



Universitat de Lleida

## MODELS UNDER UNCERTAINTY TO SUPPORT SOW HERD MANAGEMENT IN THE CONTEXT OF THE PORK SUPPLY CHAIN

Sara Verónica Rodríguez Sánchez

---

**ADVERTIMENT.** La consulta d'aquesta tesi queda condicionada a l'acceptació de les següents condicions d'ús: La difusió d'aquesta tesi per mitjà del servei TDX ([www.tesisenxarxa.net](http://www.tesisenxarxa.net)) ha estat autoritzada pels titulars dels drets de propietat intel·lectual únicament per a usos privats emmarcats en activitats d'investigació i docència. No s'autoritza la seva reproducció amb finalitats de lucre ni la seva difusió i posada a disposició des d'un lloc aliè al servei TDX. No s'autoritza la presentació del seu contingut en una finestra o marc aliè a TDX (framing). Aquesta reserva de drets afecta tant al resum de presentació de la tesi com als seus continguts. En la utilització o cita de parts de la tesi és obligat indicar el nom de la persona autora.

**ADVERTENCIA.** La consulta de esta tesis queda condicionada a la aceptación de las siguientes condiciones de uso: La difusión de esta tesis por medio del servicio TDR ([www.tesisenred.net](http://www.tesisenred.net)) ha sido autorizada por los titulares de los derechos de propiedad intelectual únicamente para usos privados enmarcados en actividades de investigación y docencia. No se autoriza su reproducción con finalidades de lucro ni su difusión y puesta a disposición desde un sitio ajeno al servicio TDR. No se autoriza la presentación de su contenido en una ventana o marco ajeno a TDR (framing). Esta reserva de derechos afecta tanto al resumen de presentación de la tesis como a sus contenidos. En la utilización o cita de partes de la tesis es obligado indicar el nombre de la persona autora.

**WARNING.** On having consulted this thesis you're accepting the following use conditions: Spreading this thesis by the TDX ([www.tesisenxarxa.net](http://www.tesisenxarxa.net)) service has been authorized by the titular of the intellectual property rights only for private uses placed in investigation and teaching activities. Reproduction with lucrative aims is not authorized neither its spreading and availability from a site foreign to the TDX service. Introducing its content in a window or frame foreign to the TDX service is not authorized (framing). This rights affect to the presentation summary of the thesis as well as to its contents. In the using or citation of parts of the thesis it's obliged to indicate the name of the author.

---

---

**MODELS UNDER UNCERTAINTY TO SUPPORT SOW  
HERD MANAGEMENT IN THE CONTEXT OF  
THE PORK SUPPLY CHAIN**



Ph. D. Thesis

Sara Verónica Rodríguez Sánchez

Supervisor: Prof. Lluís Miquel Plà Aragonés

Department of Mathematics

University of Leida

---

ISBN: 978-607-00-3166-3

**Models under uncertainty to support sow herd management in the context of the pork supply chain**

Dissertation held in February 2010

---

## Preface

In 2002 when I was studying Industrial Engineering in Mexico, I got a scholarship by the AECI (Agencia Española de Cooperación Inter-universitaria) to do a staying at the University of Lleida, Spain. The project was related to decision models applied to the Pig sector. During this period I had the opportunity to know the Prof. Lluís M. Plà Aragonés who was my tutor. Prof. Plà showed me how Operations Research can be applied to the livestock sector. It was a great experience. After two months I had to come back to Mexico to finish my degree. However, I kept in contact with the Prof. Luis Plà who motivated me to undergo a PhD, and here I am.

At the end of this project, I would like to express my deepest acknowledgement to all people who have shared with me this experience. First of all, I wish to express my deepest gratitude to my supervisor Prof. Lluís M. Plà, for leading me into the exciting field of research, for your support and patient. I am also grateful for the support and valuable input I have received from Prof. Anders R. Kristensen and Prof. Victor Albornoz. You have been my three cornerstones in this project. I feel a deepest admiration for your research but also for your life style, thanks so much. Moreover, I would like also to thank to Prof. Javier Faulin and Prof. Tina Birk Jensen for your valuable contribution to the development of the Thesis.

I would like to extend my gratitude to all the people at the Department of Mathematics, but also the secretaries, janitors and security people from the University of Lleida who I met at any time, any week day.

Nevertheless I would like to do a special dedicatory to the Prof. Silvia Miquel who always support me in the difficult times and for the peace that I felt in her house.

Moreover, along this way I have receive the support of many people specially to Family Crespo-Romero, Family Macía-Badia, Family Plà-Aldabó, Family Albonoz-Miranda and Family Dahlmann, thanks to let me stay at your warm home, at least for a while.

---

I wish to express my warmest gratitude to the farm SAT la Vall, for allowing me to enter and learn from them. Thanks to the workers who always were so nice and share their pragmatic knowledge with me.

And I keep to the end the most special acknowledgment to my family, thanks for your permanent concern throughout this period abroad and thanks to be with me inside of my heart, it was not easy to be so far from you. But you always make me feel you were next to me.

There is a long way done behind this project. It seems that I am in the summit of my dreams but I just realise that it is a saddle point, the start of a new life new dreams and new aspirations.

---

## Abstract

A noticeable change in the structure of the Spanish pork sector has been observed in recent years. Pig farms have become more and more specialized and the size of their operations has been increasing. Moreover, modern pig farms have tended to integrate and coordinate their operations into Pork Supply Chain (PSC). Thus, the overall aim of this thesis was to formulate a set of models to support sow herd management and piglet production in a pork supply chain context, giving practical answers to relevant questions often asked by decision makers. Hence, the main strategic and tactical decisions regarding sow herds and piglet production management in a pork supply chain context were considered.

Basically, four models under uncertainty were developed. The first one was a linear programming formulation of a semi-Markov model to design pig facilities. It showed that herd distribution based on physiological states and movements between facilities were useful to calculate the room needs for each sow facility. The formulation considers recent EU regulations regarding animal welfare and the impact on economic cost of housing facilities. Ongoing with animal welfare issues, a framework for the integration of clinical signs into a sow replacement model was developed. The sow replacement model used in the framework was a multi-level hierarchical Markov process using Bayesian updating. The results showed how the incorporation of clinical signs in sow replacement models led to better culling policies through more efficient detection of the weakest sows in the herd. The next two models formulated under finite time horizon showed the herd distribution or structure moving to the steady state. Temporary shocks in parameters or transitory perturbations were better represented by finite time horizon models where scenarios collected part of the uncertainty of the system. The use of Linear Programming led to incorporate herd constraints more easily than in a Markov Decision Process. Finally the two-stage stochastic programming model with recourse showed to be a suitable tool to deal with the uncertainty of the system through scenarios. Additional benefits for practical purpose were the scheduling of

---

purchasing of gilts, planning piglet production and replacement policy, all under a rolling time horizon scheme.

Nowadays, with the current structure of the sector, it is reasonable to think about models capable of solving more accurately problems involving two or more stages of the chain, and integrating them all together in some information system, in order to improve the management of the PSC. Hence the models presented in this thesis are suitable tools to deal with main strategic and tactical decisions in sow herds producing piglets in a general PSC context.

---

## Resumen

En los últimos años se ha observado un gran cambio en la estructura del sector porcino. Las explotaciones porcinas están siendo cada vez más especializadas mientras que el tamaño de sus operaciones ha ido en aumento. Además, están tendiendo a integrar y coordinar sus operaciones en cadenas de suministro (PSC en inglés). Así, el objetivo general de la tesis fue formular un conjunto de modelos para apoyar la gestión del rebaño de cerdas reproductoras y la producción de lechones dentro de un contexto de cadena de suministro, dando respuestas prácticas a preguntas relevantes a menudo cuestionadas por quienes toman las decisiones. De ahí que, las principales decisiones estratégicas y tácticas relacionadas con la gestión del rebaño en un contexto de cadena (PSC) fueron consideradas.

Básicamente, cuatro modelos bajo incertidumbre fueron desarrollados. El primero formula un programa lineal de un modelo semi-markoviano para el diseño de las instalaciones de granjas. Se mostró que la distribución de la manada basada en estados fisiológicos y movimientos entre instalaciones es útil para el cálculo de los espacios necesarios para cada instalación. La formulación consideró las recientes regulaciones de la Unión Europea relacionadas con el bienestar animal, además del coste de las instalaciones. El segundo modelo desarrolló un marco para la integración de las señales clínicas dentro de un modelo de reemplazamiento. La formulación corresponde a un proceso Markoviano jerárquico multi-nivel, con actualización de datos a través de redes Bayesianas. Los resultados mostraron cómo la incorporación de las señales clínicas ha llevado a mejores políticas de selección a través de una eficiente detección de cerdas débiles en el rebaño. Los siguientes dos modelos son formulados bajo un horizonte de tiempo finito y muestran la distribución o estructura del rebaño moviéndose hacia el estado estable. Las variaciones temporales en parámetros o perturbaciones transitorias fueron incorporadas a través de modelos de horizonte finito donde diferentes escenarios representaron la incertidumbre del sistema. El uso de Programación lineal permitió explícitamente incorporar restricciones de rebaño más fácilmente respecto a la formulación con Procesos de decisión de Markov. Finalmente el



---

modelo de programación estocástica de dos etapas con recurso mostró ser una herramienta factible para tratar con la incertidumbre del sistema a través de escenarios. Beneficios adicionales fueron la programación de cerdas jóvenes, la planeación de la producción y decisiones de reemplazo, todo bajo un esquema de horizonte de tiempo rodante.

Hoy en día, con la actual estructura del sector, es razonable pensar en modelos capaces de resolver con mayor precisión los problemas que afectan a dos o más etapas de la cadena, e integrarlos a todos en algún sistema de información, con el fin de mejorar la gestión global de la cadena. Por lo que los modelos presentados en esta tesis son herramientas adecuadas para hacer frente a las principales decisiones estratégicas y tácticas de la gestión del rebaño de cerdas reproductoras y la producción de lechones en un contexto de cadena de suministro.

---

## Resum

En els últims anys s'ha observat un canvi radical en l'estructura del sector porcí. Les explotacions porcines estan sent cada vegada més especialitzades i la grandària de les seves operacions ha anat en augment. A més, les explotacions porcines modernes estan tendint a integrar i coordinar les seves operacions en Cadenes de subministrament del sector porcí (PSC en anglès). L'objectiu general de la tesi va ser formular un conjunt de models per a donar suport a la gestió del ramat de truges reproductores i la producció de garrins dintre d'un context de cadena de subministrament, donant respostes pràctiques a preguntes rellevants sovint preguntades per qui prenen les decisions. Així les principals decisions estratègiques i tàctiques relacionades amb la gestió del ramat de truges reproductores i la producció de garrins en un context de cadena de subministrament (PSC) van ser considerades.

Bàsicament quatre models sota incertesa van ser desenvolupats. El primer va ser la formulació d'un programa lineal d'un model semi-Markovià per al disseny de les instal·lacions de granges. Es va mostrar que la distribució del ramat basada en estats fisiològics i moviments entre instal·lacions fou útil pel càlcul dels espais necessaris per a cada instal·lació. La formulació va considerar les recents regulacions de la Unió Europea relacionades amb el benestar animal i el cost de les instal·lacions. Continuant amb aspectes relacionats amb el benestar animal, es va desenvolupar un marc per a la integració dels senyals clínics dintre d'un model de reemplaçament. Aquest model correspon a un procés Markovià jeràrquic multi-nivell, que a més realitza una actualització de dades a través de xarxes Bayesianas. Els resultats van mostrar com la incorporació dels senyals clínics en els models de reemplaçament ha dut a millors polítiques de selecció a través d'una eficient detecció de truges reproductores febles en el ramat. Els dos models següents formulats sota un horitzó temporal finit han mostrat la distribució o estructura del ramat evolucionant cap a l'estat estable. Variacions temporals en paràmetres o pertorbacions transitòries van ser representades a través de models d'horitzó finit on diferents escenaris van capturar la incertesa del sistema. L'ús de Programació

---

lineal va permetre explícitament incorporar restriccions de ramat més fàcilment que en una formulació amb Processos de decisió de Markov. Finalment el model de programació estocàstica de dues etapes amb recurs va mostrar ser una eina factible per a tractar amb la incertesa del sistema a través d'escenaris. Beneficis addicionals van ser la compra de verres, la planificació de la producció i decisions de reemplaçament, tot sota un esquema d'horitzó de temps rodant.

Avui dia, amb l'actual estructura del sector, és raonable pensar en models capaços de resoldre amb major precisió els problemes que afecten a dos o més etapes de la cadena, i integrar-los a tots en algun sistema d'informació, amb la finalitat de millorar la gestió global de la cadena. Els models presentats en aquesta tesi són eines adequades per a plantar cara a les principals decisions estratègiques i tàctiques de la gestió del ramat de truges reproductores i la producció de garrins en un context de PSC.

---

## Content

Chapter 1	General introduction	1
Chapter 2	Spanish Pork supply chain context	15
Chapter 3	A Linear Programming Formulation of a Semi-Markov Model to Design Pig Facilities	47
Chapter 4	Optimal replacement policies and economic value of clinical observations in sow herds	67
Chapter 5	Modelling tactical planning decisions in breeding farms through a linear optimization model.	105
Chapter 6	A two-stage stochastic programming model for scheduling replacements in sow farms	127
Chapter 7	Conclusions and Outlook	153
Appendix	Curriculum Vitae	159

---

# General Introduction

Sara V. Rodríguez

University of Lleida, Department of Mathematics, Jaume II, 73 E-25001 Lleida, Spain

## 1 Background

The European pork industry is currently being rapidly redefined by new economic, ecological and social forces (Backus and Dijkhuizen, 2002). Globalization, consumer concerns about environment, animal welfare, food safety, food quality, technological developments, new science, and people evolving social and cultural attitudes are transforming the society in general and pork consumption, production, and markets in particular (Trienekens et al., 2009). Therefore, a noticeable change in the structure of the sector has been observed. Margin of benefit per kg of pig meat produced has been reduced in recent years. Pork enterprises are becoming more and more specialized while the size of their operations is increasing, (Balogh et al., 2009). This fact has been simultaneously performed by a general reduction of the number of pork enterprises (in particular individual farms managed by a family) but incrementing the size and overall production. To face the fierce competition on actual markets and to get competitive advantages and stability against the more often variations in gross margins derived from market prices evolution, modern pork enterprises have tended to integrate and coordinate their operations into bigger structures like the so-called Pork supply chain (figure 1) using closer vertical coordination linkages (den Ouden, 1996b).

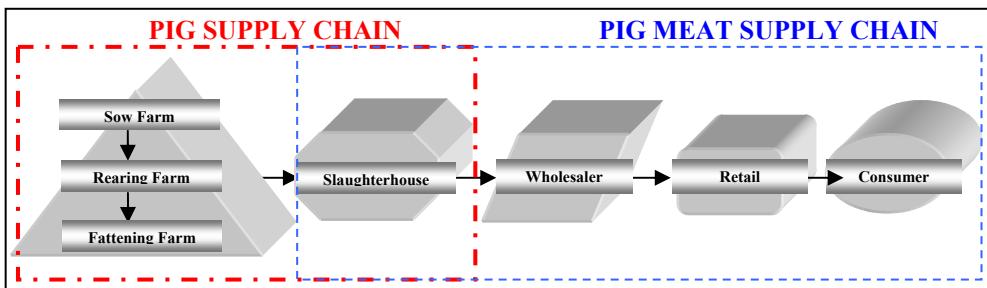


Figure 1. Distribution structure of the Pork Supply Chain.

The pork supply chain is usually managed by cooperatives of farmers or integrator companies. Within the global Pork supply chain conception two main subchains can be distinguished: pig supply chain focused on the production of pigs and the pig meat supply chain focused on the production of meat and derived products.

Slaughterhouses play a role of bridge and regulator of production between these chains. In fact, slaughterhouses represent the demand of consumers; even more true when big surfaces (that trade the biggest amount of products) treat directly with them.

## **2 Problem statement**

The PSC is basically structured in three different stages according to the final product and the activities involved; breeding, rearing and fattening. Slaughterhouses are also included to represent the demand. Specialization has permitted the existence of farms devoted to only one productive stage although different associations between these basic production units are possible. For instance there are sow farms devoted to produce piglets, rearing farms devoted to raise piglets (around of 25kg approximately) and fattening farms which devoted to raise piglets coming from rearing farms and deliver them to the slaughterhouse (with a weight around of 100kg). Integrators or cooperatives integrate and coordinate the production operations between these farms. Hence, farmers have individual contracts with integrators or cooperatives trough which they sell all their production. But at the same time they are subject to some guidelines issued by consultants or advisers belonging to or hired by the companies or cooperatives. Many times this is reflected in bonus or penalties on final reward paid to the farmer. These driving forces have transformed the sector in an agile and competitive system where changes are adopted quickly. For instance, quite recently safe food, animal welfare and environmental questions have appeared as consumer concerns. To this respect, new EU regulations are issued and they have been affecting past management practices. In June 2001 the European Ministers of Agriculture decided to approve European Directive 91-630-EEG. This directive comprises welfare prescriptions for the pig sector, among which a minimum space of sows and growing/fattening pigs is prescribed. The last has lead farmer to redesign farms [http://eur-lex.europa.eu/LexUriServ/site/en/oj/2001/l\_316/l\_31620011201en00010004.pdf accessed 15 December 2009]. Moreover, environmental legislation establishes a



maximal density or number of animals kept per hectare of cultivated land (Jongbloed, 1999). This situation had brought several challenges to the integrator companies and farmers' cooperatives who at the same time translated these challenges to individual farmers clearly stating these new requirements in contract agreements reshaping new company goals. The enforcement of this regulation for most of the farmers could turn out to be prohibitive for future expansion on herd size and derived net returns. But since small and medium-size farms have succeeded to produce at competitive prices and given present regulations, the future of farmers will depend more on their ability to enhance their economic performance by improving productive efficiency rather than increasing herd size. Productive efficiency is mainly affected by an accurate herd management. Therefore one way of improve the productive efficiency is to handle more precise information of the system to better on farm decisions and this can be performed by herd models providing or exploring optimal or quasi-optimal herd management strategies (Plà, 2007; Kristensen et al., 2008). Although most of the herd models have been developed mainly for farms without considering greater structures or production networks, they are also valid with slight modifications for building Pork supply chain management models.

Pork supply chain management models represents a new focus on how links between organizational units are set to best serve customers needs and improve the competitiveness of a supply chain as a whole (Stadtler, 2005). Aspects like the limited self life of the product, no inventory (living animals or perishable product), price variability as well as the importance played by factors such as sustainability, food quality and safety make the underlying supply chain complex and harder to manage (see Ahumada and Villalobos, 2009). Here decision models can play an important role to improve management decisions because they support the manager when exploring and understanding pork supply chain behaviour. A scarce literature related to the whole Pork supply chain models is found (den Ouden 1996a; de Castro, 2001; Bloemhof et al., 2003; Plà, 2006; Balogh et al., 2009). However, an analysis on their main parameters shows in all of these models the number of piglets (or pigs) as the central variable needed to operate the chain.

Regarding basics of pig production it is recognized that sow farms devoted to piglet production are more complex to manage than rearing or fattening farms. There, piglet production is intimately related with the reproduction process of the sow herd. But not only biologic issues affect the complexity, also market competition, contract agreements and new legislation, which are drawing new bounds in piglet production and limit management alternatives for the decision maker, manager, pig specialist or pig adviser or consultant. Consequently, several decisions models representing the productive and reproductive behaviour of the sow herd have been developed up to now (see Plà, 2007 for a survey). Nevertheless the sector is evolving to an integration of production in fewer hands and new legislation issues are regulating the activity. These aspects are not well covered by past models, although they are a reference to be taken into account. Hence, there is a need to reformulate and extend existing management models to the new context of production or even make new proposals under the pork supply chain view.

A common trait of existing sow herd management models is that they have been developed under an infinite time horizon assuring the stability of the structure of the herd (steady state distribution). Models under infinite time horizon are suitable tools for sow herd productivity assessment, evaluating sow herd performance or to analyse alternative herd management strategies, Upton (1989). For instance, simulation (Singh, 1986; Lippus et al., 1996) and Semi-markov chain models (Plà et al., 2004) have been used to plan housing facilities for sows. However, issues concerning new EU regulations on animal welfare have just to be included. Other issues are gaining relevance in the context of pig supply chain management as, limited supply of gilts, target of weekly farrowings, maximization of occupancy in lactation facilities or even seasonal variation of fertility. They are relevant for practical purpose and better modeled by models under finite time horizon where the decisions are made up front, with a rolling horizon in which the values of the variables of the problem can be updated periodically. Finite time horizon can better capture the variations of herd dynamics in terms of herd structure. The explicit inclusion of time in the formulation give a flexibility that makes these models less rigids for practical purposes and better fit for being included in a whole Pork supply chain model.

Moreover most of the published sow herd models are devoted to tackle the sow replacement problem which is one of the most important decisions in sow herd management and with a direct economic impact for the farmer (Kristensen, 1993). However this problem remains also important for the pork supply chain management because most integrators or cooperatives supplies gilts to their members. Therefore, shocks in supplies or unstable culling rates may impact on the overall chain and are wanted to be avoided for maintaining a steady flow of animals through the chain. In practice culling decisions are usually based on productive and reproductive sow performance, but also on other more qualitative traits more difficult to quantify (e.g. health status or scores on body conditions). This matter concerns also another modern hot topic, the animal welfare problem. Hence the addition of health indicators in sow replacement models seems to fulfil actual requirements in modern pork chain production. A poor health status is undesirable and risky, it can cause problems like failure to conceive, abortion and increased sow mortality which raise the production cost and, hence, reduce the profit margin. Nevertheless a poor health status is not only regarded as an economical problem, but also as a significant indicator of an animal welfare problem.

With respect to methodological improvements impacting on herd management, these are on optimisation and simulation (Plà, 2007). More specifically, as optimisation models already used in sow herd modelling, Markov decision processes have been improved to represent fairly sow herd systems. These improvements comprise multi-level hierarchical Markov process with decisions on multiple time scales, Bayesian updating and herd specific parameter estimation (Kristensen, 1988, 1993, 2004a, 2004b; Plà et al., 2001, 2003). Thus, these improvements can turn out to be efficient in the calculation of sow herd management models able also to handle optimal replacement policies enlarging the state and action sets for individual sows.

Main improvements in the simulation methodology impact on validation methods relying on statistical methods like metamodeling and optimization by simulation (Kleijnen, 1995).

It is well known the existence of variability within the main productive variables in piglet production system (e.g. prolificacy, fertility rate, conception rate and prices). Markov decision models have considered implicitly the inclusion of uncertainty in some of the parameters, however variability associated to their results is neglected. Then, this modelling approach represents risk neutral decision makers and it is the common outcome when just expectations are taken. This reveals a lack of models taking into account the uncertainty associated to the results, as was already pointed by Plà (2007).

The uncertainty of the productive and economic parameters of the farm increases with the time. For instance the selling price of piglets today can be similar to tomorrow but one month later they have been would vary greatly. The productive behaviour of the herd today can not be similar one year later some variations are registered. Then the decision maker has to make decisions today with different levels of uncertainty that conditions the future of the farm. Here another alternative to represent uncertainty in stochastic models is by considering different scenarios taking advantage of the fact that probability distributions governing the uncertain parameters are known or can be estimated (Rockafellar and Wets, 1991). In particular, one of the most common and simple approach in stochastic programming is the two stage stochastic formulation. Here the first stage lead the farmer make a decision now taking into consideration the possible scenarios or variations in the future, and the second stage acts as recourse in to compensates for any bad effects that might have been experienced as a result of the first-stage decision (see Birge and Louveaux, 1997).

Finally, actual advances in computer science and the availability of powerful solvers for solving linear programming models have lead Linear programming technique to be an important tool for solving optimization instances in different fields, in particular in sow herd management. This is so because there exist ways to formulate most of the common stochastic problems as more complex but equivalent linear programming models, getting the advantage of using reliable solvers and avoiding the implementation of a tailored-made software for a particular instance. Other advantages of linear programming relies on explicitly

incorporation of herd constraints, setting of time managing different time horizons and the possible use of non time-homogeneous parameters, as well as the calculus of shadow prices.

Explicitly incorporation of uncertainty had been pointed out as the general weakness of the original Linear Programming models, however the deterministic equivalent formulation of the stochastic programming was firstly studied by Dantzig (1955) and later deployed and complemented by several authors (see Birge and Louveaux, 1997; Ruszczyński and Shapiro, 2003; Wallace and Ziemba, 2005; Kall and Meyer, 2005).

### **3 Scope and aim**

This thesis is developed in the Spanish context of the pork supply chain. More specifically, the stress is put in the main unit governing all the chain from the beginning: the sow farms producing piglets. Then, sow herd management and piglet production are the main focus of interest. The overall objective of this research is to formulate a set of models to support sow herd management and piglet production in pork supply chain context, giving practical answers to relevant questions for decision makers. The models should be farm-specific and consequently parameters must be calculated for each farm under study. Furthermore the intention is that these models are flexible enough for a possible coordination of the flow of animals in a pork supply chain management optimization model. Therefore main strategic and tactical decisions were involved. Within this frame, the specific objectives are:

- 1) To survey the role of sow herds and piglet production models into the pork supply chain management.
- 2) To reformulate and extend Markov decision models. Different aspects concerning animal welfare respecting recent European Union regulations were included.

- 3) To explore Linear programming and modern solvers as a powerful alternative approach to compute and solve equivalent decision models under uncertainty: either semi-Markov decision models or two-stage stochastic programming models.
- 4) To develop and assess the suitability of finite time horizon models for tactical decisions on field conditions.
- 5) To approach the planning of purchase and replacement decisions by using stochastic programming models in particular two stage stochastic programming. The response of the herd structure over time was another outcome of interest in these models.

These objectives are all developed in several chapters conceived as independent papers themselves that are briefly presented in the next section (figure 2).

#### **4 Outline of the thesis**

The thesis contains results from theoretical and methodological analysis on the sow herd management and piglet production in a pig supply chain context. In figure 1 the conceptual architecture of the thesis is presented, pointing out the set of models developed and the level of decisions covered on. This thesis has been written in seven chapters, five of them with a structure of paper, embraced by the first one, this chapter gives a general introduction of the thesis and the last one that presents the general conclusions derived from the study. These seven chapters are shortly presented as follows:

**Chapter 1** presents a general introduction of the thesis. A background of the productive structure of the pork sector is given, stressing specificities dealt in this study. Later on the problem statement on which is focussed the research is presented. The scope and overall aim of the research follows. Finally the general outline of the thesis is pointed out.

**Chapter 2** set out a general survey of the role of the sow herds and piglet production system within the modern pig supply chain management and analyse the general perspective of the planning models developed for sow herd management emphasising those gaps in which the thesis is going to deep.

**Chapter 3** presents the formulation of a linear optimization model of a semi-markov decision process to design sow farm facilities. The present approach takes into account the new EU regulations regarding animal welfare, economic cost of facilities and optimal replacement strategy to design sow farms. The model is solved by an equivalent linear programming model and compared with traditional methods.

**Chapter 4** describes a framework for integration of a so-called Weak Sow Index (WSI) based on observation of several clinical signs into a sow replacement model. The novelty of the model is the inclusion of new variables concerning animal health and representing their impact on herd management. This approach aims the study of the influence of clinical signs on the optimal replacement policy. The net rewards obtained of the optimal policies under different clinical observations lead to estimate the economic value of individual clinical signs. The sow replacement model use to the framework is a multi-level hierarchical Markov process using Bayesian updating. The WSI is constructed by use of a Bayesian network. The Bayesian updating technique applied in the model was based on a dynamic linear model.

**Chapter 5** develops a linear optimization model for scheduling replacements and purchased decisions. A medium-term planning horizon based on weekly periods is considered. A highlight point of the formulation is the inclusion of a farrowing target quota and a limited supply of gilts. These issues try to represent in a more realistic way the vertical integration scheme of piglet production.

**Chapter 6** extends the LP model presented in the previous chapter into a stochastic programming model. The uncertainty in the biological parameters such as litter size, mortality rate, abortion and prices is explicitly incorporate in the formulation

thought the use of scenarios. Then a two stage stochastic programming model for scheduling replacements in a rolling finite time horizon is presented in this chapter. The decisions making at first stage are the decisions that the farmer will implement on real farm conditions while decision making at second stage attempting capture the decisions make by the farmer in different scenarios but they are not implementing. Before to reach the second stage a new runs will be done.

**Chapter 7** is devoted to the general conclusions of the thesis, mainly referred to methodological and practical issues of particular importance for the optimisation of sow herd management within the pig supply chain management context. The chapter finishes pointing out future developments.

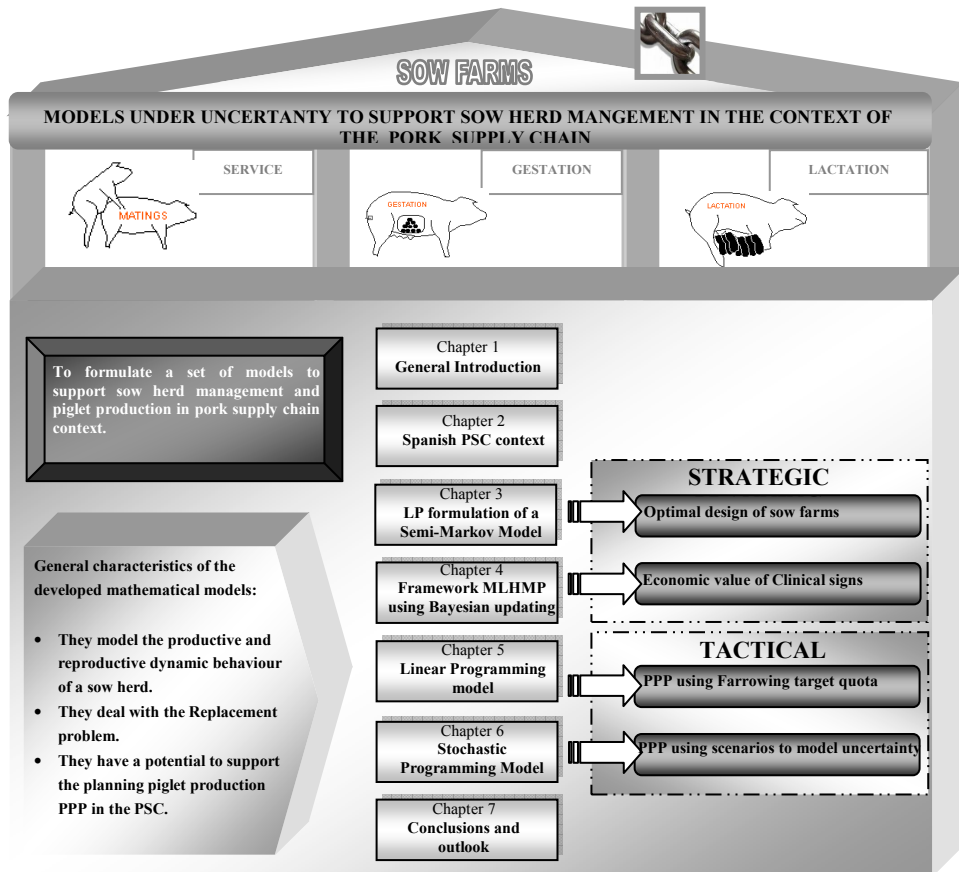


Figure 2. Conceptual architecture of the thesis.



## References

- Ahumada, O., Villalobos, J.R., 2009. Application of Planning Models in the Agri-Food Supply Chain: A review. *European Journal of Operational Research* 4, 399-412.
- Backus, G., Dijkhuizen, A.A., 2002. Kernkamp lecture: The future of the European pork chain. Allen D. Leman Swine Conference, Minnesota, USA, 8-11.
- Balogh, P., Ertsey, I., Fenyves, V., and Nagy, L., 2009. Analysis and optimization regarding the activity of a Hungarian Pig sales and Purchased Cooperation. *Studies in Agricultural Economics* 109, 35-54.
- Birge, J.R., Louveaux, F., 1997. *Introduction to Stochastic Programming*. Springer Series in Operations Research; New York: Springer.
- Bloemhof, J.M., Smeets, C.M., and van Nunen, 2003. Supply chain optimization in Animal Husbandry. Erasmus Research Institute of Management. Report Series Research in Management, ERS-2003-043-LIS.
- Dantzig, G.B., 1955. Linear Programming under Uncertainty. *Management Science* 1, 197-206.
- de Castro, R., 2001. *Optimització del procés productiu d'engreixament del porcí*. Ph.D. dissertation. University of Gerona, Spain.
- den Ouden, M., 1996a. Economic modelling of pork production-marketing chains. PhD Thesis. Wageningen Agricultural University, Netherlands.
- den Ouden, M., Dijkhuizen, A.A., Huirne, R.B.M., Zuurbier, P.J.P, 1996b. Vertical Cooperation in Agricultural Production Marketing Chains, with special Reference to Product Differentiation in Pork. *Agribusiness* 12, 277-290.
- Jongbloed, A.W., Poulsen, H.D., Dourmad, J.Y., van der Peet-Schwering, C.M.C., 1999. Environmental and legislative aspects of pig production in the Netherlands, France and Denmark. *Livestock Production Science* 58, 243-249.
- Kall, P. and Meyer, J., 2005. *Stochastic Linear Programming, Models, Theory, and Computation*. Springer.
- Kleijnen, J.P.C., 1995. Verification and validation of simulation models. *European Journal of Operational Research* 82, 145-162.
- Kristensen, A.R., 1988. Hierarchic Markov processes and their applications in replacement models. *European Journal of Operational Research* 35, 207-215.
- Kristensen, A.R., 1993. Markov decision techniques applied to the animal replacement problem. Dr.Sc. Dissertation. Dina KVL. Dep. of Animal Science and Animal Health. The Royal Veterinary and Agricultural University. Copenhagen. Denmark.

- Kristensen, A.R., Søllested, T.A., 2004a. A sow replacement model using bayesian updating in a 3-level hierarchic Markov process. I. Biological model. *Livestock Production Science* 87, 13–24.
- Kristensen, A.R., Søllested, T.A., 2004b. A sow replacement model using bayesian updating in a 3-level hierarchic markov process. II. Optimization model. *Livestock Production Science* 87, 25–36.
- Kristensen, A.R., E. Jørgensen & N. Toft. 2008. *Herd Management Science I. Basic concepts*. SI Books, ISBN: 8779970788.
- Lippus, A.C., Jalvingh, A.W., Metz, J.H.M., Huirne, R.B.M., 1996. A Dynamic Probabilistic Model for Planning Housing Facilities for Sows. *Transactions of the ASAE* 39, 1215-1223.
- Plà, L.M., 2001. A markov sow herd model for on-farm decision support. Dr.Sc. Dissertation. University of Lleida. Spain.
- Plà, L.M., Pomar, C., Pomar, J., 2003. A Markov decision sow model representing the productive lifespan of sows. *Agricultural Systems* 76, 253-272.
- Plà, L.M., Babot, D. Pomar, J., 2004. A Mathematical Model for Designing Sow Farms. *International Transactions in Operational Research* 11, 485-494.
- Plà, L.M., 2006. Tactical Supply Chain model of pig production. *Proceedings of the Second Meeting of the EURO Working group on OR in Agriculture and Forest Management. The Journal of Agricultural Science, Volume 144, Issue 05, 467-472.*
- Plà, L.M., 2007. Review of Mathematical Models for Sow Herd Management. *Livestock Science* 106, 107-119.
- Rockafellar, R.T., Wets, R.J-B., 1991. Scenarios and policy aggregation in optimization under uncertainty, *Mathematics of Operations Research* 16, 119-147.
- Ruszczynski, A., Shapiro, A., Editors 2003. *Stochastic Programming. Handbooks in Operations Research and Management Science, Vol. 10.* North-Holland, NY.
- Singh, D., 1986. Simulation-aided Capacity Selection of Confinement Facilities for Swine Production. *Transactions of the ASAE* 29, 807–815.
- Stadtler, H., 2005. Supply Chain management and advanced planning-basics, overview and challenges. *European Journal of Operational Research* 163, 575-588.
- Trienekens, J., Petersen, B., Wognum, N., and Brinkmann, D., 2009. *European pork chains.* Wagening Academic Publishers, 285pp.
- Upton, M., 1989. Livestock productivity assessment and herd growth models. *Agricultural Systems* 29, 149–164.
- Wallace, S.W. and Ziemba, W.T., Editors 2005. *Applications of Stochastic Programming.* SIAM Publications, Philadelphia.



## **Spanish Pork supply chain context**

Sara V. Rodríguez<sup>1</sup>, Lluís M. Plà<sup>1</sup>, Javier Faulin<sup>2</sup>

<sup>1</sup>University of Lleida, Department of Mathematics, Jaume II, 73 E-25001 Lleida, Spain

<sup>2</sup>Public University of Navarre, Department of Statistics and Operations Research, Los Mangnolis Building, E-31006 Pamplona, Spain

Submitted to: Supply Chain Management. An international journal.

## **Abstract**

The structure of the pork sector is changing. In the last years pig farms are becoming more and more specialized while the size of their operations is increasing. Moreover they have tended to integrate and coordinate their operations into bigger structures like the so-called Pork supply chain using closer vertical coordination linkages. This paper presents a brief description of the pork supply chain and their management. The importance and complexity of decision making are pointed out. A survey in the literature reveals planning models to sow herd management have paid little attention to issues involving the new context of production. Then some existing gaps in the literature that we believe should be addressed in the near future are identifying.

*Keywords: Pork supply chain management, sow herd models, pork sector.*

## 1 Introduction

The pig sector is one of the major economic activities of livestock production. Among all the meat production, pig meat is the first meat produced worldwide. The European Union (EU-27 members) is the second-largest producer in the world, after China. Germany, Spain, France, Poland and Denmark are the biggest EU producers. For instance, in 2008 these five countries produced more than 50% of the EU pig meat production (23.0%, 15.7%, 9.1%, 8.1% and 7.7% respectively). Nevertheless, the relevance of the pig sector not only regards production but also consumption. EU pig meat annual consumption was approximately 43kg/person/year, although this average could vary considerably among states (source: FAOstat).

The modern Pork industry has evolved greatly as a result of the globalization process, advances in technology, scientific developments, and changes in social and cultural attitudes (Trienekens et al., 2009). In most EU countries the pork business has become less attractive for traditional farmers and more for modern farm companies. Hence, a noticeable change in the EU Pork sector has been observed, which is mainly represented by a decrease in the number of pig farms while the size of the remaining or new ones has increased.

New trends, such as economic, ecological and social forces are redefining the EU pork industry (Backus and Dijkhuizen, 2002). Consumer concerns about environment, animal welfare, food safety and food quality are new challenges for the EU pig sector. More strict regulations concerning such topics have been approved and as a consequence, have impacted on production costs. Then, present competitive markets require better tackle these challenges and confront raising costs within the present markets. Moreover, the current structural changes are also motivated because many of the new technologies are capital intensive, and hence, the adoption of many of the actual production systems has been accompanied by an increase in the scale and degree of specialization of farm operations. Hence, pork production has therefore been progressively concentrated in larger and more

specialized units (Hobbs et al., 1998; de Castro, 2001; Perez et al., 2009; Taylor, 2006). EU pork industry is witnessing moving towards the integration and coordination of their operations into bigger structures like the so-called Pork supply chain (Hobbs and Young, 2000; Taylor, 2006; van der Vorst et al., 2007). Then, the profile of the typical farm is changing from family based, small-scale, independent firms to one in which larger firms are more tightly aligned across the production and distribution value chain (Porter, 1985).

Pork supply chain (PSC) is understood as a network of specialized pork units using closer vertical coordination linkages. These units are involved in different processes and activities that add value to the pig meat and address their production toward consumer demands. Current PSC structures have led modern pork units to raise global efficiency and provide them with competitive advantages and stability against the more often variations in gross margins derived from the evolution of market prices (den Ouden, 1996). Certainly, the new competitive strategy in farming today is that individual farm units no longer compete as solely autonomous entities, but rather as supply chains (Christopher, 1998). The success of a supply chain largely depends on the ability of the chain manager to effectively integrate and coordinate their units (van der Vorst et al., 2007).

In short, supply chain management is defined as the set of actions focused on finding the most effective and efficient way to coordinate and integrate the whole supply chain (Stadtler, 2008). Within the pork context, this set of actions is usually developed by cooperatives or integrator companies. *Planning* is much more important in the PSC because the production cycle is split into different production units that have to collaborate and be coordinated. This coordination refers to piglets produced in sow farms have to be transferred to rearing farms and these to fattening farms for, in the end, be slaughtered in abattoirs to satisfy customer's demand. Traditionally, judgement based on experience has been the basis for planning in the pork industry, but the complexity of a more closely vertical coordination has made it necessary to develop more formal planning methods assisted by the formulation and implementation of mathematical models. It has been recognized that quantitative models can play an important role in enhancing

the decision making process and exploiting the potential of yielding at lower costs and higher profits (Plà, 2007; Kristensen et al., 2008). Through supply chain models, researchers first, and chain managers after can better understand real PSC behaviour and how to manage it. But until now, literature on quantitative models in Pork supply chain management has been scarce. Therefore, the objectives of the paper are twofold. Firstly to describe the role of pig production in the pork supply chain mainly based on the Spanish experience. And secondly to stress the dominant importance of sow herd management regarding the coordination and planning of the whole pork supply chain management.

The organization of this paper is as follows: Section 2 presents a brief description of the pork supply chain; economic stages, vertical links and examples are pointed out. In Section 3, pork supply chain management is explained. In section 4, the importance of sow herd management in pork supply chain management is highlighted. Finally, in Section 5, discussion and some existing gaps in the literature that we believe should be addressed in the near future are identified.

## **2 The PSC structure**

PSC is composed by organizations in charge of procurement, production, processing, distribution, marketing of pig meat and derived chain products to final consumers (see figure 1). Then, PSC is referred to an integrated system that involves a set of inter-related processes in order to (1) procure raw material as feedstuffs, medicaments, gilts and semen doses; (2) employ these resources in the production of pig meat, which is the result of cooperation among different farms in the production of fattened pigs that are slaughtered in abattoirs; (3) process the pig meat and derived products; and (4) pack and distribute to costumers, making the information exchange among farm units taking part in the chain easy.

PSC can be split into two main sub-chains: *pig supply chain* focused on the production of pigs and the *pig meat supply chain* focused on the production of pig meat and derived products. Slaughterhouses play a role of bridge and regulator of production flow between these chains. In fact, slaughterhouses represent the



demand of pig meat for the pig supply chain; even truer when big surfaces trade directly with them (Figure 1).

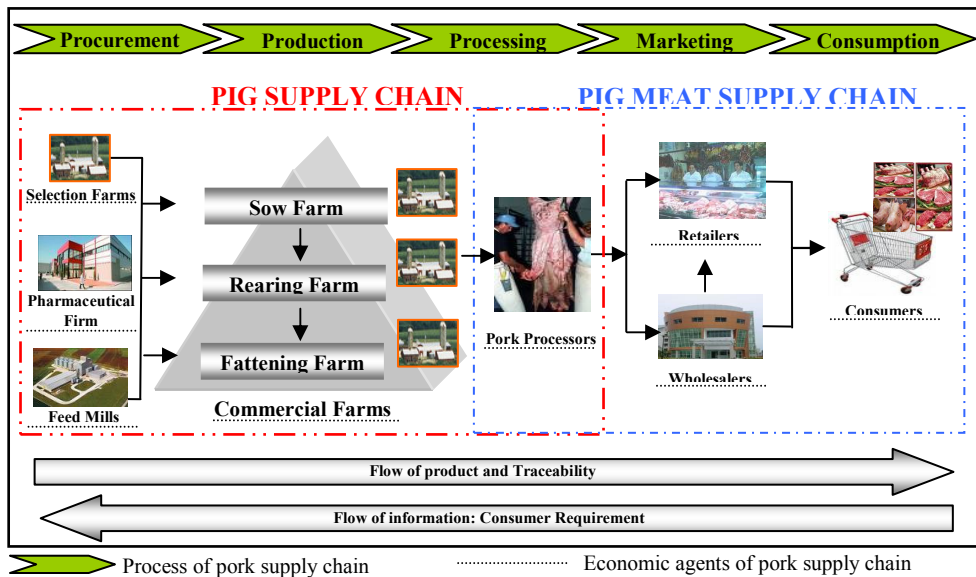


Figure 1. Simple representation of a Pork Supply Chain

### 2.1 Main processes of PSC

#### a) Procurement

It is the process of providing raw material for pig production, such as, concentrates, medicine, semen, and gilts. Feed mills, pharmaceutical firms and selection farms are the main agents at this stage. **Feed mills** supply concentrates to farmers. They must elaborate feedstuff with all required nutrients for a balanced diet. **Pharmaceutical firms** provide specialized medicine to prevention, treatment, and control of pig diseases. **Selection farms** provide the genetic basis of the commercial farms. Basically their function is to improve the genetic merit of sows and boars developing pure lines sharing traits of commercial interest, e.g. high feed conversion, fast growth, and high percentage of lean meat. Some of these traits are linked to either male or female animals, so that “male lines” and “female lines” have been developed, as well as crossing schemes serving to a variety of pig production purposes.

## **b) Production**

Pig production responds to a pyramidal model structured in three different stages according to the final product and the activities involved: breeding, rearing and fattening. Specialization has led to farms being devoted to only one productive stage, hence the existence of sow farms, rearing farms and fattening farms. Some companies may own different mixtures of these basic production units. **Sow farms** (also called farrowing farms) engage *piglet production*. There, the sow acts as a production unit, while piglets are the output. After weaning, piglets are transferred to rearing farms in order to continue the production process. **Rearing farms** involve the growing of piglets from weaning until at approximately a weight between 15 to 35 kg. **Fattening farms** involves feeding pigs from 20 kg until they are ready to slaughter, typically at the weight between 100 and 140 kg (Whittemore and Kyriazakis, 2006).

The transportation of gilts to sow farms, piglets to rearing farms and young pigs to fattening farms, is often done by trucks belonging to a transportation firm either integrated to PSC or well subcontracted by the producer or by abattoirs in the case of pigs being transferred to the slaughterhouse. The relevance of transportation is beyond health aspects. In general, it is accepted that transport activity may affect the welfare and meat quality of pigs (Perez et al., 2002).

## **c) Processing**

The largest part of the processing takes part in **slaughterhouses** where the product often heads for the market. According to Perez and de Castro (2009) the slaughtering process can be divided into four main stages: waiting time before stunning, slaughter, pork carcass classification and chilling the carcass. Afterwards, carcasses are marketed as a whole or processed further to valuable cuts (e.g. ham, cutlet, belly). After processing, distribution takes place, so the marketable product is distributed to marketing agents.

## **e) Marketing**

This is the process that deals with the selling and distributing of pig meat and derived product to the final consumer. **Wholesales and Retailers firms** including

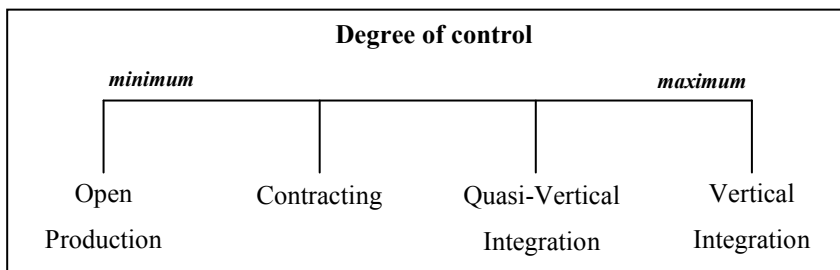
butchers are part of the marketing stage. Their main functions are: break bulk, allocating product to stores, retail packing, pricing and label display. The wholesales and retailers have direct contact with the consumers, and know their demands, so they usually pull on the rest of the chain to satisfy changing consumer preferences.

**f) Consumption.**

This is the last process of the pork supply chain. The main agent is the **Consumer**. Consumer requirements strongly influence pork supply chain behaviour. Several studies (Trienekens et al., 2009; Perez and de Castro, 2009; den Ouden, 1996; Backus and Dijkhuizen, 2002) highlighted animal welfare, environment, food quality and food safe as the major consumers concerns.

*2.2 Pork supply chain Links*

Processes in PSC are not necessarily performed by separated chain agents (Trienekens et al., 2009). Structural differences among pork supply chains in several EU countries regard the way either integrator or cooperative integrates and coordinates processes. In general, the link between integrator or cooperative and their agent is determined by the nature of the vertical coordination selected. Understanding vertical coordination as all possible economic arrangements involved in transferring resources among processes. Martinez and Reed (1996) introduced the overall structure of vertical coordination (figure 2), that can also be fixed in PSC structures.



Source: Martinez and Reed (1996).

Figure 2. Classification of vertical coordination according to the degree of control over individual agents.

*Open production and contracting* characterized EU pork markets. But today, there is instead, a trend to move towards more close vertical linkages such as *vertical cooperation* (or quasi-vertical integration) and *vertical integration* (den Ouden et al., 1996; Hobbs and Young, 2000; Perez and de Castro, 2000). This is so, because the production cost has risen, such as expenses incurred in the incorporation of consumer demands into production system (den Ouden et al. 1997a). However, they are likely to be lower in a more closely coordinated system (Veselská, 2005). The way to establish vertical coordination links is through contracting, an agreement between the integrator/cooperative firm and their agent. According to Mighell and Jones (1963), contracts can be classified into three broad groups: Market-specification contract, Production-management contract and Resource-providing contract. Under the last two contracts, for instance, pig firms devolve control over certain aspects of production in return for greater surety over access to markets or inputs and lower risk (Hobbs and Young, 2000).

Cooperatives often work under *Production-management contracts* (PMC) which entail more control over the productive system, but without full ownership. PMC allow cooperatives to specify and/or monitor production practices, input usage, and establish vertical cooperation links.

On the other hand integrator firms often work under *Resource-providing contracts* (RPC) which represent the greatest level of control over the productive system. Through RPC, integrators provide a market outlet, supervise production practices and supply resources. Basically, RPC is an arrangement in which an integrator engages pig farmers to take custody of the pigs and care for them. The pig farmer is paid a fee for the service provided.

So, resource-providing and Production-management contracts are actually the most commonly used in closer vertical coordination links. Such contracts may guarantee annual volumes for the farmers, and thereby get some reliability in the investment for improving facilities and processes. Hence, closer vertical coordination links among chain agents have led to a competitive advantage for the PSC. In fact, the strategic importance of vertical linkages was stressed by Porter (1985).

### *2.3 Examples of Pork supply chains in the European Union.*

Denmark and Spain, two of the main producers and exporters of pig meat in the European Union (<http://faostat.fao.org/>) present a very well differentiated vertical coordination links into their structure. To better understand different productive structures, the PSC coordination of both countries is going to be described.

#### *a) Vertical cooperation.*

For many years the well integrated and coordinated pork production structure of Denmark has been recognized as the cornerstone of their successful in pork markets (Hobbs et al., 1998). The Danish pork industry presents a strong and well integrated domestic infrastructure based on the co-operative system. Nowadays it has two co-operative slaughterhouses which work both abattoir and processing plant; Danish Crown and TiCan, (Hamann, 2006). Central to the Danish pork supply chain is Danske Slagterier, known in English as the Danish Bacon & Meat Council (DBMC). It is the umbrella organisation covering all Danish pork co-operatives and the pig farmers who own them. A number of companies related to the industry are also members of DBMC, see <http://www.danskeslagterier.dk>. The primary aim of DBMC is to safeguard and promote the interests of pig producers and the pork and bacon industry. It has three main tasks: 1) Research and development; 2) Sales, promotion, and information; 3) Service, disease prevention and control. In agreement to the way they structure their production, most of the agents of the DBMC establish a vertical cooperation under production management contracts.

#### *b) Vertical integration.*

In Spain, “the integrator” plays an important role in pig production. The farmers often subcontract his job and farm production to the integrator. Consequently, the integrator becomes the owner of the pig production, a characteristic related to vertical integration under resource providing contracts. The Vall Companys group provides a good example of this. The first experience of pig farm integration was in 1960, and since then the firm has grown year by year. Actually it becomes one of the biggest pork integrator in Spain. The Vall Companys group works under the

integration model: the farmer brings the facilities and his own work. The other services (animals, feed, veterinary control) are provided by the integrator. The Vall Companys group is also owner of Transegre, the logistics firm. Transegre was created due to the necessity of rigorous logistic management. Moreover recently the vertical integration of the company has been consolidated by the incorporation of several pig slaughterhouses; Patel and Cárnicas Frivall, and also by the incorporation of a new feed factory and pig artificial insemination centre providing a total guarantee of exhaustive genetic control (<http://www.vallcompanys.es>).

Besides the type of vertical coordination, there are regional, national and international pork supply chains crossing countries borders. We can take, VION Food group, an international food company with production and sales branches on all continents, as an example. VION Food group has four branches: VION Ingredients, VION Fresh Meat, VION Convenience and VION UK. Nevertheless, we will focus on VION Fresh Meat, both in Netherlands and in Germany. VION Fresh Meat is the division that produces high quality pork, beef and lamb (<http://www.vionfreshmeatnl.com>). Further examples of pork supply chains are given by Trienekens et al., (2009), specifically the pork supply chain of three large scale producing countries; 1) one in Central Europe (The Eichenhof chain, Germany), 2) one in Western Europe (Le Cochon de Bretagne, France) and 3) one in Southern Europe (The Iberian dry-cured ham chain, Spain). Moreover, the pork chain of a smaller scale producer (Greek regional pork chain, Greece), the pork chain of a large scale producing exporting country (De Groene Weg: the organic pork chain, Netherlands) and the pork chain of a relatively new EU member state (Managalica pork chain, Hungary) were also described.

### **3. PSC management**

PSC management is developed by cooperatives or integrators firms. It basically represents a new focus on how feed mills, commercial farms, slaughterhouses, wholesales and retailers to best serve customers needs and improve how the competitiveness of a supply chain are linked as a whole. Factors such as

uncertainty and the variability of related biological systems and market behaviour, in connection with new regulations concerning sustainability, food quality and safety make the underlying supply chain complex and hard to manage. Such complexity is also affected by whether the life time of the marketable product (piglets, pigs, pig meat) is finite, in which case, the opportunities to use inventory (living animals or perishable products) as a buffer against demand and transport variability are limited.

### 3.1. PSC management decisions

Decision-making is supported by planning activity (Stadler, 2005), where several levels of hierarchical decisions; strategic, tactical or operational are considered (Ahumada and Villalobos, 2009; Gupta and Maranas, 2003). Here, global trends, cost, sustainability and consumer demands represent the main key drivers for the pork value chain, see Figure 3.



Figure 3. Pork Supply Chain planning decisions, adapted from Stadler, 2005.

#### a) Strategic Level: Strategic Network Planning

Strategic or long-term planning decisions aim to identify the optimal strategic network of the supply chain (Vidal and Goetschalckx, 1997). Within the context of pork, the design of an optimal strategic network enables managers to determine the number, location, capacity and type of pigs farms, slaughterhouses, warehouses,

and retailers to be integrated, as well as the set of suppliers to be selected and the transportation channels to be used. For instance the strategic network planning tackles the problem of determining which pig-farmers are allocated to which slaughterhouse and/or which slaughterhouses are allocated to which wholesales (Bloemhof et al., 2003). In short, most of such decisions are related to location/allocation problems, (Melo, 2009; Manzini and Gebennini, 2008) emphasising the growing importance of environmental issues and economic impact. For that reason, factors such as whether pig producers locate near to land, whether slaughterhouses locate close to areas of pig production, and whether processing plants locate adjacent to abattoirs, are considered (Taylor, 2006).

### **b) Tactical Level: Master Planning Problem**

Tactical or medium-term planning decisions aim to optimally coordinate the flow of information and marketable product along the supply chain (Stadtler, 2005; Pibernik and Sucky, 2007). Specifically, *Master planning* within the pork context involves the production planning for gilts, piglets, pigs and pig meat, looking for the most efficient way to fulfil demand forecast over a medium-term planning interval. It also involves the planning for transport and logistic activities considering the whole PSC movements. So, in many cases Master planning supports contracts agreements, specifying some guidelines such as goals and targets for many farms and slaughterhouses, as well as the ordering of concentrates, and medicine, and determining the scheduling of transport and logistical activities at medium term.

### **c) Operative Level: Operative Planning**

Operative decisions are characterized by short timeframes over which chain managers address the exact sequencing of production while accounting for the various resources and timing constraints. Decisions made in the operative level enclose short term operational scheduling problems related with the flow of product or information trough chain. Problems regarding routing problems, distribution planning and transporting planning are usually founded at this level (Bookbinder, 2009). Decisions at an operative level are more oriented towards chain agent managers than PSC manger. They make an operative planning



following the guidelines of the Master planning. Hence decisions at an operative level involves tasks performed every day, such as determining sows to inseminate, sows to replace, piglets to wean, and pigs to send to slaughter.

### *3.2 Information systems serving to the pig supply chain management*

To carry out different management activities successfully, the manager must have analytical experience and access to data. The incorporation of information technologies into PSC operations has led to better quality and quantify of data collection and their management, increasingly potential information and raising the control over operations. This is so, due to huge amount of data flowing along the PSC. Among the set of information technologies, information systems (IS) are very helpful to provide different services in order to satisfy the necessity of data processing into information. VIT, BDporc, GTTT-GTE, CBK System, Pigplan, Pigwin are examples of modern IS found around Europe. At the beginning, most of these IS were oriented to collect raw data from farms and register main events that occurred there. The introduction of personal computers on farms in the 1990s complemented the original centralized IS and has led to several proposals of advanced Management Information Systems (MIS) addressed to individual farmers (Huirne, 1990). The functionality of these applications has been mainly based on important aspects, for example piglet production, the provision of feedstuffs or concentrates, breeding, sow replacement and waste disposal, which led to a significant impact on system performance. Furthermore, the adoption of such advanced tools is not clear (Kamp, 1999) in part due to complex models behind and more research oriented purpose. The trend towards PSC structures has renewed an increasing interest in MIS not only oriented to collect data but also processing and sharing information between different agents in the chain like the so called decision support systems (DSS). DSS handle refined models and methods capable of dealing with the complexity of PSC management. Practical DSS are actually a big concern for effective research. At the moment, few DSS include a formal model to analyse different decisions at different levels in the PSC being the farm level approach. AUSPIG (<http://www.auspig.csiro.au/>), PORGEP (<http://www.itp.asso.fr/>), TACT (Alsop et al., 1994), EMISP (Maliappis et al.,

1997) and DSS-IRTA (Plà et al., 2004) are examples of actual DSS in the literature. Few successful stories in practice, Kristensen (1993) and Kamp (1999) cited several reasons for that, particularly, both authors pointed towards “customized decision models” as a key element to take into account in the implementation of DSS in practice.

### *3.3 Mathematical models to support making decision in Pork supply chain*

In this section we present those studies dealing with models for the planning of activities in the Pork supply chain. Particularly, those models using optimization and simulation methodologies are considered, so thirty one research papers have been reported (Table 1, appendix A)

Pork supply chains are expected to grow in volume and size during the next years. This is so, as result of a greater integration by farmers, slaughterhouses, wholesalers and retailers. Then, most of the chain managers are looking for the development of an efficient and effective PSC in the long run. Nevertheless the literature about this topic is limited. From the list of articles in Table 1, the model developed by Bloemhof et al. (2003) is the only one model aimed at purely strategic network planning. Broadly, the research objective was to provide insight in the efficiency of distribution networks for a specific chain in the long run, by minimizing relevant costs (both transportation cost and facility cost). Regarding the characteristics of the chain, the problem responds a discrete two-level multi-period capacitated facility location problem, solved by mixed integer programming. In short, the model is focused on the distribution between farmers and slaughterhouses and the distribution between slaughterhouses and wholesalers.

On the other hand, the model developed by Plà (2006) deals with the planning of the PSC but under tactical level. It takes into consideration commercial farms and slaughterhouses as chain agents. Thus, the PSC is therefore structured in terms of number of farms per stage required to satisfy a weekly pre-stated demand. The demand is represented by the number of pigs processed by a slaughterhouse. The tactical formulation has lead seasonal effects on production to be considered.

Hence, the problem is formulated as a Linear programming problem where the objective function is to maximise revenues keeping an optimal herd structure over time.

The productive capacity in terms of quantity and quality may be diverse among chain agents. For instance, there are producers who are more committed to the quality than others. In this sense, it is needed a tool to support the distribution of pigs of different quality among slaughterhouses with different requirements for the maximization of sales revenues or an oriented production to fresh or manufactured consumption. In this context, Balogh et al. (2009) modelled a purchase and sale co-operative operating in the Northern Great Plain Region, (Hungary). For the formulation, linear programming techniques within a network model were applied. The model allowed the quantification of a number of pigs for a given farms to slaughterhouses, the maximum sales revenue, the delivery threshold prices, and an analysis of the impact co-operative members expert on sales revenue.

The current legislation and regulations imposed by the EU try to reflect people concerns about quality of animal products and the way of production, including food safety and animal welfare. These regulations have a great impact on pig production and will modify the feature of pig facilities and management tools which influence pig production cost. In this sense, Krieter (2002) evaluates different production systems in pig farming including economic, animal welfare and environmental aspects. This is done by using computer simulation. Essentially, the simulation model considers a vertically integrated system which models farrowing, weaning, fattening and slaughtering stages as well as the transportation of pigs between theses stages. He was not the only researcher interested in considering consumer's demands into pork supply chain, also den Ouden et al. (1997b) evaluates the development of pork supply chain concepts that take animal welfare concerns into account. Moreover, there is one research related to safe food, it was developed to van der Gaag et al. (2004) who develop a stochastic state-transition simulation model to simulate the spread of Salmonella from multiplier through slaughter.

The work developed by Gribkovskaia et al. (2006) falls down into operational planning. The paper is related to the transportation of pigs to slaughterhouses. So, the problem is formulated as a mixed integer programming model that combines vehicle routing and inventory control. Recently *the transportation problem* has been defined as one of the main challengers of operations research practice in agricultural value chains (Higgins, 2009). The doctoral thesis developed by Ljungberd (2006) although is applied to dairy sector; it may be used as reference for futures developments in transportation in the pork supply chain.

On the other hand, the total production cost of PSC is made up mainly by fattening, following by breeding and rearing stages, and in minor level slaughtering and processing (den Ouden et al., 1997b; Krieter, 2002). Feed cost is considered the major component. After feeding the second component of production cost is the replacement of sows. Hence, several sow herd models were found in the literature (Plà, 2007). They have been mainly developed to support sow replacement problem. Solutions based on dynamic programming (DP) and Markov decision processes (MDP) have been commonly proposed. The last seems the preferred approach given the discrete nature of the (re)production process. The dimensionality problem is an issue that has been successfully overcome in practice; however it can not be neglected in large systems. Most of these models were intended to represent the sow herd behaviour determining the effects of changes in reproduction or replacement. However, models developed by Tess (1983) and Pomar et al. (1991) besides considered reproduction or replacement also included the effect of changes in feeding and only the model developed by de Roo (1987) added genetic aspects. In general, these models were developed for research and teaching purposes, but only few of them showed a real application on practice (Plà, 2007). Another problem tackled by current models concerns delivery strategies of pigs to the slaughterhouse. The moment a pig gives the best margin of profit depends on the value of the pig derived from the live weight and the cost of intake and maintenance on the farm. The classical approach is based on DP, given that growth is a continuous time process, although others exist based on MDP and LP. Part of the complexity of the problem is due to the management of groups required. It is not possible to send pigs one by one to the slaughterhouse. On the other hand,

the variability within the group affects final reward, and the homogeneity of pigs in the group is crucial for getting good results in practice. The problem can easily become complex if additional variables such as meat quality, number of cuts, breed, freshness, for/not for human consumption and welfare issues regarding pre-slaughtering procedures are considered. Some of these variables may also be included as additional objectives. The problem of optimal marketing of slaughter pigs has been mainly studied by Chavas et al (1985); Broekmans (1992); Jørgensen (1993); Boland et al., (1993); Bailleul et al. (2000), Kure (1997); De Castro (2001); Niemi (2006); Olhlmann and Jones (2008).

Models tackled sow replacement problem (Plà, 2007) and optimal slaughter pig marketing mentioned above were deployed thinking in terms of single producers, with exception of the works developed by Kure (1997); De Castro (2001); Niemi (2006); Olhlmann and Jones (2008). However, many of them are vertically integrated in bigger companies, cooperatives or associations. And this reality should be incorporated in future research. Therefore, the optimization of the supply chain management arises naturally as a new approach to tackle actual problems of the sector. Not all the farms units are equally efficient, and competition leads managers to try to identify best practice farms and to improve the less efficient units by implementing best practices.

#### **4. Importance of sow herd management into the PSC.**

##### *4.1 Pig production in PSC models*

Due to closer coordination between agents of the PSC, the disruption of a process in a farm may not only influence their own performance but also successive linked stages. For instance, if a set of commercial sow farms suffers an epidemic of salmonellas, slaughterhouses/processors will be affected in the number of pigs to be processed. Pig production management is increasingly important in PSC because it has a cascade effect on the whole supply chain. This argument is also deducted from the analysis of several pork chain decision models (Krieter, 2002;

Bloemhof et al., 2003; Gribkovskaia et al., 2006; Plà, 2006; Balogh et al., 2009), which stated *the number of pigs* as the central variable needed to operate the chain.

At strategic level the calculation of yearly production of pigs is acceptable by multiplying the cage capacity in a region and a cycle factor i.e. the number of times the cage is occupied on average during a year, (Bloemhof et al. 2003). Nevertheless the same consideration to tactical planning may lead inefficiencies in the chain. Current information systems support manager mostly at operative level. Through registering events such as mating, farrowing and weaning date, litter size, age and others key figures. Thus, farmers can better manage individual sow behaviour at short period. Furthermore, sharing information of farm units integrated at the same company has let the chain manager knows the number of pigs flowing through the PSC at operative level. However, a lack of manager systems for the herd at tactical level is detected and for instance, no model planning the flow of pigs over the whole supply chain involving different farms is published until now.

Modern pig production is often implemented in three stages: breeding, rearing, and fattening. *Breeding* stage involves piglet production and it is more complex to manage than *rearing or fattening* stage. ***Piglet production is depending on the reproductive process of the sow*** with many events to be registered and controlled beyond the additional care that piglets require. That is why breeding stage presents more complexity than those related with the growing process.

Hence, piglet production is intimately related to the ***sow herd management*** which is becoming one of the major challenges within PSC. The effect of sow herd management through piglet production is extended to the growing process. At the beginning of the growing operations weaned piglets are grouped according to weight, age and sex and they are kept in the same group until they are sent to the slaughterhouse. As soon as pigs reach marketable weights near the end of the fattening stage, a pork producer must devise a marketing strategy to determine when to sell the pigs, which and how many pigs to sell, and to which slaughterhouse(s) selling them (Jørgensen, 1993; Boland et al., 1993; Bailleul et al., 2000; Kure, 1997; De Castro, 2001).

#### *4.2 Importance of sow herd management within the PSC.*

Sow herd management involves decisions just at the beginning of the the PSC and impacting onwards. Therefore, not only it has an effect on piglet production, but also on the subsequent stages, (Babot, 2001). For instance, the interval between farrowings can be reduced simply by a shortening the lactation length. Such a decision not only affects the current productive behaviour of the sow but also the later weaning weight of piglets that impacts on the growing process. Another example is the purchase of gilts to cover culled sows because it modifies herd structure and future expected productivity on farm. Therefore sows have important implications for technical and economic performance not only for breeding farms but also to the rest of the chain. Lower levels of piglet production bound production on fattening farms.

#### *4.3 Main sow herd management decisions.*

In sow herd management “*the design of a farm*” is one of the most important strategic decisions. Normally Spanish sow farms have three different facilities: breeding-control, pregnancy and farrowing facility. The design basically depends on the sow herd dynamic determining the maximum number of sows in mating, gestation or lactation stage. Sow herd dynamic is influenced by sow herd management strategies. For instance when the farmer allows re-matings the number of sows in service facility increases, or well the determination of a long lactation period increases the occupancy rate in the lactation facility. In both cases, the requirements in the number of facilities are affected by management strategies. Hence, housing facilities design has to present a flexible capacity to house a herd under a reasonable number of management strategies. Recently new EU regulations are issued and they have been affecting past management practices. In June 2001, the European Ministers of Agriculture approved European Directive 91-630-EEG. This directive comprises welfare prescriptions for the pig sector, among which a minimum space of sows and growing/fattening pigs is prescribed. The last has lead farmers to redesign farms [[34](http://eur-</a></p></div><div data-bbox=)

[lex.europa.eu/LexUriServ/site/en/oj/2001/l\\_316/l\\_31620011201en00010004.pdf](http://lex.europa.eu/LexUriServ/site/en/oj/2001/l_316/l_31620011201en00010004.pdf)  
accessed 15 December 2009].

On the other hand, the sow replacement problem “*Sow replacement problem*” is at this moment one of the most important challenges in sow herd management. Sow replacement is the second production cost after feeding cost (Huirne et al., 1993).

Sow replacement has a direct economic impact not only on the farmer, but also on the Pig supply chain where sow herd production is integrated. Sow replacement decisions influence the expected lifetime of sows, the annual replacement rate, the piglet production capacity and other important key figures for the planning production of pig supply chain.

In general, the sow replacement problem refers to determine when a sow must be replaced by a new one. Furthermore, their relevance is not only because it determines the structure of the herd but also the economic outputs that the farm get over time. Thus the determination of an optimal lifetime of the sow is important in order to maximise the profit from the production. The replacement decision is based on a expectation on future performance of the sow, this expectation is compared to the expected performance of a new gilt (Plà, 2007, Kristensen et al., 2008).

Today, the high complexity of sow herd management is not only due to the biological issues involved, but also by the increasing market competition, sort of contract agreements and new EU legislation, which are drawing new bounds in pig production and limit past management alternatives for the decision maker, manager, pig specialist, pig adviser or consultant. But above all is the fact that new sow herd management decisions should be coordinated with the rest of the PSC. Actually, more and more often farmers have individual contracts with cooperatives or integrators to whom, they sell all their production and being at the same time subjected to some guidelines like a limited supply of gilts, a farrowing target quota, and/or a minimum weight of piglet. The purchase of gilts is regulated within a contract agreement.



## 5 Discussion and outlook

A noticeable change in the structure of pork sector has been observed during recent years. In EU and particularly in Spain, modern pig farms have changed from family based, small-scale, independent firms to one in which larger pig firms are more tightly aligned across the production and distribution value chain. Modern firms involve more than one farm what bring higher complexity in the planning and coordination of production from farms to costumers. This new context requires new farm planning models different from that developed till now and mainly devoted to single farms. This schedule has been called PSC oriented to produce pig meat products and derivates. Scarce literature exist adopting PSC approach. But, what is clear in general, is the main role that the inventory of pigs at different stages plays in any proposed approach, (Bloemhof et al, 2003; Plà, 2006). Related to inventory over time, the flow of animals over chain is useful to forecast future production.

Regarding basics of pig production it is recognized that breeding stage devoted to piglet production is more complex to manage than rearing or fattening stages. Besides that, breeding farms impact also on the productivity and produce of the chains. These farms are the base of the systems and that is why sow herd models are common in the literature. Into breeding stage (piglet production) the main activity is the sow herd management, where the replacement decision is the major problem to tackle, Plà (2007). The sector is evolving to an integration of economic agents around PSC. The usual way of interaction is through contract agreements. New legislation issues regarding animal welfare are regulating the activity and adding complexity to the PSC management. These considerations are not well covered by past models, although they are a starting point to be taken into account. Hence, there is a need to reformulate and extend existing management models to the new context of production or even make new proposals under the pig supply chain view. For instance, most of the decision models to support sow herd management have been concentrated on steady-state studies (long run or infinite

horizon respectively, where the consideration of initial conditions turn out to be unnecessary. This is very useful to compare management strategies at equilibrium, and allows to assess sow herd productivity, evaluate sow herd performance or analyse alternative herd management strategies (Upton, 1989). However, other issues are gaining relevance in the practical context of pig supply chain management as limited supply of gilts, target of weekly farrowings with respect to the maximization of occupancy in lactation facilities or even the need to manage bad structured herds far from a steady state. But these issues are not considered or valued by infinite time horizon models. This reveals a lack of finite time horizon or short term horizon models adapted to the variability of the herd structure that in practice pig production confronts. Moreover farm-specific input parameters are essential to represent individual farm behaviour and build credibility DSS. It has only been quite recently that farm specific parameters has been used (Kristensen, 2004a, 2004b; Plà et al., 1998, 2003) whilst most of the published models assigned values calculated from general databases or extracted from the literature. This is also true for PSC models. On the other hand most of the sow herd management models have assumed input parameters homogeneous through the time. And this can be useful to strategic level but not to give tactical decision support to particular pig production system. In real world, time to time new data are collected in farms and therefore input updates or a revision of hypothesis would see reasonable and necessary as Toft (1998) pointed out. With this idea in mind Kristensen and Søllested (2004a, 2004b) used a dynamic linear model to update litter size expectations depending on previous observations. However there is still a necessity of tactical decision models to take into consideration updated parameters. It is well known the existence of variability within the main productive variables in piglet production system (e.g. prolificacy, fertility rate, conception rate and prices). Markov decision models have considered implicitly the inclusion of uncertainty in some of the parameters, however variability associated to their results is neglected. Then, this modelling approach represents risk neutral decision makers and it is the common outcome when just expectations are taken. This reveals a lack of models taking into account the uncertainty associated to the results, as was already pointed by Plà (2007). Finally the inclusion on sow herd models of variability-risk features was also of interest to practical purposes.

## References

<http://faostat.fao.org/>, last access November 28th, 2009.

- Ahumada, O., Villalobos, J.R., 2009. Application of Planning Models in the Agri-Food Supply Chain: A review. *European Journal of Operational Research* 4, 399-412.
- Allen, M.A., Stewart, T.S., 1983. A simulation model for a swine breeding unit producing feeder pigs. *Agricultural Systems* 10, 193-211.
- Alsop, J.E., Rafuse, J.B., Keith, I.R., Koopman, W., Jalvingh, A.W., Dohoo, I.R., 1994. Implementation of a computer simulation swine tactical decision, support model in Prince Edward Island. *Canadian Veterinary Journal* 35, 153-157.
- Babot, D., 2001. Gestión en empresas de producción porcina. Análisis, diagnóstico y toma de decisiones. First Edition, 228.
- Backus, G., Dijkhuizen, A.A., 2002. Kernkamp lecture: The future of the European pork chain. Allen D. Leman Swine Conference, Minnesota, USA, 8-11.
- Bailleul, P.J.D., Bernier, J.F., van Milgen, J., Sauvant, D., Pomar, C. 2000. The utilization of prediction models to optimize farm animal production systems: The case of a growing pig model. *Modelling Nutrient Utilization in Farm Animals*. J. P. McNamara, J. France and D. Beever, ed. CAB International 2000, Wallingford, Oxon, UK, 379-392.
- Balogh, P., Ertsey, I., Fenyves, V., Nagy, L., 2009. Analysis and optimization regarding the activity of a Hungarian Pig sales and Purchased Cooperation. *Studies in Agricultural Economics* 109, 35-54.
- Bloemhof, J.M., Smeets, C.M., van Nunen, 2003. Supply chain optimization in Animal Husbandry. Erasmus Research Institute of Management. Report Series Research in Management, ERS-2003-043-LIS.
- Boland, M.A., Preckel, P.V. and Schinckel, A.P., 1993. Optimal hog slaughter weights under alternative pricing systems. *Journal of Agriculture and Applied Economics* 25(2), 148-163.
- Bookbinder, J.H., and Matuk, T.A., 2009. Logistics and Transportation in Global Supply Chains: Review, Critique, and Prospects. *Tutorials in Operations Research INFORMS*. ISBN 978-1-877640-24-7.
- Broekmans, J.E., 1992. Influence of price fluctuations on delivery strategies for slaughter pigs. *Dina Nota no. 7*, Copenhagen., 24pp.

- Chavas, J.P., Kliebenstein, J. and Crenshaw, J.D., 1985. Modeling dynamical agricultural production response: The case of swine production. *American Journal of Agricultural Economics* 67(3), 636-646.
- Christopher, M., 1998. *Logistics and supply chain management. Strategies for Reducing Cost and Improving Service*. 2nd ed., London.
- de Castro, R., 2001. *Optimització del procés productiu d'engreixament del porcí*. Ph.D. dissertation. University of Gerona, Spain.
- den Ouden, M., Dijkhuizen, A.A., Huirne, R.B.M., and Zuurbier, P.J.P, 1996. Vertical Cooperation in Agricultural Production Marketing Chains, with special Reference to Product Differentiation in Pork. *Agribusiness* 12, 277-290.
- den Ouden, M., Nijsing, J.T., Dijkhuizen, A.A., Huirne, R.B.M., 1997a. Economic Optimization of pork production-marketing chains: I. Model input on animal welfare and cost. *Livestock Production Science* 48, 23-37.
- den Ouden, M., Huirne, R.B.M., Dijkhuizen, A.A., van Beek, P. 1997b. Economic Optimization of pork production-marketing chains: I. Modelling outcome. *Livestock Production Science* 48, 39-50.
- de Roo, G., 1987. A stochastic model to study breeding schemes in a small pig population. *Agricultural Systems* 25, 1-25.
- Dijkhuizen, A.A., Morris, R.S., Morrow, M., 1986. Economic optimisation of culling strategies in swine breeding herds, using the "PORKCHOP computer program". *Preventive Veterinary Medicine* 4, 341-353.
- Gribkovskaia, I., Gullberg, B.O., Hovden, K., Wallance, S.W., 2006. Optimization model for a livestock collection problem. *International Journal of Physical Distribution and Logistic Management* 36(2), 136-152.
- Gupta, A., and Maranas, C.D., 2003. Managing demand uncertainty in supply chain planning 27, 1219-1227.
- Hamann, K., 2006. An Overview of Danish Pork Industry Integration and Structure. *Advances in Pork Production* 17, 93-97.
- Higgins, A.J., Miller, C.J., Archer, A.A., Ton, T., Fletcher, C.S. and McAllister, R.R.J., 2009. Challenges of operations research practice in agricultural value chains. *Journal of the Operational Research Society*. 1-10, advance online publication, May 20, 2009; doi:10.1057/jors.2009.57.
- Hobbs, J.E., Kerr, W.A. and Klein, K.K., 1998. Creating international competitiveness through supply chain management: Danish pork. *Supply chain management* 3(2), 68-78.

- Hobbs, J.E., Young, L.M., 2000. Closer vertical co-ordination in agri-food supply chains: a conceptual framework and some preliminary evidence. *Supply chain Management: An International Journal* 5(3), 131-143.
- Huirne, R.B.M., 1990. Computerized management support for swine breeding farms. Ph.D. dissertation, Department of Farm Management, Wageningen Agricultural University, Wageningen, The Netherlands.
- Huirne, R.B., Dijkhuizen, A.A., Van Beek, P., Hendriks, Th.H.B., 1993. Stochastic dynamic programming to support sow replacement decisions. *European Journal of Operational Research* 67, 161–171.
- Jalving, A.W., Dijkhuizen, A.A., van Arendonk, J.A.M., 1992. Dynamic probabilistic modelling of reproduction and management in sow herds. General aspects and model description. *Agricultural Systems* 39, 133–152.
- Jørgensen, E., 1993. The influence of weighing precision on delivery decision in slaughter pig production. *Acta Agriculturae Scandinavica A* 43, 181-189.
- Kamp, J.A.L.M. 1999. Knowledge based systems: from research to practical application. Pitfalls and critical success factors. *Computers and Electronics in Agriculture* 22, 243-250.
- Krieter, J., 2002. Evaluation of different pig production systems including economic, welfare and environmental aspects. *Archiv für Tierzucht* 45(3), 223-235.
- Kristensen, A.R., 1993. Markov decision techniques applied to the animal replacement problem. Ph.D. dissertation. Dina KVL. Dep. of Animal Science and Animal Health. The Royal Veterinary and Agricultural University. Copenhagen. Denmark.
- Kristensen, A.R., Jørgensen, E., Toft, N., 2008. *Herd Management Science I. Basic concepts*. SI Books, ISBN: 8779970788.
- Kristensen, A.R., Søllested, T.A., 2004a. A sow replacement model using Bayesian updating in a three-level hierarchic Markov process I. Biological model. *Livestock Production Sciences* 87, 13–24.
- Kristensen, A.R., Søllested, T.A., 2004b. A sow replacement model using Bayesian updating in a three-level hierarchic Markov process II. Optimization model. *Livestock Production Sciences* 87, 25–36.
- Kure, H., 1997. Marketing Management support in slaughter pig production. Ph.D. dissertation. Department of Animal Science and Animal Health, The Royal Veterinary and Agricultural University, Copenhagen.
- Ljungberg, D., 2006. Effective Transport Systems in Food and Agricultural Supply chains. Ph.D. Dissertation. Swedish University of Agricultural Sciences, Uppsala.

- Maliappis, M.T., Yialouris, C.P., Deligeorgis, S.G., Sideridis, A.B., 1997. EMISP: an expert management Information System for pigs. First European Conference for Information Technology in Agriculture, Copenhagen, 15-18 June.
- Manzini, R., Gebennini, E., 2008. Optimization models for the dynamic facility location and allocation problem. *International Journal of Production Research* 46(8), 2061–2086.
- Marsh, W.E., 1986. Economic decision making on health and management livestock herds: examining complex problems through computer simulation. Ph.D. Dissertation. University of Minnesota, St. Paul.
- Martinez, S.W., Reed, A., 1996. From Farmers to Consumers. Vertical Coordination in the Food Industry. Economic Research Service Report, No. 720, USDA, Washington.
- Melo, M.T., Nickel, S., Saldanha-da-Gama F., 2009. Facility location and supply chain management- A review. *European Journal of Operational Research* 119, 14-34.
- Mighell, R.L., Jones, L.A., 1963. Vertical Coordination in Agriculture. Agricultural Economics Report No. 19, Farm Economics Division of Economic Research Service, US, Department of Agriculture, Washington, DC.
- Min, H., and Zhou, G., 2002. Supply Chain modelling: past, present and future. *Computers & Industrial ingeneering* 43, 231-249.
- Niemi, J.K., 2006. A dynamic programming model for optimising feeding and slaughter decisions regarding fattening pigs. *Agricultural and food science* 15, 6-121.
- Perez, C., de Castro, R., Font i Furnols, M., 2009. The pork industry: a supply chain perspective. *British Food Journal* 111(3), 257-274.
- Perez, M.P., Palacio, J., Santolaria, M.P., Aceña, M.C., Chacon, G., Calvo, J.H., Zaragoza, P., Beltran, J.A. and García-Belenger, S., 2002. Effect of transport time on welfare and meat quality in pigs. *Meat Science* 61, 425-433.
- Pettigrew, J.E., Cornelius, S.G., Eidman, V.R., Moser, R.L., 1986. Integration of factors affecting sow efficiency: a modelling approach. *Journal of Animal Science* 63, 1314–1321.
- Pibernik, R., and Sucky, E., 2007. An approach to inter-domain master planning in supply chains. *International Journal of production economics* 108, 200-212.
- Plà, L.M., 2006. Tactical Supply Chain model of pig production. Proceedings of the Second Meeting of the EURO Working group on Operational Research (OR) in Agriculture and Forest Management. *The Journal of Agricultural Science*, Volume 144, Issue 05, 467-472.
- Plà, L.M., 2007. Review of Mathematical Models for Sow Herd Management. *Livestock Science* 106, 107-119.

- Plà, L.M., Babot, D. Pomar, J., 2004. A Mathematical Model for Designing Sow Farms. *International Transactions in Operational Research* 11, 485-494.
- Plà, L.M., Conde, J., Pomar, J., 1998. Sow model for decision aid at farm level. In: Giron, F.J. (Ed.), *Applied Decision Analysis*. Kluwer Academic Publishers, Boston, 47-62.
- Plà, L.M., Pomar, C., Pomar, J., 2003. A Markov decision sow model representing the productive lifespan of sows. *Agricultural Systems* 76, 253-272.
- Pomar, C., Harris, D.L., Minvielle, F., 1991. Computer simulation model of swine production systems: III. A dynamic herd simulation model including production. *Journal of Animal Science* 69, 2822-2836.
- Porter, M.E., 1985. *Competitive Advantage*. The Free Press. New York. Chapter 1, 11-15.
- Ohlmann, J.W., and Jones, P.C., 2008. An integer programming model for optimal pork marketing. *Annals Operations Research*, ISSN 1572-9338 (Online).
- Singh, D., 1986a. Simulation of swine herd population dynamics. *Agricultural Systems* 22, 157-183.
- Stadtler, H., 2005. Supply Chain management and advanced planning-basics, overview and challenges. *European Journal of Operational Research* 163, 575-588.
- Stadtler, H., and Kilger, Ch., (Eds), 2008. *Supply Chain management and advanced planning. Concepts, models, software and case studies*. Springer, 4<sup>th</sup> edition. XVIII, 556p. Hardcover.
- Taylor, D.H., 2006. Strategic consideration in the development of lean agri-food supply chains: a case study of the UK pork sector 11(3), 271-280.
- Tess, M.W., Bennett, G.L., Dickerson, G.E., 1983. Simulation of genetic changes in life cycle efficiency of pork production. I. A bioeconomic model. *Journal of Animal Science* 56, 336-353.
- Toft, N. 1998. The dynamic aspect of the reproductive performance in the sow herd. *Dina Notat* No. 70.
- Trienekens, J., Petersen, B., Wognum, N., and Brinkmann, D., 2009. *European pork chains*. Wageningen Academic Publishers.
- Upton, M., 1989. Livestock productivity assessment and herd growth models. *Agricultural Systems* 29, 149-164.
- van der Gaag, M., Vos, F., Saatkamp, H.W., van Boven, M., van Beek, P., Huirne, R.B.M., 2004. A state-transition simulation model for the spread of salmonella in the pork supply chain. *European Journal of Operational Research* 156, 782-798.
- van der Vorst, J.G.A.J., da Silva, C.A., Trienekens, J.H., 2007. *Agro-industrial supply chain Management: Concepts and applications*. Food and Agriculture Organization of the United Nations, Rome. *Agricultural Management, Marketing and Finance*, 17.

- Veselská, E., 2005. The process of vertical coordination and its consequences within the beer commodity chain. *Agricultural Economics-CZECH* 51(9), 419-426.
- Vidal, J.C. and Goetschalckx, M., 1997. Strategic production distribution models: A critical review with emphasis on global supply chain models. *European Journal of Operational Research* 98, 1-18.
- Whittemore, C.T. and Kyriazakis, I. 2006. *Whittemore's Science and Practice of Pig Production*. Hardback, third edition, 704.



## Appendix A

Table 1. List of models for Pork supply chain management.

Author(s)	Year	Name	SC-E
Kure	1997	Optimal slaughter pig marketing.	PF-S
Den Ouden et al.,	1997a	Economic Optimization of pork production-marketing chains: I. Model input on animal welfare and cost.	PF-S-W
Den Ouden et al.,	1997b	Economic Optimization of pork production-marketing chains: II. Modelling outcome.	PF-S-W
De Castro	2001	Optimization of the fattening process.	PF-S-W
Krieter	2002	Evaluation of different pig production systems including economic, welfare and environmental aspects	PF-S
Bloemhof et al.,	2003	Supply chain optimization in Animal Husbandry	PF-S-W
Van der Gaa et al.,	2004	A state-transition simulation model for the spread of salmonella in the pork supply chain.	PF-S-R
Gribkovskaia et al.,	2005	Optimization model for a livestock collection problem	PF-D
Plà et al.	2006	Tactical Supply Chain model of pig production.	SF-FF-S
Niemi	2006	A dynamic programming model for optimising feeding and slaughter decisions regarding fattening pigs.	FF-S
Ohlmann and Jones	2008	An integer programming model for optimal pork marketing.	PF-S
Balogh, et al.,	2009	Analysis and optimization regarding the activity of a Hungarian Pig sales and Purchased Cooperation.	PF-S
Allen and Stewart	1983	A simulation model for a swine breeding unit producing feeder pigs.	SF
Tess et al.	1983	Simulation of genetic changes in life cycle efficiency of pork production I. A bioeconomic model.	SF
Dijkhuizen et al.	1986	Economic optimisation of culling strategies in swine breeding herds, using “PORKCHOP computer program”	SF
Marsh	1986	Economic decision making on health and management in livestock herds: examining complex problems through computer simulation	SF
Pettigrew et al.	1986	Integration of factors affecting sow efficiency: a modelling approach	SF
Signh	1986	Simulation of swine herd population dynamics.	SF
De Roo	1987	A stochastic model to study breeding schemes in a	SF

---

		small pig population	
Pomar et al.	1991	Computer simulation model of swine production systems: III. A dynamic herd simulation model including reproduction	SF
Jalving et al.	1992	Dynamic probabilistic modelling of reproduction and replacement management in sow herds. General aspects and model description	SF
Huirne et al.	1993	An application of stochastic dynamic programming to support sow replacement decisions	SF
Plà et al.	1998	A sow model for decision aid at farm level	SF
Plà et al.	2003	A Markov decision sow model representing the productive lifespan of herd sows	SF
Kristensen and Søllestad	2004a	A sow replacement model using Bayesian updating in a three-level hierarchical Markov process I. Biological model	SF
	2004b	A sow replacement model using Bayesian updating in a three-level hierarchic Markov process II. Optimisation model	SF
Chavas et al.,	1985	Modeling dynamical agricultural production response: The case of swine production.	FF
Broekmans	1992	Influence of price fluctuations on delivery strategies for slaughter pigs.	FF
Jørgensen	1993	The influence of weighing precision on delivery decision in slaughter pig production	FF
Boland et al.,	1993	Optimal hog slaughter weights under alternative pricing systems.	FF
Bailleul et al.	2000	The utilization of prediction models to optimize farm animal production systems: The case of a growing pig model.	FF

---

SC-E: Supply chain-agent(s); Pig farms: SF: Sow farms; FF: Fattening farms; S: slaughterhouse; W: wholesales; D: Distributors



# **A Linear Programming Formulation of a Semi-Markov Model to Design Pig Facilities**

Lluís M. Plà<sup>1</sup>, Javier Faulin<sup>2</sup>, Sara V. Rodríguez<sup>1</sup>

<sup>1</sup>University of Lleida, Department of Mathematics, Jaume II, 73 E-25001 Lleida, Spain

<sup>2</sup>Public University of Navarre, Department of Statistics and Operations Research, Los Mangnolis Building, E-31006 Pamplona, Spain

Submitted to Journal of the Operational Research Society.  
(Final version published in Journal of the Operational Research Society 60, 619-625).

## **Abstract**

Housing facilities design represents the main strategic decision in pig farms. This paper introduces a linear programming formulation of a Semi-Markov process to approach the facilities design. Thus, the Linear Programming formulation determines the optimum replacement policy and provides the equilibrium distribution of the herd along pig facilities. Then, the calculation of the associated needs of room for each sow facility is derived from sow herd distribution at equilibrium. Results show the flexibility of the model for designing pig facilities and computational advantages in the solving procedure compared to previous proposals. Furthermore, the robustness of the optimal solution is studied by means of sensitivity analysis.

*Keywords: Agriculture, Linear Programming, Planning, Replacement Policy, Semi-Markov Process.*

## 1 Introduction

Generally speaking, the problem of designing the behaviour of a probabilistic system, such as a farm, is usually confronted by farmers, swine specialists or consultants on a periodic basis. They develop that design by making decisions or choosing actions as the system evolves through time. The main question is to determine which sequence of actions leads to the optimality of the system with respect to some predetermined performance criterion. Since the farm is not static, decisions must anticipate the opportunities and costs associated with future system evolution (Chavas et al., 1985).

Mathematical models that represent herd dynamics and corresponding production behaviour are well-known tools in livestock planning (Glen, 1987). Several models simulate the main biological, physical and management factors influencing population dynamics have been widely developed in dairy production, and, less frequently, in pig production (Huirne et al., 1993). Linear programming is one of the optimisation techniques most used in livestock herd management. However, that is not the case in sow herd management (Plà, 2007). The reasons for that situation can be related to the computation time needed for solving complex models in the past few years. Frequently, researchers prefer to tailor computationally more efficient software for solving dynamic programming models or to simply use simulation. These preferences are related to the time needed to solve complex models though it is clearly favourable to the aforementioned methods.

Making the right decision in each specific situation is a hard and difficult task. That is the case of the most important strategic decisions in swine production: pig housing facilities design (Lippus et al., 1996). The stochastic nature of the swine biology and the variability in litter size makes it difficult to design facilities as Singh (1986) pointed out. Herd management models seem to be a suitable tool to analyse different management strategies concerning replacement, reproduction and the associated room needs, which finally determine the design of facilities.

Housing facilities have to present a suitable capacity to house a herd under a reasonable number of management strategies and to be respectful of European Union regulations concerning animal welfare.

Simulation (Singh, 1986; Lippus et al., 1996) and Semi-Markov chain models (Plà et al., 2004) have recently been used to plan housing facilities for pigs, but neither economic cost nor optimal replacement strategy has been considered yet. Hence, the aim of this paper is the presentation of a linear programming sow model, with a double purpose: first, to represent sow herd production through the reproduction and replacement management at the farm level and, second, to calculate room needs for each facility in which a sow farm is usually designed. The model is specific for farms and it is based on a Semi-Markov decision process, whose structure has been proposed to solve real problems in field conditions while avoiding unpractical complexities that some research models contain. In this way, the current paper can be viewed as an improvement and refinement of Plà et al.'s (2004) paper.

## 2 Mathematical Formulation: Basic Principles

Housing capacity of a sow farm depends on the expected sow herd dynamics over time. Then, herd dynamics is formulated as a Semi-Markov decision process. These models are called Semi-Markov because for fixed Markov policies the system states evolve according to a Semi-Markov process. They are characterised by the time spent in a particular state (holding time), which can follow an arbitrary probability distribution.

Let us consider a Semi-Markov process  $Y = \{Y(t), t \geq 0\}$  with finite state space  $S$  along with the embedded Markov chain  $X = \{X_n, n = 0, 1, \dots\}$ . The transition and reward structure is characterized by:

$p_{ij}$ : transition probability from  $i \rightarrow j$  of the embedded Markov chain  $X$  with generic stochastic matrix  $P = [p_{ij}]$

$\eta_{ij}$ : random time of the transition from  $i \rightarrow j$

$F_{ij}(\tau)$ : distribution function representing the conditional probability  $P(\eta_{ij} \leq \tau)$

$r_{ij}$ : instantaneous transition reward for a transition from  $i \rightarrow j$

$r'_i$ : reward rate per unit of time incurred in state  $i$

$r_i$ : expected reward in state  $i$

$\pi_i$ : expected time spent in state  $i$

Furthermore, the next assumption is considered: the transition probability matrix  $P$  has a single class of recurrent states and it is aperiodic. Therefore, it can be shown

that the rows of the limit matrix  $P^* = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^{n-1} P^k$  of the embedded Markov chain  $X$  are identical and equal to the row vector of steady state probabilities  $\pi = [\pi_1, \dots, \pi_n]$  which is determined using  $\pi = \pi P$  and the normalization condition  $\pi e = 1$ . Moreover,  $P^* = \lim_{n \rightarrow \infty} P^k$  and converges geometrically because  $P$  is aperiodic. Then:

$$g = g^r / g^t = \frac{\sum_{j \in S} \pi_j r_j}{\sum_{j \in S} \pi_j \tau_j} \quad (1)$$

where  $g$  is the average reward of the process per time unit for a given policy,  $r_j$  is the expected reward per state (i.e.  $r_j = r'_j \cdot \tau_j$ ) and  $\tau_j$  is the expected time spent in the  $j$ -state. It can be observed that only steady state probabilities of the embedded Markov process, the reward and holding time in each state are necessary for carrying out this calculation. Average reward can be calculated by standard matrix methods. Hence, note that by only using (1) for different policy comparisons between management alternatives then a specific analysis can be performed (Plà, 2007).

### 3 Sow Farm Model

According to our hypotheses, a decision maker observes the system state in a specific moment in time called the decision epoch. According to this state, she/he



chooses an action from the set of available actions in that state. The choice of that action generates two results: the decision maker receives an immediate reward (or incurs an immediate cost), and the system evolves to a new state at a subsequent point in time according to a probability distribution determined by the chosen action (Puterman, 1994).

### 3.1 State and Action Sets

The set of states is related to the places, occupied or not, where productive activity is developed on the farm. There is a strong correspondence between the sow reproductive cycle and the facilities where sows are housed. The farm design is depicted by different facilities, from which sows are moved depending on their physiological state. Service, gestation and lactation facilities are common in many farms and are also considered in this work. Hence, the sow lifespan begins when new replacement gilt is purchased and introduced into the farm awaiting the first insemination or mating, and finishes when it is culled after several reproductive cycles or with unanticipated culling or death. The main reasons for culling are the achievement of a maximum number of cycles allowed by the farmer, infertility, low productivity, abortion, accidents or illnesses. If a replacement is made immediately, the occupancy of the farm is maximized. However, this policy is unusual due to the need for a drying period in lactation facilities before they are used again and also when batch management is adopted; thus a reduction in the occupancy rate could be considered due to delay in the replacement.

Sows on a farm will be found in one of the possible states  $S = \{s_i | i=1, \dots, N\}$  depending on the sow's lifespan and sow's flow among facilities. Moreover, the set  $S$  is finite and can be split up in the form  $S = E \cup B$ , where  $E = \{e_{ijklm} | i: \text{reproductive states}, j: \text{productive cycle}, k: \text{production level}, l: \text{genetic merit and } m: \text{housing facility}\}$  and  $B = \{b_m | m: \text{housing facility}\}$ .  $B$  is the set of states representing a gap in a facility. A gap is generated after culling a sow or when the drying period is implemented. Another useful partition of  $S$  is  $S = I \cup G \cup L$ , where each subset represents states related to service, gestation and lactation facilities (i.e.  $m=1, 2$  and  $3$  respectively).

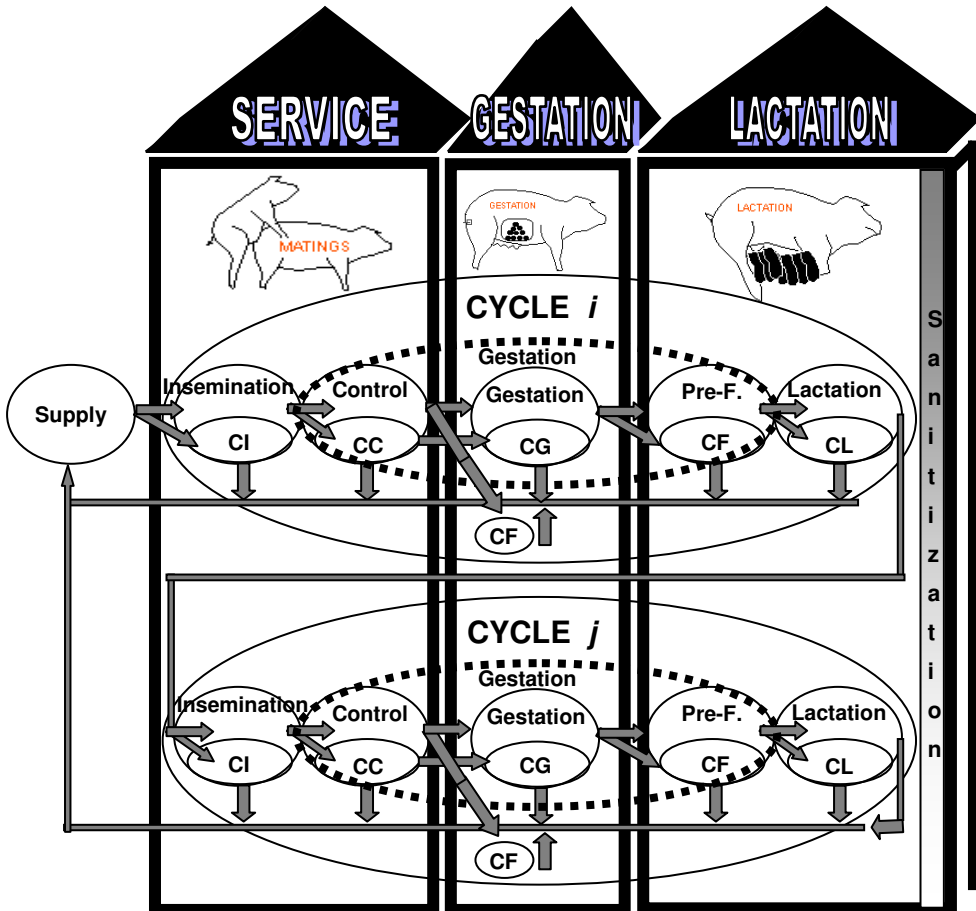
For each state, the action set  $A = \{a_i \mid i=1, \dots, |A|\}$  describes a set of possible actions, is finite and includes all possible controls that the farmer can carry out on the farm. The actions at the sow level always include the replacement as one of the alternatives, thus, two actions to solve replacement problems are usually considered,  $A = \{Keep, Replace\}$ . The state vector has been associated to a relative action vector whose components represent actions taken for each state vector component. Actions are taken by the decision function, so the action vector in a deterministic case represents application of the policy  $R$ , meaning, in mathematical terms, that the decision function is always the same (i.e. the policy is stationary). This assumption is consistent with usual management strategies of farmers who maintain their own management policy over time.

### 3.2 Transitions and Rewards

Transitions represent the system evolving from one state to another. There are two types of transitions: one connected with physiological changes in sows and the other related to sow entries or exits from a facility. Sometimes, both can occur simultaneously; for example when piglets are weaned and action “*Keep*” selected, the sow is moved to the service facility and evolves to the next state (awaiting for the first mating of the next cycle). Generally speaking, the reproductive cycle of sows starts in the service facility where sows and gilts await insemination. When pregnancy is confirmed, sows are moved to the gestation facility. Sows remain there until the week before the expected farrowing date when they are moved to the farrowing facility. Farrowing and leading lactation occurs in the farrowing facility. The end of a sow’s reproductive cycle is marked by the weaning or, occasionally, by an abortion. Then, sows are moved again to the service facility while the farrowing room is cleaned, sterilized and closed for a drying period of approximately one week (Plà et al., 2004).

Only those transitions for which there are logical flows between facilities and a biological justification (Figure 1) are considered. Whenever we have had available historical data, the maximum likelihood estimates of probability transition have

been computed (Billingsley, 1995). If action  $\{Replace\}$  is taken, then  $p(j| i, a)=1$  if  $j$  is the replacement state, and 0 otherwise.



Pre-F.: Pre-farrowing, C = State representing animals waiting for being culled in I: Insemination, C: Control, G: Gestation, L: Lactation, F: Infertility

Figure 1. Graphic representation of the sow lifespan in the farm facilities.

The model does not take into account any improvement in the genetic merit of sows to produce piglets. Thus, another hypothesis in the model is that the quality of the replacement sow is assumed to be unrelated to the fact that a particular sow is replaced, knowing that all sows have the same expected quality. The model quantifies the gains or costs obtained in a swine herd by reflecting the economic

consequences of decisions in the reward function. If action  $a$  is chosen at a decision epoch in state  $i$  and the system evolves to  $j$ , then an immediate reward  $r_t(j,a)$  is obtained. This transition occurs with probability  $p(j|i,a) = p^a_{ij}$ .

### 3.3 Holding Times

The holding times depend on the present system state in each decision epoch. Holding times can be taken as constants e. g. lactation, or a random, e.g., a normal distribution may be considered for the gestation period or the oestrus cycle as several authors recommend (Plà, 2007).

## 4 Optimality Criteria for Design the Sow Farm Facilities

### 4.1 Linear Programming Model

An infinite planning horizon involves a stationary optimal policy (Puterman, 1994). Furthermore, it is assumed that the model has stationary transition probabilities and stationary bounded rewards, such that  $(|r(i,a)| \leq M < \infty, \forall a \in A, \forall i \in S)$  where  $S$  and  $A$  are finite sets. All stationary policies have a single recurrent class and no transient states. Thus, using the average reward function as optimality criterion the linear programming model to optimise the average reward per unit of time can be formulated as follows (Tijms, 1994):

$$\begin{aligned}
 & \text{Maximise} \quad g = \sum_{i \in S} \sum_{a \in A} r(i,a) \pi(i,a) \\
 & \text{subject to} \\
 & \sum_{a \in A} \pi(j,a) - \sum_{i \in S} \sum_{a \in A} p(j|i,a) \pi(i,a) = 0 \quad j \in S \quad (2) \\
 & \sum_{i \in S} \sum_{a \in A} \pi(i,a) = 1
 \end{aligned}$$

where  $\pi(i,a)$  is the limit state distribution,  $p(j|i,a)$  the transition probabilities and  $g$  is referred to as the stationary reward per time unit. Notice that the set of constraints corresponds to the equilibrium flow of sows throughout each state,  $j \in S$ .

The last constraint is needed to completely determine the unique distribution of the herd at equilibrium. However, in our original problem each state has a different holding time, so that, the length of time in stages is not equal. Hence, using the change of the variable  $z_{ia} = \pi_{ia} / \tau_{ia}$ , derived from (2) we obtain the following linear programming model:

$$\begin{aligned}
 & \text{Maximise } \sum_{i \in S} \sum_{a \in A} r(i, a) z_{ia} \\
 & \text{subject to :} \\
 & \sum_{a \in A} z_{ia} - \sum_{i \in S} \sum_{a \in A} p(j|i, a) z_{ia} = 0 \quad j \in S \\
 & \sum_{i \in S} \sum_{a \in A} \tau_{ia} z_{ia} = 1
 \end{aligned} \tag{3}$$

where  $\tau_{ia}$  is the holding time per state and action. Then, optimal replacement policy and herd distribution at equilibrium are obtained by (3). Notice that the limit state distribution  $\pi^*_{ia} = z^*_{ia} \tau_{ia}$  corresponds to the limit distribution of the embedded Markov chain associated with the optimal replacement policy. On the other hand, the normalization of  $z^*_{ia}$  corresponds to the distribution at equilibrium of the Semi-Markov chain associated with the optimal replacement policy. Therefore, herd distribution at equilibrium can be used for planning housing facilities.

#### 4.2. The Design of Sow Farm Facilities

According to previous results, the occupation and room needs of facilities are calculated from the optimal steady state vector ( $z^*_{ia}$ ) obtained in (3). Final design of sow farm facilities implies the aggregation of state components of  $z^*_{ia}$  by facility as follows:

$$Z_m = \sum_{i \in S_m} z^*_{ia} \quad m=1,2,3 \tag{4}$$

$$X_m = \sum_{i \in E} z^*_{ia} \quad m=1,2,3 \tag{5}$$

where the subindex  $m$  represents housing facilities i.e. the value 1 represents the service facilities, the value 2 represents the gestation facilities and the value 3

represents the lactation facilities. The design of farms should prevent the overflow of capacity and account for enough crates in each facility. Note that if immediate replacement is considered and  $B$  is the empty set ( $\emptyset$ ), the occupation rate of the farm is 100% since (4) and (5) become equal. Full occupation is rather unrealistic, for instance just considering the general practice for drying period after weaning, leads to empty crates. So, in real instances, resulting housing capacity is greater than herd size. Therefore  $Z_m$  represents the distribution of crates over built facilities (occupied and non occupied), and  $X_m$  represents the herd distribution over built facilities (present number of gilts and sows). In order to determine the absolute number of crates to be built the herd size has to be fixed in  $N$ . Then, the final number of crates would be:

$$N^* = \frac{e \cdot Z}{e \cdot X} N = \frac{Z_1 + Z_2 + Z_3}{X_1 + X_2 + X_3} N \quad (6)$$

From (6) the size of each facility can be easily derived as  $N_m^* = Z_m \cdot N^*$  and corresponding occupancy of facilities:  $N_m = X_m \cdot N$  being given the occupancy rate per facility by  $OR_m = N_m / N_m^*$  and overall occupancy rate by  $OR = N / N^*$ .

## 5 A Case Study

### 5.1 Basic example

To illustrate the proposed formulation and implicit calculations involved a case study is presented. The parameters of the model are inspired by values considered as normal in Spanish conditions as reflected in the BD-Porc databank [<http://www.irta.es/bdporc/> accessed 4 September 2007], and do not correspond to a specific farm. Those parameters are presented in Appendix A. A herd size of 1000 animals, sows and gilts, was considered with a maximum lifespan allowed of 10 parities. Abortion was not allowed, rather failing conceptions were considered as culling reasons. Gilts were assumed to be purchased as needed and availability of them was not considered a constraint. Three facilities were considered according to Figure 1.

The model (3) was implemented and solved using ILOG OPL 5.0. The number of states of the set  $S=E\cup B$  is  $|E|=50$  states (the result of considering the maximum lifespan of 10 parities and 5 reproductive states per parity) and  $|B|=6$  states (the result of the culling states, one per reproductive state, plus the state representing the supply delay). Keep and replace were the actions considered by the farmer. Thus, in this case the LP model has 112 decision variables and  $|S|+1=57$  constraints. Each one of the  $|S|$  constraints represents the flow of animals through the corresponding state, taking into account the keeping or replacement decisions that the farmer can make.

Regardless of the small number of states of this example, it is similar to other models in the literature based on Markov chain models that present a higher number of states (Jalvingh et al., 1992; Lippus et al., 1996). Note that the Semi-Markov approach allows the modeller to consider less (artificial) states and gaining precision with actual holding times. Furthermore this formulation can be extended to more accurately represent the production of piglets, but it is considered unnecessary given that our central aim is the design of a farm as a strategic decision.

Results concerning herd distribution and the optimal replacement policy are shown in the Table 1. These results point out that the optimum replacement policy is to maintain a sow in the farm until the end of the 7<sup>th</sup> cycle, and the optimum average reward of the farm is 238 €/day displayed in Table 3.

With respect to the farm design, formulae (4) and (5) were applied and compiled results are presented in the Table 2. Not only was the number of crates in each facility calculated but also the minimum surface for each facility. The requirements of the Council Directive 2001/88/EC of the European Union on pigs' welfare [[http://eur-lex.europa.eu/LexUriServ/site/en/oj/2001/l\\_316/l\\_31620011201en00010004.pdf](http://eur-lex.europa.eu/LexUriServ/site/en/oj/2001/l_316/l_31620011201en00010004.pdf) accessed 8 January 2007] were taken into account.

Table 1. Distribution of the herd in equilibrium and optimal policy.

CYCLE	$X_m$			OPTIMAL POLICY		
	1	2	3	1	2	3
1	0.0189	0.2641	0.0501	Keep	Keep	Keep
2	0.0123	0.1743	0.0332	Keep	Keep	Keep
3	0.0081	0.1156	0.0221	Keep	Keep	Keep
4	0.0054	0.0780	0.0150	Keep	Keep	Keep
5	0.0037	0.0533	0.0102	Keep	Keep	Keep
6	0.0025	0.0359	0.0069	Keep	Keep	Keep
7	0.0017	0.0239	0.0046	Keep	Keep	Replace
8	0.0000	0.0000	0.0000	Replace	Replace	Replace
9	0.0000	0.0000	0.0000	Replace	Replace	Replace
10	0.0000	0.0000	0.0000	Replace	Replace	Replace
C	0.0014	0.0373	0.0216	Keep	Keep	Keep

$X_m$  Distribution of sows per cycle and facility  $m$  (1: breeding facility, 2: gestation facility and 3: lactation facility) under the optimal policy. The optimal policy prescribes an action (Keep or Replace) depending on the state of the sow. C= State representing animals waiting for being culled.

The total surface resulted in 2,772m<sup>2</sup> with an investment cost of 921,464.00€. Total investment was estimated knowing the unitary cost per facility with the purpose of reflecting the current Spanish situation. The unitary cost associated to the service and gestation facilities was 260.24 €/m<sup>2</sup>, whereas the lactation facility cost was much higher (438.91€/m<sup>2</sup>) in agreement with the extra care that piglets require.

Table 2. Facilities design of the study case.

FACILITIES	$X_m$	$Z_m$	Minimum	Total Costs
			Surface(m <sup>2</sup> )	
Service	247	250	509	132,415
Gestation	545	556	1,142	297,145
Lactation	208	258	1,121	491,904

$X_m$  herd distribution over built facilities and  $Z_m$  distribution of crates over facilities.

Results in Table 2 show that sows spend the most time in gestation, and less in service or lactation facilities as expected. However surface requirements are similar between gestation and lactation facilities while total estimated cost of lactation facility is more than half the total cost. The occupancy rate per facility was 0.99 in service, 0.98 in gestation and 0.81 in lactation, resulting in an overall occupancy



rate of 0.94, though these rates depend on the delay on replacement. For instance, the 49 empty crates were a consequence of the delay in the replacement (29%) and the drying and cleaning period (71%). The last one only affected the room requirements of the lactation facility.

The fact of considering a delay in the replacement provokes the allocation of additional room space. If the delay is reasonable, then it acts as a security margin in the facility and this way, the overflow of farm capacity is avoided. Otherwise, it would represent an overdesign of facilities and the waste of resources.

### *5.2 Sensitivity Analysis*

Room needs depend on herd management, thus, final designs should be flexible in order to allow the implementation of different management policies. This kind of robustness is important for practical purposes, therefore a sensitivity analysis on different parameters to evaluate the stability of the optimal solution in different management scenarios was performed. The case example developed in the previous section was taken as a reference basis. Thus, two scenarios were considered. The first one considered variations in fertility. The conception rate of the farm was modified systematically in all the cycles at the same time. This fact can be justified e.g. by changes in the insemination management or seasonal effects. Therefore the original values of the conception parameters were modified by  $\pm 0.05$ .

The second analysis involved variations on the culling rate at lactation, which, can be attributable to disease problems during lactation or similar problems provoking the culling of sows. Original values were modified by  $\pm 0.15$ . After the analysis it was observed that the optimal replacement policy had not changed in any of the cases.

On the other hand, there were differences between expected rewards, structure of the herd and consequently in the facilities design as shown in Table 3 and 4.

Table 3. Impact on revenues and farm capacity in each scenario.

	BASIS	Variations in Fertility		Variations in Culling Rate (Lactation)	
		+0,05	-0,05	+0,15	-0,15
		<b>g* €/day</b>	238	257 (8%)	217 (-9%)
<b>Farm capacity</b>	1064	1064 (0%)	1062 (0%)	1074 (1%)	1052 (-1%)

g\* expected revenue of the farm; ( $\Delta$  %) percentage of variation in relation to the basis.

Table 4. Sizing of each facility in each scenario.

FACILITIES	Variations in Fertility				Variations in Culling Rate (Lactation)			
	+0.05		-0.05		+0.15		-0.15	
	$X_m$	$Z_m$	$X_m$	$Z_m$	$X_m$	$Z_m$	$X_m$	$Z_m$
<b>SERVICE</b>	238 (-3.81%)	241 (-3.60%)	258 (4.13%)	260 (4.00%)	251 (1.30%)	253 (1.20%)	244 (-1.26%)	247 (-1.20%)
<b>GESTATION</b>	551 (1.15%)	562 (1.08%)	538 (-1.25%)	549 (-1.26%)	550 (0.98%)	561 (0.90%)	539 (-0.97%)	550 (-1.08%)
<b>LACTATION</b>	211 (1.52%)	261 (1.16%)	205 (-1.65%)	253 (-1.94%)	199 (-4.12%)	260 (0.78%)	216 (4.04%)	255 (-1.16%)

$X_m$  herd distribution over built facilities and  $Z_m$  distribution of crates over facilities. ( $\Delta$ %) percentage of variation in relation to the basis.

The general trend in the previous data was as expected. For instance, the improvement of fertility is associated with greater revenue derived from the higher number of sows in the lactation facility. That is to say, that better conception rate implies fewer sows in service facilities to reach the same number of crates occupied in lactation. Furthermore, low conception rates imply an increase of culled animals due to low fertility and result in more gilts in the service facility. These changes impact on the herd distribution over facilities and final productivity, but occupancy rate remains at in similar values.

The results, when the culling rate in lactation decreased, show that the herd distribution over lactation facility tends to increase (4.04%). So, the herd distribution over service and gestation facilities tends to decrease to 1.26% and 0.97% respectively. This fact is explained by the reduction of the number of gilts in the herd. Those changes tended to have a smaller effect on the overall capacity of the farm tending to be smaller (1.13%). Similarly, an increase of the culling rate

provokes an augmentation in the service facilities and a loss of productivity as revealed by minor revenues decreases. The largest decrease per facility was observed in lactation facilities as the number of stalls decreased by 22%. Therefore, an increase of 4% over occupancy rate of lactation facilities was observed.

## 6 Conclusions

The linear programming formulation presented in this paper is useful for supporting the strategic decision of designing sow herd facilities. Previous approaches dealing with this topic were simulation models; though their results were based on average results derived from the steady state. Our model represents computational savings from a more efficient calculation of the optimal replacement policy and sow herd distribution at equilibrium (steady state), and useful information to design facilities as well as to support tactical decisions. Moreover, the description of herd dynamics by a Semi-Markov decision process based on physiological states and movements between facilities allows our model to be flexible in dealing with the representation of animal behaviour or room needs. Moreover, the relaxation of the immediate replacement assumption makes our model more realistic, resulting in a farm with greater capacity or a lower occupation rate and reducing the risk of overflow.

The availability of linear programming solvers makes handling the calculations involved in these kinds of models easier. Similarly, a future incorporation into decision support systems developed for practical purposes is also possible. The formulation of more complex systems is feasible by just conveniently redefining the parameters the Semi-Markov decision model. Such a model provides more insight into the technical and economic consequences of the design of sow herd facilities, reproduction performances, prices and replacement policies. Concerning the robustness of the optimal solution it can be concluded that the use of high performance-related parameters provides a more flexible design of housing facilities. Thus, it allows the farmer to have enough capacity in the lactation facility. Accordingly, slight declines in production never provoke problems with respect the occupation of facilities.

## References

- Billingsley, P., 1995. Probability and Measure. Wiley, New York.
- Chavas, J.P., Kliebenstein, J., Crenshaw, T.D., 1985. Modeling Dynamic Agricultural Production Response: The Case of Swine Production. American Journal of Agricultural Economics 67, 636-646.
- Glen, J.J., 1987. Mathematical Models in Farm Planning: A Survey. Operations Research 35, 641-666.
- Huirne, R.B., van Beek, P., Hendriks, T.H., Dijkhuizen, A.A., 1993. Stochastic Dynamic Programming to Support Sow Replacement Decisions. European Journal of Operational Research 67, 161-164.
- Jalvingh, A.W., Dijkhuizen, A.A., van Arendonk, J.A.M., 1992. Dynamic Probabilistic Modelling of Reproduction and Management in Sow Herds. General Aspects and Model Description. Agricultural Systems 39, 133-152.
- Lippus, A.C., Jalvingh, A.W., Metz, J.H.M., Huirne, R.B.M., 1996. A Dynamic Probabilistic Model for Planning Housing Facilities for Sows. Transactions of the ASAE 39, 1215-1223.
- Plà, L.M., Babot, D. Pomar, J., 2004. A Mathematical Model for Designing Sow Farms. International Transactions in Operational Research 11, 485-494.
- Plà, L.M., 2007. Review of Mathematical Models for Sow Herd Management. Livestock Science 106, 107-119.
- Puterman, M.L., 1994. Markov Decision Processes. Wiley, New York.
- Singh, D., 1986. Simulation-aided Capacity Selection of Confinement Facilities for Swine Production. Transactions of the ASAE 29, 807-815.
- Tijms, H.C., 1994. Stochastic Models: An Algorithmic Approach. John Wiley & Sons, New York.

## Appendix A: Case Study Parameters

The parameters of the case study are enumerated here. Rewards were established for each state and action as shown in Table 5. Income included sales of culled sows to the slaughterhouse (160 €/sow) and sales of weaned piglets (see Table 6). Variable costs were related to feeding cost, which was calculated depending on daily intake, reproduction state and feed type. The purchased cost of gilts was fixed at 180 €/gilt. The fixed cost was in relation to the investment cost per facility. A depreciation of facilities,  $\xi_i$ , over 20 years was considered. The result was represented by the values of 0.06€/gilt and 0.08€/sow in the service and gestation facilities, and 0.23€/gilt and 0.27€/sow in the lactation facility, according to minimum room requirements detailed in the Council Directive 2001/88/EC of the European Union on pigs welfare.

Table 5. Economic input per model (€).

State	A = Keep	A = Replace
Insemination	$\lambda(-0.36 \tau_i + 3.10) - \xi_i$	$\lambda(-0.36 \tau_i + 3.10 - 20) - \xi_i$
Control	$\lambda (-0.36 \tau_i) - \xi_i$	$\lambda (-0.36 \tau_i - 20) - \xi_i$
Gestation	$\lambda (-0.45 \tau_i) - \xi_i$	$\lambda (-0.45 \tau_i - 20) - \xi_i$
Pre-farrowing	$\lambda (-0.45 \tau_i) - \xi_i$	$\lambda (-0.45 \tau_i - 20) - \xi_i$
Lactation	$\lambda (-0.8 \tau_i + LWI) - \xi_i$	$\lambda (-0.8 \tau_i + LWI - 20) - \xi_i$
Culling of insemination	$\lambda (-0.36 \tau_i - 20) - \xi_i$	$\lambda (-0.36 \tau_i - 20) - \xi_i$
Culling of control	$\lambda (-0.36 \tau_i - 20) - \xi_i$	$\lambda (-0.36 \tau_i - 20) - \xi_i$
Culling of gestation	$\lambda (-0.45 \tau_i - 20) - \xi_i$	$\lambda (-0.45 \tau_i - 20) - \xi_i$
Culling of pre-farrowing	$\lambda (-0.45 \tau_i - 20) - \xi_i$	$\lambda (-0.45 \tau_i - 20) - \xi_i$
Culling of lactation	$\lambda (-0.8 \tau_i - 20) - \xi_i$	$\lambda (-0.8 \tau_i - 20) - \xi_i$
Delay of Supplying	$\lambda (-0.36 \tau_i - 20) - \xi_i$	$\lambda (-0.36 \tau_i - 20) - \xi_i$
Culling by infertility	$\lambda (-0.36 \tau_i - 20) - \xi_i$	$\lambda (-0.36 \tau_i - 20) - \xi_i$

A: Action,  $\tau_i$ : Holding times per state  $i$ ,  $\lambda$ : Discount factor.

LWI: Litter weaned income,  $\xi_i$ : Cost per facility considering a 20-year-time horizon.

Table 6. Incomes per litter weaned and sold by cycle (€),LWI: Litter weaned income.

Cycle	1	2	3	4	5	6	7	8
LWI	269.44	305.92	342.40	342.40	342.40	342.40	305.92	232.96

The holding times are shown in the Table 7, although they can change according to the management policy expected for the farm operation.

Table 7. Holding times in days ( $\tau_i$ ) by state  $i$ .

State ( $i$ )	$\tau_i$
Insemination	7
Control	28
Gestation	80
Pre-farrowing	7
Lactation	28
Culling of insemination	14
Culling of control	28
Culling of gestation	62
Culling of lactation	17
Delay of supplying	0
Culling by infertility	0

The main transition probabilities are showed in the Table 8. Culling rates per state are considered to affect with the same intensity over all cycles.

Table 8. Main transition probabilities considered in the basic model.

Cycle	Fertility	Culling		
		Insemination	Gestation	Lactation
1	0.90	0.02	0.08	0.20
2	0.92	0.02	0.08	0.20
3	0.92	0.02	0.08	0.20
4	0.94	0.02	0.08	0.20
5	0.95	0.02	0.08	0.20
6	0.93	0.02	0.08	0.20
7	0.92	0.02	0.08	0.20
8	0.90	0.02	0.08	0.20



# **Optimal replacement policies and economic value of clinical observations in sow herds**

Sara V. Rodríguez<sup>1</sup>, Tina Birk Jensen<sup>2</sup> and Lluís M. Plà<sup>1</sup>

<sup>1</sup>University of Lleida, Department of Mathematics Jaume II, 73 E-25001 Lleida, Spain

<sup>2</sup>Department of Large Animal Sciences, University of Copenhagen, Grønnegårdsvej 2, DK-1870 Frederiksberg C, Denmark

To submitte: Preventive Veterinary Medicine.



## **Abstract**

Even though several sow replacement models have been already published, the problem with representation of the health status of sows has not yet been treated satisfactorily. This paper presents a framework for integration of a so-called Weak Sow Index (WSI) based on observation of several clinical signs into a sow replacement model. The objective was to study the influence of clinical signs on the optimal replacement policy in herds with different risk factors for health disorders. A second objective was to estimate the economic value of observation of individual clinical signs. Bayesian networks were used to estimate the WSI from observed clinical signs, and the optimization of the replacement policy was done in a multi-level hierarchical Markov decision process. The WSI value was shown to have a high influence on the optimal replacement policy, allowing better classification of sow regarding health status. It was also shown that the economic value of the WSI is higher in a high risk than a low risk herd. The lowest economic value of a clinical sign was associated with “unwilling to stand”, while the highest economic value was associated with “vulva bite”.

*Keywords; optimal replacement; Hierarchical Markov models; Bayesian updating; Weak sow index; health status*

## 1 Introduction

The sow replacement problem is today one of the most important challenges in sow herd management having a direct economical impact not only for the farmer, but also for the Pig supply chain where sow herd production is integrated. Sow replacement decisions influence the expected lifetime of sows, the annual replacement rate, the piglet production capacity and other important key figures for the planning production of pig supply chain. Basically, the sow replacement problem is defined as a management decision determining the optimal time to replace a sow, based on the characteristics of individual sows (e.g. parity, litter size and conception rate) (Kristensen, 1994). In order to support the decision on when to cull a sow, several quantitative models have been developed and described in the literature (Plà, 2007). Generally, those models try to model the dynamic production of sows taking into account the most representative and directly observable variables such as the conception rate, the litter size and the genetic merit.

Until now no replacement models have incorporated information in regard to the health status of sows. This is probably due to difficulties in observing and recording variables representing the health status, such as clinical signs and physical and behavioural abnormalities, in a systematic and operational way. However, the incorporation of health indicators in sow replacement models seems to be crucial since a poor health status (defined as the occurrence of clinical signs and/or the presence of physical or behavioural abnormalities) have a high impact on sow farm production thus playing an important role in the economic viability of sow farms. Moreover, a poor health status can cause failure to conceive, abortion and increased sow mortality which raise the production cost and, hence, reduce the profit margin (Straw et al., 2006). A poor health status is not only regarded as an economical problem, but also as a significant animal welfare problem. From 2000 to 2007 the average annual sow mortality in Denmark increased from 11% to 15% (Danish Pig Production, 2007). Coping with increased mortality among sows is therefore given high priority in Danish livestock research. Moreover, optimal

replacement policies incorporating animal health aspects are becoming increasingly important in modern sow farming.

The aim of this study is to incorporate information about the health status of individual sows into an existing sow replacement model developed by Kristensen and Søllested (2004a and b). The health status of a sow is described as an index (called the Weak Sow Index (WSI)) quantifying significant clinical signs into one numerical value. Hence, the effect of the health status of sows in both the gestation and lactation period on the optimal replacement policies will be studied.

The outline of the paper is as follows. First the development of the WSI will be described and the integration of the WSI into the replacement model presented. To illustrate a potential application of the model, two cases will be used to calculate the economic value of observing clinical signs when deciding on when to replace a sow, hence, balancing the value of information against the labour requirements.

## **2 Materials and Methods**

### *2.1 The existing replacement model*

The existing replacement model for sows was originally developed by Kristensen and Søllested (2004a and b). The replacement model is a multi-level hierarchical Markov process using Bayesian updating. In general, a hierarchical model is an infinite stage Markov decision process with parameters defined in a special way, but nevertheless in accordance with all usual rules and conditions relating to such processes.

The basic idea of the hierarchic structure is that stages of the process can be expanded to a so-called child process, which again may expand stages further to new child processes leading to multiple levels. By using hierarchical modelling more detailed models can be solved. For a detailed description of the concept, reference is made to Kristensen and Jørgensen (2000).

The properties (state variables) represented in the original model by Kristensen and Søllested (2004a and b) were the litter size potential of the sow calculated by Bayesian updating and the number of re-matings. Furthermore, the age of the sow was represented through the hierarchical structure. The Bayesian updating technique applied in the model was based on a dynamic linear model as described by West and Harrison (1997). The transition probabilities were based on a litter size model of which the parameters were fitted as described by Toft and Jørgensen (2002).

In order to incorporate the health status of sows in the existing replacement model, we first of all developed a WSI for a sow quantifying various clinical signs into one numerical value.

## *2.2 Development of the WSI*

### *2.2.1 General structure of the WSI*

The WSI is constructed by use of a Bayesian network consisting of a set of herd level variables,  $H$ , a set of sow specific variables,  $S$ , representing potentially observed clinical signs, a sow specific variable  $n$  representing the parity, and finally, a sow specific variable  $w$  representing the weak sow index. The network further has the property that, for any variable  $x \in H$ ,  $x$  is d-separated from  $w$  given  $S \cup \{n\}$ . For a detailed explanation of the concept of d-separation, reference is made to Jensen (2001).

The practical implications of the d-separation in this particular case are that the WSI,  $w$ , is calculated exclusively from clinical observations at sow level, i.e. a subset of  $S$ , but since the prevalence of the clinical signs depends on herd specific conditions, the distribution of  $w$  will be herd specific. Thus, the transition probabilities describing the dynamic properties of  $w$  will be herd specific as well. All variables in the Bayesian network are discrete. The herd level variables are categorical with a number of distinct states characterizing the production system

(for instance the variable “herd size” has the state space {small, medium, large}). The clinical signs are typically binary or ordinal. The WSI,  $w$ , is numerical, and for convenience, it is modelled as a number of discrete numerical levels.

Two versions of the network have been constructed: One for the gestation period and one for the lactation period. In general, the WSI represents the risk of a sow to be involuntarily culled; however the interpretation of involuntary culling differs for pregnant and lactating sows. For the pregnant sow, “involuntary culling” represents a pool of sows that are dead, euthanized or sent to slaughter unexpectedly (due to clinical signs and/or physical and behavioural problems). For the lactating sows, “involuntary culling” includes only sows that are either dead or euthanized.

### *2.2.2 Data analysis and construction of the WSI*

Both models were developed using data from 3541 pregnant and 1347 lactating sows, randomly selected from 34 Danish sow herds. Each sow was individually examined for 16 clinical signs (e.g. lameness {no, mild, severe}, body condition score {thin, normal, lean} and vulva bite {no, yes}), and all clinical signs were recorded on a qualitative scale. After the clinical examination, farmers recorded the replacements of sows (euthanization, sudden death or sent to slaughter) and the reasons for these actions. Only replacement information recorded maximum three months after the clinical examination, were used for the construction of the WSI models. Jensen et al. (submitted) described the data collection at both sow and herd level in details.

Explanatory factor analysis with principal axis factoring (PAF) was used to identify and characterise the underlying correlation structure of the clinical variables for the lactating and the pregnant sows, respectively (Jensen et al., submitted). These factors incorporated a number of clinical variables that shared a common structure, and which were used to develop the causal structure of the WSI model. The two versions of the WSI models were modelled as Bayesian networks, combining the clinical signs, the latent factors and the probability of involuntary culling. To identify and estimate significant links in the model, logistic regression

analyses were performed with “involuntary culling” as outcome variable and the latent factors, as well as clinical variables not included in the factors, as explanatory variables.

For the pregnant sows, three different factors extracted the clinical variables which described most of the variation of data. We interpreted these factors as: “Pressure marks”, “Wounds” and “Lameness”. Only the factor: “Lameness” (which included the clinical signs lameness {no, mild, severe} and unwillingness to stand {no, yes}) appeared to be significantly associated with the outcome variable: “Involuntary culling” ( $p=0.01$ ). Moreover, the clinical sign: vulva bite {no, yes}, which did not load high on any of the three factors appeared to be significantly associated with “Involuntary culling” ( $p<0.05$ ). Vulva bite was therefore included in the WSI model for pregnant sows (Figure 1). Thus, the set of sow variables for the pregnancy period was  $S = \{\text{lameness, unwillingness to stand, vulva bite}\}$ . The latent factor “Lameness” was only used as an aid to specify the structure of the conditional probability table of the WSI variable,  $w$ . It is therefore shown with dashed border in the figure.

Only two factors were found for the lactating sows, which were interpreted as “Pressure marks” and “Wounds”. The two latent factors did not show any statistical significance in regard to “Involuntary culling”. However, the clinical signs: Vulva colour {no, yes}, shoulder ulcer {no, scar, ulcer} and body condition score {thin, normal, lean} all had a significant effect on “involuntary culling” ( $P<0.05$ ), and were therefore included in the WSI for lactating sows (Figure 2). Thus, the set of sow variables for the lactation period was  $S = \{\text{vulva color, shoulder ulcer, body condition score}\}$ .

Information about the herds (e.g. herdsize {less than 400 sows, 400-600 sows, more than 600 sows}, feeding system {electronic sow feeding, feeding boxes, competition based feeding} and deep bedding {yes, no} (pregnant sow)) was finally included in the WSI models. Based on logistic analyses, the herd variables: herd size and feeding system were found to affect the clinical variables: lameness and unwillingness to stand, whereas deep bedding was found to influence the

occurrence of vulva bite in a sow. Hence, these herd risk factors were included in the WSI for the pregnant sows (Figure 1). In other words, the set of herd level variables for the pregnancy period was  $H = \{\text{herd size, feeding system, deep bedding}\}$ .

For the lactating sows, herd size and feeding system were found to influence shoulder ulcer only, and were consequently included in the WSI model for lactating sows (Figure 2). Thus, the set of herd level variables for the lactation period was  $H = \{\text{herd size, feeding system}\}$ .

No effect of parity was found in any of the analyses. Nevertheless, it was decided to keep parity in both networks in order to illustrate the full concept of the developed framework. The conditional probabilities of the WSI,  $w$ , were just defined independently of the parity.

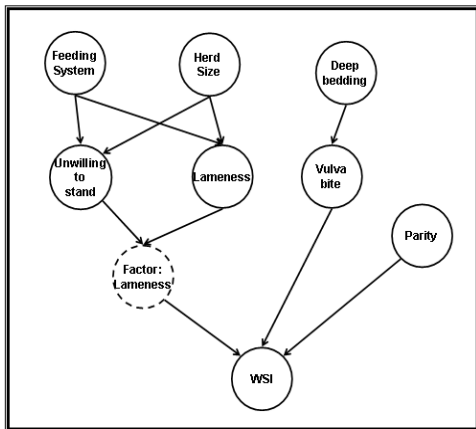


Figure 1. The WSI for the pregnant

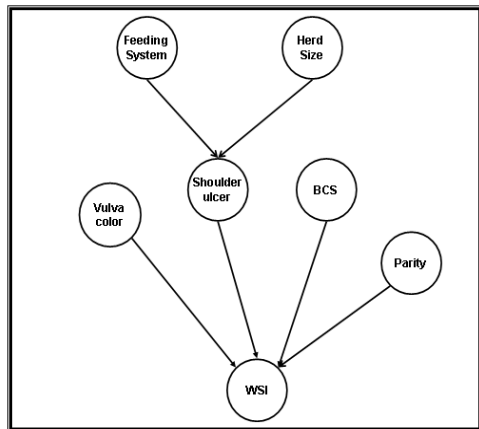


Figure 2. The WSI for the lactating

The parameters used for calculation of the conditional probabilities of the Bayesian networks are shown in Appendix A. The Bayesian networks were implemented by use of the Esthauge LIMID software system<sup>1</sup>.

<sup>1</sup> [www.esthauge.dk](http://www.esthauge.dk)

### 2.3 Use of the Bayesian networks for calculation of the WSI

It is assumed that all variables in  $H$  are observed in every herd, whereas the sow specific variables being observed depend on an *observation policy* defining the clinical examinations done in the herd. Denote as  $E_h \subseteq S$  the set of sow specific clinical variables being observed in herd  $h$ . A corresponding configuration (or evidence set) of  $E_h$  observed for sow  $s$  is denoted as  $e_{hs}$ .

In principle, the WSI for a sow is found by entering the observed values of the sow variables (the clinical signs) and then propagating. If all sow variables are observed (i.e. if  $E_h = S$ ), the value of  $w$  is known with certainty. If fewer sow level variables are observed (i.e. if  $E_h \subset S$ ), an estimate for the WSI is still available from the Bayesian network, but the precision will be lower. In the replacement model, the weak sow index is expressed relatively as the deviation from the herd average (defined by the values of the herd level variables in  $H$ ). A formal description is given in the following subsections.

#### 2.3.1 The gestation period

Denote as  $w_{ns}^G$  the WSI for the gestation period of sow  $s$  at parity  $n$  in herd  $h$ . We may model the value as a sum of an underlying herd mean  $\mu_{hn}^G$  and a random term  $A_{ns}^G$ :

$$w_{ns}^G = \mu_{hn}^G + A_{ns}^G \quad (1)$$

where the sow specific variable  $A_{ns}^G$  has the properties  $E(A_{ns}^G) = 0$  and  $V(A_{ns}^G) = \sigma_{Ghn}^2$ .

The herd mean,  $\mu_{hn}^G$  of Eq. (1), is found as



$$\mu_{hn}^G = E(w | h, n) = \sum_{j=1}^J P(w = w_j | h, n) w_j, \quad (2)$$

where, now,  $h$  is interpreted as the configuration of  $H$  representing the observed values of the herd level variables, and  $w_j$  is the value corresponding to the  $j$ th state of  $w$ . The expected value is simply found by inserting  $h$  and  $n$  as evidence in the Bayesian network and then propagating. The variance  $\sigma_{Ghn}^2$  between sows in the herd is found analogously.

For a given configuration  $e_{hs}$  of the observation set, the estimated (relative) WSI,  $\hat{w}_{ns}^G$ , of the sow will be defined as

$$\hat{w}_{ns}^G = E(w_{ns}^G | e_{hs}, n) - \mu_{hn}^G. \quad (3)$$

The precision of this estimate depends, as mentioned, on the herd observation policy described by  $E_h$ . For convenience the standard deviation will be denoted as  $\sigma_{Gn}$ , where

$$\sigma_{Gn}^2 = Var(w_{ns}^G | E_h, n). \quad (4)$$

Thus  $\sigma_{Gn}$  may be regarded as the standard deviation of the observation error. It should be noticed, that this approach assumes variance homogeneity over the configurations of  $E_h$ . For the special case where all sow specific variables (described in section 2.1) have been observed (i.e. if  $E_h = S$ ) there will be no observation error, implying that  $\sigma_{Gn} = 0$ .

The conditional expectation and variance in Eqs. (3) and (4) are found by inserting the evidence into the Bayesian network followed by a propagation. Thus, assuming normal distributions,

$$(w_{ns}^G | e_{hs}, n) \sim N(\mu_{hn}^G + \hat{w}_{ns}^G, \sigma_{Gn}^2). \quad (5)$$

Eq. (5) will later be used as basis for deduction of the transition probabilities of the replacement model.

### 2.3.2 The lactation period

The (relative) WSI for the lactation period,  $\hat{w}_{ns}^L$ , is modeled completely analogously.

### 2.4 Dynamics of the WSI

The observed WSIs,  $\hat{w}_{1s}^G, \hat{w}_{1s}^L, \dots, \hat{w}_{ns}^G, \hat{w}_{ns}^L$ , of an individual sow are assumed to be autocorrelated as a first order autoregressive time series. Thus, the autocorrelation coefficient between  $\hat{w}_{ns}^G$  and  $\hat{w}_{ns}^L$  is denoted as  $\rho_{GLn}$ , and the corresponding autocorrelation coefficient between  $\hat{w}_{ns}^L$  and  $\hat{w}_{n+1,s}^G$  is denoted as  $\rho_{LGn}$ . These coefficients will be used later for calculation of the transition probabilities of the replacement model.

### 2.5. Integration of the WSI into the replacement model

#### 2.5.1 State variables for the WSI

The WSI state variables will in the replacement model be represented at  $2k + 1$  levels,  $-k, \dots, 0, \dots, k$ , where  $-k$  corresponds to a very weak sow, 0 to a sow at the herd average for the parity (and stage of cycle), and  $+k$  corresponds to a very strong sow.

Hence, in the replacement model, the WSI of a parity  $n$  sow,  $s$ , is represented by state variables as follows:

**In the gestation period:** The estimated WSI for present gestation period,  $\hat{w}_{ns}^G$ .

**In the lactation period:** The estimated WSI for present lactation period,  $\hat{w}_{ns}^L$ .

**For parity  $n > 1$ :** The estimated WSI for previous lactation period,  $\hat{w}_{n-1,s}^L$ . This state variable is necessary for representation of the autocorrelation between the WSI in previous lactation period and present pregnancy period.

Thus, combining these new state variables with those included by Kristensen and Søllested (2004a and b) results in a hierarchical model with 3 levels defined as follows:

**Founder process:** Infinite time horizon.

**Stage:** Stage length is equal to the life span of a sow in the herd.

**State space:** Only one dummy state is defined.

**Action space:** Only one dummy action is defined.

**Child level 1:** Finite time horizon.

**Stage:** Stage length is equal to a reproductive cycle from weaning to weaning. Stage number equals parity.

**State space:** Depends on parity:

**Parity 1:** Only one dummy state is defined.

**Parity >1:** Two state variables are defined:

- Litter size potential (21 levels).
- WSI of previous lactation period ( $2k+1$  levels).

An additional state representing a culled sow is added.

The number of states equals  $21 \times (2k+1) + 1$ .

**Action space:** Mating method: 2 options that for instance represent “Normal mating” and “Artificial insemination” as described by Kristensen and Søllested (2004a and b).

**Child level 2:** Finite time horizon.

**Stage:** Stage length is equal to the duration of “Mating” (stage 1), “Gestation” (stage 2) or “Lactation” (stage 3).

**State space:** Depends on stage:

**Stage 1, “Mating”:** Three states reflecting health status: “Healthy”, “Diseased” and “Dead”. The “Diseased” state is used to

represent involuntarily culled sows which can still be slaughtered, whereas “Dead” represents dead and euthanized sows.

**Stage 2, “Gestation”:** “Pregnant” with a combination of WSI level of the current gestation period. Two additional states representing a “Diseased” and “Infertile” sows are added. The number of states equals  $(2k+1) + 2$ .

**Stage 3, “Lactation”:** Two state variables are defined:

- “Litter size”, present parity (20 levels).
- WSI of present lactation period ( $2k+1$  levels).

Two additional states representing a “Diseased” and “Dead” sow are added. The number of states thus equals  $20 \times (2k+1) + 2$ .

**Action space:** Depends on stage:

**Stage 1, “Mating”:** Mating policy: Allow 1,...,5 matings before culling for infertility if the sow is “Healthy”. If the sow is “Diseased” or “Dead”, only one dummy action is defined.

**Stage 2, “Gestation”:** Only one dummy action is defined.

**Stage 3, “Lactation”:** Two actions defined: “Keep the sow” and “Replace the sow at weaning”. But if the sow is “Diseased” or “Dead”, only one dummy action is defined.

The model has been implemented as a plug-in to the MLHMP software system presented by Kristensen (2003).

### *2.5.2 Consequences for the sow of the WSI in the gestation period*

The effect of the WSI is that it increases the probability of death/euthanization/send to slaughter due to a poor health status. Each level of the (current) WSI will be associated with a value on the logistic scale reflecting directly the probability of death/euthanization/send to slaughter due to a poor health status.

Thus, the state space at child level 2 will, as described above, have a special state called “Dead” reflecting that the sow dies/is euthanized. The value of the dead sow is zero, but there will be a disposal cost. The “Dead” state will be defined for the mating period and the nursing period.

The “Diseased” state will in the extended model only be used for involuntarily culled sow which can be sent to slaughter (and thus have a full slaughter value).

### *2.5.3 Consequences for the sow of the WSI in the lactation period*

The effect of the WSI in the lactation period is that it increases the probability of death/euthanization, but does not include the probability of the event “send to slaughter” due to a poor health status. Hence, each level of the (current) WSI will be associated with a value on the logistic scale reflecting directly the probability of death/euthanization.

An additional effect of the WSI of the lactation period, is that it influences the conception rate of the following mating period so that a strong sow has a higher conception rate.

## *2.6 Description of the parameters*

### *2.6.1 Information needs for model construction*

The basic idea behind the existing sow replacement model described by Kristensen and Søllested (2004a and b) is that a herd specific model is constructed. The model is based on the prices and biological conditions of the particular herd in question. Thus, to the extent possible, all parameters describing litter size, variance components, conception, involuntary culling and piglet mortality are estimated from data originating from the herd. The extended model taking the WSI into account will be based on the same principle. The biological basis for inclusion of the WSI is provided through the Bayesian networks described in Section 2.2. Furthermore, the probabilities of the Bayesian networks and the necessary auto

correlation coefficient for the WSI from the gestation to the lactation period  $\rho_{GLn}$  are presented in Appendix A. Due to very few repeated measurements from the lactation period to the next gestation period, it was not possible to estimate the corresponding autocorrelation coefficient,  $\rho_{LGn}$ . As a starting point it was therefore decided to assume that  $\rho_{LGn} = \rho_{GLn}$ .

In order to adapt the model to the conditions of a specific herd, we also need:

- Information about the values of all variables in  $H$  for the herd.
- Information about the observation policy both gestation and lactation period (i.e. identification of  $E_h \subseteq S$ ).

In that way, the specific conditions of a particular herd are taken into account.

## 2.7 Transition probabilities

### 2.7.1 The WSI probabilities

The transition probabilities from state  $i$  to state  $j$  express the combined probabilities of the transitions reflected in the values of the state variables belonging to state  $i$  and those belonging to state  $j$ . As was described in the model structure the state variables reflect traits like litter size potential, pregnancy status and WSI, and the final transition probabilities are calculated as the product of the individual transition probabilities for the state variables in question.

The probabilities related to transitions in litter size potential, observed litter size and conception are calculated as described in the original articles by Kristensen and Søllested (2004a and b). For the extended model, we furthermore need the probabilities related to transitions in WSI. In the following, all formulas refer to the same sow, so the index  $s$  for sow will be skipped for convenience. For the model specification, the transition probabilities listed in Table 1 are needed for the WSI. The precise calculation of those probabilities is described in details in Appendix B.

Table 1. Transition probabilities for the WSI.

Probability	Explanation
$P(\hat{w}_1^G)$	Initial distribution of the estimated WSI of a pregnant gilt
$P(\hat{w}_1^L   \hat{w}_1^G)$	Conditional probability distribution of the estimated WSI of a lactating gilt given the estimated WSI of the gestation.
$P(\hat{w}_n^G   \hat{w}_{n-1}^L)$	Conditional probability distribution of the estimated WSI of a pregnant sow given the estimated WSI of the previous lactation period.
$P(Diseased   \hat{w}_{n-1}^L)$	Conditional probability of entering the “Diseased” state at the beginning of the mating period given the WSI of previous lactation.
$P(Dead   \hat{w}_{n-1}^L)$	Conditional probability of entering the “Dead” state at the beginning of the mating period given the WSI of previous lactation.
$P(Diseased   \hat{w}_1^G)$	Conditional probability of entering the “Diseased” state at the beginning of the lactation period given the WSI of the gestation period.
$P(Dead   \hat{w}_1^G)$	Conditional probability of entering the “Dead” state at the beginning of the lactation period given the WSI of the gestation period.

As mentioned, the final transition probabilities are a combination of the transition probabilities originating from the WSI and those from the litter size model and the mating policy model. They are all defined at Child level 2 of the model. A process at child level 2 has 3 ordinary stages for mating, gestation and lactation, respectively, and in addition an initial dummy stage holding the probability distribution over states of the mating stage. For a gilt (parity 1) only the information defined by the states at child level 2 is available, whereas sows from parity 2 and higher are also characterized by the state variables defined for child level 1 (the WSI of previous lactation period and the litter size potential).

In the following description we shall denote as  $p_{ij}^d(n)$  the probability of transition from state  $i$  at stage  $n$  to state  $j$  at stage  $n+1$  under decision  $d$ .

### 2.7.2 Initial state probabilities (Stage 0)

For a gilt no WSI information is available before mating, so the probabilities are defined as in the current model described by Kristensen and Søllested (2004a and b). For parity 2 and higher, information about WSI of the previous lactation period is also available. The information is stored in the Child level 1 state.

The initial transition probabilities  $p_{1j}^1(0)$  define probabilities to stage  $j$  of the mating stage, where  $j \in \{\text{“Healthy”}, \text{“Diseased”}, \text{“Dead”}\}$ . For  $j = \text{“Healthy”}$ , the probability equals the probability of the sow not being involuntarily culled. For the two other probabilities, the calculation is described in details in Appendix B.

### 2.7.3 Mating period (Stage 1)

Ignoring the special states, “Diseased” and “Dead” there is only one state  $i$  to consider, i.e.  $i = \text{“Healthy”}$ . The 5 actions “Allow  $d$  matings” must define transition probabilities to states representing different values of WSI. Let  $d \in \{1, \dots, 5\}$  be the action of the mating period, and let  $j \in \{1, \dots, 2k+3\}$  be the state of the gestation period. The value of  $j$  corresponds directly to level of WSI. If the model has more than one mating method, the actual mating method is known from the decision at Child level 1.

For a gilt, no information about previous WSI is available. The calculation is rather simple. Defining the events  $S$  and  $C$  corresponding to survival and conception, respectively, the transition probability  $p_{ij}^d(1)$  is then, for  $j < 2k + 2$ , calculated as the product of the probability of conception, the probability of survival (neither dead nor diseased) and the probability of observing a certain WSI level in the gestation period. The probabilities of conception and survival are the same as described by Kristensen and Søllested (2004a and b), and the probability of WSI level is described in Appendix B. The probabilities to “Infertile” and “Diseased” are as described by Kristensen and Søllested (2004a and b).

For a parity 2 sow (and higher), information about the old WSI of the sow (from previous lactation period) is also available. The information is stored in the Child level 1 state. This value influences the conception rate and the WSI of the gestation period. The transition probability  $p_{ij}^d(1)$  at parity  $n$  is again, for  $j < 2k + 2$ , calculated as the product of three separate probabilities (conception, survival and WSI level). The only difference are that the transition probability of the WSI level



and the conception rates are now conditioned on WSI level of previous lactation period. Appendix B describes how to calculate the conditional probability. Probabilities of the two states “Infertile” and “Diseased” are calculated in the same way as for parity 1.

### 2.7.3 Gestation period (Stage 2)

Ignoring the special states (“Diseased” and “Dead”) the other states are described directly by their WSI level. Since the possible influence of previous parity WSI in the lactation period is ignored, the procedure is the same for first and higher parities. The probabilities link to states  $j$  of the lactation period, where a state is described by present litter size and WSI.

The transition probabilities for the gestation period are calculated as the product of the conditional probability of observing a given litter size (given litter size potential known from child level 1), the conditional probability of survival (given current WSI level) and the conditional probability of observing a given WSI level in the lactation period (given the current WSI level). The transition probability related to litter size is as described by Kristensen and Søllested (2004a and b), whereas the two other conditional probabilities are described in Appendix B. The calculation of the probabilities to states “Dead” and “Diseased” is described in Appendix B.

### 2.7.4 Lactation period (Stage 3)

Here the state  $i$  is described by combined values of litter size and WSI. For the action “**Keep**”, the states, that the probabilities link to, are the Child level 1 states of next parity. A destination state  $j^1$  is described by combined values of updated litter size potential  $m_{j^1}$  and old WSI,  $\hat{w}_{m_{j^1}}$ . The transition is deterministic in the sense that there exists a target state  $j^1$  of which  $p_{ij^1}^d(3)=1.0$ . Thus the problem reduces to determining this  $j^1$ , which is the state  $j$  having the following properties:

1.  $\hat{w}_{nj_1} = \hat{w}_{ni}^L$ : The WSI at Child level 1 is a simple memory variable.
2.  $m_{j_1}$  is identical to the old litter size potential  $m_{i_1}$  updated by the present litter size  $l_{ni}$ .

For the highest parity the transition probabilities define a deterministic transition to the founder stage. Under the action “**Replace**” the process goes to state “Replaced” at Child level 1 with probability 1.0.

### 2.8 Rewards

The rewards of the model are calculated in the same way as it was described by Kristensen and Søllested (2004b), except for the states ”Diseased” and “Dead”. If the sow is dead (or has been euthanized), the farmer does not receive any income, but will have to pay a cost for disposal of the dead body. If the sow is diseased, it is sent to slaughter immediately, and the reward equals the slaughter price.

## 3 Example

In order to illustrate the formulation and produced output of the model (highlighting the effect of the WSI on the replacement problem), the effect of WSI on the replacement policy and the economic value of clinical observations are presented. A hypothetical sow herd with two scenarios corresponding to different herd risk levels is used for illustration.

### 3.1 Parameters, basic scenarios

The parameters of the model are inspired by values considered as normal under Danish conditions. Hence, settings from a commercial herd “Herd A” described by Kristensen and Søllested (2004b) were selected to represent a typical commercial Danish sow herd to be part of this study. Nevertheless, the prices were updated, Table 2.

Table 2. Price conditions (in DKK) used for the commercial herd “Herd A” described by Kristensen and Søllested (2004b).

Description	Unit	Price
Feed in the mating period	Fes	1.20
Feed in the gestation period	Fes	1.20
Feed in the lactation period	Fes	1.20
Feed for piglets	Kg	1.73
Basic piglet price	Piglet	196.21
Price per mating, method 1	Mating	20.00
Price per mating, method 2	Mating	30.00
Price of gilt for replacement	Gilt	1300.00
Slaughter price of sow	Kg live weight	4.72
Disposal cost, dead sow	Sow	198.50

In order to adapt the model to the conditions of “Herd A”, information about the values of all variables in  $H$  and information about the observation policy are given. Two cases have been considered; one case where the values of all variables in “Herd A” define a high risk level of involuntary culling and another case where the values define a low risk level of involuntary culling both in the gestation and lactation period. Hence, a high risk level (HR) is defined as a herd with a large herd size ( $>600$  sows), using electronic sow feeding and with deep bedding in the pens. Contrary, a low risk level (LR) is a small herd ( $<400$  sows) with feeding boxes and with no deep bedding in the pens.

The specific observation policy in both cases was assumed to be complete  $E_h = S$ , meaning that all clinical examinations were done in both the gestation and the lactation period. Therefore, no observation error was considered.

From the structure and probabilities of the resulting Bayesian networks, mean and standard deviation both gestation and lactation period were obtained, Table 3. In addition to these parameters, the necessary auto correlation coefficients for the WSI were provided. From the gestation to the lactation period it was estimated to  $\rho_{GLn}=0.1145$  (see Appendix A), and it was assumed that the auto correlation coefficient for the WSI from previous lactation period to current gestation period,  $\rho_{LGn}$  takes the same value as  $\rho_{GLn}$ .

Table 3. Parameters obtained from the Bayesian network structure of the current example

Parameter	Explanation	High risk level	Low risk level
$\mu_{hn}^G$	Herd mean for parity $n$ WSI in the gestation period	-2.97	-3.16
$\mu_{hn}^L$	Herd mean for parity $n$ WSI in the lactation period	-3.84	-3.97
$\sigma_{Ghn}$	Standard deviation of WSI between gestating sows	0.34	0.26
$\sigma_{Lhn}$	Standard deviation of WSI between lactation sows	0.64	0.55

The WSI state variables were represented at  $-2, \dots, 0, \dots, +2$  levels so that  $k=2$ . The resulting state space was 130,628 states considering a maximum lifespan of 12 parities. The parameters described above were given as input to the replacement model incorporating WSI information.

### 3.2 Effect of WSI on the replacement policy

In this section optimization of the extended model taking the WSI into account was carried out for both risk levels. Hence, optimal replacement policies maximizing average net returns over time were obtained. For the high risk herd the optimal replacement policy implied no culling based on litter size and WSI before the 6<sup>th</sup> parity. From the 6<sup>th</sup> parity the culling for low litter size or low WSI appeared and increased through the 7<sup>th</sup> parity. In fact, culling actions were present in 40% and 71% of the defined states of the 6<sup>th</sup> and the 7<sup>th</sup> parity, respectively. After the 8<sup>th</sup> parity all sows were culled independently of litter size when the WSI was obtained. While with the low risk level the same pattern was observed except that the culling action was present in 39% and 72% of the defined states of the 6<sup>th</sup> and the 7<sup>th</sup> parity, respectively. The expected economic net returns (DKK per sow per year) obtained from the replacement policy was 2,742 under low herd risk and 2,540 under high risk, thus being 3% higher in low risk than high risk herd. Typically the average herd size of a Danish breeding farm is 1000 sows. So, such percentage on the net returns would in economic term represent 68,000 (DKK/year). The consequences of the two resulting policies were compared through technical and economic key figures calculated by Markov chain simulations. It is; associated to

the optimal solution of the problem there is a Markov chain defining herd dynamics over time under optimal replacement policy. So, this chain was used to perform simulations. In table 4 technical and economic results from chain simulations are presented. From the results, the probability of dead/euthanized over gilt was 4% higher in high risk than low risk herd is observed. As a consequence lower average culling age in high risk than in low risk herd was found (Table 4).

Table 4. Technical and economical results from chain simulations.

Key Figure	High risk level	Low risk level
Net returns, DKK per piglet weaned	117.77	119.95
Piglets weaned per sow per year	20.99	21.18
Piglets weaned per litter	9.22	9.23
Average age (parity) at culling	4.89	5.10
Voluntary annual culling rate	20%	21%
Involuntary annual culling rate	14%	13%
Voluntary annual culling over gilt	0.43	0.47
Involuntary annual culling over gilt	0.30	0.30
Dead/Euthanized over gilt	0.27	0.23

To illustrate the effect of the WSI on the replacement problem, the distribution of the WSI of voluntarily culled sows was computed. Figure 3 shows the results. As it is seen, the WSI plays an important role when sows are selected for culling.

The figures described above are calculated by means of the optimization and simulation facilities of the MLHMP software (Kristensen, 2003).

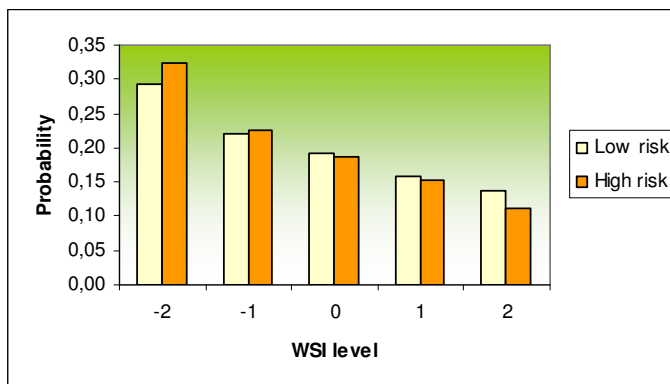


Figure 3. Distribution of the WSI of voluntarily culled sows on Herd A.

### 3.3. Economic value of clinical observations

As mentioned, the WSI characterizes the health status of an individual sow using available information from simple clinical examinations at sow level. In the first part of the study it was assumed that all sow level variables defining the WSI were observed, i.e.  $E_h = S$ . Since, however, examination of a clinical sign can be expensive (in terms of time consumption and money) some farmers would probably choose to observe fewer clinical signs. In that case, the given observation policy would be incomplete,  $E_h \subset S$ , but, due to the Bayesian network approach, the value of WSI is still calculated although with less precision. Hence the aim of this section is to identify clinical signs of major economic importance of individual sows to define the observation policy.

#### 3.3.1 Definition of scenarios to test

To identify the economic value of clinical observations, 15 scenarios were defined. Each scenario is represented by a specific observation policy, Table 5.

Table 5. Description of 15 observation policies.

scenario	Description of the observation scenario
1	All clinical signs are observed (default)
2	a) Unwilling to stand
3	b) Lameness
4	c) Vulva bite
5	d) BSC
6	e) Shoulder ulcer
7	f) Vulva color
8	a) Unwilling to stand
9	b) Lameness
10	c) Vulva bite
11	d) BSC
12	e) Shoulder ulcer
13	f) Vulva color
14	Only clinical signs of gestation period are observed.
15	Only clinical signs of lactation period are observed.

Each scenario was run for both a high risk (HR) and a low risk (LR) herd. Scenarios HR-1 and LR-1 are identical to the cases presented in the previous section. They will be used as references for the other scenarios.

### 3.3.2 Results from the example

The economic net returns of the set of scenarios are presented in Figure 4 as deviations from HR-1 and LR-1, respectively. As it is seen, the lowest reduction on the economic net returns is from the scenario 2, meaning that if a farmer wishes to reduce the number of clinical signs to observe by one, he should refrain from observing “unwilling to stand”. In contrast, the most expensive single clinical sign to leave out is “vulva bite”. The same pattern is found in both high and low risk herd. Nevertheless the high risk herd presents higher losses than low risk.

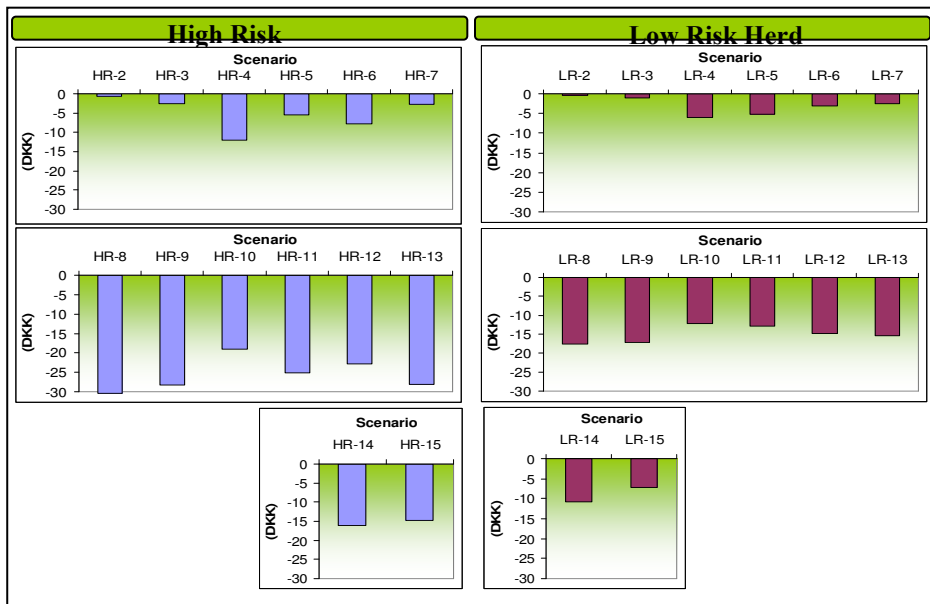


Figure 4. Reduction on the Net returns (DKK/sow/ year) perceived by the different scenarios regarding the base.

No clinical examination of the sign “shoulder ulcer” (HR-6) presents a higher reduction than no clinical examination of the sign “BSC” (HR-5). However this pattern is different for low risk level. If only one clinical sign can be observed, it is

clear that the observation of “Vulva bite” gives the best outcome with the lowest reduction on expected rewards, while the highest reduction is coming from the observation of the single sign “unwilling to stand”.

Moreover an analysis on HR-14, LR-14 HR-15 and LR-15 showed that the observation of the set of clinical signs in the lactation period caused a lower decrease on net returns than the observation of the set of clinical in the gestation period.

Figure 5 presents the distributions of WSI for voluntarily culled sows for the scenarios HR-14, LR-14 HR-15 and LR-15. It seems that observation of the set of clinical signs in the gestation period allows for a better detection of weak sows than the set of clinical signs in the lactation period.

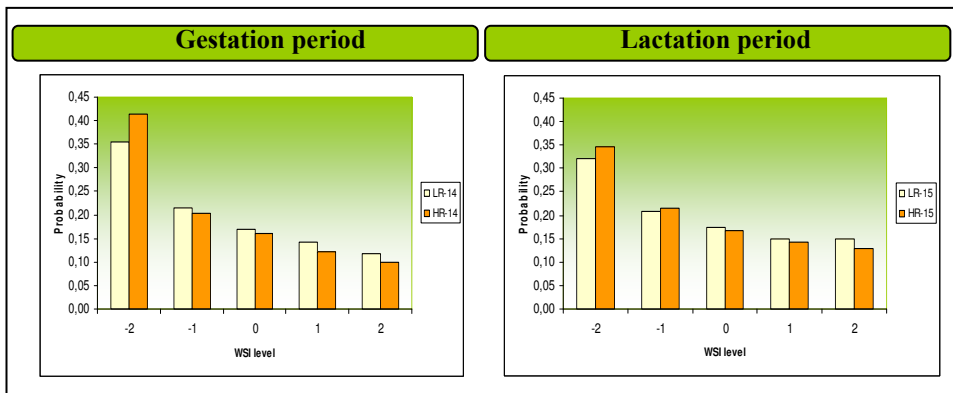


Figure 5. Distribution of the WSI over voluntarily culling regarding HR-14, LR-14 HR-15 and LR-15.

#### 4 Sensitivity Analysis, autocorrelation

Due to too few repeated measurements, the corresponding auto correlation coefficient for the WSI from previous lactation to current gestation  $\rho_{LGn}$  could not be estimated from data. It was therefore, arbitrarily assumed that it was equal to  $\rho_{GLn}$ . Hence, the aim of this section is to test and discuss the effect of  $\rho_{LGn}$  of



WSI on the voluntarily culling policy. For such purpose, original value of autocorrelation is modified by  $\pm 50\%$  in both types of herd (LR and HR).

In Table 6 the economic results of the optimization model are shown. A limited impact on results is observed. On the other hand, Figure 6 shows the estimates of the WSI for the three different correlations. As it is seen, the size of this autocorrelation coefficient only marginally influences the distributions. The effect of a 50% of variation in the auto correlation coefficient for the WSI from previous lactation period to current gestation period,  $\rho_{LGn}$  does not have any bigger repercussion on the WSI estimates and economic rewards.

Table 6. Sensitivity Analysis, value of autocorrelation coefficient. Net rewards.

Scenario	$\rho_{LGn}$	HR (DKK/sow /year)	LR (DKK/sow /year)
+50%	$\Delta$ 50%	2473.86	2540.22
Base	Base	2472.36	2539.98
-50%	$\nabla$ 50%	2471.88	2539.75

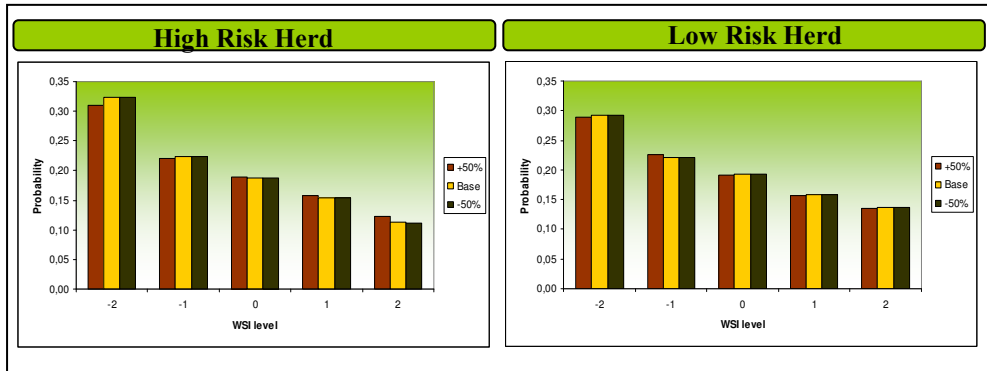


Figure 6. Distributions of the WSI over voluntarily culling regarding scenarios table 5.

## 5 Discussion and Conclusion

Even though several computer-based simulation and optimization models have been developed for breeding and culling decision support on pig farms, the effects of health status on optimal breeding and replacement policies have not been thoroughly studied. In this study, a framework was developed for incorporating

weak sow index information into an existing hierarchical Markov decision process model, which optimizes breeding and replacement decisions for sow herds.

The framework with integration of Bayesian networks to combine information from several different clinical signs into one numerical value, the Weak Sow Index, has turned out to be an efficient tool being able also to handle incomplete observation strategies and even evaluate their economic value. The herd level variables included in the Bayesian networks furthermore enable us to create herd specific models reflecting the risk factors of the individual herd. The presented framework could easily be extended to include more clinical observations without a combinatorial explosion of the size of the state space. It therefore seems to be a powerful technique in dealing with health properties in replacement models.

The WSI value was shown to have a high influence on the optimal replacement policy, allowing better classification of sow regarding the health status. Hence, better replacement policies can be set keeping strong sows and replacing weak sows as it was shown. It was also seen that the economic value of the WSI is higher in a high risk than a low risk herd. The total economic net returns were 3% higher in a low risk herd than in a high risk herd. Regarding clinical observations, it was determined that observation of the whole set of signs allows better estimation of the WSI, and as a consequence higher economic net returns were obtained. But in the case that the complete observation of all clinical signs can be prohibitive, it was shown that the lowest economic value of a clinical sign was associated with “unwilling to stand”, while the highest economic value was associated with “vulva bite”. Using this information the farmer can better establish observation policies to develop the WSI value and as a consequence compare the economic benefit to the costs of observing the clinical signs.

## References

- Danish Pig Production. 2007. Annual Report 2007.  
<http://www.danishpigproduction.dk/index.aspx?id=ca5c27e6-1cc3-414a-8b3a-eba11b740ff0> (accessed)
- Jensen, F.V., 2001. Bayesian networks and decision graphs. Springer-Verlag, New York.
- Jensen, T.B., Bonde, M.K., Kongsted, A.G., Toft, N., Sørensen, J.T., The interrelationships between clinical signs and their effect on involuntary culling among pregnant sows in group- housing systems. Submitted to *Animal*.
- Kristensen, A.R., 1994. A survey of Markov decisions programming techniques applied to the animal replacement problem. *European Review of Agricultural Economics* 21, 73–93.
- Kristensen A.R., Jørgensen, E., 2000. Multi-level hierarchic Markov processes as a framework for herd management support. *Annals of Operations Research* 94, 69-89.
- Kristensen, A.R., 2003. A general software system for Markov decision processes in herd management applications. *Computers and Electronics in Agriculture* 38, 199–215.
- Kristensen, A.R., Søllested, T.A., 2004a. A sow replacement model using bayesian updating in a 3-level hierarchic Markov process. I. Biological model. *Livestock Production Science* 87, 13–24.
- Kristensen, A.R., Søllested, T.A., 2004b. A sow replacement model using bayesian updating in a 3-level hierarchic markov process. II. Optimization model. *Livestock Production Science* 87, 25–36.
- Plà, L.M., 2007. Review of mathematical models for sow herd management. *Livestock Science* 106, 107–119.
- Straw, E.B., Zimmerman, J.J., D’Allaire S., Taylor, D.J., 2006. *Diseases of Swine*. Blackwell Publishing Professional, Iowa, ninth edition, 153.
- Toft, N., Jørgensen, E., 2002. Estimation of farm specific parameters in a longitudinal model for litter size with variance components and random dropout. *Livestock Production Science* 77, 175–185.
- West, M., and Harrison, J., 1997. *Bayesian Forecasting and Dynamic Models*. Springer series in Statistics. Second edition.

**Appendix A**

*A.1 Presentation of the parameter estimates used for calculating the WSI probabilities.*

Table A.1: Parameter estimates from the logistic analyses used to calculate the probabilities for the WSI for the pregnant sow.

<b>Outcome variable</b>	<b>Explanatory variable</b>	<b>Parameter estimate</b>
Involuntary culling	Intercept	-2.5718
	Factor: Lameness <sup>a</sup>	0.334
	Vulva bite	
	Yes	-0.7417
	No	0
Unwilling to stand	Intercept	-0.467
	Herd size	
	Small (< 400 sows)	-0.2852
	Average (400-600 sows)	-1.0944
	Large (> 600 sows)	0
	Feeding system	
	Electronic sow feeding	0.6146
	Individual based feeding	-0.5601
Competition based feeding	0	
Lameness	Intercept 1	-1.0925
	Intercept 2	-0.1875
	Herd size	
	Small (< 400 sows)	-0.7532
	Average (400-600 sows)	-0.9281
	Large (> 600 sows)	0
	Feeding system	
	Electronic sow feeding	-0.2754
	Individual based feeding	-0.7973
	Competition based feeding	0
Vulva bite	Intercept	-1.2666
	Deep bedding	
	No	-0.841
	Yes	0

a: The factor loading for the clinical sign: “lameness” was 0.51 and the factor loading for the clinical sign: “unwilling to stand” was 0.44.

Table A.2: Parameter estimates from the logistic analyses used to calculate the probabilities for the WSI for the lactating sow

Outcome variable	Explanatory variable	Parameter estimate
Involuntary culling	Intercept	-0.8098
	Shoulder ulcer	
	No scars or ulcers	-0.9763
	Scar	-1.101
	Ulcer	0
	Body condition score	
	Below average	1.3086
	Average	0.3006
	Above average	0
	Vulva colour	
	No	-2.6063
	Yes	0
	Shoulder ulcer	Intercept 1
Intercept 2		-0.9467
Herd size		
Small (< 400 sows)		0.259
Average (400-600 sows)		1.2783
Large (> 600 sows)		0
Feeding system		
Electronic sow feeding		0.8261
Herd size*feeding system	Individual based feeding	-0.1208
	Competition based feeding	0
	Small * Electronic sow feeding	-0.8214
	Small * Individual feeding	-0.4492
	Small * Competition feeding	0
	Average * Electronic sow feeding	-2.2932
	Average * Individual feeding	-0.5723
	Average * Competition feeding	0
	Large * Electronic sow feeding	0
	Large * Individual feeding	0
Large * Competition feeding	0	

The effect of the WSI on the conception rate was  $\psi=0.2224$  (see Appendix B.5). The dead fraction of gestation period parameter ( $\xi=0.61$ ) was also estimated (see Appendix B.3.1).

## Appendix B

### B.1. Parameters needed for calculation of the transition probabilities

A list of the parameters, their symbols and sources are summarized in Table B.1.

Table B.1: Parameter needs for calculation of transition probabilities.

Parameter	Explanation	Source
$\mu_{hn}^G$	Herd mean for parity $n$ WSI in the gestation period	BN
$\mu_{hn}^L$	Herd mean for parity $n$ WSI in the lactation period	BN
$\sigma_{Ghn}$	Standard deviation of WSI between gestating sows	BN
$\sigma_{Lhn}$	Standard deviation of WSI between lactation sows	BN
$\sigma_{Gn}$	Standard deviation of the observation error of WSI for gestating sows when $E_h \subset S$	BN
$\sigma_{Ln}$	Standard deviation of the observation error of WSI for lactating sows when $E_h \subset S$	BN
$\rho_{GLn}$	Auto correlation between WSI in the gestation period and the subsequent lactation period of parity $n$ .	DA
$\rho_{LGn}$	Auto correlation between WSI in the lactation period of parity $n$ and the subsequent gestation period of parity $n+1$	GU
$\sigma_{GOhn}$	Total standard deviation of the observed WSI in the gestation period	ID
$\sigma_{LOhn}$	Total standard deviation of the observed WSI in the gestation period	ID
$\sigma_{GLhn}$	Standard deviation of the forecast error for WSI in the lactation period	ID
$\sigma_{LGhn}$	Standard deviation of the forecast error for WSI in the lactation period	ID
$k$	Number of WSI levels from $-k$ to $k$	DC
$\xi$	Fraction of death animals in gestation period	DA
$\psi$	Effect of WSI on conception rate	DA
$w_{ni}^{G-}$	Lower limit for $i$ of estimated WSI in the gestation period of parity $n$ (in herd $h$ )	ID
$w_{ni}^G$	Mean for level $i$ of estimated WSI in the gestation period of parity $n$ (in herd $h$ )	ID
$w_{ni}^{G+}$	Upper limit for level $i$ of estimated WSI in the gestation period of parity $n$ (in herd $h$ )	ID
$w_{ni}^{L-}$	Lower limit for level $i$ of estimated WSI in the lactation period of parity $n$ (in herd $h$ )	ID

$w_{ni}^L$	Mean limit for level $i$ of estimated WSI in the lactation period of parity $n$ (in herd $h$ )	ID
$w_{ni}^{L+}$	Upper limit for level $i$ of estimated WSI in the lactation period of parity $n$ (in herd $h$ )	ID

---

Source: BN=Bayesian network; DA=Data analysis; GU=Guess; ID=Indirectly from the other values; DC=Decided.

## B.2. A model for the transition probabilities

### B.2.1 Levels of the state variables representing WSI

For each level  $i \in \{-k, \dots, 0, \dots, k\}$  of a WSI state variable, a lower limit  $w_i^-$ , a mean value  $w_i$  and an upper limit  $w_i^+$  are calculated under the assumption that the WSI (on the logistic scale) is normally distributed. The values depend on stage of cycle and parity, but for simplicity further indexes are omitted. These values are determined under the standard assumption that all  $2k+1$  states have the same probability. It should be noticed, that  $w_{-k}^- = -\infty$ ,  $w_0 = 0$  and  $w_k^+ = \infty$ . For the determination of these level delimiters it must be remembered that the total variance of  $\hat{w}_n^G$  is  $\sigma_{GOhn}^2 = \sigma_{Ghn}^2 + \sigma_{Gn}^2$  and accordingly for  $\hat{w}_n^L$ .

In the following, a level  $i$  is also identified by its mean,  $w_i$ .

### B.2.2 The initial distribution of $\hat{w}_n^G$ for a gilt

Since the WSI of gilt is the first one observed, no prior knowledge is available, and

$$P(\hat{w}_{1i}^G) = \Phi\left(\frac{w_{1i}^{G+}}{\sigma_{GOhn}}\right) - \Phi\left(\frac{w_{1i}^{G-}}{\sigma_{GOhn}}\right) \quad (6)$$

where  $\Phi$  is the distribution function of the standard normal distribution.

B.2.3 The conditional distribution of  $\hat{w}_n^L$  given  $\hat{w}_n^G$  for a gilt or sow

It is assumed that  $P(\hat{w}_n^L | \hat{w}_{n-1}^L, \hat{w}_n^G) = P(\hat{w}_n^L | \hat{w}_n^G)$  even for  $n > 1$ .

A simple first order auto regressive model is assumed for the WSI. Let  $\rho_{GLn}$  be the correlation coefficient between the WSI in the gestation period and the subsequent lactation period.

Thus,

$$w_n^L = \mu_{hn}^L + \frac{\rho_{GLn} \sigma_{Lhn}}{\sigma_{Ghn}} (w_n^G - \mu_{hn}^G) + \eta_{GLn}, \quad (7)$$

where  $\eta_{GLn} \sim N(0, (1 - \rho_{GLn}^2) \rho_{Lhn}^2)$ . Recalling from Eq.(5) that, for a given sow with observed clinical signs.

$$w_n^G = \mu_{hn}^G + \hat{w}_n^G + \varepsilon_{Gn} \quad (8)$$

where  $\varepsilon_{Gn} \sim N(0, \sigma_{Gn}^2)$ . Accordingly,

$$w_n^L = \mu_{hn}^L + \hat{w}_n^L + \varepsilon_{Ln} \quad (9)$$

where  $\varepsilon_{Ln} \sim N(0, \sigma_{Ln}^2)$ . Substituting (8) and (9) into (7) and reducing gives us,

$$\hat{w}_n^L + \varepsilon_{Ln} = \frac{\rho_{GLn} \sigma_{Lhn}}{\sigma_{Ghn}} (\hat{w}_n^G - \varepsilon_{Gn}) + \eta_{GLn} \quad (10)$$

By further reduction the following expression is obtained

$$\hat{w}_n^L = \frac{\rho_{GLn} \sigma_{Lhn}}{\sigma_{Ghn}} \hat{w}_n^G + \varepsilon_{GLn} \quad (11)$$

where  $\varepsilon_{GLn} = \eta_{GLn} + \frac{\rho_{GLn} \sigma_{Lhn}}{\sigma_{Ghn}} \varepsilon_{Gn} - \varepsilon_{Ln}$  implying that  $\varepsilon_{GLn} \sim N(0, \sigma_{GLn}^2)$ , where



$$\sigma_{GLhn}^2 = \left( 1 + \rho_{GLn}^2 \left( \frac{\sigma_{Gn}^2}{\sigma_{Ghn}^2} - 1 \right) \right) \sigma_{Lhn}^2 + \sigma_{Ln}^2 \quad (12)$$

It is now possible to specify the transition probabilities from WSI =  $i$  in the gestation period to WSI=  $j$  in the lactation period:

$$P(\hat{W}_{nj}^L | \hat{W}_{ni}^G) = \Phi \left( \frac{W_{nj}^{L+} - \frac{\rho_{GLn} \sigma_{Lhn}}{\sigma_{Ghn}} \hat{W}_{ni}^G}{\sigma_{GLhn}} \right) - \Phi \left( \frac{W_{nj}^{L-} - \frac{\rho_{GLn} \sigma_{Lhn}}{\sigma_{Ghn}} \hat{W}_{ni}^G}{\sigma_{GLhn}} \right) \quad (13)$$

#### B.2.4 The conditional distribution of $\hat{w}_n^G$ given $\hat{w}_{n-1}^L$ for a gilt or sow

Analogously, the transition probabilities from WSI= $i$  in the lactation period to WSI=  $j$  in the next gestation period become:

that

$$P(\hat{W}_{nj}^G | \hat{W}_{n-1,i}^L) = \Phi \left( \frac{W_{nj}^{G+} - \frac{\rho_{LGn} \sigma_{Ghn}}{\sigma_{Lhn}} \hat{W}_{n-1,i}^L}{\sigma_{LGhn}} \right) - \Phi \left( \frac{W_{nj}^{G-} - \frac{\rho_{LGn} \sigma_{Ghn}}{\sigma_{Lhn}} \hat{W}_{n-1,i}^L}{\sigma_{LGhn}} \right) \quad (14)$$

### B.3. Transition probabilities to State “Dead”

#### B.3.1. Gestation Period

The probability follows directly from the estimated WSI which is considered to be a direct estimate for the involuntarily culling (both death and send to slaughter probability) on the logistic scale. Since it has defined that low relative values of WSI means a weak sow (and high values accordingly refer to strong sows) it is defined the corresponding logistic value  $y_{ni}^G$  for a parity  $n$  sow with WSI index level  $i$  in the gestation period as

$$y_{ni}^G = \mu_{hn}^G - \hat{w}_{ni}^G \quad (15)$$

Thus, the probability of involuntarily culling becomes

$$P(Dead, Diseased | \hat{w}_{ni}^G) = \frac{1}{e^{-y_{ni}^G} + 1} \quad (16)$$

The data analysis performed by Jensen et al. (submitted) showed no correlation between the fraction of deed/euthanized sows among the involuntarily culled sows. A fixed fraction of  $\xi = 0.61$  was found.

Thus, the probability of death becomes

$$P(Dead | \hat{w}_{ni}^G) = \xi \times P(Dead, Diseased | \hat{w}_{ni}^G) \quad (17)$$

### B.3.2. Lactation Period

The probability follows directly from the estimated WSI which is considered to be a direct estimate for the death probability on the logistic scale. Since it has defined that low relative values of WSI means a weak sow (and high values accordingly refer to strong sows) it is defined the corresponding logistic value  $y_{ni}^L$  for a parity  $n$  sow with WSI index level  $i$  in the lactation period as

$$y_{ni}^L = \mu_{hn}^L - \hat{w}_{ni}^L \quad (18)$$

Thus, the probability of death becomes

$$P(Dead | \hat{w}_{ni}^L) = \frac{1}{e^{-y_{ni}^L} + 1} \quad (19)$$

#### B.4. Transition probabilities to State “Diseased”

For the gestation period, the diseased sows are just the involuntarily culled sows that are not dead/euthanized. Thus,

$$P(\text{Diseased} | \hat{w}_{ni}^G) = (1 - \xi) \times P(\text{Dead}, \text{Diseased} | \hat{w}_{ni}^G) \quad (20)$$

In the lactation period this state in the version of the replacement model as was described by Kristensen and Søllested (2004a and b) corresponds to a sow being involuntarily culled either because of death/euthanization or premature slaughtering. However in the current model with WSI this state only corresponds to premature slaughtering, since the state “Dead” corresponds to death/euthanization.

Thus, the probability of entering the “Diseased” state must be reduced by the probability of entering the “Dead” state. In the current version of the model, the probability of involuntary culling only depends on parity and stage (i.e. mating, gestation, lactation), but it seems more logical to let the total probability of involuntary culling (“Diseased” and “Dead”) depend on the WSI.

The applied procedure for the gestation stage is as follows:

- Denote the current probability of involuntary culling for a parity  $n$  sow in the gestation period as  $q_n^L$  and the corresponding logistic value as  $z_n^L$ , i.e.,

$$z_n^L = \ln \frac{q_n^L}{1 - q_n^L} \quad (21)$$

- Define  $q_n^L$  as the probability of involuntary culling for a sow in WSI state  $i = 0$  (i.e. a sow with an average WSI).

- For other WSI states  $i \in \{-k, \dots, 0, \dots, k\}$ , the logistic value of the probability of involuntary culling is adjusted by the numerical value of the WSI state, i.e. the logistic value  $z_{ni}^L$  for WSI state  $i$  becomes

$$z_{ni}^L = z_n^L - \hat{w}_{ni}^L \quad (22)$$

Thus the probability of involuntary culling becomes

$$P(Dead, Diseased | \hat{w}_{ni}^L) = \frac{1}{e^{-z_{ni}^L} + 1} \quad (23)$$

Finally, the probability of entering the “Diseased” state becomes:

$$P(Diseased | \hat{w}_{ni}^G) = P(Dead, Diseased | \hat{w}_{ni}^G) - P(Dead | \hat{w}_{ni}^G) \quad (24)$$

#### B.5. Adjustment of the conception rate

The conception rate as defined in the model by Kristensen and Søllested (2004a,b) is in the present extended model adjusted for the influence of the WSI of the most recent lactation period. The procedure is that for WSI state  $i = 0$  (an average sow), the original conception rate is used. Denote as  $y_n$  the logistic transform of this conception rate for parity  $n$ . For other WSI states, the logistic transform is adjusted linearly in the WSI:

$$y_{ni} = y_n + \psi \cdot \hat{w}_{ni}^G \quad (25)$$

where the coefficient in the data analysis by Jensen et al. (2009) was estimated as  $\psi = 0.2224$ . Finally, the adjusted conception rate is calculated as

$$(conception \ rate | w_{ni}^G) = \frac{1}{e^{-y_{ni}} + 1} \quad (26)$$



# **Modelling tactical planning decisions in breeding farms through a linear optimization model**

Sara V. Rodríguez-Sánchez<sup>1</sup>, Victor M. Albornoz-Sanhueza<sup>2</sup> and Lluís M. Plà-Aragónés<sup>1</sup>

<sup>1</sup>University of Lleida, Department of Mathematics Jaume II, 73 E-25001 Lleida, Spain

<sup>2</sup>Departamento de Industrias, Universidad Técnica Federico Santa María, Campus Santiago, Av. Santa María, 6400 Santiago, Chile

Submitted to Agricultural Systems

## **Abstract**

This paper deals with tactical planning decisions for breeding farms producing piglets through a linear optimization model. A medium-term planning horizon based on weekly periods is considered. The proposed model maximizes the profit of the farm and takes into account sow herd dynamic behaviour, housing facilities, reproduction management, available stocks and a target quota of weekly weaning to integrate piglet production into the pig supply chain management. As result, an optimal replacement policy of sows and schedules purchases of gilts during the whole planning horizon are provided. The model is solved using a modelling language software, in combination with a general purpose linear optimization solver. The article also discusses results obtained from a sensitivity analysis performed to assess the suitability of the model approach. Finally, the conclusions and future extensions of the work are presented.

*Keywords: Tactical decisions; Planning; Sow herd management; Replacement problem;*

## 1 Introduction

Nowadays pig production is very competitive. In Spain for instance, pig production has evolved from familiar to industrial structure which is characterized by a production concentrated in bigger and more specialized pig production units. Usual units are for instance breeding farms producing piglets, rearing farms producing young pigs and fattening farms producing pigs to be slaughtered. This kind of organisation has encouraged individual farms to be specialized in a single phase of production. In this context, most of the piglet production is done on breeding farms owned by a big company or by independent farmers who sell all piglet production by contract to a company usually called integrator (see Ouden et al., 1996).

Sow farms devoted to piglet production are more complex to manage than rearing or fattening farms. There, piglet production is intimately related with the reproduction process and different factors others than feeding may affect final results. Furthermore recent EU regulations concerning pig welfare (affecting for instance housing facilities for sows or fixing a minimum lactation period) reduced or bound the margin of benefit that individual farms could have attained years ago. Hence optimization models can play an important role in farm management, and may raise farm productivity by improving the quality of farm management decisions enhancing the competitive position of farms.

Decisions on farm are taken at operational, tactical and strategic levels as discussed by Jalvingh (1992). Tactical decisions on sow breeding farms have been well covered by research studies and several models have been developed to support this type of decisions as was pointed out by Plà (2007). Nevertheless, they were developed for teaching and research purposes more than practical ones. Then, some important details with practical relevance for weekly operations on farm had been left aside. For instance, tactical models with finite time horizon seem to be a better option to support decision making tasks in practical conditions, as it is going to be shown in this paper. Different operations are performed on a breeding farm, most of them after having grouped animals in batches or bands. In general a weekly



basis period is adopted to rationalise the daily work on sow farms when bands management is adopted. Thus, for instance inseminations are scheduled Thursday and Tuesday, weaning on Wednesday and so on. Also, purchases of gilts and culling of animals are not effective daily. Culled animals are also grouped one day a week to be transported to the slaughterhouse and replaced by young breeding animals or gilts. Replacement decision is especially important because determines future productivity of the herd. This is so because along the age-structure of the herd, gilts and old sows are less productive than young or medium age sows. In addition, the age-structure of the herd may be affected by sanitary aspects as the sensibility to diseases outbreaks or passive immunization by contact between young and mature sows. On the other hand, a fixed scheduling of replacement and purchases have not sense in the long term because the dynamics in a commercial sow herd is variable (regular annual replacement rates are easily around 50%). Also, seasonal variations in reproductive performance and pig meat demand are frequently observed and may strongly affect herd dynamics and net revenues impacting in the replacement policy. Then, the scheduling should be adapted depending on actual state of the herd and expected market conditions. When scheduling, farmers try to take into account future possible variations, and hence the scheduling is for the medium term, i.e. periods under a year. Also it is very important to point out that globalisation makes piglet production connected with the production of other agents of the pig supply chain, for instance when they are part of a vertical integration scheme. In view of that, it is common a target quota of weaning or farrowing get established by the integrator to attain a steady flow of animals through the chain.

The objective of this paper is to formulate and solve a Linear Programming model for scheduling replacements and purchases in sow farms producing piglets in a finite time horizon. Replacement and purchase decisions are two of the most important tactical decisions on sow farms (Dijkhuizen et al., 1986; Huirne et al., 1993). They are sensible to changes from time to time due to variations in prices or in reproductive performances of sows. Furthermore, they not only impact on actual results but also determine herd structure over time and consequently future production. The model is tailored to specific individual farm conditions. It includes

the productive and reproductive behaviour of a group of breeding sows over time where piglets are the commercial product. Hence, the herd model is mainly focused on reproduction and replacement management of sows. The objective function maximizes profits constrained in different aspects like the purchase of gilts, the replacement of sow or the efficient occupancy of facilities according to reproduction performances and farrowing goals (given that farrowing facilities are the most expensive in sow farms).

The remainder of this paper is organized as follows. The description of the deterministic model related to operations in sow farms is presented in section 2. Results are presented in section 3 including a discussion and a sensitivity analysis in section 4. Conclusions and future work are drawn in section 5.

## **2 The linear optimization model**

In this section, the linear programming model used to determine the optimal purchase and replacement policy for a given planning horizon is described. The proposed model considers a medium term planning horizon, divided into a set of weekly period,  $T$ , and maximizes the total profit of the production plan,  $\Phi$ , while satisfying a set of constraints that mainly concern the sow herd dynamics behavior and the facilities capacity. Hence, the model includes equations representing herd dynamics over time in order to describe the inter-temporal behaviour under different replacement policies. The inventory at the beginning and at the end of the planning horizon is stated.

This representation considers different reproductive cycles in the sow lifespan, assuming that at the end of it a sow is sold to the slaughterhouse and replaced by a purchased gilt. Moreover, the piglet production model contributes to decide in *which* cycle a sow should be culled, as well as in *which* state *inside* the cycle. Additionally, the model indicates how many gilts must be purchased to achieve targets of farrowing or weaning. So, herd size has not to be constant as most of the models published until now require (Plà, 2007).

The model also leads to a maximal use of lactation facilities through the target of farrowing per week. This target permits a better coordination and management of piglet production into large pig supply chains.

The formulation of a model representing sow herd management through the reproduction and replacement management at farm level leads to consider the following decision variables that characterize the herd structure of the farm at any given time period:

$X_{t,c,g}$  = number of sows in gestation state at period  $t$ , cycle  $c$ , gestation week  $g$ ,

$Y_{t,c,l}$  = number of sows in lactation state at period  $t$ , cycle  $c$ , lactation week  $l$ ,

$Z_{t,c}$  = number of sows in mating state at period  $t$ , cycle  $c$ ,

$ZR_{t,c,r,k}$  = number of sows in control state at period  $t$ , cycle  $c$ , waiting for the insemination attempt  $r$ , at the week  $k$ ,

$UL_{t,c,l}$  = number of replaced sows at the end of the lactation state at period  $t$ , cycle  $c$ , at lactation week  $l$ ,

$UZ_{t,c,r}$  = number of replaced sows at the end of the mating state at period  $t$ , cycle  $c$ , waiting for the insemination attempt  $r$ ,

$AB_{t,c}$  = number of sows with abortion at period  $t$ , cycle  $c$ ,

Hence, the proposed model maximizes the total profit of the production plan given by the following function:

$$\begin{aligned} \text{Maximise } \Phi^d = & \sum_{t \in T} \sum_{c \in C} r p_t \cdot \gamma_{t,c} \cdot Y_{t,c,l} + \sum_{t \in T} \sum_{c \in C} \left( r y_{t,c} \cdot UL_{t,c} + \sum_{r \in Nr} r z_{t,c} \cdot UZ_{t,c,r} + r x_{t,c} \cdot AB_{t,c} \right) \\ & - \sum_{t \in T} \sum_{c \in C} \left( c z_{t,c} \cdot Z_{t,c} + \sum_{r \in Nr} \sum_{k \in Sr} c z r_{t,c,k} \cdot ZR_{t,c,r,k} + \sum_{g \in Sg} c x_{t,c,g} \cdot X_{t,c,g} + \sum_{l \in Sl} c y_{t,c,l} \cdot Y_{t,c,l} \right) \end{aligned} \quad (1)$$

where the objective function in (1) represents the maximization of the total profit over the finite time horizon,  $T$ , that is, the addition of profits for each period of time,  $t \in T$ , and reproductive cycle,  $c \in C$ . In this context, profit is the difference between incomes obtained by sales and the different costs of production incurred. Incomes are considering from two sources. The first one, sales of piglets sold per sow depending on period and reproductive cycle,  $\gamma_{t,c}$ , assuming an unitary price per

period of  $rp_t$  (€/piglet). The second one, sales to the slaughterhouse of culled sows assuming an unitary price (€/kg per live weight). Hence, the replaced sows have different selling value regarding individual live weight which is affected by cycle and reproductive state. Then,  $rz_{t,c}$ ,  $rx_{t,c}$  and  $ry_{t,c}$  are the corresponding selling value for replaced sow on mating, gestation and lactation states respectively. Culled sows include also sows with abortion,  $AB_{t,c}$ . Production costs considered are feeding cost of sows and piglets, labour, insemination and veterinary expenses. These costs are calculated per period and summarised per sow (€/head/week) being in gestation ( $cx_{t,c,g}$ ), lactation ( $cl_{t,c,l}$ ), waiting for pregnancy detection ( $czr_{t,c,k}$ ) or waiting for insemination ( $cz_{t,c}$ ), given that  $k \in Sr$ ,  $g \in Sg'$  and  $l \in Sl$  are the weeks before pregnancy test, after pregnancy confirmation and lactation respectively. Thus, the whole set of gestation weeks is represented by  $Sg = Sr \cup Sg'$ , where  $Sr = \{1,2,3\}$  and  $Sg' = \{4 \text{ to } 16\}$  and this partition is more useful to account for occupation of housing facilities. Finally  $r \in Nr$  is the number of repetitions (failed conceptions). According to fore above incomes and costs described, corresponding coefficients of the objective function are set for each decision variable. For instance, the coefficient  $cz_{t,1}$  involves feeding, insemination and labour but also takes into account the purchase cost of a new gilt.

The objective function is affected by several constraints determining feasible herd management strategies. These constraints are enumerated from (2)-(26). Therefore, constraints (2)-(5) describe the initial stock at  $t=1$ . They account for the initial herd distribution of sows over different states and can be adapted to any particular situation either a starting farm or an existing one.

$$Z_{1,c} = z0_c \quad c \in C - \{1\} \quad (2)$$

$$ZR_{1,c,r,k} = zr0_{c,r,k} \quad c \in C \quad r \in Nr \quad k \in Sr \quad (3)$$

$$X_{1,c,g} = x0_{c,g} \quad c \in C \quad g \in Sg' \quad (4)$$

$$Y_{1,c,l} = y0_{c,l} \quad c \in C \quad l \in Sl \quad (5)$$

where:

$z0_c$  = initial stock of animals in mating state at cycle  $c$ ,

$zr0_{c,r,k}$  = initial stock of animals in control state at cycle  $c$ , waiting for the insemination attempt  $r$  and week  $k$ ,

$x0_{c,g}$  = initial stock of animals in gestation state at cycle  $c$  and gestation week  $g$ ,

$y0_{c,l}$  = initial stock of animals in lactation state at cycle  $c$  and lactation week  $l$ ,

The knowledge of population dynamic is crucial because the proposed model represents the future state of the sow farm in terms of the present state. All possible transitions of sows evolving from one state to another are represented in constraints (6)-(13). They represent the herd dynamics, i.e., the flow of sows throughout the different states over time. More specifically, constraint (6) refers to the number of sows at the first mating state. Constraints (7)-(9) refers to sows under control stage (waiting the pregnancy confirmation), if the sow is no pregnant a re-mating is done. Constraints (10) and (11) refer to the flow among gestation states while constraints (12) and (13) refer to the flow among lactation states.

$$Z_{t,c} = Y_{t-1,c-1,r} - UL_{t-1,c-1,l-1} \quad t \in T - \{1\} \quad c \in C - \{1\} \quad l \in Sl - \{1\} \quad (6)$$

$$ZR_{t,c,1,1} = Z_{t-1,c} \quad t \in T - \{1\} \quad c \in C \quad (7)$$

$$ZR_{t,c,r,1} = (1 - \beta_{t-1,c,r-1}) \cdot ZR_{t-1,c,r-1,3} - UZ_{t-1,c,r-1} \quad t \in T - \{1\} \quad c \in C \quad r \in Nr - \{1\} \quad (8)$$

$$ZR_{t,c,r,k} = ZR_{t-1,c,r,k-1} \quad t \in T - \{1\} \quad c \in C \quad r \in Nr \quad k \in Sr - \{1\} \quad (9)$$

$$X_{t,c,4} = \sum_{r \in Nr} \beta_{t-1,c,r} \cdot ZR_{t-1,c,r,3} \quad t \in T - \{1\} \quad c \in C \quad (10)$$

$$X_{t,c,g} = \alpha_{t-1,c,g-1} \cdot X_{t-1,c,g-1} \quad t \in T - \{1\} \quad c \in C \quad g \in Sg' - \{4\} \quad (11)$$

$$Y_{t,c,1} = \alpha_{t-1,c,g^*} \cdot X_{t-1,c,g^*} \quad t \in T - \{1\} \quad c \in C \quad (12)$$

$$Y_{t,c,l} = Y_{t-1,c,l-1} - UL_{t-1,c,l-1} \quad t \in T - \{1\} \quad c \in C \quad l \in Sl - \{1\} \quad (13)$$

where:

$\alpha_{t,c,g}$  = survival rate of gestation at period  $t$ , cycle  $c$  and gestation week  $g$ ,

$\beta_{t,c,r}$  = survival rate of mating at period  $t$ , cycle  $c$  and waiting for the mating attempt  $r$ ,

Normally, Spanish sow farms have three different facilities: breeding-control, pregnancy and farrowing facility. Breeding facility is where sows are inseminated and controlled in order to confirm the pregnancy (more or less three weeks after the

insemination). Once the pregnancy is positively confirmed sows are moved to the pregnancy facility. Otherwise, it is considered that conception has failed and they remain in the same facility for a subsequent re-insemination, according to a maximum number of attempts that is part of the management policy. Farrowing facility is where farrowing and weaning operations are done. So, before farrowing (normally one week), pregnant sows are moved to the farrowing facility and sows remain there until weaning (normally 3 weeks after farrowing). Constraints (14) – (16) correspond to the limited capacity in number of sows of the three facilities considered, being  $cb$  the capacity in the service facility,  $cp$  the capacity in the pregnancy facility and  $cf$  the capacity in the farrowing facility.

$$\sum_{c \in C} Z_{t,c} + \sum_{c \in C} \sum_{r \in Nr} \sum_{k \in Sr} ZR_{t,c,r,k} \leq cb \quad t \in T \quad (14)$$

$$\sum_{c \in C} \sum_{g \in Sg^{-\{g^*\}}} X_{t,c,g} \leq cp \quad t \in T \quad (15)$$

$$\sum_{c \in C} X_{t,c,g^*} + \sum_{c \in C} \sum_{l \in Sl} Y_{t,c,l} \leq cf \quad t \in T \quad (16)$$

Constraints (17) and (18) refer to smooth variation in purchases from week to week,  $dz$ , and fixing bounds for the minimum,  $lz$ , and maximum,  $uz$ , number of gilts allowed to be purchased per period.

$$|Z_{t+1,1} - Z_{t,1}| \leq dz \quad t \in T - \{t^*\} \quad (17)$$

$$lz \leq Z_{t,1} \leq uz \quad t \in T - \{t^*\} \quad (18)$$

Similarly to constraints (2)-(5), constraints (19)-(22) determine the imposed inventory of animals at the end of the planning horizon, representing the continuity of the farm beyond the end of the finite time horizon considered. The imposed values are computed apart and tend to the herd structure at equilibrium or long term herd structure, according to some complementary study and methodology used by the authors (Plà et al., 2008).

$$\sum_{c \in C} Z_{t^*,c} \geq zf \quad (19)$$

$$\sum_{c \in C} ZR_{t^*,c,r,k}^* \geq zrf_{r,k} \quad k \in Sr \quad r \in Nr \quad (20)$$

$$\sum_{c \in C} X_{t^*,c,g}^* \geq xf_g \quad g \in Sg' \quad (21)$$

$$\sum_{c \in C} Y_{t^*,c,l}^* \geq yf_l \quad l \in Sl \quad (22)$$

where:

$zf$  = number of animals in mating state at the end of the planning horizon,

$zrf_{r,k}$  = number of animals in control state at the end of the planning horizon waiting for the insemination attempt  $r$  and week  $k$ ,

$xf_g$  = number of animals in gestation state at the end of the planning horizon at gestation week  $g$ ,

$yf_l$  = number of animals in lactation state at the end of the planning horizon at lactation week  $l$ ,

Important decisions refer to culled sows at specific states as constraints (23) and (24) represent. Only animals in the breeding and lactation facilities are replaced and they are culled before being transferred to the next facility. No casualties are considered effective in the gestation facility.

$$UL_{t,c^*,l} \leq Y_{t,c^*,l} \quad t \in T \quad l \in Sl \quad (23)$$

$$UZ_{t,c,3} = (1 - \beta_{t,c,3}) \cdot ZR_{t,c,3,3} \quad c \in C \quad t \in T \quad (24)$$

In connection with gestation, abortions may occur; hence constraint (25) refers to gestating sows suffering an abortion and culled thereafter of the herd. This is a culling reason often adopted by pig specialists in Spain.

$$(1 - \alpha_{t,c,10}) \cdot X_{t,c,10} \geq AB_{t,c} \quad c \in C \quad t \in T \quad (25)$$

Finally, complementing constraint (16) involving lactation facilities, constraint (26) is added referring to the target of farrowing,  $fq$ . Usually  $fq$  value is specified by an agreement between the farmer and the integrator through a contract and it is related

with the capacity of lactation facilities and band management. Furthermore these constraints are only considered beyond an initial subset of periods  $T_1$ , assuring the fulfilment of the target quota.

$$\sum_{c \in C} Y_{t,c,1} \leq fq \quad t \in T - T_1 \quad (26)$$

### 3 Computational results

In this section a case study is presented in order to illustrate the suitability and advantages of the proposed optimization model. Basic parameters were estimated using the maximum likelihood method. The statistical data were taken from standard values under Spanish conditions and recorded in the BD-Porc databank (national record keeping system hosted at <http://www.irta.es/bdporc/>, accessed 14 May 2008), and do not correspond to a specific farm. While some other parameters were taken from the literature. The algebraic modelling language ILOG OPL 6.1 was used with CPLEX 11.2 as the linear optimization solver for implementing and solving the different models developed.

#### 3.1 Basic case

In this study an initial herd size of 2,055 sows was considered a regular size to represent a typical commercial Spanish sow herd. Initial herd distribution was selected from an arbitrary farm. It was established that initial and final herd distribution (related with (2)-(5) and (19)-(22) constraints, respectively) were the same. A maximum number of 8 parities were allowed as sow lifespan. Parity was considered to finish with a weaning or an abortion. The actual capacity of the lactation facility was of 500 crates. Hence, a rate of 100 farrowing/week represented a measure of the weekly work load expected by the farmer. The maximum number of allowed inseminations was three, beyond that infertility was considered a culling reason, just like an abortion. The problem was solved assuming a planning horizon of  $T=156$  weeks, approximately 3 years. This way,



occasional perturbations caused by border conditions (i.e. final inventory) are avoided. Parameters regarding fertility are described in Table 1. Nevertheless the conception rate declines on warm periods. It is assumed based on expert advice a decrease of 3% in the months of June and September and 5% in the months of July and August.

Mortality rate of gestation period was observed around 1.5% (modelled all casualties together at the 16<sup>th</sup> week), while abortion rate was around 1% (modelled all abortions at the 10<sup>th</sup> week). Table 2 describes the daily feed intake of breeding sows. Figures were extracted from Kyriazakis and Whittemore (2006) and values applied for each state updated weekly proportionally to the duration of corresponding state. Furthermore in Table 3 is made known the number of piglets weaning per reproductive cycle.

Table 1. Main fertility rates ( $\beta$ ) of mating state at  $t$ = time period,  $c$ = cycle,  $r$ = insemination attempt.

Cycle	Mating		
	$B(t,c,1)$	$\beta(t,c,2)$	$\beta(t,c,3)$
1	0.890	0.863	0.837
2	0.924	0.896	0.869
3	0.931	0.903	0.876
4	0.934	0.896	0.879
5	0.922	0.884	0.867
6	0.912	0.885	0.858
7	0.906	0.878	0.852
8	0.903	0.875	0.849

Table 2. Daily feed intake of breeding sows, (Kyriazakis and Whittemore, 2006).

State	Feed intake (Kg/day)
Mating	[3.0 - 3.5]
Pregnancy	
Week 1-4	[2.1 - 2.6]
Week 4-12	[2.1 - 2.9]
Week 12-16	[2.1 - 3.7]
Farrowing	[2.0 - 2.5]
Suckling	[4.0 - 8.0]

Table 3. Number of piglets weaning per reproductive cycle,  $\gamma_c$ .

Cycle	1	2	3	4	5	6	7	8
$\gamma$	10.17	10.30	10.82	11.18	11.09	10.74	10.38	9.24

Unitary sale price parameters were extracted from the main auction market of pigs in Spain: MercoLleida (<http://www.mercolleida.com>), settled in the same area where the farm was supposed to be operating. In particular, these data are registered and available in the Department of Agriculture from the autonomous government of Catalonia (<http://www20.gencat.cat/portal/site/DAR/menuitem.3645c709047c363053b88e10b031e1a0/?vgnextoid=3fc4361d78b24110VgnVCM1000000b0c1e0aRCRD&vgnextchannel=3fc4361d78b24110VgnVCM1000000b0c1e0aRCRD&vgnextfmt=default>, accessed 16 February 2009) in Spain. Years considered for this study were 2005, 2006 and 2007. For instance, figure 1 shows the evolution week by week of the slaughterhouse value (€/kg live weigh) obtained from replaced sows. It is observed along time some seasonal pattern with lower prices in October and November. From June to August better prices are registered due to the increment in demand during summer time.

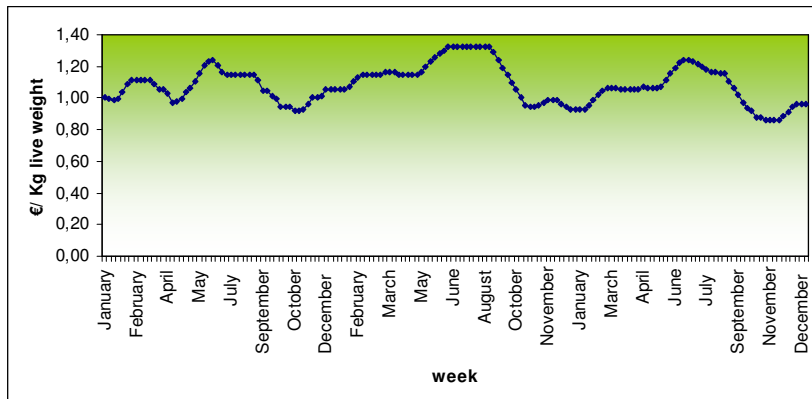


Figure 1. Weekly price offered by the slaughterhouse to the producers per a replaced sow (€/Kg live weight).

Figure 2 describes week by week the behaviour of the price of piglets (€/piglet) regardless the weight of them (i.e. around 20 kg). In 2007 a remarkable decrease in prices is observed but also the impact of seasonal effect leading to the lowest

annual prices after summer time. For modelling purpose, the purchase value of gilts was calculated considering the same price per kg as that fixed for the slaughterhouse (figure 1).

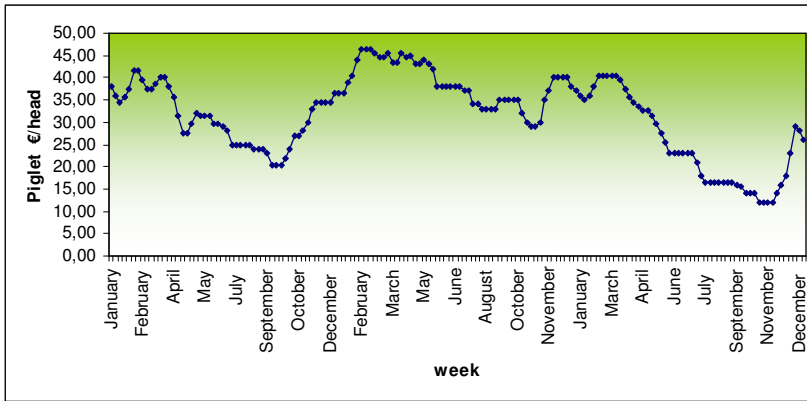


Figure 2. Weekly price offered by the slaughterhouse to the producers per piglet (€/head). Type: Piglet 20kg.

In Table 4 the weights of sow per cycle and reproductive state are given. The smooth variation in purchases from week to week, as well as minimum and maximum bounds of gilts allowed to be purchased were set in  $dz=5$ ,  $lz=5$  and  $uz=50$ , respectively. Feeding prices parameters were obtained from the Annual Statistics of the Agricultural sector, 2007, (<http://www.mapa.es/es/estadistica/pags/anuario/introduccion.htm>) edited by the former Spanish Ministry of Agriculture, Food and Fisheries and shown in Table 5.

Table 4. Weight of a sow according to their physiological state. (Kristensen and Søllested, 2004).

Cycle	Weight at mating	Weight at farrowing	Weight at weaning
1	157	203	183
2	181	227	203
3	205	251	223
4	219	265	236
5	236	276	250
6	252	290	258
7	259	297	268
8+	267	295	271

Table 5. Average yearly price of feed (€/100kg).

Price/year	2005	2006	2007
Piglet	35.27	37.00	42.07
Sow	20.18	20.64	24.31

Moreover the price described above, an average insemination cost per insemination was fixed in 3€.

### *Results and discussion*

The size of the model corresponding to basic case is presented in Table 6. The number of new animals entering to the farm (purchased gilts) are determined in agreement to facilities size, herd replacement and reproduction policies.

Table 6. Report of the size of the Linear Programming Model.

CPLEX	Value
Variables	36,469
Constraints	42,433
Non-zero-coefficients	115,968

Results in figure 3 show the behaviour of the herd size and the pattern of the purchase scheduling of new gilts over time. It is observed how the herd size is going up during the first 21 weeks until the steady-state is reached. These variations in herd size observed at the beginning of the time horizon are mainly a consequence of the initial herd distribution and later adjustments.

Although herd size reaches and remains in steady-state, this is not in contradiction with adaptive variations of herd composition over time as response to changes in the environment. For instance, the effect of low fertility in summer provokes a higher purchase rate just before the arrival of this season and anticipating negative effects on fertility while herd size is rather constant.

The occupancy rate per farrowing facilities was 100% from the 21<sup>th</sup> week onwards (figure 4) according with the stability in herd size and the target of farrowing per week. This proves the rational behaviour of taking the maximum profit of lactation

facilities from where piglets are produced and given that they represent the main source of income in the farm.

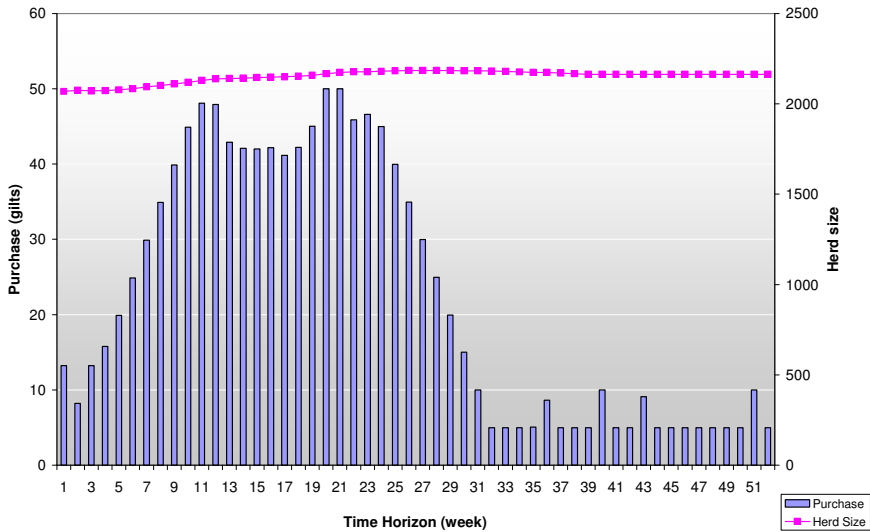


Figure 3. Representation of purchase scheduling and the behaviour of herd size over time.

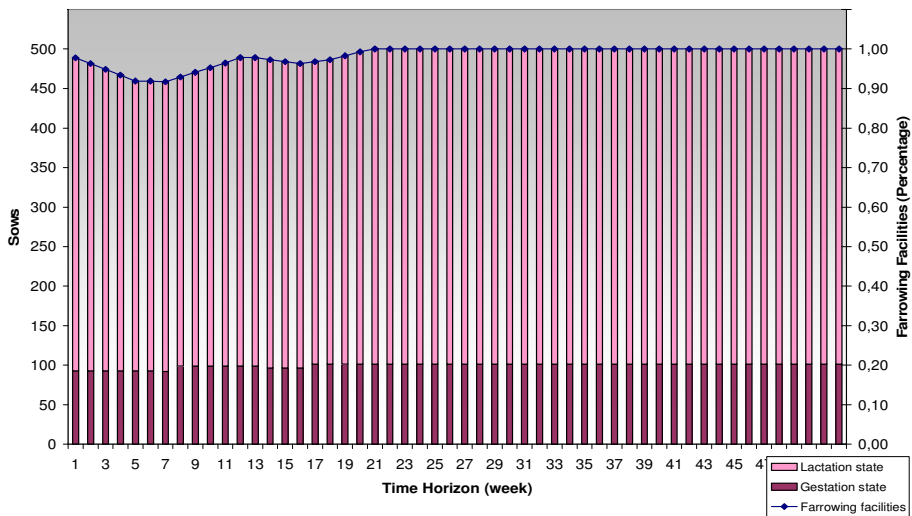


Figure 4. Representation of the behaviour occupancy rate of farrowing facilities.

Figure 5 illustrates the effect of summer in the rate of re-mating over total first mating. It is shown how the rate increases in summer till a 16% just when the

fertility rate is the lowest. Otherwise the regular value of the re-mating rate out of the summer season varies between 8 and 11% when average fertility is around 91%.

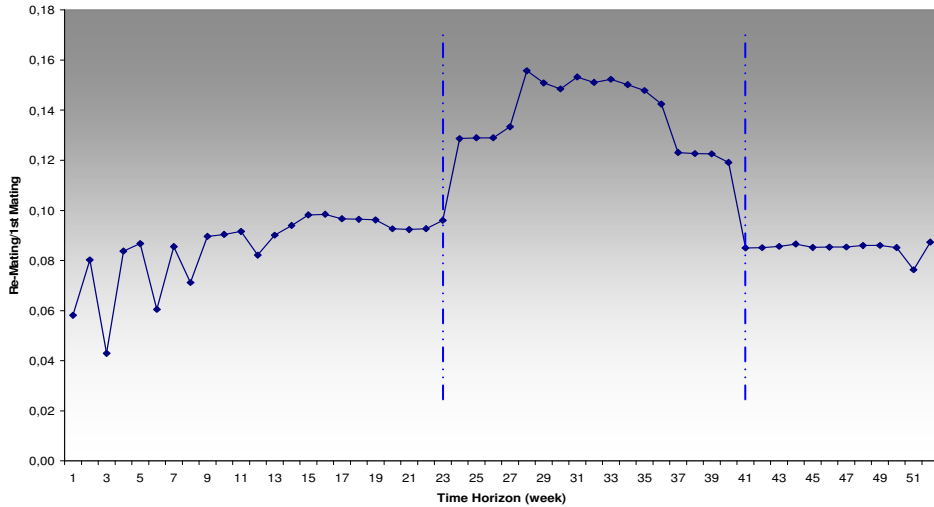


Figure 5. Behaviour of the re-mating over first mating along time horizon.

The optimum replacement policy implies not culling before parity 5, from parity 5 through 7 the culling on each period depended on the conditions presented by the farm but at no time the sows were keeping beyond 7<sup>th</sup> parity. Hence the expected average economic reward of the farm over time was 3,335 thousand (€/156 weeks), with a computation time around 40 sec.

### 3.2 Sensitivity analysis

It is well known the high variability of the fertility rate,  $\beta_{t,c,r}$  of a herd and since it has a direct impact on the productivity of the farm, it is important to value their impact on model performances.

The model described in section 3 was taken as a base. Hence, two cases more were added to this base case: the optimistic and pessimistic case. These cases are defined by the introduction of variation on values of  $\beta_{t,c,r}$  parameter of the base case. The variations applied is calculated by 3% of corresponding standard deviations that in

the optimistic case represents an increment and in the pessimistic case a decrement in respective parameter.

Results for each case concerning herd size and purchase scheduling are compared and shown in figure 6.

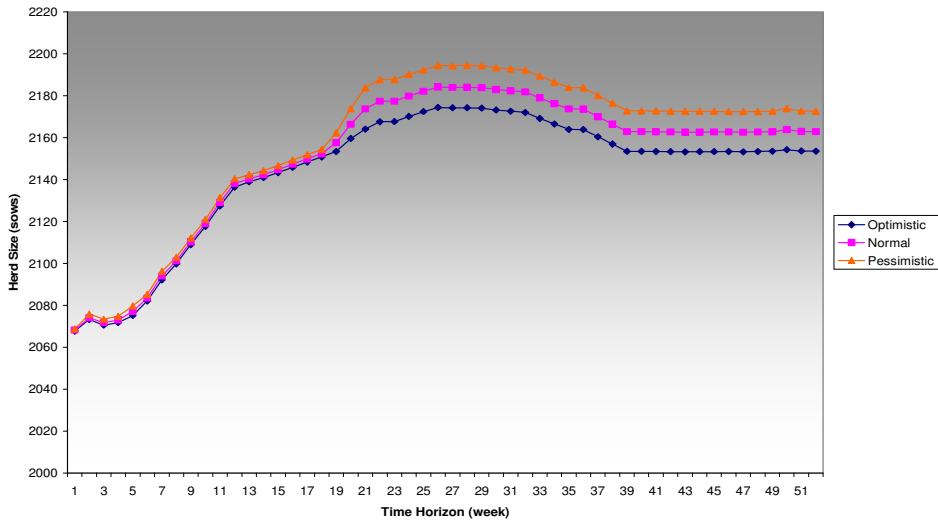


Figure 6. Comparison of the purchase scheduling behaviour. Productivity cases.

The optimistic case showing a high productivity requires lower level of purchase against the pessimistic case (with low productivity) which needs a higher level of purchase to satisfy the target of farrowing. This makes vary the replacement rate from 11% to 14% that is translated in an extra replacement cost for the latter case. In relation with that lower replacement rate are associated with smaller breeding facilities and smaller herd size and vice versa.

Another effect is the farmer workload that in the pessimistic case is much higher due to the number of inseminations to perform for attaining the same target of farrowings than those performed in the optimistic case. The impact on the profit is similar in absolute terms for both cases, but slightly greater in the pessimistic one, table 7.

Table 7. Comparison of the expected profit regarding the optimistic and pessimistic case, (thousands of €).

	<b>Base</b>	<b>Optimistic Case</b>	<b>Pessimistic Case</b>
Φ €/year	3,335	3,345	3,325
Difference vs Base		Δ0.03%	∇0.03%

These economic results are complemented with an additional calculation assuming the best and worst series of available market prices extracted from available data, table 8. It is important to note the extra advantage for farms benefiting of good prices in comparison with the moderate impact of lower prices. This may serve to explain why the sector overcome long market crisis.

Table 8. Comparison of the expected profit regarding the best and worst series of market prices, (thousands of €).

	<b>Base</b>	<b>Upper Prices</b>	<b>Lower Prices</b>
Φ €/year	3,335	4,631	3,025
Difference vs Base		Δ39%	∇9%

#### 4 Conclusions

In this work is formulated a linear programming model for planning piglet production in a breeding farm. This approach is shown to be suitable to adapt and prepare the system to variations in the short term. Furthermore the incorporation of a farrowing target quota allows modelling the role of the integrator in piglet production. It is demonstrated the different response of the herd when farm specific parameters are different and the benefits of an equilibrated herd structure. This shows the need of analytical tools for improve integrator understanding. Thus, the use of mixed integer linear programming for short term decision support in this paper constitutes a valuable tool in the field of pig production.

Finally, we leave open the extension of this methodology to a stochastic formulation with the inclusion of different scenarios capturing part of the



uncertainty of the system related for instance with prices and litter size. Such extensions would require the development of suitable numerical strategies of models with integer recourse variables, which are, in general, more complex than the ones examined in this paper.

---

## References

- Dijkhuizen, A.A., Morris, R.S., Morrow, M., 1986. Economic optimisation of culling strategies in swine breeding herds, using the “PORKCHOP computer program”. *Preventive Veterinary Medicine* 4, 341-353.
- Huirne, R.B.M., Dijkhuizen, A.A., van Beek, P., Hendriks, Th.H.B., 1993. Stochastic dynamic programming to support sow replacement decisions. *European Journal of Operational Research* 67, 161-171.
- Jalvingh, A.W., Dijkhuizen, A.A., van Arendonk, J.A.M., 1992. Dynamic probabilistic modelling of reproduction and management in sow herds. General aspects and model description. *Agricultural Systems* 39, 133-152.
- Kristensen, A.R., Søllested, T.A., 2004. A sow replacement model using bayesian updating in a 3-level hierarchic markov process. i. Biological model. *Livestock Production Science* 87, 13–24.
- Kyriazakis, I., Whittemore, C., 2006. *Whittemore’s Science and Practice of Pig Production*. Wiley-blackwell, 704pp.
- Plà, L.M., 2007. Review of Mathematical Models for Sow Herd Management. *Livestock Science* 106, 107-119.
- Plà, L.M., Faulin, J., Rodríguez, S.V., 2008. A Linear Programming formulation of a semi-markov model to design pig facilities. *Journal of the Operational Research Society* 60, 619-625.
- den Ouden, M., Dijkhuizen, A.A., Huirne, R.B.M., Zurbier, P.J.P., 1996. Vertical Cooperation in Agricultural Production Marketing Chains, with Special Reference to Product Differentiation in Pork. *Agribusiness* 12, 277-290.



# **A two-stage stochastic programming model for scheduling replacements in sow farms**

Sara V. Rodríguez<sup>1</sup>, Victor M. Albornoz<sup>2</sup> and Lluís M. Plà<sup>1</sup>

<sup>1</sup>University of Lleida, Department of Mathematics Jaume II, 73 E-25001 Lleida, Spain

<sup>2</sup>Departamento de Industrias, Universidad Técnica Federico Santa María, Campus Santiago, Av. Santa María 6400, Santiago, Chile

Submitted to TOP.

(Final version published in TOP 17, 171-179).

## **Abstract**

This paper presents the formulation and resolution of a two-stage stochastic linear programming model with recourse for sow farms producing piglets. The proposed model considers a medium-term planning horizon and specifically allows optimal replacement and schedule of purchases to be obtained for the first stage. This model takes into account sow herd dynamics, housing facilities, reproduction management, herd size with initial and final inventory of sows and uncertain parameters such as litter size, mortality and fertility rates. These last parameters are explicitly incorporated via a finite set of scenarios. The proposed model is solved by using the algebraic modelling software OPL Studio from ILOG, in combination with the solver CPLEX to solve the linear models resulting from different instances considered. The article also presents results obtained with previous deterministic models assessing the suitability of the stochastic approach. Finally, the conclusions drawn from the study including an outlook are presented.

*Keywords: Replacement; Planning; Stochastic programming; Sow herd management;*

## 1 Introduction

Traditional pig production in Spain was based on small familiar farrowing - to finish farms, but this is undergoing a rapid change as in the rest of the European Union (EU). Nowadays, production is being concentrated in bigger and specialised pig production units. In Spain, commercial pig production tends to be divided into three different phases according to the final product and the activities involved. The first phase relates to farms producing piglets, the second one to those producing feeder pigs and the third one to those producing fattened pigs. This division provokes a specialisation in farming activities by phase and gives additional efficiency gains as Rowland et al. (1998) already pointed out. The first phase represented by sow farms producing piglets is the most important because of the complexity of the reproduction process and the caring that piglets need before being sold or transferred to a different unit in the next phase.

The sow herd structure is central to maintain a steady production over time and replacement is the most crucial decision, not only due to consequences on piglet production, but also for being the main production cost (Jalvingh et al., 1992; Huirne et al. 1993). Furthermore, EU regulations concerning pig welfare have reduced profitability margins and increased competition. As consequence the interest in the optimisation or in improvements of management strategies by implementing suitable decisions in sow farms have been increasing constantly.

Improvements in modelling the decision-making process on sow farms to represent fairly the system have been done and thus, advances have been obtained in solving or circumventing methodological problems related to complex models (e.g. Kristensen, 1988, 1993). Most of them are related to Markov chain and simulation models (see Plà, 2007) but none with stochastic programming in a finite time horizon. Researchers have the benefit of advances in computing, database and solving software which enables farming systems to be described in greater detail and with greater ease (Kingwell, 1996). Taking advantage of this situation the aim of the model presented in this paper is to provide a complimentary analytical tool

to specialists and advisers serving to better decisions around the culling and replacement of sows. This is done scheduling purchases of gilts and culling less productive sows in the way that herd structure is preserved and productivity levels maintained. In the present paper, this scheduling is the result of solving an optimization model that can also represent explicitly the uncertainty present in the system.

After the seminal papers of Dantzig (1955) and Beale (1955), most of the references concerning optimization models in the presence of uncertainty come under the name of Stochastic Programming that allows to explicitly incorporate the uncertainty of the parameters in the model formulation, see Birge and Louveaux (1997), Ruszczyński and Shapiro (2003) and Wallace and Ziemba (2005), among others. In particular, a two-stage stochastic program with recourse is an important class of models in stochastic programming and widely used in multiperiod planning problems. In such models, two kinds of decision variables there exist. The first-stage decisions represent proactive decisions whose values are not conditioned by any particular realization of the uncertain parameter. In this paper, these decisions are simply related to the purchase of gilts and replacement of sows in the more immediate planning period. On the other hand, the second-stage or recourse variables represent reactive decisions made in recourse or response to compensate for the decision made in the first stage after materializing the uncertainty and correspond to the remaining decision variables in the proposed model.

The objective of this paper is precisely to formulate and solve a two-stage stochastic programming model for scheduling replacements in sow farms. The model includes the productive and reproductive behaviour of a group of breeding sows over time where piglets are the commercial product. Hence, the herd model is mainly focused on reproduction and replacement management of sows. This model maximizes profits carrying out an efficient occupancy of farrowing facilities because these are the most expensive and as a consequence have a high impact in (or bound) the production. The remainder of this paper is organized as follows. The description of the problems related to operations in sow farms is presented in section 2. This is followed by a deterministic model in section 3 that will serve as a

base for the formulation of a more elaborated model incorporating parameters under uncertainty using a two-stage stochastic programming model in section 4. Results of both models and discussion are presented in section 5. Conclusions and future work are drawn in section 6.

## **2 Scheduling replacement and medium time decisions**

Normally Spanish sow farms have three different facilities: breeding-control, pregnancy and farrowing facility. The breeding facility is where sows are inseminated and controlled in order to confirm the pregnancy (more or less three weeks after the insemination). Once the pregnancy is positively confirmed they are moved to the pregnancy facility. Otherwise, it is considered that conception has failed and they remain in the breeding facility for subsequent re-inseminations, according to a number of attempts that must be determined optimally. The farrowing facility is where farrowing and weaning operations are done. So, before farrowing (normally one week), pregnant sows are moved to the farrowing facility and sows remain there until weaning (normally 3 weeks after farrowing).

Different operations are performed on a sow farm. In general a weekly basis period is adopted to rationalise the daily work. Thus, for instance inseminations are scheduled Tuesdays and Thursdays, weanings on Wednesdays and so on. Also, purchases of gilts and culling of animals are not carried out daily, they are also grouped one day a week to send and receive breeding animals. Replacement is especially important because it determines future productivity of the herd. This is so because along the age-structure of the herd gilts and old sows are less productive than young or medium age sows. In addition, the age-structure of the herd is also related with sanitary aspects as the sensibility to diseases outbreaks or passive immunization by contact between young and mature sows. Scheduling makes no sense in the long term because the population dynamics in a commercial sow herd is high (regular annual replacement rates are easily around 50%). Also, seasonal variations in reproductive performance and changes in pig meat demand are observed and may affect net revenues. Then scheduling should be modified depending on actual state of the herd and expected market conditions. When



scheduling, farmers take into account future possible variations, and scheduling consider a medium term planning horizon. Different replacement models concerning sow farms or other livestock species can be found in the literature (Jalvingh et al., 1992; Kristensen, 1993; Plà, 2007). However, most of them are developed for research purposes and then the details with practical relevance for daily operations on farms are left aside.

### 3 The deterministic model

In this section, a deterministic model used to determine the optimal purchase and replacement policy for a given planning horizon is briefly described, a detailed version can be found in Rodríguez et al. (2008). The deterministic model assumes that we know with certainty all the parameters present in the model. This assumption is very restrictive but will serve as a base for a more complex extension, in the next section, that replaces some of the uncertain parameters through a finite set of scenarios in the model formulation. In the model, the knowledge of population dynamic is crucial because the model represents the future state of the sow farm in terms of a present state. More precisely, the life span of the sow is divided into different states in a way that all possible transitions of sows evolving from one state to another can be represented. Assuming a given scenario for all the parameters in the life span of the sow, the deterministic model considers the following notation.

#### Set and indexes

$C = \{c\}$  Set of number of cycles,

$Sg = \{g\}$  Set of number of gestation week,

$Sl = \{l\}$  Set of number of lactation week,

$T = \{t\}$  Set of periods,

$Nr = \{r\}$  Set of repetitions of insemination,

$Sr = \{k\}$  Set of insemination waiting week,

#### Parameters

$rp_t$  = price (€/head) offered per piglet at period  $t$ ,

- $ru_t$  = price (€/head) offered by the slaughter for a replaced sow at period  $t$ ,  
 $rz_{t,c}$  = cost (€/week) to keep a sow in the insemination state at period  $t$  and cycle  $c$ ,  
 $rzr_{t,c,k}$  = cost (€/week) to keep a sow in the breeding-control state at period  $t$ , cycle  $c$  and waiting week  $k$ ,  
 $rx_{t,c,g}$  = cost (€/week) to keep a sow in the gestation state at period  $t$ , cycle  $c$  and gestation week  $g$ ,  
 $rl_{t,c,l}$  = cost (€/week) to keep a sow in the lactation state at period  $t$ , cycle  $c$  and lactation week  $l$ ,  
 $\alpha_{t,c,g}$  = survival rate of gestation at period  $t$ , cycle  $c$  and gestation week  $g$ ,  
 $\beta_{t,c,r}$  = survival rate of insemination at period  $t$ , cycle  $c$  and waiting for the insemination attempt  $r$ ,  
 $\gamma_{t,c}$  = number of piglets at period  $t$  and cycle  $c$ ,  
 $cb$  = number of boxes in breeding facility,  
 $cp$  = number of boxes in pregnancy facility,  
 $cf$  = number of boxes in farrowing facility,  
 $z0_c$  = initial stock of animals in insemination state at cycle  $c$ ,  
 $zr0_{c,r,k}$  = initial stock of animals in control state at cycle  $c$ , waiting for the insemination attempt  $r$  and week  $k$ ,  
 $x0_{c,g}$  = initial stock of animals in gestation state at cycle  $c$  and gestation week  $g$ ,  
 $y0_{c,l}$  = initial stock of animals in lactation state at cycle  $c$  and lactation week  $l$ ,  
 $zf$  = number of animals in insemination at the end of the planning horizon,  
 $zrf_{r,k}$  = number of animals in control at the end of the planning horizon waiting for the insemination attempt  $r$  and week  $k$ ,  
 $xf_g$  = number of animals in gestation at the end of the planning horizon at cycle  $c$ ,  
 $yf_l$  = number of animals in lactation at the end of the planning horizon at lactation week  $l$ ,  
 $lz$  = lower bound of gilt purchase,  
 $uz$  = upper bound of gilt purchase,  
 $dz$  = maximum variation between gilt purchase of two consecutive periods,

### Decision Variables

- $Y_{t,c,l}$  = number of sows in lactation state at period  $t$ , cycle  $c$ , lactation week  $l$ ,  
 $X_{t,c,g}$  = number of sows in gestation state at period  $t$ , cycle  $c$ , gestation week  $g$ ,

$Z_{t,c}$  = number of sows in insemination state at period  $t$ , cycle  $c$ ,

$ZR_{t,c,r,k}$  = number of sows in control state at period  $t$ , cycle  $c$ , waiting for the insemination attempt  $r$ , at the week  $k$ ,

$UL_{t,c}$  = number of replaced sows at the end of lactation state at period  $t$ , cycle  $c$ ,

$UZ_{t,c,r}$  = number of replaced sows at the end of the insemination state at period  $t$ , cycle  $c$ , waiting for the insemination attempt  $r$ ,

$AB_{t,c}$  = number of sows with abortion at period  $t$ , cycle  $c$ ,

The proposed deterministic model maximizes the total profit of the production plan and the non-negative feasible solutions must satisfy a set of constraints that mainly concern the population dynamic behaviour-r and capacity constraints, given by the following optimization problem:

$$\begin{aligned} \text{Maximise } \Phi^d = & \sum_{t \in T} \sum_{c \in C} r p_t * \gamma_{t,c} * Y_{t,c,l} + \sum_{t \in T} \sum_{c \in C} r u_t * UL_{t,c} + \sum_{t \in T} \sum_{c \in C} \sum_{r \in Nr} r u_t * UZ_{t,c,r} + \\ & \sum_{t \in T} \sum_{c \in C} r u_t * AB_{t,c} - \sum_{t \in T} \sum_{c \in C} r z_{t,c} * Z_{t,c} - \sum_{t \in T} \sum_{c \in C} \sum_{r \in Nr} \sum_{k \in Sr} r z_{t,c} * ZR_{t,c,r,k} - \quad (1) \\ & \sum_{t \in T} \sum_{c \in C} \sum_{g \in Sg-Sr} r x_{t,c,g} * X_{t,c,g} - \sum_{t \in T} \sum_{c \in C} \sum_{l \in Sl} r l_{t,c,l} * Y_{t,c,l} \end{aligned}$$

s.t.

$$Z_{1,c} = z0_c \quad c \in C - \{1\} \quad (2)$$

$$ZR_{1,c,r,k} = zr0_{c,r,k} \quad c \in C \quad r \in Nr \quad k \in Sr \quad (3)$$

$$X_{1,c,g} = x0_{c,g} \quad c \in C \quad g \in Sg \quad (4)$$

$$Y_{1,c,l} = y0_{c,l} \quad c \in C \quad l \in Sl \quad (5)$$

$$Z_{t,c} = Y_{t-1,c-1,l} - UL_{t-1,c-1} \quad t \in T - \{1\} \quad c \in C - \{1\} \quad (6)$$

$$ZR_{t,c,1,1} = Z_{t-1,c} \quad t \in T - \{1\} \quad c \in C \quad (7)$$

$$ZR_{t,c,r,1} = (1 - \beta_{t-1,c,r-1})ZR_{t-1,c,r-1,3} - UZ_{t-1,c,r-1} \quad t \in T - \{1\} \quad c \in C \quad r \in Nr - \{1\} \quad (8)$$

$$ZR_{t,c,r,k} = ZR_{t-1,c,r,k-1} \quad t \in T - \{1\} \quad c \in C \quad r \in Nr \quad k \in Sr - \{1\} \quad (9)$$

$$X_{t,c,4} = \sum_{r \in Nr} \beta_{t-1,c,r} ZR_{t-1,c,r,3} \quad t \in T - \{1\} \quad c \in C \quad (10)$$

$$X_{t,c,g} = \alpha_{t-1,c,g-1} X_{t-1,c,g-1} \quad t \in T - \{1\} \quad c \in C \quad g \in Sg - Sr - \{4\} \quad (11)$$

$$Y_{t,c,1} = \alpha_{t-1,c,g} * X_{t-1,c,g} \quad t \in T - \{1\} \quad c \in C \quad (12)$$

$$Y_{t,c,l} = Y_{t-1,c,l-1} \quad t \in T - \{1\} \quad c \in C \quad l \in Sl - \{1\} \quad (13)$$

$$\sum_{c \in C} Z_{t,c} + \sum_{c \in C} \sum_{r \in Nr} \sum_{k \in Sr} ZR_{t,c,r,k} \leq cb \quad t \in T \quad (14)$$

$$\sum_{c \in C} \sum_{g \in Sg - Sr - \{g^*\}} X_{t,c,g} \leq cp \quad t \in T \quad (15)$$

$$\sum_{c \in C} X_{t,c,g^*} + \sum_{c \in C} \sum_{l \in Sl} Y_{t,c,l} \leq cf \quad t \in T \quad (16)$$

$$|Z_{t+1,1} - Z_{t,1}| \leq dz \quad t \in T - \{t^*\} \quad (17)$$

$$lz \leq Z_{t,1} \leq uz \quad t \in T - \{t^*\} \quad (18)$$

$$\sum_{c \in C} Z_{t^*,c} \geq zf \quad (19)$$

$$\sum_{c \in C} ZR_{t^*,c,r,k} \geq zrf_{r,k} \quad k \in Sr \quad r \in Nr \quad (20)$$

$$\sum_{c \in C} X_{t^*,c,g} \geq xf_g \quad g \in Sg - Sr \quad (21)$$

$$\sum_{c \in C} Y_{t^*,c,l} \geq yf_l \quad l \in Sl \quad (22)$$

$$UL_{t,c^*} \leq Y_{t,c^*,l^*} \quad t \in T - \{t^*\} \quad (23)$$

$$UZ_{t,c,r^*} = (1 - \beta_{t,c,3}) ZR_{t,c,3,r^*} \quad c \in C \quad t \in T \quad (24)$$

$$(1 - \alpha_{t,c,10}) * X_{t,c,10} = AB_{t,c} \quad c \in C \quad t \in T \quad (25)$$

The objective function in (1) represents the maximization of the total profit, that is, the addition of profits for each period of time and reproductive cycle per decision variable. Profit is the difference between incomes obtained by sales and the different costs of production incurred. Incomes consider the sales of piglets and sales to the slaughterhouse of replaced sows while, the costs of production include feeding costs of sows and piglets, labour, insemination and veterinary expenses. According to incomes and costs affecting each decision variable coefficients of the objective function are set. For instance, the coefficient  $rz_{t,1}$  involves feeding, insemination and labour but also takes into account the purchase cost of a new gilt. Constraints (2)-(5) describes the initial stock at  $t=1$ . They account for the initial herd distribution of sows over different states and can be adapted to any particular

situation either a starting farm or an existing one. Constraints (6)-(13) represent the herd dynamics, i.e., the flow of sows throughout the different states over time. More specifically, constraint (6) refers to the number of sows at the first insemination state. Constraints (7)-(9) refers to states of sows being under control for pregnancy or a new insemination, the so-called repetition. Constraints (10) and (11) refer to the flow among gestation states. Constraints (12) and (13) refer to the flow among lactation states. Constraints (14) – (16) are the facilities capacity constraints. Three different facilities are considered depending on the state of the sow with respect to the reproductive cycle, i.e. breeding-control, gestation and farrowing facilities. Constraints (17) and (18) refer to how purchases are limited. Then, a smooth variation in purchases from week to week was allowed (17) and a minimum and maximum amount of new gilts purchased (18) is fixed per week. Constraints (19)-(22) determine the imposed inventory of animals at the end of the planning horizon, representing the continuity of the farm beyond the end of the finite time horizon considered and imposed values that tend to the stationary or long term optimal decisions level, according to some complementary studies and methodologies used by the authors (Plà et al., 2008). Constraints (23) and (24) refer to culled sows. Constraint (25) refers to gestating sows suffering an abortion and culled from the herd.

#### **4 Stochastic programming model**

The proposed optimization model of the previous section includes, in practice, some parameters often known with uncertainty. The classical approach to face the uncertainty consists in replacing the stochastic parameters by their expected values in a deterministic model as the previous one. However, the optimal solution achieved in this way might not be sufficiently representative of the reality and it does not take into account the variability of these parameters with respect to their expected value. Thus, among the different methodologies for production planning under uncertainty (Mula et al., 2006), the previous deterministic model is reformulated as a stochastic optimization program, which maximize the expected value of the farm's profit. In what follows, a two-stage stochastic optimization

model is stated to the corresponding production planning problem, where we look for a *first-stage* optimal decisions mainly related to the purchase of gilts and replacement of sows in the first  $T_1$  weeks of the planning horizon and a *second-stage* decisions related to optimal policy in the rest of the planning horizon as a recourse policy. The proposed model is based on previous papers by Escudero et al. (1993), Albornoz and Contesse (1999), Gupta and Maranas (2003), Alonso-Ayuso et al. (2005) and Albornoz and Canales (2006).

The model includes the uncertainty in the dynamic behaviour parameters, future price and cost parameters modeled by means of a given finite set of scenarios. Let consider additional sets and indexes to those given in the previous section:

$S = \{s\}$  finite set of scenarios,

$T_1 \subset T$  subset of  $T$  corresponding to the periods of the first stage,

and the following notation for those parameters now defined by scenarios:

$p_s$  = probability for scenario  $s$ .

$rp_{t,s}$  = price (€/head) offered per piglet at period  $t$  and scenario  $s$ ,

$ru_{t,s}$  = price (€/head) offered by the slaughter of a replaced sow at period  $t$  and scenario  $s$ ,

$rz_{t,c,s}$  = cost (€/week) to keep a sow in the insemination state at period  $t$ , cycle  $c$  and scenario  $s$ ,

$rZR_{t,c,k,s}$  = cost (€/week) to keep a sow in the breeding-control state at period  $t$ , cycle  $c$ , waiting week  $k$  and scenario  $s$ ,

$rx_{t,c,g,s}$  = cost (€/week) to keep a sow in the gestation state at period  $t$ , cycle  $c$ , gestation week  $g$  and scenario  $s$ ,

$rl_{t,c,l,s}$  = cost (€/week) to keep a sow in the lactation state at period  $t$ , cycle  $c$ , lactation week  $l$  and scenario  $s$ ,

$\alpha_{t,c,g,s}$  = survival rate of gestation at period  $t$ , cycle  $c$ , gestation week  $g$  and scenario  $s$ ,

$\beta_{t,c,r,s}$  = survival rate of insemination at period  $t$ , cycle  $c$ , waiting for the insemination attempt  $r$  and scenario  $s$ ,

$\gamma_{t,c,s}$  = number of piglets at period  $t$  cycle  $c$  and scenario  $s$ .

Associated with each scenario  $s \in S$ , there is a given weight or probability  $p_s$ . Once the scenarios are settled, all the non-negative decision variables in the model are defined by scenarios according to the following notation:

$Y_{t,c,l,s}$  = number of sows in lactation at period  $t$ , cycle  $c$ , lactation week  $l$  and scenario  $s$ ,

$X_{t,c,g,s}$  = number of sows in gestation at period  $t$ , cycle  $c$ , gestation week  $g$  and scenario  $s$ ,

$Z_{t,c,s}$  = number of sows in insemination at period  $t$ , cycle  $c$  and scenario  $s$ ,

$ZR_{t,c,r,k,s}$  = number of sows in control at period  $t$ , cycle  $c$ , waiting for the insemination attempt  $r$ , at the week  $k$  and scenario  $s$ ,

$UL_{t,c,s}$  = number of replaced sows at the end of lactation state at period  $t$ , cycle  $c$  and scenario  $s$ ,

$UZ_{t,c,r,s}$  = number of replaced sows at the end of the insemination state at period  $t$ , cycle  $c$ , waiting for the insemination attempt  $r$  and scenario  $s$ ,

$AB_{t,c,s}$  = number of sows with abortion at period  $t$ , cycle  $c$  and scenario  $s$ ,

In spite of the fact that all the decision variables are defined by scenarios, the decisions related to the more immediate planning period  $T_1$  satisfy an additional set of constraints, known as the *non-anticipativity* constraints. These constraints impose to concerned decision variables a value that does not depend on any particular scenario realization. These decisions are called *here-and-now* decision variables and guarantee identical first-stage decisions (in particular sow replacement and purchase of gilts the  $T_1$  period) for all the scenarios considered. Therefore the decision variables for the rest of the planning period  $T - T_1$ , are called wait-and-see or *recourse* variables, whose values depend on the corresponding scenario realization that provide the flexibility needed to deal with uncertainty according to the number of periods to be included in the first stage.

The resulting model is actually a two-stage stochastic linear program with recourse, whose extended deterministic equivalent program is given in the extensive form by the following optimization problem:

$$\begin{aligned}
 \text{Maximise } \Phi^S = & \sum_{s \in S} p_s * \left( \sum_{t \in T} \sum_{c \in C} r p_{t,s} * \gamma_{t,c,s} * Y_{t,c,l^*,s} + \sum_{t \in T} \sum_{c \in C} r u_{t,s} * U L_{t,c,s} + \sum_{t \in T} \sum_{c \in C} \sum_{r \in Nr} r u_{t,s} * U Z_{t,c,r,s} + \right. \\
 & \sum_{t \in T} \sum_{c \in C} r u_{t,s} * A B_{t,c,s} - \sum_{t \in T} \sum_{c \in C} r z_{t,c,s} * Z_{t,c,s} - \sum_{t \in T} \sum_{c \in C} \sum_{r \in Nr} \sum_{k \in Sr} r z_{t,c,k,s} * Z R_{t,c,r,k,s} - \\
 & \left. \sum_{t \in T} \sum_{c \in C} \sum_{g \in Sg-Sr} r x_{t,c,g,s} * X_{t,c,g,s} - \sum_{t \in T} \sum_{c \in C} \sum_{l \in Sl} r l_{t,c,l,s} * Y_{t,c,l,s} \right) \quad (26)
 \end{aligned}$$

s.t.

$$Z_{1,c,s} = z0_c \quad c \in C - \{1\} \quad s \in S \quad (27)$$

$$Z R_{1,c,r,k,s} = z r 0_{c,r,k} \quad c \in C \quad r \in Nr \quad k \in Sr \quad s \in S \quad (28)$$

$$X_{1,c,g,s} = x0_{c,g} \quad c \in C \quad g \in Sg \quad s \in S \quad (29)$$

$$Y_{1,c,l,s} = y0_{c,l} \quad c \in C \quad l \in Sl \quad s \in S \quad (30)$$

$$Z_{t,c,s} = Y_{t-1,c-1,l^*,s} - U L_{t-1,c-1,s} \quad t \in T - \{1\} \quad c \in C - \{1\} \quad s \in S \quad (31)$$

$$Z R_{t,c,1,1,s} = Z_{t-1,c,s} \quad t \in T - \{1\} \quad c \in C \quad s \in S \quad (32)$$

$$Z R_{t,c,r,1,s} = (1 - \beta_{t-1,c,r-1,s}) Z R_{t-1,c,r-1,3,s} - U Z_{t-1,c,r-1,s} \quad t \in T - \{1\} \quad c \in C \quad r \in Nr - \{1\} \quad s \in S \quad (33)$$

$$Z R_{t,c,r,k,s} = Z R_{t-1,c,r,k-1,s} \quad t \in T - \{1\} \quad c \in C \quad r \in Nr \quad k \in Sr - \{1\} \quad s \in S \quad (34)$$

$$X_{t,c,4,s} = \sum_{r \in Nr} \beta_{t-1,c,r,s} Z R_{t-1,c,r,3,s} \quad t \in T - \{1\} \quad c \in C \quad s \in S \quad (35)$$

$$X_{t,c,g,s} = \alpha_{t-1,c,g-1} X_{t-1,c,g-1,s} \quad t \in T - \{1\} \quad c \in C \quad g \in Sg - Sr - \{4\} \quad s \in S \quad (36)$$

$$Y_{t,c,1,s} = \alpha_{t-1,c,g^*,s} X_{t-1,c,g^*,s} \quad t \in T - \{1\} \quad c \in C \quad s \in S \quad (37)$$

$$Y_{t,c,l,s} = Y_{t-1,c,l-1,s} \quad t \in T - \{1\} \quad c \in C \quad l \in Sl - \{1\} \quad s \in S \quad (38)$$

$$\sum_{c \in C} Z_{t,c,s} + \sum_{c \in C} \sum_{r \in Nr} \sum_{k \in Sr} Z R_{t,c,r,k,s} \leq cb \quad t \in T \quad s \in S \quad (39)$$

$$\sum_{c \in C} \sum_{g \in Sg - Sr - \{g^*\}} X_{t,c,g,s} \leq cp \quad t \in T \quad s \in S \quad (40)$$

$$\sum_{c \in C} X_{t,c,g^*,s} + \sum_{c \in C} \sum_{l \in Sl} Y_{t,c,l,s} \leq cf \quad t \in T \quad s \in S \quad (41)$$

$$|Z_{t+1,1,s} - Z_{t,1,s}| \leq dz \quad t \in T - \{t^*\} \quad s \in S \quad (42)$$

$$lz \leq Z_{t,1,s} \leq uz \quad t \in T - \{t^*\} \quad s \in S \quad (43)$$

$$\sum_{c \in C} Z_{t^*,c,s} \geq zf \quad s \in S \quad (44)$$

$$\sum_{c \in C} Z R_{t^*,c,r,k,s} \geq z r f_{r,k} \quad k \in Sr \quad r \in Nr \quad s \in S \quad (45)$$



$$\sum_{c \in C} X_{t,c,g,s}^* \geq x f_g \quad g \in Sg - Sr \quad s \in S \quad (46)$$

$$\sum_{c \in C} Y_{t,c,l,s}^* \geq y f_l \quad l \in Sl \quad s \in S \quad (47)$$

$$UL_{t,c^*,s} \leq Y_{t,c^*,l^*,s} \quad t \in T - \{t^*\} \quad s \in S \quad (48)$$

$$UZ_{t,c,r^*,s} = (1 - \beta_{t,c,3,s}) ZR_{t,c,3,r^*,s} \quad c \in C \quad t \in T \quad s \in S \quad (49)$$

$$(1 - \alpha_{t,c,10,s}) * X_{t,c,10,s} = AB_{t,c,s} \quad c \in C \quad t \in T \quad s \in S \quad (50)$$

$$Z_{t,c,s} = Z_{t,c,1} \quad t \in T_1 \quad c \in C \quad s \in S \quad (51)$$

$$ZR_{t,c,r,k,s} = ZR_{t,c,1} \quad t \in T_1 \quad c \in C \quad s \in S \quad (52)$$

$$X_{t,c,g,s} = X_{t,c,g,1} \quad t \in T_1 \quad c \in C \quad s \in S \quad (53)$$

$$Y_{t,c,l,s} = Y_{t,c,l,1} \quad t \in T_1 \quad c \in C \quad s \in S \quad (54)$$

$$UL_{t,c,s} = UL_{t,c,1} \quad t \in T_1 \quad c \in C \quad s \in S \quad (55)$$

$$UZ_{t,c,r,s} = UZ_{t,c,r,1} \quad t \in T_1 \quad c \in C \quad s \in S \quad (56)$$

The objective function in (26) represents the maximization of the expected profits. As in the deterministic model constraints (27)-(30) describe the initial stock at  $t=1$ . The initial stock is the same for each scenario since only express the initial stocks of animals in farm considered. Constraints (31)-(38) represent herd's dynamics. Constraints (39)-(41) are the facilities' capacity constraints. Constraints (42) and (43) refer to how purchases are limited. Constraints (44)-(47) determine the imposed inventory of animals at the end of the planning horizon, where each scenario tends to its own long term optimal decisions level. Constraints (48) and (49) refer to culled sows. Constraint (50) refers to gestating sows suffering an abortion and culled from the herd. Constraints (51)-(56) refer to the *non-anticipativity* constraints, therefore only decision variables in the first stage, i.e. with  $t \in T_1$ , are involved.

## 5 Computational results

In order to illustrate the suitability of the proposed deterministic model (1)-(25) and the corresponding stochastic extension (26)-(56), a case study is presented. Basic

parameters of the study were taken from standard values under Spanish conditions and recorded in the BD-Porc databank (national record keeping system hosted at <http://www.irta.es/bdporc/>, accessed 14 May 2008), and do not correspond to a specific farm. An initial herd size of 2330 sows was considered a regular size. A maximum lifespan of 8 parities was allowed. The effective capacity of lactation facility was of 500 crates. The maximum number of allowed insemination was three, beyond that number it was considered as a culling reason, just like an abortion. The time horizon was 52 weeks (approximately one year).

Models were implemented and solved using the algebraic modelling language ILOG OPL 6.1 and the solver CPLEX 11.2 respectively on a laptop computer (Intel Centrino Duo T5600 at 1.83 GHz and 1Gb RAM).

### 5.1 Basic example. Deterministic model

Strictly speaking, decision variables of either models representing the number of sows ought to be integer and non-negative variables. However, given the computational time consumed for calculations in preliminary tests when all decision variables related to sows were considered integer, as well as the pertinent changes in the model's constraints to be included, make the pure integer model inappropriate for practical purposes (Rodríguez et al. 2008). As a consequence, only those decision variables corresponding to the first four periods were declared as integer and the rest as real variables. Beyond this limitation, these four periods represent the roller horizon where decisions must be implemented before new environmental changes could be appreciated or taken into account. Specific parameters of the linear programming model (1)-(25) are detailed in Appendix A. Figures corresponding to the actual instance that was solved are presented in Table 1.

Table 1. Report of the size of the Linear Programming Model (Deterministic).

CPLEX	Value
Variables	11957
Constraints	12897
Non-zero-coefficients	36679

The occupancy rate of the farrowing facilities was more than 0.95 over the time horizon of planning (Figure 1) and reached the full occupancy before the half time horizon was reached. This shows the rational behaviour of taking the maximum profit of lactation facilities given that piglets represent the main source of income for the farm. The optimum average reward of the farm was 699 thousands of euros.

The optimal replacement policy indicates to keep a sow until the end of the 6th cycle. Results provided the scheduling of the purchase and optimal replacement policy week by week.

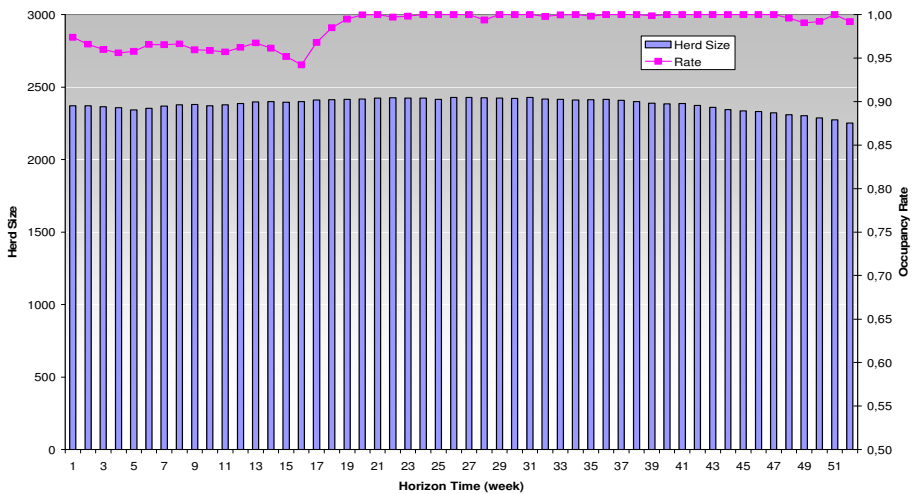


Figure 1. Representation of the behaviour occupancy rate of Farrowing facilities and herd size over time. Deterministic Model (Base).

The initial (2)-(5) and final (19)-(22) herd distribution of the 2330 sows along different reproductive states was fixed. These distributions were important because no value resulted in a feasible program.

Final herd distribution was selected from a nearer distribution to the ideal steady-state distribution of the herd (Plà et al., 2008) while initial herd distribution was arbitrary selected from those that made solvable the problem. These distributions may affect the herd size over time as shown in Figure 1 and the pattern of the purchase scheduling.

*Sensitivity analysis*

It is known that variation over time in the dynamic parameters ( $\alpha_{t,c,g}$ ,  $\beta_{t,c,r}$  and  $\gamma_{t,c}$ ) of a realistic biological system like a pig farm can be very high. Therefore, to prepare the extension of the model into a stochastic linear programming model and to value the impact of the uncertainty of dynamic parameters on model performances two additional cases were considered. The optimistic case, where dynamic parameters were increased a 5%, and the pessimistic case where dynamic parameters were reduced by 5%. Results concerning the purchase scheduling are shown in Figure 2.

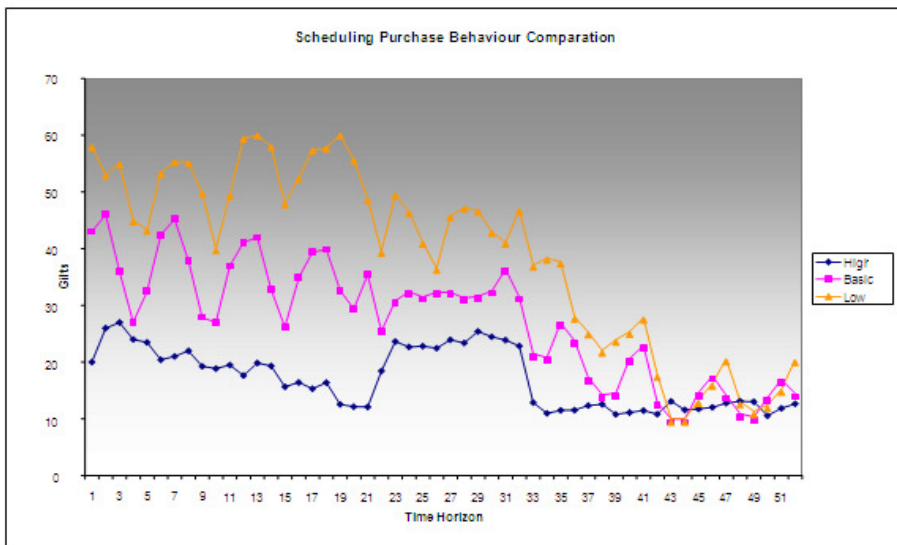


Figure 2 Representation of the behaviour of gilts purchase scheduling. Models: Base, High (+5%) and Low (-5%) productivity.

The case showing a high productivity requires lower purchases against the case with low productivity and more purchases of gilts needed to maintain lactation facilities near full occupation. The occupancy rate of lactation facilities in the three cases was more than 0.95 along the horizon planning. However, the productivity of this facility varies among cases due to herd structure and composition of this occupancy. For instance, it was observed that the herd with low productivity has a higher rate of sows in gestation state occupying the lactation facility than the others

cases. This is a logical result since more pregnant sows and less lactating sows lead to a low production. Comparing the optimal solution achieved in the deterministic model (1)-(25) for the three cases considered, it can be concluded that the uncertainty inherent to the model it is not corresponded with that observed in real systems.

As is shown in Table 2 changes of 5% in the dynamic parameters provoke changes of more than the 25% in the maximum revenue. The overall profit ranges from 699 to 888 thousands of euros. Therefore, it would seem appropriate to extend the model into a new one dealing properly with this uncertainty.

Table 2. Deterministic model: expected profit of the 3 cases in thousands of €.

	<b>Base Productivity</b>	<b>Low Productivity</b>	<b>High Productivity</b>
$\Phi$ €/year	699	494	888
Difference vs Base	0	-29%	+27%

### 5.2 Basic example. Stochastic model

Stochastic model formulation requires the generation of a set of scenarios  $S$ . To illustrate and assess the suitability of the stochastic approach, three scenarios were defined in this example. Then, the uncertain parameters ( $\alpha_{t,c,g,s}$ ,  $\beta_{t,c,r,s}$  and  $\gamma_{t,c,s}$ ) were considered to be modelled by scenario. Therefore, the optimistic, normal and pessimistic scenarios were defined in correspondence with the values of high, average and low productivity respectively. Time horizon was of 52 weeks as with the deterministic example and  $T_1 = \{1,2,3,4\}$ . The resolution of this formulation (26)-(56) give an optimal profit (RP) of 664 thousands of €/year. The results shown lactation facilities occupancy was maximized (see Figure 3) as can be observed in Figure 1 for the deterministic model. However in that case the different behaviour for each scenario is observed and reveals the extra effort in the pessimistic scenario to take the maximum profit of lactation facilities.

The optimistic scenario instead reaches the maximum occupancy of the lactation facility sooner.

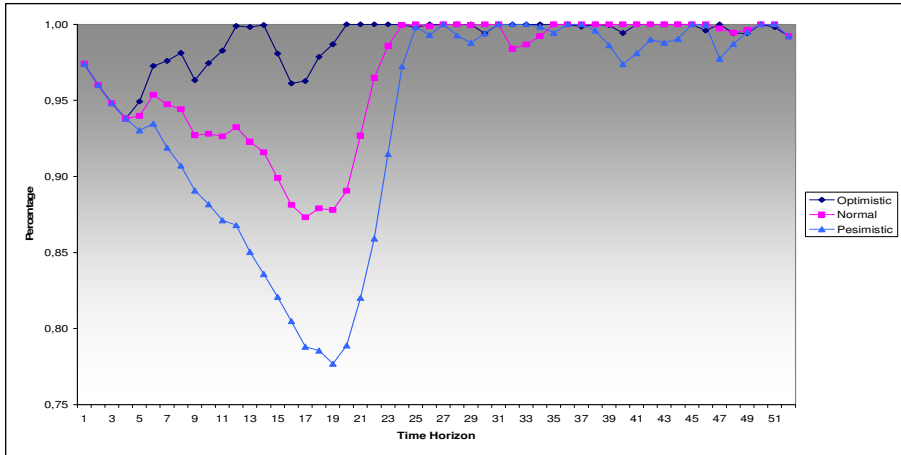


Figure 3. Behaviour of the occupancy rate of Lactation Facilities with 3 scenarios.

Concerning the herd size behaviour (see figure 4) shows how scenarios with high productivity need to maintain a lower size than the rest showing the regulation role of lactation facilities.

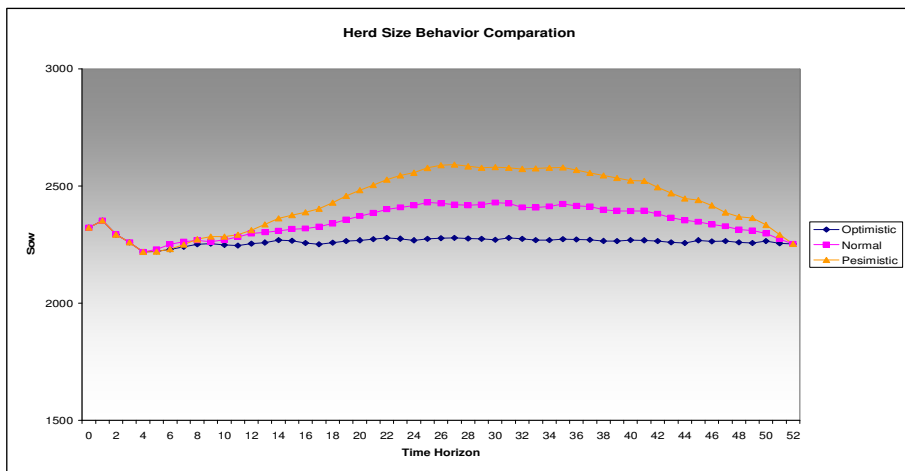


Figure 4. Representation of the Herd Size behaviour regarding three scenarios.

Furthermore, depending on the initial and final inventory, the optimistic scenario shows greater capability to reach a steady state sooner. With respect to the scheduling of purchases, again it is shown how the scenario affects the need for a supply of more gilts to the farm in the worst scenarios. Scenarios affecting

negatively production require a higher replacement rate of sows which is translated in more gilts being purchased (Figure 5).

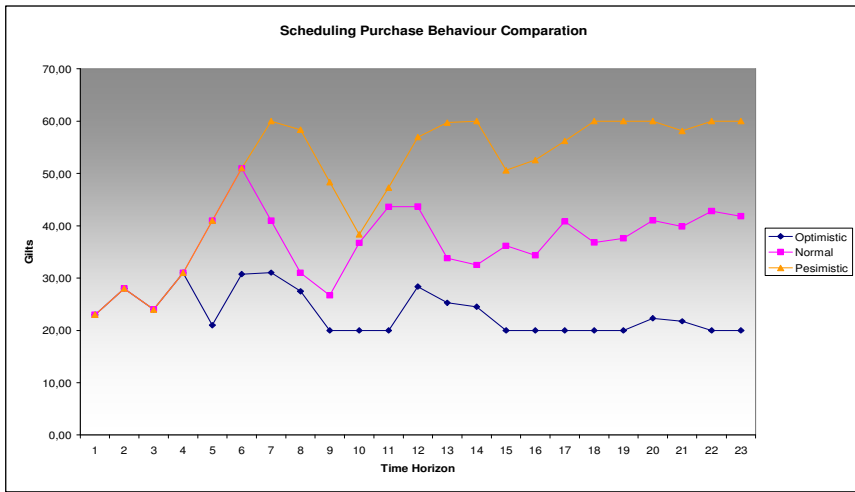


Figure 5. Representation of the Scheduling Purchase Behaviour regarding three scenarios.

Furthermore, if purchase scheduling of optimal solutions is compared for the first four periods, the stochastic model shows a better behaviour under practical point of view with lesser variations than the deterministic model (see Figures 2 and 5). This is a direct consequence of the inclusion of uncertainty by scenarios at the second stage of the stochastic model. In addition, just to analyse the importance of time horizon on outcomes for the first 52 weeks different instances for  $T=78, 104, 130$  and 156 were solved (Table 3).

It is observed that the time horizon has a very little influence on the first 52 weeks because in all instances the objective function never reports differences greater than a 0.08%. Even less is the impact on the expected profit for the first stage period (0.02% as maximum). Another aspect of interest was to see the impact of different number of weeks considered in the first stage. Therefore, new instances were solved for a different range of weeks in the first stage. The increment of weeks in the first stage showed a linear reduction in the profit and as it is shown in Figure 6 an increment in earlier purchases of gilts.

Table 3. Report of the size of the Stochastic Linear Programming Model.

	Time horizon				
	52	78	104	130	156
Constraints	38847	56709	75571	92433	110295
Variables	38689	58033	77377	96721	116065
Nonzero coef.	114005	169073	224141	279209	334277
Solving Time	0.52	1.48	4.55	10.09	17.69
Index (52)	100	99.94	99.92	99.92	99.93
Index(T1)	100	99.99	99.99	99.98	99.99

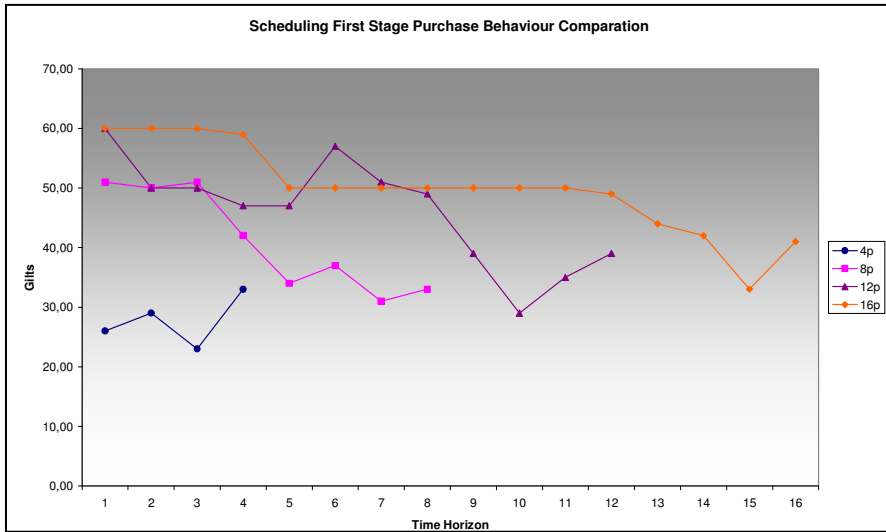


Figure 6. Purchase of gilts at first stage.

Inspecting the solution of the deterministic models with respect to the stochastic one, with the first 4 weeks as the first-stage, we compute the expected value of perfect information (EVPI), defined through the following expression:

$$EVPI = \sum_{s \in \Omega} p_s \Phi^s - RP$$

being  $\Phi^s$  the optimal value of the deterministic model (1)-(25) when it was solved (separately) for each scenario  $s$  in  $\Omega$  and  $RP$  the optimal value of the stochastic model (26)-(56). For our study the  $EVPI=694-664 = 30$  thousands of euros. EVPI measures the value of knowing the future with certainty. This is how much the farmer would be ready to pay this year to obtain perfect information about the



population dynamic behaviour. Additionally, the Value of the Stochastic Solution (VSS) was computed. Roughly speaking, it measures how good or bad results to use the optimal solution of the stochastic model instead of the deterministic one. Then, the Value of the Stochastic Solution is defined as  $VSS = RP - EEV$ , where  $EEV$  is the expected value assuming expected yields and expected parameters fixing the optimal values at the first stage. In our case, the  $VSS = 664 - 647 = 17$  thousands of euros, this is the cost of ignoring uncertainty in choosing a decision.

## 6 Conclusions

In this study we formulated a two-stage stochastic programming model with recourse, for planning a piglet production farm. The approach is shown to be suitable to deal with the uncertainty present in the system and provides better solutions than any other of the deterministic models presented for managing the replacement of sows. The stochastic model incorporates the uncertainty associated with the biological system and provides an optimum replacement decision for the immediate time horizon (first stage) which does not depend on each particular scenario considered in the problem. Incorporating integer variables only at the first stage a better representation of the system was achieved and computational problems derived from complex pure integer programming models were avoided. Future uncertainties are taken into account through a finite set of scenarios and considered in the second stage.

It is demonstrated the different response of the herd not only when specific farm parameters are different but also when scenarios are uncertain. This shows the need for analytical tools for consultants and pig specialists to give better advice to their costumers. Thus, the use of stochastic programming in this paper constitutes a valuable tool to support decision-making in the field of pig production. Finally, we intend to extend this methodology developing a refined set of scenarios and considering more than two stages. Such extensions would require the development of suitable numerical strategies of models with integer recourse variables, which are, in general, more complex than the ones examined in this paper.

## References

- Albornoz, V.M., Canales, C., 2006. Total allowable catch for managing squat lobster fishery using stochastic nonlinear programming. *Computers & Operations Research* 33, 2113-2124.
- Albornoz, V.M., Contesse, L., 1999. Modelos de Optimización Robusta un Problema de Planificación Agregada de la Producción bajo Incertidumbre en las Demandas. *Investigación Operativa* 7(3), 1-15.
- Alonso-Ayuso, A., Escudero, L., Ortuño, M.T., 2005. Modelling production planning and scheduling under uncertainty. In *Applications of Stochastic Programming*. Wallace, S.W., Ziemba, W.T., (Editors). SIAM Publications, Philadelphia.
- Beale, E.M.L., 1955. On Minimizing a Convex Function Subject to Linear Inequalities. *Journal of the Royal Statistical Society, Series B* 17, 173-184.
- Birge, J.R., Louveaux, F.V., 1997. *Introduction to Stochastic Programming*, Springer-Verlag, New York.
- Chavas, J.P., Kliebenstein, J., Crenshaw, Th.D., 1985. Modeling dynamic agricultural production response: The case of swine production. *American Journal of Agricultural Economics* 67, 636-646.
- Dantzig, G.B., 1955. Linear Programming under Uncertainty. *Management Science* 1, 197-206.
- Escudero, L., Kamesan, P., King, A., Wets, R., 1993. Production Planning via Scenarios Modelling. *Annals of Operations Research* 43, 311-335.
- Glen, J.J., 1987. Mathematical models in farm planning: a survey. *Operations Research* 35: 641-666.
- Gupta, A., Maranas, C.D., 2003. Managing demand uncertainty in supply chain planning. *Computers and Chemical Engineering* 27, 1219-1227.
- Huirne, R.B., Dijkhuizen, A.A., van Beek, P., Hendriks, Th.H.B., 1993. Stochastic dynamic programming to support sow replacement decisions. *European Journal of Operational Research* 67, 161-171.
- Jalvingh, A.W., Dijkhuizen, A.A., van Arendonk, J.A.M., 1992. Dynamic probabilistic modelling of reproduction and management in sow herds. General aspects and model description. *Agricultural Systems* 39, 133-152.
- Kingwell, R., 1996. Programming models of farm supply response: the impact of specification errors. *Agricultural Systems* 50, 307-324.
- Kristensen, A.R., 1988. Hierarchic Markov processes and their applications in replacement models. *European Journal of Operational Research* 35, 207-215.

- Kristensen, A.R., 1993. Markov decision techniques applied to the animal replacement problem. Dr.Sc. Dissertation. Dina KVL. Dep. of Animal Science and Animal Health. The Royal Veterinary and Agricultural University. Copenhagen. Denmark.
- Kristensen, A.R., Søllested, T.A., 2004a. A sow replacement model using bayesian updating in a 3-level hierarchic Markov process. I. Biological model. *Livestock Production Science* 87, 13–24.
- Mula, J., Poler, R., García-Sabater, J.P., Lario, F.C., 2006. Models for production planning under uncertainty: A review, *International Journal of Production Economics* 103, 271–285.
- Plà, L.M., 2005. A Stochastic Model for Planning Swine Facilities. In *Proceedings of the 2005 Winter Simulation Conference, Orlando*, 2378-2384.
- Plà, L.M., 2007. Review of Mathematical Models for Sow Herd Management. *Livestock Science* 106, 107-119.
- Plà, L.M., Babot, D., Pomar, J., 2004. A mathematical model for designing and sizing sow farms. *International Transactions in Operations Research* 11, 485-494.
- Plà, L.M., Faulin, J., Rodríguez, S., 2008. A linear programming formulation of a semi-markov model to design pig facilities. *Journal of Operational Research* 60, 619-625.
- Puterman, M.L., 1994. *Markov Decision Processes*. Wiley, New York.
- Singh, D., 1986. Simulation of swine herd population dynamics. *Agricultural Systems* 22, 157-183.
- Rodríguez, S.V., Albornoz, V.M., Plà, L.M., 2008. A mixed-integer linear programming model for scheduling replacements and purchases in sow farms. Technical Report, Mathematic Department, University of Lleida.
- Rowland, W.W., Langemeier, M.R., Schurle, B.W., Featherstone, A.M., 1998. A nonparametric efficiency analysis for a sample of Kansas swine operations. *Journal of Agricultural and Applied Economics* 30, 189-199.
- Ruszczynski, A., Shapiro, A., Editors 2003. *Stochastic Programming*. Handbooks in Operations Research and Management Science, Vol. 10. North-Holland, New York.
- Wallace, S.W. and Ziemba, W.T., Editors 2005. *Applications of Stochastic Programming*. SIAM Publications, Philadelphia.
- White, D.J., 1988. Mean, Variance, and probabilistic criteria in finite Markov Decision Processes: a review. *Journal of the Optimisation Theory and Applications* 56, 1-29.

## Appendix A

Table 1. Daily feed intake of sows according to their physiological state (Kristensen and Søllested, 2004).

State	Feed (FEs)	Comments
Mating period	3.5-4.0	
Pregnancy		
Week 1-12	2.1-2.7	adjusted to physical condition of individual sows adjusted to physical condition of individual sows
Week 12-16	3.0-4.0	
Farrowing	2.0-2.5	
Suckling	4.0-7.0	Sows fed ad libitum

Daily feed intake has been transferred from kg to FEs (1 FE=NE of 1kg barley ~13MJ DE).

Table 2. Unitary prices of different concepts related with costs and incomes in sow production.

Concept	Price	Comments
Feed	0.18 €/kg	
Mating	2 €	
Veterinary	1 €	
Piglet	32.5 €/head	Sold to a rearing farm
Gilt	200 €/head	Supplied by another farm
Sow	180 €/head	Sold to the slaughterhouse

Table 3. Main transition probabilities considered in the basic model,  $(1-\alpha(t,c,10))$  represents the probability of abortion.

Cycle	Insemination			Gestation		
	$\beta(t,c,1)$	$\beta(t,c,2)$	$\beta(t,c,3)$	$\alpha(t,c,10)$	$\alpha(t,c,16)$	$\alpha(t,c,g-\{10,16\})$
1	0.93	0.90	0.87	0.94	0.85	1.00
2	0.93	0.90	0.87	0.94	0.85	1.00
3	0.93	0.90	0.87	0.94	0.85	1.00
4	0.93	0.89	0.87	0.94	0.85	1.00
5	0.92	0.88	0.86	0.94	0.85	1.00
6	0.91	0.87	0.86	0.94	0.85	1.00
7	0.90	0.86	0.85	0.94	0.85	1.00
8	0.90	0.86	0.85	0.94	0.85	1.00

Table 4. Expected number of piglets weaned per sow per cycle,  $\gamma_{t,c}$ .

Cycle	1	2	3	4	5	6	7	8
$\gamma$	7.6	8.6	9.5	9.5	9.5	8.6	5.7	5.7



## **Conclusions and Outlook**

Sara V. Rodríguez

University of Lleida, Department of Mathematics, Jaume II, 73 E-25001 Lleida, Spain

The existence of big companies and cooperatives acting in a competitive and global market make more complex the production process. The creation of PSC to integrate pig production is more and more common, therefore the formulation of new decision models adapted to this new reality is needed. In Chapter 1, the main objective of this research was presented: the formulation of a set of models to support sow herd management and piglet production in the PSC context, in order to be able to give practical answers to relevant questions often asked by decision makers. Furthermore the idea was to make these models flexible enough for a possible coordination of the flow of animals in a PSC management model.

Therefore main strategic and tactical decisions of the production process were involved. Within this frame, the specific objectives were:

- 1) To survey the role of sow herds and piglet production models into the PSC management.
- 2) To reformulate and extend Markov decision models. Different aspects concerning animal welfare respecting recent European Union regulations were included.
- 3) To explore Linear programming and modern solvers as a powerful alternative approach to compute and solve equivalent decision models under uncertainty: either semi-Markov decision models or two-stage stochastic programming models.
- 4) To develop and assess the suitability of finite time horizon models for tactical decisions on field conditions.
- 5) To approach the planning of purchase and replacement decisions by using stochastic programming models in particular two stage stochastic programming. The response of the herd structure over time was another outcome of interest in these models.

In Chapter 2 a survey of the PSC management and the role of sow herd models was done. One of the main gaps detected in sow herd models was the need to represent in a more realistic way the vertical integration scheme of piglet production into the PSC context. Moreover, new constraints have appeared and have to be considered in modern formulations. For instance, the need to incorporate issues related to new EU regulations on animal welfare or bounds detailed in contract agreements (such as a weekly limited supply of gilts or farrowing target quota) and other derived from them (like the maximization of the occupancy of lactation facilities). Other traditional parameters already considered in past models were also candidates to be improved as those representing seasonal variations of fertility rate in summer seasons or prices' volatility. Chapters 3 to 6 were deployed keeping in mind the coverage of some of these gaps. Hence Chapter 3 was aimed at the development of a linear programming model to design sow farm facilities taking into consideration new EU regulation concerning animal welfare. Chapter 4 was aimed at the study of the influence of clinical signs on the optimal replacement policy in sow herds with different risk factors for health disorders. Chapter 5 was aimed at the formulation of a linear programming model taking into account several constraints derived from common practices in sow farms that belong to a PSC. Hence, the piglet production system under limited supply of gilts, a farrowing target quota and the maximization of the occupancy of lactation facilities were modelled and optimised. Later on the former proposal was extended in Chapter 6. The extension consisted in the formulation of a two stage stochastic programming formulation with three basic scenarios. The use of the model was intended for a rolling finite time horizon.

Hence from the previous chapters it can be conclude that:

- 1) In pig farms, the optimal solution of a Semi-Markov decision process based on physiological states and movements between facilities is associated with a steady flow of animals between facilities (i.e. herd structure at equilibrium). This herd distribution is useful in strategic decisions like this of sizing housing facilities on sow farms (cf. Chapter 3).



- 2) The incorporation of clinical signs in sow replacement problems leads to slightly better culling policies through better detecting the weakest sows of the herd. Here the developed framework based on multi-level hierarchical Markov process using Bayesian updating to integrate the weak sow index has turned out to be an efficient tool to take into account health status of individual sows to optimize breeding and replacement decisions (cf. Chapter 4).
- 3) Unstructured herd structure moving to the steady state or temporary shocks in parameters or transitory perturbations is better represented by finite time horizon models (cf. Chapter 5 and 6).
- 4) Finite time horizon models are useful to adapt the model to changing environments rather common in the competitive pig sector. Furthermore, they permit the study of the convergence to the steady state and tracking the state of the herd at each period of time (cf. Chapter 5 and 6).
- 5) The linear programming formulation is easier to explicitly incorporate herd constraints, more specifically limited supply of gilts, farrowing target quota, capacity of the facilities and initial conditions of the herd. These would be more difficult or impossible to manage with semi-Markov decision models (cf. Chapter 5 and 6). On the other hand, the use of specific LP solvers allows researchers to concentrate in the formulation and analysis of the model. Hence, it is a powerful alternative approach to either semi-Markov decision models or two-stage stochastic programming models (cf. Chapter 3, 5 and 6).
- 6) The two-stage stochastic programming model with recourse is a suitable way to deal with the uncertainty of the system through scenarios. Main variables affected by associated uncertainty are litter size, conception rate, mortality, abortion and prices. Additional benefits are the scheduling of gilts' purchase, planning piglet production and replacement decisions, all under a rolling horizon scheme (cf. Chapter 6).

In addition to the conclusions of the thesis some ideas guiding future projects can be derived. For instance, although the stress in this thesis is put on sow farms models presented can be extended easily to represent other sort of farms and cover the whole pig supply chain. On the other hand, the spreading of sensors on farms (e.g. electronic identification or biosensors and automatic feeding machines) makes the volume of available data to increase and this may put problems about how to extract useful information to the farmer and to the firms. Another weakness of present situation is that there has been detected a lack of decision support systems (DSS) or in general software adopting models to support pig supply chain management task in general and pig farm tasks in particular. DSS are important tools to deliver research deployments on herd modelling aiding decision makers. It is predictable the important role that internet can play in this context. This is a challenging task in future research. A not less important issue is the development of finite time horizon models capable of dealing with transient (not stable) farms or set of them (e.g. pig chains). Hence, the explicit incorporation of time in the formulation allows the researcher to incorporate variations in parameters affected by time like prices over time and variations on fertility rate on summer seasons. The number of parameter affected by uncertainty and the approche of scenario analysis may lead to an extension of the two-stage stochastic programming deployed in this thesis and extended to a multi-stage programming model. Besides that, complexities of models considering integer variables could be another interesting contribution for future works.

Finally, the new reality for the pig sector integrated into Pork supply chains has brought new challenges to the operational research methods in practice. Location and allocation models have many potential applications for developing existing chains in order to reduce cost of transport and gas emissions. Nowadays, with the current structure of the sector, it is reasonable to think about models capable of solving more accurately problems involving two or more stages of the chain, and integrating them all together in some information system, in order to improve the management of the PSC. Hence the models presented in this thesis are suitable tools to deal with main strategic and tactical decisions in sow herds producing piglets in a general PSC context.



**CURRICULUM VITAE**

Sara Verónica Rodríguez Sánchez was born on July 24, 1981 in Hidalgo, Mexico. In 1998 she entered “Universidad Autónoma del Estado de Hidalgo” to study industrial engineering and finished her Bachelor of Science (BSc) in 2002. From 2003 till 2004, she worked as industrial engineer in a firm devoted to produce snack where she realised the relevance of supply chains in today’s business. At the end of 2004 she enrolled in the “Engineering” PhD program at University of Lleida located in Spain. During this period she visited the “Universidad Técnica Federico Santa María” (UTFSM) and the University of Copenhagen for scientific research. She is a board member of the EURO working group Operations Research in Agriculture and Forest Management (ORAFM, <http://orafm.org/>).

**PUBLICATIONS***Articles in Scientific Journals*

Pla, L.M., Faulin, J., Rodríguez, S.V., 2008. A Linear Programming formulation of a semi-markov model to design pig facilities. *Journal of the Operational Research Society* 60, 619-625.

Rodríguez, S.V., Albornoz, V., Pla, L.M., 2009. A two stage Stochastic programming model for scheduling replacements in sow farms. *TOP* 17, 171-179.

*Main contributions in Conferences*

Rodríguez, S.V., and Plà, L.M., 2006. Optimization of the livestock production, facing the replacement problem. XIII Latin-Iberoamerican Congress on Operational Research CLAIO. Montevideo, Uruguay, November 27-30.

- Rodríguez, S.V. and Plà, L.M., 2007. Ideas around steady state herd management models. XXII European Conference on Operational Research EURO. Prague, Czech Republic, July, 8-11.
- Rodríguez, S.V., and Plà, L.M., Modelo de Programación Lineal para el diseño de granjas porcinas”, V Seminar USM on Operations Research, Universidad Técnica Federico Santa María, Valparaiso, Chile, October, 26, 2007.
- Rodríguez, S.V., and Plà, L.M., Convergence of markov decision process and derived application in sow herd management models. VII Chilean Congress of Operations Research, OPTIMA'07. Puerto Montt, Chile, November, 21-23, 2007. (Poster).
- Rodríguez, S.V., Albornoz, V., Plà, L.M., 2008. On planning production in swine farms under uncertainty. International Workshop on Operational Research, IWOR. Madrid, Spain, June, 5-7.
- Rodríguez, S.V., Flores V., Plà, L.M., 2008. Simulation of a Swine Farm under uncertainty. Winter Simulation Conference. Miami, USA, December, 13-16.
- Rodríguez, S.V., Albornoz, V., Plà, L.M., 2009. Modelo de Programación Estocástica en Apoyo a la Planificación Táctica de Granjas Reproductoras Porcinas, VII Seminar USM on Operations Research, Universidad Técnica Federico Santa María, Santiago, Chile, April, 3.
- Rodríguez, S.V., Jensen, T., Plà, L.M., Kristensen, A., 2009. Clinical signs for characterization of individual sows: The weak sow Index II. Optimal replacement policies and economic value of clinical observations. XXII European Conference on Operational Research EURO. Bonn, Germany, July, 5-8.

*Visit with Scientific Research purpose*

Research Centre: Departamento de industrias, Universidad Técnica Federico Santa María, Chile.

Date: 10/10/2007 to 07/12/2007 and 25/02/2009 to 15/05/2009.

Research Centre: Large Animals Sciences, University of Copenhagen, Denmark.

Date: 24/08/2008 to 30/10/2008 and 08/10/2009 to 08/11/2009.

*Schools on Operational Research*

Attendance:

- I-Math Intensive School on Mathematical Programming and its Applications, 2008. Spain, February, 28-02.
- Stochastic Summer School, Italy, November, 23-28.

Attendance and Contributions:

- Rodríguez, S.V., and Plà, L.M., 2005. Modelo markoviano para apoyar la toma de decisiones en explotaciones porcinas. XI Latin-American Summer Workshop on Operations Research, Colombia, July, 25-29.
- Rodríguez, S.V., Albornoz, V. Plà, L.M., 2009. Modelo markoviano para apoyar la toma de decisiones en explotaciones porcinas. Euro Summer School, Spain, July, 25-8.
- Rodríguez, S.V., Albornoz, V. Plà, L.M., 2009. Modelo markoviano para apoyar la toma de decisiones en explotaciones porcinas. XIV Latin-American Summer Workshop on Operations Research, Mexico, August, 9-14.

Impreso en los Talleres de:



Tel. (33) 3615 – 9271  
[www.groppelibros.com.mx](http://www.groppelibros.com.mx)

Hospital No. 2295 - A  
Col. Ladrón de Guevara  
Guadalajara, Jalisco