

In this dissertation, we study projective posets, that is, partially ordered sets of reals such that the ordering and the incompatibility relation are definable subsets of the real plane in the sense of Descriptive Set Theory. We are interested in projective posets as forcing notions.

First, we study Martin's axiom restricted to projective posets. We build a finite support iteration of ccc projective posets with length a cardinal which is not a successor of a cardinal of countable cofinality and show that forcing with it produces a model of set theory where Martin's axiom for projective posets holds and the continuum is greater than  $\aleph_1$ . Then, using this iteration, we show that Martin's axiom for projective posets is weaker than Martin's axiom. We first Levy-collapse a weakly compact cardinal  $\kappa$  onto  $\omega_1$ . This allows us to apply a version of a well-known theorem of K. Kunen and to prove a theorem that essentially says that Martin's axiom for projective posets is consistent, modulo a weakly-compact cardinal, with the existence of almost any kind of uncountable structure that Martin's axiom forbids as, for instance, Suslin trees, non-strong gaps in the Baire space or entangled sets of reals. Finally, we show that in some cases we do not need to collapse a weakly-compact cardinal in order to obtain the consistency with Martin's axiom for projective posets of statements like "No set of reals is a  $\mathcal{Q}$ -set". As a consequence, we get that Martin's axiom for projective posets does not imply Martin's axiom for  $\sigma$ -centered posets.

The last chapter of the dissertation is devoted to the generic absoluteness properties for projective ccc posets. We study the preservation under projective ccc forcing extensions of the property of  $L(\mathbb{R})$  of being a Solovay model. A Solovay model over some model  $V$  is the  $L(\mathbb{R})$ , the smallest transitive model of Zermelo-Fraenkel set theory that contains all the ordinals and all the reals, of some generic extension of  $V$  via the Levy-collapse of an inaccessible cardinal. We show that the preservation of being a Solovay model over some model  $V$  under forcing notions that do not collapse  $\omega_1$  implies a very strong form of generic absoluteness. Then, we show that this property is preserved by every  $\Sigma_3^1$  (with parameters) ccc forcing extension. We also show that this is the optimal preservation result, i.e., it does not hold for  $\Delta_4^1$  (without parameters) ccc forcing notions. We also introduce a new family of large cardinals, the definable-Mahlo cardinals and extend all these results to the higher projective classes of ccc posets and to all projective ccc posets. We obtain, as a consequence of all these results an exact equiconsistency result for generic absoluteness under projective ccc forcing notions.