k)^2$, and as the supply function $s(k) = y(k)v(k)$ with $y(k) = x_1(k)^2 + x_2(k)^2 + u(k)$. The dissipation rate function to be considered is the one given in (5.24) with μ a positive constant acting as a damping injection coefficient. Function ϕ will be used in the passivation of the error dynamics e, so it will be considered as $\phi(e, \alpha(e, 0))$, a function which is always positive and it is zero for the equilibrium state $e = (0, 0)^T$, then Theorem 6.4 can be applied.

Then, the ESDI control scheme takes the following form,

Step 1 Computation of the control which strictly passifies the error system with $v = 0$, i.e, $u(k) = \alpha(e(k), 0)$.

Step 2 Computation of the stabilizing control $u^*(k)$. From (6.12), the auxiliary dynamics takes the following form

$$\xi_1(k+1) = \left[x_1(k)^2 + x_2(k)^2 + u(k)\right]\cos[x_2(k)] - f_1(e(k),\alpha(e(k),0)) \tag{6.21}$$

$$\xi_2(k+1) = \left[x_1(k)^2 + x_2(k)^2 + u(k)\right]\sin[x_2(k)] - f_2(e(k),\alpha(e(k),0)) \tag{6.22}$$

with

$$\begin{aligned} f_1(e(k),\alpha(e(k),0)) &= \left[e_1(k)^2 + e_2(k)^2 + \alpha(e(k),0)\right]\cos[e_2(k)] \\ f_2(e(k),\alpha(e(k),0)) &= \left[e_1(k)^2 + e_2(k)^2 + \alpha(e(k),0)\right]\sin[e_2(k)] \end{aligned}$$

Since our goal is to stabilize ξ_1 to a constant value $\overline{x}_1$, then, $u^*(k)$ will be obtained from (6.21) as

$$u^*(k) = \frac{\overline{x}_1 - \left[x_1(k)^2 + x_2(k)^2\right]\cos[x_2(k)] + f_1(e(k),\alpha(e(k),0))}{\cos[x_2(k)]} \tag{6.23}$$

Step 3 Computation of the auxiliary dynamics using $u^*(k)$

$$\xi_2(k+1) = \left[x_1(k)^2 + x_2(k)^2 + u^*(k)\right]\sin[x_2(k)] - f_2(e(k),\alpha(e(k),0)) \tag{6.24}$$

Step 4 Computation of the system state

$$\begin{aligned} x_1(k+1) &= \left[x_1(k)^2 + x_2(k)^2 + u^*(k)\right]\cos[x_2(k)] \\ x_2(k+1) &= \left[x_1(k)^2 + x_2(k)^2 + u^*(k)\right]\sin[x_2(k)] \end{aligned}$$

In Sections 6.7 and 6.8, this stabilization scheme will be applied for the stabilization of system (5.23) using the two last feedback dissipativity methodologies presented in Chapter 5 for the case of feedback passivity.

The next four sections are based upon the results given in this chapter concerning the study of the stability and stabilization of dissipative systems, and upon the results posed in Chapter 5 concerning the study of the feedback dissipativity problem in discrete-time systems. Section 6.5 is devoted to the application of the ESDI methodology to the DC-to-DC buck converter making use of the feedback dissipativity scheme given in Section 5.2, which will be regarded as **Method 1**. Section 6.6 also applies the ESDI stabilization method to the buck example, however, the feedback dissipativity technique used in order to obtain control $\alpha(e,0)$ is the SG algorithm as Section 5.3 proposed. The feedback dissipativity methodology which uses the SG algorithm will be regarded as **Method 2**. Sections 6.7 and 6.8 present the application of the ESDI control scheme to the academic nonlinear example given by (5.23). On the one hand, Section 6.7 will use the feedback dissipativity method presented in Section 5.4.1 in which (V,s)-dissipativity is seen as a "perturbation" of the energy invariance situation. This feedback dissipativity methodology will be called in the sequel **Method 3**. On the other hand, Section 6.8 will use the feedback dissipativity method presented in Section 5.4.2 in which (V,s)-dissipativity is achieved by means of the "perturbation" of the losslessness situation. This feedback dissipativity methodology will be regarded as **Method 4** from now an so on. In the following four sections, simulations results are presented in order to illustrate the control action proposed. The admissible values for the constants appearing in the feedback dissipativity schemes for both examples are analyzed. In the examples, the feedback dissipativity methods are restricted to the feedback passivity case.

6.5 Stabilization of the buck converter. Feedback dissipativity by means of Method 1

This section applies the ESDI stabilization methodology to the buck example. The control $\alpha(e,0)$ will be obtained by **Method 1** presented in Section 5.2. Simulations results are presented for two different cases. On the one hand, a fixed normalized load γ will be considered, then the constants of the model (a, b, c, γ) depending on the physical parameters of the system will be considered to be constants. On the other hand, the robustness of the controller under load variations will be illustrated by means of simulations presented for different values of γ.

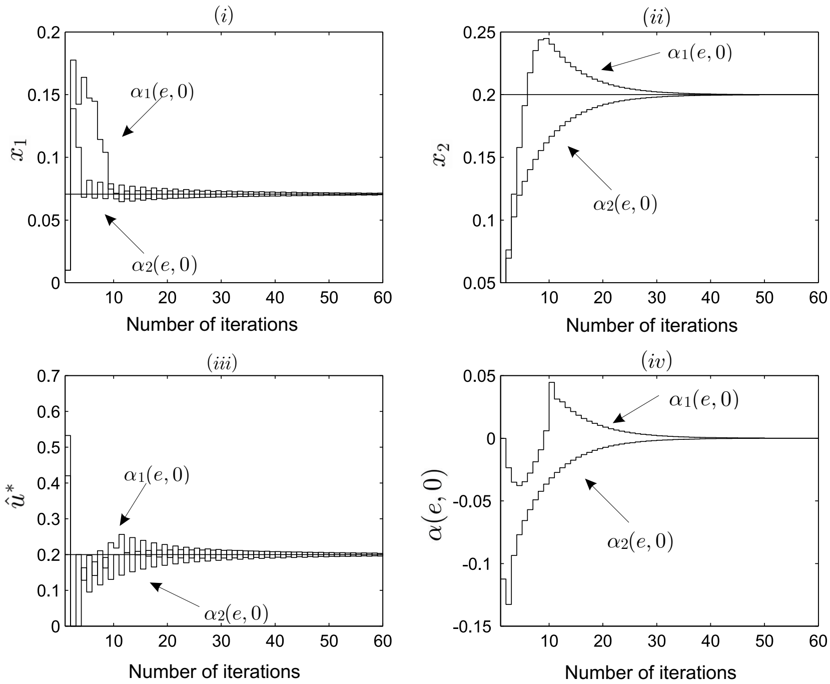

Figure 6.3: Stabilized system response for (5.13) with $\bar{x}_2 = 0.2$, $\mu = 0.135$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$ (i) Normalized current x_1 (ii) normalized voltage x_2 (iii) stabilizing control $\hat{u}^*$ (iv) passifying controls for $\alpha_1(e,0)$ and $\alpha_2(e,0)$.

Consider the ESDI scheme presented in Section 6.4.2. The feedback dissipativity **Method 1** will be applied in **Step 1** resulting in:

Step 1 Computation of the control $\alpha(e,0)$ from

$$V(f(e,\alpha(e,0))) - V(e) = -\phi(e,\alpha(e,0)) \tag{6.25}$$

which results in a second-order equation of the form

$$a_\alpha \alpha^2(e,0) + b_\alpha(e)\alpha(e,0) + c_\alpha(e) = 0 \tag{6.26}$$

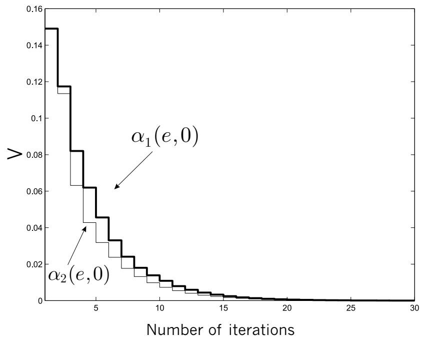

Figure 6.4: Storage energy associated to the error dynamics of the stabilized system (5.13) obtained for $\alpha_1(e,0)$ and $\alpha_2(e,0)$, with $\bar{x}_2 = 0.2$, $\mu = 0.135$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$.

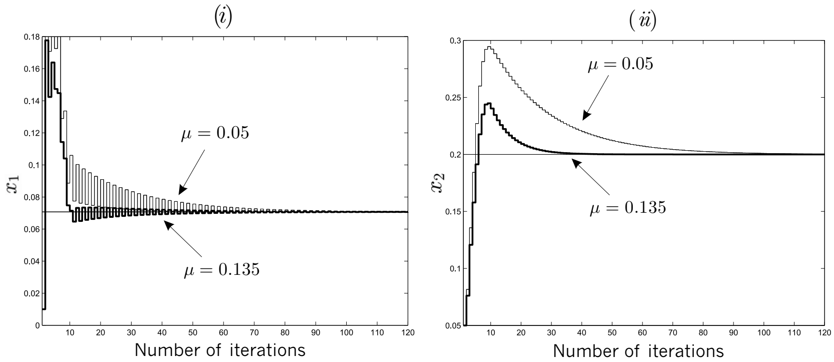

Figure 6.5: Stabilized system variables of system (5.13) for different values of μ with $\bar{x}_2 = 0.2$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$ and $\alpha_1(e,0)$ (i) x_1 for $\mu = 0.135$ and $\mu = 0.05$ (ii) x_2 for $\mu = 0.135$ and $\mu = 0.05$.

with

$$\begin{aligned} a_\alpha &= \left(\frac{1}{2}+\mu\right)\left[\gamma^2(-a+1)^2+b^2(\gamma^2+1)+(-c+1)^2\right]+\gamma(bc-ab) \\ b_\alpha(e_1,e_2) &= [\gamma(-a+1)+b](ae_1-be_2)+[-\gamma b-c+1](be_1+ce_2) \\ c_\alpha(e_1,e_2) &= \left(\frac{1}{2}+\mu\right)\left[e_1^2(a^2+b^2)+e_2^2(b^2+c^2)\right]+(bc-ab)e_1e_2- \\ &\quad -\frac{1}{2}(e_2^2+e_1^2) \end{aligned}$$

with $e = (e_1, e_2)^T = (x_1 - \xi_1, x_2 - \bar{x}_2)^T$, $\xi = (\xi_1, \xi_2)^T$. Passivation conditions (5.4) and (5.5), are transformed for stabilization purposes in conditions (6.4)-(6.5) which are met for this example if,

$$a_\alpha\alpha^2(e,0)+b_\alpha(e)\alpha(e,0)+c_\alpha(e)=0, \quad 2a_\alpha\alpha(e,0)+b_\alpha(e)\neq 0 \qquad (6.27)$$

for some (e,α). If conditions (6.27) are satisfied, α exists. This α can be obtained from the explicit solution of (6.26), then two possible solutions for $\alpha(e,0)$ are obtained:

$$\alpha_1(e,0) = \frac{-b_\alpha + \sqrt{b_\alpha^2 - 4a_\alpha c_\alpha}}{2a_\alpha}, \qquad \alpha_2(e,0) = \frac{-b_\alpha - \sqrt{b_\alpha^2 - 4a_\alpha c_\alpha}}{2a_\alpha} \quad (6.28)$$

For these solutions to be real, it is necessary to assure that $\varphi(e_1,e_2) = b_\alpha^2 - 4a_\alpha c_\alpha \geq 0$, which will be achieved by means of the value of μ. At a first sight, the values of μ proposed in Section 5.2.2.3 for the passivation problem cannot be used, due to the fact that the domain of definition of the errors e_1, e_2 are different to the ones given for the states x_1, x_2. Let write function $\varphi(e_1,e_2)$ as a quadratic form $\varphi(e_1,e_2) = (e_1,e_2)^T P(e_1,e_2)$ with P a symmetric positive definite matrix. Therefore,

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{pmatrix}$$

with

$$\begin{aligned}
p_{11} &= b_1^2 a^2 + b_2^2 b^2 + 2b_1 b_2 ab + 2a_\alpha \left[1 - 2\left(\frac{1}{2} + \mu\right)(a^2 + b^2)\right] \\
p_{12} &= -b_1^2 ab + b_2^2 bc + b_1 b_2 ac - b_1 b_2 b^2 - 2a_\alpha(bc - ab) \\
p_{22} &= b_1^2 b^2 + b_2^2 c^2 - 2b_1 b_2 bc + 2a_\alpha \left[1 - 2\left(\frac{1}{2} + \mu\right)(b^2 + c^2)\right] \\
b_1 &= \gamma(1-a) + b \\
b_2 &= -\gamma b - c + 1
\end{aligned}$$

and a, b, c, γ the constants of the model. The two eigenvalues of P are positive with $\mu < 0.137817$ and $\mu < 0.296853$, respectively; consequently, with $0 < \mu < 0.137817$ the function $\varphi(e_1,e_2) > 0$ and the controls $\alpha_1(e,0)$ and $\alpha_2(e,0)$ exist. The admissible values of μ are the same as the ones given in Section 5.2.2.3.

6.5.1 Consideration of a fixed load

The control design method given in the previous section has been applied to (5.13) with the following parameters obtained from a real physical system (see data in Section 5.2.2):

$$\gamma = 0.3535533906, a = 0.9406416964, b = 0.3254699438, c = 0.8255706942$$

and a sampling period of $T = 0.3535533906$. The fixed points of the system (5.13) are $\bar{x}_1 = \gamma\hat{\bar{u}}$, $\bar{x}_2 = \hat{\bar{u}}$. The dissipation rate function is considered as in (5.16). The admissible values for constant μ are $\mu \in (0, 0.137817)$, as it was established before. Considering $\bar{x}_2 = 0.2$, with $x_0 = (0.01, 0.05)^T$, $\xi_{10} = 0.01$ as initial conditions for x and ξ_1, respectively, and $\mu = 0.135$, we obtain the system response presented in Figure 6.3. Control $\hat{u}^*$ has been constrained to $[0,1]$. The system converges to its fixed point.

The system responses obtained with $\alpha_1(e,0)$ and $\alpha_2(e,0)$ are slightly different. The difference between considering $\alpha_1(e,0)$ or $\alpha_2(e,0)$ can be appreciated in the type of transient response for the obtained output voltage x_2. Using the latter, a first-order type system response for the voltage without overshooting is obtained. Both responses have the same settling time. The states x_1, x_2, the stabilizing control $\hat{u}^*$, and the passifying controls $\alpha_1(e,0)$ and $\alpha_2(e,0)$ are depicted in Figure 6.3. The storage energy associated to the error system is given in Figure 6.4.

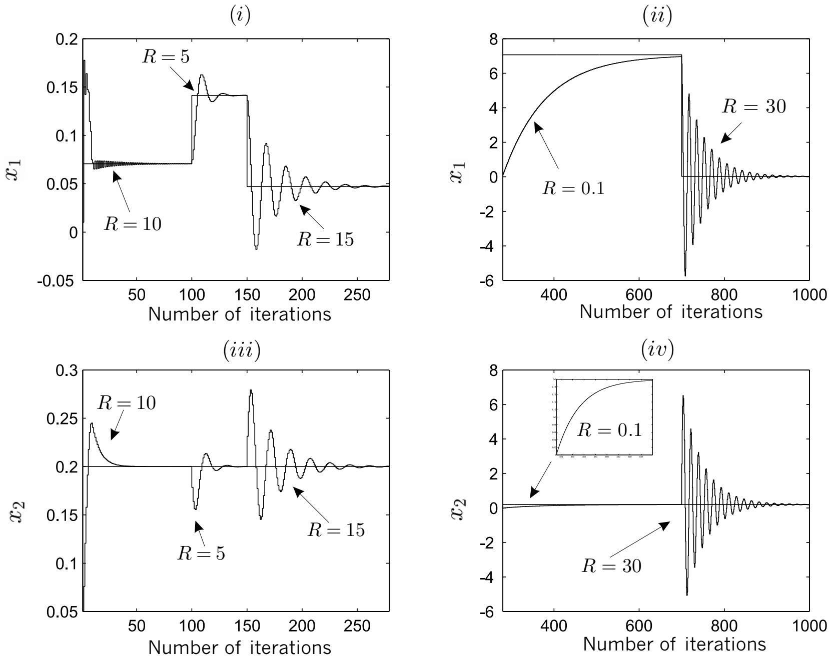

Figure 6.6: Buck response with $\alpha_1(e,0)$ under variations in the load with $\mu = 0.135$, $\bar{x}_2 = 0.2$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$ (i) x_1 in the first 280 iterations (ii) x_1 in the iterations $280-1000$ (iii) x_2 in the first 280 iterations (iv) x_2 in the iterations $280-1000$.

The influence of μ in the response of the system can be analyzed from Figure 6.5. Comparing the responses of the closed-loop system for different values of μ, the importance of the value of this constant can be appreciated. Constant μ represents the damping injection to the system: the smaller μ is, the smaller the dissipation rate is and the slower the convergence to the fixed point is. The smaller μ is, the greater the overshooting is. The problem is that μ cannot be as big as we would like to, due to the fact that the radicand of the square root in $\alpha_1(e,0)$ or $\alpha_2(e,0)$ can be negative. Figure 6.5 compares the responses for the stabilized system using $\alpha_1(e,0)$ with $\mu = 0.135$ and $\mu = 0.05$. Similar results are obtained with $\alpha_2(e,0)$.

6.5.2 Control robustness under variations in the load

In this section, the robustness of the control proposed under variations in the load will be illustrated by means of some simulations. The physical nominal load parameter $R = 10\,\Omega$ will be changed, then, different values for the constants appearing in (5.13) will be obtained, however, the nominal control obtained with $R = 10\Omega$ and the associated a, b, c, γ will stabilize the system. The different values of R considered are the following ones, with their corresponding model constants:

- $R_1 = 5\,\Omega$:

$$\begin{aligned}
\gamma_1 &= 0.70710678112, \\
a_1 &= 0.94298581996, b_1 = 0.30635311757, c_1 = 0.726361453087
\end{aligned}$$

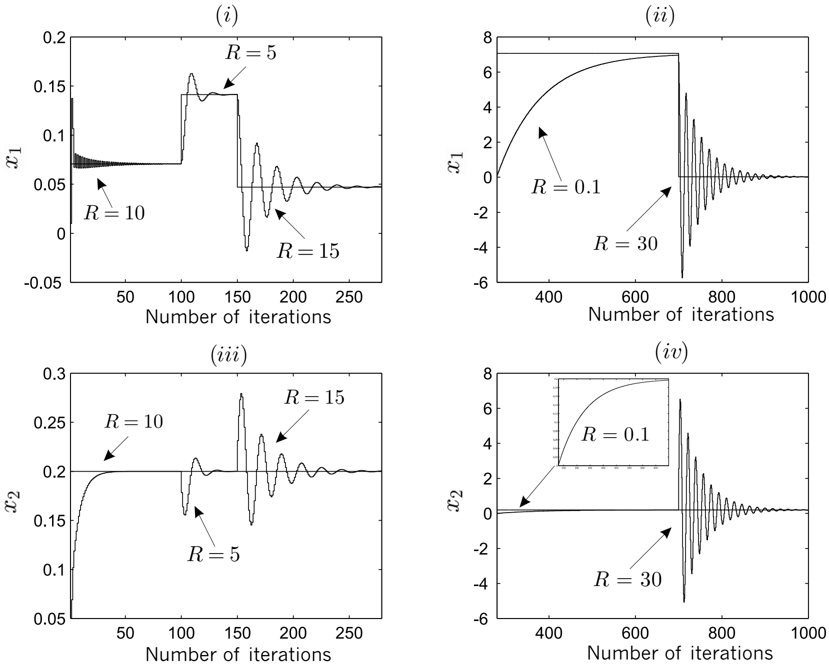

Figure 6.7: Buck response with $\alpha_2(e,0)$ under variations in the load $\mu = 0.135$, $\bar{x}_2 = 0.2$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$ (i) x_1 in the first 280 iterations (ii) x_1 in the iterations $280-1000$ (iii) x_2 in the first 280 iterations (iv) x_2 in the iterations $280-1000$.

- $R_2 = 15\,\Omega$:

$$\begin{aligned} \gamma_2 &= 0.235702260395, \\ a_2 &= 0.939827710025, b_2 = 0.3322005265, c_2 = 0.861527295018 \end{aligned}$$

- $R_3 = 0.1\,\Omega$:

$$\begin{aligned} \gamma_3 &= 35.35533906, \\ a_3 &= 0.99083583715, b_3 = 0.02804741903, c_3 = -0.000790172329 \end{aligned}$$

- $R_4 = 30\,\Omega$:

$$\begin{aligned} \gamma_4 &= 0.1178511302, \\ a_4 &= 0.93899673245323, b_4 = 0.3391197492, c_4 = 0.89903108674 \end{aligned}$$

Figures 6.6 and 6.7 depict the controlled states x_1 and x_2 for the different models corresponding to different values of the load R and for the two solutions of $\alpha(e,0)$. The system is stabilized to $(\gamma_i\bar{x}_2, \bar{x}_2)$, with $i = 1,...,4$ by means of the control obtained for the nominal load $R = 10\,\Omega$. The fixed point of the system changes if the normalized load γ changes, as this is the case when R is changed, however, the voltage component remains.

6.6 Stabilization of the buck converter. Feedback dissipativity by means of Method 2

As it was made in Section 6.5, the ESDI control scheme proposed in Section 6.4.2 will be applied to the buck converter example, with the difference that the method in order to obtain control $\alpha(e,0)$ will be **Method 2**, i.e., the feedback dissipativity methodology based upon the SG algorithm proposed in Chapter 5. The passifying controller to use is the static-type one. The only step in the ESDI design different from the ones presented in Section 6.4.2 is **Step 1**:

Step 1 Computation of control $\alpha(e,0)$. The control goal function proposed in Chapter 5 as (5.43) for the error dynamics takes the following form, considering $v=0$ for stabilization purposes:

$$Q_d(e,\alpha(e,0),0) = V(f(e,\alpha(e,0))) - V(e) - s(h(e,\alpha(e,0)),0) + \phi(e,\alpha(e,0)) \tag{6.29}$$

which results in a second-order equation in α:

$$Q_d(e,\alpha(e,0),0) = a_\alpha \alpha^2(e,0) + b_\alpha(e)\alpha(e,0) + c_\alpha(e), \tag{6.30}$$

with

$$\begin{aligned}
a_\alpha &= \eta\left[\left(\frac{1}{2}+\mu\right)[\gamma_b^2(-a+1)^2 + b^2(\gamma_b^2+1) + (-c+1)^2] + \right. \\
&\quad + \left. \gamma_b(bc-ab)\right] \\
b_\alpha(e_1,e_2) &= \eta\left\{\left[\gamma_b(-a+1)+b\right](ae_1-be_2) + \left(-\gamma_b b - c + 1\right)(be_1+ce_2)\right\} \\
c_\alpha(e_1,e_2) &= \eta\left\{\left(\frac{1}{2}+\mu\right)[e_1^2(a^2+b^2) + e_2^2(b^2+c^2)] + (bc-ab)e_1e_2 - \right. \\
&\quad \left. -\frac{1}{2}(e_1^2+e_2^2)\right\}
\end{aligned}$$

with γ_b the normalized load and $e=(e_1,e_2)^T = (x_1-\xi_1, x_2-\bar{x}_2)^T$ the error dynamics.

As it was analyzed in Chapter 5, the value of parameter μ will be such that assures Q_d to be positive. Since the domains of definition of the variables have changed, the positiveness of Q_d must be studied again for e_1, e_2 and $\alpha(e,0)$. In order to assure Q_d to be positive, it will be written as a quadratic form $(e_1,e_2,\alpha)^T P(e_1,e_2,\alpha)$ with P a symmetric positive definite matrix. Then,

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{12} & p_{22} & p_{23} \\ p_{13} & p_{23} & p_{33} \end{pmatrix}$$

with

$$\begin{aligned}
p_{11} &= \eta\left[\left(\frac{1}{2}+\mu\right)(a^2+b^2)-\frac{1}{2}\right]\\
p_{12} &= \frac{\eta}{2}(bc-ab)\\
p_{13} &= \frac{\eta}{2}\left\{\left[\gamma_b(1-a)+b\right]a+(-\gamma_b b-c+1)b\right\}\\
p_{22} &= \eta\left[\left(\frac{1}{2}+\mu\right)(b^2+c^2)-\frac{1}{2}\right]\\
p_{23} &= \frac{\eta}{2}\left\{(-\gamma_b b-c+1)c-\left[\gamma_b(1-a)+b\right]b\right\}\\
p_{33} &= \eta a_\alpha
\end{aligned}$$

and a, b, c, γ_b the constants of the model. The eigenvalues of P are positive for $\mu > 0$, $\mu > 0.296853$ and $\mu > 0.13819$, respectively, then the admissible values of μ to consider in order to have $Q_d > 0$ are $\mu > 0.296853$. This values coincides with the ones given in Section 5.3.3. In this case, $Q_d(e,\alpha(e,0),0)$ with $\mu > 0.296853$ is assured to be a convex function with the variables defined on compact sets, then, the assumptions in order to apply the SG algorithm are verified and $\alpha(e,0)$ is obtained,

$$\alpha(e(k),0) = -\frac{\gamma_{cs} b_\alpha(e(k))}{1+2\gamma_{cs} a_\alpha}$$

with γ_{cs} a positive constant chosen in such a way to ensure the fixed point of the controlled error system, i.e., $(0,0)$, to be asymptotically stable. It can be checked that this goal is achieved with $\mu > 0.296853$ and $\gamma_{cs} > 0$.

The system is stabilized and consequently, it converges to the desired fixed point $(\gamma_b\bar{x}_2,\bar{x}_2)$, with $\bar{x}_2$ as the desired voltage, but the transient system response is rather dependent of the values of parameters γ_{cs} and μ. On the one hand, the smaller γ_{cs} is, the more oscillating the system response is and the higher the overshooting and the settling time are, regardless of the fact the slightly smaller the control $\hat{u}^*$ is and the smaller the peak value of control $\alpha(e,0)$ is. See Figures 6.8, 6.9. On the other hand, the higher μ is, the more oscillating the system response is and the slower the system converges to the equilibrium, regardless of the fact the greater the peak value of the stabilizing control $\hat{u}^*$ is and the less the control $\alpha(e,0)$ is in its transient state. Control $\hat{u}^*$ has been constrained to $[0,1]$. See Figures 6.10, 6.11.

6.7 Stabilization of an academic nonlinear example. Feedback dissipativity through Method 3

In this section, the ESDI method will be applied to the nonlinear example (5.23) as presented in Section 6.4.3. Control $\alpha(e,0)$ will be obtained by means of the feedback dissipativity methodology presented in Section 5.4.1. We will be based on the passifying scheme proposed in Section 5.4.1.2 for the example, where the applications conditions for the methodology were analyzed, and on Proposition 5.2, now considering $u^*(x)$ as $\alpha^*(e)$ and $\delta u^*(x,u^*,v)$ as $\delta\alpha^*(e,\alpha^*,0)$, and having $\alpha(e,0) = \alpha^*(e)+\delta\alpha^*(e,\alpha^*,0)$. Here, the sets where the errors and the controls $\alpha^*(e)$, $\delta\alpha^*(e,\alpha^*,0)$ are defined are considered the same as the ones used for the states and controls $u^*(x)$ and $\delta u^*(x,u^*,v)$,

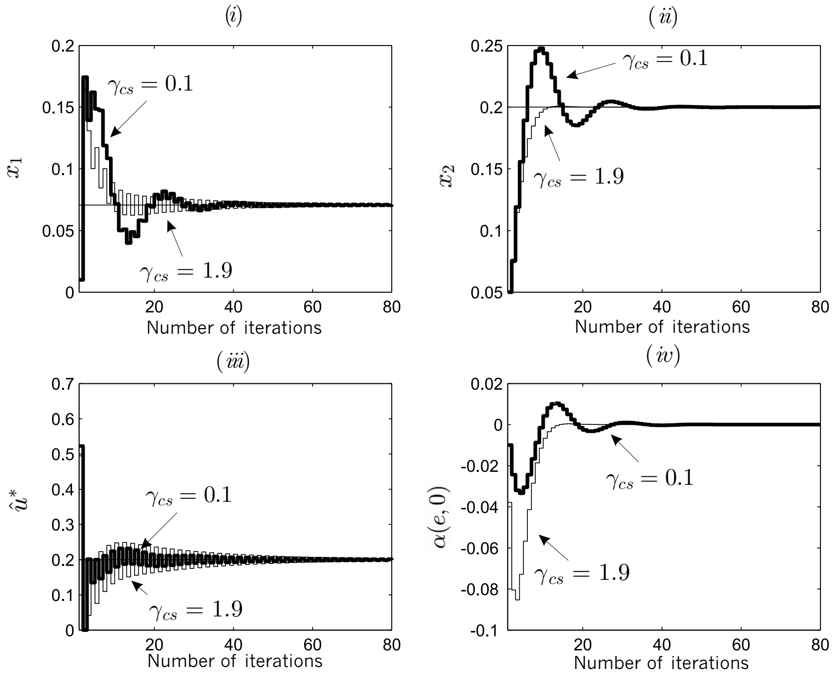

Figure 6.8: Stabilized system response of (5.13) for different values of γ_{cs}: $\gamma_{cs} = 0.1$ and $\gamma_{cs} = 1.9$ with $\bar{x}_2 = 0.2$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$, $\mu = 0.3$ (i) x_1 (ii) x_2 (iii) stabilizing control $\hat{u}^*$ (iv) passifying control $\alpha(e,0)$ obtained by means of the SG algorithm.

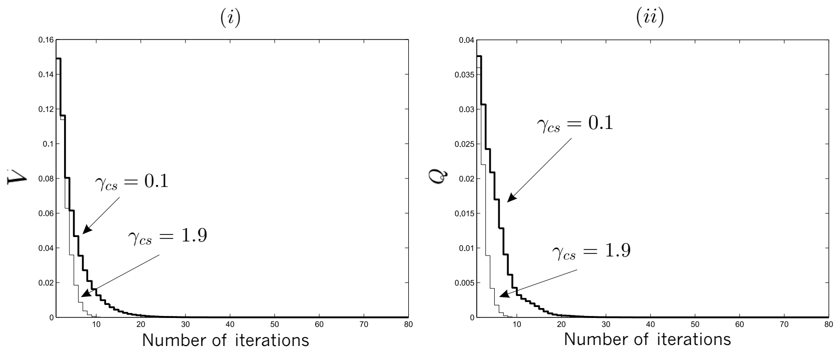

Figure 6.9: $V(e)$ and $Q(e,\alpha(e,0),0)$ for the stabilized system (5.13) by means of the SG for different values of γ_{cs}: $\gamma_{cs} = 0.1$ and $\gamma_{cs} = 1.9$ with $\bar{x}_2 = 0.2$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$, $\mu = 0.3$ (i) Storage energy function associated to the error dynamics $V(e)$ (ii) control goal function $Q(e,\alpha(e,0),0)$.

therefore, the analysis given for parameter μ in Section 5.4.1.2 is valid for μ appearing in the passifying control of the error dynamics in this section.

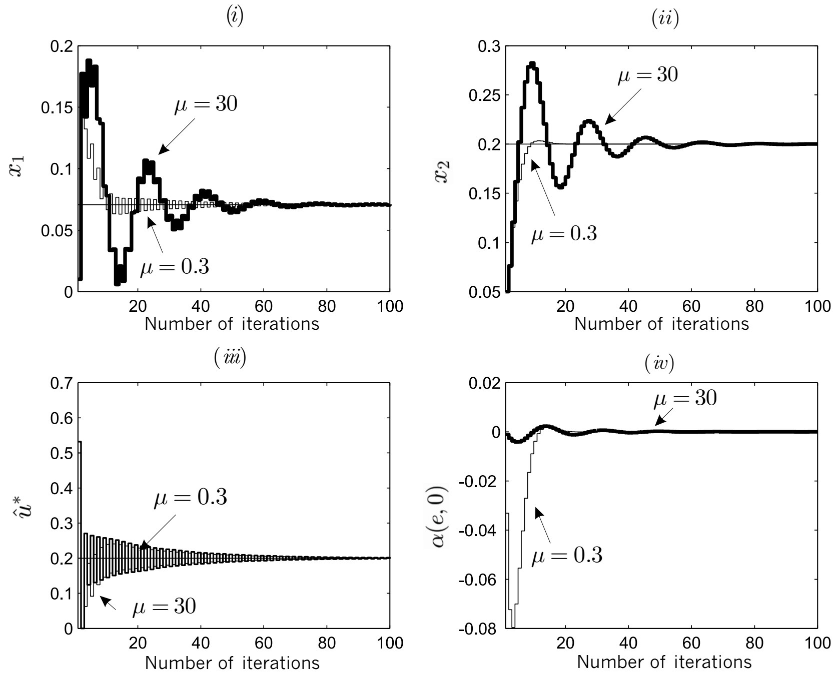

Figure 6.10: Stabilized system response of system (5.13) for different values of μ: $\mu = 0.3$ and $\mu = 30$ with $\bar{x}_2 = 0.2$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$, $\gamma_{cs} = 1$ (i) x_1 (ii) x_2 (iii) stabilizing control $\hat{u}^*$ (iv) passifying control $\alpha(e,0)$ obtained by means of the SG algorithm.

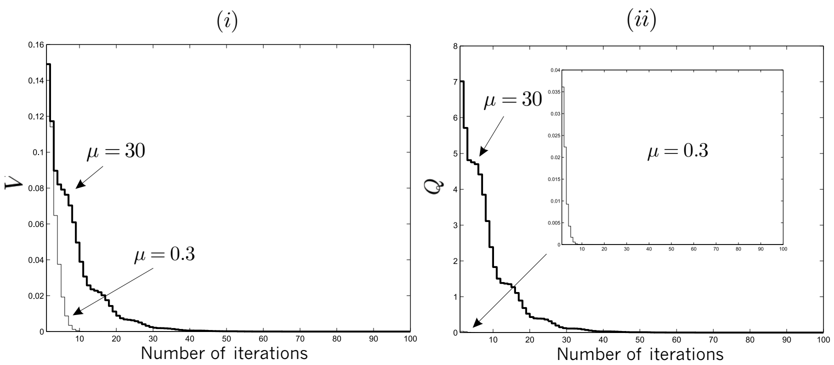

Figure 6.11: $V(e)$ and $Q(e,\alpha(e,0),0)$ for the stabilized system (5.13) by means of the SG algorithm for different values of μ: $\mu = 0.3$ and $\mu = 30$ with $\bar{x}_2 = 0.2$, $x_0 = (0.01, 0.05)^T$, $\xi_{01} = 0.01$, $\gamma_{cs} = 1$ (i) Storage energy function $V(e)$ (ii) control goal function $Q(e,\alpha(e,0),0)$.

6.7.1 Computation of the error passifying control

Consider the ESDI control scheme proposed for the nonlinear example (5.23) in Section 6.4.3. The four steps given are considered, with **Step 1** taking the following form:

Step 1 Computation of $\alpha(e,0) = \alpha^*(e) + \delta\alpha^*(e,\alpha^*,0)$. Let $\mathscr{E}_1 = [-\varepsilon_{x_1}, \varepsilon_{x_1}] \times [-\varepsilon_{x_2}, \varepsilon_{x_2}]$ and $\mathscr{E}_2 = [-\varepsilon_u, \varepsilon_u]$, with ε_{x_1}, ε_{x_2} and ε_u positive constants. Consider $\alpha^* : \mathscr{E}_1 \to \mathscr{E}_2$ and $\delta\alpha^* : \mathscr{E}_1 \times \mathscr{E}_2 \times \mathscr{E}_2 \to \mathscr{E}_2$ with $|\delta\alpha^*| \leq \rho$, and $\rho > 0$. Then, controls $\alpha^*(e)$ and $\delta\alpha^*(e,\alpha^*,0)$ are calculated as follows:

1. Computation of the control $\alpha^*(e)$ which makes the storage energy V be invariant along the orbits of the error dynamics, i.e.,

$$V(f(e,\alpha^*(e))) - V(e) = 0, \;\; \forall e \in \mathscr{E}_1 \tag{6.31}$$

Equation (6.31) results in the following second-order equation in α^*,

$$a_{\alpha^*}(\alpha^*)^2(e) + b_{\alpha^*}(e)\alpha^*(e) + c_{\alpha^*}(e) = 0 \tag{6.32}$$

with

$$\begin{array}{rcl} a_{\alpha^*} & = & 1 \\ b_{\alpha^*}(e_1,e_2) & = & 2(e_1^2+e_2^2) \\ c_{u^*}(e_1,e_2) & = & (e_1^2+e_2^2)^2 - (e_1^2+e_2^2) \end{array}$$

Sufficient conditions for the existence of control $\alpha^*(e)$ are (5.69) and (5.70) for $x_0 = e_0$, $u_0^* = \alpha^*(e_0) = \alpha_0^*$, which are met for this example if,

$$a_{\alpha^*}(\alpha^*)^2(e) + b_{\alpha^*}(e)\alpha^*(e) + c_{\alpha^*}(e) = 0, \tag{6.33}$$

$$2a_{\alpha^*}\alpha^*(e) + b_{\alpha^*}(e) \neq 0 \tag{6.34}$$

for some (e,α^*). If conditions (6.33)-(6.34) are satisfied for some (e,α^*), a control $\alpha^*(e)$ satisfying (6.31) exists. This α^* can be obtained from the explicit solution of (6.32), then, it is necessary to assure that $b_{\alpha^*}^2 - 4a_{\alpha^*}c_{\alpha^*} \geq 0$, which will be always achieved since this function is always positive. Then, two solutions for control $\alpha^*(e)$ are obtained,

$$\alpha_1^*(e) = \frac{-b_{\alpha^*} + \sqrt{b_{\alpha^*}^2 - 4a_{\alpha^*}c_{\alpha^*}}}{2a_{\alpha^*}}, \;\; \alpha_2^*(e) = \frac{-b_{\alpha^*} - \sqrt{b_{\alpha^*}^2 - 4a_{\alpha^*}c_{\alpha^*}}}{2a_{\alpha^*}} \tag{6.35}$$

2. Computation of the control $\delta\alpha^*(e,\alpha^*,0)$,

$$\delta\alpha^*(e_1,e_2,\alpha^*,0) = -\frac{\mu\left[(e_1^2+e_2^2)^2 + (\alpha^*)^2 + e_1^2 + e_2^2\right]}{2\alpha^*(1+\mu) + 2\left(e_1^2+e_2^2\right)} \tag{6.36}$$

with $\mu > 0$, and control $\alpha^*(e_1,e_2)$ as defined in (6.35). For the existence of control $\delta\alpha^*$, it is also needed to assure that,

$$2\alpha^*(e_1,e_2)(1+\mu) + 2\left(e_1^2+e_2^2\right) \neq 0 \tag{6.37}$$

3. There are two solutions for control $\alpha(e,0)$ which passifies the error dynamics with $v = 0$,

$$\begin{array}{rcl} \alpha_1(e,0) & = & \alpha_1^*(e) + \delta\alpha^*(e,\alpha_1^*,0) \\ \alpha_2(e,0) & = & \alpha_2^*(e) + \delta\alpha^*(e,\alpha_2^*,0) \end{array}$$

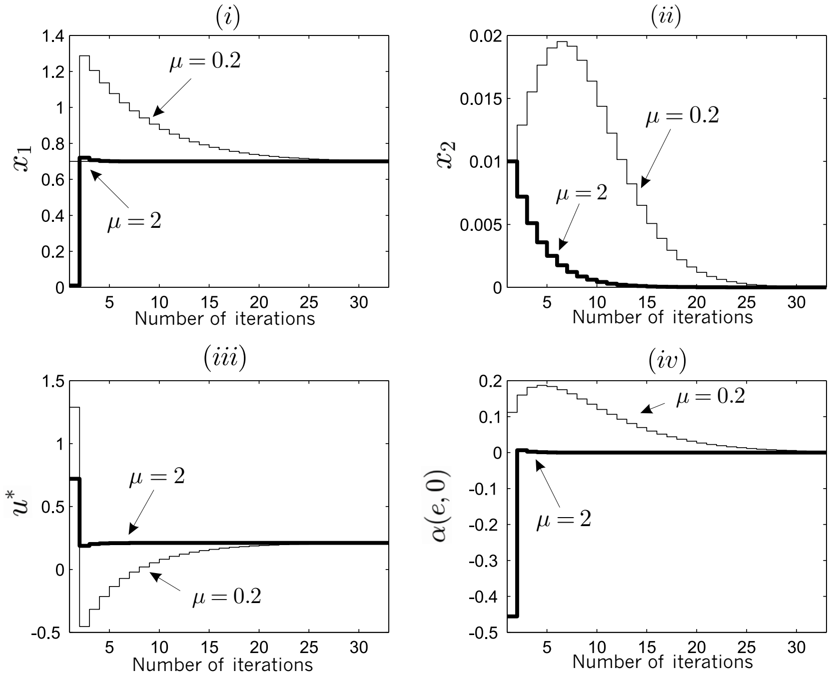

Figure 6.12: Stabilized system response of (5.23) for the steady state $(\bar{x}_1 = 0.7, 0)^T$ with $x_0 = (0.01, 0.01)^T$, $\xi_{01} = 0.01$, for different values of μ: $\mu = 0.2$, $\mu = 2$ (i) x_1 (ii) x_2 (iii) stabilizing control $u^*(x,e)$ (iv) passifying control for the error dynamics and $v = 0$, $\alpha_1(e,0) = \alpha_1^*(e) + \delta\alpha^*(e, \alpha_1^*, 0)$.

The applicability condition (5.72) of this passifying method takes the following form,

$$\left| \frac{-\mu\left[(e_1^2+e_2^2)^2+(\alpha^*)^2+e_1^2+e_2^2\right]-(1+\mu)\min[(\delta\alpha^*)^2]}{2\alpha^*(1+\mu)+2\left(e_1^2+e_2^2\right)} \right| \leq \rho \qquad (6.38)$$

with $\rho > 0$ and sufficiently small.

Remark 6.4 *When the fixed point of the error system is achieved, i.e., $e_1 = 0$, $e_2 = 0$, α^* becomes zero and the denominator of $\delta\alpha^*$ also becomes zero. This is not a problem for the stabilization method, due to the fact that the value for the passifying control $\delta\alpha^* + \alpha^*$ in the steady state is zero, so the value of zero for the passifying control is introduced without the need of computing it by means of the equation of $\delta\alpha^*$. This problem would be overcome for $s(h(x,u),v)$ such that $\frac{\partial s(h(x,u),0)}{\partial u} \neq 0$.*

Figure 6.12 illustrates the result of the ESDI stabilization scheme using the passifying control $\alpha_1(e,0)$. The desired fixed point of the system $(\bar{x}_1, 0)$ is asymptotically stable. Results are similar using control $\alpha_2(e,0)$ instead of $\alpha_1(e,0)$.

The response of the stabilized system depends on the parameter μ appearing in the dissipation rate function $\phi(e, \alpha(e,0))$. Parameter μ acts as the damping coefficient of the system. The influence of this parameter is illustrated in Figure 6.12. It can be noticed that the greater μ is the faster the system response is and the less overshooting the system has, however, the higher the peak value of the control $\alpha(e,0)$ is. The smaller μ is, the slower the system converges to $(\bar{x}_1,0)$ and the greater the peak value of the stabilizing control $u^*(x,e)$ is.

6.7.2 Analysis of the approximations

As it was pointed out in Section 5.4.1.2, for the validity of control $\alpha(e,0)$ it is necessary to analyze the approximations made for functions $V(f(e,\alpha(e,0)))$, $\phi(e,\alpha(e,0))$ and $s(h(e,\alpha(e,0)),v)$. As $v=0$, the function $s(h(e,\alpha(e,0)),0)=0$ and there is no need to study its first-order Taylor approximation.

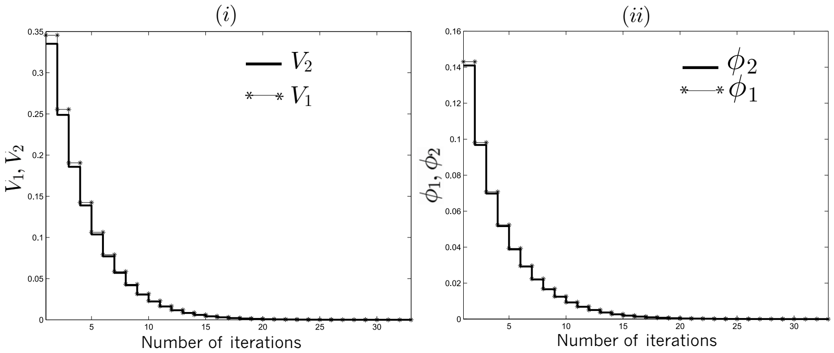

Figure 6.13: Analysis of the approximations for $V(f(e,\alpha_1(e,0)))$ and $\phi(e,\alpha_1(e,0))$ with $x_0 = (0.01,0.01)^T$, $\bar{x}_1 = 0.7$, $\xi_{01} = 0.01$, $\mu = 0.2$ (i) V_1, V_2 (ii) ϕ_1, ϕ_2.

Consider functions V_1, V_2, ϕ_1 and ϕ_2 calculated in Section 5.4.1.2, in which x is substituted by e, $u^*(x)$ by $\alpha^*(e)$ and $\delta u^*(x,u^*,v)$ by $\delta\alpha^*(e,\alpha^*,0)$. As $\delta\alpha^*$ tends to zero in the steady state ($e=0$), the approximation errors are zero and the approximations made are valid (see Remark 5.16). In Figure 6.13, functions V_1,V_2 and ϕ_1, ϕ_2 are compared graphically, for $\mu=0.2$, with $\alpha_1(e,0)$. It can be noticed that unless the steady state is reached, V_2 is not equal to V_1 and ϕ_1 is not equal to ϕ_2. When the system has reached its steady state $\delta\alpha^*=0$, we have that $V_2=V_1$, $\phi_1=\phi_2$. In the transient response, the larger in modulus control $\delta\alpha^*$ is, the worse the approximation is and the larger the differences V_2-V_1, $\phi_1-\phi_2$ are. This is illustrated by the fact that for the stabilized system response with $\mu=0.2$, the approximations V_2 and ϕ_2 are better than the approximations given for $\mu=2$.

6.8 Stabilization of an academic nonlinear example. Feedback dissipativity through Method 4

In this section, the ESDI method will be applied to the nonlinear example (5.23). Control $\alpha(e,0)$ will be obtained by means of the feedback dissipativity methodology presented in Section 5.4.2. We will be based on the passifying scheme proposed in Section 5.4.2.2 for this example, where the applications conditions for the methodology were analyzed, and on Proposition 5.4, now considering $u^*(x,v)$ as $\alpha^*(e,0)$ and $\delta u^*(x,u^*,v)$

as $\delta\alpha^*(e,\alpha^*,0)$, and having $\alpha(e,0) = \alpha^*(e,0) + \delta\alpha^*(e,\alpha^*,0)$. The sets where the errors and the controls $\alpha^*(e)$, $\delta\alpha^*(e,\alpha^*,0)$ are defined are considered the same as the ones used for the states and controls $u^*(x)$ and $\delta u^*(x,u^*,v)$, therefore, the analysis given for parameter μ in Section 5.4.2.2 is valid for μ appearing in the passifying control of the error dynamics in this section. Consider the sets where the states and the controls are defined given in Section 6.4.3.

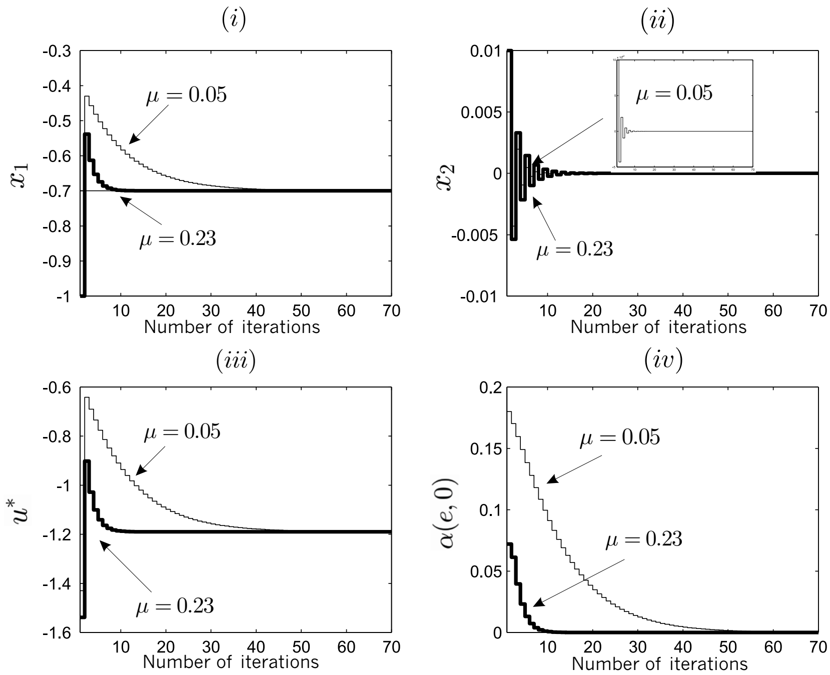

Figure 6.14: Stabilized system response of (5.23) for the steady state $(\bar{x}_1 = -0.7, 0)^T$ with $x_0 = (-1, 0.01)^T$, $\xi_{01} = 0.01$, for different values of μ: $\mu = 0.05$, $\mu = 0.23$ (i) x_1 (ii) x_2 (iii) stabilizing control $u^*(x,e)$ (iv) passifying control for the error dynamics and $v = 0$, $\alpha_1(e,0) = \alpha_1^*(e,0) + \delta\alpha^*(e,\alpha_1^*,0)$.

6.8.1 Computation of the error passifying control

Consider the ESDI control scheme proposed for the nonlinear example (5.23) in Section 6.4.3. The four steps given are considered with **Step 1** taking the following form:

Step 1 Computation of $\alpha(e,0) = \alpha^*(e,0) + \delta\alpha^*(e,\alpha^*,0)$. Let

$$\mathscr{E}_1 = [-\varepsilon_{x_1}, \varepsilon_{x_1}] \times [-\varepsilon_{x_2}, \varepsilon_{x_2}],\ \mathscr{E}_2 = [-\varepsilon_u, \varepsilon_u],$$

with ε_{x_1}, ε_{x_2} and ε_u positive constants. Consider $\alpha^* : \mathscr{E}_1 \times \mathscr{E}_2 \to \mathscr{E}_2$ and $\delta\alpha^* : \mathscr{E}_1 \times \mathscr{E}_2 \times \mathscr{E}_2 \to \mathscr{E}_2$ with $|\delta\alpha^*| \leq \rho$, and $\rho > 0$. Then, controls $\alpha^*(e,0)$ and $\delta\alpha^*(e,\alpha^*,0)$ are calculated as follows:

1. Computation of the control $\alpha^*(e,0)$ which makes the error dynamics (V,s)-lossless, with $v = 0$, i.e.,

$$V(f(e,\alpha^*(e,0))) - V(e) = s(h(e,\alpha^*(e,0)),0),\ \forall e \in \mathscr{E}_1 \tag{6.39}$$

Since feedback passivity is considered, for our example $s(h(e,\alpha^*(e,0)),0) = 0$, then equation (6.39) results in equation (6.31) and two solutions of $\alpha^*(e,0)$ are obtained: $\alpha_1^*(e,0)$ and $\alpha_2^*(e,0)$ as defined in (6.35).

2. Computation of the control $\delta\alpha^*(e,\alpha^*,0)$,

$$\delta\alpha^*(e_1,e_2,\alpha^*,0) = -2\mu(\alpha^* + e_1^2 + e_2^2) \tag{6.40}$$

with control α^* as defined in (6.35).

3. There are two solutions for control $\alpha(e,0)$ which passifies the error dynamics with $v = 0$,

$$\begin{aligned}\alpha_1(e,0) &= \alpha_1^*(e,0) + \delta\alpha^*(e,\alpha_1^*,0)\\ \alpha_2(e,0) &= \alpha_2^*(e,0) + \delta\alpha^*(e,\alpha_2^*,0)\end{aligned}$$

The applicability conditions (5.96)-(5.97) for this example are taken into account and the parameter μ will be bounded. As it was obtained in Section 5.4.2.2, the admissible values of μ for the passifying method to be valid are,

$$0 < \mu \le \min(\overline{\mu}_1, \overline{\mu}_2) \tag{6.41}$$

with

$$\overline{\mu}_1 = \left[\frac{-\varepsilon_u}{2(\varepsilon_{x_1}^2 + \varepsilon_{x_2}^2) + \varepsilon_u}\right]^2, \quad \overline{\mu}_2 = \frac{\rho}{4(\varepsilon_{x_1}^2 + \varepsilon_{x_2}^2) + 2\varepsilon_u}$$

Then, for these values of μ the method is valid for a sufficiently small ρ.

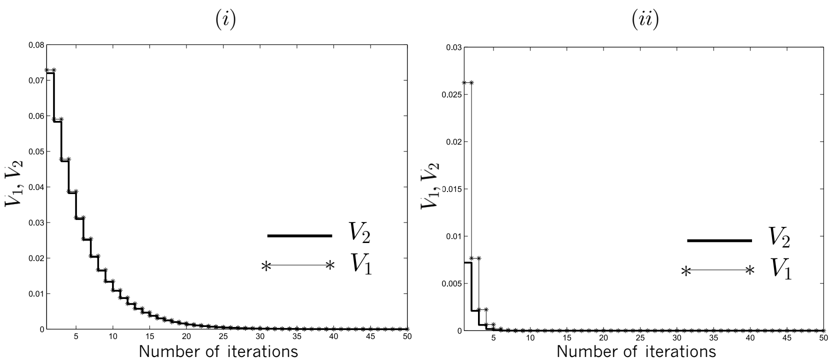

Figure 6.15: Analysis of the approximations for $V(f(e,\alpha_1(e,0))$ with $x_0 = (-1, 0.01)^T$, $\overline{x}_1 = -0.7$, $\xi_{01} = 0.01$, for different values of μ (i) V_1, V_2 for $\mu = 0.05$ (ii) V_1, V_2 for $\mu = 0.23$.

The presented ESDI control scheme is applied to system (5.23) using the passifying control for the error dynamics $\alpha_1(e,0)$, and the system converges to its desired fixed point $(\overline{x}_1, 0)$. It is considered $\varepsilon_{x_1} = 1$, $\varepsilon_{x_2} = 0.01$, $\varepsilon_u = 1.9$, $\rho = 1.8$ and the values for the upper bounds of μ are given by $\overline{\mu}_1 = 0.2373195$ and $\overline{\mu}_2 = 0.2307574$, consequently, admissible values of μ for which the passifying method is valid are $\mu \in (0, \overline{\mu}_2]$. The response of the system for $\alpha_1(e,0)$ and different values for the constant μ is depicted in Figure 6.14. Changes in the constant μ influence the response of the stabilized system.

6.8.2 Analysis of the approximations

Now, the approximation made for functions $V(f(e,\alpha(e,0)))$ and $s(h(e,\alpha(e,0)),v)$ are analyzed. As $v=0$, the function $s(h(e,\alpha(e,0)),0)=0$ and there is no need to study its first-order Taylor approximation.

Consider functions V_1, V_2 calculated in Section 5.4.1.2, in which x is substituted by e, $u^*(x,v)$ by $\alpha^*(e,0)$ and $\delta u^*(x,u^*,v)$ by $\delta\alpha^*(e,\alpha^*,0)$. As $\delta\alpha^*$ tends to zero in the steady state ($e=0$), the approximation errors are zero and the approximations made are valid (see Remark 5.22). In Figure 6.15, functions V_1,V_2 are compared graphically, for different values of μ: $\mu=0.05$, $\mu=0.23$, with $\alpha_1(e,0)$. It can be noticed that unless the steady state is reached, V_2 is not equal to V_1. When the system has reached its steady state $\delta\alpha^*=0$, $V_2=V_1$ is obtained. In the transient response, the worse the approximation is, the larger the difference V_2-V_1 is. The smaller in modulus the value of $\delta\alpha^*$ is, the better the approximation is, that is why the stabilized system response with $\mu=0.23$ gives the worst approximation V_2, however, with $\mu=0.23$ V_1-V_2 gets zero sooner due to the fact that the response of the system is faster than the response with $\mu=0.05$.

6.9 Conclusions and further research

In this chapter, some stability properties of (V,s)-dissipative and feedback dissipative nonlinear discrete-time systems have been analyzed. Sufficient conditions under which a class of feedback dissipative systems is locally stabilizable have been proposed. The dissipativity-based ESDI controller design methodology has been extended to general nonlinear single-input single-output systems in the discrete-time domain. This control scheme has been validated in the stabilization of the voltage in a proposed discrete-time model for the DC-to-DC buck converter and the stabilization of a nonlinear academic example. The different feedback dissipativity methodologies proposed in Chapter 5, restricted to the passivation case, have been used in the ESDI control design given for the two examples. The controller constants have been bounded for the validity of the methods. For the case of the buck converter, the proposed control scheme has been illustrated to be robust under variations in the load parameter.

The main contribution of this chapter has been the extension of the ESDI controller design method for the stabilization of nonlinear discrete-time systems, in addition to the application of the new feedback dissipativity techniques proposed in Chapter 5 for stabilization purposes.

As future work, it would be interesting to explore other kinds of stability different to Lyapunov stability for the stabilization of dissipative nonlinear discrete-time systems, such as contraction analysis, see *(Lohmiller, 1998)* [95] and *(Lohmiller and Slotine, 1998)* [96]. This fact implies to give another kind of dissipativity characterizations different from the ones which use storage functions seen as Lyapunov functions.