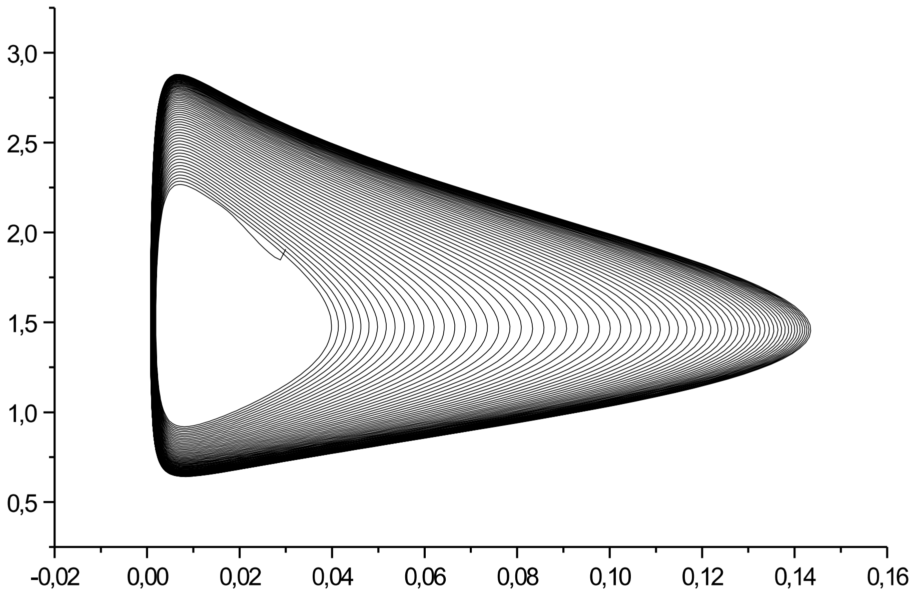

Figure 6.14: The curve (r, p) for $k_2 = 150, v_0 = 1, r_0 = 0.03, \Delta t = 0.1$ and $N = 10000$

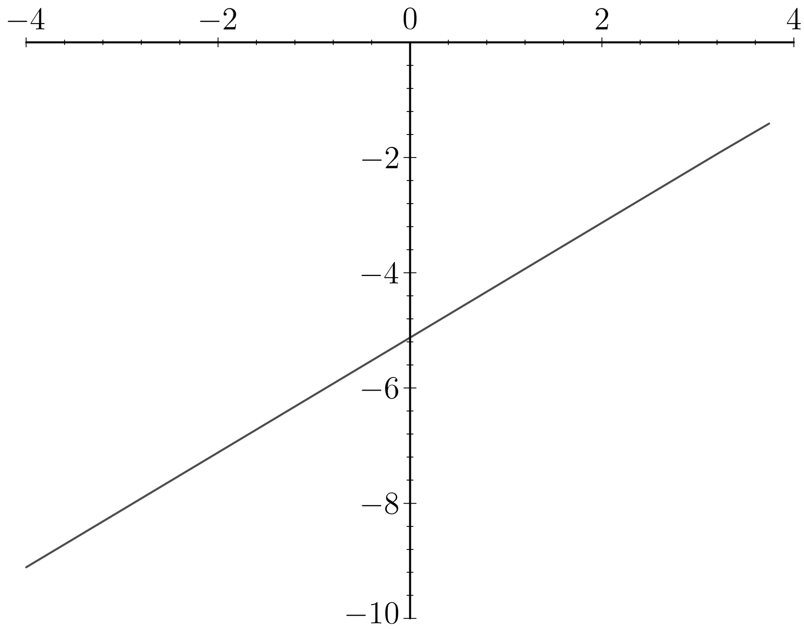

Figure 6.15: The curve $\Phi_{\alpha,M_1,M_2}(\omega)$ for $k_2 = 150$

Remarks

- Firstly, in this example we note that the stability of the steady state (u_e, v_e, r_e) does not depend on k_1 (for all $k_2 < \tilde{k}_2$, the implicit scheme shows that this point is stable independently on k_2).

- Changing k_2 from 60 to 80, we only note a significant change of the value of M_2'. This is coherent with the method used in Sect. 4.2.

- For big values of Δt (e.g., $2 \leq \Delta t \leq 6$), the convergence of our program is assured only if we take (u_0, v_0, r_0) near to the coexistence equilibrium solution (u_e, v_e, r_e). Every time the initial condition is very far from (u_e, v_e, r_e), it is necessary to work with a small steptime Δt.

- The black spots, which appear on some of the figures, can be explained by the fact that near the bifurcation, the absolute value of the real part of the dominant eigenvalue of the linearization at the coexistence equilibrium point is very small. In other words, the solutions move very slowly nearer this equilibrium or nearer the periodic orbit obtained via Hopf bifurcation.

6.2.3 Second example

Now, we take $b = 10$. In each the following cases which we shall study, the curve $\Phi_{\alpha,M_1,M_2}(\omega) = (M_1'(\omega), M_2'(\omega)), \omega > 0$ issues from the second quadrant. So, unlike in the first case we shall take k_1 as a variable and we shall fix the other parameters in order to study the loss of stability of the coexistence equilibrium point (u_e, v_e, r_e) via Hopf bifurcation. Thus, we set $k_2 = 1, m_1(0) = 6, m_2(0) = 3, m_1(\infty) = 0.1, m_2(\infty) = 0.5, c_3 = 15$ and $r_c = 5$.

Like in the first example, the numerical computations show that there exists a constant $\tilde{k}_1 \simeq 105$ such that if $k_1 < \tilde{k}_1$ then (u_e, v_e, r_e) is stable. Otherwise, it is unstable.

Later, we will draw the curve of the number of the consumer population of the adults and the curve of the total predator populations as a function of the amount of the resources. We will also give the curve $\Phi_{\alpha,M_1,M_2}(\omega), \omega \geq 0$ which shows the stability and instability regions.

In this example we will study four cases varying the parameter k_1. Two cases before bifurcation occurs and the other after the bifurcation. On the other hand, we note that for $k_1 < 10$ the coexistence equilibrium point (u_e, v_e, r_e) attracts quickly the solutions of (2.1).

Case 1: $k_1 = 10$

The numerical computations, made with Maple, show that in this case, $p_e = 13.98072283$, $v_e = 2.220461273$, $\beta = r_e = .3398311846, M_1 = 1.441423757, M_2 = 2.365906711$, $M_1' = -3.049860501, M_2' = -1.392643142$ and $\alpha = -1.019493554$

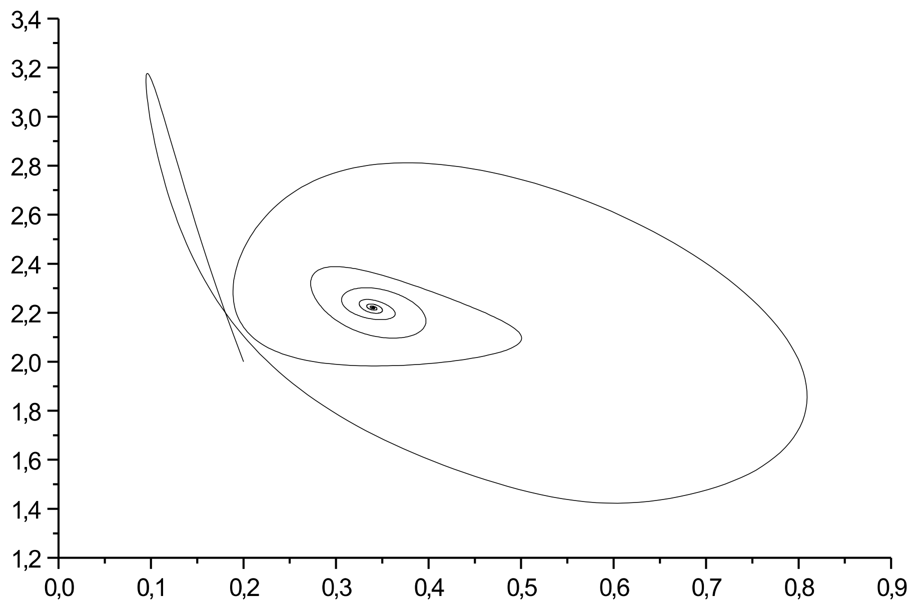

Figure 6.16: The curve (r, v) for $k_1 = 10, v_0 = 2, r_0 = 0.2, \Delta t = 0.01$ and $N = 10000$

Figure 6.16, Figure 6.17 and Figure 6.18 enlighten that the coexistence equilibrium solution (u_e, v_e, r_e) is stable for $k_1 = 10$. Moreover, they show that the solution goes quickly to this point. Indeed, the eigenvalue of the linearization at this equilibrium with the greatest real part is $\lambda = -.4404649107 + 3.365096094i$.

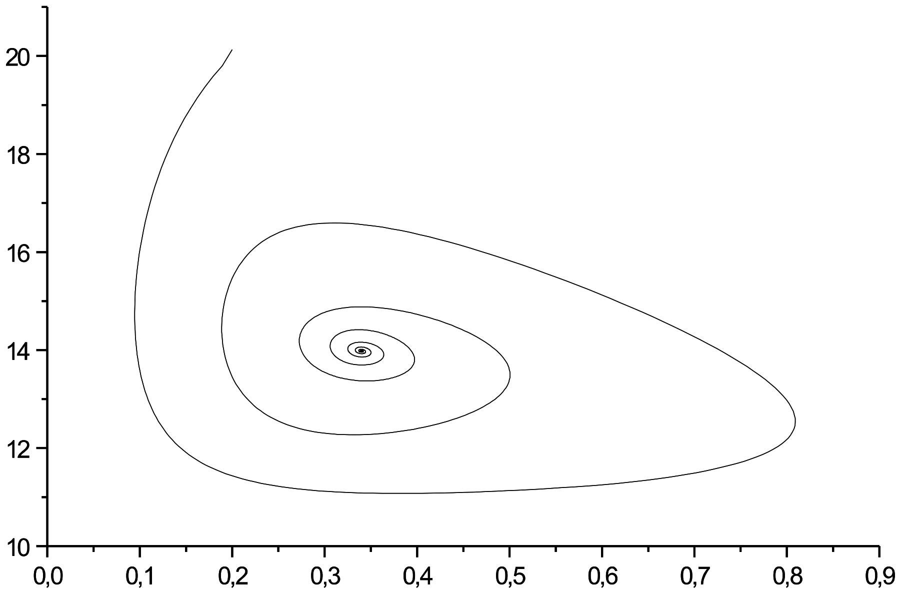

Figure 6.17: The curve (r, p) for $k_1 = 10, v_0 = 2, r_0 = 0.2, \Delta t = 0.01$ and $N = 10000$

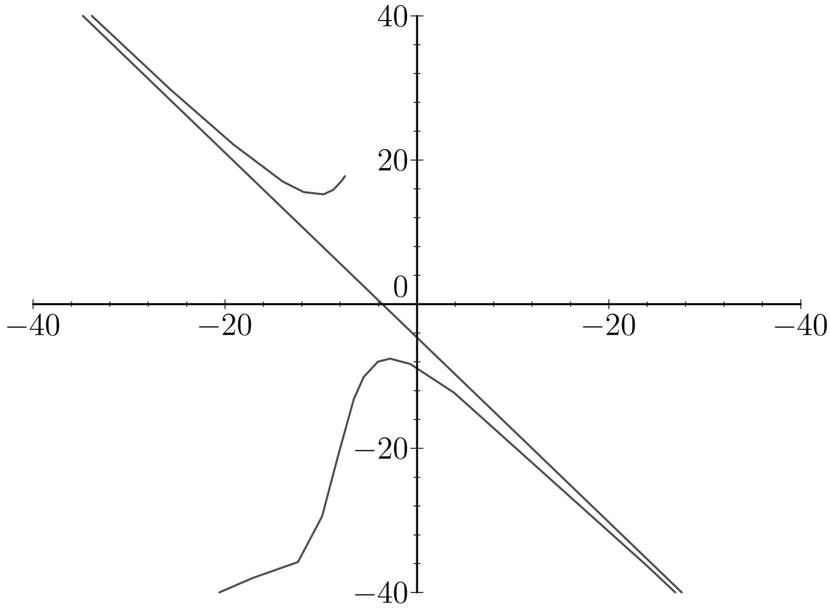

Figure 6.18: The curve $\Phi_{\alpha,M_1,M_2}(\omega)$ for $k_1 = 10$

Case 2: $k_1 = 100$

From the numerical computations, it follows, for $k_1 = 100$, that $\alpha = -.1255338786$ and the coexistence equilibrium is such that $p_e = 14.87463806, v_e = 2.208429230, r_e = .04184462619$. In this case, the death rates of the consumer populations and their derivatives applied to r_e satisfy $M_1 = 1.238015728, M_2 = 2.899590052, M_1' = -21.95050502$ and $M_2' = -2.303212966$.

Notice that, as functions of the amount of the available resources r with $k_1 = 100$,

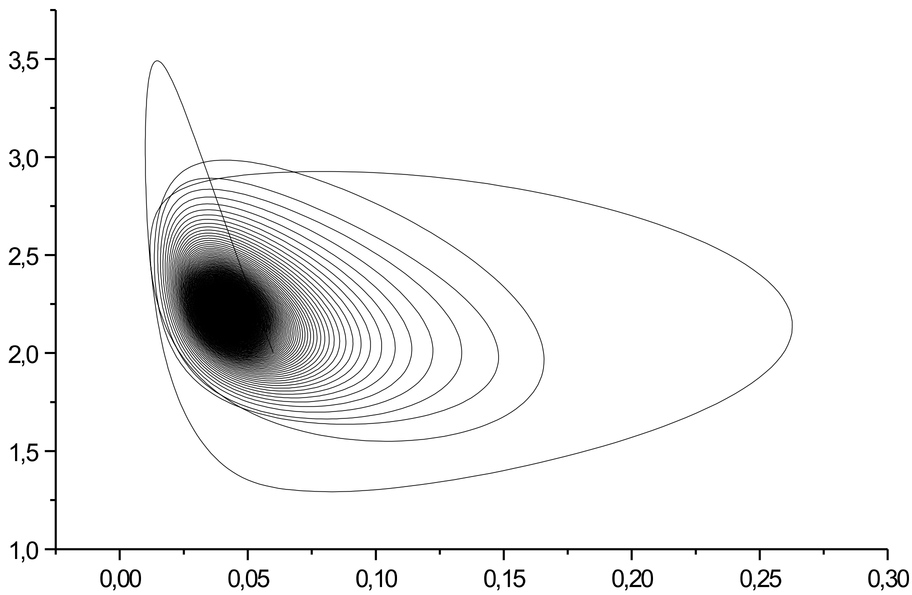

Figure 6.19: The curve (r, v) for $k_1 = 100, v_0 = 2, r_0 = 0.06, \Delta t = 0.01$ and $N = 100000$

the number of the adults of the consumer population, v, is drawn in Figure 6.19 and the total populations of the consumer is plotted in Figure 6.20. In Figure 6.21, the curve $\Phi_{\alpha,M_1,M_2}(\omega) = (M_1'(\omega), M_2'(\omega)), \omega > 0$ is plotted when $k_1 = 100$.

These figures show clearly that in this case the coexistence steady state (u_e, v_e, r_e) is still stable and attracts slowly the solutions causing the black spot. Indeed, the real part of the dominant eigenvalue is small, $\lambda = -.004825265149 + 3.156355016i$. Notice that this causes the black spot near (r_e, v_e) in Figure 6.19 and near (r_e, p_e) in Figure 6.20.

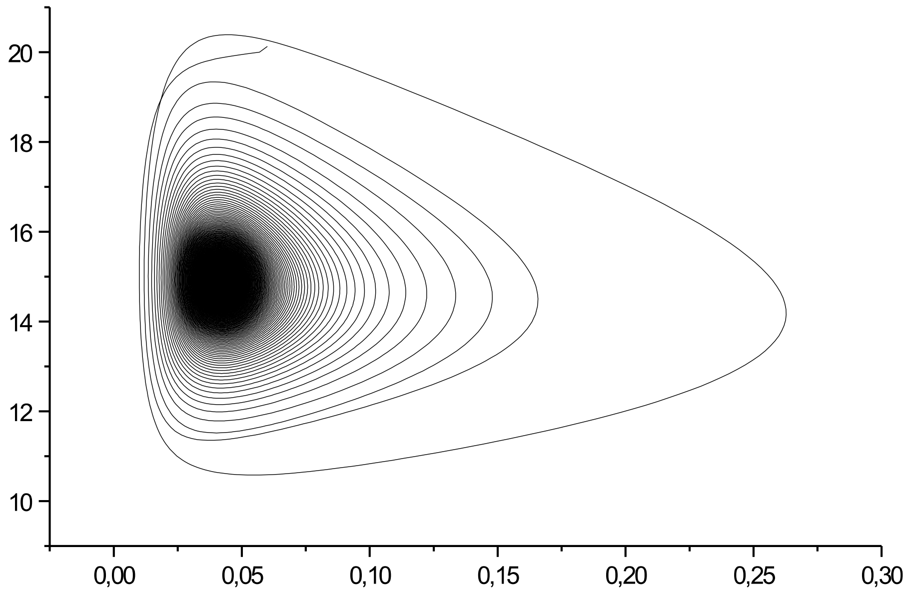

Figure 6.20: Curve (r, v) for $k_1 = 100, v_0 = 2, r_0 = 0.06, \Delta t = 0.01$ and $N = 100000$

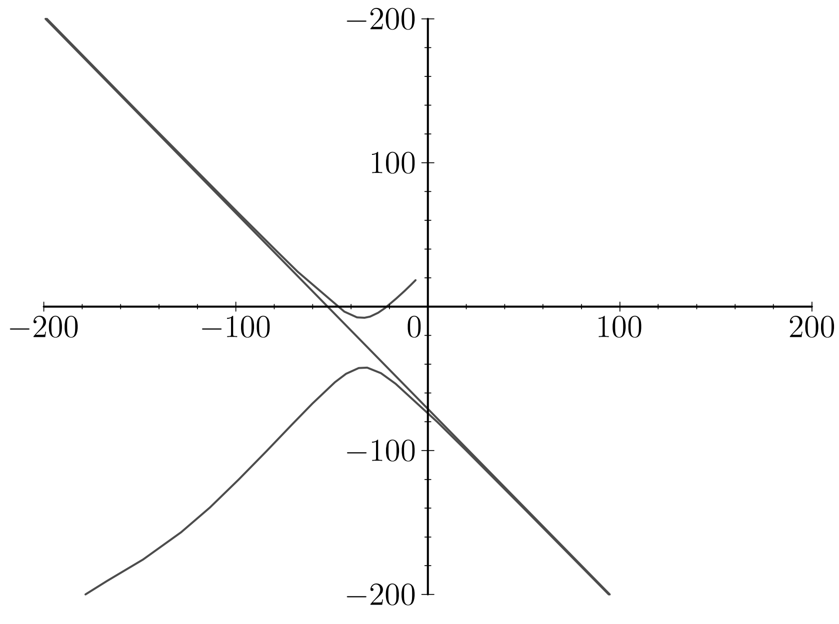

Figure 6.21: The curve $\Phi_{\alpha,M_1,M_2}(\omega)$ for $k_1 = 100$

Case 3: $k_1 = 110$

The numerical computations show, for this case, that $\alpha = -.1144868214$, $p_e = 14.88568446, v_e = 2.207866706, r_e = .03816227380, M_1 = 1.235084673, M_2 = 2.908101375$, $M_1' = -24.02133789$ and $M_2' = -2.319580894$.

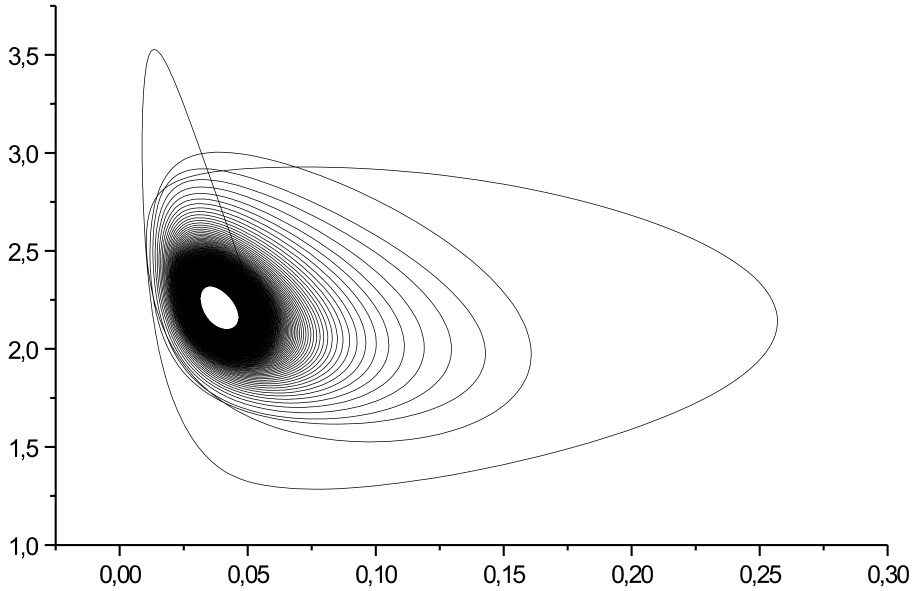

Figure 6.22: The curve (r, v) for $k_1 = 110, v_0 = 2, r_0 = 0.06, \Delta t = 0.01$ and $N = 100000$

From Figure 6.24, we can see that (M_1', M_2') belongs to the instability region. Then, the coexistence equilibrium solution (u_e, v_e, r_e) is instable in this case. This is explained also by the fact that the eigenvalue of the linearization at this equilibrium with the greatest real part is $\lambda = .0007220614914 + 3.150975605i$. Moreover, Figure 6.22 and Figure 6.23 show that the crossing of (M_1', M_2') to the instability region generates a Hopf bifurcation of a periodic solution which is stable. On the other hand, we can also see that a small change in $k_1 (e.g. 100 \leq k_1 \leq 110)$, only causes an important change of the derivative of the death rate of the adults, M_1'.

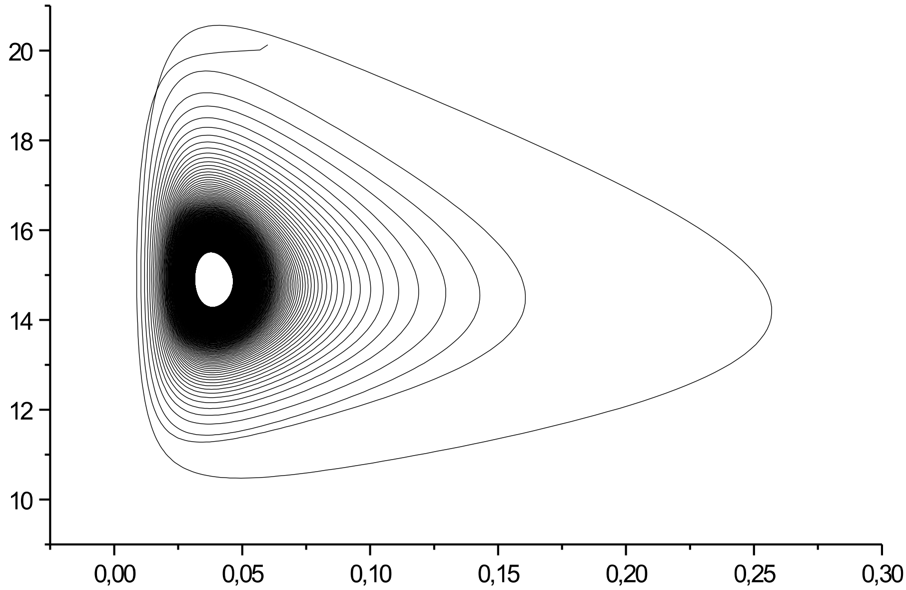

Figure 6.23: The curve (r, p) for $k_1 = 110, v_0 = 2, r_0 = 0.06, \Delta t = 0.01$ and $N = 100000$

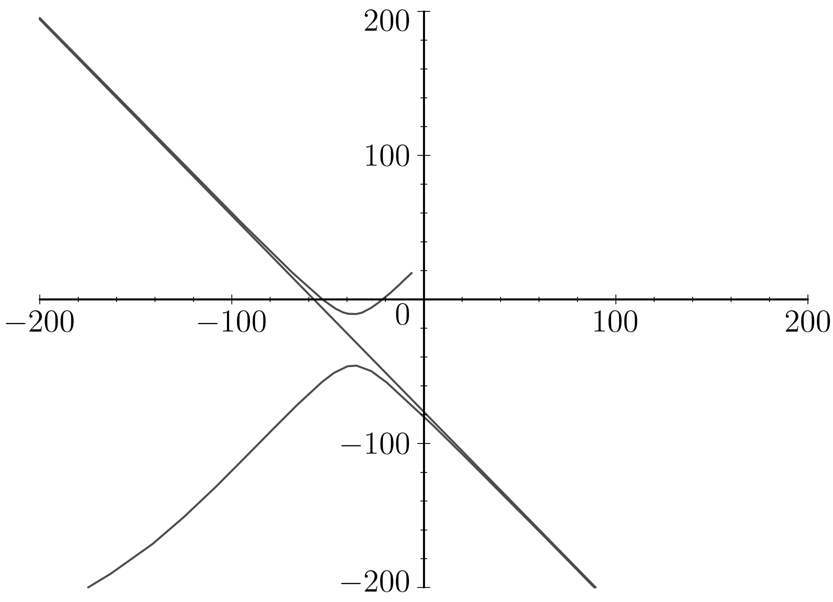

Figure 6.24: The curve $\Phi_{\alpha,M_1,M_2}(\omega)$ for $k_1 = 110$

Case 4: $k_1 = 200$

The numerical computations show now that $p_e = 14.93625670$, $v_e = 2.205138460$, $\beta = r_e = .02130385649, M_1 = 1.221508552, M_2 = 2.947851328, M_1' = -42.63665871, M_2' = -2.396790450$ and $\alpha = -.06391156947$. This allows to draw the curve $\Phi_{\alpha,M_1,M_2}(\omega) = (M_1'(\omega), M_2'(\omega)), \omega > 0$ which shows the stability and instability regions.

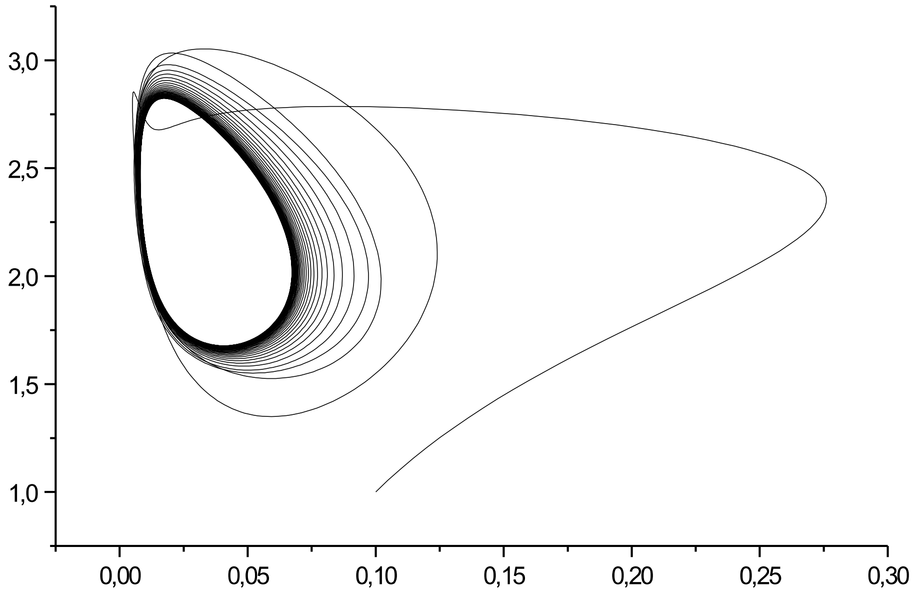

Figure 6.25: The curve (r, v) for $k_1 = 200, v_0 = 1, r_0 = 0.1, \Delta t = 0.01$ and $N = 10000$

Figure 6.25, Figure 6.26 and Figure 6.27 reflect that the coexistence equilibrium solution (u_e, v_e, r_e) is unstable for $k_1 = 200$. Moreover, there exists a stable periodic solution enclosing (u_e, v_e, r_e) and attracting the solutions of (2.1). In this case, the eigenvalue of the linearization at this equilibrium with the greatest real part is $\lambda = .02617703924 - 3.125117440i$.