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OPTICAL SOLITONS IN QUADRATIC NONLINEAR MEDIA AND APPLICATIONS TO ALL-OPTICAL SWITCHING AND ROUTING DEVICES

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Appendix A

Equation for linear pulse propagation

When finite delay effects are present in the structure they may be included into the wave equation setting $\overrightarrow{\mathcal{P}}_{NL} = 0$. One obtains

$$\nabla^2 \overrightarrow{\mathcal{E}} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\overrightarrow{\mathcal{E}} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{\infty}\chi^{(1)}(t-t_1)\,\overrightarrow{\mathcal{E}}(t_1)\,dt_1 = 0\,. \tag{A.1}$$

To see how this impacts on pulse propagation the electric field is conveniently expressed through

$$\overrightarrow{\mathcal{E}}(z,t) = \frac{\widehat{e}}{2}\left[q(z,t)\,e^{jk_0 z}e^{-j\varpi_0 t} + c.c\right]\,, \tag{A.2}$$

where $q(z,t)$ is assumed to be a slowly varying function in both variables.Substituting into the wave equation one arrives to

$$e^{-j\varpi_0 t}\left\{\frac{\partial^2 q}{\partial z^2} + 2jk_0\frac{\partial q}{\partial z} - k_0^2 q - \frac{1}{c^2}\left[\frac{\partial^2 q}{\partial t^2} - 2j\varpi_0\frac{\partial q}{\partial t} - \varpi_0^2 q\right]\right\} - \frac{1}{c^2}\Lambda=0, \tag{A.3}$$

where Λ is given by

$$\Lambda = \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{\infty}\chi^{(1)}(t-t_1)\,q(z,t_1)\,dt_1\right]. \tag{A.4}$$

Considering that

$$\int_{-\infty}^{\infty}\chi^{(1)}(\alpha)\,e^{j\varpi_0\alpha}e^{j\varpi\alpha}d\alpha = \chi^{(1)}(\varpi+\varpi_0)\,, \tag{A.5}$$

one may work out the Λ expression to yield

$$\Lambda = \frac{\partial^2}{\partial t^2}\left[\frac{e^{-j\varpi_0 t}}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\chi^{(1)}\left(\varpi+\varpi_0\right)e^{-j\varpi(t-t_1)}d\varpi\, q\left(z,t_1\right)dt_1\right], \tag{A.6}$$

which allows to solve the integral in t_1 so that Λ is expressed

$$\Lambda = \frac{-1}{2\pi}\int_{-\Delta\varpi/2}^{\Delta\varpi/2}\left(\varpi+\varpi_0\right)^2\chi^{(1)}\left(\varpi+\varpi_0\right)Q\left(z,\varpi\right)e^{-j(\varpi+\varpi_0)t}d\varpi, \tag{A.7}$$

where explicit use has been made of the fact that since $q\left(z,t\right)$ was assumed slowly varying in t, its Fourier transform $Q\left(z,\varpi\right)$ is restricted to an $\Delta\varpi$ frequency band such that $\frac{\Delta\varpi}{\varpi_0}\sim\epsilon << 1$.

In that small frequency band the electric susceptibility frequency dependence is assumed to be well described by a Taylor expansion around the carrier frequency ϖ_0 in which terms up to second order are considered, namely

$$\chi^{(1)}\left(\varpi_0+\varpi\right)\simeq\chi^{(1)}\left(\varpi_0\right)+\left(\chi^{(1)}\right)'|_{\varpi_0}\varpi+\frac{1}{2}\left(\chi^{(1)}\right)''|_{\varpi_0}\varpi^2.... \tag{A.8}$$

Substitute that into (A.7) to get

$$\Lambda = \frac{-1}{2\pi}\left[\begin{array}{c}\chi^{(1)}\left(\varpi_0\right)\int\left(\varpi+\varpi_0\right)^2Q\left(z,\varpi\right)e^{-j(\varpi+\varpi_0)t}d\varpi+\\ \left(\chi^{(1)}\right)'|_{\varpi_0}\int\varpi\left(\varpi+\varpi_0\right)^2Q\left(z,\varpi\right)e^{-j(\varpi+\varpi_0)t}d\varpi+\\ \frac{1}{2}\left(\chi^{(1)}\right)''|_{\varpi_0}\int\varpi^2\left(\varpi+\varpi_0\right)^2Q\left(z,\varpi\right)e^{-j(\varpi+\varpi_0)t}d\varpi\end{array}\right], \tag{A.9}$$

where a clearer expression has been obtained by omitting the integral limits. Picking up terms up to second order in ϖ one obtains

$$\Lambda = e^{-j\varpi_0 t}\left[\begin{array}{c}\chi^{(1)}\left(\varpi_0\right)\left(-\varpi_0^2 q-2j\varpi_0\frac{\partial q}{\partial t}+\frac{\partial^2 q}{\partial t^2}\right)+\left(\chi^{(1)}\right)'|_{\varpi_0}\left(-j\varpi_0^2\frac{\partial q}{\partial t}+2\varpi_0\frac{\partial^2 q}{\partial t^2}\right)\\ +\frac{1}{2}\left(\chi^{(1)}\right)''|_{\varpi_0}\varpi_0^2\frac{\partial^2 q}{\partial t^2}\end{array}\right]. \tag{A.10}$$

A result which introduced into (A.1) yields

$$\left(-k_0^2+\frac{\varpi_0^2}{c^2}\left(1+\chi^{(1)}\left(\varpi_0\right)\right)\right)q+2jk_0\frac{\partial q}{\partial z}+\frac{\partial^2 q}{\partial z^2}+ \tag{A.11}$$

$$+j\frac{\varpi_0}{c^2}\left(2\left(1+\chi^{(1)}(\varpi_0)\right)+\left(\chi^{(1)}\right)'|_{\varpi_0}\right)\frac{\partial q}{\partial t}-\frac{1}{c^2}\left(\left(1+\chi^{(1)}(\varpi_0)\right)+2\left(\chi^{(1)}\right)'|_{\varpi_0}+\frac{\varpi_0^2}{2}\left(\chi^{(1)}\right)''|_{\varpi_0}\right)\frac{\partial}{\partial}$$

where to leading order it is satisfied

$$k_0^2 = k^2(\varpi_o) = \frac{\varpi_0^2}{c^2}\left(1+\chi^{(1)}(\varpi_0)\right) = k^2(\varpi), \tag{A.12}$$

whereupon the dispersion relation is expressed

$$k^2(\varpi) = \frac{\varpi^2}{c^2}\left(1+\chi^{(1)}(\varpi)\right). \tag{A.13}$$

Using that, the equation to the next magnitude order yields

$$2jk_0\frac{\partial q}{\partial z} + j\left(k^2\right)'|_{\varpi_0}\frac{\partial q}{\partial t} - \frac{1}{2}\left(k^2\right)''|_{\varpi_0}\frac{\partial^2 q}{\partial t^2} = 0. \tag{A.14}$$

Assuming that $\frac{(k)'|_{\varpi_0}}{k_0} << 1$ one may write

$$j\frac{\partial q}{\partial z} + jk'|_{\varpi_0}\frac{\partial q}{\partial t} - \frac{1}{2}k''|_{\varpi_0}\frac{\partial^2 q}{\partial t^2} = 0. \tag{A.15}$$

With the usual definitions $v_g = \left(k'|_{\varpi_0}\right)^{-1}$, the above reads in standard form

$$j\frac{\partial q}{\partial z} + j\frac{1}{v_g}\frac{\partial q}{\partial t} - \frac{1}{2}k''\frac{\partial^2 q}{\partial t^2} = 0. \tag{A.16}$$

In a frame of reference moving with velocity v_g, $\overline{z} = t + z/v_g$, finally one obtains for the equation governing pulse propagation in linear media

$$j\frac{\partial q}{\partial \overline{z}} = \frac{1}{2}k''\frac{\partial^2 q}{\partial t^2}. \tag{A.17}$$

Appendix B

Color scale

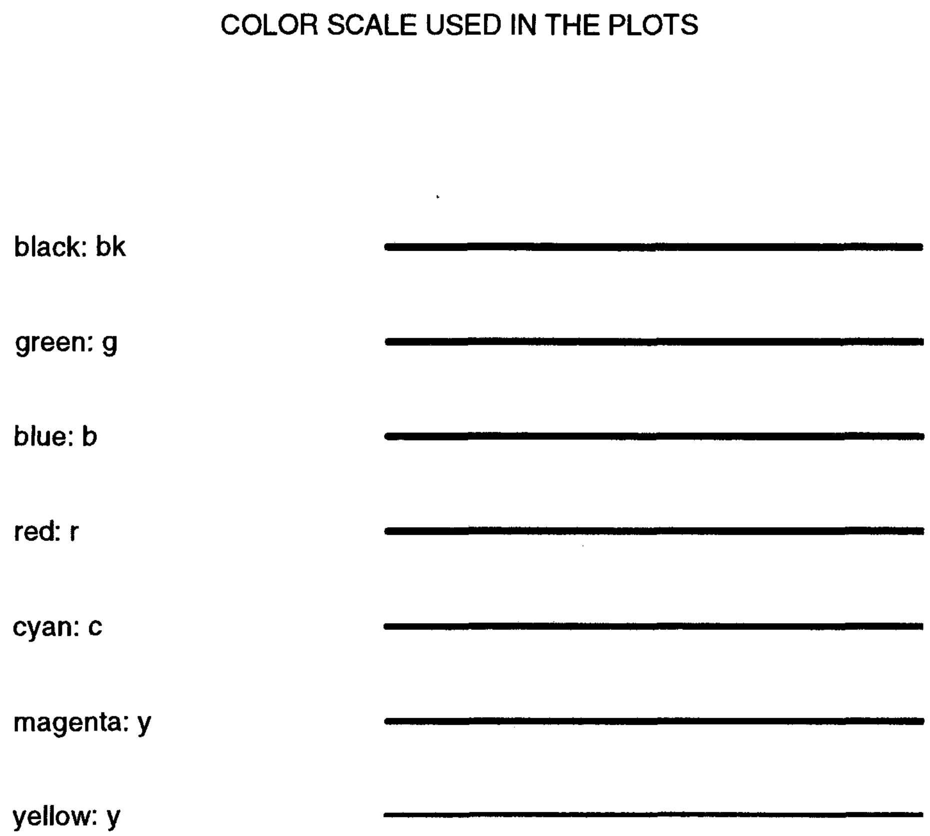

Figure B-1: Key for the color scales used in the Thesis.