

## Chapter 9

# Conclusions and future work

This dissertation has been devoted to dissipativity-related concepts and applications in the nonlinear discrete-time setting. Dissipativity formalism has been shown as an appealing physical-oriented approach which has succeeded in the nonlinear control area and has appeared to be as a powerful tool for analyzing systems behaviour.

For the discrete-time case, many dissipativity-related problems remain unsolved. Our work has aimed to explore some non-solved problems in the nonlinear discrete-time dissipativity-based control theory and in the study of the properties of nonlinear discrete-time dissipative systems.

The main problem attracting our attention has been the establishment of conditions for a nonlinear discrete-time system to be rendered dissipative via static state feedback, which, as far as we know, had not been reported before. This is the main contribution of this dissertation.

Two other main contributions or parts into which our results have been divided are: on the one hand, the extension of the necessary and sufficient conditions existing in the literature regarded as KYP conditions to a class of dissipative and lossless nonlinear discrete-time systems of general form. On the other hand, the use of dissipative and feedback dissipative systems properties for stabilization purposes, given in the proposal of the nonlinear discrete-time version of the ESDI dissipativity-based control methodology.

Other secondary goals in the dissipativity properties exploration in discrete-time systems have been achieved, mainly: the study of the relative degree and zero dynamics of passive nonlinear discrete-time systems, some conclusions about passivity preservation under feedback and parallel interconnections, some notes on the non-preservation and preservation of dissipativity, and its special case of passivity, under sampling, in addition, dissipativity frequency-domain properties have been used and related to some of the most important frequency-based feedback stability criteria. Furthermore, the feedback dissipativity and dissipativity-based control results have been applied to solve the regulation problem in a discrete-time model with physical interpretation: the DC-to-DC buck converter, whose open-loop response has been improved by means of the use of some of the stabilization methods proposed.

This last chapter sums up and collects all the conclusions and future work given at the end of the preceding chapters. Some additional suggestions for future work are given, and some non-reported results are briefly pointed out as studied but not closed problems.

## 9.1 Conclusions

The study of dissipativity given in this dissertation has been concentrated in the state-space or internal description representation of systems. In this framework, the following main results have been achieved. These results can be classified into five parts:

1. **The characterization of dissipative MIMO nonlinear discrete-time systems of general form.** The KYP conditions existing in the literature have been extended to a class of nonlinear MIMO dissipative discrete-time systems which are non-affine in the control input. The class of dissipativity characterized is regarded as *QSS*-dissipativity, that is, dissipative systems whose storage and supply functions,  $V$  and  $s$ , satisfy that  $V(f(x, u))$  and  $s(h(x, u), u)$  are quadratic in  $u$ . Necessary and sufficient conditions for the characterization of *QSS*-lossless discrete-time systems which are non-affine in the control input are also given. The existing conditions presented in previous works are strictly contained in our characterization.
2. **The feedback dissipativity problem in the nonlinear discrete-time setting.** Two approaches are proposed to deal with this topic:
  - (a) **The feedback dissipativity problem through the fundamental dissipativity inequality.** The feedback dissipativity problem has been solved for SISO nonlinear discrete-time non-affine in the control input systems by means of four methodologies based on the fundamental dissipativity equality. Sufficient conditions under feedback dissipativity is possible have been proposed. The first approach proposes the control achieving feedback dissipativity as the explicit solution of the dissipativity equality. The second one uses the speed-gradient algorithm in its discrete-time version in order to achieve what has been regarded as quasi- $(V, s)$ -dissipativity. The notion of quasi- $(V, s)$ -dissipativity includes the concept of  $(V, s)$ -dissipativity. The third and fourth methodologies are of approximate type and are based upon a first-order Taylor approximation of the fundamental dissipativity inequality. Sufficient conditions under which the approximation considered is valid have been posed. In these two last methodologies, feedback dissipativity is seen as a “perturbation” of the storage energy invariance or the system losslessness situations.
  - (b) **The feedback passivity problem through the properties of the relative degree and zero dynamics of the non-passive system.** The problem of rendering a system passive via state feedback is solved for a class of MIMO nonlinear discrete-time systems which are affine in the control input using the properties of the relative degree and the zero dynamics of the non-passive system. It is an extension to the passivity case of the results reported in the literature for the losslessness feedback problem. The class of systems for which feedback passivity has been solved is regarded as *QS*-passive systems, i.e.,  $V$ -passive systems for which  $V(f(x) + g(x)u)$  is quadratic in  $u$ , with  $V$  the storage energy function.
3. **The dissipativity-based stabilization problem in nonlinear discrete-time systems.** The dissipativity-based controller design methodology of the ESDI has been

extended to general nonlinear SISO discrete-time systems, in addition to, the analysis of some stability properties of a class of dissipative and feedback dissipative SISO nonlinear discrete-time systems. Furthermore, sufficient conditions under which a class of feedback dissipative systems is stabilizable have been proposed.

4. **Some dissipativity topics in nonlinear non-affine in the input continuous-time systems.** The fact of treating general discrete-time systems has allowed us to extend some dissipativity-related definitions for the case of general continuous-time systems of the form

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= h(x(t), u(t))\end{aligned}$$

The main contributions on this topic are:

- (a) The study of the feedback dissipativity problem for nonlinear non-affine SISO systems based upon the fundamental dissipativity equality.
  - (b) The use of the feedback dissipativity results in order to extend the ESDI controller design method to the case of non-affine SISO nonlinear systems.
5. **Other properties of dissipative and passive discrete-time systems.** Other properties of dissipative and passive discrete-time systems which have not been broadly treated in the literature before, have been also considered. This is the case of the study of the special features that the relative degree and the zero dynamics of passive and dissipative systems present, the analysis of passivity preservation under sampling or under feedback and parallel interconnections. Some notes about the preservation of other kinds of dissipativity, such as OSP and ISP, under block interconnections have also been pointed out. The frequency-domain properties of passivity and  $(Q, S, R)$ -dissipativity have been used to illustrate some of the mentioned characteristics and to relate dissipativity to some of the most important frequency-based feedback stability criteria in the discrete-time domain, namely: Tsytkin's, Popov's and the circle criteria.

Furthermore, the feedback dissipativity, feedback passivity and dissipativity-based stabilization methodologies have been illustrated by means of two examples: an academic nonlinear discrete-time example and a discrete-time model of the DC-to-DC buck converter. The open-loop performance of the buck converter has been improved by means of some of the DBC methodologies proposed, and one of the proposed dissipativity-based control schemes has been illustrated to be robust under variations in the load parameter.

In the rest of the sections of this chapter, some ideas which we have started to work on are given. They are presented in order to show that dissipativity field in the discrete-time setting has a lot of open problems and appealing ideas to explore. This future work is divided into five main motivating work groups:

1. Future work on dissipativity characterization.
2. Future work on feedback dissipativity.
3. Future work on dissipativity-based stabilization.
4. Chaos and feedback dissipativity.

5. Future work on studying the positive realness property of passive and dissipative nonlinear systems.

## 9.2 Future work on dissipativity characterization

### 9.2.1 Improving the Taylor approximations used

The main problem in our dissipativity characterization is that due to the fact that we have intended to characterize systems as much general as possible, some restrictions on the class of dissipativity treated must have given. The class of dissipative systems we have concentrated on have been the  $QSS$  ones, whose storage and supply functions are such that  $V(f(x, u))$  and  $s(h(x, u), u)$  are quadratic in  $u$ . These conditions were needed in order to show the sufficiency of the KYP conditions proposed in Chapter 4. Higher order approximations in order to cover a broad class of dissipative systems may be used.

Since the domains of the states and the inputs in most control systems are compact sets, from the *Stone-Weierstrass Theorem*, functions  $V(f(x, u))$  and  $s(h(x, u), u)$  can be uniformly approximated by polynomials. The KYP conditions presented in Chapter 4 covers a first step in this direction due to the fact  $QSS$ -dissipative, respectively,  $QSS$ -lossless systems, in which  $V(f(x, u))$  and  $s(h(x, u), u)$  have been approximated by polynomials of second order in  $u$ . A way of extending our results may be by means of the use of higher order polynomial approximations and studying the conditions that dissipative systems must fulfil with this kind of storage and supply functions.

### 9.2.2 Using discrete-time tools in order to treat dissipativity

The results presented in this dissertation are basically extensions of ideas existing in the dissipativity formalism in the continuous-time case, this can be the reason why these results appear to be limited in some cases. It would be interesting to take advantage of the discrete-time dynamics properties in order to understand dissipativity and energy concepts. Some mathematical tools defined in the discrete-time domain may be used.

A research line we have initiated in which we have not achieved concluding results yet is the use of a discrete geometric tool in order to interpret dissipativity concepts: the controlled invariant distributions, see (*Grizzle, 1985*) [46]. A first step in this direction, indeed, has given in the two approximate-like feedback dissipativity methodologies which see dissipativity as the “perturbation” of the stored energy invariance or the losslessness situations of the original system. We think that these procedures of achieving feedback dissipativity may have a geometric interpretation by means of the controlled invariant distributions. This fact would lead us to give new definitions of dissipativity.

### 9.2.3 Frequency-domain characteristics of quasi- $(V, s)$ -dissipativity

Along this dissertation the frequency-domain characteristics of passive and  $(Q, R, S)$ -dissipative systems have been exploited for different purposes. Different classes of dissipativity have appeared to present distinctive frequency-domain properties.

Several classes of dissipativity have been defined in this dissertation, mainly,  $QSS$ -dissipativity and quasi- $(V, s)$ -dissipativity. In the case of linear discrete-time systems, an attractive problem to handle would be the analysis of the frequency-domain properties

of these kinds of dissipativity. A class of  $QSS$ -dissipative systems is given by  $(Q, S, R)$ -dissipative LTI systems of the form

$$\begin{aligned}x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k)\end{aligned}$$

with storage functions of the form  $V = x^T Px$ , with  $P$  a symmetric positive definite matrix. The frequency-domain features of other kinds of  $QSS$ -dissipative systems would be interesting to study.

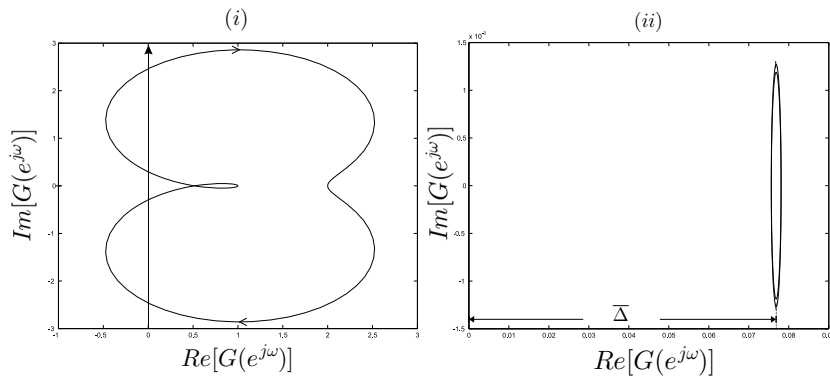
Concerning quasi- $(V, s)$ -dissipative systems, we could start to study the implications of quasi- $V$ -passivity in the frequency-domain characteristics of the system. The example of the buck example passified by means of the SG-based feedback passivity method proposed in Chapter 5 can illustrate the properties of the Nyquist of a quasi- $V$ -passive system. The Nyquist plot of the non-passive original system can be compared with the quasi- $V$ -passive system obtained after the feedback passivity control scheme is applied. The two pulse transfer functions involved are presented as follows. On the one hand, the pulse transfer function for the state-space description (5.13) with  $y = x_2 + \hat{u}$ :

$$G_b(z) = \frac{z^2 - 1.707z + 0.9394}{z^2 - 1.766z + 0.8825} \quad (9.1)$$

and, on the other hand, the pulse transfer function for the passified system (5.54) with the constants model defined in (5.14):

$$G_b(z) = \frac{0.07794(z^2 - 1.766z + 0.8825)}{z^2 - 1.763z + 0.8789} \quad (9.2)$$

The Nyquist plots of (9.1) and (9.2) are depicted in Figure 9.1. The Nyquist of the quasi- $V$ -passive system lies on the right-hand side half plane, but it does not touch the axis  $Re[G(e^{j\omega})] = 0$ . The Nyquist plot begins at  $Re[G(e^{j\omega})] = \bar{\Delta}$ . A detailed analysis is required in order to relate this constant to the quasi- $(V, s)$ -dissipativity definition.



**Figure 9.1:** (i) Nyquist diagram for the exact discretized model of the buck converter with  $y = x_2 + \hat{u}$  (ii) Nyquist diagram for the passified buck converter by means of the SG-based feedback passivity method.

## 9.3 Future work on feedback dissipativity

### 9.3.1 Feedback dissipativity through the properties of the relative degree and the zero dynamics

Feedback passivity has been studied by means of the properties of the relative degree and the zero dynamics of nonlinear MIMO affine-in-the-input discrete-time passive systems. In the same line, the properties of the relative degree and zero dynamics of dissipative general systems could be studied and used in order to achieve feedback dissipativity.

The most immediate extension of the feedback passivity results given in Chapter 7 is to nonlinear discrete-time systems non-affine in the control input. After that, the feedback dissipativity problem could be analyzed. The first obstacle in achieving this goal is the study of the properties of the relative degree of general dissipative discrete-time systems. As it was pointed out in preceding chapters, a dissipative system, in general, has relative degree greater than zero. The dissipative systems in which our studies have been initiated are *QSS*-dissipative systems with relative degree one, satisfying the ZIO property.

### 9.3.2 Feedback dissipativity for the MIMO case

The feedback dissipativity methodologies proposed in Chapter 5 have been given for the SISO case. It would be interesting to extend these results to the MIMO case.

Although we have not studied the problem of feedback dissipativity in MIMO systems for the discrete-time case, we have obtained some preliminary results in the passivation of MIMO nonlinear continuous-time systems of the form:

$$\begin{aligned}\dot{x} &= f(x) + G(x)u, \quad G(x) = (g_1(x), \dots, g_m(x)) \\ y &= h(x)\end{aligned}$$

The storage energy function is proposed to be a vector Lyapunov-like function (see for example *(Retchkiman and Silva-Navarro, 1998)* [144]) and the feedback passivity conditions are given in terms of conditions on the vector field  $G(x)$ . The decomposition of the vector field  $f$  in a dissipative, non-dissipative and invariant part is used, based on the approach given in *(Sira-Ramírez, 1998)* [159] for the SISO case. These ideas are pretended to be explored for the discrete-time case.

### 9.3.3 Iterative-like methods

The feedback dissipativity problem in the discrete-time case can be also seen as a optimization oriented problem. A control goal function is proposed, as the one proposed for the SG-based feedback dissipativity approach, and any iterative-like method can be used for driving this control goal function to zero. Here, the feedback quasi-dissipativity problem would be treated.

### 9.3.4 Frequency-based feedback dissipativity for linear systems

The positive realness property and frequency-domain implications of dissipativity and passivity in linear discrete-time systems may be used to treat the problem of feedback

dissipativity or passivation in the linear discrete-time setting. Therefore, the problem to solve will be the establishment of conditions for a transfer function to be positive real.

## 9.4 Future work on dissipativity-based stabilization

### 9.4.1 Alternative methods to ESDI. Contractive analysis

The ESDI dissipativity-based stabilization control scheme would be interesting to be extended to the MIMO case. For this goal, the feedback dissipativity problem for MIMO discrete-time systems would be solved. Otherwise, a more attractive future work in the dissipativity-based stabilization field is exploring other kinds of stability different to Lyapunov stability for the stabilization of dissipative nonlinear discrete-time systems, such as contraction analysis, see (Lohmiller, 1998) [95] and (Lohmiller and Slotine, 1998) [96]. Some preliminary studies have been made considering  $V(x(k+1))$  as a contractive function, in addition to identifying the symmetric part and skew-symmetric parts of a discrete-time system corresponding to its non-dissipative and conservative energy part, respectively.

### 9.4.2 Interconnection of dissipative discrete-time systems

In Chapter 8, passivity preservation under feedback and parallel interconnection was studied. It would be interesting to study the preservation of other kinds of dissipativity under interconnection, in addition to, the stability properties of passive and dissipative interconnected discrete-time systems. The stability property is weaker than the dissipativity or passivity one, therefore, it would be interesting to study if the interconnection of dissipative discrete-time systems is stable.

### 9.4.3 Use of the feedback dissipativity methodologies proposed for the stabilization of other kinds of DC-to-DC power converters

Dissipativity-based stabilization appears to be an appropriate practical oriented control scheme in order to stabilize DC-to-DC power converters, since it gives a valuable physical interpretation of the controller design.

The feedback dissipativity and feedback passivity methodologies, in addition to the dissipativity-based stabilization schemes proposed in this dissertation would be interesting to be applied to other DC-to-DC power converters more complex than the buck converter. The boost converter is a DC-to-DC converter which has attracted a lot of attention in the power electronics field. This system is bilinear and non-minimum phase when the voltage is considered as the system output, as its more important characteristics. The most important drawback in the control of the boost converter is the non-minimum phase property. Some preliminary studies have been made on this topic. A second-order approximate discrete-time model of this converter has been passified by means of the methodology presented in Chapter 7. A more detailed analysis on the results obtained is needed.

On the other hand, following the research line of application of the feedback dissipativity and dissipativity-based stabilization results to the control of power converters,

other discrete-time models describing the DC-to-DC buck converter would be interesting to be obtained.

The tracking problem would be also interesting to be studied from the dissipativity viewpoint.

## 9.5 Chaos and dissipativity

Along the research done in this dissertation, some chaotic behaviour and strange attractors have appeared when some nonlinear examples were passified. Since, in our feedback dissipativity controls different constants appear, the presence of chaos varying these constants is not surprising.

An interesting open problem to explore is the study of the relations between chaos and dissipativity in the discrete-time domain, including studying if the conditions that a chaotic system has to meet in order to be feedback dissipative change.

## 9.6 Studying the positive realness of passive and dissipative nonlinear systems.

Frequency-domain properties of dissipative systems can be used in order to derive from the dissipativity formalism the most important frequency-based feedback stability criteria, such as Popov's or the circle criteria.

In Chapter 8, some brief notes were given on this topic. From the implications of  $(Q, S, R)$ -dissipative LTI discrete-time systems in the frequency domain, Tsytkin's criterion and some cases of the discrete-time version of the circle criterion were derived. A challenging problem would be to advance in the study of the frequency-domain properties of other more general classes of dissipativity in order to derive from the dissipativity formalism the stability conditions given by the Popov's and the circle criteria. This problem appears to be complicate and difficult, but very attractive.

The conclusion of this dissertation is that there is much work to do in the discrete-time dissipativity field and there are still many questions to answer.