Universitat Autònoma de Barcelona

PhD Thesis

PID Control

Servo/regulation performance and robustness issues

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CERTIFIES:

That the thesis entitled: **"PID Control: Servo/regulation performance and robustness issues"**, by Orlando Arrieta Orozco, presented in partial fulfillment of the requirements for the degree of Doctor Engineer, has been developed and written under his supervision.

Dr. Ramon Vilanova i Arbós
Bellaterra, September 2010

About PID Control[1]

"There's an apocryphal story you might have heard. A brilliant graduate student was working at a prestigious institution under a famous professor of control theory. This ingenious student managed to solve several of the deepest longstanding problems of control theory, developing a nonlinear, adaptive control algorithm that was guaranteed to converge globally, under extremely general conditions of noise and modeling uncertainty, to a controller that represented the best possible trade-offs among stability, robustness, and performance, both transient and steady state. All that remained to be done was the computer implementation. Unfortunately, the computational burden was immense, and years passed before a sufficiently powerful computer could be harnessed to perform the massive computations. Finally, the algorithm was implemented, and a group of distinguished researchers, all experts in the most advanced methods and theories of control, waited expectantly for the ultimate controller. When the computations were finished, the answer appeared: PID."

[1]Taken from: Dennis S. Bernstein, IEEE Control System Magazine, Vol.26, No.1, February 2006, p.8.

Acknowledgments

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Esta tesis representa la culminación de una etapa que inicié algunos años atrás y que ha estado marcada por todo tipo de experiencias, que me han ayudado a crecer y madurar como persona.

Mi gratitud a Dios, a mis padres y a mi hermana, ya que sin ellos esta aventura hubiese sido imposible. Su esfuerzo, apoyo y cariño, hacen que les deba todo lo que hoy día soy y por eso, a ellos está dedicada esta tesis.

Agradezco a mis amigos y demás personas conocidas en Costa Rica, quienes desde la distancia me han expresado siempre su apoyo.

Com a homenatge, vull expressar unes paraules en la llengua de la terra que m'ha acollit durant tots aquests anys de doctorat, Catalunya.

Voldria mostrar el meu reconeixement a molts companys i companyes de la Universitat Autònoma de Barcelona, amb els que he compartit moltíssimes coses i que m'han donat el seu suport i, en alguns casos, una inoblidable amistat.

A en Ramon, el meu "Jefe", vull agrair-li especialment la seva ajuda, dedicació, paciència i confiança, que el fan ser el millor tutor que qualsevol podria trobar-se per a fer un doctorat.

Finalmente, mi reconocimiento a todas aquellas personas que de una u otra forma han marcado mi vida, algunas de ellas ya no están en este mundo, pero sus recuerdos permanecerán conmigo para siempre.

Orlando
Barcelona, Septiembre de 2010

Abstract

The great evolution that control systems have had in the last years, has brought the need to make a more precise control, taking into account all possible situations that can be presented. Within these possibilities, the control system's performance - as well as - the robustness must be considered as important attributes that every control-loop has to consider.

From the performance side, considering the two possible modes of operation for the system, the requirements have to include good disturbance rejection (regulation-control) and set-point tracking (servo-control), what represents by itself a *trade-off* between these both considerations.

Moreover, if we look from the system's robustness point of view, due the process variations, it is an important aspect that should be included explicitly in the design stage. However, the accomplishment of the corresponding robustness specification is not always verified, therefore affecting the *trade-off* between performance and robustness.

This thesis presents an approach that faces with a problem that takes into account the above considerations. The aim is to provide solutions to improve the general behavior of a control system, with a One-Degree-of-Freedom (1-DoF) Proportional-Integral-Derivative (PID) controller structure.

The proposal is focused from two points of view. In the first part, the analysis is conducted from the perspective of the *operating mode* (either servo or regulation mode) of the control-loop and *tuning mode* of the controller. When the operating mode is different from the one selected for tuning, the performance of the optimal tuning settings can be degraded. Obviously both situations can be present in any control system and in this context, a general approach for servo/regulation control is provided to improve the performance on both operation modes. This is formulated from the optimal controller parameters for set-point and regulation tuning methods, and looking for an *intermediate* tuning between these settings.

Considering the importance of robustness, in the second part the purpose is to design a control strategy that does not depends of the extreme tunings (for servo and regulation), and also that includes robustness considerations, in an explicit way. Therefore, it is formulated a combined servo/regulation index, to evaluate the system's performance, and incorporating a robustness constraint. The accomplishment of the claimed robustness is checked and then, the PID controller gives a good performance with also a precise and certain robustness degree.

The results conducted to several PID tunings, that use the robustness or the degradation of the performance, as the design parameter. In both cases the aim is to meet the resulting selected value for the design, providing as much as possible the best value for the other characteristic (robustness or performance, depending of the case).

As a main contribution, it is also presented a balanced performance/robustness PID tuning for the best *trade-off*, between the robustness increase and the consequent loss in the optimality degree of the performance.

Contents

Chapter 1

Introduction

1.1 A short glimpse on PID control

Since their introduction in 1940 (Babb, 1990; Bennett, 2000) commercial *Proportional - Integrative - Derivative* (PID) controllers have been with no doubt the most extensive option that can be found on industrial control applications (Åström and Hägglund, 2001). Their success is mainly due to its simple structure and to the physical meaning of the corresponding three parameters (therefore making manual tuning possible). This fact makes PID control easier to understand by the control engineers than other most advanced control techniques. In addition, the PID controller provides satisfactory performance in a wide range of practical situations.

With regard to the design and tuning of PID controllers, there are many methods that can be found in the literature over the last sixty years. Special attention is made of the *IFAC workshop PID'00 - Past, Present and Future of PID Control*, held in Terrassa, Spain, in April 2000, where a glimpse of the state-of-the-art on PID control was provided. Moreover, because of the widespread use of PID controllers, it is interesting to have simple but efficient methods for tuning the controller.

Recently, tuning methods based on optimization approaches with the aim of ensuring robust stability have received attention in the literature (Ge *et al.*, 2002; Toscano, 2005). Also, great advances on optimal methods based on stabilizing PID solutions have been achieved (Silva *et al.*, 2002; Pedret *et al.*, 2002; Ho and Lin, 2003). However these methods, although effective, use to rely on somewhat complex numerical optimization procedures and do not provide autotuning rules. Instead, the tuning of the controller is defined as the solution of the optimization problem.

In fact, since the initial work of Ziegler and Nichols (Ziegler and Nichols, 1942), an intensive research has been done, developing autotuning methods to determine the PID controller parameters (Skogestad, 2003; Åström and Hägglund, 2004; Kristiansson and Lennartson, 2006). It can be seen that most of them are concerned with feedback controllers which are tuned either with a view to the rejection of disturbances (Cohen and Coon, 1953; López *et al.*, 1967) or for a well-damped fast response to a step change in the controller set-point (Rovira *et al.*, 1969; Martin *et al.*, 1975; Rivera *et al.*, 1986).

Moreover, in some cases the methods considered only the system performance (Ho *et al.*, 1999), or its robustness (Åström and Hägglund, 1984; Ho *et al.*, 1995; Fung *et al.*, 1998). However, the most interesting cases are the ones that combine performance and robustness, because they face with all system's requirements (Ho *et al.*, 1999; Ingimundarson *et al.*, (n.d.); Yaniv and Nagurka, 2004; Vilanova, 2008).

O'Dwyer (O'Dwyer, 2003) presents a complete collection of tuning rules for PID controllers, which show their abundance.

The previous cited methods study the performance and robustness jointly in the control design. However, no one treats specifically the performance/robustness *trade-off* problem, nor consider in the formulation the servo/regulation *trade-off* or the interacting between all of these variables. Therefore, it can be stated as the major novel feature in this research work.

1.2 Objective and background

Taking into account that in industrial process control applications, it is required a good load-disturbance rejection (usually known as *regulatory-control*), as well as, a good transient response to set-point changes (known as *servo-control* operation), the controller design should consider both possibilities of operation.

Moreover, it is important that every control system provides a certain degree of robustness, in order to preserve the closed-loop dynamics, to possible variations in the process. Therefore, the robustness issue should be included within the multiple *trade-offs* presented in the control design and it must be solved on a balanced way.

With respect to performance, the Two-Degree-of-Freedom (2-DoF) formulation is aimed at trying to met both objectives. Two closed-loop transfer functions can be adjusted independently and the design is usually state for optimal regulation operation and suboptimal for servo-control (Araki and Taguchi, 2003). This suboptimal behavior is achieved using a set-point weighting factor, as an extra tuning parameter, that gives the second Degree-of-Freedom to improve the tracking action (Araki and Taguchi, 1998; Taguchi and Araki, 2000).

Many tuning methods for this kind of PID controllers have been formulated over the last years (Åström and Hägglund, 2004; Leva and Bascetta, 2007; Bascetta and Leva, 2008; Alfaro *et al.*, 2009), and also some particular applications of the 2-DoF formulation based on advanced optimization algorithms have been developed (Kim, 2002; Kim, 2004; Zhang *et al.*, 2002).

Despite the above, the servo and regulation demands cannot be simultaneously satisfied with a One-Degree-of-Freedom (1-DoF) controller, because the resulting dynamic for each operation mode is different and it is possible to choose just one for an optimal solution.

Considering the previous statement, the studies have focused only in fulfilling one of the two requirements, providing tuning methods that are optimal to servo-control or to regulation-control. However, it is well known that if we optimize the closed-loop transfer function for a step-response specification, the performance with respect to load-disturbance attenuation can be very poor and vice-versa (Arrieta *et al.*, 2010). Therefore, it is desirable to get a compromise design, between servo/regulation, by using 1-DoF controller.

The proposed methods consider 1-DoF PID controllers as an alternative when *explicit* Two-Degree-of-Freedom (2-DoF) PID controllers are not available. Therefore, it could be stated that the proposed tunings can be used when both operation modes may happen and it could be seen as an *implicit* 2-DoF approach (because the design takes into account both objectives, servo and regulation modes).

With respect to the robustness issue, during the last years, there has been a perspective change of how to include the robustness considerations. In this sense, there is variation from the classical Gain and Phase Margin measures to a single and more general quantification of robustness, such as the Maximum of the Sensitivity function magnitude.

Taking also into account the importance of the explicit inclusion of robustness into the design, the aim is to look for an optimal tuning for a combined servo/regulation index, that also guarantees a robustness value, specified as a desirable Maximum Sensitivity requirement.

The research line of this thesis, follows the above idea and some previous work can be found as background in (Arrieta, 2007; Arrieta and Vilanova, 2007*a*; Arrieta and Vilanova, 2007*b*; Arrieta and Vilanova, 2007*c*; Arrieta and Vilanova, 2007*d*).

1.3 Thesis outline

In this thesis, the contents are restricted to include only the main contributions and ideas, shown as *highlights*. This manuscript is by no means intended to be self contained, however reviews and bibliographical references are provided for sake of clarity and to help the understanding of this work. Evidently, to read and follow this document, some knowledge of control systems' theory is needed.

The thesis is divided in three parts and the contents are organized as follows:

Part I: Combined servo/regulation operation for PID controllers

In this first part, the aim is to look for an *intermediate* tuning that combining existing optimal settings for set-point and load-disturbance tuning modes, improves the overall operation of the system, therefore taking into account both servo and regulation operation modes.

Chapter 2. This chapter introduces all the general framework to formulate the proposed problem statement. Important concepts and aspects like the control system configuration or tuning and operation modes are shown. Also, a motivation example is provided.

Chapter 3. In this chapter, the Performance Degradation concept is introduced and then, a general description of the followed procedure to find an *intermediate* tuning for balanced servo/regulation operation is presented, in order to improve the overall performance of the system (also using weighting factors). The advantages of the proposal are shown by some simulation examples.

Chapter 4. As an extension of the general approach for servo/regulation control operation, the idea is applied for unstable and integrating processes, achieving simple tunings that allow to improve the system's behavior.

Part II: Robustness and performance trade-off for PID controllers

In the second part of this thesis, the idea is still to provide a good servo/regulation performance for the system. However, in this case, the proposed method is formulated from the beginning as an optimization problem for combined performance (not from the extreme existing tunings), including also the robustness property as a constraint.

Chapter 5. This chapter begins with the main aspects and framework for the approach, stating the used performance and robustness considerations and the optimization problem setup. The proposed robust based PID control design is shown and tested against other tuning methods.

Chapter 6. It is proposed a PID design based on the optimality degree of the system's performance. The resulting tuning looks for the robustness increase, selecting an allowed degradation value for the performance. This approach is different but complementary to the one presented in chapter 5.

Chapter 7. This chapter concludes the second part providing a balanced performance/robustness PID design. The formulation looks for the best compromise between the robustness increase and the consequent loss in the optimality degree of the performance.

Part III: Concluding remarks

This final part presents a summary of the main results and conclusions of the thesis, as well as, some future work and research to be conducted.

Chapter 8. Finally, the conclusion remarks and main contributions are pointed, jointly with the proposals for future research.

1.4 List of publications

The thesis has generated the following journal papers:

- O. Arrieta, R. Vilanova. *Performance Degradation Analysis of Tuning Modes: Application to an Optimal PID Tuning.* International Journal of Innovative Computing, Information and Control, Vol.6, No.10, 2010, pp. 4719-4729.
- O. Arrieta, A. Visioli, R. Vilanova. *PID autotuning for weighted servo/ regulation control operation.* Journal of Process Control, Vol. 20 (4), 2010, pp. 472-480.
- O. Arrieta, R. Vilanova, A. Visioli. *PID tuning for servo/regulation control operation for unstable and integrating processes.* Journal of Industrial & Engineering Chemistry Research, 2010. (Submitted)
- O. Arrieta, R. Vilanova. *PID tuning rules for servo/regulation performance and robustness issues.* ISA Transactions Journal, 2010. (Submitted)

Also, these are the main papers presented in international conferences:

- O. Arrieta, A. Visioli, R. Vilanova. *Improved PID Autotuning for balanced control operation.* ETFA09, 14th IEEE International Conference on Emerging Technologies and Factory Automation, Mallorca - Spain, September 22-26, 2009.
- O. Arrieta, A. Ibeas and R. Vilanova. *Stability Analysis for the Intermediate Servo/Regulation PID Tuning.* ETFA09, 14th IEEE International Conference on Emerging Technologies and Factory Automation, Mallorca - Spain, September 22-26, 2009.
- R. Vilanova, V.M. Alfaro, O. Arrieta, C. Pedret. *Analysis of the claimed robustness for PI/PID Robust Tuning Rules.* MED10, 18th IEEE Mediterranean Conference on Control and Automation, Marrakech - Morocco, June 23-25, 2010, pp. 658-662.

- O. Arrieta, R. Vilanova. *Arbitrary robustness achievement for PID tuning.* 18th IFAC World Congress, Milan - Italy, August 28-September 2, 2011. (Submitted)
- O. Arrieta, R. Vilanova. *Optimality and robustness analysis, a simple balanced PID tuning.* 18th IFAC World Congress, Milan - Italy, August 28-September 2, 2011. (Submitted)

Part I

Combined servo/regulation operation for PID controllers

Chapter 2

Materials and methods

Within the wide range of approaches to autotuning, optimal methods have received special interest. These methods provide, given a simple model process description -such as a First-Order-Plus-Dead-Time (FOPDT) model- settings for optimal closed-loop responses (Zhuang and Atherton, 1993).

For One-Degree-of-Freedom (1-DoF) controllers, it is usual to relate the tuning method to the expected operation mode for the control system, known as *servo* or *regulation*.

Therefore, controller settings can be found for optimal set-point or load-disturbance responses. This fact allows better performance of the controller when the control system operates on the selected tuned mode but, a degradation in the performance is expected when the tuning and operation modes are different. Obviously there is always the need to choose one of the two possible ways to tune the controller, for set-point tracking or load-disturbances rejection. In the case of 1-DoF PID, tuning can be optimal just for one of the two operation modes.

2.1 Control system configuration

We consider the unity-feedback system shown in Fig. 2.1, where P is the process and C is the (1-DoF PID) controller.

The variables of interest can be described as follows:

- y is the process output (controlled variable).

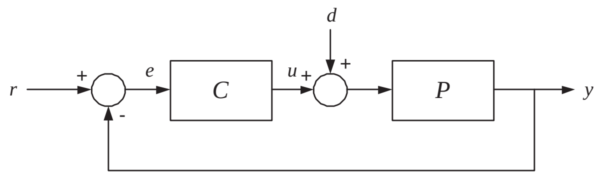

Figure 2.1: The considered feedback control system.

- u is the controller output signal.
- r is the set-point for the process output.
- d is the load-disturbance of the system.
- e is the control error $e = r - y$.

Also, the process P is assumed to be modelled by a FOPDT transfer function of the form

$$P(s) = \frac{K}{1 + Ts} \mathrm{e}^{-Ls} \tag{2.1}$$

where K is the process gain, T is the time constant and L is the dead-time. This model is commonly used in process control because is simple and describes the dynamics of many industrial processes approximately (Åström and Hägglund, 2006).

The availability of FOPDT models in the process industry is a well known fact. The generation of such model just needs for a very simple step-test experiment to be applied to the process. This can be considered as an advantage with respect to other methods that need a more *plant demanding* experiment such as methods based on more complex models or even data-driven methods where a sufficiently rich input needs to be applied to the plant. From this point of view, to maintain the need for plant experimentation to a minimum is a key point when considering industrial application of a technique.

In this context, a common characterization of the process parameters is done in terms of the normalized dead-time $\tau = L/T$ (Visioli, 2006). On the other hand, the ideal 1-DoF PID controller with derivative time filter is considered

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + (T_d/N)s}\right) \tag{2.2}$$

where K_p is the proportional gain, T_i is the integral time constant and T_d is the derivative time constant. The derivative time noise filter constant N usually takes values within the range 5-33 (Åström and Hägglund, 2006; Visioli, 2006). Without loss of generality, here we will consider $N = 10$ (Zhuang and Atherton, 1993).

2.2 Servo and regulation operation modes

Considering the closed-loop system of Fig. 2.1 the process output is given by

$$y(s) = \underbrace{\frac{C(s)P(s)}{1 + C(s)P(s)} r(s)}_{servo-control} + \underbrace{\frac{P(s)}{1 + C(s)P(s)} d(s)}_{regulatory-control} \tag{2.3}$$

The process output y depends of its two input signals, r and d and from that, the system can operate in two different modes, known as *servo-control* or *regulatory-control*. In the first case, the control objective is to provide a good tracking of the signal reference r, whereas in the second case is to maintain the output variable at the desired value, despite possible disturbances in d.

For the design of the control system, both operation modes must be considered, however depending on the controller structure (e.g. 1-DoF PID), it is not always possible to specify different performance behaviors for changes in the set-point and load-disturbances.

For the servo operation mode, disturbances are not considered ($d(s) = 0$), then (2.3) takes the form

$$y_{sp}(s) \doteq \frac{C(s)P(s)}{1 + C(s)P(s)} r(s) \tag{2.4}$$

For regulation operation mode, no changes in the set-point reference are supposed (e.g. $r(s) = 0$), then, process output would be

$$y_{ld}(s) \doteq \frac{P(s)}{1 + C(s)P(s)} d(s) \tag{2.5}$$

2.3 Set-point and load-disturbance tuning modes

Controller tuning is one of the most important aspects in control systems. For the selection of this, it is necessary to take into account some aspects like: the controller structure, the information that is available for the process and the specifications that the output has to fulfill.

The analysis presented in this work is focused on the Integral Square Error (ISE) criteria, which is one of the most well known and most often used (Åström and Hägglund, 1995), however, the general analysis could be developed in terms of any other performance criterion. A formulation of the performance index is

$$J = \int_0^\infty e(t)^2 dt \tag{2.6}$$

The optimization of (2.6) is considered, subject to the control system configuration shown in Fig. 2.1 where the controller $C(s)$ takes the explicit form of a 1-DoF PID controller (2.2).

When the settings for optimal set-point (servo-control) response are considered, the controller parameters are adjusted according to the following formulae (Zhuang and Atherton, 1993)

$$\begin{aligned} K_p &= \frac{a_1}{K}\tau^{b_1} \\ T_i &= \frac{T}{a_2 + b_2\tau} \\ T_d &= a_3 T \tau^{b_3} \end{aligned} \tag{2.7}$$

and for the optimal load-disturbance (regulatory-control) response

$$\begin{aligned} K_p &= \frac{a_1}{K}\tau^{b_1} \\ \frac{1}{T_i} &= \frac{a_2}{T}\tau^{b_2} \\ T_d &= a_3 T \tau^{b_3} \end{aligned} \tag{2.8}$$

where the corresponding values of a_i and b_i are given in Table 2.1 (Zhuang and Atherton, 1993).

Note that due to the fitting procedure, the tuning expressions do not include the whole range of τ, therefore split in two, resulting in different constants for each parameter.

Table 2.1: Optimal PID settings for set-point (sp) and load-disturbance (ld)

τ **range**	**0.1 - 1.0**		**1.1 - 2.0**	
Tuning	SP	LD	SP	LD
a_1	1.048	1.473	1.154	1.524
b_1	-0.897	-0.970	-0.567	-0.735
a_2	1.195	1.115	1.047	1.130
b_2	-0.368	-0.753	-0.220	-0.641
a_3	0.489	0.550	0.490	0.552
b_3	0.888	0.948	0.708	0.851

2.3.1 Motivation example

In order to show the performance of the previously presented settings and how this can degrade when the controller is not operating according to the tuned mode, an example is provided. This motivates the analysis to be presented in the next sections.

Consider the following plant transfer function, taken from (Zhuang and Atherton, 1993), and the corresponding FOPDT approximation

$$P_1(s) = \frac{\mathrm{e}^{-0.5s}}{(s+1)^2} \approx \frac{\mathrm{e}^{-0.99s}}{1+1.65s} \tag{2.9}$$

The application of the ISE tuning formulae for optimal set-point and load-disturbance responses provides the PID parameters shown in Table 2.2.

Fig. 2.2 shows the performance of both settings when the control system is operating in both, servo and regulation mode. It can be appreciated that the load-disturbance response of the set-point tuning is closer to the optimal regulation one than the load-disturbance tuning to the optimal servo tuning. Therefore the observed Performance Degradation is larger for the

Table 2.2: Motivation example - PID controller parameters for P_1

tuning	K_p	T_i	T_d
$set - point(sp)$	1.657	1.694	0.513
$load - disturbance(ld)$	2.418	1.007	0.559

load-disturbance tuning. From a global point of view, it will seem better to choose the set-point settings.

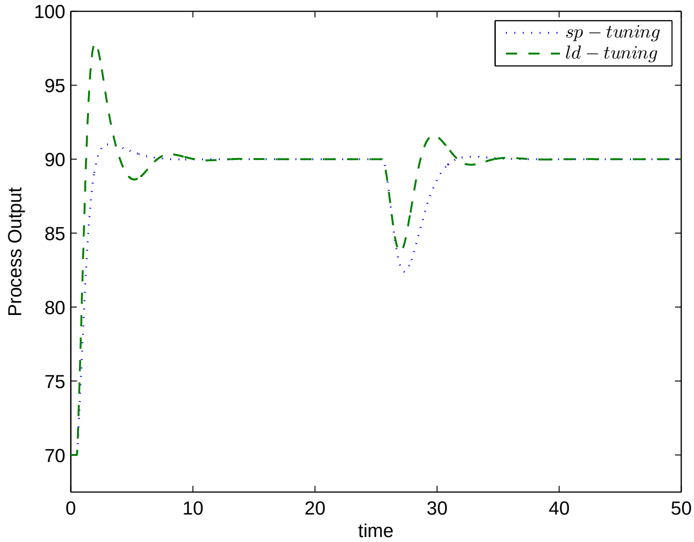

Figure 2.2: Motivation example - Process responses for servo and regulation for system P_1.

2.4 Problem statement

If the control-loop has always to operate on one of the two possible operation modes (servo or regulator) the tuning choice will be clear. However, when both situations are likely to occur, it may not be so evident which are the most appropriate controller settings.

The analysis to answer the problem concentrates on the Performance Degradation index which provides a quantitative evaluation of the controller settings with respect to the operation mode and the main objective is to reduce it.

Here, the question "*How to improve the performance when the system operates also in a different mode that it was tuned for?*" is treated by searching an *intermediate* tuning for the controller, between both optimal parameters settings for set-point and load-disturbance, in order to reduce the global Performance Degradation index.

Also, the selection of the servo/regulation *trade-off* tuning can be made to achieve a balanced performance behavior between the operation modes.

Chapter 3

General approach for servo/regulation operation

3.1 Performance Degradation of the control system

The performance of the control system is measured in terms of a performance index that takes into account the possibility of an operation mode different from the selected one. This motivates the redefinition of the performance index (2.6) as

$$J_x(z) = \int_0^\infty e(t, x, z)^2 dt \tag{3.1}$$

where x denotes the *operating mode* of the control system and z the selected operating mode for tuning, i.e., the *tuning mode*. Thus, we have $x \in \{sp, ld\}$ and $z \in \{sp, ld\}$, where sp states for set-point (servo) tuning and ld for load-disturbance (regulator) tuning. Obviously, for one specific process it has to be verified that:

$$\begin{aligned} J_{sp}(sp) &\leq J_{sp}(ld) \\ J_{ld}(ld) &\leq J_{ld}(sp) \end{aligned}$$

Performance will not be optimal for both situations. The Performance Degradation measure helps in the evaluation of the loss of performance with respect to their optimal value (Arrieta and Vilanova, 2007*a*). Performance Degradation, $PD_x(z)$, will be associated to the *tuning mode* - z - and tested on the, opposite, *operating mode* - x -. According to this, the Performance Degradation of the load-disturbance tuning, $PD_{sp}(ld)$, will be defined as

$$PD_{sp}(ld) \doteq \left| \frac{J_{sp}(ld) - J_{sp}(sp)}{J_{sp}(sp)} \right| \tag{3.2}$$

whereas the Performance Degradation associated to the set-point tuning, $PD_{ld}(sp)$, will be

$$PD_{ld}(sp) \doteq \left| \frac{J_{ld}(sp) - J_{ld}(ld)}{J_{ld}(ld)} \right|. \tag{3.3}$$

Note that, because the controller settings expressed through (2.7) and (2.8) have explicit dependence on the process normalized dead-time τ, it is worth taking into account that, for the PID application, the Performance Degradation will also depend on τ.

Fig. 3.1 shows the performance analysis for the normalized dead-time ranges where PID controller settings (set-point and load-disturbance) are provided by (Zhuang and Atherton, 1993).

Note also that Performance Degradation is a decreasing function of the normalized dead-time, taking very high values for processes with small normalized dead-time.

The final decision for the choice of the appropriate tuning mode will depend on the importance for the system operation as servo or regulation modes. However, if both situations are likely to occur, Fig. 3.1 suggests a set-point based tuning is to be preferred, because it provides less Performance Degradation than load-disturbance tuning.

3.2 Controller's search space

The tuning approaches presented in Section 2.3 can be considered extremal situations. The controller settings are obtained by considering exclusively one mode of operation. This may generate, as it has been shown in the previous section, quite poor performance if the non-considered situation happens. This fact suggests to analyze if, by loosing some degree of optimality with respect

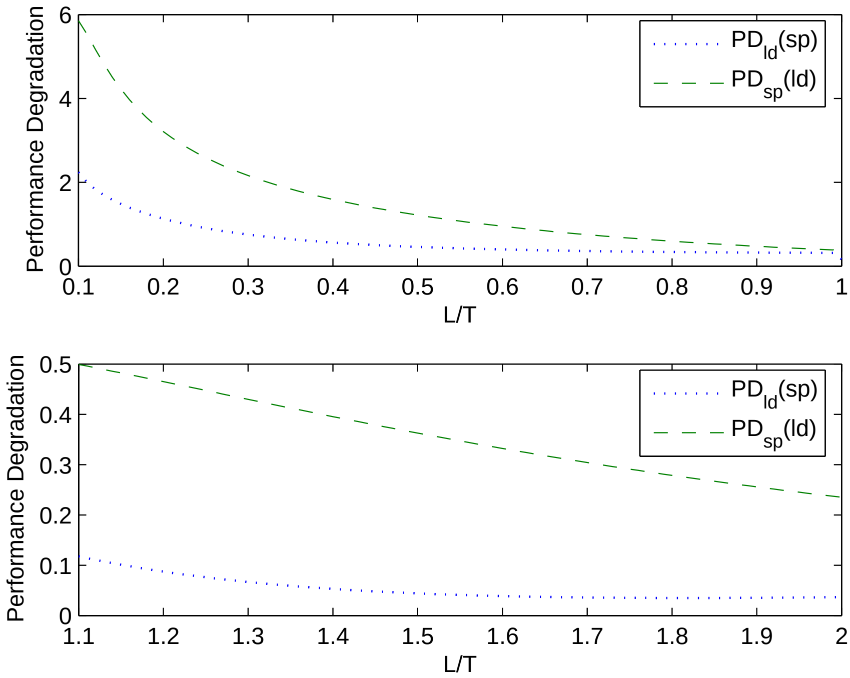

Figure 3.1: Performance Degradation of set-point (sp) and load-disturbance (ld) tunings for ISE criteria with respect to the normalized dead-time τ.

to the tuning mode, the Performance Degradation can be reduced when the operation is different to the selected one for tuning.

Based on this observation we suggest to look for an *intermediate* controller. In order to define this exploration, we need to define the search-space and the overall Performance Degradation index to be minimized (Arrieta and Vilanova, 2010). Obviously the solution will depend on how this factors are defined.

The search of the controller settings that provide a *trade-off* performance for both operating modes could be stated in terms of a completely new optimization procedure. However, we would like to take advantage of existing autotuning formulae (like (2.7) and (2.8)), in order to keep the procedure, as well as the resulting controller expression, in similar simple terms. Therefore, the resulting controller settings could be considered as an extension of the optimal ones. On this basis we define a controller settings family parameterized in terms of a vector as

$$\overline{\gamma} = [\gamma_1, \gamma_2, \gamma_3] \tag{3.4}$$

where γ_i is a variable for each controller parameter (K_p, T_i, T_d) that allows searching for the *intermediate* tuning. The values for this factor are restricted to $\gamma_i \in [0,1] \quad i = 1,2,3$. Also, the set-point tuning will correspond to a contour constraint for each $\gamma_i = 0$, whereas the load-disturbance tuning corresponds to $\gamma_i = 1$. Fig. 3.2 shows graphically the procedure and the application for the 1-DoF PID controller tuning.

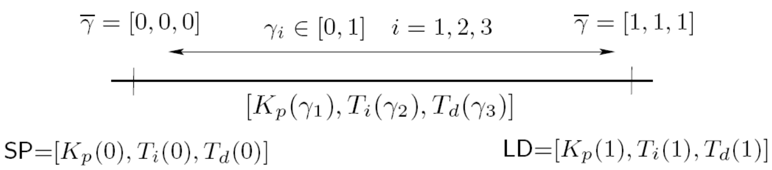

Figure 3.2: $\overline{\gamma}-$*tuning* procedure for the search of the *intermediate* controller.

The controller settings family $[K_p(\gamma_1), T_i(\gamma_2), T_d(\gamma_3)]$, can be expressed, in a more general form, as

$$\begin{aligned} K_p(\gamma_1) &= \mathbf{f_{K_p}}(\gamma_1; K_p^{ld}, K_p^{sp}) \\ T_i(\gamma_2) &= \mathbf{f_{T_i}}(\gamma_2; T_i^{ld}, T_i^{sp}) \\ T_d(\gamma_3) &= \mathbf{f_{T_d}}(\gamma_3; T_d^{ld}, T_d^{sp}) \end{aligned} \tag{3.5}$$

where $\gamma_i \in [0,1] \quad i = 1,2,3$ and $[K_p^{sp}, T_i^{sp}, T_d^{sp}]$ and $[K_p^{ld}, T_i^{ld}, T_d^{ld}]$ stand for the set-point and load-disturbance settings for $[K_p, T_i, T_d]$ respectively. Also, every $\overline{\gamma}$ transition has to satisfy the contour constraints with the form

$$
\begin{aligned}
K_p^{sp} &= \mathbf{f_{K_p}}(0; K_p^{ld}, K_p^{sp}) \\
K_p^{ld} &= \mathbf{f_{K_p}}(1; K_p^{ld}, K_p^{sp}) \\
\\
T_i^{sp} &= \mathbf{f_{T_i}}(0; T_i^{ld}, T_i^{sp}) \\
T_i^{ld} &= \mathbf{f_{T_i}}(1; T_i^{ld}, T_i^{sp}) \\
\\
T_d^{sp} &= \mathbf{f_{T_d}}(0; T_d^{ld}, T_d^{sp}) \\
T_d^{ld} &= \mathbf{f_{T_d}}(1; T_d^{ld}, T_d^{sp})
\end{aligned} \tag{3.6}
$$

Taking (3.5) as the general formulation, the controller parameters can be generated by a linear evolution between the settings from the set-point tuning to the load-disturbance one and the other way around. Therefore,

$$
\begin{aligned}
K_p(\gamma_1) &= \gamma_1 K_p^{ld} + (1-\gamma_1) K_p^{sp} \\
T_i(\gamma_2) &= \gamma_2 T_i^{ld} + (1-\gamma_2) T_i^{sp} \\
T_d(\gamma_3) &= \gamma_3 T_d^{ld} + (1-\gamma_3) T_d^{sp}
\end{aligned} \tag{3.7}
$$

3.2.1 Parametric stability analysis

Here, the objective is to introduce the stability analysis of the closed-loop generated by the controller defined by (3.7) in terms of the vector $\overline{\gamma}$ (Arrieta *et al.*, 2009*a*).

Stabilizing region for a PID controller

Here, the main relevant results obtained by Silva *et. al.* in (Silva *et al.*, 2002) are reproduced in order to have a clear idea of the applied methodology.

First, consider that the PID controller is expressed with its three gains as

$$K_c = K_p, \quad K_i = K_p/T_i, \quad K_d = K_p T_d \tag{3.8}$$

Then, we can cite the following *theorem*.

Theorem 3.2.1 *(Silva* et al.*, 2002): The range of K_c values for which a given open-loop stable plant, with a transfer function as (2.1), can be stabilized using a PID controller in the structure depicted in Fig. 2.1 is given by*

$$-\frac{1}{K} < K_c < \frac{1}{K}\left[\frac{T}{L}\alpha_1 \sin(\alpha_1) - \cos(\alpha_1)\right] \tag{3.9}$$

where α_1 is the solution of the equation:

$$\tan(\alpha) = -\frac{T}{T+L}\alpha \tag{3.10}$$

in the interval $(0, \pi)$. For K_c values outside this range, there are no stabilizing PID controllers. The complete stabilizing region is given by (see Fig. 3.3):

1. *For each $K_c \in (-(1/K), 1/K)$, the cross-section of the stabilizing region in the (K_i, K_d) space is the trapezoid* **T**.
2. *For $K_c = 1/K$, the cross-section of the stabilizing region in the (K_i, K_d) space is the triangle* **Δ**.
3. *For each $K_c \in (1/K, K_u := 1/K[(T/L)\alpha_1 \sin(\alpha_1) - \cos(\alpha_1)])$, the cross-section of the stabilizing region in the (K_i, K_d) space is the quadrilateral* **Q**.■

The parameters m_j, b_j, w_j, $j = 1, 2$ necessary for determining the boundaries of **T**, **Δ** and **Q** can be determined using the following equations

$$m_j \doteq m(z_j) \tag{3.11}$$
$$b_j \doteq b(z_j) \tag{3.12}$$
$$m(z) \doteq \tfrac{L^2}{z^2} \tag{3.13}$$
$$b(z) \doteq -\tfrac{L}{Kz}\left[\sin(z) + \tfrac{T}{L} z \cos(z)\right] \tag{3.14}$$

where z_j, $j = 1, 2, \ldots$ are the positive-real solutions of

$$KK_c + \cos(z) - \frac{T}{L} z \sin(z) = 0 \tag{3.15}$$

arranged in ascending order of magnitude.

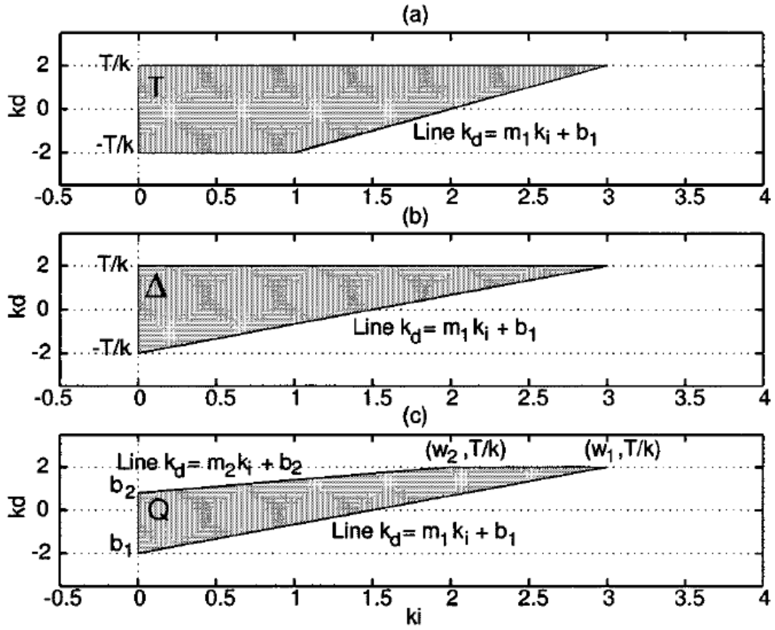

Figure 3.3: The stabilizing regions of (K_i, K_d) for: (a)$-(1/K) < K_c < 1/K$, (b)$K_c = 1/K$ and (c)$1/K < K_c < K_u$.

Stabilizing region for the *intermediate* PID controller's family

Our stability analysis is based on the demonstration that for each frozen value of $\overline{\gamma}$ the so defined *intermediate* PID controller gains lie within the corresponding stability polyhedral described in Section 3.2.1 by Theorem 3.2.1. To simplify the proof, the following procedure is proposed.

Step 1 Firstly, verify that for all frozen value of $\gamma_1 \in [0, 1]$, the proportional gain $K_c(\gamma_1)$ guarantees the existence of a stabilizing PID controller. Otherwise, there would be an *intermediate* controller parametrization that would make the unstable the closed-loop.

Step 2 Since Step 1 guarantees the existence of a PID controller that ensures the stability of the closed-loop, for each value of $\gamma_1 \in [0, 1]$ the corresponding stability region described by Theorem 3.2.1 and denoted by

R_{γ_1} can be considered.

Step 3 In this step, the intersection of all the stabilizing regions is calculated and checked to be non-empty, $\mathbf{R} = \bigcap_{\gamma_1 \in [0,1]} R_{\gamma_1} \neq \emptyset$.

Step 4 In the last step, the values of remaining controller parameters gains $K_i(\gamma_2)$, $K_d(\gamma_3)$ are plotted for a mesh of (γ_2, γ_3) generated on $[0,1] \times [0,1]$. The resulting graph is verified to completely lie within the stability region $\mathbf{R}$ guaranteeing then the stability for each frozen value of vector $\overline{\gamma}$. Finally, for a sufficiently small value of $1/N$, the closed-loop system is interpreted as a singularly perturbed system for which the Tikhonov's Theorem guarantees the preservation of stability.

In this way, following the above steps, the subsequent theorem can be proved:

Theorem 3.2.2 *The* intermediate *controller given by (2.2) and (3.7) asymptotically stabilizes the system (2.1) provided that the border values are given by (2.7), (2.8) and Table 2.1,* $1/N$ *is sufficiently small and* $\gamma_i \in [0,1]$ *for* $i = 1, 2, 3$.

Proof: Following the steps introduced above, we will show that the proposed border values for K_c, namely K_{c1}, K_{c2}, satisfy equation (3.9), guaranteeing then the existence of a stabilizing PID controller. Equation (3.10) can be rewritten as

$$\tan(\alpha) = -\frac{1}{1+\tau}\alpha \tag{3.16}$$

For each value of $\tau \in [0.1, 1.0] \cup [1.1, 2.0]$, Fig. 3.4 shows the maximum allowed proportional gain, given by the right hand side of (3.9) and the other gains given by the tuning equations (2.7) and (2.8) for the border parameters.

Since the controller values are obtained from a convex linear combination of the border values, it can be directly deduced from Fig. 3.4 that equation (3.9) is satisfied for all admissible normalized dead-time. In conclusion, there always exists a stabilizing PID controller.

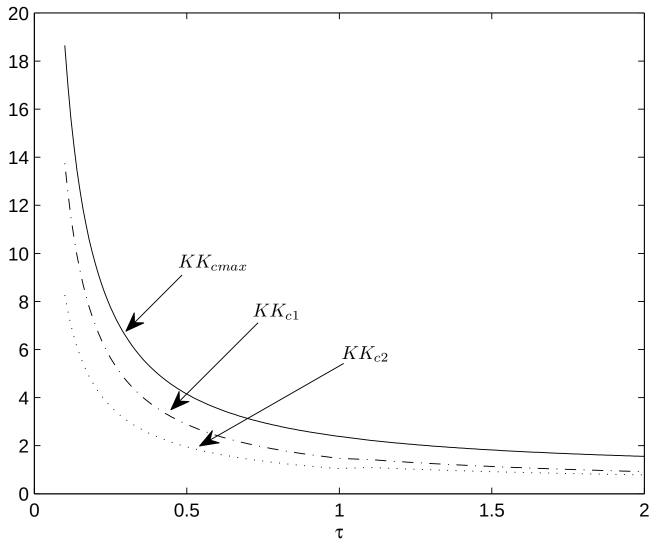

Figure 3.4: Maximum allowable proportional gain and values obtained by the tuning method.

Hence, from Theorem 3.2.1, there exist non-empty regions describing the set of stabilizing PID controllers. In particular, Table 3.1 shows the type of regions for each value of the normalized dead-time and from this, we can say that

- For the $\tau \in [0.1, 1.0]$ there is not change in the stabilizing region, being the type $\mathbf{Q}$ for the whole range.
- For the $\tau \in [1.1, 2.0]$ the stabilizing regions between the two possible extremes could change, what means that a new region from the intersection of them has to be obtained.

Furthermore, the intersection of such stabilizing regions is non-empty. To verify this, denote by $\{\mathbf{T}_k\}_{k=1}^{n_1}$, $\{\mathbf{\Delta}_k\}_{k=1}^{n_2}$, $\{\mathbf{Q}_k\}_{k=1}^{n_3}$, the sets of potential stabilizing regions.

Table 3.1: Stabilizing regions for set-point(sp) and load-disturbance(ld)

τ **range**	**0.1 - 1.0**	
Tuning	set-point	load-disturbance
Stabilizing Region	$\mathbf{Q}$, $\forall\tau \in [0.1, 1.0]$	$\mathbf{Q}$ $\forall\tau \in [0.1, 1.0]$
τ **range**	**1.1 - 2.0**	
Tuning	set-point	load-disturbance
Stabilizing Region	$\mathbf{Q}$, $\forall\tau \in [1.1, 1.29[$ $\boldsymbol{\Delta}$, $\tau = 1.29$ $\mathbf{T}$, $\forall\tau \in]1.29, 2.0]$	$\mathbf{Q}$, $\forall\tau \in [1.1, 1.77[$ $\boldsymbol{\Delta}$, $\tau = 1.77$ $\mathbf{T}$, $\forall\tau \in]1.77, 2.0]$

Every element of the set $\{\mathbf{T}_k\}_{k=1}^{n_1}$ share the left-hand border while the right hand one is defined by the straight line $K_d = m_1 K_i + b_1$. For every admissible normalized dead-time and any allowable KK_c product, the solution to equation (3.15) is finite which implies that the slope of the above border line is finite. Hence, its maximum value can attained and is finite which implies that there always exists a common point in all the sets and $\cap\{\mathbf{T}_k\}_{k=1}^{n_1} \neq \emptyset$.

Similar arguments show that $\cap\{\boldsymbol{\Delta}_k\}_{k=1}^{n_2} \neq \emptyset$ and $\cap\{\mathbf{Q}_k\}_{k=1}^{n_3} \neq \emptyset$. Finally, the intersection of all set is non-empty since from Fig. 2.1, $\mathbf{Q} \subset \boldsymbol{\Delta} \subset \mathbf{T}$ implying that $\cap\{\mathbf{Q}_k\}_{k=1}^{n_3} \subset \cap\{\boldsymbol{\Delta}_k\}_{k=1}^{n_2} \subset \cap\{\mathbf{T}_k\}_{k=1}^{n_1}$ and therefore, $\cap\{\mathbf{Q}_k\}_{k=1}^{n_3} \cap \{\boldsymbol{\Delta}_k\}_{k=1}^{n_2} \cap \{\mathbf{T}_k\}_{k=1}^{n_1} \neq \emptyset$.

The proof is completed depicting the remaining controller parameter variation and verifying that the plotted polygon lies within the intersection. This step will be illustrated by an example.

Thus, the theorem is proved for the ideal PID controller. Finally, for a sufficiently small $1/N > 0$ the closed-loop is still stable by applying the Tikhonov's Theorem (Khalil, 2002) to the resulting singularly perturbed system, hence concluding the proof of the Theorem. ■

Example

We consider the controlled process (2.9) shown before as a motivation example. Table 2.2 shows the PID controller parameters for the system using the ISE tuning formulae (Zhuang and Atherton, 1993) for optimal set-point and load-disturbance.

From the PID parameters, the whole controller family parameters (3.7), as well as, the equivalent expressions (3.8) can be obtained. From Table 3.1, it can be seen that for system (2.9) the shape region is **Q**, see Fig. 3.3 (step 1).

Then, applying equations (3.11) to (3.15) we can look for the set of all R_{γ_1} stabilizing regions (step 2) and from this, the intersection for the set, that results in the most restricted region (step 3).

Fig. 3.5 shows the resulting stabilizing region **R** and the variation of K_i and K_d parameters for all $\overline{\gamma}$ set between 0 and 1 (step 4).

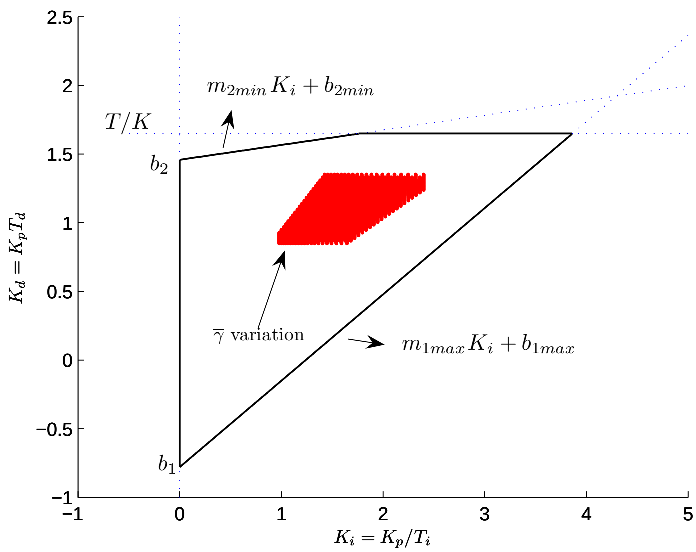

Figure 3.5: The stabilizing region for the system (2.9) and the defined polygon for the $\overline{\gamma}$ variation.

We can say that the closed-loop system would be stable for all the parameters (3.7), because the resulting polygon for $\overline{\gamma}$ variation is into the stabilizing region **R**.

3.3 Overall Performance Degradation

Now, in order to define a global Performance Degradation (PD) index, the previously defined terms (3.2) and (3.3) need to be extended. Note that the Performance Degradation was associated to the *tuning mode*, therefore tested against the opposite *operating mode*. Now, for every combination of $\overline{\gamma}$ the Performance Degradation needs to be measured with respect to both operating modes (because the corresponding $\overline{\gamma}-tuning$ does not necessarily corresponds to an operating mode). Hence,

- $PD_{sp}(\overline{\gamma})$ will represent the Performance Degradation of the $\overline{\gamma} - tuning$ on servo operating mode.

$$PD_{sp}(\overline{\gamma}) = \left| \frac{J_{sp}(\overline{\gamma}) - J_{sp}(sp)}{J_{sp}(sp)} \right| \tag{3.17}$$

- $PD_{ld}(\overline{\gamma})$ will represent the Performance Degradation of the $\overline{\gamma} - tuning$ on regulation operating mode.

$$PD_{ld}(\overline{\gamma}) = \left| \frac{J_{ld}(\overline{\gamma}) - J_{ld}(ld)}{J_{ld}(ld)} \right| \tag{3.18}$$

From the above Performance Degradation definitions, the overall Performance Degradation is introduced and interpreted as a function of $\overline{\gamma}$. There may be different ways to define the $PD(\overline{\gamma})$ function, depending on the importance associated to every operating mode (e.g. applying weighting factors to each component). However, every definition must satisfy the following contour constraints

$$PD(\overline{\gamma}) = \begin{cases} PD_{ld}(sp) & \text{for } \overline{\gamma} = [0,0,0] \\ PD_{sp}(ld) & \text{for } \overline{\gamma} = [1,1,1] \end{cases}$$

The most immediate definition would be

$$PD(\overline{\gamma}) = PD_{ld}(\overline{\gamma}) + PD_{sp}(\overline{\gamma}) \tag{3.19}$$

This expression represents a compromise, or a balance, between both losses of performance (Arrieta and Vilanova, 2007*b*; Arrieta *et al.*, 2009*b*).

3.4 Weighted Performance Degradation

As it has been mentioned before, the greatest loss of performance occurs when the load-disturbance tuning operates on servo mode. Therefore, $PD_{sp}(\overline{\gamma})$ will be the largest component of the global expression of $PD(\overline{\gamma})$ and in the opposite side $PD_{ld}(\overline{\gamma})$ the smallest one. This causes that the percentage reduction of PD that can be obtained from the PD_{ld} side is smaller than the one for the PD_{sp} part. A balanced reduction of $PD(\overline{\gamma})$ from both Performance Degradations is possible by introducing weighting factors associated to each operating mode (Arrieta *et al.*, 2010). This idea can be applied rewriting (3.19) as

$$WPD(\overline{\gamma};\alpha) \doteq \alpha PD_{ld}(\overline{\gamma}) + (1-\alpha)PD_{sp}(\overline{\gamma}) \tag{3.20}$$

that we call Weighted Performance Degradation (*WPD*) index, where $\alpha \in [0,1]$ is the weight factor and indicates which one of the two possible operation modes is preferred or more important.

One way to express the importance between both operation modes, could be the total time that the system operates in each one of them. For example, a system that operates the 75% of the time as a regulator (or viceversa 25% as a servo), $\alpha = 0.75$. However, the α parameter allows to make a more general choice for the preference of the system operation (not only taking into account the time for each operation mode).

Note also that (3.20) with $\alpha = 0.50$, represents an equivalent expression to the one obtained previously in (3.19) that gives the same significance for both operation modes.

The *intermediate* tuning will be determined by proper selection of $\overline{\gamma} = [\gamma_1, \gamma_2, \gamma_3]$. This choice will correspond to the solution of the following optimization problem,

$$\overline{\gamma}_{op} \doteq [\gamma_{1op}, \gamma_{2op}, \gamma_{3op}] = \arg\left[\min_{\overline{\gamma}} WPD(\overline{\gamma};\alpha)\right] \tag{3.21}$$

It is obvious that $\alpha = 0$ means

$$WPD(\overline{\gamma};0) = PD_{sp}(\overline{\gamma}) \tag{3.22}$$

and of course the $\overline{\gamma}_{op}$ that minimizes the Performance Degradation for servo operation mode (3.22), is the one that corresponds to the set-point tuning ($\overline{\gamma} = [0, 0, 0]$). On the other side, $\alpha = 1$ is equivalent to

$$WPD(\overline{\gamma}; 1) = PD_{ld}(\overline{\gamma}) \tag{3.23}$$

and the tuning that minimizes the Performance Degradation for regulation operation (3.23) is the load-disturbance tuning that equals to $\overline{\gamma} = [1, 1, 1]$.

The optimal values (3.21) jointly with (3.7), give a tuning formula that provides a worse performance than the optimal settings operating in the same way but also a lower degradation in the performance when the *operating mode* is different from the *tuning mode*.

Remark: It is important to note that the presented procedure has just considered the performance with respect to the proposed Performance Degradation index. Other closed-loop characteristics such as stability robustness, tolerance to parameter variation, etc. are not taken into account. How to consider additional closed-loop characteristics is a subject that will be presented later in Part II of this thesis. However, it is clear that in order to include such characteristics into consideration, they need to be part of the original extreme tunings (Arrieta and Vilanova, 2007*c*).

3.5 Optimization procedure

To provide the possibility to specify any possible combination between both operation modes, the index (3.20), with an appropriated weight factor α and subjected to the optimization (3.21), gives the suitable γ_i values that provide the PID tuning according to (3.7).

However, from a more practical point of view it is unusual and very difficult to say for example, that the regulation mode, in a control system, has the 63% of the importance (that means the 37% for the servo). With this respect, we can establish a categorization in order to make the analysis simpler and also to help the choice of the weight factor. Therefore, depending on the operation for the control system, we can identify the following general cases:

- Operation only as a servo that means $\alpha = 0$.
- Operation only as a regulator that means $\alpha = 1$.

- Same importance for both system operation modes, servo and regulation, that is equivalent to $\alpha = 0.50$.
- More importance for the servo than the regulation operation, that can be expressed by $\alpha = 0.25$.
- More importance for regulator than servo, that can be indicated as $\alpha = 0.75$.

This broad classification allows for a qualitative specification of the control system operation.

Here, the optimization was performed using genetic algorithms (Mitchell, 1998), taking problem (3.21) as the *fitness function.* The implementation was performed using MATLAB 7.6.0(R2008a)® for a *population size* of 20 and a maximum number of *generations* of 50.

The optimal solution was found for $\alpha = \{0.25, 0.50, 0.75\}$. As we said before for $\alpha = \{0, 1\}$, as extreme situations, the optimal tunings are the ones related to set-point and load-disturbance presented previously in Section 2.3.

It is worth to say that at first the optimization was performed by considering an enlarged search space for the $\overline{\gamma}$ vector, however, for the rare cases in which the optimal γ_i parameters were outside the interval $[0, 1]$, the value of the objective function was practically the same and, therefore, it is preferred to constraint the search space in order to provide a bounded controller's family that is easier to understand as are presented of an *intermediate* controller.

3.6 Intermediate tuning for balanced servo/regulation operation

What is provided in this section is a procedure about how to find an *intermediate* tuning for the controller that improves the overall performance of the system, considered as a *trade-off* between servo and regulation operation modes. The settings are determined from the combination of the optimal ones for set-point and load-disturbance, presented in (Zhuang and Atherton, 1993), and taking into account the balance between the importance of each one of the operation modes for the control system (servo or regulation).

3.6.1 Autotuning rules

Tuning relations (3.7) allow to select γ_i values on the basis of *trade-off* performance for both operating modes. However, it would be desirable an automatic methodology to choose this set of parameters without the need to run the whole Weighted Performance Degradation analysis.

In order to pursue the previous idea, by repeating the problem optimization posed in (3.21) for the three weighting factors and different values of the normalized dead-time τ, we can find an optimal set for each γ_i parameter. For each one of these groups, it is possible to approximate a function to determine a general procedure that allows to find the suitable values for the γ_i's, that provide the best *intermediate* tuning. Results are adjusted to the general expression as

$$\gamma_i(\tau) = a + b\tau + c\tau^2 \qquad \tau \in [0.1, 1.0] \cup [1.1, 2.0] \tag{3.24}$$

where a, b and c are given in Table 3.2, according to the weighting factor α and for each γ_i and τ range. Fig. 3.6 shows the followed procedure for $\alpha = 0.50$ and the γ_1 case. Note that, the range of τ is split in two because it is part of the formulation for the original extreme tunings.

Table 3.2: $\overline{\gamma}_\alpha - $ *autotuning* settings

	τ **range**	**0.1 - 1.0**			**1.1 - 2.0**		
	constant	a	b	c	a	b	c
	γ_1	0.082	0.074	0.138	0.021	0.040	-0.006
$\alpha = 0.25$	γ_2	0.896	-1.238	0.854	0.097	-0.723	0.173
	γ_3	0.332	-0.592	0.508	0.323	-0.183	0.033
	γ_1	0.093	0.547	-0.106	1.162	-1.258	0.406
$\alpha = 0.50$	γ_2	0.920	-0.540	0.206	2.222	-2.184	0.639
	γ_3	0.831	-1.197	0.548	-0.436	0.941	-0.334
	γ_1	0.108	0.566	0.067	2.197	-2.529	0.774
$\alpha = 0.75$	γ_2	0.869	-0.271	0.129	1.312	-1.021	0.296
	γ_3	0.211	0.701	-0.683	-0.987	1.791	-0.579

Equation (3.24) for each γ_i along with the settings (3.7) provide what we call here $\overline{\gamma}_\alpha - $ *autotuning* for weighted servo/regulation operation, that is one

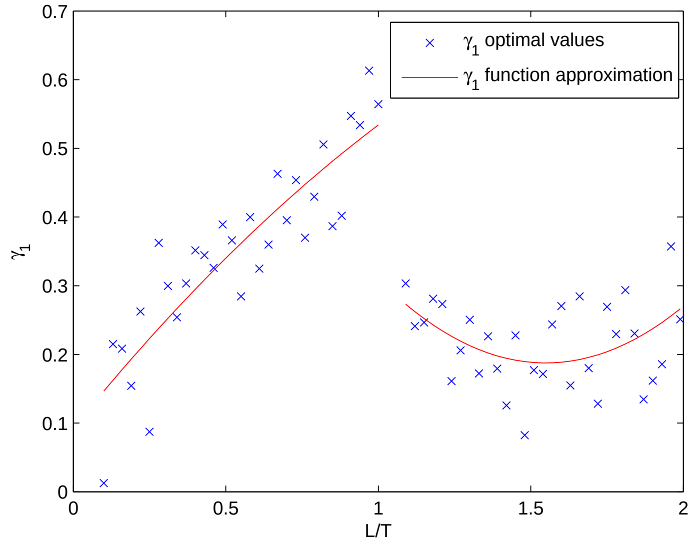

Figure 3.6: Optimal set for γ_1 parameter and the corresponding approximated function for $\alpha = 0.50$

of the contributions of this work.

3.7 Examples

This section presents several examples to illustrate how the implementation of the $\overline{\gamma}_\alpha - autotuning$ improves the performance of the closed-loop system respect to the both operation modes.

In all the examples it is supposed that the process output can vary in the 0 to 100% normalized range and that in the normal operation point, the controlled variable has a value close to 70%.

3.7.1 Example 1

Let us to consider the system (2.9), shown before as a *Motivation Example*.

Table 3.3 shows the PID controller parameters for the system (2.9) using the (Zhuang and Atherton, 1993) method and the proposed $\overline{\gamma}_\alpha - autotuning$ with $\alpha = \{0.25, 0.50, 0.75\}$. Moreover, the corresponding system outputs responses to a 20% set-point change followed by a -20% load-disturbance change, are shown in Fig. 3.7 for the following tuning methods: set-point, load-disturbance and $\overline{\gamma}_\alpha - autotuning$ with its three possible scenarios. The control signal is not shown for the sake of brevity, however it can be easily guessed that it would be smoother when the value of α is lower (see Example 3).

Table 3.3: Example 1 - PID controller parameters for P_1

tuning	K_p	T_i	T_d
$set - point(sp)$	1.657	1.694	0.513
$load - disturbance(ld)$	2.418	1.007	0.559
$\overline{\gamma}_{\alpha=0.25} - autotuning$	1.791	1.378	0.520
$\overline{\gamma}_{\alpha=0.50} - autotuning$	1.949	1.234	0.527
$\overline{\gamma}_{\alpha=0.75} - autotuning$	2.016	1.177	0.531

It can be seen that the proposed $\overline{\gamma}_\alpha - autotuning$ gives lower performance than the optimum settings when the system operates in the same way as it was tuned. However, higher performance can be obtained for the whole system operation (regulatory-control and servo-control), when an *intermediate* controller is used.

Table 3.4 shows the Performance Degradation values calculated from (3.17) and (3.18) and also the *WPD* index (3.20) for each tuning. The below side of the table presents the improvement in percentage that can be achieved with each one of the $\overline{\gamma}_\alpha - autotuning$ respect to the extreme tunings (set-point and load-disturbance).

All the values confirm the fact that, in global terms, when both operating modes could appear and taking into account the importance that the control-loop is operating as servo or regulation mode, the proposed $\overline{\gamma}_\alpha - autotuning$ is the best choice to tune the PID controller in order to get less Performance Degradations.

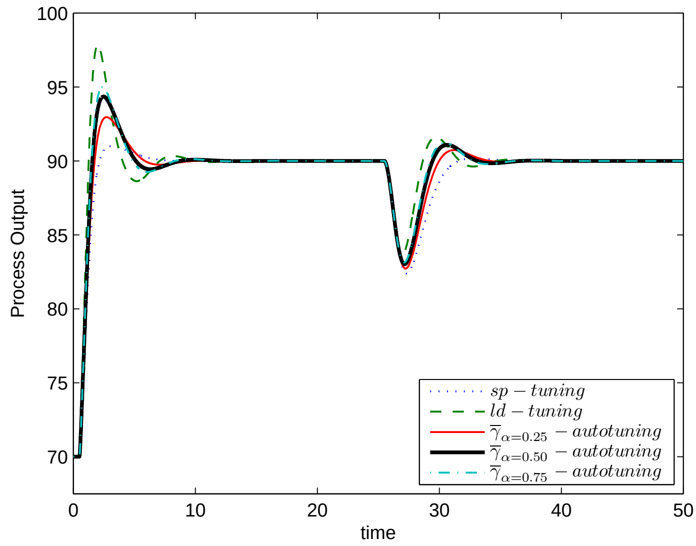

Figure 3.7: Example 1 - Process responses for servo and regulation for system P_1.

3.7.2 Example 2

In order to add completeness to the comparison, a case-study example is provided. We consider the isothermal Continuous Stirred Tank Reactor (CSTR), as the one in Fig. 3.8, where the isothermal series/parallel Van de Vusse reaction (Van de Vusse, 1964; Kravaris and Daoutidis, 1990) is taking place. The reaction can be described by the following scheme

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C \qquad (3.25)$$
$$2A \xrightarrow{k_3} D$$

Doing a mass balance, the system can be described by the following model

Table 3.4: Example 1 - PD and WPD values for the system P_1 and the improvement obtained with $\overline{\gamma}_{\alpha} - autotuning$

tuning	PD_{sp}	PD_{ld}	$WPD_{\alpha=0.25}$	$WPD_{\alpha=0.50}$	$WPD_{\alpha=0.75}$
$set-point(sp)$	-	0.3951	0.0988	0.1976	0.2964
$load-disturbance(ld)$	0.9496	-	0.7123	0.4748	0.2374
$\overline{\gamma}_{\alpha=0.25} - autotuning$	0.0336	0.1662	0.0668	-	-
$\overline{\gamma}_{\alpha=0.50} - autotuning$	0.1088	0.0376	-	0.0732	-
$\overline{\gamma}_{\alpha=0.75} - autotuning$	0.1578	0.0009	-	-	0.0401
improvement in % of					
$\overline{\gamma}_{\alpha=0.25} - autotuning$	96.46%(ld)	57.93%(sp)	32.39%(sp)	-	-
			90.62%(ld)	-	-
$\overline{\gamma}_{\alpha=0.50} - autotuning$	88.54%(ld)	90.49%(sp)	-	62.95%(sp)	-
			-	85.58%(ld)	-
$\overline{\gamma}_{\alpha=0.75} - autotuning$	83.38%(ld)	99.77%(sp)	-	-	86.47%(sp)
			-	-	83.11%(ld)
(respect to)					

$$
\begin{aligned}
\frac{dC_A(t)}{dt} &= \frac{F_r(t)}{V}\left[C_{Ai} - C_A(t)\right] - k_1 C_A(t) - k_3 C_A^2(t) \\
\frac{dC_B(t)}{dt} &= -\frac{F_r(t)}{V} C_B(t) + k_1 C_A(t) - k_2 C_B(t)
\end{aligned}
\qquad (3.26)
$$

where F_r is the feed flow rate of product A, V is the reactor volume which is kept constant during the operation, C_A and C_B are the reactant concentrations in the reactor, and k_i $(i = 1, 2, 3)$ are the reaction rate constants for the three reactions.

In this case, the variables of interest are: the concentration of B in the reactor (C_B as the controlled variable), the flow through the reactor (F_r as the manipulated variable), and the concentration C_{Ai} of A in the feed flow (whose variation can be considered as the disturbance). The kinetic parameters are chosen to be $k_1 = 5/6\ min^{-1}$, $k_2 = 5/3\ min^{-1}$, and $k_3 = 1/6\ l\ mol^{-1}\ min^{-1}$. Also, is assumed that the nominal concentration of A in the feed (C_{Ai}) is $10\ mol\ l^{-1}$ and the volume $V = 700\ l$.

Using (3.26) and the parameters values, the characterization of the steady-state for the process can be obtained as it is shown in Fig.3.9, for three concentrations of C_{Ai}, where is easy to see the non-linearity of the system.

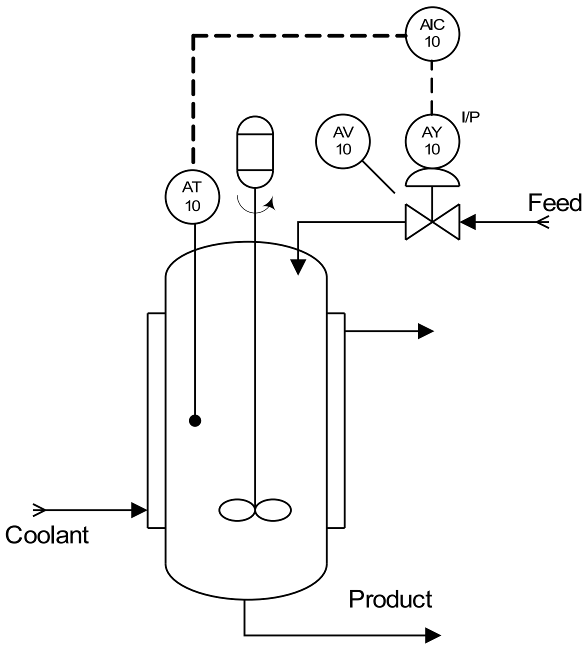

Figure 3.8: Example 2 - CSTR System

Initially, the system is at the steady-state (therefore the operational point) with $C_{Ao} = 2.9175\ mol\ l^{-1}$ and $C_{Bo} = 1.10\ mol\ l^{-1}$. From this, the measurement range for C_B can be selected from 0 to 1.5714 mol/l and the capacity for the control valve with a maximum flow of 634.1719 l/min (variation range of the flow from 0 to 634.1719 l/min) (Arrieta *et al.*, 2008). The signals (y, u, r) will be in percentage (0 to 100%).

The sensor-transmitter element takes the form

$$y(t)_{\%} = \left(\frac{100}{1.5714}\right) C_B(t) \tag{3.27}$$

and the control valve with a linear flow characteristic,

$$F_r(t) = \left(\frac{634.1719}{100}\right) u(t)_{\%} \tag{3.28}$$

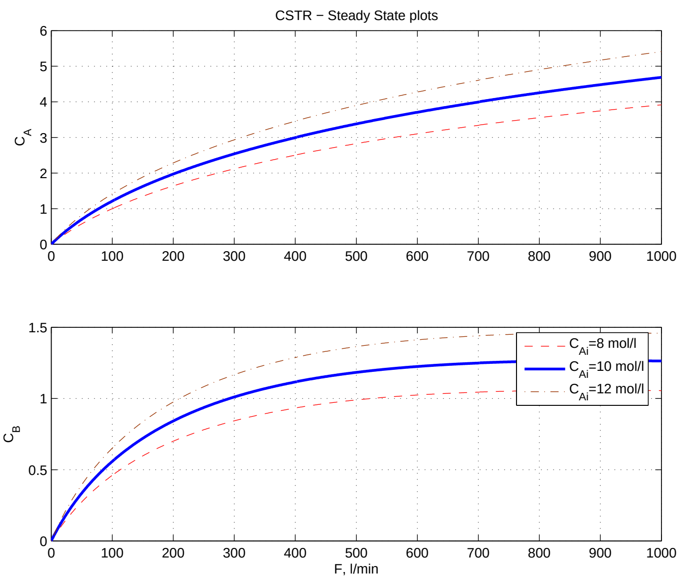

Figure 3.9: Example 2 - Steady-State characterization for the reactor (3.26)

Fig. 3.10 shows the steady-state characterization, taking into account elements represented by (3.27) and (3.28). This is called *set actuator-process-sensor* and from this, it is clear the choice for the operational point as, $r_o = 70\%$ and $u_o = 60\%$.

It is assumed that changes in the set-point would be not larger than 10% and the possible disturbance in C_{Ai}, can variate around $\pm 10\%$. In Fig. 3.11 the process output can be seen (including the sensor and the control valve) and also the FOPDT model for a step change in the process input ($y_u(t)$).

Using the identification method (Alfaro, 2006), the determined FOPDT model is

$$P_3(s) \approx \frac{0.3199\mathrm{e}^{-0.5289s}}{0.6238s + 1} \tag{3.29}$$

From (3.29), the application of the ISE tuning formulae for optimal set-point and load-disturbance responses and also the *intermediate* $\overline{\gamma}_\alpha$*−autotuning*

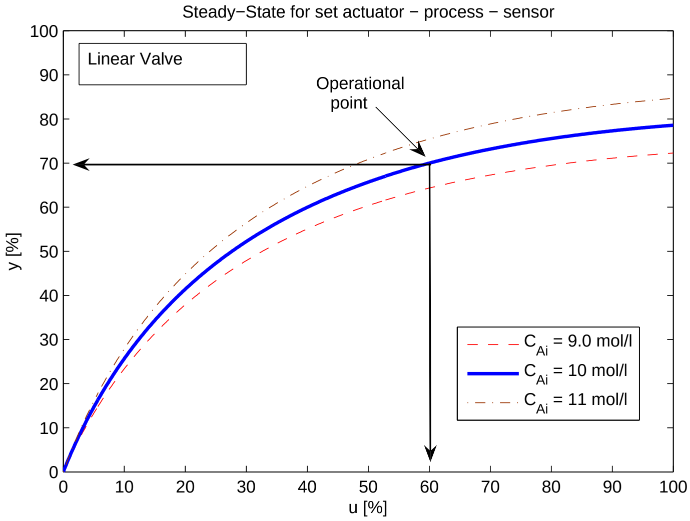

Figure 3.10: Example 2 - Steady-State characterization for the set actuator-process-sensor

provide the parameters for the PID controller that are shown in Table 3.5.

Table 3.5: Example 2 - PID controller parameters for P_3

tuning	K_p	T_i	T_d
$set - point(sp)$	3.799	0.707	0.264
$load - disturbance(ld)$	5.404	0.494	0.293
$\overline{\gamma}_{\alpha=0.25} - autotuning$	4.190	0.609	0.269
$\overline{\gamma}_{\alpha=0.50} - autotuning$	4.570	0.577	0.270
$\overline{\gamma}_{\alpha=0.75} - autotuning$	4.820	0.551	0.273

Process outputs of the closed-loop system are shown in Fig. 3.12, first for a set-point step change of -10%, follows by a disturbance of +10% and finally a new change in the set-point of +5%, all these situations using the three tuning modes (set-point, load-disturbance and $\overline{\gamma}_{\alpha} - autotuning$). Also, the control signal ($u(t)$) can be seen. It appears that, as expected, the control signal is

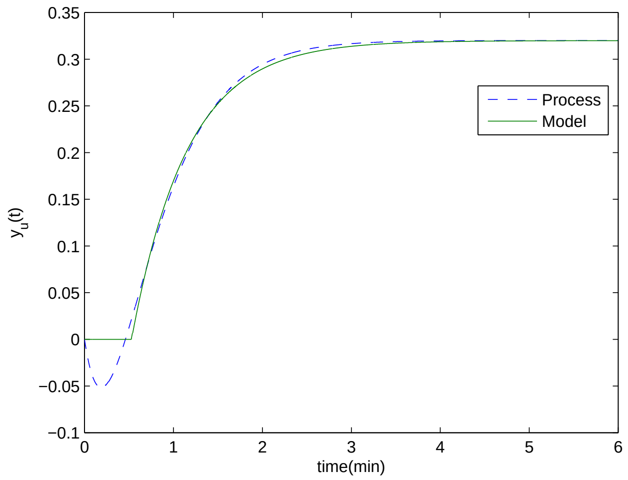

Figure 3.11: Example 2 - Reaction curve for process (3.26) and FOPDT model 3.29.

smoother for lower values of α.

A more comprehensive picture of the process outputs, for the set-point change, is shown in Fig. 3.13. In this case, it can be seen that, as expected, the set-point tuning gives the better performance for servo operation mode. Furthermore, the $\overline{\gamma}_\alpha - autotuning$ provides a lower degradation, respect to the optimal, than the load-disturbance tuning.

The detail of load-disturbance attenuation is in Fig. 3.14. Similarly to the previous case, the load-disturbance tuning is the one that gives better performance for regulation operation and the performance degradation of the set-point tuning is greater than the three cases for $\overline{\gamma}_\alpha - autotuning$.

In general terms, it can be confirmed that the $\overline{\gamma}_\alpha - autotuning$ gives a better overall performance when the system operates in both servo and regulation modes. Also, the control signal for the *intermediate* tuning seems to be *smoother* than that provided by the optimal regulation settings.

Table 3.6 shows the *PD* and *WPD* indices and the improvement that can be achieve for each case of the $\overline{\gamma}_\alpha - autotuning$.

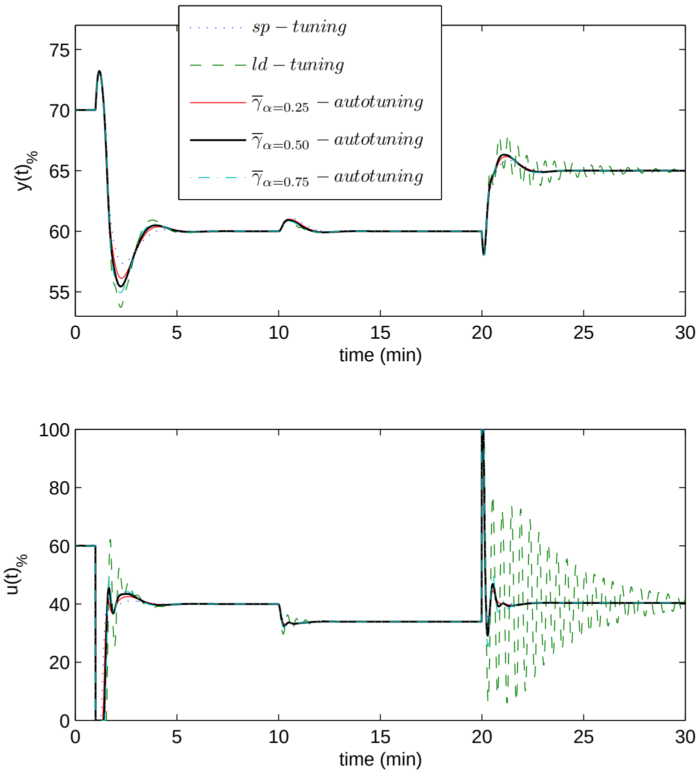

Figure 3.12: Example 2 - Process responses for servo and regulation for non-linear control system.

3.8 Summary

In process control it is very usual to have changes in the set-point, as well as in the disturbance. This causes the need to face with both servo and regulatory control problems. For 1-DoF PID controllers, when the tuning objective is different to the real system operation, a degradation in the performance is expected and it can be evaluated. A reduction in the overall Performance Degradation can be obtained by searching an *intermediate* controller between

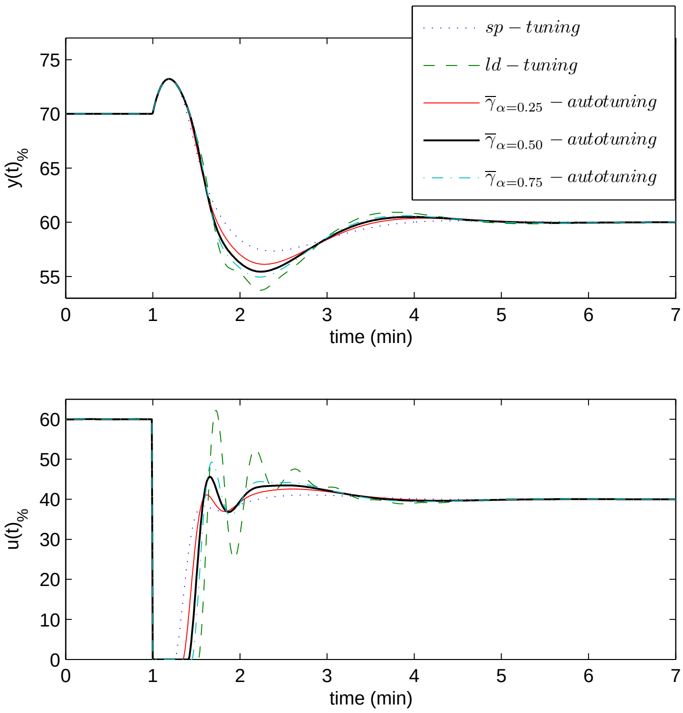

Figure 3.13: Example 2 - Process responses for the non-linear control system operating as servo.

the optimal ones proposed for set-point and load-disturbance tunings.

Autotuning formulae have been presented with the aim to minimize the Weighted Performance Degradation, expressed as a combination depending of the balance between the total time that the system operates in servo and regulation modes. This is one of the main contributions of this work because it is a novel feature that allows to select the tuning according to a general qualitative specification of the control system operation.

Results are given for PID controllers, in order to get results closer to industrial applications. The examples have shown the improvement obtained with each one of the $\overline{\gamma}_\alpha - autotuning$ cases.

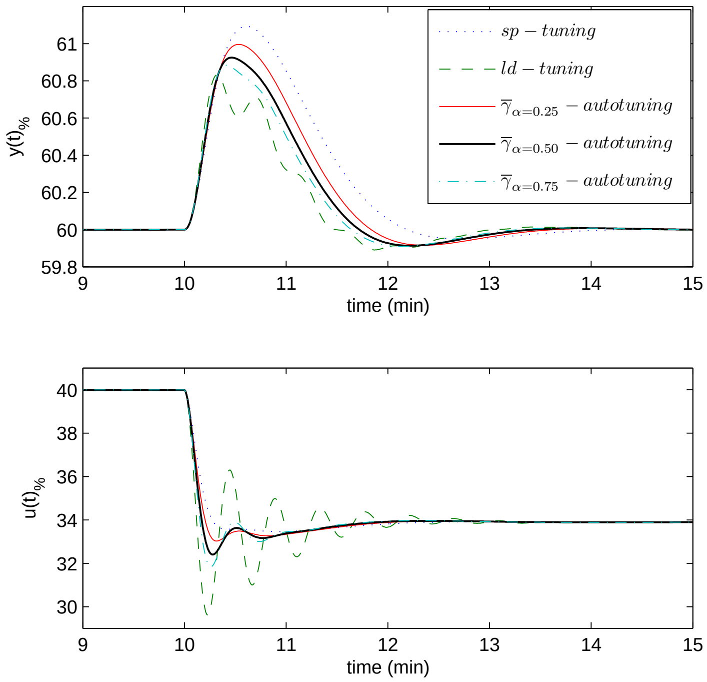

Figure 3.14: Example 2 - Process responses for the non-linear control system operating as regulator

Even if the results were presented and exemplified using the ISE performance criteria, it could be possible to reproduce a similar methodology for other PID controller tuning, like the one that uses derivative action just applied to the process output, or to other PID tunings with different performance objectives.

Table 3.6: Example 2 - *PD* and *WPD* values for the non-linear CSTR system and the improvement obtained with $\overline{\gamma}_\alpha - autotuning$

tuning	PD_{sp}	PD_{ld}	$WPD_{\alpha=0.25}$	$WPD_{\alpha=0.50}$	$WPD_{\alpha=0.75}$
$set-point(sp)$	-	0.3284	0.0821	0.1642	0.2463
$load-disturbance(ld)$	0.5316	-	0.3987	0.2658	0.1329
$\overline{\gamma}_{\alpha=0.25} - autotuning$	0.0192	0.1398	0.0493	-	-
$\overline{\gamma}_{\alpha=0.50} - autotuning$	0.0706	0.0470	-	0.0588	-
$\overline{\gamma}_{\alpha=0.75} - autotuning$	0.1334	0.0038	-	-	0.0362
improvement in % of					
$\overline{\gamma}_{\alpha=0.25} - autotuning$	96.39%(ld)	57.73%(sp)	39.95%(sp)	-	-
			87.63%(ld)	-	-
$\overline{\gamma}_{\alpha=0.50} - autotuning$	86.72%(ld)	85.99%(sp)	-	64.19%(sp)	-
			-	77.88%(ld)	-
$\overline{\gamma}_{\alpha=0.75} - autotuning$	74.91%(ld)	99.15%(sp)	-	-	85.30%(sp)
			-	-	72.76%(ld)
(respect to)					

Chapter 4

Application to unstable and integrating processes

4.1 Considerations

Much of the effort in control systems has been concentrated on the application to stable systems, while quite a few of the important chemical processing units in industrial and chemical practices are open-loop unstable processes that are known to be difficult to control, especially when there exists a time delay, such as in the case of continuous stirred tank reactors, polymerization reactors and bio-reactors which are inherently open-loop unstable by design (Sree and Chidambaram, 2006).

Clearly, the tuning of controllers to stabilize these processes and to achieve adequate disturbance rejection is critical. Moreover, integrating processes are very frequently encountered in process industries and many researchers have suggested that for the purpose of designing a controller, a large number of chemical processes could be modelled using an integrating process with time delay. Consequently, there has been much interest in the literature in the tuning of industrially standard PID controllers for open-loop unstable systems as well as for integrating processes.

In fact, several papers can be found in the literature that deals with the tuning of unstable (Lee *et al.*, 2000; Visioli, 2001; Sree *et al.*, 2004; Vivek and Chidambaram, 2005; Panda, 2009) and integrating processes (Lee *et al.*, 2000; Visioli, 2001; Chen and Seborg, 2002; Chidambaram and Sree, 2003; Ali and Majhi, 2010). There is, however, a common problem with the tuning of PI/PID controllers for such systems: the tunings are usually devoted to the servo or regulation operation and may exhibit a significant performance degradation when operating on the tuning mode they were not designed for. This is also observed when operating with stable systems, but becomes really a problem for unstable and integrating processes. A simple look at the existing literature shows that the performance is highly dependent of using the appropriate tuning mode. (O'Dwyer, 2003) presents a collection of tuning rules for PID controllers for stable, unstable and integrating processes.

Based on this observation, in this chapter the purpose is to provide an alternative way of addressing the tuning of unstable and integral processes in order to alleviate the aforementioned situation and to provide a better overall performance. The approach constitutes and extension of the method presented in section 3.6 for stable systems (Arrieta *et al.*, 2010). The idea is to find an *intermediate* tuning for the controller that improves the overall performance of the system, considered as a *trade-off* between servo and regulation operation modes. The settings are determined from the combination of the optimal ones for set-point and load-disturbance, presented in (Visioli, 2001), and taking into account the balance between the importance of each one of the operation modes for the control system (servo or regulation).

4.2 General framework

We consider again the unity-feedback system shown in Fig. 2.1, where P is an unstable system assumed to be modelled by

$$P(s) = \frac{K}{Ts-1}\mathrm{e}^{-Ls} \tag{4.1}$$

or if we have an integrating process, the model will be

$$P(s) = \frac{K}{s}\mathrm{e}^{-Ls} \tag{4.2}$$

In both cases, K is the process gain and L is the dead-time. For unstable system (4.1), T is the time constant. These models are commonly used because are capable of satisfactorily modelling the dynamics of unstable and integrating processes.

Let us consider for controller C, the ideal One-Degree-of-Freedom (1-DoF) PID controller, that is considered as

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right) \tag{4.3}$$

where K_p is the proportional gain and T_i and T_d are the integral and derivative time constants, respectively.

The analysis presented here, is an extension of the Performance Degradation idea, adapting all the aspects and considerations to the cases of unstable an integrating systems. We rely once more on the Integral Square Error (ISE) criteria (2.6). In this case the settings are determined from a combination of the optimal ones for set-point and load-disturbance, presented in (Visioli, 2001), and taking into account the balance between the preference of each one of the operation modes for the control system.

Tables 4.1 and 4.2 show the tuning formulas for unstable and integrating system respectively, where the resulting settings are optimal to the ISE criteria (Visioli, 2001).

Table 4.1: Tuning rules for optimal ISE set-point and load-disturbance for unstable processes

PID parameter	set-point	load-disturbance
K_p	$1.32/K(L/T)^{-0.92}$	$1.37/K(L/T)^{-1}$
T_i	$4.00(L/T)^{0.47}T$	$2.42(L/T)^{1.18}T$
T_d	$3.78T(1-0.84(L/T)^{-0.02})/(L/T)^{-0.95}$	$0.60(L/T)T$

4.3 Tuning rules for unstable processes

Following the previously developed procedure for stable systems and adapting it to the unstable case, we say that the controller settings family $[K_p(\gamma_1^u)$, $T_i(\gamma_2^u)$, $T_d(\gamma_3^u)]$ will be generated by a linear evolution from the parameters

Table 4.2: Tuning rules for optimal ISE set-point and load-disturbance for integrating processes

PID parameter	set-point	load-disturbance
K_p	$1.03/KL$	$1.37/KL$
T_i	-	$1.49L$
T_d	$0.49L$	$0.29L$

for the set-point tuning to the load-disturbance one and the other way around (as relations (3.7)). Therefore,

$$\begin{aligned} K_p(\gamma_1^u) &= \gamma_1^u K_p^{ld} + (1-\gamma_1^u)K_p^{sp} \\ T_i(\gamma_2^u) &= \gamma_2^u T_i^{ld} + (1-\gamma_2^u)T_i^{sp} \\ T_d(\gamma_3^u) &= \gamma_3^u T_d^{ld} + (1-\gamma_3^u)T_d^{sp} \end{aligned} \tag{4.4}$$

Once again, repeating the problem optimization posed in (3.21) for the three weighting factors and different values of the normalized dead-time $\tau = L/T$, we can find an optimal set for each γ_i^u parameter. For each one of these groups, it is possible to approximate a function to determine a general procedure that allows to find the suitable values for the γ_i^u's, that provide the best *intermediate* tuning. Results are adjusted according to

$$\gamma_i^u(\tau) = a^u + b^u\tau + c^u\tau^2 \tag{4.5}$$

where a^u, b^u and c^u are given in Table 4.3, according to the weighting factor α and for each γ_i^u.

Equation (4.5) for each γ_i^u along with the settings (4.4) provide what we call here $\overline{\gamma}_\alpha^u - tuning$ for unstable processes offering a weighted servo/regulation operation.

4.3.1 Illustrative example

Consider the following unstable system represented by

$$P_4(s) = \frac{1}{s-1}\mathrm{e}^{-0.2s} \tag{4.6}$$

Table 4.3: $\overline{\gamma}^u_\alpha$-tuning settings for unstable systems

	$\alpha = 0.25$			$\alpha = 0.50$			$\alpha = 0.75$		
constant	a^u	b^u	c^u	a^u	b^u	c^u	a^u	b^u	c^u
γ_1^u	0.544	-1.631	2.194	0.629	-0.801	1.009	0.711	0.061	-0.324
γ_2^u	0.807	0.100	-1.491	0.787	1.026	-2.513	0.687	2.173	-4.026
γ_3^u	0.660	-0.019	0.293	0.718	0.268	-0.580	0.547	1.265	-2.116

The application of the ISE tuning formulae of (Visioli, 2001), as well as, the $\overline{\gamma}^u_\alpha - tuning$, provides the PID parameters shown in Table 4.4. Fig. 4.1 shows the control system performance for the two possible operation modes for the above tuning methods.

Table 4.4: Unstable process - PID controller parameters for P_4

tuning	K_p	T_i	T_d
$set-point(sp)$	5.803	1.877	0.109
$load-disturbance(ld)$	6.850	0.362	0.120
$\overline{\gamma}^u_{\alpha=0.25} - tuning$	6.123	0.715	0.116
$\overline{\gamma}^u_{\alpha=0.50} - tuning$	6.336	0.526	0.117
$\overline{\gamma}^u_{\alpha=0.75} - tuning$	6.547	0.442	0.117

It can be seen that the proposed $\overline{\gamma}^u_\alpha - tuning$ gives lower performance than the optimum settings when the system operates in the same way as it was tuned. However, higher performance can be obtained for the whole system operation (regulatory-control and servo-control), when the *intermediate* controller is used.

Table 4.5 shows the *PD* and *WPD* indices and the improvement, in percentage, that can be achieve for each case of the $\overline{\gamma}^u_\alpha - tuning$ respect to the extreme tunings (set-point and load-disturbance).

4.4 Tuning rules for integrating processes

Now, we analyze the case for integrating systems represented as (4.2). For this case, it must be taken into account that the method presented in (Visioli, 2001)

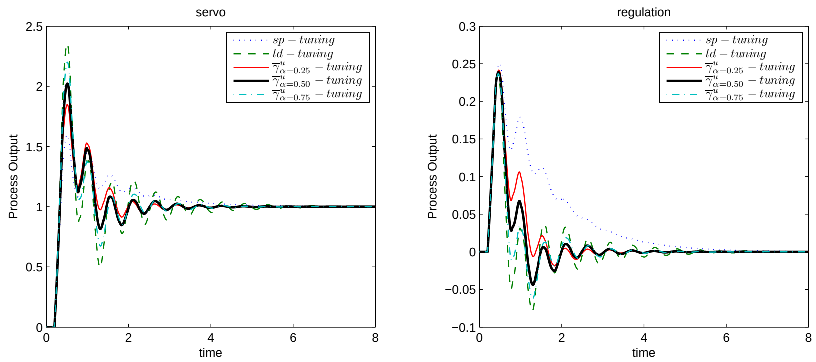

Figure 4.1: Unstable process - Servo and regulation control responses for system P_4.

Table 4.5: Unstable process - PD and WPD values for the system P_4 and the improvement obtained with $\overline{\gamma}_{\alpha}^{u} - tuning$.

tuning	PD_{sp}	PD_{ld}	$WPD_{\alpha=0.25}$	$WPD_{\alpha=0.50}$	$WPD_{\alpha=0.75}$
$set - point(sp)$	-	1.8829	0.4707	0.9414	1.4122
$load - disturbance(ld)$	0.6788	-	0.5091	0.3394	0.1697
$\overline{\gamma}_{\alpha=0.25}^{u} - tuning$	0.1207	0.3190	0.1703	-	-
$\overline{\gamma}_{\alpha=0.50}^{u} - tuning$	0.2524	0.0993	-	0.1759	-
$\overline{\gamma}_{\alpha=0.75}^{u} - tuning$	0.4201	0.0088	-	-	0.1116
improvement in % of					
$\overline{\gamma}_{\alpha=0.25}^{u} - tuning$	82.21%(ld)	83.06%(sp)	63.82%(sp)	-	-
			66.55%(ld)	-	-
$\overline{\gamma}_{\alpha=0.50}^{u} - tuning$	62.81%(ld)	94.73%(sp)	-	81.32%(sp)	-
			-	48.18%(ld)	-
$\overline{\gamma}_{\alpha=0.75}^{u} - tuning$	38.10%(ld)	99.54%(sp)	-	-	92.10%(sp)
			-	-	34.23%(ld)
(respect to)					

uses a PD (Proportional-Derivative) controller, when the expected operation for the system is servo-control. This kind of controller could be seen as a PID controller with the integral part disabled, that is equivalent to $T_i \to \infty$.

Regarding to the above consideration, the generation of the controller's parameters family like (4.4), has the difficulty to make a transition in the integral time between T_i^{ld} and infinite.

In order to adjust the general procedure considering the foregoing concerns, we obtain the PID controller's gains as

$$K_c = K_p, \quad K_i = K_p/T_i, \quad K_d = K_p T_d \tag{4.7}$$

after that, the controller's family is generated with these gains according to

$$\begin{array}{rcl} K_c(\gamma_1^i) & = & \gamma_1^i K_c^{ld} + (1-\gamma_1^i)K_c^{sp} \\ K_i(\gamma_2^i) & = & \gamma_2^i K_i^{ld} \\ K_d(\gamma_3^i) & = & \gamma_3^i K_d^{ld} + (1-\gamma_3^i)K_d^{sp} \end{array} \tag{4.8}$$

Then, the original PID parameters can be obtained by applying an inverse concept of (4.7), like

$$K_p = K_c, \quad T_i = K_c/K_i, \quad T_d = K_d/K_c \tag{4.9}$$

With this change, it is possible to achieve a suitable transition in the PID parameters, maintaining the idea of $\overline{\gamma}^i \in [0,1]$.

Once again, if we repeat the optimization problem posed in (3.21) for the three weighting factors and different values of dead-time L, we can find an optimal set for each γ_i^i parameter. In this case, unlike the case of unstable systems, the optimal values are practically constant and for that reason, the method is approximated with fixed values for each γ_i^i. Table 4.6 gives the corresponding constant values depending only of the weighting factor α.

Table 4.6: $\overline{\gamma}_\alpha^i$-tuning values for integrating systems.

	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$
γ_1^i	0.5591	0.7064	0.8274
γ_2^i	0.3906	0.5702	0.7532
γ_3^i	0.5903	0.7421	0.8731

Summarizing, for integrating systems, the suitable values for $\overline{\gamma}^i$ along with the settings (4.8) provide the $\overline{\gamma}_\alpha^i - tuning$, for weighted servo/regulation operation.

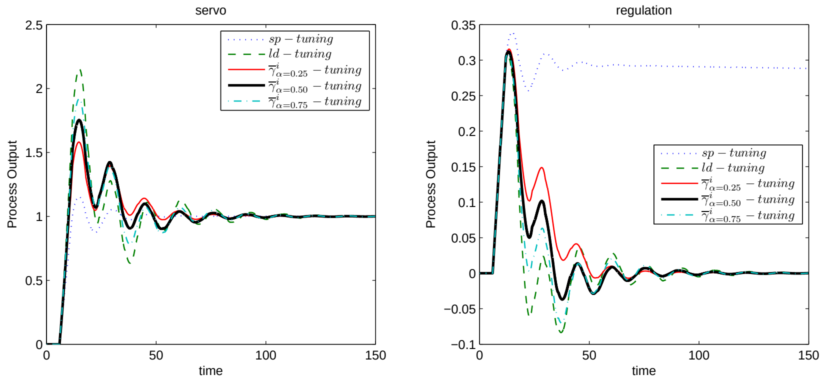

Figure 4.2: Integrating process - Servo and regulation control responses for system P_5.

4.4.1 Illustrative example

Consider the following integrating process represented by

$$P_5(s) = \frac{0.0506}{s}\mathrm{e}^{-6s} \tag{4.10}$$

Table 4.7 shows the PID controller parameters for the system (4.10) using the (Visioli, 2001) method and the proposed $\overline{\gamma}^i_\alpha - tuning$ with $\alpha = \{0.25, 0.50, 0.75\}$. Fig. 4.1 shows the control system output for the servo and regulation operation modes.

Table 4.7: Integrating process - PID controller parameters for P_5

tuning	K_p	T_i	T_d
$set - point(sp)$	3.393	-	2.940
$load - disturbance(ld)$	4.513	8.940	3.540
$\overline{\gamma}^i_{\alpha=0.25} - tuning$	4.019	20.384	3.363
$\overline{\gamma}^i_{\alpha=0.50} - tuning$	4.184	14.536	3.448
$\overline{\gamma}^i_{\alpha=0.75} - tuning$	4.319	11.361	3.552

It can be confirmed that the $\overline{\gamma}^i_\alpha - tuning$ gives a better performance when the system operates in both servo and regulation modes. Special attention has to be put to the set-point tuning operating in regulation mode because being the controller PD type (without integral action), a static error is expected and consequently the Performance Degradation is infinite.

Table 4.8 shows the *PD* and *WPD* indices and the improvement, that can be achieve for each case of the $\overline{\gamma}^i_\alpha - tuning$.

Table 4.8: Integrating process - *PD* and *WPD* values for the system P_5 and the improvement obtained with $\overline{\gamma}^i_\alpha - tuning$.

tuning	PD_{sp}	PD_{ld}	$WPD_{\alpha=0.25}$	$WPD_{\alpha=0.50}$	$WPD_{\alpha=0.75}$
$set-point(sp)$	-	∞	∞	∞	∞
$load-disturbance(ld)$	0.9742	-	0.7307	0.4871	0.2436
$\overline{\gamma}^i_{\alpha=0.25} - tuning$	0.2810	0.4611	0.3260	-	-
$\overline{\gamma}^i_{\alpha=0.50} - tuning$	0.4443	0.1682	-	0.3063	-
$\overline{\gamma}^i_{\alpha=0.75} - tuning$	0.6393	0.0468	-	-	0.1950
improvement in % of					
$\overline{\gamma}^i_{\alpha=0.25} - tuning$	71.16%(ld)	∞(sp)	∞(sp)	-	-
			55.38%(ld)	-	-
$\overline{\gamma}^i_{\alpha=0.50} - tuning$	54.39%(ld)	∞(sp)	-	∞(sp)	-
			-	37.12%(ld)	-
$\overline{\gamma}^i_{\alpha=0.75} - tuning$	34.38%(ld)	∞(sp)	-	-	∞(sp)
			-	-	19.95%(ld)
(respect to)					

4.5 Comparative Study

In this section the proposed $\overline{\gamma}_\alpha - tuning$ method with $\alpha = 0.50$ (balanced servo and regulation operation) is compared with other well known PID tuning methods for unstable and integrating systems.

4.5.1 Unstable process example

Let us consider the First-Order-Delayed-Unstable-Process (FODUP) with the following transfer function

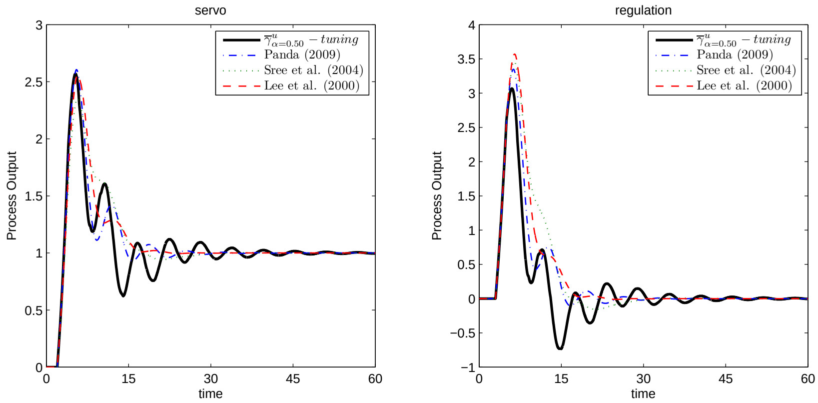

Figure 4.3: Unstable process - Servo and regulation control responses for system P_6.

$$P_6(s) = \frac{4}{4s-1}\mathrm{e}^{-2s} \tag{4.11}$$

Table 4.9 shows the PID parameters obtained with different tuning methods for system (4.11). Also in Fig. 4.3 it is possible to see the process outputs for the servo and regulation cases.

Table 4.9: Unstable process - PID controller parameters for P_6

tuning	K_p	T_i	T_d
$\overline{\gamma}^u_{\alpha=0.50} - tuning$	0.654	6.662	1.188
(Panda, 2009)	0.653	10.420	0.908
(Sree *et al.*, 2004)	0.571	11.122	1.025
(Lee *et al.*, 2000)($\lambda = L$)	0.606	11.732	0.840

Table 4.10 gives the performance criteria (3.1) for servo (J_{sp}) and regulation (J_{ld}) operation modes, as well as, the associated Performance Degradation indices ((3.17) to (3.20)) for each tuning.

All the values confirm the fact that, in global terms, when both operating modes could appear and taking into account the importance that the control-

Table 4.10: Unstable process - Performance and Performance Degradation indices for system P_6.

tuning	J_{sp}	J_{ld}	PD_{sp} [1]	PD_{ld} [2]	$WPD_{\alpha=0.50}$
$\overline{\gamma}^u_{\alpha=0.50} - tuning$	8.9776	26.3352	0.0837	0.0728	0.0783
(Panda, 2009)	9.2178	34.3572	0.1127	0.3997	0.2562
(Sree *et al.*, 2004)	9.4747	46.7897	0.1437	0.9061	0.5249
(Lee *et al.*, 2000)($\lambda = L$)	9.9005	44.4132	0.1951	0.8093	0.5022

[1] Calculated using equation (3.17) with $J_{sp}(sp) = 8.2839$.
[2] Calculated using equation (3.18) with $J_{ld}(ld) = 24.5470$.

loop is operating as servo or regulation mode, the proposed $\overline{\gamma}^u_\alpha - tuning$ is the best choice to tune the PID controller in order to get less Performance Degradations.

4.5.2 Integrating process example

A distillation column separates a small amount of a low-boiling material from the final product. This technique is very common in chemical processes for the separation of mixed fluids. The bottom level of the distillation column is controlled by adjusting the steam flow rate. The process for the level control system is usually represented by an integrating model as

$$P_7(s) = \frac{0.2}{s}\mathrm{e}^{-7.4s} \tag{4.12}$$

The PID controller parameters, for the different tuning methods, are shown in Table 4.11 and in Fig. 4.4 there are the process outputs for system (4.12) operating in servo and regulation modes.

In Table 4.12, there are the performance criteria (3.1) for servo (J_{sp}) and regulation (J_{ld}) operation modes, as well as, the associated Performance Degradation indices ((3.17) to (3.20)) for each tuning.

From all the data, it is possible to see that even that the servo performance of the proposed tuning is lower than the provided for the other tunings, the general behavior (taking into account both operation modes) of the control system, tuned with $\overline{\gamma}^i_\alpha - tuning$, is better. This means achieving the best value for the Weighted Performance Degradation index (WPD).

Table 4.11: Integrating process - PID controller parameters for P_7

tuning	K_p	T_i	T_d
$\overline{\gamma}^i_{\alpha=0.50} - tuning$	0.858	17.928	4.253
(Ali and Majhi, 2010)	0.696	23.458	3.626
(Chidambaram and Sree, 2003)	0.834	33.300	3.330
(Lee *et al.*, 2000)($\lambda = L$)	0.675	27.099	2.620

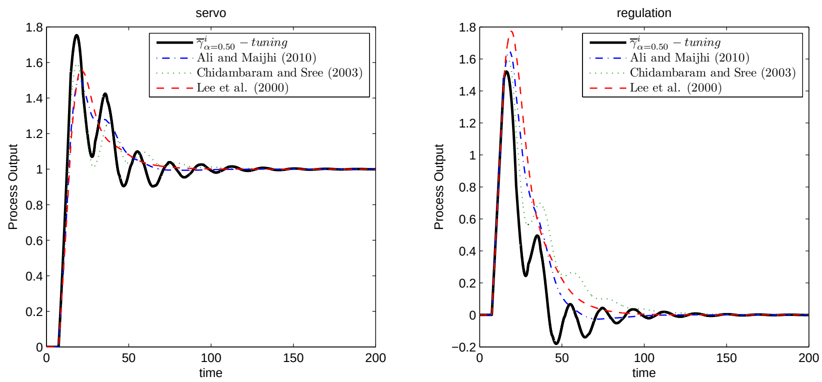

Figure 4.4: Integrating process - Servo and regulation control responses for system P_7.

Table 4.12: Integrating process - Performance and Performance Degradation indices for system P_7.

tuning	J_{sp}	J_{ld}	${PD_{sp}}^1$	${PD_{ld}}^2$	$WPD_{\alpha=0.50}$
$\overline{\gamma}^i_{\alpha=0.50} - tuning$	14.4299	22.4365	0.4443	0.1684	0.3063
(Ali and Majhi, 2010)	13.0946	36.8297	0.3106	0.9179	0.6142
(Chidambaram and Sree, 2003)	12.3686	33.7810	0.2380	0.7591	0.4985
(Lee *et al.*, 2000)($\lambda = L$)	13.3858	44.9949	0.3398	1.3430	0.8414

[1] Calculated using equation (3.17) with $J_{sp}(sp) = 9.9912$.
[2] Calculated using equation (3.18) with $J_{ld}(ld) = 19.2036$.

4.6 Summary

The control of stable processes by using PI/PID controllers is by now a well established and understood problem (even new methods and approaches are continuously appearing). However, when the process has integrating or unstable characteristics the problem becomes much more difficult. Several proposals have appeared in the literature presenting different approaches to tackle the problem

These procedures are usually related to servo or regulation control problems, however, in process control it is very usual to have changes in the set-point as well as in the disturbance. A common drawback of such approaches is the high loss of performance if the other operation mode is used.

An approach for providing a unique tuning for unstable and integrating processes, depending on the importance given to the system operations in servo and regulation modes, is presented. It combines tunings for both operation modes in such a way that the performance degradation is *traded-off*. Results are given for PID controllers, in order to get results closer to industrial applications and the examples have shown the improvement that can be achieved with the proposal.

Part II

Robustness and performance trade-off for PID controllers

Chapter 5

Robust based PID control design

5.1 Motivation and framework

Robustness is an important attribute for control systems, because the design procedures are usually based on the use of low-order linear models identified at the closed-loop operation point. Due to the non-linearity found in most of the industrial process, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements, for the control system. Therefore, the design of the closed-loop control system must take into account the system performance to load-disturbance and set-point changes and its robustness to variation of the controlled process characteristics, preserving the well-known *trade-off* between all these variables.

The general procedure, presented in chapter 3, depends on a direct way of the extreme tunings, robustness considerations are therefore not taken into account explicitly. Of course, if the original extreme tunings include any closed-loop characteristic, for example stability or robustness, they would be reflected in the resulting *intermediate* tuning. In this sense, the purpose here is to go forward and to propose an approach that takes into account the robustness issue, in an explicit way at the design stage.

Another aspect that is constrained by the extreme tunings is the PID controller type. On this direction, it is desirable to use a PID controller that applies the derivative part just to the feedback signal, in order to avoid extreme instantaneous change in the controller output signal when a set-point step change occurs (Åström and Hägglund, 2006). This kind of controller is commonly called ISA-PID and it is the most dominating control structure present in industrial control applications.

Taking into account the above statements, we consider the feedback control system shown in Fig. 5.1, where $P(s)$ is the controlled process, $C(s)$ is the controller, $r(s)$ is the set-point, $u(s)$ is the controller output signal, $d(s)$ is the load-disturbance and $y(s)$ is the system output.

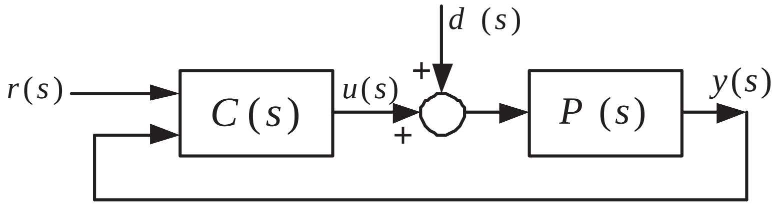

Figure 5.1: Closed-loop control system.

Now, the output of the ISA-PID controller is given by

$$u(s) = K_p \left(1 + \frac{1}{T_i s}\right) e(s) - K_p \left(\frac{T_d s}{1 + (T_d/N)s}\right) y(s) \tag{5.1}$$

where $e(s) = r(s) - y(s)$ is the control error, K_p is the controller static gain, T_i the integral time constant, T_d the derivative time constant and the derivative filter constant N is taken $N = 10$ as it is usual practice in industrial controllers.

5.1.1 Performance

One way to evaluate the performance of control systems is by calculating a cost function based on the error, i.e. the difference between the desired value (set-point) and the actual value of the controlled variable (system's output). Of course, as larger and longer in time is be the error, the system's performance will be worse.

In this sense, a common reference for the evaluation of the controller performance, is a functional based on the integral of the error like: Integral-Square-Error (ISE) (2.6), or Integral-Absolute-Error (IAE).

Some approaches had used the ISE criterion, because its definition allows an analytical calculation for the index (Zhuang and Atherton, 1993; Visioli, 2001). However, nowadays can be found in the literature that IAE is the most useful ans suitable index to quantify the performance of the system (Chen and Seborg, 2002; Skogestad, 2003; Åström and Hägglund, 2006; Kristiansson and Lennartson, 2006; Tan *et al.*, 2006). It can be used explicitly in the design stage or just as an evaluation measure.

The formulation of the criterion is stated as

$$IAE \doteq \int_0^\infty |e(t)|\,dt = \int_0^\infty |r(t) - y(t)|\,dt \tag{5.2}$$

where the index can be measure for changes in the set-point or in the load-disturbance.

5.1.2 Robustness

As an indication of the system *robustness* (relative stability) the Sensitivity Function peak value will be used. The control system Maximum Sensitivity is defined as

$$M_s \doteq \max_\omega |S(j\omega)| = \max_\omega \frac{1}{|1 + C(j\omega)P(j\omega)|} \tag{5.3}$$

and recommended values for M_s are typically within the range 1.4 - 2.0 (Åström and Hägglund, 2006).

The use of the maximum sensitivity as a robustness measure, has the advantage that lower bounds to the Gain, A_m, and Phase, ϕ_m, margins (Åström and Hägglund, 2006) can be assured according to

$$A_m > \frac{M_s}{M_s - 1}$$

$$\phi_m > 2\sin^{-1}\left(\frac{1}{2M_s}\right)$$

Therefore, to assure $M_s = 2.0$ provides what is commonly considered minimum robustness requirement (that translates to $A_m > 2$ and $\phi_m > 29^o$, for $M_s = 1.4$ we have $A_m > 3.5$ and $\phi_m > 41^o$).

5.2 Optimization problem formulation

From the above definitions for performance and robustness specifications, there appears the need to formulate a joint criteria that faces with the *trade-off* between the performance for servo and regulation operation and also that takes into account the accomplishment of a robustness level.

5.2.1 Servo/Regulation trade-off

As it was exposed before, there is a *trade-off* behavior between the dynamics for servo and regulation control operation modes. It is not enough just to consider the tuning mode, it is also necessary to include the system operation in the controller's design.

Using some of the exposed ideas, we can say that J_x^z represents the criteria (5.2) taking into account the operation mode x, for a tuning mode z. From this, we can post the following definitions:

- J_r^r is the value of performance index for the set-point tuning operating in servo-control mode.
- J_d^r is the value of performance index for the set-point tuning operating in regulatory-control mode.
- J_r^d is the value of performance index for the load-disturbance tuning operating in servo-control mode.
- J_d^d is the value of performance index for the load-disturbance tuning operating in regulatory-control mode.

Obviously J_r^r is the optimal value for servo-control operation, J_r^o, and J_d^d is the optimal one for regulation, J_d^o. An *intermediate* tuning between servo and regulation operation modes should have higher values than the optimal ones, when the tuning and operation modes are the same, but the indexes would be lower when the modes are different. So, for each operation mode we have the following relationships,

$$J_r^o \doteq J_r^r \leq J_r^{rd} \leq J_r^d$$
$$J_d^o \doteq J_d^d \leq J_d^{rd} \leq J_d^r$$

where J_r^{rd} and J_d^{rd} are the performance values of the *intermediate* tuning for servo and regulation control operation, respectively.

The previous ideas can be represented graphically, the results are shown in Fig. 5.2, where the performance indexes are plotted in the $J_r - J_d$ plane.

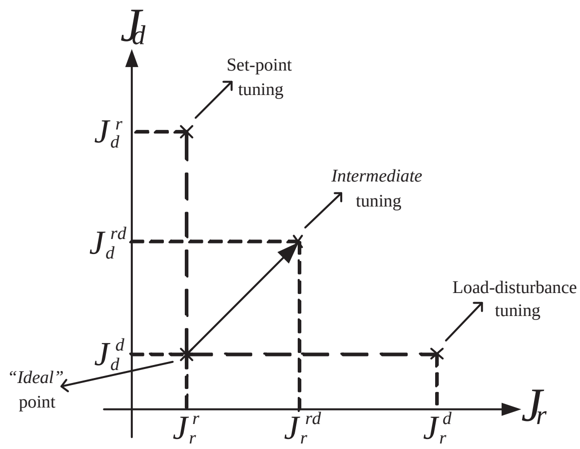

Figure 5.2: Plane $J_r - J_d$.

It can be seen that the point (J_r^r, J_d^d) is the "ideal" one because it represents the minimum performance values taking both possible operation modes, servo and regulation, into account. However, this point is unreachable due the differences in the dynamics for each of the objectives of the control operation modes. Therefore our efforts must go towards getting the minimum resulting distance, meaning the best balance between the operation modes.

On this way, a cost objective function is formulated in order to get closer, as much as possible, the resulting point (J_r^{rd}, J_d^{rd}), to the "ideal" one, (J_r^o, J_d^o). Therefore,

$$J_{rd} = \sqrt{\left(J_r^{rd} - J_r^o\right)^2 + \left(J_d^{rd} - J_d^o\right)^2} \tag{5.4}$$

where J_r^o and J_d^o are the optimal values for servo and regulation control respectively, and J_r^{rd}, J_d^{rd} are the performance indexes for the *intermediate* tuning considering both operation modes. In Fig. 5.2, index (5.4) is represented by the arrow between the "ideal" point and the corresponding to the *intermediate* tuning.

From the above analysis, the optimization problem setup considers the model's normalized dead-times, τ, in the range $0.1 \leq \tau \leq 2.0$, to obtain the PID controller optimum parameters such that

$$\overline{p}_o := [K_{po}, T_{io}, T_{do}] = \arg\left[\min_{\overline{p}} J_{rd}\right] \tag{5.5}$$

where $\overline{p}$ is the PID controller parameters vector. Here, optimization is done using genetic algorithms technique.

The aim of minimization (5.5) is to achieved a balanced performance for both operation modes of the control system.

5.2.2 Robustness constraint criterion

The cost functional (5.4) proposed before, even though face with the *trade-off* problem between the operation modes of the system, just takes into account characteristics of performance. However, there is a need to include a certain robustness for the control-loop.

In that sense, we want to use (5.3) as a robustness measure. So, the optimization problem (5.5) is subject to a constraint of the form

$$|M_s - M_s^d| = 0 \tag{5.6}$$

where M_s and M_s^d are the Maximum Sensitivity and the desired Maximum Sensitivity functions respectively. This constraint tries to guarantee the selected robustness value for the control system.

5.3 Servo/regulation PID tunings with robustness consideration

From the previous formulation, we look for a tuning rule that face with the *trade-off* problem between the performance for servo and regulation modes and providing, at the same time, a certain degree of robustness (if necessary).

5.3.1 PID tuning for specified robustness levels

As it has been stated, we solve the optimization problem (5.5) subject to constraint (5.6). In that sense, a broad classification can be established, using specific values for M_s, within the suggested range between $1.4 - 2.0$. This will allow a qualitative specification for the control system robustness. So, the rating can be described as

- Low robustness level - $M_s = 2.0$
- Medium-low robustness level - $M_s = 1.8$
- Medium-high robustness level - $M_s = 1.6$
- High robustness level - $M_s = 1.4$

According to this principle, the above mentioned four values for M_s are used here as desirable robustness, M_s^d into the robustness constraint (5.6), for the problem optimization (5.5). Additionally, an unconstrained optimization is done, that can be seen as the M_s^d free case.

In order to provide results for autotuning methodology, the optimal sets for the PID parameters with the corresponding desired robustness, are approximated in equations for each controller's parameter. This fitting procedure looks for simple expressions that allow for an homogenized set, to preserve the simplicity and completeness of the approach.

Therefore, the resulting controller parameters will be, expressed just in terms of the FOPDT process model parameters (2.1) as

$$K_pK = a_1\tau^{b_1} + c_1$$
$$\frac{T_i}{T} = a_2\tau^{b_2} + c_2 \tag{5.7}$$
$$\frac{T_d}{T} = a_3\tau^{b_3} + c_3$$

where the constants a_i, b_i and c_i are given in Table 5.1, according to the desired robustness level for the control system.

It is important to note that, although there may be other tuning equations that provide a good fit, the choice of the proposals (5.7) represents an option to retain the simplicity that is searched, because all the robustness levels and controller's parameters can be expressed in the same form and only changing the constants, according to each case as in Table 5.1.

Table 5.1: PID settings for servo/regulation tuning with robustness consideration

constant	M_s^d free	$M_s^d = 2.0$	$M_s^d = 1.8$	$M_s^d = 1.6$	$M_s^d = 1.4$
a_1	1.1410	0.7699	0.6825	0.5678	0.4306
b_1	-0.9664	-1.0270	-1.0240	-1.0250	-1.0190
c_1	0.1468	0.3490	0.3026	0.2601	0.1926
a_2	1.0860	0.7402	0.7821	0.8323	0.7894
b_2	0.4896	0.7309	0.6490	0.5382	0.4286
c_2	0.2775	0.5307	0.4511	0.3507	0.2557
a_3	0.3726	0.2750	0.2938	0.3111	0.3599
b_3	0.7098	0.9478	0.7956	0.8894	0.9592
c_3	-0.0409	0.0034	-0.0188	-0.0118	-0.0127

In the literature, there are many control designs that include robustness in the formulation stage and even more, in some cases the consideration is regarded as a parameter design directly. However, none of these methods check the accomplishment of the claimed robustness and this should be an aspect that deserves much attention.

The deviation of the resulting value of M_s with respect to the specified target has a direct influence (as a *trade-off*) in the performance of the system (Vilanova *et al.*, 2010).

In order to guarantee the selected robustness, the constraint stated in (5.6) forces the optimization problem to fulfill the fixed value M_s^d and for this, the minimum of the performance index J_{rd} is achieved.

Here, the resulting robustness, applying the proposed methodology, is compared to the desired one, in order to check the accomplishment of the claimed robustness. Fig. 5.3 shows that the robust tuning has a very good accuracy for the M_s values for all the range of processes, therefore assuring that performance is the best one that can be achieved for that robustness value.

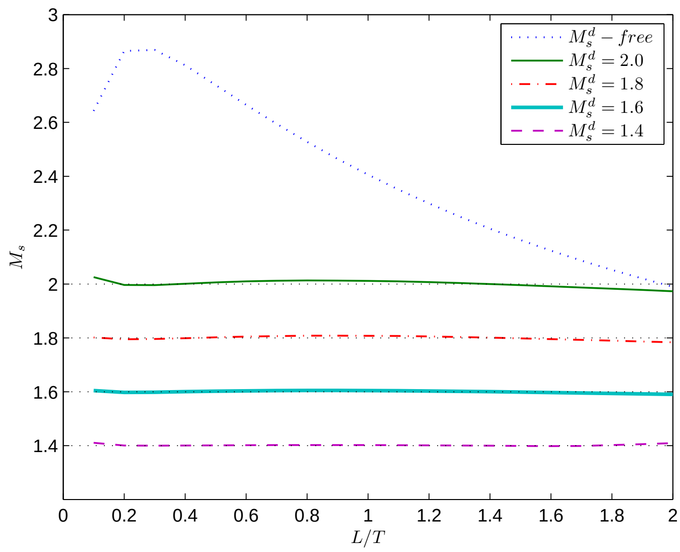

Figure 5.3: Accomplishment for each claimed robustness level.

From the very well known performance-robustness *trade-off*, the increase of the system's robustness from the M_s^d-free case (no robustness constraint), is reflected in a deterioration of the system's performance, and vice-versa. Similar to Fig. 5.3, where it can be seen the robustness increasing, in Fig. 5.4 it is shown how the performance index J_{rd} varies, for each one of the proposed robustness levels.

If we use the information of Fig. 5.3 and Fig. 5.4, and the unconstrained case as the starting point, it is possible to see that for each selected level, the

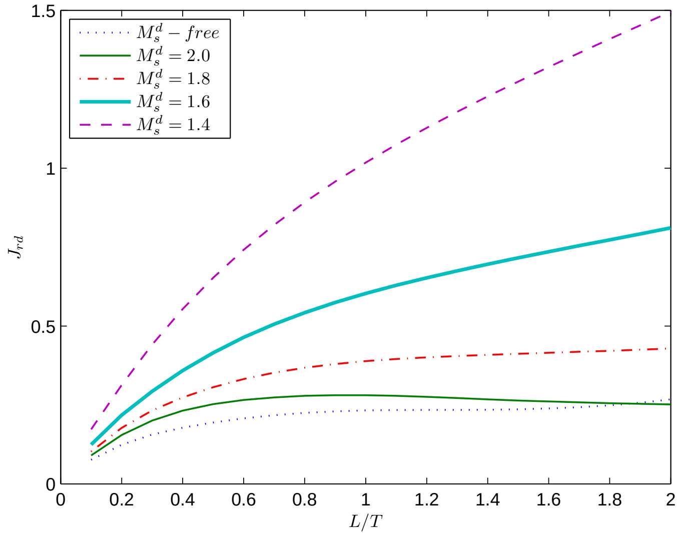

Figure 5.4: Combined index J_{rd} for each robustness level tuning.

robustness is improved achieving smaller values for M_s, but at the same time having larger values (i.e worse) for the performance index J_{rd}.

It is also important to note that, the relation between the loss of performance and the robustness increase (for each level of M_s) is nonlinear, neither for the τ range. For example, in Fig. 5.4 the difference between the performance for cases $M_s^d = 1.8$ and $M_s^d = 1.6$, is much lower than the one for $M_s^d = 1.6$ and $M_s^d = 1.4$, despite that the levels are equally separated.

In general terms, it is possible to say that the robustness requirements are fulfilled, facing at the same time, to the performance servo/regulation *trade-off* problem.

5.3.2 PID tuning for an arbitrary specific robustness value

With the aim to give more completeness to the previous exposed tuning method, an extension of the approach is presented. We want to take advantage of the simplicity and homogeneity offered by tuning (5.7), in order to exploit in the same lines, for a simple tuning rule that allows to specify, on an explicit way, the robustness parameter value within the range $M_s \in [1.4, 2.0]$.

Maybe, it is true that from a more practical point of view, it is difficult to have an specification of a $M_s = 1.57$ and this fact leads to the establishment of qualitative levels. However, a generic M_s^d tuning method can be used as a comparison tool, setting the same robustness (e.g. from a robust profile for all range of τ), and looking for the behavior of the performance.

Once again, the above idea is possible just because the followed fitting procedure was conceived to have the same form for each tuning and also for each parameter. Thus, the final expression for the controller's parameters that we are looking for is

$$p_i(\tau, M_s^d) = a_i(M_s^d)\tau^{b_i(M_s^d)} + c_i(M_s^d) \tag{5.8}$$

where $i = 1, 2, 3$ indicates the corresponding controller's parameter, p, and constants are expressed as functions of M_s^d. Therefore, from Table 5.1 each constant a_i, b_i and c_i are generated from a a generic second order M_s^d dependent polynomial as

$$\begin{aligned}
a_1 &= -0.3112(M_s^d)^2 + 1.6250(M_s^d) - 1.2340 \\
b_1 &= 0.0188(M_s^d)^2 - 0.0753(M_s^d) - 0.9509 \\
c_1 &= -0.1319(M_s^d)^2 + 0.7042(M_s^d) - 0.5334 \\
a_2 &= -0.5300(M_s^d)^2 + 1.7030(M_s^d) - 0.5511 \\
b_2 &= -0.1731(M_s^d)^2 + 1.0970(M_s^d) - 0.7700 \\
c_2 &= -0.0963(M_s^d)^2 + 0.7899(M_s^d) - 0.6629 \\
a_3 &= 0.1875(M_s^d)^2 - 0.7735(M_s^d) + 1.0740 \\
b_3 &= 0.3870(M_s^d)^2 - 4.7810(M_s^d) + 4.9470 \\
c_3 &= 0.1331(M_s^d)^2 - 0.4733(M_s^d) + 0.4032
\end{aligned} \tag{5.9}$$

Parameters (5.7) joint with (5.9) allow to determine the PID controller for any arbitrary value M_s^d in the range $[1.4, 2.0]$.

As it was said before, the accomplishment of the selected value for M_s is an aspect that must be checked. Fig. 5.5 shows the resulting M_s values obtained with the generic tuning (5.7) and (5.9), for the previously stated robustness levels and also for an intermediate value $M_s^d = 1.7$ (not included in the initial data).

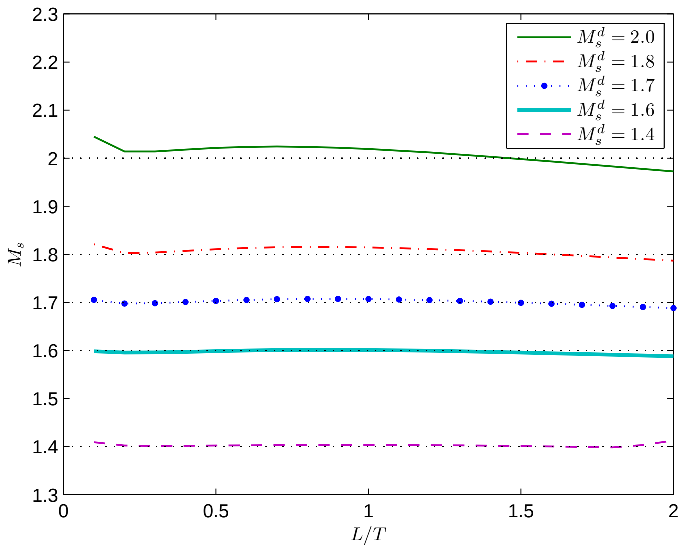

Figure 5.5: Accomplishment for the claimed robustness value.

5.4 Comparative examples

This section presents two kinds of examples in order to evaluate the characteristics of the proposed tuning rule. First example is an analysis not only for a specific process, but for the whole set of plants provided in their range of validity, $\tau \in [0.1, 2.0]$, in order to show the global advantages that the proposal can provide.

Then, the other example is for a specific plant, providing the control system time responses and some evaluation data. It is supposed that the process output can vary in the 0 to 100% normalized range and that in the normal operation point, the controlled variable has a value close to 70%.

5.4.1 Complete tuning case

The robust tuning rules that can be found in the literature consider different specifications for M_s. They range from the considered minimum robustness; $M_s = 2.0$; to a high robustness; $M_s = 1.4$.

Here, we compare the tuning proposed in Section 5.3 with the following methods:

- AMIGO method (Åström and Hägglund, 2004) provides tuning with a design specification of $M_s = 1.4$.

- Kappa-Tau ($\kappa - \tau$) method (Åström and Hägglund, 1995) provides tuning with a design specification of $M_s = 1.4$ and $M_s = 2.0$.

- Tavakoli method (Tavakoli *et al.*, 2005) provides tuning with a design specification of $M_s = 2.0$.

Fig. 5.6 shows the achieved M_s values for 1.4 and 2.0 cases, for the compared tuning rules. With this information and the one in Fig. 5.3, it seems that the proposed tuning is the option that provides the best accuracy for the selected robustness.

With the aim to establish a more precise and quantitative measure of the claimed robustness accomplishment for the whole range of models, the next index is stated

$$I_{M_s} \doteq \int_{\tau_o}^{\tau_f} |M_s^r(\tau) - M_s^d(\tau)| d\tau \tag{5.10}$$

where M_s^d and M_s^r are the desired and resulting M_s values, respectively. As I_{M_s} is smaller, the accuracy is better. In Table 5.2 there are the values (5.10) for the analyzed tuning rules.

Now, from the plots in Figs. 5.3 and 5.6, and the measured values (5.10) in Table 5.2, it is possible to say that the proposed PID tuning (using the levels classification), is the one that provides the best accomplishment between the desirable and the achieved robustness.

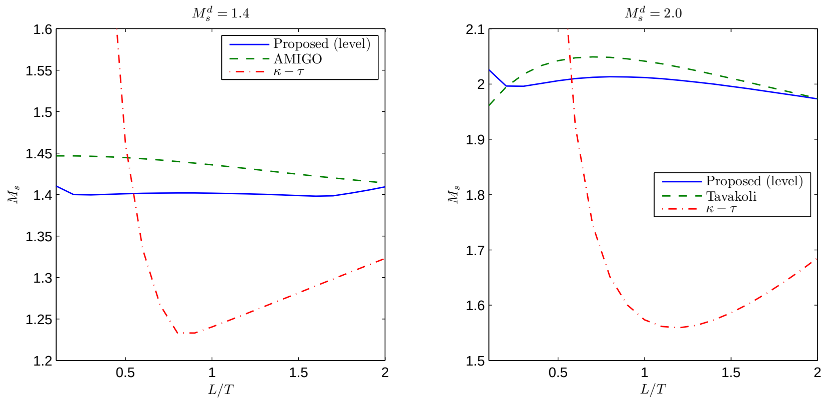

Figure 5.6: Comparative for claimed robustness.

Table 5.2: Claimed robustness accomplishment I_{M_s} for different tuning rules

Tuning	M_s^d	I_{M_s}
AMIGO	1.4	0.0634
$\kappa - \tau$	1.4	4.9672
$\kappa - \tau$	2.0	10.2377
Tavakoli	2.0	0.0519
Proposed (levels)	1.4	0.0035
	1.6	0.0066
	1.8	0.0111
	2.0	0.0186
Proposed (generic)	1.4	0.0054
	M_s^{AMIGO}	0.0028 (M_s^{AMIGO})
		0.0628 (1.4)

Once the robustness accomplishment has been verified, it is important to see the resulting performance for the compared tuning rules. Fig. 5.7 shows the combined servo/regulation performance index (5.4).

For the $M_s^d = 2.0$ it is obvious that the proposed tuning is the one that provides the best robustness accomplishment and at the same time, the best achievable performance. However, for $M_s^d = 1.4$ case, even the proposal is

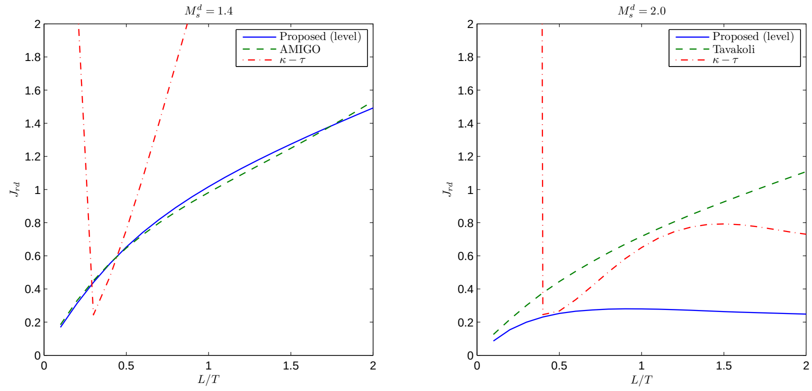

Figure 5.7: J_{rd} index for the compared tuning rules.

more accurate for the claimed robustness, Fig. 5.7 shows that AMIGO tuning method has values slightly lower for J_{rd} index compared to the proposed tuning. Precisely, because AMIGO method does not fulfill the robustness requirements, having a somewhat lower robustness, is that it achieves better results for performance. This fact strongly confirms the importance of the relation between robustness and performance variations.

Generic PID tuning method (5.7) joint with (5.9) allows to determine the controller parameters for any value M_s^d in the range $[1.4, 2.0]$. So, it can set flat profiles for robustness as, $M_s^d = 1.4$ (similarly to the level classification case), or variable ones as functions like $M_s^d = \mathbf{f_{M_s}}(\tau)$.

From the above and in order to have a fair comparison, we can take advantage of the generic PID tuning, reproducing a profile with the same robustness that AMIGO tuning provides, $M_s^d = M_s^{AMIGO}(\tau)$. Then, for the same robustness the idea is to look for the best performance. Fig. 5.8 shows the results.

It can be seen that the proposed generic tuning, is very accurate for the two selected robustness profiles. If we compare the proposal in the two situations (levels and generic), for the $M_s^d = 1.4$ the resulting performance is practically the same. However, the most interesting thing is when the generic tuning is set with the AMIGO values (M_s^{AMIGO}). As said before, AMIGO provides slightly lower values for J_{rd} index when the comparison is with the specified robustness levels, but if we use the generic approach to have exactly the same robustness

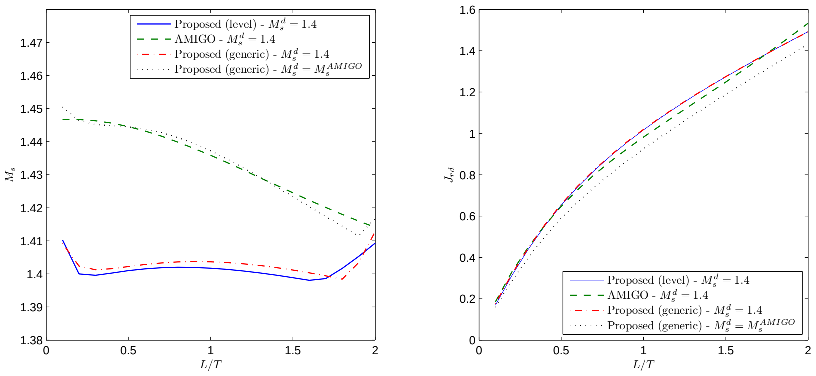

Figure 5.8: Robustness and performance for AMIGO and proposed tuning rules.

behavior, it is easy to see from Fig. 5.8, that there is an improvement in the performance component.

5.4.2 Particular process case

Consider the following fourth order controlled process

$$P_8(s) = \frac{1}{\prod_{n=0}^{3}(\sigma^n s + 1)} \tag{5.11}$$

with $\sigma = \{0.25, 0.50, 1.0\}$ taken from (Åström and Hägglund, 2000). Using a two-point identification procedure (Alfaro, 2006) FOPDT models were obtained whose parameters are show in Table 5.3. These parameters will be the ones used for tuning the PID controllers.

Table 5.3: Particular processes - FOPDT model parameters for P_8

σ	K	T	L
0.25	1.0	1.049	0.298
0.50	1.0	1.247	0.691
1.00	1.0	2.343	1.861

Table 5.4: Particular processes - PID controller parameters for P_8

		$\sigma = 0.25$			$\sigma = 0.50$			$\sigma = 1.00$		
Tuning	M_s^d	K_p	T_i	T_d	K_p	T_i	T_d	K_p	T_i	T_d
AMIGO	1.4	1.784	0.709	0.137	1.012	1.079	0.296	0.767	2.326	0.751
$\kappa - \tau$	1.4	2.982	0.832	0.200	0.867	1.233	0.320	0.486	2.432	0.630
$\kappa - \tau$	2.0	5.712	0.654	0.165	1.719	1.151	0.285	1.030	2.555	0.633
	1.4	1.745	0.751	0.100	0.978	1.083	0.239	0.737	2.274	0.646
Proposed	1.6	2.323	0.811	0.094	1.300	1.193	0.215	0.979	2.544	0.566
(levels)	1.8	2.779	0.836	0.094	1.552	1.227	0.206	1.167	2.635	0.529
	2.0	3.153	0.866	0.091	1.761	1.261	0.200	1.324	2.709	0.523
Proposed	1.4	1.749	0.754	0.112	0.981	1.087	0.254	0.739	2.283	0.675
(generic)	M_s^{AMIGO}	1.888	0.767	0.110	1.055	1.112	0.247	0.790	2.338	0.653

Table 5.5: Particular processes - Controller robustness

		$\sigma = 0.25$		$\sigma = 0.50$		$\sigma = 1.00$	
Tuning	M_s^d	M_s^r	$\lvert M_s^d - M_s^r\rvert$	M_s^r	$\lvert M_s^d - M_s^r\rvert$	M_s^r	$\lvert M_s^d - M_s^r\rvert$
AMIGO	1.4	1.446	0.046 (3.3%)	1.444	0.044 (3.1%)	1.440	0.040 (2.9%)
$\kappa - \tau$	1.4	2.922	1.522 (108.7%)	1.382	0.018 (1.3%)	1.234	0.166 (11.9%)
$\kappa - \tau$	2.0	12.370	10.370 (518.5%)	2.062	0.062 (3.1%)	1.655	0.345 (17.3%)
	1.4	1.400	0.000 (0.0%)	1.401	0.001 (0.1%)	1.402	0.002 (0.1%)
Proposed	1.6	1.598	0.002 (0.1%)	1.603	0.003 (0.2%)	1.605	0.005 (0.3%)
(levels)	1.8	1.796	0.004 (0.2%)	1.804	0.004 (0.2%)	1.808	0.008 (0.4%)
	2.0	1.995	0.005 (0.3%)	2.008	0.008 (0.4%)	2.013	0.013 (0.6%)
Proposed	1.4	1.401	0.001 (0.1%)	1.403	0.003 (0.2%)	1.404	0.004 (0.3%)
(generic)	M_s^{AMIGO}	1.445	0.001 (0.1%)	1.444	0.000 (0.0%)	1.441	0.001 (0.1%)

Table 5.4 shows the PID controller parameters whereas in Table 5.5 the specified (M_s^d) and achieved (M_s^r) robustness of the control system, are presented when the controller is tuned with Kappa-Tau ($\kappa - \tau$) (Åström and Hägglund, 1995) and AMIGO (Åström and Hägglund, 2006) rules and the corresponding parameters obtained with the proposed method (levels and generic). The accuracy in terms of $|M_s^d - M_s^r|$ is also shown, where it is possible to say that, when qualitative robustness levels are specified, the proposed method is closer to the desired values than the AMIGO or Kappa-Tau tuning rules.

The control system's performance for servo J_r, regulation J_d and the combined index J_{rd}, are provided in Table 5.6 for each tuning and each case of process P_8.

The analysis to determine the best option to tune the controller, must be

Table 5.6: Particular processes - Controller performance

		$\sigma = 0.25$			$\sigma = 0.50$			$\sigma = 1.00$		
Tuning	M_s^d	J_r	J_d	J_{rd}	J_r	J_d	J_{rd}	J_r	J_d	J_{rd}
AMIGO	1.4	0.8606	0.4465	0.4524	1.7067	1.1961	0.8184	4.1494	3.2979	1.7021
$\kappa - \tau$	1.4	0.7047	0.2892	0.2324	1.7382	1.4626	1.0560	5.0165	5.0094	3.6171
$\kappa - \tau$	2.0	0.5561	0.1183	0.0513	1.4597	0.7366	0.3189	3.7911	2.6407	0.9560
Proposed (levels)	1.4	0.8274	0.4611	0.4379	1.6958	1.2238	0.8336	4.1888	3.3712	1.7852
	1.6	0.7115	0.3513	0.2784	1.5135	0.9669	0.5188	3.8318	2.7597	1.0776
	1.8	0.6551	0.3006	0.2028	1.4488	0.8254	0.3689	3.7905	2.4555	0.8104
	2.0	0.6175	0.2746	0.1584	1.4095	0.7396	0.2809	3.7861	2.2600	0.6750
Proposed (generic)	1.4	0.8336	0.4633	0.4439	1.6980	1.2246	0.8356	4.1882	3.3691	1.7830
	M_s^{AMIGO}	0.8005	0.4297	0.3968	1.6485	1.1541	0.7495	4.0915	3.2142	1.6004

done using Tables 5.5 and 5.6 jointly. First, the target robustness has to be fulfilled with a small bounded error and then, for same robustness values the tuning with the minimum performance will be selected.

For example, if we concentrate in $\sigma = 1.00$ and $M_s^d = 1.4$ case, Kappa-Tau method has the worst robustness accomplishment (11.9%), whereas AMIGO and proposed (level) tunings have small values (2.9% and 0.1% respectively). In this first step, we can rule out $\kappa - \tau$ tuning and continue with the other ones. Then, looking for the performance behaviour, it is possible to see that AMIGO gives a smaller index value J_{rd} than the proposed level tuning. However, if we use the generic tuning (5.9), in order to relax the accuracy of the claimed robustness equal to the AMIGO (means $M_s^d = M_s^{AMIGO}$), it is possible to see that the achieved performance index is the best one. So, as a conclusion the proposed tuning should be the best choice to obtain the PID parameters.

The control system's and controller's outputs for each level of the proposed method are shown in Fig. 5.9 for the case of $\sigma = 0.50$, whereas Fig. 5.10 compares the Kappa-Tau, AMIGO and the level proposed method, for the $\sigma = 1.00$ and $M_s^d = 1.4$ case, with also the proposed method using the generic approach for two values of M_s^d, as exposed above. In both figures, the responses reflect the previous comments, although in some cases because the similarity of the curves, the information contained in the Tables is necessary.

5.5 Summary

In process control, it is very important to guarantee some degree of robustness, in order to preserve the closed-loop dynamics, to possible variations in the

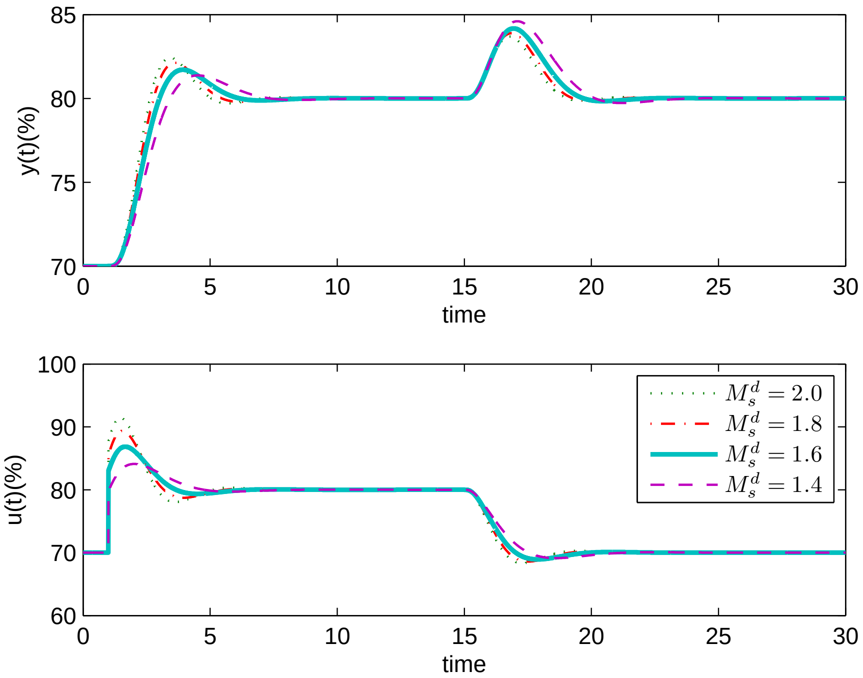

Figure 5.9: Particular process - Proposed method ($\sigma = 0.50$).

control system. Also, at the same time, it must be provided the best achievable performance for servo and regulation operation.

All of the above specifications, lead to have different *trade-offs*, between performance and robustness or between servo and regulation modes, that must be solved on a balanced way. Here, we looked for a PID controller tuning rule that faces to the general problem. This tuning is optimal, as much as possible, to a proposed performance index that takes into account both system operation modes, including also a certain degree of robustness, specified as a desirable Maximum Sensitivity value.

Autotuning formulae have been presented for two approaches. First, robustness is established using a qualitative levels classification and then, the idea is extended to an issue that offers a generic expression, to allow the

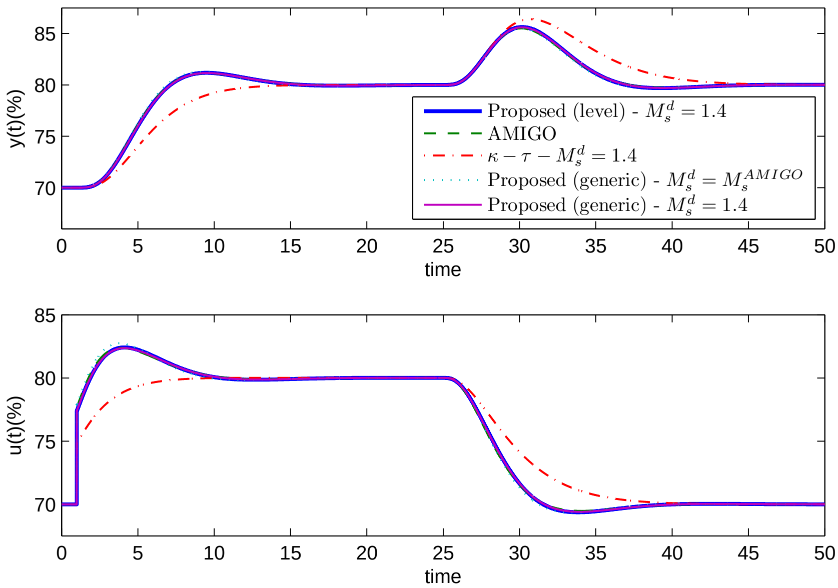

Figure 5.10: Particular process - Control system responses for $M_s^d = 1.4$ case($\sigma = 1.00$).

specification in terms of any value of robustness in the range $M_s \in [1.4, 2.0]$. Moreover, taking into account the performance/robustness *trade-off*, the accuracy of the claimed robustness is a point that has been verified, achieving flat curves for the resulting values. In short, both approaches are two of the main contributions presented in this thesis.

Chapter 6

Optimality based PID control design

6.1 General aspects

The analysis exposed in Chapter 5, shows the interaction between performance and robustness in control systems. It is possible to say that, an increase of robustness implies an optimality loss in the performance (i.e. a degradation), with respect to the one that can be achieved without any robustness constraint. From this,it is possible to define the degree of optimality of the constrained case, with respect to the unconstrained one (that is, the optimum).

In order to quantify the degree of optimality, the following index is proposed

$$I_{Perf} \doteq \frac{J_{rd}^{o}}{J_{rd}^{rdM_s}} \tag{6.1}$$

where J_{rd}^{o} is the optimal index value (5.4), using the tuning (5.7) for no constraint of M_s (first column of Table 5.1), that means the best one that can be achieved. Then, $J_{rd}^{rdM_s}$ is the value of index (5.4) for the cases where the tuning has a robustness constraint.

Note that (6.1) is normalized in the $[0, 1]$ range, where $I_{Perf} = 1$ means a perfect optimality and, as much as the robustness is increased, the index $J_{rd}^{rdM_s}$ will increase and, consequently, $I_{Perf} < 1$, meaning an optimality reduction.

The degree of optimality that each control system has when a desirable value of M_s is stated, can be evaluated taking advantage of the generic tuning rule presented before in section 5.3.2. For each value of $M_s^d \in [1.4 - 2.0]$, the optimality degree (6.1) can be obtained.

Motivation example

In order to show how the optimality degree of a system varies, as a function of the robustness variable M_s, a motivation example is provided.

Consider system (5.11), $P_8(s)$, with $\sigma = 0.50$, from the example in section 5.4.2. For this particular process, the idea is to use each value within the range $M_s^d \in [1.4 - 2.0]$, as a design parameter of tuning (5.7) and (5.9). Then, it is possible to calculate a set of degree of optimality values (6.1) and compare them with respect to the ones for AMIGO, Kappa-Tau and Tavakoli tuning methods.

Fig. 6.1 illustrates the results, where it is shown the optimality index I_{Perf} with the corresponding achieved robustness. It is possible to see that the singletons for the compared tunings, are below the line that represents the generic proposed tuning for the M_s^d range. This means that the proposal is the option that provides better degree of optimality for any value of robustness.

In addition, note that the horizontal variation of the points represents the error between the desired and the resulting robustness. The points for AMIGO, Kappa-Tau and Tavakoli methods, should be located on the axes corresponding to $M_s = 1.4$ and $M_s = 2.0$, but they are not, because they do not accomplish the robustness constraint.

The line for the proposed tuning, in Fig. 6.1, represents a limit. For each choice of robustness M_s^d, the intersection point with the line, indicates the best degree of optimality that can be achieved. From the other side, if the choice is a certain degree of optimality, the intersection provides the best robustness value that can be selected.

This analysis has been done for one specific process (P_8), however, the idea is to reproduce the same procedure in order to generalize the results for any possible process.

From above, we want to repeat the idea of Fig. 6.1, but now for the whole range of family plants within $\tau \in [0.1, 2.0]$. So, for each τ, we take advantage of the possibilities of the proposed tuning (5.7) and (5.9), to get the PID

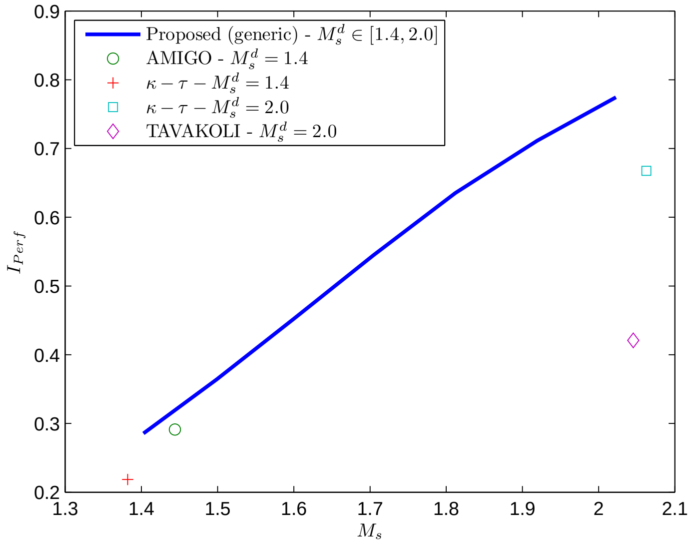

Figure 6.1: Process P_8 - Index I_{Perf} values for different tunings.

parameters for any value of $M_s^d \in [1.4 - 2.0]$ and then, compute the degree of optimality using (6.1). Fig. 6.2 shows the I_{Perf} variation, as a function of M_s^d, for some values of the normalized dead-time, τ.

Note that, as an example, the horizontal line indicates when the degree of optimality is 55%. With the intersection points between this line and the curves correspondig to the I_{Perf} variation for each τ, it is possible to determine a set of desired robustness that is related with this degree of optimality ($I_{Perf} = 0.55$). This set of $M_s^d(\tau)$ can be seen as a profile that the tuning should follow to satisfy the degree of optimality. Fig. 6.3 shows the detail where also, just to clarify, there is the previous case of $M_s^d = 1.7$, that can be considered as a flat profile.

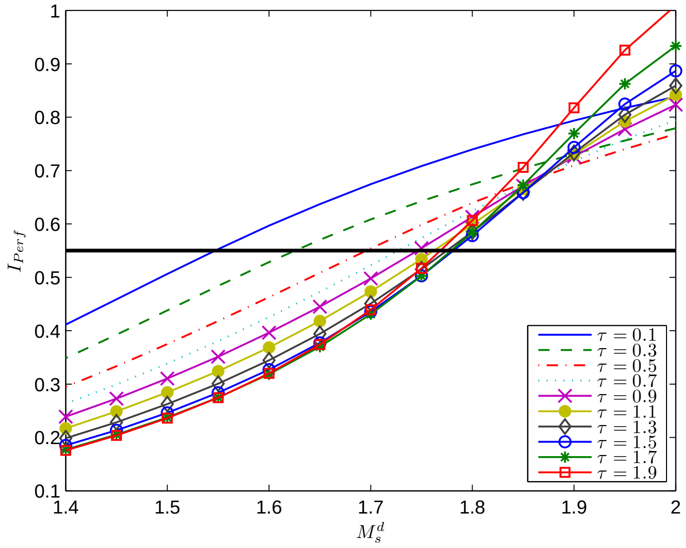

Figure 6.2: Variation of the index I_{Perf}.

6.2 PID tunings with performance optimality degree

As it has been shown above, there is a relation (*trade-off*) between the degree of optimality and the increase in the system's robustness. It is possible therefore to find the corresponding M_s^d value for any specific optimality, as a point (M_s^d, I_{Perf}).

Following the above idea for all plants in the range $\tau \in [0.1, 2.0]$, fixing a certain degree of optimality we can look for the corresponding set of M_s^d values. So, using the robustness profile in tuning (5.7) jointly with the generic form (5.9), the controller's parameters $[K_p, T_i, T_d]$, can be obtained.

Here, with the aim to facilitate the understanding of the general idea and taking into account that, it could be easier to specify a certain degradation, than a degree of optimality (i.e. an optimality loss), we redefine (6.1) as,

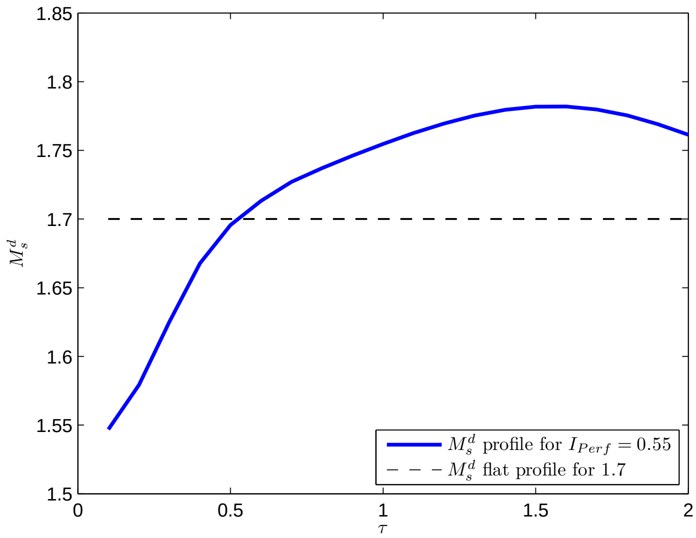

Figure 6.3: M_s^d profile corresponding to a 55% optimality degree.

$$Deg \doteq 1 - I_{Perf} \tag{6.2}$$

In this sense, a desirable optimality degree of 60% can be interpreted as a 40% of degradation. The general concept is exactly the same, but just the way of interpretation is changed.

Using a similar idea to the one exposed in Section 5.3, we look for a tuning methodology that uses the degradation as a parameter design, in order to increase the robustness of the system.

6.2.1 PID tuning for fixed performance degradation levels

The previous exposed procedure tries to achieve that, allowing a degradation in the performance, the system's robustness can be increased.

As it was stated before for robustness, a broad classification is a very easy and practical way to understand a problem formulation. In this sense, now it will be used the degradation of the system performance, Deg as a parameter to establish the levels.

To fix these degradation levels, we see information provided in Fig. 6.2. Here, the aim is to obtain profiles of M_s^d for the range of $\tau \in [0.1, 2.0]$. Therefore, the selected optimality degree level must intersect each one of the curves corresponding to each one of the plants.

Fulfilling the above, to get a degree of optimality higher than 75%, the range of considered robustness should be extended to values greater than $M_s^d = 2.0$, but this value is the minimum acceptable robustness and for that, this option is not studied. From the other side, to have an optimality degree lower than 45% the robustness values must be minor than $M_s^d = 1.4$, that is considered as a high robustness level, therefore decrease the degree of optimality to less than 45% (meaning a degradation more than 55%), is not justified.

Then, the range of application was established as $Deg \in [0.25, 0.55]$ and therefore the classification as,

- Low degradation - $Deg = 0.25$
- Medium-low degradation - $Deg = 0.35$
- Medium-high degradation - $Deg = 0.45$
- High degradation - $Deg = 0.55$

As it was said above, for each stated degradation level and each τ, it is found the corresponding M_s^d. Then, the set of robustness values determines the M_s^d profile, that is used in the proposed generic tuning (5.7) and (5.9), of section 5.3.2, to determine sets for each parameter of the PID controller, K_p, T_i and T_d. Therefore, with all the parameters sets, the tuning rule can be formulated (by fitting).

Once again, following a similar idea to that described for the M_s^d case, the aim is to provide a simple tuning and for that, we take advantage of the good fitting that equations (5.7) provide. So, the sets for each PID parameter and for each degradation level, are approximated to fit the corresponding equations form.

Table 6.1: PID tuning settings for allowed performance degradation

constant	$Deg^a = 0$	$Deg^a = 0.25$	$Deg^a = 0.35$	$Deg^a = 0.45$	$Deg^a = 0.55$
a_1	1.1410	0.8787	0.7490	0.6292	0.5252
b_1	-0.9664	-0.9280	-0.9348	-0.9444	-0.9492
c_1	0.1468	0.2033	0.2669	0.3195	0.3494
a_2	1.0860	0.8154	0.8664	0.8871	0.8755
b_2	0.4896	0.6431	0.6033	0.5847	0.5830
c_2	0.2775	0.4502	0.3874	0.3466	0.3275
a_3	0.3726	0.2794	0.2757	0.2804	0.2949
b_3	0.7098	0.8765	0.8698	0.8471	0.8123
c_3	-0.0409	-0.0149	-0.0070	-0.0037	-0.0055

The tuning rule remains the expressed form in (5.7), but the a_i, b_i and c_i constants are given in Table 6.1, according to the *allowed degradation* level in the system's performance, Deg^a.

Table 6.1 shows, in the first column, the case for $Deg^a = 0$, that is exactly the same to the one in Table 5.1 for M_s^d free (without any constraint), but it is included here in order to give completeness to the approach. Note also that, to keep the same tuning expressions (5.7) provides even more uniformity and simplicity to the proposed approach.

The evaluation of the above proposed tuning rule has to be done taking into account both performance and robustness issues. In order to study the system's performance, in Fig. 6.4 there are the indexes J_{rd}, for each case of Deg^a. Note that, with this information and using (6.1) and (6.2), it is possible to find the resulting degradation values, as they are shown in Fig. 6.5.

Even the accuracy between the achieved degradation (6.2) and the selected in Deg^a, is not as good as the one obtained for the M_s^d case, we can say that the tuning accomplishment is good enough, specially for the first τ values range $[0.1, 1.5]$. For $\tau > 1.5$, the deviation is due to the successive approximations for the controller parameters and also, it is a region of processes that is difficult to control using a PID, because it represents systems dominated by the dead-time.

Once more, it is important to see how the changes in the performance (due

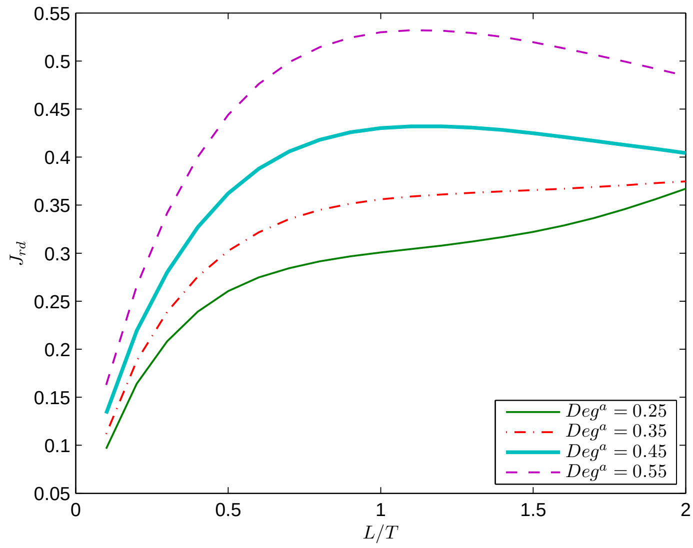

Figure 6.4: Combined index J_{rd} for each degradation level tuning.

to the imposed degradation), affects the achieved robustness for the system. Fig. 6.6 shows this evaluation, where the optimality decreases (i.e. degradation increases), the robustness of the system grows up. This is an important aspect because these M_s values represent the profile that should be accomplished in order to achieve the fixed degradation (meaning a certain degree of optimality).

It can be seen, all results are in agreement regarding to the well known performance/robustness ratio.

6.2.2 PID tuning for an arbitrary performance degradation

Analogously to the tuning presented in Section 5.3.2, for arbitrary values of M_s^d, we want to give here a formulation that allow us to specify any value for the allowed degradation. Because the approach is based on the information provided by the fixed degradation levels, the range of validity is within $Deg^a \in [0.25, 0.55]$.

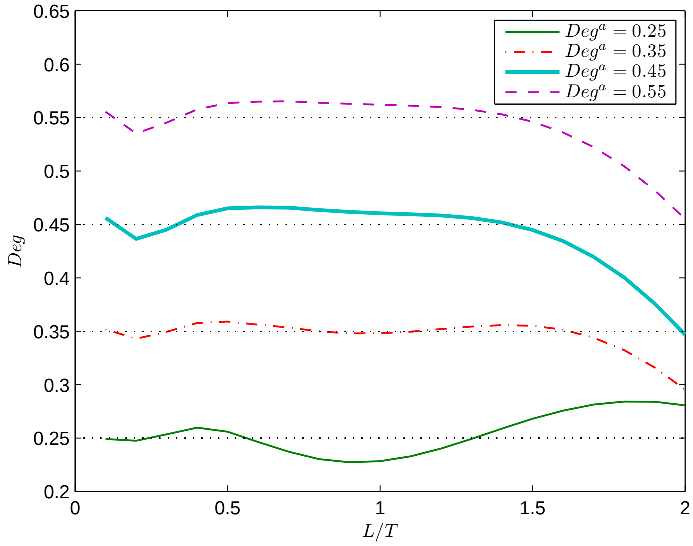

Figure 6.5: Accomplishment of the fixed degradation level tuning.

It is important to emphasize that, this extension is just possible because the simplicity and the homogeneity of the PID tuning parameters (5.7), that remains the same expression for each one of the fixed degradation levels.

Once again, the aim is to provide a generic formulation in order to give completeness, as much as possible. Therefore, because each controller parameter has the same form, we look for a general equation as,

$$p_i(\tau, Deg^a) = a_i(Deg^a)\tau^{b_i(Deg^a)} + c_i(Deg^a) \tag{6.3}$$

where $i = 1, 2, 3$ indicates the corresponding controller's parameter, p, and constants are expressed as functions of Deg^a. Then, from Table 6.1, the constants a_i, b_i and c_i with their respective Deg^a value, are fitted to a second order polynomial as

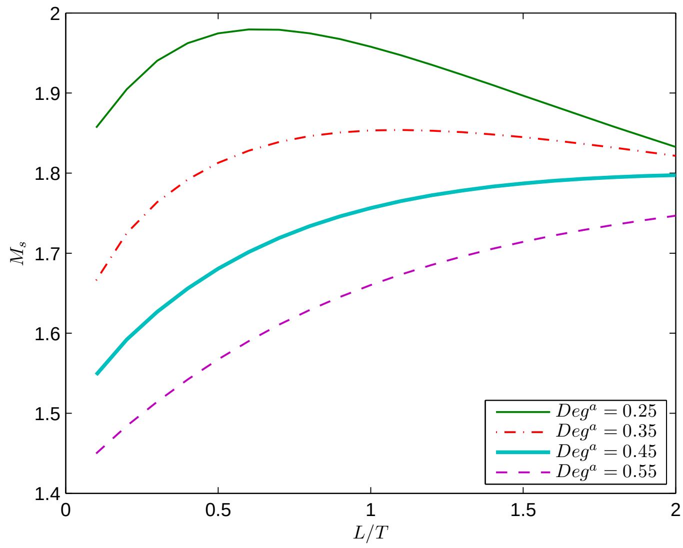

Figure 6.6: Achieved robustness M_s for each degradation level tuning.

$$
\begin{aligned}
a_1 &= 0.6425(Deg^a)^2 - 1.6940(Deg^a) + 1.2620 \\
b_1 &= 0.0500(Deg^a)^2 - 0.1132(Deg^a) - 0.9024 \\
c_1 &= -0.8425(Deg^a)^2 + 1.1650(Deg^a) - 0.0359 \\
a_2 &= -1.5650(Deg^a)^2 + 1.4530(Deg^a) + 0.5499 \\
b_2 &= 0.9525(Deg^a)^2 - 0.9609(Deg^a) + 0.8236 \\
c_2 &= 1.0930(Deg^a)^2 - 1.2830(Deg^a) + 0.7026 \\
a_3 &= 0.4550(Deg^a)^2 - 0.3128(Deg^a) + 0.3292 \\
b_3 &= -0.7025(Deg^a)^2 + 0.3467(Deg^a) + 0.8339 \\
c_3 &= -0.2425(Deg^a)^2 + 0.2255(Deg^a) - 0.0561
\end{aligned}
\tag{6.4}
$$

Specifically, parameters (5.7) jointly with the resulting constants (6.4), provide the PID controller tuning that choosing an arbitrary degradation value for the system performance, increases the robustness, as much as possible, for the prescribed degree of optimality.

Table 6.2: Particular process - PID controller parameters for P_8 ($\sigma = 0.50$)

Tuning	Deg^a	K_p	T_i	T_d
Proposed (levels)	0.25	1.723	1.257	0.189
	0.35	1.568	1.240	0.197
	0.45	1.418	1.216	0.207
	0.55	1.269	1.182	0.221
$\kappa - \tau$ ($M_s^d = 2.0$)	-	1.719	1.151	0.285
Proposed (generic)	$Deg^{\kappa-\tau}$	1.529	1.234	0.199

6.2.3 Evaluation example

In order to evaluate the proposal, we take again advantage of system (5.11), $P_8(s)$, with $\sigma = 0.50$, shown in section 5.4.2 and also in this chapter, as a *Motivation example*.

From the FOPDT model and using tuning (5.7) for each fixed degradation level in Table 6.1, the PID parameters can be obtained. Moreover, the proposed generic tuning (6.4), for arbitrary values of allowed degradation is compared (setting the same degradation value), with the Kappa-Tau ($\kappa - \tau$) (Åström and Hägglund, 1995) for the $M_s = 2.0$ case. Tunings are shown in Table 6.2.

Comparisons with other tunings like AMIGO, Kappa-Tau for $M_s = 1.4$ or Tavakoli, are not included because, as it can be seen in Fig. 6.1, the optimality degree values that they provide are outside the validity range of the proposed tuning, $I_{Perf} \in [0.45, 0.75]$ or seen from the allowed degradation point of view, $Deg^a \in [0.25, 0.55]$.

Table 6.3 gives the performance and robustness values provided by each tuning. Also, in Fig. 6.7 the control system's and controller's output are shown for each allowed degradation level, whereas in Fig. 6.8, it is possible to see the comparison between the $\kappa - \tau$ tuning and the proposed settings with the specific value of degradation.

From the two approaches for the proposal, it can be concluded that the levels version has a good accuracy with respect to the selected value for the allowed performance degradation, giving at the same time an increase in the robustness. Concerning to the proposed tuning for an arbitrary value of Deg^a,

Table 6.3: Particular process P_8 ($\sigma = 0.50$) - Controller evaluation

Tuning	Deg^a	Deg^r	M_s^r	J_r	J_d	J_{rd}
Proposed (levels)	0.25	0.2508	1.9780	1.4229	0.7568	0.3024
	0.35	0.3575	1.8217	1.4421	0.8208	0.3609
	0.45	0.4657	1.6923	1.4755	0.8954	0.4392
	0.55	0.5645	1.5800	1.5283	0.9871	0.5438
$\kappa - \tau$ ($M_s^d = 2.0$)	-	0.3325	2.0626	1.4597	0.7366	0.3189
Proposed (generic)	$Deg^{\kappa-\tau}$	0.3379	1.8474	1.4378	0.8086	0.3491

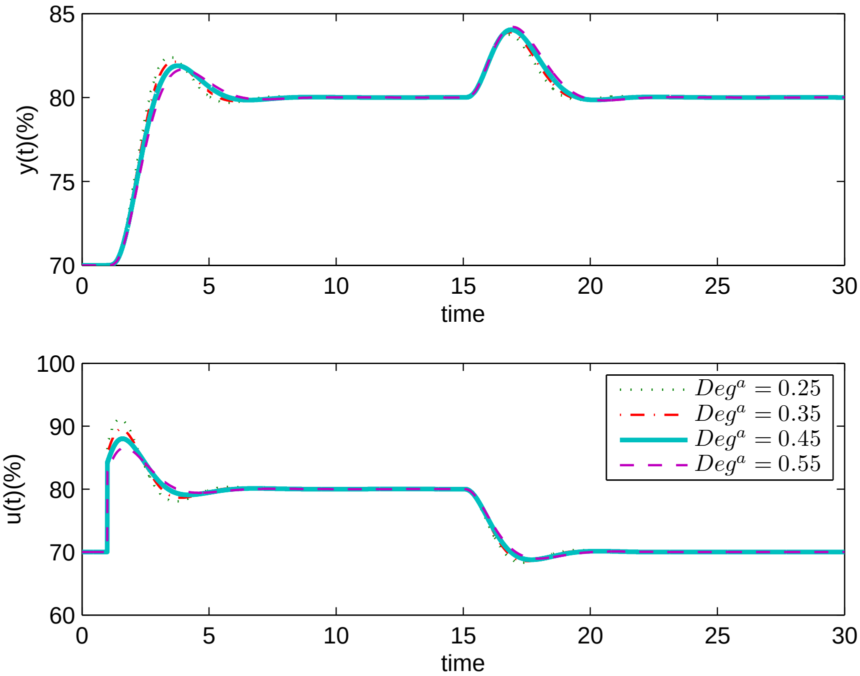

Figure 6.7: Particular process P_8 - Proposed method ($\sigma = 0.50$).

from Table 6.3 it is possible to see that compared to $\kappa-\tau$, the achieved perfor-

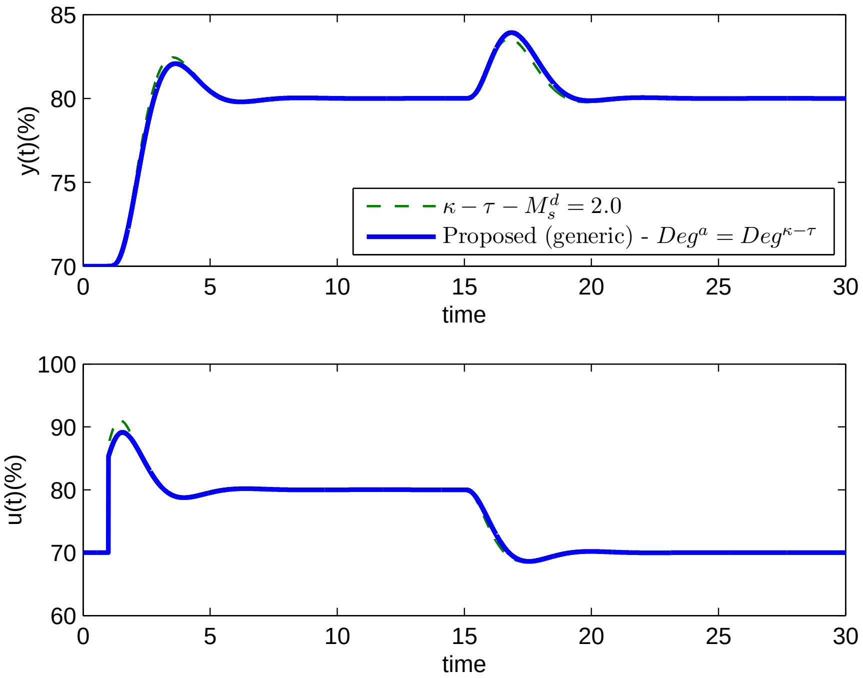

Figure 6.8: Particular process P_8 - Control system responses ($\sigma = 0.50$).

mance is practically the same (because they have the same degradation value), however, in the proposed tuning the robustness is much better. So, concluding, for same performance, the proposed tuning provides greater robustness, making it a better option to tune the controller.

6.3 Summary

The control system's *trade-off* between performance and robustness, can be studied from two points of view. First, as it was shown in Chapter 5, selecting a desirable value for robustness and facing to the resulting performance degradation.

In this chapter, we formulated the problem from the other side, selecting an allowed performance degradation in order to get a higher robustness, respect to the case with zero degradation. The proposal is presented for some degradation levels (qualitative classification), and also for generic specific values of degradation within the range 25%-55%.

Results are presented as autotuning formulae, maintaining the same simplicity shown before for other proposed PID tuning approaches. The example shows the accuracy and the benefits of the contribution.

Chapter 7

Balanced performance/robustness PID design

Approaches presented in chapters 5 and 6 show two different ways to face with the *trade-off* problem between performance and robustness issues. First, it was proposed a tuning method that choosing a desired robustness M_s^d, provides the best servo/regulation performance that can be achieved for the system. In the second case, the idea is to reduce the degree of optimality of the system's performance, in order to increase the robustness. So, the tuning proposes to select a certain degradation for the performance, that at the same time gives the largest robustness increase.

Both approaches provide good results from the point of view they were formulated and, of course, they are different, but complementary. However, it is necessary to go beyond and look for an intermediate idea, that provides a balance between the degree of optimality and the robustness increase.

7.1 Robustness increase measure

From the above, index (6.1) provides an idea about the degree of optimality for the system's performance. Here, we need to state a similar concept but with the aim to quantify the robustness increase. Then, we compare the achieved robustness of the constrained case with respect to the unconstrained one, that is the optimum in performance but with lower robustness.

Therefore, the following index is proposed

$$I_{Rob} \doteq \left| \frac{M_s^o - M_s^{J_{rd}}}{M_s^o} \right| \tag{7.1}$$

where M_s^o represents the robustness value achieved by tuning (5.4) without any constraint for M_s (first column of Table 5.1), and $M_s^{J_{rd}}$ is the robustness value for the cases where the robustness is constrained (i.e. increased). Then, (7.1) is a normalized index, where as much as it grows, the robustness increase will be larger.

Once more, we want to take advantage of the generic tuning rule, presented in section 5.3.2, in order to evaluate the robustness increase for the whole range of family plants within $\tau \in [0.1, 2.0]$. Then, for each τ and each value $M_s^d \in [1.4, 2.0]$, it is possible to find the PID parameters and obtain index (7.1). This procedure is analogous to the presented in the previous chapter for I_{Perf} index.

Fig. 7.1 shows the I_{Rob} variation, as a function of M_s^d, for some cases of the normalized dead-time, τ. Each line represents the relative robustness increase for a particular process (value of τ).

Note that in Fig. 7.1, we could define a certain value (level), for the robustness increase and with the intersection points, determine a suitable robustness profile (analogous to the procedure for I_{Perf} in section 6.1). However, this idea has the same objective to the tuning presented in chapter 5, that is to increase the robustness of the system and provide at the same time, the best servo/regulation performance. Here, as we stated before, the aim is to look for a balanced tuning.

7.2 Robustness/Performance balance

Taking into account that indexes (6.1) and (7.1) are both normalized into the range $[0, 1]$, therefore having the same scale, if we put the information of Fig. 6.2 and Fig. 7.1 all together, the intersection points for each pair of curves, can be interpreted as equilibrium points. These, represent when the degree of optimality is consistent to the robustness increase, and vice-versa.

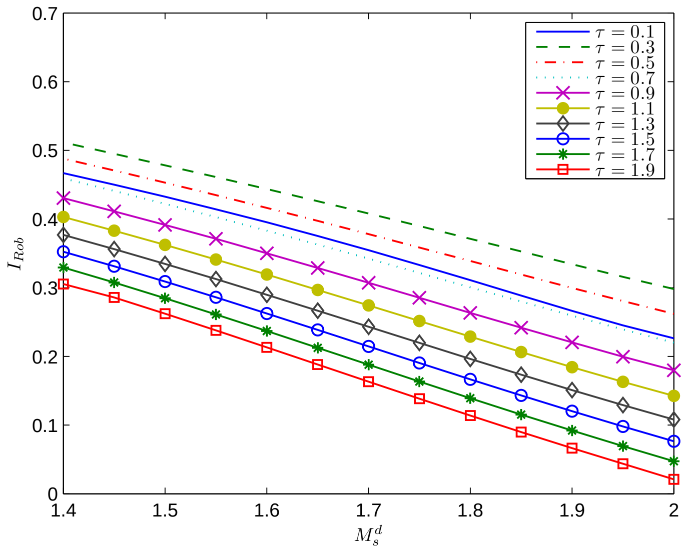

Figure 7.1: Variation of the index I_{Rob}.

In Fig. 7.2, there are just few cases for the model normalized dead-times τ, where it is possible to see the consideration exposed before.

With the intersection points between the corresponding pairs of curves, we can determine the suitable set of desired robustness, that provides the best balance between performance optimality degree and robustness increase. This set of $M_s^d(\tau)$ determines the robustness profile for the best robustness/performance compromise, for all the τ range. See Fig. 7.3.

7.3 Balanced PID tuning

As it has been explained before, with information provided by indexes I_{Perf} and I_{Rob}, it is possible to obtain the corresponding M_s^d value, that achieves the best compromise between the loss of the optimality degree and the robustness increase.

The set of desired robustness values for all the plants within $\tau \in [0.1, 2.0]$,

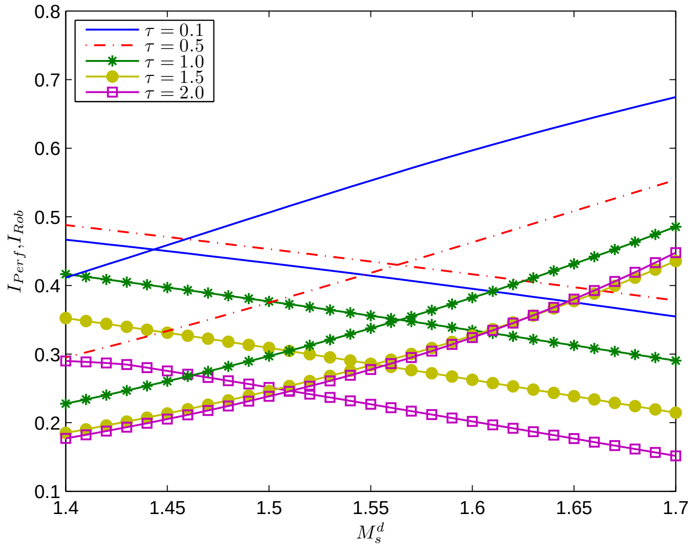

Figure 7.2: I_{Perf} and I_{Rob} variation for some values of τ.

is a M_s^d profile that, as before, can be used in tuning (5.7) and (5.9), in order to get the PID controller parameters.

Then, following the same aim to propose a simple and homogeneous tuning, the set for each parameter is fitted to the general tuning expressions (5.7). Therefore,

$$\begin{aligned} K_pK &= 0.6776\tau^{-0.8630} + 0.1162 \\ \frac{T_i}{T} &= 0.9950\tau^{0.4016} + 0.1564 \\ \frac{T_d}{T} &= 0.2998\tau^{0.9760} + 0.0110 \end{aligned} \tag{7.2}$$

Tuning (7.2) is the one that provides the best compromise/balance between the robustness increase and the resulting loss of optimality degree for the system's performance. It is important to note that also, because this performance

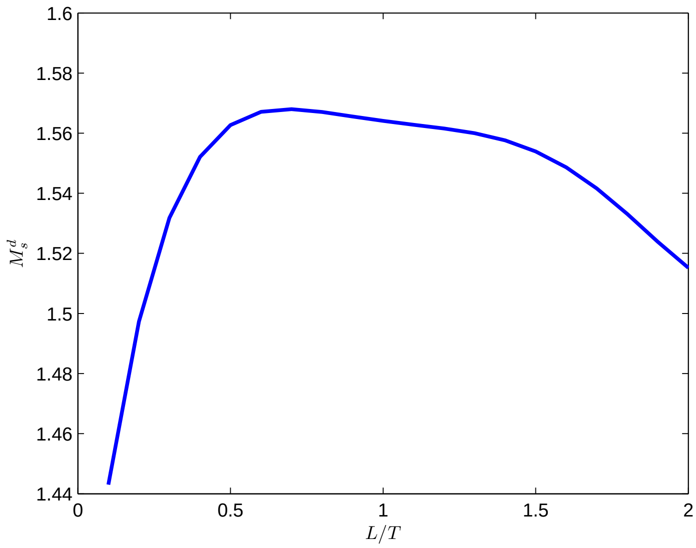

Figure 7.3: M_s^d profile for the best robustness/performance balance.

index formulation, the *trade-off* problem between servo and regulation control operation modes, is included.

7.4 Tuning evaluation

In this section, the intention is to evaluate the performance and robustness features for the proposed balanced tuning. Also, it is desirable to establish a more precise quantification of the balance concept, stated before. Therefore, we define the following index,

$$I_B \doteq \int_{\tau_o}^{\tau_f} |I_{Perf}(\tau) - I_{Rob}(\tau)| d\tau \tag{7.3}$$

where the idea is to measure the difference between the robustness increase and the degree of optimality, for the whole range of plants $\tau \in [0.1, 2.0]$. Note that, when the index (7.3) is low, it means a good balance between the robustness increase and the corresponding price for performance optimality. A high value of I_B indicates a lack of balance for the studied indexes.

In Fig. 7.4, there are the robustness and servo/regulation performance evaluation for the proposed balanced tuning and compared with the unconstrained case, equivalent to M_s^d free and/or $Deg^a = 0$. There, it can be seen the robustness increase, as well as the consequent loss of degree of optimality for the system's performance.

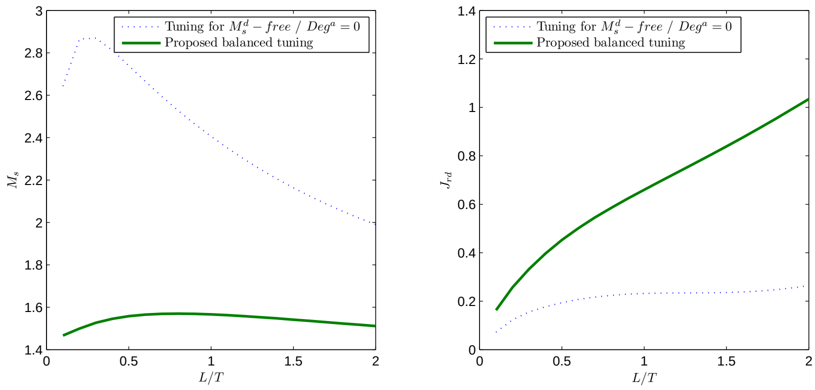

Figure 7.4: Robustness and performance values for proposed tuning.

In addition, Fig. 7.5 shows I_{Perf} and I_{Rob} indexes for the proposed balanced tuning (7.2), where the similarity between the values indicates that, for the whole τ range, the robustness increase is equivalent to the loss of performance's optimality, therefore having a balance. It means a "fair price between what is paid and what is got".

Table 7.1 shows the balanced index I_B, for each one of the proposed tuning rules and also compared with AMIGO and Kappa-Tau methods.

It is possible to see that, apart from the case for proposed balanced tuning that has obviously the best I_B index, tuning with $M_s^d = 1.6$ is the one that provides lower value for the balanced index (7.3). This has a lot of sense because, if we look in Fig. 7.3, the robustness profile in almost the cases are within the range $[1.5, 1.6]$.

Moreover, AMIGO tuning has a better I_B index than its counterpart for

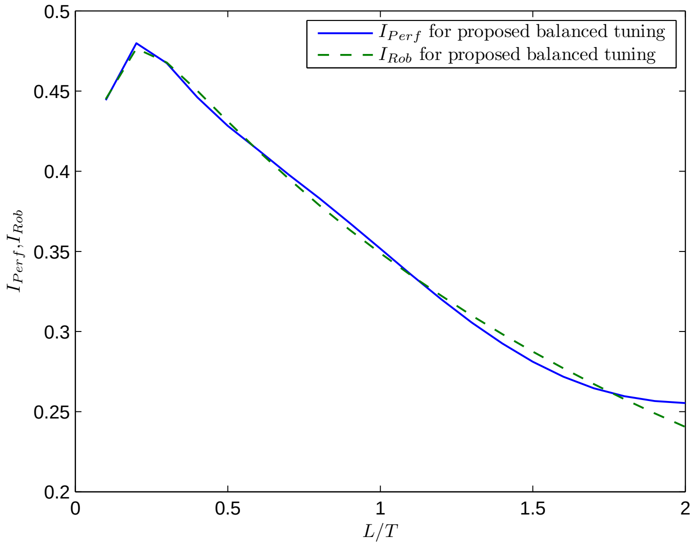

Figure 7.5: I_{Perf} and I_{Rob} indexes for the proposed balanced tuning.

proposed $M_s^d = 1.4$. However, this is again for the previously exposed non-accomplishment of the specified robustness for the AMIGO method. If we use the generic tuning for M_s^d, in order to reproduce the robustness profile for AMIGO ($M_s^d = M_s^{AMIGO}$), it is possible to see in Table 7.1, that this resulting tuning provides a better robustness/performance balance.

7.5 Comparison example

We will use again system (5.11), $P_8(s)$, with $\sigma = 0.50$, in order to evaluate the proposed balanced tuning.

From the FOPDT model information and applying the proposed balanced tuning (7.2), it is possible to obtain the PID parameters as: $K_p = 1.244$, $T_i = 1.174$ and $T_d = 0.224$. Here, the idea is to evaluate the robustness and performance provided for the balanced tuning and also to compare with: AMIGO and proposed generic tuning for $M_s^d = M_s^{AMIGO}$ and Kappa-Tau

Table 7.1: Balanced index 7.3 for different tuning rules

Tuning	Criteria design	I_B
AMIGO	$M_s^d = 1.4$	0.2900
$\kappa - \tau$	$M_s^d = 1.4$	2.1373
$\kappa - \tau$	$M_s^d = 2.0$	3.6303
	$M_s^d = 1.4$	0.3163
	$M_s^d = 1.6$	0.1416
Proposed	$M_s^d = 1.8$	0.7011
	$M_s^d = 2.0$	1.3228
	$M_s^d = M_s^{AMIGO}$	0.2512
	$Deg^a = 0.25$	1.0513
Proposed	$Deg^a = 0.35$	0.7987
	$Deg^a = 0.45$	0.5482
	$Deg^a = 0.55$	0.2956
Proposed	Balanced-I_B	0.0070

Table 7.2: Particular process P_8 ($\sigma = 0.50$) - Controller tuning evaluation

Tuning	Criteria design	I_{Perf}	I_{Rob}	$\lvert I_{Perf} - I_{Rob}\rvert$
AMIGO	$M_s^d = 1.4$	0.2913	0.4651	0.1738
Proposed	$M_s^d = M_s^{AMIGO}$	0.3186	0.4650	0.1464
$\kappa - \tau$	$M_s^d = 2.0$	0.6675	0.2359	0.4316
Proposed	$Deg^a = Deg^{\kappa-\tau}$	0.6621	0.3156	0.3465
Proposed	Balanced-I_B	0.4202	0.4211	0.0009

($M_s = 2.0$) and the proposed tuning for an arbitrary degradation value of $Deg^a = Deg^{\kappa-\tau}$. Controller's parameters for these last tuning methods can be found in Tables 5.4 and 6.2.

In Table 7.2, the indexes (6.1) and (7.1) are shown, as well as the corresponding difference $|I_{Perf} - I_{Rob}|$, that indicates the system robustness/performance balance. Just to remember, index I_{Perf} indicates the degree of optimality for the system, so it is desired high values near to one. Then, for the system's robustness increase I_{Rob}, it must be as large as possible.

The comparison shown in Table 7.2 provides that, as expected, AMIGO and proposed generic tuning for $M_s^d = M_s^{AMIGO}$ have an equal I_{Rob} value, because their robustness is the same. However, the proposed M_s^d tuning has a higher performance optimality degree than AMIGO, therefore achieving a better balance (less $|I_{Perf} - I_{Rob}|$ value).

For the Kappa-Tau and proposed tuning with $Deg^a = Deg^{\kappa-\tau}$, we have an analogous result because, for similar I_{Perf} index values, the robustness increase achieved by proposed tuning for arbitrary Deg^a values, is higher than the one for $\kappa - \tau$ method and therefore, the balance is better for the proposal.

Finally, the last row of Table 7.2 gives the evaluation for the proposed balanced tuning (7.2), that has obviously the best balance calculation, meaning a similar values for I_{Perf} and I_{Rob}. For this system, it is possible to see that the best balance can be obtained when the performance degree of optimality and the robustness increase, are values around the 40%.

In Fig. 7.6, there are the system's and controller's outputs for three cases of the proposed tuning. It is possible to see that, responses for the balanced tuning are between the ones that provide the best values for I_{Perf} and I_{Rob} indexes.

7.6 Summary

In this chapter a new PID tuning methodology is proposed, with the aim to achieve the best possible balance between robustness and performance issues. The problem was faced using the results presented in chapters 5 and 6.

The proposal is presented as an autotuning rule, where just the FOPDT model information is needed to calculate the PID controller parameters. This proposed balanced tuning looks for the best compromise between the robustness increase and the consequent reduction in the performance optimality for the system.

An important aspect of the proposed balanced tuning is that, it remains the same expression for the controller's parameters than the previous proposed ones for M_s^d and Deg^a, providing a complete and homogeneous set of options to tune the PID.

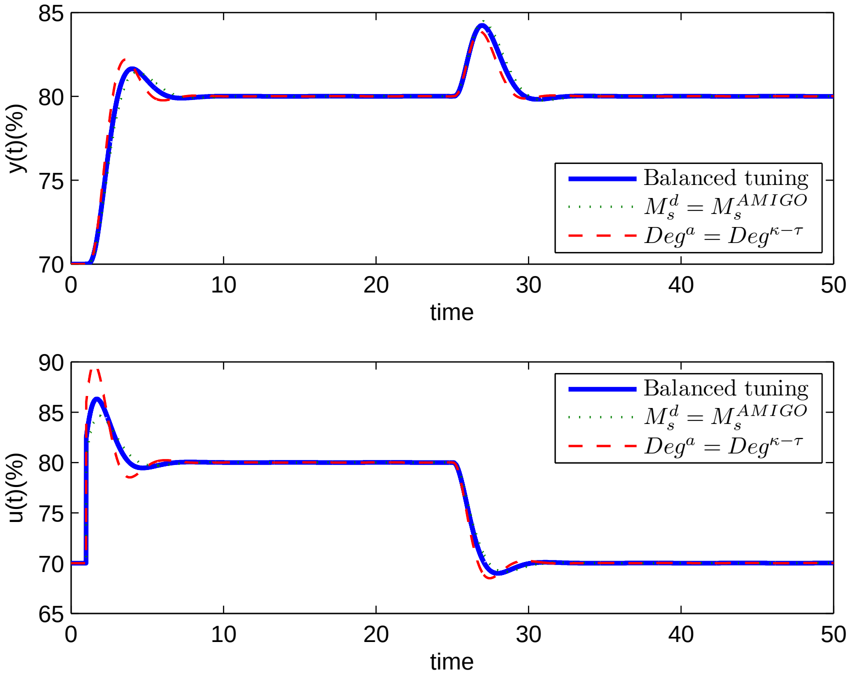

Figure 7.6: Particular process P_8 - Proposed methods ($\sigma = 0.50$).

Part III

Concluding remarks

Chapter 8

Conclusions and future work

This thesis faces to various and very important aspects of PID control, such as performance and robustness. The main objective is to provide answers to the very well known *trade-off* problems, between performance for servo and regulation operation and between performance and robustness features.

From the above, the proposed tuning methodologies take into account these considerations, in order to improve the general behavior of the control system.

This chapter concludes the work presented in this thesis and gives some clues for future work and research.

8.1 Conclusions and contributions

New autotuning methods for PID controllers have been presented. These methods use the information provided by a FOPDT model of the process, to obtain the controller's parameters. The design looks for the essential requirements of any control system as combined servo/regulation performance and robustness with respect to model uncertainties. The formulation of the design problems were divided in two approaches (corresponding to the two parts of this document).

Part I: Combined servo/regulation operation for PID controllers

It is presented a general procedure to find an *intermediate* tuning for servo/regulation control operation, in order to reduce the performance degradation when the system operates in a different way that it was tuned for. This approach is stated in terms of the extreme optimal tunings for each operation mode and weighting factors can be used, as a qualitative specification, to declare a preference between both operation modes.

The resulting *intermediate* tuning provides lower performance than the optimum settings when the tuning and the operation modes are equal. However, in general terms, if we take into account the whole system operation (servo and regulation), it is possible to achieve the highest performance when the proposed *intermediate* tuning is used.

In addition of the above idea, the general procedure was extended to unstable an integrating processes showing that for this application, the proposal is even better because the expected performance degradation is very high, when the tuning and operation modes are different. Therefore, the improvement that can be achieved using the *intermediate* tuning, results in a good option to obtain the PID controller parameters.

Part II: Robustness and performance trade-off for PID controllers

The proposed approaches include the consideration of robustness, in an explicit way into the design stage. First, it was presented a tuning rule that selecting a desired robustness M_s^d, provides the best servo/regulation performance (5.4), that can be achieved for the system. The accomplishment of the claimed robustness was verified, obtaining almost negligible deviation values, therefore the PID controller gives a good performance with also a precise and certain robustness degree.

In the second case, it is proposed an approach that, even different, is complementary to the previous one. The idea is to allow some reduction in the optimality degree of the system's performance, in order to achieve the largest increase in the robustness. Some evaluations show the good results for the proposed tunings compared with other well-known PID methods.

Finally, it was desirable to look for an intermediate approach that provides a balance between the degree of optimality and the robustness increase and in this sense, a well-balanced PID tuning was presented (7.2).

A very important aspect of this second part is that, all PID tuning methods were parametrized using the same form for the equations of controller's parameters (5.7). This allows to maintain simple and homogeneous expressions that can be implemented easily.

8.2 Future work and research

Some ideas to extend or to explore new opportunities from this research study, can be followed in future works.

- **Process model information.** The proposed tunings were based on the information provided by a FOPDT model. To develop the same procedure for Second-Order-Plus-Dead-Time (SOPDT) models, could improve the whole system's behavior. Moreover, although in the first part the results were presented for unstable and integrating processes cases, could be interesting to follow the proposed ideas in the second part for these kind of systems and even more, to study the non-minimum phase systems (inverse response).

- **PID controller structure.** Taking into account the little-known of fractional PID controllers, could be very interesting to explore the servo/ regulation dynamics, as well as the robustness, in this kind of PID. For its novelty, it is a point that can receive much attention.

  Another approach is to obtain the four parameters of a 2-DoF PID controller just with a single optimization of index (5.4). This could improve the actual and well-known procedure of two stages (first one for regulation and then for servo). A systematic way to determine when a PID should be of one or two degrees of freedom (1-DoF/2-DoF) is very welcome.

- **Control configuration.** Considering that, usually control systems operate in regulation mode and set-point changes can be predicted, the formulation of an adaptive re-tuning design between servo/regulation tuning modes could be investigated. Also, the researched topic could be studied for multivariable systems, specifically for TITO processes, considering that it is a system that follows a set-point but has continuous disturbances, produced by the interaction effects between the control-loops.

- **Real implementation.** The proposed PID controller tunings implemented by simulation in this work, can be tested and performed in real plants.

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