Universitat Autónoma de Barcelona

Doctoral Thesis

Essays on Empirical Asset Pricing

Author: *Xiang Zhang*

Supervisor: *Abhay Abhyankar*

A thesis submitted in fulfillment of the requirements
for the degree of
Doctor of Philosophy
in the subject of
Economics

in the

The International Doctorate in Economic Analysis Program
Departament d'Economia i d'História Económica

June 2013

UNIVERSITAT AUTÓNOMA DE BARCELONA

Abstract

Thesis Advisor: Professor Abhay Abhyankar
Departament d'Economia i d'História Económica

Doctor of Philosophy

Essays on Empirical Asset Pricing

by Xiang ZHANG

This thesis consists of three essays on empirical asset pricing around three themes: evaluating linear factor asset pricing models by comparing their misspecified measures, understanding the long-run risk on consumption-leisure to investigate their pricing performances on cross-sectional returns, and evaluating conditional asset pricing models by using the methodology of dynamic cross-sectional regressions.

The first chapter is "Comparing Asset Pricing Models: What does the Hansen–Jagannathan Distance Tell Us?". It compares the relative performance of some important linear asset pricing models based on the Hansen–Jagannathan (HJ) distance using data over a long sample period from 1952–2011 based on U.S. market. The main results are as follows: first, among return-based linear models, the Fama–French (1993) [1] five-factor model performs best in terms of the normalized pricing errors, compared with the other candidates. On the other hand, the macro-factor model of Chen, Roll, and Ross (1986) [2] five-factor is not able to explain industry portfolios: its performance is even worse than that of the classical CAPM. Second, the Yogo (2006) [3] non-durable and durable consumption model is the least misspecified, among consumption-based asset pricing models, in capturing the spread in industry and size portfolios. Third, the Lettau and Ludvigson (2002) [4] scaled consumption-based CAPM (C-CAPM) model obtains the smallest normalized pricing errors pricing gross and excess returns on size portfolios, respectively, while Santos and Veronesi (2006) [5] scaled C-CAPM model does better in explain the return spread on portfolios of U.S. government bonds.

The second chapter ("Leisure, Consumption and Long Run Risk: An Empirical Evaluation") uses a long-run risk model with non-separable leisure and consumption, and studies its ability to price equity returns on a variety of portfolios of U.S. stocks using data from 1948–2011. It builds on early work by Eichenbaum et al. (1988) [6] that explores the empirical properties of intertemporal asset pricing models where the representative agent has utility over consumption and leisure. Here we use the framework in Uhlig (2007) [7], that allows for a stochastic discount factor with news about long-run growth in consumption and leisure. To evaluate our long-run model, we assess its performance relative to standard asset pricing models in explaining the cross-section of returns across size, industry and value-growth portfolios. We find that the long-run consumption-leisure model cannot be rejected by the J–statistic and it does better than the standard consumption-based CAPM, the Yogo durable consumption and Fama–French three-factor models. We also rank the normalized pricing errors using the HJ distance: our model has a smaller HJ distance than other candidate models. Our paper is the first, as far as we are aware, to use leisure data with adjusted working hours as a measure of leisure i.e., defined as the difference between a fixed time endowment and the observable hours spent on working, home production, schooling, communication, and personal care (Yang (2010) [8]).

The third essay: "Empirical Evaluation of Conditional Asset Pricing Models: An Economic Perspective" uses dynamic Fama–MacBeth cross-sectional regressions and tests the performance of several important conditional asset pricing models when allowing for time-varying price of risk. It compares the performance of conditional asset pricing models, in terms of their ability to explain the cross-section of returns across momentum, industry, value-growth and government bond portfolios. We use the new methodology introduced by Adrian et al. (2012) [9]. Our main results are as follows: first we find that the Lettau and Ludvigson (2001) conditional model does better than other models in explaining the cross-section of momentum and value–growth portfolios. Second we find that the Piazessi et al. (2007) consumption model does better than others in pricing the cross-section of industry portfolios. Finally, we find that in the case of the cross-section of risk premia on U.S. government bond portfolios the conditional model in Santos and Veronesi (2006) outperforms other candidate models. Overall, however, the Lettau and Ludvigson (2001) model does better than other candidate models. Our main contributions here is using a recently developed method of dynamic Fama–MacBeth regressions to evaluate the performance of leading conditional CAPM (C-CAPM) models in a common set of test assets over the time period from 1951–2012.

Acknowledgements

I became interested in empirical asset pricing as a master student at UAB while taking the course taught by Abhay Abhyankar. Working on a senior thesis on that topic, I had read many related papers and became captivated by the intimate connection between theory and evidence at the frontier of empirical asset pricing. Abhay has led me step by step in the process of becoming an academic: first equipping me with the necessary tools through a research assistantship, then teaching me to carry out a research project from start to finish through a joint paper (Chapter 1), and finally navigating me through my "job market paper" (Chapter 2) and the third essay (Chapter 3). I thank Abhay for his father-like kindness and strict requirements for my research.

Michael Creel has contributed to my intellectual development, particularly in econometrics. What began as a tutor of mine in my first year developed into a three-year research on application of the general moment estimator. His helps led me to write Chapter 1 and Chapter 2 of the thesis. I thank Michael for his constant encouragement and support during my graduate studies.

Jordi Caballé and Francisco Peñaranda have given me invaluable guidance and support on the work contained in the thesis. I also want to thank others who have provided helpful feedback on parts of the thesis. Chapter 1 has benefited from comments by Cesare Robotti, Raymond Kan, Motohiro Yogo, and Rosario Crinó, and seminar participants at Arne Ryde Workshop, RIEM, IFS in Southwestern University of Finance and Economics, QQE 2011, EEFS 2011, and Rimini Quantitative Finance Workshop. Chapter 2 has benefited from comments by Phil Dybvig, Mark Loewenstein, Michael Brennan, Albert Marcet, Hossein Asgharian, Marvin Goodfriend, Tan Wang, Harald Uhlig and seminar participants at Centre for Finance in University of Gothenburg, International 2012 Paris Finance Meeting, SAEe2012, Institute of Financial Studies, RIEM in Southwestern University of Finance and Economics and Arne Ryde Workshop. Chapter 3 has benefited from comments by Abhay Abhyankar and Michael Creel.

I am very grateful for financial support through the Spanish Ministry of Science and Innovation through grant ECO2008-04756 (Grupo Consolidado-C) and FEDER in 2009-2013. Without the generous support, I would not have been able to complete my doctoral work in three years.

What I will most miss about graduate school are the daily interactions with interesting and nice colleagues. In particular, I will miss the not always productive conversations with Pinghan Liang, Dimitrios Bermperoglou, and Emanuel Alfranseder. I learned a lot about economics and life from these guys. Special thanks to Jingwen Liu for bearing the downs with me through the final two years. Lastly, I thank my parents for a constant source of wisdom and encouragement.

To my parents, Kai Zhang and Yixiu Lv, and my daughter, Elisa Xinyuan Zhang

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Chapter 1

Comparing Asset Pricing Models: What does the Hansen–Jagannathan Distance Tell Us?

1.1 Introduction and Motivation

The purpose of this paper is to compare the relative performance of some important linear asset pricing models based on the Hansen–Jagannathan (HJ) distance using data over a long sample period from 1952–2011 based on U.S. market.

Such comparisons, in the prior literature (for example, Lettau and Ludvigson (2001)[10], Lustig and Van Nieuwerburgh (2005)[11], and Parker and Julliard (2005)[12]) rely on tests of pricing errors for individual models using purely statistical criteria such as Fama–MacBeth cross-sectional regression or the Hansen–Singleton J–statistics (1982). We see that most asset pricing models are rejected by these statistical tests: this is not surprising given that most of them are misspecified. Thus, such comparisons of individual models, even using the same test assets and data, does not help us to understand why the models fail. In contrast, the HJ measure is a test of the degree of misspecification from the "true" model that correctly prices the data. In this paper, we use this measure, based on new econometric methods proposed by Hansen and Jagannathan (1997) [13], which allow us to compare across models and to choose the one that prices the chosen assets with the "best".

We compare the performance of the following linear factor models. As a benchmark we use Fama–French three-factor model [14] which is based on firm characteristics. Next we use a set of important consumption-based based models with linearized discount factors. Finally, we also compare models by incoporating conditioning information; this allows us to further compare the performance of conditional versus unconditional models. We use a common set of test assets used in the literature; the 25 Fama–French size/book-to-market, 30 industry, 10 deciles portfolios and a set of US government bonds.

Our main results are as follows.

First, among return-based linear models, the Fama–French (1993) [1] five-factor model performs best in terms of the normalized pricing errors, compared with the other candidates. On the other hand, the macro-factor model, the Chen, Roll, and Ross (1986) [2] five-factor, is not able to explain industry portfolios: its performance is even worse than CAPM. Given the test portfolios, the Fama–French factor residuals of the size-value portfolios is tiny, because it is likely to produce betas that line up with expected returns.

Second, the Yogo (2006) [3] non-durable and durable consumption model is the least misspecified, among consumption-based asset pricing models, to capture industry and size effects. Small stocks deliver relatively low returns during recessions, when durable consumption falls sharply, which explain the cross-sectional variation in the equity premium. Furthermore, the non-durable and durable consumptions display a pronounced lead-lag structure, which can price industry portfolios well (Kroenche et al. (2013) [15]).

Third, pricing performances on conditional models are unstable but better than unconditional models; Lettau and Ludvigson (2002) [4] scaled consumption-based CAPM (C-CAPM) model obtains the smallest normalized pricing errors pricing gross and excess returns on size portfolios, respectively, while Santos and Veronesi (2006) [5] scaled C-CAPM model is better to explain gross yields on U.S. government bonds.

Forth, through a multiple comparison test, our results show that above models are tested as the relatively less misspecified ones. This is achieved by incorporating the appropriate null hypotheses leading to simpler model comparison tests. In the existing literature, the null hypothesis states that whether or not the HJ distance is equal to zero. When two models' misspecified measures are both not rejected by the null hypothesis, however, we cannot tell which one is relatively better. With the practice of imposing the null hypotheses in constructing the test statistics based on asymptotic arguments, our simpler comparison method has obtained the same result as Gospodinov, Kan and Robotti (2012)[16], which presents a general statistical framework for estimation, testing and comparison of asset pricing models using the unconstrained HJ distance measure.

Our work is related to and builds upon Hodrick and Zhang (2001) [17] who also evaluate the specification errors of several empirical asset pricing models. In their paper, they use the traditional HJ distances, J–statistics and supLM test as the statistical criteria in order to test the model specifications. However, their results do not allow for inference about which model is the relatively less misspecified. In other work, Wang (2005) [18] compares asset pricing models among eight proposed factors and eight proposed conditioning variables for explaining the cross section of stock returns. Actually scaled factor models have smaller HJ distances than non-scaled factor models, since by doubling the number of parameters, a scaled factor model uses additional degrees of freedom in the minimization problem and is better able to fit the data.

The rest of the paper consists of Section 1.2 which introduces the HJ distance and the multiple comparison tests. Section 1.3 describes the candidate models. Section 1.4 presents data and the empirical analysis. The final section summarizes the findings.

1.2 Test Methodology

In the paper, we assume that the risk-free rate R_t^f is observed and m_{t+1} presents the admissible stochastic discount factor (SDF). Any tradable asset with payoff x_{t+1} must satisfy the pricing formula

$$p_t = E_t[m_{t+1}x_{t+1}], \tag{1.1}$$

where E_t denotes the expectation conditional on the information known at time t.

1.2.1 Hansen–Jagannathan Distance

How to examine the pricing error on the portfolios that are most mispriced by a given model? Hansen and Jagannathan (1997) [13] develop a measure of degree of misspecification of an asset pricing models. This measure is defined as

$$min_{m\in\aleph} \|m - y\|,$$

the least squares distance between the family of stochastic discount factors that price all the assets correctly m and the stochastic discount factor associated with an asset pricing model y. Figure 1.1 shows a direct image that the HJ distance is the least squared distance between any point along the admissible SDF line and the cross point between these two orthogonal lines (the payoffs line).

Now we assume that the proposed SDF y_{t+1} can be approximated as a linear function of factors

$$y_{t+1} = \theta' f_{t+1}. \tag{1.2}$$

Following the pricing equation, we define $R_t = [R_{1,t}, R_{2,t}, ..., R_{N,t}]'$ being the gross returns on N assets, and let

$$\alpha_t(\theta) = R_t y_t(\theta) - I_N = R_t \theta f_t' - I_N, \tag{1.3}$$

where $\alpha_t(\theta)$ is the vector of pricing errors. Hence, the maximum pricing error per unit norm of any portfolio of N assets (HJ distance) is given by

$$\delta^2 = E[(\alpha_t(\theta))'][E(R_t R_t')]^{-1} E[\alpha_t(\theta)]. \tag{1.4}$$

The HJ distance measure is equivalent to a GMM estimator with the moment condition $E[\alpha_t(\theta)] = 0$ and the weighting matrix $[E(R_t R_t')]^{-1}$, which is different from the optimal matrix (see Appendix on sample estimates and tests on Hansen–Jagannathan distance).

1.2.2 Modified Hansen–Jagannathan Distance

If excess returns are used to measure model misspecification, one cannot specify a proposed SDF in a way such that it can be zero for some values of θ; when excess returns using the moment restriction does not separately the parameters θ in equation (1.3), since the GMM errors for the parameter pair (θ_0, θ), where θ_0 stands for the constant term, are proportional to the GMM errors for the parameter pair $(k\theta_0, k\theta)$, for any scalar k. Kan and Robotti (2008) [19] suggest defining the SDF as a linear function of the demeaned factors in order to avoid the affine transformation problem. Hence, the modified HJ distance is defined as

$$\delta^2_{mod} = min_\theta E[\alpha_T(\theta)'] V_{22T}^{-1} E[\alpha_T(\theta)], \tag{1.5}$$

where V_{22T}^{-1} is the covariance matrix of the test portfolios.

1.2.3 Constrained Hansen–Jagannathan Distance

It is possible for an SDF to price all the test assets correctly and yet to take on negative values with positive probability. This case happens when these exist arbitrage opportunities among test portfolios (e.g. derivatives on test assets) and it could be problematic to set the SDF to price payoffs. Therefore, it is necessary to constrict the admissible SDFs being non-negative.

Following Gospodinov, Kan and Robotti (2010) [20] mechanism, the vector of gross returns on N assets at t is denoted by R_t, and the corresponding costs of these N assets at $t-1$ are q_{t-1}, where $E[q_{t-1}] \neq 0$. Empirically, we can solve the constrained HJ distance as

$$\delta_+^2 = min_{m_t, t=1,...,T} \frac{1}{T} \sum_{t=1}^{T} (y_t - m_t)^2, \tag{1.6}$$

subject to

$$\frac{1}{T} \sum_{t=1}^{T} m_t R_t = \bar{q},$$
$$m_t \geq 0, t = 1, ..., T,$$

where y_t denotes the candidate SDF and m_t stands for admissible SDF in the set $\aleph_+$.

1.2.4 Testing for Multiple Comparisons

The traditional HJ distance test provides no method for comparing HJ distances statistically, i.e., HJ^1 may be less than HJ^2; are they statistically different from one another once we account for sampling error?

Let $\delta_{j,T}^2$ denote the squared HJ distance for model j. Taking a benchmark model, e.g., the model with smallest squared HJ distance among $j = 1, ..., K$ competing models, and denoting

$$\delta_{1,T}^2 = min(d_{j,T}^2)_{j=1}^K. \tag{1.7}$$

The null hypothesis states

$$H_0 : d_{1,T}^2 - d_{2,T}^2 \leq 0,$$

where $d_{2,T}^2$ is the competing model with the next smallest squared distance. Now we define the test statistic as $T^W = max_{2,...,5} \sqrt{T}(d_{1,T}^2 - d_{j,T}^2)$, based on White (2003) [21]. The distribution of T^W is computed via block bootstrap (Chen and Ludvigson (2009) [22]). Need to mention, the justification for the bootstrap rests on the existence of a multivariate, joint, continues, limiting distribution for the set $(d_{j,T}^2)_{j=1}^K$ under the null.

By repeated sampling, the bootstrap estimates of the p-value is

$$p_W = \frac{1}{B}\sum_{b=1}^{B} I_{(T^{W,b}>T^W)}, \tag{1.8}$$

where B is the number of bootstrap samples and $T^{W,b}$ stands for White's original bootstrap test statistic. If the null is true, the historical value of T^W should not be unusually large, given sampling error. Given the distribution of T^W, reject the null if its historical value, T^W, is greater than the 95th percentile of the distributions for T^W. At a 5 % level of significance, we reject the null if p_W is less than 0.05, but do not reject otherwise. Furthermore, we robust check these multiple comparison results using Chi-squared test (Gospodinov et al. (2012) [16]).

1.3 Description of the Candidate Models

We focus on linear asset pricing models given their popularity in the literature. However, how to select the set of candidate models seems to be beyond the scope of any econometric methods.

1.3.1 Return-Based Linear Factor Asset Pricing Models

CAPM: Sharpe (1964) and Lintner (1965) develop the Capital Asset Pricing Model (CAPM), in which the expected excess return on an asset equals the market risk θ of the asset times the expected excess return on market portfolio,

$$y_{t+1}^{CAPM} = \theta_0 + \theta R_{t+1}^{eM}, \tag{1.9}$$

where R_{t+1}^{eM} denotes excess returns on the market portfolios.

FF3: Fama and French (1992) [14] document the role of size and book/market ratio characteristics in the cross-section of expected stock returns,

$$y_{t+1}^{FF3} = \theta_0 + \theta_1 R_{t+1}^{eM} + \theta_2 SMB_{t+1} + \theta_3 HML_{t+1}, \tag{1.10}$$

where SMB denotes the size effect and HML is the book-to-market ratio effect.

FF5: Fama and French (1993) [1] state that the five-factor model can explain stocks and bonds better than the three-factor model,

$$y_{t+1}^{FF5} = \theta_0 + \theta_1 R_{t+1}^{eM} + \theta_2 SMB_{t+1} + \theta_3 HML_{t+1} + \theta_4 TERM_{t+1} + \theta_5 DEF_{t+1}, \quad (1.11)$$

where $TERM$ and DEF stand for the maturity risk and the default risk factors.

CCR5: Chen et al. (1986) [2] develop a macroeconomic factor model based on the Arbitrage Pricing Theory (APT) [23],

$$y_{t+1}^{CRR5} = \theta_0 + \theta_1 MP_{t+1} + \theta_2 UI_{t+1} + \theta_3 DEI_{t+1} + \theta_4 UTS_{t+1} + \theta_5 UPR_{t+1}, \quad (1.12)$$

where MP is the growth rate of industrial production, UI is the unexpected inflation, DEI is defined as the change in expected inflation, the term premium UTS, and UPR the default premium.

1.3.2 Consumption-Based Linear Factor Asset Pricing Models

C-CAPM: the consumption-based CAPM (Lucas (1978) and Breeden (1979)) states as

$$y_{t+1}^{CCAPM} = \theta_0 + \theta_1 c_{t+1}^{ndur}, \quad (1.13)$$

where c_{t+1}^{ndur} is the growth rate of non-durable consumption.

Yogo: the durable consumption CAPM of Yogo (2006) [3] is

$$y_{t+1}^{YOGO} = \theta_0 + \theta_1 R_{t+1}^{eM} + \theta_2 c_{t+1}^{ndur} + \theta_3 c_{t+1}^{dur}, \quad (1.14)$$

where R_{t+1}^{eM} is the excess returns on market portfolios and c_{t+1}^{dur} denotes the consumption growth rate of durable goods.

PST: the consumption-housing CAPM of Piazzesi et al. (2007) [24] states as

$$y_{t+1}^{PST} = \theta_0 + \theta_1 c_{t+1}^{nh} + \theta_2 s_{t+1}, \quad (1.15)$$

where c_{t+1}^{nh} is the growth rate of non-housing consumption and s_{t+1} denotes the log non-housing consumption expenditure share.

1.3.3 Linear Scaled Factor Asset Pricing Models

LL: the conditional consumption CAPM of Lettau and Ludvigson (2002) [4] shows that the consumption-wealth ratio can capture the time-varying risk premiums,

$$y_{t+1}^{LL} = \theta_0 + \theta_1 c_{t+1}^{ndur} + \theta_2 cay_t + \theta_3 c_{t+1}^{ndur} cay_t, \tag{1.16}$$

where c_{t+1}^{ndur} is the growth rate of non-durable consumption and cay_{t-1} is the consumption-wealth ratio.

SPST: the scaled consumption-housing CAPM of Piazzesi et al. (2007) [24] finds that while the non-housing expenditure ratio changes, the composition risk which relates changes in asset prices also changes,

$$y_t^{SPST} = \theta_0 + \theta_1 c_{t+1}^{ndur} + \theta_2 s_t + \theta_3 c_{t+1}^{ndur} s_t, \tag{1.17}$$

where s_t is the non-housing consumption expenditure share.

LVN: the scaled collateral-consumption CAPM of Lustig and Van Nieuwerburgh (2005) [11] shows the ratio of housing wealth to human wealth changes the conditional distribution of consumption growth across households in a model with collateralized borrowing and lending,

$$y_{t+1}^{LVN} = \theta_0 + \theta_1 c_{t+1}^{ndur} + \theta_2 my_t + \theta_3 c_{t+1}^{ndur} my_t, \tag{1.18}$$

where my_t is the housing collateral ratio.

SV: the scaled C-CAPM with the labor income of Santos and Veronesi (2006) [5] introduce the labor income to consumption ratio to be the conditional variable,

$$y_{t+1}^{SV} = \theta_0 + \theta_1 R_{t+1}^m + \theta_2 (R_{t+1}^m \cdot s_t^w) + \theta_3 R_{t+1}^W + \theta_4 (R_{t+1}^W \cdot s_t^w), \tag{1.19}$$

where R_{t+1}^m is the return on non-human, or financial wealth which is proxy by a market portfolio returns, R_{t+1}^W, is proxy by labor income growth, s_t^w denotes the ratio of labor income to consumption.

1.4 Preliminary Analysis of Data

1.4.1 Data Descriptions

For the financial data, Fama–French three factors and test portfolios, such as Fama–French 25 portfolios sorted by size and book-to-market ratio, 30 industry portfolios, and 10 deciles portfolios, are available on the Professor French's webpage. Seven different maturities U.S. government bonds are from "The CRSP U.S. Treasury Database".

For consumption-based asset pricing models, quarterly consumption data are from the National Income and Product Accounts (NIPA). The non-durable consumption in C-CAPM defines as the sum of real personal consumption expenditures on non-durable goods and services, including food, clothing and shoes, housing, utilities, transportation, and medical care. Yogo's (2006) durable-consumption consists of items such as motor vehicles, furniture and appliances, and jewelry and watches. Non-housing consumption, the consumption-housing CAPM of Piazzesi et al. (PST 2007), is measured by the non-durables consumption but excludes services such as shoes, clothing and housing. All consumption stocks are divided by population.

The factors in conditional models include: (i) the aggregate consumption-to-wealth ratio cay_t in Lettau and Ludvigsons (LL 2001) conditional C-CAPM (available on Ludvigson's website); (ii) the housing collateral ratio $mymo_t$ in Lustig and Van Nieuwerburgh's (LVN 2004) conditional C-CAPM; $mymo_t$ is computed by the ratio of collateralizable housing wealth to non-collateralizable human wealth, which are from the Historical Statistics for the US (Bureau of the Census) and the Flow of Funds data (Federal Board of Governors); (iii) the labor income-to-consumption ratio s_t^w in Santos and Veronesi's (SV 2004) conditional CAPM; labor income comes from the same database in Lettau and Ludvigson (2001), and (v) the non-housing consumption expenditure share s_t in Piazzesi et al. (SPST 2007) conditional C-CAPM; the expenditure share relies on per-period dollar expenditures on the item in NIPA.

1.4.2 Empirical Results on Return-Based Models

It is well documented in the literature that the CAPM fails to explain small growth portfolios[1]. Meanwhile, Lewellen, Nagel and Shanken (2010) [25] state that the Fama–French factor model is able to explain equity portfolios because it captures the characteristics on firms.

[1]see Merton (1973), Roll (1977), Banz (1981), Basu (1983), Reinganum (1981), Chan, Chen and Hsieh (1985), Bhandari (1988), Gibbons (1982), Shanken (1985), Fama and French (1992, 1993, 1995, 1996), and Jagannathan and McGrattan (1995).

From Table 1.1 to Table 1.3, we have ranked candidates in terms of the HJ, the modified, the unconstrained, and the constrained HJ distance measures. In every test portfolios section, the first row shows that model which obtains the largest normalized pricing errors; the second row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are first tested by the null hypothesis that the HJ distance is equal to zero, and then are tested by the null hypothesis the least misspecified candidate has the smallest normalized pricing errors via block bootstrapping.

For gross returns of all test portfolios, the Fama–French five-factor model obtains smaller HJ, modified HJ, unconstrained, and constrained distance measures than the other three models: its normalized pricing errors are the smallest and are robust among different estimating methods compared with the CAPM, the Fama–French three-factor, and the Chen, Roll, and Ross five-factor. On the other hand, the CAPM has the largest normalized pricing errors among the three sample test portfolios. Here, through the multiple comparison test, we cannot statistically reject the null hypothesis that the Fama–French five-factor has the smallest distance measure.

While explaining excess returns on assets, the Fama–French five-factor performs better than others again, but the Chen, Roll, and Ross five-factor macro model is not able to capture the industry effect (it is even worse than the traditional CAPM). Interestingly, the traditional HJ distance test is misleading in this case. For instance, the least and the most misspecified ones both have p-values larger than 5% for the test. This means that we cannot statistically reject the null hypothesis that both of their HJ distance measures are zero. However, after implementing multiple comparison tests, all bootstrapping p-values for the rankings are greater than 0.05: we cannot statistically reject the null hypothesis that the least misspecified model outperforms the others because of their relatively smaller normalized pricing errors.

Overall, the Fama–French five-factor model performs best in terms of the normalized pricing errors, compared with the other candidates. The result maintains among the four HJ distance measures. On the other hand, the traditional CAPM cannot successfully prices payoffs on cross-section assets; the macro-factor model, the Chen, Roll, and Ross five-factor, is not able to explain industry portfolios: its performance is even worse than CAPM.

1.4.3 Empirical Results on Consumption-Based Models

The consumption-based CAPM has been criticized by the literature for the low correlation between the consumption growth and equity returns. In this part, we treat the consumption-based CAPM as the benchmark model. The Chen, Roll, and Ross five-factor macro model is also included in order to compare a pure macro-factor model with macro-derived models (the Yogo non-durable and durable consumption and the Piazzesi, Schneider, and Tuzel housing consumption model).

According to tables from 1.4 to 1.6, the Chen, Roll, and Ross macro-factor model statistically dominates the others, getting the smallest normalized pricing errors when explaining payoffs on the 25 Fama–French size–value and the yields on U.S. government bonds; the consumption-based CAPM is able to explain these two portfolios better than the CAPM.

The Yogo non-durable and durable model outperforms the others in capturing the industry and size effects, except for the case when the Chen, Roll, and Ross five-factor model performs well in pricing the gross returns on 30 industry portfolios in terms of the traditional HJ distance. On the other hand, the consumption-based CAPM is not able to capture these effects, and even the CAPM performs relatively better. Again, rather than testing whether the HJ distance is equal to zero, we show that all p-values on multiple comparisons tests are statistically larger than 5%, therefore we cannot reject the null hypothesis that the Chen, Roll, and Ross five-factor, and the Yogo models have the smallest pricing errors, compared to the others when explaining excess and gross returns on specific assets.

Overall, the Chen, Roll and Ross macro-factor model outperforms others to explain returns on 25 Fama–French size-value stocks and yields on U.S. government bonds; the consumption-based CAPM outperforms the CAPM. Moreover, the Yogo non-durable and durable consumption model is the least misspecified model to capture industry and size effects.

1.4.4 Empirical Results on Scaled Consumption-Based Models

Here, the consumption-based CAPM and Chen, Roll and Ross macro-factor model are chosen as the benchmark.

From Table 1.7 to 1.9, all conditional C-CAPMs outperform the C-CAPM in explaining cross sectional payoffs, except for the Santos and Veronesi scaled C-CAPM with labor income in pricing payoffs to size portfolios. In particular, in explaining payoffs on 25 Fama–French size–value portfolios, the Chen, Roll and Ross macro-factor model has smaller normalized pricing errors than the others in terms of the unconstrained and the constrained HJ distance measures; the Santos and Veronesi conditional C-CAPM can perform well in pricing gross returns on 25 Fama–French size–value portfolios in terms of the HJ distance, and the scaled housing consumption-based model is able to explain excess returns well in terms of the modified HJ distance.

While capturing the industry effect, the Lettau and Ludvigson conditional CAPM obtains the smallest normalized pricing errors; the labor income scaled C-CAPM outperforms others in pricing its excess returns. On the other hand, the labor income scaled C-CAPM fails to explain the size effect, while the Lettau and Ludvigson conditional C-CAPM dominates others in getting the smallest pricing errors. A special case happens when the Chen, Roll, and Ross five-factor model is used to price gross returns on size portfolios: it performs best.

In U.S. government bond portfolios, the labor income scaled C-CAPM outperforms the others in explaining gross yields, while the scaled housing consumption and the macro-factor models are outstanding in explaining net yields in terms of the modified and the unconstrained HJ distance measures, respectively.

Overall, the pricing performances on conditional models is unstable, meanwhile the Chen, Roll and Ross macro-factor model outperforms to explain size–value stocks, gross returns on size portfolios and net yields on U.S. government bond portfolios. Through the multiple comparison test, the chosen models are the least misspecified ones to explain specific test portfolios.

1.4.5 Economic Interpretations

When all models are misspecified, the HJ distance measure gives the statistic criteria on the normalized pricing errors to explain asset returns. In this section, we have analyzed the economic reasons that why those models outperform others to explain specific test portfolios.

For most return-based models, factors are obtained directly from the financial market. For instance, size and book-to-market ratio factors are well-known to explain Fama–French size-value portfolios, hence, Fama and French three- and five- factors explain more than 90% of the time-series variation in portfolios' returns and more than 75% of the cross-sectional variation in their average returns. Given those features, it is reasonable for the Fama–French factor model to obtain a relative low value of HJ distance, because it is likely to produce betas that line up with expected returns; given the test portfolios, the Fama–French factor residuals of the size-value portfolios is tiny.

For consumption-based asset pricing models, the intuition on a 'successful' consumption factor is different from using financial factors to explain equity returns. Small stocks and value stocks deliver relatively low returns during recessions, which explain their high average returns relative to big stocks and growth stocks. When utility is nonseparable in non-durable and durable consumption (or housing consumption) and the elasticity of substitution between the two consumption goods is sufficiently high, marginal utility rises when durable consumption (or housing consumption) falls. Therefore stock returns are unexpectedly low at business cycle troughs, when durable consumption (or housing consumption) falls sharply, which explain the cross-sectional variation and the countercyclical variation in the equity premium. There is a little data difference between durable consumption in Yogo (2006) and housing consumption in Piazzesi et al. (2007): NIPA provides a direct measure of service flow for real estate, whereas it only reports expenditure on other durables.

For the conditional consumption-based models, the fact that scaled factor models have smaller HJ distances than non-scaled factor models comes from two sources. Mainly, the conditioning information reduces the pricing errors by allowing the prices of risks to vary with the business cycle. Then, by doubling the number of parameters, a scaled factor model uses additional degrees of freedom in the minimization problem and is better able to fit the data. This better fit may be spurious, though, as small sample biases may worsen. Another important issue is the stability of the model's parameters. If the conditional version is correctly specified and captures the dynamics in risk premiums, it will outperform the unconditional model. However, if the implied time-varying risk premiums are inherently misspecified because we choose the wrong conditioning variable, this false model may still appear to work well in small samples since it uses additional degrees of freedom. Ghysels (1998) finds that conditional models are fragile and may have bigger pricing errors than unconditional models. How to compare the conditional asset pricing models is worthy to investigating in future research.

1.5 Conclusions

Multi-factor linear asset pricing models play an important role in evaluating portfolio performances and cost-of-capital applications for practitioners. In this paper, we apply various HJ distance measures to understand which linear factor model outperforms to explain cross-sectional financial assets, and to seek an economic interpretation of the specifications that appears most promising.

We find that the Fama–French five-factor is ranked top in terms of misspecified measures in explaining the Fama–French size-value and these equities combined with seven government bond portfolios. When pricing returns on industrial-sorted assets, the Yogo durable consumption model performs better than other consumption-based models. For the conditional consumption-based models, the Lettau and Ludvigson (2001) outperforms others in explaining the gross returns on size portfolios, and Santos and Veronesi conditional CAPM with labor income behaves better to explain the excess returns. Through a multiple comparison test, we show all rankings maintain among gross and excess returns in terms of several distance measures. At last, we explain the economic reason why some models are least misspecified. Moreover, the SDFs of those least misspecified candidates are quite volatile and have clear financial market cycle patterns: some of them capture the periods of financial market crashes.

The paper also empirically investigates conditional asset pricing models while scaling risk factors as 'conditioning down' the dynamic pricing equation as (1.1). As Cochrane (2001) [26] emphasizes, the conditioning information of economic agents may not be observable, and one cannot omit it in making inferences about the behavior of conditional moments.

There are two solutions. One is to identify the conditional Euler equation, but the identification of the conditional mean in the Euler equation requires knowing the joint distribution of m_{t+1} and the set of test asset returns R_{t+1}.

Scaling factors is one way to incorporate conditioning information into the pricing kernel. Lettau and Ludvigson (2001) [10] therefore used the terms "scaling" and "conditioning" interchangeably when referring to models with scaled factors even though the models were estimated and tested on unconditional Euler equation moments. An unfortunate consequence may have been to create the case that scaled factor models for the conditional asset pricing models may have been mis-impression, since the conditional beta is always derived from conditional Euler equation moments (scaling returns), whether or not the pricing kernel includes scaled factors. We will work on this topic in future research.

Table 1.1: **Return-based Models via HJ and Modified HJ Distance**

Notes: The table reports the Hansen–Jagannathan (HJ) distance, modified HJ distance measures and their tests. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:			Traditional HJ Test	
Fama–French 25 Portfolios			*30 Industrial-sorted Portfolios*	
HJ	Modif. HJ		HJ	Modif. HJ
CAPM	CAPM		CAPM	CRR5
0.6591	0.6714		0.4086	0.4786
(0)	(0)		(0)	(0)
FF5	**FF5**		**FF5**	**FF5**
0.5329	0.5452		0.3755	0.437
(0)	(0)		(0)	(0)
10 Deciles Portfolios			*Fama–French 25 plus 7 Gov. Bonds*	
HJ	Modif. HJ		HJ	Modif. HJ
CAPM	CAPM		CAPM	CAPM
0.1406	0.1523		0.6949	0.7133
(0)	(0)		(0.29)	(0.43)
FF5	**FF5**		**FF5**	**FF5**
0.1372	0.1417		0.6034	0.615
(0)	(0)		(0)	(0)
Panel B:			Multiple Comparison	
H_0 :	*HJ (FF-25)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.608
H_0 :	*MHJ (FF-25)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.5608
H_0 :	*HJ (Ind-30)*	FF5 < FF3	< CRR5 < CAPM	p_{value}: 0.5704
H_0 :	*MHJ (Ind-30)*	FF5 < FF3	< CAPM < CRR5	p_{value}: 0.7194
H_0 :	*HJ (Dec-10)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.5782
H_0 :	*MHJ (Dec-10)*	FF5 < FF3	< CRR5 < CAPM	p_{value}: 0.6312
H_0 :	*HJ (Gov-Bond)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.5782
H_0 :	*MHJ (Gov-Bond)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.6312

Table 1.2: **Return-based Models via Unconstrained HJ Distance**

Notes: The table reports the unconstrained Hansen–Jagannathan (HJ) distance measure and its test for both gross and excess returns. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:		Traditional HJ Test	
Fama–French 25 Portfolios		*30 Industrial-sorted Portfolios*	
Gross	Excess	Gross	Excess
CAPM	CAPM	CAPM	CRR5
0.6591	0.5574	0.4086	0.4317
(0)	(0)	(0.511)	(0.28)
FF5	**FF5**	**FF5**	**FF5**
0.5329	0.4787	0.3755	0.4004
(0.2)	(0.247)	(0.51)	(0.44)
10 Deciles Portfolios		*Fama–French 25 plus 7 Gov. Bonds*	
Gross	Excess	Gross	Excess
CAPM	CAPM	CAPM	CAPM
0.1406	0.1505	0.6949	0.5807
(0.88)	(0.92)	(0.01)	(0)
FF5	**FF5**	**FF5**	**FF5**
0.1372	0.1403	0.6034	0.5239
(0.58)	(0.69)	(0.08)	(0.12)

Panel B:		Multiple Comparison		
H_0 :	*GR (FF-25)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.478
H_0 :	*ER (FF-25)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.4698
H_0 :	*GR (Ind-30)*	FF5 < FF3	< CRR5 < CAPM	p_{value}: 0.4766
H_0 :	*ER (Ind-30)*	FF5 < FF3	< CAPM < CRR5	p_{value}: 0.4802
H_0 :	*GR (Dec-10)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.5148
H_0 :	*ER (Dec-10)*	FF5 < FF3	< CRR5 < CAPM	p_{value}: 0.5218
H_0 :	*GR (Gov-Bond)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.449
H_0 :	*ER (Gov-Bond)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.4562

Table 1.3: **Return-based Models via Constrained HJ Distance**

Notes: The table reports the constrained Hansen–Jagannathan (HJ) distance measure and its test for both gross and excess returns. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:			Traditional HJ Test	
Fama–French 25 Portfolios			*30 Industrial-sorted Portfolios*	
Gross	Excess		Gross	Excess
CAPM	CAPM		CAPM	CAPM
0.674	0.607		0.4109	0.447
(0.0324)	(0)		(0.5207)	(0.476)
FF5	**FF5**		**FF5**	**FF5**
0.577	0.506		0.3769	0.3896
(0.1261)	(0.134)		(0.5593)	(0.467)
10 Deciles Portfolios			*Fama–French 25 plus 7 Gov. Bonds*	
Gross	Excess		Gross	Excess
CAPM	CAPM		CAPM	CAPM
0.1448	0.1505		0.726	0.698
(0.6036)	(0.567)		(0.0658)	(0)
FF5	**FF5**		**FF5**	**FF5**
0.1372	0.1407		0.6504	0.61
(0.5757)	(0.4)		(0.105)	(0.2)
Panel B:			Multiple Comparison	
H_0 :	*GR (FF-25)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.36
H_0 :	*ER (FF-25)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.4698
H_0 :	*GR (Ind-30)*	FF5 < FF3	< CRR5 < CAPM	p_{value}: 0.625
H_0 :	*ER (Ind-30)*	FF5 < FF3	< CRR5 < CAPM	p_{value}: 0.4802
H_0 :	*GR (Dec-10)*	FF5 < FF3	< CAPM < CRR5	p_{value}: 0.63
H_0 :	*ER (Dec-10)*	FF5 < FF3	< CAPM < CRR5	p_{value}: 0.5218
H_0 :	*GR (Gov-Bond)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.6
H_0 :	*ER (Gov-Bond)*	FF5 < CRR5	< FF3 < CAPM	p_{value}: 0.4562

Table 1.4: **Consumption-based Models via HJ and Mod. HJ Distance**

Notes: The table reports the Hansen–Jagannathan (HJ) distance, modified HJ distance measures and their tests on consumption-based asset pricing models. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:		Traditional HJ Test	
Fama–French 25 Portfolios		*30 Industrial-sorted Portfolios*	
HJ	Modif. HJ	HJ	Modif. HJ
CAPM	CAPM	CCAPM	CCAPM
0.6591	0.6714	0.4095	0.5085
(0)	(0)	(0)	(0.4)
CRR5	**CRR5**	**CRR5**	**Yogo**
0.6314	0.6453	0.3766	0.4364
(0)	(0)	(0)	(0)
10 Deciles Portfolios		*Fama–French 25 plus 7 Gov. Bonds*	
HJ	Modif. HJ	HJ	Modif. HJ
CCAPM	CCAPM	CAPM	CAPM
0.1617	0.1646	0.6949	0.7133
(0.55)	(0.62)	(0.29)	(0.43)
Yogo	**Yogo**	**CRR5**	**CRR5**
0.1263	0.1385	0.6706	0.6887
(0)	(0)	(0)	(0)

Panel B:		Multiple Comparison		
H_0 :	*HJ (FF-25)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.9156
H_0 :	*MHJ (FF-25)*	CRR5 < Piza <	Yogo < CCAPM < CAPM	p_{value}: 0.9148
H_0 :	*HJ (Ind-30)*	CRR5 < Yogo <	Piza < CAPM < CCAPM	p_{value}: 0.8514
H_0 :	*MHJ (Ind-30)*	Yogo < CRR5 <	CAPM < Piza < CCAPM	p_{value}: 0.9368
H_0 :	*HJ (Dec-10)*	Yogo < CRR5 <	CAPM < Piza < CCAPM	p_{value}: 0.8082
H_0 :	*MHJ (Dec-10)*	Yogo < CRR5 <	CAPM < Piza < CCAPM	p_{value}: 0.8376
H_0 :	*HJ (Gov-Bond)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.9024
H_0 :	*MHJ (Gov-Bond)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.9052

Table 1.5: **Consumption-based Models via Unconstrained HJ Distance**

Notes: The table reports the unconstrained Hansen–Jagannathan (HJ) distance measure and its test for both gross and excess returns for consumption-based asset pricing models. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:			Traditional HJ Test	
Fama–French 25 Portfolios			*30 Industrial-sorted Portfolios*	
Gross	Excess		Gross	Excess
CAPM	CAPM		CCAPM	CCAPM
0.6591	0.5574		0.4095	0.4533
(0)	(0)		(0.522)	(0.23)
CRR5	**CRR5**		**Yogo**	**Yogo**
0.5554	0.4944		0.3957	0.3999
(0.29)	(0.32)		(0.606)	(0.66)
10 Deciles Portfolios			*Fama–French 25 plus 7 Gov. Bonds*	
Gross	Excess		Gross	Excess
CCAPM	CCAPM		CAPM	CAPM
0.1617	0.1624		0.6949	0.5807
(0.81)	(0.94)		(0.01)	(0)
Yogo	**Yogo**		**CRR5**	**CRR5**
0.1263	0.1372		0.6294	0.5416
(0.79)	(0.88)		(0.07)	(0.1)
Panel B:			Multiple Comparison	
H_0 :	*GR (FF-25)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.4412
H_0 :	*ER (FF-25)*	CRR5 < Piza <	Yogo < CCAPM < CAPM	p_{value}: 0.299
H_0 :	*GR (Ind-30)*	Yogo < CRR5 <	Piza < CAPM < CCAPM	p_{value}: 0.6554
H_0 :	*ER (Ind-30)*	Yogo < CAPM <	CRR5 < Piza < CCAPM	p_{value}: 0.7358
H_0 :	*GR (Dec-10)*	Yogo < CRR5 <	CAPM < Piza < CCAPM	p_{value}: 0.7486
H_0 :	*ER (Dec-10)*	Yogo < CRR5 <	CAPM < Piza < CCAPM	p_{value}: 0.749
H_0 :	*GR (Gov-Bond)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.5018
H_0 :	*ER (Gov-Bond)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.3706

TABLE 1.6: **Consumption-based Models via Constrained HJ Distance**

Notes: The table reports the constrained Hansen–Jagannathan (HJ) distance measure and its test for both gross and excess returns for consumption-based asset pricing models. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:		Traditional HJ Test	
Fama–French 25 Portfolios		*30 Industrial-sorted Portfolios*	
Gross	Excess	Gross	Excess
CAPM	CAPM	CCAPM	CCAPM
0.674	0.56	0.4116	0.467
(0.0324)	(0.04)	(0.5126)	(0.678)
CRR5	**CRR5**	**Yogo**	**Yogo**
0.6192	0.54	0.3984	0.405
(0.0937)	(0.0708)	(0.5853)	(0.593)
10 Deciles Portfolios		*Fama–French 25 plus 7 Gov. Bonds*	
Gross	Excess	Gross	Excess
CCAPM	CAPM	CAPM	CCAPM
0.169	0.56	0.726	0.467
(0.7473)	(0.045)	(0.0658)	(0.04)
Yogo	**Yogo**	**CRR5**	**CRR5**
0.128	0.1376	0.684	0.62
(0.8)	(0.3)	(0.0687)	(0.051)

Panel B:		Multiple Comparison		
H_0 :	*GR (FF-25)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.545
H_0 :	*ER (FF-25)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.299
H_0 :	*GR (Ind-30)*	Yogo < CRR5 <	Piza < CAPM < CCAPM	p_{value}: 0.64
H_0 :	*ER (Ind-30)*	Yogo < CRR5 <	Piza < CAPM < CCAPM	p_{value}: 0.7358
H_0 :	*GR (Dec-10)*	Yogo < CAPM <	CRR5 < Piza < CCAPM	p_{value}: 0.75
H_0 :	*ER (Dec-10)*	Yogo < CAPM <	CRR5 < Piza < CCAPM	p_{value}: 0.749
H_0 :	*GR (Gov-Bond)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.5
H_0 :	*ER (Gov-Bond)*	CRR5 < Yogo <	Piza < CCAPM < CAPM	p_{value}: 0.3706

Table 1.7: **Cond. Consumption-based Models via HJ and Mod. HJ Dist.**

Notes: The table reports the Hansen–Jagannathan (HJ) distance, modified HJ distance measures and their tests on conditional consumption-based asset pricing models. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:			Traditional HJ Test	
Fama–French 25 Portfolios			*30 Industrial-sorted Portfolios*	
HJ	Modif. HJ		HJ	Modif. HJ
CCAPM	CCAPM		CCAPM	CCAPM
0.6502	0.6623		0.4095	0.5058
(0)	(0)		(0)	(0.4)
SV	**SPiza**		**LL**	**SV**
0.577	0.5983		0.346	0.3936
(0.177)	(0)		(0)	(0)
10 Deciles Portfolios			*Fama–French 25 plus 7 Gov. Bonds*	
HJ	Modif. HJ		HJ	Modif. HJ
SV	SV		CCAPM	CCAPM
0.1622	0.1969		0.6858	0.7075
(0.681)	(0.948)		(0.383)	(0.721)
LL	**LL**		**SV**	**SPiza**
0.1345	0.1402		0.6212	0.6473
(0.134)	(0.02)		(0.99)	(0)
Panel B:			Multiple Comparison	
H_0 :	*HJ (FF-25)*	SV < SPiza < LVN <	CRR5 < LL < CCAPM	p_{value}: 0.4368
H_0 :	*MHJ (FF-25)*	SPiza < SV < LVN <	CRR5 < LL < CCAPM	p_{value}: 0.2394
H_0 :	*HJ (Ind-30)*	LL < SV < CRR5 <	LVN < SPiza < CCAPM	p_{value}: 0.379
H_0 :	*MHJ (Ind-30)*	SV < LL < CRR5 <	LVN < SPiza < CCAPM	p_{value}: 0.6688
H_0 :	*HJ (Dec-10)*	LL < CRR5 < LVN <	SPiza < CCAPM < SV	p_{value}: 0.2738
H_0 :	*MHJ (Dec-10)*	LL < LVN < CRR5 <	SPiza < CCAPM < SV	p_{value}: 0.3564
H_0 :	*HJ (Gov-Bond)*	SV < SPiza < LVN <	CRR5 < LL < CCAPM	p_{value}: 0.4946
H_0 :	*MHJ (Gov-Bond)*	SPiza < SV < LVN <	CRR5 < LL < CCAPM	p_{value}: 0.2738

Table 1.8: **Cond. Consumption-based Models via Unconst. HJ Dist.**

Notes: The table reports the unconstrained Hansen–Jagannathan (HJ) distance measure and its test for both gross and excess returns for conditional consumption-based asset pricing models. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:			Traditional HJ Test	
Fama–French 25 Portfolios			*30 Industrial-sorted Portfolios*	
Gross	Excess		Gross	Excess
CCAPM	CCAPM		CCAPM	CCAPM
0.6502	0.5522		0.4095	0.4533
(0.014)	(0)		(0.522)	(0.23)
CRR5	**CRR5**		**LL**	**SV**
0.5554	0.4944		0.346	0.3663
(0.2926)	(0.315)		(0.851)	(0.856)
10 Deciles Portfolios			*Fama–French 25 plus 7 Gov. Bonds*	
Gross	Excess		Gross	Excess
SV	SV		CCAPM	CCAPM
0.1622	0.1932		0.6858	0.5776
(0.483)	(0.7)		(0.02)	(0.01)
LL	**LL**		**SV**	**CRR5**
0.1345	0.1388		0.6212	0.5416
(0.87)	(0.94)		(0.05)	(0.1)
Panel B:			Multiple Comparison	
H_0 :	*GR (FF-25)*	CRR5 < SV < SPiza <	LVN < LL < CCAPM	p_{value}: 0.5434
H_0 :	*ER (FF-25)*	CRR5 < SPiza < SV <	LVN < LL < CCAPM	p_{value}: 0.5516
H_0 :	*GR (Ind-30)*	LL < SV < CRR5 <	LVN < SPiza < CCAPM	p_{value}: 0.3982
H_0 :	*ER (Ind-30)*	SV < LL < CRR5 <	LVN < SPiza < CCAPM	p_{value}: 0.8678
H_0 :	*GR (Dec-10)*	LL < CRR5 < LVN <	SPiza < CCAPM < SV	p_{value}: 0.4624
H_0 :	*ER (Dec-10)*	LL < LVN < CRR5 <	SPiza < CCAPM < SV	p_{value}: 0.6484
H_0 :	*GR (Gov-Bond)*	SV < SPiza < CRR5 <	LVN < LL < CCAPM	p_{value}: 0.7398
H_0 :	*ER (Gov-Bond)*	CRR5 < SPiza < SV <	LVN < LL < CCAPM	p_{value}: 0.5596

Table 1.9: **Cond. Consumption-based models via Constrained HJ Dist.**

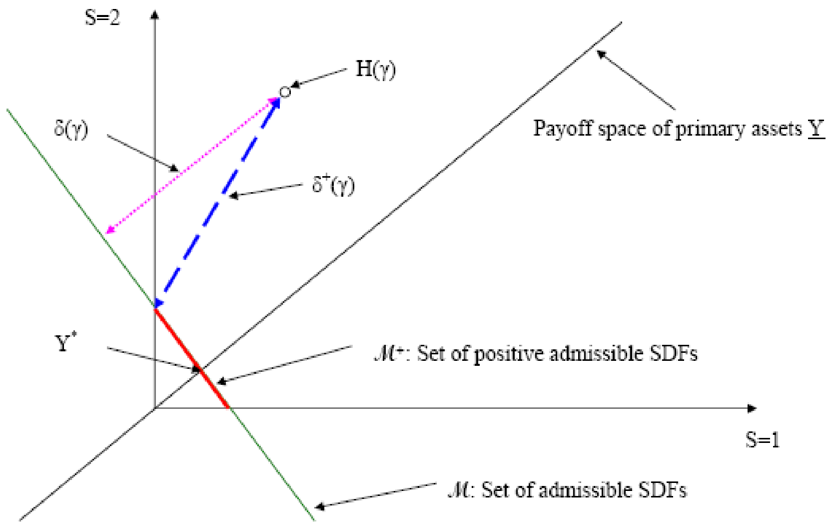
Notes: The table reports the constrained Hansen–Jagannathan (HJ) distance measure and its test for both gross and excess returns for conditional consumption-based asset pricing models. In Panel A, HJ distance measures and the traditional HJ test are shown. In every test portfolios section, the third row shows that model which obtains the largest normalized pricing errors; the sixth row gives the least misspecified (the one that gets the smallest normalized pricing errors). All distance measures are tested by the null hypothesis that the HJ distance is equal to zero, the p-values are reported below the distance measure, respectively. In Panel B, multiple comparison for all models are reported. The null hypothesis that the least misspecified one has the smallest distance measure is tested via block-bootstrapping 5000 times.

Panel A:		Traditional HJ Test	
Fama–French 25 Portfolios		*30 Industrial-sorted Portfolios*	
Gross	Excess	Gross	Excess
CCAPM	CCAPM	CCAPM	CCAPM
0.6688	0.5788	0.4116	0.4616
(0.0443)	(0.05)	(0.5126)	(0.6)
CRR5	**CRR5**	**LL**	**SV**
0.6192	0.5523	0.3648	0.3863
(0.0937)	(0)	(0.7223)	(0.70)
10 Deciles Portfolios		*Fama–French 25 plus 7 Gov. Bonds*	
Gross	Excess	Gross	Excess
CCAPM	CCAPM	CCAPM	CCAPM
0.169	0.1979	0.721	0.6163
(0.7473)	(0.6745)	(0.0677)	(0.586)
CRR5	**CRR5**	**SV**	**CRR5**
0.1448	0.1499	0.6688	0.628
(0.6036)	(0.70)	(0.0394)	(0.034)

Panel B:			Multiple Comparison	
H_0 :	*GR (FF-25)*	CRR5 < SV < SPiza <	LL < LVN < CCAPM	p_{value}: 0.445
H_0 :	*ER (FF-25)*	CRR5 < SV < SPiza <	LL < LVN < CCAPM	p_{value}: 0.5516
H_0 :	*GR (Ind-30)*	LL < SV < CRR5 <	LVN < SPiza < CCAPM	p_{value}: 0.315
H_0 :	*ER (Ind-30)*	LL < SV < CRR5 <	LVN < SPiza < CCAPM	p_{value}: 0.8678
H_0 :	*GR (Dec-10)*	CRR5 < SPiza < LL <	SV < LVN < CCAPM	p_{value}: 0.595
H_0 :	*ER (Dec-10)*	CRR5 < SPiza < LL <	SV < LVN < CCAPM	p_{value}: 0.6484
H_0 :	*GR (Gov-Bond)*	SV < SPiza < CRR5 <	LL < LVN < CCAPM	p_{value}: 0.7
H_0 :	*ER (Gov-Bond)*	SV < SPiza < CRR5 <	LL < LVN < CCAPM	p_{value}: 0.5596

FIGURE 1.1: **Hansen–Jagannathan Distance**

Notes: Figure shows that the Hasen–Jagannathan (HJ) distance is the least squared distance between any point along the admissible SDF line and the cross point between these two orthogonal lines (the payoffs line).
Here it assumes that there are only two states on nature, then the payoffs line is a combination of the payoff on each state. The circle point is the proposed stochastic discount factor (SDF), the purple line denotes the HJ (unconstrained) distance and the blue dash line shows the constrained HJ distance.



Chapter 2

Leisure, Consumption and Long Run Risk: An Empirical Evaluation

2.1 Introduction

This paper investigates the ability of a long-run risks asset pricing model, with non-separable leisure and consumption, to explain the cross-section of asset returns. Using two long-run factors estimated via a VAR, news about future consumption growth and news about leisure, we find that the model does well, relative to other consumption-based models, in explaining the risk-return profile of size, industry and value-growth portfolios.

We build on a rich prior literature starting with Eichenbaum et al. (1988) [6] that explores the empirical properties of intertemporal asset pricing models where the representative agent has utility over consumption and leisure. In this paper we use the framework in Uhlig (2007) [7], which allows for a stochastic discount factor with news about long-run growth in consumption and leisure. An important class of models, introduced by Bansal and Yaron (BY (2005) [27]) rely on long-run consumption growth factors to explain aggregate stock market behavior. The BY model requires a highly persistent consumption process. However, the persistence of such a predictable component in consumption growth is hard to measure in the data. In contrast, leisure data shows more persistence and is correlated with equity returns. As a result introducing non-separable leisure and consumption into the long run model leads to nontrivial interaction between consumption and leisure choice and heightens the volatility of the stochastic discount factor (SDF).

Our paper is related to recent work in Yang (2010) [8]. He defines leisure as the difference between a fixed time endowment and the observable hours spent on working, home production, schooling, communication, and personal care. Our work uses instead adjusted working hours as a measure of leisure. This allows us to avoid misleading results when applying a VAR[1]. In other work, Dittmar and Palomino (2010) [29] also investigate the role of labor income risk in the non-separable consumption-leisure model. This paper directly studies the leisure effect in the pricing kernel instead of the long-run relations between real wage, labor and consumption.

To evaluate our long-run model, we assess the performance with two measures competing against standard asset pricing models to explain size, industry and value-growth portfolios. Through cross-sectional regressions, we find that the long-run consumption-leisure model cannot be rejected by the J–statistics: it obtains zero pricing errors. Meanwhile the consumption-based CAPM, the Yogo durable consumption and Fama–French three-factor models do not achieve. We also rank the normalized pricing errors by Hansen–Jagannathan (HJ) distance misspecified measures: our model statistically obtains smaller HJ distance than other candidates.

Furthermore, we find that the equity premium arises, if assets are highly exposed to long-run consumption risk and long-run leisure risk. Intuitively, while the consumption growth falls, the leisure increases and decreases the price of long-run risks. Hence, compositing high long-run consumption and high long-run leisure with a negative sign, stocks will be asked for more compensation. To empirically test our model, we conduct time-series regressions, and evidence that more risky stocks obtain higher long-run consumption betas and higher negative long-run leisure betas.

In the rest of the present paper, we first evidence some stylized facts from the data in Section 2.2. Section 2.3 introduces the basic model and derives the SDF. Section 2.4 specifies the estimation method and describes the data. In Section 2.5, we show the empirical results. Section 2.6 concludes.

2.2 Stylized Facts

2.2.1 Leisure and Consumption

Figure 2.2 shows the average weekly leisure, which takes into account demographic and sectoral movements[2]. On average, the amount of leisure is about 44 hours per week

[1] Francis and Ramey (2009)[28] point that the demographic effect in working hours gives conflicting results on the effects of technology shocks on working hours via the VAR estimation

[2] The activities with the highest enjoyment scores (sex, playing sports, etc.) are those that one would generally classify as leisure (Figure 2.1 shows a survey reported in Robinson and Godbey (1999)).

and strongly counter-cyclical. Non-durable consumption and services is shown in Figure 2.3. Leisure growth exhibits large oscillations during the pre-WWII period, but stays relatively tranquil during the post-war period in Figure 2.4. The leisure growth stays low most of time, but it becomes large occasionally, especially after the financial crisis period of 2007. The volatility of leisure growth is about 27.96% (35.22% and 27.04% for demographic, productivity and Tornqvist adjusted measures).

Panel A in Table 2.1 reports the significant first order autocorrelations of leisure as 0.38. As in Bansal and Yaron (2005) [27], the variance ratio test is performed on both its log and realized volatility of innovations. Panel B reports the variance ratio test for leisure. If it is an i.i.d., then the ratio should be equal to 1, but the ratio for leisure is higher than unity implies that a positive autocorrelation dominates. Panel C investigates the time-varying volatility of leisure, since it shows that the adjusted variance ratios are all below unity, which, together with the decreasing variance ratios, provides evidence of a negative serial correlation in the realized volatility. Panel D explores the predictive relations between the realized growth and the price–dividend ratio. The results indicate that realized leisure growth in the future is predicted by the log price–dividend ratio,

$$\Delta_{l,t+j} = a_j + b_j(p_t - d_t) + \eta_{t+j}, \tag{2.1}$$

with negative slopes. Conversely, log price–dividend ratios in the future are also predicted by the realized leisure growth,

$$(p_{t+j} - d_{t+j}) = a_j + b_j\Delta_{l,t} + \eta_{t+j}, \tag{2.2}$$

also with negative slopes. Both sets of results are statistically strong.

Table 2.2 presents the empirical properties for quarterly per capita nondurable consumption and services. Panel A reports a significant first order autocorrelation of 0.28, which is close to that reported in Bansal and Yaron (2005) [27]. Notably, the autocorrelations for leisure and consumption exhibit patterns that are very similar both qualitatively and quantitatively. Panel B and Panel C show that the short run consumption is close to a random walk, but the volatility is time-varying when the time horizon increases. Panel D provides further evidence for the positive predictive relations between the realized consumption growth and the price–dividend ratio. These results are similar to those reported in Bansal et al. (2005) [30] for nondurable consumption growth.

2.2.2 Leisure, Consumption and Asset Returns

We firstly conduct a Granger causality test to examine the lead-lag relation between leisure or consumption and asset returns

$$r_t = c_1 + \alpha_1 r_{t-1} + \alpha_2 r_{t-2} + \beta_1 g_{i,t-1} + \beta_2 g_{i,t-2} + u_{r,t},$$

where r_t is the quarterly market excess return and $g_{i,t}$ is the quarterly leisure or consumption growth rate at date t. We then conduct an F test of the following null hypothesis: $H_0 : \beta_1 = \beta_2 = 0$. Similarly, we estimate,

$$g_{i,t} = c_2 + \gamma_1 g_{i,t-1} + \gamma_2 g_{i,t-2} + \eta_1 r_{t-1} + \eta_2 r_{t-2} + u_{i,t},$$

and then conduct an F test of the null hypothesis $H_0 : \eta_1 = \eta_2 = 0$. The $p-$value of the Granger causality test of consumption (leisure) Granger-causing returns is 0.2 (0.5), asset returns predict future consumption and leisure growths.

We then calculate the correlation between leisure and asset returns over different horizons in Figure 2.9, while the analysis of consumption is reported in Figure 2.8. For instance, the correlation between cumulative consumption growth and cumulative excess market returns,

$$corr(\sum_{j=1}^{k} g_{l,t+j}, \sum_{j=1}^{k} r_{t+j})$$

increases from -0.1 for $k = 1$ to -0.78 for $k = 20$ quarters. To further confirm the stronger correlation over the long run, we perform a band-pass filtering analysis (e.g., Baxter and King (1999) [31]). The band-pass filter is used to extract the low-frequency and high-frequency components of leisure (consumption) and asset returns. For leisure data, lower frequencies have the higher correlation with asset returns, the correlation is 0.42 for lower frequencies and 0.17 for higher frequencies (with cycles between 2 and 12 quarters) for quarterly data.

Because we focus on the long run (low frequency) relation between consumption, leisure and returns, the most convenient way to proceed is to use bivariate spectral analysis in the right part of Figure 2.8 and Figure 2.9. To be more specific, the coherence of the leisure or consumption growth rate and stock market returns at frequency λ measures the correlation between leisure or consumption and returns at frequency λ. When the frequency is λ, the corresponding length of the cycle is $1/\lambda$ quarters. To identify the sign of the correlation, the cospectrum needs to be examined. The cospectrum at frequency λ can be interpreted as the portion of the covariance between consumption or leisure growth and asset returns that is attributable to cycles with frequency λ. The slope of the phase spectrum at any frequency λ is the group delay at frequency λ and precisely measures the number of leads or lags between leisure or consumption and asset returns. When this slope is positive, leisure or consumption leads the market returns. The left panel in the right part of Figure 2.9 plots the results for the quarterly data, while the right panel for the annual data. The coherence between the quarterly leisure growth rate and quarterly excess market returns is much higher at low frequencies than at high frequencies. Therefore the comovement between leisure growth and asset market returns is much stronger at low frequencies.

2.3 The Model

2.3.1 Epstein–Zin Preferences with Leisure

The representative household has preference such as the Epstein-Zin (E–Z) utility function

$$V_t = U(C_t, \Phi_t L_t) + \beta \left[E_t[(V_{t+1})^{1-\upsilon}]\right]^{1/(1-\upsilon)}, \tag{2.3}$$

where C_t presents the aggregate consumption, L_t denotes the aggregate leisure and Φ_t is the long run trend at time t. β is the subjective discount rate, υ denotes the curvature of the E–Z utility, $\upsilon \in \Re$, $\upsilon \neq 1$[3].

To specify the utility, the intra-temporal utility U_t is given by

$$U_t = \frac{(C_t^{\alpha} \cdot (\Phi_t L_t)^{1-\alpha})^{1-\eta}}{1-\eta}, \tag{2.4}$$

where α is the expenditure share of consumption, for instance, a 1% drop in consumption results in an α% drop in the consumption-leisure composite. η denotes the risk aversion on the temporal utility.

Hence, the preference function becomes

$$V_t = \frac{(C_t^{\alpha} \cdot (\Phi_t L_t)^{1-\alpha})^{1-\eta}}{1-\eta} + \beta \left[E_t[(V_{t+1})^{1-\upsilon}]\right]^{1/(1-\upsilon)}. \tag{2.5}$$

The expected $V(.)$ function is twisted by the coefficient $1-\upsilon$. When $\upsilon = 0$, the preference given by the above reduces to special case of expected utility. The main advantage of

[3] Note that, traditionally, E–Z preferences over consumption and leisure streams have been written as

$$V_t = [U(C_t, L_t)^{1-\rho} + \beta(E_t V_{t+1}^{1-\tilde{\upsilon}})^{\frac{1-\rho}{1-\tilde{\upsilon}}}]^{\frac{1}{1-\rho}}$$

but by setting $\widetilde{V} = V^{1-\rho}$ and $1-\upsilon = \frac{1-\tilde{\upsilon}}{1-\rho}$.

the recursive preference is that it allows for greater flexibility in modeling risk aversion and the intertemporal elasticity of substitution. In the recursive preference above, the intertemporal elasticity of substitution over deterministic consumption paths is exactly the same as in the expected utility, but the household's risk aversion with respect to gambles can be amplified (or attenuated) by the additional parameter υ. Importantly, the intra-temporal Cobb–Douglas (C–D) form can be treated as a normalization of the preference function; shifting the intercept of the felicity function, i.e., $U(.)$, will not affect economic choices[4]. Furthermore, the C–D form ensures the felicity function $U(C, \Phi L)$ does not depend on Φ for (C, L) locally around (C, L, Φ) up to a second-order approximation[5].

Two cases are distinguished: treating consumption and leisure as a single composite good, or not. The household's risk aversion to gambles can be amplified (or attenuated) by the additional parameter υ as $\eta + \upsilon(1-\eta)$ (consumption and leisure as a composite good). If non-separable consumption and leisure (not a single composite commodity), the consumption-only coefficient of relative risk aversion (RRA) and intertemporal elasticity of substitution (IES) will depend on α as $\eta\alpha + \upsilon(1-\eta)\alpha$ and the inverse of $1 - \alpha(1-\eta)$ [6].

To proceed towards asset pricing, the budget constraint states

$$C_t + S_t = R_t S_{t-1} + W_t N_t, \tag{2.6}$$

where S_t is the wealth invested in some asset with a gross return (measured in consumption units) of R_t from period $t-1$ to t, W_t is the real wage and N_t presents the labor supply. The standard pricing formula or the stochastic discount factor (SDF) can be computed by the Euler equation with respect to consumption,

$$\Lambda_t = \beta E_t[\Lambda_{t+1} R_{t+1}], \tag{2.7}$$

where Λ_t is the Lagrange multiplier on the budget constraint.

This paper assumes log consumption is trend-stationary, e.g., the log consumption is equal to a linear trend $K \cdot t$ and an AR(1) process $\epsilon_{t+1} + \rho\epsilon_t$, where $|\rho| < 1$. It is more

[4] see Uhlig (2007) [7] Assumption 3.

[5] Based on C–D form and [7] Proposition 2, $\chi = \eta/(1-\eta)\bar{U}$.

[6] Swanson (2012) [32] has proofed that the risk aversion of the recursive preference is given by

$$R^S(S;\theta) = \frac{-V_{11}(S;\theta)}{V_1(S;\theta)} + \upsilon \frac{V_1(S;\theta)}{V(S;\theta)},$$

where S denotes the household's beginning-of-period assets in her budget constraint, θ is exogenous to the household and V stands for the value function.

convenient to restate the investment problem in terms of the detrended variables

$$\Gamma_t = \frac{\Phi_t}{\Phi_{t-1}}, \tag{2.8}$$

$$\widetilde{V}_t = \frac{V_t}{\Phi_t}, \tag{2.9}$$

$$\widetilde{C}_t = \frac{C_t}{\Phi_t}, \tag{2.10}$$

$$\widetilde{S}_t = \frac{S_t}{\Phi_t}. \tag{2.11}$$

The maximization of the problem can be rewritten as

$$max V_0, \tag{2.12}$$

subject to

$$\widetilde{V}_t = U(\widetilde{C}_t, L_t) + \widetilde{\beta}\left[E_t[(\Gamma_{t+1}/\bar{\Gamma})^{1-\upsilon}(\widetilde{V}_{t+1})^{1-\upsilon}]\right]^{1/(1-\upsilon)},$$

$$\widetilde{C}_t + \widetilde{S}_t = \frac{R_t}{\Gamma_t}\widetilde{S}_{t-1} + W_t N_t,$$

where $\widetilde{\beta}$ stands for $\beta\bar{\Gamma}^{1-\eta}$, $\widetilde{\Omega}_t$ denotes the Lagrange multiplier for the detrended $V(.)$ function constraint, and $\widetilde{\Lambda}_t$ is the Lagrange multiplier for the detrended budget constraint. The first-order conditions are

$$\frac{\partial}{\partial \widetilde{V}_t} : \widetilde{\Omega}_t = \widetilde{\Omega}_{t-1}\left[\frac{E_{t-1}[(\frac{\Gamma_t}{\bar{\Gamma}})^{1-\upsilon}(\widetilde{V}_t^{1-\upsilon})]^{\frac{1}{1-\upsilon}}}{\widetilde{V}_t}\right]^{\upsilon}\left(\frac{\Gamma_t}{\bar{\Gamma}}\right)^{1-\upsilon}, \tag{2.13}$$

$$\frac{\partial}{\partial \widetilde{C}_t} : \widetilde{\Lambda}_t = (1-\widetilde{\beta})\widetilde{\Omega}_t U_1(\widetilde{C}_t, L_t), \tag{2.14}$$

$$\frac{\partial}{\partial \widetilde{S}_t} : \widetilde{\Lambda}_t = \widetilde{\beta} E_t\left[\frac{\widetilde{\Lambda}_{t+1}}{\Gamma_{t+1}} R_{t+1}\right]. \tag{2.15}$$

2.3.2 Log-linearizing the First Order Conditions

We demonstrate the mechanisms working in the model via approximate analytical solutions. Small letters are used to denote the log-linear deviation of a variable from its steady state, where $c = log(\widetilde{C}) - log(\bar{\widetilde{C}})$, $l = log(L) - log(\bar{L})$, $\zeta = log(\Gamma) - log(\bar{\Gamma})$ and $u = log(U(\widetilde{C}, L)) - log(\bar{\widetilde{U}})$.

The second order Taylor expansion of the felicity function $U(.)$ yields

$$u \approx c + \kappa \cdot l - \frac{1}{2}\eta_{cc} \cdot c^2 + (\eta_{cl,l} - \kappa) \cdot c \cdot l - \frac{1}{2} \cdot \kappa \cdot (\eta_{ll} + 1 - \kappa) \cdot l^2. \tag{2.16}$$

To provide some further intuition on κ, consider a stochastic neoclassical growth model such as C–D production function, where wages times labor equals the share of labor times output[7]. The usual first order condition with respect to leisure then shows κ to be the ratio of the expenditure shares for consumption to leisure.

The $V(.)$ function will be log-linearized to

$$v_t = c_t + \kappa l_t + \widetilde{\beta} E_t \left[\frac{\upsilon - \eta}{\psi} \zeta_{t+1} + v_{t+1} \right], \tag{2.17}$$

where $\psi = \frac{\upsilon-\eta}{1-\eta}\frac{\bar{V}}{\bar{V}+\chi}$ measuring the degree of curvature in departing from the benchmark expected discounted utility framework, and $\chi = \frac{\eta}{1-\eta}\bar{U}$. Note that $\psi = \upsilon - \eta$ in the steady state path. The equation shows that v_t can be related back to the observables, i.e., to c_t, l_t, as well as ζ_t.

Equations (2.13), (2.14) and (2.15) log-linearize to

$$\varpi_t - \varpi_{t-1} = -\psi(v_t - E_{t-1}[v_t]) + (1-\upsilon)(\zeta_t - E_{t-1}[\zeta_t]) + (1-\eta)E_{t-1}[\zeta_t], \tag{2.18}$$

$$\lambda_t - \varpi_t = 1 - \eta_{cc} c_t + (\eta_{cl,l} - \kappa) l_t, \tag{2.19}$$

$$0 = E_t[\lambda_{t+1} - \lambda_t + r_{t+1} - \zeta_{t+1}], \tag{2.20}$$

where ϖ denotes the log Lagrange multiplier for the detrended $V(.)$ function constraint, and λ stands for the log Lagrange multiplier for the detrended budget constraint.

2.3.3 The Stochastic Discount Factor (SDF)

The corresponding SDF can be written as

$$m_{t,t+1} = \lambda_{t+1} - \lambda_t - \zeta_{t+1}, \tag{2.21}$$

[7] Introduce

$$\eta_{cc} = -\frac{U_{CC}(\bar{C},\bar{L})\bar{C}}{U_C(\bar{C},\bar{L})}, \eta_{ll} = -\frac{U_{LL}(\bar{C},\bar{L})\bar{L}}{U_L(\bar{C},\bar{L})},$$
$$\eta_{cl,c} = \frac{U_{CL}(\bar{C},\bar{L})\bar{C}}{U_L(\bar{C},\bar{L})}, \eta_{cl,l} = \frac{U_{CL}(\bar{C},\bar{L})\bar{L}}{U_C(\bar{C},\bar{L})}.$$
$$\kappa = \frac{U_L \bar{L}}{U_C \bar{C}} = \frac{\eta_{cl,l}}{\eta_{cl,c}}.$$

.

for the log-deviation of the SDF

$$M_{t,t+1} = \widetilde{\beta}\frac{\widetilde{\Lambda}_{t+1}}{\widetilde{\Lambda}_t\Gamma_{t+1}}. \tag{2.22}$$

Combining equations from (2.16) to (2.20) derives the SDF[8]. The log SDF relates the pricing kernel to macroeconomics variables as following:

$$\begin{aligned} m_{t,t+1} = & -\eta_{cc}\cdot(c_{t+1}-c_t) + (\eta_{cl,l}-\kappa)\cdot(l_{t+1}-l_t) \\ & - \upsilon\cdot(\zeta_{t+1} - (1+\frac{\eta}{\upsilon})\cdot E_t[\zeta_{t+1}]) \\ & - \psi\cdot[\sum_i^\infty (E_{t+1}-E_t)\widetilde{\beta}^i c_{t+i}] \\ & - \psi\cdot[\sum_i^\infty (E_{t+1}-E_t)\widetilde{\beta}^i \kappa l_{t+i}] \\ & - (\upsilon-\eta)\cdot[\sum_i^\infty (E_{t+1}-E_t)\widetilde{\beta}^i \zeta_{t+i}]. \end{aligned} \tag{2.23}$$

The SDF depends on the framework of temporal utility, i.e., η_{cc} the risk aversion with respect to consumption[9], $\eta_{cl,l}$ the preference parameter denoting the non-separable characteristics in consumption and leisure, κ the ratio of the expenditure shares for consumption and leisure, η the risk aversion of the temporal utility, and the curvature of the E–Z recursive utility function υ. Note that, when coming to the expected utility ($\psi = 0$ and $\upsilon = \eta$), the SDF will be deduced from the function of second moments of the utility function and the shock scaled by the risk aversion as

$$m_{t,t+1} = \eta_{cc}\cdot(c_{t+1}-c_t) + (\eta_{cl,l}-\kappa)\cdot(l_{t+1}-l_t) - \eta\cdot(\zeta_{t+1}-\zeta_t), \tag{2.24}$$

here we assume that ζ can be predictable.

If consumption and leisure are not non-separable, i.e., $\eta_{cl,l} = 0$ and $\kappa = 0$, then the SDF can be rewritten as

$$m_{t,t+1} = \eta_{cc}\cdot(c_{t+1}-c_t) - \eta\cdot(\zeta_{t+1}-\zeta_t), \tag{2.25}$$

where the expected trend $\zeta_{t+1} - \zeta_t$ is scaled by η in Abel (1990) [33][10]. Thus, given the parameters of the recursive utility function, the SDF can be explained by six macro

[8] The details on algebra are shown in the Appendix.

[9] Due to the E–Z formulation, the role for η_{cc} will be the characterization of intertemporal substitution, rather than risk aversion.

[10] Provided $log\Phi$ is cointegrated with the log of total factor productivity.

variables: news on consumption, leisure, the trend, and their corresponding short-term growth rates.

2.3.4 The Return–Risk in the Model

Let r_t be the log gross return of any asset $r_t = log(1+R_t)$ and r_t^f be the log risk-free rate at time t. We assume that conditionally on information at date t, $m_{t,t+1}$ and r_{t+1} are jointly log-normally distributed, conditional on information up to and including t. The asset pricing formula (2.7) can be rewritten as

$$0 = log(\bar{M}\bar{R}) + E_t[m_{t,t+1}] + E_t[r_{t+1}] + \frac{1}{2}(\sigma_{m,t}^2 + \sigma_{r,t}^2 + 2\rho_{m,r,t}\sigma_{m,t}\sigma_{r,t}). \tag{2.26}$$

For the risk-free rate, i.e., for an asset with $\sigma_r^2 = 0$,

$$r_t^f = -log(\bar{M}) - E_t[m_{t,t+1}] - \frac{1}{2}\sigma_{m,t}^2. \tag{2.27}$$

To specify the SDF as factors

$$r_t^f = -log(\bar{M}) + \eta_{cc} \cdot E_t[c_{t+1} - c_t] - (\eta_{cl,l} - \kappa) \cdot E_t[l_{t+1} - l_t] + \eta E_t[\zeta_{t+1}] - \frac{1}{2}\sigma_{m,t}^2. \tag{2.28}$$

Furthermore, the risk premium on any asset is stated as

$$\begin{aligned} E_t[r_{t+1}] - r_{t+1}^f + \frac{\sigma_t^2}{2} - \frac{r_t^f}{2} = {} & \eta_{cc} \cdot Cov_t(r_{t+1}, \Delta c_{t+1}) \\ & - (\eta_{cl,l} - \kappa) \cdot Cov_t(r_{t+1}, \Delta l_{t+1}) \\ & + \eta \cdot Cov_t(r_{t+1}, \Delta\zeta_{t+1}) \\ & + \psi \cdot Cov_t(r_{t+1}, \sum_{j=0}^{\infty} \widetilde{\beta}^j c_{t+1+j}) \\ & + \psi \cdot \kappa \cdot Cov_t(r_{t+1}, \sum_{j=0}^{\infty} \widetilde{\beta}^j l_{t+1+j}) \\ & + (\upsilon - \eta) \cdot Cov_t(r_{t+1}, \sum_{j=0}^{\infty} \widetilde{\beta}^j \zeta_{t+1+j}). \end{aligned} \tag{2.29}$$

To explain (2.29), we rewrite the equation into risk and risk-price representation (beta-representation)[11].

$$\begin{aligned} E_t[r_{t+1} - r_{t+1}^f] + \frac{\sigma_t^2}{2} - \frac{r_t^f}{2} &= \eta_{cc} \cdot \sigma_c^2 \cdot \beta_c \\ &- (\eta_{cl,l} - \kappa) \cdot \sigma_l^2 \cdot \beta_l \\ &+ \eta \cdot \sigma_\zeta^2 \cdot \beta_\zeta \\ &+ \psi \cdot \sigma_{\epsilon_c}^2 \cdot \beta_{\epsilon_c} \\ &+ \psi \cdot \kappa \cdot \sigma_{\epsilon_l}^2 \cdot \beta_{\epsilon_l} \\ &+ (\upsilon - \eta) \cdot \sigma_{\epsilon_\zeta}^2 \cdot \beta_{\epsilon_\zeta}. \end{aligned} \tag{2.30}$$

Long-run risks on consumption and leisure arise because investors care about future leisure and future consumption growth. The expected future variable will affect the investors' current behavior through inter-temporal consumption smoothing. That's why the inter-temporal elasticity of substitution (κ and η) enters the price of risk of the risk factors. Moreover, the preference parameters (η_{cc}, $\eta_{cl,l}$ and κ) denoting the non-separability between consumption and leisure also goes into the risk price for current and long-run leisure terms.

2.4 Test Methodology

2.4.1 State of the Economy

The log-linearized equilibrium of the economy can be expressed in a state space form as follows:

$$X_t = GX_{t-1} + Hu_t, \tag{2.31}$$

where X_t is a $n \times 1$ vector of a constant state variable and time varying state variables, and u_t is an $m \times 1$ vector of innovations to economic shocks and measurement errors.

$$Y_t = UX_t + Vu_t, \tag{2.32}$$

where Y_t is a $k \times 1$ vector of observable variables and ϵ_t is a vector of measurement errors. This vector typically includes growth rates of non-cointegrated $I(1)$ observable variables,

[11] Define

$$\beta_a = \frac{Cov(a_{t+1}, r_{t+1})}{\sigma_a^2}$$

as the risk loadings for factors therefore the risk price will be $parameters \cdot \sigma_a^2$, and $\frac{\sigma_t^2}{2} - \frac{r_t^f}{2}$ is a term adjusting for the 'Jensen effect'.

error correction variables from cointegrating relationships, and $I(0)$ observable variables. The matrix G has eigenvalues less than one in absolute values except for a single unit eigenvalue associated with a constant state variable. Let $E_t[.] = E[.|\{X_{t-s}, Y_{t-s}\}_{s=0}^{\infty}]$ be the conditional expectation operator for the agents in the economy, and $E_{Y,t}[.] = E[.|\{Y_{t-s}\}_{s=0}^{\infty}]$ be the conditional expectation operator for the econometrician who only observes the present and past realizations of the observables. I assume that u_t and ϵ_t satisfy $E_{t-1}u_t$, $E_{t-1}u_t u_t^{'} = I$, $E_{t-1}u_{t+i}u_{t+i}^{'} = 0$ for $i \neq j$.

The news on observables at time $t+s$ to the shocks at time t can be calculated by the equation as follows:

$$E_t[Y_{t+s}] - E_{t-1}[Y_{t+s}] = \begin{cases} Vu_t & for \quad s = 0, \\ UG^{s-1}Hu_t & for \quad s \geq 1. \end{cases} \tag{2.33}$$

Suppose that consumption growth, $\Delta c_t = log(C_t) - log(C_{t-1})$, is g-th variable in Y. Then, the news on consumption growth is

$$E_t[\Delta c_{t+s}] - E_{t-1}[\Delta c_{t+s}] = \begin{cases} e_g^{'}Vu_t & for \quad s = 0, \\ e_g^{'}UG^{s-1}Hu_t & for \quad s \geq 1, \end{cases} \tag{2.34}$$

where e_g is the $k \times 1$ selection vector with one in the g-th place and zeros elsewhere.

Here X_t contains in particular the log of the ratio of consumption to long run trend growth (c_t/Φ_t), the log leisure l_t, the log of the de-meaned growth of the long-run trend $\zeta_t = log(\Phi_t) - log(\Phi_{t-1})$, and the excess returns $r_t - r_t^f$ as the first, the second, the third, and the forth variables. We allow for heteroskedasticity in the innovations but not in the VAR coefficients. The news about consumption, leisure and the long-run trend are now given by (for $s \geq 1$)

$$E_t[\Delta c_{t+s}] - E_{t-1}[\Delta c_{t+s}] = e_1^{'}UG^sHu_t, \tag{2.35}$$

$$E_t[l_{t+s}] - E_{t-1}[l_{t+s}] = e_2^{'}UG^sHu_t, \tag{2.36}$$

$$E_t[\zeta_{t+s}] - E_{t-1}[\zeta_{t+s}] = e_3^{'}UG^sHu_t, \tag{2.37}$$

2.4.2 News Shock Identification

The result from the equation (2.30) shows that expected log excess returns of an asset depend on the asset's betas with economic shocks. We assume that the state space forms (2.31) and (2.32) are invertible. When a state space form is invertible, the state variables can be expressed as a weighted sum of the current and past realizations of observables, and an economic shock can be expressed as a linear combination of the VAR innovations of the observables. The identification of these shock components and the resulting asset-pricing implications critically depends on two features in the model, the multivariate structure of predictability in all state variables and the recursive utility form.

Hence, the set of information variables need to have predictive power beyond that of lagged growth on consumption, leisure and the trend in the VAR system[12]. These three predictor variables can be motivated as follows. First, the per capita consumption tracks the business cycle, and there are a number of reasons why expected returns on the stock market could co-vary with the business cycle. Second, leisure can be motivated by the dynamic model itself and its data characteristics. Third, in order to investigate the role of the long-run trend in the data, we proceed somewhat artificially as follows. Constructing ζ_t as the transformation of the growth rate on past aggregate consumption

$$\zeta_t = \rho_\zeta \zeta_{t-1} + (1 - \rho_\zeta)(logC_t - logC_{t-1}) \tag{2.38}$$

with $\zeta_1 = (logC_1 - logC_0)$. Find ρ_ζ such that

$$\Delta c_t = logC_t - logC_{t-1} - \zeta_t \tag{2.39}$$

has zero autocorrelation: $\rho_\zeta = 0.71$. We also try alternatives for this trend, per $\rho_\zeta = 0.9$ and $\rho_\zeta = 0.5$. Here ζ_t can be interpreted as a medium frequency filter to consumption; consumers care about the lagged value of aggregate consumption which differs from habit formation in that it is independent of an individual consumer's own consumption.

We also put excess returns as elements in VAR (Hansen et al. (2008) [35] and Malloy et al. (2009) [36]). We choose Fama–French 25 size- and book-to-market- portfolios as test assets, then the VAR differs across test assets to avoid spurious correlation between a given test asset and innovations. Because our VARs differ across test assets, therefore the conditional expectations for consumption, leisure and the trend are different depending on the excess return of interest so that we do not obtain a consistent model for dynamics across the test assets. There is no obvious bias in this procedure, but the varying variable dynamics across test assets are somewhat unappealing. To address the impact of estimating a separate VAR for each test asset, we repeat the approach above and estimate a 'mean' VAR in order to see the effects on estimating risks and prices of risks.

[12] Barsky and Sims (2011) [34] propose the identification methodology of two technology shocks in a structural VAR analysis. They put the first variable as the measure of technology.

Now the SDF can be rewritten as the representation of the infinite order VAR innovations of Y_t spanning the space of economic shocks

$$m_{t,t+1} = \vec{a}^{'} u_{t+1} - \vec{b}^{'} U(I-G)^{-1} H u_{t+1}, \tag{2.40}$$

where the vectors $\vec{a}, \vec{b}$ and, additionally, the vector $\vec{e}_4$, are defined by

$$\vec{a} = \begin{pmatrix} -\eta_{cc} \\ \eta_{cl,l} - \kappa \\ -\eta \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}, \vec{b} = \begin{pmatrix} \psi(1-\beta) \\ \psi(1-\beta)\kappa \\ \upsilon - \eta \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}, \vec{e}_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}. \tag{2.41}$$

Equation (2.40) is similar as the equation (3) in Hansen et al. (2008) [35]. In their paper, the SDF is written as

$$m_{t,t+1} = \mu_m + U_m X_t + \vec{b}^{'} (I - \beta G)^{-1} u_{t+1}, \tag{2.42}$$

where the vectors $\vec{b}$ is defined as

$$\vec{b} = \begin{pmatrix} \psi(1-\beta) \\ \psi(1-\beta)\kappa \\ \upsilon - \eta \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}. \tag{2.43}$$

Here we can rewrite $\vec{b}^{'} (I - \beta G)^{-1}$ as $\lambda(\beta)$. The vector $\lambda(\beta)$ is the discounted impulse response of consumption to each of the respective components of the standardized shock vector u_{t+1}. As emphasized by Bansal and Yaron (2005) [27], the contribution of the discounted response to the stochastic discount factor makes consumption predictability a potentially potent way to enlarge risk prices, even over short horizons. Further the term $\gamma\lambda(\beta)$ captures the "bad beta" of Campbell and Vuolteenaho (2004) [37] except that they measure shocks using the market return instead of aggregate consumption.

In this paper, $\lambda(\beta)$ reflects the discounted impulse response of consumption, leisure and the trend to the standardized shocks. Since the consumption and leisure reacts oppositely to the shock, therefore both of them decreases risk prices if the long run leisure risk is predominated. Otherwise, the discounted response to the stochastic discount factor enlarges risk prices if the long run consumption risk is predominated. Hence, this flexible feature helps to explain the cross-sectional equity premiums. Need to mention, this feature also creates an important measurement challenge in implementation. Under the alternative interpretation suggested by Anderson et al. (2003) [38], $\vec{b}'\lambda(\beta)$ is the contribution to the induced prices because investors cannot identify potential model misspecification that is disguised by shocks. In considering how big the concern is about model misspecification, we ask that it could be ruled out with historical carefully accounted leisure data and a misspecificied measure test (Hansen–Jagannathan distance).

2.4.3 Data Description

The quarterly sample of 1948–2011 is used after intersecting the data on leisure, nondurable consumption, and asset markets.

2.4.3.1 Asset Returns

We use three test assets, Fama–French 25 size- and book-to-market sorted portfolios, 25 size- and momentum portfolios, and 30 industry portfolios on the left-hand side of the unconditional first order condition, the equation (2.30). We use the three-month T-bill rate from FRED over the period January 1948 to December 2011 as the riskless rate of interest.

2.4.3.2 Consumption

The real level series for nondurables and services is obtained by accumulating the log growth rates, and taking the exponent of the resulting series with the first value of the series normalized to one. The series is then rescaled in the base year (2005). To generate the per capita series, we use a filtered version of the quarterly population series. To deal with the seasonality issue, the population series we use is the exponent of the Hodrick–Prescott trend of the logarithm of the original population series.

2.4.3.3 Leisure

To calculate leisure data, we collect working, schooling and home production, communication and personal care hours, respectively.

Time spent working includes paid hours in the private sector, as well as hours worked for the government (either voluntarily or involuntarily) and unpaid family labor[13].

For school hours, we base the calculation on information from the Digest of Education Statistics (DES) and Historical Statistics for college enrollment data from 1940 on. Hours per day attended for secondary school are based on time-use studies and AHTUS and 2003 BLS ATUS. The hours spent by college students are taken from the time-diary studies from Babcock and Marks (2010) [39].

Home production data comes from AHTUS 1965, 1975, 1985 and BLS 2003–2010. Commuting time is also considered: in the absence of firm evidence, we assume that commute times were 10 percent of total hours worked, equal to the average during the last 40 years of the sample. The time-use studies of high school and college students from the late 1920s to the present all suggest school commute times approximately equal to 10 percent of total time spent on school and homework. Thus we use a constant 10 percent commute time for this activity as well. For personal care time, we subtract 77 hours from the time endowment. This number is very similar to the estimates for all individuals ages 15 and up in the time-use surveys of the 2000s.

2.5 Basic Results

2.5.1 Data Analysis

Innovations are extracted via Vector Autocorrelation (VAR) representation for filtered per capita consumption, leisure and the transformed past aggregate consumption trend.

The figures 2.10 and 2.11 show different news for four variables in the bootstrapping impulse response functions. In particular, a bad shock increases leisure growth and decreases consumption growth, which is consistent with theory.

[13]The inclusion of government workers and unpaid family labor is consistent with the post-World War II US Department of Labor, Bureau of Labor Statistics (BLS) labor series, as well as the fact that gross domestic product includes the output of these workers.

2.5.1.1 A Linear-factor Asset Pricing Model

In this part, results are reported under unconditional covariance among variables; a linear-factor asset pricing model, the equation (2.30), explains cross-sectional returns on assets.

To avoid models that researchers would never consider in practice, we narrow down the focus to our new long-run factor, the consumption-based CAPM, and the Fama–French three-factor modelsto price Fama–French 25 size- and book-to-market ratio portfolios. Since the Fama–French three-factor model obtains a 'factor-structure' pattern, it is chosen as the benchmark[14].

Our new factor model is stated as:

1. The model with long-run consumption–leisure trend three factors:

$$y_{t+1}^{N3} = \theta_0 + \theta_1 lrc_{t+1} + \theta_2 lrl_{t+1} + \theta_3 lrg_{t+1}, \tag{2.44}$$

where growths on per capita consumption, leisure and the consumption trend are represented by factors lrc_{t+1}, lrl_{t+1} and lrg_{t+1} denoting long-run growth among these variables[15].

2. Consumption-based CAPM (C-CAPM):

$$y_{t+1}^{CCAPM} = \theta_0 + \theta_1 c_{t+1}^{ndur}, \tag{2.45}$$

where c_{t+1}^{ndur} is the growth rate of non-durable consumption.

3. Fama and French (1993) [1] (FF3) document the role of size and book/market in the cross-section of expected stock returns, and show that CAPM are not supported by the data

$$y_{t+1}^{FF3} = \theta_0 + \theta_1 R_{t+1}^{eM} + \theta_2 SMB_{t+1} + \theta_3 HML_{t+1}, \tag{2.46}$$

where R_{t+1}^{eM}, SMB_{t+1} and HML_{t+1} stand for the market return, size and book-to-market ratio factors, respectively.

We turn to the main findings of the paper now. This subsection first looks at how portfolios load on these factors, and then we examine whether the factor loadings are significantly priced. For the purpose of comparison, the factor exposures in benchmark models are reported. Table 2.3 and Table 2.4 explain the portfolios' exposures to C-CAPM and Fama–French three-factor models via multivariate and univariate time-series

[14] Lewellen, Nagel and Shanken (2009) advocate this solution to the problem.

[15] To identify preference parameters in equation (31), I also estimate a six-factor model: $y_{t+1}^{N6} = \theta_0 + \theta_1 \Delta c_{t+1} + \theta_2 \Delta l_{t+1} + \theta_3 g_{t+1} + \theta_4 lrc_{t+1} + \theta_5 lrl_{t+1} + \theta_6 lrg_{t+1}$.

regressions. Each row in the tables from left to right represents the size portfolios from small to big in a given book-to-market category. Each column from top to bottom corresponds to lowest to highest book-market quintiles for a given size category. There is a weak correlation between average returns and the exposures to the market factor. The SMB factor captures the size effect and the HML factor lines up with returns on the book-and-market dimension. Almost all factor loadings are statistically significant at block bootstrapping 5% level. Likewise, Table 2.5 and Table 2.6 show the performance on our linear-factor model.

The long-run leisure factor explains the size effect, while it does negatively line up well with returns on the book-to-market effect; the estimated long-run consumption betas behave oppositely to the long-run leisure. The consumption trend lines up with returns on the size- and book-to-market effects like the long-run consumption betas. With univariate regression, the trend captures size and book-to-market effects, simultaneously. The long-run consumption factor obtains these characteristic, so does the long-run leisure. To sum up, the factor loadings are significant for our new factor model: the long-run factors can capture the book-and-market and size spreads.

In order to further examine whether the risks measured by these factor loadings are significantly priced, we can look at the results of cross-sectional regressions. We consider two alternative formulations (with and without the constant term) to assess the models' ability to capture the cross-section of average returns. The constant term should be zero according to theory; a non-zero large constant term indicates that a model cannot price the assets on average. A non-zero λ_0 can also be interpreted as a zero-beta rate different from the risk free rate that is imposed[16]. In some specifications, the sign and magnitude of the estimated risk premia are found to depend on the inclusion of the constant.

Table 2.7 gives us the prices of risk, R^2, and pricing error tests without the constant term for all candidates. The first row ('mean') shows the λs (prices of risk) for factors after average innovations are taken from the VAR. The second row ('vector') gives the results when 25 'cells' innovations are used to price the risks on Fama–French 25 portfolios, since we put each test portfolio as the fourth element in the VAR estimation. From the third to the eighth rows, various standard errors (i.i.d., Shanken, and GMM) and bootstrapping (5000 times) upper–lower bands are shown. If the estimated standard errors stay between the upper and the lower band, it means the estimates are not statistically significant. The long-run factors statistically can price risks on assets well, comparing the same performances on market return and book-to-market loadings in the Fama–French three-factor model. Besides, the trend factor is able to explain the risk,

[16] Burnside (2011) [40] shows that the constant can be interpreted as the model's pricing error for the risk free rate.

though its factor loadings are small via time-series regressions. It should be mentioned that the upper–lower bands here are computed using 'mean' innovations: the 'vector' upper–lower bands are not reported. However, the risk prices are still significant for all long-run factors. Similar results are shown in Table 2.8. Then the R^2s (Adjusted R^2s) are 79.4% (76.6%) and 89.8% (88.5%) for our long-run three-factor models and Fama–French three-factor, respectively. Without the constant term, the Fama–French three-factor model still has higher explanatory powers than our model: the difference is about 10%, while C-CAPM obtains the lowest. In the last three columns of the table, we test the pricing errors with Alpha tests: the null hypothesis states that the pricing errors are zero. Our model cannot be rejected by the hypothesis, while pricing errors are significantly not equal to zero for the other candidates.

Tables 2.9 and 2.10 give us the direct image from the cross-sectional regression with a constant term, in which the λ_0s for the two models are quite small for the zero-beta portfolio and are also insignificant. Like the results without a constant term, the long-run factor loadings significantly price risks of assets. R^2s (Adjusted R^2s) on our factor models obtain less explanatory power for the benchmark. According to the Alpha tests, the null hypothesis, which states that the pricing errors are zero, cannot be rejected for our model, but the C-CAPM and Fama–French three-factor model obtains significant pricing errors.

To avoid the 'generated regressors' problem, the cross-sectional regression by GMM is reported in Table 2.11. The long-run factors statistically significantly price risks. The J–statistics in the last two columns cannot be rejected, in which the null hypothesis states that the model is 'valid'. In the literature, those linear factor models are usually rejected by this statistical test[17]. From Table 2.11, we can conclude that our factor model is valid: we cannot statistically reject the null hypothesis. This method statistically proves such results from Table 2.5 to Table 210.

Using equation (2.30), we can identify the preference parameters under the unconditional covariance matrix, such as κ, υ and η. A six-factor model $y^{N6}_{t+1} = \theta_0 + \theta_1 \Delta c_{t+1} + \theta_2 \Delta l_{t+1} + \theta_3 g_{t+1} + \theta_4 lrc_{t+1} + \theta_5 lrl_{t+1} + \theta_6 lrg_{t+1}$ is estimated via time-series and cross-sectional regressions[18]. However, the last four significant estimates empirics are of help in identifying the preference parameters κ (the consumption-leisure ratio), ψ (the curvature in E–Z), υ and η (both characterize RRA and IES) in equation (2.30). Therefore the estimated RRA and IES are equal to 5.7832 and 1.4807 for 'vector' innovations (26.9072 and 0.4061 for 'mean' innovations) after running a cross-sectional regression without the constant term; while with the constant term, the applied RRA and IES are 2.0235

[17] If there are over-identifying restrictions, the test statistics are known to over-reject the null hypothesis.

[18] To save space, the tables are not reported in this paper.

and 1.13102 for 'vector' innovation (11.6050 and 0.5032 for 'mean' innovations). In this case we empirically prove that IES is not the inverse of RRA like the expected utility function does.

2.5.1.2 Hansen–Jagannathan Distance and Multiple Comparison Test

Because of drawbacks to the R^2, we apply various Hansen–Jagannathan (HJ) distances to evaluate the pricing performances and to test the rankings. The basic motivation of the HJ distance is to supply a method to find the least misspecified one among some candidates[19].

The basic HJ, the modified HJ (Kan and Robotti (2008) [19]), the unconstrained HJ, and the constrained HJ (Gospodinov et al. (2010) [20]) are used to rank the misspecification of the candidates. The long-run three-factor model outperforms the other two across these four distance measures. In addition, the statistical test will be that the distance measure is equal to zero as the null hypothesis. As a result, we cannot reject the null hypothesis for all HJ distance measures. Besides, we apply the block-bootstrapping multiple comparison test in the last four rows to test whether the rank stays statistically true. The null hypothesis states that the winner obtains the minimum HJ distance among others. The last four rows of Table 2.12 show that the statistical p values are 0.2488, 0.3636, 0.2456 and 0.533, respectively, for our consumption-leisure-trend three-factor model cannot be used to reject the null hypothesis across four distance measures. In other words, the model statistically gets the least misspecified measures compared to the other pricing models.

2.5.2 Additional Robustness Checks

In this section, we extend the candidate models and test portfolios, i.e., the Fama–French 25 size and momentum, and 30 Industry-sorted portfolios.

For the candidate models, the following will be considered:

The Yogo non-durable and durable consumption model

$$y_{t+1}^{Yogo} = \theta_0 + \theta_1 c_{t+1}^{ndur} + \theta_2 c_{t+1}^{dur}, \tag{2.47}$$

where c_{t+1}^{ndur} denotes durable consumption growth.

[19] Hansen and Jagannathan (1997) [13] develop a measure of the degree of misspecification of an asset pricing model. This measure is defined as $min_{m \in \aleph} \|m - y\|$, the least squares distance between the family of stochastic discount factors that price all the assets correctly and the stochastic discount factor associated with and an asset pricing model.

The Fama–French 25 size and momentum, which are constructed monthly, are the intersections of five portfolios formed on size (market equity, ME) and five portfolios formed on prior (2–12) returns. The 30 industry portfolios are NYSE, AMEX, and NASDAQ industry portfolios based on its four-digit SIC code at that time, whose returns are from July of t to June of $t+1$.

To save space, we only report the important results, i.e., the Alpha tests, J tests, R^2s, and HJ misspecified distance measures.

Table 2.13 shows that four models explain the Fama–French 25 size- and momentum-portfolios. The second and the third columns present R^2s, consumption-leisure four-factor model obtains higher values on both R^2 and adjusted R^2. The results with a constant term are reported in the brackets below; the Fama–French three-factor gets more explanatory power than the others. The fourth to the tenth columns show the results of the Alpha and Chi-square tests for pricing errors. Higher p values indicate that we cannot reject the null hypothesis, that the pricing errors are zero. Like the result in pricing the Fama–French 25 size and book-to-market ratio portfolios, the Fama–French three-factor model statistically has non-zero pricing errors, though our long-run three-factor does not. The last four columns represent misspecification measures; our long-run models statistically obtain the smallest measures among the candidates, in other words, the model stays the least misspecified.

The 30 industry portfolios are priced by candidate models in Table 2.14. Different from the above portfolios, the Fama–French three-factor obtains the highest R^2s whether there is a constant term or not. All the factor models statistically get zero pricing errors to explain industry-sorted assets via the Alpha and Chi square tests. The results on the misspecification measures show that the Fama–French three-factor statistically outperforms the others.

2.6 Concluding Remarks

In his discussion of the empirical evidence on market efficiency, Fama (1991) writes: '...and relates the behavior of expected returns to the real economy in a rather detailed way.' In this paper, we have exhibited a model that meets Fama's objectives and, empirically, helps to explain the cross-sectional equity premiums.

The non-separable consumption and leisure with Epstein–Zin preference allows the leisure dynamics to interact with the consumption to generate interesting asset pricing implications. Comparing with a model with consumption only, incorporating leisure reduces the price of long-run risk and leisure acts as a ‘hedge’. Hence, stocks with predominated long run leisure risk will be relatively less risky and bear smaller average returns. Empirical results show that growth (big) stocks obtain lower negative long run leisure betas but smaller long run consumption betas, and vice visa for value (small) stocks.

In concluding the paper, we point out some limitations and thus possible extensions of this study. The neoclassical framework in the model can be extended to link asset prices with other types of intangible capital, e.g., human capital and organizational capital. Empirically, the correlation between human capital, organizational capital, and physical capital and their relations with the cross-section of equity returns is worth investigating further.

TABLE 2.1: **An Initial Leisure Analysis**

Notes: The table reports the autocorrelation of leisure growth in Panel A. Panel B shows the variance ratio test. If the growth is i.i.d., then the ratio should be equal to 1. A higher (lower) than unity ratio implies positive (negative) autocorrelation dominates. Panel C investigates time-varying volatility. Without time-varying volatility, the adjusted variance ratios would be flat with respect to the horizon, and stay close to 1. Panel D runs regressions: $\Delta_{l,t+j} = a_j + b_j(p_t - d_t) + \eta_{t+j}$ and $(p_{t+j} - d_{t+j}) = a_j + b_j\Delta_{l,t} + \eta_{t+j}$ to see the predictive power with leisure growth and real price-dividend ratio.
** denote statistical significance at 5% level.

Panel A. Autocorrelation

	Leisure Growth
Variable	0.3835**
	0.0581
Contant	0.00157**
	-0.00037

Panel B. Variance Ratio of Log Leisure

Horizon	VR	AdVR	Bootstrap Percentiles (90%, 95%, 99%)		
2	1.3912	1.5986	1.0186	1.0218	1.0285
5	2.1893	2.8203	0.6956	0.7229	0.7741
10	2.8579	2.0407	0.3712	0.3925	0.4407

Panel C. Variance Ratio of Vol. of Leisure

Horizon	VR	AdVR	Bootstrap Percentiles (90%, 95%, 99%)		
2	0.5604	0.5486	1.0133	1.0145	1.0169
5	0.5220	0.5203	0.7021	0.7287	0.7770
10	0.5002	0.5000	0.3790	0.3993	0.4497

Panel D. Predictability Regressions

Horizon	Leisure Growth predicted by p-d		p-d predicted by Leisure Growth	
	b	R^2	b	R^2
1	-1.1511**	0.0970	-0.0171**	0.0970
3	-1.1657**	0.0202	-.01735**	0.0202
5	-1.1901**	0.0208	-.01751**	0.0208

TABLE 2.2: **An Initial Consumption Analysis**

Notes: The table reports the autocorrelation of average non-durable consumption growth in Panel A. Panel B shows the variance ratio test. If the growth is i.i.d., then the ratio should be equal to 1. A higher (lower) than unity ratio implies positive (negative) autocorrelation dominates. Panel C investigates time-varying volatility. Without time-varying volatility, the adjusted variance ratios would be flat with respect to the horizon, and stay close to 1. Panel D runs regressions: $\Delta_{c,t+j} = a_j + b_j(p_t - d_t) + \eta_{t+j}$ and $(p_{t+j} - d_{t+j}) = a_j + b_j\Delta_{c,t} + \eta_{t+j}$ to see the predictive power with average consumption growth and real price-dividend ratio.
** denote statistical significance at 5% level.

Panel A. Autocorrelation

	Ave. Non-durable Growth
Variable	0.2875**
	0.0604
Contant	0.1111**
	0.0335

Panel B. Variance Ratio of Log Consumption

Horizon	VR	AdVR	Bootstrap Percentiles (90%, 95%, 99%)		
2	1.1732	1.0986	1.0134	1.0147	1.0170
5	1.8449	1.0203	0.6986	0.7227	0.7703
10	2.1823	2.0407	0.3777	0.3974	0.4432

Panel C. Variance Ratio of Vol. of Consumption

Horizon	VR	AdVR	Bootstrap Percentiles (90%, 95%, 99%)		
2	1.0361	1.03	1.0134	1.0147	1.0175
5	1.5517	1.5123	0.6998	0.7251	0.7775
10	1.5750	1.5623	0.3742	0.3954	0.4448

Panel D. Predictability Regressions

Horizon	Non-dur Growth predicted by p-d		p-d predicted by Non-dur Growth	
	b	R^2	b	R^2
1	2.2189**	0.0723	0.0326**	0.0723
3	2.2189**	0.0723	0.0326**	0.0723
5	2.2347**	0.0725	.03243**	0.0725

TABLE 2.3: **Exposure to Benchmark Candidates (Multivariate)**

Notes: The table shows the exposures to consumption-based CAPM and Fama–French three-factor models when explain Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate βs through time series regressions. It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns and repeating the 3-step estimations for 5000 times, then report the upper and the lower band for estimated standard variances. Here the model is estimated with multivariate betas.
** denote block bootstrapping statistical significance at 5% level.

	Growth	2	3	4	Value
			Average Excess Returns %		
Small	-0.42	1.36	1.66	2.27	2.60
2	0.24	1.33	1.96	2.13	2.42
3	0.62	1.52	1.62	1.98	2.22
4	0.92	1.14	1.86	1.83	1.97
Big	0.89	1.01	1.37	1.30	1.37
Panel A : Fama–French Three-factor Model					
			β_M		
Small	0.5601**	0.4353**	0.3034**	0.2802**	0.3885**
2	0.5642**	0.4141**	0.3545**	0.3528**	0.4464**
3	0.5347**	0.4097**	0.3489**	0.3937**	0.372**
4	0.5194**	0.4382**	0.4325**	0.4149**	0.5078**
Big	0.4764**	0.3982**	0.3471**	0.3948**	0.469**
			β_{SMB}		
Small	1.4302**	1.2243**	1.0849**	1.0146**	1.0798**
2	1.0454**	0.9104**	0.7409**	0.6751**	0.7535**
3	0.7391**	0.5929**	0.5019**	0.4055**	0.5859**
4	0.4145**	0.3291**	0.2271**	0.2465**	0.3469**
Big	-0.198**	-0.1526**	-0.1903**	-0.1784**	-0.1296**
			β_{HML}		
Small	-0.3683**	-0.0562**	0.0884**	0.1927**	0.4392**
2	-0.4304**	-0.0706	0.0883**	0.3018**	0.5251**
3	-0.5056**	-0.0638**	0.1374**	0.3418**	0.4734**
4	-0.5097**	-0.0374**	0.1831**	0.2897**	0.5031**
Big	-0.3825**	-0.0692**	0.1339**	0.3458**	0.4681**
Panel B : Consumption-based CAPM					
			β_{Nonc}		
Small	0.0104	0.0154	0.0022	0.0038	0.0097
2	-0.0025	-0.0029	-0.0023	0.0001	0.0052
3	-0.0026	-0.0051	-0.0069	-0.004	-0.0033
4	-0.0084	-0.0096	-0.0063	-0.0041	0.0067
Big	-0.0094	-0.0153	-0.0146	-0.0087	-0.0013

TABLE 2.4: **Exposure to Benchmark Candidates (Univariate)**

Notes: The table shows the exposures to consumption-based CAPM and Fama–French three-factor models when explain Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate βs through time series regressions. It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns and repeating the 3-step estimations for 5000 times, then report the upper and the lower band for estimated standard variances. Here the model is estimated with univariate betas.
** denote block bootstrapping statistical significance at 5% level.

	Growth	2	3	4	Value
			Average Excess Returns %		
Small	-0.42	1.36	1.66	2.27	2.60
2	0.24	1.33	1.96	2.13	2.42
3	0.62	1.52	1.62	1.98	2.22
4	0.92	1.14	1.86	1.83	1.97
Big	0.89	1.01	1.37	1.30	1.37
Panel A : Fama–French Three-factor Model					
			β_M		
Small	1.001**	0.7843**	0.5975**	0.5431**	0.6426**
2	0.9042**	0.6768**	0.5523**	0.5087**	0.5998**
3	0.7972**	0.5828**	0.4744**	0.4697**	0.4841**
4	0.6915**	0.5345**	0.476**	0.4521**	0.5497**
Big	0.4629**	0.3631**	0.2791**	0.3069**	0.3813**
			β_{SMB}		
Small	1.8063**	1.5158**	1.2876**	1.2015**	1.3383**
2	1.4244**	1.1878**	0.9778**	0.9103**	1.0505**
3	1.0987**	0.8673**	0.735**	0.6678**	0.8333**
4	0.7639**	0.6225**	0.516**	0.5232**	0.6851**
Big	0.1221**	0.1141**	0.0416**	0.0847**	0.1827**
			β_{HML}		
Small	-0.4905	-0.1515	0.0214	0.1307	0.3542
2	-0.5525	-0.1606	0.0114	0.2254	0.4287
3	-0.6206	-0.1521	0.0623	0.2575	0.3931
4	-0.6207	-0.1309	0.091	0.2013	0.3948
Big	-0.4827	-0.153	0.061	0.2628	0.3692
Panel B : Consumption-based CAPM					
			β_{Nonc}		
Small	0.0104	0.0154	0.0022	0.0038	0.0097
2	-0.0025	-0.0029	-0.0023	0.0001	0.0052
3	-0.0026	-0.0051	-0.0069	-0.004	-0.0033
4	-0.0084	-0.0096	-0.0063	-0.0041	0.0067
Big	-0.0094	-0.0153	-0.0146	-0.0087	-0.0013

TABLE 2.5: **Exposure to Long-run Three Factors (Multivariate)**

Notes: The table shows the exposures to consumption-leisure-trend three-factor model when explain Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate βs through time series regressions. It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns and repeating the 3-step estimations for 5000 times, then report the upper and the lower band for estimated standard variances. Here the model is estimated with multivariate betas.
** denote block bootstrapping statistical significance at 5% level, * denote block bootstrapping statistical significance at 1% level

	Growth	2	3	4	Value
			Average Excess Returns %		
Small	-0.42	1.36	1.66	2.27	2.60
2	0.24	1.33	1.96	2.13	2.42
3	0.62	1.52	1.62	1.98	2.22
4	0.92	1.14	1.86	1.83	1.97
Big	0.89	1.01	1.37	1.30	1.37
			β_{LrC}		
Small	13.247**	10.391**	10.732**	16.669**	18.946**
2	11.865**	8.349**	6.914**	11.801**	12.897**
3	6.84**	7.664**	5.935**	7.434**	8.07**
4	4.00**	5.41**	7.866**	6.743**	11.77**
Big	-4.498**	3.559**	4.967**	6.676**	13.228**
			β_{LrL}		
Small	-9.451**	-7.538**	-8.18**	-5.22**	-7.06**
2	-5.941**	-5.643**	-5.525**	-5.36**	-4.575**
3	-8.689**	-5.927**	-4.76**	-3.89**	-7.29**
4	-3.364**	-3.87**	-3.35**	-3.738**	-3.163**
Big	-9.856**	-5.44**	-3.552**	-5.024**	-5.77**
			β_{LrT}		
Small	-0.1505**	-0.0739**	-0.0707**	-0.074**	-0.0459**
2	-0.1244**	-0.0949**	-0.0372**	-0.0668**	-0.0705**
3	-0.1507**	-0.1217**	-0.1108**	-0.0803**	-0.0653**
4	-0.106**	-0.0878**	-0.1219**	-0.0982**	-0.0476**
Big	-0.0938**	-0.0571**	-0.0402**	-0.0952**	-0.0503**

TABLE 2.6: **Exposure to Long-run Three Factors (Univariate)**

Notes: The table shows the exposures to consumption-leisure-trend three-factor model when explain Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate βs through time series regressions. It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns and repeating the 3-step estimations for 5000 times, then report the upper and the lower band for estimated standard variances. Here the model is estimated with univariate betas.
** denote block bootstrapping statistical significance at 5% level, * denote block bootstrapping statistical significance at 1% level

	Growth	2	3	4	Value
			Average Excess Returns %		
Small	-0.42	1.36	1.66	2.27	2.60
2	0.24	1.33	1.96	2.13	2.42
3	0.62	1.52	1.62	1.98	2.22
4	0.92	1.14	1.86	1.83	1.97
Big	0.89	1.01	1.37	1.30	1.37
			β_{LrC}		
Small	0.260**	0.673**	0.026**	1.232**	0.675**
2	-0.901**	-0.017**	0.51**	1.017**	0.417**
3	-1.825**	-0.436**	0.045**	0.483**	-0.298**
4	-1.870**	-0.296**	-0.106**	-0.097**	-0.012**
Big	-1.128**	-0.838**	-0.647**	-0.593**	-0.522**
			β_{LrL}		
Small	-0.389**	-0.56**	-0.494**	-0.633**	-0.474**
2	-0.323**	-0.525**	-0.582**	-0.578**	-0.332**
3	-0.037**	-0.297**	-0.238**	-0.258**	-0.13**
4	0.059**	-0.219**	-0.139**	-0.29**	-0.087**
Big	-0.471**	-0.37**	-0.236**	-0.271**	-0.293**
			β_{LrT}		
Small	-0.0129**	0.0004**	0.0071**	0.0096**	0.0107**
2	-0.0001**	0.0046**	0.012**	0.0141**	0.0148**
3	-0.0057**	0.0052**	0.0122**	0.0085**	0.0079**
4	-0.0017**	0.0071**	0.0074**	0.0075**	-0.0016**
Big	-0.0056**	0.0016**	0.007**	0.0097**	-0.004**

TABLE 2.7: **Cross-section Results Without Constant on Fama–French 25 Portfolios (Multivariate)**

Notes: The table gives the risk price for Fama–French three-factor, consumption-based CAPM and the consumption-leisure-trend three-factor model when explaining Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate λs through time series and cross-sectional regressions simultaneously. It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns while repeating the 3-step estimations for 5000 times. Each bracket reports the upper and the lower bands for estimated standard variances. Here the model, without constant term, is estimated with multivariate betas. The null hypothesis on χ^2 test is the pricing errors are equal to zeros.
The last four columns show statistics like R^2, adjusted R^2, Alpha statistics, χ^2 statistics and their p values. For Alpha statistics, RMSE, MAE, and Shanken correction are shown. For χ^2 statistics, the table shows i.i.d., Shanken and GMM cases, respectively.
** denote block bootstrapping statistical significance at 5% level.

		Fama–French Three-factor			**CCAPM**	**Long-run Three-factor**						
		λ_M	λ_{SMB}	λ_{HML}	λ_{NonC}	λ_{LrC}	λ_{LrL}	λ_{LrT}	R^2	Alpha	χ^2	p_{value}
1	λ mean	0.025	0.003	0.020					0.898	0.005	94.683	0
	λ vector	0.025	0.003	0.020					0.885	0.004	75.990	0
	se i.i.d.	0.008**	0.004**	0.004**						1.246	64.455	0
		[0.024 0.08]	[0.016 0.046]	[0.016 0.063]							92.160	0
	se Shanken	0.009**	0.004**	0.005**								
		[0.048 0.316]	[0.037 0.207]	[0.034 0.285]								
	se GMM_0	0.009**	0.004**	0.005**								
		[0.047 0.293]	[0.036 0.195]	[0.033 0.274]								
	se GMM_{12}	0.007**	0.004**	0.005**								
		[0.048 0.313]	[0.033 0.221]	[0.036 0.264]								
2	λ mean				-0.418				0.042	0.015	103.437	0
	λ vector				-0.418				0.002	0.014	63.761	0
	se i.i.d.				0.162**					1.622	80.017	0
					[0.221 0.704]						121.204	0
	se Shanken				0.205**							
					[0.269 2.849]							
	se GMM_0				0.191**							
					[0.26 3.023]							
	se GMM_{12}				0.19**							
					[0.273 2.956]							
3	λ mean					-0.137	-0.138	-4.385	0.794	0.007	88.684	0
	λ vector					-0.134	-0.137	-13.597	0.766	0.006	10.182	0.985
	se i.i.d.					0.027**	0.027**	1.401**		8.710	12.920	0.935
						[0.082 0.332]	[0.082 0.332]	[0.185 0.608]			17.993	0.706
	se Shanken					0.079**	0.079**	4.102**				
						[0.242 1.052]	[0.242 1.053]	[0.646 1.698]				
	se GMM_0					0.086**	0.086**	4.012**				
						[0.233 1.059]	[0.233 1.06]	[0.602 1.73]				
	se GMM_{12}					0.092**	0.092**	3.64**				
						[0.213 0.933]	[0.213 0.934]	[0.536 1.652]				

TABLE 2.8: **Cross-section Results Without Constant on Fama–French 25 Portfolios (Univariate)**

Notes: The table gives the risk price for Fama–French three-factor, consumption-based CAPM and the consumption-leisre-trend three-factor model when explaining Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate λs through time series and cross-sectional regressions simultaneously. It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns while repeating the 3-step estimations for 5000 times. Each bracket reports the upper and the lower bands for estimated standard variances. Here the model, without constant term, is estimated with univariate betas. The null hypothesis on α test is the pricing errors are equal to zeros.
The last four columns show statistics like R^2, adjusted R^2, Alpha statistics, χ^2 statistics and their p values. For Alpha statistics, RMSE and MAE are shown. For χ^2 statistics, the table shows i.i.d. and GMM cases, respectively.
** denote block bootstrapping statistical significance at 5% level.

		Fama–French Three-factor			**CCAPM**	**Long-run Three-factor**						
		λ_M	λ_{SMB}	λ_{HML}	λ_{NonC}	λ_{LrC}	λ_{LrL}	λ_{LrT}	R^2	Alpha	χ^2	p_{value}
1	λ mean	0.034	-0.006	0.024					0.898	0.005	94.683	0
	λ vector	0.034	-0.006	0.024					0.885	0.004	64.455	0
	se i.i.d.	0.01**	0.005**	0.005**							92.160	0
		[0.027 0.087]	[0.018 0.049]	[0.017 0.071]								
	se GMM_0	0.012**	0.005**	0.006**								
		[0.055 0.35]	[0.044 0.21]	[0.033 0.296]								
	se GMM_{12}	0.009**	0.006**	0.006**								
		[0.055 0.339]	[0.039 0.2]	[0.034 0.264]								
2	λ mean				-0.418				0.042	0.015	103.437	0
	λ vector				-0.418				0.002	0.014	80.017	0
	se i.i.d.				0.162**						121.204	0
					[0.221 0.704]							
	se GMM_0				0.191**							
					[0.26 3.023]							
	se GMM_{12}				0.19**							
					[0.273 2.956]							
3	λ mean					7.923	-8.089	-5.657	0.794	0.007	88.684	0
	λ vector					-4.198	1.750	-15.099	0.766	0.006	12.920	0.935
	se i.i.d.					2.516**	2.528**	1.454**			17.993	0.706
						[32.911 963.495]	[32.975 963.745]	[0.179 0.796]				
	se GMM_0					8.315**	8.361**	4.421**				
						[113.82 2785.769]	[114.076 2786.668]	[0.505 2.194]				
	se GMM_{12}					7.236**	7.291**	4.158**				
						[103.93 2891.486]	[104.14 2892.26]	[0.451 2.294]				

TABLE 2.9: **Cross-section Results With Constant on Fama-French 25 Portfolios (Multivariate)**

Notes: The table gives the risk price for Fama–French three-factor, consumption-based CAPM and the consumption-leisure-trend three-factor model when explaining Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate λs through time series and cross-sectional regressions simultaneously. It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns while repeating the 3-step estimations for 5000 times. Each bracket reports the upper and the lower bands for estimated standard variances. Here the model, with constant term, is estimated with multivariate betas. The null hypothesis on α test is the pricing errors are equal to zeros.
The last four columns show statistics like R^2, adjusted R^2, Alpha statistics, χ^2 statistics and their p values. For Alpha statistics, RMSE, MAE, and Shanken correction are shown. For χ^2 statistics, the table shows i.i.d., Shanken and GMM cases, respectively.
** denote block bootstrapping statistical significance at 5% level.

			Fama–French Three-factor			**CCAPM**			**Long-run Three-factor**						
			λ_M	λ_{SMB}	λ_{HML}		λ_{NonC}		λ_{LrC}	λ_{LrL}	λ_{LrT}	R^2	Alpha	χ^2	p_{value}
1	λ mean	0.027	-0.033	0.002	0.012							0.723	0.003	62.839	0
	λ vector	0.027	-0.033	0.002	0.012							0.684	0.003	51.463	0
	se i.i.d.	0.005	0.01**	0.004**	0.004**								1.221	55.391	0
		[0.004 0.006]	[0.022 0.045]	[0.014 0.034]	[0.015 0.04]									76.061	0
	se Shanken	0.005	0.011**	0.004**	0.004**										
		[0.005 0.014]	[0.035 0.106]	[0.023 0.078]	[0.026 0.086]										
	se GMM_0	0.005	0.01**	0.004**	0.004**										
		[0.005 0.0133]	[0.032 0.106]	[0.022 0.07]	[0.025 0.077]										
	se GMM_{12}	0.005	0.01**	0.004**	0.004**										
		[0.005 0.013]	[0.034 0.108]	[0.023 0.069]	[0.024 0.078]										
2	λ mean					0.015	0.111					0.016	0.006	78.381	0
	λ vector					0.015	0.111					-0.026	0.005	75.105	0
	se i.i.d.					0.005	0.185						1.044	75.621	0
						[0.004 0.005]	[0.156 0.348]							85.787	0
	se Shanken					0.005	0.189								
						[0.004 0.011]	[0.17 0.588]								
	se GMM_0					0.005	0.193								
						[0.004 0.011]	[0.166 0.566]								
	se GMM_0					0.004	0.181								
						[0.003 0.01]	[0.167 0.57]								
3	λ mean							0.020	-0.104	-0.103	2.238	0.328	0.005	66.743	0
	λ vector							0.016	0.033	0.031	0.425	0.232	0.004	24.446	0.272
	se i.i.d.							0.004	0.027	0.027	0.955**		2.730	35.156	0.027
								[0.004 0.006]	[0.024 0.183]	[0.024 0.183]	[0.202 0.536]			55.156	0
	se Shanken							0.007	0.045**	0.045**	1.559**				
								[0.006 0.016]	[0.055 0.318]	[0.055 0.318]	[0.422 0.995]				
	se GMM_0							0.006	0.044**	0.044**	1.74**				
								[0.006 0.016]	[0.054 0.302]	[0.054 0.302]	[0.359 0.876]				
	se GMM_{12}							0.006	0.044**	0.044**	1.76**				
								[0.006 0.016]	[0.045 0.299]	[0.045 0.299]	[0.36 0.867]				

Table 2.10: **Cross-section Results With Constant on Fama–French 25 Portfolios (Univariate)**

Notes: The table gives the risk price for Fama–French three-factor, consumption-based CAPM and the consumption-leisure-trend three-factor model when explaining Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate λs through time series and cross-sectional regressions simultaneously. It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns while repeating the 3-step estimations for 5000 times. Each bracket reports the upper and the lower bands for estimated standard variances. Here the model, with constant term, is estimated with univariate betas. The null hypothesis on α test is the pricing errors are equal to zeros.
The last four columns show statistics like R^2, adjusted R^2, Alpha statistics, χ^2 statistics and their p values. For Alpha statistics, RMSE and MAE are shown. For χ^2 statistics, the table shows i.i.d. and GMM cases, respectively.
** denote block bootstrapping statistical significance at 5% level.

		Fama–French Three-factor				**CCAPM**		**Long-run Three-factor**							
			λ_M	λ_{SMB}	λ_{HML}		λ_{NonC}		λ_{LrC}	λ_{LrL}	λ_{LrT}	R^2	Alpha	χ^2	p_{value}
1	λ mean	0.027	-0.040	0.013	0.008							0.723	0.003	62.839	0
	λ vector	0.027	-0.040	0.013	0.008							0.684	0.003	55.391	0
	se i.i.d.	0.005	0.012**	0.005**	0.005**									76.061	0
		[0.004 0.006]	[0.025 0.053]	[0.017 0.037]	[0.015 0.041]										
	se GMM_0	0.005	0.012**	0.005**	0.005**										
		[0.005 0.013]	[0.037 0.131]	[0.026 0.086]	[0.027 0.083]										
	se GMM_{12}	0.005	0.013**	0.006**	0.005**										
		[0.005 0.013]	[0.037 0.133]	[0.025 0.084]	[0.025 0.081]										
2	λ mean					0.015	0.111					0.016	0.006	78.381	0
	λ vector					0.015	0.111					-0.026	0.005	75.621	0
	se i.i.d.					0.005	0.185							85.787	0
						[0.004 0.005]	[0.156 0.348]								
	se GMM_0					0.005	0.193								
						[0.004 0.011]	[0.166 0.566]								
	se GMM_{12}					0.004	0.181								
						[0.003 0.01]	[0.167 0.57]								
3	λ mean							0.020	-0.249	0.155	1.524	0.328	0.005	66.743	0
	λ vector							0.016	-18.988	17.377	-1.925	0.232	0.004	35.156	0.027
	se i.i.d.							0.004	1.903**	1.911**	0.973**			55.156	0
								[0.004 0.006]	[21.788 438.019]	[21.785 437.9]	[0.23 0.684]				
	se GMM_0							0.006	2.638**	2.65**	1.824**				
								[0.006 0.016]	[58.41 762]	[58.424 762]	[0.398 1.3]				
	se GMM_{12}							0.006	2.574**	2.578**	1.782**				
								[0.006 0.015]	[54.8 761.749]	[54.825 761.63]	[0.397 1.403]				

TABLE 2.11: **GMM Results on Fama–French 25 Portfolios**

Notes: The table gives the risk price for Fama–French three-factor, consumption-based CAPM and the long-run consumption-leisure-trend three-factor model when explain Fama–French 25 size and book/market ratio portfolios. To estimate the model, we extract innovations from VAR firstly, then estimate λs through two-stage GMM (Cochrane, 2000). It should be noted that the multi-step procedure causes 'generated regressors' problem in the estimation. The standard errors are obtained by bootstrapping the VAR errors and returns while repeating the 2-step estimations for 5000 times. Each bracket reports the upper and the lower bands for estimated standard variances. The null hypothesis on χ^2 test (J–statistics) is the pricing errors are equal to zeros.
The last three columns show statistics like Alpha statistics, χ^2 statistics and their p values. For Alpha statistics, GMM and GMM with 12 lags correction are shown, respectively. For J–statistics, the table shows GMM and GMM with 12 lags cases.
** denote block bootstrapping statistical significance at 5% level.

		Fama–French Three-factor			**CCAPM**	**Long-run Three-factor**					
		λ_M	λ_{SMB}	λ_{HML}	λ_{NonC}	λ_{LrC}	λ_{LrL}	λ_{LrT}	Alpha	J Stat.	p_{value}
					Panel A : First Stage GMM						
1	λ mean	0.025	0.003	0.020					0.005	64.455	0
	λ vector	0.025	0.003	0.020					0.004	92.160	0
	se GMM_0	0.009**	0.004**	0.006**							
		[0.048 0.297]	[0.036 0.194]	[0.035 0.283]							
	se GMM_{12}	0.006**	0.005**	0.007**							
		[0.049 0.312]	[0.033 0.223]	[0.036 0.259]							
2	λ mean				-0.418				0.015	80.017	0
	λ vector				-0.418				0.014	121.204	0
	se GMM_0				0.197**						
					[0.27 3.082]						
	se GMM_{12}				0.184**						
					[0.273 2.927]						
3	λ mean					-0.137	-0.138	-4.385	0.007	12.920	0.935
	λ vector					-0.134	-0.137	-13.597	0.006	17.993	0.706
	se GMM_0					0.1**	0.1**	4.265**			
						[0.246 1.141]	[0.247 1.142]	[0.653 1.744]			
	se GMM_{12}					0.104**	0.104**	4.009**			
						[0.208 0.981]	[0.208 0.982]	[0.562 1.67]			
					Panel B : Second Stage GMM						
1	λ mean	0.020	0.000	0.015					0.007	94.761	0
	λ vector	0.020	0.000	0.015					0.006		
	se GMM_{12}	0.005**	0.003**	0.004**							
		[0.021 0.076]	[0.013 0.042]	[0.016 0.052]							
2	λ mean				0.236				0.014	55.969	0
	λ vector				0.236				0.013		
	se GMM_{12}				0.087*						
					[0.092 0.438]						
3	λ mean					-0.032	-0.032	-0.710	0.013	18.554	0.673
	λ vector					-0.089	-0.088	-2.735	0.012		
	se GMM_{12}					0.063	0.063	1.399**			
						[0.05 0.24]	[0.05 0.24]	[0.15 0.513]			

TABLE 2.12: **Hansen–Jagannathan Distance Comparisons**

Notes: We apply various Hansen–Jagannathan (HJ) distance to evaluate pricing performances and to test the rankings (Zhang, 2011). The basic motivation of HJ distance supplies a method in order to find the least mis-specified one among candidates. Hansen and Jagannathan (1997) develop a measure of degree of misspecification of an asset pricing models. This measure is defined as: $min_{m \in \aleph} \|m - y\|$ the least squares distance between the family of stochastic discount factors that price all the assets correctly and the stochastic discount factor associated with and an asset pricing model.
The basic HJ, modified HJ (Kan and Robotti, 2008), unconstrained HJ and constrained HJ (Gospodinov,Kan and Robotti, 2010) are used to rank misspecified measures on candidate models.
Candidates are: consumption-based CAPM, Fama–French three-factor, and long-run three-factor models.

		CCAPM			**Fama–French Three-factor**			**Long-run Three-factor**		
		Misspe.	Stat.	p_{value}	Misspe.	Stat.	p_{value}	Misspe.	Stat.	p_{value}
1	HJ	0.54**	73.25	0						
	HJ_M	0.642**	103.437	0						
	HJ_U	0.54**	0.292	0						
	HJ_C	0.53**	0.192	0						
2	HJ				0.523**	68.749	0			
	HJ_M				0.614**	94.683	0			
	HJ_U				0.523**	0.274	0			
	HJ_C				0.511**	0.174	0.012			
3	HJ							0.511**	65.531	0
	HJ_M							0.594**	88.684	0
	HJ_U							0.511**	0.261	0.012
	HJ_C							0.507**	0.13	0.01
	HJ		$H_0:$			$LR3 < FF3 < CCAPM$			p_{value}:0.2488	
	HJ_M		$H_0:$			$LR3 < FF3 < CCAPM$			p_{value}:0.3636	
	HJ_U		$H_0:$			$LR3 < FF3 < CCAPM$			p_{value}:0.2456	
	HJ_C		$H_0:$			$LR3 < FF3 < CCAPM$			p_{value}:0.533	

TABLE 2.13: R^2, **Alpla, and** χ^2 **Tests on Fama–French 25 Size and Momentum**

Notes: The second and the third columns present R^2s and adjusted R^2s. Results with the constant term are reported in the brackets below. From the forth to the tenth columns show results on Alpha and χ^2 tests for pricing errors. Higher p values stands for we cannot reject the null hypothesis on pricing errors are equal to zeros. The last four columns represent misspecified measures.

	R Square(W Constant)		P_{value} **on** α **Test(W Constant)**				P_{value} **on J Test**			**Mis. Measure**			
	R^2	Adj R^2	i.i.d.	Shank	GMM_0	GMM_{12}	GMM_0	GMM_{12}	GMM_{12} 2	HJ	MHJ	HJU	HJC
Yogo	0.584	0.5679											
Fama–French 3	0.6528	0.6054											
Long-Run 3	0.6269	0.576											
	(0.227)	(0.1)											
	(0.917)	(0.9051)											
	(0.3828)	(0.2947)											
Yogo			0	0	0	0							
Fama–French 3			0	0	0	0							
Long-Run 3			0	0.914	0.643	0.092							
			(0)	(0)	(0)	(0)							
			(0)	(0.28)	(0.405)	(0)							
			(0)	(0.999)	(0.969)	(0.656)							
Yogo							0	0	0				
Fama–French 3							0	0	0				
Long-Run 3							0.643	0.963	0.589				
Yogo										0.590	0.731	0.600	0.582
Fama–French 3										0.581	0.714	0.581	0.572
Long-Run 3										0.571	0.696	0.571	0.561

TABLE 2.14: R^2, **Alpla, and** χ^2 **Tests on 30 Industry-sorted**

Notes: The second and the third columns present R^2s and adjusted R^2s. Results with the constant term are reported in the brackets below. From the forth to the tenth columns show results on Alpha and χ^2 tests for pricing errors. Higher p values stands for we cannot reject the null hypothesis on pricing errors are equal to zeros. The last four columns represent misspecified measures.

	R Square(W Constant)		P_{value} **on** α **Test(W Constant)**				P_{value} **on J Test**			**Mis. Measure**			
	R^2	Adj R^2	i.i.d.	Shank	GMM_0	GMM_{12}	GMM_0	GMM_{12}	GMM_{12} 2	HJ	MHJ	HJU	HJC
Yogo	0.315	0.291											
Fama–French 3	0.786	0.762											
Long-Run 3	0.746	0.718											
	(0.165)	(0.135)											
	(0.283)	(0.200)											
	(0.021)	(-0.092)											
Yogo			0.189	0.868	0.662	0							
Fama–French 3			0.268	0.488	0.337	0							
Long-Run 3			0.211	0.995	0.964	0.516							
			(0.673)	(0.728)	(0.643)	(0.000)							
			(0.664)	(0.685)	(0.644)	(0.041)							
			(0.700)	(0.741)	(0.743)	(0.019)							
Yogo							0.662	0	0				
Fama–French 3							0.337	0	0				
Long-Run 3							0.964	0.516	0.486				
Yogo										0.352	0.376	0.352	0.340
Fama–French 3										0.332	0.352	0.332	0.327
Long-Run 3										0.317	0.334	0.317	0.310

FIGURE 2.1: **Enjoyment of Various Activities in 1985**

9.3	Sex	6.9	
9.2	Play sports	6.8	
9.1	Fishing	6.7	**Second job**
9	Art, music	6.6	**Cook, work at home, shop**
8.9	Bars, lounges	6.5	
8.8	Play with kids, hug and kiss	6.4	**Child care, help adults**
8.7		6.3	**Work commute**
8.6	Talk/read to kids	6.2	
8.5	Sleep, church, attend movies	6.1	**Dress**
8.4		6	Pet care, **classes**
8.3	Read, walk	5.9	**Errands**
8.2	Work break, meals out, visit	5.8	**Housework**
8.1		5.7	
8	Talk with family	5.6	
7.9	Lunch break	5.5	**Home repair, grocery shopping**
7.8	Meal at home, TV, read paper	5.4	
7.7	Knit, sew	5.3	**Homework**
7.6		5.2	**Pay bills, iron**
7.5	Recreational trip	5.1	
7.4		5	**Yardwork**
7.3	Hobbies	4.9	**Clean house, dishes**
7.2	**Baby care**, exercise, meetings	4.8	**Laundry**
7.1	Gardening	4.7	**Child health, doctor, dentist**
7	**Work, homework help, bathe**	4.6	**Car repair shop**

Notes: Categories in bold are classified as nonleisure activities. Baby care is included only because most of the time use surveys do not distinguish baby care from other child care. Pet care is classified as a leisure activity because early time use surveys did not include it as household production.

Source: Robinson and Godbey 1999, Appendix O.

FIGURE 2.2: **Average Weekly Hours of Leisure**

Notes: Figure shows the average weekly leisure which takes accounts demographic and sector movements: adjusted leisure hours takes the demographic between different age cohorts into account; efficiency leisure hours consider about the productivity (real wage) among age cohorts; Tornqvist leisure hours use time-varying weights.

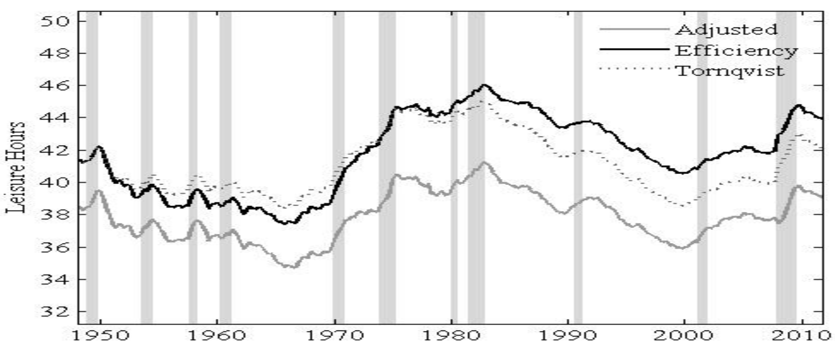


FIGURE 2.3: **Non-durable Consumption and Services**

Notes: Figure shows non-durable consumption and services.

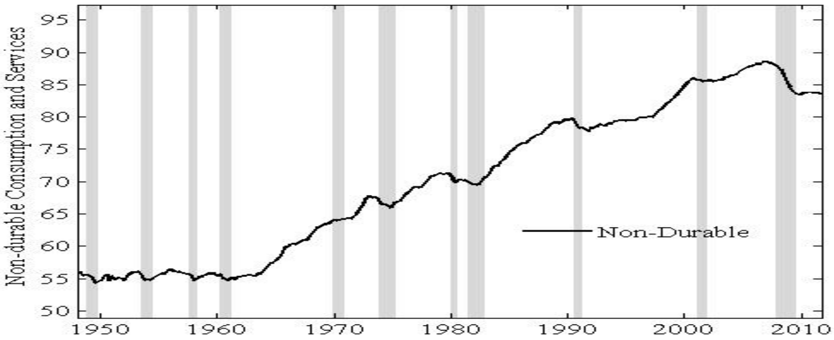


FIGURE 2.4: **The Growth on Hours of Leisure**

Notes: Figure shows the growth rate on average weekly leisure hours, which take the first order difference between log leisure hours.

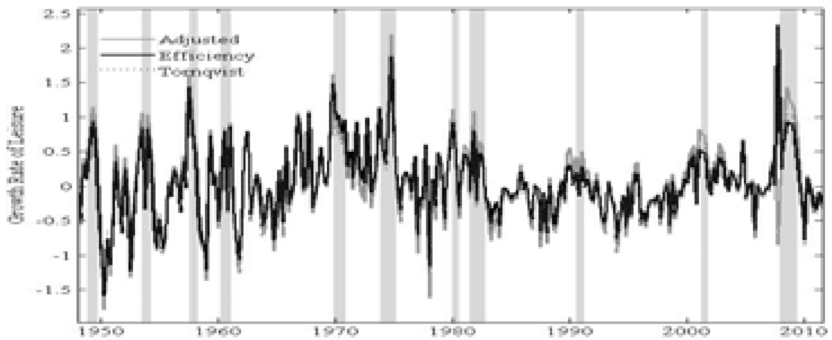


FIGURE 2.5: **The Growth on Non-durable Consumption and Services**

Notes: Figure shows the growth rate on non-durable consumption and services, which take the first order difference between log consumption and services.

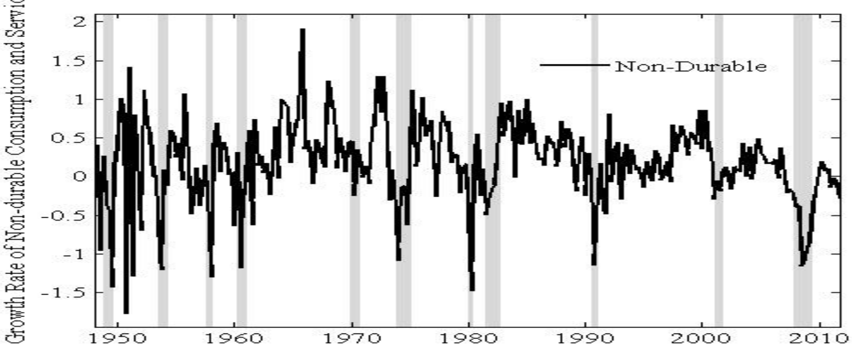


Figure 2.6: **Leisure Growth and Equity Returns**

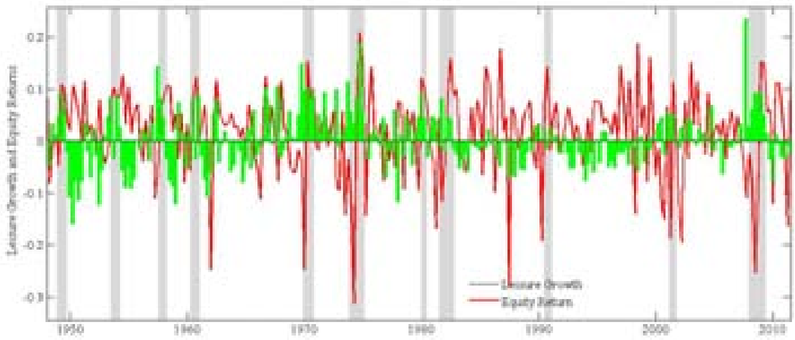


Figure 2.7: **Per Capita Consumption Growth and Equity Returns**

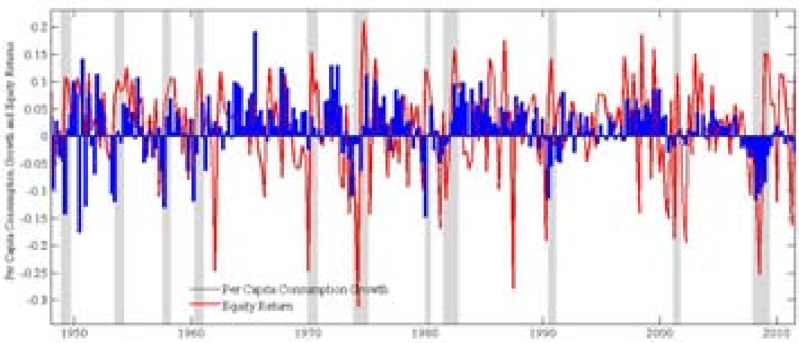


FIGURE 2.8: **Correlation Analysis for Consumption**

Notes: Figure shows the correlation analysis for quarterly and annual consumption with asset returns. The left part calculates the correlation between cumulative consumption growth and cumulative excess market returns, $corr(\sum_{j=1}^{k} g_{c,t+j}, \sum_{j=1}^{k} r_{t+j})$. The right part shows the bivariate spectral analysis for the quarterly data in the first column and the annual data in the second column. The coherence measures the correlation between consumption and returns at frequency λ. When the frequency is λ, the corresponding length of the cycle is $1/\lambda$ quarters. The cospectrum at frequency λ can be interpreted as the portion of the covariance between consumption growth and asset returns that is attributable to cycles with frequency λ. The slope of the phase spectrum at any frequency λ precisely measures the number of leads or lags between consumption and asset returns. When this slope is positive, consumption leads the market returns.

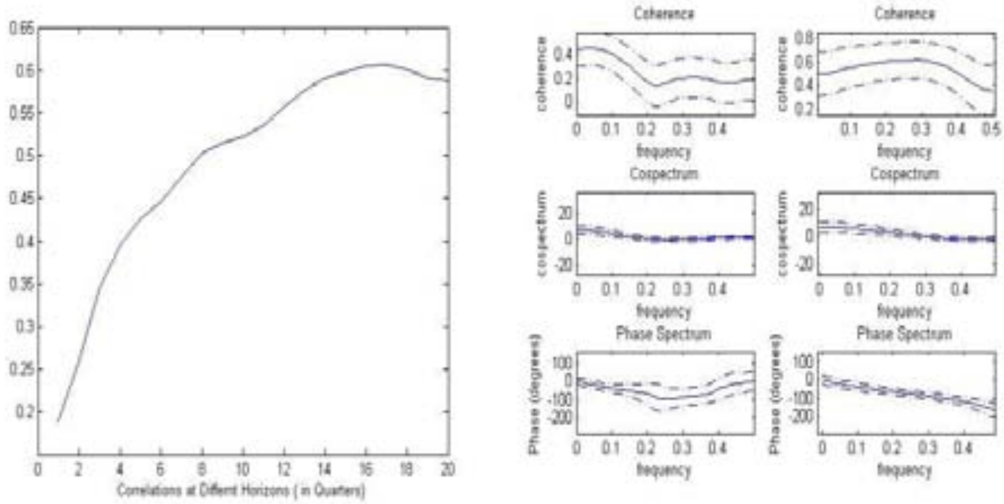


FIGURE 2.9: **Correlation Analysis for Leisure**

Notes: Figure shows the correlation analysis for quarterly and annual leisure with asset returns. The left part calculates the correlation between cumulative leisure growth and cumulative excess market returns, $corr(\sum_{j=1}^{k} g_{l,t+j}, \sum_{j=1}^{k} r_{t+j})$. The right part shows the bivariate spectral analysis for the quarterly data in the first column and the annual data in the second column. The coherence measures the correlation between leisure and returns at frequency λ. When the frequency is λ, the corresponding length of the cycle is $1/\lambda$ quarters. The cospectrum at frequency λ can be interpreted as the portion of the covariance between leisure growth and asset returns that is attributable to cycles with frequency λ. The slope of the phase spectrum at any frequency λ precisely measures the number of leads or lags between leisure and asset returns. When this slope is positive, leisure leads the market returns.

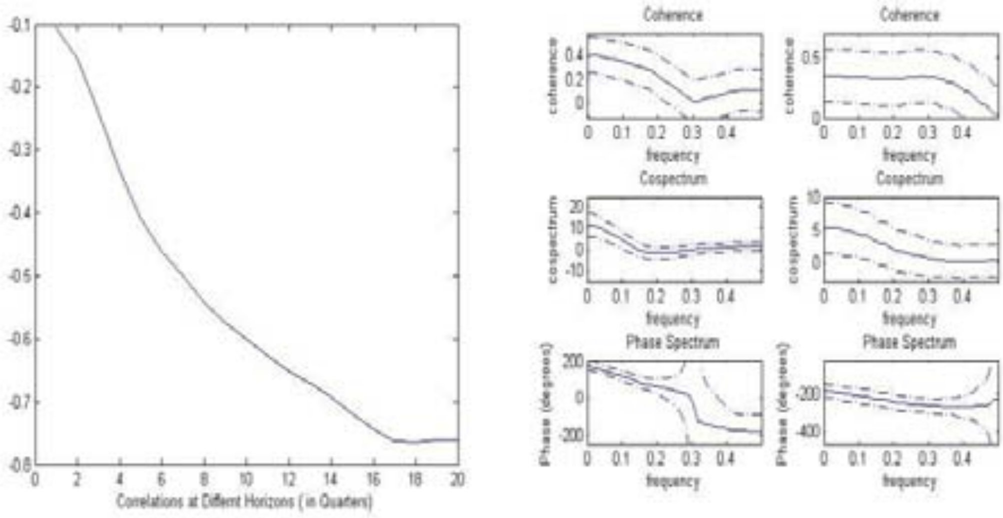


FIGURE 2.10: **Impulse Responses Functions (Growth)**

Notes: Figure shows the impulse responses functions for all growth variables in VAR to a shock. The green line stands for point estimates and the dash line denotes the 20% and 80% bands.

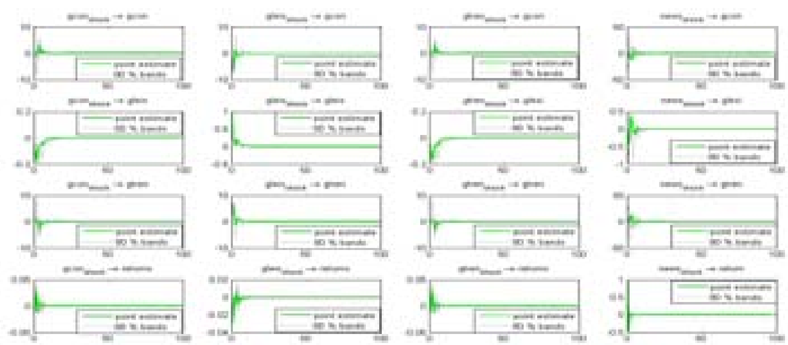
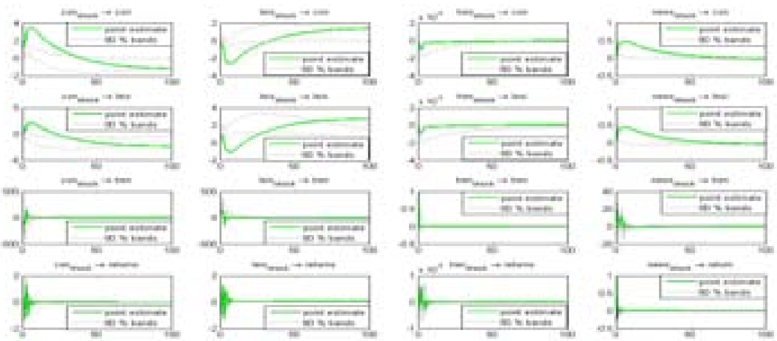


FIGURE 2.11: **Impulse Responses Functions (Log Level)**

Notes: Figure shows the impulse responses functions for all log variables in VAR to a shock. The green line stands for point estimates and the dash line denotes the 20% and 80% bands.



Chapter 3

Empirical Evaluation of Conditional Asset Pricing Models: An Economic Perspective

3.1 Introduction

A large recent literature attempts to explain various cross sectional anomalies with conditional models that have economically motivated conditioning variables (Ludvigson (2011) [41]). These conditional variables represent a measure of the state in the economy, and reflect the changing information sets of agents, while the prices on financial assets vary.

In general, asset pricing models with nonlinear stochastic discount factors (SDF) can be written as approximate linear multifactor models by multiplying out the conditioning variables and the fundamental factors (Cochrane (2001) [26] and Ludvigson (2011) [41]). For example, the conditional non-linear pricing kernel, like consumption-based CAPM (C-CAPM) with power utility, can be approximated as the function of consumption growth with time-varying weights[1]. We refer to these as linear scaled consumption-based models because the pricing kernel is the function of consumption growth and scaling factors.

[1] Brandt and Chapman (2011) [42] argues that the nonlinear time-varying risk premium matters for testing conditional models, but the nonlinear evidence is empirically weak.

These scaled consumption-based models offer a convenient way to represent state-dependency in the pricing kernel. In this case we can model the dependence of parameters in the SDF on the current information set. This dependence can be specified by scaling factors with instruments that summarize the state of the economy according to some models (Jagannathan and Wang (1996) [43], Lettau and Ludvigson (2001) [10], Lustig and Van Nieuwerburgh (2005) [11], Santos and Veronesi (2006) [5], and Piazessi et al. (2007) [24]).

However, most of these papers have applied the static Fama–MacBeth cross-sectional regression to estimate and test parameters. This paper makes two key contributions to the literature on investigating conditional asset pricing models. First, we use a recently developed method of dynamic Fama–MacBeth regressions to evaluate the performance of leading conditional CAPM (C-CAPM) models in a common set of test assets over the time period from 1951–2012. Second, we show that scaling factors that obtain persistence and slow-moving characteristics are of great help to explain the cross section of returns on value-growth and momentum.

To empirically apply the dynamic cross-sectional regression, we firstly estimated economic shocks to the state variables via a vector autoregression (VAR). At the second stage asset returns are regressed in the time series on lagged state variables and their contemporaneous innovations, generating predictive slopes and risk betas for each test asset. At the third stage prices of risk are obtained by running a static cross sectional (Fama–MacBeth) regression of the stacked predictive slopes onto the stacked betas.

There are several advantages to use the dynamic cross-sectional regression introduced by Adrian et al. (2012) [9]. Mainly, it focuses more on estimating and testing time-varying risk premiums, given state variables (according to their economic environment). Lewellen and Nagel (2006) [44] provide a simple test from short-window regressions in which each quarter's conditional alpha and beta are directly estimated. However, the high frequency of macro variables is hard to obtain. Finally, Ghysels and Goldberger (2012) [45] use MIDAS instruments (high frequency returns data and low frequency consumption data) to proxy the information set; we instead replicate and extend data used in several published papers.

Our main results are as follows. First, we find that the Lettau and Ludvigson (2001) conditional C-CAPM model does better than the Piazessi et al. (2007), Santos and Veronesi (2006) and Lustig and Van Nieuwerburgh (2005) models in explaining the cross-section of momentum and value-growth portfolios. Second, we find that the Piazessi et al. (2007) consumption model does better than others in pricing the cross-section of industry portfolios. Finally, we find that in the case of the cross-section of risk premia on U.S. government bond portfolios the conditional model in Santos and Veronesi (2006) outperforms other candidate models. Overall, however, the Lettau and Ludvigson (2001) model does better than other candidate models. Intuitively, the consumption-wealth ratio *cay* (Lettau and Ludvigson (2001)) is slow-moving and persistent across the time (Figure 1). In contrast, the collateral housing ratio (Lustig and Van Nieuwerburgh (2005)) and the labor income ratio (Santos and Veronesi (2006)) have the clear decreasing and increasing trend patterns, respectively.

The rest of this paper is organized as follows. Section 3.2 introduces the conditional asset pricing model and their SDF. Section 3.3 describes the test methodology. Section 3.4 presents data and Section 3.5 gives the empirical analysis. The final section summarizes the findings.

3.2 The Model

The basic conditional asset pricing formula describes the prices in terms of conditional moments which come from the first order Euler equation of investors.

$$p_{i,t} = E_t(m_{t+1}x_{i,t+1}), \tag{3.1}$$

where $p_{i,t}$ denotes the price of the ith portfolio at time t, $x_{i,t+1}$ presents the payoff on the ith portfolio at time $t+1$, m_{t+1} is the log stochastic discount factor (SDF), and E_t denotes the expectation conditional on the information known at time t.

We assume that there are a class of economies with complete financial markets and no arbitrage opportunities. The general form of the pricing kernel or the SDF states as following:

$$m_{t+1} = log(M_{t+1}) = -r_t^f + \frac{1}{2}\Lambda_t^{'}\Lambda_t - \Lambda_t^{'}\epsilon_{t+1}, \tag{3.2}$$

where r_t^f denotes the one-period risk-free rate at time t, Λ_t presents a K-vector of pricing kernel functions at time t, and ϵ_{t+1} is the vector of normalized common factor innovations with zero means and unit variances. We refer to Λ_t as risk premium functions and elements of Λ_t are nonlinear function of information available to the market at time t. We rewrite the pricing kernel as

$$m_{t+1} = -r_t^f + \frac{1}{2}\Lambda_t^2 - \Lambda_t w^{-1}\upsilon_{t+1}, \tag{3.3}$$

where $\upsilon_{t+1} = \delta^{'}\epsilon_t$ denotes the shock at time t, δ is a vector of constants and $w = (\delta^{'}\Sigma\delta)^{1/2}$. Here, Σ is the identity matrix.

Equation (3.2) and Equation (3.3) show that the SDF depends on the risk-free rate, the square risk premiums and innovations on an economy. First, the risk-free rate is negative to the SDF,

$$E_t[M_{t+1}R_{i,t+1}] = 1, \tag{3.4}$$

$$E_t[M_{t+1}] = \frac{1}{R^f_{t+1}}, \tag{3.5}$$

where M_{t+1} stands for the exponential SDF, $R_{i,t+1}$ is the gross return on the ith asset and R^f_{t+1} denotes the risk-free rate at date $t+1$.

Second, the square risk premiums can be defined the maximum Sharpe Ratio (SR) process, denoting S_t

$$S_t = \frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})} = \sqrt{\Lambda_t^2}. \tag{3.6}$$

The third component is the weighted innovations of an economy. For instance, a conditional CAPM (C-CAPM) sets up the state space as the market return (the consumption growth), the risk-free rate, and the conditional variable. Here, we assume a single risk premium on a single composite shock, and it is easy to extend to the case that the standard linear conditioning approximation carries over into a multifactor setting. Here, the market portfolio (the consumption growth) plays a specific role in the pricing kernel, but it is not true of all prominent pricing models[2]. Besides, the fact that the shock, υ, is normally distributed which implies that m_t is conditionally lognormal.

We also assume that asset returns and pricing kernel have a joint lognormal distribution, conditional on the current realizations of the market and the factors. We can apply the specific SDF to Equation (3.1). Hence,

$$E_t(r_{i,t+1}){+}E_t(m_{t+1}){+}\frac{1}{2}\left[var_t(r_{i,t+1}) + var_t(m_{t+1}) + 2cov_t(r_{i,t+1}, m_{t+1})\right] = 0, \forall i = 1, ..., N, \tag{3.7}$$

where $r_{i,t+1}$ denotes the log gross return to asset i at time $t+1$. The adjusted return premium to asset i is

$$E_t(r_{i,t+1}) - r_t^f + \frac{1}{2}var_t(r_{i,t+1}) = \Lambda_t cov_t(r_{i,t+1}, w^{-1}\upsilon_{t+1}), \forall i = 1, ..., N. \tag{3.8}$$

For the β-representation,

$$E_t(r_{i,t+1}) - r_t^f + \frac{1}{2}var_t(r_{i,t+1}) = \Lambda_t \beta_{i,t}, \tag{3.9}$$

[2]Bansal and Yaron (2005) [27] show that innovations plays significant role in the pricing kernel.

where

$$\beta_{i,t} = \frac{cov_t(r_{i,t+1}, \upsilon_{t+1})}{w^2}. \tag{3.10}$$

The above equation states that given the w, the conditional covariance matrix between innovations of the state space and future returns determine the risk of the asset i at date t. If the future macro news highly co-moves to the future returns on asset i, then the asset obtains high betas.

Here, the risk premium function is a nonlinear function on state variables. Brandt and Chapman (2011) [42] argue that the nonlinear time-varying risk premium matters for testing conditional models. However, they found that there is weak evidence the linear approximation will be largely affected according to the pricing error. Moreover, Jagannathan and Wang (1996) [43] study the linear form of Equation (3.9) in terms of the conditional CAPM

$$E[r_{i,t+1}|I_t] = \lambda_{0,t} + \lambda_{1,t}\beta_{i,t}, \tag{3.11}$$

where $\beta_{i,t}$ is the conditional beta of asset i defined as $\beta_{i,t} = \frac{cov(r_{i,t}, r_{m,t}|I_t)}{var(r_{m,t}|I_t)}$, and $r_{m,t}$ denotes the log returns on market portfolios. Moreover, they proxy for the conditional market risk premium $\lambda_{1,t}$ as variables that help to predict the business cycle, the yield spread between $BAA-$ and $AAA-$ rated bonds[3].

In this paper, we implement the general form of the SDF (Equation (3.3)) to explain several test portfolios. The time-varying risk premiums are estimated and tested by the dynamic cross-sectional regression stated below.

3.3 Test Methodology

We estimate and test prices of risk via an extended static cross-sectional asset pricing estimators (Fama–MacBeth setting) where prices of risk vary with observed state variables.

We assume that the dynamics of a $K \times 1$ vector of state variables X_t evolves according to the following vector autoregressive process:

$$X_{t+1} = \zeta + \Phi X_t + \upsilon_{t+1}, t = 1, ..., T, \tag{3.12}$$

[3]Stock and Watson (1989) and Bernanke (1990) find that the best single variable in predicting business cycles is the spread between the six-month commercial paper rate and six-month Treasury bill rate.

where initial condition X_0. For now I only assume that

$$B[\upsilon_{t+1}|F_t] = 0,\ V[\upsilon_{t+1}|F_t] = \Sigma_{\upsilon,t}, \tag{3.13}$$

where F_t denotes the information set at time t.

According to Equation (3.3), the log SDF can be defined as

$$m_{t+1} = -r_t^f + \frac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\Sigma_{\upsilon,t}^{-1/2}\upsilon_{t+1}, \tag{3.14}$$

where r_t^f denotes the risk-free rate at time t, λ_t is a $K \times 1$ vector assumed to be an affine function of the state variables X_t

$$\lambda_t = \Sigma_{\upsilon,t}^{-1/2}(\lambda_0 + \Lambda_1 X_t). \tag{3.15}$$

Need to mention, if $\Lambda_1 = 0$, then the prices of risk are constant. According to Equation (3.15), risk premiums are time-varying and depend on the state X_t of an economy at date t (Ferson and Harvey (1991) [46] and Ferson and Harvey (1999) [47]). Furthermore, factors follow a first order VAR.

Now we start to define holding period returns in excess of the risk free rate r_t^f of asset i by $r_{i,t+1}^e$. According to Equation (3.9), the beta representation of expected returns can be rewritten as

$$E_t[r_{i,t+1}^e] = \beta_{i,t}^{'}(\lambda_0 + \Lambda_1 X_t), \tag{3.16}$$

where $\beta_{i,t}$ is a time-varying exposure vector. Then excess returns can be decomposed as an expected and an unexpected component

$$r_{i,t+1}^e = \beta_{i,t}^{'}(\lambda_0 + \Lambda_1 X_t) + (r_{i,t+1}^e - E_t[r_{i,t+1}^e]). \tag{3.17}$$

The unexpected excess return $r_{i,t+1}^e - E_t[r_{i,t+1}^e]$ can be further decomposed into the innovations of the states, and a return pricing error $e_{i,t+1}$ that is conditionally orthogonal to the state innovations

$$r_{i,t+1}^e - E_t[r_{i,t+1}^e] = \gamma_{i,t}^{'}(X_{t+1} - E_t[X_t]) + e_{i,t+1}, \tag{3.18}$$

where $\gamma_{i,t} = \beta_{i,t}$. Therefore the excess returns of ith at time $t+1$ can be written as

$$r_{i,t+1}^e = \beta_{i,t}^{'}(\lambda_0 + \Lambda_1 X_t) + \beta_{i,t+1}^{'}\upsilon_{t+1} + e_{i,t+1}, t = 1, ..., T, \tag{3.19}$$

where $e_{i,t+1}$ is a return pricing error that is conditionally orthogonal to the state innovations. The above equation states that the excess return on ith asset depends on the expected return, $\beta^{'}_{i,t}(\lambda_0 + \Lambda_1 X_t)$, the component that is conditionally correlated with innovations of the states, $\beta^{'}_{i,t+1}\upsilon_{t+1}$, and the pricing error $e_{i,t+1}$.

Equation (3.19) shows that the expected excess return on ith asset is determined by the time-varying exposure vector and risk premiums ($\beta^{'}_{i,t}(\lambda_0 + \Lambda_1 X_t)$), and the innovations of an economy ($\beta^{'}_{i,t+1}\upsilon_{t+1}$). This form is different from the traditional β-representation which states that the expected excess returns depend on $\beta^{'}_{i,t}\lambda_t$.

To estimate parameters, we rewrite equations into the matrix version

$$r_e = B^{'}\Lambda Z_- + B^{'}V + E, \tag{3.20}$$

$$X = \Psi Z_- + V, \tag{3.21}$$

where $\Psi = [\zeta, \psi]$, $\Lambda = [\lambda_0, \Lambda_1]$, $X = [X_1, ..., X_T]$, $X_- = [X_0, ..., X_{T-1}]$, $Z_- = [\upsilon_T, X^{'}_-]^{'}$, E and V are matrices formed by $e_{i,t}$ and υ_t.

In order to economically test performances among models, we assume that prices of risk are affine functions of lagged state variables. The estimator can be described as follows. In the first stage, economic shocks to the state variables are obtained from a vector autoregression (VAR).

$$\hat{\Psi} = XZ^{'}_-(Z_-Z^{'}_-)^{-1}. \tag{3.22}$$

Then, asset returns are regressed in the time series on lagged state variables and their contemporaneous innovations, generating predictive slopes and risk betas for each test asset.

$$\hat{B} = (\hat{V}\hat{V}^{'})^{-1}\hat{V}r^{'}_e, \tag{3.23}$$

where $\hat{V}$ is the estimated innovations from the VAR.

In the third stage, prices of risk are obtained by running a cross sectional regression of the stacked predictive slopes onto the stacked betas.

$$\hat{\Lambda} = (\hat{B}\hat{B}^{'})^{-1}\hat{B}r_e Z^{'}_-(Z_-Z^{'}_-)^{-1}. \tag{3.24}$$

This cross sectional regression is based on OLS regressions, but it is more efficient when the variances of test assets are equal and on GLS generalization. Moreover, to avoid the error-in-variable problem because of estimated V and B in the second and the third stage, we re-estimate the B using $\Lambda\hat{Z}_- + \hat{V}$ instead of using $\hat{V}$ in Equation (3.12). Adrian et al. (2012) [9] find that the re-estimation step sharpens the estimation and inference about the risk premia $B^{'}\Lambda Z_{-}$ and the conditional pricing errors $B^{'}V + E$.

Given the asymptotic distributions of the estimators, we use the Wald test for the null hypothesis that a given row of Λ is equal to zero. The Wald statistic is

$$W_{\Lambda'} = \hat{\Lambda^i}^{'} \hat{\Sigma}_{\Lambda^i}^{-1} \hat{\Lambda}^i \xrightarrow{a} \chi^2(k), \tag{3.25}$$

where Λ^i denotes the ith row of Λ and k is the degree of freedom in a chi-square distribution.

Importantly, the true state space is unobservable and can be different to specific models. Therefore we firstly assume that the state space form (Equation (3.12)) is invertible. Then state variables can be expressed as a weighted sum of the current and past realizations of observables, and an economic shock can be expressed as a linear combination of the VAR innovations of observables. The identification of these shock components and the resulting asset pricing implications critically depends on the multivariate structure of predictability in all state variables; the set of information variables need to have predictive power beyond that of lagged state variables in the VAR system[4].

3.4 Data and Conditional Variables

The quarterly sample of 1952–2012 is used after intersecting the data on conditional variables used by several published papers and asset markets.

We use four test assets, for instance, Fama–French 25 size and book-to-market sorted portfolios, 25 size and momentum portfolios, 30 industry portfolios and U.S. government bond portfolios with different maturities on the left-hand side of the pricing formula. All the equity returns can be downloaded from Professor French's website, and government bonds yields data are from "The CRSP US Treasury Database". We use the three-month T-bill rate from FRED over the period January 1952 to December 2012 as the risk-free rate.

Lettau and Ludvigson (2001) [10] (LL) introduce *cay* (the consumption-wealth ratio) as the residual in which aggregate consumption, asset holdings, and labor income share a common long-term trend. They show that the log consumption-wealth ratio predicts asset return because it is a function of expected future returns on the market portfolio.

[4]Barsky and Sims (2011) [34] propose the identification methodology of two technology shocks in a structural VAR analysis. They put the first variable as the measure of technology.

Lustig and Van Nieuwerburgh (2005) [11] (LVN) choose *mymo* (the housing collateral ratio) as the conditional variable. They find that the ratio of housing wealth to human wealth changes the conditional distribution of consumption growth across households in a model with collateralized borrowing and lending. Intuitively, when the housing collateral ratio is low, households demand larger risk compensation, because the housing collateral ratio predicts aggregate stock returns.

Piazzesi et al. (2007) [24] (PST) find the conditional variable α (the non-housing expenditure ratio), since the composition risk which relates changes in asset prices also to changes in expenditure shares. During recession, investors expect higher future consumption; hence, they try to sell stocks today to increase current consumption. Therefore stock prices go down in bad times.

Santos and Veronesi (2006) [5] (SV) introduce s the labor income to consumption ratio as the conditional variable. They extend the standard consumption asset pricing model where consumption is funded by labor income, and that allows for tractable and interpretable formulas for prices and returns. The model shows that fluctuations in the fraction of consumption funded by labor income results in stock return predictability both in the time series and the cross-section.

3.5 Basic Empirical Results

3.5.1 The Main Result

Since we allow for time varying prices of risk by applying the estimator suggested above, therefore we test Λ_1 is statistically significant and pricing errors are reduced in economically meaningful or not.

Consistent with LL, we assume that there is no time variation in conditional second moment, for instance, $\beta_t = \beta$ and $V[R^M_{t+1}|F_t] = \Sigma_M, \forall t$ or $V[\Delta c_{t+1}|F_t] = \Sigma_{\Delta c}, \forall t$. Furthermore, we assume that risk-free rate is time-varying.

In the LL model, the vector of state variables becomes

$$X_t = (R^M_t, R^f_t, R^f_t \cdot cay_t)', \tag{3.26}$$

where cay_t stands for the consumption-wealth ratio at date t.

Since the state space depends on the market portfolios return, risk-free rate and the consumption-wealth ratio scaled by the risk-free rate, therefore it is more general and we allow time variation in risk premia which comes from all elements of the state space instead of *cay* alone. Table 1 reports the estimated risk premiums for factors via the four-stage OLS and GLS estimators to explain return-spreads on test portfolios, i.e., the Fama-French 25 size and book-to-market ratio, 30 industry, 25 size and momentum and government bonds portfolios.

We obtain the similar result while pricing return-spreads on size/book-to-market ratio sorted and size/momentum sorted portfolios. The last column shows that *cay* leads to the time-varying market risk premium among size, book-to-market ratio, industry, and momentum sorted portfolios, except for bonds portfolios. The impact of $R^f \cdot cay$ on the price of market risk is weak when explaining the industry effect and government bonds. For the time-varying risk-free rate, its impact on the price of market risk is the same as the consumption-wealth ratio.

In sum, equity premia are time-varying in size, book-to-market ratio and momentum sorted portfolios because of time-varying price of market risk (the 'market' portfolio). This market risk premium can be explained by the time-varying market portfolio return, the risk-free rate and the consumption-wealth ratio. To be more specific, the time-varying expected return can be accounted for by a time-varying risk-free rate or monetary policy shocks (Cochrane and Piazzesi (2005) [48]). Moreover, LL shows that the log consumption-wealth ratio has predictive power for equity premia.

Table 1 also compares the two different estimators - the four stage OLS and GLS. We find that the point estimates for the prices of risk are different from these two estimators. The reason is due to the weighting in the efficient estimation approach.

Consumption-based asset pricing models suggest that risk premiums could vary alongside changes in economic conditions. Investors decide at any time how much to save, how much to consume, and prefer to have a steady consumption stream. Therefore, investors view assets as hedging products which may help them smooth their consumption stream, and then rationally require a higher risk premium on assets that are correlated positively with business conditions. For the conditional C-CAPM, we specify the fundamental factor as the consumption growth. The time-varying price of consumption risk comes from observable economic information such as the risk-free rate and the housing collateral ratio. Hence, the time-varying risk premium is related to the covariance of the risky asset return with the consumption growth and the state variables themselves related to the growth rate of wealth. For instance, LVN shows that household demand a larger compensation while the housing collateral ratio is low, because the housing collateral ratio predicts aggregate stock returns. Given the risk exposure, the expected risk premium changes across time, because the borrowing and lending constraints are time varying. In this case we define the vector of state variables as

$$X_t = (\Delta c_t, R_t^f, R_t^f \cdot mymo_t)', \tag{3.27}$$

where $mymo_t$ denotes the housing collateral ratio at date t. The last column in Table 2 reports that the price of risk is time-varying when pricing all the test portfolios via the GLS estimator. When looking at individual λ_0 and Λ_1, all parameters are not significant; there is no difference to use conditional or unconditional for explaining expected returns. But while combining these three variables together, the time-varying price of risks explains return-spreads on test portfolios.

Unlike the risk premium comes from the housing collateral lending constraint, PST show that the risk premium is time-varying because of changes in expenditure shares between non-housing and housing consumption. First, the housing share forecasts excess returns on stocks. Second, if investors can substitute between non-housing and housing consumption, it will increase the downward pressure on stock prices in severe recessions while the share of housing consumption becomes low. According to the PST economy, the vector of state variables is

$$X_t = (\Delta c_t, R_t^f, R_t^f \cdot \alpha_t)', \tag{3.28}$$

where α_t is the non-housing expenditure ratio at date t.

Table 3 shows that the elements on Λ_1 are significantly different from zeros; time variation in prices of risk is due to time varying risk-free rates and time variation in the non-housing expenditure ratio scaled by the risk-free rate. When looking at individual elements, the time variation in prices of risk comes from the time-varying risk-free rate and the expenditure shares to explain the industry effect. Need to mention, in the PST model, the substitution between non-housing and housing consumption leads to changes in their expenditure shares without the solvency constraints. To be more specific, the SDF in the LVN model is

$$m_{t+1} = m_{t+1}^a g_{t+1}^\gamma, \tag{3.29}$$

where m_{t+1}^a denotes the IMRS of a representative agent who consumes non-durable consumption and housing services, and g_{t+1}^γ is the liquidity factor contributed by the solvency constraints.

The Λ_t in the LVN model is different from that in the PST model which states α_t scaled by g_t^γ. After comparing the individual elements in Λ_1 for both the LVN and the PST models, the solvency constraints may not be of help to explain the time variation in prices of risk in the industry portfolios.

At last, the labor income conditional CAPM model is investigated. SV show that since the consumption is funded by labor income, fluctuations in the fraction of consumption funded by labor income results in stock return predictability both in the time series and the cross-section. The vector of the state variables becomes

$$X_t = (\Delta c_t, R_t^f, R_t^f \cdot s_t)', \tag{3.30}$$

where s_t denotes the labor income to consumption ratio at date t.

The last column in Table 4 shows that time variation in prices of risk is due to the labor income scaled by the risk free rate for its element on Λ_1 is significantly different from zero while pricing return spreads on 30 industry portfolios and government bonds. For individual elements in Λ_1, the time-varying labor income is better to capture the time variation in prices of risk for bond portfolios, but not equity portfolios. While looking at the individual elements, the time varying labor income leads to the time varying risk premiums to explain government bonds only.

In sum, the conditional housing consumption model (Piazzesi et al. (2007)) without the solvency constraints is able to explain the time variation in prices of risk when pricing return spreads on the 30 industry portfolios. Moreover, the conditional labor income CAPM model (Santos and Veronesi (2006)) outperforms other conditional models, i.e., Lettau and Ludvigson (2001), Lustig and Van Nieuwerburgh (2005), and Piazzesi et al. (2007), to explain time-varying prices of risk while pricing return premiums on U.S. government bonds portfolios.

3.5.2 Robustness Check

3.5.2.1 Other Conditional Variables

In this part, we investigate the conditional CAPM and C-CAPM with the same state variables. Firstly we put def_t the default rate, which is defined as the yield spread between $BAA-$ and $AAA-$ rated bonds and helps to predict the business cycle (Jagannathan and Wang (1996) [43]) as one element of the state variables.

In Table 5 and Table 6, we report the results based on conditional CAPM and C-CAPM models. The last columns in both tables show that the market portfolios return and the consumption growth scaled by the default rate that predicts the business cycle help to explain the time variation in prices of risk.

When looking at the individual elements in Λ_1, the time variation of prices of risk is due to the time varying market portfolio return and the risk-free rate but not the default rate while pricing return spreads on size, book-to-market ratio and momentum sorted stocks. On the other hand, this conditional CAPM with the default rate is not able to capture the industry effect and risk spreads on government bonds. For the conditional C-CAPM with the default rate, the time variation in prices of risk is due to the time varying risk-free rate and the default rate when pricing return spreads on size, book-to-market ratio and momentum sorted portfolios. Different from the conditional CAPM, the prices of risk associated with the default rate feature time variation while pricing the risk spreads on government bonds; the time variation in prices of risk is due to the time-varying the risk-free rate and the default rate, or the monetary policy.

Moreover, we also choose the dividend yield diy_t, the ratio between dividends and equity prices, into the state space, because an impressive list of academic papers documents a statistical relationship between the dividend yield and risk premiums.

Results for the conditional CAPM and C-CAPM with the dividend yields are shown in Table 7 and Table 8, in which elements Λ_1 are jointly significantly different from zeros via the GLS estimator. While pricing return spreads on size, book-to-market ratio sorted stocks and government bonds, the dividend yield in the conditional CAPM features the time variation in prices of risk, but it is not able to achieve the same result in the conditional C-CAPM. Moreover, adding the dividend yield into conditional models are better to explain the time variation in prices of risk while explaining the industry effect on stocks, but the conditional models with the default rate feature time variation in prices of risk when pricing return spreads on size, book-to-market ratio sorted equities and government bond portfolios. Furthermore, comparing the conditional CAPM with C-CAPM, there exist big differences on the pricing performance with the dividend yield, but not much with the default rate.

3.5.2.2 Conditional Variables and the Business Cycle

In this part, we show that the 'successful' conditional variables are slow-moving and exhibit cyclical patterns related to the business cycle. Especially, their behavior seems consistent with the economic intuition derived from inter-temporal asset pricing models: economic recessions should trigger higher risk premiums. Here, the recession period data comes from the NBER.

From the main result part, we find that both the consumption-wealth ratio *cay* and the non-housing and housing expenditure ratio α help the CAPM and the CCAPM to explain equity premiums and yields on government bonds. Figure 1 shows that these two variables are slow-moving and persistent across the time, except that there is an increasing trend on the expenditure ratio from 1952 to 1958. Moreover, the collateral housing ratio and the labor income ratio have the clear decreasing and increasing trend patterns, respectively. For the default rate and the dividend yield, there are huge volatiles during the subprime financial crisis period.

3.6 Conclusion

In this paper, we investigate several conditional asset pricing models via the dynamic cross-sectional regression in which considering time-varying prices of risks. Given the constant risk exposure, the paper finds that conditional models usually have time-varying prices of risks via the GLS estimator. To be more specific, the conditional CAPM outperforms the conditional C-CAPM to explain the time-varying risk premiums on equity returns, because the time-varying price of the market risk is found to be significantly different from zero, while the time-varying price of the consumption risk is not. Among the conditional C-CAPM, the labor income leading to the time-varying price of consumption risk makes the conditional CCAPM explain yields on government bonds, and the time-varying price of the expenditure ratio risk between non-housing and housing consumption helps to capture the industry effect.

Moreover, the paper finds that the conditional variables which have the macroeconomic background are better than the conditional variables directly coming from the financial market, like the dividend yield. Second, the conditional C-CAPM can compete again with the conditional CAPM to explain the industry effect if the conditional variables are slow-moving and persistent across the time.

Table 3.1: **Lettau–Ludvigson - Price of Risk Estimates**

Notes: This table reports estimates for the price of risk parameters λ_0 and Λ_1 in the conditional CAPM with the state variables as R^M, R^f, and $R^f \cdot cay$. Two different estimators, 4-step OLS and GLS, are shown. The last column reports Wald statistics for the null of a respective row of Λ_1 being equal to zero. The sample period is 1952Q1-2012Q3. ** and *** denote statistical significance at 5% and 1% level, respectively.

	λ_0	Λ_{R^M}	Λ_{R^f}	$\Lambda_{R^f \cdot cay}$	χ^2 test
Panel A. Fama-French 25 Size and B/M					
4S-OLS					
R^M	0.2908	-0.5342**	-49.3765	48.8409	63.6877***
R^f	-0.0628**	-0.0099	3.5560	-3.3076	52.643***
$R^f \cdot cay$	-0.0673**	-0.0110	3.8612	-3.5953	51.2602***
4S-GLS					
R^M	0.1045	-0.4651**	-45.7816**	45.5024**	108.742***
R^f	-0.0156**	-0.0244**	-1.6653	1.6826	141.26***
$R^f \cdot cay$	-0.0176**	-0.0260**	-1.5958	1.6211	142.5***
Panel B. 30 Industry					
4S-OLS					
R^M	0.3251	-0.3383	-47.7998	46.3921	70.8743***
R^f	-0.0285	-0.0037	5.8899	-5.8303**	28.5278
$R^f \cdot cay$	-0.0324	-0.0049	6.2435**	-6.1709**	28.8159
4S-GLS					
R^M	0.2554**	-0.2678	-57.1052**	54.9663**	99.874***
R^f	-0.0206**	-0.0082	2.3979	-2.4075	99.26***
$R^f \cdot cay$	-0.023**	-0.0093	2.5844	-2.5863	101.94***
Panel C. Fama-French 25 Size and Momentum					
4S-OLS					
R^M	0.2908	-0.5342**	-49.3765	48.8409	63.6877***
R^f	-0.0628**	-0.0099	3.5560	-3.3076	52.643***
$R^f \cdot cay$	-0.0673**	-0.0110	3.8612	-3.5953	51.2602***
4S-GLS					
R^M	0.1045	-0.4651**	-45.7816**	45.5024**	108.7418***
R^f	-0.0156**	-0.0244**	-1.6653	1.6826	141.2614***
$R^f \cdot cay$	-0.0176**	-0.026**	-1.5958	1.6211	142.518***
Panel D. Government Bonds					
4S-OLS					
R^M	0.0754	-0.5816	-19.5286	16.0253	18.9408***
R^f	0.0008	-0.0072	1.9758	-1.9561**	9.3555
$R^f \cdot cay$	0.0006	-0.0050	2.2721**	-2.2396**	8.3124
4S-GLS					
R^M	0.0735**	-0.5621	1.7803	-3.5781	29.7681***
R^f	0.0009	-0.0047	1.3610	-1.3884	12.6542
$R^f \cdot cay$	0.0008	-0.0033	1.5218	-1.5506	13.1034

Table 3.2: **Lustig–Van Nieuwerburgh - Price of Risk Estimates**

Notes: This table reports estimates for the price of risk parameters λ_0 and Λ_1 in the conditional CCAPM with the state variables as Δc, R^f, and $R^f \cdot mymo$. Two different estimators, 4-step OLS and GLS, are shown. The last column reports Wald statistics for the null of a respective row of Λ_1 being equal to zero. The sample period is 1952Q1-2012Q3. ** and *** denote statistical significance at 5% and 1% level, respectively.

	λ_0	$\Lambda_{\Delta c}$	Λ_{Rf}	$\Lambda_{Rf \cdot mymo}$	χ^2 test
Panel A. Fama-French 25 Size and B/M					
4S-OLS					
Δc	0.1222**	0.3385	-0.1291	0.1599	34.4358
R^f	-0.0435**	0.1621	0.1941	-0.1620	17.5260
$R^f \cdot mymo$	-0.033**	0.1674	0.1303	-0.0844	14.8941
4S-GLS					
Δc	0.082**	0.1521	0.2040	-0.1776	107.989***
R^f	-0.028**	0.0812	0.1154	-0.1138	47.333***
$R^f \cdot mymo$	-0.0178**	0.0678	0.1153	-0.1178	51.371***
Panel B. 30 Industry					
4S-OLS					
Δc	0.0434	-0.4173	-0.4029	0.2855	30.3999
R^f	-0.0563**	0.1557	-0.2530	0.1680	19.5395
$R^f \cdot mymo$	-0.0517**	0.0680	-0.1594	0.1320	20.9877
4S-GLS					
Δc	0.0282**	-0.1396	-0.3861	0.4179	65.663***
R^f	-0.0256**	0.0224	-0.0875	0.0154	58.362***
$R^f \cdot mymo$	-0.022**	0.0053	-0.0638	0.0226	88.494***
Panel C. Fama-French 25 Size and Momentum					
4S-OLS					
Δc	0.1222**	0.3385	-0.1291	0.1599	34.4358
R^f	-0.0435**	0.1621	0.1941	-0.1620	17.5260
$R^f \cdot mymo$	-0.033**	0.1674	0.1303	-0.0844	14.8941
4S-GLS					
Δc	0.082**	0.1521	0.2040	-0.1776	107.989***
R^f	-0.028**	0.0812	0.1154	-0.1138	47.333***
$R^f \cdot mymo$	-0.0178**	0.0678	0.1153	-0.1178	51.371***
Panel D. Government Bonds					
4S-OLS					
Δc	-0.0004	-0.3052	0.0500	-0.0194	4.796
R^f	-0.0003	0.0555	-0.1139	0.1237	38.6563***
$R^f \cdot mymo$	0.0003	0.0091	-0.0780	0.0301	20.0299***
4S-GLS					
Δc	-0.0074	-0.1300	-0.0219	1.2731	10.4983
R^f	-0.0001	0.0615	-0.0993	0.0566	96.514***
$R^f \cdot mymo$	0.0003	0.0327	-0.0932	0.0370	17.9014**

Table 3.3: **Piazzesi–Schneider–Tuzel - Price of Risk Estimates**

Notes: This table reports estimates for the price of risk parameters λ_0 and Λ_1 in the conditional CCAPM with the state variables as Δc, R^f, and $R^f \cdot \alpha$. Two different estimators, 4-step OLS and GLS, are shown. The last column reports Wald statistics for the null of a respective row of Λ_1 being equal to zero. The sample period is 1952Q1-2012Q3. ** and *** denote statistical significance at 5% and 1% level, respectively.

	λ_0	$\Lambda_{\Delta c}$	Λ_{R^f}	$\Lambda_{R^f \cdot \alpha}$	χ^2 test
Panel A. Fama-French 25 Size and B/M					
4S-OLS					
Δc	0.0771	0.0327	-4.7065	5.8912	35.5018
R^f	-0.0805**	0.3058	-4.6698	6.1353	13.0211
$R^f \cdot \alpha$	-0.0617**	0.2380	-3.5321	4.6439	13.1349
4S-GLS					
Δc	0.0685**	0.0554	4.8790	-6.2055	126.706***
R^f	-0.0235**	0.0963	2.0777	-2.6017	29.7112
$R^f \cdot \alpha$	-0.0184**	0.0751	1.5752	-1.9713	30.1657
Panel B. 30 Industry					
4S-OLS					
Δc	0.1409**	-0.1279	4.4280	-6.1001	20.6477
R^f	-0.0715**	0.1508	-4.3220	5.4254	46.2027**
$R^f \cdot \alpha$	-0.0556**	0.1177	-3.3772	4.2385	46.7921**
4S-GLS					
Δc	0.0352**	-0.0514	1.1942	-1.7031	91.5112***
R^f	-0.0299**	0.0341	-5.618**	7.1097**	54.0947***
$R^f \cdot \alpha$	-0.0233**	0.0262	-4.358**	5.5146**	54.9517***
Panel C. Fama-French 25 Size and Momentum					
4S-OLS					
Δc	0.0771	0.0327	-4.7065	5.8912	35.5018
R^f	-0.0805**	0.3058	-4.6698	6.1353	13.0211
$R^f \cdot \alpha$	-0.0617**	0.2380	-3.5321	4.6439	13.1349
4S-GLS					
Δc	0.0685**	0.0554	4.8790	-6.2055	126.706***
R^f	-0.0235**	0.0963	2.0777	-2.6017	29.7112
$R^f \cdot \alpha$	-0.0184**	0.0751	1.5752	-1.9713	30.1657
Panel D. Government Bonds					
4S-OLS					
Δc	0.0021	-0.5934	23.4092	-30.1410	2.9332
R^f	-0.0007	0.0957	-4.5025	5.7871	19.0809***
$R^f \cdot \alpha$	-0.0006	0.0736	-3.2881	4.2240	22.1662***
4S-GLS					
Δc	-0.0073	-0.1292	1.2683	-0.6780	10.4435
R^f	0.0002	0.0592	-3.0213	3.7638	14.2071**
$R^f \cdot \alpha$	0.0002	0.0470	-2.2312	2.7787	17.3288**

TABLE 3.4: **Santos–Veronesi - Price of Risk Estimates**

Notes: This table reports estimates for the price of risk parameters λ_0 and Λ_1 in the conditional CCAPM with the state variables as Δc, R^f, and $R^f \cdot labor$. Two different estimators, 4-step OLS and GLS, are shown. The last column reports Wald statistics for the null of a respective row of Λ_1 being equal to zero. The sample period is 1952Q1-2012Q3. ** and *** denote statistical significance at 5% and 1% level, respectively.

	λ_0	$\Lambda_{\Delta c}$	Λ_{R^f}	$\Lambda_{R^f \cdot labor}$	χ^2 test
Panel A. Fama-French 25 Size and B/M					
4S-OLS					
Δc	0.1140	-0.3076	4.2447	-4.5577	26.3951
R^f	-0.0747**	-0.0373	5.2559	-5.3560	8.7605
$R^f \cdot labor$	-0.073**	-0.0338	5.2860	-5.3854	8.8095
4S-GLS					
Δc	0.077**	-0.0494	32.5992	-33.2021	110.578***
R^f	-0.0255**	0.0340	11.0193	-11.2051	31.7282
$R^f \cdot labor$	-0.025**	0.0338	10.7943	-10.9763	31.7982
Panel B. 30 Industry					
4S-OLS					
Δc	0.1398**	-0.0925	-19.3746	19.3546	22.7256
R^f	-0.076**	0.1378	-17.2344	17.4668	17.7678
$R^f \cdot labor$	-0.074**	0.1363	-17.0168	17.2459	17.7868
4S-GLS					
Δc	0.0364**	-0.0179	-20.0873	20.3038	89.1027***
R^f	-0.0298**	0.0225	-7.4298	7.4975	44.5743**
$R^f \cdot labor$	-0.0291**	0.0224	-7.3140	7.3802	44.307**
Panel C. Fama-French 25 Size and Momentum					
4S-OLS					
Δc	0.114	-0.3076	4.2447	-4.5577	26.3951
R^f	-0.0747**	-0.0373	5.2559	-5.3560	8.7605
$R^f \cdot labor$	-0.073**	-0.0338	5.2860	-5.3854	8.8095
4S-GLS					
Δc	0.0771**	-0.0494	32.5992	-33.2021	110.578***
R^f	-0.0255**	0.0340	11.0193	-11.2051	31.7282
$R^f \cdot labor$	-0.0251**	0.0338	10.7943	-10.9763	31.7982
Panel D. Government Bonds					
4S-OLS					
Δc	0.0021	-0.4675	36.2953	-37.1102	2.2019
R^f	-0.0008	0.0703	3.6970	-3.7708	12.7437
$R^f \cdot labor$	-0.0008	0.0685	3.3144	-3.3821	13.6856**
4S-GLS					
Δc	-0.0132	-0.1441	90.5931	-91.2787	9.9359
R^f	0.00007	0.0756**	-3.4390	3.4385	9.9709
$R^f \cdot labor$	0.00002	0.0731**	-3.4188	3.4197	10.2357

TABLE 3.5: **The Default Rate in CAPM - Price of Risk Estimates**

Notes: This table reports estimates for the price of risk parameters λ_0 and Λ_1 in the conditional CAPM with the state variables as R^M, R^f, and $R^f \cdot def$. Two different estimators, 4-step OLS and GLS, are shown. The last column reports Wald statistics for the null of a respective row of Λ_1 being equal to zero. The sample period is 1952Q1-2012Q3. ** and *** denote statistical significance at 5% and 1% level, respectively.

	λ_0	Λ_{R^M}	Λ_{R^f}	$\Lambda_{R^f \cdot def}$	χ^2 test
Panel A. Fama-French 25 Size and B/M					
4S-OLS					
R^M	0.7619**	-0.5187**	0.3655	0.6313	52.8147***
R^f	-0.079**	-0.0465**	0.0201	-4.3452	16.0094
$R^f \cdot def$	0.0080	0.0011	-0.0153	0.5042	16.3525
4S-GLS					
R^M	0.1682**	-0.4507**	1.1260	-5.8654	129.4754***
R^f	-0.0161**	-0.0272**	0.0567	-1.6456	119.2444***
$R^f \cdot def$	0.0018**	0.0002	-0.0043	0.1902	123.3678***
Panel B. 30 Industry					
4S-OLS					
R^M	1.2571**	0.0549	-1.6813	56.2197	53.1442**
R^f	-0.0395	-0.0078	-0.1519	0.5858	33.9809
$R^f \cdot def$	0.0113**	0.0050	-0.0254	0.8636	19.6458
4S-GLS					
R^M	0.3975**	-0.1356	-0.8557	32.0535	127.9018***
R^f	-0.018**	-0.0092	-0.0970	0.2323	114.9072***
$R^f \cdot def$	0.0023**	0.0014	-0.0111**	0.3025	94.6947***
Panel C. Fama-French 25 Size and Momentum					
4S-OLS					
R^M	0.7619**	-0.5187**	0.3655	0.6313	52.8147***
R^f	-0.079**	-0.0465**	0.0201	-4.3452	16.0094
$R^f \cdot def$	0.0080	0.0011	-0.0153	0.5042	16.3525
4S-GLS					
R^M	0.1682**	-0.4507**	1.1260	-5.8654	129.4754***
R^f	-0.0161**	-0.0272**	0.0567	-1.6456	119.2444***
$R^f \cdot def$	0.0018**	0.0002	-0.0043	0.1902	123.3678***
Panel D. Government Bonds					
4S-OLS					
R^M	0.0547	-0.4832	-0.9673	19.0502	33.2754***
R^f	0.0011	-0.0118	-0.0522	-0.3606	888.8275***
$R^f \cdot def$	-0.0002	0.0019	0.0073	0.0333	52.0599***
4S-GLS					
R^M	0.0596	-0.4510	-1.5519	52.6596	31.9123***
R^f	0.0007	-0.0036	-0.0805**	0.4054	1169.3691***
$R^f \cdot def$	0.00004	-0.0002	0.0048	-0.2049	69.5571***

TABLE 3.6: **The Default Rate in CCAPM- Price of Risk Estimates**

Notes: This table reports estimates for the price of risk parameters λ_0 and Λ_1 in the conditional CCAPM with the state variables as Δc, R^f, and $R^f \cdot def$. Two different estimators, 4-step OLS and GLS, are shown. The last column reports Wald statistics for the null of a respective row of Λ_1 being equal to zero. The sample period is 1952Q1-2012Q3. ** and *** denote statistical significance at 5% and 1% level, respectively.

	λ_0	$\Lambda_{\Delta c}$	Λ_{Rf}	$\Lambda_{Rf \cdot def}$	χ^2 test
Panel A. Fama-French 25 Size and B/M					
4S-OLS					
Δc	0.148**	0.0526	-0.0444	10.7087	22.5840
R^f	-0.0761**	0.0618	-0.0136	-1.2940	21.4755
$R^f \cdot def$	-0.0019	-0.0287**	-0.0151	0.1522	27.0656
4S-GLS					
Δc	0.0817**	0.0803	0.1085	7.5892**	100.24***
R^f	-0.0271**	0.0596	0.0065	-1.6943	78.3705***
$R^f \cdot def$	-0.0001	-0.0100	-0.0103**	0.0910	62.3771***
Panel B. 30 Industry					
4S-OLS					
Δc	0.1972**	-0.1611	-0.2950	11.4202	17.6129
R^f	-0.0654**	0.1548	-0.1284	-0.5279	30.5069
$R^f \cdot def$	-0.0023	-0.0011	-0.0084	0.1392	58.1819**
4S-GLS					
Δc	0.0324**	-0.1052	-0.0734	6.8577	77.3256***
R^f	-0.0284	0.0200	-0.0833	-0.4771	168.0809***
$R^f \cdot def$	0.0008	-0.0018	-0.0058	0.0635	74.9255***
Panel C. Fama-French 25 Size and Momentum					
4S-OLS					
Δc	0.148**	0.0526	-0.0444	10.7087	22.5840
R^f	-0.0761**	0.0618	-0.0136	-1.2940	21.4755
$R^f \cdot def$	-0.0019	-0.0287**	-0.0151	0.1522	27.0656
4S-GLS					
Δc	0.0817**	0.0803	0.1085	7.5892**	100.24***
R^f	-0.0271**	0.0596	0.0065	-1.6943	78.3705***
$R^f \cdot def$	-0.0001	-0.0100	-0.0103**	0.0910	62.3771***
Panel D. Government Bonds					
4S-OLS					
Δc	-0.0017	-0.2905	0.1912	-10.4710	12.3685***
R^f	-0.0002	0.0582	-0.0498	-0.3470	967.2988***
$R^f \cdot def$	-0.0001	0.0040	0.0092	-0.0998	66.4262***
4S-GLS					
Δc	-0.0087	-0.1406	0.8969	-19.1171	11.1954***
R^f	-0.0001	0.0646**	-0.0613**	-0.3449	1058.0613***
$R^f \cdot def$	-0.0001	0.0067	0.0056	-0.2885	68.4467***

TABLE 3.7: **The Dividend Yields in CAPM - Price of Risk Estimates**

Notes: This table reports estimates for the price of risk parameters λ_0 and Λ_1 in the conditional CAPM with the state variables as R^M, R^f, and $R^f \cdot diy$. Two different estimators, 4-step OLS and GLS, are shown. The last column reports Wald statistics for the null of a respective row of Λ_1 being equal to zero. The sample period is 1952Q1-2012Q3. ** and *** denote statistical significance at 5% and 1% level, respectively.

	λ_0	Λ_{R^M}	Λ_{R^f}	$\Lambda_{R^f \cdot diy}$	χ^2 test
Panel A. Fama-French 25 Size and B/M					
4S-OLS					
R^M	0.3543	-0.4350	2.5981	-2.9047	63.4735**
R^f	-0.1824	-0.0508	0.4193	-0.5394	4.3935
$R^f \cdot diy$	-0.0454	-0.0061	0.2176	-0.3861	7.4874
4S-GLS					
R^M	0.1514**	-0.3675**	1.9310	-1.1976	148.86***
R^f	-0.0124**	-0.0227	0.1352	-0.0810	137.9062***
$R^f \cdot diy$	-0.0021	-0.0036	0.0943	-0.1596	85.6888***
Panel B. 30 Industry					
4S-OLS					
R^M	-0.1371	-0.5039	4.5781	-10.5677	55.8174***
R^f	-0.1909**	-0.0659	0.3429	-0.5721	72.1136***
$R^f \cdot diy$	-0.0714**	-0.0246	0.1179	-0.1808	64.5528***
4S-GLS					
R^M	0.2968**	-0.2661	0.5164	-2.1983	133.9929***
R^f	-0.0251**	-0.0236**	0.0522	-0.2687	107.9627***
$R^f \cdot diy$	-0.0076**	-0.0093**	0.0000	-0.0595***	77.7709
Panel C. Fama-French 25 Size and Momentum					
4S-OLS					
R^M	0.3543	-0.4350	2.5981	-2.9047	63.4735***
R^f	-0.1824	-0.0508	0.4193	-0.5394	4.3935
$R^f \cdot diy$	-0.0454	-0.0061	0.2176	-0.3861	7.4874
4S-GLS					
R^M	0.1514**	-0.3675**	1.9310	-1.1976	148.86***
R^f	-0.0124**	-0.0227	0.1352	-0.0810	137.9062***
$R^f \cdot diy$	-0.0021	-0.0036	0.0943	-0.1596	85.6888***
Panel D. Government Bonds					
4S-OLS					
R^M	0.1429**	-1.2915**	1.0351	-10.8598	25.1649***
R^f	0.0018	-0.0166	0.0606	-0.3621	148.522***
$R^f \cdot diy$	0.0006	-0.0067	0.0044	-0.0267	11.1948***
4S-GLS					
R^M	0.087**	-0.5979	0.3583	-7.8625	29.7583***
R^f	0.0012	-0.0058	0.0231	-0.3243**	237.0354***
$R^f \cdot diy$	0.0009	-0.0077	0.0183	-0.1103	13.4406***

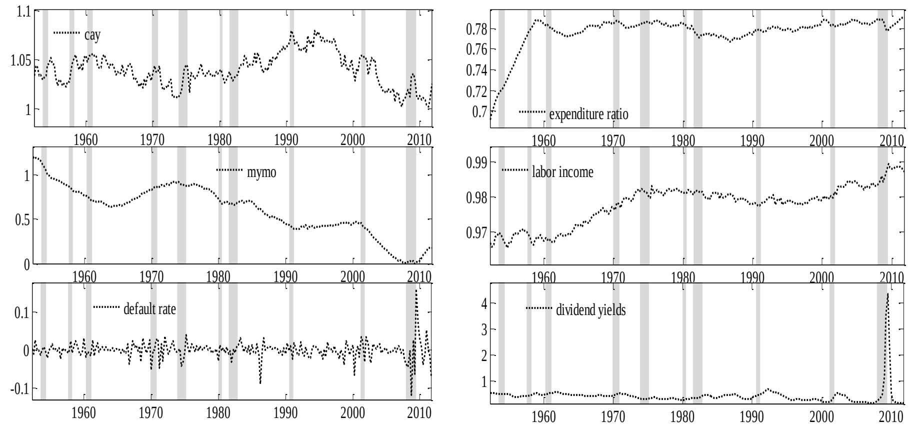
TABLE 3.8: **The Dividend Yields in CCAPM - Price of Risk Estimates**

Notes: This table reports estimates for the price of risk parameters λ_0 and Λ_1 in the conditional CCAPM with the state variables as Δc, R^f, and $R^f \cdot diy$. Two different estimators, 4-step OLS and GLS, are shown. The last column reports Wald statistics for the null of a respective row of Λ_1 being equal to zero. The sample period is 1952Q1-2012Q3. ** and *** denote statistical significance at 5% and 1% level, respectively.

	λ_0	$\Lambda_{\Delta c}$	Λ_{Rf}	$\Lambda_{Rf \cdot diy}$	χ^2 test
Panel A. Fama-French 25 Size and B/M					
4S-OLS					
Δc	0.144**	-0.1392	-0.2673	0.3144	29.9614
R^f	-0.053**	0.2384	0.2300	-0.3998	30.6444
$R^f \cdot diy$	-0.0233	0.1457	0.1774	-0.3744	14.8429
4S-GLS					
Δc	0.0767**	0.0214	-0.2555	0.8022	113.6867***
R^f	-0.0263**	0.0933	0.0959	-0.1686	51.5121**
$R^f \cdot diy$	-0.0094**	0.0505	0.0876	-0.2045	34.2185
Panel B. 30 Industry					
4S-OLS					
Δc	0.1455**	-0.1321	-0.4257	0.0825	20.7774
R^f	-0.0742**	0.0869	-0.0399	-0.2480	49.2292**
$R^f \cdot diy$	-0.0288**	0.0018	-0.0385	-0.0110	45.6**
4S-GLS					
Δc	0.0389**	-0.0615	-0.1222	-0.0704	93.8483***
R^f	-0.0292**	0.0056	0.0095	-0.2557	101.4132***
$R^f \cdot diy$	-0.0089**	-0.0185	-0.0205	-0.0371***	74.6352***
Panel C. Fama-French 25 Size and Momentum					
4S-OLS					
Δc	0.144**	-0.1392	-0.2673	0.3144	29.9614
R^f	-0.053**	0.2384	0.2300	-0.3998	30.6444
$R^f \cdot diy$	-0.0233	0.1457	0.1774	-0.3744	14.8429
4S-GLS					
Δc	0.0767**	0.0214	-0.2555	0.8022	113.6867***
R^f	-0.0263**	0.0933	0.0959	-0.1686	51.5121***
$R^f \cdot diy$	-0.0094**	0.0505	0.0876	-0.2045	34.2185
Panel D. Government Bonds					
4S-OLS					
Δc	0.0065	-0.4770	0.7143	-3.0279	3.0114
R^f	-0.0004	0.0671	0.0420	-0.2051	204.7159***
$R^f \cdot diy$	-0.0005	0.0424	-0.0020	0.0617	16.7791**
4S-GLS					
Δc	-0.0109	-0.2744	0.6418	1.0646	10.6713
R^f	0.0001	0.0733**	0.0348	-0.2915**	219.729***
$R^f \cdot diy$	0.0001	0.0432	0.0041	-0.1007	12.6088

FIGURE 3.1: **Conditional Variables**

Notes: Figure shows that the time series on conditional variables, such as the consumption-wealth rato, *cay*, the non-housing and housing expenditure ratio, α, the collaterial ratio, *mymo*, the labor income share, *s*, the default rate, *def* and the dividend yields *diy*. Here, the recession period data comes from the NBER. The sample period is 1952Q1-2012Q3.



Appendix A

Appendix on Chapter 1

A.1 Sample Estimates on Hansen–Jagannathan (HJ) Distance

In sample estimation, if the test portfolios are in gross returns, we define

$$D_T = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial \alpha_t(\theta)}{\partial \theta} = \frac{1}{T}R^{'}f, \tag{A.1}$$

$$g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T}\alpha_t(\theta) = D_T\theta - I_N, \tag{A.2}$$

$$G_T = \frac{1}{T}\sum_{t=1}^{T}R_t R_t^{'} = \frac{1}{T}R^{'}R, \tag{A.3}$$

where

$$R = [R_1, R_2, ..., R_T]^{'},$$

$$f = [f_1, f_2, ..., f_T].$$

The sample analog of the HJ distance is thus

$$\delta_T = \sqrt{min_\theta g_T(\theta))' G_T^{-1} g_T(\theta)}. \tag{A.4}$$

Taking the derivative of the above equation

$$D_T^{'} G_T^{-1} g_T(\theta) = 0, \tag{A.5}$$

which gives an analytic expression for the sample minimizer

$$\hat{\theta} = (D_T' G_T^{-1} D_T)^{-1} D_T' G_T^{-1} I_N. \tag{A.6}$$

From Hansen(1982) [49] the asymptotic variance of $\hat{\theta}$ is given by

$$var(\hat{\theta}) = \frac{1}{T}(D_T' G_T^{-1} D_T)^{-1} D_T' G_T^{-1} \Omega_T G_T^{-1} D_T (D_T' G_T^{-1} D_T)^{-1}, \tag{A.7}$$

where if the data is serially uncorrelated, the estimate of the variance matrix of pricing errors is give by

$$\Omega_T = \frac{1}{T}\sum_{t=1}^{T} \alpha_t(\hat{\theta})\alpha_t(\hat{\theta})'. \tag{A.8}$$

The estimator $\hat{\theta}$ is equivalent to a GMM estimator defined by Hansen(1982) with the moment condition $E[g(\theta)] = 0$ and the weighting matrix G^{-1}.

If the test portfolios are in excess returns, we define

$$y_{t+1}(\theta) = 1 - \theta' f_{t+1}, \tag{A.9}$$

$$E_t[y_{t+1}(\theta)R_{t+1}] = 0_N, \tag{A.10}$$

the estimates of risk premiums will change into

$$\hat{\theta} = -(D_t' G_t^{-1} D_t)^{-1} D_t' G_t^{-1} \bar{R}_t, \tag{A.11}$$

where $\bar{R}_t$ is the average excess return across N.

A.2 Testing Hansen–Jagannathan Distance

If the weighting matrix is optimal in the sense of Hansen (1982) [49], then $T\delta_T^2$ is asymptotically a random variable of χ^2 distribution with $N - K$ freedom, where is the dimension of θ.

However, if G is generally not optimal, $T\delta_T^2$ is not asymptotically a random variable of χ^2. Instead, under the hypothesis that the SDF prices the returns correctly, the sample HJ distance follows:

$$T[\hat{\delta^2}] \xrightarrow{d} \sum_{j=1}^{N-K} a_j \chi^2(1), \tag{A.12}$$

where $\chi^2(1)$ are independent chi-squared random variables with one degree of freedom, and a_j are $N-K$ nonzero eigenvalues of the matrix A given by

$$A = \Omega^{\frac{1}{2}} G^{\frac{-1}{2}} [I_N - (G^{\frac{-1}{2}})' D(D' G^{-1} D)^{-1} D' G^{\frac{-1}{2}}](G^{\frac{1}{2}})'(\Omega^{\frac{1}{2}})'. \tag{A.13}$$

Here $\Omega = E[\alpha_t \alpha_t^{'}]$ denotes the variance of pricing errors, and $D = E(R_t^{'} f_t)$. The $\frac{1}{2}$ means the upper-triangle matrices from the Cholesky decomposition. As long as we have a consistent estimate Ω_T of the matrix Ω, we can estimate the matrix A by replacing Ω and G by Ω_T and G_T, respectively. Under the hypothesis that the SDF prices the returns correctly, The Ω can be estimated consistently by $\Omega_T = T^{-1} \sum_{t=1}^{T} [\alpha_t \alpha_t^{'}]$.

Following Jagannathan and Wang (1996) [43], to adjust for the small sample bias, we use Monte Carlo method to calculate the empirical distribution of HJ distance (under the null hypothesis). First, draw $M \otimes (N-K)$ independent random variables from $\chi^2(1)$ distribution. Then, we calculate $u_j = \sum_{i=1}^{N-K} a_i \chi^2(1)$. Here M is the number of simulation. Then the empirical p-value of the HJ distance is

$$p_{\hat{HJ}} = \frac{1}{M} \sum_{j=1}^{M} I_{(u_j \geq T[HJ_T(\theta_T)]^2)}, \tag{A.14}$$

where $I(.)$ is an indicator function which equals one if the expression in the brackets is true and zero otherwise.

A.2.1 Testing Constrained Hansen–Jagannathan Distance

To test the constrained HJ distance, we follow Gospodinov, Kan and Robotti (2010)[1]. They state an asset pricing model is correctly specified if there exists a $\theta \in \Gamma$ such that $y_t(\theta) \in \aleph^+$, which implies that $\iota = 0_N$ and $\delta_+ = 0$; the model is misspecified if $y_t(\theta) \notin \aleph^+$ for all $\theta \in \Gamma$, which implies that $\delta_+ > 0$.

They show that

(a) if $\delta_+ = 0$, the pricing model is correctly specified,

$$T\hat{\delta}_+^2 \xrightarrow{A} \sum_{t=1}^{N-K} \varsigma_i \upsilon_i, \tag{A.15}$$

[1] Gospodinov, Kan and Robotti (2010) [20], On the Hansen–Jagannathan distance with a no-arbitrage constraint, Proposition 3.

where the υ_i are independent chi-squared random variables with one degree of freedom and the ς_i are the eigenvalues of

$$A = P^{'}U^{-\frac{1}{2}}SU^{-\frac{1}{2}}P, \tag{A.16}$$

with $S = \sum_{j=-\infty}^{\infty} E[(x_t y_t(\theta^*) - q_{t-1})(x_{t+j}y_{t+j}(\theta^*) - q_{t+j-1})^{'}]$, $D = E[x_t \frac{\partial y_t(\theta^*)}{\partial \theta^{'}}]$, $U = E[x_t x_t^{'}]$, and P being an $N \times (N-K)$ orthonormal matrix whose columns are orthogonal to $U^{-\frac{1}{2}}D$. This is the same as traditional HJ distance test.

(b)if $\delta_+ > 0$, the pricing model is misspecified,

$$\sqrt{T}(\hat{\delta_+^2} - \delta_+^2) \xrightarrow{A} N(0, \upsilon), \tag{A.17}$$

where $\upsilon = \sum_{j=-\infty}^{\infty} E[(\varphi_t(\lambda^*) - \delta_+^2)(\varphi_{t+j}(\lambda^*) - \delta_+^2)^{'}]$ and $\delta = [\theta^{'}, \iota^{'}]$.

To conduct inference, the variance matrix should be replaced by consistent estimator. In sample, we can replace A with $\hat{A}$, and $\hat{U} = \frac{1}{T}\sum_{t=1}^{T} x_t x_t^{'}$, we also can obtain $\hat{S}$ using a nonparametric heteroskedasticity and autocorrelation consistent estimator.

A.3 Entropy and the Filtered Pricing Kernel

In the absence of arbitrage opportunities, there exists a pricing kernel, y_{t+1}, or the stochastic discount factor (SDF), such that the equilibrium price, p_{it}, of any asset i delivering a future payoff R_{it+1}, is given by the Lucas pricing equation

$$E_\mu[y_t(Z,\theta)R_t^e] \equiv \int y_t(Z,\theta)R_t^e d\mu = 0, \tag{A.18}$$

where $R_t^e \in R^N$ is a vector of excess returns on different tradable assets, E is the unconditional rational expectation operator, μ is the unconditional physical probability measure, and Z stands for the factor variables.

We define the candidate SDFs, factorized as

$$y_t = yb_t(Z,\theta) \times \varphi_t, \tag{A.19}$$

where $yb_t(Z,\theta)$ is a known benchmark non-negative model specific function of data observable (Z) at time t and the parameter vector $\theta \in R^k$, and φ_t is a potentially unobservable component. Then this implies that for any set of tradable assets, the following vector of Euler equations must hold in equilibrium

$$E_\mu[yb_t(Z,\theta)\varphi_t R_t^e] \equiv \int yb_t(Z,\theta)\varphi_t R_t^e d\mu = 0, \tag{A.20}$$

where $R_t^e \in R^N$ is the vector of excess returns on different tradable assets.

The paper assumes that

Ψ and μ are both sigma-finite;

Ψ is absolutely continuous with respect with μ;

φ_t is a measurable function taking values in $[0,\infty)$.

Under weak regularity conditions, the above pricing restrictions for a SDF can be rewritten as

$$\begin{aligned} 0 &= \int yb_t(Z,\theta)\varphi_t R_t^e d\mu \\ &= \int yb_t(Z,\theta)R_t^e d\Psi = E^\Psi[yb_t(Z,\theta)R_t^e], \end{aligned} \tag{A.21}$$

where $\varphi_t = \frac{d\Psi}{d\mu}$ is the Radon–Nikodym derivative of Ψ with respect with μ.

$$\begin{aligned} \hat{\Psi} &= argmin_\Psi D(\Psi||\mu) \\ &= argmin_\Psi \int log(\frac{d\Psi}{d\mu})d\Psi, \end{aligned} \tag{A.22}$$

subject to

$$E^\Psi[yb_t(Z,\theta)R_t^e] = 0.$$

Note also that $D(\Psi||\mu)$ is always non-negative and has a minimum at zero that is reached when Ψ is identical to μ (a.e.). The above is a relative entropy minimization under the asset pricing restrictions coming from the Euler equations, $D(\Psi||\mu)$ is the Kullback–Leibler information criterion (KLIC) distance from μ to Ψ. That is, we can estimate the unknown measure Ψ as the one that adds the minimum amount of additional information needed for the pricing kernel to price assets.

Furthermore, we need $D(\Psi||\mu)$ to be a continuous function, which means that a mere relabeling of states should not change the value of D. Suppose $\hat{\mu}$ were uniformly distributed on a subset of m outcomes (zeros elsewhere), while $\hat{\Psi}$ is also uniform, but on only n of those outcomes, where $n \leq m$. Then D should be increasing in m and should be decreasing in n.

A.3.1 The Least Misspecified SDF and Economic Cycles

We present the difference between the least and the most misspecified candidates via entropy across the business cycle and financial market bull–bear periods.

We define the candidate SDFs, factorized as

$$y_t = yb_t(Z, \theta) \times \varphi_t, \tag{A.23}$$

where $yb_t(Z, \theta)$ is a known benchmark non-negative model specific function of data observable (Z) at time t and the parameter vector $\theta \in R^k$, and φ_t is a potentially unobservable component. Then this implies that for any set of tradable assets, the following vector of Euler equations must hold in equilibrium

$$E_\mu[yb_t(Z, \theta)\varphi_t R_t^e] \equiv \int yb_t(Z, \theta)\varphi_t R_t^e d\mu = 0, \tag{A.24}$$

where $R_t^e \in R^N$ is the vector of excess returns on different tradable assets[2].

The recession period data comes from the NBER and the financial market crashes and bull periods data come from Mishkin and White (2002) [50]. Here, financial market crashes means a 20% drop in the market; speed is another feature. Therefore, we look at declines over windows of three months and one year.

At three months, the Dow additionally identifies several more months in the 1930–1932 slide, November and December 1987, October, November, and December 1937, and June 1962. Similarly, the S&P500 finds crashes in 1930–1933, 1937, 1962, 1974, and 1987, with June 1940 narrowly missing at 19.2%. For the NASDAQ, there are crashes in a three-month window in 1974, 1987, 1990, 2000, 2001, and 2008.

For the Dow Jones, declines in the 12 month window in excess of 20% picks out 1930–1933, 1937, 1938, 1970, 1974, and 1988. Using the S&P500 and a 12 month window there are crashes ending in 1930–1933, 1937, 1938, 1941, 1947, 1970, 1974, and 1975. Looking at the NASDAQ for one year crashes, we pick out the same years as using a three month window, plus 1973, 1975, 1982, 1983, and 1984.

[2]For estimating φ, see details on Entropy in Appendix.

Figure A.1 illustrates the time series of the filtered SDF on the Fama–French five-factor model and the CAPM while explaining payoffs on the Fama–French 25 size and book-to-market ratio sorted portfolios. The green dashed line plots the component of the SDF that is a parametric function of CAPM, $yb(\theta, t) = R^m_{t+1}$. The blue line plots the filtered SDF on the Fama–French five-factor, which is the product of the unobservable component of the SDF, φ_t. The grey shaded areas represent NBER-dated recessions, the red line stands for the financial market crashes, and the aquamarine line marks the financial market bull periods. The figure reveals two main points. The filtered SDF is much volatile than the benchmark. For instance, the filtered Fama–French five-factor obtains much larger variance (1.25/0.02) than the CAPM. Second, the peaks, the periods which are 2.5 points above, are correlated with the financial market crash and bull periods, i.e., the correlation between the peak of the filtered SDF and the financial bear–bull periods is 83.34%. Figure A.2 shows that the Fama–French five-factor model varies sharply comparing with the CAPM when explaining bond portfolios. Moreover, the correlation between the filtered SDF and the financial bear–bull periods stays at 88.89%. In order to reconcile higher risk premiums of equities, individuals must have implausibly time-varying pricing kernels. Therefore individuals have a high discount rate during the financial crashes and a lower value for the bull periods.

Figure A.3 shows movements of the filtered SDF when pricing the 10 Deciles portfolios. The green dashed line plots the benchmark consumption-based CAPM. The least misspecified Yogo durable consumption model is plotted in blue. The grey shaded areas represent the NBER-dated recessions, the red line stands for the financial market crashes and the aquamarine line for the financial market bull periods. While obtaining the same time-varying volatile for the filtered Yogo durable consumption SDF, its correlation with the financial market crashes and bull periods is nearly 92.31%.

Figures A.4 and A.5 show that the difference between the least and the most misspecified SDFs in the 30 industry portfolios. The Lettau and Ludvigson conditional model (the blue line) varies sharply comparing with the consumption-based CAPM (C-CAPM). Moreover, the Lettau and Ludvigson model captures the financial crashes in 1962, in 1974, in 1987, and its correlation with financial market crashes is 76.2%. The Santos and Veronesi labor income conditional model is plotted by the blue line in Figure A.5. The correlation between its filtered SDF and the financial bear and bull periods is 81.25%. Here the Santos and Veronesi SDF is filtered by using mimicking portfolios, i.e., the excess returns on market portfolios (the growth rate of consumption), then leading it to the nested model with CAPM (C-CAPM). To avoid the information loss when transferring macro variables to financial data, we check the bound on SDFs of the least and the most misspecified models.

A.4 A Nonparametric Method for Canonical Valuation

This canonical valuation method proceeds in three parts. First, time series of past underlying consumptions, and other macroeconomic variables are used to compute the empirical probability distribution $\hat{\varphi}$. Second, we will describe and justify use of the maximum entropy principle of information theory and its numerous successful applications, to transform the estimated probability distribution $\hat{\varphi}$ into an estimate $\hat{\varphi}^*$ of the unknown measure φ^*.

Note first that, normalizing the sequence $[\varphi_t]_{t=1}^T$ to lie in the unit simplex Δ^T, which is like $\widetilde{\varphi}_t = [\frac{\varphi_t}{\sum_{t=1}^T \varphi_t}]$

$$\Delta = (\widetilde{\varphi}_1, \widetilde{\varphi}_2, ..., \widetilde{\varphi}_T) : \widetilde{\varphi}_t \geq 0, \sum_{t=1}^{T} \widetilde{\varphi}_t = 1. \tag{A.25}$$

The solution of the estimation problem also solves the following optimization

$$[\hat{\widetilde{\varphi}}_t]_{t=1}^T = argmax \frac{1}{T} \sum_{t=1}^{T} In \widetilde{\varphi}_t, \tag{A.26}$$

subject to

$$[\hat{\widetilde{\varphi}}_t]_{t=1}^T,$$

$$\sum_{t=1}^{T} yb_t(x, \alpha) R_t^e \hat{\widetilde{\varphi}}_t = 0.$$

But the objective function above is simply the non-parametric likelihood of Owen (1988, 1991, 2001) maximized under the asset pricing restrictions for a vector of asset returns.

First, given an integer $N >> 0$, distribute to the various points in time $t = 1, ..., T$, at random and with equal probabilities, the value $1/N$ in N independent draws. That is, draw a series of values (probability weights) $[\widetilde{\varphi}]_{t=1}^T$ given by

$$\widetilde{\varphi}_t := \frac{n_t}{N}, \tag{A.27}$$

when n_t measures the number of times that the value $1/N$ has been assigned to time t. Second, check whether the drawn series $[\widetilde{\varphi}]_{t=1}^T$ satisfies the pricing restriction

$$(1/T) \sum_{t=1}^{T} yb_t(x, \theta) R_t^e \widetilde{\varphi}_t = 0$$

. If it does, use this series as the estimator of $[\widetilde{\varphi}_t]_{t=1}^T$, and it does not, draw another series. A more efficient way of finding an estimate for $\widetilde{\varphi}_t$ would be to choose the most likely $[\widetilde{\varphi}_t]_{t=1}^T$ of the above procedure. This can be done by noticing that the distribution of the $\widetilde{\varphi}_t$ is, by construction, the multinomial distribution with support given by the data sample. Therefore, the likelihood of any particular sequence $[\widetilde{\varphi}_t]_{t=1}^T$ is

$$L([\widetilde{\varphi}_t]_{t=1}^T) = \frac{N!}{n_1!n_2!...n_T!} \times T^{-N} = \frac{N!}{N\widetilde{\varphi}_1!N\widetilde{\varphi}_2!...N\widetilde{\varphi}_T!} \times T^{-N}. \qquad (A.28)$$

This implies that the most likely value for $[\widetilde{\varphi}_t]_{t=1}^T$ would be the maximizer fo the log likelihood

$$InL([\widetilde{\varphi}_t]_{t=1}^T) \propto \frac{1}{N}(InN! - \sum_{t=1}^T In(N\widetilde{\varphi}_t!)). \qquad (A.29)$$

Since the above procedure of assigning probability weights will become more and more accurate as N grows bigger, we would ideally like to have $N \to \infty$. In this case one can show that

$$lim_{N\to\infty} InL([\widetilde{\varphi}_t]_{t=1}^T) = -\sum_{t=1}^T \widetilde{\varphi}_t In\widetilde{\varphi}_t. \qquad (A.30)$$

Therefore, taking into account the constraint for the pricing kernel, the maximum likelihood estimate of the time series of $\widetilde{\varphi}_t$ would solve (Hobson, 1997)

$$[\hat{\widetilde{\varphi}}_t]_{t=1}^T = argmax - \sum_{t=1}^T \widetilde{\varphi}_t In\widetilde{\varphi}_t, \qquad (A.31)$$

subject to

$$[\hat{\widetilde{\varphi}}_t]_{t=1}^T \in \Delta^T, \sum_{t=1}^T yb_t(x,\theta)R_t^e\widetilde{\varphi}_t = 0.$$

However, the solution of the above MLE problem is also the solution of the relative entropy minimization problem (see e.g. Csiszar(1975)). That is, the KLIC minimization problem we propose is equivalent to maximizing the likelihood in an unbiased procedure for finding the $\widetilde{\varphi}_t$ component of the pricing kernel.

Figure A.1: **Fama–French Five-factor in Fama–French 25 Portfolios**

Notes: The Figure shows the different characteristics of the most and the least misspecified candidates in explaining return spreads on Fama–French 25 size and book-to-market ratio portfolios. The green dashed line plots the component of the SDF that is a parametric function of CAPM, $yb(\theta, t) = R^m_{t+1}$. The blue line plots the filtered SDF on the Fama–French five-factor, which is the product of the unobservable component of the SDF, φ_t. The grey shaded areas represent NBER-dated recessions, the red line denotes the financial market crashes, and the aquamarine line marks the financial market bull periods.

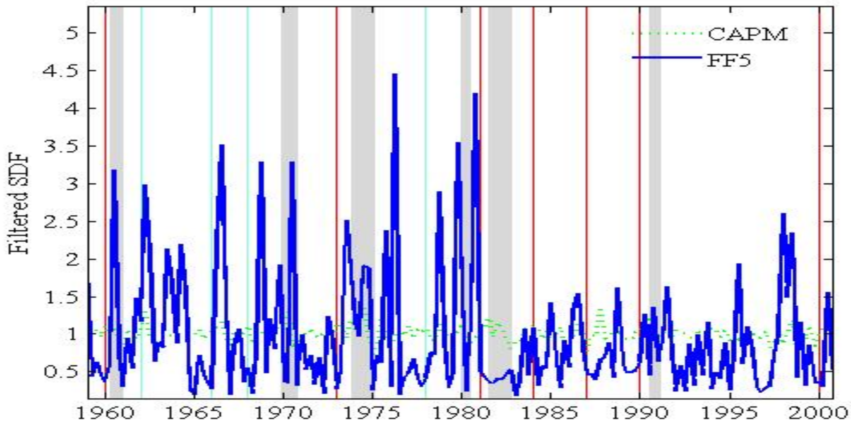


Figure A.2: **Fama–French Five-factor in FF 25 plus Gov. Bonds**

Notes: The Figure shows the different characteristics of the most and the least misspecified candidates in explaining risk spreads on government bonds portfolios. The green dashed line plots the component of the SDF that is a parametric function of CAPM, $yb(\theta, t) = R^m_{t+1}$. The blue line plots the filtered SDF on the Fama–French five-factor, which is the product of the unobservable component of the SDF, φ_t. The grey shaded areas represent NBER-dated recessions, the red line denotes the financial market crashes, and the aquamarine line marks the financial market bull periods.

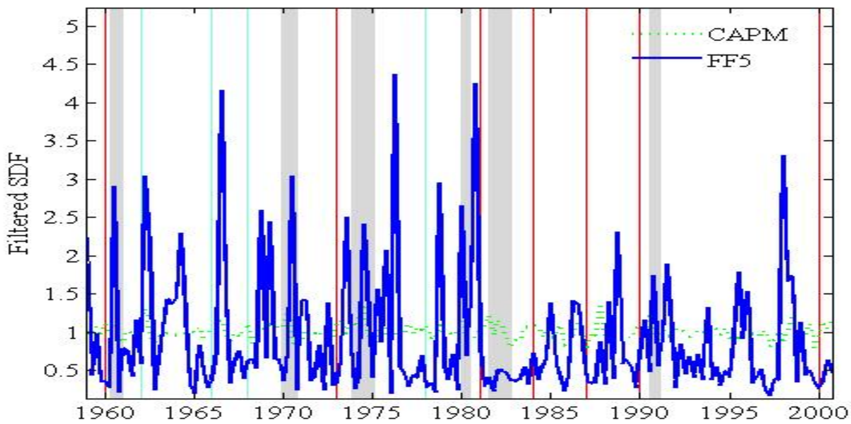


Figure A.3: **Yogo in 10 Deciles Portfolios**

Notes: The Figure shows the different characteristics of the most and the least misspecified candidates in explaining return spreads on the 10 deciles portfolios. The green dashed line plots the component of the SDF that is a parametric function of CCAPM, $yb(\theta, t) = \Delta c_{t+1}$. The blue line plots the filtered SDF on the Yogo non-durable- and durable-consumption growth factors, which is the product of the unobservable component of the SDF, φ_t. The grey shaded areas represent NBER-dated recessions, the red line denotes the financial market crashes, and the aquamarine line marks the financial market bull periods.

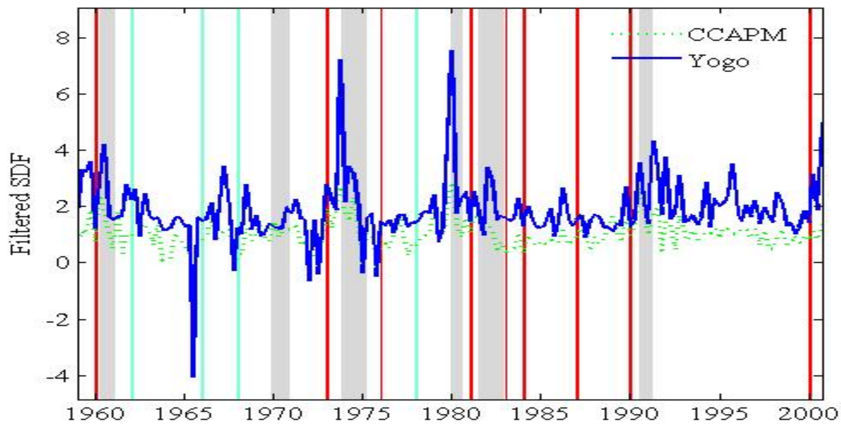


Figure A.4: **Lettau and Ludvigson in 30 Industry Portfolios**

Notes: The Figure shows the different characteristics of the most and the least misspecified candidates in explaining return spreads on the 30 industry portfolios. The green dashed line plots the component of the SDF that is a parametric function of CCAPM, $yb(\theta, t) = \Delta c_{t+1}$. The blue line plots the filtered SDF on the Lettau and Ludvigson conditional consumption-based factors, which is the product of the unobservable component of the SDF, φ_t. The grey shaded areas represent NBER-dated recessions, the red line denotes the financial market crashes, and the aquamarine line marks the financial market bull periods.

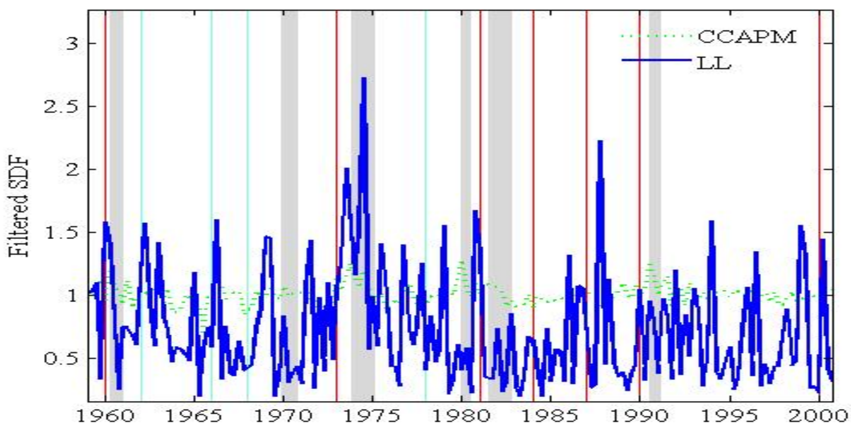
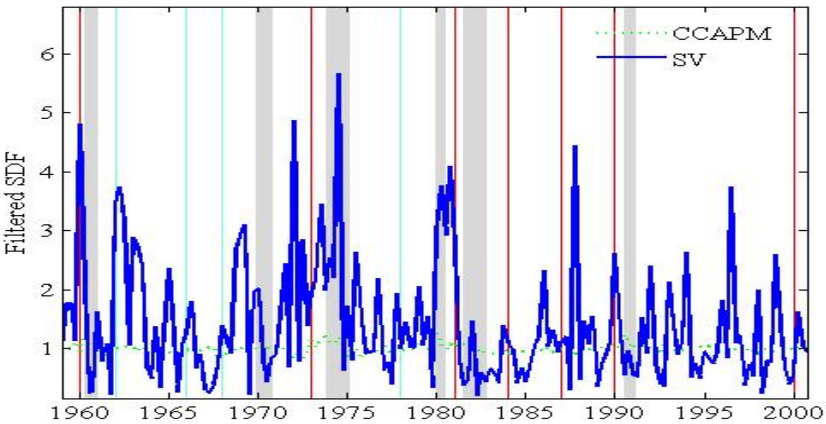


FIGURE A.5: **Santos and Veronesi in 30 Industry Portfolios**

Notes: The Figure shows the different characteristics of the most and the least misspecified candidates in explaining return spreads on the 30 industry portfolios. The green dashed line plots the component of the SDF that is a parametric function of CCAPM, $yb(\theta, t) = \Delta c_{t+1}$. The blue line plots the filtered SDF on the Santos and Veronesi conditional labor income factors, which is the product of the unobservable component of the SDF, φ_t. The grey shaded areas represent NBER-dated recessions, the red line denotes the financial market crashes, and the aquamarine line marks the financial market bull periods.



Appendix B

Appendix on Chapter 2

B.1 Unemployment

In this section, we investigate the relationship between leisure and unemployment rate. The basic intuition is that when the bad state comes (i.e. a negative technology shock), leisure absorbs the most of foregone working hours. While the unemployment rate increases, the rate at a vacancy is filled decreases, because job creation flows hampers for the marginal costs of hiring fails to decline fast in recessions. As the marginal costs of hiring fail to decline to shrink profits, the cash flows become even smaller while productivity falls. Furthermore, since wages are inelastic, reducing on profits becomes even further, the incentives of hiring are suppressed and job creation flows stifled.

Here we choose seasonally adjusted U–3 unemployment rate as the data which has been tracked by the Bureau of Labor Statistics since 1948. Seasonal adjustment is a statistical technique which eliminates the influences of weather, holidays, the opening and closing of schools, and other recurring seasonal events from economic time series. This permits easier observation and analysis of cyclical, trend, and other nonseasonal movements in the data. By eliminating seasonal fluctuations, the series becomes smoother. Furthermore, the U–3 unemployment rate takes a fairly narrow view of who qualifies as "unemployed" and excludes people who are considered "discouraged" and have given up looking for work (which are included in the U-6 unemployment rate).

Figure B.1 shows that there is a high correlation between leisure time and the unemployment rate. Evidences have been found by Aguiar et al. (2012) [51] in which between the pre-recessionary period (2006–2008) and the recession (2009–2010), 61 percent of the decline in total market work hours is accounted for by the increase in the proportion of unemployed in the population, 13 percent by the decrease in labor force participation, and 26 percent by the decline in market work hours per employed person. Besides, they show that roughly two-thirds of the increase in leisure time is associated with the decline in market work at the business cycle frequency are concentrated in television watching and sleeping.

Specifically, at the individual level a one-hour decline in market work is offset by a 25.8 percent increase in non-market work and a 59.7 percent increase in leisure. Conditional on being employed, a one-hour decline in market work is offset by a 27.3 percent increase in non-market work and by a 60.7 percent increase in leisure. Conditional on not being unemployed, a one-hour decline in market work is offset by a 25.8 percent increase in non-market work and by a 60.2 percent increase in leisure. Conditional on being unemployed, a one-hour decline in market work is offset by a 25.9 percent increase in non-market work and by a 51.1 percent increase in leisure.

B.2 Home production

In theory, a model in which home production provides a substitute to market consumption is equivalent to a model without home production but in which consumption and leisure are substitutable. Shifts in relative prices cause households to substitute goods and time not only intertemporally between periods but also intratemporally between the market and the home sector. Intratemporal substitution introduces a powerful amplification channel to hours worked in response to changes in market productivity which is absent from the standard real business cycle model. The first central issue of models with home production is that they typically assume a high degree of substitution of time between the market and the home sector over the business cycle. However, there has been little systematic evidence that the substitution of time across sectors in these models is consistent with the actual behavior of the households during recessions.

Here we show the correlations not only between asset returns and home production, but also between leisure and returns.

a. Correlations

Figure B.2 shows the rolling correlations for home production hours and leisure time. Like studying the rolling window correlation between consumption, leisure and asset returns, we implement the methodology to home production hours. The red line denotes leisure-equity relationship and the blue line is the home production-equity correlation. In the short run, leisure obtains lower correlation with equity returns. After 15 quarters (around 4 years), home production decreases the correlation with equity returns while leisure time keeps 40%, while aggregate consumption obtains 30%.

b. Movements during the recession periods

Figure B.3 shows that how do working, production and leisure hours change during the recession periods. The recession periods definition comes from NBER, trough periods include 1945 September, 1949 September, 1954 April, 1958 March, 1961 January, 1970 October, 1975 February, 1980 June, 1982 October, 1991 February, 2001 October, 2009 May. Home production hours stay the same during the recession, though before that time home production negatively correlates to working hours; when there is shock, home production will change sharply, but go back to the trend quickly. Leisure time absorbs much working hours and most of time they are substitutable for each other.

FIGURE B.1: **The Correlation between Leisure and the Unemployment**

Notes: Figure shows the rolling window correlation between the leisure and the unemployment rate. Seasonally adjusted U–3 unemployment rate has been chosen as the data which is tracked by the Bureau of Labor Statistics since 1948. Seasonal adjustment is a statistical technique which eliminates the influences of weather, holidays, the opening and closing of schools, and other recurring seasonal events from economic time series.

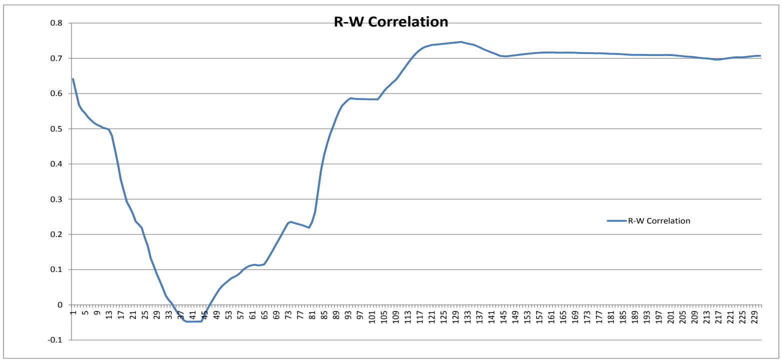


FIGURE B.2: **Home Production, Leisure and Equity Returns**

Notes: Figure shows the rolling window correlation between leisure–returns and home production–returns. The red line denotes leisure-equity relationship and the blue line is the home production-equity correlation.

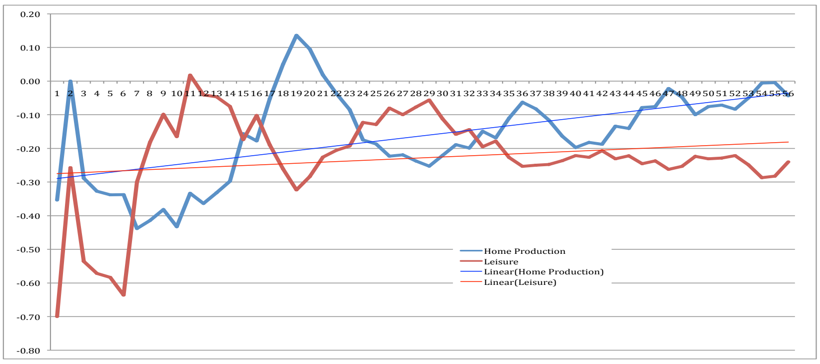


Figure B.3: **Working, Home Production and Leisure**

Notes: Figure shows that how do working, production and leisure hours growth change during the recession periods. The recession periods definition comes from NBER, trough periods include 1945 September, 1949 September, 1954 April, 1958 March, 1961 January, 1970 October, 1975 February, 1980 June, 1982 October, 1991 February, 2001 October, 2009 May.

Quarter	Work Hours	Home Production	Leisure	Growth Work	Growth Home	Growth Leisure
1949	22	24	41	-1.3461	***1.2776***	0.1018
1949.25	21	24	42	-0.8422	0.0000	0.4807
1949.5	21	24	42	**-1.6652**	*0.0000*	**0.9321**
1949.75	21	24	42	**-1.1370**	*0.0000*	**0.6227**
1950	21	24	42	0.1848	0.3172	-0.2030
1950.25	21	24	42	1.9051	0.0000	-1.0538
1953.75	23	24	40	**-1.3013**	*0.0000*	**0.8386**
1954	23	25	39	**-0.9000**	*1.1717*	**-0.3319**
1954.25	22	25	40	**-1.3243**	*0.0000*	**0.8305**
1954.5	22	25	40	**-0.4078**	*0.0000*	**0.2522**
1954.75	23	25	40	**0.9164**	*0.0000*	**-0.5691**
1955	23	25	39	1.4727	-0.3278	-0.8853
1955.25	23	25	39	0.7399	0.0000	-0.4768
1957.75	23	25	39	**-2.1842**	*0.0000*	**1.4273**
1958	22	25	39	**-2.5485**	*0.8951*	**0.8039**
1958.25	22	25	40	**-0.7154**	*0.0000*	**0.4414**
1958.5	22	25	39	**0.5335**	*0.0000*	**-0.3287**
1958.75	22	25	39	**1.1875**	*0.0000*	**-0.7419**
1959	23	25	39	0.9477	0.0188	-0.8572
1959.25	23	25	38	1.8496	0.0000	-1.2063
1960.5	23	25	39	**-0.1339**	*0.0000*	**0.0873**
1960.75	23	25	39	**-1.2325**	*0.0000*	**0.7943**
1961	22	25	39	**-0.5908**	*0.4355*	**-0.3458**
1961.25	22	25	39	**-1.3723**	*0.0000*	**0.8636**
1961.5	22	25	39	**0.4614**	*0.0000*	**-0.2882**
1961.75	22	25	39	**1.1532**	*0.0000*	**-0.7299**
1962	23	25	38	1.1308	-0.0469	-1.0594
1962.25	23	25	38	0.9188	0.0000	-0.6046
1970	23	23	39	**-1.2205**	*-1.4175*	**1.4797**
1970.25	23	23	40	**-1.5192**	*0.0000*	**0.9621**
1970.5	22	23	40	**-1.6670**	*0.0000*	**1.0287**
1970.75	22	23	41	**-1.3017**	*0.0000*	**0.7843**
1971	22	23	41	-0.5395	-1.4204	0.962
1971.25	22	23	41	-0.4817	0.0000	0.2819
1974.75	21	21	44	**-1.5484**	*0.0000*	**0.8149**
1975	20	21	44	**-2.9256**	*-1.1166*	**1.8757**
1975.25	20	21	45	**-1.1427**	*0.0000*	**0.5617**
1975.5	20	21	45	**-0.0821**	*0.0000*	**0.04**
1975.75	20	21	44	**0.6742**	*0.0000*	**-0.3298**
1976	20	21	45	0.6124	-0.4699	0.1287
1976.25	20	21	45	-0.4157	0.0000	0.2048
1980	21	21	44	**-0.8227**	*0.3045*	**0.4574**
1980.25	20	21	45	**-1.8255**	*0.0000*	**0.9177**
1980.5	20	21	45	**-0.8825**	*0.0000*	**0.4347**
1980.75	20	21	45	**0.6164**	*0.0000*	**-0.3031**
1981	20	21	45	-0.1548	0.2205	0.1662
1981.25	20	21	45	-0.9515	0.0000	0.4651
1982.25	20	21	46	0.4365	0.0000	-0.2048
1982.5	19	21	46	**-1.0222**	*0.0000*	**0.4775**
1982.75	19	21	46	**-0.9171**	*0.0000*	**0.4223**
1983	19	21	46	0.4088	0.2099	-0.1443
1983.25	19	21	46	0.5380	0.0000	-0.2484
1990.75	21	21	44	-0.3623	0.0000	0.1935
1991	21	22	44	-0.7815	1.0004	-0.1328
1991.25	21	22	44	**-0.5153**	*0.0000*	**0.2717**
1991.5	21	22	44	**0.0189**	*0.0000*	**-0.01**
1991.75	21	22	44	**-0.1877**	*0.0000*	**0.0985**
1992	21	22	44	-0.2962	0.6263	-0.1723
1992.25	21	22	44	0.4315	0.0000	-0.2265
2001.25	23	22	41	-0.8624	0.0000	0.5263
2001.5	22	22	41	-0.8147	0.0000	0.4905
2001.75	22	22	41	**-0.8182**	*0.0000*	**0.4863**
2002	22	23	41	-0.8094	0.7940	-0.0133
2002.25	22	23	41	0.2030	0.0000	-0.1191
2009	20	22	44	**-2.0022**	*0.0900*	**0.9055**
2009.25	19	22	44	**-1.8840**	*0.0000*	**0.9084**
2009.5	19	22	45	**-1.0437**	*0.0000*	**0.4924**
2009.75	19	22	45	**-0.3071**	*0.0000*	**0.1435**
2010	19	22	45	0.1604	-0.2252	-0.3432

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