

Sobolev Inequalities: Isoperimetry and Symmetrization
Walter Andrés Ortiz Vargas

The first part of the thesis is devoted to obtain a Sobolev type embedding result for Besov spaces defined on a doubling metric space. This will be done by obtaining pointwise estimates between the special difference $f_\mu^{**}(t) - f_\mu^*(t)$ (called oscillation of f_μ^*) and the X -modulus of smoothness defined by

$$E_X(f, r) := \left\| \int_{B(x,r)} |f(x) - f(y)| d\mu(y) \right\|_X.$$

(here f_μ^* is the decreasing rearrangement of f , $f_\mu^{**}(t) = \frac{1}{t} \int_0^t f_\mu^*(s) ds$, for all $t > 0$ and X a rearrangement invariant space on Ω).

In the second part of the thesis, to obtain symmetrization inequalities on probability metric spaces that admit a convex isoperimetric estimator which incorporate in their formulation the isoperimetric estimator and that can be applied to provide a unified treatment of sharp Sobolev-Poincaré and Nash type inequalities.