Chapter 8

Brief notes on dissipativity and passivity implications in the discrete-time setting

8.1 Introduction

This chapter is endeavored to motivate the use and further research of dissipative and passive discrete-time systems exploiting their frequency-domain characteristics. Our aim will be to answer the question what dissipativity may be for. For this purpose, some important features and implications of the dissipativity and passivity properties in the discrete-time setting are collected. For the linear case, an important tool will be used as a way of illustrating the special properties that dissipative and passive systems exhibit: the frequency-domain interpretation of passivity and dissipativity by means of the positive realness property of a transfer function.

This chapter is out of the spirit of the preceding chapters of this dissertation, it is thought as a results informing and collecting chapter; nevertheless, it is different from the background review Chapter 2, since some small contributions are given and some ideas which have not been completely exploited before are presented from another viewpoint. Some kind of formalism will be lost in order to gain in understanding. All the implications of dissipativity and passivity given in this chapter are considered as approaches to explore and study in a deeper way in the future.

As it was stated in Chapter 2, dissipativity and passivity implications in dynamical continuous-time systems have been broadly studied. However, a great variety of problems concerning dissipativity and passivity in the discrete-time setting have not attracted as significant attention as in the continuous-time domain. This is the case of the study of the frequency-domain implications of dissipativity, the interconnection of dissipative and passive discrete-time systems or the study of absolute stability by means of the dissipativity approach in nonlinear discrete-time systems.

In this chapter, passivity preservation under feedback and parallel interconnections is studied. Some notes on the preservation of some classes of dissipativity under feedback and parallel interconnections are also given. Furthermore, the frequency-domain properties of dissipativity are related to some of the most important frequency-based nonlinear feedback stability criteria in the discrete-time domain, such as Popov's, Tsypkin's and the circle criteria.

The preservation of dissipativity and passivity under sampling is also treated. Some comments about the difficulties that passivity implies in the discrete-time domain will be remarked in order to make notice the problems involved in the discrete-time study of dissipativity-related properties.

It must be pointed out that in the sequel, when we refer to passivity and dissipativity, we will refer to V-passivity and (V,s)-dissipativity, respectively.

The chapter is organized as follows. Section 8.2 revisits the frequency-domain characteristics of dissipative and passive systems which will be used in the sequel. Section 8.3 is devoted to the preservation of dissipativity and passivity under sampling. Section 8.4 studies the interconnection of passive discrete-time systems. Section 8.5 collects the most important existing results in nonlinear feedback stability analysis and relates to them the dissipativity frequency-domain properties of discrete-time systems. Two examples are used in order to illustrate all these properties: the continuous-time and the discrete-time models of the DC-to-DC buck converter given in Chapter 5, and the continuous-time and the discrete-time models for a simple *RC*-circuit. These examples are linear in order to use the frequency-domain characteristics of dissipativity and passivity. Conclusions and suggestions for future work are given in the last section.

8.2 Frequency-domain characteristics

The formalization of dissipativity and passivity concepts in connection with the frequency-domain properties of dissipative and passive linear systems was presented in Chapter 2 by means of the KYP lemma. The frequency-domain interpretation of passivity for linear systems is given by means of the positive realness property of a transfer function. Passivity is equivalent to positive realness, see for the discrete-time case *(Hitz and Anderson, 1969)* [58]. Dissipativity is also equivalent to the positive realness of a transfer function see *(Goodwin and Sin, 1984)* [45]. Moreover, in the linear case, the connection between dissipativity and frequency-domain stability concepts is given. In this section, the most important results concerning these points, which will be used in the rest of the chapter, will be shown.

Dissipativity merges the input-output behaviour features of a system and its internal description or state-space representation. For the linear case, this is given by the KYP lemma (see Figure 8.1) which establishes the equivalence between the state-space dissipativity properties of a system and its frequency-domain characteristics by means of positive real transfer functions.

As it has been pointed out before, the relations between passivity and dissipativity with the positive realness property was proposed in the discrete-time setting by Hitz and Anderson, and Goodwin and Sin, respectively. The results given in the discrete-time setting are preceded by the relations between passivity and dissipativity with the positive realness property in the continuous-time domain. Some important works appearing in

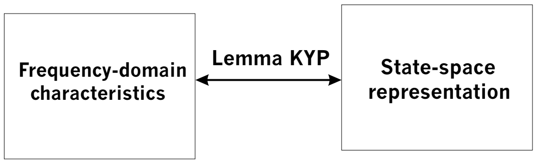

Figure 8.1: Merging state-space and frequency-domain dissipativity properties.

the literature devoted to this point in the linear case are presented here. The concept of strict positive realness is introduced as an equivalent concept to strict passivity.

The concept of positive realness is known to come from the circuit-theoretic analysis, this is formalized for single-input single-output (SISO) linear systems of the form $\dot{x} = Ax + Bu, y = Cx$ in *(Anderson, 1967)* [3] where necessary and sufficient conditions for the transfer function matrix $G(s)$ to be strictly positive real are presented. These conditions are used in *(Desoer and Vidyasagar, 1975)* to establish the equivalence between output dissipativity and strict positive realness. The equivalence between strict positive realness and output dissipativity for SISO systems of the form $\dot{x} = Ax + Bu, y = Cx + Du$ is given in *(Anderson, 1967)* [4] from a circuit analysis viewpoint and in *(Desoer and Vidyasagar, 1975)*. This equivalence is given for the multiple-input multiple-output (MIMO) case for outputs with $D \neq 0$ in *(Arimoto and Naniwa, 2000)* [9], see references therein. In *(Arimoto and Naniwa, 2000)* [9], the equivalence between strict positive realness, output dissipativity, and what is called learnability, related to the invertibility of the system, is stood out. It must be pointed out that from the existing works in the last four decades about positive realness, there appears to be no consensus for the definition for strict positive realness, see for example *(Wen, 1988)* [177] where the frequency-domain conditions for strict positive realness are unified for the MIMO case.

The interest of studying positive realness properties arises from their implications in systems stability. Positive real systems have played a major role in stability theory. An interesting linear control problem which has attracted broad attention is the *positive real control problem* which consists in designing a controller which renders the closed-loop transfer function positive real, see *(Sun and Shim, 1994)* [167] and references therein, *(Joshi and Gupta, 1996)* [64] and *(Safonov et al., 1987)* [146] which is an interesting tutorial paper explaining the relationships between positive real transfer functions, sector problems and the small-gain problem. The study of the positive real control problem is motivated because robust stability can be guaranteed provided an appropriated closed-loop system strictly positive real *(Bao et al., 1996)* [10], *(Kelkar and Joshi, 1998)* [71]; in these last two mentioned works the positive real control problem is considered as a passivation problem. The robust quadratic dissipative control problem, i.e., the problem of rendering an uncertain linear system (Q,S,R)-dissipative and asymptotic stable is treated in *(Xie et al., 1998)* [184]. All these works are in the framework of continuous-time systems. The discrete-time counterpart of these results is given in *(de Souza and Xie, 1992)* [164], *(Souza et al., 1993)* [165] for the positive control problem and in *(Tan et al., 1999)* [170], *(Tan et al., 2000)* [171] for the quadratic dissipative control problem and the robust quadratic dissipative control problem, respectively. Some examples of works which apply the frequency-domain implications of dissipativity and passivity in the discrete-time case for control purposes are: *(Colgate and Schenkel, 1994, 1997)*

[18, 17], *(Tsai, 1996)* [172]), *(Albertos, 1993)* [1], devoted to the study of passivity in sampled-data systems, which will be further mentioned along this chapter.

In this section, the most important results, which will be used in the rest of the chapter, referring the relations between passivity and dissipativity with the positive realness property will be collected. The results given for the continuous-time case will be also presented due to the fact that discrete-time results are derived from them.

8.2.1 The continuous-time case

Dissipativity and passivity concepts and the study of the relations between them and positive realness comes from the circuit-theoretic analogy, see for example *(Zames, 1966)* [190]. The concept of positive real transfer functions is originated in the continuous-time setting in network theory as the frequency-domain formulation of the fact that the time integral of the energy input to a passive network must be positive, in other words, a linear time-invariant passive circuit, having positive resistance, inductance, and capacitance values, has a positive real impedance function. Then, a MIMO continuous-time passive system with an input and an output defined by the variables u and y, respectively, is such that for each time interval $[0,T]$, see *(Popov, 1973)* [141], *(Takahashi, 1975)* [169],

$$\begin{aligned}\int_0^T y^T(t)u(t) &= \left\{\text{energy storage at time T}\right\}+\\ &\quad +\left\{\text{energy dissipation in time [0,T]}\right\}-\\ &\quad -\left\{\text{initial energy storage}\right\}\\ &\geq -\gamma_0^2,\ \forall T>0 \end{aligned} \tag{8.1}$$

where γ_0^2 is a positive constant (initial energy storage) which only depends on the initial state of the system. Condition (8.1) is also regarded as hyperstability condition *(Popov, 1973)* [141]. Consider a MIMO LTI system, and let $G(s)$ be the transfer function associated to the system. If (8.1) is satisfied then,

$$H(\mathrm{j}\omega) = G^*(\mathrm{j}\omega) + G^T(\mathrm{j}\omega) \geq 0 \tag{8.2}$$

for all real ω for which $\mathrm{j}\omega$ is not a pole of an element of $G(s)$ and $H(\mathrm{j}\omega)$ is Hermitian, with $*$ denoting the operation of complex conjugation, and H is Hermitian in the sense that $(H^T)^* = H$. Relation (8.2) is one of the conditions to be satisfied for $G(s)$ to be a positive real transfer function, see Theorem 2.1 and *(Hitz and Anderson, 1969)* [58], *(Popov, 1973)* [141].

For the SISO case, the positive real condition (8.2) is rewritten as

$$Re\left[G(\mathrm{j}\omega)\right] \geq 0 \tag{8.3}$$

for all real ω for which $\mathrm{j}\omega$ is not a pole of $G(s)$.

Corollary 8.1 (Tsai, 1996) *[172] If a SISO LTI continuous-time system is passive then the following inequality holds*

$$Re[G(s)] \geq 0,\ \textit{whenever}\ Re(s) \geq 0$$

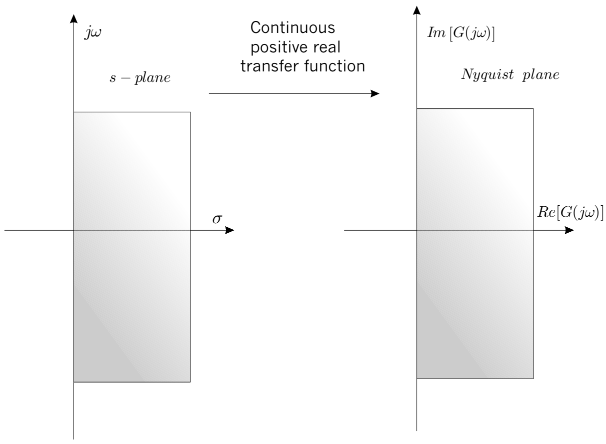

Figure 8.2: Property of a SISO linear time-invariant continuous-time passive system.

In a graphical and intuitive way, this property of passive SISO LTI systems can be illustrated, see Figure 8.2. A continuous-time passive transfer function maps points in the right-half plane of the s-plane to the right-half plane in the Nyquist plot. The Nyquist plot of a passive system lies on the right-half plane, which means that an infinite gain proportional control can be introduced without destabilizing the system. In addition, positive real transfer functions do not have poles on the right-half-s-plane and their poles lying on $Re(s) = 0$ are simple with positive real residues.

Positiveness is also considered as a stability passive energetic system characteristic, which is also regarded as hyperstability, see *(Popov, 1973)* [141], *(Takahashi, 1975)* [169].

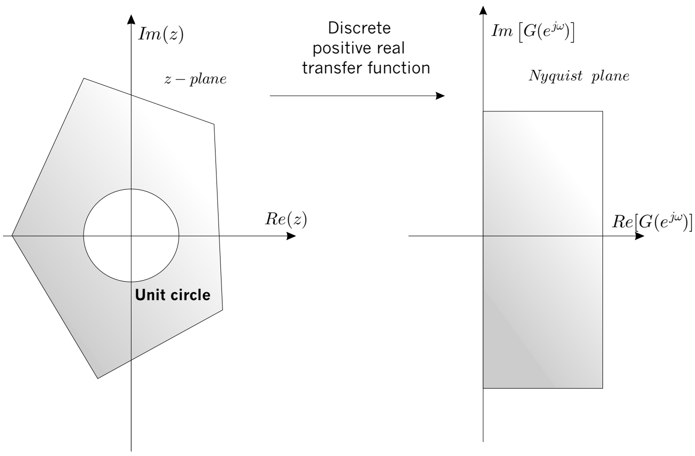

Figure 8.3: Property of a SISO linear time-invariant discrete-time passive system.

8.2.2 The discrete-time case

8.2.2.1 The passivity case

For the discrete-time case, condition (8.1) is rewritten in the following way, which is also called hyperstability condition *(Popov, 1973)* [141], *(Takahashi, 1975)* [169],

$$\sum_{k=1}^{N} y^T(k)u(k) \geq -\gamma_0^2 \tag{8.4}$$

For MIMO LTI systems, condition (8.4) implies that,

$$G^*(e^{j\omega}) + G^T(e^{j\omega}) \geq 0 \tag{8.5}$$

$\forall$ real ω at which $G(e^{j\omega})$ exists, in addition to have a square matrix $G(z)$ of real-rational functions whose elements are analytic in $\mid z \mid > 1$ such that the poles of the elements of $G(z)$ on $\mid z \mid = 1$ are simple and if $z_0 = e^{j\omega_0}$, ω_0 real, is a pole of an element of $G(z)$, and if K is the residue matrix of $G(z)$ at $z = z_0$, the matrix $Q = e^{-j\omega_0}K$ is nonnegative definite Hermitian *(Hitz and Anderson, 1969)* [58]. These positive realness conditions were presented in Chapter 2 in Theorem 2.2 or discrete-time KYP lemma, where they were related to the state-space passivity conditions.

As it is known, for the SISO case, the real positiveness of the linear block $G(z)$ means that the real part of $G(z)$ is positive for all z outside the unit circle in the z-plane. This property can be easily identified via the Nyquist diagram of the associated transfer function of the system, which is confined in the right-hand side half of the Nyquist plane. This can be illustrated in a graphical way. A discrete-time passive transfer function can be also seen as a map which maps points outside the unit circle to the right-half plane of the z-Nyquist plane. This fact is illustrated in Figure 8.3. In addition, positive real transfer functions do not have poles with modulus greater than one, and their poles lying on $|z| = 1$ are simple with positive real residues. These features will be used in the sequel.

8.2.2.2 The dissipativity case

The generalized version of the KYP lemma, for (Q,S,R)-dissipative discrete-time systems is given in *(Goodwin and Sin, 1984)* [45]. As in the passivity case, (Q,S,R)-dissipative systems can be also identified by the positive realness of a transfer function, see Lemma 2.1 in Chapter 2. The most useful consequences of this fact are the constraints on $G(e^{j\omega})$ for SISO systems, which will be used in this chapter to analyze the absolute stability of discrete-time systems. The characteristics of the Nyquist plot of $G(e^{j\omega})$ for (Q,S,R)-dissipative SISO systems are presented depending on the form of the supply function *(Goodwin and Sin, 1984)* [45]. Two cases are analyzed: Q being negative and $Q = 0$. On the one hand, if $Q < 0$, the Nyquist plot of $G(e^{j\omega})$ lies inside the circle with center $S/\mid Q \mid$ and radius $(1/\mid Q \mid)\sqrt{S^2 + R \mid Q \mid}$. On the other hand, if $Q = 0$, the Nyquist plot of $G(e^{j\omega})$ lies to the right (if $S > 0$) or to the left (if $S < 0$) of the vertical line $Re\, z = -R/2S$. Figures 8.4 and 8.5 show the characteristics of the Nyquist plot of $G(e^{j\omega})$ for the four different classes of dissipativity: passivity, ISP, OSP and VSP (see Chapter 2). ε and δ are positive constants used in Chapter 2 for the definition of ISP, OSP and VSP.

In the next two sections, two linear SISO continuous-time systems will be considered. They are passive for an appropriate output, and consequently, their Nyquist plots

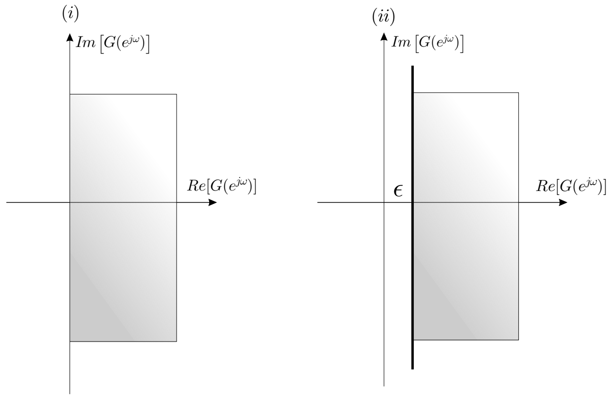

Figure 8.4: Frequency-domain properties for dissipative discrete-time LTI systems (i) Passivity (ii) ISP. Shaded part is allowable for $G(e^{j\omega})$.

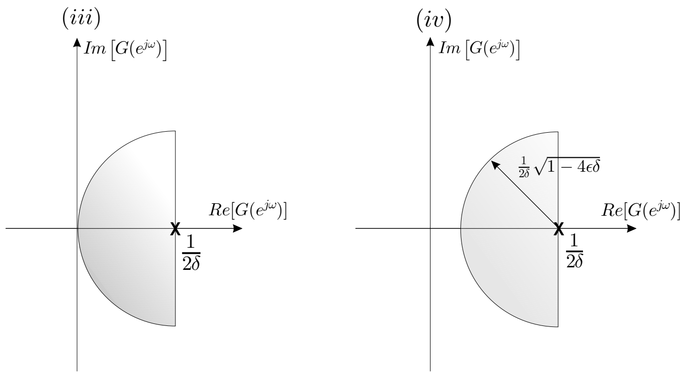

Figure 8.5: Frequency-domain properties for dissipative discrete-time LTI systems (iii) OSP (iv) VSP. Shaded part is allowable for $G(e^{j\omega})$.

lie on the right-side half. The two examples to be considered are a *RC* network and a model for the DC-to-DC buck converter analyzed in Chapter 5. The passive linear discrete-time models obtained from the presented continuous-time models are postponed to Section 8.3 where some discrete-time systems are obtained via the discretization of the passive continuous-time models of the *RC* network and the buck converter.

8.2.3 Example 1. A simple circuit

Let the electric circuit of Figure 8.6. This system can be described by the following ordinary differential equation,

$$\frac{dv(t)}{dt} = -\frac{1}{RC}v(t) + \frac{1}{C}i(t) \tag{8.6}$$

Considering the energy stored in the capacitance as the system storage energy function V, we have that,

$$V = \frac{1}{2}Cv^2 \tag{8.7}$$

Taking the current $i(t)$ as the input to the system, and the voltage $v(t)$ as its output, from (8.7) and claiming the definition of a passive system, system (8.6) is concluded to be passive. This passive system will be shown to have a positive real transfer function whose Nyquist plot lies on the right-hand side.

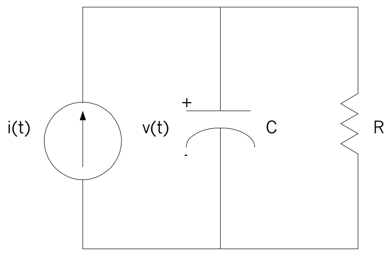

Figure 8.6: A simple electric passive system.

The transfer function associated to the state-space representation (8.6) is the following one,

$$\frac{V(s)}{I(s)} = G(s) = \frac{1/C}{s+1/(RC)} = \frac{1000}{s+100} \tag{8.8}$$

obtained for $C = 0.001F$, $R = 10\Omega$. The Nyquist plot of (8.8) is shown in Figure 8.7.

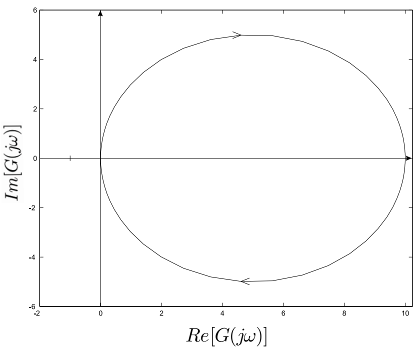

Figure 8.7: Nyquist plot of the continuous-time model for the RC electric network.

8.2.4 Example 2. The buck converter model

Let the continuous-time normalized average model of the DC-to-DC buck converter (5.10) with the following physical parameters: $V_{in} = 407V, L = 1mH, C = 80\mu F, R = 10\Omega$, $\gamma = 0.3535533906$. The energy associated to the system given by (5.15) is used as the storage function V. Considering the normalized current x_1 as the output, the system is passive. The system is concluded not to be passive considering the normalized voltage x_2 as the system output. These passivity and non-passivity properties will be illustrated by means of the Nyquist plot of these systems. Then, two transfer functions are obtained from system (5.10),

$$G_{ci}(s) = \frac{X_1(s)}{\hat{U}(s)} = \frac{s+0.3536}{s^2+0.3536s+1} \quad (8.9)$$

$$G_{cv}(s) = \frac{X_2(s)}{\hat{U}(s)} = \frac{1}{s^2+0.3536s+1} \quad (8.10)$$

where $G_{ci}(s)$ is the transfer function representing the relation between the input $\hat{u}$ and the output x_1; $G_{cv}(s)$ is the transfer function representing the relation between the input $\hat{u}$ and the output x_2. As it was expected, the Nyquist plot for (8.9) has the characteristics of a positive real transfer function, whereas the one corresponding to (8.10) does not (see Figure 8.8). In addition, all the poles of $G_{ci}(s)$ have real parts strictly negative, therefore $G_{ci}(s)$ is a positive real transfer function.

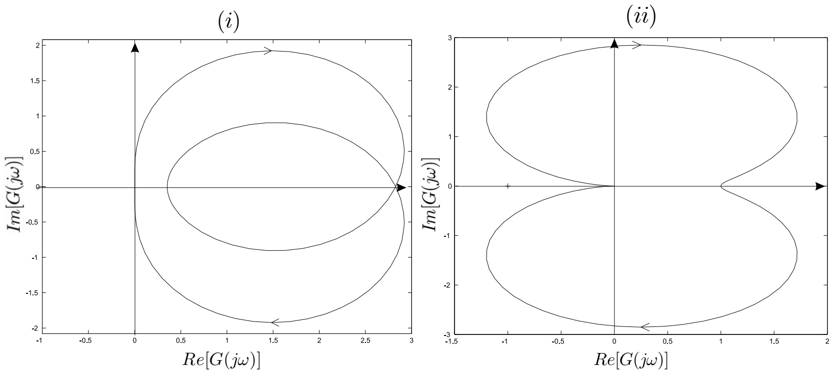

Figure 8.8: Nyquist plots for the transfers functions derived from the continuous-time state-space normalized average model of the DC-to-DC buck converter (i) The Nyquist plot of (8.9) lies on the right-hand side (ii) The Nyquist plot of (8.10) does not lie on the right-hand side.

8.3 Dissipativity preservation under sampling

8.3.1 Dissipativity under sampling

If a dissipative system is discretized, the discretized model is not assured to keep on being dissipative. The preservation of dissipativity or passivity under sampling has not been studied in a deep way. The work *(de la Sen, 2000)* [150] treats the problem of how to preserve the property of positive realness through discretization. As we know, the KYP Lemma offers the equivalence between positive realness and passivity in the linear setting. Therefore, the mentioned work could be considered as the study of preservation of passivity under sampling. The study of passivity of linear sampled-data systems is also given in *(Colgate and Schenkel, 1994)* [18]. The work of *(Kristiansen and Egeland, 2000)* [78] also treats the problem of preservation of passivity, however, for systems described by partial differential equations. A recent work which studies the preservation of the dissipation inequality under sampling for nonlinear systems is given by *(Nešić et al., 2000)* [123] and *(Laila and D. Nešić, 2001)* [79]. They also give an unified framework for the design of nonlinear digital controllers using the emulation method, considering static controllers *(Nešić et al., 2000)* [123] and dynamic controllers *(Laila and D. Nešić,* *2001)* [79]. Then, as it is known, in general, dissipativity and passivity are not preserved under sampling.

A very interesting work, which can be considered as an application of the concepts of passivity in order to stabilize a discrete-time linear system is the one presented by *(Tsai, 1996)* [172]. Frequency-domain characteristics of passive systems are exploited, and, what it is more challeging, a brief, but important comment about passivity under sampling is given. It is concluded that there are discretization methods which do not preserve passivity and there are some which do. A continuous-time system may be converted to the discrete-time setting without compromising passivity by using the trapezoid-rule transformation, i.e.,

$$s = \frac{2}{T}\frac{z-1}{z+1}, \tag{8.11}$$

with $z = e^{sT}$, and T the sampling period. The transformation (8.11) is also called the bilinear transformation from consideration of its mathematical form.

For the dissipative case, a way of illustrating that the trapezoid-rule transformation preserves dissipativity under sampling is the following one. Let a continuous-time system to be (V,s)-dissipative, which means that

$$V(x(t_f)) - V(x(t_0)) \leq \int_{t_0}^{t_f} s(y(\tau),u(\tau))d\tau, \tag{8.12}$$

in particular,

$$V(x(k+1)) - V(x(k)) \leq \int_{kT}^{(k+1)T} s(y(\tau),u(\tau))d\tau \tag{8.13}$$

If the integral is approximated by the trapezoid rule, it is obtained,

$$\frac{T[s(y(k+1),u(k)) + s(y(k),u(k))]}{2}, \tag{8.14}$$

which gives the bilinear transformation in the linear case. Here, as usual, u is supposed to be constant in a sampling period. The relation (8.14) will be valid depending on how good (8.14) approximates the integral appearing in (8.13), this fact will depend on the system treated and the form of the supply function s. In the linear case, it can be concluded that passivity is preserved under sampling using the trapezoid-rule transformation.

By means of two linear examples and using their passivity frequency-domain characteristics, it will be illustrated that the sampled-data model obtained from a continuous-time model by means of the exact discretization does not preserve passivity. The discretization of a linear system obtained by the trapezoid-rule transformation will be illustrated to preserve passivity. The two examples considered are the passive circuit presented in Section 8.2.3 and the discrete-time model for the buck converter used in Section 8.2.4.

8.3.2 Example 1. A passive circuit

Let consider the model for the electric circuit shown in Figure 8.6, corresponding to the state-space description (8.6). The first clue which alerts us about the possible loss of passivity through discretization is that the balance of energy may be lost under sampling. In Figure 8.9, it is depicted that for the exact discretization of system (8.6), for which the input is kept constant through each sampling interval, the stored energy ($V = \frac{1}{2}Cv^2$) is slightly less than the stored energy in the continuous-time model. This fact suggests that the discretized system may lose the passivity property. As it is pointed out in *(Colgate and Schenkel, 1994)* [18], sampling produces a time delay and a loss of information which may make the system lose its passivity property.

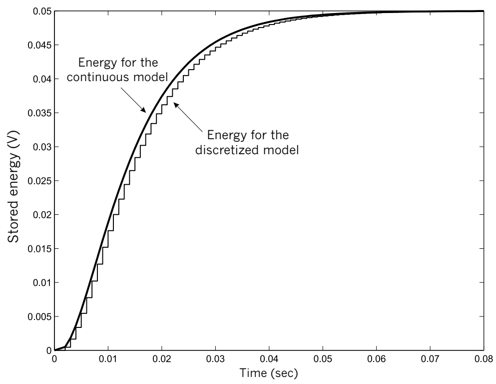

Figure 8.9: Comparison of the stored energy for the continuous-time circuit and its discretized model. The stored energy for the discretized model is slightly less than the one obtained for the continuous-time system.

Let us obtain the exact discretization of system (8.6). Consider the current i constant for each interval $[kT,(k+1)T], \forall k \geq 0$, and T the sampling period time. Taking $t_0 = 0$, $t_f = T$ and $v(k+1)$ as the solution to the Cauchy problem in $t = T$ having as initial condition $v(k)$. Then,

$$\begin{aligned} v(k+1) &= e^{\frac{-T}{RC}} v(k) + \frac{1}{C}\int_0^T e^{\frac{-T}{RC}(T-s)} B\, ds\, i(k) = \\ &= e^{\frac{-T}{RC}} v(k) + R(1 - e^{\frac{-T}{RC}}) i(k) \end{aligned} \tag{8.15}$$

The pulse transfer function for this system with v as the output has the following form,

$$G_{edc}(z) = \frac{R(1 - e^{\frac{-T}{RC}})}{z - e^{\frac{-T}{RC}}} \tag{8.16}$$

which for $C = 0.001F$, $R = 10\Omega$, $T = 0.001sec$, is such that

$$G_{edc}(z) = \frac{0.9516}{z - 0.9048} \tag{8.17}$$

As it is depicted in Figure 8.10, the Nyquist plot of (8.17) does not correspond to the Nyquist plot of a positive real pulse transfer function, and consequently, the discretized system (8.17) is not passive. In conclusion, the exact discretization of the system does not preserve, in general, the passivity property.

On the other hand, using the trapezoid-rule transformation or bilinear transformation (8.11), the transfer function (8.8) is converted into

$$G_{tdc}(z) = \frac{1000Tz + 1000T}{(2 + 100T)z + (100T - 2)}$$

which for $T = 0.001$ takes the form

$$G_{tdc}(z) = \frac{0.47619(z+1)}{z-0.9048} \tag{8.18}$$

The Nyquist diagram of (8.18) lies on the right-hand side, see Figure 8.10, in addition, the system has a fixed point $x = 0$ being asymptotically stable. In conclusion, the trapezoid-rule transformation preserves positive realness through discretization, in other words, the linear discrete-time resulting system is passive.

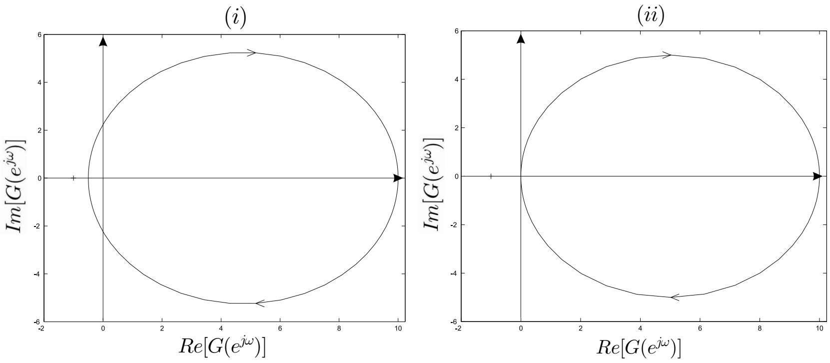

Figure 8.10: (i) Nyquist diagram for the exact discretization of the passive circuit (ii) Nyquist diagram for the discretized passive circuit using the trapezoid-rule transformation.

8.3.3 Example 2. The buck converter model

The results obtained with the continuous-time buck converter model (5.10) shown to be passive with respect to the current as the system output, are equivalent to the ones presented for the passive *RC*-circuit of the previous section.

First, consider the exact discretization of the original continuous-time model (5.10) with the physical parameters used in Section 8.2.4. The pulse transfer function obtained from the state-space description (5.13) with $y = x_1$, $T = 0.3535533906$ and $a = 0.9406416964, b = 0.3254699438, c = 0.8255706942, \gamma = 0.3535533906$ is the following one:

$$G_{edb}(z) = \frac{0.34646(z-0.8813)}{z^2-1.766z+0.8825} \tag{8.19}$$

The system represented by (8.19) converges to its fixed point, but it is not passive, as it is illustrated by its Nyquist plot (Figure 8.11), which is not completely in the right-hand side.

Applying the trapezoid-rule transformation on system (8.9) the following transfer function in z is obtained,

$$G_{tdb}(z) = \frac{T(2+aT)z^2+2aT^2z+T(aT-2)}{(4+2aT+T^2)z^2+(2T^2-8)z+(4-2aT+T^2)}, \tag{8.20}$$

where $a = 0.3536$. Considering as sampling period time T=0.3535533906, (8.20) takes the form

$$G_{tdb}(z) = \frac{0.17173(z+1)(z-0.8823)}{z^2-1.771z+0.8857} \tag{8.21}$$

System (8.21) is passive (as it is shown in Figure 8.11) and converges to its fixed point.

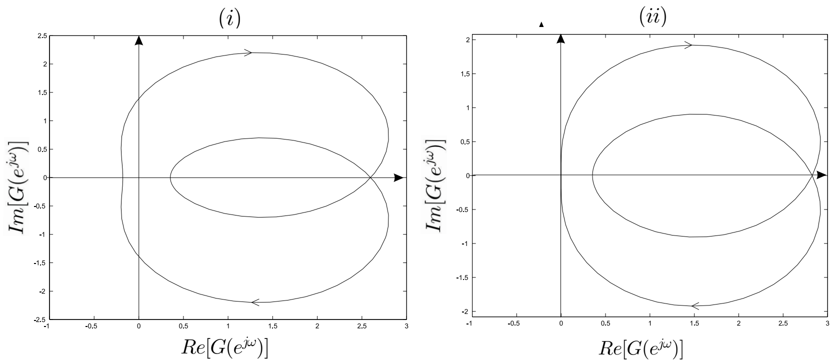

Figure 8.11: (i) Nyquist diagram for the exact discretized model of the buck converter (ii) Nyquist diagram for the discretized buck converter using the trapezoid-rule transformation, $y = x_1$.

Remark 8.1 *It is interesting to notice in the examples shown in this section that the relative degree of passive continuous-time systems is one, see (8.8) and (8.9), otherwise, the relative degree for the associated passive pulse transfer functions have relative degree zero (as it was established in Chapter 7), see (8.18) and (8.21).*

To conclude with, the discretization of a linear system by means of the trapezoid-rule transformation preserves the passivity property of the continuous-time system, due to the fact that the positive realness property is preserved and the stability property of the system is also preserved under sampling (the A-stability preservation under sampling is treated in *(Tsai, 1996)* [172]). In general, passivity is not preserved under exact discretization.

There exist some works in the literature which exploit the preservation of passivity under sampling and the frequency-domain passivity implications for stability purposes, most of these works are devoted to hybrid dynamical systems applied to robotics systems, see for example the works of Colgate and coworkers *(Colgate and Schenkel, 1994, 1997)* [18, 17], *(Tsai, 1996)* [172]). Another interesting study of sampled-data passive systems is given by *(Albertos, 1993)* [1]. The study of passivity frequency-domain properties related to stability in sampled-data systems has been also studied, see as a recent work *(Okuyama and Takemori, 1996)* [125], for more information and related works on this topic see Section 8.5.

8.4 Passivity implications in interconnected systems

One of the most important passivity results is that a negative feedback loop consisting of two passive systems is passive. In addition, under an additional detectability condition, this feedback system is also stable. This result is well known for continuous-time systems *(Sepulchre et al., 1997)* [152], but it has not been broadly exploited for the discrete-time case.

Passivity and dissipativity properties have been used in the framework of interconnected discrete-time systems for stability analysis purposes, see for example, *(Wu and*

Desoer, 1970) [182], *(Desoer and Vidyasagar, 1975)* [25]. However, the study of passivity preservation under block interconnection has aroused less attention in the discrete-time setting than in the continuous-time case (as recent works in the continuous-time setting, *(Bao et al., 1996)* [10], *(Kelkar and Joshi, 1998)* [71]). In the seminal work *(Popov, 1973)* [141], among other things, the interconnection of passive systems is studied by means of the introduction of the concept of hyperstability, that is, a closed-loop system consisting of a linear system with a nonlinear block in the feedback path is hyperstable when the nonlinear block satisfies a passivity-like characteristic and the linear block is positive real. This result is given either for the discrete-time or the continuous-time case.

The purpose of this section is to show an alternative way in studying whether the feedback and the parallel interconnections (given in Figure 8.12) of two discrete-time passive systems result in a passive system. It is inspired by the continuous-time results given in *(Sepulchre et al., 1997)* [152]. A linear example is used to illustrate the conclusions given.

8.4.1 Feedback and parallel interconnections of passive discrete-time systems

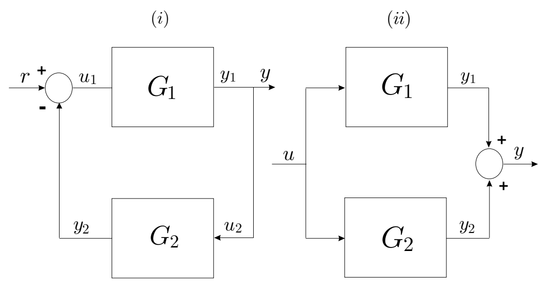

Figure 8.12: (i) Feedback interconnection (ii) Parallel interconnection.

Theorem 8.1 (Navarro-López et al., 2002) *[120] Consider the systems G_1 and G_2 (linear or nonlinear) to be passive. Then, the systems resulting from the feedback and the parallel interconnections of systems G_1 and G_2 are passive.*

Proof.

Let x_1 states of G_1, and x_2 states of G_2. Taking into account the dissipativity definition given in Chapter 2 (Definition 2.5), and particularizing it for the passivity case, i.e., $s(y,u) = y^T u$, it is concluded that if G_1 and G_2 are passive, then there exist two storage functions $V_1(x_1)$ and $V_2(x_2)$, such that

$$V_1(x_1(k+1)) - V_1(x_1(k)) \leq y_1^T u_1 \tag{8.22}$$

$$V_2(x_2(k+1)) - V_2(x_2(k)) \leq y_2^T u_2 \tag{8.23}$$

A new state vector is defined as $x := (x_1, x_2)$, which will be the new state vector for the interconnected systems, and a new positive definite storage function V is also considered

$$V(x) := V_1(x_1) + V_2(x_2) \tag{8.24}$$

For the feedback interconnection (i), one has

$$V(x(k+1)) - V(x(k)) \leq y_1^T u_1 + y_2^T u_2$$

Taking into account that $u_2 = y_1$, $u_1 = r - y_2$, it follows that $y_1^T(r - y_2) + y_2^T y_1 = y_1^T r$. Consequently,

$$V(x(k+1)) - V(x(k)) \leq y_1^T r,$$

that is, the feedback interconnected system is passive.

For the parallel interconnection, the output of the system is $y_1 + y_2 = y$. If G_1 and G_2 are passive,

$$V_1(x_1(k+1)) - V_1(x_1(k)) \leq y_1^T u \tag{8.25}$$

$$V_2(x_2(k+1)) - V_2(x_2(k)) \leq y_2^T u \tag{8.26}$$

Adding (8.25) and (8.26), it is obtained

$$V(x(k+1)) - V(x(k)) \leq (y_1 + y_2)^T u = y^T u,$$

i.e., the system corresponding to the parallel interconnection is passive.

■

Remark 8.2 *Following the same procedure, it can be easily checked that the property of OSP for supply functions of the form (2.23) is preserved under feedback block interconnection. Besides, ISP for supply functions as in (2.23) is preserved under parallel interconnection.*

8.4.2 An example. Interconnection of passive linear discrete-time systems

For SISO LTI dynamics, a way of illustrating that the feedback and parallel interconnections of two passive systems result in a passive system is by means of the positive realness property of the transfer function of the interconnected resulting systems.

The corresponding transfer functions for systems (i) and (ii) given in Figure 8.12 are the following ones

$$G_f(z) = \frac{Y(z)}{R(z)} = \frac{G_1(z)}{1 + G_1(z)G_2(z)} \tag{8.27}$$

$$G_p(z) = \frac{Y(z)}{U(z)} = G_1(z) + G_2(z) \tag{8.28}$$

with $G_f(z)$ the transfer function for system (i) and $G_p(z)$ the corresponding one for (ii). Consider the discretized normalized model of the buck converter (8.21) obtained by means of the trapezoid-rule or bilinear transformation presented in Section 8.2.4. This system was proved to be passive and now will be connected to itself by means of a negative feedback and a parallel interconnection. The transfer functions for the feedback and parallel interconnections of system (8.21), respectively, take the form:

$$G_f(z) = \frac{k_1(z+1)(z-a_1)(z^2 - a_2 z + a_3)^2}{(z^2 - a_4 z + a_5)(z^2 - a_6 z + a_7)(z^2 - a_2 z + a_3)} \tag{8.29}$$

$$G_p(z) = \frac{k_2(z+1)(z-a_1)(z^2 - a_2 z + a_3)}{(z^2 - a_2 z + a_3)^2} \tag{8.30}$$

with

$k_1 = 0.16681$ $k_2 = 0.34345$ $a_1 = 0.8823$ $a_2 = 1.771$ $a_3 = 0.8857$
$a_4 = 1.787$ $a_5 = 0.8357$ $a_6 = 1.648$ $a_7 = 0.9385$

The Nyquist diagrams for (8.29) and (8.30) are presented in Figure 8.13. They both correspond to positive real transfer functions or to passive systems, due to the fact that their Nyquist diagrams lies on the right-hand side and they do not have poles with modulus greater than one. Transfer functions (8.29) and (8.30) corresponds to asymptotically stable systems to their fixed points with relative degree 0.

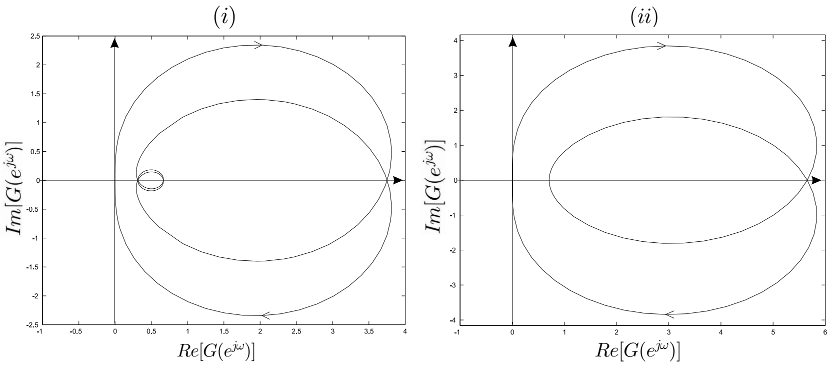

Figure 8.13: Nyquist plots for the feedback and parallel interconnections of the discretized normalized model of the buck converter obtained by means of the trapezoid-rule transformation (i) Nyquist plot for the feedback interconnection (ii) Nyquist plot for the parallel interconnection.

Remark 8.3 *The properties of relative degree zero and passive zero dynamics shown for linear discrete-time V-passive systems in Chapter 7 are accomplished by the passive or positive real transfer function (8.21) and its feedback and parallel interconnections (8.29) and (8.30).*

In conclusion, the preservation of passivity under feedback and parallel interconnection has been proven for discrete-time systems. This property has been illustrated by a linear example and its passivity frequency-domain characteristics. Some notes about the preservation of other kinds of dissipativity, such as OSP and ISP, under block interconnection have also been briefly pointed out.

8.5 Dissipativity implications in nonlinear feedback systems stability

The study of the stability of non-linear systems using frequency-based criteria is well known for systems of the form presented in Figure 8.14: a linear system $G(s)$ with a single non-linear gain $F(y)$ in the feedback path.

This section is devoted to present the most important existing stability criteria for this kind of systems and relate dissipativity to all these ones. Three frequency-domain stability criteria will be studied: Popov's, the circle and Tsypkin's criteria. All the results are presented for the SISO case.

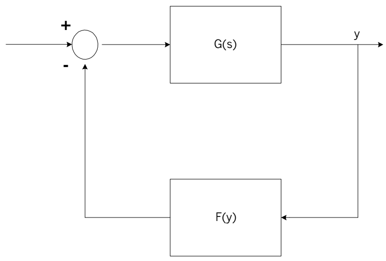

Figure 8.14: Linear system with a non-linear feedback.

8.5.1 Popov's criterion

Numerous authors have been interested in the study of the stability of systems of the form presented in Figure 8.14. Most of them looked for stability criteria based on Lyapunov's direct method. A Romanian mathematician Popov, established a stability criterion based upon the frequency response of the linear part *(Popov, 1973)* [141]. The geometric interpretation of Popov's criterion lead to a similar criterion to the Nyquist criterion for linear systems. The study of the asymptotic and global stability of systems of the form presented in Figure 8.14 is also called *absolute stability* or Lur'e problem. The problem of absolute stability has been studied in connection to passivity and real positiveness concepts, see *(Zames, 1966)* [190, 191] as a seminal work and as a recent one *(Paré et al., 2001)* [136] where the stability analysis combines passivity, Lyapunov and Popov's stability theories.

Now, the main ideas of Popov's criterion will be introduced in such a way they will be able to be used in the connection between this stability method and the frequency-domain stability implications of dissipativity. For more details see *(Khalil, 1996)* [76].

Theorem 8.2 (Aleixandre-Campos and Alonso-Romero) *[2] The system with non-linear feedback shown in Figure 8.14 is absolutely stable if*

a) *The non-linear function F satisfies*

a.1. $F(0) = 0$

a.2. $0 < \frac{F(y)}{y} < b$, $\forall y \neq 0$ *with b a positive constant.*

b) *There exists some constant* α *such that,*

$$(1 + \alpha s)G(s) + \frac{1}{b}$$

is a positive real transfer function.

■

The most interesting feature of Popov's theorem is that it does not impose any restriction on the system order. Besides, it only uses the transfer function of the linear part,

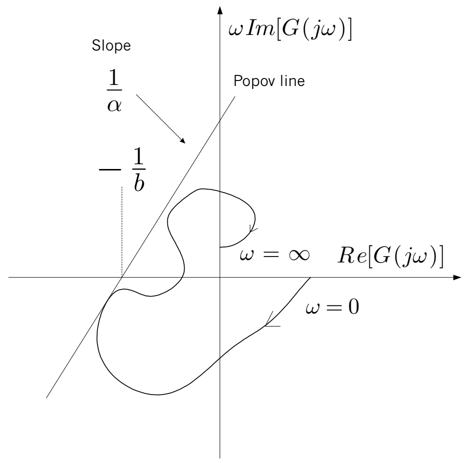

Figure 8.15: Geometric interpretation of Popov's criterion.

and does not require the mathematic expression of the non-linear funtion, only its linear bounds.

Popov's criterion appears to be useful when its geometric interpretation is established. As it has been shown, the condition on the transfer function of the linear part requires the existence of a constant α, such that

$$Re\left[(1+\alpha j\omega)G(j\omega)+\frac{1}{b}\right] \geq 0,\ \forall w \tag{8.31}$$

Popov's theorem also establishes the residues for

$$(1+\alpha s)G(s)+\frac{1}{b}$$

in the poles with zero real part to be real and positive, $G(s)$ not to have poles with positive real part, and the poles on the imaginary axis to be simple. These three conditions are easy to check over $G(s)$ directly. It is not the case of condition (8.31). Fortunately, the geometric interpretation for (8.31) is direct, and in addition, it gives a value for α. Let consider the following function,

$$M(j\omega) = Re\left[G(j\omega)\right] + j\omega Im\left[G(j\omega)\right],\ \omega > 0 \tag{8.32}$$

whose polar plot for $\omega \in [0,\infty)$ is called modified polar plot of $G(j\omega)$. Condition (8.31) sets that a straight line must exist with an arbitrarily chosen, but fixed slope through the point $\left(-\frac{1}{b}+j0\right)$, such that the modified polar plot of $G(j\omega)$ lies to the right of this line, see Figure 8.15. The slope of this straight line, which is tangent to the polar plot $M(j\omega)$, is precisely $\frac{1}{\alpha}$.

8.5.2 The circle criterion

The circle criterion, proposed in *(Zames, 1966)* [191] (see also *(Franklin et al., 1990)* [42], *(Khalil, 1996)* [76]), gives a sufficient condition for the stability of a system $G(s)$ with a non-linear function gain $F(y)$ in the feedback path (see Figure 8.14).

Suppose the function $F(y)$ to fall in a sector bounded by two straigh lines with slopes a and b,

$$a \leq \frac{F(y)}{y} \leq b$$

This situation is presented in Figure 8.16.

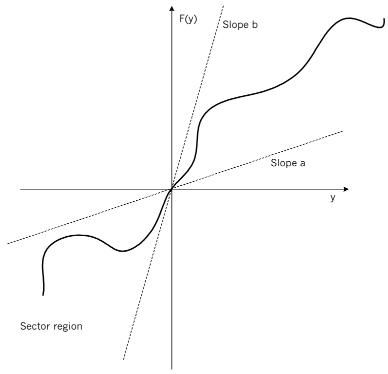

Figure 8.16: Sector bounded non-linear function.

The sector region will determine a circular region C in the complex plane, that is, *(Tsai, 1996) [172]*

$$C_1(a,b) = \left\{(U,V) : ab\left[\left(U + \frac{1}{2a} + \frac{1}{2b}\right)^2 + V^2 - \left(\frac{1}{4a} - \frac{1}{4b}\right)\right] \leq 0\right\}, \quad \forall a, b \neq 0 \tag{8.33}$$

or

$$C_2(a,b) = \left\{(U,V) : U \leq -\frac{1}{b}\right\}, \ \forall a = 0, b \neq 0 \tag{8.34}$$

or

$$C_3(a,b) = \left\{(U,V) : U \geq -\frac{1}{a}\right\}, \ \forall a \neq 0, b = 0, \tag{8.35}$$

where $U = Re[G(\mathrm{j}\omega)]$ and $V = Im[G(\mathrm{j}\omega)]$. In every case, the circular region C excludes the origin on the complex plane (U,V), i.e., $0 \notin C(a,b)$. Regions (8.33), (8.34), and (8.35) are represented in Figures 8.17, 8.18 and 8.19. Note that when a or b are zero, the critical disk is converted into a critical straight line.

Theorem 8.3 (Zames, 1966) *[191] The system shown in Figure 8.14 is absolutely stable if the Nyquist plot of* $G(\mathrm{j}\omega)$ *does not touch or intersect the circular region C.*

■

Another way of expressing the circle criterion is by means of considering the positive realness of a transfer function depending on $G(s)$, a and b.

Theorem 8.4 (Aleixandre-Campos and Alonso-Romero) *[2] The system shown in Figure 8.14 is absolutely stable if,*

a) $a < \frac{F(y)}{y} < b,\ \forall y.$

b) *The transfer function*

$$F(s) = \frac{1 + bG(s)}{1 + aG(s)}$$

is positive real.

■

Now, the geometric interpretation of condition b) of Theorem 8.4 will be presented. On the complex plane $(Re[G(j\omega)], Im[G(j\omega)])$, the points $-\frac{1}{a}$ and $-\frac{1}{b}$ are considered. The segment between these two points is taken as the diameter of a circle C (as was drawn in Figure 8.17). Condition $Re[F(j\omega)] \geq 0,\ \forall \omega$ can be written as follows,

$$Re\left[\frac{1/b + G(j\omega)}{1/a + G(j\omega)}\frac{b}{a}\right] \geq 0,\ \forall \omega \tag{8.36}$$

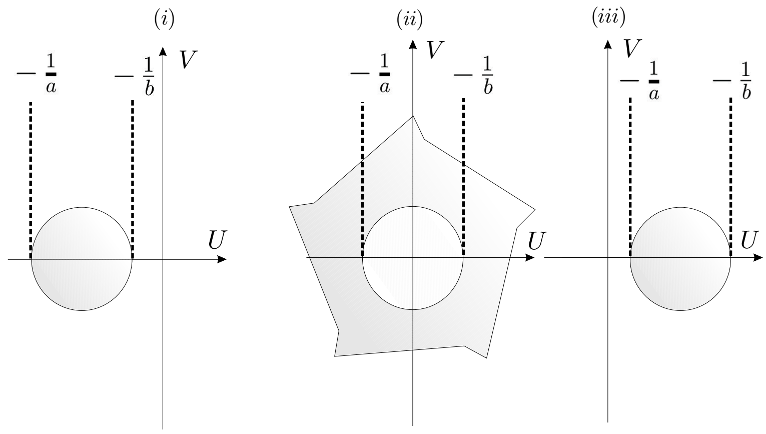

Figure 8.17: $C_1(a,b)$ is the shaded region, $a,b \neq 0$ (i) $a,b > 0$ (ii) $a > 0$, $b < 0$ (iii) $a,b < 0$.

There are, then, three possibilities:

1. a and b have the same sign. In this case, the region C or critical disk, is the interior of the circle, therefore the Nyquist diagram for $G(s)$ must not have any point inside C (cases (i) and (iii) on Figure 8.17).

2. a and b have different signs. Now, C will be the outside of the circle, and, consequently the Nyquist diagram of $G(s)$ must be located on the inside of C (case (ii) on Figure 8.17).

3. Some of the constants are zero. The critical disk is converted into a critical line which the $G(s)$ Nyquist plot must not cross (Figures 8.18 and 8.19).

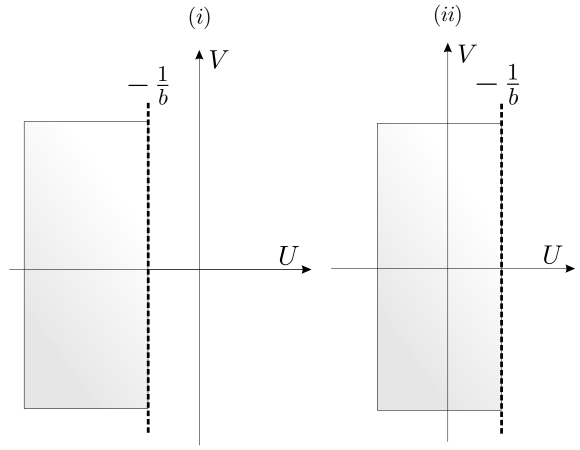

Figure 8.18: $C_2(a,b)$ is the shaded region, $a=0$, $b\neq 0$ (i) $a=0$, $b>0$ (ii) $a=0$, $b<0$.

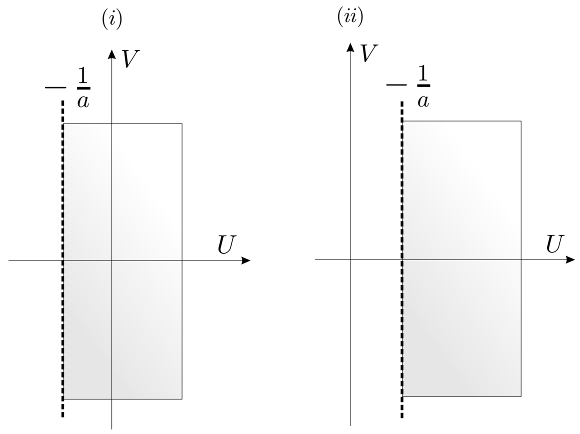

Figure 8.19: $C_3(a,b)$ is the shaded region, $b=0$, $a\neq 0$ (i) $b=0, a>0$ (ii) $b=0$, $a<0$.

In case a and b have different signs, these two conditions are met if the Nyquist plot of $G(s)$ remains inside C. However, if a and b have the same sign, it is necessary to apply the argument principle, which will establish that the Nyquist plot of $G(s)$ must encircle C in counterclockwise so many times as the number of unstable poles of $G(s)$.

When $a\rightarrow b$, the critical disk tends to a critical point, and the circle criterion "tends" to the Nyquist criterion. It must be pointed out that the circle criterion is much more restrictive than the Nyquist one. Besides, it can not be forgotten that the circle criterion conditions (as the Popov's criterion ones) are sufficient but not necessary, so if they are not met it does not imply un-stability.

The discrete-time version of the circle criterion is presented for the case of $a>0$ in *(Franklin et al., 1990)* [42], and implies the same conditions presented above with the difference that the transfer function to be analyzed in the frequency domain is $G(e^{j\varpi T})$, with T the sampling period time. It can be noticed that the discretization of a transfer function is inherited where the equivalence $z = e^{sT}$ is considered.

8.5.3 Absolute stability in nonlinear sampled-data systems. Tsypkin's criterion

Besides the discrete-time version of the circle criterion given in *(Franklin et al., 1990)* [42], which has not been widely used and has not been given for all the cases of the circle criterion, some of the main results concerning the study of absolute stability in the discrete-time setting will be mentioned. These results corresponds to sampled-data systems and they are not so numerous as the ones existing for the continuous-time case.

The absolute stability is studied in the case of sampled-data control systems, i.e., systems of the form depicted in Figure 8.20: a closed-loop system containing a linear plant, a sampler and zero-order hold and a single-valued nonlinearity. Traditional Popov's-type criteria appear in the literature using function analytic methods, see *(Jury and Lee, 1964)* [65, 66], *(Jury and Lee, 1966)* [67], *(Szegö, 1964)* [168]. As recent works on this topic, the works of *(Haddad and Bernstein, 1994)* [50] and *(Okuyama and Takemori, 1996)* [125] can be underlined; the former exploits results from passivity and positive realness theory and the latter gives a robust stability criterion derived from a norm condition in the frequency domain. Other discrete-time absolute stability studies to point out are *(Ramarajan and Rao, 1972)* [143], *(Sharma and Singh, 1981)* [154] and *(Hagiwara et al., 1998)* [51].

In order to analyze the stability of discrete-time systems containing a feedback nonlinearity, Tsypkin's criterion *(Tsypkin, 1962)* [173] appears to be the closest analogous to Popov's criterion, which is most used for analyzing such systems in the continuous-time setting. Tsypkin's criterion is derived from the passivity formalism in terms of input-ouput properties and from an operator viewpoint in *(Wu and Desoer, 1970)* [182]. A recent Tsypkin's-type criterion reported in the literature for MIMO systems containing an arbitrary number of sector-bounded nonlinearities is the one given by *(Kapila and Haddad, 1996)* [69] which provides a Lyapunov-kind analysis based on the KYP conditions.

Now, in a schematic way, the main ideas of Tsypkin's criterion will be revisited. Let the system shown in Figure 8.20.

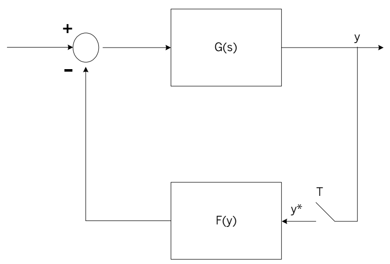

Figure 8.20: Sampled-data control system with non-linear gain in the feedback path.

Theorem 8.5 (Tsai, 1996) *[172] System shown in Figure 8.20 is said to be absolutely stable if,*

a) *The nonlinear function F satisfies*

a.1. $F(0) = 0$

a.2. $0 \leq \frac{F(y)}{y} < b, \; \forall y \neq 0$

b) *The discrete transfer function of $G(s)$, i.e., $G^*(j\omega T)$, is such that*

$$Re\left[G^*(j\omega T)\right] > -\frac{1}{b}, \; \forall \omega \;\; 0 \leq \omega \leq \frac{\omega_s}{2} \tag{8.37}$$

with b a constant, ω_s the sampling frequency, and T the sampling period time.

■

The geometric interpretation of condition (8.37) is illustrated in Figure 8.21.

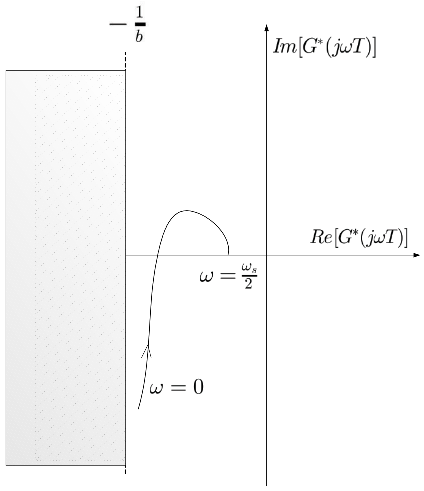

Figure 8.21: Illustration of Tsypkin's criterion in the Nyquist plane. Shaded part is not allowable for the Nyquist plot of $G^*(j\omega T)$ in order that the whole system is absolute stable.

8.5.4 Implications of dissipativity and passivity in feedback discrete-time systems stability

The study of stability of nonlinear systems using frequency criteria instead of Lyapunov's direct method has been proposed for linear systems with a non-linearity in the feedback path. These methods, mainly, Popov's, Tsypkin's and the circle criteria establish stability criteria based upon the frequency response of the linear part. It is proposed *(Popov, 1973)* [141] that if the transfer function corresponding to the linear block is positive real or passive and the non-linearity satisfies a Popov-like inequality, i.e., it is a sector bounded nonlinear function, then the resulting closed-loop system is said to be absolutely stable.

This section tries to present the valuable importance that dissipativity and passivity concepts may have in the stability analysis of nonlinear interconnected systems. The most interesting and remarkable property of passivity is that in linear systems (either discrete or continuous), the positive realness characteristic is equivalent to the passivity property, and in addition, it presents highly interesting stability properties in the frequency domain.

Since the geometric interpretation of stability criteria such as Popov's, the circle and Tsypkin's ones are based on the positive realness of a transfer function, and a particular emplacement of the Nyquist plot, dissipativity formalism can be considered to have interesting relations with these stability criteria. Indeed, a passive nonlinear function has the property of falling in sector $[0,\infty)$ *(Franklin et al., 1990)* [42], consequently, the passivity property increases the validity of Popov's, the circle and Tsypkin's criteria. If a sector bounded non-linearity is passive, its sector boundaries are augmented in comparison to the boundaries proposed in the mentioned stability criteria.

In *(Goodwin and Sin, 1984)* [45], the generalized KYP or Discrete Positive Real Lemma is proposed for (Q,S,R)-dissipative discrete-time linear systems, see Lemma 2.1 in Chapter 2. In addition, the characteristics of the Nyquist plot of $G(e^{j\omega})$ for single-input single-output (Q,S,R)-dissipative linear systems are presented depending on the form of the supply function. These characteristics depending on the value of matrices Q, R and S were presented in Section 8.2.2.2.

From the characteristics of the Nyquist plot of $G(e^{j\omega})$, the frequency-domain dissipativity properties could be considered as the generalization of the stability conditions of the mentioned criteria for the discrete-time setting.

Tsypkin's criterion for nonlinear sampled-data systems establishes that the closed-loop system consisting of a linear transfer function with a nonlinear function in the feedback path is absolutely stable if the nonlinear function falls in a sector bounded by two straight lines with slopes 0 and b, and the Nyquist plot of the discrete transfer function lies to the right of the vertical line $Re\,z = -1/b$ *(Tsai, 1996)* [172], see Figure 8.21. Considering (Q,S,R)-dissipative systems, it is easy to check that the geometric interpretation of the Tsypkin's criterion in the framework of the frequency domain is a special case of (Q,S,R)-dissipativity with $Q=0$, $S=I/2$, $R=I/b$.

The **circle criterion** gives a sufficient condition for the absolutely stability of a linear system with a nonlinear function gain in the feedback path wich falls in a sector bounded by two straight lines with slopes a and b. This class of system will be absolutely stable if the Nyquist plot of the transfer function associated to the linear block does not intersect a region C defined by the points $(-1/a+0j)$ and $(-1/b+j0)$. In case $a,b\neq 0$ the region C will be a circle. On the other hand, if $a=0, b\neq 0$ or $b=0, a\neq 0$, the critical disk is converted into a critical line which the Nyquist plot must not cross, see Figures 8.17, 8.18, 8.19.

The discrete-time version of the circle criterion is obtained from the continuous-time result for the case of $a>0$, and using $z=e^{j\omega T}$, with T the sampling period, see *(Franklin et al., 1990)* [42]. Considering the frequency-domain characteristics of dissipativity, the conditions that the linear block of the nonlinear system under consideration must accomplish can be seen as different classes of dissipativity. For example, the case of having $a=0, b\neq 0$ corresponds to the (Q,S,R)-dissipativity case considering $Q=0$, $S=I/2$, $R=I/b$ where the Nyquist plot of the transfer function corresponding to the linear part lies to the right of the vertical line $Re\,z=-1/b$. The case of having $b=0, a\neq 0$

corresponds to the (Q,S,R)-dissipativity case with $Q = 0$, $S = -I/2$, $R = -I/a$ where the Nyquist plot of the transfer function corresponding to the linear part lies to the left of the vertical line $Re\,z = -1/a$. When the critical region corresponds to the interior or the outside of the circle determined by the points $(-1/a + 0\mathrm{j})$ and $(-1/b + 0\mathrm{j})$, the stability conditions proposed by the circle criterion may also be obtained from the dissipativity frequency-domain properties, considering supply functions of the form (2.23) with Q negative.

From the dissipativity characterization in the frequency domain, an extension of the **Popov's stability criterion** to the discrete-time setting may be obtained, however, a more complicated analysis than the one made for Tsypkin's and circle criteria is required, we think that another kind of supply functions different to (2.23) should be proposed for this purpose.

8.6 Conclusions and future work

Some implications of dissipativity and passivity properties for the discrete-time case have been presented, mainly: the preservation of passivity under feedback and parallel interconnections, and the study of the preservation of passivity under sampling; some notes on the preservation of dissipativity under sampling have also been pointed out. Dissipativity characterization in the frequency domain has been used to illustrate the preservation of passivity under feedback and parallel interconnections and under sampling by means of linear examples. The frequency-domain characteristics of dissipative systems have also been used to present dissipativity as an interesting tool for the study of systems stability in the discrete-time setting, and it can be considered as a way for obtaining frequency-based stability criteria types, such as: Tsypkin's, the circle and Popov's criteria, for discrete-time systems.

The most important conclusions given in this chapter are the following ones. In general, passivity is not preserved under sampling. For the linear case, the discretization of a system by means of the trapezoid rule preserves the passivity property of the continuous-time system. In addition, passivity is preserved under feedback and parallel interconnections. Some notes about the preservation of other kinds of dissipativity, such as OSP and ISP, under block interconnection have also been pointed out. Finally, dissipativity is seen as an alternative to study nonlinear feedback systems stability.

The contribution of this chapter is the way of using passivity and dissipativity. The existing frequency-domain characterization of passivity and (Q,S,R)-dissipativity has been used to illustrate dissipativity consequences in systems properties. The properties studied in this chapter have not been treated broadly in the literature for the discrete-time case, indeed, the proposals made in Section 8.5.4, related to the fact of using dissipativity to study the frequency-domain stability properties of nonlinear discrete-time systems have not attracted significant attention in the literature.

There are a lot of interesting problems to point out as further research. First of all, it will be interesting to define what a positive real discrete-time system is in the nonlinear setting. Second, a deeper analysis of the preservation of passivity and dissipativity under sampling for nonlinear systems is needed. Referring the study of dissipativity under block interconnection, it must be underlined that we have heuristic results which illustrate that some kinds of dissipativity are preserved under block interconnections, however, we have not proven it in a formal way yet. So, it would be interesting to study dissipativity preservation under block interconnection. Finally, it would be interesting to derive in a formal way Popov's and the circle criteria from the dissipativity formalism. This fact implies to explore the frequency-domain characteristics of other kinds of dissipativity different to passivity and (Q,S,R)-dissipativity.

All the attracting characteristics of passive and dissipative systems can be exploited in the discrete-time setting jointly other controls, such us, adaptive control, robust control, iterative learning control, or in order to solve the tracking control problem using the equivalence between invertibility, output dissipativity and positive realness, *(Arimoto and Naniwa, 2000)* [9].

This chapter has shown some of the interesting properties that dissipative and passive systems exhibit, they motivate the transformation of a system which is not dissipative or passive into a dissipative or passive one.