Optimal Fiscal Policy, Limited Commitment and Learning

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To my parents, Alessandro and Patrizia,
to my brother, Massimo,
to my grandparents, Ivo and Elsa,
to my girlfriend, Sofia,
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Abstract

This thesis is about how fiscal authority should optimally set distorting taxes. Chapter 1 deals with the optimal fiscal policy problem when the government has an incentive to default on external debt. Chapter 2 deals with the optimal fiscal policy problem when households do not know how government sets taxes. The main conclusion I get is that, in each of these two contexts, the tax smoothing result, which is the standard result in the optimal taxation literature, is broken. When governments do not have a commitment technology taxes respond to the incentives to default; when agents have partial information about the underlying economic model, taxes depend on their beliefs about it.

Resumen

Esta tesis trata sobre cómo la autoridad fiscal debe fijar los impuestos distorsivos de manera óptima. El Capítulo 1 analiza el problema de la política fiscal cuando el gobierno tiene un incentivo a hacer default a su deuda externa. El Capítulo 2 trata sobre el problema de la política fiscal cuando los agentes no conocen cómo el gobierno fija las tasas impositivas. La principal conclusión que obtengo es que, en ambos contextos, el resultado de suavidad de las tasas, que es estándar en la literatura de imposición óptima, se rompe. Cuando los gobiernos no tienen una tecnología de compromiso, los impuestos responden a los incentivos de default; cuando los agentes poseen información parcial sobre el modelo subyacente de la economía, los impuestos dependen de sus expectativas sobre los mismos.

Foreword

An important question in macroeconomics is how policymakers should set distorting taxes. Before the 80's the literature did not address this issue, as most papers simply ruled out distorting taxes. When lump-sum taxes are available, the Ricardian equivalence holds and fiscal policy is irrelevant. The seminal paper by Barro (1979) was the first introducing distorting taxation and the paper by Lucas and Stokey (1983) provides a fully microfounded answer to the question.

After Lucas and Stokey (1983) the optimal fiscal policy literature has been developing over time (e.g. Aiyagari, Marcet, Sargent and Seppala (2002), Chari, Christiano and Kehoe (1994a), Chari and Kehoe (1998), Faraglia, Marcet and Scott (2006, 2007), Marcet and Scott (2008), Scott (1999a, 2007)). One strong conclusion coming out of these models is that a benevolent government should set distorting taxes roughly constant over time and across states of nature.

This conclusion has been derived under two main assumptions. The first one is that governments have a commitment technology. The second one is that households have full information about the underlying economic model.

None of these two assumptions seems very realistic. Governments may be unable to make binding commitments to future policies: this problem is particular relevant in open economy models, where countries can find optimal not to repay their external debt. The Russian and Ecuadorian crises in 1999 and the Argentinean one in 2001 are clear examples of how fragile the assumption of full commitment by governments is. On the other hand, there are many examples of structural shocks (e.g. the current financial crisis, the stage three of the European Monetary Union, the Easter Europe transition towards a capitalistic system) during which agents may not know exactly how fiscal policymakers behave. A natural question arises then: What is the optimal fiscal policy once we relax these assumptions?

In this thesis I offer an answer to this question. In particular, Chapter 1 deals with the optimal fiscal policy problem in a small open economy model with two countries, the Home Country and the Rest of the World. I relax the full commitment assumption supposing that each country may find it convenient not to pay back the debt contracted with the other. The benevolent government in the Home Country can finance an exogenous public expenditure shock either through labour income taxes, or through domestic debt or through external transfers with the Rest of the World. The market of internal transfers is regulated by a contract which reflects the fact that none of the countries can commit to pay back the external debt. In particular, the contract between the government in the Home Country and the Rest of the World is designed so that ι) at period 0 there is no net transfer of wealth between the two countries and $\iota\iota$) in equilibrium at any point in time, both countries repay the external debt.

Analytical results suggest that the presence of limited commitment alters substantially the dynamics of the fiscal variables with respect to the full commitment case.

Whereas under full commitment the optimal distorting labour income tax is perfectly flat, under limited commitment it responds strongly to the incentives to default of both countries. When the Home Country would have an incentive to default, the tax rate decreases in order to allow for higher consumption, which prevents the Home Country from defaulting in equilibrium. On the other hand, when the Rest of the World would have an incentive to default, the tax rate increases to pay back the external debt. As an implication, the volatility of the tax rate under limited commitment is higher than under full commitment. This result is in line with the empirical evidence on tax rate volatility in small open economies with limited commitment, for example Argentina. Moreover, this model can explain another feature of the data, which is that fiscal policy in small open economies is often procyclical.

Chapter 2 deals with the optimal fiscal policy problem when households do not know the tax rate rule followed by the government. Optimal behaviour requires households to forecast their own marginal utility of consumption one-period-ahead. As expectations about tax rate tomorrow are not model consistent, expectations about marginal utility of consumption are neither. As a consequence, the saving decision is inefficient. I this sense learning introduces an additional distortion in the economy; on one hand taxes are distorting and on the other hand expectations are distorted. The government chooses the fiscal plan which minimizes the losses associated to both distortions.

There are three main results. First, the government uses fiscal policy in order to manipulate expectations. When agents are initially pessimistic, the government sets low taxes in order to stimulate the economy and to reduce their unjustified pessimism. The converse is true when agents are initially optimistic. Second, in the long run agents form fully rational expectations as they correctly understand how the fiscal policymaker sets taxes. Third, the long run level of debt depends on the initial expectations held by agents: economies populated by pessimistic households converge to a rational expectations equilibrium characterized by a positive amount of government debt, whereas economies populated by optimistic households converge to a rational expectations equilibrium characterized by a negative amount of government debt.

The analysis is related to important fiscal policy issues. First, there is an ongoing debate about the effectiveness of a fiscal stimulus to handle the recent crisis. Whereas some economists argue that lower taxes would allow for a quicker recovery, other economists and policymakers are worried about an excessive accumulation of government debt. Is a fiscal stimulus a good idea or not? The answer would depend crucially on expectations: if households are "wrongly" pessimistic about the future, then a fiscal stimulus is a good policy option. But if they are not, then a fiscal stimulus is a bad idea idea because it can generate endogenous cyclical fluctuations. Second, as the government increases debt in order to stabilize out-of-equilibrium expectations, the desirability of imposing debt limits, as emphasized in the literature (e.g. Chari and Kehoe (2004)) is less clear cut than under the rational expectations framework. Moreover, debt limits may fail to discriminate between "good" and "bad" governments. Finally, some tests used in the literature to discriminate the financial markets structure (see inter alia Scott (2007), Marcet and Scott (2008), and Faraglia et al. (2006)) can mix evidence of learning with evidence of incomplete markets.

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1 Optimal Fiscal Policy in a Small Open Economy with Limited Commitment

1.1 Introduction

A fundamental question in macroeconomics is how a policymaker has to set distortionary taxes in order to finance an exogenous public expenditure shock. The answer to this question depends on both the degree of openness of the economy and on the commitment technology of the government to fulfill its obligations towards foreign investors.

Consider a setup in which a small open economy, which we call the *home country*, can trade assets with the rest of the world. The government of the home country has to collect revenues optimally in order to finance an exogenous stream of public expenditure, while the rest of the world is subject to no shock. In this case, a benevolent government of the home country would set taxes roughly constant over the business cycle. When a bad shock hits the economy, the government can borrow from abroad and pay back the debt later on, when the economy faces instead a good shock. In this way, the possibility to do risk-sharing with the rest of the world implies that the deadweight losses associated to distortionary taxation are minimized. In the extreme case in which the rest of the world is risk neutral, the optimal tax rate is perfectly flat and all fluctuations in public expenditure can be absorbed by international capital flows. It follows that, at least from a theoretical point of view, tax volatility in small open economies should be lower than the tax volatility in large or closed economies, thanks to the insurance role played by international borrowing and lending.

Nevertheless, this conclusion does not seem to be validated in the data. Table 1.1 shows some statistics for government expenditure and average tax rate series in Argentina and in the USA.[1]. Although the variability of the government expenditure series is roughly the same in the two countries, tax rates in Argentina are much more volatile than in the USA: the standard deviation of the series for Argentina is almost 60% higher than the one for the US economy. As can be seen from Table 1.2, this empirical evidence applies to other countries as well, for the same sample period.

In this paper we introduce sovereign risk into a standard optimal fiscal policy open economy model as the one described before by relaxing the assumption of full commitment from the home country and the rest of the world towards their contracted obligations. We show that this framework provides a theoretical justification for the

[1] The series for USA are from the Bureau of Economic Analysis of the US Department of Commerce. In the case of Argentina, the data we use is from the IMF, INDEC and Ministerio de Economia. We use quarterly data of current government expenditure net of interest payments plus gross government investment as a measure of government expenditure, and total tax revenues plus contributions to social security as a measure of total tax revenues. The average tax rate is calculated as the ratio between tax revenues over GDP. Due to reliability/availability of data for Argentina, we use data for the period 1993 – 2005.

Table 1.1: Fiscal variables for the USA and Argentina

	USA		ARG	
	Govt. expend.	Tax rate	Govt. expend.	Tax rate
Mean	0.1755	0.1850	0.1704	0.1828
St. deviation	0.0092	0.0130	0.0098	0.0214
Coef. of variation	0.0525	0.0704	0.0573	0.117

Table 1.2:

Country	Tax rate coefficient of variation
Bulgaria	0.104
Guatemala	0.136
Nicaragua	0.139
Venezuela	0.13

tax rate volatility observed in small open economies that have commitment problems to repay their external obligations.

In the model the home country is populated by risk adverse households. The fiscal authority has to finance an exogenous public expenditure shock either through distortionary labor income taxes or by issuance of internal and/or international debt. The rest of the of world is inhabited by risk-neutral agents that receive a constant endowment and have to decide how much to consume and how much to borrow/lend in the international capital market. We assume that neither the government in the home country nor the rest of the world can commit to pay back the debt contracted among themselves.

A contract, signed by the two countries, regulates international capital flows. The terms of the contract depend on the commitment technology available to the two parts to honor their external obligations. When both countries can fully commit to stay in the contract in all states of nature, the only condition to be met is that *ex-ante* there is no exchange of net wealth among them. Instead, when the countries may at some point decide to leave the contract, further conditions need to be imposed. In particular, since default takes place if the benefit a country obtains from staying in the contract is smaller than its outside option, the contract must specify an adjustment in the allocation necessary to rule out default in equilibrium.

We show that the presence of sovereign default risk, i.e., the possibility that a country may exit the contract with the other country, limits the amount of risk-sharing among countries. Consequently, the classical tax-smoothing result is broken since now the optimal tax rate depends on the incentives to default of both countries. In particular, when the home country wants to exit the contract it has to be compensated so that the benefits of staying in it equal the value of its outside option. Therefore, consumption and leisure have to increase, and the tax rate decreases. On the other hand, when the rest of the world has incentives to default, the tax rate in the home country increases to pay back the external debt and induce the rest of the world not to leave the contract.

In our model, the home country has incentives to exit the contract when the realization of the public expenditure shock is low. There are two reasons behind this feature. First, the benefits from staying in the contract decrease, since in this case the home country has to repay its foreign debt. The second reason relies on our definition of the outside option for the home country. We assume that, if the home country defaults, its government is forced to run a balanced-budget thereafter. The outside option is the expected life-time utility of the households under this fiscal policy plan. When the shock is low, the tax rate is low as well, so the outside option increases. Therefore, an important corollary of the analysis is that the optimal fiscal policy is pro-cyclical: tax rates decrease when the country has incentives to leave the contract, and this happens when public consumption is low. This conclusion is in line with recent evidence for developing countries (see e.g. Ilzetzki and Vegh (2008) and Cuadra and Sapriza (2007)).

Some possible alternative explanations for the high volatility of tax rates observed in developing economies rely on the quality of their institutions and the sources of tax collection. It is argued that developing countries are more prone to switches in political and economic regimes that, almost by definition, translate into unstable tax systems. Moreover, in booms these countries' often tax heavily those economic sectors that are responsible for the higher economic activity[2]. As a consequence, when economic conditions deteriorate, necessarily tax revenues go down dramatically. We are aware that these considerations are relevant sources of tax variability and that our study does not incorporate them in the analysis. However, we do not intend to provide an exhaustive description of such sources. In this sense, by focusing on sovereign risk and incomplete international capital markets as causes for the high tax rate volatility of developing economies we are carrying out a partial analysis of the phenomenon.

In the recent years there have been some attempts to add default to dynamic macroeconomic models. A number of papers (Arellano (2008), Aguiar and Gopinath (2006), Hamann (2004)) have introduced sovereign default in otherwise standard business cycle models in order to quantitatively match some empirical regularities of small open developing economies. More specifically, they adapt the framework of Eaton and Gersovitz (1981) to a dynamic stochastic general equilibrium model. These models are usually able to explain with relative success the evolution of the interest rate, current account,

[2] As an example, in the recent years Argentina has been experiencing rapid export-led growth, mainly due to exports of commodities such as soya. In this period, the government's main source of tax revenues has come from taxation of these exports.

output, consumption and the real exchange rate. Nevertheless, since they all consider endowment economies, they fail to capture the effects of default risk over the taxation scheme. Our contribution is to extend the analysis to be able to characterize the shape of fiscal policy and the links between the risk of default and taxes in a limited commitment framework.

Many papers have introduced the idea of limited commitment to study many important issues. Among others, Kehoe and Perri (2002) introduce credit arrangement between countries to reconcile international business cycle models with complete markets and the data, Krueger and Perri (2006) look at consumption inequality, Chien and Lee (2008) look at capital taxation in the long-run, Marcet and Marimon (1992) study the evolution of consumption, investment and output, and Kocherlakota (1996) analyzes the properties of efficient allocations in a model with symmetric information and two-sided lack of commitment. To our knowledge, none of them has focused on the impact of the possibility of default on the volatility of optimal taxation.

The closest papers to ours are probably those by Cuadra and Sapriza (2007), Pouzo (2008) and Scholl (2009). The first paper focuses on matching some stylized facts in developing countries, namely the positive correlation between risk premia and the level of external debt, higher risk premia during recessions and the procyclicality of fiscal policy in developing economies. The second paper studies the optimal taxation problem in a closed economy under incomplete markets allowing for default on internal debt. Finally, the third paper analyzes the problem of a donor that has to decide how much aid to give to a government that has an incentive to use these external resources to increase its own personal consumption without decreasing the distortive tax income it levies on private agents.

We differentiate from these papers along various dimensions. In the first place, we consider the full commitment solution instead of the time-consistent one. We do this to isolate the effect of endogenously incomplete markets on the optimal fiscal plan, while giving the government all the usual tools to distribute the burden of taxation across periods and states of the world. In particular, in our framework there is a complete set of state-contingent bonds the government can issue internally. This has important implications for consumption smoothing as it allows the government to distribute the burden of taxation across states. Finally, in contrast with the assumption in Scholl (2009), we focus on the scenario in which the government of the small open economy is benevolent, i.e., its objective is to maximize the expected life-time utility of its citizens.

The rest of the paper proceeds as follows. Section 1.2 describes the model. Section 1.3 shows how the optimal fiscal plan is affected by the possibility of default in the case study of a perfectly anticipated one-time fiscal shock. In section 1.4 we solve the model for the general case of correlated government expenditure shock. Section 1.5 is devoted to show that our economy can be reinterpret as one in which the government can issue debt subject to debt limits, both on internal and external debt. Section 1.6 offers some empirical evidence on the relationship between tax volatility and default risk. Section

1.7 concludes.

1.2 The Model

We assume that the economy is constituted by two countries: the home country (HC) and the rest of the world (RW). The HC is populated by risk-averse agents, which enjoy consumption and leisure, and by a benevolent government that has to finance an exogenous public expenditure shock either by levying distortionary taxes, by issuing state-contingent internal bonds, or by receiving transfers from the RW. The RW is populated by risk-neutral agents that receive a fixed endowment each period. These resources can be either consumed or lent to the HC. Being an endowment economy without shocks, there is no government activity in this country.

a The contract

The government of the HC can do risk-sharing with the RW by contracting transfers[3]. Let T_t be the amount of transfers received by the HC at time t. There are three conditions that have to be met by $\{T_t\}_{t=0}^{\infty}$.

First, the expected present discounted value of transfers exchanged with the RW must equal zero:

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{1.1}$$

where β is the discount factor of households in the RW and the HC. This condition rules out the possibility that the government of the HC uses resources from the RW for reasons other than the risk-sharing one. In other words, we do not allow for net redistribution of wealth between countries at time 0. We call this condition the *fairness condition*, since it implies that *ex-ante* the contract is fair from an actuarial point of view[4].

If we assumed that the two parts in the contract have full commitment to pay back the debt contracted with each other, equation (1.1) would be the only condition regulating international flows. The allocations compatible with this situation will be our benchmark for comparison purposes. However, when the government in the HC does not have a commitment technology, it may decide to leave the contract if it finds it profitable to do so. Denote by V_t^a the value of the government's outside option, i.e., the

[3]In section 1.5 we show that these transfers can be reinterpreted as bonds traded in the international capital market.

[4]This condition implies that the contract is actuarially fair only if the RW has full commitment. This is due to the fact that, if the RW has limited commitment, the risk-free interest rate will not always be $1/\beta$ (see Section 1.5 for further details). This condition is useful because it allows us to pin down the allocations. However, one can impose other similar conditions that will yield different allocations.

expected life-time utility of households in the HC if the government leaves the contract, and by V_t the continuation value associated to staying in the contract in any given period t. Then, in order to rule out default in equilibrium, the following condition has to be satisfied

$$V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V_t^a \ \forall t \tag{1.2}$$

This condition constitutes a *participation constraint for the HC*. We assume that, if the government chooses to leave the contract at any given period, it remains in autarky from then on. Moreover, when the government defaults on its external obligations, it also default on its outstanding domestic debt. Consequently, the government is forced to run a balanced budget thereafter[5]. Alternative assumptions to identify the costs of default could be made, for example that the government cannot use external funds, but it still has access to the domestic bonds market to smooth the distortions caused by the expenditure shock. We have chosen the current specification for two reasons. First, this allows us to keep the problem tractable, both from an analytical and a numerical point of view. Second, this specification is consistent with the interpretation that the government is subject to debt limits, as shown in section 1.5.

Similar to the case of the HC, the RW also lacks a commitment technology and can potentially exit the contract at any point in time. Therefore, we need to impose a *participation constraint for the RM*:

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \ \forall t \tag{1.3}$$

This condition is analogous to (1.17) and states that, at each point in time and for any contingency, the expected discounted value of future transfers the HC is going to receive cannot exceed an exogenous threshold value $\underline{B}$. This restriction, together with the fairness condition, poses an upper limit on how much indebted the RW can get.

As long as conditions (1.1), (1.17) and (1.18) are satisfied, the government of the HC can choose any given sequence $\{T_t\}_{t=0}^{\infty}$ to partially absorb its expenditure shocks.

b Households in the HC

Households in the HC derive utility from consumption and leisure, and each period are endowed with one unit of disposable time. The production function is linear in labor and one unit of labor produces one unit of the consumption good. Therefore, wages $w_t = 1 \ \forall t$.

The representative agent in the HC maximizes her expected lifetime utility

[5] It follows that the only state variable influencing the outside option is the government expenditure shock. Therefore $V_t^a = V_t^a(g_t)$.

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to the period-by-period budget constraint

$$b_{t-1}(g_t) + (1-\tau_t)(1-l_t) = c_t + \sum_{g^{t+1}|g^t} b_t(g_{t+1}) p_t^b(g_{t+1}) \tag{1.4}$$

where c_t is private consumption, l_t is leisure, $b_t(g_{t+1})$ denotes the amount of bonds issued at time t contingent on the government shock in period $t+1$, τ_t is the flat tax rate on labor earnings and $p_t^b(g_{t+1})$ is the price of a bond contingent on the government shock realization in the next period.

The optimality condition with respect to the state-contingent bond is:

$$p_t^b(g_{t+1}) = \beta \frac{u_{c,t+1}(g_{t+1})}{u_{c,t}} \pi(g^{t+1}|g^t) \tag{1.5}$$

.

where $\pi(g^{t+1}|g^t)$ is the conditional probability of the government expenditure shock. Combining the optimality conditions with respect to consumption and leisure we obtain the intratemporal condition

$$1 - \tau_t = \frac{u_{l,t}}{u_{c,t}} \tag{1.6}$$

c Government of the HC

The government finances its exogenous stream of public consumption $\{g_t\}_{t=0}^{\infty}$ by levying a distortionary tax on labor income, by trading one-period state-contingent bonds with domestic consumers and by contracting transfers with the RW. The government's budget constraint is

$$g_t = \tau_t(1-l_t) + \sum_{g^{t+1}|g^t} b_t(g_t+1) p_t^b(g_t+1) - b_{t-1}(g_t) + T_t \tag{1.7}$$

d Equilibrium

We proceed to define a *competitive equilibrium with transfers* in this economy.

Definition 1 *A competitive equilibrium with transfers is given by allocations $\{c, l\}$, a price system $\{p^b\}$, government policies $\{g, \tau^l, b\}$ and transfers T such that*[6]*:*

[6] We follow the notation of **?** and use symbols without subscripts to denote the one-sided infinite sequence for the corresponding variable, e.g., $c \equiv \{c_t\}_{t=0}^{\infty}$.

1. *Given prices and government policies, allocations satisfy the household's optimality conditions* (2.3), (2.10) *and* (1.6).

2. *Given the allocations and prices, government policies satisfy the sequence of government budget constraints* (1.7).

3. *Given the allocations, prices and government policies, transfers satisfy conditions* (1.1), (1.17) *and* (1.18).

4. *Allocations satisfy the sequence of feasibility constraints:*

$$c_t + g_t = 1 - l_t + T_t \tag{1.8}$$

e Optimal policy

The government of the HC behaves as a benevolent Ramsey Planner and chooses tax rates, bonds and transfers $\{c_t, b_t, T_t\}_{t=0}^{\infty}$ in order to maximize the representative household's life-time expected utility, subject to the constraints imposed by the definition of competitive equilibrium.

Before studying the consequences of introducing default in terms of the optimal fiscal plan, it is instructive to analyze the benchmark scenario in which both the government in the HC and the RW have a full commitment technology.

Full commitment

If both the HC and the RW can commit to honor their external obligations in all states of nature, then conditions (1.17) and (1.18) need not be specified in the contract. Then, the problem of the Ramsey planner is

$$\max_{\{c_t, l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

s.t.

$$b_{-1} u_{c,0} = E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)) \tag{1.9}$$

$$c_t + g_t = 1 - l_t + T_t \tag{1.10}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{1.11}$$

The optimality conditions for $t \geq 1$ are:

$$u_{c,t} + \Delta(u_{cc,t} c_t + u_{c \cdot t} + u_{cl,t}(1 - l_t)) = \lambda \tag{1.12}$$

$$u_{l,t} + \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda \tag{1.13}$$

where λ is the multiplier associated with constraint (1.11), and Δ is the multiplier associated with the implementability condition (1.9). The next proposition characterizes the equilibrium.

Proposition 1 *Under full commitment, consumption, labor and taxes are constant* $\forall t \geq 1$*. Moreover, if* $b_{-1} = 0$*,* $b_t(g_{t+1}) = 0$ $\forall t$*,* $\forall g_{t+1}$ *and the government perfectly absorbs the public expenditure shocks through transfers* T_t*.*

Proof.

Using optimality conditions (1.12) and (1.13) we have two equations to determine two unknowns, c_t and l_t, given the lagrange multipliers λ and Δ. Since these two equations are independent of the current shock g_t, the allocations are constant $\forall t \geq 1$. From the intratemporal optimality condition of households (1.6) it can be seen that the tax rate τ_t^l is also constant $\forall t \geq 1$[7]. Finally, the intertemporal budget constraint of households at time 0 (equation (1.9)) can be written as

$$\frac{1}{1-\beta}(u_c c - u_l(1-l)) = 0$$

Notice that, for any given time $t+1$, domestic bond holdings $b_t(g_{t+1})$ are obtained from the intertemporal budget constraint of households in that period, i.e.,

$$b_t(g_{t+1})u_{c,t+1} = E_{t+1}\sum_{j=0}^{\infty}\beta^j(u_{c,t+1+j}c_{t+1+j} - u_{l,t+1+j}(1 - l_{t+1+j}))$$

However, since the allocations are constant over time, it is the case that

$$b_t(g_{t+1})u_c = E_{t+1}\sum_{j=0}^{\infty}\beta^j(u_c c - u_l(1-l)) = \frac{1}{1-\beta}(u_c c - u_l(1-l)) = 0$$

Therefore, $b_t(g_{t+1}) = 0$ $\forall g_{t+1}$ and, from the feasibility constraint (1.10) it follows that all fluctuations in g_t must be absorbed by T_t.

■

Proposition 1 illustrates the effect of full risk-sharing on the optimal fiscal policy plan: being consumption and leisure constant along the business cycle, the optimal tax rate is constant as well. The government in the HC uses transfers from the RW to absorb completely the exogenous shock. When g_t is higher than average, the government uses transfers to finance its expenditure; conversely, when g_t is below average, the

[7]Notice that at $t = 0$ the optimality conditions of the Ramsey planner differ from those at $t \geq 1$. This is the standard source of time-inconsistency in this type of problems.

government uses the proceeds from taxation to pay back transfers received in the past[8]. The RW, which is a risk neutral agent, provides full insurance to the domestic economy.

Limited Commitment

We consider the case in which neither the government in the HC nor the RW can commit to repay external debt. The problem of the Ramsey planner is identical to the one in the previous section, but now conditions (1.17) and (1.18) have to be explicitly taken into account:

$$\max_{\{c_t,l_t,T_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to

$$c_t + g_t = (1 - l_t) + T_t \tag{1.14}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = u_{c^1,0}(b_{-1}) \tag{1.15}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{1.16}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{1.17}$$

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \ \forall t \tag{1.18}$$

Since the participation constraint at time t (1.17) includes future endogenous variables that influence the current allocation, standard dynamic programming results do not apply directly. To overcome this problem we apply the approach described in Marcet and Marimon (2009) and write the Lagrangean as:

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [&(1 + \gamma_t^1) u(c_t, l_t) - \psi_t(c_t + g_t - (1 - l_t) - T_t) \\ &- \mu_t^1 (V_t^a) + \mu_t^2(\underline{B}) - \Delta(u_{c,t} c_t - u_{l,t}(1 - l_t)) - T_t(\lambda + \gamma_t^2)] + \Delta(u_{c,0}(b_{-1}))\end{aligned}$$

where

$$\gamma_t^1 = \gamma_{t-1}^1 + \mu_t^1$$

[8]In Appendix A.1 we study the case in which the utility function is logarithmic in its two arguments. In such a case, it is easy to see that transfers behave exactly as described here.

$$\gamma_t^2 = \gamma_{t-1}^2 + \mu_t^2$$

for $\gamma_{-1}^1 = 0$ and $\gamma_{-1}^2 = 0$. Δ is the Lagrange multiplier associated to equation (1.15), ψ_t is the Lagrange multiplier associated to equation (1.14), λ is the Lagrange multiplier associated to equation (1.16), μ_t^1 is the Lagrange multiplier associated to equation (1.17) and μ_t^2 is the Lagrange multiplier associated to equation (1.18). γ_t^1 and γ_t^2 are the sum of past Lagrange multipliers μ^1 and μ^2 respectively, and summarize all the past periods in which either constraint has been binding. Intuitively, γ^1 and γ^2 can be thought of as the collection of past compensations promised to each country so that it would not have incentives to leave the contract.

In can be shown that, for $t \geq 1$[9], the solution to the problem stated above is given by time-invariant policy functions that depend on the *augmented* state space $\mathcal{G} \times \Gamma^1 \times \Gamma^2$, where $G = \{g_1, g_2, \ldots, g_n\}$ is the set of all possible realizations of the public expenditure shock g_t and Γ^1 and Γ^2 are the sets of all possible realizations of the costate variables γ^1 and γ^2, respectively. Therefore,

$$\begin{bmatrix} c_t \\ l_t \\ T_t \\ \mu_t^1 \\ \mu_t^2 \end{bmatrix} = H(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) \qquad \forall t \geq 1$$

More specifically, the government's optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = 0 \tag{1.19}$$

$$u_{l,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = 0 \tag{1.20}$$

$$\psi_t = \lambda + \gamma_t^2 \tag{1.21}$$

Other optimality conditions are:

$$c_t + g_t = (1 - l_t) + T_t \tag{1.22}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{1.23}$$

$$\mu_t^1 (E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) - V^a(g_t)) = 0 \tag{1.24}$$

[9]Once again, for $t = 0$ the FOCs of the problem are different. Applying Marcet and Marimon (2009), the problem only becomes recursive from $t \geq 1$ onwards.

$$\mu_t^2(E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} - \underline{B}) = 0 \tag{1.25}$$

$$\gamma_t^1 = \mu_t^1 + \gamma_{t-1}^1 \tag{1.26}$$

$$\gamma_t^2 = \mu_t^2 + \gamma_{t-1}^2 \tag{1.27}$$

$$\mu_t^1 \geq 0 \tag{1.28}$$

$$\mu_t^2 \geq 0 \tag{1.29}$$

Two observations are worth mentioning. First, from (1.19), (1.20) and (1.21) it is immediate to see that now the presence of γ_{t-1}^1 and γ_{t-1}^2 makes the allocations state-dependent. Moreover, being γ_{t-1}^1 and γ_{t-1}^2 functions of all the past shocks hitting the economy, the allocations are actually history-dependent. Second, the presence of these Lagrange multipliers makes the cost of distortionary taxation state-dependent. While in the full-commitment case this cost is constant over time and across states, in the limited commitment case it changes depending on the incentives to default that the HC and the RW have[10]. We will discuss this is further detail in section 1.5.

The next proposition characterizes the equilibrium for a logarithmic utility function.

Proposition 2 *Consider a utility function logarithmic in consumption and leisure and separable in the two arguments:*

$$u(c_t, l_t) = \alpha * log(c_t) + \delta * log(l_t) \tag{1.30}$$

with $\alpha > 0$ and $\delta > 0$. Define $t < t'$:

1. *If the participation constraint (1.17) binds such that $\gamma_t^1 < \gamma_{t'}^1$, then $c_t < c_{t'}$, $l_t < l_{t'}$ and $\tau_t > \tau_{t'}$.*

2. *If the participation constraint (1.18) binds such that $\gamma_t^2 < \gamma_{t'}^2$, then $c_t > c_{t'}$, $l_t > l_{t'}$ and $\tau_t < \tau_{t'}$.*

Proof. See Appendix A.2. ■

Proposition 2 states the way the allocations and tax rates adjust in order to make the contract incentive-compatible for the HC and the RW. The optimal tax rate decreases whenever constraint (1.17) is binding and increases when constraint (1.18) binds

[10]It can be shown that, in the full commitment case, this cost is given by Δ, while in the limited commitment one is determined by $\frac{\Delta}{1+\gamma_t^1}$.

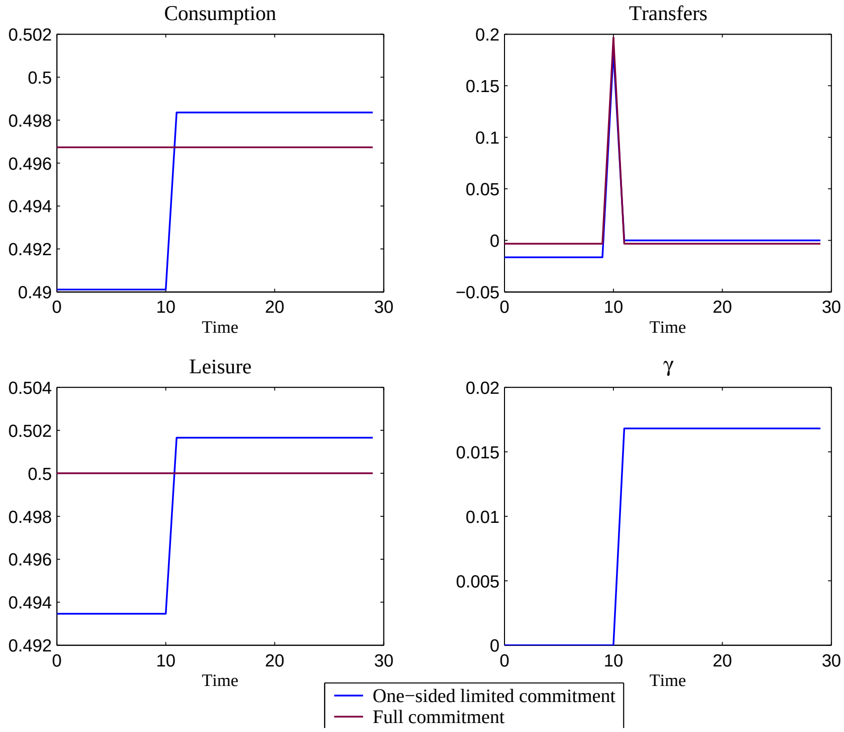

Figure 1.1: Example: $g_t = 0$ for $t \neq T$, and $g_T > 0$

instead[11]. Since the government in the HC has incentives to leave the contract when the government expenditure shock is low, because the the value of the ouside option in that case is high, the model implies a procyclical fiscal policy: the tax rate decreases following a low realization of the public expenditure process and increases when a the realization instead is high. This conclusion is in line with some recent empirical evidence for developing countries (see Ilzetzki and Vegh (2008) and Cuadra and Sapriza (2007)).

1.3 An example of labor tax-smoothing

In order to understand better the impact of limited commitment on the ability of the government to smooth taxes, in this section we analyze the case study of a perfectly anticipated government expenditure shock. Suppose that government expenditure is known to be constant and equal to 0 in all periods except in T, when $g_T > 0$. In order to simplify the analysis, throughout this section we assume that only the HC can default, while the RW has a commitment technology. Moreover, we assume that $b_{-1} = 0$ and that households have a logarithmic utility function as (1.30).

Since equilibrium allocations depend on γ_t^1, understanding the dynamics of the

[11]Notice that it cannot be the case that the two participation constraints bind at the same time.

Table 1.3: Parameter values

Preferences	$\alpha = \delta = 1$	
Intertemporal discount factor	$\beta = 0.98$	
Government expenditure	T = 10	$g_T = 0.2$

incentives to default is crucial. The next proposition states that, given the assumptions previously made, the participation constraint (1.17) binds only at $t = T + 1$[12].:

Proposition 3 *Suppose that government expenditure is known to be constant and equal to* 0 *in all periods except in* T*, when* $g_T > 0$*. Assume further that* $b_{-1} = 0$*. Then, the participation constraint* (1.17) *binds at exactly* $T + 1$*.*

Proof. See Appendix A.3. ■

From the results of Proposition 2 we can characterize the allocations for $t < T+1 \leq t'$. Given that $\gamma_t^1 < \gamma_{t'}^1$, it follows that $c_t < c_{t'}$, $l_t < l_{t'}$ and $\tau_t > \tau_{t'}$[13]. The limited commitment by the government exerts a permanent effect on the tax rate and alters its entire dynamics, since the tax rate level after the shock is permanently lower than before the shock.

The intuition for this result is as follows. Since at $T + 1$ the continuation value of staying in the contract has to increase in order to prevent default, utility of households in the HC has to increase. By the intratemporal optimality condition, a positive tax rate implies that the marginal utility of consumption is higher than the marginal utility of leisure. Therefore, increasing consumption is relatively more efficient than increasing labor and, as a consequence, the tax rate decreases.

a The example in numbers

In this section we solve numerically the example depicted above. Table 1.3 contains the parameters values used in the simulation.

Figures 1.1 and 1.2 show the evolution of the allocations c_t, l_t, the tax rate τ_t, international capital flows T_t, domestic bonds b_t and the costate variable γ_t^1. We compare the allocations with limited commitment to the ones under full commitment by the government towards the RW. There are two forces determining the dynamics of the economy. On one side the government has to finance the higher and expected expenditure outflow at T in the most efficient way; on the other, the participation constraint

[12]The reader may wonder why the participation constraint binds just after the shock. The reason is that agents know the bad shock will happen in T, so this decreases the outside option value in every period before the shock effectively takes place. Once the shock is over, the autarky value goes up.

[13]In Appendix A.5 we show that $\Delta < 0$ in this case.

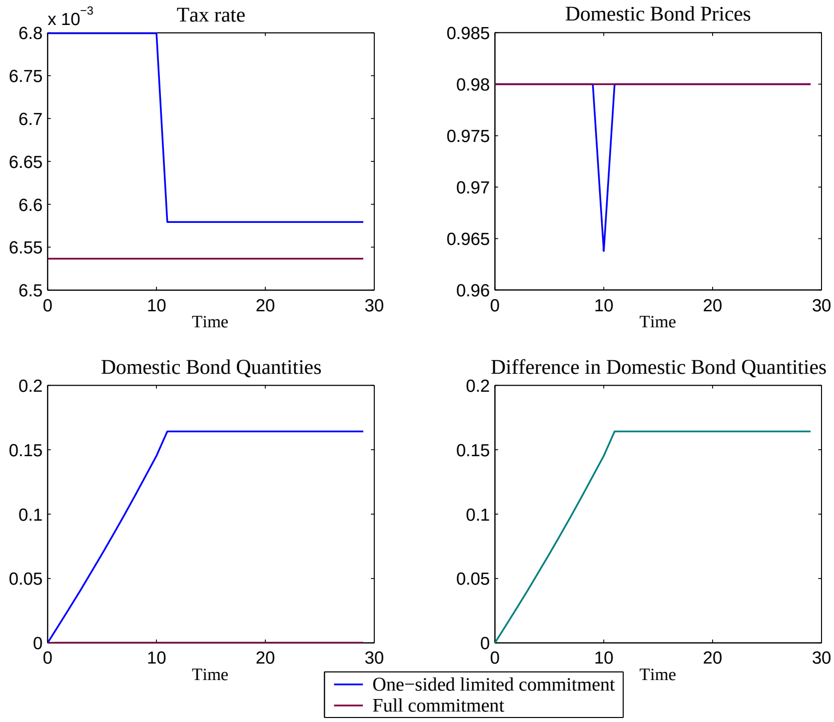

Figure 1.2: Example: $g_t = 0$ for $t \neq T$, and $g_T > 0$

has to be satisfied. For $t \leq T$ the higher and expected shock at T keeps the continuation value of autarky low, and for this reason leaving the contract is not optimal. Therefore, before T the government accumulates assets towards the RW, and uses them to finance part of the high expenditure outflow in T. The remaining part is covered both through tax revenues and through transfers from the RW. After the high shock has taken place, the outside option value increases. In order to prevent default, the government lowers the tax rate to allow domestic households to enjoy a higher level of consumption and leisure. Moreover, from $T + 1$ onwards, the transfer with the RW adjusts to guarantee that the expected present discounted value of net international flows is zero.

Notice the difference with the full commitment scenario, where the allocations are constant and transfers absorb completely the shock. The high inflow in period T is repaid forever by the government through small outflows after the shock. Taxes remain constant *even in period* T and do not react to the shock at all. The limited commitment technology constraints the amount of insurance offered by international capital markets, and perfect risk-sharing among countries is no longer possible. Consequently, the negative expenditure shock has to be absorbed through external debt and higher tax revenues in the initial periods.

Table 1.4: Parameter values

Preferences	$\alpha = \delta = 1$
Intertemporal discount factor	$\beta = 0.98$
Government expenditure process	$g_t = g * + \rho^g g_{t-1} + \epsilon_t$
$g*$	$0.1820 * 0.33$
ρ^g	0.9107
σ_g^2	$0,1320 * 0.0607$
$\underline{B}$	0.031
$b_{-1} = b_{-1}^G$	0

1.4 Numerical results

In section 1.2 we have characterized the equilibrium allocations arising from the Ramsey planner's maximization problem under two different scenarios. First, we studied the case in which the two parts in the contract have full commitment. In this case, we have seen that the risk-neutral households of the RW fully insure the HC and, consequently, consumption, labor and tax rates are perfectly constant in the RW. When both parts have limited commitment, however, it is no longer possible to do perfect risk-sharing and the allocations are no longer constant.

In this section we proceed to solve the model numerically assuming that the government spending follows an $AR(1)$ process. We calibrate the parameters of this process to the argentinean economy. We use quarterly series of current government expenditure net of interest payments plus gross government investment as our measure of government expenditure for the period 1993-I to 2005-IV.

Given that we need to calibrate the process for government expenditure, we estimate an AR(1) process in levels for the argentinean data. We find that, for the broader measure of real government expenditure, $\hat{\rho} = 0,9107$ for a specification as

$$g_t = \alpha + \rho g_{t-1} + \epsilon_t$$

We also need to obtain a value for the variance of the shock associated to g_t. Since the variance of g_t and, similarly, of ϵ_t are influenced by the units in which government expenditure is measured, in order to make meaningful international comparison across countries we need to find a statistic that is not influenced by neither the currency in which expenditure is denominated nor the size of the government itself. We therefore use the coefficient of variation (CV), defined as

$$CV = \frac{\text{Std. Dev}}{\text{Mean}}$$

In the data for Argentina, $CV = 0,1320$. We estimate the mean of g_t as the value

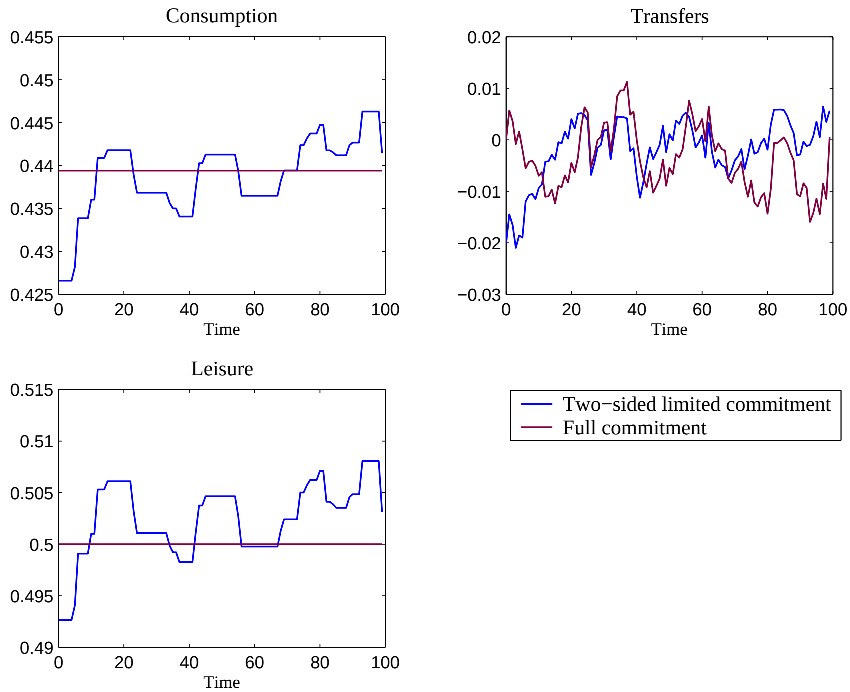

Figure 1.3: Two-sided limited commitment - Allocations

of g_t in steady state, given the mean of $\frac{g_t}{GDP_t}$ in the data. This value is $\overline{\frac{g}{GDP}} = 0,182$. Since our problem does not have a well defined steady state, we consider, as others in the literature, that $1 - l_t = \frac{1}{3}$ in steady state. Then $\overline{\frac{g}{GDP}} = \overline{\frac{g}{1-l}} = 0,182$. Therefore $\overline{g} = 0,33{*}0,182 = 0,0607$. Finally, the variance of $g_t = (0,1320{*}0.0607)^2 = 0,0000641$. We obtain the variance of ϵ_t in the following way:

$$\sigma_\epsilon^2 = \sigma_g^2(1 - \rho^2)$$

Figures 1.3, 1.4 and 1.5 show the allocations, co-state variables and fiscal variables respectively for the calibrated government expenditure shock, for the case in which both the government and the international institution have limited commitment (blue line). For comparison purposes, we show the same variables under full commitment (red line). Compared to the case in which international flows allow the government to smooth completely the distortion in the consumption-leisure choice, under partial commitment the current realization of the shock influences the equilibrium. Tables 1.5 and 1.6 summarize some statistics for the allocations and the fiscal variables for the cases of full and limited commitment.

Three observations are worth making. First, while the average values of the allocations are roughly the same in the two frameworks, their variance is much higher under partial commitment. Second, the allocations under full commitment are uncorrelated with the government expenditure shock; however, under partial commitment this correlation is negative. The reason is that an increase in public consumption in-

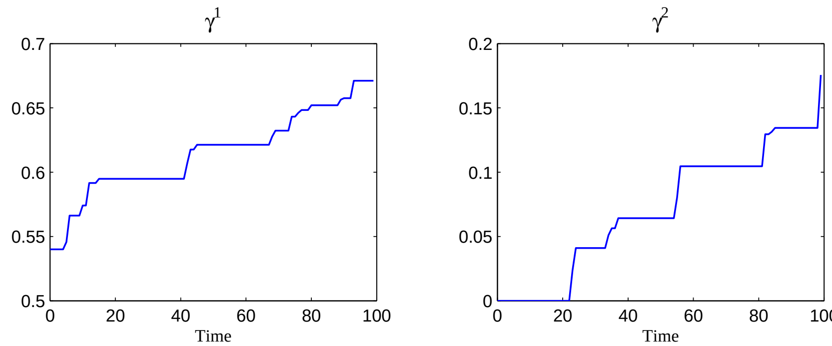

Figure 1.4: Two-sided limited commitment - Co-state variables

Table 1.5: Statistics of allocations for the first 50 periods

	Partial Comm.			Full Comm.		
	Mean	St.Dev.	Autocorr.	Mean	St.Dev.	Autocorr.
consumption	.43	.004	.92	.44	0	1
leisure	.5	.003	0.6	.92	0	1
labor tax rate	.13	.001	.92	.12	0	1
international flows	-.005	.006	.86	-.01	.003	.7

duces the standard crowding out effect to operate and, additionally, the participation constraint of the RW to be binding. This second channel reinforces the decrease in private consumption due to the first effect. Third, there is a positive correlation between consumption in the HC and the Lagrange multiplier associated to the participation constraint of the HC. The same is true for the correlation between international flows and the incentives to default. In periods in which it is optimal for the government of the HC to leave the contract, it receives a positive amount of transfers from abroad so as to adjust the continuation value of staying in the contract upwards to equate it with the continuation value associated to the outside option. Thus, default does not take place in equilibrium.

1.5 Borrowing constraints

In this section we show that it is possible to reinterpret the problem depicted in the previous sections as one in which the the HC and the RW trade one-period state-contingent bonds in the international financial market, but their trading is limited by borrowing constraints. To do so, we follow the strategy proposed by Alvarez and Jermann (2000) and Abraham and Cárceles-Poveda (2009)[14]. We show that, if we

[14]In the Appendix we show that the government's problem coincides with the one of an international institution in charge of distributing resources among the HC and the RW, taking into account the

Table 1.6: Correlation with past promises

	Partial Comm.		Full Comm.	
	$Corr(x, \gamma^1)$	$Corr(x, g)$	$Corr(x, \gamma^1)$	$Corr(x, g)$
consumption	.5	-48	-	0
labor tax rate	-.97	.36	-	0

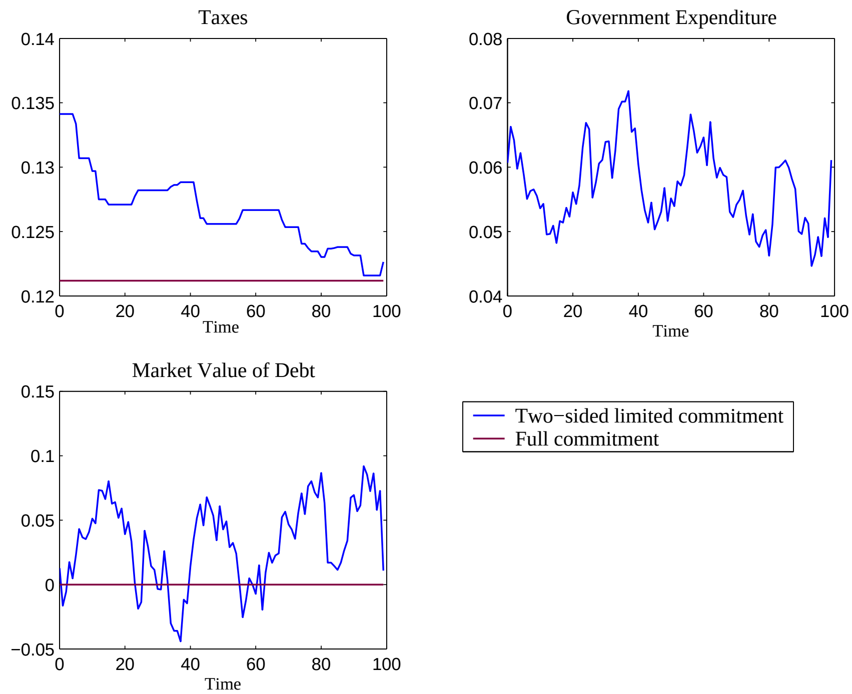

Figure 1.5: Two-sided limited commitment - Fiscal variables

impose limits only on international borrowing, the allocations in the two setups do not coincide. An additional constraint on the value of domestic debt that the government of the HC can issue is required.

In what follows, we will denote with a superscript 1 variables corresponding to the HC, and with superscript 2 variables corresponding to the RW. $Z_t^1(g_{t+1})$ is the international bond bought at t by the government of the HC contingent on next period's realization of the government expenditure shock. Symmetrically, call $Z_t^2(g_{t+1})$ the international bond bought at t by households of the RW contingent on next period's realization of the government expenditure shock. Denote the price of these bonds by $q_t(g_{t+1})$, and assume that there are lower bounds, denoted by $A_t^1(g_{t+1})$ and $A_t^2(g_{t+1})$, on the amount of bonds that the government of the HC and the households in the RW

aggregate resource constraint, the implementability condition of the HC, and the fact that countries have limited commitment. Therefore, the problem laid out in section 1.2 can be thought of one in which a central planner determines the constrained efficient allocations.

can hold, respectively.

In the current setup, the problem of the households in the HC is exactly identical to the one described in section b, so we do not reproduce it here. The problem of the government in the HC is slightly different from the one in previous sections. In order to finance its public expenditure, in addition to distortionary taxes on labor income and domestic bonds, now the government has available one-period state-contingent bonds traded with the RW. The budget constraint of the government is:

$$g_t + \sum_{g^{t+1}|g^t} Z_t^1(g_{t+1})q_t(g_{t+1}) - Z_{t-1}^1(g_t) = \tau_t(1-l_t) + \sum_{g^{t+1}|g^t} b_t(g_{t+1})p_t^b(g_{t+1}) - b_{t-1}(g_t) \quad (1.31)$$

The government faces a constraint on the amount of debt that can issue in the international financial market:

$$Z_t^1(g_{t+1}) \geq A_t^1(g_{t+1}) \quad (1.32)$$

The problem of households in the RW that trade bonds with the government in the HC now is

$$\max_{\{c_t^2, T_t^2\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t c_t^2$$

$$y + Z_{t-1}^2(g_t) = c_t^2 + \sum_{g_{t+1}} q_t(g_{t+1}) Z_t^2(g_{t+1}) \quad (1.33)$$

$$Z_t^2(g_{t+1}) \geq A_t^2(g_{t+1}) \quad (1.34)$$

Notice that the RW is also constraint in the amount of debt it can trade with the RW. The optimality conditions of this problem are equation (1.33) and

$$q_t(g_{t+1}) = \beta(1+\gamma_t^2)\pi(g^{t+1}|g^t) \quad (1.35)$$

$$\gamma_t^2(Z_t^2(g_{t+1}) - A_t^2(g_{t+1})) = 0$$

$$\gamma_t^2 \geq 0$$

where γ_t^2 is the Lagrange multiplier associated to the borrowing constraint (1.34).

Definition 2 *A competitive equilibrium with borrowing constraints is given by allocations* $\{c^1, c^2, l\}$*, a price system* $\{p^b, q\}$*, government policies* $\{g, \tau, b\}$ *and international bonds* $\{Z^1, Z^2\}$ *such that:*

1. *Given prices and government policies, allocations* c *and* l *satisfy the HC household's optimality condition* (2.3), (2.10) *and* (1.6).

2. *Given the allocations and prices, government policies and bonds Z^1 satisfy the sequence of government budget constraints* (1.31) *and borrowing constraints* (1.32).

3. *Prices q and bonds Z^2 satisfy the RW optimality conditions* (1.35) *and* (1.34).

4. *Allocations satisfy the sequence of feasibility constraints:*

$$c_t^1 + g_t + \sum_{g^{t+1}|g^t} Z_t^1(g_{t+1})q_t(g_{t+1}) = 1 - l_t + Z_{t-1}^1(g_t) \tag{1.36}$$

$$c_t^2 + \sum_{g^{t+1}|g^t} Z_t^2(g_{t+1})q_t(g_{t+1}) = y + Z_{t-1}^2(g_t) \tag{1.37}$$

5. *International financial markets clear:*

$$Z_t^1(g_{t+1}) + Z_t^2(g_{t+1}) = 0$$

We need to specify borrowing constraints that prevent default by prohibiting agents from accumulating more contingent debt than they are willing to pay back, but at the same time allow as much risk-sharing as possible. Define first

$$V_t^1(Z_{t-1}^1(g_t), g_t) = u(c_t^1, l_t) + \beta E_{t|g_t} V_{t+1}^1(Z_t^1(g_{t+1}), g_{t+1})$$

$$V_t^2(Z_{t-1}^2(g_t), g_t) = c_t^2 + \beta E_{t|g_t} V_{t+1}^2(Z_t^2(g_{t+1}), g_{t+1})$$

Then we define the notion of borrowing constraints that are not too tight:

Definition 3 *An equilibrium has borrowing constraints that are not too tight if*

$$V_{t+1}^1(A_t^1(g_{t+1}), g_{t+1}) = V_{t+1}^a \;\; \forall t \geq 0, g_{t+1}$$

and

$$V_{t+1}^2(A_t^2(g_{t+1}), g_{t+1}) = \underline{B} \;\; \forall t \geq 0, g_{t+1}$$

where V_{t+1}^a and $\underline{B}$ are defined as in section a.

We continue to assume that the government of the HC behaves as a benevolent Ramsey Planner and chooses tax rates, domestic and international $\{\tau_t, b_t, Z_t^1\}_{t=0}^{\infty}$ in order to maximize the representative household's life-time expected utility, subject to the constraints imposed by the definition of competitive equilibrium with borrowing constraints. We can write the problem of the government as

The problem of the government in the HC is

$$\max_{\{c_t^1, l_t, Z_t^1\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^1, l_t) \tag{1.38}$$

s.t.

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c^1,t} c_t^1 - u_{l,t}(1 - l_t)) = b_{-1} u_{c^1,0} \tag{1.39}$$

$$1 - l_t + Z_{t-1}^1(g_t) = c_t^1 + g_t + \sum_{g_{t+1}} q_t(g_{t+1}) Z_t^1(g_{t+1}) \tag{1.40}$$

$$Z_t^1(g_{t+1}) \geq A_t^1(g_{t+1}) \tag{1.41}$$

Proposition 4 *When the only borrowing constraints imposed on the competitive equilibrium are* (1.32) *and* (1.34)*, the allocations satisfying* (1.19)-(1.21) *do not solve the government's problem* (1.38)-(1.41).

Proof.

The proof is immediate. Taking the first order conditions

$$u_{c_t^1} - \Delta(u_{c_t^1} c_t^1 + u_{cc_t^1} c_t^1 + u_{c^1 l,t}(1 - l_t)) = \lambda_{1,t} \tag{1.42}$$

$$u_{l_t} - \Delta(u_{c^1 l,t} c_t^1 + u_{l,t} l_t + u_{ll,t}(1 - l_t)) = \lambda_{1,t} \tag{1.43}$$

Clearly, the allocations satisfying equations (1.42)-(1.43) cannot coincide with the solution of the system of equations (1.19)-(1.20), because in the latter case the weight attached to the term $u_{c_t^1} c_t^1 + u_{cc_t^1} c_t^1 + u_{c^1 l,t}(1 - l_t)$ is constant and equal to Δ, while in the former it is given by $\frac{\Delta}{1+\gamma_t^1}$ and varies over time. ■

This proposition states that the economy with transfers cannot be reinterpreted as an economy in which there are international bond markets and limits to international debt issuance only.

From the proof of the proposition, it is again evident what has already been pointed out in section e. When there is full commitment, the cost of distortionary taxation is given by the Lagrange multiplier associated to the implementability constraint, Δ. This cost is constant due to the presence of complete bond markets. However, when we relax the assumption of full commitment and consider instead the case in which the government of the HC has limited commitment, this cost becomes state-dependent and is given by $\frac{\Delta}{1+\gamma_t^1}$. The reason for this is that now the government faces endogenously incomplete international bond markets. Since allocations and tax rates vary permanently every time the participation constraint of the HC binds, so does the burden of taxation.

The previous discussion leads us to impose borrowing constraints on the value of domestic debt in addition to the constraints on international debt[15]. The next proposition

[15]Sleet (2004) also defines a borrowing constraint in terms of the value of debt.

states that, in this case, it is possible to establish a mapping between the economy with transfers and the one with borrowing constraints on domestic as well as international debt:

Proposition 5 *Suppose that, in addition to constraints* (1.32) *and* (1.34)*, we impose a lower bound on the value of state contingent domestic debt (expressed in terms of marginal utility)*

$$b_{t-1}^{1}(g_t)u_{c^1,t} = E_t \sum_{j=0}^{\infty} \beta^j (u_{c^1,t+j}c_{t+j}^1 - u_{l,t+j}(1 - l_{t+j})) \leq B_{t-1}(g_t) \qquad (1.44)$$

Then the allocations solving the system (1.19)-(1.21) *also solve the government's problem* (1.38)-(1.41)*.*

Proof. See Appendix A.6 ■

This result provides a rationale for our specification of the outside option of the government in the HC. In section 1.2 we assumed that if the government defaulted, it would lose access to the international and domestic bond markets and would remain in financial autarky thereafter. It seems natural then to impose constraints on the amount of debt that it can issue in both markets.

1.6 Stylized Facts

In this section we want to check if in the data tax rate volatility is affected by the availability of external sources to finance domestic shocks. We use the Emerging Markets Bond Index (EMBI) to measure the degree of insurance against internal shocks governments in emerging markets can get offshore. EMBI tracks total returns for traded external debt instruments, and gives a measure of the riskiness of the sovereign bonds issued by a country. We compute the annual average of EMBI for 6 emerging economies (Argentina, Mexico, Nigeria, Venezuela, Panama, Peru) from 1995 until 2001 and we look at the relationship with the average tax rate volatility referring to the same period.[16][17] The idea is that the higher the (mean) EMBI specific to a country, the higher the perceived investment risk that investors from abroad associate to that country, and the lower the amount of international flows the country can use to hedge against government revenues shocks.

Figure 1.6 plots this relationship. The horizontal axis measures tax rate standard deviation, and the vertical axis measures the EMBI. The graph shows that the higher the EMBI the more the government has to vary taxes to satisfy its budget constraint:

[16] We restrict our analysis to this period and to these countries for a question of availability of data.

[17] We consider a broad measure of tax rate as we define it as the ratio between total government revenues over GDP.

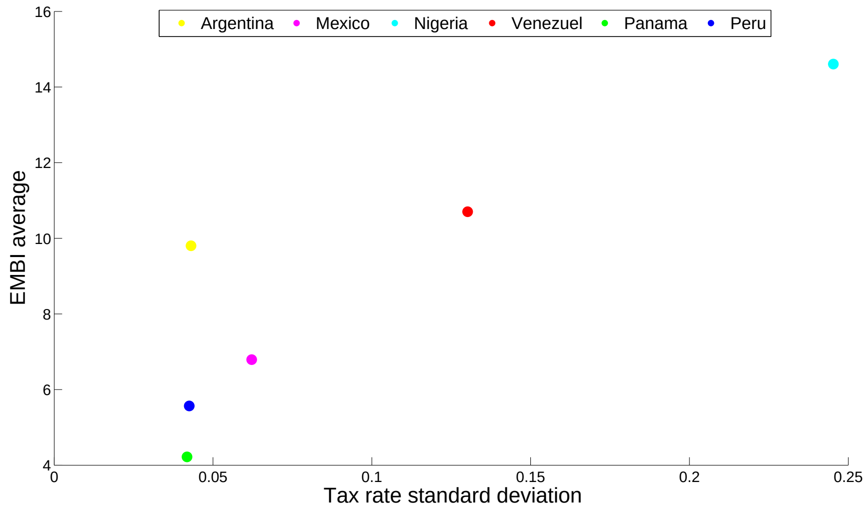

Figure 1.6:

in a sense the government can rely less on international debt to minimize the taxation distortion.

Although it supports the main result of this paper, the evidence we suggest is very preliminary. Data on fiscal variables for emerging economies are difficult to obtain and, when available, the series are either very short or they are not always reliable. For this reason we can not perform any time series analysis. Apart from this problem, what really matters for tax rate variability in the model is the availability of international lending/borrowing and it is not obvious how to measure this variable. As we use EMBI, which refers to emerging markets, we cannot run any cross-sectional estimation as there are too few observations. Nevertheless using some other indicator for limited commitment across countries would allow us to address the empirical estimation of the model in a more formal way. We leave this issue as future research.

1.7 Conclusions

A key issue in macroeconomics is the study of the optimal determination of the tax rate schedule when the government has to finance (stochastic) public expenditure and only has available distortionary tools[18]. Under this restriction, a benevolent planner seeks to minimize the intertemporal and intratemporal distortions caused by taxes. Since consumption should be smooth, a general result is that taxes should also be smooth across time and states.

When considering a small open economy that can borrow from international risk-

[18]When the government can levy non-distortionary taxes, such as lump-sum taxes, the Ricardian Equivalence holds and the first best can be achieved.

neutral lenders and both parts can fully commit to repay the debt, this result is amplified because there is perfect risk-sharing. Consumption and leisure are perfectly flat, thus the tax rate is also flat. The domestic public expenditure shock is absorbed completely by external debt and there is no role for internal debt. In light of this result, one would expect small open economies to have less volatile tax rate schedules than large economies. However, the available data seems to contradict this intuition.

When we relax the assumption of full commitment from both the small open economy and from international lenders towards their international obligations, perfect risk-sharing is no longer possible. The presence of limited commitment hinders the ability of the government to fully insure against the public expenditure shock through use of international capital markets. Consequently, the government has to resort to taxes and internal debt in order to absorb part of the shock, and it is no longer possible to have constant allocations and tax rates.

Our simulation results show that the volatility of the tax rate increases substantially when there is limited commitment. Moreover, fiscal policy is procyclical: when the government expenditure shock is low, the country has incentives to leave the contract with the international institution. Therefore, taxes should decrease in order to allow consumption and leisure to be higher and, in this way, increase household utility. On the contrary, when the government expenditure is high, taxes need to be high as well to repay the external debt.

These two features of our model are in line with what we observe in the data for developing countries, which are the ones more likely to lack a commitment technology towards international obligations. In section 1.6 we confirm this by looking at some stylized facts regarding fiscal variables, public debt and sovereign risk. We find that there seems to be a positive relation between the volatility of the average tax rate and that of a measure of the country risk premium.

The results presented in the paper suggest that the volatility and cyclicality of tax rates observed in developing countries is not necessarily an outcome of reckless policy-making, as one could think a priori. We have shown that, in order to establish the optimal fiscal policy plan in small developing countries, it is important to take into account the degree of commitment that the economy has towards its external obligations, as this element is crucial in determining the extent of risk-sharing that can be achieved.

A question that remains unanswered is where the allocations converge in the long run. It could be the case that the participation constraints continue to bind in some states of nature, even in the long run. In that case γ^1 and γ^2 diverge to infinity. The other possibility is that the economy arrives to a point in which neither the HC nor the RW has further incentives to leave the contract, and from that moment onwards c, l, τ, γ^1 and γ^2 remain constant. In that case, there is partial risk-sharing only in the short-run. This is an issue that we plan to address in the future.

Finally, the theoretical results outlined in the paper suggest a new mechanism to check in the data for developing countries. Our claim is that governments that are suspected to have commitment problems in repaying their debt necessarily have to set more volatile taxation schemes than more reliable ones. We have presented some very preliminar evidence that this might be supported by the data, but evidently a deeper analysis is called for.

2 Optimal Fiscal Policy when Agents are Learning

2.1 Introduction

> ”*... there is no reason and no occasion for any American to allow his fears to be aroused or his energy and enterprise to be paralyzed by doubt or uncertainty... It is true that the national debt increased sixteen billion dollars... you will be told that the Government spending program of the past five years did not cause the increase in our national income... But that Government spending acted as a trigger, a trigger to set off private activity.”* . (Franklin Delano Roosevelt, On the Recession (1938))

> ”*Just this week, we learned that retails sales have fallen off a cliff, and so industrial production. All signs point to an economic slump that will be nasty, brutish-and long. How nasty?... the unemployment rate will go above 7 percent, and quite possibly above 8 percent... And how long? It could be very long indeed... there is a lot the federal government can do for the economy... Now it is not the time to worry about the deficit.”* . (Paul Krugman, (2008))

An important issue in public finance theory is how to collect revenues to pay for government expenditures. When lump-sum transfers are not available, fiscal authorities must resort to taxes which distort people's decisions and move the economy away from the first-best. The optimal taxation literature (e.g. Lucas and Stokey (1983), Chari, Christiano and Kehoe (1994b), Chari and Kehoe (1999)) focuses on identifying the tax profile which minimises the associated distortionary costs. The key insight of this literature is that under complete markets the pay-off of the portfolio of state-contingent bonds works as an insurance device. As a consequence, the tax rate should be smooth and respond very little to shocks.

This conclusion has been derived in a framework in which agents fully understand the problem faced by the government. As they know the problem, they know the solution too, so that their expectations about future tax rates are model-consistent. This requires full information about the model. In many real-world situations the agents' knowledge may not be so deep. Then two questions arise: 1) What happens if the government pursues rational expectations optimal policies but the private sector's expectations are not rational? 2) What is the optimal tax policy if the government recognises that the private sector does not have rational expectations?

The motivation for this work is twofold. The first aspect is normative. As most of the models on optimal taxation assume rational expectations, it is important to check whether they suggest policy recommendations which are robust to alternative expectation formation mechanisms. In this chapter I show that the optimal fiscal policy under rational expectations, implemented in a set-up in which agents are boundedly rational, actually generates a suboptimally high volatility in private consumption and leisure. The second motivating factor is that complete markets models with rational expectations are at odds with the empirical evidence on fiscal variables. In this chapter I outline a very simple model that can bridge part of the gap between the data and the theoretical predictions of the complete markets framework.

I consider a closed production economy with no capital and infinitely lived agents. I start assuming that public spending is an exogenous shock, as it is the usual reference point in the public finance literature. Later on I extend the analysis to the case in which the government decides the amount of public consumption. The problem of the household is to maximise her lifetime expected utility subject to her flow budget constraint. The only difference between this framework and the standard optimal fiscal policy one is that agents do not have model-consistent expectations. They act like econometricians and to forecast next period's contingent marginal utility of consumption they use a weighted average of past values of it. Given the realisation of the shock, each period they update their belief about the marginal utility of consumption contingent on that specific realisation.

The government is benevolent and chooses distortionary taxes on labour income and state-contingent debt to maximise households' expected utility, subject to the feasibility constraint, households' optimality conditions and the way in which they update their beliefs.

I find that the government should set fiscal variables to manipulate private agents' expectations. To give an intuition, assume that the public expenditure is constant and that the government has zero initial wealth. Under rational expectations, the optimal fiscal rule prescribes a balanced budget: the government sets the tax rate to collect enough revenues to finance expenditure. When agents do not have rational expectations, this fiscal rule is still feasible, but it will imply a much longer time for agents to learn the tax rate than if the government followed an expectation-dependent fiscal plan. When agents are pessimistic, (i.e. they expect the one-step-ahead tax rate to be higher than they would expect it to be if they were fully rational) government optimality conditions require current expenditure to be financed mainly through debt: in this way the low current tax rate induces agents to revise downwards their expectations about the next period's tax rate.[1] In the long-run the tax rate is higher than in a rational expectations framework because the government has to finance the interest paid on a positive amount of debt.[2]

In this sense the agents' initial beliefs have an effect on the long-run mean value of the tax rate and debt: the more pessimistic (optimistic) the agents are, the higher is

[1]One implication of this result is that restricting how much a government can become indebted can delay the learning process.

[2]The analysis is symmetric for the case of optimistic agents.

the government debt (wealth) in the long-run. One implication of this result is that the model can help explain the wide dispersion across countries in the level of government debt and tax rate.

As expectations are not model-consistent, taxes are less smooth than under rational expectations. The reason is that the government has to minimise the welfare costs associated with distortionary taxes on one hand, and with distorted expectations on the other. When expectations are rational only the first distortion is present, and to minimise the associated losses taxes have to be smooth. But when both distortions are present, this is no longer optimal. The case-study of a perfectly anticipated war is a clear example of the tension between the two conflicting goals the government wants to achieve, tax smoothing on one hand and manipulation of beliefs on the other. Under rational expectations it is optimal for the government to accumulate assets before the war and sell them during the war-time. In this way the tax rate is constant in all periods before and after the war. By contrast, in a learning framework pessimistic agents do not trust the promises made by the government of higher-than-expected future consumption. The government sets low tax rates to manipulate agents' expectations, accumulating less assets (than in a RE framework) before the war. As a consequence, the war is financed issuing more debt than in a RE framework. The tax rate after the big shock is much higher than before.[3]

Since tax rates and debt have a unit-root behaviour, bounded rationality affects the power of some widely used tests to check for market completeness and debt sustainability. In line with Marcet and Scott (2008) I find that looking at the behaviour of debt is a much more reliable way to test the bond market structure than looking at the behaviour of tax rates. Similarly, the standard unit-root test in the debt/GDP ratio used to discriminate between responsible and non-responsible governments can be misleading, since it may cause a fiscal policy plan to be declared unsustainable when instead it is sustainable by construction. Augmenting this test to include the primary surplus in the regressors is a sharper way to distinguish the optimal and sustainable fiscal policy from an unsustainable policy.

Finally, I extend the model to the case in which the government chooses public consumption. I find that, when agents are pessimistic, the fiscal authority increases public spending above the rational expectations level, financing it mainly through debt. This conclusion is in line with some proposals to deal with the recent distress.

Many authors have studied the impact of learning on monetary policy design, either when the central bank follows some ad hoc policy rules (see *inter alia* Orphanides and Williams (2006), Preston (2005a,b, 2006), Preston and Eusepi (2007b,a)) or when it implements the optimal monetary policy (see *inter alia* Evans and Honkapohja (2003, 2006), Molnar and Santoro (n.d.)). Perhaps surprisingly, fiscal policy has received much

[3]Manipulating expectations can explain why a benevolent government should run a deficit during peacetime periods, an implication that the Lucas and Stokey (1983) does not have and for which has been criticized.

less attention. Evans, Honkapohja and K. (2007) study the interest rate dynamic learning path in a non-stochastic economy in which the fiscal authority credibly announces a future change in government purchases. Karantounias, Hansen and Sargent (2007) and Svec (2008) study the optimal fiscal policy when agents do not trust the transition probabilities of the public expenditure suggested by their approximating model. Up to my knowledge, this is the first chapter studying the influence of learning on fiscal policy design.

The chapter proceeds as follows. Section 2.2 studies the consequences of implementing the bond policy function under rational expectations when agents are learning. Section 2.3 solves for the optimal fiscal policy under learning. In Section 2.4 I characterise the fiscal plan restricting the government expenditure shock to a specific form. Section 2.5 gives some policy implications. In section 2.6 I extend the basic model to the case of endogenous government expenditure. Section 2.7 deals with the problem of discriminating between a complete markets model with learning and an incomplete markets model with rational expectations. Section 2.8 focuses on debt sustainability and debt limits. In section 2.9 I report some stylized facts about fiscal variables and agents' sentiment and use US data in order to test the model. Section 2.10 concludes.

2.2 The Model

I consider an infinite horizon economy where the only source of aggregate uncertainty is represented by a government expenditure shock.[4] Time is discrete and indexed by $t = 0, 1, 2....$ In each period $t \geq 0$ there is a realisation of a stochastic event $g_t \in G$. The history of events up and until time t is denoted by $g^t = [g_t, g_{t-1}, g_{t-2}, ..., g_0]$. The conditional probability of g^r given g^t is denoted by $\pi(g^r|g^t)$. For notational convenience, I let $\{x\} = \{x(g^t)\}_{g^t \in G}$ represent the entire state-contingent sequence for any variable x throughout the chapter.

In section a I briefly review the Lucas and Stokey (1983) model. The economy is populated by a representative household and a government. To finance an exogenous stream of public consumption, the government levies a proportional tax on labour income and has access to a complete set of one-period state-contingent bonds. Both the household and the government have rational expectations. The solution to this model states policy rules for labour tax rate and bond-holdings which maximise households' welfare subject to the restriction that taxes are distortionary. In section b I assume that, although the government follows the bond-holdings policy rule as in section a households' expectations are not rational; I show that in this case the optimal fiscal policy under rational expectations translates into sub-optimal volatility of the allocation.

[4]In section 2.6 I extend the analysis to the case in which the fiscal authority chooses the amount of public consumption.

a Rational expectations by both the government and agents

Consider a production economy where the technology is linear in labour. The household is endowed with 1 unit of time that can be used for leisure and labour. Output can be used either for private consumption or public consumption. The resource constraint is

$$c_t + g_t = 1 - l_t \tag{2.1}$$

where c_t, l_t and g_t denote respectively private consumption, leisure, and public consumption.

The problem of the household is to maximise his lifetime discounted expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \tag{2.2}$$

subject to the period-by-period budget constraint

$$b_{t-1}(g_t) + (1-\tau_t)(1-l_t) = c_t + \sum_{g^{t+1}|g^t} b_t(g_{t+1}) p_t^b(g_{t+1}) \tag{2.3}$$

where β is the discount factor, τ_t is the state-contingent labour tax rate and $b_t(g_{t+1})$ denotes the amount of bonds issued at time t contingent on period $t+1$ government shock at the price $p_t^b(g_{t+1})$. $vb_t \equiv -\sum_{g^{t+1}|g^t} b_t(g_{t+1}) p_t^b(g_{t+1})$ is defined as the value of government debt.

The household's optimality condition are

$$1 - \tau_t = \frac{u_{l,t}}{u_{c,t}} \tag{2.4}$$

$$p_t^b(g_{t+1}) = \beta \frac{u_{c,t+1}(g_{t+1})}{u_{c,t}} \pi(g^{t+1}|g^t) \tag{2.5}$$

together with the budget constraint 2.3.

The government pursues an optimal taxation approach: given an initial amount of inherited debt, b_{-1}^g, she chooses the sequence of tax rates and state-contingent bonds to maximise consumer's welfare. The solution to this dynamic optimal taxation problem is called a Ramsey plan. Lucas and Stokey (1983) show that under complete markets and rational expectations the Ramsey plan has to satisfy the following restriction

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1-l_t)) = u_{c,0} b_{-1} \tag{2.6}$$

which can be thought of as the intertemporal consumer budget constraint with both prices and taxes replaced by the households' optimality conditions, (2.4) and (2.5). Constraint (2.6) is the implementability condition. The Ramsey plan satisfies

$$\tau_t = T(g_t, b_{-1}^g) \forall t > 0 \tag{2.7}$$

$$b_t^g(g_{t+1} = \bar{g}) = D(\bar{g}, b_{-1}^g)\forall t > 0 \tag{2.8}$$

$$v b_t^g = V(g_t, b_{-1}^g)\forall t > 0 \tag{2.9}$$

The allocation is a time invariant function of the only state variable in this model, g_t. The initial holding of government bonds matters for the allocation because it determines the value of the Lagrange multiplier attached to the implementability condition. The state-contingent bond holding is a time invariant function and does not depend on the current state of the economy, and the market value of debt is influenced by the current shock only through variations in the state-contingent interest rates.

b The government behaves as in Lucas and Stokey (1983) but agents are boundedly rational

I assume now that agents are learning fiscal policy: they have perfect knowledge about their own decision problem, in the sense that they correctly understand their own objective function and constraints, but they do not know the problem that the other agents in the economy, including the government, have to solve.

The representative agent's problem is to maximise the lifetime utility

$$\tilde{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to the flow budget constraint equation (2.3). $\tilde{E}_0$ denotes the agent's subjective expectations.

Given the tax rate, equation (2.4) gives the combination of consumption and leisure. It is still open how much the agent consumes and saves. Given the price of the state-contingent bond, the inter-temporal optimality condition

$$p_t^b(g_{t+1}) = \beta \frac{\tilde{u}_{c,t+1}(g_{t+1})}{u_{c,t}} \pi(g^{t+1}|g^t) \tag{2.10}$$

.

dictates that the optimal consumption choice depends on the forecast of next-period marginal utility of consumption. Equation (2.10) looks very similar to equation (2.5), with the only difference that now $\tilde{u}_{c,t+1}(g_{t+1})$ is the non-rational expectation, conditional on the information up to time t, about next-period state-contingent marginal utility of consumption.

For simplicity, and to be consistent with the analysis carried out in the rest of the chapter, I restrict the government expenditure shock to follow a 3-state Markov chain with the following transition probabilities

$$P = \begin{pmatrix} \pi_{L,L} & \pi_{L,M} & \pi_{L,H} \\ \pi_{M,L} & \pi_{M,M} & \pi_{M,H} \\ \pi_{H,L} & \pi_{H,M} & \pi_{H,H} \end{pmatrix}$$

$\pi_{i,j}$ is the probability of moving from state i to state j in one period, for $i = L, M, H$ and $j = L, M, H$.

Given the law of motion for the shock, each period agents have to forecast three values, one for each realisation of the shock. Let $\gamma_t^i \equiv \tilde{u}_{c,t+1}(g_{t+1} = g^i)$ for $i = L, M, H$. Beliefs evolve over time according to the following scheme:

$$\gamma_t^i = \begin{cases} \gamma_{t-1}^i + \alpha_t(u_{c,t}(g_t = g_i) - \gamma_{t-1}^i), & \text{if } g_t = g_i \\ \gamma_{t-1}^i, & \text{if } g_t = g_j \end{cases} \tag{2.11}$$

with $i = L, M, H$. α_t represents the weight of the forecasting errors when updating the estimates. In this section we consider two standard specifications for the gain α_t, namely $\alpha_t = \frac{1}{t}$ and $\alpha_t = \alpha$.

I assume that the government implements the policy sequences $\{b_{t+j}(g_{t+j+1})\}_{j=0}^{\infty}$ coming out of (2.8). The time-line of the events is the following. At the beginning of period t, agents observe the realisation of the shock g_t and the tax rate set by the government at t: using all the information up to $t-1$, they form their expectations about next-period state-contingent marginal utility of consumption and decide how much to consume.

The question is whether, given a sufficient amount of data, this equilibrium converges to the rational expectations equilibrium. The criterion adopted to judge convergence under a recursive least-squares algorithm is the expectational stability of rational expectations equilibrium, called E-stability by Evans and Honkapohja (2001). The following proposition holds:

Proposition 6 *Suppose that the government does not make the fiscal plan contingent on agents' beliefs and follows the bond fiscal rule which is optimal under rational expectations. Assume that agents' utility is separable into consumption and leisure, logarithmic in consumption and linear in leisure. Assume moreover that the initial bond holding is zero. Then the rational expectations equilibrium is learnable.*

Proof. We relegate the proof to the appendix. ■

Even when the rational expectation equilibrium is learnable, along the transition path, the fluctuations in the allocation can be quite large with respect to those under rational expectations. Conditioning on the government implementing the rational expectations bond policy rule, we compute the standard deviation of consumption and leisure when agents learn and when they have rational expectations. To do this we simulate the model.

We assume the utility function

$$u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + log(l_t) \tag{2.12}$$

and set $\beta = 0.95$, $g_L = 0$, $g_M = 0.1$, $g_H = 0.2$, $\pi_{i,i} = 0.94$, $\pi_{i,j} = 0.03\ \forall i, j = L, M, H$.

For this parametrisation, the ratio of the standard deviation of consumption when agents are learning to the one when agents have rational expectations is equal to 4.5. This value is averaged across 50000 simulations. The intuition for this result hinges on the implication of tax smoothing in terms of bond portfolio management. To smooth taxes over time and across states the government holds sizable state-contingent bond positions. This policy generates large wealth variation that in turns magnifies the effect of distorted beliefs. The more correlated the shock, the larger the state-contingent bond holdings, and the more distorted beliefs reflect into consumption volatility.

The main implication is that "teaching a lesson", in the sense of imposing the optimal rational expectations policy, is not a good recipe for welfare maximisation. Implementing a fiscal policy plan without taking into account the way in which not fully rational agents form their expectations induces a stream of consumption and leisure that is much more volatile than if agents formed model-consistent expectations. Making the policy plan contingent on agents' beliefs is superior, in terms of welfare, than obstinately performing the optimal fiscal policy under the rational expectations paradigm.

The analysis also raises some doubts about recent developments in the debt management literature, according to which a government should hold extreme debt positions. The first reason to hold such positions relates to the possibility of recovering the complete markets outcome; almost every equilibrium allocation under complete markets can be replicated through an appropriate position in non-contingent bonds at different maturities.[5] Since the welfare is higher under complete markets than under incomplete markets, the optimal debt management is the one that allows the complete markets outcome to be reached. Due to the low variability of next-period bond prices across realisations of shock today, Angeletos (2002) and Buera and Nicolini (2004) show that the government can implement the complete markets allocation holding very extreme positions in bonds with different maturities.

The second reason for debt volatility is to make the full commitment solution time consistent. Persson, Persson and L. (1987) consider a model with nominal frictions

[5]Under incomplete markets with only one period bonds, it is not possible to achieve the complete market solution because the implementability condition is replaced with an infinite sequence of period-by-period budget constraints. Aiyagari et al. (2002) for example show that in case of i.i.d. shock, implementing the complete market solution, which generates i.i.d. primary surplus, would imply an explosive path for debt.

where the government in charge has an incentive to engage in a surprise inflation to erode the nominal inherited debt. The unique maturity structure of debt implementing the full commitment solution requires the government in t to leave to its successor a positive amount of nominal bond holdings.

2.3 Optimal Fiscal Policy When Agents Are Learning

The previous section shows that the optimal fiscal policy under rational expectations can lead to very bad outcomes under learning. This suggests that learning should be incorporated in the design of optimal fiscal policy: in this section we study the problem of a government which internalises the fact that agents do not have rational expectations.

The households' optimality condition, which we repeat for convenience, are

$$\frac{u_{l,t}}{u_{c,t}} = 1 - \tau_t \tag{2.13}$$

$$p_t^b(g_{t+1}) = \beta \frac{\tilde{u}_{c,t+1}(g_{t+1})}{u_{c,t}} \pi(g^{t+1}|g^t) \tag{2.14}$$

The implementation of equation (2.14) requires agents to forecast their own state-contingent consumption one-period-ahead. This approach of modeling boundedly rational behaviour may seem strange at first glance, but it is commonly used in the learning literature (see Evans, Honkapohja and K. (2003), Carceles-Poveda and Giannitsarou (2007), Milani (2007) among many others). It is a very useful short-cut to model households' lack of knowledge about market determined variables, which are outside of agents' control although they are relevant to their decision problem. In the current setup, non-rational expectations about future consumption can be interpreted as non rational expectations about the tax policy rule followed by the government. In fact, considering the next-period equivalent of equation (2.13), agents understand that consumption at $t+1$ depends on the tax rate the government will set at $t+1$; as far as expectations about tax rate are not-model consistent, expectations about consumption are neither.

We simplify even further the analysis in section b assuming that the government expenditure shock can take only two realisations, g_H and g_L, with $g_H¿g_L$. Let $\gamma_t^i \equiv \tilde{u}_{c,t+1}(g_{t+1} = g^i)$ for $i = H, L$. As before, agents update their beliefs according to the following scheme

$$\gamma_t^i = \begin{cases} \gamma_{t-1}^i + \alpha_t(u_{c,t}(g_t = g_i) - \gamma_{t-1}^i), & \text{if } g_t = g_i \\ \gamma_{t-1}^i, & \text{if } g_t = g_j \end{cases} \tag{2.15}$$

with $i = H, L$ and where α_t follows an exogenous law of motion.[6]

[6] In Appendix B.10 we discuss a measure of the 'quality' of this learning scheme.

Definition 4 *A competitive equilibrium with boundedly rational agents is an allocation* $\{c_t, l_t, g_t\}_{t=0}^{\infty}$, *state-contingent beliefs about one-step-ahead marginal utility of consumption* $\{\gamma_t^i\}_{t=0}^{\infty}$ *for* $i = H, L$, *a price system* $\{p_t^b\}_{t=0}^{\infty}$ *and a government policy* $\{g_t, \tau_t, b_t\}_{t=0}^{\infty}$ *such that (a) given the price system, the beliefs and the government policy the households' optimality conditions are satisfied; (b) given the allocation and the price system the government policy satisfies the sequence of government budget constraint* (2.3)*; and (c) the goods and the bond markets clear.*

Let

$$x_t = [\gamma_t^H I(g_{t+1} = g_H) + \gamma_t^L I(g_{t+1} = g_L)] \tag{2.16}$$

where I is the indicator function and define

$$A_t \equiv \prod_{k=0}^{t} \frac{x_{k-1}}{u_{c,k}} \tag{2.17}$$

Taking logs to both sides we get

$$logA_t = \sum_{k=0}^{t} (log(x_{k-1}) - log(u_{c,k})) \tag{2.18}$$

The log of A_t is the sum of the log-differences between expected and actual marginal utility of consumption from period 0 to period t. This variable has a very natural interpretation as the sum of all past forecast errors agents have made up to period t in predicting next-period log consumption. Under rational expectations, this variable is constant and equal to 1, while under learning it is not, unless the initial beliefs coincide with the rational expectations ones.

Using households' optimality conditions to substitute out prices and taxes from the government budget constraint, Lucas and Stokey (1983) show that under complete markets and rational expectations the competitive equilibrium imposes one single intertemporal constraint on allocations. Using a similar argument, we show that under complete markets and bounded rationality the following result holds.

Proposition 7 *Assume that for any competitive equilibrium* $\beta^t A_t u_{c,t} \to 0$ *a.s.*[7] *Given* b_{-1}, γ_{-1}^H, γ_{-1}^L, *a feasible allocation* $\{c_t, l_t, g_t\}_{t=0}^{\infty}$ *is a competitive equilibrium if and only if the following constraint is satisfied*

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = A_0 u_{c,0} b_{-1} \tag{2.19}$$

with initial condition $A_{-1} = 1$

[7]Using the results of Proposition 9 we show that this is actually the case.

Proof. We relegate the proof to the appendix. ■

Equation (2.19) is the bounded rationality version of the intertemporal constraint on the allocation derived by Lucas and Stokey (1983) in a rational expectations framework. The difference between equations (2.19) and (2.6) arises through the effect that out-of-equilibria expectations exert on state-contingent prices. As expectations are not model-consistent, the primary surplus at time t, expressed in terms of marginal utility of consumption, is weighted by the product of ratios of expected to actual marginal utility from period 0 till period t.

a The government problem

Using the *primal approach* to taxation we recast the problem of choosing taxes and state-contingent bonds as a problem of choosing allocations maximising households' welfare over competitive equilibria. At this point a clarification is needed. When the households and the benevolent government share the same information, they maximise the same objective function. But when the way in which they form their expectations differ, as in this setup, their objective functions differ as well. Therefore it is no longer obvious which objective function the government should maximise. In what follows we assume that she maximises the representative consumer's welfare *as if* he were rational. Two reasons justify this assumption. First, as agents form model-consistent expectations in the long-run, in the long-run agents are going to be rational. Second, the government understands how agents behave and form their beliefs, and it understands that these beliefs are distorted. Consequently, it uses this information to give the allocation which is best for them from an objective point of view. This is consistent with a paternalistic vision of the government.[8]

Definition 5 *The government problem under learning is*

$$\max_{\{c_t,l_t,\gamma_t^H,\gamma_t^L,A_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to

$$E_0 \sum_{t=0}^{\infty} \beta^t A_t (u_{c,t} c_t - u_{l,t}(1-l_t)) = A_0 u_{c,0} b_{-1} \tag{2.20}$$

$$A_t = A_{t-1} \frac{\gamma_{t-1}^H I(g_t = g^H) + \gamma_{t-1}^L I(g_t = g^L)}{u_{c,t}} \tag{2.21}$$

$$\gamma_t^i = \begin{cases} \gamma_{t-1}^i + \alpha_t (u_{c,t}(g_t = g_i) - \gamma_{t-1}^i), & \text{if } g_t = g_i \\ \gamma_{t-1}^i, & \text{if } g_t = g_j \end{cases} \tag{2.22}$$

$$c_t + g_t = 1 - l_t \tag{2.23}$$

[8]The same assumption is made in Karantounias et al. (2007).

Equation (2.20) constraints the allocation to be chosen among competitive equilibria. Equation (2.21) is the recursive formulation for A_t, obtained directly from equation (2.17). Equation (2.22) gives the law of motion of beliefs. Equation (2.23) is the resource constraint. Since A_t and γ_t^i for $i = L, H$ have a recursive structure, the problem becomes recursive adding A_{t-1} and γ_{t-1}^i for $i = L, H$ as state variables.

Leaving the details about the derivation in appendix B.6, first order necessary conditions[9] with respect to consumption and leisure impose that

$$\begin{aligned} & u_{c,t} + \Delta A_t(u_{cc,t}c_t + u_{c,t}) - \lambda_{1,t}\alpha_t u_{cc,t} I(g_t = g^H) \\ & - \lambda_{2,t}\alpha_t u_{cc,t} I(g_t = g^L) - \Delta \frac{u_{cc,t}}{u_{c,t}} E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1 - l_{t+j})) = \lambda_{3,t} \end{aligned} \tag{2.24}$$

$$u_{l,t} + \Delta A_t(u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda_{3,t} \tag{2.25}$$

The first term on the left side of equation (2.24) represents the benefit for the government from increasing consumption by one unit. The second one measures the impact of the implementability constraint on the allocation, weighted by the distortion A_t represented by non rational expectations. The third and fourth terms reflect the fact that the government takes into account how agents update their expectations on the basis of the current consumption. The last term on the left represents the derivative of all future expected discounted primary surpluses with respect to current consumption. This is because from equation (2.19) each primary surplus (in terms of marginal utility) at $t + j, \forall j \geq 0$ is pre-multiplied by the product of past ratios of expected to actual marginal utility. In choosing optimal consumption today, the government is implicitly choosing the factor at which all future primary surpluses are discounted through its effect on A_t. The term on the right is the shadow value of output. A similar interpretation holds for the optimality condition with respect to leisure, equation (2.25).

Several comments are necessary. First, the optimal allocation is history-dependent through the presence of A_{t-1}: differently from Lucas and Stokey (1983), the allocation is not any more a time-invariant function of the current realisation of the government shock only, but depends on what happened in the past. Second, in appendix B.7 we show that the optimality conditions in a complete markets and rational expectations framework are a special case of equation (2.24) and (2.25). Third, using the recursive formulation of A_t, the intertemporal budget constraint at t

$$b_{t-1}A_t u_{c,t} = E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1 - l_{t+j}))$$

[9]As standard in the optimal fiscal policy literature, it is not easy to establish that the feasible set of the Ramsey problem is convex. To overcome this problem in our numerical calculations we check that the solution to the first-order necessary conditions of the Lagrangian is unique.

and combining (2.24) and (2.25) the optimal allocation satisfies the following equations:

$$u_{c,t} + \Delta A_t(u_{cc,t}(c_t - b_{t-1}) + u_{c,t}) - \lambda_{1,t}\alpha_t u_{cc,t} I(g_t = g^H) - \lambda_{2,t}\alpha_t u_{cc,t} I(g_t = g^L) = u_{l,t} + \Delta A_t(u_{l,t} - u_{ll,t}(1 - l_t)) \tag{2.26}$$

Equation (2.26) looks very similar to the first-order condition with respect to consumption in the incomplete markets model of Aiyagari et al. (2002). In facts in both frameworks the excess burden of taxation is not constant, although for different reasons. In the absence of a full set of state-contingent bonds, as in Aiyagari et al. (2002), the excess burden of taxation is time-varying because of the incomplete insurance offered by the financial bonds: since the interest payment on last period debt is fixed across realisations of the current government shock, the government in each period has to adjust the stream of all future taxes to ensure solvency.[10] In a complete markets model with learning, what makes the excess burden of taxation time-varying is the cost of issuing state-contingent debt. Although market completeness implies that in each period the government can fully insure against expenditure shocks, the state contingent interest rates change as time goes by because agents' expectations change. When agents stop updating their beliefs because the forecast error is zero, $A_{t+j} = A_{t-1} \forall j \geq 0$, and the excess burden of taxation becomes constant again.

Equation (2.24) expresses the actual marginal utility of consumption as a function of agents' beliefs about it. Figure 2.1 shows this relation for a log-log utility function and a given value of A_{t-1}.[11] The left panel displays the actual marginal utility of consumption contingent on the government expenditure shock being low (average with respect to the expected marginal utility of consumption contingent on the government expenditure shock being high), as a function of the previous period belief, γ_{t-1}^L. The right panel display the same for the government expenditure shock being high. Figures 2.2 and 2.3 show the tax rate and the state-contingent bond policy functions which guarantee that the convergence between actual and expected marginal utility holds. The tax rate is a decreasing function of the previous period expected marginal utility; symmetrically, the state-contingent bond is an increasing function of it.

2.4 Some examples

In order to characterise the optimal fiscal policy in the framework we are studying, in what follows we consider some examples restricting the government expenditure shock to a specific form.

[10] In a complete market framework with rational expectations the excess burden of taxation is constant because the variable which adjusts to ensure solvency is the pay-off of the portfolio of contingent bonds.

[11] The shape of the mapping is robust to different values of this variable.

a Constant government expenditure

Consider the case in which the government expenditure is known to be constant and the initial amount of bond holdings is zero. The Lagrangian associated to the government problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [u(c_t, l_t) + \Delta A_t(u_{c,t}c_t - u_{l,t}(1-l_t)) \\ + \lambda_{1,t}(\gamma_t - (1-\alpha_t)\gamma_{t-1} - \alpha_t u_{c,t}) + \lambda_{2,t}(1 - l_t - c_t - g)] - \Delta A_0 u_{c,0} b_{-1} \tag{2.27}$$

where the notation is the same as before and $x_t = \gamma_t$.

The optimality conditions $\forall t \geq 0$ are:

$$u_{c,t} + \Delta A_t(u_{cc,t}c_t + u_{c,t}) - \lambda_{1,t}\alpha_t u_{cc,t} - \\ \Delta \frac{u_{cc,t}}{u_{c,t}} \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1-l_{t+j})) = u_{l,t} + \Delta A_t(u_{l,t} - u_{ll,t}(1-l_t)) \tag{2.28}$$

$$\lambda_{1,t} - \beta(1-\alpha_t)\lambda_{1,t+1} + \beta \Delta b_t A_t = 0$$

Equation (2.28) gives the mapping T between agents' beliefs about (marginal utility of) consumption and actual (marginal utility of) consumption. In the next proposition we characterise the properties of this mapping.

Proposition 8 *Assume the utility function*

$$u(c_t, l_t) = log c_t + l_t \tag{2.29}$$

and that the gain α_t is small.

Then, in the set $\gamma_{t-1} > 0$ the mapping T: $R_+ \rightarrow R_+$ has the following properties:

- *T is increasing and concave.*
- *T has one fixed point.*
- *The least squares learning converges to it.*

Proof. We relegate the proof to the appendix. ■

Proposition 8 shows that the expected marginal utility converges to actual one, so that in the long-run agents' expectations are model-consistent and the forecast error is zero. The next proposition characterises the value towards which expectations converge.

Proposition 9 *Given an initial value for the government bond holding b_{-1} the allocation under learning does not converge to the allocation under rational expectations implied by the same initial bond holding. However, for any initial belief held by agents, there exists a b_{-1} such that*

$$lim_{t\to\infty} c_t^L(\gamma_{-1}) = c_t^{RE}(b_{-1}) \tag{2.30}$$

Proof. We relegate the proof to the appendix. ■

For any initial belief about the marginal utility of consumption, there is always an initial level of government wealth such that the allocation under learning converges to the one under rational expectations starting with that initial government wealth. Figure 2.4 shows this relation assuming that in equation (2.27) $b_{-1} = 0$. Given the parameters values used, the solution of the Ramsey problem under rational expectations implies that the marginal utility of consumption is constant and equal to 2.5. For all values of initial belief higher (lower) than this reference value, the learning allocation coincides with the solution of a Ramsey problem in an economy populated by rational agents and endowed with a positive (negative) initial government debt.

Policy implications

The example of constant government consumption highlights the impact of expectations on the optimal fiscal plan. Under rational expectations, the only distortion is the one associated to taxes. In order to smooth this distortion over time, taxes are set to balance the government budget every period. In this way agents can enjoy a perfectly constant allocation. By contrast under learning, there are two distortions in the economy, one associated with taxes and the other one associated with agents' expectations. Therefore, although the government could follow a balanced-budget rule, it decides not to do it because in this way it would not minimise the *overall* distortions. To influence out-of-equilibria expectations the government animates initially pessimistic agents setting a low tax rate at the beginning and financing the public consumption with debt. As time goes by, the tax rate has to increase in order to ensure government solvency.[12]. This stabilization policy is resistant to a selection of robustness checks. For example, it holds if 1) we suppose that agents use lagged value of marginal utility of consumption to update their current beliefs, 2) the government has access to consumption taxes instead of labour ones.

Figures 2.5, 2.6 and 2.7 offer a graphical interpretation of the result. The solid lines show the optimal fiscal plan under rational expectations. Whereas the dashed lines show the optimal fiscal plan when agents adopt a constant gain algorithm to update

[12]The analysis is symmetric for the case of initially optimistic agents

their belief while supposing that in the initial period the expected consumption is lower than the actual consumption prevailing at $t = 0$.[13]

b A single big shock

Consider the case in which expenditure is constant in all periods apart from T, when $g_T > g_t$. Both the government and households know the entire path of the expenditure, so that the shock in T is perfectly anticipated. Under rational expectations the government runs a positive primary surplus from period 0 to $T-1$, using it to buy bonds. At T the government finances the high public consumption level by selling the accumulated assets and possibly by levying a tax rate on labour income. From period $T+1$ onwards the tax rate is just sufficient to cover the expenditure and to service the interest on the bonds issued at T. By contrast, in an economy populated by pessimistic agents, the government can accumulate less assets because it has to stimulate the economy to manipulate expectations. The big shock at T is financed by increasing debt much more than under rational expectations. Figures 2.8-2.10 illustrate the optimal plan under rational expectations (the solid lines) and under learning (the dashed lines), assuming $T = 10$, $g_t = 0.1$ and $g_T = 0.2$.

Policy implications

The example of a perfectly anticipated war is useful for two reasons. First, it clarifies how the tax smoothing result is altered by the presence of boundedly rational agents. Under rational expectations the government spreads over time the cost of financing the war in T thorough distortionary taxes. As a result, the tax rate is perfectly constant in all periods before and after the war: taxes are smooth in the sense that they have a smaller variance than a balanced-budget rule would imply. By contrast, when agents are learning, they do not trust the promises made by the fiscal authority in terms of future consumption. The government uses taxes and debt to correct agents' distorted expectations, in a way that the tax rate is more volatile than under rational expectations.

The example is relevant also because it reconciles the complete markets framework with the empirical evidence that during peacetime periods countries run a primary deficit. The Lucas and Stokey (1983) model is unable to fit this evidence, as the government runs a primary surplus to accumulate assets before the war.

[13] Since for optimistic agents the evolution of the system is symmetric, we do not report it.

c Cyclical shocks

Suppose that

$$\begin{aligned} g_{t'} &= g_H \quad \forall t' = j \times H \leq \overline{T} \\ g_t &= g_L \quad otherwise \end{aligned}$$

with $j = 1, 2, ..., \frac{\overline{T}}{H}$. H is the length of time over two subsequent bad shocks and $\overline{T}$ is the last period in which a bad shock can occur. The rational expectations policy recipe is the same as before: the tax rate is constant in all periods when $g_t = g_L$ and increases very little when the bad shock hits the economy, due to the assets the government accumulates during the good shock periods. Under learning with pessimistic agents, before the first realisation of the bad shock the tax rate is lower than under RE and increases between any two subsequent bad shocks, generating resources devoted to reducing debt, which increases whenever the bad shock occurs. After the last bad realisation of the shock, the tax rate falls and then gradually increases over time to ensure intertemporal solvency.

d Bad shock of unknown duration

Suppose that the shock can take two realisations, g_L and g_H, with the following transition probabilities matrix

$$P = \begin{pmatrix} 1 & 0 \\ \pi_{H,L} & \pi_{H,H} \end{pmatrix}$$

where $\pi_{i,j}$ is the probability that tomorrow the shock is in state j, being today in state i, with $g_t = g_H$ at $t = 0$. This example corresponds to an absorbing Markov chain, where the low realisation of the shock is the absorbing state and the high one is the transient state. Under rational expectations, the government finances the bad shocks partly through taxes and partly by issuing debt. Numerical results, not reported here, confirm the role of fiscal policy as stabiliser of expectations: the accumulation of public debt is higher and longer under learning than under rational expectations, the difference being due to the opportunity of inducing the agents to revise their expectations downwards.

e Serially correlated shock

Suppose that the shock can take two realisations, g_L and g_H, with the following transition probabilities matrix

$$P = \begin{pmatrix} \pi_{L,L} & \pi_{L,H} \\ \pi_{H,L} & \pi_{H,H} \end{pmatrix}$$

We set $g_L = 0.05$,$g_H = 0.1$ and $\pi_{H,H} = \pi_{L,L} = 0.8$.[14] As in the previous examples, we assume a discount factor equal to 0.95 and a gain parameter equal to 0.02.[15]

Table 2.2 summarises some statistics for the allocation and the fiscal variables under rational expectations. Table 2.3 summarises the same statistics under learning after convergence of beliefs for initially pessimistic and optimistic agents. Reported values are average across 1000 simulations. Comparing the two tables we can observe that in the long run initially pessimistic (optimistic) agents consume less (more) than if they had been rational. This result is in line with the examples in sections a and b. Since at the beginning consumers were pessimistic, the accumulation of debt necessary to induce them to revise upwards their expectations about consumption requires higher taxes in the long run than under rational expectations. Because of this, consumption is lower and leisure is higher (than under rational expectations). The average primary surplus is higher as well. In this sense we can say that beliefs are self-fulfilling in the long run: the lower is the initial expected consumption, the lower the actual consumption after convergence. Exactly the opposite is true with initially optimistic agents.

Table 2.4 shows the same statistics for the system during the first 30 periods of transition, under rational expectations and initially pessimistic agents. Although all the endogenous variables are more volatile before convergence than after convergence, the market value of government debt and the labour tax rate are the most volatile. For example, the tax rate volatility before convergence is double that after convergence. This is due to the fact that the government implements an expectation-dependent fiscal plan. When beliefs are distorted, fiscal variables react to correct this distortion. As time passes and agents' expectations become model-consistent, the government stops using fiscal variables to influence distorted beliefs.

2.5 Policy Implications

The analysis in section 2.4 has characterized the optimal fiscal plan that a benevolent government should implement when agents are learning. With respect to the rational expectations framework, bounded rationality introduces a new distortion in the economy. The government takes into account the way in which agents form their expectations and realises that these expectations are distorted. The optimal fiscal plan minimises the distortions associated with both taxes and expectations. Stabilising out-of-equilibria expectations requires setting low taxes when agents are initially pessimistic

[14]For the case of i.i.d shock case the results are very similar to those with a serially correlated shock, and therefore they are not reported.

[15]The choice of the updating parameter is not easy because it requires a trade-off between filtering noises and tracking structural changes. Milani (2007) estimates a New-Keynesian model and finds that the best fitting specification has a gain coefficient in a range between 0.015 and 0.03. Orphanides and Williams (2004) find that a value for k in the range $0.01 - 0.04$ fits the expectations data from the Survey of Professional Forecasters better than using higher or lower values. Evans et al. (2007) also use the same value for k.

and high ones when they are initially optimistic. This has a cost and a benefit. The cost is represented by the fact that taxes are less smooth, as the examples in sections a and b clarify. But the benefit is that under the expectation-dependent fiscal plan agents learn the tax rate policy rule much faster than under the rational expectations optimal fiscal plan. Figure 2.11 illustrates this point graphically. The solid line shows the next-period marginal utility of consumption forecast error made by the agents when the government follows the rational expectations recipe compared to the case when she implements the optimal policy under learning, represented by the dashed line. Agents are initially pessimistic and the time period is one year. The lower (than RE) tax rate set at the beginning following the optimal fiscal policy plan induces agents to correct their pessimism much faster than if the fiscal policy suggested by Lucas and Stokey (1983), which is the optimal one under rational expectations, were followed.

The way in which the government should use fiscal variables to manipulate agents' distorted expectations in some sense resembles a standard Keynesian-inspired stabilisation policy. However, it is important to stress that the government should not stimulate economic activity indiscriminately. Actually it is very important to implement the right policy at the right moment. In what follows we show that an expansionary fiscal policy, implemented when agents' expectations require a restrictive one, generates a sub-optimal volatility in the system.

Suppose for simplicity that the public consumption shock is constant and that the government wants to animate the economy when agents are optimistic. To this aim, it implements the following tax-rate rule

$$\tau_t = \begin{cases} \tau_t^{pess}, & \forall t \leq T \\ \xi_t \tau_t^{pess} + (1-\xi_t)\tau_t^{bb}. & \forall t > T \end{cases} \tag{2.31}$$

According to equation (2.31) the fiscal authority stimulates the economy till period T setting the tax rate at the (low) optimal level when agents are pessimistic, τ_t^{pess}, and that from T onwards sets the tax rate as a weighted average between this value and the one which raises enough revenues to pay-back both the interests on the inherited debt and the current government expenditure shock, τ_t^{bb}. The weight is given by $\xi_t = k^{t-T}$, with $0 < k < 1$. In order to ensure that the transversality condition is not violated, it is necessary to impose the restriction that the weight ξ_t goes to zero quickly. Otherwise the revenues raised through distortionary taxes would not be enough to finance the interests on the debt that the government has accumulated. Therefore, in order to rule-out Ponzi-schemes, the parameter k is set small enough to ensure that the fiscal plan which belongs to the class of the feasible ones.

The dashed lines in figure 2.14 show the optimal tax rate and bond holdings when agents are optimistic and the government implements the fiscal policy taking into account that they are optimistic. Whereas the solid lines show the same variables when agents are optimistic and the government implements the rule given by equation 2.31.

The dashed lines in figure 2.15 show the consumption, leisure and forecast error when agents are optimistic and the government implements the optimal fiscal plan conditioning on this, while the solid lines show the same variables under the wrong stimulus.

Two observations are worth noting. First, the allocation is more volatile under the wrong stimulus, and consequently households' welfare is lower. In order to quantify these losses we utilise numerical methods. We assume the utility function:

$$log(c_t) + log(l_t) \tag{2.32}$$

and set $\beta = 0.95$, $g = 0.1$, $T = 24$ and $k = 0.7$. Given this parametrisation, the welfare losses in terms of consumption-equivalence units of animating the economy when the optimum requires depressing it is equal to 0.2 percent. Increasing T and/or k increases the welfare losses. Using the same parameters as before but with $T = 25$, the welfare losses become equal to 0.25; similarly with $k = 0.78$ the welfare losses is equal to 0.27. The intuition is that the higher is T and/or the higher is k, the more the fiscal policy is expansionary instead of being restrictive, as it should be since agents are optimistic.

The sub-optimally higher volatility generated by an unjustifiable government's desire to stimulate the economy translates into a longer time for agents to learn. While under the optimal fiscal plan the forecast error is zero after 50 years, under the wrong stimulus it fluctuates much more and it is still not zero after 300 years. In conclusion, setting a low tax rate to stimulate the economy when the opposite is required inefficiently induces instability into the system. While a government can accumulate debt even when it is not necessary, a responsible government will only accumulate it when necessary.

2.6 Endogenous Government Spending

Up to now we have considered public consumption as a completely exogenous shock. This assumption seems quite restrictive, as governments can decide how much to spend. In order to add some realism to the analysis, in this section we consider the same model as in subsection a but we allow the government to choose the amount of public spending. We find that this extension corroborates the policy implications outlined in section 2.4.

We assume the utility function

$$u(c_t, l_t, g_t) = log(c_t) + log(l_t) + \alpha log(g_t)$$

with $\alpha < 1$. The Lagrangian associated to the government problem is

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [&u(c_t, l_t, g_t) + \Delta A_t (u_{c,t} c_t - u_{l,t}(1 - l_t)) \\ &+ \lambda_{1,t}(\gamma_t - (1 - \alpha_t)\gamma_{t-1} - \alpha_t u_{c,t}) + \lambda_{2,t}(1 - l_t - c_t - g_t)] - \Delta A_0 u_{c,0} b_{-1}\end{aligned}$$

The dashed lines in figures 2.12 and 2.13 show the rational expectations equilibrium, whereas the solid lines show the learning optimal allocation when agents are pessimistic. In line with the previous results, at the beginning the government chooses an expansionary fiscal policy, setting higher public spending than under rational expectations and financing it mainly through debt.

2.7 Testing Complete Versus Incomplete Markets

In section 2.3 we have shown that the first order condition with respect to consumption in a complete markets model with learning looks very similar to the one in an incomplete markets model with rational expectations because in both cases, although for different reasons, the excess burden of taxation changes over time.

Assessing whether markets are complete or incomplete is not an obvious issue, since there are theoretical justifications in both directions: while transaction costs and limited commitment push in favour of market incompleteness, the possibility of replicating the complete markets equilibrium through a portfolio of bonds with different maturities favours market completeness. The tests proposed in the literature to discriminate between complete and incomplete markets (see inter alia Scott (2007), Marcet and Scott (2008), and Faraglia et al. (2006)) are based on two discriminating features between the two regimes:

1. Under complete markets fiscal variables (tax rate and market value of debt) inherit the serial properties of the underlying shocks hitting the economy, while under incomplete markets they have a unit-root component.

2. Under complete markets the market value of debt and the primary deficit co-move negatively, while under incomplete markets they co-move positively.

Tests based on the first feature are persistence tests, and those based on the second are impact tests. The aim of this section is to show that the tests belonging to the first category are not able to discriminate between an incomplete markets model and a complete markets model when agents have non-rational expectations: in particular we argue that these tests would be prone to accept the wrong hypothesis that markets are incomplete if, in fact, agents learn and markets are complete. The reason is simply that learning creates persistence in the system.[16]

[16] One way to compare persistence under learning and under rational expectations is to look at autoregressions of tax rates in the two frameworks when the shock is *i.i.d.*. Table 2.5 shows that while under rational expectations the coefficient on the lagged tax rate is close to zero and not statistically significant, under learning it is high and statistically significant.

We replicate persistence tests proposed in Scott (1999a, 2007) and Faraglia et al. (2007) to check market completeness. The first test is based on the presence of unit root in the labour tax rate. Assume that the government expenditure shock is stationary; under complete markets the labour tax rate is stationary, while under incomplete markets it contains a unit root. We simulate the model described in Section 2.3 and we apply the Augmented Dickey Fuller test to the tax rate using the first 50 periods of data. Out of 1000 simulations, the probability of accepting the unit-root test, and therefore of concluding (erroneously) that markets are incomplete, is equal to 0.999.

The second test is to estimate whether the excess burden of taxation has a unit root: under rational expectations and complete markets the excess burden of taxation is constant, while under rational expectations and incomplete markets it has a unit root, as shown in Aiyagari et al. (2002). Using the same sample period as before, the probability of accepting the unit-root test, and therefore of concluding (erroneously) that markets are incomplete, is equal to 0.922.

The third test in based on the fact that under complete markets the market value of debt and the primary deficit have the same persistence, while under incomplete markets the first is more persistent than the second. This result does not hold in a boundedly rationality framework. The top panel in Fig. 2.16 displays the persistence of the debt/GDP ratio, the primary surplus/GDP ratio and the government expenditure shock when agents have rational expectations and markets are complete. The bottom panel displays the same variables when agents are boundedly rational and markets are complete.[17]

Results in figure 2.16 would induce one to accept, once again erroneously, the hypothesis of market incompleteness.

However a model with complete markets and learning is *not* observationally equivalent to a model with incomplete markets and rational expectations. Actually impact tests, already used in the literature, are able to capture the differences between the two frameworks. Consider for example the co-movement between primary deficit and government debt. Independently of the way in which agents form their expectations, under complete markets this co-movement is negative, while under incomplete markets it is positive, and so is in data. Therefore, it seems important to consider a model with both incomplete markets and learning. We leave this for future research.

To conclude, in line with Marcet and Scott (2008) we find that looking at the behaviour of debt is a much more reliable way to test the bond market structure than looking at the behaviour of tax rate.

[17] To measure the persistence of a variable, say y, we use the k-variance ratio, defined as

$$P_y^k = \frac{Var(y_t - y_{t-k})}{kVar(y_t - y_{t-1})}$$

2.8 Debt Sustainability and Debt Limits

The literature has recently emphasised the opportunity of imposing limits on the amount of debt a government can accumulate.[18] In a context of non-rational agents the long run market value of debt depends on the initial beliefs held by agents: the higher the initial pessimism in the economy, the higher is the long-run level of debt. Since this debt accumulation is "good", in the sense that it allows for convergence between actual and expected marginal utility, there is not necessarily a correspondence between keeping the debt/GDP ratio low and optimal fiscal policy considerations. Moreover, debt limits may fail to discriminate between "good" and "bad" governments. Consider two countries, hit by the same realisation of the government expenditure shock which differ only as to the vector of initial beliefs. Figure 2.17 shows the probability that the debt limit (set equal to 60 per cent of steady state GDP) is binding for the two countries conditioning on the fact that each of the two governments implements the optimal fiscal plan taking as given the initial degree of pessimism. Since the first country is populated by less pessimistic agents than the second, the long-run debt level is lower in the first than in the second. But this does not mean that the government in the first country is more responsible than the one in the second just because it accumulated less debt. The only reason for the difference in the long-run debt level is that in the first country initial beliefs were less distorted than in the second, and the government had to intervene less to correct them.

The main advantage of debt constraints is that they are helpful in ensuring sustainability of fiscal policy.[19] Assessments of debt sustainability performed by international institutions are usually based on medium-term simulations (generally 5-10 years) of the debt/GDP ratio. A declining trend in debt/GDP ratio is interpreted as a signal that the government follows a sustainable fiscal policy, whereas an increasing one raises doubts about intertemporal solvency. In a model with boundedly rational agents assessing sustainability is particularly cumbersome, exactly because at the beginning government debt displays a trend.

Suppose that an agency wants to test for the presence of unit root in the debt/GDP ratio, in which case the fiscal policy plan is declared unsustainable. We show that actually this test can perform very poorly if the government follows the optimal fiscal policy plan when agents are learning.

Suppose that agents in the economy are pessimistic and that to cover the government expenditure shock the government at the beginning uses debt, which is financed through government revenues that are increasing over time. The agency is asked to evaluate the government solvency and to do that it applies an Augmented Dickey Fuller test on the

[18] In Chari and Kehoe (2004), debt constraints are beneficial if the monetary authority cannot commit to solve the time inconsistency problem of deflating the nominal debt issued by the fiscal authorities of the member states.

[19] A set of tests has been proposed by the literature to check sustainability, among others by Hamilton and Flavin (1986), Trehan and Walsh (1991) and Bohn (1998).

market value of debt using the first 50 periods of observations.

$$\frac{debt_t}{GDP_t} = \alpha + \beta_T^{OLS} \frac{debt_{t-1}}{GDP_{t-1}} + \gamma_T^{OLS} (\frac{debt_{t-1}}{GDP_{t-1}} - \frac{debt_{t-2}}{GDP_{t-2}}) + \epsilon_t \tag{2.33}$$

Over 1000 simulations, the probability that the agency would declare the fiscal plan to be unsustainable when instead it is sustainable by construction is equal to 0.697.[20] The reason for this is that debt is used to manipulate agents' expectations: since these are persistent, they impart persistence to debt as well. In other words, bounded rationality increases the lack of power of unit root tests. Suppose now that the agency applies the Augmented Dickey Fuller test to the following equation

$$\begin{aligned}(\frac{debt}{GDP})_t = \alpha^{OLS} + \beta_T^{OLS}(\frac{debt}{GDP})_{t-1} + \gamma_T^{OLS}(\frac{s}{GDP})_t + \delta_T^{OLS}\Delta(\frac{debt}{GDP})_{t-1} + \\ + \nu_T^{OLS}\Delta(\frac{s}{GDP})_{t-1} + \mu_T^{OLS}\Delta(\frac{s}{GDP})_{t-2} + \epsilon_t\end{aligned} \tag{2.34}$$

where we added two lagged difference terms of the primary surplus/GDP ratio to obtain white noise residuals. In this case the probability of getting the wrong answer of debt unsustainability is be equal to 0.163, much lower than before. We conclude that one way to disentangle between persistence and sustainability of debt is to consider the evolution of primary surpluses.

2.9 Some Stylized Facts

The idea that fiscal variables depend on agents' expectations is consistent with some stylized facts. Figure 2.18 shows the government deficit/GDP ratio (dashed line), the growth rate of output (dotted line) and the two-periods-lagged Consumer Sentiment Index (CCI), in deviation from its sample mean, (solid line) for the US economy. Data are quarterly and refer to the period 1960-2007. Periods of optimism can be identified as those in which the deviation of the CCI from its sample mean is positive. There is a strong negative correlation between the fiscal deficit/GDP ratio and the level of optimism: roughly, while the growth rate of the economy does not show any substantial variation, during phases of optimism the deficit/GDP ratio decreases, to increase during phases of pessimism.

In Europe as well a similar evidence holds. Table 2.1 shows for each country the correlation between the deficit/GDP ratio at t and the level of optimism at $t-j$, each column referring to a different j.[21] Data cover the period 1985-2006. Two observations

[20] The result is robust to a number of alternative specifications: adding lagged difference terms of the dependent variable the probability would be equal to 0.77; including a time trend would lower the probability to 0.652.

[21] The Consumer Confidence Index database for Europe can be freely downloaded from the website $http://ec.europa.eu/economy_finance/db_indicators/surveys9185_en.htm$

are important. First, Apart for Czech Republic, Hungary and Spain, the correlation between the current value of the primary deficit/GDP ratio and the j-periods-lagged Consumer Confidence Index is negative, implying that the more optimistic were agents' j periods ahead, the lower the current deficit/GDP ratio. Second, in general the correlation decreases with the time distance.

In this section we want to test our theory empirically. In particular, we want to check if, in the data, the government debt responds to agents' expectations in a way that when agents are pessimistic (optimistic) the government issues more (less) debt (and lowers (increases) distortionary taxes) to finance its expenditure, in order to induce agents to revise their expectations upwards (downwards). In the model outlined in section 2.3, debt at time t depends on the contemporaneous government expenditure shock, on agents' expectations formed at $t-1$ over the marginal utility of consumption at time t, and on the history of all past forecast errors up to $t-1$.

While data on public consumption are easily available, there is no direct series for the history of forecast errors. In order to construct this variable, we proceed in the following way. We use quarterly data contained in the Survey of Professional Forecasters to have the mean one quarter forecast of the Gross Domestic Product.[22] The forecast error in each quarter is defined as the difference between the actual Gross Domestic Product realised at the survey date and the Gross Domestic Product forecast for the same period, normalised by the actual value.[23] In order to work with stationary series, we use the total federal government debt/GDP ratio, and the government expenditure/GDP ratio. Government expenditure includes Federal Consumption Expenditures and Gross Investment. Data are quarterly and cover the period 1980 : 1-2008 : 1 for the U.S. economy.

We estimate the following equation

$$\Delta log(\frac{D}{Y})_t = \alpha_0 + \alpha_1 log(\frac{D}{Y})_{t-1} + \alpha_2 \gamma_t + \alpha_3 log(\frac{G}{Y})_t + \alpha_4 \nu_{t-1} + \alpha_5 log(A_{t-1}) + \epsilon_t \quad (2.35)$$

where γ_t is the growth rate of real GDP from period $t-1$ till t, $(\frac{D}{Y})_t$ and $(\frac{G}{Y})_t$ are the face value of the debt/GDP ratio and the government expenditure/GDP ratio respectively. ν_t is the forecast error and $A_t = \sum_{k=1980:1}^{t} \nu_k, \forall t = 1980 : 1, 2008 : 1$. By construction, a positive forecast error means that agents in the last period were in some sense pessimistic, since they expected that the economy would have performed worse than it actually did. In terms of the model above, $\nu_t > 0$ is equivalent to $\gamma_{t-1} > u_{c,t}$. Table 2.6 summarises the estimation results. Since the forecast errors are highly correlated, we use the Newey-West standard error estimation. α_1, α_2 and α_3 are statistically significant and their sign is in line with evidence of market incompleteness:

[22]Data can be freely downloaded from the webpage http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/

[23]Before 1992 agents had to forecast the Gross National Product and after 1992 the Gross Domestic Product. The forecast error has been computed taking this difference into account.

an higher government expenditure, or an higher inherited debt, increases the current debt. More importantly, α_4 and α_5 are statistically significant too, and their sign in coherent with the analysis carried over in the chapter. In particular, the higher the pessimism in $t-1$, or the more pessimistic have been agents in the past, the higher the debt in t.[24] This result seems to indicate that in U.S. fiscal authorities follow an expectation-dependent plan in a way similar to the one proposed in this chapter.

2.10 Conclusions

Lucas and Stokey (1983) model prescribes to set smooth taxes along the business cycle and to hold sizable state-contingent bond positions. This debt management generates large wealth variation that in turns magnifies the effect of distorted beliefs. As a consequence, small deviations from rational expectations translate into sub-optimal volatility of consumption. Reformulating the government problem to take into account how agents form their expectations leads to very different conclusions in terms of the optimal fiscal policy plan.

There are two main results. The first one is that the policymaker should manipulate agents' beliefs by setting low (high) taxes in a context of pessimism (optimism). This conclusion seems to be supported by the data, and in line with some recent suggestions to handle the up-to-date distress. Moreover, because of the role played by the initial beliefs, this model can account for the wide dispersion across countries in the level of government debt and tax rate, as far as one is willing to accept that different countries have different initial beliefs.

The second one is that the complete market solution under learning is history-dependent. This fact makes assessing market completeness more challenging, since unit-root test in the tax rate can mix evidence of non rational expectations with evidence of market incompleteness. In line with Marcet and Scott (2008) we find that looking at the behaviour of debt is a much more reliable way to test the bond market structure. Also gauging debt sustainability is more complicated because of the persistence induced by agents' expectations.

Several important issues are still open question. First, for simplicity we restrict the analysis to one-period state-contingent bonds. It may be interesting to extend the analysis to the incomplete markets framework where the government can issue bonds at different maturities. Second, we suppose that while agents do not know how aggregate variables are determined, the government has full information about the structure of the economy. Other authors (*inter-alia*, Primiceri (2005) and Cogley and Sargent (2005)) followed the opposite approach. It would be interesting to analyse the case in which neither the households know the government policy rules nor the government knows the

[24] The results are robust to the measure of the forecast error: similar conclusions can be drawn if the forecast error is defined as the difference between actual GDP and one-year ahead expected GDP.

households' response to these rules. Third, as in several papers on optimal taxation we abstract from monetary issues. On the other hand, the literature studying the impact of learning on the monetary policy design abstracts from fiscal policy considerations, such as distortionary taxes. A natural step would be to unify these two strands and to understand how the interaction of fiscal and monetary policy can help agents to form their expectations. We leave these issues to a future exercise.

Table 2.1: Correlation between deficit/GDP ratio and j-periods-lagged agents optimism. The first column refers to $j = 1$,the second to $j = 2$, and the third to $j = 3$

Country	$j = 1$	$j = 2$	$j = 3$
Belgium	-0.43	-0.37	-0.28
Czech Republic	0.05	0.04	-0.29
Denmark	-0.55	-0.49	-0.53
Germany	-0.07	-0.44	-0.19
Estonia	-0.6	-0.57	-0.17
Ireland	-0.64	-0.3	0.14
Greece	-0.36	-0.18	0.41
France	-0.63	-0.55	-0.23
Italy	-0.37	-0.36	-0.26
Hungary	0.55	-0.03	-0.15
Netherland	-0.45	-0.69	-0.5
Austria	-0.32	-0.14	-0.24
Portugal	-0.22	-0.4	-0.65
Finland	-0.21	-0.32	-0.42
Spain	0.1	0.02	0.11
Sweden	-0.7	0.08	0.41
UK	-0.8	-0.78	-0.6

Table 2.2: Statistics of the allocation under rational expectations

	Mean	St.Dev.	Autocorr
consumption	0.423	0.01	0.6
leisure	0.5	0.01	0.6
labor tax rate	0.16	0.003	0.6
market value of debt	0.04	0.03	0.6
primary surplus	0.003	0.02	0.6

Table 2.3: Statistics under learning after convergence of beliefs

	Initially pessimistic agents			Initially optimistic agents		
	Mean	St.Dev.	Autocorr	Mean	St.Dev.	Autocorr
consumption	.416	.015	.6	.45	1e-4	.6
leisure	.51	.01	.6	.47	1e-4	.6
labor tax rate	.18	.01	.6	.04	3e-6	.6
market value of debt	.38	.04	.6	-0.35	3e-4	.6
primary surplus	.015	.02	.6	-.056	.02	.6

Table 2.4: Statistics under RE and learning for the first 30 periods

	Rational expectations			Initially pessimistic agents		
	Mean	St.Dev.	Autocorr	Mean	St.Dev.	Autocorr
consumption	.42	.012	.53	.43	.015	.62
leisure	.5	.012	.53	.49	.014	.73
labor tax rate	.16	.003	.526	.14	.03	.95
market value of debt	.04	.03	.526	.28	.08	.86
primary surplus	.001	.02	.526	-.006	.02	.73

Table 2.5: OLS estimates and t-statistics (in parenthesis) with i.i.d. government shock

	α	β	R^2
$\tau_t^{RE} = \alpha + \beta\tau_{t-1}^{RE} + \varepsilon_t$	0.1562 (7.0467)	-0.0171 (-0.1207)	0.9996
$\tau_t^{L} = \alpha + \beta\tau_{t-1}^{L} + \varepsilon_t$	0.0288 (2.3923)	0.8213 (10.7910)	0.9960

Table 2.6: Newey-West standard error estimates and t-statistics (in parenthesis)

α_0	α_1	α_2	α_3	α_4	α_5	R^2_{adj}
-0.0 (-0.15)	-0.12 (-6.5)	-1.92 (-5.32)	0.24 (10.97)	0.18 (3.27)	0.16 (6.43)	0.57

Table 2.7: $\pi^{\epsilon,T}$

$T\backslash\epsilon$	0.04	0.03	0.02
5	1	.4	0
10	1	.8	0
15	1	1	.06
20	1	1	.4

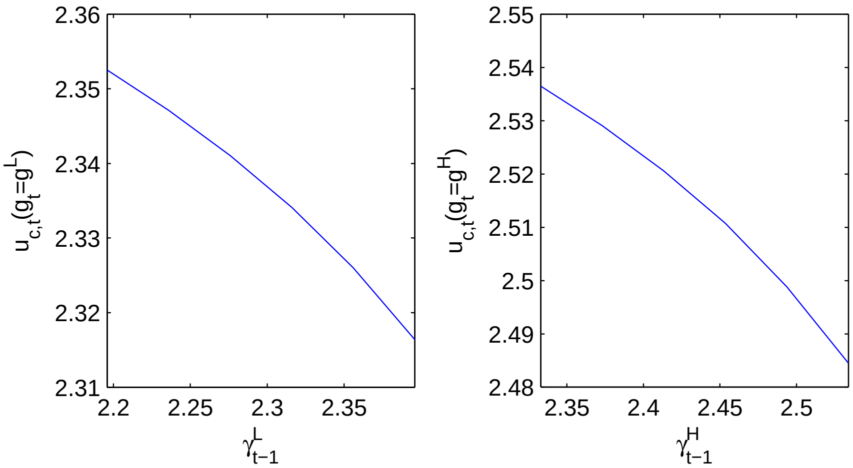

Figure 2.1: T-mapping

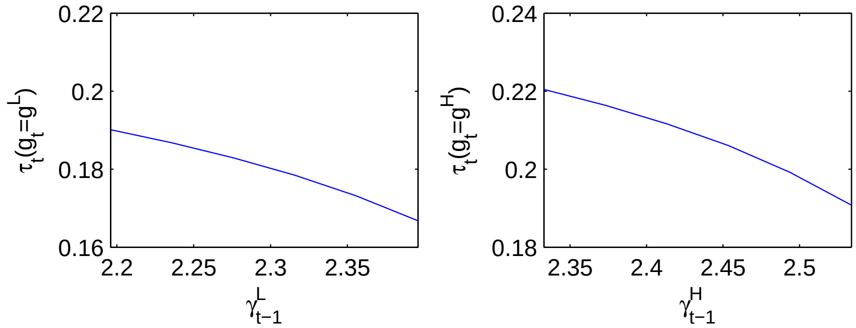

Figure 2.2: Tax policy function

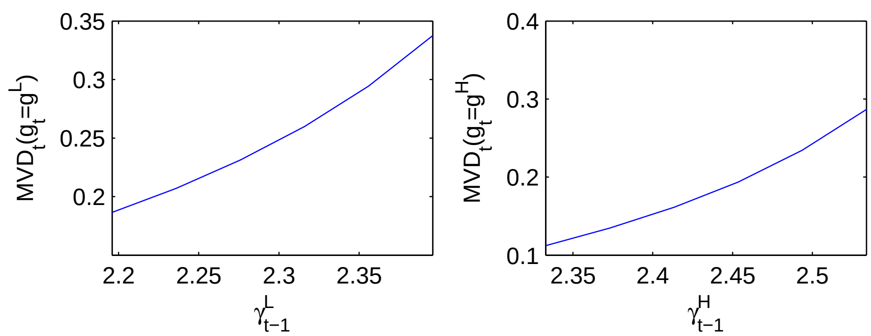

Figure 2.3: Bond holdings policy function

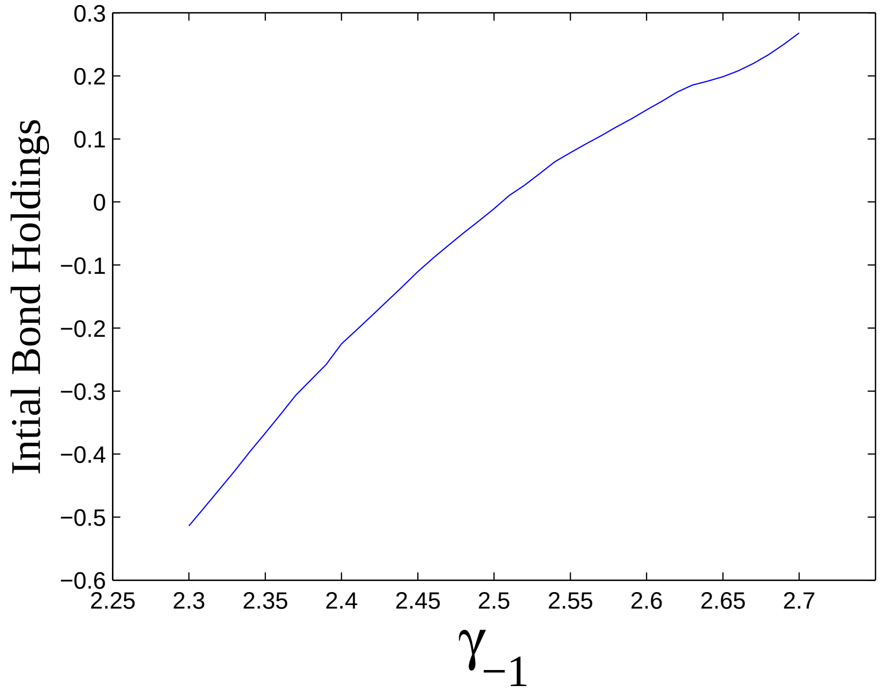

Figure 2.4: Beliefs and initial debt

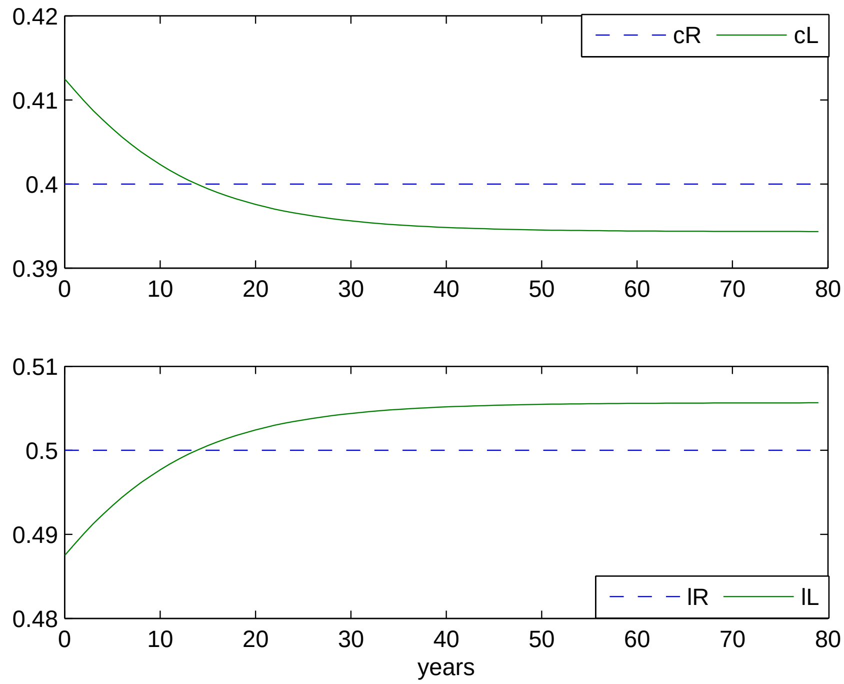

Figure 2.5: Consumption and leisure under RE and under learning dynamics

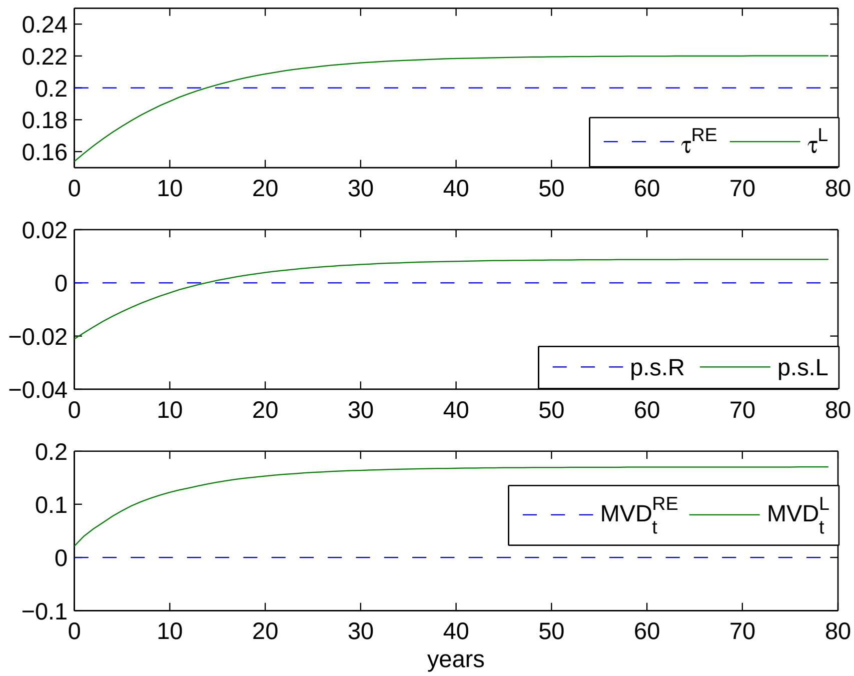

Figure 2.6: Taxes, primary surplus and debt under RE and under learning dynamics

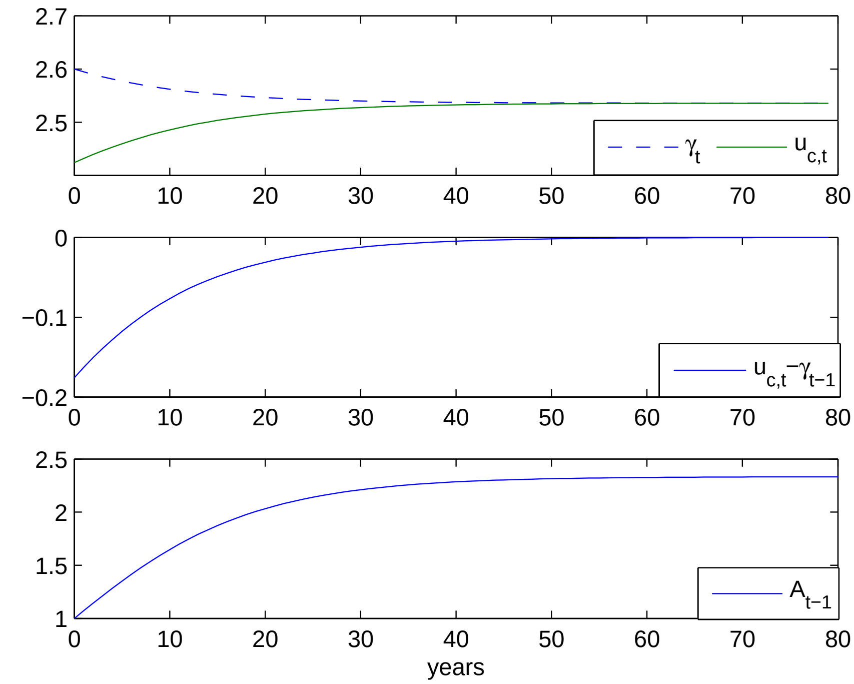

Figure 2.7: Forecast errors, history and non convergence to the RE values

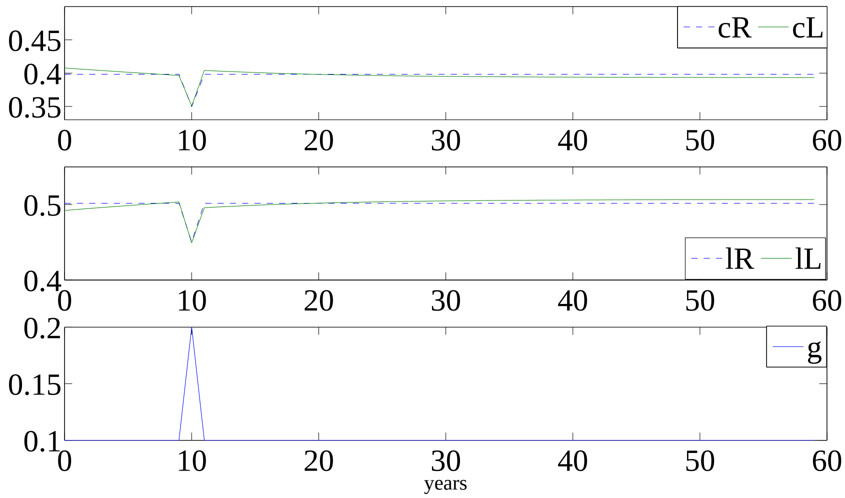

Figure 2.8: Consumption and leisure under RE and under learning dynamics

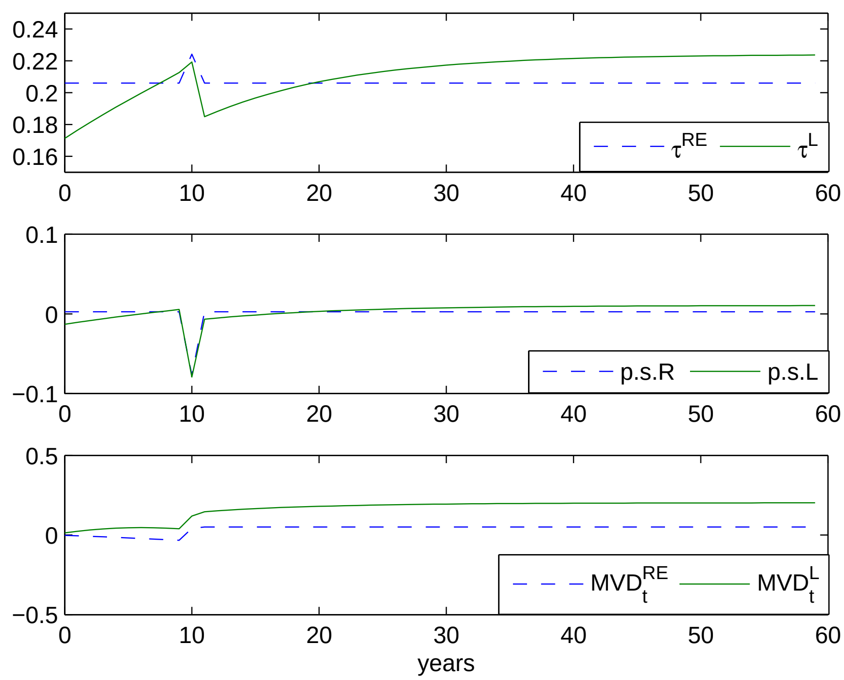

Figure 2.9: Taxes, primary surplus and debt under RE and under learning dynamics

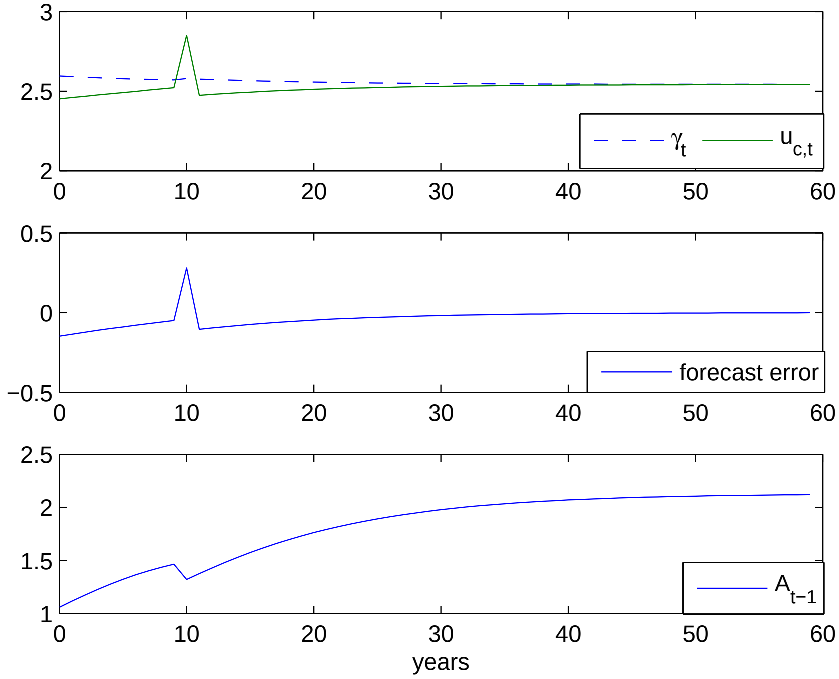

Figure 2.10: Forecast errors, history and non convergence to the RE values

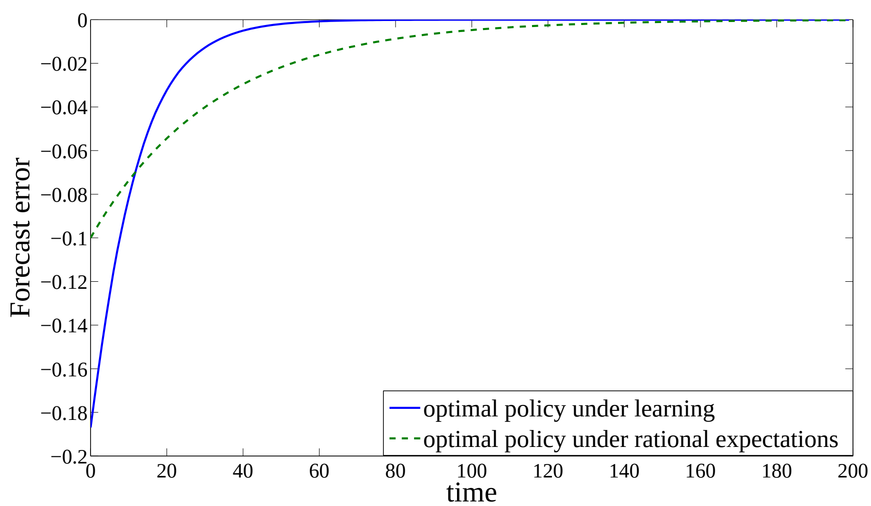

Figure 2.11: Forecast Error

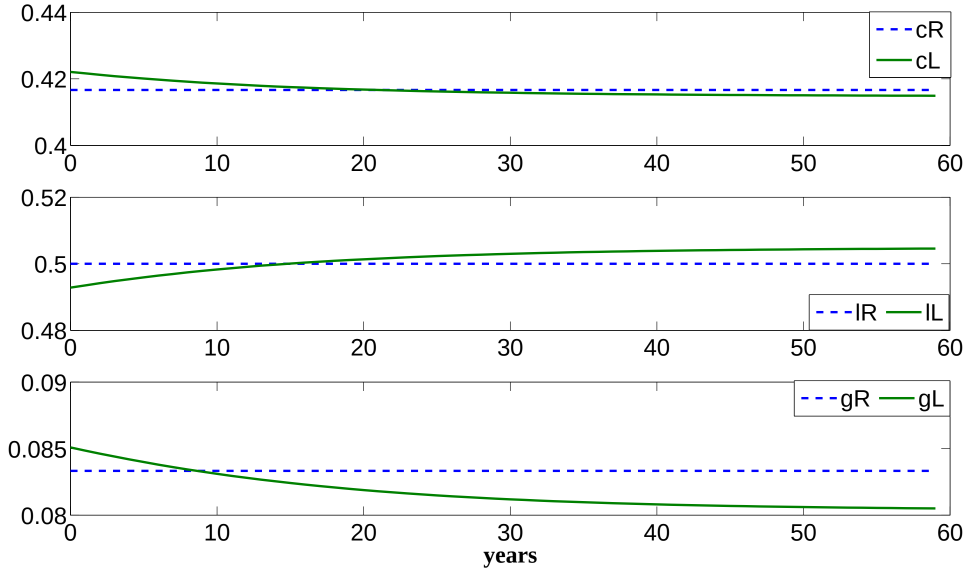

Figure 2.12: Consumption and leisure under RE and under learning dynamics

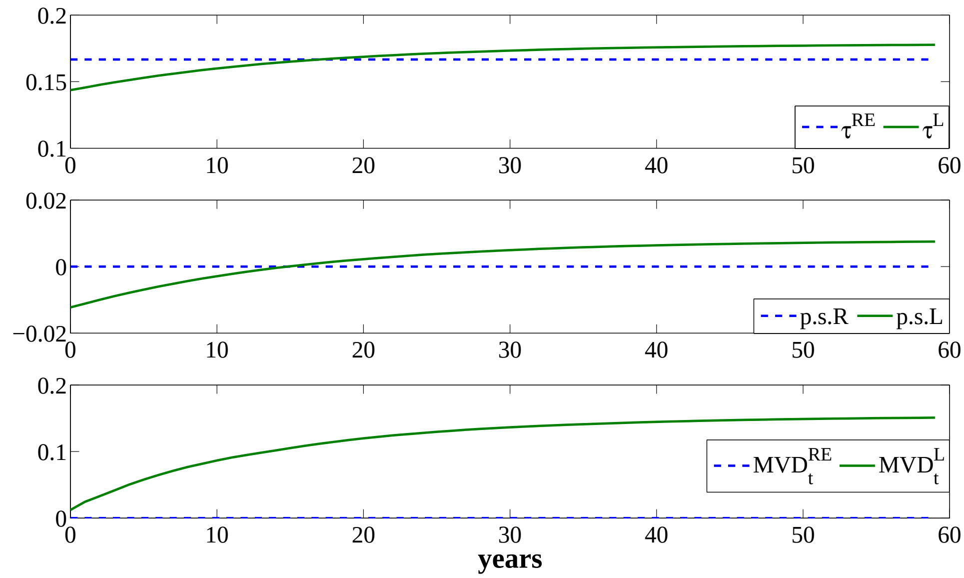

Figure 2.13: Taxes, primary surplus and debt under RE and under learning dynamics

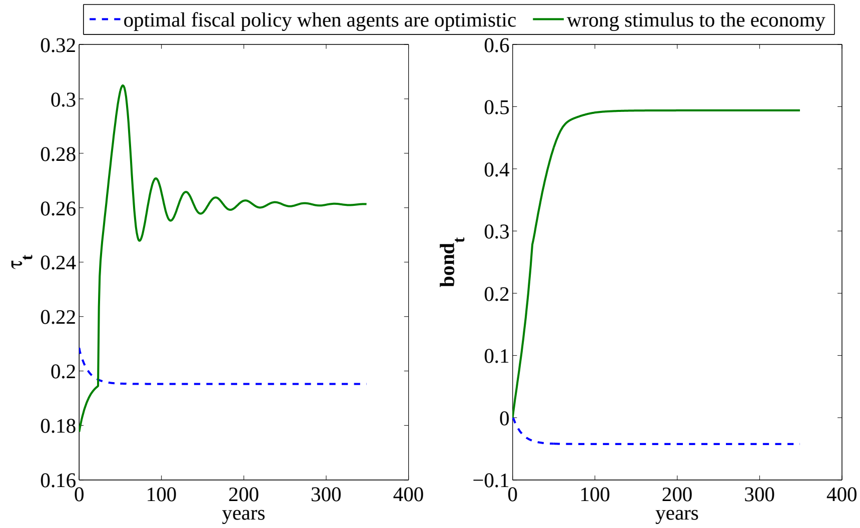

Figure 2.14:

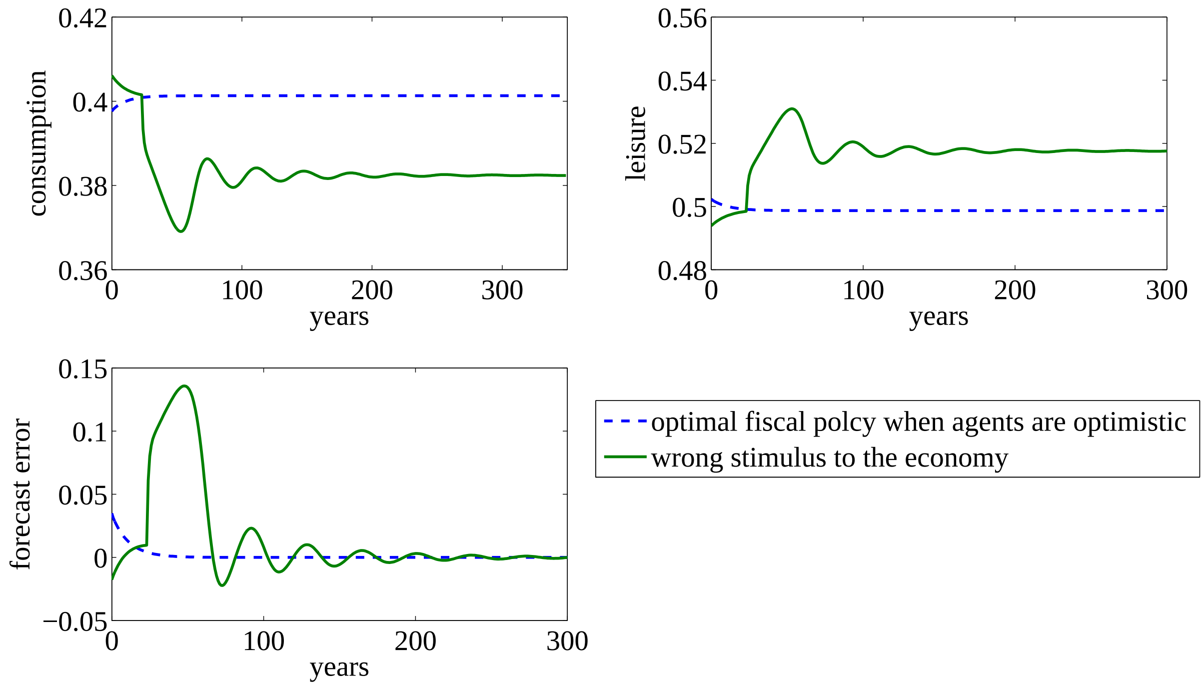

Figure 2.15:

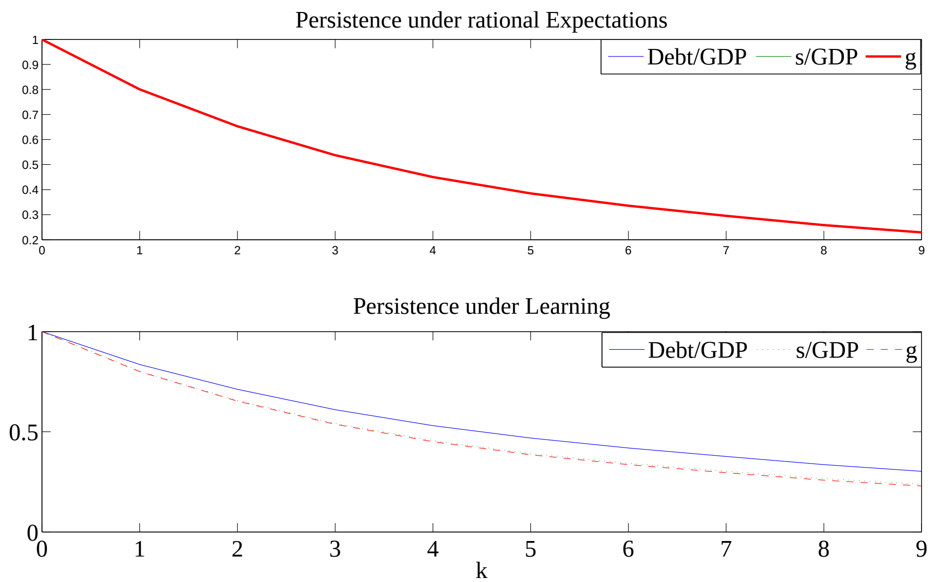

Figure 2.16: Top Panel: RE framework; Bottom Panel: Learning framework

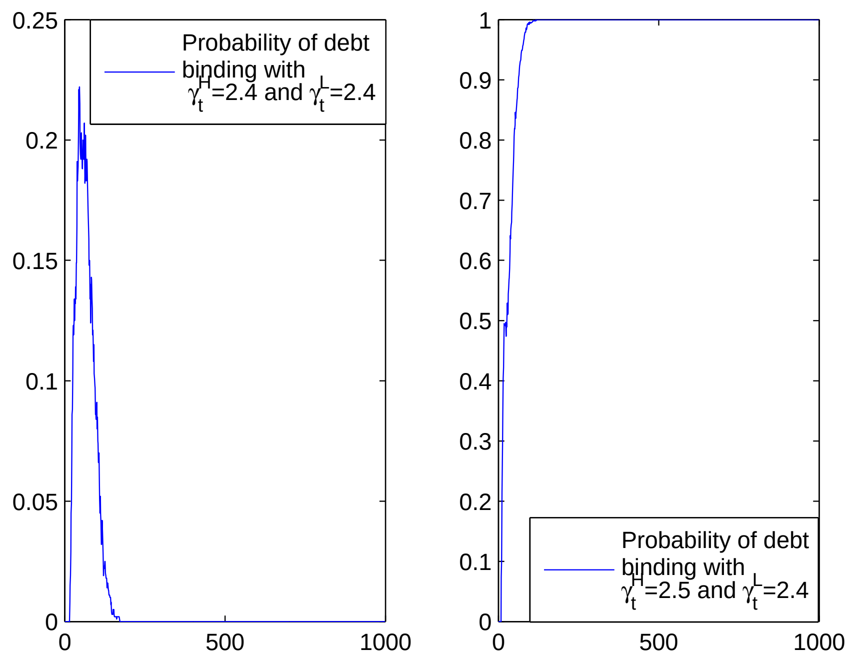

Figure 2.17: Comparison of probabilities of binding debt limits

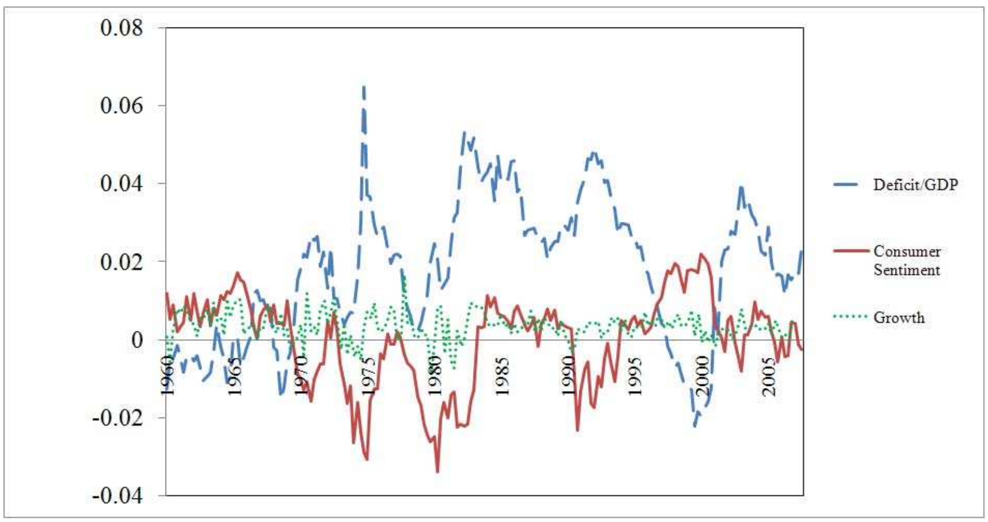
0.08
0.06
0.04
0.02
0
-0.02
-0.04
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
Deficit/GDP
Consumer Sentiment
Growth

Figure 2.18:

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A Appendix to Chapter 1

A.1 Optimal policy under full commitment: logarithmic utility

In this section we show a particular case of Proposition 1 when the utility function of households is logarithmic both in consumption and leisure, which corresponds to the utility function used for the numerical exercises in the paper.

Consider a utility function of the form:

$$u(c_t, l_t) = \alpha * log(c_t) + \delta * log(l_t)$$

with $\alpha > 0$ and $\delta > 0$. Assume that initial wealth $b_{-1} = 0$. Then the allocations and government policies can be easily computed from the optimality conditions (1.9) to (1.13). From the intertemporal budget constraint of households (1.9) it can be derived that:

$$l = \frac{\delta}{\alpha + \delta} \tag{A.1}$$

Plugging in this expression in (1.12), $c = \frac{\alpha}{\lambda}$. Combining this expression for consumption, together with (A.1), (1.10) and (1.11) we arrive to the following expression:

$$\frac{1}{1-\beta}\left(\frac{\alpha}{\lambda} - 1 + \frac{\delta}{\alpha+\delta}\right) + E_0 \sum_{t=0}^{\infty} \beta^t g_t = 0$$

Define the last term of the previous expression as

$$E_0 \sum_{t=0}^{\infty} \beta^t g_t \equiv \frac{1}{1-\beta}\tilde{g}$$

where $\tilde{g}$ is known at $t = 0$. Then

$$\lambda = \frac{\alpha + \delta}{1 - \frac{\alpha+\delta}{\alpha}\tilde{g}}$$

Substituting in the expression for c, we obtain

$$c = \frac{\alpha - (\alpha + \delta)\tilde{g}}{\alpha + \delta} \tag{A.2}$$

From the feasibility constraint (1.10), transfers are given by the difference between the actual realization of public expenditure g_t and its expected discounted value $\tilde{g}$:

$$T_t = g_t - \tilde{g} \tag{A.3}$$

Finally, from the intratemporal optimality condition of households (1.6) we can obtain an expression for the tax rate:

$$\tau = \frac{\delta(\alpha + \delta)}{\alpha}\tilde{g} \tag{A.4}$$

A.2 Proof of Proposition 2

In order to prove Proposition 2, we first need to establish some intermediate results. We begin with a discussion about the sign of Δ, the Lagrange multiplier associated to the intertemporal budget constraint in the Ramsey planner's problem.

a The Ramsey problem with limited commitment

For ease of exposition, we will assume that only the HC has limited commitment. Since the problem of the Ramsey planner is identical to the one in section e, but without imposing constraint 1.18, we do not reproduce it here.

The optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = 0 \tag{A.5}$$

$$u_{l,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = 0 \tag{A.6}$$

$$\psi_t - \lambda = 0 \tag{A.7}$$

$$c_t + g_t = (1 - l_t) + T_t \tag{A.8}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{A.9}$$

$$\mu_t^1(E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) - V^a(g_t)) = 0 \tag{A.10}$$

$$\gamma_t^1 = \mu_t^1 + \gamma_{t-1}^1 \tag{A.11}$$

$$\mu_t^1 \geq 0 \tag{A.12}$$

Multiplying equations (A.5) and (A.6) by c_t and $-(1-l_t)$ respectively, and summing:

$$
\begin{aligned}
&(1+\gamma_t^1-\Delta)(u_{c,t}c_t - u_{l,t}(1-l_t)) - \psi_t(c_t-(1-l_t)) \\
&-\Delta\underbrace{(u_{cc,t}c_t^2 - 2u_{cl,t}(1-l_t)c_t + u_{ll,t}(1-l_t)^2)}_{A_t} = 0
\end{aligned} \tag{A.13}
$$

Notice that given that the utility function is strictly concave, expression A is strictly negative. By a similar procedure we can write down an equivalent expression at $t=0$:

$$
\begin{aligned}
&(1+\gamma_0^1-\Delta)(u_{c,0}(c_0-b_{-1}) - u_{l,0}(1-l_0)) - \psi_0(c_0-(1-l_0)-b_{-1}) \\
&-\Delta\underbrace{(u_{cc,0}(c_0-b_{-1})^2 - 2u_{cl,0}(1-l_0)(c_0-b_{-1}) + u_{ll,0}(1-l_0)^2)}_{A_0} = 0
\end{aligned} \tag{A.14}
$$

Multiplying (A.13) by $\beta^t\pi(s^t)$[1], summing over t and s^t and adding expression (A.14):

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t(1+\gamma_t^1-\Delta)(u_{c,t}c_t - u_{l,t}(1-l_t)) - (1+\gamma_0-\Delta)u_{c,0}b_{-1} \\
&-\Delta Q - E_0\sum_{t=0}^{\infty}\beta^t\psi_t(c_t-(1-l_t)) + \psi_0 b_{-1} = 0
\end{aligned}
$$

where Q is the expected value of the sum of negative quadratic terms A_t. Using the implementability constraint (1.15) and the resource constraint (1.14) we obtain equation (A.15)

$$
\begin{aligned}
&E_0\sum_{t=0}^{\infty}\beta^t(\gamma_t^1-\gamma_0^1)(u_{c,t}((1-l_t)+T_t-g_t) - u_{l,t}(1-l_t)) \\
&-\Delta Q + E_0\sum_{t=0}^{\infty}\beta^t\psi_t(g_t-T_t) + \psi_0 b_{-1} = 0
\end{aligned} \tag{A.15}
$$

For later purposes, using the intratemporal optimality condition of households (1.6) we can reexpress this equation as[2].

$$
E_0\sum_{t=0}^{\infty}\beta^t(\gamma_t^1-\gamma_0^1)u_{c,t}(\tau_t(1-l_t)-g_t+T_t) - \Delta Q + E_0\sum_{t=0}^{\infty}\beta^t\lambda g_t + \lambda b_{-1} = 0 \tag{A.16}
$$

Notice that, in the case of full commitment, expression (A.15) simplifies to

[1] $\pi(s^t)$ is the probability of history s^t taking place given that the event s_0 has been observed.

[2] Notice that if the participation constraint was never binding, then $\gamma_t^1 = \gamma_0^1 = 0$ and we would recover an identical condition to the one obtained in the Lucas and Stokey (1983) model.

$$-\Delta Q + \lambda \left(E_0 \sum_{t=0}^{\infty} \beta^t g_t + b_{-1} \right) = 0 \tag{A.17}$$

Since $\lambda = \psi_t > 0\ \forall t$, it is straightforward to see that when the present value of all government expenditures exceeds the value of any initial government wealth, the Lagrange multiplier $\Delta < 0$.

In the presence of limited commitment, however, there is an extra term involving the costate variable γ_t^1 which prevents us from applying the same reasoning. Nevertheless, we will show that this is the case for the specific example of section 1.3, and we will assume this result extends to the general setup. In the numerical exercise we perform in section 1.4 we confirm that this assumption holds.

We show now under which conditions $\Delta = 0$. Setting $\Delta = 0$, from equations (A.5) and (A.6) we know that

$$u_{c,t}(1+\gamma_t) = u_{l,t}(1+\gamma_t) \tag{A.18}$$

$$u_{c,t} = u_{l,t} \tag{A.19}$$

This last expression and equation (1.6) in the text imply that $\tau_t = 0\ \forall t$. Inserting these results into equation (A.16):

$$E_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t - \gamma_0)(u_{c,t}(T_t - g_t) + E_0 \sum_{t=0}^{\infty} \beta^t \lambda (g_t - T_t) + \lambda b_{-1} = 0$$

Using (A.18)

$$\Rightarrow E_0 \sum_{t=0}^{\infty} \beta^t (-\gamma_t + \gamma_0 + 1 + \gamma_t) u_{c,t}(g_t - T_t) + u_{c,0}(1+\gamma_0) b_{-1} = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_{c,t}}{u_{c,0}} (g_t - T_t) = -b_{-1} \tag{A.20}$$

We can rewrite (A.20) as

$$\sum_{t=0}^{\infty} \sum_{s^t} p_t^0 (g_t - T_t) = -b_{-1} = b_{-1}^g \tag{A.21}$$

where p_t^0 is the price of a hypothetical bond issued in period 0 with maturity in period t contingent on the realization of s_t. Equation (A.21) states that when the government's initial claims b_{-1}^g against the private sector equal the present-value of all future government expenditures net of transfers, the Lagrange multiplier Δ is zero. Since the government does not need to resort to any distortionary taxation, the household's present-value budget does not exert any additional constraining effect on welfare maximization beyond what is already present through the economy's technology.

Finally, we will follow Ljungqvist and Sargent (2000) and assume that if the government's initial claims against the private sector were to exceed the present value of future government expenditures, the government would return its excess financial wealth as lump-sum transfers and Δ would remain to be zero.

b Proof of Proposition 2

We begin by proving the first part of the Proposition. Given a logarithmic utility function as (1.30), optimality conditions (1.19) to (1.21) become

$$\frac{\alpha}{c_t}(1+\gamma_t^1) - (\lambda+\gamma_t^2) - \Delta\left(-\frac{\alpha}{c_t^2}c_t + \frac{\alpha}{c_t}\right) = 0$$

$$\Longrightarrow c_t = \frac{\alpha(1+\gamma_t^1)}{\lambda+\gamma_t^2} \tag{A.22}$$

$$\frac{\delta}{l_t}(1+\gamma_t^1) - (\lambda+\gamma_t^2) - \Delta\left(\frac{\delta}{l_t} + \frac{\delta}{l_t^2}(1-l_t)\right) = 0$$

$$\Longrightarrow l_t = \frac{\delta(1+\gamma_t^1) \pm \sqrt{\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}}{2(\lambda+\gamma_t^2)} \tag{A.23}$$

Notice from equation (A.23) that if $\Delta < 0$ then we need to take the square root with positive sign in order to have $l_t > 0$. To show that consumption and leisure increase with γ_t^1, we take the derivatives of c_t and l_t with respect to γ_t^1

$$\frac{\partial c_t}{\partial \gamma_t^1} = \frac{\alpha}{\lambda+\gamma_t^2} > 0$$

$$\frac{\partial l_t}{\partial \gamma_t^1} = \frac{\delta + \left(\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)\right)^{-\frac{1}{2}}\delta^2(1+\gamma_t^1)}{2(\lambda+\gamma_t^2)} > 0$$

We can write the intratemporal optimality condition of households (1.6) as

$$\tau_t = \frac{u_{c,t} - u_{l,t}}{u_{c,t}} = 1 - \frac{\delta c_t}{\alpha l_t} \tag{A.24}$$

Given $t < t'$, assume $\gamma_t^1 < \gamma_{t'}^1$ while $\gamma_t^2 = \gamma_{t'}^2$. Now we compare the tax rates at t and t', and show that τ_t decreases with γ_t^1 by contradiction. Then, using (A.24)

$$\tau_{t'} - \tau_t = \frac{\delta}{\alpha}\left(\frac{c_t}{l_t} - \frac{c_{t'}}{l_{t'}}\right) > 0$$

It follows that it must be the case that $c_t l_{t'} - c_{t'} l_t > 0$. After some algebra this condition translates into

$$\left(\frac{1+\gamma_t^1}{1+\gamma_{t'}^1}\right)^2 > \frac{\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}{\delta^2(1+\gamma_{t'}^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}$$

$$(1+\gamma_t^1)^2 > (1+\gamma_{t'}^1)^2$$

which is clearly a contradiction. Thus, τ_t increases with γ_t^1.

Now we proceed to prove the second part of the Proposition. We can immediately check that c_t decreases with γ_t^2 by taking partial derivates:

$$\frac{\partial c_t}{\partial \gamma_t^2} = -\frac{\alpha(1+\gamma_t^1)}{(\lambda+\gamma_t^2)^2} < 0$$

Suppose l_t is an increasing function of γ_t^2. Then the partial derivative of l_t w.r.t γ_t^2 must be positive

$$\frac{\partial l_t}{\partial \gamma_t^2} = \frac{-2\Delta\delta(\lambda+\gamma_t^2)A^{-\frac{1}{2}} - \delta(1+\gamma_t^1) - A^{\frac{1}{2}}}{4(\lambda+\gamma_t^2)^2} > 0$$

where $A = \delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)$. This last expression implies that

$$-2\Delta\delta(\lambda+\gamma_t^2) > \delta(1+\gamma_t^1)A^{\frac{1}{2}} + A$$

Then,

$$2\Delta\delta(\lambda+\gamma_t^2) - \delta^2(1+\gamma_t^1)^2 > A^{\frac{1}{2}}\delta(1+\gamma_t^1)$$

Since the left hand side of the previous expression is negative, while the right hand side is positive, this statement is clearly a contradiction. Then it must be the case that l_t is a decreasing function of γ_t^2.

Finally, suppose that $t' > t$, $\gamma_{t'}^2 > \gamma_t^2$ but $\gamma_{t'}^1 = \gamma_t^1$. Assume that τ_t is a decreasing function of γ_t^2. Then, using (A.24), it must be the case that

$$u_{c,t'}u_{l,t} < u_{c,t}u_{l,t'}$$

This implies that

$$\frac{\delta(1+\gamma_t^1) + \sqrt{\delta^2(1+\gamma_t^1)^2 - 4\delta\Delta(\lambda+\gamma_{t'}^2)}}{2(\lambda+\gamma_{t'}^2)}\frac{\alpha(1+\gamma_t^1)}{\lambda+\gamma_t^2} < \frac{\delta(1+\gamma_t^1) + \sqrt{\delta^2(1+\gamma_t^1)^2 - 4\delta\Delta(\lambda+\gamma_t^2)}}{2(\lambda+\gamma_t^2)}\frac{\alpha(1+\gamma_t^1)}{\lambda+\gamma_{t'}^2} \tag{A.25}$$

Simplifying and remembering that $\Delta < 0$, the previous inequality is a contradiction. Therefore, τ_t increases with γ_t^2. This completes the proof.

A.3 Proof of Proposition 3

Notice first that at $t = 0$ and for $\gamma_0^1 = 0$, the continuation value of staying in the contract has to be (weakly) greater than the value of the outside option (financial autarky):

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \geq \sum_{t=0}^{\infty} \beta^t u(c_{t,A}, l_{t,A}) \tag{A.26}$$

The reason for this statement is that, for the government, subscribing the contract with the rest of the world represents the possibility to do risk-sharing and, consequently, to smooth consumption of domestic households. Since utility is concave, a smoother consumption path translates into a higher life-time utility value. Obviously, this result hinges on the fact that the initial debt of the government is zero and that equation (1.1) must hold[3].

Now we show that equation (1.17) holds with strict inequality for $1 \leq t \leq T$. It is important to bear in mind that the allocations could change in time only due to a different γ_t^1. Since $\gamma_{t-1}^1 \leq \gamma_t^1 \; \forall t$, then $u(c_{t-1}) \leq u(c_t)$. Assume that $\mu_1^1 > 0$. This implies that, if μ_1^1 was equal to zero, the PC would be violated, that is,

$$\begin{aligned} u(c_0, l_0) + \sum_{t=2}^{T-1} \beta^{t-1} u(c_t, l_t) + \beta^{T-1} u(c_T, l_T) + \sum_{t'=T+1}^{\infty} \beta^{t'-1} u(c_{t'}, l_{t'}) \\ < \sum_{t=0}^{T-2} \beta^t u(c_A, l_A) + \beta^{T-1} u(c_{A'}, l_{A'}) + \sum_{t=T}^{\infty} \beta^t u(c_A, l_A) \end{aligned} \tag{A.27}$$

Equation (A.26) can be rewritten as

$$\begin{aligned} \sum_{t=0}^{T} \beta^t u(c_t, l_t) + \sum_{t'=T+1}^{\infty} \beta^{t'} u(c_{t'}, l_{t'}) \\ > \sum_{t=0}^{T-1} \beta^t u(c_A, l_A) + \beta^T u(c_{A'}, l_{A'}) + \sum_{t=T+1}^{\infty} \beta^t u(c_A, l_A) \end{aligned} \tag{A.28}$$

Subtracting (A.28) from (A.27):

$$\begin{aligned} &\beta[u(c_2, l_2) - u(c_1, l_1)] + \beta^2[u(c_3, l_3) - u(c_2, l_2)] + \ldots + \beta^{T-1}[u(c_T, l_T) - u(c_{T-1}, l_{T-1})] + \\ &\beta^T[u(c_{T+1}, l_{T+1}) - u(c_T, l_T)] + \beta^{T+1}[u(c_{T+2}, l_{T+2}) - u(c_{T+1}, l_{T+1})] + \ldots \\ &< \beta^{T-1}[u(c_{A'}, l_{A'}) - u(c_A, l_A)] + \beta^T[u(c_A, l_A) - u(c_{A'}, l_{A'})] \end{aligned} \tag{A.29}$$

[3]If, for example, the initial level of government debt b_{-1} was very high, then the government could find it optimal to default on this debt and run a balanced budget thereafter. On the other hand, if condition (1.1) was not imposed, then the contract could mean a redistribution of resources from the HC to the RW that could potentially lead the HC to have incentives not to accept the contract.

Reordering terms we arrive at:

$$\beta \underbrace{[u(c_2,l_2)-u(c_1,l_1)]}_{\geq 0} + \beta^2 \underbrace{[u(c_3,l_3)-u(c_2,l_2)]}_{\geq 0} + \ldots + \beta^{T-1} \underbrace{[u(c_T,l_T)-u(c_{T-1},l_{T-1})]}_{\geq 0} +$$
$$+ \beta^T \underbrace{[u(c_{T+1},l_{T+1})-u(c_{T-1},l_{T-1})]}_{\geq 0} + \beta^{T+1} \underbrace{[u(c_{T+2},l_{T+2})-u(c_{T+1},l_{T+1})]}_{\geq 0} + \ldots$$
$$< \underbrace{[u(c_{A'},l_{A'})-u(c_A,l_A)]}_{<0} \underbrace{(\beta^{T-1}-\beta^T)}_{>0} \tag{A.30}$$

Expression (A.30) is clearly a contradiction, since the left hand side of the inequality is greater or equal to 0, but the right hand side is strictly smaller than 0. We conclude then that it cannot be that $\mu_1^1 > 0$. Therefore, equation(1.17) is not binding in period $t = 1$. The same reasoning can be extended to periods $t = 2, 3, \ldots, T$. Therefore, $\gamma_t^1 = \gamma_0^1 = 0$ for $t = 1, 2, ..., T$ and the allocations $\{c_t\}_{t=0}^T$, $\{l_t\}_{t=0}^T$ are constant.

Notice that, from $T+1$ onwards, $g_t = 0$ so the allocations do not change. Therefore, $\gamma_t^1 = \gamma_{T+1}^1$ for $t = T+2, T+3, \ldots, \infty$.

Finally, we show that $\mu_{T+1}^1 > 0$[4]. We prove this by contradiction. Assume that $\mu_{T+1} = 0$. From the previous discussion, this implies that $\gamma_t^1 = 0 \ \forall t$. Then the allocations are identical to the case of limited commitment, and from the results of section A.1, we know that $T_t < 0$ for $t \neq T$ and $T_T > 0$. Thus, from the feasibility constraint (1.14) we can see that $c_{T+1} < c_A$ and $l_{T+1} < l_A$. But this implies that utility $u(c_{T+1}, l_{T+1}) < u(c_A, l_A)$, so

$$\frac{1}{1-\beta} u(c_{T+1}, l_{T+1}) < \frac{1}{1-\beta} u(c_A, l_A)$$
$$\sum_{j=0}^{\infty} \beta^j u(c_{T+1+j}, l_{T+1+j}) < \sum_{j=0}^{\infty} \beta^j u(c_A, l_A)$$

which clearly contradicts with the fact that $\mu_{T+1}^1 = 0$. Therefore, it must be the case that $\mu_{T+1}^1 > 0$. This completes the proof.

A.4 Proof that $\Delta < 0$ in Section 1.3

Since in the example of Section 1.3 we have a full analytical characterization of the equilibrium, it is possible to determine the sign of Δ.

Given our assumption about the government expenditure shock and the result of Proposition 3, equation (A.16) can be written as

[4]Notice that, given that our shock in this example is not a Markov process, neither γ_t nor the allocations c_t and l_t are time-invariant functions of the state variables g_t, γ_{t-1} but, on the contrary, the depend on t.

$$\sum_{t=T+1}^{\infty} \beta^t(\gamma_{T+1} - \gamma_0)u_{\bar{c}}(\bar{\tau}(1-\bar{l}) + \bar{T}) - \Delta Q + \beta^T \lambda g_T = 0 \tag{A.31}$$

where $\bar{c}, \bar{l}, \bar{\tau}$ and $\bar{T}$ are the constant allocations and fiscal variables from $t = T+1$ onwards. In order to determine the sign of the first term of the previous expression, we recall the period by period budget constraint of the government for $t \geq T+1$:

$$(\beta - 1)\bar{b}^G = \bar{\tau}(1-\bar{l}) + \bar{T}$$

The sign of the first term of equation (A.16) depends on whether government bonds are positive or negative after the big shock has taken place. From equation (1.15)

$$\sum_{j=0}^{T} \beta^t(u_{\tilde{c}}\tilde{c} - u_{\tilde{l}}(1-\tilde{l})) + \sum_{j=T+1}^{\infty} \beta^t(u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l})) = 0$$
$$\Rightarrow \frac{1-\beta^{T+1}}{1-\beta}\left(\alpha - \frac{\delta}{\tilde{l}}(1-\tilde{l})\right) + \frac{\beta^{T+1}}{1-\beta}\left(\alpha - \frac{\delta}{\bar{l}}(1-\bar{l})\right) = 0 \tag{A.32}$$

where $\tilde{c}$ and $\tilde{l}$ are the constant allocations from $t = 0$ to $t = T$. We know that the participation constraint binds in period $T+1$ and consequently $\bar{l} > \tilde{l}$. But this implies that

$$\alpha - \frac{\delta}{\tilde{l}}(1-\tilde{l}) < 0$$
$$\alpha - \frac{\delta}{\bar{l}}(1-\bar{l}) > 0 \tag{A.33}$$

because the two terms of (A.32) have to add up to zero. Now we recover b_t for $t \geq T+1$ from the intertemporal budget constraint (1.15) of households at time $T+1$:

$$u_{\bar{c}}\bar{b} = \sum_{j=0}^{\infty} \beta^j(u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l})) = \frac{1}{1-\beta}(u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l}))$$
$$= \frac{1}{1-\beta}\left(\alpha - \frac{\delta}{\bar{l}}(1-\bar{l})\right) > 0$$

If $\bar{b} > 0$, $\bar{b}^G < 0$ so the first term in equation (A.31) is positive. But then from this equation it is immediate to see that $\Delta < 0$.

A.5 The International Institution Problem and the Government Problem: Equivalence of Results

Suppose that there exists an international financial institution that distributes resources among the HC and the RW, taking into account the aggregate feasibility constraint, the implementability condition (1.15), and participation constraints (1.17) and (1.18). The Lagrangian associated to the international institution is

$$\max_{\{c_t^1,c_t^2,T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t(\alpha u(c_t^1,l_t^1)+(1-\alpha)u(c_t^2)+$$
$$+\tilde{\mu}_{1,t}(E_t\sum_{j=0}^{\infty}\beta^j u(c_{t+j}^1,l_{t+j}^1)-V_t^{1,a})+\tilde{\mu}_{2,t}(\underline{B}-E_t\sum_{j=0}^{\infty}\beta^j T_{t+j})+ \tag{A.34}$$
$$-\tilde{\Delta}(u_{c^1,t}c_t^1-u_{l_{1_t}}(1-l_t^1))+\tilde{\psi}_t(c_t^1+c_t^2+g-(1-l_t^1+y)))$$

where α is the Pareto weight that the international institution assigns to the HC. Since by assumption households in the RW are risk-neutral, $u(c_t^2)=c_t^2$. The feasibility constraint in the RW implies that $c_t^2=y-T_t$. Substituting this into B.1 and applying **?** we can recast problem B.1 as

$$\max_{\{c_t^1,c_t^2,T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t((\alpha+\tilde{\gamma}_{1,t})u(c_t^1,l_t^1)+(1-\alpha)(y-T_t)+$$
$$-\tilde{\mu}_{1,t}V_t^{1,a}+\tilde{\mu}_{2,t}\underline{B}-\tilde{\gamma}_{2,t}T_t)-\tilde{\Delta}(u_{c^1,t}c_t^1-u_{l_{1_t}}(1-l_t^1))+\tilde{\psi}_t(c_t^1+g-(1-l_t^1+T_t)) \tag{A.35}$$

Dividing each term by α does not change the solution, since α is a constant. Let $\frac{\tilde{\gamma}_t^i}{\alpha}\equiv\gamma_t^i$, for $i=1,2$, $\frac{\tilde{\Delta}}{\alpha}\equiv\Delta$ and $\frac{\tilde{\psi}_t}{\alpha}\equiv\psi_t$.

The first-order conditions are

$$u_{c,t}(1+\tilde{\gamma}_t^1)-\psi_t-\Delta(u_{cc,t}c_t+u_{c,t}-u_{cl,t}(1-l_t))=0 \tag{A.36}$$

$$u_{l,t}(1+\gamma_t^1)-\psi_t-\Delta(u_{cl,t}c_t+u_{l,t}-u_{ll,t}(1-l_t))=0 \tag{A.37}$$

$$\psi_t=\frac{1-\alpha}{\alpha}+\gamma_t^2 \tag{A.38}$$

where $c_t\equiv c_t^1$. Posing $\lambda\equiv\frac{1-\alpha}{\alpha}$ makes the system of equations (A.36)-(A.38) coincide with (1.19)-(1.21).

A.6 Proof of Proposition 5

The Lagrangean for the government in the HC can be written as

$$\mathcal{L}=E_0\sum_{t=0}^{\infty}\beta^t\{u(c_t^1,l_t^1)+(\Delta+\Lambda_{1,t})(u_{c^1,t}c_t^1-u_{l^1,t}(1-l_t^1))-\lambda_{1,t}B_{t-1}(g_t)+\lambda_{2,t}(-A_t^1(g_{t+1})$$
$$+T_t^1(g_{t+1}))+\lambda_{3,t}(1-l_t^1-\sum_{g_{t+1}}q_tT_t^1(g_{t+1})+T_{t-1}^1-c_t^1-g_t)-\Delta b_{-1}u_{c^1,0}\}$$

where $\Lambda_{1,t} = \Lambda_{1,t-1} + \lambda_{1,t}$.

The optimality conditions are

$$u_{c^1,t} + (\Delta + \Lambda_{1,t})(u_{cc^1,t} c_t^1 + u_{c^1,t}) = \lambda_{3,t} \tag{A.39}$$

$$u_{l^1,t} + (\Delta + \Lambda_{1,t})(u_{l^1,t} - u_{ll^1,t}(1 - l_t^1)) = \lambda_{3,t} \tag{A.40}$$

$$q_t(g_{t+1}) = \frac{\beta \lambda_{3,t+1}(g_{t+1})\pi((g_{t+1})) + \lambda_{2,t}}{\lambda_{3,t}} \tag{A.41}$$

Denote by $\overline{T}_t^2$ the limit on transfers received by the RW, in the original problem of section 1.2. The problem of the household in the RW is

$$\max_{\{c_t, T_t^2\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{A.42}$$

s.t.

$$T^2(g_{t+1}) > \overline{T}_t^2 \tag{A.43}$$

$$y + T_{t-1}^2(g_t) = c_t^2 + \sum_{g_{t+1}} q_t T_t^2(g_{t+1}) \tag{A.44}$$

The Lagrangean for this problem can be written as

$$\begin{aligned}\mathcal{L} =& E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t^2) + \lambda_t \left(y - \sum_{g_{t+1}} q_t T_t^2(g_{t+1}) + T_{t-1}^2 - c_t^2 \right) \\ &+ \gamma_t^2 (T^2(g_{t+1}) - A_t^2(g_{t+1}))\}\end{aligned}$$

The first order conditions are

$$u_{c^2,t} = \lambda_t \tag{A.45}$$

$$q_t = \frac{\beta u_{c^2,t+1}\pi((g_{t+1})) + \gamma_t^2}{u_{c^2,t}} \tag{A.46}$$

It is easy to show that equations (A.39) and (A.40) coincide with equations (1.19) and (1.20) with

$$\frac{\Delta}{1 + \gamma_t^1} = (\Delta + \Lambda_{1,t}). \tag{A.47}$$

Proposition 5 says that a binding participation constraint in the HC can be seen as a binding constraint on the value of domestic debt the government can issue.

The price of the bond exchanged between the two countries is equal to

$$q_t(g_{t+1}) = max\{\frac{\beta u_{c^2,t+1}\pi((g_{t+1})) + \gamma_t^2}{u_{c^2,t}}, \frac{\beta\lambda_{3,t+1}(g_{t+1})\pi((g_{t+1})) + \lambda_{2,t}}{\lambda_{3,t}}\} \tag{A.48}$$

B Appendix to Chapter 2

B.1 A Simple Endowment Economy

We analyse here a non-stochastic endowment economy in which households do not have fully rational expectations. The purpose of the exercise is to identify the fiscal policy design which implements the first-best allocation given non-rational expectations.[1]

In the literature there are two approaches to modelling the behaviour of agents endowed with non-rational expectations: the Euler Equation Approach (henceforth EEA) and the Infinite Horizon Approach (henceforth IHA). In the first, households' decisions depend on one-period-ahead forecasts of future variables appearing in the Euler Equation, while in the second approach they depend on the forecasts regarding outcomes arbitrarily far in the future, insofar as these forecasts satisfy standard probability laws. Without entering into the discussion of which of the two approaches explains better the behaviour of not fully-informed agents, in this section we use both of them to model agents' behaviour and we show that the fiscal policy design is independent of the approach used.

B.2 Modelling learning according to the Euler Equation Approach

The representative household maximises its lifetime utility

$$\widetilde{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{B.1}$$

subject to the flow budget constraint

$$b_{t-1} + y - T_t = c_t + p_t^b b_t \tag{B.2}$$

where $y \equiv 1 - g$ is the exogenous and constant output, b_t is the amount of bonds issued by the government at t at the price p_t^b, and T_t is the lump-sum tax in period t.

The Euler equation for consumption is

$$p_t^b = \beta \frac{u_{c,t+1}}{u_{c,t}} \tag{B.3}$$

Since taxes are lump-sum, the Ricardian Equivalence holds, and there are infinite ways, equivalent in terms of allocations, of financing the public expenditure shock. Among these, we focus on the following policy rules:

[1] Since the results do not change once uncertainty is introduced, we focus only on the non-stochastic framework for clarity.

$$
\begin{aligned}
b_t &= b \forall t & \text{(B.4)} \\
T_t &= T \forall t & \text{(B.5)}
\end{aligned}
$$

The equilibrium in the goods market requires consumption to be equal to output, $c_t = y$. Since agents do not know either that the resource constraint has to hold in any period (since this is one equilibrium condition they have to learn) or that agents are identical, their expectations about consumption tomorrow do not coincide with those that fully-informed agents would have. It follows that the interest rate is not equal to the interest rate under rational expectations, β^{-1}. The question we pose is: given the quantity of bonds issued by the government under rational expectations, what is the fiscal policy in terms of lump-sum taxes which induces agents to consume the exact amount of goods implied by the resource constraint? The following proposition answers the question:

Proposition 10 *Assume that the government fixes the amount of bonds at the RE level, $b^{RE} > 0$. Then the fiscal policy has to be expansionary (restrictive) whenever agents' expectations over one-step-ahead marginal utility of consumption are higher than the expectations fully rational agents would have.*

Proof. The proof is immediate. Under RE, combining the flow budget constraint with the resource constraint we get

$$T^{RE} = (1 - \beta) b^{RE} \tag{B.6}$$

Let $\gamma_t \equiv \widetilde{E}_t u_{c,t+1}$. Combining equation (B.2) after substituting for the agents' optimality condition with the resource constraint we get

$$T_t^L = (1 - \beta \frac{\gamma_t}{u_c}) b^{RE} \tag{B.7}$$

Subtracting equation (B.7) from (B.6) we obtain

$$T^{RE} - T_t^L = b^{RE} \beta (\frac{\gamma_t}{u_c} - 1) \tag{B.8}$$

Insofar as agents are pessimistic, i.e. $\gamma_t > u_c$, $T^{RE} > T_t^L$. Symmetrically, when agents are optimistic, i.e. $\gamma_t < u_c$, $T^{RE} < T_t^L$.

■

If the belief over the next period marginal utility coincides with the one under rational expectations, the equilibrium tax under learning would also coincide with that prevailing under rational expectations. For pessimistic (optimistic) agents, i.e. for agents expecting their marginal utility to be higher (lower) than it will actually be, the government has to reduce (increase) taxes relative to the RE benchmark. The reason is the following. Pessimistic agents expect future taxes to be higher than today, and therefore to smooth consumption they want to save more and consume less today: an expansionary fiscal policy is required to clear the goods market. The further away the initial belief is compared with that under rational expectations, the higher is the difference between taxes under rational expectations and under learning.

B.3 Modelling learning according to the Infinite Horizon Approach

In section B.2, only one-period-ahead expectations over marginal utility matter in order to pin down agents' optimal choice. In a series of papers (see Preston (2005a) and Preston (2005b) among others) Preston shows two possible shortcomings intrinsic to the EEA approach. The first is that the expectation of next period consumption based on the regression of past observations on aggregate shocks does not necessarily coincide with the one implied by forwarding the optimal consumption rule, unless agents have already converged to the rational expectations equilibrium; in other words, the EEA is not micro founded. The second problem is that agents do not take into account their wealth, and therefore they would violate ex ante their transversality condition.

In this section we show that, even when agents make long-run forecasts about future macroeconomic conditions, the tax policy which ensures the goods market clearing shares the same qualitative features as the one highlighted in the previous section.

As before, households' problem is to maximise equation (B.1) subject to (B.2). Using forward substitution in the flow budget constraint (B.2), using (B.3) and imposing the transversality condition

$$\lim_{T\to\infty}\Big(\prod_{k=0}^{T}\frac{1}{r_{t+k}}\Big)b_{t+T}=0 \tag{B.9}$$

we get

$$b_{t-1}=\frac{1}{1-\beta}c_t-y+T_t-\sum_{j=1}^{\infty}\Big(\prod_{k=0}^{j-1}\frac{1}{r^e_{t+k}}\Big)(y-T^e_{t+j}) \tag{B.10}$$

where r^e_{t+k} and T^e_{t+j} denote agents' expectations regarding the interest rate prevailing from $t+k$ to $t+k+1$ and the tax at $t+j$.

Let $r^e_{t+k}\equiv\tilde{R}$ and $T^e_{t+j}\equiv\tilde{T}$. The optimal consumption rule is therefore given by

$$c_t=(1-\beta)\{b_{t-1}+y-T_t+\frac{1}{r_t}\{\frac{\tilde{R}}{\tilde{R}-1}\}(y-\tilde{T})\} \tag{B.11}$$

Optimal consumption is an increasing function of the current wealth, b_{t-1} and a decreasing function of both current and expected interest rates and of current and expected endowments net of taxes. As before, we want to characterise the fiscal policy which ensures equilibrium in the goods market, assuming that the bond policy function is given by equation (B.4). The next proposition, similar in spirit to proposition 10, answers the question.

Proposition 11 *Assume that the government fixes the amount of bonds at the RE level, $b^{RE} > 0$, and that $\tilde{R} > 1$. Then the higher are the expected tax liabilities, the lower is the current tax rate the government has to set.*

Proof. Forwarding equation (B.11) the expected one-period-ahead consumption is

$$\widetilde{E}_t c_{t+1} = (1-\beta)(b + y - \tilde{T} + \frac{1}{\tilde{R}}\frac{\tilde{R}}{\tilde{R}-1}(y - \tilde{T}) \tag{B.12}$$

Assuming a logarithmic utility function the interest rate is given by

$$\frac{1}{r_t} = \beta \frac{c_t}{(1-\beta)(b + y - \tilde{T} + \frac{1}{\tilde{R}}\frac{\tilde{R}}{\tilde{R}-1}(y - \tilde{T})} \tag{B.13}$$

Inserting (B.13) into equation (B.11) we get

$$c_t = \frac{b + (y - \tilde{T})\frac{\tilde{R}}{\tilde{R}-1}}{b + (y - \tilde{T})\frac{\tilde{R}}{\tilde{R}-1}(1-\beta)}(1-\beta)(b + y - T_t) \tag{B.14}$$

It is immediate to show that if $\tilde{T} = (1-\beta)b$ and $\tilde{R} = \frac{1}{\beta}$, which is the REE, then automatically $c_t = y \forall t$. Rearranging terms in equation (B.14) we express the current tax liabilities as a function of agents expectations' about future taxes and interest rates,

$$T_t = b + y - \frac{\left(b + (y - \tilde{T})\frac{\tilde{R}}{\tilde{R}-1}(1-\beta)\right) y}{(1-\beta)\left(b + (y - \tilde{T})\frac{\tilde{R}}{\tilde{R}-1}\right)} \tag{B.15}$$

Taking the derivatives of equation (B.15) with respect to $\tilde{T}$, after some algebra we get

$$\frac{\partial T_t}{\partial \tilde{T}} = -\frac{\tilde{R}}{\tilde{R}-1}(1-\beta)by < 0 \tag{B.16}$$

■

B.4 Proof of Proposition b

Let the perceived law of motion be equal to

$$\hat{u}_{c,t} = d_{t-1}\hat{g}_{t-1} + \hat{\epsilon}_t \tag{B.17}$$

Substituting prices from the households' optimality conditions into the period by period budget constraint and loglinearizing around the rational expectations equilibrium we get

$$\bar{b}u_{\bar{c}}\hat{b}_{t-1} + \bar{b}u_{\bar{c}\bar{c}}\bar{c}\hat{c}_t = (u_{\bar{c}\bar{c}}\bar{c} + u_{\bar{c}})\bar{c}\hat{c}_t - (u_{\bar{l}\bar{l}}(1-\bar{l}) - u_{\bar{l}})\bar{l}\hat{l}_t + \beta\bar{b}u_{\bar{c}}\tilde{E}_t\hat{u}_{c,t+1} + \beta\bar{b}u_{\bar{c}}E_t\hat{b}_t \tag{B.18}$$

where $\bar{x} = x^{stst}$.

Substituting for the bond policy function under rational expectations and for the resource constraint we get

$$\hat{c}_t = \frac{[u_{\bar{l}\bar{l}}(1-\bar{l}) - u_{\bar{l}}]\bar{g}\hat{g}_t - \bar{b}u_{\bar{c}}\psi^b_{RE}(1-\beta\rho_g)\hat{g}_t + \beta\bar{b}u_{\bar{c}}d_{t-1}\hat{g}_t}{\bar{c}[\bar{b}u_{\bar{c}\bar{c}} - (u_{\bar{c}\bar{c}}\bar{c} + u_{\bar{c}}) + u_{\bar{l}} - u_{\bar{l}\bar{l}}(1-\bar{l})]} \tag{B.19}$$

from which follows that

$$\frac{\partial \hat{u}_{c,t}}{\partial d_{t-1}} = \frac{-\beta\bar{b}u_{\bar{c}}\hat{g}_t}{\bar{c}[\bar{b}u_{\bar{c}\bar{c}} - (u_{\bar{c}\bar{c}}\bar{c} + u_{\bar{c}}) + u_{\bar{l}} - u_{\bar{l}\bar{l}}(1-\bar{l})]} \tag{B.20}$$

being $\hat{u}_{c,t} = -\hat{c}_t$. Assuming $u_{c,t} = log(c_t) + l_t$, the right side of equation (B.20) becomes $\frac{-\beta\bar{b}}{-\bar{b}+\bar{c}^2}$. The E-stability condition is satisfied iff $\frac{-\beta\bar{b}}{-\bar{b}+\bar{c}^2} < 1$. The intertemporal budget constraint implies that in steady state it has to be the case that $\bar{b}(1-\beta) = \bar{c}\bar{l}$. Hence using the resource constraint the learnability condition becomes

$$c > \frac{1-\bar{g}}{2} \tag{B.21}$$

Because of the assumed utility function, we can show that the inequality in (B.21) is always satisfied. Using the government FOC with respect to consumption we get that $c_t = \frac{1}{1+\Delta}$, where Δ is the Lagrange multiplier associated with the implementability condition $E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t}c_t - u_{l,t}(1-l_t) = b_{-1}u_{c,0}$. $E_0 \sum_{t=0}^{\infty} \beta^t l_t = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Delta-(1+\Delta)g_t}{1+\Delta} = 0$, where the first equality holds for the specific utility function and the second one holds assuming that $b_{-1} = 0$. If $g_0 = \bar{g}$, $E_0 \sum_{t=0}^{\infty} \beta^t g_t = \frac{\bar{g}}{1-\beta}$, from which $\Delta = \frac{\bar{g}}{1-\bar{g}}$. Inserting into optimal consumption under rational expectations we get that $c_t = (1-\bar{g}) > \frac{1-\bar{g}}{2}$.

B.5 Proof of Proposition 7

First we show that constraints (2.3), (2.13) and (2.14) imply (2.19).

Consider the period-by-period budget constraint after substituting for the household optimality conditions:

$$b_{t-1}(g_t) = \frac{u_{c,t}(g_t)s_t(g_t)}{u_{c,t}(g_t)} + \beta\frac{\gamma^H_t b_t(H)}{u_{c,t}(g_t)}\pi(H) + \beta\frac{\gamma^L_t b_t(L)}{u_{c,t}(g_t)}\pi(L) \tag{B.22}$$

where $s_t \equiv c_t - \frac{u_{l,t}}{u_{c,t}}(1-l_t)$, $b_t(i)$ for $i = H, L$ is the amount of state-contingent bond holdings, and γ^i_t for $i = H, L$ are the (state contingent) marginal utilities that agents expect in the next period and $\pi(i) = \pi(g_{t+1} = g^i|g_t)$ for $i = H, L$. Since the fiscal authority has full information about the economy, in t it will issue state-contingent bonds such that the budget constraint in the next period is satisfied for any realisation of the government shock. Forwarding (B.22) one period we get:

$$b_t(H) = \frac{u_{c,t+1}(H)s_{t+1}(H)}{u_{c,t+1}(H)} + \beta\frac{\gamma^H_{t+1}b_{t+1}(H)}{u_{c,t+1}(H)}\pi(H) + \beta\frac{\gamma^L_{t+1}b_{t+1}(L)}{u_{c,t+1}(H)}\pi(L) \tag{B.23}$$

$$b_t(L) = \frac{u_{c,t+1}(L)s_{t+1}(L)}{u_{c,t+1}(L)} + \beta\frac{\gamma^H_{t+1}b_{t+1}(H)}{u_{c,t+1}(L)}\pi(H) + \beta\frac{\gamma^L_{t+1}b_{t+1}(L)}{u_{c,t+1}(L)}\pi(L) \tag{B.24}$$

Substituting (B.23) and (B.24) into (B.22), and multiplying both sides by $x_{t-1} \equiv [\gamma^H_{t-1}I(g_t = g^H) + \gamma^L_{t-1}I(g_t = g^L)]$

$$\begin{aligned} b_{t-1}(g_t)x_{t-1} = u_{c,t}s_t\frac{x_{t-1}}{u_{c,t}} + \frac{x_{t-1}}{u_{c,t}}\beta\{&\frac{\gamma^H_t}{u_{c,t+1}(H)}[u_{c,t+1}(H)s_{t+1}(H)+ \\ &\beta\gamma^H_{t+1}b_{t+1}(H)\pi(H) + \beta\gamma^L_{t+1}b_{t+1}(L)\pi(L)]\pi(g_{t+1} = g^H|g_t) + \frac{\gamma^L_t}{u_{c,t+1}(L)}[u_{c,t+1}(L)s_{t+1}(L)+ \\ &\beta\gamma^H_{t+1}b_{t+1}(H)\pi(H) + \beta\gamma^L_{t+1}b_{t+1}(L)\pi(L)]\pi(g_{t+1} = g^L|g_t)\} \end{aligned} \tag{B.25}$$

Define $W_t = \frac{x_{t-1}}{u_{c,t}}$. Keeping substituting forward we get

$$b_{t-1}(g_t)x_{t-1} = E_t\sum_{j=0}^{\infty}\beta^j\prod_{k=t}^{t+j}W_k u_{c,t+j}s_{t+j} \tag{B.26}$$

Define

$$\tilde{A}_{t+j} \equiv \prod_{k=t}^{t+j}W_k \tag{B.27}$$

and multiply each side of (B.26) by $H_{t-1} \equiv \prod_{k=0}^{t-1}W_k$. We get

$$b_{t-1}(g_t)x_{t-1}H_{t-1} = E_t\sum_{j=0}^{\infty}\beta^j A_{t+j}u_{c,t+j}s_{t+j} \tag{B.28}$$

where $A_{t+j} = H_{t-1}\prod_{k=t}^{t+j}W_k$.
Notice that the A_{t+j} has a recursive formulation given by:

$$\begin{aligned} A_{t+j} &= \prod_{k=0}^{t-1}W_k \times \prod_{k=t}^{t+j}W_k = \prod_{k=0}^{t+j}W_k = \prod_{k=0}^{t+j-1}W_k \times \frac{x_{t+j-1}}{u_{c,t+j}} = \\ &= A_{t+j-1}\frac{x_{t+j-1}}{u_{c,t+j}} \end{aligned} \tag{B.29}$$

It follows that

$$A_t \equiv H_t = H_{t-1}\frac{x_{t-1}}{u_{c,t}} \tag{B.30}$$

To prove the reverse implication, take any feasible allocation $\{c_{t+j}, l_{t+j}\}_{j=0}^{\infty}$ that satisfies equation (2.19). Then it is always possible to back out the state-contingent bond holding such that the period-by-period budget constraint is satisfied.

From equation (B.28), define

$$b_{t-1}(g_t) = E_t \sum_{j=0}^{\infty} \beta^j A_{t+j} u_{c,t+j} s_{t+j} \frac{1}{x_{t-1} H_{t-1}} \tag{B.31}$$

It follows that

$$b_t(g_{t+1}) = E_{t+1} \sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j} \frac{1}{x_t H_t} \tag{B.32}$$

$$\begin{aligned} b_{t-1}(g_t) &= \frac{A_t u_{c,t} s_t}{x_{t-1} H_{t-1}} + E_t \sum_{j=1}^{\infty} \beta^j A_{t+j} u_{c,t+j} s_{t+j} \frac{1}{x_{t-1} H_{t-1}} = \\ &= \frac{A_t u_{c,t} s_t}{x_{t-1} H_{t-1}} + \beta E_t \sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j} \frac{1}{x_{t-1} H_{t-1}} = \\ &= \frac{A_t u_{c,t} s_t}{x_{t-1} H_{t-1}} + \frac{\beta}{x_{t-1} H_{t-1}} E_t \{ x_t H_t [\frac{\sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j}}{x_t H_t}] \} = \\ &= \frac{A_t u_{c,t} s_t}{x_{t-1} H_{t-1}} + \frac{\beta}{x_{t-1} H_{t-1}} E_t \{ x_t H_t [E_{t+1} \frac{\sum_{j=0}^{\infty} \beta^j A_{t+1+j} u_{c,t+1+j} s_{t+1+j}}{x_t H_t}] \} = \\ &= \frac{A_t u_{c,t} s_t}{x_{t-1} H_{t-1}} + \frac{\beta}{x_{t-1} H_{t-1}} E_t \{ x_t H_t B_t \} = \end{aligned} \tag{B.33}$$

Using (B.30) we get

$$\begin{aligned} b_{t-1}(g_t) &= s_t + \frac{\beta}{u_{c,t}} E_t ([\gamma_t^H I(g_{t+1} = g^H) + \gamma_t^L I(g_{t+1} = g^L)] b_t(g_{t+1})) = \\ &= s_t + \frac{\beta}{u_{c,t}} [\gamma_t^H b_t(g_{t+1} = g^H) \pi(g_{t+1} = g^H | g_t) + \gamma_t^L b_t(g_{t+1} = g^L) \pi(g_{t+1} = g^L | g_t)] \end{aligned} \tag{B.34}$$

B.6

Attach the multipliers Δ, $\beta^t \pi_t(g^t) \lambda_{1,t}(g^t)$, $\beta^t \pi_t(g^t) \lambda_{2,t}(g^t)$, $\beta^t \pi_t(g^t) \lambda_{3,t}(g^t)$ and $\beta^t \pi_t(g^t) \lambda_{4,t}(g^t)$ to constraints (2.20), (2.22) for $i = H, L$, (2.23) and to (2.21).

The Lagrangian is

$$\begin{aligned} \mathcal{L} =& E_0 \sum_{t=0}^{\infty} \beta^t \{ u(c_t, l_t) + \Delta (A_t (u_{c,t} c_t - u_{l,t}(1 - l_t))) \\ &+ \lambda_{1,t} ((\gamma_t^H - \gamma_{t-1}^H) I(g_t = g_L) + (\gamma_t^H - (1 - \alpha_t) \gamma_{t-1}^H - \alpha_t u_{c,t}) I(g_t = g_H)) \\ &+ \lambda_{2,t} ((\gamma_t^L - \gamma_{t-1}^L) I(g_t = g_H) + (\gamma_t^L - (1 - \alpha_t) \gamma_{t-1}^L - \alpha_t u_{c,t}) I(g_t = g_L)) \\ &+ \lambda_{3,t} (1 - l_t - c_t - g_t) \} + \lambda_{4,t} (A_t - A_{t-1} \frac{\gamma_{t-1}^H I(g_t = g^H) + \gamma_{t-1}^L I(g_t = g^L)}{u_{c,t}}) - \Delta A_0 u_{c,0} b_{-1} \end{aligned}$$

Assuming $b_{-1} = 0$, the first-order necessary conditions $\forall t \geq 0$ are:

- c_t:

$$
\begin{aligned}
&u_{c,t} + \Delta A_t(u_{cc,t}c_t + u_{c,t}) - \lambda_{1,t}\alpha_t u_{cc,t} I(g_t = g^H) \\
&- \lambda_{2,t}\alpha_t u_{cc,t} I(g_t = g^L) + u_{cc,t}\lambda_{4,t}A_{t-1}\frac{\gamma_{t-1}^H I(g_t = g^H) + \gamma_{t-1}^L I(g_t = g^L)}{u_{c,t}^2} = \lambda_{3,t}
\end{aligned} \tag{B.35}
$$

- l_t:

$$
u_{l,t} + \Delta A_t(u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda_{3,t} \tag{B.36}
$$

- γ_t^H:

$$
\begin{aligned}
&\lambda_{1,t} - \beta E_t\{\lambda_{1,t+1} I(g_{t+1} = g^L) + (1 - \alpha_{t+1})\lambda_{1,t+1} I(g_{t+1} = g^H) + \\
&+ \frac{\lambda_{4,t+1}A_t}{u_{c,t+1}} I(g_{t+1} = g^H)\} = 0
\end{aligned} \tag{B.37}
$$

- γ_t^L:

$$
\begin{aligned}
&\lambda_{2,t} - \beta E_t\{\lambda_{2,t+1} I(g_{t+1} = g^H) + (1 - \alpha_{t+1})\lambda_{2,t+1} I(g_{t+1} = g^L) + \\
&+ \frac{\lambda_{4,t+1}A_t}{u_{c,t+1}} I(g_{t+1} = g^L)\} = 0
\end{aligned} \tag{B.38}
$$

- A_t:

$$
\Delta(u_{c,t}c_t - u_{l,t}(1 - l_t)) + \lambda_{4,t} - \beta E_t\lambda_{4,t+1}\frac{\gamma_t^H I(g_{t+1} = g^H) + \gamma_t^L I(g_{t+1} = g^L)}{u_{c,t+1}} \tag{B.39}
$$

From equation (B.39)

$$
\lambda_{4,t} = -\Delta(u_{c,t}c_t - u_{l,t}(1 - l_t)) + \beta E_t\lambda_{4,t+1}\frac{\gamma_t^H I(g_{t+1} = g^H) + \gamma_t^L I(g_{t+1} = g^L)}{u_{c,t+1}} \tag{B.40}
$$

Multiplying both sides by A_t we get

$$
\begin{aligned}
\lambda_{4,t}A_t &= -\Delta A_t(u_{c,t}c_t - u_{l,t}(1 - l_t)) + \beta E_t\lambda_{4,t+1}\frac{A_t\gamma_t^H I(g_{t+1} = g^H) + \gamma_t^L I(g_{t+1} = g^L)}{u_{c,t+1}} = \\
&= -\Delta A_t(u_{c,t}c_t - u_{l,t}(1 - l_t)) + \beta E_t\lambda_{4,t+1}A_{t+1}
\end{aligned} \tag{B.41}
$$

where the last equality follows from equation (2.21).

Iterating forward we obtain

$$\lambda_{4,t}A_t = -\Delta E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1 - l_{t+j})) \tag{B.42}$$

Inserting (B.42) into (B.35) we get

$$\begin{aligned} &u_{c,t} + \Delta A_t(u_{cc,t}c_t + u_{c,t}) - \lambda_{1,t}\alpha_t u_{cc,t} I(g_t = g^H) \\ &- \lambda_{2,t}\alpha_t u_{cc,t} I(g_t = g^L) - \Delta \frac{u_{cc,t}}{u_{c,t}} E_t \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1 - l_{t+j})) = \lambda_{3,t} \end{aligned} \tag{B.43}$$

B.7

Under rational expectations the following equalities hold

$$\begin{aligned} \gamma^H_{t-1} &= u_{c,t}(g_t = g^H)\forall t \\ \gamma^L_{t-1} &= u_{c,t}(g_t = g^L)\forall t \end{aligned}$$

which implies that $A_t = A_{t-1} = 1 \forall t$. The Lagrangian collapses to

$$\begin{aligned} \mathcal{L} = &E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t, l_t) + \Delta(u_{c,t}c_t - u_{l,t}(1 - l_t)) + \lambda_{1,t}(\gamma^H_t - \gamma^H_{t-1}) + \lambda_{2,t}(\gamma^L_t - \gamma^L_{t-1}) \\ &+ \lambda_{3,t}(1 - l_t - c_t - g_t)] - \Delta u_{c,0} b_{-1} \end{aligned}$$

The first-order conditions with respect to γ^H_t and γ^L_t are

- γ^H_t :

$$\lambda_{1,t} = \beta E_t \lambda_{1,t+1} \tag{B.44}$$

- γ^L_t :

$$\lambda_{2,t} = \beta E_t \lambda_{2,t+1} \tag{B.45}$$

which imply that the only solution is $\lambda_{1,t} = \lambda_{2,t} = 0$. The first-order condition with respect to consumption and leisure are

$$u_{c,t} + \Delta(u_{cc,t}c_t + u_{c,t}) = \lambda_{3,t} \tag{B.46}$$

$$u_{l,t} + \Delta(u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda_{3,t} \tag{B.47}$$

which are exactly the optimality conditions found in a rational expectations framework (see Lucas and Stokey (1983)) in which expectations do not depend on the current consumption level and in which there is no distortion into agents' beliefs that the government has to manipulate optimally.

B.8 Proof of Proposition 8

The period-by-period budget constraint implies that the following equality

$$b_{t-1}A_{t-1}\gamma_{t-1} = \sum_{j=0}^{\infty} \beta^j A_{t+j}(u_{c,t+j}c_{t+j} - u_{l,t+j}(1 - l_{t+j})) \tag{B.48}$$

Equation (2.28) can be written as

$$\begin{aligned} &u_{c,t} + \Delta A_t(u_{cc,t}c_t + u_{c,t}) - \lambda_{1,t}\alpha_t u_{cc,t} - \\ &\Delta \frac{u_{cc,t}}{u_{c,t}} b_{t-1}A_{t-1}\gamma_{t-1} = u_{l,t} + \Delta A_t(u_{l,t} - u_{ll,t}(1 - l_t)) \end{aligned} \tag{B.49}$$

The optimal policy under rational expectations is to run a balanced budget. Assuming that agents' initial belief over marginal utility of consumption is close to the actual marginal utility under rational expectations, $A_{t-1} \approx 1$. Moreover, beliefs close to rational expectations implies that also the optimal fiscal policy under learning is close to that under rational expectations, therefore $b_{t-1} \approx 0$. Under this assumption and the one that α_t is small enough, equation (2.28) can be rewritten as

$$u_{c,t} + \Delta A_t(u_{cc,t}c_t + u_{c,t}) \approx u_{l,t} + \Delta A_t(u_{l,t} - u_{ll,t}(1 - l_t)) \tag{B.50}$$

Using equation (2.29) we get that

$$u_{c,t} = \frac{2\Delta\gamma_{t-1}A_{t-1}}{-1 + \sqrt{1 + 4\Delta A_{t-1}\gamma_{t-1}}} \tag{B.51}$$

1. Taking the first derivative of (B.51) we get

$$\frac{\partial u_{c,t}}{\partial \gamma_{t-1}} = \frac{-1 + \sqrt{1 + 4\Delta\gamma_{t-1}} - 2\Delta\gamma_{t-1}\frac{1}{\sqrt{1+4\Delta\gamma_{t-1}}}}{(-1 + \sqrt{1 + 4\Delta\gamma_{t-1}})^2} \tag{B.52}$$

   which is positive being $\Delta > 0$ and $\gamma_{t-1} > 0$.
   The second derivative with respect to γ_{t-1} is equal to

$$\frac{\partial^2 u_{c,t}}{\partial \gamma_{t-1}^2} = \frac{\frac{4\Delta^2\gamma_{t-1}}{\sqrt{x}x}(-1 + \sqrt{x})^2 - \frac{4\Delta}{\sqrt{x}}(-1 + \sqrt{x})(-1 + \sqrt{x} - \frac{2\Delta\gamma_{t-1}}{\sqrt{x}})}{(-1 + \sqrt{x})^4} \tag{B.53}$$

   where $x = \sqrt{1 + 4\Delta\gamma_{t-1}}$. After some algebra, it can be shown that equation (B.53) is negative, being $\Delta > 0$ and $\gamma_{t-1} > 0$

2. Imposing $T(\gamma^*, A_{t-1}) = \gamma^*$ we get that the fixed point is given by

$$\gamma^* = 1 + \Delta A_{t-1} > 0 \tag{B.54}$$

3. Imposing $\gamma_{t-1} < \frac{3}{4\Delta}$ implies that $-1 + \sqrt{1 + 4\Delta\gamma_{t-1}} < 1$. It follows that the learnability condition

$$\frac{\partial u_{c,t}}{\partial \gamma_{t-1}} \mid_{\gamma^*} < 1 \tag{B.55}$$

is satisfied because

$$-\frac{2\Delta\gamma^*}{\sqrt{1 + 4\Delta A_{t-1}}} < 0 \tag{B.56}$$

B.9 Proof of Proposition 9

Suppose that if in period t $u_{c,t} - \gamma_{t-1} < 0$, then $u_{c,t+1} - \gamma_t < 0$ as well. This condition is satisfied for small enough values of α_t.

First notice that consumption under rational expectations is a special case of the previous equation when the agents' belief about today's marginal utility coincides with the actual marginal utility and the product of the past ratios between expected and actual marginal utility is 1.

$$c^{RE} = c^L(u_{c,t}, 1, b_{-1}) \tag{B.57}$$

Consumption under learning converges to the one under rational expectations if and only if

$$lim_{t\to\infty} log A_t = 0 \tag{B.58}$$

since under rational expectations $logA_t \equiv 0$. When the government expenditure shock is constant, the law of motion for A_t is given by

$$A_t = A_{t-1}\frac{\gamma_{t-1}}{u_{c,t}} \tag{B.59}$$

Substituting backwards in the definition of (B.59) and taking log we get that

$$lim_{t\to\infty} log A_t = lim_{t\to\infty} \sum_{j=0}^{t} log\frac{\gamma_{j-1}}{u_{c,j}} \tag{B.60}$$

Using the result in proposition 8 we know that the expected marginal utility converges to the actual one. Define N the time when this happens. Then

$$lim_{t\to\infty} log A_t = \sum_{j=0}^{N} log\frac{\gamma_{j-1}}{u_{c,j}} \tag{B.61}$$

Being the finite sum of finite numbers, $logA_t$ converges to a strictly positive value for initial pessimistic belief and to a strictly negative value for initial optimistic belief.

Since the implementability condition under learning does not converge to the one under rational expectations, the allocation does not either.

To show the second part of the proposition, assume for simplicity the same utility function as in equation (2.29). Equation (B.47) implies that

$$c_t = \frac{1}{1+\Delta} \forall t \geq 1 \tag{B.62}$$

At $t = 0$, for any given b_{-1}, consumption under RE is equal to

$$c_0 = \frac{1 + \sqrt{(1 + 4\Delta b_{-1}(1+\Delta))}}{2(1+\Delta)} \tag{B.63}$$

Imposing the resource constraint, the implementability condition $\sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t - u_{l,t}(1 - l_t)) = u_{c,0} b_{-1}$ implies that

$$\begin{aligned} & l_0 + \sum_{t=1}^{\infty} \beta^t l_t = 1 - c_0 - g + \frac{\beta}{1-\beta}(1 - \frac{1}{1+\Delta} - g) = \\ & \frac{1-g}{1-\beta} - \frac{\beta}{1-\beta}\frac{1}{1+\Delta} = b_{-1} \frac{4(1+\Delta)^2 + (1 + \sqrt{(1 + 4\Delta b_{-1}(1+\Delta))^2}}{2(1 + \sqrt{(1 + 4\Delta b_{-1}(1+\Delta)))(1+\Delta)}} \end{aligned} \tag{B.64}$$

Denote the positive root of equation (B.64) as $\Delta^* = \Delta(b_{-1})$. Inserting Δ^* into equation (B.62) we get

$$c_t = \frac{1}{1+\Delta^*} = \frac{1}{1+\Delta(b_{-1})} \tag{B.65}$$

Let $c^L = lim_{t\to\infty} c_t^L(\gamma_{-1})$. The initial holding of bonds such that the allocation under learning converges to the one under rational expectations starting with that amount of bond is defined by the equation

$$c^L = \frac{1}{1+\Delta(b_{-1})} \tag{B.66}$$

B.10

To discuss the quality of the learning equations used by agents to predict one-step-ahead state-contingent marginal utility of consumption, we use the Epsilon-Delta Rationality criterion (EDR), as formalized in Marcet and Nicolini (2003). Define

$$\pi^{\epsilon,T} \equiv P(\frac{1}{T}\sum_{t=0}^{T}[u_{c,t} - \gamma_{t-1}]^2 < \frac{1}{T}\sum_{t=0}^{T}[u_{c,t} - E_{t-1}u_{c,t}]^2 + \epsilon)$$

which is a function of ϵ, δ and T. $E_{t-1}u_{c,t}$ denote the expectations of an agent who knows the whole economic structure of the model. The learning mechanism (2.15) with $\alpha_t = \alpha$ satisfies EDR for (ϵ,δ,T) if $\pi^{\epsilon,T} \geq 1 - \delta$. Table 2.7 shows this $\pi^{\epsilon,T}$ for different values of ϵ (across columns) and T (across rows). Reported values are computed out of 1000 simulations. Since the discount factor is equal to 0.95, each period corresponds to one year. After 10 years, there is an 80 percent probability that the prediction error made by boundedly rational agents is at most 3 percent higher than the prediction error made by fully rational agents. We conclude that along the transition agents use a learning scheme that generate quite good forecasts.