Chapter 7

Summary and conclusions

The inclusive distribution $(1/\sigma_{\text{tot}})(d\sigma/dx)$ for charged particles has been measured by the ALEPH experiment for hadronic events of all flavours and enriched samples in light flavours, c quarks and b quarks. In addition, the transverse and longitudinal distributions were measured and, together with information from identified gluon jets, used to constrain the gluon fragmentation function.

A global analysis of these measurements and results from other experiments at lower centre-of-mass energies has been carried out in the framework of next-to-leading order QCD. Scaling violations in the time-like domain between $\sqrt{s}$ = 22 GeV and $\sqrt{s}$ = 91.2 GeV are observed in agreement with QCD predictions. The data are found to be consistent with one universal coupling constant describing the evolution of the fragmentation functions between $\sqrt{s}$ = 22 GeV and $\sqrt{s}$ = 91.2 GeV. At the same time, the shape of the fragmentation function for gluons, light flavours, c and b quarks were determined from the data alone.

The strong coupling constant measured here from scaling violations is consistent with other determinations [?, ?] at one fixed energy based on global event shape variables. Expressed at the scale M_Z, the measured value is $\alpha_s(M_Z) = 0.126 \pm 0.009$.

The main single contribution to the error on α_s comes from the dependence on the factorization scale chosen. Next-to-next-to-leading order calculations of the coefficient functions and splitting kernels would decrease this source of error. The overall error is bigger than for some other determinations of the strong coupling constant [48, ?] mainly because all non-perturbative effects (in the value of the fragmentation functions at one energy and in their evolution) have been taken directly from data, without relying on the quantitative predictions of the Monte Carlo models.

Appendix A

Electro-weak cross-sections

The computation of the scaled energy inclusive distributions from the fragmentation functions need the knowledge of the electro-weak relative cross sections in order to weight the different flavour contributions species at each particular energy. This section describes this factors that are then introduced in formula ().

The flavour weights are defined through

$$w_i(s) = \frac{\sigma_i(s)}{2\sigma_u(s) + 3\sigma_d(s)} \tag{A.1}$$

where it has been taken into account that only five active flavours exist in the whole energy range used in the analysis.

The relative cross sections $r_i(s)$ are given by the electroweak theory and are given by

$$\sigma_i(s) = \sigma_i^{(v)}(s) + \sigma_i^{(a)}(s) = \frac{4\pi\alpha^2}{s}\left(r_i^{(v)}(s) + r_i^{(a)}(s)\right) \tag{A.2}$$

with

$$\begin{aligned} r_i^{(v)}(s) &= q_i^2 + \frac{s}{(M_Z^2 - s)^2 + M_Z^2\Gamma_Z^2} \cdot \\ & \left[2q_i v_e v_i \frac{M_Z^2 - s}{4s_w^2 c_w^2} + (v_e^2 + a_e^2)v_i^2 \frac{s}{16 s_w^4 c_w^4}\right] \qquad \text{(A.3)} \\ r_i^{(a)}(s) &= \frac{s}{(M_Z^2 - s)^2 + M_Z^2\Gamma_Z^2}(v_e^2 + a_e^2)(a_i^2)\frac{s}{16 s_w^4 c_w^4} \qquad \text{(A.4)} \end{aligned}$$

where s_w and c_w denote the sine and cosine of the weak mixing angle, q_i the charge of the respective quarks and, the vector and axial couplings for electrons, u-quarks and d-quarks are give in table A.

v_e	$=$	$-\frac{1}{2}+2s_w^2$	a_e	$=$	$-\frac{1}{2}$
v_u	$=$	$\frac{1}{2}-\frac{4}{3}s_w^2$	a_u	$=$	$\frac{1}{2}$
v_d	$=$	$-\frac{1}{2}+\frac{2}{3}s_w^2$	a_d	$=$	$-\frac{1}{2}$

Table A.1: Vector and axial couplings for electrons, u- and d- type quarks used in the computation of the relative electro-weak cross sections in equation (A.2).

Figure A.1 plots the relative electroweak cross sections as function of the centre-of-mass energy. The large variation in the different proportions between the low and LEP energies can be seen.

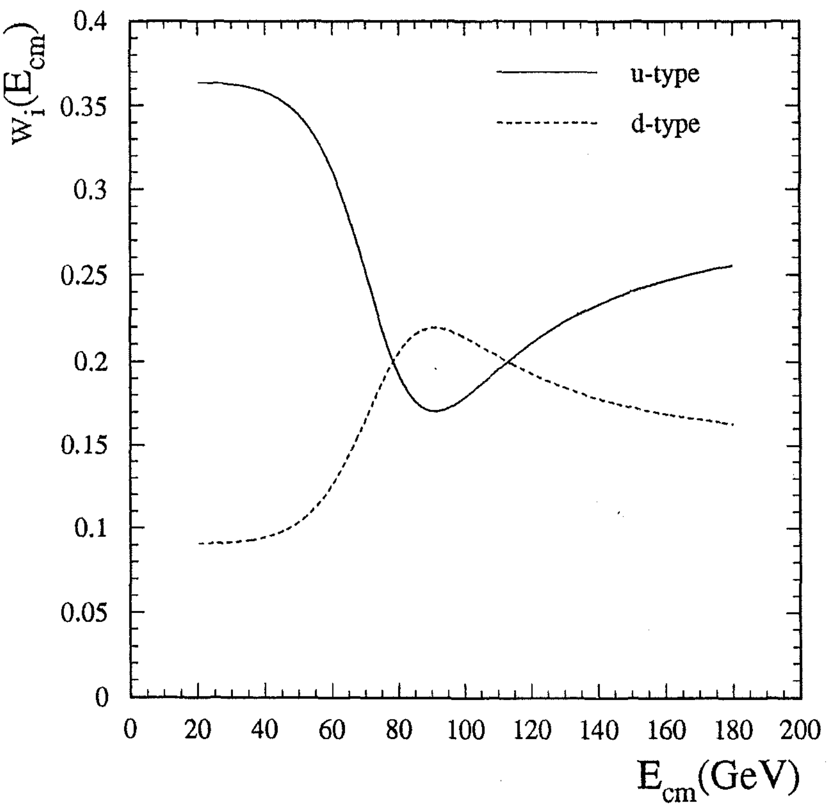

Figure A.1: Relative Electro-weak cross-section as function of the cetre-of-mass energy. The two contributions for u-type quarks (continuous line) and d-type quarks (dashed line) are plotted.

Appendix B

Coeficient functions and Splitting Kernels

This appendix list a collection of the next-to-leading formula necessary for the calculation of the cross-sections from the fragmentation functions and their energy evolution.

B.1 Coeficient functions

This section contains the expressions for the coefficient functions that relate the effective fragmentation functions with the scaled energy measurable cross-sections though equation (2.34). According to reference [49] they are given by

$$C_g^L(z, \alpha_s(\mu_F), \mu_F^2/s) = a(\mu_F^2) 2\frac{1-z}{z} \tag{B.1}$$

$$C_q^L(z, \alpha_s(\mu_F), \mu_F^2/s) = a(\mu_F^2) \tag{B.2}$$

$$\begin{aligned} C_g^T(z, \alpha_s(\mu_F), \mu_F^2/s) &= a(\mu_F^2) \left\{ \frac{1+(1-z)^2}{z} \cdot \right. \\ &\left. \cdot \left[\ln(1-z) + 2\ln z - \ln\frac{\mu_F^2}{s} \right] - 2\frac{1-z}{z} \right\} \end{aligned} \tag{B.3}$$

$$\begin{aligned} C_q^T(z, \alpha_s(\mu_F), \mu_F^2/s) &= \delta(1-z) \left[1 + a(\mu_F^2) \left(\frac{2\pi^2}{3} - \frac{9}{2} \right) \right] \\ &-a(\mu_F^2) \ln\frac{\mu_F^2}{s} \left(\frac{1+z^2}{1-z} \right)_{(+)} \end{aligned}$$

$$+a(\mu_F^2)\left[\frac{3}{2}(1-z)-\frac{3}{2}\left(\frac{1}{1-z}\right)_{(+)}\right.$$
$$\left.+\,2\frac{1+z^2}{1-z}\ln z+(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_{(+)}\right] \quad \text{(B.4)}$$

where $a(\mu_F^2)$ reads for the couplant constant defined in (2.12

The subscript (+) in eq. B.4 specifies a regularization procedure for the integral over the spliting functions which is singular at $z=1$ defined as

$$\int_0^1 dz\,[f(z)]_{(+)}g(z)=\int_0^1 dz\,f(z)\,(g(z)-g(1)) \quad \text{(B.5)}$$

which, in the usual case that the lower limit of the integral is not 0 but some variable x, translates into

$$\int_x^1 dz\,[f(z)]_{(+)}g(z)=\int_x^1 dz\,\left[f(z)(g(z)-g(1))\right]-g(1)\int_0^x dz\,f(z) \quad \text{(B.6)}$$

The convolution integrals that are to be performed in (B.4) are straightforward, with the exception of the integral over $C_Q^T(z)$. After expanding the (+) regularizations one obtains

$$\begin{aligned}\int_x^1 dz P_Q^T(z)D_Q(\frac{x}{z},\mu_F^2) &= D_Q(x,\mu_F^2)(1+a(\mu)F_\delta)\\ &+ a(\mu)\int_x^1 dz\left[D_Q(\frac{x}{z},\mu_F^2)F_R-D_Q(x,\mu_F^2)F_S\right] \quad \text{(B.7)}\end{aligned}$$

with

$$F_\delta = \frac{2\pi^2}{3}-\frac{9}{2}-\frac{3}{2}\ln(1-x)+\ln^2(1-x)-\ln\frac{\mu_F^2}{s}\left[2\ln(1-x)+\frac{3}{2}\right] \quad \text{(B.8)}$$
$$F_R = \frac{3}{2}(1-z)+\frac{2(1+z^2)}{1-z}\ln z+\frac{1+z^2}{1-z}\left[\ln(1-z)+\ln\frac{\mu_F^2}{s}\right]-\frac{3}{2}\frac{1}{1-z} \quad \text{(B.9)}$$
$$F_S = \frac{2}{1-z}\left[\ln(1-z)+\ln\frac{\mu_F^2}{s}\right]-\frac{3}{2}\frac{1}{1-z} \quad \text{(B.10)}$$

B.2 Evolution splitting kernels

This sections gives the expressions of the splitting kernels used in the evolutions equations (2.38). They are given separately for the Non-singlet and Singlet parts of the fragmentation functions evolution.

B.2.1 Evolution of Flavour Non-Singlet Fragmentation Functions

The formulae for the evolution of the non-singlet parts of the fragmentation functions can be found in reference [50]. From there, the NLO splitting function governing the evolution of non-singlet fragmentation functions is obtained as

$$P_N(z, \alpha_s(\mu_R), \mu_R^2/s) = \left[\mathcal{P}^{\mathcal{N}}\right]_{(+)} + a^2(\mu_R) 2\delta(1-z) \int_0^1 dz P^N_{q\bar{q}}(z), \tag{B.11}$$

being the expression for $\mathcal{P}^{\mathcal{N}}$

$$\mathcal{P}^{\mathcal{N}}(s) = a(\mu_R) P_{qq} + a^2(\mu_R^2)\left(P^N_{qq} + P^N_{q\bar{q}} + P_{qq} b_0 \ln\frac{\mu_R}{s}\right) \tag{B.12}$$

where

$$\begin{aligned} P^N_{qq} &= P_F + X P_G + Z P_n \\ P^N_{q\bar{q}} &= (2-X) P_A, \end{aligned} \tag{B.13}$$

and

$$\begin{aligned} P_F &= 2P_{qq}\ln x \ln(1-x) + \left(\frac{3}{1-z} - 3z - 5\right)\ln z + \left(\frac{1+z}{2} - 2P_{qq}\right)\ln^2 z - 5(1-z) \\ P_G &= P_{qq}\left(\frac{1}{2}\ln^2 z + \frac{11}{6}\ln z - \frac{\pi^2}{6} + \frac{67}{18}\right) + (1+x)\ln z + \frac{20}{3}(1-z) \\ P_n &= -P_{qq}\left(\frac{2}{3}\ln z + \frac{10}{9}\right) - \frac{4}{3}(1-z) \\ P_A &= S_2 M_{qq} + (1+x)\ln z + 2(1-z) \end{aligned} \tag{B.14}$$

In the above expressions, the values of b_0, X and Z are the ones given in equations (2.20) and (2.12) and a shorthand notation for the first order the Alrarelly-Parisi spliting Kernels

$$P_{qq} \equiv P_{qq}(z) = \frac{1+z^2}{1-z} \tag{B.15}$$

$$P_{qg} \equiv P_{qg}(z) = \frac{1+(1-z)^2}{z} \tag{B.16}$$

$$P_{gq} \equiv P_{qq}(z) = z^2 + (1-z)^2 \tag{B.17}$$

$$P_{gg} \equiv P_{qq}(z) = -z^2 + z - 2 + \frac{1}{z(1-z)} \tag{B.18}$$

was introduced together with the definition

$$M_{ij} \equiv M_{ij}(z) = P_{ij}(-z). \tag{B.19}$$

B.2.2 Evolution of the Flavour Singlet Fragmentation Functions

The singlet splitting kernel can be found in reference [51]. The ones comming from the diagonal parts are singular at $z = 1$. Thus they are regularized according to

$$\begin{aligned} P_{GG}(z) &= \frac{1}{z}\left[z\hat{P}_{GG}(z)\right]_{(+)} - \delta(1-x)\int_0^1 dy\, y P_{GQ}(y) \\ P_{QQ}(z) &= \frac{1}{z}\left[z\hat{P}_{QQ}(z)\right]_{(+)} - \delta(1-z)\int_0^1 dy\, y P_{QG}(y) \end{aligned} \tag{B.20}$$

which With the definition of the (+) regularization, equation (B.6) allow to write the evolution equations for the singlet fragmentation functions (equation (2.42)) as

$$\begin{aligned} s\frac{d}{ds}G(x,s) &= \int_x^1 dz \left\{\hat{P}_{GG}(z)\left[G(\frac{x}{z},s) - zG(x,s)\right]\right. \\ &\left. + P_{GQ}(z)\left[S(\frac{x}{z},s) - zG(x,s)\right]\right\} \\ &- G(x,s)\int_0^x dz\, z\left[\hat{P}_{GG}(z) + P_{GQ}(z)\right] \\ s\frac{d}{ds}S(x,s) &= \int_x^1 dz \left\{P_{QG}(z)\left[G(\frac{x}{z},s) - zS(x,s)\right]\right. \\ &\left. + \hat{P}_{QQ}(z)\left[S(\frac{x}{z},s) - zS(x,s)\right]\right\} \\ &- S(x,s)\int_0^x dz\, z\left[P_{QG}(z) + \hat{P}_{QQ}(z)\right] \end{aligned} \tag{B.21}$$

where only convolution integrals remain. The terms in these evolution equations are given by

$$\hat{P}_{QQ} = a(\mu_R)\, P_{\rm qq} + a^2(\mu_R)\left(R_{\rm qq} + XS_{\rm qq} + ZT_{\rm qq} + P_{\rm qq}b_0 \ln\frac{\mu_R}{s}\right) \tag{B.22}$$

$$P_{QG} = a(\mu_R)\, P_{\rm qg} + a^2(\mu_R)\left(R_{\rm qg} + XS_{\rm qg} + P_{\rm qg}b_0 \ln\frac{\mu_R}{s}\right) \tag{B.23}$$

$$\begin{aligned} P_{GQ} &= a(\mu_R)\, 2ZP_{\rm gq} \\ &+ a^2(\mu_R)\left(ZR_{\rm gq} + ZXS_{\rm gq} + ZZT_{\rm gq} + 2ZP_{\rm gq}b_0 \ln\frac{\mu_R}{s}\right) \end{aligned} \tag{B.24}$$

$$\begin{aligned} \hat{P}_{GG} &= a(\mu_R)\, 2XP_{\rm gg} \\ &+ a^2(\mu_R)\left(ZR_{\rm gg} + XXS_{\rm gg} + ZXT_{\rm gg} + 2XP_{\rm gg}b_0 \ln\frac{\mu_R}{s}\right) \end{aligned} \tag{B.25}$$

with the following definitions in the fermion-fermion spliting

$$R_{\rm qq} = z - 1 - (\frac{3}{2} - \frac{1}{2}z)\ln z + \frac{1+z}{2}\ln^2 z$$

$$+P_{qq}\left[\frac{3}{2}\ln z-2\ln^2 z+2\ln z\ln(1-z)\right]+2S_2M_{qq} \qquad (B.26)$$

$$S_{qq} = \frac{14}{3}-\frac{14}{3}z+P_{qq}\left[\frac{11}{6}\ln z+\frac{1}{2}\ln^2 z-\frac{\pi^2}{6}+\frac{67}{18}\right]-S_2M_{qq} \qquad (B.27)$$

$$T_{qq} = -\frac{52}{3}+\frac{28}{3}z+\frac{112}{9}z^2-\frac{40}{9z}+(2+2x)\ln^2 z$$
$$-(10+18z+\frac{16}{3}z^2)\ln z-P_{qq}\left[\frac{2}{3}\ln z+\frac{10}{9}\right] \qquad (B.28)$$

For the fermion-gluon splitting they are

$$R_{qg} = -\frac{1-z}{2}+4z-\frac{16-x}{2}\ln z+2x\ln(1-z)+\frac{2-z}{2}\ln^2 z$$
$$+P_{qg}\left[\ln^2(1-z)+4\ln z\ln(1-z)-8S_1-\frac{4\pi^2}{3}\right] \qquad (B.29)$$

$$S_{qg} = S_2M_{qg}+\frac{62}{9}-\frac{35}{18}z+\frac{44}{9}z^2+(2+12z+\frac{8}{3}z^2)\ln z-2z\ln(1-z)$$
$$-(4+x)\ln^2 z+P_{qg}\left[\frac{17}{18}-2\ln z\ln(1-z)-3\ln z-\frac{3}{2}\ln^2 z\right.$$
$$\left.-\ln^2(1-z)+8S_1+\frac{7\pi^2}{6}+\right] \qquad (B.30)$$

The gluon-fermion splitting is described by

$$R_{gq} = -2+3z-(7-8z)\ln z-4\ln(1-z)+(1-2z)\ln^2 z$$
$$-2P_{gq}\left[2\ln z\ln(1-z)+\ln^2 z+\right.$$
$$\left.\ln^2(1-z)+\ln(1-z)-\ln z-8S_1-\pi^2+5\right] \qquad (B.31)$$

$$S_{gq} = 2S_2M_{gq}-\frac{152}{9}+\frac{166}{9}z-\frac{40}{9z}-(\frac{4}{3}+\frac{76}{3}z)\ln z+4\ln(1-z)+(2+8z)\ln^2 z$$
$$+P_{gq}\left[\frac{178}{9}+8\ln z\ln(1-z)-\ln^2 z-\frac{4}{3}\ln z\right.$$
$$\left.+\frac{10}{3}\ln(1-z)+2\ln^2(1-z)-16S_1-\frac{7\pi^2}{3}+\right] \qquad (B.32)$$

$$T_{gq} = -\frac{8}{3}-P_{gq}\left[\frac{16}{9}+\frac{8}{3}\ln z+\frac{8}{3}\ln(1-z)\right] \qquad (B.33)$$

and the gluon-gluon splitting by

$$R_{gg} = -4+12z-\frac{164}{9}z^2+\frac{92}{9z}$$
$$+\left[10+14z+\frac{16}{3}(z^2+\frac{1}{z})\right]\ln z+2(1+z)\ln^2 z \qquad (B.34)$$

$$S_{gg} = \frac{27}{2}(1-z)+\frac{134}{18}(z^2-\frac{1}{z})+(\frac{11}{3}-\frac{25}{3}z+\frac{44}{3z})\ln z-4(1+z)\ln^2 z$$

$$+P_{gg}\left[4\ln z\ln(1-z) - 3\ln^2 z + \frac{22}{3}\ln z - \frac{\pi^2}{3} + \frac{67}{9}\right] + 2S_2 M_{gg} \quad \text{(B.35)}$$

$$T_{gg} = 2(1-z) + \frac{26}{9}(z^2 - \frac{1}{z}) - \frac{4}{3}(1+z)\ln z - P_{gg}\left[\frac{20}{9} + \frac{8}{3}\ln z\right] \quad \text{(B.36)}$$

In the expressions above S_1 and S_2 are defined as

$$S_1 = -Li_2(1-z) = \int_0^{1-z} \frac{dx}{x}\ln(1-x) \quad \text{(B.37)}$$

$$S_2 = -Li_2(\frac{1}{1+z}) + \frac{1}{2}\ln^2 z - \ln^2(1+z) + \frac{\pi^2}{6} \quad \text{(B.38)}$$

where $Li_2(x)$ is the dilogarithm function defined as

$$Li_2(x) = -\int_0^x \frac{\ln(1-t)}{t}\,dt = -\int_0^1 \frac{\ln(1-xt)}{t}\,dt \quad \text{(B.39)}$$

Appendix C

Cross section tables

This appendix gives the list of cross-sections measured with ALEPH. The scaled energy distribution for all flavours is listed in table C. Tables C,C and C list the corresponding distributions for light (uds), c, and b enriched flavours. Table ?? list the gluon distribution measured in three jet symmetric events and tables C and C the transverse and longitudinal cross-sectionns.

For all the except for the gluon distribution, the bin-to-bin errors are specified. They are separated in statistical and systematic error. For the systematic, the three contributions considered (the limited statistics of the Monte Carlo used for the correction, the selection cut variation and the Monte Carlo model dependence) are listed explicitely.

Interval	$\frac{1}{N_{ev}}\frac{dn^{tr}}{dx}$	E_{stat}	$E_{binsys}(= E_{MCstat} \pm E_{cut} \pm E_{MCmodel})$
0.008–0.012	501.3	0.4	7.8 (= 0.25 ±7.6 ±1.8)
0.012–0.020	392.69	0.24	0.97 (= 0.15 ±0.48 ±0.83)
0.020–0.030	274.81	0.18	0.99 (= 0.11 ±0.39 ±0.91)
0.030–0.040	191.05	0.15	0.48 (= 0.09 ±0.25 ±0.40)
0.040–0.050	139.94	0.13	0.44 (= 0.08 ±0.19 ±0.39)
0.050–0.060	107.33	0.11	0.30 (= 0.07 ±0.12 ±0.27)
0.060–0.070	85.09	0.10	0.17 (= 0.06 ±0.07 ±0.14)
0.070–0.080	68.96	0.09	0.19 (= 0.07 ±0.06 ±0.17)
0.080–0.090	56.81	0.08	0.13 (= 0.05 ±0.06 ±0.11)
0.090–0.100	47.875	0.075	0.095 (= 0.047 ±0.049 ±0.066)
0.100–0.120	37.655	0.047	0.074 (= 0.030 ±0.036 ±0.057)
0.120–0.140	28.061	0.041	0.046 (= 0.026 ±0.037 ±0.013)
0.140–0.160	21.379	0.035	0.054 (= 0.022 ±0.039 ±0.031)
0.160–0.180	16.661	0.031	0.042 (= 0.020 ±0.030 ±0.022)
0.180–0.200	13.233	0.028	0.027 (= 0.018 ±0.020 ±0.004)
0.200–0.225	10.376	0.022	0.023 (= 0.014 ±0.017 ±0.009)
0.225–0.250	7.928	0.019	0.020 (= 0.012 ±0.007 ±0.014)
0.250–0.275	6.197	0.017	0.016 (= 0.011 ±0.006 ±0.011)
0.275–0.300	4.874	0.015	0.012 (= 0.010 ±0.006 ±0.004)
0.300–0.325	3.862	0.014	0.013 (= 0.009 ±0.003 ±0.009)
0.325–0.350	3.054	0.012	0.018 (= 0.008 ±0.016 ±0.003)
0.350–0.375	2.461	0.011	0.009 (= 0.007 ±0.002 ±0.006)
0.375–0.400	1.995	0.010	0.011 (= 0.006 ±0.007 ±0.007)
0.400–0.430	1.5555	0.0079	0.0059 (= 0.0049±0.0025±0.0021)
0.430–0.460	1.2122	0.0070	0.0081 (= 0.0043±0.0038±0.0057)
0.460–0.490	0.9400	0.0061	0.0051 (= 0.0038±0.0031±0.0014)
0.490–0.520	0.7346	0.0054	0.0091 (= 0.0034±0.0071±0.0046)
0.520–0.550	0.5631	0.0047	0.0048 (= 0.0029±0.0038±0.0005)
0.550–0.600	0.4098	0.0031	0.0025 (= 0.0019±0.0015±0.0008)
0.600–0.650	0.2572	0.0025	0.0033 (= 0.0015±0.0024±0.0018)
0.650–0.700	0.1719	0.0020	0.0027 (= 0.0012±0.0023±0.0007)
0.700–0.750	0.1041	0.0015	0.0026 (= 0.0009±0.0023±0.0007)
0.750–0.800	0.0606	0.0011	0.0027 (= 0.0007±0.0026±0.0004)
0.800–0.900	0.0262	0.0005	0.0032 (= 0.0003±0.0032±0.0002)
0.900–1.000	0.0048	0.0002	0.0020 (= 0.0001±0.0020±0.0001)

Table C.1: All flavour inclusive cross section for charged particles measured at $\sqrt{s} = 91.2\,\mathrm{GeV}$. The erorrs listed are the statistical and the systematic bin-to-bin errors. The three sources of systematic uncertainties are specified. A normalization error of 1% has to be added in quadrature everywhere as specified in section 5.4.1.

Interval	$\frac{1}{N_{ev}}\frac{dn^{tr}}{dx}$	E_{stat}	$E_{binsys}(= E_{MCstat} \pm E_{cut} \pm E_{MCmodel})$
0.008–0.012	492.6	0.4	7.3 (= 0.2 ±7.1 ±1.9)
0.012–0.020	383.1	0.2	1.2 (= 0.1 ±0.8 ±0.9)
0.020–0.030	266.5	0.2	1.2 (= 0.1 ±0.6 ±1.0)
0.030–0.040	184.38	0.15	0.72 (= 0.09 ±0.54 ±0.48)
0.040–0.050	134.60	0.12	0.64 (= 0.08 ±0.42 ±0.47)
0.050–0.060	103.06	0.11	0.46 (= 0.07 ±0.32 ±0.32)
0.060–0.070	81.42	0.10	0.39 (= 0.06 ±0.34 ±0.17)
0.070–0.080	66.11	0.09	0.34 (= 0.05 ±0.27 ±0.20)
0.080–0.090	54.76	0.08	0.20 (= 0.05 ±0.17 ±0.10)
0.090–0.100	46.05	0.07	0.17 (= 0.05 ±0.14 ±0.09)
0.100–0.120	36.50	0.05	0.13 (= 0.03 ±0.11 ±0.07)
0.120–0.140	27.468	0.039	0.074 (= 0.025 ±0.069 ±0.009)
0.140–0.160	21.199	0.035	0.061 (= 0.022 ±0.053 ±0.022)
0.160–0.180	16.730	0.031	0.044 (= 0.019 ±0.031 ±0.024)
0.180–0.200	13.359	0.028	0.035 (= 0.017 ±0.029 ±0.007)
0.200–0.225	10.644	0.022	0.053 (= 0.014 ±0.050 ±0.010)
0.225–0.250	8.249	0.020	0.042 (= 0.012 ±0.037 ±0.016)
0.250–0.275	6.532	0.017	0.037 (= 0.011 ±0.033 ±0.013)
0.275–0.300	5.196	0.016	0.031 (= 0.010 ±0.028 ±0.007)
0.300–0.325	4.136	0.014	0.026 (= 0.009 ±0.023 ±0.009)
0.325–0.350	3.323	0.012	0.030 (= 0.008 ±0.029 ±0.005)
0.350–0.375	2.678	0.011	0.022 (= 0.007 ±0.019 ±0.010)
0.375–0.400	2.190	0.010	0.024 (= 0.006 ±0.022 ±0.009)
0.400–0.430	1.717	0.008	0.018 (= 0.005 ±0.017 ±0.003)
0.430–0.460	1.353	0.007	0.017 (= 0.004 ±0.014 ±0.007)
0.460–0.490	1.0564	0.0064	0.0084 (= 0.0039±0.0072±0.0019)
0.490–0.520	0.8313	0.0057	0.0159 (= 0.0035±0.0143±0.0059)
0.520–0.550	0.6513	0.0050	0.0079 (= 0.0031±0.0072±0.0012)
0.550–0.600	0.4835	0.0033	0.0099 (= 0.0021±0.0096±0.0005)
0.600–0.650	0.3093	0.0026	0.0040 (= 0.0016±0.0027±0.0025)
0.650–0.700	0.2079	0.0021	0.0032 (= 0.0013±0.0027±0.0010)
0.700–0.750	0.1283	0.0017	0.0037 (= 0.0010±0.0034±0.0011)
0.750–0.800	0.0768	0.0012	0.0022 (= 0.0008±0.0020±0.0005)
0.800–0.900	0.0323	0.0006	0.0022 (= 0.0003±0.0022±0.0002)
0.900–1.000	0.0059	0.0002	0.0015 (= 0.0001±0.0015±0.0002)

Table C.2: uds-enriched inclusive cross section for charged particles measured at $\sqrt{s} = 91.2$ GeV. The flavour composition is of 78.9%, 14.5% and 6.6% of *uds*-,*c*-,and *b*-quarks, respectively, as specified in table ??. The erorrs listed are the statistical and the systematic bin-to-bin errors. The three sources of systematic uncertainties are specified. A normalization error of 1% has to be added in quadrature everywhere as specified in section 5.4.1.

Interval	$\frac{1}{N_{ev}}\frac{dn^{tr}}{dx}$	E_{stat}	E_{binsys}	($= E_{MCstat}$	$\pm E_{cut}$	$\pm E_{MCmodel}$)
0.008–0.012	509.1	1.3	7.5	(= 0.8	±7.2	±1.8)
0.012–0.020	397.3	0.8	1.3	(= 0.5	±1.0	±0.8)
0.020–0.030	279.0	0.6	1.2	(= 0.3	±0.7	±0.9)
0.030–0.040	196.08	0.50	0.67	(= 0.29	±0.45	±0.40)
0.040–0.050	143.38	0.42	0.74	(= 0.24	±0.60	±0.36)
0.050–0.060	109.81	0.37	0.44	(= 0.21	±0.26	±0.28)
0.060–0.070	86.86	0.33	0.30	(= 0.19	±0.20	±0.13)
0.070–0.080	70.05	0.29	0.28	(= 0.17	±0.17	±0.14)
0.080–0.090	57.84	0.27	0.31	(= 0.15	±0.24	±0.13)
0.090–0.100	48.26	0.24	0.20	(= 0.14	±0.13	±0.06)
0.100–0.120	37.93	0.15	0.13	(= 0.09	±0.08	±0.06)
0.120–0.140	28.63	0.13	0.10	(= 0.08	±0.07	±0.02)
0.140–0.160	21.87	0.12	0.10	(= 0.07	±0.06	±0.04)
0.160–0.180	16.77	0.10	0.08	(= 0.06	±0.06	±0.02)
0.180–0.200	13.322	0.090	0.081	(= 0.053	±0.058	±0.019)
0.200–0.225	10.471	0.072	0.051	(= 0.043	±0.028	±0.007)
0.225–0.250	7.809	0.062	0.050	(= 0.036	±0.031	±0.017)
0.250–0.275	6.059	0.054	0.044	(= 0.032	±0.027	±0.013)
0.275–0.300	4.794	0.049	0.053	(= 0.029	±0.044	±0.008)
0.300–0.325	3.846	0.043	0.047	(= 0.026	±0.036	±0.013)
0.325–0.350	3.089	0.039	0.047	(= 0.024	±0.040	±0.007)
0.350–0.375	2.375	0.034	0.030	(= 0.020	±0.022	±0.004)
0.375–0.400	1.876	0.031	0.028	(= 0.018	±0.020	±0.008)
0.400–0.430	1.436	0.024	0.020	(= 0.014	±0.014	±0.001)
0.430–0.460	1.088	0.021	0.017	(= 0.012	±0.010	±0.008)
0.460–0.490	0.871	0.019	0.016	(= 0.011	±0.011	±0.003)
0.490–0.520	0.656	0.016	0.016	(= 0.010	±0.012	±0.004)
0.520–0.550	0.484	0.014	0.010	(= 0.008	±0.006	±0.001)
0.550–0.600	0.3479	0.0089	0.0099	(= 0.0053	±0.0084	±0.0008)
0.600–0.650	0.2157	0.0070	0.0060	(= 0.0041	±0.0041	±0.0017)
0.650–0.700	0.1321	0.0055	0.0050	(= 0.0032	±0.0037	±0.0008)
0.700–0.750	0.0900	0.0045	0.0060	(= 0.0028	±0.0053	±0.0006)
0.750–0.800	0.0387	0.0028	0.0034	(= 0.0015	±0.0030	±0.0003)
0.800–0.900	0.0173	0.0013	0.0013	(= 0.0007	±0.0010	±0.0001)
0.900–1.000	0.00324	0.00041	0.00049	(= 0.00022	±0.00042	±0.00009)

Table C.3: c-enriched inclusive cross section for charged particles measured at $\sqrt{s} = 91.2\,\text{GeV}$. The flavour composition is of 38.2%, 35.1% and 26.7% of *uds*-,*c*-,and *b*-quarks, respectively, as specified in table ??. The erorrs listed are the statistical and the systematic bin-to-bin errors. The three sources of systematic uncertainties are specified. A normalization error of 1% has to be added in quadrature everywhere as specified in section 5.4.1.

Interval	$\frac{1}{N_{ev}}\frac{dn^{tr}}{dx}$	E_{stat}	$E_{binsys}(= E_{MCstat} \pm E_{cut} \pm E_{MCmodel})$
0.008–0.012	541.3	1.0	9.2 (= 0.6 ±9.0 ±1.6)
0.012–0.020	435.8	0.6	1.1 (= 0.4 ±0.5 ±0.9)
0.020–0.030	312.3	0.5	1.3 (= 0.3 ±0.8 ±0.9)
0.030–0.040	219.34	0.41	0.54 (= 0.24 ±0.41 ±0.25)
0.040–0.050	161.88	0.35	0.49 (= 0.20 ±0.40 ±0.19)
0.050–0.060	125.34	0.31	0.27 (= 0.18 ±0.12 ±0.15)
0.060–0.070	100.30	0.27	0.34 (= 0.16 ±0.28 ±0.12)
0.070–0.080	80.37	0.24	0.25 (= 0.14 ±0.15 ±0.15)
0.080–0.090	66.11	0.22	0.24 (= 0.13 ±0.08 ±0.19)
0.090–0.100	56.16	0.20	0.15 (= 0.12 ±0.09 ±0.02)
0.100–0.120	42.49	0.12	0.16 (= 0.07 ±0.13 ±0.06)
0.120–0.140	30.29	0.11	0.09 (= 0.06 ±0.05 ±0.04)
0.140–0.160	22.22	0.09	0.11 (= 0.05 ±0.08 ±0.06)
0.160–0.180	16.341	0.078	0.077 (= 0.046 ±0.059 ±0.021)
0.180–0.200	12.173	0.067	0.046 (= 0.039 ±0.017 ±0.016)
0.200–0.225	9.166	0.052	0.037 (= 0.031 ±0.019 ±0.009)
0.225–0.250	6.489	0.044	0.036 (= 0.025 ±0.020 ±0.016)
0.250–0.275	4.869	0.038	0.025 (= 0.022 ±0.010 ±0.007)
0.275–0.300	3.589	0.033	0.034 (= 0.019 ±0.018 ±0.022)
0.300–0.325	2.689	0.028	0.024 (= 0.016 ±0.005 ±0.017)
0.325–0.350	2.069	0.025	0.015 (= 0.014 ±0.005 ±0.002)
0.350–0.375	1.589	0.022	0.021 (= 0.013 ±0.011 ±0.012)
0.375–0.400	1.201	0.019	0.012 (= 0.011 ±0.003 ±0.003)
0.400–0.430	0.880	0.015	0.010 (= 0.008 ±0.004 ±0.003)
0.430–0.460	0.705	0.013	0.013 (= 0.008 ±0.006 ±0.009)
0.460–0.490	0.487	0.011	0.007 (= 0.006 ±0.002 ±0.002)
0.490–0.520	0.3566	0.0093	0.0068 (= 0.0055 ±0.0011 ±0.0038)
0.520–0.550	0.2548	0.0079	0.0081 (= 0.0046 ±0.0067 ±0.0004)
0.550–0.600	0.1543	0.0047	0.0047 (= 0.0027 ±0.0037 ±0.0010)
0.600–0.650	0.0768	0.0033	0.0022 (= 0.0019 ±0.0012 ±0.0003)
0.650–0.700	0.0348	0.0021	0.0022 (= 0.0012 ±0.0018 ±0.0004)
0.700–0.750	0.0164	0.0014	0.0017 (= 0.0008 ±0.0015 ±0.0001)
0.750–0.800	0.0074	0.0009	0.0013 (= 0.0005 ±0.0012 ±0.0002)
0.800–0.900	0.00160	0.00025	0.00023 (= 0.00013±0.00018±0.00006)
0.900–1.000	0.00030	0.00005	0.00009 (= 0.00004±0.00006±0.00003)

Table C.4: b-enriched inclusive cross section for charged particles measured at $\sqrt{s} = 91.2\,\text{GeV}$. The flavour composition is of 2.2%, 7.3% and 90.5% of *uds*-,*c*-,and *b*-quarks, respectively, as specified in table ??. The erorrs listed are the statistical and the systematic bin-to-bin errors. The three sources of systematic uncertainties are specified. A normalization error of 1 % has to be added in quadrature everywhere as specified in section 5.4.1.

Interval	$\frac{1}{N_{ev}}\frac{dn^{tr}}{dx}$	Error
0.00–0.05	61.19	2.24
0.05–0.10	40.22	1.46
0.10–0.15	16.87	0.91
0.15–0.25	6.87	0.47
0.25–0.35	1.86	0.19
0.35–0.55	0.40	0.09
0.55–0.80	0.04	0.03

Table C.5: Gluon scaled energy distribution measured in three jet symetric events.

Interval	$\frac{1}{N_{ev}}\frac{dn^{tr}}{dx}$	E_{stat}	$E_{binsys}(= E_{MCstat} \pm E_{cut} \pm E_{MCmodel})$
0.008–0.012	377.2	0.9	52.5 (= 0.5 ±34.0 ±40.0)
0.012–0.020	314.9	0.5	4.4 (= 0.3 ± 4.4 ± 0.7)
0.020–0.030	234.5	0.4	2.4 (= 0.2 ± 2.0 ± 1.2)
0.030–0.040	170.1	0.3	1.0 (= 0.2 ± 0.8 ± 0.6)
0.040–0.050	127.50	0.25	0.72 (= 0.15 ± 0.57 ± 0.42)
0.050–0.060	99.29	0.22	0.61 (= 0.13 ± 0.44 ± 0.40)
0.060–0.070	79.61	0.19	0.60 (= 0.12 ± 0.43 ± 0.40)
0.070–0.080	65.23	0.17	0.42 (= 0.11 ± 0.34 ± 0.22)
0.080–0.090	54.09	0.16	0.39 (= 0.10 ± 0.17 ± 0.33)
0.090–0.100	45.72	0.14	0.23 (= 0.09 ± 0.16 ± 0.15)
0.100–0.120	36.22	0.09	0.16 (= 0.06 ± 0.11 ± 0.09)
0.120–0.140	27.12	0.08	0.31 (= 0.05 ± 0.29 ± 0.09)
0.140–0.160	20.77	0.07	0.11 (= 0.04 ± 0.09 ± 0.02)
0.160–0.180	16.32	0.06	0.16 (= 0.04 ± 0.15 ± 0.04)
0.180–0.225	11.323	0.033	0.089 (= 0.021 ± 0.083 ± 0.026)
0.225–0.275	6.943	0.025	0.055 (= 0.015 ± 0.052 ± 0.011)
0.275–0.325	4.319	0.020	0.027 (= 0.012 ± 0.023 ± 0.008)
0.325–0.400	2.474	0.012	0.018 (= 0.008 ± 0.015 ± 0.008)
0.400–0.600	0.8439	0.0043	0.0068 (= 0.0027± 0.0056± 0.0027)
0.600–0.800	0.1516	0.0018	0.0067 (= 0.0011± 0.0065± 0.0014)
0.800–1.000	0.0175	0.0005	0.0057 (= 0.0003± 0.0057± 0.0004)

Table C.6: Transverse inclusive cross section for charged particles measured at $\sqrt{s} = 91.2\,\mathrm{GeV}$. The erorrs listed are the statistical and the systematic bin-to-bin errors. The three sources of systematic uncertainties are specified. A normalization error of 1% has to be added in quadrature everywhere as specified in section 5.4.1.

Interval	$\frac{1}{N_{ev}}\frac{dn^{tr}}{dx}$	E_{stat}	$E_{binsys}(= E_{MCstat} \pm E_{cut} \pm E_{MCmodel})$
0.008–0.012	123.6	0.5	16.4 (= 0.4 ±3.1 ±16.1)
0.012–0.020	77.2	0.3	2.8 (= 0.2 ±2.3 ± 1.5)
0.020–0.030	40.6	0.2	1.6 (= 0.1 ±0.8 ± 1.4)
0.030–0.040	21.1	0.1	1.2 (= 0.1 ±0.2 ± 1.1)
0.040–0.050	12.42	0.12	0.81 (= 0.09 ±0.13 ± 0.80)
0.050–0.060	8.00	0.10	0.58 (= 0.08 ±0.15 ± 0.55)
0.060–0.070	5.40	0.09	0.40 (= 0.07 ±0.08 ± 0.39)
0.070–0.080	3.81	0.08	0.31 (= 0.06 ±0.09 ± 0.29)
0.080–0.090	2.74	0.07	0.20 (= 0.05 ±0.05 ± 0.18)
0.090–0.100	2.14	0.06	0.20 (= 0.05 ±0.08 ± 0.17)
0.100–0.120	1.43	0.04	0.14 (= 0.03 ±0.04 ± 0.13)
0.120–0.140	0.90	0.03	0.10 (= 0.03 ±0.07 ± 0.07)
0.140–0.160	0.569	0.026	0.060 (= 0.022 ±0.021 ± 0.052)
0.160–0.180	0.393	0.024	0.082 (= 0.018 ±0.068 ± 0.042)
0.180–0.225	0.259	0.012	0.037 (= 0.012 ±0.025 ± 0.025)
0.225–0.275	0.115	0.008	0.024 (= 0.008 ±0.015 ± 0.018)
0.275–0.325	0.055	0.005	0.013 (= 0.005 ±0.011 ± 0.006)
0.325–0.400	0.0293	0.0032	0.0076 (= 0.0034 ±0.0058 ± 0.0040)
0.400–0.600	0.0067	0.0007	0.0023 (= 0.0012 ±0.0008 ± 0.0017)
0.600–0.800	0.00041	0.00020	0.00094 (= 0.00033±0.00087± 0.00008)
0.800–1.000	−0.0002	0.0002	0.0011 (= 0.0002 ±0.0011 ± 0.00003)

Table C.7: Longitudinal inclusive cross section for charged particles measured at $\sqrt{s} = 91.2$ GeV. The erorrs listed are the statistical and the systematic bin-to-bin errors. The three sources of systematic uncertainties are specified. A normalization error of 1% has to be added in quadrature everywhere as specified in section 5.4.1.

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