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METHODOLOGY FOR OPTIMAL DESIGN OF EFFICIENT AIR TRANSPORT NETWORKS IN A COMPETITIVE ENVIRONMENT

PhD Thesis

PhD Candidate:
César Trapote-Barreira

Director:
Dr. Francesc Robusté

Civil Engineering PhD Program
Civil Engineering School of Barcelona
Technical University of Catalonia - BarcelonaTech

Barcelona, November 2015

Abstract

Air transport network design is a key factor of efficiency in a very competitive industry. Even though airline profitability is paramount, the system approach considers the passenger experience and the social improvements in order to achieve a global optimum.

Airline network design principles are derived from an analytical approach: the thesis shows how a few variables and their interactions are able to explain the key parameters in network design at the strategic level. *Point to point* services show supremacy in simple networks and for high and balanced demand, *stopover* configuration is adequate for long range routes with low demand, and *hub and spoke* operations outperform the others for large networks, when demand is low or frequencies are high, but it is more sensitive to delay propagation.

The analytical approach allows forecasting the performance of new airlines entering the sector. For the model predictions to be accurate enough, fixed costs are introduced in aircraft ownership and labor costs (as opposed to variable ones, as they are considered in the industry), since they have an important impact on the profit and loss account.

Once the design guidelines have been derived, the thesis formulates a more realistic airline network planning model based on linear mathematical programming, which is solved with a combination of the Complete Enumeration Algorithm and the Exhaustive Search Algorithm (both algorithms provide the exact solution or global optimum for any problem statement). The model includes fleet assignment, aircraft routing and crew scheduling. While the exact techniques are appropriate for small airlines, they are outperformed by a Tabu Search Algorithm for larger (realistic) problems.

Air transportation growth and airport congestion (brought about by *hub and spoke* operations) may affect delays in a kind of snowball or bullwhip effect; however the analysis of the airline network complexity makes it possible to increase the resilience of the operations. Better airline network design, planning and efficient algorithms are key assets to provide reliability for airlines and to reduce the need for extra resources allocated in time buffers (flight schedules) and/or in extra aircraft on the ground on “reserve” to recover flight plans. While padding improves passenger quality perception by increasing the airline costs, an active control of flight schedule may achieve the same good quality perception with smaller costs.

Airlines’ competitive environment is analyzed with game theory: a Stakelberg model for two competing airlines shows that a war on frequencies or fares damages both airlines. A Cournot model proposes a navigation fee and congestion charge according to correct utilization of capacity. A round-the-world stopover strategy between main hubs (as it already exists in maritime transportation) is proposed as future research.

Dr. Francesc Robusté

Key words: *Air transport network design, airline network planning, airline competition, airline operations research, fleet assignment, aircraft routing, crew scheduling, Tabu search, game theory, Cournot model, network complexity, reliability, resilience, cost optimization.*

Resumen

El diseño de redes de transporte aéreo es un factor clave de eficiencia en una industria altamente competitiva. A pesar de que la rentabilidad es de suma importancia, el sistema considera la experiencia del usuario y los beneficios sociales para obtener un óptimo global.

Un enfoque analítico permite derivar principios de diseño de redes de transporte aéreo: la tesis muestra cómo unas pocas variables y sus interacciones explican los factores clave del diseño de la red a nivel estratégico. Los servicios punto-a-punto muestran supremacía en las redes simples y para demandas altas y compensadas, una configuración con escalas es adecuada para rutas lejanas con poca demanda, y las operaciones *hub and spoke* mejoran las dos estrategias anteriores para redes grandes, cuando la demanda es baja o cuando las frecuencias son altas, pero son más sensibles a la propagación de demoras.

El enfoque analítico permite prever el comportamiento de nuevas compañías aéreas entrando en el sector. El modelo incluye costes fijos de propiedad del avión y laborales (al contrario de las hipótesis habituales de la industria, que trabaja con costes variables), puesto que tienen un impacto importante en la cuenta de resultados.

Una vez se han derivado guías de diseño, la tesis formula un planteamiento más realista del diseño de redes de transporte aéreo basado en programación matemática lineal, que se resuelve con una combinación del Algoritmo de Enumeración Completa y el Algoritmo de Búsqueda Exhaustiva (ambos proporcionan la solución exacta o un óptimo global para cualquier planteamiento del problema). El modelo incluye asignación de flotas, rutas de aeronaves y programación de tripulaciones. Mientras que los algoritmos exactos son apropiados para aerolíneas pequeñas, los problemas más grandes necesitan Búsqueda Tabú.

El crecimiento del transporte aéreo y la congestión en los aeropuertos (a veces propiciada por las operaciones *hub&spoke)* pueden afectar las demoras con un efecto de bola de nieve o látigo; sin embargo el análisis de la complejidad de la red aérea puede incrementar la resiliencia de las operaciones. Un buen diseño de la red aérea, una buena planificación y unos algoritmos eficientes, son aspectos clave para proporcionar fiabilidad a las aerolíneas y así reducir los recursos inactivos asociados a “colchones de tiempo” (en los horarios de los vuelos) y/o en aviones “de reserva” en la plataforma para recuperar planes de vuelo. El “acolchado” mejora la percepción de la calidad por parte del pasajero, pero con un control activo del horario de vuelos puede conseguirse la misma percepción con costes menores.

El entorno competitivo de las aerolíneas se analiza con teoría de juegos: un modelo de Stakelberg para dos aerolíneas competidoras muestra que una guerra de frecuencias o tarifas es perjudicial para ambas aerolíneas. Un modelo Cournot propone una tasa de navegación y de congestión según la correcta utilización de la capacidad. Vuelos (de la misma alianza) a lo largo del mundo con escalas en los *hubs* principales se proponen como investigación futura.

Dr. Francesc Robusté

Palabras clave: *diseño de redes de transporte aéreo, planificación de red en líneas aéreas, competencia entre aerolíneas, investigación operativa, asignación de flotas, programación de tripulaciones, búsqueda Tabú, teoría de juegos, modelo de Cournot, complejidad de la red, fiabilidad, resiliencia, optimización de costes.*

Acknowledgements

I would like to thank my dissertation adviser, Francesc Robuste, for his continuous guidance in the field transport engineering research. He has also shown to me the convenience of using analytical methods to understand transport, business and system dynamics.

Furthermore, I am greatly indebted to my colleagues during my stay at UPC. They shared their knowledge and discussed with me the concepts and techniques of this research. This research was financially supported by Airnet Project, which we proposed some years ago to Ministerio de Ciencia e Innovación when I carried out an entrepreneurial process to build the air transport division at Cenit. Also, I thank Aeroports de Catalunya for supporting the project with their interest. The final weeks of document writing and dissemination benefited from support from Abertis Chair at BarcelonaTech.

In addition, I would like to thank to all colleagues, scientists and professionals that I have found in this trip because our discussions have increased my world.

Finally, I give this dissertation as a dedication to my sister, Ana, and my parents, Dora and Floren. Without their support and love, I would not have achieved this goal. You give sense to my life.

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1 Introduction and objectives

Air transportation growth rate have been 6% in the recent years (ICAO, 2015), even during economic recession, and main airports duplicate traffic every fifteen years approximately. Congestion, delays and fares are three key concepts that passengers, airlines and airports pay attention to. The introduction of low cost carriers (LCC) in the markets promotes the end of an era as it was established with Liberalization Act of 1978.

In this era, full cost carriers (FCC) or “legacy carriers” have dominated the network design problem and they have focused on pricing models to capture value from customers, working with hub and spoke networks to cut costs and alliances to increase their networks. Today, the new era is characterized by new organization cultures with different cost structures that LCC improves. The fact is that cost available seat kilometre (CASK) for this airlines is substantially smaller than it is for legacy carriers. But, there is a reason why LCC cannot run long haul business model at the moment: low CASK is possible when some factors confluent in the network design (high level of resources utilization, predictability, commercialization channels, etc.), but airlines operating long haul require a different utilization of resources and other commercialization strategies. There is a set of key indicators related to the network and the strategy that explain performance well.

The aim of this thesis is to contribute to network design of air transport taking into consideration the efficiency and the competition. The work is going to be developed from three areas of science: logistics, game theory and complexity. A plus of value is obtained when the interactions between these approaches are understood.

Airlines are managed paying attention to bottom line of profit and loss account. This is a survival exercise and it is possible to check how this industry is far from an attractive business with high returns of capital investment but companies move large revenues. So, improving financial statements requires controlling costs and maximizing revenues.

The main objectives of this thesis are to contribute to network analysis with analytical approach to strategyc decision making and planning, to develop an algorithm for airline network planning, which will be able to solve large problems with less computational effort, to develop indices for the analyisis of complexity in airlines and provide active control for delays and, finally, to analyse the impact of competitivity in network and routes performance.

This thesis is organized as follows. Chapter 2 describes airline network design, integrating routing and resources assignment, where the goal is to minimize the total cost. A methodology is outlined based on an analytical model to understand the effect of key network parameters when a set of OD pairs could be served with point-to-point, hub-and-spoke or stopover operation. This approach lets build models with less parameters, which is very useful for strategic analysis.

Chapter 3 focus on Operation Research models for large problems with realistic flight scheduling. Particularly, a Tabu Search Algorithm (TSA) is implemented to solve sophisticated scenarios and improve the limits of the analytical approach (i.e. heterogeneous flight lengths and realistic schedules).

The complexity of the airline network has a high impact on operation and cost structure. Furthermore, airline network operation could be understood like a problem in the field of dynamic complex systems. Especially, reliability has direct impact on airlines and passengers: costs reliability could be improved through implementing simple network configurations to avoid reactive propagation of delays, scheduling flights with buffers of time or parking extra aircrafts on bases for recovering disruptions. The reliable network design principles are discussed in Chapter 4. An application for a real airline is going to be developed to demonstrate the utility of this approach.

Reducing costs in airlines is a very challenging task, however this is not a guarantee of success. Selecting right markets is a critical factor of success and it is due to necessity of mobilizing enough demand to achieve good load factors. In a very competitive environment and consolidated industry, there are not a lot of markets or segments for discovering. Sometimes low cost suppliers mobilize demands that legacy carriers cannot serve, but sometimes equilibrium of frequencies and prices determines the market share for competing airlines. Chapter 5 pays attention to this problem, applying game theory to induce principles of network design.

In addition, Chapter 5 deals with theory of commons in air transport industry too. In practice, airlines compete with frequencies in some markets. Especially, if there are some operators at the same airport competing for demand, then the game usually is carried out in terms of frequency (prices could be similar and/or margins could be small). Supplying high frequencies has advantages for passengers but if that is not accompanied by good load factors, then the system is not taking advantages of airports capacity and delays appear. This situation is not efficient. This chapter analyses the previous contributions and proposes a Cournot model to design a mechanism to improve the efficiency.

Chapters 2 to 5 are self-contained even if implies the repetition of some information. This is due to the fact that each of these chapters are conceived as different scientific papers (some of them were submitted at the moment).

Finally, Chapter 6 closes this thesis with some conclusions. Furthermore, possible future lines of research are summarized.

2 Airline network design principles

2.1 Introduction

This chapter analyses principles of airline network design based on key parameters and their interactions. For this purpose the work is structured in a concise review of previous works, a definition of specific objectives and development of analytical model to understand the key aspects of airline network design. Some experiments are carried out to test the model and to extract the main conclusions.

2.1.1 State of the art

Two kinds of airline network are studied by Vany and Garges (1972): point-to-point (PP) and hub-and-spoke (HS). They study the interaction between network configuration and fleet assignment for understanding operating cost. First, they find that HS networks supply higher frequencies that compensate higher travel time values. Secondly, this structure builds a feeder network that improves load factor in wide body aircrafts and achieves efficient assignments. Current practices constates that LCC designs radial networks (whithout connections) because that has some advantages in terms of coordination, less delay propagation, etc.

Later, Gordon (1975) carries out a mathematical and empirical exploration about interaction between scale economies and network structure, studying air transport and other modes. His studies conclude that fully connected transportation networks are rare because of the existence of scale economies. The greater the scale economies, the less connected the network shape and the more concentrated the traffic pattern also, congestion at nodes should result in a more connected network. In addition, he demonstrated that if network configuration and cost function are given, then these are an output of supply-demand or cost-service equilibrium. Hansen (1990) also studies this dynamic equilibrium and he modelizes it applying game theory.

Gordon and De Neufville (1973) propose a model for designing air transport network. One of the main conclusions was HS network let operators minimize costs and improve reliability. However, PP network supplies a higher quality service from the passenger's point of view. Then, Ghodrial (1983) developed an equilibrium model considering competition between airlines and customer's preferences about routing strategies: he found that airlines could take advantage of operating HS despite the fact that they have to pay externalities originated by congestion. Kanafani and Ghodrial (1984) demonstrated that hubbing is inelastic front airport fees and airports could find some potential benefits too. Also, Kanafani and Hansen (1985) researched the great effects of hubbing in airlines productivity. Philips (1986) shows statistical information related to airline's operating strategies that strengthen their hubs. Furthermore, he found that if an airline dominates a hub, then it is in better position to defence their market against competitors. O'Kelly (1986) determined optimal location for one or two hubs minimizing total distance (weighted by flow) and he indicated the interest of studying accurately transition networks.

Jeng (1987) disserts about HS and PP networks, applying analytical models and continuus aproximations that provides a powerful tool to explain equilibriums and causal relations. This line of research was initiated by Daganzo and Newell (1986) for the analysis of logistic distribution in several scenarios (HS, PP, peddling). The advantage is that it requires less parameters in the model and it allows researchers to develop a conceptual model for better capturing interactions. In particular, Jeng considers a configuration around a circle or a small network of different circle configurations, which allows him to estimate an average length for different flights and simplify computation. Later, Lederer (1997) introduced similar model for other configurations and he included a simplified routes with stopovers.

Swan and Adler (2006) disaggregate aircraft operating costs into various cost categories and provide background for an engineering approach used to compute a generalized aircraft trip cost function that varies with seat capacity and distance. Usually, cost per available seat kilometer indicator (CASK) is deducible from airline's financial reports, but it is associated to average flight length and average fleet size. This is typical in some studies and it is possible to find the basis of that in studies about estimations of flight direct costs (Bailey, 1985; Belobaba, 2009). The authors consider that 50% of total trip cost is related to aircraft, 30% to staff and 20% to distribution.

Finally, it is interesting to show a table of comparative values of a profit and loss account (PLA) for different kind of airlines. Some of most relevant key performance indicators (KPI) for airline's industry are estimated directly from this rows and that fact has two main concepts: first, aggregated values derived from PLA do not consider particularities of operating strategies and, for this reason, more than one KPI are needed to induce conclusions; secondly, network is strategic and there is a process to design it that influences the values of KPIs. There is not a definitive model for airlines network or operating strategy.

Table 2.1. Financial statements for main airlines in Europe.
Source: CAPA, 2012.

	LCC			FCC		
EUR million	**2011**	**2012**	**% change**	**2011**	**2012**	**% change**
Revenue	10,723	12,346	15.1%	89,805	96,205	7.1%
Operating profit	1,047	1,174	12.2%	956	488	-48.9%
Operating margin %	9.8	9.5	-0.3	1.1	0.5	-0.6
Fuel cost	3,299	4,130	25.2%	22,145	26,130	18.0%
Fuel as % of revenues	30.8	33.4	2.7	24.7	27.2	2.5
Ex fuel cost	6,377	7,042	10.4%	63,220	65,874	4.2%
Total costs	9,676	11,172	15.5%	88,850	95,717	7.7%
Net profit	823	1,015	23.3%	-579	-946	63.3%
ASK bn	221	237	7.0%	1,014	1,037	2.3%
RPK bn	182	198	8.3%	796	829	4.1%
Pax m	158	171	8.0%	363	375	3.3%
Load Factor %	82.5	83.5	0.9	78.5	79.9	1.4
Average sector km	1,155	1,158	0.3%	2,191	2,209	0.8%
RASK EUR cent	4.85	5.22	7.6%	8.86	9.28	4.7%
CASK EUR cent	4.38	4.72	7.9%	8.76	9.23	5.3%
CASK ex fuel EUR cent	4.01	4.00	-0.4%	6.55	6.65	1.6%
Fuel CASK EUR cent	2.08	2.34	12.9%	2.18	2.52	15.3%
	EasyJet, Norwegian, Ryanair, Vueling Airlines			Aer Lingus, AF-KLM, Airberlin, Alitalia, Finnair, IAG, Lufthansa Group, SAS, Turkish Airlines		

2.1.2 Objective

The main objective is to understand pros and cons of different network configurations. For this purpose, different goals are proposed.

The specific objectives of this chapter are the following:

1. To understand the relation between cost function and network performance.
2. To understand the effect of key network parameters when a set of OD pairs could be served with PP, HS or stopover (SO) operation.
3. To develop an analyitical model to analyse the relation between the main parameters.

2.2 Problem statement

Network configuration is a strategic concept for many carriers, especially for airlines. Operational plans and financial previsions come after knowing routing structure. And this network configuration has a strong impact on passenger travel experience because at the same level of resources, travel time will be different.

There are three basic configurations of airline networks (see figure 2.1). First, point-to-point (PP) network that is the first model airlines operate since the beginnings of commercial aviation. Nowadays, legacy carreirs, charters or low cost airlines operate this kind of network totally or partially. The main advantage is simplicity because operators can avoid problems of coordination. However, it is necessary a minimum load factor to achieve the breakeven point.

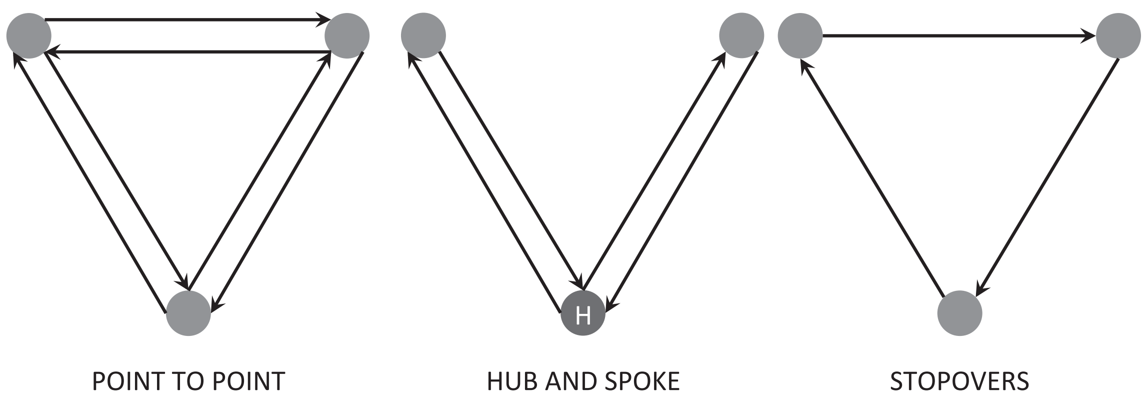


Figure 2.1. Idealized configurations of airline network.

Secondly, hub-and-spoke (HS) is the configuration that airlines implemented after the liberalization of the industry and the main advantage is that operators achieve scale economies through consolidation of flows at hub, achieving better load factors and ratio between fixed cost and seats. An argument against this is that passengers have to travel longer distances (although they may take advantage of higher frequencies).

Thirdly, stopover (SO) network is similar to peddling in logistics or bus operation in a city. In airline industry is not usual because the main disadvantage is that the cost of one stop is very high (airport fees, turnaround time, handling cost, fuel in climbing phase, etc.). However, if demand is low, then this system (with just one stopover near the origin or near the destination) let operators improve better load factors.

Finally, airlines can operate pure strategies (LCC usually operate PP) or more than one (FCC usually supply flights with and without connection). In practice, SO network is unfrequent but some airlines operate it in long haul routes where some technical stop is mandatory.

2.2.1 Basic assumptions

The analytical model is developed with a set of basic assumptions. These permit to simplify some characteristics of airline operation (to be developed in chapter 3 with operation research methods) and it is absolutely necessary to understand some basic causal dynamics.

Given a set of n nodes (airports), there is demand (d_{ij}) between each pair and every node can be served by only one configuration at the same time: PP, HS or SO.

There is only one hub and process of passenger transfers takes a minimum connection time (MCT) plus extra time for scheduling reasons. Total time on ground is bigger than turnaround time (TAT) that is conventional for all configurations. Hubs work with time windows (bank) where planes arrive and depart coordinately. The following figure shows an idealized operation and allows estimating average time for this operation. In the analytical model a parameter time of connection in a hub (t_H) is added to total travel time for HS configuration, which is very simple but improves the clarity of the model. Also, time of flight (t_F) and turnaround time (t_G) at the airport are going to be considered one parameter (t_L, time of leg, with t_L=t_F + t_G) with the same spirit of simplification. Furthermore, airport has enough capacity to not consider restrictions.

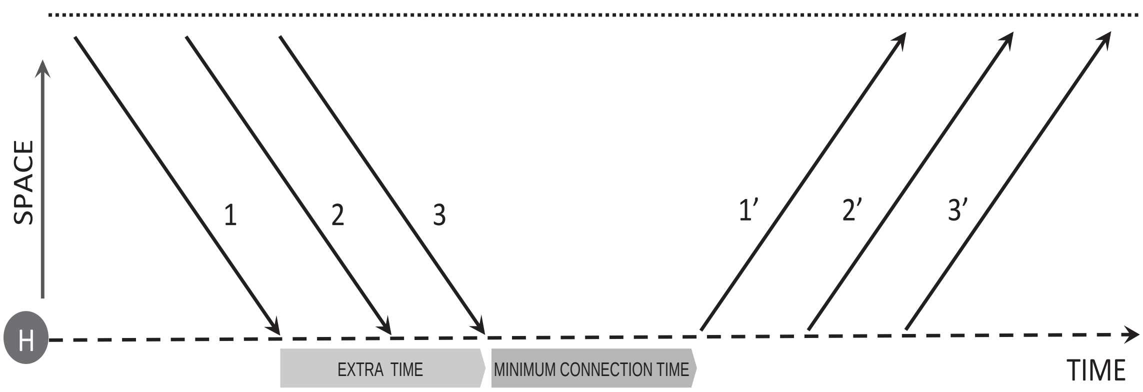


Figure 2.2. Connection window for hub-and-spoke configuration.

Demand is ineslastic with respect to time and cost; then, the average schedule delay (t_W) can be calculated as half of average flight headway (h). In future chapters conditions of competition will be modelled and then demand will be a parameter for equilibrium. At this stage, the model does not achieve this level of complexity.

Stage length between each pair of airports can be different (l_{ij}). For first scenarios this is considered in average terms for all pairs (l), which is impossible for more than three airports, however is very clear to understand some operating aspects that usually are misunderstood. For scenarios with high level of complexity these distances could be different: the performance can be summarized by the average and the coefficient of variation. In this scenario, the period of calculation is a key parameter that allows considering fixed costs more accurately.

2.2.2 Objective function

Measuring the network performance with only one indicator is quite difficult, but the exercise of simplification is addressed to this point. This optimization problem can be reduced to a Lagrangian using Lagrange multipliers. Then, total cost (C) is the design parameter and the objective function (Eq. 2.1). Usually previus works have been using this concept for optimization problems.

Total cost (C) equals to total operator cost (C_O) plus total passengers cost (C_P) (Eq. 2.1):

$$C = C_O + C_P \tag{2.1}$$

2.2.3 Flight time

Functional relationship between stage length and flight time follows equation (2.2). Stage length between two airports can be obtained by calculation of Great-Circle Distance (GCD), however this chapter is not focused on real locations and stage length (l) is an input parameter. Time is not vectorial space with distance because extra time (t_0) is required to achieve cruise speed (s) and flight level.

$$t_F = t_0 + \frac{l}{s} \tag{2.2}$$

The difference between two commercial aircrafts can be significant depending on propulsion. That is Airbus 3xx family is quite similar but it is very different of ATR family. Finally, it is not a goal to analyse how aircraft performance impacts on flight time, but it is really interesting because different aircrafts have different speeds and fuel consumptions. Also, there is a trade off between increasing costs and decreasing crew resources with speed.

Figure 2.3 shows time-distance function for aircrafts A330 and A380 and it is obvious that differences are very little (estimated with a flight time calculator). The independent coefficient is related to time that aircraft needs to achieve cruise speed and the dependent coefficient is proportional to the inverse of cruise speed (900km/h for A330, 945km/h for A380). Definitively, for purpose of this analysis the performance of aircraft A330 is considered the reference.

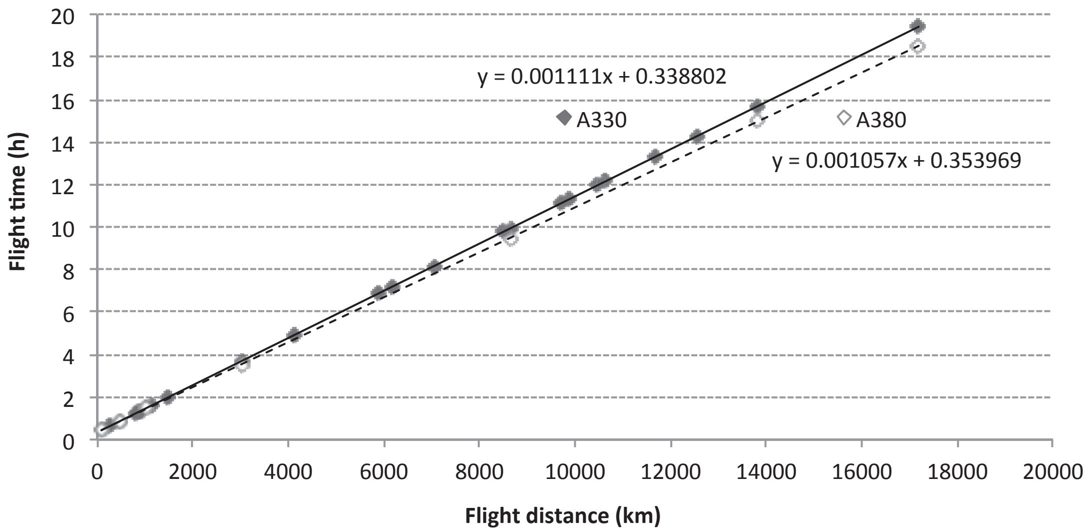


Figure 2.3. Time-distance function for different aircrafts.

2.2.4 Operator cost

Total operating cost per unit of time (tipically, a day) is the major concern for the airlines. Usually, airlines use CASK as KPI to measure the unitary cost and it is defined as the ratio between operating expenses and available seat-kilometre.

Operating expenses include (Belobaba, 2009): aircraft operating costs, aircraft servicing costs, traffic and passenger service costs, commercial expenses (promotion and sales) and other costs (general expenses, etc.). Then, aircraft operating costs and aircraft servicing costs have impact at level of network configuration. Obviously, network configuration is not independent of revenues but if demand is inelastic, then it is possible to simplify and achieve clear results.

Airline operating cost breakdown considers: flight direct operating costs, ground operating costs and system operating costs. Based on DOT Form 41 (USA), the distribution follows the rule 50-30-20%. Network configuration sums up more than 60% of operating costs. From functional perspective, airline's organization has no impact on costs when network is analysed but it has impact for pricing definition and for CASK determination (see table 2.1). Especially, fixed costs have inertia to change and they are a critical factor when competition between airlines is high.

Flight operating costs (FOC) is a KPI for airlines that includes all costs related to aircraft flying operations. It is measured in monetary units per block-hour (block-time is time spent between the arrival time at gate, when handling put blocks on wheels, and departure from gate, when handling put blocks off). Typical breakdown of FOC sum up: crew, fuel, maintenance and ownership. Then, different stage lengths and utilization by different airlines or network configurations result in substantial variations in block-hour costs for same aircraft type. Also, differences in crew (union contracts, seniority, nights out of base…), maintenance and ownership costs raise variations too.

Routes with long stage length use large aircrafts that have high FOC. However utilization (block-hour per day), seats and average stage increase too. At the same time, these routes require more crew pairs per plane (due to turnover), which spend some nights away from their bases, and, consequently, the costs increase too. Finally, if this cost is divided by seats capacity per plane, the resultanting indicator (FOC/seat-hour) decreases with average stage length (Belobaba, 2007). In terms of CASK, the figure (Fig. 2.4) shows some values for different airlines of Europe.

The operator cost is calculated for a period of time. Then, given a set of routes in this period of time and their flight times:

- Fuel cost depends on the number of flights, distance and aircraft capacity.
- Crew cost depends on the number of flights, fleet size, aircraft capacity, distance (time flight), crew workday and scheduling strategy.
- Maintenance cost has a variable part and a fixed part at the same time. Usually, financial auditors tend to assume total maintenance cost as variable or imputable fixed cost with the objective to analyse better profitability of routes.
- Ownership cost is a fixed cost (previous works consider this a variable cost because they do not dissert about utilization factors but this hypothesis is imprecise).
- Air navigation services cost depends on the distance and aircraft capacity.
- Airport services cost depends on the aircraft size (in this section, passengers fees are not considered).
- Air navigation services and airport services are estimated together in this section.

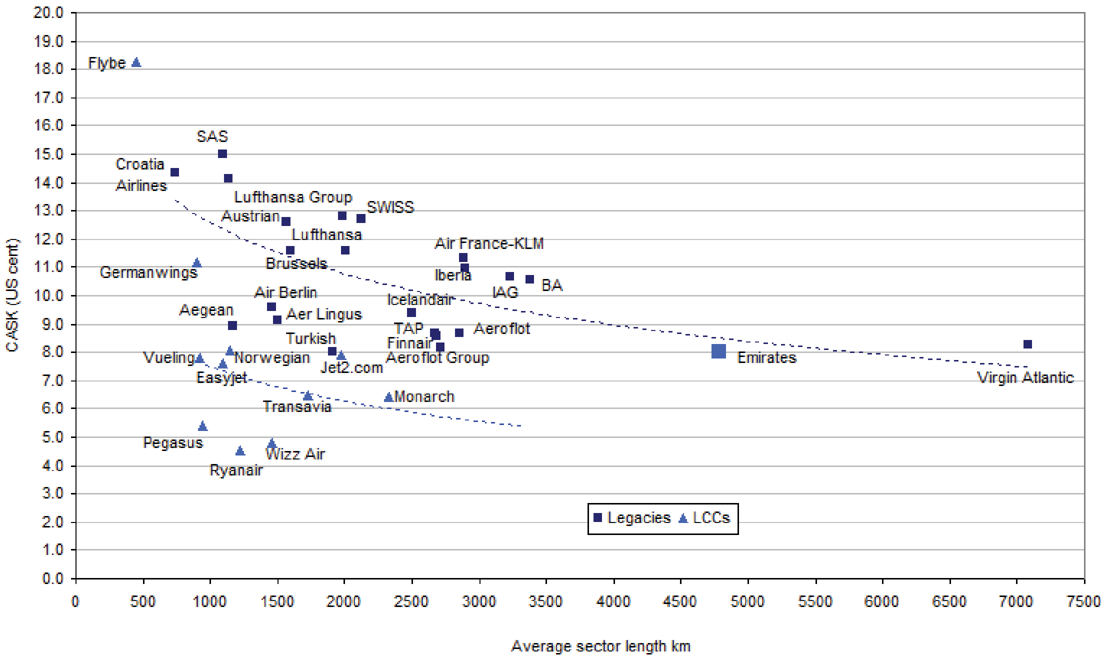


Figure 2.4. CASK versus average sector length for airlines.

Soure: CAPA, 2012.

Total operator cost is calculated with Equation 2.3.

$$C_O = \sum_i (c_{K,i} + c_{M,i} + c_{N,i}) + t_C \left(\sum_j c_{W,j} + \sum_r \xi(\mathrm{r}) c_{E,r}\right) \quad (2.3)$$

Given one airline network (S) with n airports, served by R routes, I flights, J aircrafts and V crew sets. Each route is served by one aircraft. The following variables are identified: i, index of flight (i=1,…,I) that goes from airport x to airport y (x, y = 1,…, n); $c_{K,i}$, fuel cost for flight i; $c_{M,i}$, maintenance cost for flight i; c_N, navigation and airports charges for flight i. This first term only depends on flight performance and number of flights, for this reason they are variable costs. The second term are related to resources that airline has to hold with long contracts (unit of time) and the cost is independent of utilization (kilometres or number of flights). Then, t_C is the cycle time or period of time of analysis (days); j, index of aircraft (j=1,…,J); $c_{W,j}$, ownership cost for aircraft j; $c_{E,r}$, crew cost associated with route r; $\xi(r)$, factor based on route r to consider extra allocation of crew sets for long haul flights.

Each element of the breakdown follows a linear expression that depends on aircraft capacity. Then, fuel cost is defined as $c_K = p_K k(q) t_F = p_K(k_0 + k_1 q) t_F$, where p_K is the price of fuel per kilogram, t_F is the flight time and k(q) is a linear function to consider relationship of consumption (kilograms of fuel) with aircraft size in terms of seat capacity. In further developments the following notation is assumed: $k'(q) = p_K k(q)$. Maintenance cost is defined as $c_M = m(q) t_F = (m_0 + m_1 q) t_F$, where function m(q) is linear with aircraft capacity and expresses the cost of maintenance per hour of flight. Navigation fee is defined as c_N (EUR per flight), which is assumed as a constant. Ownership cost is defined as $c_W = w(q) = (w_0 + w_1 q)$, which is linear with aircraft capacity. Finally, crew cost is defined as $c_E = e(q) = (e_0 + e_1 q)$ and it is linear with aircraft size.

Appendix 1 estimates each concept of aircraft operating cost breakdown (defining variables and units). The estimations are made following previous works (Radnoti, 2002, Belobaba, 2007, IATA, 2009, Cook and Tanner, 2011). Only some considerations about divisibility of some costs are diferent with the aim of working better with utilization factors, according to previous explanations.

Other works have estimated the same cost functions with non-linear models (Wei and Hansen, 2003), which are very interesting. However this approach makes difficult to develop analyitical models and arrive to compact formulation (they use numerical algorithms since the beginning).

2.2.5 Passenger cost

Total passeger cost (Eq. 2.4) is integrated by cost of different components of travel time (t_T): schedule delays (t_W), connection times (t_H) and line-haul times (t_F) for all passengers (D) inside planes and weigthed by value of time (θ), considering all flights inside the period of analysis. Value of time is estimated with values between EUR 47 and EUR 60 per hour[1] (Eurocontrol, 2013) and is assumed to be linear with time. Each phase of travel has different weight from passenger perspective: additional parameters (α, β, γ) are considered to calculate t_F (Eq. 2.5).

$$C_P = \theta D t_T \tag{2.4}$$

$$t_T = \alpha t_S + \beta t_F + \gamma t_H \tag{2.5}$$

This work assumes values for (α, β, γ) according with Jeng (1987). Then $\alpha < \beta < \gamma$ and reasonable values are assumed for these fractions based on this relationship. Conceptually, value of 1 is assumed for γ since this is the highest value of time people can give to transfers. Jeng proposed a value of 2/3 for β and 1/3 for α (based on Hensher, 1977). These assumptions permit to add line-haul time (t_F), schedule delay (t_S, related to the opportunity cost of waiting time) and connection time (t_H, related to the opportunity cost of connecting time at hub).

2.3 Analytical model

This section aims to develop a model based on continuum approximations to analyze airline network structures. This concept was developed to study commuting, congestion and minimum costs pahts problems (Wardrop, 1971). The idealized model approach has been applied to scheduling, location and zoning problems (Newell, 1973; Cooper, 1972; Vaughan, 1984). More recently it has been used to examine many-to-one and many-to-many logistic problems (Estrada, 2007).

In the field of air transport, Jeng (1987) applied the model for network configuration. Focused on PP and HS network, considering homogeneus demand and circle spatial configurations (where the average distance only depends on the radius).

One of the advantages is that these models require less computational effort because complexity of the network is low. So, they are convenient for sensitivity analysis and are very useful in strategic planning, usually leading to qualitive insights.

[1] EUR/USD = 1.12 (average value from March.2015 to November.2015).

OR models let operators consider all details to evaluate accurately the performance of the definitive network and design the implementation. This point is critical for small networks because the variation of one aircraft in size fleet has a strong financial impact. Both approaches are complementary: good strategic layout based on continuous approximations can be easily fine tuned with numerical methods (Robusté et al., 1990)

This section presents some extreme cases with idealized configurations (PP, HS, SO) with the objective to understand their applicability. Also, other experiments are analysed to understand the influence of some parameters.

2.3.1 Relationship between stage length and crew cost

Practitioners know that the operating cost for long haul business model is much more expensive than for short haul business model. It is independent of network configuration, but it depends on the capacity of flights. For this reason, FCCs can survive in the era of low cost without changing drastically their structure organization (at the moment).

This section presents an idealized configuration of a single route with I flights between a set of n airports (figure 2.5). The route starts at airpot 1 and goes to airport 2, 3,...,n. Finally, it ends at airport 1 again and takes t_R. There are I flights connecting all the airports and flight time takes t_F (h) and turnaround time takes t_G (h), both of them sum up t_L (h). It is a simplification and average value for more sophisticated networks. Also, t_C is total cycle time, which is composed by k times t_R (in figure $n = 2$) and $t_\emptyset$ (if in a cycle there is some time without flights), then $t_C = kt_R + t_\emptyset$. Crew assigned to a route can work for a maximum time t'_e and need a minimum time of rest t'_u.

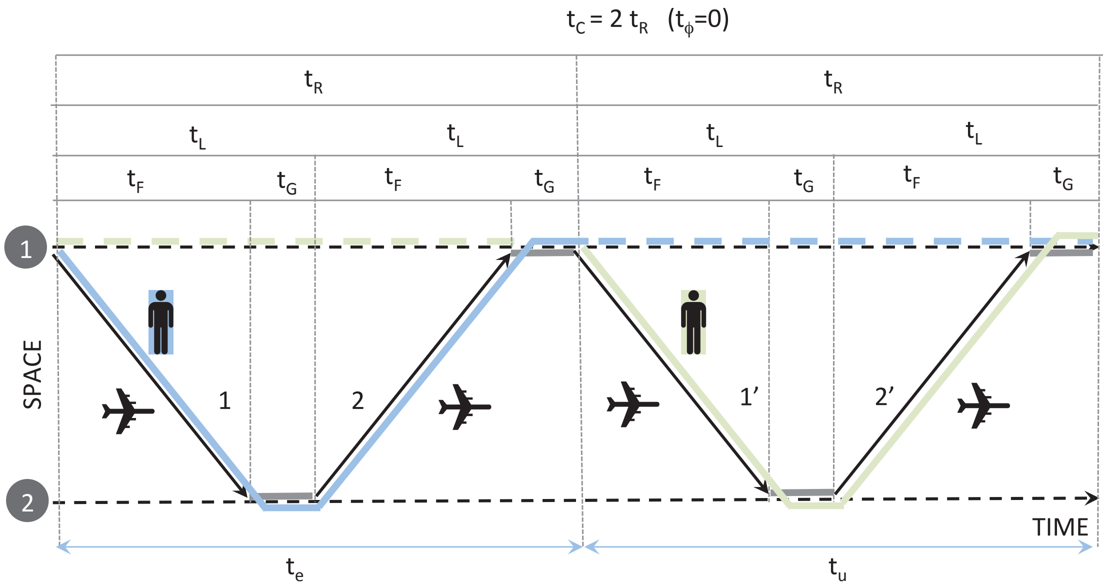


Figure 2.5. Route configuration for crew assignment experiment.

It is necessary to determine if this crew can work without spending any rest out of base (this situation increases operating cost and requires extra crew). Then, the total worktime (t_e) for this route is given by Eq. 2.6, where $[\cdot]^-$ is a function that takes integer part of the argument by deffect.

$$t_e = \frac{t_R}{n}\left[\frac{nt'_e}{t_R}\right]^- \tag{2.6}$$

It is possible to calculate how many bases are needed in this route (Eq. 2.7). Where $Num(\cdot)$ is a function that takes the numerator after reducing the fraction $\frac{t_R}{t_e}$. It is equivalent to find two integer numbers which ratio is equivalent to ratio between t_R and t_e and, then, take the numerator.

$$b = Num\left(\frac{t_R}{t_e}\right) \tag{2.7}$$

Also, $Den(t_R/t_e)$ is a function that takes the denominator and it returns the number of cycles that crews need to come back to their bases.

Finally, total crew sets for this route are estimated by equation 2.8. This simple formulation avoids to use large OR programs if network configuration verifies some conditions.

$$\xi = 1 + b\left[\frac{t'_u}{bt_e}\right]^+ \tag{2.8}$$

Observe, $\xi = 1$ if only one base is necessary and crew can rest at their base (typical plan for low cost airlines or regional airlines). Therefore, crew cost can be estimated directy with formulation of Appendix 1. However, if not, crew cost has to be multiplied by the output to increase the operating cost (see Eq. 2.3). Also, if extra-time $t_\emptyset$ exists at the end of cycle and let crews finish their rest ($t_C = nt_R + t_\phi > t_e + t_u$, where t_u is real time of rest that it can be greater than t'_u because if there is remaining time until next departure, the crew members can spend and extra time in this situation. But, generally, $t_u = t'_u$), for sure the pattern is replicable and only one crew should be necessary (it is one of LCC's principles).

Figure 2.6 presents a numerical experiment to analyse the impact of stage length in allocation of crew. Total flight plus ground time t_L increases from 1 hour to 9 hours. Also, $t'_e + t'_u = 24$ hours, only two scenarios are presented (serie for t'_e=12h and t'_e=24h –with purpose to show that if crew don't rest, then only one crew set is required-). Observe that there is not a gradual transition from one crew to three. Requeriments of crew are a wave function that depends on coordination of flight hours and worktime. In conclusion, airlines in the long haul business run higher operating costs than short haul airlines.

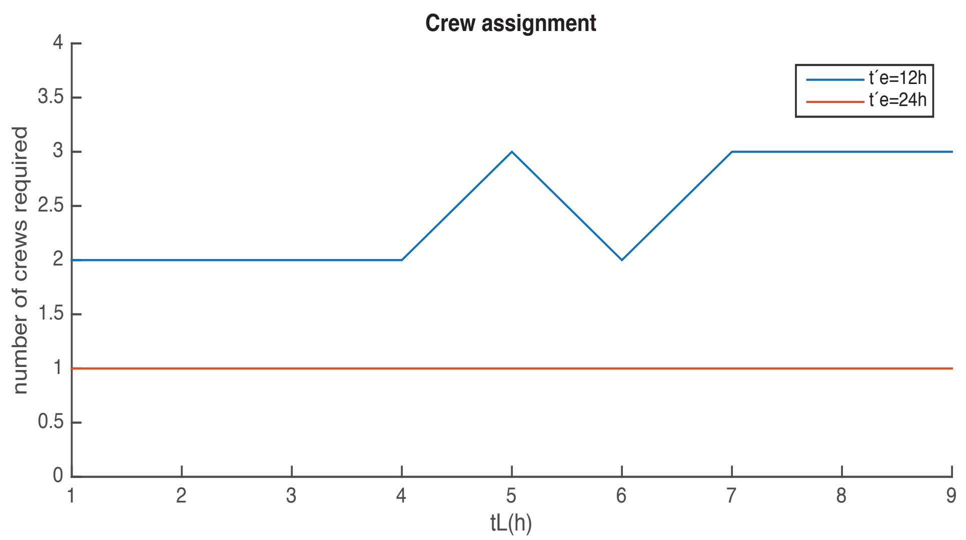


Figure 2.6. Crew assignment with analytical model.

Appendix 1 explains that large aircrafts are often assigned to long haul routes and these require extra crew (Figure A1.6). This assignment is due to rotation factors explained above.

The utilitzation of resources varies with the same parameters (Figure 2.7). Logically, when more resources are needed for the same route, the airline achieves worse utilization factors.

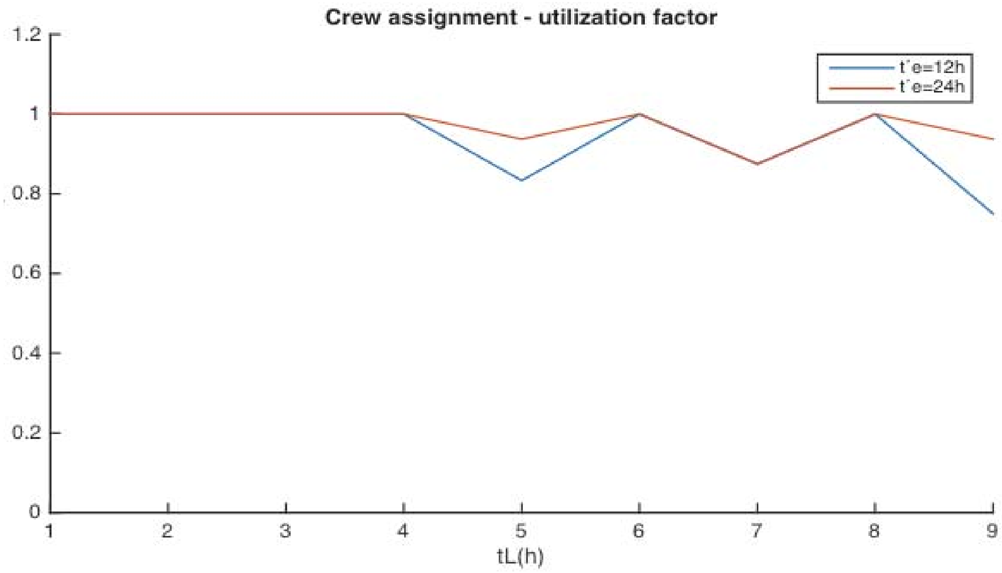


Figure 2.7. Crew utilization factor for the assignment with analytical model.

The same experiment for more than two airports results in an important size of crews (Figure 2.8 with $t_R = 10h$, $t'_e = 16h$, $t'_u = 8h$,). These crews spend some nights away from their bases, which is an extra cost for airlines. This test presents the main reason because LCC operates shorthaul and each aircraft comes back to its base every night. In this way, one aircraft only needs two crews every day (duty time of 8 hours each one).

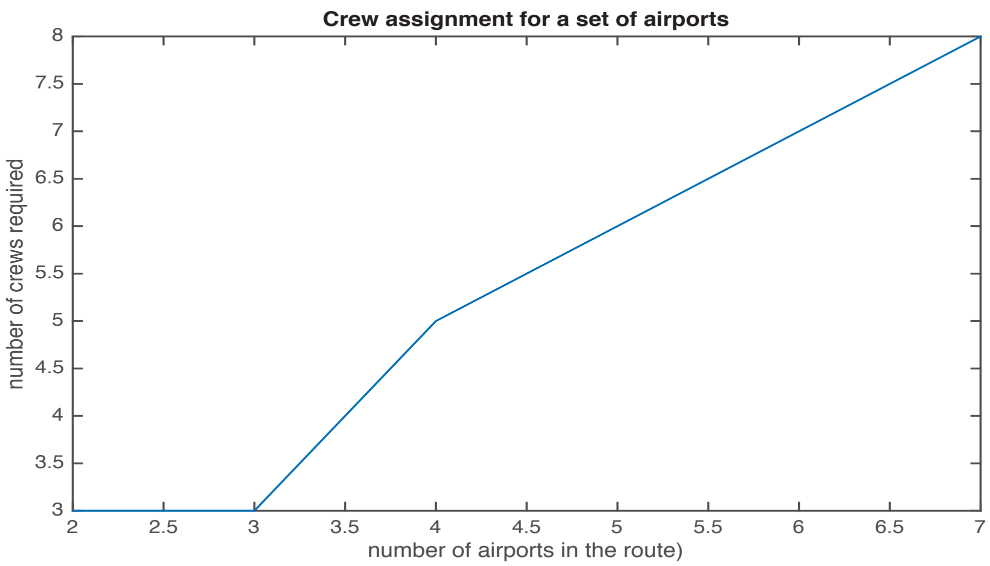


Figure 2.8. Crew requeriments for a range of airports in the same route.

Finally, for real flight schedules with high diversity of stage lengths the assignment is complex and this fact justifies large OR models. The main goal for the company is to reduce the cost of human labour and the better utilitzation of all resources. However, this analytical model is very interesting for strategic evaluations of networks or routes. Especially, evaluation of start-up airlines requires simple models to develop business plans trading-off accuracy and agility.

2.3.2 Point to point network (PP)

The network (S) is operated with PP configuration. A set of n airports are connected with a fleet of J aircrafts and $J\xi$ crews for a period of analysis t_C=1day (simplification). In this period, there is a frequency of f ($f > 0$) expeditions per day and headway (hours) is $h = 24/f$. Total cost can be calculated by the following expression (Eq. 2.9a). All the partial costs for the operator are calculated with the functions indicated in Appendix 1. Furthermore, fleet size is estimated by Equation 2.9b and it takes into consideration two scenarios: first, fleet size is conditionated by time-espace coverage problem; secondly, fleet size is determinated by the relationship between demand, aircraft size and expeditions.

$$C_T = \mathrm{n}(n-1)f\left(t_F\left(p_K k(q) + m(q)\right) + \mathrm{c}_N\right) + \mathrm{J}(\mathrm{w(q)} + \xi\mathrm{e(q)}) + \theta dn(n-1)\left(\frac{12\alpha}{f} + \beta t_F\right) \tag{2.9a}$$

$$\mathrm{J} = \max\left\{\left[n(n-1)\frac{t_L}{2t_e}f\right]^+, \left[n(n-1)\frac{t_L}{t_e}\frac{d}{q}\right]^+\right\} \tag{2.9b}$$

If J tends to be a large number, it can be estimated as a real number and further analytical developments are possible without important errors. The function $[\cdot]^+$ takes the integer part of the argument by excess.

If the first term of Eq. 2.9b is binding, optimality can be developed (notation, $k'(q) = p_K k(q)$).

$$\frac{\partial C_{T,1}}{\partial f} = \mathrm{n}(n-1)(t_F(k'(q) + m(q)) + \mathrm{c}_N) + \frac{n(n-1)t_L}{2t_e}(\mathrm{w(q)} + \xi\mathrm{e(q)}) - \theta dn(n-1)\frac{12\alpha}{f^2} \tag{2.10a}$$

$$\frac{\partial C_{T,1}}{\partial q} = \mathrm{n}(n-1)t_F f(k'_1 + m_1) + \mathrm{n(n-1)}\frac{t_L}{2t_e}f(w_1 + \xi e_1) \tag{2.10b}$$

The condition of optimum is given by the condition of zero for parcial derivatives $\left(\frac{\partial C_T}{\partial f}, \frac{\partial C_T}{\partial q}\right) = (0,0)$. Then, equation 2.10b cannot be zero because f > 0 (bondary condition). Consequently, aircraft size q is a function of daily frequency to satisfy demand.

However, if the second term of Eq. 2.9b is binding, then optimality is as follows:

$$\frac{\partial C_{T,2}}{\partial f} = \mathrm{n}(n-1)(t_F(k'(q) + m(q)) + c_N) - \theta dn(n-1)\frac{12\alpha}{f^2} \tag{2.11a}$$

$$\frac{\partial C_{T,2}}{\partial q} = \mathrm{n}(n-1)t_F f(k'_1 + m_1) + \mathrm{n(n-1)}\frac{t_L}{t_e}d\left((w_1 + \xi e_1)\frac{1}{q} - (w(q) + \xi e(q))\frac{1}{q^2}\right) \tag{2.11b}$$

Both expressions define an implicit system of equations. It is not linear, but the optimal solution (f^*, q^*) exists and can be achieved enforcing the zero.

$$(f^*, q^*) = \left\{\frac{\partial C_{T,2}}{\partial f} = 0, \frac{\partial C_{T,2}}{\partial q} = 0\right\} \tag{2.12}$$

In conclusion, if the network is very large, the fleet is determined by a cinematic problem. In this case, the cost grows with frequency and fixed cost dominates the problem. Airlines tend to choose big planes and minimize number of expeditions, which is typical for long haul. However, if demand is critical (small and crowded networks), then variable costs dominate the problem, enforcing airlines to trafe-off frequency and fleet size. Finally, some numerical experiments are carried out in section 2.4. Figure 2.9 shows a surface of network costs when airport size and demand between airports vary between specific limits.

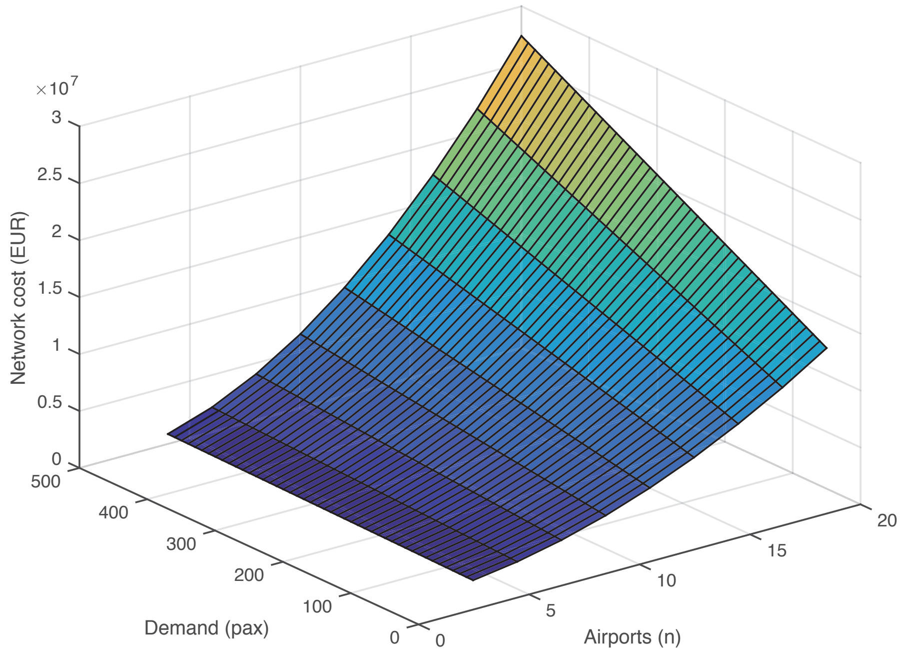


Figure 2.9. Point-to-Point network costs for variations of number of airports and daily demand.

2.3.3 Hub and spoke network (HS)

The network (S) is operated with HS configuration. A set of n airports are connected with a fleet of J aircrafts and $J\xi$ crews for a period of analysis t_C=1day (simplification). In this period, there is a frequency of f ($f > 0$) expeditions and headway (hours) is $h = 24/f$. One more assumption more is taken: hub capacity is not limited, and then all aircrafts arrive and depart at the same moment.

Then, the following condition is enforced: $t_G \rightarrow t_H,\ t_L = t_F + t_H$.

The total cost can be calculated by the following expression (Eq. 2.13a). Also, fleet size is estimated by Equation 2.13b.

$$C_T = 2(n-1)f\left(t_F\left(p_K k(q) + m(q)\right) + \mathrm{c}_N\right) + \mathrm{J}(\mathrm{w(q)} + \xi \mathrm{e(q)}) + \theta d n(n-1)\left(\frac{12\alpha}{f} + \beta 2 t_F + \gamma t_H\right) \quad (2.13a)$$

$$\mathrm{J} = \max\left\{\left[2(n-1)\frac{t_L}{t_e}f\right]^+, \left[2(n-1)^2\frac{t_L}{t_e}\frac{d}{q}\right]^+\right\} \quad (2.13b)$$

If J is a large enough, it can be estimated as a real number and further analysis is possible without important errors.

If the first term of Eq. 2.13b is binding, there is no optimal solution different that minimum frequency and maximum capacity. The problem is dominated by time-space coverage. However, in the other case, new expressions are found:

$$\frac{\partial C_{T,3}}{\partial f} = 2(n-1)(t_F(k'(q)+m(q))+c_N) - \theta dn(n-1)\frac{12\alpha}{f^2} \tag{2.14a}$$

$$\frac{\partial C_{T,3}}{\partial q} = 2(n-1)t_F f(k'_1+m_1) + 2(\mathrm{n}-1)^2\frac{t_L}{2t_e}d\left((w_1+\xi e_1)\frac{1}{q} - (w(q)+\xi e(q))\frac{1}{q^2}\right) \tag{2.14b}$$

Both expressions define an implicit equations system that is not linear, but the optimal solution (f^*, q^*) exists and can be achieved enforcing the zero (Eq. 2.15).

$$(f^*, q^*) = \left\{\frac{\partial C_{T,3}}{\partial f} = 0, \frac{\partial C_{T,3}}{\partial q} = 0\right\} \tag{2.15}$$

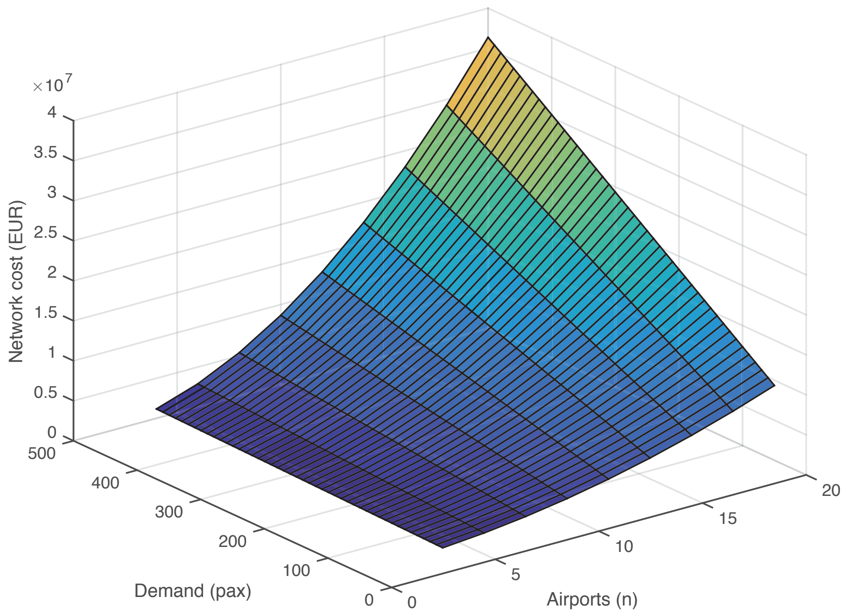


Figure 2.10. Hub-and-spoke network costs for variations of number of airports and daily demand.

2.3.4 Stopover network (SO)

The network (S) is operated with SO configuration. A set of n airports are connected with a fleet of J aircrafts and $J\xi$ crews for a period of analysis t_C=1day (simplification). In this period, there is a frequency of f ($f > 0$) expeditions and headway (hours) is $h = 24/f$.

An important difference is that passengers now spend more time in line-haul because they have to do more steps and stay at plane in turnarounds, depending on the routing strategy.

Total cost can be calculated by the following expression (Eq. 2.16a). Also, fleet size is estimated by Equation 2.16b.

$$C_T = \eta_f(t_F(p_K k(q) + m(q)) + c_N) + J(w(q) + \xi e(q)) + \theta dn(n-1)\left(\frac{12\alpha}{f} + \beta\frac{n}{2}t_L\right) \quad (2.16a)$$

$$J = \max\left\{\left[\frac{\eta_f}{n}\right]^+, \left[n(n-1)\quad \frac{\eta_f}{n}\frac{d}{q}\right]^+\right\} \quad (2.16b)$$

$$\eta_f = \begin{Bmatrix} nf & nt_L \le 2t_e \\ n\varsigma\rho & nt_L > 2t_e,\ f \le \varsigma \\ n\varsigma\rho\left[\frac{f}{\varsigma}\right]^+ & nt_L > 2t_e,\ f > \varsigma \end{Bmatrix} \quad (2.16c)$$

$$\varsigma = \left[\frac{2t_e}{t_L}\right]^- \quad (2.16d)$$

$$\rho = \left[\frac{n}{\varsigma}\right]^- \quad (2.16e)$$

Where, η_f is an expression that allows calculating airline's total number of legs in the network and $[\cdot]^-$ is a function that takes the integer part of the argument by deffect. Observe that SO configuration is difficult because it implicates a problem of logistics (shipments many-to-many with time windows). Analytical expressions were formultated by Daganzo (1994) and they apply here to estimate costs. The main problem of this configuration is that if an airplane is not able to visit all airports within the time windows of service (t_C or less, supply adapted to demand), then the airline has to supply several routes (less than in a PP network, but characterisc for many-to-many configuration). Finally, in these conditions there is a structural frequency due to overlapping of routes.

For further developments in this work, the less costly scenario is assumed. If J is large enough, it can be estimated as a real number and further analysis is possible without important errors. In this case is very likely that only the second term is binding and the solution becomes:

$$\frac{\partial C_{T,4}}{\partial f} = n(t_F(k'(q) + m(q)) + c_N) - \theta dn(n-1)\frac{12\alpha}{f^2} \quad (2.17a)$$

$$\frac{\partial C_{T,4}}{\partial q} = nt_F f(k'_1 + m_1) + n^2(n-1)\frac{t_L}{2t_e}d\left((w_1 + \xi e_1)\frac{1}{q} - (w(q) + \xi e(q))\frac{1}{q^2}\right) \quad (2.17b)$$

Again, optimal solution (f^*, q^*) exists and can be achieved enforcing the zero:

$$(f^*, q^*) = \left\{\frac{\partial C_{T,4}}{\partial f} = 0, \frac{\partial C_{T,4}}{\partial q} = 0\right\} \quad (2.18)$$

Finally, this configuration is less expensive in variable cost for the operator if demand d is low and it needs to consolidate it in stopover configuration. This strategy is typical for transit operators (i.e. urban bus). In contrast, if demand is high the fleet increases and fleet and crew fixed costs balance the increasing frequencies. Note that in this equilibrium the passenger experience is less important (in order to determine f* and q* does not appear the line-haul time) and the problem is dominated by the number of stops that aircrafts do.

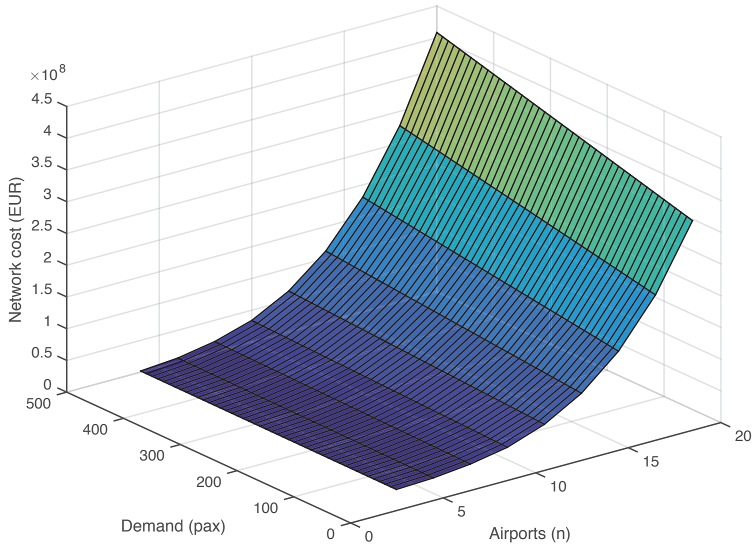


Figure 2.11. Stopover network costs for variations of number of airports and daily demand.

2.3.5 Comparison for 3-node network

This section develops a comparison for a 3-node network and some specific values ($n = 3, t_L = 2h, t_e = 8h, t_F = 1{,}5h, t_G = 0{,}5h, t_H = 0{,}5h, \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \gamma = 1, \theta = 60\ €/h, t_C = 1d$, c_N=c'_N/t_F). Then, three expressions are shown and allow understanding some implications.

$$C_{T,PP} = 9f(k'(q) + m(q) + c'_N) + \frac{3d}{4q}(\mathrm{w(q)} + \xi\mathrm{e(q)}) + 360d\left(\frac{4}{f} + 1\right) \qquad (2.19a)$$

$$C_{T,HS} = 6f(k'(q) + m(q) + c'_N) + \frac{d}{q}(\mathrm{w(q)} + \xi\mathrm{e(q)}) + 360d\left(\frac{4}{f} + 2{,}5\right) \qquad (2.19a)$$

$$C_{T,PP} = 4{,}5f(k'(q) + m(q) + c'_N) + \frac{9d}{4q}(\mathrm{w(q)} + \xi\mathrm{e(q)}) + 360d\left(\frac{4}{f} + 2\right) \qquad (2.19a)$$

Formulation applied to 3-node case shows that variable costs (fuel and maintenance) decreases with strategies that reduce the links in the network. Obviously, these strategies are successful if frequency is well controlled; ensuring that aircrafts do not become excessively small. Observe that fixed costs increases with strategies, for this reason, airlines tend to increase the capacity of aircrafts. Observe that variable cost depends on flight time while fixed cost depends on flight and turnaround time; these are block-hours and service-hours respectively.

Especially, for SO configuration the key of success is to operate aircrafts with large capacity (related to demand) and few stops. In contrast, for this strategy, if aircraft is large and few routes are necessary, extra crew (ξ) increases too much and benefits of this planning fail for human labour costs.

In addition, passenger costs increase as far as strategies beneficate airlines. For any given frequency and aircraft capacity, HS configuration presents a good trade-off between operator and passenger costs. PP can compite very well if airlines can operate more frequencies (smaller aircrafts), exists enough demand for direct service and connection times penalizes HS configuration (high values for connection time, descoordination of scheduling, congestion at hub).

This model considers inelastic demand, so if airlines do not transfer the benefits in cost intro attractive fares for passengers, it is not clear that demand will prefer some strategies that penalizes passenger travel time with connections or long line-hauls.

2.4 Numerical experimentation

Some experiments are carried out in this section to understand the sensitiveness of the models presented in previous section.

2.4.1 3-node network configuration vs frequency and capacity

The first test shows a sensitivity analysis for 3-node network when frequency of different configurations varies from 1 expedition to 10 expeditions per day. Flight time is 1.5 hours and turnaround is 0.5 hours. For duty times of 8 hours and supply windows of 16 hours, the costs have the performance presented in Figure 2.12.

Observe that costs are soft functions for point-to-point and hub-and-spoke because frequencies are taken by fractions of unit. However, stopover costs are calculated with an approximation of fleet that considers increments to cover all routes and provide frequencies desired in the time windows especificated. This situation is consistent and for high frequencies SO configuration cannot compete with PP or HS networks.

PP configuration presents supremacy for this scenario. But, there is a border for frequency equals to 5 expeditions per day, for greater values HS is better than SO. The reason is that for low level of frequencies, it is possible to construct routes with peddling strategy because the network is small (few airports and short hauls). But, if the number of expeditions per day increases over this limit, it is necessary to replicate the network with more resources bad utilized and for this scenario HS can provide the same service with less cost.

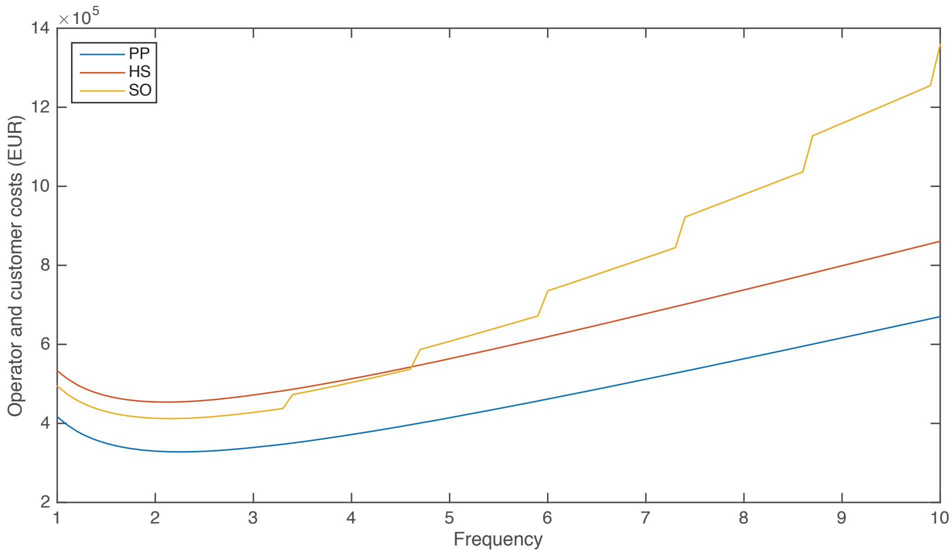


Figure 2.12. Relationship between cost and frequency for 3-node network configuration.

If the 3-node network is analysed against variations of flight time, considering that demand can travel in only one expedition because aircraft capacity is done and is large enough. Then, PP configuration has supremacy. Only for small networks SO is better than HS, the only reason is that HS needs to consolidate flows of passengers from many origins (3 node network is too small).

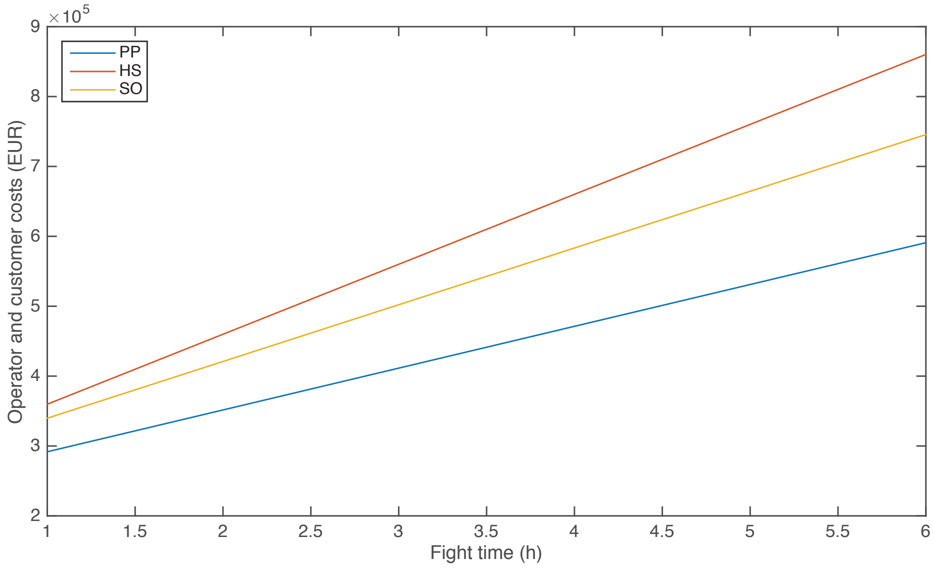


Figure 2.13. Relationship between cost and flight time for 3-node network configuration.

Demand is a critical parameter for network design. Considering a flight time of 1.5 hours and no restrictions on frequency, Figure 2.14 shows the relationship between cost of network and demand level. Observe that it is a very small network and there are no enough airports or stage length compared with demand level to maintain high-frequency network as HS.

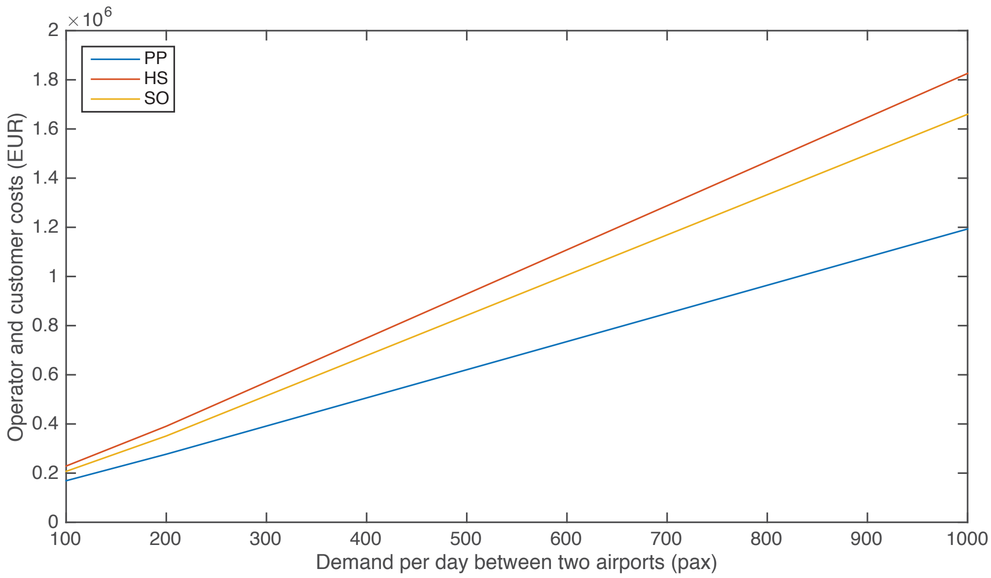


Figure 2.14. Relationship between cost and demand for 3-node network configuration (2exp/d, t_F 1.5h).

However, it is surprising the better performance of SO network regarding HS (Fig. 2.14). If minimum frequency per day (5 exp/day) and flight time increases (4h) (large network and high level of service), taking into account that SO is a network that needs more fleet to cover all demands, consequently its costs grow up (see Figure 2.15).

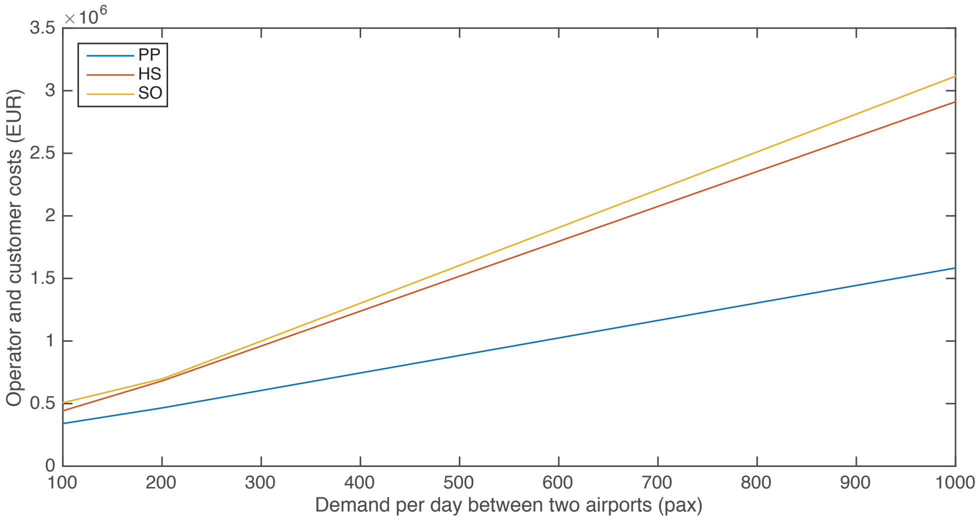


Figure 2.15. Relationship between cost and demand for 3-node network configuration (5exp/d, t_F 4h).

For small networks with demand enough to achieve good load factors, PP is the best option and HS strategy is not competitive.

2.4.2 n-node network configuration vs frequency and capacity

Large networks are suitable for hub-and-spoke operations. This section shows that HS configuration is recommended for servicing a set of many airports. Figure 2.16 shows the relationship between costs for operator and passengers and number of airports when level of demand is low (few passengers per day between each pair of airports, average flight time of 4 hours and airlines decide consolidate passengers and choose better aircraft). SO network runs high costs in this configuration and it is not represented in the Figure 2.16. HS can reduce 40% of PP costs.

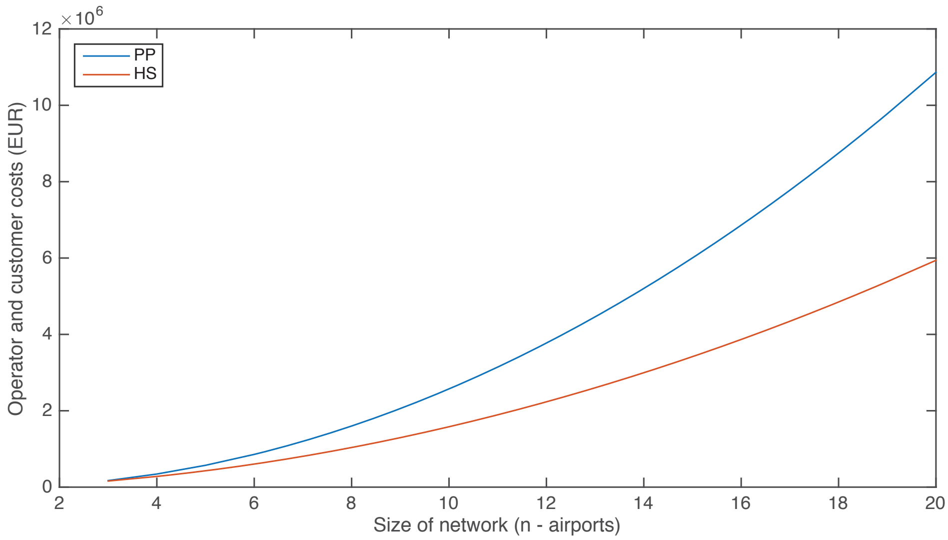


Figure 2.16. Relationship between cost and size of network.

2.5 Conclusions

A simple analytical model allows calculating costs for airlines and passengers and understanding trade-offs. One of the main contributions is to present a differenciation of variable and fixed cost, that it is important to understand impacts on profit and loss account or margins. Previous works based on analytical models have not considered this difference, assigining resources in an inefficient way. Obviously, analytically determination of fleet size and crew resources is not easy.

Furthermore, utilization factor of fleet and crews are a critical success factor for airlines. Long haul routes have the main problem that it is easy to take advantatge of airplanes (they do not need to rest) but not of pilots or cabin crews. As an example, only one route between BCN and JFK needs three crew sets to be operated, while only one is need from DUB. Increments of onw aircraft or three crew pairs could not be significatively for big legacy carrier, however, however it is very significative for small and start-up airlines. This is the reason why legacy carriers hold long haul markets (especially operating from United Kingdom or Ireland) and obtain enough margins to compete with new potential players, the LCC (for them increase fleet or crew is not easy).

In addition, three configurations reveal some considerations. First, SO configuration has more disadvantages than advantages. Only with long routes and few stops airline could take profit of this configuration, always with large aircraft compared to demand and for only one expedition. However, HS presents a good trade-off between variable and fixed costs, also between operator and passenger costs. In this case, consolidation of flows let airlines achieve scale economies and they can transfer the advantage to passenger with low fares or high frequencies, which is a critical success factor to hold the market against LCC, which operates PP networks. HS can reduce cost about 40% compared with PP for the same large network. However, connection time is critical for this configuration and competition in terms of frequency only constraints capacity at large airports and increments levels of primary and reactionary delays. Finally, PP configuration is the most interesting network when the problem is dominated by time-space coverage (demand is enough to achieve a good load factor for their decision of frequency and aircraft size –Ryanair, Easyjet, etc.-).

3 Tabu Search Algorithm for airline network planning

3.1 Introduction

This section is focused on development of an OR model for airline network planning that allows airlines operators to assign resources to flight schedule. This assignment method is useful to calculate costs accurately.

This chapter is structured in three sections. First, a review of the state of the art regarding previous studies that have analysed airline network design and planning with operation research methods. This review facilitates to concret the objectives of this chapter. Second, the problem statement: mathematical models based on linear or integer programming have been used to provide solutions to this problem. Third, this chapter presents a Tabu Search Algorithm (TSA) to assign resources with better performance measured in computational effort. Finally, the algorithm is tested with some numerical experiments based on real flight schedules.

3.1.1 State of the art

Airline industry traditionally has used operations research for operations planning. Some problems are well known by researchers, for example: minimum cost flow problem, multi-commodity network flow problem, traveling salesman problem, etc. All of them constitute a large base for many developments of operations research. Methods like mathematical linear (integer or mixed) programming, exhaustive searching methods, heuristics or metaheuristics provide several references in this industry or other industries with similar goals.

O'Kelly and Miller (1994) analyze with mathematical linear programming (MLP) different patterns of hubbing location, applying several hypothesis about direct routes or hubbing strategies for solving the location problem. Furthermore, Jaillet et al. (1996) study demand and aircraft size interaction, designing flexible network configuration.

Campbell (1994) proposes a review of hub location problems. He considers pure strategies (PP or HS), because this situation does not exist in practice. Also, he proposes networks with different mixes of both strategies and analyses the improvement of performance.

Contribution of MLP for this kind of problems was overpassed by heuristics. Tipically, for large problems exact methods have high computation costs. For example, research lines proposed by Abdinour et al. (1998), Skorin-Kapov (1994), Newell (1973) or O'Kelly (1986, 1994) are examples of good optimization methodologies and techniques.

Bazargan (2005) proposes a methology to study airline opertions and scheduling with MLP models. These models let operators to maximize demand captures and minimize operating costs. A basic assumption of the linearity is to consider unitary costs independent of network size or network performance, but this assumption introduces errors for small networks or particular business models.

Banhart et al. (2003) presents an overview of several important areas of operations research applications in the air transport industry. Specific areas covered are: the various stages of aircraft and crew schedule planning, revenue management and the planning and operations of airports. For each of these areas, the authors provide a historical perspective on OR contributions.

Finally and very useful in this work, Estrada (2007) proposes a Tabu Serch Algorithm for logistics distribution where analyses different strategies of shippments (direct, cross-docking and peddling). Improvements between 7% and 12% are achieved with this metaheuristic compared to other heuristics. Later, Roca-Riu et al. (2012) continue this work applying TSA to transit networks. However, the sophistication of TSA compared with other algorithms is important and it has an extra cost related to effort of codification the algorithm.

3.1.2 Objectives

The aim of this chapter is to continue the network design problem; starting from the assumption that network configuration is well defined based on methodologies of chapter 1. Now, the main goal is to carry out a fine tunning of this design to allocate resources well and calculate accurately their cost in real (non idealized) problems. Especific objectives are:

1. To develop a mathematical framework to define the problem of assignment in different phases for fleet and crew.
2. To develop a Complete Enumeration Algorithm (CEA) and Exhaustive Search Algorithm (ESA) to have a base of comparison with other developments.
3. To develop a Tabu Search Algorithm (TSA) to solve sophisticated scenarios and improve the limits of the analytical approach and/or complete enumeration algorithms (i.e. heterogeneous flight lengths).
4. To accurately calculate the costs of the network and allocate resources for real flight schedulings.

3.2 Problem statement

This chapter assumes that the network configuration and fleet planning has been decided by the airline in previous stage of decision-making, associated with chapter one. Here the main problem is to assign resources accurately and estimate real costs. This is a tactical phase.

Flight scheduling is the starting point for all other airline planning and operations. An airline's decision to offer certain flights will mainly depend on market demand forecasts, available aircraft operationg characteristics, available manpower, regulations and the behaviour of competing airlines. The number of airports and flight frequencies served by an airline usually expresses and measures the physical size of the airline network (Janic, 2000). For large air carriers, the flight scheduling group and route development may contain more than 30 employees (Kuzminski, 1999).

The schedule construction phase begins with the route system. The cities in the airline network determine the route system. The economics of an air carrier are driven by its route system. All the short and long term costs attributed to fleet, labor contracts and operations are tied to the route systems of an airline.

There are two types of route development activites: strategic and tactical. Strategic aims to design the topology of network and analytical models like it was presented in chapter one are suitable. However, tactical development focuses on short-term changes of schedule and routes. This is done by constantly monitoring markets, competitors and operations. The tactical strategy includes adding, dropping flights and making changes to city-pair markets and their frequencies.

Flight schedule construction creates a complex system with a large number of variables in the model. Due to its complexity it is difficult to formulate the complete scheduling construction problem as a mathematical model. In practice, airlines managers prefer plan separately fleet assignment, routing and crew assignments.

This process is descomposed into sub problems with less complexity, which are solved sequentially (losing certain optimality of solutions but earning robustness). These are presentend in different subsections. For these ones and further developments in this chapter some flight schedules are presented as a support for the explanations and for testing algorithms. They are real flight schedules that some airlines' managers provided for this thesis, however due to agreement of confidentiality the company name is not mentioned.

The algorithms presented in this chapter start with a space of feasible solutions achieved by complete enumeration algorithm. Then, two strategies are developed: first, complete evaluation; second, searching strategy with TSA. The optimality is conditioned by the effectiveness of the complete enumeration algorithm.

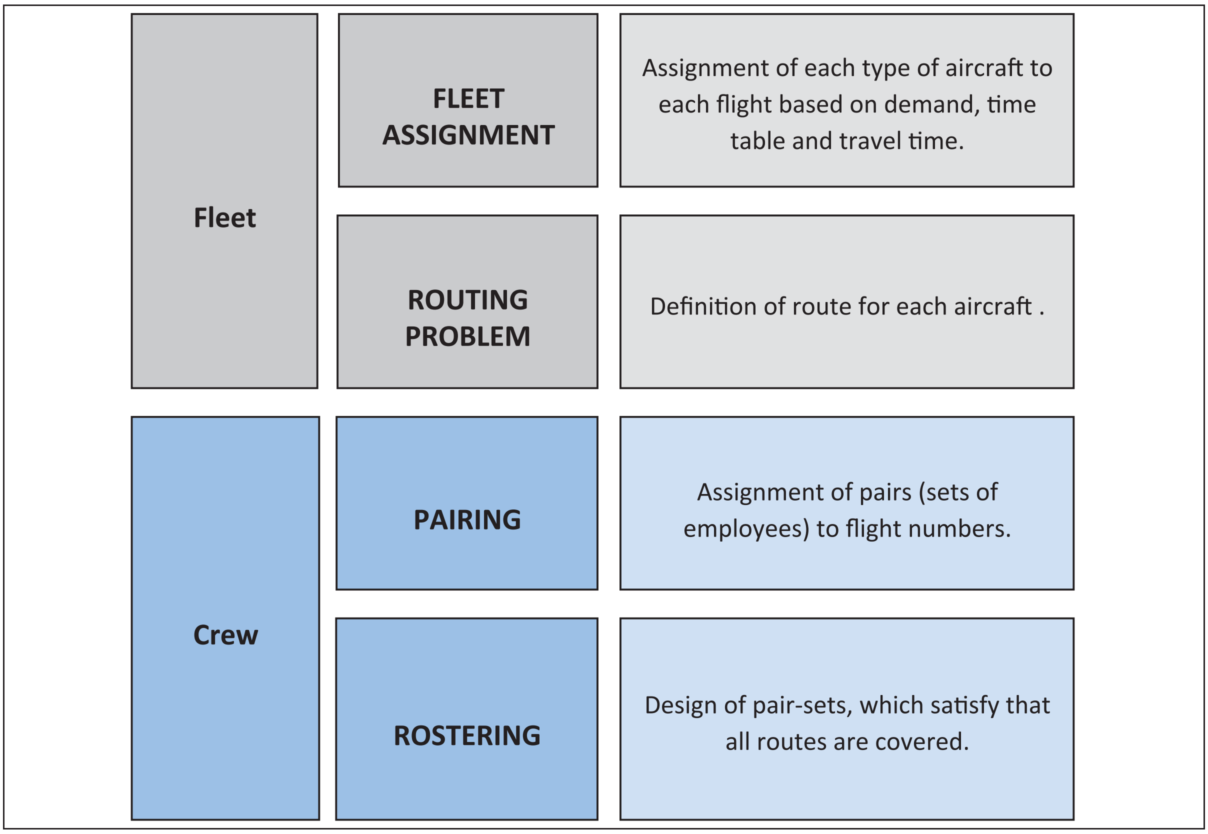


Figure 3.1. Methodological structure of the airline assignment problem.

3.2.1 Fleet assignment

Following the construction of a flight schedule, given its corresponding network, the next step is to assign the right fleet type to each flight in the schedule. The task of fleet assignment is to match each aircraft type with a particular route in the schedule. It should be noted that this phase of planning concerns only the fleet type and not a particular aircraft. The goal of fleet assignment is to assign as many flight segments as possible in a schedule to one or more fleet types, while optimizing the objective function and meeting several operational constraints.

Airlines typically operate a number of different fleet types. Each fleet type has different characteristics and costs, such as seating capacity, landing weights, crew, maintenance, and fuel. Maintenance cost is a major factor that persuades airlines to be less diverse when planning for their fleet. Fleet diversity requires the airlines to have skilled crew and personnel for each fleet type, plan for different maintenance checks, and have less flexibility in replacing an aircraft when a failure occurs.

A major concern in formulating the fleet assignment problem is keeping track of the fleet at different airports at any given point in time. For this purpose models adopt a time-space network (Figure 3.2), which is a representation of aircraft's trajectories. Observe that a wrap-around link is a ground link that connects the last node to the first node in a given city. These arcs normally represent the aircraft that stay overnight in an airport, and connect the last arrival to the next day's departure flight.

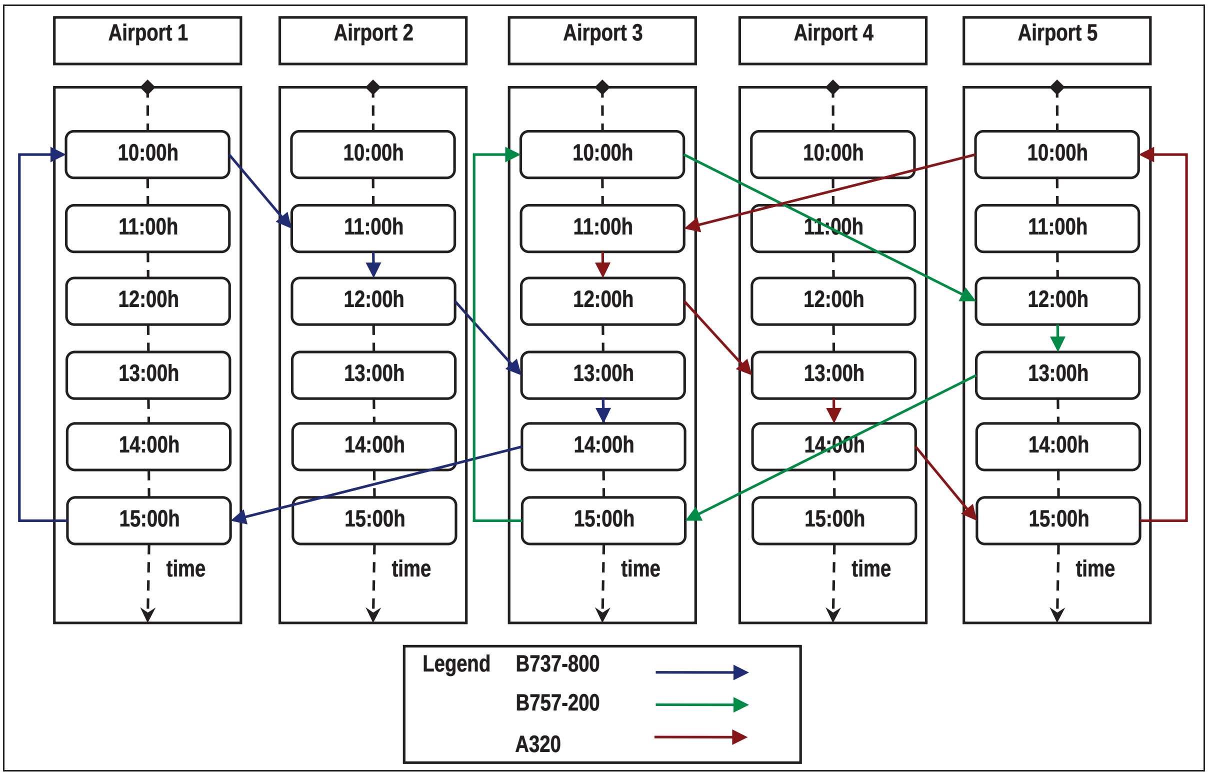


Figure 3.2. Time-space diagram for flight schedule.

The fleet assignment problem is basically formulated as a multi-commodity network problem (Hillier and Lieberman, 2006). Each node represents supply/demand, which can be satisfied through a diverse fleet. The model seeks to minimize the total cost by assigning the most appropriate fleet type to each flight leg. The constraints ensure that each flight is assigned to a particular fleet type, and that the number of aircraft for each fleet does not exceed the number of available aircraft.

The mathematical model takes as objective function the total cost (C_T) of the network (Eq. 3.1a), which is the sum of variable cost of each flight.

$$C_T = \sum_i c_i \tag{3.1a}$$

The cost per flight c_i includes two parts: operating cost for each flight ($c_{O,i}$) and spill cost for each flight ($c_{S,i}$) that are defined in the following paragraphs.

$$c_i = c_{O,i} + c_{S,i} \tag{3.1b}$$

Operating cost is evaluated following expressions presented in the Appendix 1 and it is expressed by Eq. 3.2.

$$c_{O,i} = c_{K,i} + c_{M,i} + c_{N,i} \tag{3.2}$$

Given one airline network (S) served by R routes and I flights, the following variables are identified: i, index of flight (i=1,…, I) that goes from airport x to airport y (x, y = 1,…, n); $c_{K,i}$, fuel cost for flight i; $c_{M,i}$, maintenance cost for flight i; $c_{N,i}$, navigation and airports charges for flight i. This cost is evaluated at variable costs only (because the fixed cost is the same independently of the assignment).

The spill cost is the revenue of lost passengers due to insufficient aircraft capacity. It can be calculated as product of expected spill passengers (total passengers that are going to be out of aircraft) times the revenue of available seat per kilometer.

The recapture rate represents the percentage of passengers that were spilled, but could be accommodated or recaptured on other flights by the same airline. That is, if a passenger cannot get a seat on a specific flight, the airline offers earlier or later flights to the passenger for consideration. If the passenger accepts the offer for another flight, then this passenger is considered as recaptured passenger.

In addition, the spill cost for flight i is expressed by this expression (Eq. 3.3), where for I given flights (i=1,…, I), w_i is the spill passenger (expected demand minus capacity of aircraft for flight i), y_i is the yield for flight i (RASK per stage length) and $\widehat{\rho_i}$ is the complementary to recapture rate.

$$c_{S,i} = w_i y_i \widehat{\rho_i} \tag{3.3}$$

Finally, the mathematical model for fleet assignment is given by Eq. 3.4 and Eq. 3.5, it is based on previous work done by Bazargan (2005).

$$\text{Min Z} = \sum_i \sum_a c_{i,a} x_{i,a} \tag{3.4}$$

Subject to:

$$\sum_a x_{i,a} = 1 \qquad \forall\ i = 1, \dots, I \quad a = 1, \dots, A \qquad (3.5a)$$

$$G_{k-,a} + \sum_i S_{i,k} x_{i,a} = G_{k,a} \qquad \forall\ a = 1, \dots, A \quad k = 1, \dots, N \qquad (3.5b)$$

$$\sum_k G_{k,a} \le A_a \qquad \forall\ a = 1, \dots, A \quad k = 1, \dots, N \qquad (3.5c)$$

$$x_{i,a} \in \{0,1\} \qquad \forall\ i = 1, \dots, I \quad a = 1, \dots, A \qquad (3.5d)$$

$$G_{k,a} \in \mathbb{N}^+ \qquad \forall\ a = 1, \dots, A \quad k = 1, \dots, N \qquad (3.5e)$$

Where, there are A types of aircraft (a=1,…, A) and N airports (k=1,…, N). The decision variable, $x_{i,a}$ is 1 if fleet type a is assigned to flight i. The parameters, $c_{i,a}$ is the total cost of assigning fleet type a to flight i and it is avaluated with Eq. 3.1b; $G_{k,a}$ represents the number of aircraft type a at ground k at any given step of calculation; $G_{k-,a}$ is the same but an step before and $S_{i,k}$ is a counter to balance flights arriving at node (+1 if flight i arrives at node k, -1 if it is a departure).

Observe that equation 3.5a is a constraint of coverage, 3.5b is an equation of balance (mass at any airport), 3.5c respect the fleet size and 3.5 and 3.6 constraint the values of decision variables.

3.2.2 Aircraft routing

Aircraft routing is a critical success task and it is so important that many airlines has outsourcing for crew assignment but never for routing. This activity is the process of assigning each individual aircraft (identificated by tail number) within each fleet to flight legs. The goal of this activity is minimize operating cost. There are three major constraints: all flights are covered by one aircraft, aircraft load balance and maintenance requirements (aircrafts visit maintenance bases periodically).

The problem defined here is based on Kabbani (1992) and it starts with the generation of all aircrafts routing generation and then a decision variable takes the value 1 or 0 to choose each route for the final solution.

Turnaround time is a new variable in the generation of routes and it is a required time that any aircraft spends on ground at every airport in its route to service passengers and baggage. This is one of the main differences between analytical or strategic models with OR models: the last ones allow customized times for each flight and airport, which is more realistic.

Usually, routing is determined for a period of time equal to the cycle time. At the end of that, all aircrafts and crew come back at their base. Usually, LCC works with one day cycle times and it allows managers to reduce cost derivated of extra crew out of bases (accommodation expenses and subsistence) and other charges at airports. However, long haul services require long periods.

Routing generation is a critical task because it creates a space of feasible solutions. It is the starting point for Complete Enumeration Algorithm (CEA), Exhaustive Search Algorithm (ESA) and Tabu Search Algorithm (TSA).

The logical for this routing generator is as follow:

1. Consider the set of bases where the aircrafts are available to start the operation.
2. Take all flights in the scheduling, one by one, as current-flight and iniciating a route (current-route), only if they are contained in set of bases.
3. Memorize the arrival time as current-time and airport as current-airport.
4. Add corresponding turnaround time and update current-time.
5. For all flights in scheduling that verify: (i) flight departs from current-airport and (ii) flight departs later than current-time; consider all alternatives adding flights to current-route, duplicate the route and create a set that is saved in memory. Close the current route (if it ends where it starts, save it) and take the next one and continue with point (3) and (4).
6. At the end, no flights are available to continue. Close current-route (if it ends where it starts, save it) and take the next one from the list.
7. Filter routes with restrictions: for a cycle time all routes start and end at the same airport, those are bases. Control cycle time.
8. The set of routes is generated.

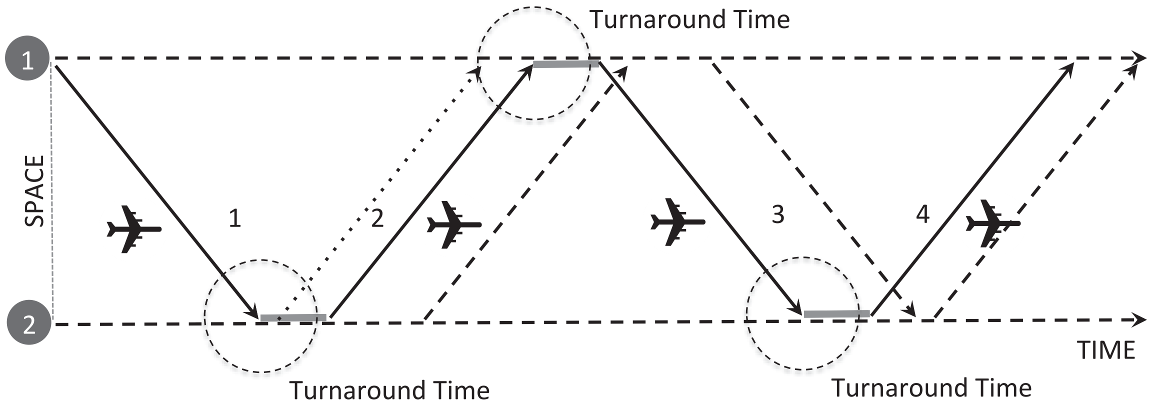


Figure 3.3. Routing generation strategy.

Cost of routes are evaluated at this moment, basically this cost is the sum of each cost of flight and cost of ownership for fleet (Eq. 3.6).

$$c_r = \sum_{i \in I_r} \left(c_{K,i} + c_{M,i} + c_{N,i} \right) + t_C \hat{c}_{W,r} \quad (3.6)$$

Given a set of R routes and a fleet of aircrafts, the operating cost for each route is defined by these parameters: the variable costs related to fuel ($c_{K,i}$), maintenance ($c_{M,i}$), navigation ($c_{N,i}$), which are defined above and they are considered for flights included in each route ($\forall\ i\ \in I_r$, where I_r is the set of flights for route r) and $\hat{c}_{W,r}$ is the unit ownership cost per unit of time for aircraft assigned to route r, for period t_C.

Finally, the mathematical model is defined as follows:

$$\text{Min Z} = \sum_r c_r x_r \quad (3.7)$$

Subject to:

$$\sum_r u_{i,r} x_r = 1 \qquad \forall\ i = 1, \dots, I \quad r = 1, \dots, R \quad (3.8a)$$

$$\sum_r x_r \leq J \qquad \forall\ r = 1, \dots, R \quad (3.8b)$$

$$x_r \in \{0,1\} \qquad \forall\ r = 1, \dots, R \quad (3.8c)$$

Where, the decision variable is x_r with r=1,…,R, being R number of routes at the feasible solution space. The parameters: c_r is the total cost of route r evaluated with Eq. 3.6; $u_{i,r}$ is 1 if flight i is assigned to route r and 0 otherwise, and this variable is a matrix derivated from feasible solution space.

Observe that equation 3.8a is a constraint of coverage that enforce each flight is covered by one and only one route, 3.8b is a constraint that restricts the number of routes to available number of aircrafts for this fleet-type. Finally, 3.8c indicates the set of values of the integer decision variables.

Finally, a different model could be carried out integrating fleet assignment and aircraft routing. It could be easy to enumerate each route for different type of aircrafts and evaluate their cost. However, the total computation cost could increase excessively but not the utility. The reason is that airlines tend to assign fleet in a different stage of the problem related to market needs.

3.2.3 Crew scheduling

Crew scheduling involves the process of identifying sequences of flight legs and assigning crew to them (pilots and cabin crew). Crew scheduling, like aircraft routing, is normally performed after the fleet-assignment process. This task allows airlines to calculate accurately the cost associated with labor costs.

Crew scheduling is one of the most computationally intensive combinatorial problems (Barnhart 2003a, 2003b). Furthermore, this scheduling problem is typically solved in two phases, crew pairing and crew rostering. Depiste of the existence of algorithms and research about the integrated problem, industry tends to maintain the problem separated because has some advantages at operational level.

First phase is to develop crew pairing. This is a sequence of flight legs, within the same fleet, that starts and ends at the same crew base. The sequence of crew pairing must satisfy constraints such as regulations or contracts. In practice, operators try to coordinate aircraft route and crew route because they can manage better operations when delays or disruptions happen.

The objective of crew pairing is to find a set of pairings that covers all flights and minimizes the total crew cost. This phase is impersonal and crew rostering, second phase, is in charge to assign each specific crew member to these pairs.

The particularity of crew scheduling is that people need to rest when they accumulate some working hours. Then, regulation is complex because the last of workday depends on flight time and number of flights. European Commission dictates the Flight Time Limitations, which are a rules for regulate the system. A duty is a tipical workday of a crew and rest is the period between two duties. A duty or workday (in this work) has a maximum of time permited, and a rest has a minimum. If the crew has to change of airplane in an airport, then they have a sit connection, which is a period of time limitated to do this change.

Key factors of success in this assignment are to balance well workload, minimize rests out of home, try to maintain crews at the same aircraft as much as possible and minimize total number of crews.

The problem is affordable with CEA and it is necessary the pair generation tool. The generation process is based on rules and regulations. It starts with a crew base and adds all the feasible flight legs according to the specified rules. It finally ends up at the same crew base from which it started.

The rules are related to total daily flight time, minimum and maximum sit-connection times, total pair time and all of them start at bases where crew are assigned (their "home"). Sequences of flights end where they start. The logical aspects are the same than for route generation. However, only sit connection-time constraint is different.

Also, to ensure that pairs spend the maximum time inside the same aircraft, there is a rule of generation that is realistic. Pairs have to arrive to aircraft some minutes before it takes-off and they can exit the plane some minuts after it lands, because they have to prepare the plane for new passengers. Then, this time can be assumed to be one half of their flight turnaround time or half part of the minimum turnaround time (Ryanair is the fastest airline in ground and spends about 20 minutes). So, for this work minimum sit connection-time is 20 minutes. Furthermore, a vector of priorities is constructed considering aircraft tail number: if more than one option is possible, then take the one whose aircraft tail number is the same.

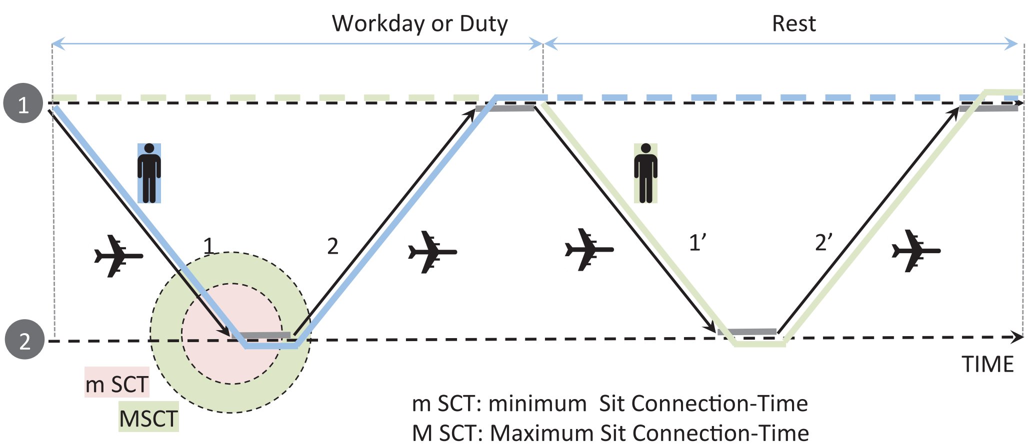


Figure 3.4. Pairing generation strategy.

Costs of pairs are evaluated based on next expression (Eq. 3.9):

$$c_p = \sum_i (c_{E,i} \cdot \mathrm{t}_{L,i} / \bar{\mathrm{t}}_E) \qquad (3.9)$$

Operating cost for each pair c_p is defined by these parameters: given a set of pairs (P, p=1…P) composed by (I) flights. The cost per day $c_{E,i}$ is known by Appendix 1, however here is necessary by hour of service time and this value is achievable dividing by average service time $\bar{\mathrm{t}}_E$ (Appendix 1). The variable $\mathrm{t}_{L,i}$ is the total time of stage length plus the turnaround time for flight I, which are needed to evaluate variable costs.

Finally, the mathematical model is defined as follows:

$$\text{Min Z} = \sum_p c_p x_p \qquad (3.10)$$

Subject to:

$$\sum_p u_{i,p} x_p = 1 \qquad \forall\ i = 1, \dots, I \quad p = 1, \dots, P \qquad (3.11a)$$

$$b_{lower,h} \le \sum_p d_{h,p} x_p \le b_{upper,h} \qquad \forall\ h = 1, \dots, H \qquad (3.11b)$$

$$x_p \in \{0,1\} \qquad \forall\ p = 1, \dots, P \qquad (3.11c)$$

Where, the decision variable is x_p with p=1,…,P, being P the number of pairs at the feasible solution space (obtained with pairing generator). The parameters: c_p is the total cost of pair p evaluated with Eq. 3.9; $u_{i,p}$ is 1 if flight i is assigned to pair p and 0 otherwise, and this variable is a matrix derivated from feasible solution space.

Observe that equation 3.11a is a constraint of coverage that enforce that each flight is covered by one and only one pair, 3.11b is a constraint that ensures that the selected flight pairings stay within the available number of crew members at each home base. Finally, 3.11c indicates the set of values of the integer decision variable.

The second phase is crew rostering which is a typical problem of labor schedule and does not have any interaction with routing strategies for airlines. The main objective is to ensure that all crew members has the same workdays in a long period of time (month, semester or year). Due to this fact, this part is not developed in this work because does not contribute to improve the research in airline network design.

3.3 Complete Enumeration Algorithm

The Complete Enumeration Algorithm (CEA) has the objective to design a process to define all possible candidates that could be the solution for an assignment problem (regarding fleet, routing or crew pairing in this work). The following description citates routing problem, but it is identical for crew pairing.

Given a set of candidates $\{y_j,\ j = 1 \dots m\}$ to receive an assignation (flights, routes or pairs that are given by problem statement). Combinatorial process defines matrix $\boldsymbol{X} = \{x_{i,j}\}\ i = 1 \dots n, j = 1 \dots m, x_{i,j} \in \{0,1\}$, whose rows define element by element if one candidate is considered in the potential solution (i.e. if candidates are feasible routes, values can be 0 or 1 to reject or accept this route in the potential solution).

The size of feasible solutions space depends on capacity constraints and it is an output of combinatorial problem. Given a set of elements {0,1}, the element 1 can be selected V times and 0 can be selected W times, with $v = 1 \dots q$, $w = m - 1 \dots m - q$. For each pair of (v, w), there is a problem of permutations with repetitions. Then, it is necessary to solve this problem of permutations q times. Each time, a number of n_v potential solutions are generated and $n = \sum_{v=1}^{q} n_v$.

The process to create matrix $\boldsymbol{X}$ consists of creating a seed, $\boldsymbol{s} = [1\ 1\ 1\ \dots\ 1\ 0\ \dots\ 0]$ with $|\boldsymbol{s}| = J$ and R components. The permutation of its components results in different variations of this seed. These variations are rows for matrix $\boldsymbol{X}$.

If m is very large and q is about one half, then n results a large number and computationally is difficult to generate all the solutions. However, if this is not the case, then it is possible to enumerate the space and it is a good strategy to test TSA.

The logical procedure to enumerate all candidates for this space is defined by this pseudo-code (Figure 3.5).

Set of candidates $\{y_j,\ j = 1 \dots m\}$
Set of values {0, rejected, 1, accepted}

For v=1 to q
  $w = m - v$
  $n_v = P_m^{v,w} = \frac{m!}{v!w!}$
  $s = [1\ \dots\ 1\ 0\ \dots\ 0]$ (there are v 1s and w 0s)
  $S = P(s)$ (function of permutations)
  $X = [X;\ S]$
Next v
$[n, m] = size(X)$

Figure 3.5. Structure of Complete Enumeration Algorithm.

If an airline has 3 or 4 types of aircrafts (A320, A330, A380,…), then the set {0,1} has to change to {1,2,3,…}. In this case, the problem is extensive and permutations are defined by general expression $P_m^{a,b,c,\dots}$, where a, b,c… are how many times appears each element of set {1,2,3…}, respectively. In this work, subset is limited to two items because of computational cost.

The computational cost of this method is very high. If m varies between 2 and 10, being v=w (m ever) and v=w+1 (m odd), Figure 3.6 shows the evolution of dimension of feasible solutions space. Usually, in the airline industry m is a large number and for this reason the industry adopted advanced OR techniques very early.

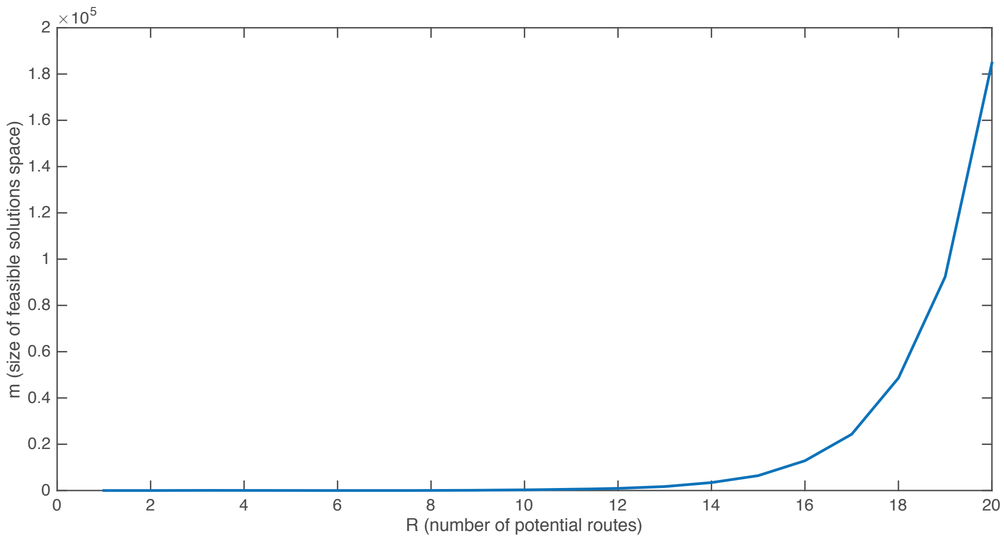


Figure 3.6. Relationship between size of feasible solutions and number of potential routes.

3.4 Exhaustive Search Algorithm

The aim of Exhaustive Search Algorithm (ESA) is to choose the best candidate within the feasible solution space, which minimizes objective function Z (Eq. 3.4, 3.7 or 3.10). The description of ESA is focused on aircraft routing problem and crew pairing (for fleet assignment is the same).

Given a set of candidates $\{y_j,\ j = 1 \dots m\}$ to receive an assignation, each of them indicates which items of a subset $\{v_i, i = 1 \dots n\}$ are covered. The feasible solutions space created with CEA is $\boldsymbol{X} = \{x_{kj}\}, k = 1 \dots t$. For each one of the elements in the feasible solution space, the cost is calculated as $z_k = \sum_j c_j x_{i,j}$. Vector $\boldsymbol{c} = \{c_j\}, j = 1 \dots m$ indicates the cost related to each candidate.

Also, vector $\boldsymbol{g} = \{g_k\}$ evaluates the set of constraints for each candidate $k = 1 \dots t$ in each particular model (fleet, routing or pairing). If x_k verifies all of them, $g_k = 1$; else, $g_k = 0$. For aircraft routing it is easy, $g_k = 1 \leftrightarrow \sum_r u_{i,r} x_{k,r} = 1\ \forall\, i$. On the other case, for crew pairing, more than one condition has to be satisfied simultaneously 3.11a and 3.11b (equations and inequations).

Termination criterion consists on: (1) z_k is the minimum value found in searching process, (2) that verifies constraints $g_k = 1$.

```
Set of feasible solutions {x_{k,r}, k = 1 ... t, r = 1 ... m}
Initial_time
Initialize minval;
x_solution=[];

For k=1 to t
     z_k=z(x_k);
    g_k=g(x_k);
    if g_k=1 and z_k<minval
         minval=z_k;
         x_solution=x_k;
    end
Next k
End_time
Solution: x_solution
Value of objective function: minval
Iterations: k
Time of computation = End_time – Initial_time
```

Figure 3.7. Structure of Exhaustive Search Algorithm.

3.5 Tabu Search Algorithm

This section focuses on Tabu Search Algorithm (TSA) applied to airline network planning. The objective is to develop a tool that has less computational cost than ESA.

3.5.1 TSA concepts and applications

TSA is a metaheuristic that has ability to overcome the limitations of local optimality, but the strategy has no stochastic elements as simulated annealing has. TSA combines the aggressiveness of descent methods and diversity (the ability to explore the solution space extensively) of simulated annealing.

Furthermore, this metaheuristic search method uses memory structures to direct an efficient and effective search of a solution spaces associated with large complex constrained optimization problems. Extensive and detailed discussions of TSA abound. One such discussion is contained in Glover and Laguna (1997). In essence TSA starts from an initial solution, defines a “neighborhood” which can be reached from the current solution by a “move,” a simple change to the current solution. A move’s value is the associated change in the objective function value.

One use of the memory structures is to control the search by forbidding “tabu moves” that would return the search to previously visited solutions for a specific number of iterations, the tabu tenure. Various strategies may be adopted to improve the search. For example, an aspiration criterion can be employed to override tabu restrictions in specified circumstances. Intensification strategies can be used to concentrate the search in the vicinity of “good solutions,” while diversification strategies are used to encourage the search to proceed to a different area of the solution space.

In adaptive and reactive tabu search (Battiti and Techiolli 1994), search parameters like the tabu tenure are automatically adjusted based on the quality of the search. Adaptive Tabu Search (ATS) myopically decrements (increments) the tenure based on whether the objective function improves (disimproves). Reactive tabu search (RTS) changes the tabu tenure according a more global set of decision rules. In RTS the history of solutions visited is maintained during the search and is used to check if the search has been restricted in an “attractor basin” residing in the solution space. RTS also uses various mechanisms to escape from chaotic attractor basins once they are identified.

Laguna et al. (1991) applied TS methods on a single machine-scheduling problem and described a TS-hybrid method that employs both swap and insert move. Barnes et al. (1993) solved the multiple-machine weighted flow time problem using static TS. Compared to the branch and bound method, their computational experiments showed that TS is superior to branch and bound in the quality of solutions and the time needed to obtain a solution. Also, there is only a modest growth in the computational effort required to acquire the solution, as the number of jobs and machines get larger.

Battiti (1994) presented a reactive TSA method, which adapts the size of tabu tenure in response to the search history. The tenure was increased when configurations were repeated and reduced in the absence of such repetitions.

Barnes (1995) applied TS to solve the job shop-scheduling problem. Starting from the best solution rendered by a set of 14 heuristic dispatching solutions, it iteratively moves to another feasible solution by reversing the order of two adjacent critical path operations performed by the same machine. Laguna (1995) presented a TS method to solve the multilevel generalized assignment problem, which used ejection chains to construct the candidate list of moves at each iteration of the solution approach. Carlton and Barnes (1996) used the reactive TSA to solve the TSP with time windows. Their experiments showed that the reactive TSA is robust across a wide range of problem types.

Lokketangen (1998) solved general zero-one mixed integer programming problems using TSA. González-Velarde (2002) used TSA employing ejection chains to solve graph coloring problem. Nanry (2000) used reactive TSA to solve a pickup and delivery problem with the constraints of vehicle capacity and customer time windows. Korycinski (2003) combined TSA within a classification algorithm. Reactive tabu search was used to select features in hyperspectral data analysis to improve classification accuracy. Scrich (2004) applied TSA to the problem of scheduling jobs in a flexible job shop with the objective of minimizing total tardiness. Improved solutions were found in neighborhood generated by the critical paths of the jobs in a disjunctive graph representation.

Barnes et al. (2004) used group theoretic TSA to solve the aerial fleet refueling problem. They applied group theory to partitioning and ordering combinatorial problems; combined with dynamic search methodologies, the algorithm was shown to be effective and efficient. Crino (2004) also used group theoretic TS to solve the theater distribution vehicle routing and scheduling problem. Harwig (2006) used an adaptive TSA to solve 2-dimensional orthogonal packing problems. Using a very efficient dynamic move neighborhood strategy the method quickly finds excellent near-optimal solutions, Kinney (2007) developed a group theoretic TSA to solve the unicost set covering problem by partitioning the solution space into orbits and a reactive TSA procedure based on both inter-orbit and intra-orbit swap was used to explore the neighborhood.

3.5.2 TSA definition

TSA is defined by a set of concepts that are described below.

The starting set of problem

The problem formulation for TSA starts with the definition of a strategy to solve the problem. In this work, TSA has to search the optimal combination of candidates within a feasible solutions space. Therefore, it is going to start with output provided by generation algorithms (i.e. routing generator algorithm or pairing generator algorithm, which enumerate all possible routes or pairs given a flight schedule and conditions). There is a set of candidates $\{y_j,\ j = 1 \dots m\}$.

TSA requires an initial solution or seed, because it is a combinatorial algorithm. Therefore, considering the number of resources that airline hold for each problem, seed is $\boldsymbol{s} = [1\ 1\ 1\ \dots\ 1\ 0\ \dots\ 0]$ with $|\boldsymbol{s}| = r$ and m components. Permutations of this original seed become new candidates to be evaluated with the objective function.

The number of total candidates that TSA can evaluate depends on termination criterion determined by user. Then, parameter K is the maximum number of iterations allowed and the set of total candidates tested can be expressed by $\{x_k\}$.

The objective function

Objective function Z is the operator cost for each problem. The general expression is $Z = \sum_j c_j x_j$. First, in case of fleet assignment, Equation 3.4 applies to the problem. Secondly, if aircraft routing is the target, then Equation 3.7 is the candidate. Finally, for crew pairing, Equation 3.10 is the choice. Also, the potential solution is defined as $x = \{x_j\}$, with j=1…m; and it was generated with permutations over $\boldsymbol{s}$.

A second evaluation of function is necessary, the verification of constraints. Then, given a set of constraints (Eq. 3.5, 3.8, 3.11), the constraint function $G = g(x)$ evaluates the set of constraints for each candidate x that the algorithm creates. If x verifies constraints, $g(x) = 1$; else, $g(x) = 0$. For aircraft routing it is easy, $\mathrm{g} = 1 \leftrightarrow \sum_j u_{i,j} x_j = 1 \ \forall\, i$. On the other case, for crew pairing, more than one condition has to be satisfied simultaneously 3.11a and 3.11b (equations and inequations).

TSA works with different criteria and it consists on: (1) if $Z = z(x)$ is the minimum value found in the searching process and verifies constraints $g(x) = 1$, then the solution is saved in memory. If not, tabu tenure is applied. (2) If x does not verify constraints $g(x) = 0$, then tabu tenure is applied directly but this solution is saved in the memory of infeasible solutions.

The neighbourhood definitions

Given a $\boldsymbol{x^0} = \boldsymbol{s}$ initial solution, with (z_k, g_k) evaluations. The neighbourhood is created by permutating the elements of the seed. There are three mechanisms:

(1) Simple swap. This action is defined by the permution of two components of a given x. In particular, given a vector **x**, with $x_j\ j = 1 \dots m$; before the permutation the situation is $x_v = 1, x_w = 0$, aflter that $x_v = 0$ and $x_w = 1$.

(2) Complete swap. This action consists in a total permutation of all the elements of x. Applying a conventional algorithm for randomly variation of x's indexes. The objective of this kind of movement is to overpass local optimums, when the improvements of Z are small applying criterion (1).

(3) Simple elimination. This action is an extra mechanism to achieve better assignments of resources. If a good solution is achieved, simple elimination proposes to eliminate one of the candidates assigned ($x_v = 1 \rightarrow x_v = 0$). Then, some more iterations can be run with criteria (1) and (2).

Move evaluations

The evaluation of movements requires the complete evaluation of objective function Z and constraint function G, specified in the objective function description. Sometimes, researchers can propose the partial evaluation because the function is linear and can add the contributions of each permutation. It is possible in this approach for functions Z and G. For function Z it is eassy to do, however it is not for G; due to the variation of forms related to the specific problem to be solved (constraints are different for fleet assignment, aircraft routing or crew pairing). The aim of this TSA is to be useful for three particular problems; therefore the evaluation of the objective function is complete.

TSA attribute

The concept of Tabu Attributes is defined as follows: if two elements of x are permutated, it is forbidden to permutate again for tabu tenure iterations. The objective is to avoid falling in local optima.

For example, suppose, in the current solution $x = [1\ 1\ 0\ 0\ 0]$ route 2 is activated, then simple swap changes the situation to $x' = [1\ 0\ 1\ 0\ 0]$. Now, route 2 is not activated but route 3 is. TSA attribute avoids to execute $x_2 \underset{p(2,3)}{\longleftrightarrow} x_3$ again, for tabu tenure iterations.

A vector is employed where the value of $tabu\_list[v]$ indicates the earliest iteration at which the element v may again be moved to any other position. Each time a move is executed, the $tabu\_list[]$ for all flights moved is updated.

Tabu tenure

Tabu tenure is the number of iterations for which a permutation is forbidden. Tabu tenure can take different values depending on characteristics of TSA. If tabu tenure is a constant, the TSA is simple. If tabu tenure takes different values depending on the number of repetitions that some permutation is proposed, it is known as Reactive TSA.

Reactive TSA was employed and the tabu memory structure was extensively used to control the search and to adjust the search parameters based on the quality of the search. The search quality is determined by the frequency of revisiting previously visited solutions. The simplest way to identify the solution is to compare the solutions with history of all previously visited solutions. The visit information includes the number of repeated visits and the iteration that each of the previous visits occurred.

In particular, if one solution does not verify constraint (g(x)=0), then tabu tenure = K (maximum number of iterations for TSA). It is equal to forbid this solution for all iterations. If g(x)=1, then apply ordinary value of tabu tenure.

The procedure is as follows. Search for the objective function value of the solution s in the solution history. If it is not found, then this solution has never been visited and the solution s is added to the solution history. Otherwise, among all solution history records with this objective function value, determine if the hash value Hash(s) is already present. If not, s has never been visited and its hash value is added to the solution history. If found, s is being revisited. Update the revisit information.

In practice, it is very difficult that two solutions have the same value for objective function. Then, it is a common practice to use this value as identicator to compare candidates. However, sometimes coincidences happen, and then there are references of previous works of researchers who used additional functions to improve characterization of solutions (i.e. hash indicator). For this work, these kinds of strategies are not implemented, but for further lines of research could be interesting to implement in TSA code.

Finally, tabu tenure is adjusted in the following way: If a solution is revisited within a specified number of iterations (max_cycle), then tenure is increased by a predetermined factor to diversify the search. A moving average of the iteration intervals between the solution revisits is calculated to track the recent revisitation cycle length in the search history. If tabu tenure has not been increased for more iterations than this moving average, then tabu tenure is decreased to avoid excessive increase in tenure and to intensify the search. Finally, TSA determines that all possible moves are tabu and none satisfy the aspiration criterion, then the tabu tenure is decreased, with the first solution on the elite list of solutions selected as the new incumbent solution and the tabu memory structure is reinitialized.

3.5.3 TSA algorithm

TSA begins from an initial starting solution. The search and its result will vary with each initial starting solution. A good initial approximation can improve time performance for searching process.

The algorithm maintains a memory structure which records solutions, attributes of the solutions encountered (objective function value) and the iteration number(s) in which the solution was visited. It also maintains an elite solution list of good solutions. The memory structure is used to determine if search is trapped in an attractor basin.

The corresponding parameters in Reactive TSA could be defined as follows: (1) rep = 3 (number of repetitions to be considered as “frequent” solution); (2) max_cycle = 2 (If a solution is repeated in less than 2 iterations since the last repeated solution, increase the tabu tenure).

The history of solutions visited is maintained during the search. If 2 solutions are visited more than 3 times each in the recent search history, the search is said to be trapped in an attractor basin. In this case, an escape process is performed clearing all tabu memory structures and a sequence of escape moves are performed to lead to a markedly different region of the solution space. In this problem, the escape mechanisms were implemented and tried: perform the most disimproving neighborhood move for a specified number of iterations.

The pseudocode of the main TSA program is presented in Appendix 2.

3.6 Numerical experiment

This section applies previous developments to a real flight scheduling. For this task an airline has provide real data, however due to the confidentiality agreement the name of the airline is not revealed.

Both techniques described and developed algorithms are applied to the same problem test. Finally, results are compared.

3.6.1 Test 1. Sensitivity analysis of computational cost for CEA-ESA

This numerical experiment demonstrates the evolution of computational cost for CEA-ESA when size flight scheduling increases.

First, given a set of flights with a variable size of network (Table 3.1), an analysis of number of feasible solutions is developed. Applying CEA for each size sub-problem it is possible to determine relationship between set size provided by CEA and network size. It is possible to observe that space dimension increases very fast. However, all subproblems are suitable for futher tests.

Table 3.1. Flight schedule – FS1.

Number of flight	Departure Airport	Schedule Departure Time	Arrival Airport	Schedule Arrival Time
1	1	8	2	10
2	2	10.5	1	12.5
3	1	9	3	12
4	3	12.5	1	15.5
5	1	10	4	15
6	1	14	2	16
7	2	16.5	1	18.5
8	4	16	1	21
9	1	16.5	2	18
10	2	18.5	1	20

Note: hours expressed in decimal system (i.e. 10:30 h = 10.5h).

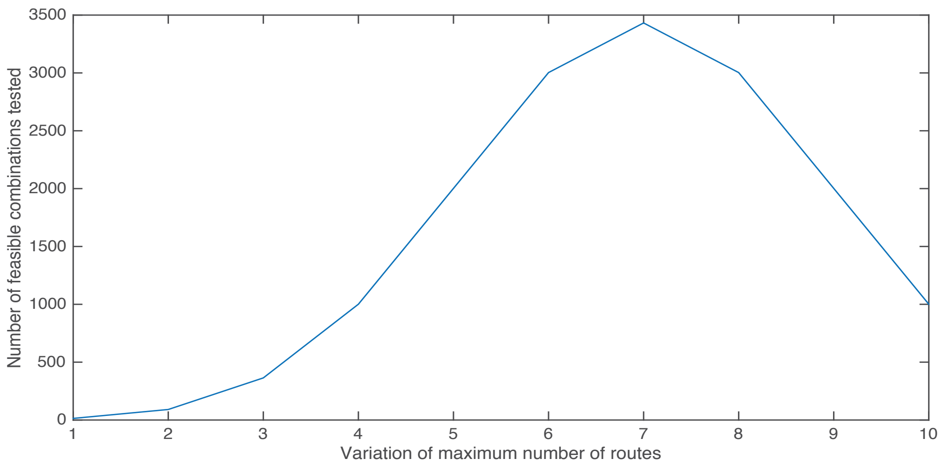


Figure 3.8. Feasible solutions vs. maximum number of routes for CEA-ESA-FS1.

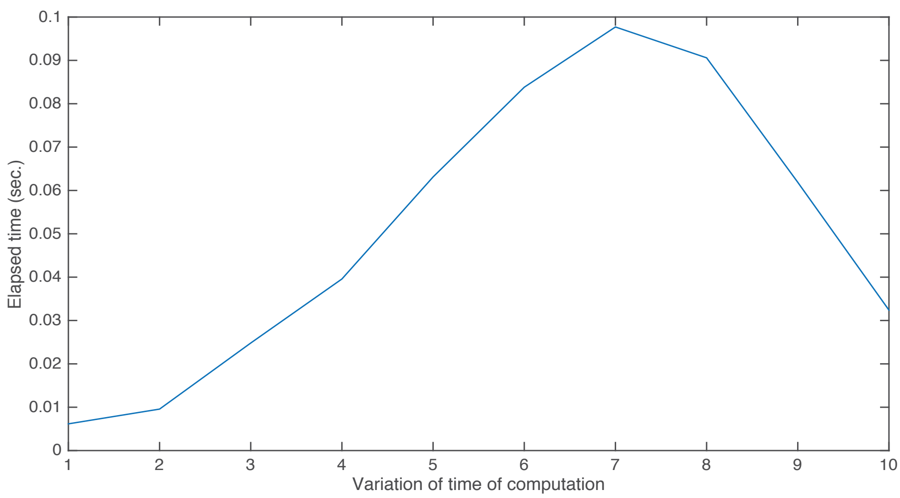


Figure 3.9. Computation time vs. maximum number of routes for CEA-ESA-FS1.

For example, this flight schedule has an optimum with 3 routes (flights: 1 2 6 7; 2 3 4 9 10; 5 8) and a total cost of EUR 131,450 per day.

Flight schedule FS2 (Table 3.2) shows other real case and it is solved applying CEA-ESA and Figure 3.10 and 3.11 show the evolution of time required for that with different fleet size.

Table 3.2. Flight schedule – FS2.

Number of flight	Departure Airport	Schedule Departure Time	Arrival Airport	Schedule Arrival Time
1	4	7.42	8	12.92
2	1	8.17	4	10.67
3	6	9.17	4	12.17
4	4	9.50	1	12.00
5	4	12.50	2	14.00
6	1	13.17	4	15.67
7	8	14.00	4	19.50
8	6	14.50	4	17.50
9	2	15.00	4	16.50
10	4	15.17	6	18.17
11	4	18.08	1	20.58
12	4	18.17	6	21.17

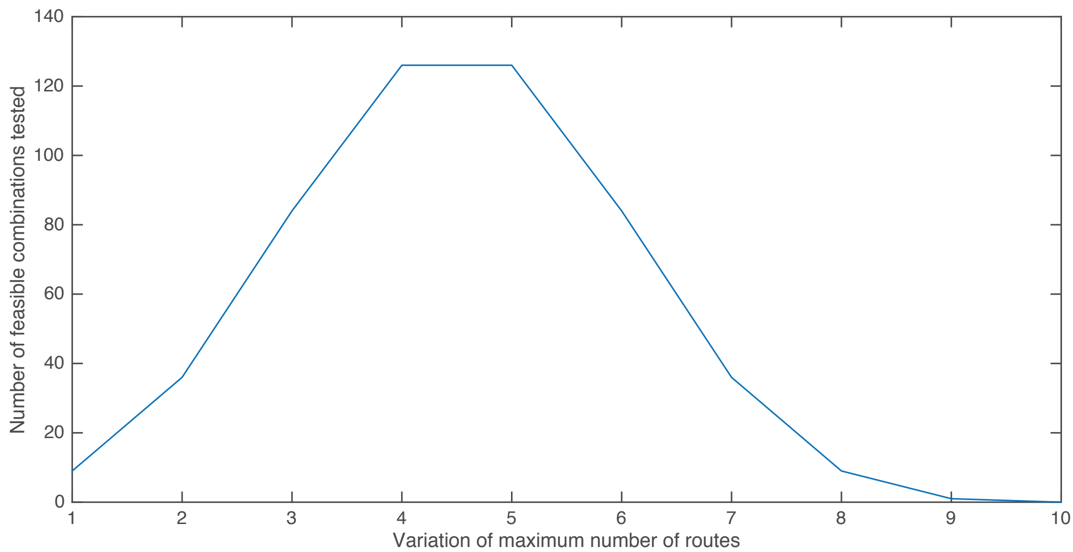


Figure 3.10. Feasible solutions vs. maximum number of routes for CEA-ESA-FS2.

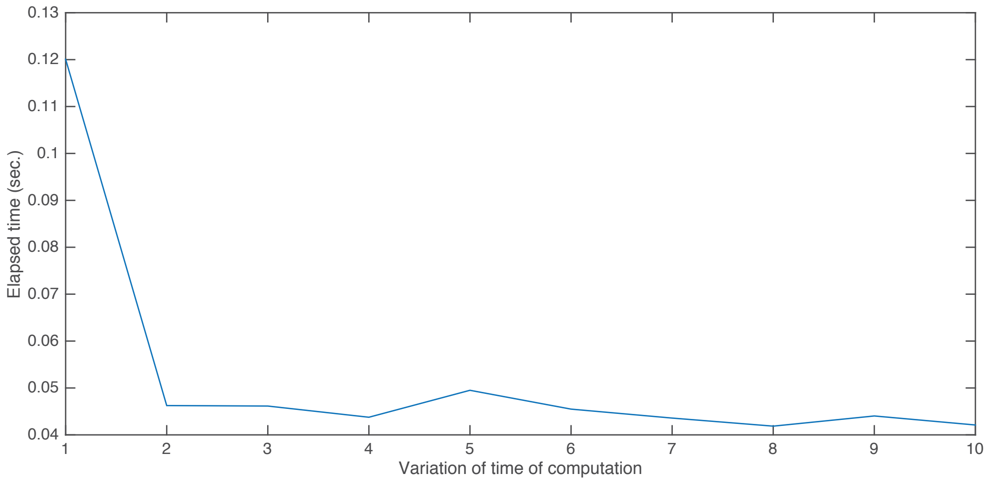


Figure 3.11. Computation time vs. maximum number of routes for CEA-ESA-FS2.

3.6.2 Test 2. Sensitivity analysis of computational cost for TSA

This numerical experiment demonstrates the evolution of computational cost for TSA for the same set of sub-problems generated in the precedent section.

Figure 3.12 shows the time of computation to solve flight schedule FS1 (Table 3.1) with a maximum of 200 iterations and maximum routes feasibles between 1 and 10. Automatically, comparison between CEA-ESA and TSA is possible. Observe that for small problems, TSA does not provide advantages because there are not many options to evaluate, then it is easy to evaluate them all.

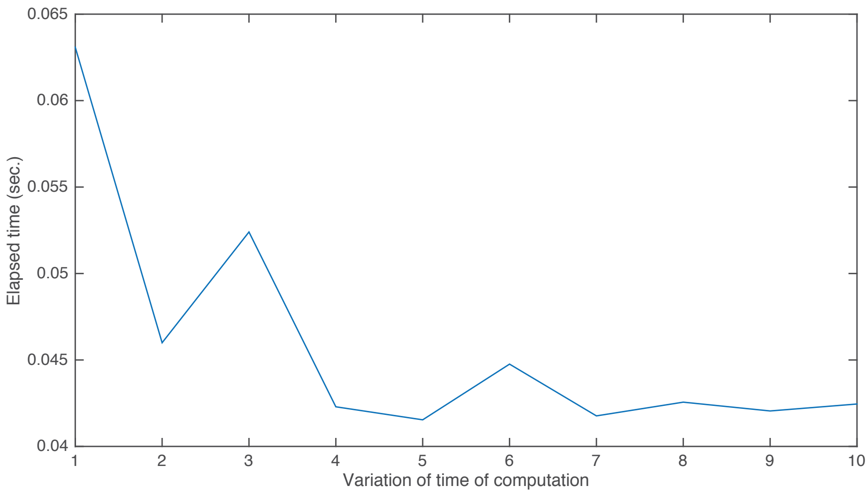


Figure 3.12. Computation time vs. maximum number of routes for TSA-FS1.

The main difference is to evaluate large flight schedules, when CEA-ESA is unfeasible or very costly. The following figure (Figure 3.13) shows computation time for FS2 with 100 iterations (enough to achive solutions for small problems).

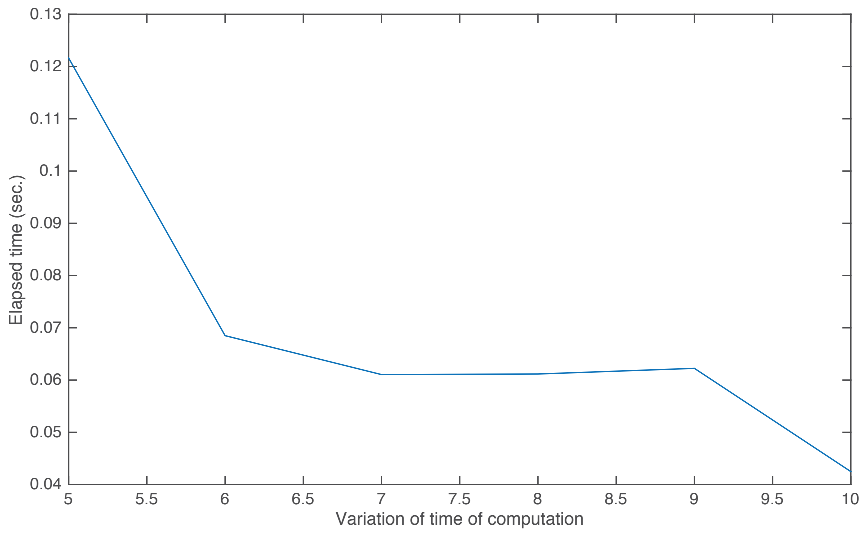


Figure 3.13. Computation time vs. maximum number of routes for TSA-FS2.

However, the main difference is that TSA allows solving large flight schedules like FS3 (Table 3.3). This flight schedule is too large and CEA-ESA is not valid. Then TSA is the alternative. For small number of iterations, the algorithm cannot achieve solutions. It is due because the number of feasible routes is high and the fleet small (permutations of 1s and 0s with low increment of value are done). Finally, the algorithm provides solutions but the main advantatge is computation time, being considerably larger than in previous tests.

Table 3.3. Flight schedule – FS3.

Number of flight	Departure Airport	Schedule Departure Time	Arrival Airport	Schedule Arrival Time
1	2	5.25	4	6.75
2	4	5.33	3	6.33
3	4	6.58	5	12.08
4	4	6.67	2	8.17
5	5	7.00	4	12.50
6	8	7.08	4	12.58
7	3	7.25	4	8.25
8	7	7.50	4	9.50
9	2	9.00	4	10.50
10	4	9.08	7	11.08
11	4	9.58	6	12.58
12	4	10.00	8	15.50
13	4	11.00	3	12.00
14	5	11.75	4	17.25
15	7	12.33	4	13.33
16	3	13.42	4	14.42
17	4	13.58	1	16.08

18	4	14.08	7	16.08
19	4	14.25	3	15.25
20	4	15.00	5	20.50
21	1	17.00	4	19.50
22	7	17.17	4	19.17
23	6	17.25	4	20.25
24	5	17.33	4	22.83
25	8	17.42	4	22.92
26	3	17.50	4	18.50
27	4	18.00	5	23.50
28	4	18.50	8	24.00
29	4	20.00	7	22.00
30	4	20.50	2	22.00

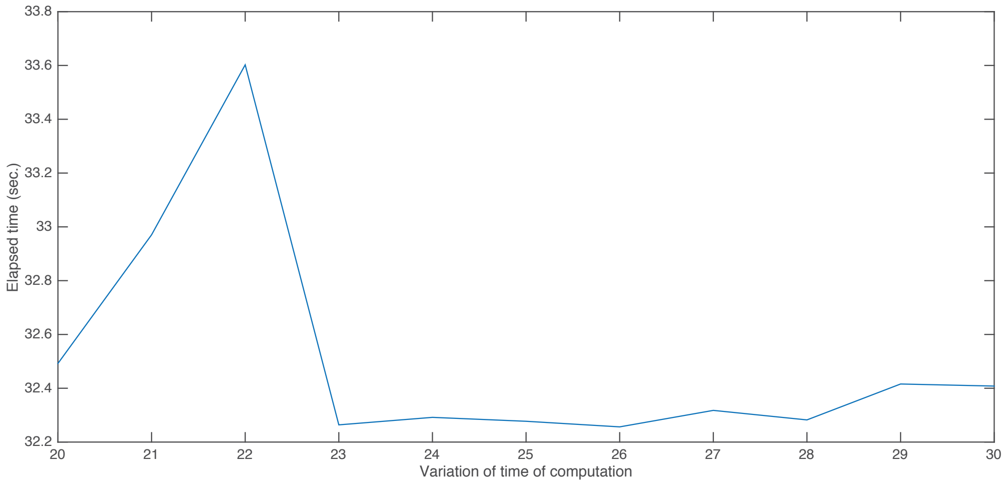


Figure 3.14. Evolution of time of computation with maximum number of routes for TSA-FS3.

3.6.3 Test 3. Applying CEA-ESA and TSA to the same schedule

Given a real flight scheduling operated by the airline with only one aircraft type (Table 3.4), the outputs for this problem are given in Table 3.5.

Table 3.4. Flight schedule – FS4.

Number of flight	Departure Airport	Schedule Departure Time	Arrival Airport	Schedule Arrival Time
1	1	8	2	10
2	2	10.5	1	12.5
3	1	9	3	12
4	3	12.5	1	15.5
5	1	10	4	15
6	1	14	2	16

7	2	16.5	1	18.5
8	4	16	1	21
9	1	16.5	2	18
10	2	18.5	1	20
11	1	17.5	3	20.5
12	3	21	1	24
13	1	12.5	2	14
14	2	14.5	1	16
15	1	20.5	2	22
16	2	22.5	1	24
17	1	11	5	14
18	5	15	3	16.5
19	3	17.5	5	19
20	5	20	1	23

Table 3.5. Outputs for both algorithms.

Maximum number of flights	**10**	**12**	**14**	**16**	**20**
TSA time	0.0656	0.2471	0.7224	3.9294	143.82
Max iteration	100	1000	3000	20,000	400,000
Tabu tenure	10	10	100	500	1,000
Feasible routes	14	19	28	63	68
Max teration	364	969	20,475	595,665	10,424,128
CEA-ESA time	0.0464	0.0705	0.6774	31.3584	785.9020
Routes	[2 10 11]	[5 11 15 16]	[2 15 18 26]	[3 33 37 54]	[3 33 38 55 56]
Flights	[1 2 6 7; 3 4 9 10; 5 8]	[1 2 11 12; 3 4 9 10; 5 8; 6 7]	[1 2 6 7; 3 4 11 12; 5 8; 13 14 9 10]	[1 2 6 7 15 16; 3 4 11 12; 5 8; 13 14 9 10]	[1 2 6 7 15 16; 3 4 11 12; 5 8; 13 14 9 10; 17 18 19 20]
Fleet	3	4	4	4	5
Crew pairs (sit 2h)	6	7	8	8	10
Total Cost (assuming variable cost, 10^5 EUR)	1.3145	1.6026	1.7648	1.9271	2.3774
Total Cost (assuming variable and fixed costs, 10^5 EUR)	1.2858	1.5509	1.7144	1.8080	2.2316

Note: both algorithms achieve the same solution.

Results indicate that TSA is more efficient for large problems because it needs fewer iterations (maximum allowed, but it usually finds solutions before arriving to the maximum) and less effort in computation time. However, unitary time is not so efficient because it has to try some permutations until it finds valid ones (not tenure). This particularity is specific of the code implemented for this work and it accepts some improvements (future research lines). Moreover, considering costs as variable ones simplify formulations in codes and developments, but at level of codification some improvements can be achieved introducing both parts (variable and fixed costs). There are some differences in total cost for operations when costs are considered accurately. Finally, while CEA-ESA requires fleet size before computation, TSA accepts good initial approximations and it makes a fine tunning, reducing fleet if it is possible (within a small interval).

3.7 Conclusions

The airline network planning problem is a large and complex problem because there are a lot of variables and key parameters that control the system. If this problem is compared against airline network design at strategic level (with analytical models) it is obvius that the degree of sophistication increases. Therefore, managing large variables and detailed flight schedules difficults understanding cause-effect relationships in the system. To analyse one scenario it is necessary to design accurately a large set of variables, a lot of parameters and one or two heuristics. If the goal that airline manager searchs is to define operating details, these methods are totally justified. On the other hand, if network configuration is the goal, then analytical models suit better because sensitivity analysis can be carried out with better balance of value and effort.

It is very important to remind that airlines run business in aggressive environment and they have to supply high frequencies and low fares (if they do not have monopolistic or oligopolistic positions in their markets). Therefore, cutting costs are very important in this industry and a variation of one aircraft in airline’s fleet has a strong impact on profit and losses account. Indentical consideration is valid for crew members because they are fixed costs from financial perspective. From this point of view, these algorithms can achieve good level of compacity for routing and pairing, reducing the number of resources that previous estimations could be provided by analytical methods.

Routing and crew pairing are similar problems. The main difference is that pairing problem has hard constraints in the feasible route generation problem and finally more crew pairings are needed to satisfy fligh schedule with higher costs.

Both algorithms developed in this section (ESA and TSA) have been very useful to solve airline network planning problem. On one hand, for small problems the combination of CEA and ESA is right because computation time is affordable and it achieves global optima. Moreover, it makes unnecessary to pay attention to determining the metaheuristics parameters. On the other hand, if the network is large or computational time has to be short, then applying TSA is better choice. Particularly, the TSA developed in this section starts with the same seed than CEA starts; therefore, comparisons are easy to be carried out.

TSA proposed here is robust because the searching mechanism is simple, however its cost could be high compared with other TSA estructures. For example, starting with a set of candidate routes and permutating flights could be an alternative for further developments.

Finally, the proposed strategy holds the traditional approach of separating the whole problem in different steps. Obviously, the integrated problem is more interesting from computation point of view. However, practices are mandatory in this field: airlines prefer to manage these four problems separately because they can supervise any one of them independently. In practice, airlines usually tend to outsource the crew scheduling, but never the fleet assignment or aircraft routing.

4 Airline network complexity

4.1 Introduction

Complex systems have always existed, but complexity has gone from something found mainly in large systems, such as cities, to something that affects a lot of common things and organizations. Systems that used to be separate are now interconnected and interdependent, which means more complexity in many cases. Furthermore, technology revolution has been a factor to increase level of complexity.

Complex organizations are far more difficult to manage than merely complicated ones. It is harder to predict what will happen, because complex systems interact in unexpected ways. Also, it is harder to make sense of things, because the degree of complexity may lie beyond cognitive limits. And it is harder to place bets, because the past behavior of a complex system may not predict its future behavior. In a complex system the outlier is often more relevant than the average.

On one hand, complicated systems have many moving parts, but they operate in patterned ways. It is possible to make accurate predictions about how a complicated system will behave. For example, flying a commercial airplane involves complicated but predictable steps. On the other hand, complex systems, by contrast, are imbued with features that may operate in patterned ways but whose interactions are continually changing.

Three properties determine the complexity of an environment. The first, multiplicity, refers to the number of potentially interacting elements. The second, interdependence, relates to how connected those elements are. The third, diversity, has to do with the degree of their heterogeneity.

It is possible to understand both simple and complicated systems by identifying and modeling the relationships between the parts; the relationships can be reduced to clear, predictable interactions. It is not possible to understand complex systems in this way, because all the elements are interacting continuously and unpredictably.

This chapter aims to develop an analysis of the airline network from the point of view of complexity science. It is structured in three sections. First, includes an introduction to complex theory and its applications to airline industry. Second, a network analysis applying complexity indices is presented. Finally, a methodology of reliability estimation and cost associated to delay propagation is developed.

4.1.1 State of the art

Network analysis has already a long history in operations research and quantitative social science research. In the past, much attention has been paid to shortest-route algorithms (for example, the traveling salesman problem), where the spatial configuration of networks was put in the centre of empirical investigation. Integer programming, linear and nonlinear programming turned out to offer a proper analytical toolbox. In recent years, there are several new trends; in particular, the rise of hub-and-spoke systems in liberalized networks, the emergence of dynamic adjustments to new competitive conditions and the increase in complexity in international networks.

Furthermore, it appears that in the past decades many social, spatial and economic systems show an organized pattern characterized by network features, such as transportation, telecommunication, information or energy systems. As a consequence, much attention has recently been paid to the study of network properties emerging in many social, spatial and economic fields, as witnessed by the vast amount of literature published in the past years (Barthélemy, 2003; Gorman and Kulkarny, 2004; Reggiani and Nijkamp, 2006).

Air transport shows indeed clear network features, which impact on the way single airline carriers operate (Button et al., 2000). The abundant scientific literature on airline networks has addressed this topic in terms of theoretical modelling and empirical measurements on different typologies of airline network configurations. This strand of recent research aimed to measure the network structure in relation to the new trends in airline business strategies denoted as 'low cost' principles. Low cost carriers developed rather fast after the deregulation policy, by acquiring a competitive network advantage on traditional airlines, which consequently seemed to reorganise rapidly their airline network to respond to the new market dynamics.

In this context, interesting research has emerged that mainly addressed the issue of describing and classifying networks by means of geographical concentration indices of traffic or flight frequency (Caves et al., 1984; Toh and Higgins, 1985; Reynolds-Feighan, 1994, 2001). These measures, such as the Gini concentration index or the Theil index, provide a proper measure of frequency or traffic concentration of the main airports in a simple and well-organized network. However, if a real-world network structure is complex, including multi-hub or mixed point-to-point and hub-spokes connections, the concentration indices may record high values for all types of structure, but fail to clearly discriminate between different network shapes (Alderighi et al., 2007). There is a need for a more appropriate measurement of connectivity structures in complex networks.

However, indices derivated from spatial network analysis are static and often a dynamic perspective of network is necessary. One of the main problems in field of airline management is delay propagation. Delays have a strong impact on operational reliability and these impacts directly on profit and loss account and passenger experience.

Delay propagation is the result of different factors, including the lack of coordination of airline flight schedules, finely tuned airline flight schedules with little slack to dampen delay propagation, high levels of congestion preventing re-accommodation of delayed flights, or high aircraft load factors preventing timely re-accommodation of passengers who misconnect or whose flights are cancelled. All combine to create passenger disruptions and lengthy passenger waits that exceed the levels of flight delays.

According to Eurocontrol (2014) there were 1.7% more flights per day in the reference area than in 2013. However, data received directly from airlines by CODA describing delays from all-causes illustrated a stable situation for the network during the year. First, the average delay per delayed (ADD) flight of 26 minutes per flight; this was a small decrease of 2.6% when compared to 2013 where the ADD was 26.7 minutes. Secondly, this small improvement was offset by a small increase of 1.3 points to 37.4% of flights delayed on departure (>=5 minutes). Thirdly, the share of reactionary delay was 44% of delay minutes reported compared to 45% in 2013. Finally, regarding arrival delay, the average delay per delayed flight on arrival from all-causes was 27.2 minutes per flight in 2014. Then, the percentage of delayed flights increased by 0.7 percentage points to 34.3% and operational cancellations remained stable at 1.5% of planned flights.

Delays cause immense losses to the Air Traffic System, a situation that will worsen in the near future if traffic increases. Models and methods allowing stakeholders to characterize mechanisms behind delay propagation, to forecast network congestion, and to optimize planning and operational practices to mitigate delays are thus of great relevance.

Researchers who have been studying the performance of ATM have done a significant effort to identify the causes for initial or primary delays (Rupp, 2007). These primary delays can in turn trigger a cascade of secondary delays as was noted in (Beatty et al., 1999; Jetzki, 2009) by the introduction of a ripple effect. Because of the inherent complexity of the mechanisms that produce and boost delay spreading, different modeling techniques were proposed. A first line of research focused on simulating the air traffic system as a network of queues without considering information on aircraft schedules (Schaefer and Miller, 2001). A second line of research was devoted to analytical approximations for modeling the airport as a dynamic queuing system with varying demand and service rate (Malone, 1995). Another analytical queuing model was used in (Pyrgiotis et al., 2013). In this work, airports were modeled as dynamic queues and implemented in a network. The authors ran the model in a network of 34 airports with a specific algorithm that accounts for downstream propagation of delays. An additional body of work uses statistical tools to predict the delay patterns observed in the data. Such techniques could be classified into traditional linear regression models (Churchill et al., 2007), artificial neural networks (Sridhar et al., 2009) and Bayesian networks (Xu et al., 2005).

Technologically driven transport systems are characterized by a networked structure connecting operation centers and by dynamics ruled by pre-established schedules. Schedules impose serious constraints on the timing of the operations; they condition the allocation of resources and define a baseline to assess system performance. Technical, operational or meteorological issues affecting some flights give rise to primary delays. When operations continue, such delays can propagate, magnify and eventually involve a significant part of the network. Metrics have been defined to quantify the level of network congestion. The results indicate that there is a non-negligible risk of systemic instability even under normal operating conditions. Passenger and crew connectivity were also identified as the most relevant internal factor contributing to delay spreading.

4.1.2 Objective

The main objective of this chapter is to describe a network in terms of complexity and to estimate the cost of managing this complexity. Then, specific objectives are:

1. To ropose a set of indices to describe airline network complexity based on previous works about complexity of networks.
2. To apply these indices to a real airline network and derivate some highlights.
3. To understand the reliability of network in terms of delay propagation and estimate the cost.

4.2 Indices of complexity

This section aims to investigate the scientific potential and applicability of a series of network connectivity/concentration indices, in order to properly typify and map out complex airline network configurations. Specifically, several network indicators will be adopted and tested to describe the main properties of airline systems.

The goals of this section are then: first, to detect the extent to which the real network configuration is close to typical network models that evolved over time and, secondly, to examine how concentration measures can point to the different network topologies.

Many economic activities are currently characterized by network characteristics with a high degree of complexity, since their processes and outcomes depend not only on the choices of the single agents but also on the dynamic – often nonlinear – interactions between them in a continuous dynamic interplay (Reggiani and Nijkamp, 2006). A clear example of a complex spatial-economic network is the geographical network of the air transport industry: understanding its peculiarities and responding to these features can bring about substantial advantages for both consumers and producers (Button and Stough, 2000). The focus of this work is not the whole air transport industry; really, it is focused on one airline and how indices can provide information for decision making processes.

Modelling complex networks is also a great challenge: on one side, the topology of the network is governing the complex connectivity dynamics; on the other side, the functional-economic relationships in such networks might also depend on the type of connectivity structure. The understanding of these two interlinked network aspects may be instrumental for capturing and analysing airline network patterns.

4.2.1 Problem statement

The analysis and representation of complex systems as complex networks has been a growing trend in the last years (Strogatz, 2001). Moreover, complex networks have been crucial in order to understand many emergent phenomena in systems with a large number of interacting actors. Such formalism has been successfully applied to the study of many transportation systems: railways (Kurant and Thiran., 2006), subways (Latora and Marchiori, 2002) and air transport (Zanin and Lillo, 2013).

There are some some standard metrics that are usually considered when a network has to be characterized and some of them are presented in following subsections.

Degree index

Airline networks may exhibit simple or complex topologies. Networks have been given several definitions in the framework of graph theory. In this context it is useful to outline some indicators most frequently used to represent the network shape.

Considering the adjacency matrix $\boldsymbol{A} = \{a_{ij}\}$ associated with the network (a_{ij} = 1 if the node i is connected with the node j and a_{ij} = 0 otherwise), the degree is the number of links connected to a node (Eq. 4.1):

$$\mathrm{k}_i = \sum_j a_{ij} \tag{4.1}$$

Many generalizations of the standard topological metrics for weighted networks were introduced, in order to take into account the strength of the connection in topological measures (Barrat et al., 2004).

The most common metrics measured in weighted networks is the strength (Eq. 4.2), where w_{ij} is the weight on the link (i, j) and a_{ij} is the element of adjacency matrix. These weights are flights from airport i to airport j, however it could be number of passengers or tonnes of cargo. This metric is a generalization of the degree of a node and in transportation networks is usually a measure of the traffic handled by a node (passengers, freight or operations). As well as the degree, also the strength is usually power-law distributed in many real world networks.

$$s_i = \sum_j w_{ij} a_{ij} \tag{4.2}$$

Figure 4.1 presents a degree index (left) for a set of ten nodes that they constitute a point-to-point configuration. For strength index (right), flights have been created randomly, with the aim of show the behaviour. Both figures represent a non-concentrated network.

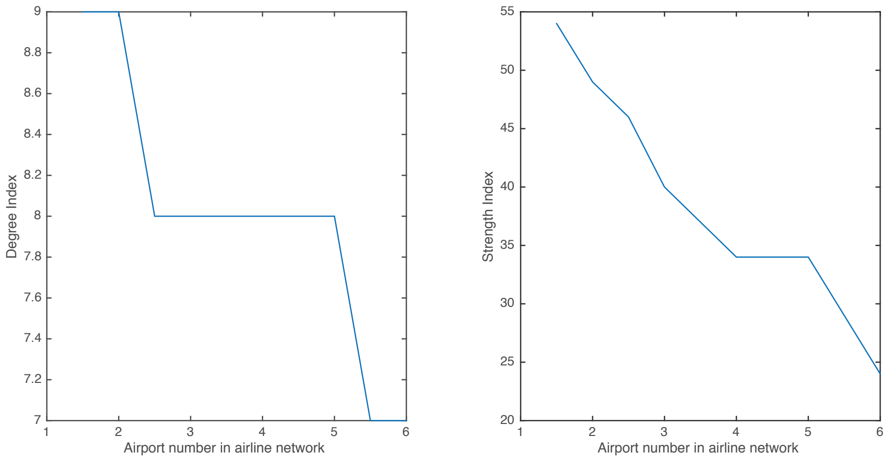


Figure 4.1. Degree and strength index for point-to-point network.

When the same indices are analysed in a hub-and-spoke network, the concentration increases. Figure 4.2 presents distributions, degree and strength, for a set of ten nodes. Observe that the probability of be well connected is high for the first airport in the network and the others are significatively less connected. If degree is measured in terms of links and not flights (weights) this evidence is clearer. Furthermore, it could be very interesting to compare strength for flights (supply) with strength for passengers (demand).

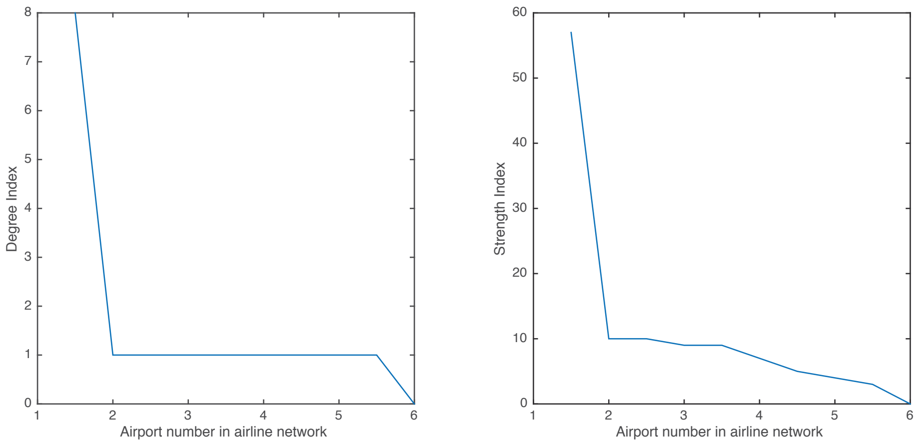


Figure 4.2. Degree and strength index for hub-and-spoke network.

Network degree distributions

The vertex degree distribution or network degree distribution is one of the key concepts to understand network configuration. This function determines how nodes are connected and it is defined as probability of finding nodes with k links. This index is defined as follows (Eq. 4.3), where N(k) is the number of nodes with k connections and n is total number of nodes.

$$\mathrm{P(k)} = \frac{N(k)}{n} \tag{4.3}$$

Furthermore, the distribution of the degree $P(k)$ is a metric that can give interesting information about the air transport system. Considering an airline, its network distribution follows a power-law indicating the presence of a relevant number of hubs (highly connected nodes that are particulary important for the network) (Caldarelli, 2007). However, $P(k)$ can be peaked in spatial networks and that makes this distribution less interesting for their characterization.

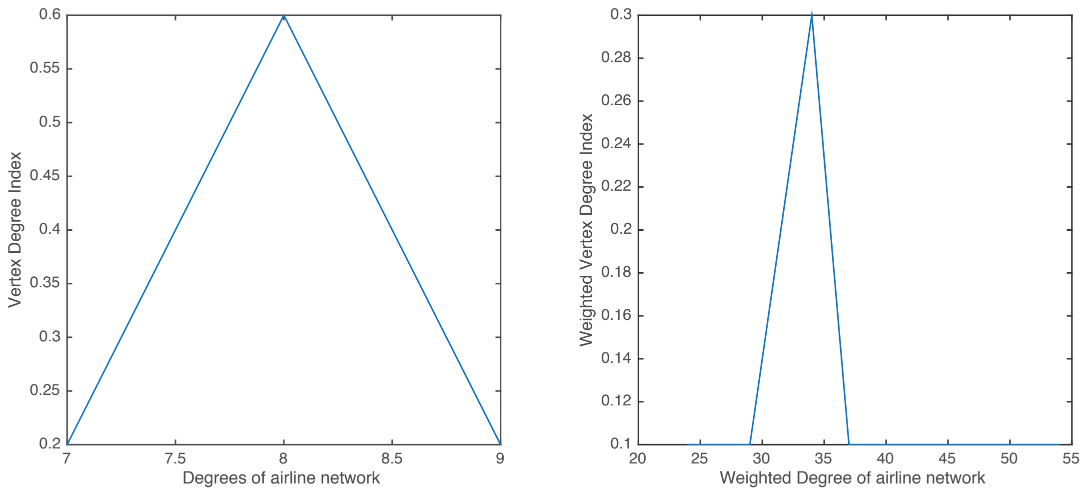


Figure 4.3. Vertex degree distribution for point-to-point network.

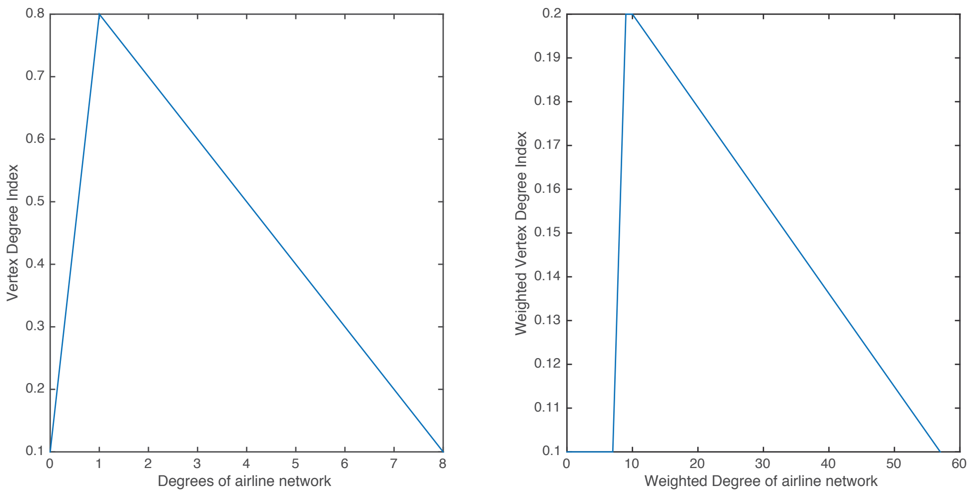


Figure 4.4. Vertex degree distribution for hub-and-spoke network.

Clustering coefficient

The clustering coefficient is defined by Equation (4.4), where k_i is the degree of the node i and E_i is the number of links connecting the neighbors of i. This index gives information about the spatial structure of the network and depends on the number of triangles present in the network.

$$\mathrm{C}_i = \frac{E_i}{k_i(k_i-1)} \tag{4.4}$$

The weighted clustering coefficient is the generalization of Equation (4.4) that measures the local cohesiveness taking into account the intensity of the connections of the triplets:

$$\mathrm{C}_w(\mathrm{i}) = \frac{1}{s_i(k_i)-1} \sum_{jh} \frac{w_{ij}+w_{ih}}{2} A_{ij} A_{ih} A_{jh} \tag{4.5}$$

For hub and spoke networks (pure) cluster coefficient is zero because is not possible to evaluate C. In complex network theory a value of zero is assigned for isolated nodes. However, Kaiser (2008) proposes a correction for coefficient of these nodes (not considered here).

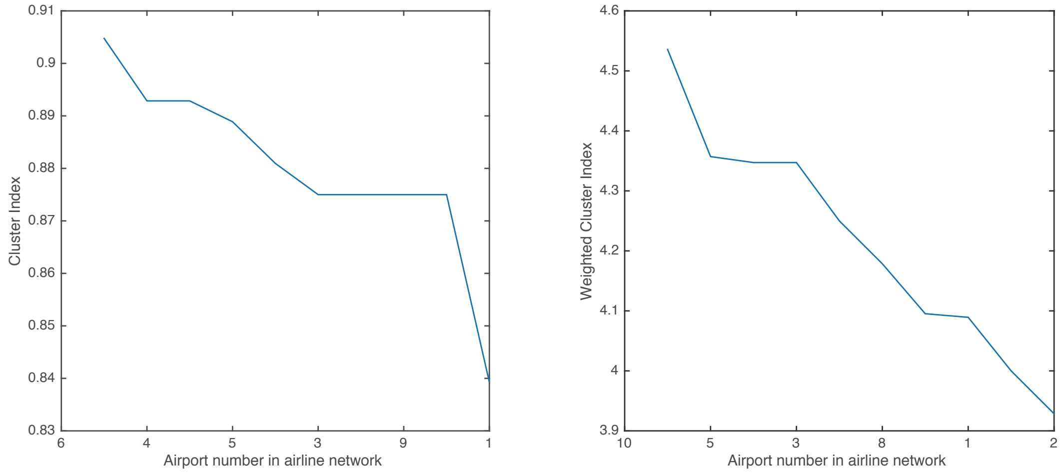


Figure 4.5. Cluster coefficient for point-to-point network.

Average nearest-neighbors degree

The average nearest-neighbors degree gives information about classes of degree. This index is related to the correlations between the degrees of connected vertices (Eq. 4.6). Where $\Gamma(i)$ is the set of the neighbors of i. This quantity can identify the presence of assortative or disassortative mixing of the degree (nodes with large degree tend to connect to other high degree nodes or with low degree nodes, depending on its behavior as k grows).

$$\mathrm{k}_{nn,i} = \frac{1}{k_i} \sum_{j\in\Gamma(i)} k_j \tag{4.6}$$

These metrics concern to the topological structure of the network, but they disregard the information about the weights of the links. Weights are a valuable instrument to characterize a network since they describe the intensity of a connection. In case of pure airline networks, the differences are significatively as it is possible to observe in Figures (4.6) and (4.7).

The weighted generalization for average nearest-neighbors degree is defined as follows (4.7):

$$\mathrm{k}_{nn,i}^{w} = \frac{1}{s_i}\sum_j A_{ij} w_{ij} k_j \tag{4.7}$$

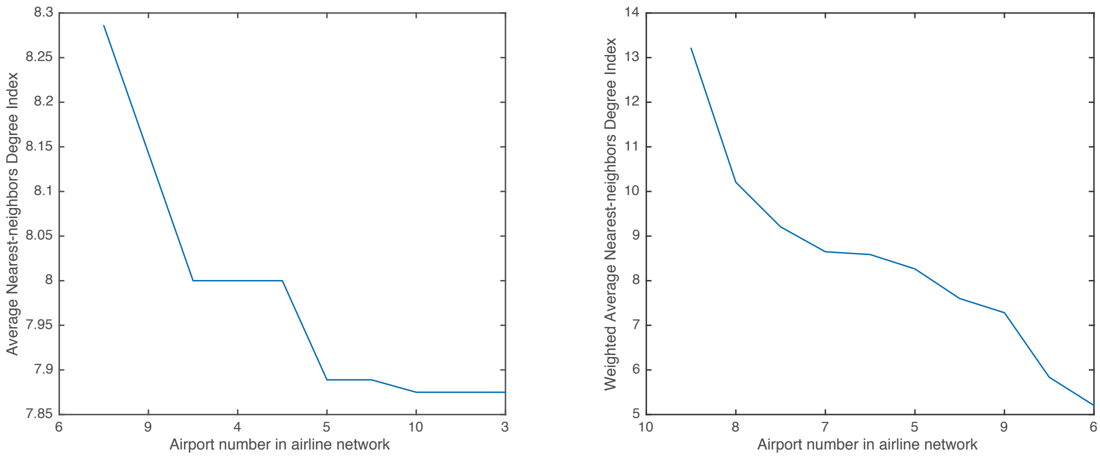


Figure 4.6. Average nearest-neighbors degree for point-to-point network.

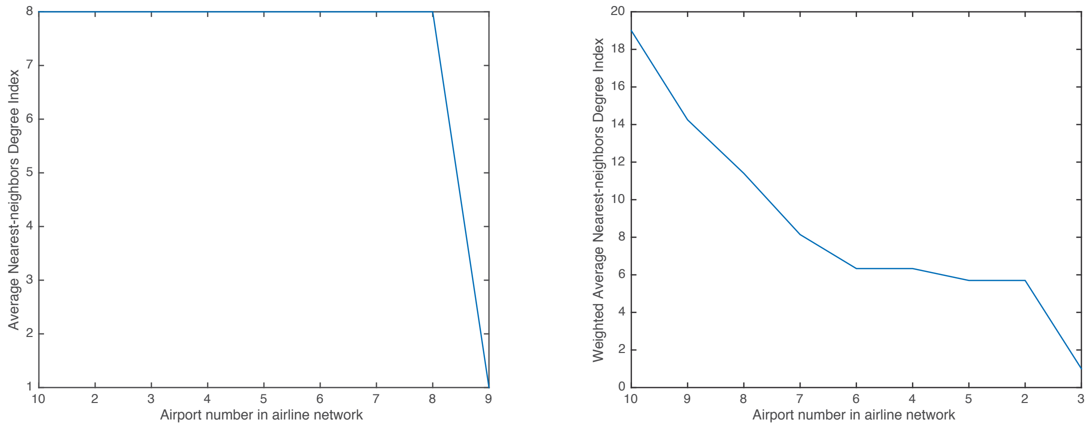
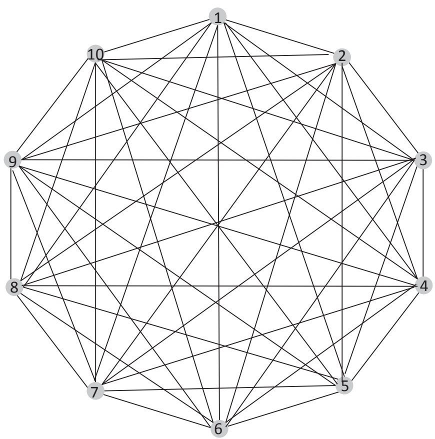


Figure 4.7. Average nearest-neighbors degree for hub-and-spoke network.



		Arrival airport									
Flights		1	2	3	4	5	6	7	8	9	10
Departure airport	1	0	9	9	7	8	8	8	10	10	7
	2	9	0	6	4	6	8	4	9	8	9
	3	6	1	0	8	10	2	10	5	7	6
	4	2	1	8	0	4	4	8	10	10	5
	5	3	7	4	0	0	10	2	10	6	5
	6	7	6	9	10	3	0	6	5	1	0
	7	4	5	5	2	5	10	0	4	5	5
	8	4	1	4	3	8	0	7	0	10	4
	9	3	0	7	8	6	5	4	3	0	1
	10	2	0	3	2	0	2	2	0	8	0

Figure 4.8a. Point-to-point network for test.

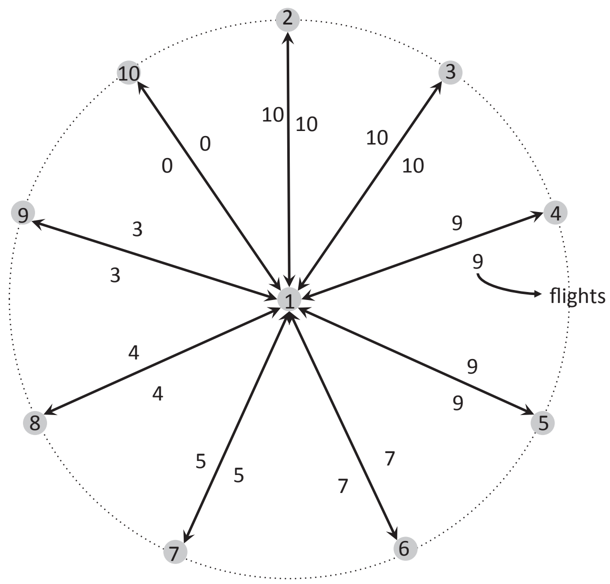


Figure 4.8b. Hub-and-spoke network for test.

Gini index

A central issue in networks characterization is the identification of the most important nodes according to some given criterion. Usually this is done by means of centrality metrics used to rank the nodes of the network. The degree of the nodes is one of the most natural metric of centrality that can be considered but it could lead to misleading classifications since low degree nodes could be important since they may be bridges connecting different part of the network (Eq. 4.1).

Gini index is a measure of geographical concentration. Equation (4.8) defines Gini Index (Cento, 2006), where f_i, f_j are the number of weekly flights from airports i and j (ranked in increasing order; n is the number of airports in the network; $\mu = \sum_i f_i/n$).

$$G = \frac{\sum_i \sum_j |f_i - f_j|}{2n \quad \sum_i f_i} \tag{4.8}$$

For previous examples, Gini Index has been calculated. Point-to-point network has a G=0.1115 and hub-and-spoke has a G = 0.9000. Furthermore, a simple experiment is carried out and a simple point-to-point network with 10 nodes is defined, all of them are interconnected and flights are simulated with uniform distribution. Progressively, flights for pair of airports (i,j) are cancelled and are migrated to hub-and-spoke system (evolutionning through mix network) until obtainning pure configuration. Gini index changes in this process and Figure 4.9 shows this evolution. Gini for hub-and-spoke does not achieve value of one because flights are well distributed in the feeders.

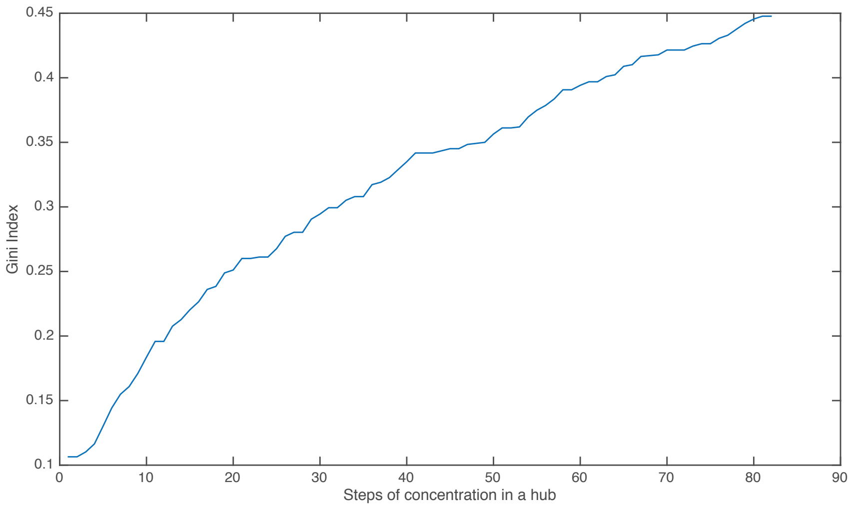


Figure 4.9. Evolution of Gini Index from PP network to HS network.

4.2.2 Case study

This section presents an empirical and short study case applying indices of complexity to data of real airlines. Due confidentiality agreement the identity of these airlines are not revealed.

Airline 1 is a short-medium haul low cost carrier and it is small and a start-up project. For this reason, the number of flights for one day is large but not comparable with airline 2 (set of flight data is presented in Appendix 3). Airline 2 is a large low cost carrier and one of the most important LCC in Europe.

Figure 4.10 shows the degree index and strength index for airline 1. This airline manages a point-to-point network but very concentrated in three bases around Europe. For more than 300 flights in one week and more than 60 airports, both indices are presented here.

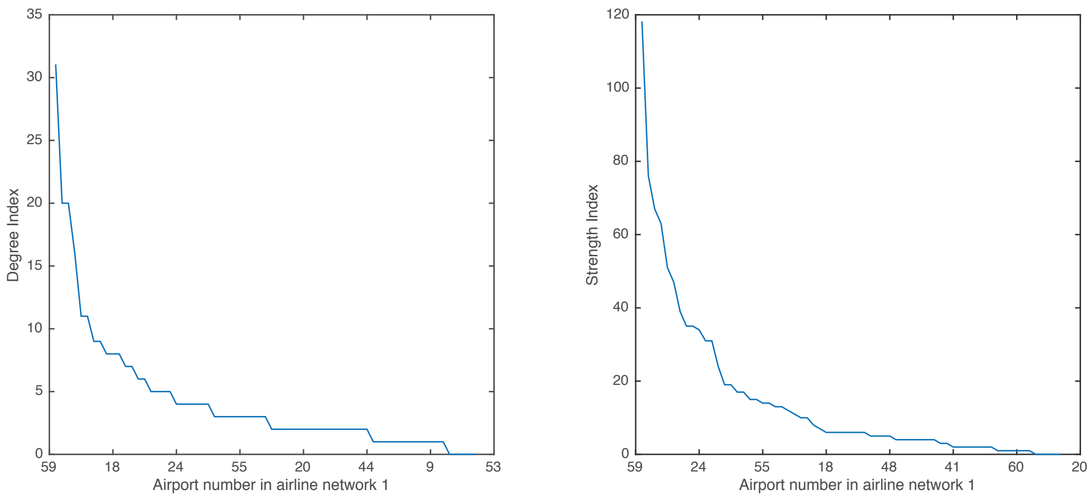


Figure 4.10. Degree and strength index for Airline 1.

Airline 2 is a LCC and it is focused on short and medium haul, but it manages more than 180 airports (more than 70 bases) and more than 2,000 relations each day. The degree and strength indices are shown in Figure 4.11. The level of concentration is higher than concentration of airline 1 and it is due to the fact that airline 2 dominates some markets and it supplies high frequencies in some routes very profitables.

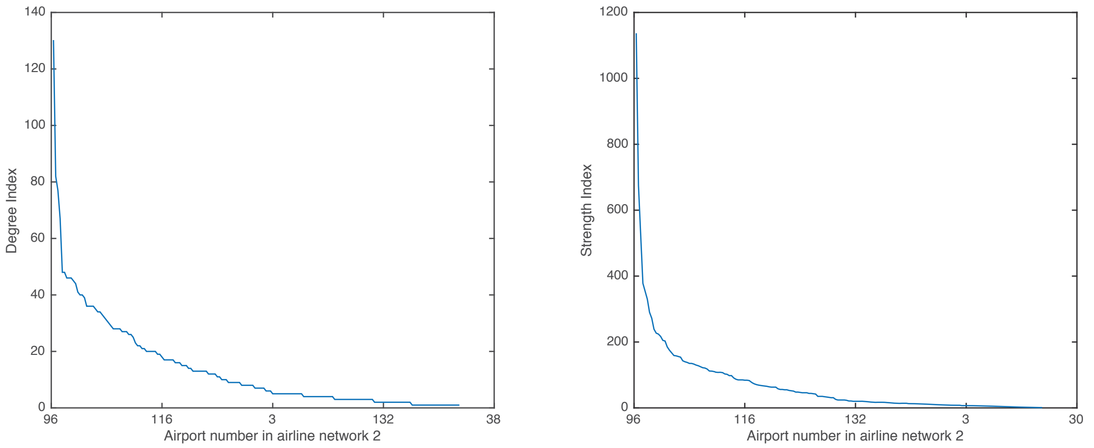


Figure 4.11. Degree and strength index for Airline 2.

It is not surprising that both low cost airlines show distributions relatively concentrated despite of the fact they manage routes point to point. First of all, from a business point of view, airlines have the need to concentrate main operations in a few bases because it is easier to control the cost. Furthermore, it is easier to give reliability to the network. Second, one of the critical aspects it is to not provide connections for passengers, this issue allows managers to not transfer delays in connection times. In practice, low cost airlines manage the best indices of puntuallity.

Regarding network degree distribution (vertex and weighted vertex degree), figures 4.12 and 4.13 show distributions for both airlines. These present peaks are due to assymetric level of supply and spatial distribution, especially for large airlines (this fact can be related with the schedule design).

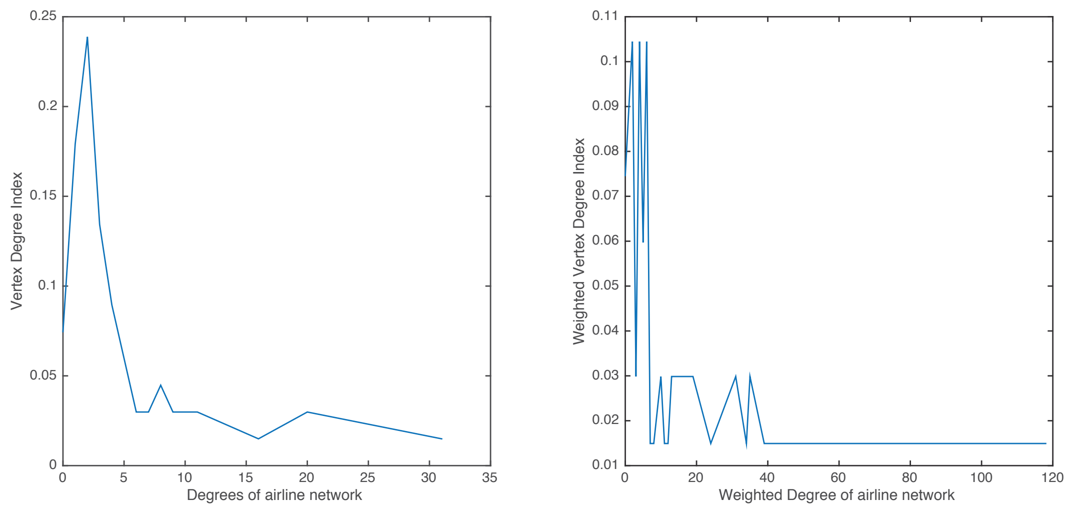


Figure 4.12. Network degree distribution for Airline 1.

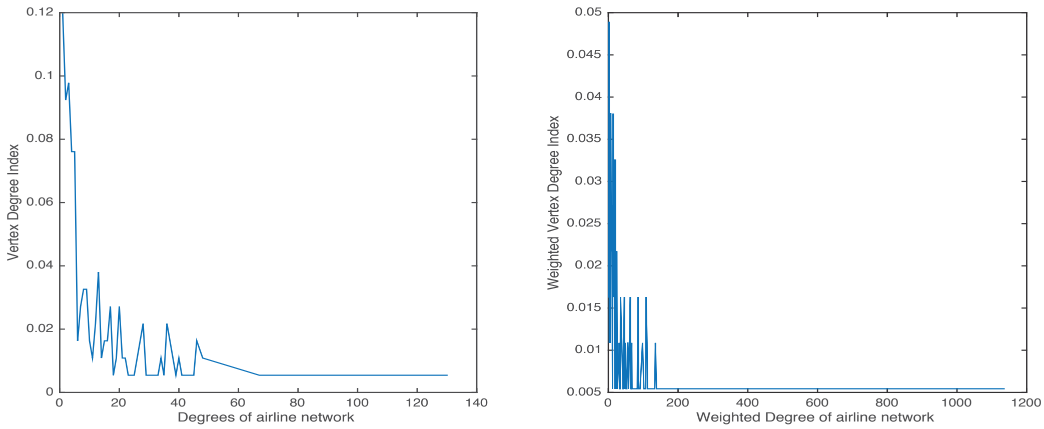


Figure 4.13. Network degree distribution for Airline 2.

Cluster cofficients are more interesting for this analysis and comparatively; airline 1 presents a distribution with fewer tendencies to clusterization. The main reason is that this airline manages only a few bases around Europe and less flights than airline 2. With density and radial configurations, these networks tend to create clusters.

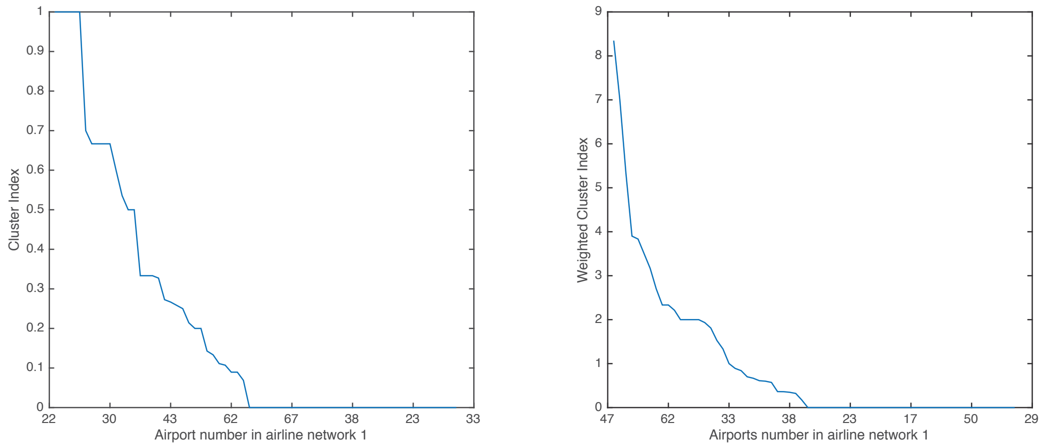


Figure 4.14. Distribution of cluster coefficient for Airline 1.

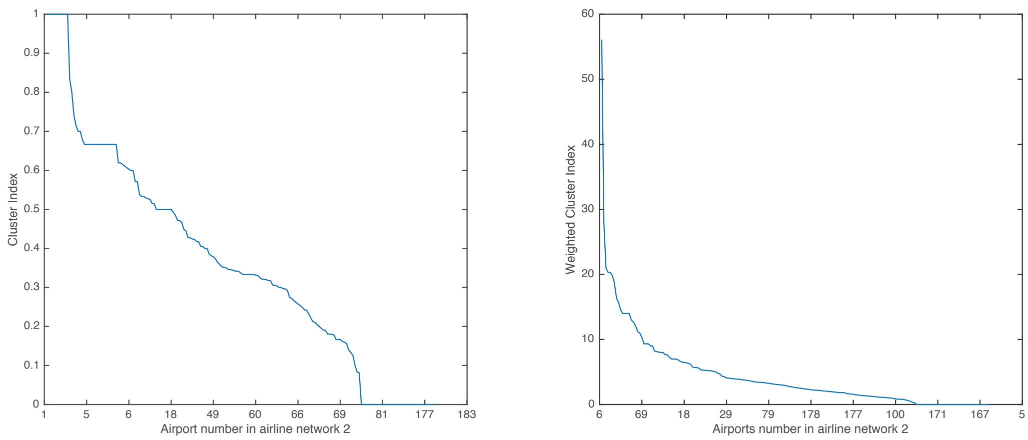


Figure 4.15. Distribution of cluster coefficient for Airline 2.

Average nearest-neighbors degree for airline 1 and 2 are presented in following figures. As degree index and cluster coefficient, this metric allows recognized patterns of concentration much more important for airline 2 than airline 1.

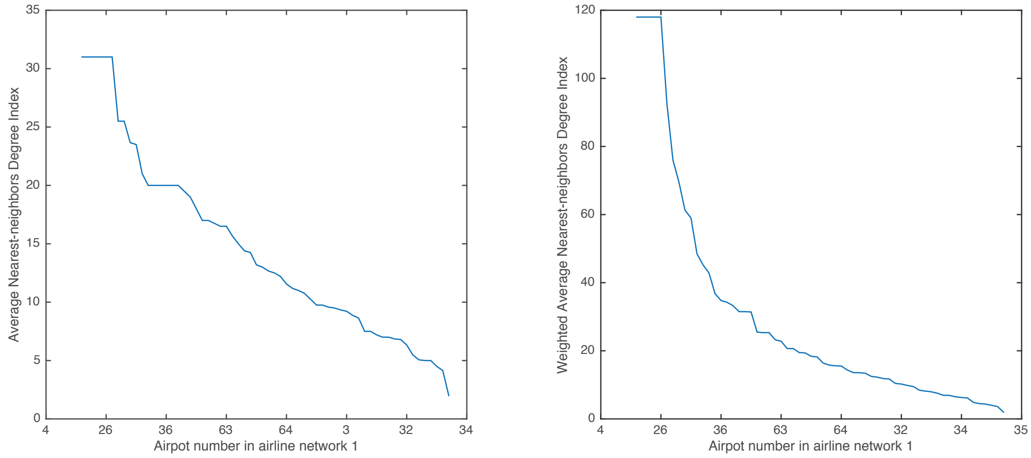


Figure 4.16. Average nearest-neighbors degree for Airline 1.

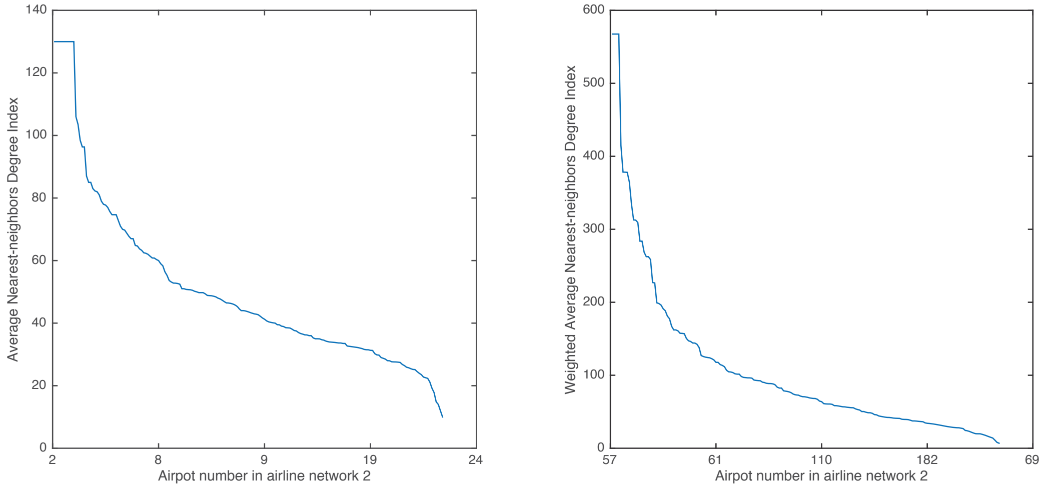


Figure 4.17. Average nearest-neighbors degree for Airline 2.

Finally, Gini index has been calculated for both airlines, being G_1=0.6310 and G_2=0.6422. Taking into consideration that they are low cost airlines with point-to-point configurations, the level of concentration is very important. This fact highlights the conclusion that if low cost and point-to-point airlines manage better levels of realiability is due to the fact of not providing connections at their main bases.

4.3 Delay propagation

Direct costs originated by delays amounted in Europe to 1,250 million euros during 2010 according to the European airline delay cost references values report from the Westminster University (Cook and Tanner, 2011). Understanding delays propagation in the airport network starting from primary events is thus of high economic relevance for air transport stakeholders. When a disruption is presented, airline managers try to minimize the impact by revisiting flight schedule as quickly as possible. Several factors can mitigate or recover the original plan, however usually the original plan is not achieved. The system when is perturbed is not reversible at 100% because appears complex dynamics.

Several studies analysed statistical data to find cause-effects relations between air transport schedules and the reactionary delays distributions in the network. Furthermore, optimization of airline schedules is an important field of research, where the general objective is to mitigate the spreading of delays. In addition, propagation trees are a useful tool for tracking the propagation originated at some point in the network and evaluate the impact and the cost (Beatty, 1999).

Finally, delay management is incorporated to airline operational process strategically. There are two mechanisms. First, introducing buffers of time in the schedule (buffering or padding) and holding the aircraft at main bases to recover the service if the perturbation is very high. Both of them impacts negatively in profit and losses account. Leasing of B777 is near USD 1.2 million per month without taking into consideration human labor necessary to manage these extra resources. Airline industry is focused on cost control because it is one of keys of competition.

This section is focused on understanding the relationship between delay propagation and overcost for airline. A mathematical model is developed for this purpose which is linked to results of Chapter 1.

4.3.1 Problem statement

Let us consider a flight scheduling and aircraft routing. This is the basis for a propagation tree scheme.

Delays follow statistical distributions. First, flight time can present an average delay μ_F per one hour of last and a statistical deviation σ_F, considering $N(\mu_F, \sigma_F) = N(1.78,\ 3.30)$ in minutes per hour. Secondly, turnaround time can be delayed in the same way, with average delay μ_G per one hour of ground time and σ_G statistical deviation, $N(\mu_G, \sigma_G) = N(2.32,\ 3.43)$ in minutes per hour. This basic assumption improves simplicity and it is consistent from a conceptual point of view.

Airlines can plan or test their flight schedules with characteristic flight and turnaround times (Eq. 4.9a, 4.9b). Where k is the parameter of reliability that they want to considerer to hedge their operation. Finally, these expressions are transformed to other ones more useful (involving characteristic delays of desing φ, ρ).

$$\widehat{\mathrm{t}_F} = \mathrm{t}_F(1 + \mu_F + k\sigma_F) \rightarrow \widehat{\mathrm{t}_F} = \mathrm{t}_F(1 + \varphi) \tag{4.9a}$$

$$\widehat{\mathrm{t}_G} = \mathrm{t}_G(1 + \mu_G + k\sigma_G) \rightarrow \widehat{\mathrm{t}_G} = \mathrm{t}_G(1 + \rho) \tag{4.9b}$$

Airports and airlines have the possibility to recover part of delays operating some factors. However, this work does not consider this aspect, so the propagation is considered inelastic through the same route. Furthermore, if the airport is congested, it can transfer delays between flights inside of different routes. This transferability is modelled with a parameter that depends on level of utilitzation of the airport and hubbing practices of the airline. Airline network configuration is a key aspect to understand delays propagation.

4.3.2 Analytical approach

This section recovers the formulation of airline costs for a given network configuration of Chapter 1. From strategical point of view, a planner could be interested to preevaluate operational cost taking into consideration different values of flight and turnaround characteristics times.

For point-to-point network, the formulation involving Eq. 2.9a and 2.9b results in Eq. 4.10 (a,b).

$$C_T = \mathrm{n}(n-1)f\left(t_F(1+\varphi)\left(p_K k(q) + m(q)\right) + \mathrm{c}_N\right) + \mathrm{J}(\mathrm{o(q)} + \xi \mathrm{e(q)}) + \theta dn(n-1)\left(\frac{12\alpha}{f} + \beta t_F(1+\varphi)\right) \quad (4.10a)$$

$$\mathrm{J} = \max\left\{\left[n(n-1)\frac{t_L(1+\theta)(1+\rho)}{2t_e} f\right]^+, \left[n(n-1)\frac{t_L(1+\varphi)(1+\rho)}{t_e}\frac{d}{q}\right]^+\right\} \quad (4.10b)$$

For point-to-point network, the formulation involving Eq. 2.13a and 2.13b results in Eq. 4.11 (a,b).

$$C_T = 2(n-1)f\left(t_F(1+\varphi)\left(p_K k(q) + m(q)\right) + \mathrm{c}_N\right) + \mathrm{J}(\mathrm{o(q)} + \xi \mathrm{e(q)}) + \theta dn(n-1)\left(\frac{12\alpha}{f} + \beta 2t_F(1+\varphi) + \gamma t_H(1+\rho)\right) \quad (4.11a)$$

$$\mathrm{J} = \max\left\{\left[2(n-1)\frac{t_L(1+\varphi)(1+\rho)}{t_e} f\right]^+, \left[2(n-1)^2\frac{t_L(1+\varphi)(1+\rho)}{t_e}\frac{d}{q}\right]^+\right\} \quad (4.11b)$$

It is important to take into account that airline cost associated to flight time is linear. For passengers the linearity appears too. However, ownership and labor costs for airlines is not linear and it can make necessary to overallocate resources and employees in the network to manage delays. Especially, hub and spoke network has double impact because passengers have to absorb delays due to connection times. In fact, regarding resources, hub and spoke configuration has a plus of cost due reliability because there is factor of 2 controlling the expression for resources. In the situations where the hub and spoke configuration is not the best option against point-to-point (networks with enough critical demand to justify direct expeditions), delays penalize passengers and the airline.

Figure 4.18 shows a sensitivity analysis for characteristic delay and cost of network when point-to-point configuration is analysed (assuming parameters of chapter 1). And Figure 4.19 shows the relationship between costs of network for hub and spoke configuration when characteristics delays are involved in the planning phase.

Both figures have been calculated with the parameters of Appendix 1 and considerations explained in Chapter 1. Also, additional data are: a set of 10 airports, flight time 2h and turnaround time 0.5h (before delays), 180 passengers per aircraft, demand between airports 1,000 pax.

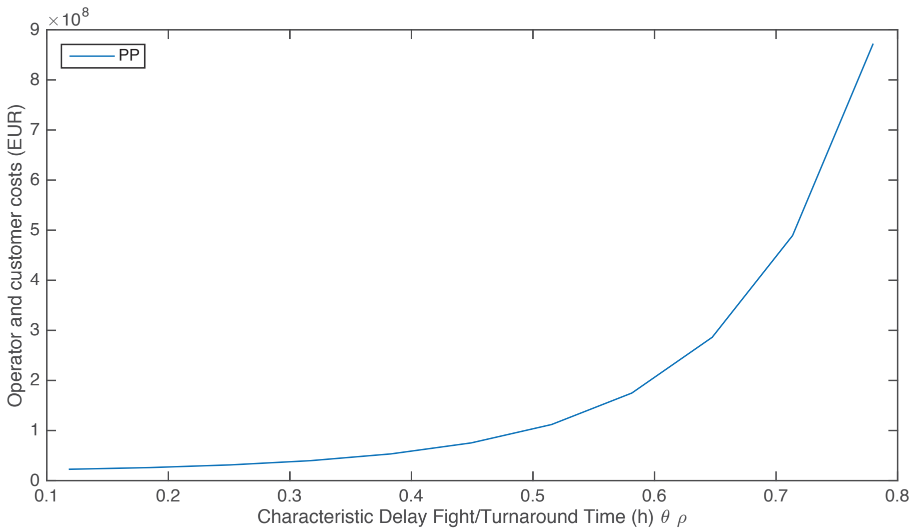


Figure 4.18. Relationship between characteristic delay and network cost for PP configuration.

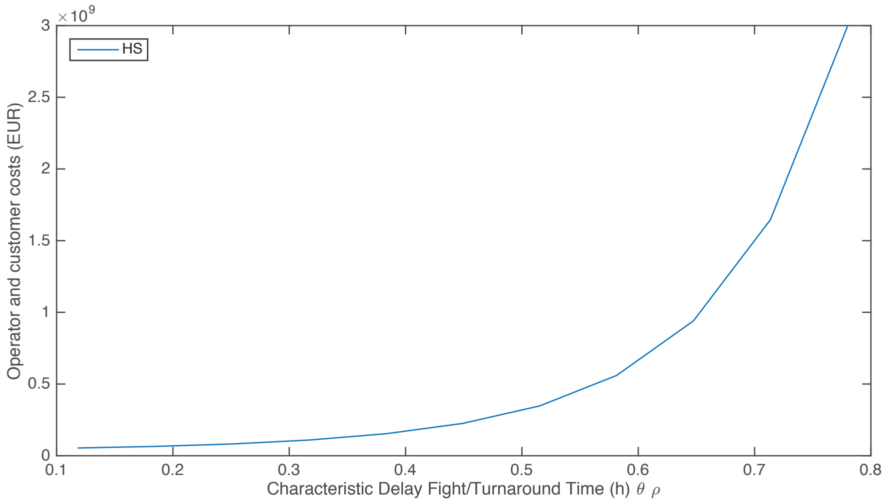


Figure 4.19. Relationship between characteristic delay and network cost for HS configuration.

4.3.3 Algorithm to improve reliability

This section presents an algorithm to control airline planning and to manage delay propagation.

Flight scheduling and routing

First, a flight scheduling is considered (FS), which is planning taking into consideration just the time needed to flight between two scheduled airports and the turnaround time. When delays appear, this FS is degenerated to FS’, which is the timetable after the perturbation. Airline has the option to design a flight schedule FS* with padding to be reliable in front delay propagation.

Figure 4.20 shows this concept, where serie of continuous line (SDT_i, SAT_i) is original FS. The serie with discontinuous line (SDT'_i, SAT'_i) represents FS'. Finally, airline plans FS* that is represented by serie with line of dots (SDT^*_i, SAT^*_i).

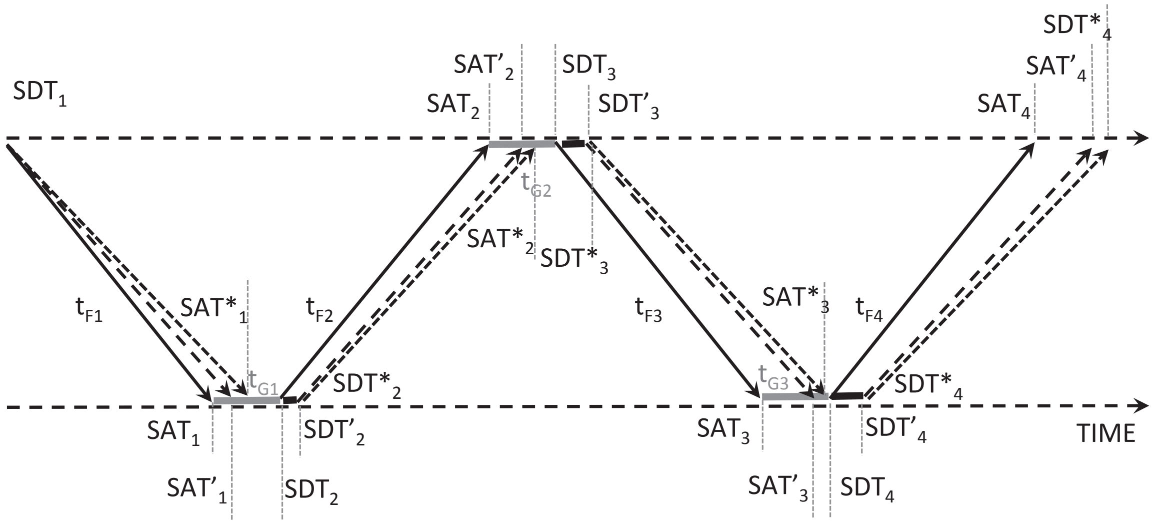


Figure 4.20. Flight schedule with propagation of delays.

This FS* can be the same flight schedule in origin with departure and arrival time adjusted to new conditions or it can be designed *adhoc*. This work considers the adjusted version that consists on introducing extra time for flight time and turnaround time to absorb mean values of delays and one or more times the standard deviation. Then, these rules are characterized by Equations 4.9a and 4.9b, where parameter K decides the level of coverage that airline wants to have and it results in different values of φ and ρ. It is not allowed any value of K because fleet is constrained.

Second, a set of routes $R = \{r_j\}$ are considered and it is related to first assignment done for a flight schedule, where each route r_j containts a sequence of flights that are served by the same plane.

Cost of operation considering delays

First, operating costs increase with delays and padding, but in a different way. For delays not planned, the cost for airline is evaluated in terms of variable costs (because fixed costs not vary in this scenario) and them can be estimated with expressions of Appendix 1. However, a coefficient of penalization is considered because the perturbation enforces to reassign resources and extend duty time for crew ($\Delta C_O = \nu \cdot C_O(\Delta t)$, considering a standard value of $\nu = 1.0$ for this work).

Second, passengers experiment a penalization in their travel time because extra time not expected is necessary. Then, passenger cost is estimated as proportion of value of time related to this increment of time ($\Delta C_P = \varpi\theta\Delta t$, considering a standard value of $\varpi = 1.0$ for this work).

If padding is considered, then more resources are going to be needed at planning phase (or not) but these costs are going to be balanced by savings on penalization. Furthermore, passengers are going to be beneficiated with more reliability.

Backup of resources for reliability

Airlines usually take into consideration the need of having a backup of resources in main bases to recover the planned service or mitigate perturbations in flight scheduling. When flight schedule is delayed and these delays exceeds a threshold, airline can make the decision of breaking a route into two sub-routes and the second one departs on time with the backup. In this kind of operation, eventually, a ferry flight can be necessary to send it to the airport where this is necessary.

These kinds of flights are not desirable by companies because of the cost. However if penalization increases, then this strategy can be interesting. Furthermore, costs of delays are variable costs, but backup resources are fixed and variable costs. Therefore, this extra cost (and padding strategy) decreases margins in profit and loss account, this is the cost of reliability and it can make the difference between having negative o positive bottom line at the end of the year.

Algorithm of control

Given a flight schedule and quatification of time for flights and turnarounds. An algorithm of control is in charge to monitorize the development of operation. This is to take into account the current departure time and arrival time for all flights. Considering that if a flight is delayed, the following flight that is covered with the same plane will be delayed to because there are transferability of delays, except that padding strategy can absorb this delay. Furthermore, in case of hubs, transferability of delays between different airplanes is possible because there is the coordination of connections.

The algorithm considers each flight of FS. However, it starts computing all delays according with distributions (section 4.3.1) and it applies a mechanism of propagation (described above) and control. This mechanism lets operator introduce actions to recover the plan or mitigate delays. For each arrival, the algorithm evaluates the prevision of delays and it can decide:

i. To mobilize a backup. If predicted delay in a route exceeds a threshold (τ), one backup is mobilized, recovering the original SDT for the following flight. Only if airline has this resource in the base, else the airline can send it where is necessary with an extra cost.
ii. To reorganize routes. If other route accumulates less delays and the swap is allowed (coincidence of flights at the same airport at the same time). Changing routes has the cost of reallocating crew and flights at the end of the cycle with ferry flights or recover the original end of routes if it is possible (very improbable because this option propagates delays). This selection is done applying Tabu Search principles.
iii. To permit the delay propagation. If other alternatives are not possible, then it is necessary to accept the delay propagation. This alternative would give the option to work at level of flight plan and turnaround operations to absorbe part of the delay (which is not the goal of this work).

The total cost of operation with compensations (or penalizations) is evaluated and it defines the criteria to make the final decision.

The algorithm runs a simulator based on probability distributions. At the beginning all flights are included in a future event list. All of them are considered as events not condicionated. The schedule departure creates an evaluation of time flight and turnaround time, which is a conditionated event, and this evaluation modifies flight schedule for this flight arrival. Then, this arrival time fires a conditionated event, which is in charge to decide which of the three options are the choice, therefore the fligh schedule is perturbated FS_P.

Finally, for each pair of variables K and number of aircrafts in the backup, the operator finds a trade-off to improve reliability of their flight scheduling.

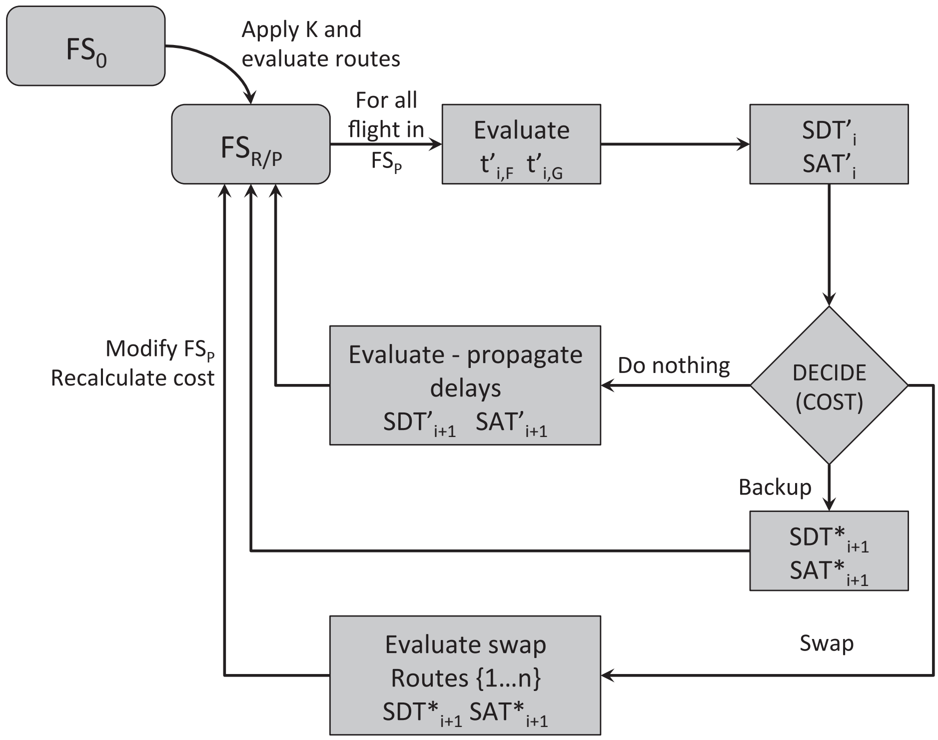


Figure 4.21. Basic flow of algorithm of delay control.

Numerical experimentation

Given a flight scheduling FS_0 and distributions for flight delays $N(\mu_F, \sigma_F) = N(1.78, 3.30)$ and turnaround delays $N(\mu_G, \sigma_G) = N(2.32, 3.43)$, both in minutes per hour. The airline decides the padding and estimates FS_R for different scenarios (Table 4.1), which produces different routing strategies.

Table 4.1. Scenarios for delay propagation.

	Airport		Scenario 1. No delays considered		Scenario 2. Delays considered with k=0.		Scenario 3. Delays considered with k=1.		Scenario 4. Delays considered with k=2.		Scenario 5. Delays considered with k=3.	
Num. Flight	**Depar-ture**	**Arri-val**	**SDT**	**SAT**	**SDT***	**SAT***	**SDT***	**SAT***	**SDT***	**SAT***	**SDT***	**SAT***
1	1	2	8.0	10.0	8.00	10.06	8.00	10.17	8.00	10.28	8.00	10.39
2	2	1	10.5	12.5	10.58	12.64	10.72	12.89	10.86	13.14	10.99	13.38
3	1	3	9.0	12.0	9.00	12.09	9.00	12.25	9.00	12.42	9.00	12.58
4	3	1	12.5	15.5	12.61	15.70	12.80	16.06	13.00	16.41	13.19	16.77
5	1	4	10.0	15.0	10.00	15.15	10.00	15.42	10.00	15.70	10.00	15.97

6	1	2	14.0	16.0	13.16	15.22	13.43	15.60	13.71	15.99	13.99	16.38
7	2	1	16.5	18.5	15.74	17.80	16.15	18.32	16.57	18.85	16.98	19.37
8	4	1	16.0	21.0	16.19	21.34	16.52	21.94	16.85	22.55	17.18	23.16
9	1	2	16.5	18.0	16.22	18.28	16.60	18.77	16.99	19.27	17.38	19.77
10	2	1	18.5	20.0	18.80	20.85	19.32	21.49	19.85	22.13	20.37	22.76
Routing strategy			{r1:1-2-6-7; r2:3-4-9-10; r3:5-8}									
Demand			180 passengers per flight (180 seats).									

A preliminary evaluation of cost of delays is carried out without applying the algorithm of delay propagation (without mechanisms of active control, Table 4.2).

Table 4.2. Results of scenarios for delay propagation without active control.

Indicator	**Scenario 1. No delays considered**	**Scenario 2. Delays considered with k=0.**	**Scenario 3. Delays considered with k=1.**	**Scenario 4. Delays considered with k=2.**	**Scenario 5. Delays considered with k=3.**
Fleet	3	3	3	3	3
Original cost planned	$1.3689\cdot10^5$	$1.4066\cdot10^5$	$1.4754\cdot10^5$	$1.5443\cdot10^5$	$1.6131\cdot10^5$
Real cost	$1.4094\cdot10^5$	$1.4117\cdot10^5$	$1.4235\cdot10^5$	$1.4407\cdot10^5$	$1.4579\cdot10^5$
Accumulated delay for airline (h)	1.09	0.38	0	0	0
Extra cost of delays for airline	$4.9942\cdot10^3$	$1.7347\cdot10^3$	0	0	0
Accumulated delay for passengers (h – total)	605.69	300.44	36.49	6.79	0
Cost of delays for passengers	$3.6342\cdot10^4$	$1.8026\cdot10^4$	$2.1897\cdot10^3$	407.6793	0

Note: costs are expressed in EUR.

Applying the algorithm of control delay with backup fleet, some disruptions are eliminated or mitigated. Table 4.3 shows results if different value of threshold time is considered to activate backup resource.

Results indicates reliability has an extra cost for airline in terms of extra resources in back up to recover the service or mitigate delays or the cost of padding (allocating more time for each operation). However, this operating strategy has advantatges for airline and for passenger. Obviously, if airline has not to pay penalizations, the strategy only generates costs. Passengers could be the part beneficiated of this strategy because they save a lot of time.

Table 4.2. Results of scenarios for delay propagation without active control.

Indicator	**Scenario 1. No delays considered**	**Scenario 2. Delays considered with k=0.**	**Scenario 3. Delays considered with k=1.**	**Scenario 4. Delays considered with k=2.**	**Scenario 5. Delays considered with k=3.**
Treshold	15min				
Fleet (backup required)	3 (1)	3 (1)	3 (0)	3 (0)	3 (0)
Cost of backup (per day)	$8.7864 \cdot 10^3$	$8.7864 \cdot 10^3$	0	0	0
Backup entries	1 (base)	1 (base)	0	0	0
Swaps executed	0	0	0	0	0
Original cost planned	$1.3689 \cdot 10^5$	$1.4066 \cdot 10^5$	$1.4754 \cdot 10^5$	$1.5443 \cdot 10^5$	$1.6131 \cdot 10^5$
Real cost	$1.4030 \cdot 10^5$	$1.4080 \cdot 10^5$	$1.4235 \cdot 10^5$	$1.4407 \cdot 10^5$	$1.4579 \cdot 10^5$
Accumulated delay for airline (h)	0.58	0.07	0	0	0
Extra cost of delays for airline	$2.5815 \cdot 10^3$	338.86	0	0	0
Accumulated delay for passengers (h – total)	419.08	191.81	36.49	6.79	0
Cost of delays for passengers	$2.5145 \cdot 10^4$	$1.1509 \cdot 10^4$	$2.1897 \cdot 10^3$	407.6793	0
Treshold	10min				
Fleet (backup required)	3 (1)	3 (1)	3 (0)	3 (0)	3 (0)
Cost of backup (per day)	$8.7864 \cdot 10^3$	$8.7864 \cdot 10^3$	0	0	0
Backup entries	1 (base)	1 (base)	0	0	0
Swaps executed	0	0	0	0	0
Original cost planned	$1.3689 \cdot 10^5$	$1.4066 \cdot 10^5$	$1.4754 \cdot 10^5$	$1.5443 \cdot 10^5$	$1.6131 \cdot 10^5$
Real cost	$1.4000 \cdot 10^5$	$1.4080 \cdot 10^5$	$1.4235 \cdot 10^5$	$1.4407 \cdot 10^5$	$1.4579 \cdot 10^5$
Accumulated delay for airline (h)	0.34	0.07	0	0	0
Extra cost of delays for airline	$1.5574 \cdot 10^3$	338.86	0	0	0
Accumulated delay for passengers (h – total)	333.31	191.81	36.49	6.79	0
Cost of delays for passengers	$1.9999 \cdot 10^4$	$1.1509 \cdot 10^4$	$2.1897 \cdot 10^3$	407.6793	0

Note: costs are expressed in EUR.

4.4 Conclusions

To sum up, some ideas are highlighted. First, regarding indices of complexity, airline networks are fascinating examples of emerging complex and interacting structures, which may evolve in a competitive environment under liberalized market conditions. They may exhibit different configurations, especially if a given carrier has developed alliances and has extended their service network. However, this work has analysed two simple networks of low cost carriers. One of them is a new entrant in the market and the other is a big carrier.

The network presented is characterized by an important concentration of the activity. The most important reason is the necessity to consolidate strong bases and give financial support to development plan. This fact is consistent with the ways of early entrants that manage large networks today. Furthermore, network carriers of case study exhibit a hierarchical structure despite the service point to point.

Results for indices of complexity and performance of cost of reliability for networks are linked. LCC airlines present concentration but not much as hub-and-spoke. Their level of propagation of delays is low in practice and analyitical model confirms this fact with theoretical approach. Furthermore, it is consistent with indices of complexity.

The results obtained are interesting and invite to further developments to understand in parallel aspects related to customers and demand. Furthermore, a weighted network analysis with flow of passengers could be interesting to understand how topology of links is well related to demand patterns.

Second, related to propagation of delays, the model developed to improve reliability shows that airlines have to assume higher costs due reactionary delays. These could be very important at the end of each route if there are not mechanisms to mitigate or eliminate delay propagation. This delay generates costs for airlines and for passengers. Airlines assume more costs because they have to react against perturbations of flight schedule. Passengers assume delays not planned that affect departures and arrivals, being the part most affected. However, introducing padding, back-up resources or re-scheduling improves reliabity and this has a cost. Back-up mechanism obligates to assume more fixed cost but the improvement of quality is very interesting. Re-scheduling is difficult if there are not coincidences of planes at the airport. Padding runs fine always but the cost is very important. This strategy runs especially fine for point-to-point networks if ferry flights are not allowed, while hub-and-spoke could take profit of back-up resources in hubs.

5 Implications of airline competition for network design

5.1 Introduction

Airline competition is a key to understand dynamics in the air transportation industry. Operators try to find new routes to improve the bottom line in profit and loss account, basically capturing new demand at good level of prices to increase margins. However, if there is enough demand and the operator do not put barriers to new entrants, competition becomes a reality and, finally, airline business model only is sustainable if costs are low.

This chapter proposes a theoretical approach to competition in airlines with game theory. From this analysis some conclusions are outlined regarding network configuration and key parameters of supply.

5.1.1 State of the art

This section presents a short review of the state of the art through different works that have studied airlines competition. Usually, previous works apply microeconomic models and game theory.

Game theory allows modelling games with n-players where each of them tries to maximize their profit function (Von Neuman and Morgestern, 1943). The main previous works show equilibrium models for frequencies and prices as key parameters of supply. Furthermore, some of them introduce a congestion charge.

Hansen (1990) is one of the most representative publications in this field. He develops a model for airlines competition and applyes it to USA. The model is based on non-cooperative game for n-players, airlines that want to maximize their benefits. There are two types of airlines: hub-and-spoke and point-to-point operators. The model makes some assuptions –simplifications- regarding decision variables and uses US DOT data from 1985. He found equilibrium very similar with real situation at that moment. Furthermore, it allows outlining conclusions related to viability of hubs in competitive environments, protection of markets and users’ preferences (point-to-point is twice desirable for passengers in US markets).

Other researchers like Oum et al. (1995), Hong and Harker (1992), Adler (2001) or Martín and Román (2003) work with game theory. For example, Martín and Román (2003) analyse location problem through competitive game in two stages. First, airlines decide sequencially their hub location and, secondly, they compete supplying direct or connection services through their hubs. First stage outputs impact on second stage, basically they induce a market share that competitors hold. The model is appliyed to South-Atlantic market before Open Skies agreement.

Flores-Fillol (2009) has worked on topic about airlines competition and network structure. Some conclusions are presented about airlines decisions with low cost structures and network configuration. He finds an asymetric equilibrium in which an operator adopts a point to point network and the other a hub and spoke network. He finds that it is easy to achieve an excess of supply in hub and spoke networks.

Later, the same author (Flores-Fillol, 2010) presents an additional research working on congestion at hubs. The main objective for this paper is the analyisis of feasibility of congestion charge. He wants to demonstrate that this charge can support the mitigation of negative effects for passengers. Other approaches to this topic were outlined before, for example, Hansen (2002) had worked on deterministic queue modelling for parallel runway in Los Angeles International Airport.

Ryerson and Hansen (2011) study the network configuration effects on fuel consumption, flight planning and flight schedules with padding (allocating time-buffers between flights to mitigate perturbations or delays). Also, Wei and Hansen (2006, 2007) apply game theory to understand how airlines compete with frequency and aircraft size when market is duopolistic. They propose an analysis oriented to decision-making process.

Wang et al. (2010) proposes three different games for competition between ship carriers. These are Nash Equilibrium, Stackelberg game and deterrence game. He takes into account the utility based on fare and time of service. The evaluations of marketshare and potential attractiveness of each competitor are carried out with one algorithm oriented to integer variables.

Gallego (1994) proposes a dynamic pricing of perishable assets under competition. Fares decisions are made at strategic level and capacity is allocated among fares. He assumes that demands for different fares are independent. Also, low fare carriers impose few or no restrictions. Therefore, dynamic pricing integrates pricing and capacity allocation and he finds revenue under competition.

Finally, Trapote (2008) applies a metaheuristic algorithm to allocate frequencies in a new hub (BCN) that compite with current European infraestructures and operators and analyse the sustainability of this operation. This game is based on frequencies equilibrium with the assumption that a minimum break-even load factor is enough to preserve the route.

5.1.2 Objective

The main objective of this chapter is to present a theoretical approach to airlines competition with game theory. Models could be very complicated in real conditions, for this reason this is a line of research with high perspetives for the future and a lot of questions to be solved. The goal is to propose a simple analytical model based on formulations of Chapter 1 and achive outputs to recommend or to understand the basis of competition.

Furthermore, this work aims to be a resource for policy makers and airlines’ managers. Understand how airline configuration allows different pricing strategies or taking more profit from resources. Moreover, competition enforces operators to supply more frequencies at fewer prices, which is better for passengers. However, some inefficiency is achieved by oversupply when demand is very elastic to supply.

5.2 Problem statement

5.2.1 Principles of game theory

First of all, the problem is defined in the frame of theory game. Fudenberg and Tirole (1991) explain the theory for dynamic games with complete information. Stackelberg’s game is a two-stage model that is simple because the model assumes only two players and each of them decides supply’s parameters as a reaction of potential decisions of each other.

The basis of the problem is two players $i = 1, 2$, those decide the quantity of product they introduce in the market q_i. Price is defined by law of supply-demand, $P(Q) = a - Q$. Then profit for each player is $\pi_i(q_i, q_j) = q_i(P(Q) - c)$, where c is the marginal cost for one unit of product (this kind of models usually assumes zero fixed costs). Also, $Q = q1 + q2$.

Finding the solution for this game requires a back-inductive process. First, reaction of player 2 is calculated finding the maximum of Equation 5.1 (for any decision of player 1):

$$\max_{q_2 \geq 0} \pi_2(q_1, q_2) = \max_{q_2 \geq 0} q_2[a - q_1 - q_2 - c] \tag{5.1}$$

Then, solution for this model is $R_2(q_1) = (a - q_1 - c)/2$, with $q_1 < a - c$. And player 1 can anticipate this reaction, optimizing the problem:

$$\max_{q_1 \geq 0} \pi_1\big(q_1, R_2(q_1)\big) = \max_{q_1 \geq 0} q_1 \left(\frac{a - q_1 - c}{2}\right) \tag{5.2}$$

Finally, solution for Stackelberg's duopoly is presented by Equation (5.3).

$$(q_1^*, q_2^*)_S = \left(\frac{a-c}{2}, \frac{a-c}{4}\right) \tag{5.3}$$

Company 2 (player 2) is worse than company 1. Furthermore, it is worse in Stackelberg's game than it is in Cournot's game. This one is static and with complete information, and each company produces $(a - c)/3$ (see Eq. 5.4 and 5.5). Therefore, companies produce more quantity of services with Stackelber's game than with Cournot's model. Moreover, they produce much more quantity than social optimum or monopoly.

$$\begin{cases} \max_{q_1 \geq 0} \pi_1(q_1, q_2) = \max_{q_1 \geq 0} q_1[a - q_1 - q_2 - c] \\ \max_{q_2 \geq 0} \pi_2(q_1, q_2) = \max_{q_2 \geq 0} q_2[a - q_1 - q_2 - c] \end{cases} \tag{5.4, 5.5}$$

$$(q_1^*, q_2^*)_C = \left(\frac{a-c}{3}, \frac{a-c}{3}\right) \tag{5.6}$$

Gibbons (1992) presents a Cournot's model to search social optimum, based on Hardin (1968). For this problem, the social optimum is $Q = (a - c)/2$. The proposal of Gibbons and Hardin presents an interesting perspective to analyse regulatory frameworks for airlines.

In game theory, the information is the key. If player i has more information and other players know that, then this situation induces to poor solutions for player i. In Stakelberg's game, it is possible to drive a different approach. Suppose that company 1 anticipates the new entrant (company 2) and it decides q_1. Company 2 decides $q_2 = (a - c)/2$. But company 1 can anticipate this decision and it really decides $q_1 = 3(a - c)/8$ but not its Stackelberg's quantity $(a - c)/2$. Then, there is only one Nash's equilibrium: both players decide $q = (a - c)/3$; which is the equilibrium for Cournot's game.

5.2.2 Basic assumptions

Operating cost for airline

This section presents the basic assumptions for further developments. Related to costs, game theory uses marginal costs. However, the aim of the work is to show analytical and simple models to understand interactions between key parameters. For this reason, an extra exercise of simplification is developed and costs presented in Appendix 1 are reduced to variable costs. Then, costs depend on stage length and size aircraft (average cost for ownership and crew staff are estimated). This assumption allows derivating expressions to achieve useful equations.

$$c(t_F, t_G, q) = t_F\big(p_k k(q) + m(q)\big) + (t_F + t_G)(\widehat{w}(q) + \hat{e}(q)) + cn \tag{5.7}$$

$$C = f \cdot c(t_F, t_G, q) \tag{5.8}$$

Where, $c(t_F, t_G, q)$ is the unitary cost of one flight whose stage length corresponds to flight time and turnaround time when aircraft provides q seats. It is estimated as variable cost. Parameter p_k is cost of fuel, $k(q)$ is fuel consumption per block-hour for aircraft with q seats, $m(q)$ is the maintenance cost per block-hour. Furthermore, $\widehat{w}(q)$ and $\hat{e}(q)$ are ownership and crew operating costs per block-hour, which really are imputable fixed costs but here are assumed as variable costs to operate easily (based on statistics appendix 1). Finally, cn is navitagion and landing charges. Therefore, the operator cost (C) here is linear with frequency (f) weighted by unitary cost (c).

Demand and market share

Total demand for air transport service depends on the utility of this service and the social characteristics of markets where service is provided. Wei and Hansen (2005) proposes an estimation based on $D = exp(K_m)I^{\rho}U^{\zeta}$, where K_m represents characteristics of market m, I is an income function, S is an satisfaction function (Daganzo, 1979) evaluated as $S = \sum_j exp\big(V_{jm}\big)$ and ρ, ζ are a parameters of calibration. Utility V_{jm} depends on frequencies, travel time, price and other parameters. Then demand is variable with quality and quantity of service.

The model presented here is quite simple with the aim to capture cause-effect dynamics for airline decicions. Demand is estimated as a part of total supply provided in the market of analysis (Eq. 5.9). Where D is total demand, α is a parameter that represents the elasticity of demand against supply and Q is supply or total quantity of seats ($Q = \sum_j f_j q_j$, where f_j is the frequency for alternative j and q_j is the capacity of aircrafts).

$$D = \alpha Q \tag{5.9}$$

Related to market share of airlines, Hansen (1990), Martín (2003), Garrow (2010) or Atasoy (2012) apply the logit model (5.10). Market share is estimated from passenger itinerary, travel time and average fare.

$$S_{im} = \frac{e^{u_{im}}}{\sum_j e^{u_{jm}}} \tag{5.10}$$

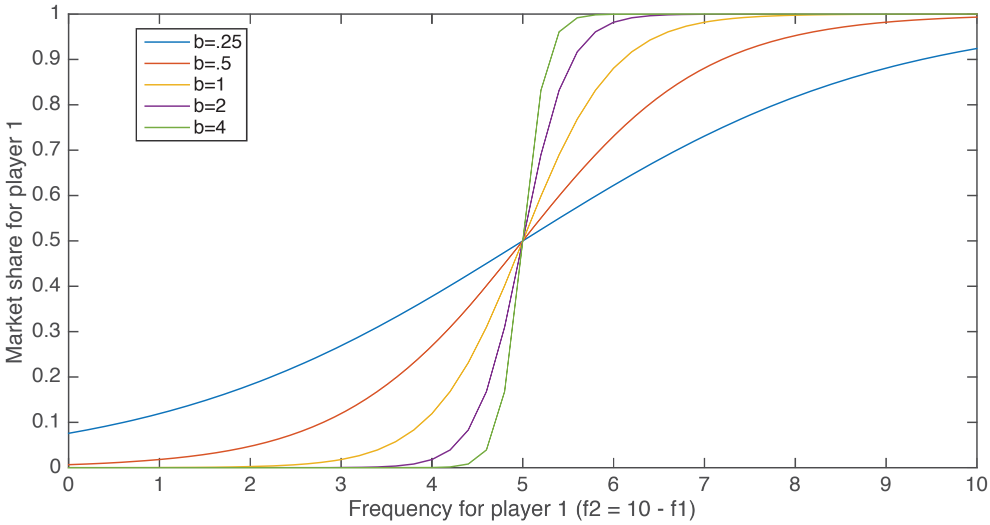


Figure 5.1. Marketshare distribution with logit model.

Note: logit model assumes utility function linear with frequency and weighted with parameter b.

Where, S_{im} is the share related to alternative i in market m (that is supply between two cities or airports), u_{im} is the deterministic (measurable) utility for passengers and for alternative i in market m. Hansen (1990) proposes a utility function that is a linear combination of average fare, stage length and logarithm of frequency. Furthermore, Martín (2003) uses a very similar expression. However, Atasoy (2012) formulates a linear combination of average fare and travel time.

In general, passenger's utility is based on generalized costs. This is a linear combination of average fare plus cost of travel time.

$$u(p, t_T) = p + \theta t_T = p + \theta(\alpha t_W + \beta t_F + \gamma t_C) \tag{5.11}$$

Where, $u(p, t_T)$ is the utility for passenger, p is the average fare, θ is the value of time, t_T is the travel time perceibed by passenger $(\alpha t_W + \beta t_F + \gamma t_C)$, being this the weighted addition of waiting time, flight time and connection time.

However, Flores-Fillol (2009) simplifies the market-share model and proposes a linear function between demand carried by an airline with difference of frequencies and prices for both competitors. He works in a strictly theoretical framework and achieves a compact formulation.

Finally, Belobaba (2009) proposes a different approach to market-share model (Eq. 5.12), based on S-shape function. This function involves obtaining the ratio between frequency for alternative i in market m (elevated to parameter a) and the addition of frequencies for all alternatives (elevated each of them to parameter a). Parameter a controls the curvature to the function.

$$S_{im} = \frac{(f_{im})^a}{\Sigma_j (f_{jm})^a} \tag{5.12}$$

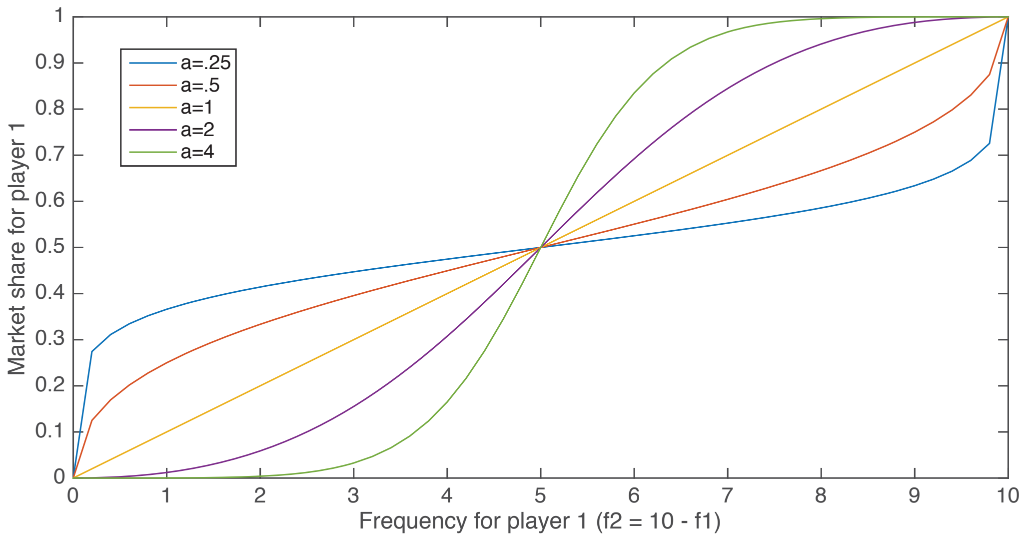


Figure 5.2. Marketshare with S-shape function.

This work denvelops models with this formulation, with the aim of achieving compact formulation.

Law of prices

Prices follow the rules of supply-demand laws and a linear relationship is proposed in this work.

$$p(D) = P - \lambda D \tag{5.13}$$

$$p(Q) = P - \lambda \alpha Q \xrightarrow{q_j = q \ \forall j} p(Q) = P - \lambda q \textstyle\sum_v f_v \tag{5.14}$$

Where, P is the maximum price allowed in the system (upper this limit, demand does not want to travel in this market). Function $p(Q)$ controls price depending on capacity supplied ($Q = \sum_v f_v q_v$, when aircraft size is the same or very similar for all competitors is possible to write q).

Profit

Airline i in the market m has the profit indicated by Equation 5.15 (where –i represents all the competitors excepts for i). Finally, Q, D, S and p depends on players' decisions $(f_{i\,m}, f_{-i\,m})$.

$$\pi_{i\,m}(f_{i\,m}, f_{-i\,m}) = D \cdot S_{i\,m} \cdot p(D) - f_{i\,m} c_{i\,m} \tag{5.15}$$

5.3 Modelling airlines competition

This section proposes an analysis of competition from different models of game theory.

5.3.1 Stackelberg's model for airlines competition

This section approaches the problem of airlines competition with a simple Stackelberg's game. The assumptions are that two airlines compete in the same market (one route with identical flight time and turnaround time for both, for this reason index m are omitted) with different unitary cost for each competitor (c_1, c_2). Assuming that q (size of aircraft measured in seats) is fixed and exogenous variable, average unitary cost per seat is calculated ($\hat{c}_1$, $\hat{c}_2$).

Each airline has to decide frequency (f_1, f_2) and the model considers and it is equivalent to decide (Q_1 and Q_2). Stakelberg's decision for airline 2 front any decision of airline 1 is as follows.

$$\max_{f_2} \pi_2 \;= \max_{f_2}\; D\, S_2 \;\; [P - \lambda\, D] - f_2\;\, c_2 \tag{5.16}$$

Applying S-Shape function (Eq.5.12), the objective function is as follows:

$$\max_{f_2} \pi_2 \;= \max_{f_2}\; \alpha q\,\left(f_1 \;\; + f_2 \;\right) \frac{(f_2)^a}{(f_1)^a + (f_2)^a} \left[P - \alpha\lambda\, q\,\left(f_1 \;\; + f_2 \;\right)\right] - f_2\;\, c_2 \tag{5.17}$$

Finally, the optimum decision for airline 2, expresed in terms of Q_1, is as follows:

$$R_2(Q_1) \leftrightarrow \frac{d\pi_2}{df_2} = 0 \tag{5.18}$$

If it is assumed that a=1 (market share follows a linear model), then the problem is solved analytically.

$$R_2(Q_1) = \frac{1}{2}\left(\frac{\alpha P - \hat{c}_2}{\alpha^2 \lambda} - Q_1\right) \tag{5.19}$$

Equation 5.19 is subject to no exceeding the quantity that creates losses for the company. Therefore, maximization of profit for airline 1 follows as:

$$\max_{f_1} \pi_1 \;= \max_{f_1}\; D\,\frac{f_1}{f_1 + f_2}\; [P - \alpha\lambda\, q\,(f_1 + f_2)] - f_1 c_1 \tag{5.20}$$

$$f_1^* \leftrightarrow \frac{d\pi_1}{df_1} = 0 \tag{5.15}$$

Finally, both optimal strategies for competitors are obtained in terms of Q_i.

$$(Q_1^*, Q_2^*) = \left(\frac{\alpha P - 2\widehat{c_1} + \widehat{c_2}}{2\alpha^2 \lambda}, \quad \frac{\alpha P + 2\widehat{c_1} - 3\widehat{c_2}}{4\alpha^2 \lambda}\right) \tag{5.16}$$

To conclude, if costs for airline 1 and 2 are equal ($c = c_1 + c_2$, $\hat{c} = \hat{c}_1 + \hat{c}_2$,), then decisions follow proportions of original Stackelberg's game. Note that airline 1 has a dominant position in the market and more market share. However, total supply provided by two competitors is higher than monopolistic supply, with the avantadge for customers of fewer prices. Furthermore, if parameter$\alpha = 1$, it is not necessary to compete with oversupply to capturate demand, aircrafts travel full of passengers for both competitors. Then, α is equivalent to average load factor and it is structural due to competition. On the other hand, if costs are different, then they play an important role to manage the equilibrium of the system. If second competitor runs lower costs, it has more opportunities to survive in the market.

5.3.2 Cournot's model for airline competition

This section approaches the problem of airlines competition with a simple Cournot's game. The assumptions are that two airlines compete in the same market (one route with identical flight time and turnaround time for both) with different unitary cost for each competitor (c_1, c_2).

Each airline has to decide frequency (f_1, f_2) and the model considers that fleet type is the same (capacity q for both). Cournot's decision for airline 1, 2 is as follows.

$$\max_{f_k} \pi_k = \max_{f_k} D \cdot S_k \left[P - \lambda\, q \left(f_1 + f_2\right)\right] - f_k\, c_k \tag{5.17}$$

Applying S-Shape function (Eq.5.12) with parameter a=1, demand equation (5.9) and optimizing for all competitors, it is possible to achieve the Nash equilibrium for Cournot's model.

$$(Q_1^*, Q_2^*) = \left(\frac{\alpha P - 2\widehat{c_1} + \widehat{c_2}}{3\alpha^2 \lambda}, \quad \frac{\alpha P + \widehat{c_1} - 2\widehat{c_2}}{3\alpha^2 \lambda}\right) \tag{5.18}$$

Cournot's game is less aggressive than Stackelberg's game, it is due to difference between static and dynamic approach. Cournot represents the convergence of different statges of competition in a dynamic scenario. Finally, both competitors agree an equilibrium that provides less supply than Stackelberg's game and is better balanced. Observe that Cournot's model has less dependency of costs than Stackelberg's game, especially for second supplier. Figure 5.3 shows profit for both players when they decide their strategies in the same interval and operate the same costs. The equilibrium is a pair of frequencies that provides the best profit that they can achieve in this competitive environment.

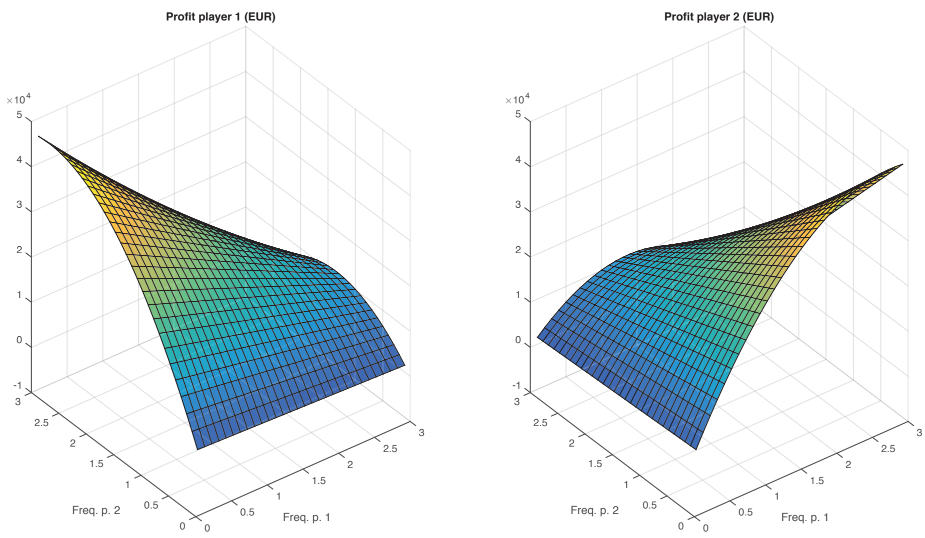


Figure 5.3. Profits for both players with Cournot's game.

5.3.3 Cournot's model for airline competition with general S-Shape function

This section aims to present expression for Cournot's model when S-Shape function, which models market share for airlines, is not simplified and exponents are involved in formulations until the end. Interaction of demand preferences is higher than developments carried out in previous sections. Developing Equation 5.17 is possible to achive next steps (supply in terms of Q).

$$\max_{Q_i} \pi_i \; = \max_{Q_i} \; \alpha Q \; \frac{Q_i^a}{\sum_j Q_j^a} \, [P - \alpha\lambda Q] - \hat{c}_i \;\; Q_i \tag{5.19}$$

Enforcing conditions for Nash Equilibrium, the following expression models the performance of competition in this scenario.

$$[P - 2\alpha\lambda Q]Q_i^a \sum_j Q_j^a \; + a[P - \alpha\lambda Q]QQ_i^a \sum_{j,j\neq i} Q_j^a = \frac{\hat{c}_i}{\alpha}\left(\sum_j Q_j^a\right)^2 \tag{5.20}$$

The problem can be solved numerically because involves non-linear formulation. A simple method is developed and it involves a Complete Enumeration Algorithm (CEA) and Executive Search Algorithm (ESA). First, given a range of frequencies available, CEA proposes a complete space of strategies for both players (j=1,2) and evaluates the profit function for them. Then, ESA carries out a process of scanning and for each pair of strategies (f_1, f_2or Q_1 , Q_2), it looks for a maximum for both functions at the same time. Finally, this element, if it exists, corresponds to Paretto's optimum in Cournot's model (Dresher, 1961, Dixit, 2004, Wang, 2014).

5.3.4 Cournot's model for airline competition with congestion charge

This section aims to present expression for Cournot's model for preventing congestion at networks. The fundamentals of this idea are that operators could be incetivated to oversupply capacity to attract demand when prices could high, competition hard and operating costs low (related to prices). Then, main airports or TMA sectors could be used intensively, introducing mechanisms for delays generation or propagation.

Given an air transport facility (i.e. airport) which could be modelled as queueing system M/M/1, the relationship between level of utilitzation of system capacity and average delay follows a non-linear function, which performance for levels of full utilization indicates average times of delay that tends to infinite (see Figure 5.4). At an airport or other system, this situation could be favoured by competition, especially at hubs where hub-and-spoke carriers offer enough supply to attract demand.

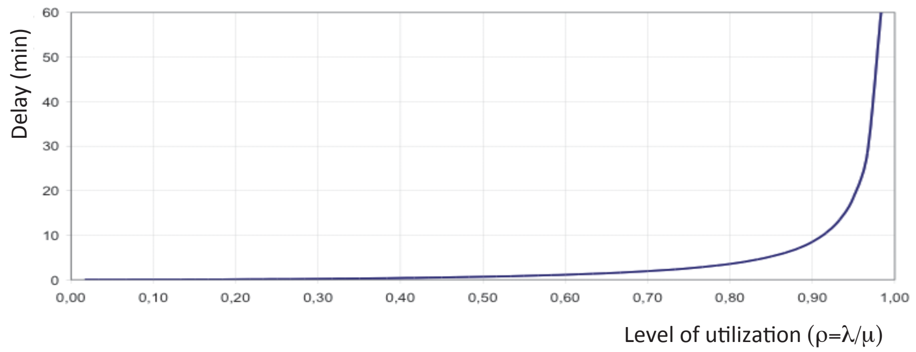


Figure 5.4. Average time of service related to level of utilization of single server with M/M/1.

Oum et al. (1995) proposes a socially optimal pricing of airports in a hub-and-spoke network, taking into account explicity the fact that demands for airport services at hub and spoke are complementary. One of the models is focused in pricing each airport independently, following previous works (Carlin and Park, 1970, Gillen et al., 1987, Morrison, 1983, Oum and Zhang, 1990), and it proposes a external congestion costs f(D/K), which is a function of demand/capacity ratio. This is the social marginal cost pricing.

Flores-Fillol (2010) makes a similar proposal and he aims to apply congestion fee at hub airports due this tendency to propagate delays. With a detailed linear combination of different marginal costs related to frequency of different players, basically a new mechanism is proposed based on the idea of capacity over-utilization.

This section proposes an additional cost for flights that is a monotone increasing function of supply. Considering, η basic fee for utilization, μ as capacity of the system, n number of players, α elasticity demand-supply, Equation 5.21 shows the additional term.

$$cn = \eta \left[\frac{1}{\alpha}\frac{f_i}{\mu}\right]^{1/n} \tag{5.21}$$

Introducing this expression in Equation 5.17, general form of profit is achieved.

$$\max_{Q_i} \pi_i \; = \max_{Q_i} \; \alpha Q_i \left[P - \alpha\lambda Q\right] - \hat{c}_i \; Q_i - \eta \left[\frac{1}{\alpha}\frac{Q_i}{\mu q}\right]^{1/n} \tag{5.22}$$

Enforcing conditions for equilibrium, the following expression models the performance of competition in this scenario and new cost is added to the expression, working as marginal cost that penalizes increasing frequencies.

$$\alpha \left[P - \alpha\lambda Q\right] - \alpha^2 \lambda Q_i \; - \hat{c_i} - \frac{\eta}{\alpha n \mu q}\left[\frac{1}{\alpha}\frac{Q_i}{\mu q}\right]^{1/n - 1} = 0 \tag{5.23}$$

The problem can be solved numerically because involves non-linear formulation. The algorithm presented in previous section suits fine to this problem.

5.4 Numerical experimentation

A set of numerical experiments is presented in this section. They correspond to previous sections and are commented to clarify the variables and parameters that take part of the sensitivity analysis.

Some experiments are based on operating costs analysed in Appendix 1 and other experiments are based on costs of empirical analysis carried out by CAPA (2012), which shows CASK of EUR 4.7c for LCC and EUR 9.3c for FCC.

5.4.1 Analysis of Stackelberg's model for LCC and FCC

Two airlines are considered with costs (CASK) defined in Table 2.1 (approximately, CASK EUR 4.7c for point-to-point LCC and CASK EUR 9.3c for network or FCC). LCC is new entrant in the market. Both of them manage the same route with aircrafts of 180 seats, flight time of 1 hour. At maximum price of EUR 200, EUR 2 decrases for each 10 passengers (table 5.1 summarizes data set).

Table 5.1. Set of data for Stackelberg's model on LCC and FCC competition.

Variable / parameter	Value
P (EUR)	200.00
λ (EUR/PAX)	0.20
α	0.95
t_F (h)	1.00
t_G (h)	0.50
c_1 (cEUR)	[9.3 ... 4.7]
c_2 (cEUR)	[4.7 ... 9.3]

The analysis presents the evolution of the system if costs change between player 1 and 2, being always player 2 the new entrant.

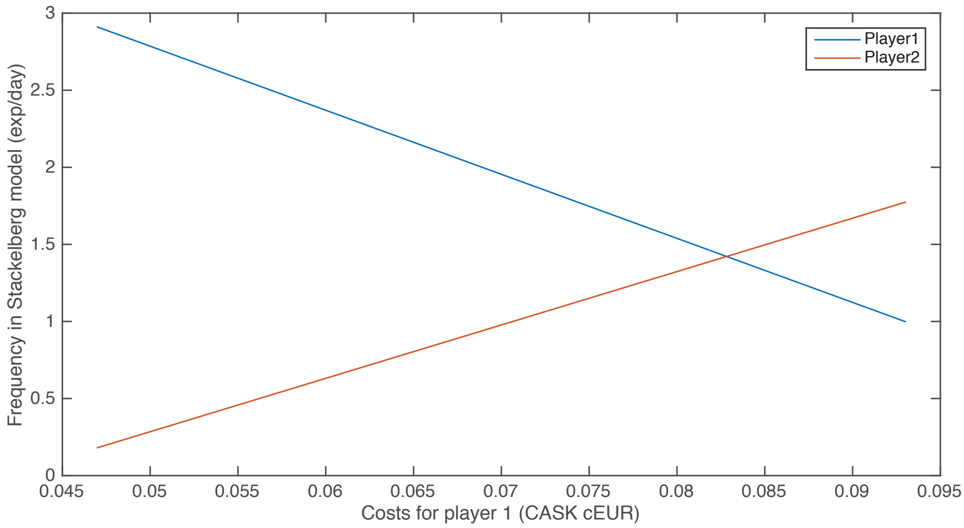


Figure 5.5. Evolution of supply with Stackelberg's model on LCC and FCC competition.

If player 1 is in the market with high CASK (blue line at the right of the figure), second player with low CASK (red line at the right) has high potential of success if it enters in the market, despite of being second operator. This fact explains how LCC have achieved market share thanks to low costs. However, if player one has lower costs, it can maintain a dominant market share, taking advantatge of being the first enter in the market. The point of balance is at CASK EUR 8.2c for player 1, and EUR 5.8c for player 2. Therefore, observe that costs are the critical factor for airlines and it is very difficult to take adavantages of markets operating costly value propositions.

5.4.2 Analysis of Cournot's model for LCC and FCC

This experiment shows a Cournot's game between two airlines. Both of them operate the same route and data is summarized in Table 5.1.

Cournot model is very interesting because explains a point of convergence for both players at long term if they not change their strategies. Observe Figure 5.6, one player dominates the market while its costs are lower than others. The market is balanced only if two players operate with the same costs, indepently of their business model. This simple model reflects very well the reality and shows why full cost carriers have been restructuring their companies to operate at the same level of costs. Then, low cost model explains a philoshopy about running companies independently of customers value proposition, when prices are ditacted by the market.

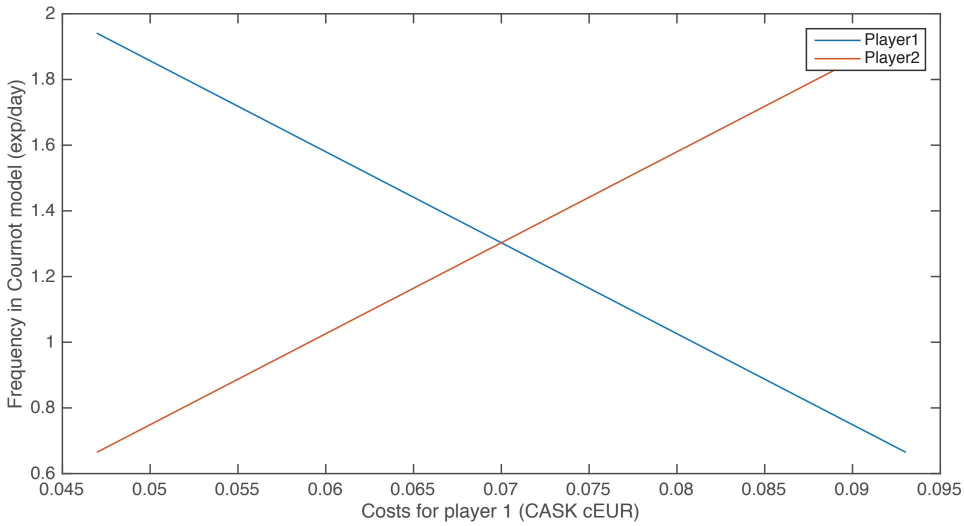


Figure 5.6. Evolution of supply with Cournot's model when operating costs changes for LCC and FCC.

5.4.3 Sensitivity analysis for parameter α

Parameter α introduces in the model the elasticity of demand in front supply. In some markets, airlines attract passengers supplying more frequencies or seats. If markets hold this performance, it is more probable that airlines tend to compete offering more capacity. Therefore, the system tends to congestion. Figure 5.7 shows the evolution of frequency provided by one player considering Cournot's model (game between two players, both operate same supply). The parameters considered for this experiment are summarized in Table 5.2.

Table 5.2. Set of data for sensitivity analysis of parameter α.

Variable / parameter	**Value**
P (EUR)	100.00
λ (EUR/PAX)	0.20
α	[0.50 … 1.00]
t_F (h)	1.00
t_G (h)	0.50
c_1 (cEUR)	4.00
c_2 (cEUR)	4.00

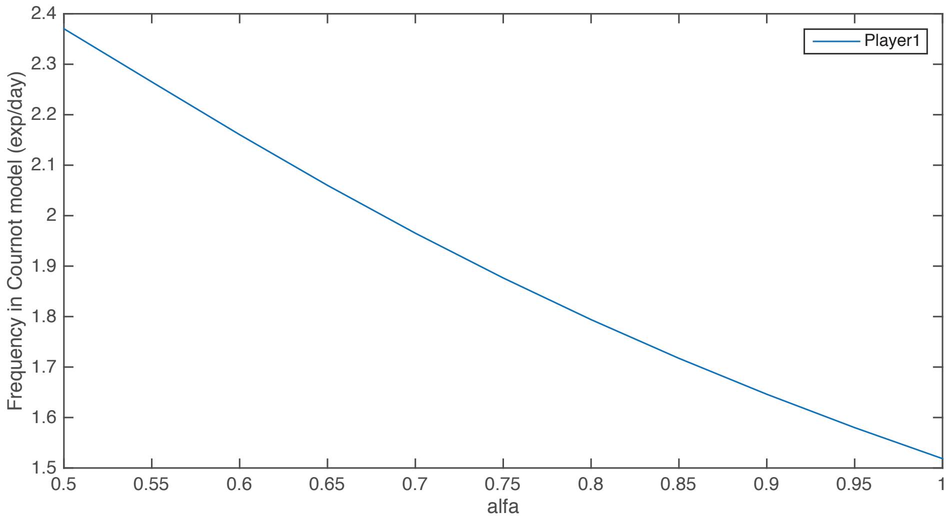


Figure 5.7. Evolution of frequencies with variation of parameter α with Cournot's game.

5.4.4 Sensitivity analysis for parameter λ

Parameter λ introduces in the model the elasticity price-demand. It is the parameter that controls inverse demand law (less price suppose more demand). If this parameter is high, increasing frequencies and attracting more demand, airlines enforce fewer prices. Then, the goal is to achieve equilibrium with low frequencies to hold high prices and maximize profits. The problem happens if costs for both competitors are very different. Figure 5.8 shows the evolution of frequency provided by one player considering Cournot's model (game between two players, both operate same supply). The parameters considered for this experiment are summarized in Table 5.3.

Table 5.3. Set of data for sensitivity analysis of parameter λ.

Variable / parameter	**Value**
P (EUR)	100.00
λ (EUR/PAX)	[0.02 … 0.20]
α	0.95
t_F (h)	1.00
t_G (h)	0.50
c_1 (cEUR)	4.00
c_2 (cEUR)	4.00

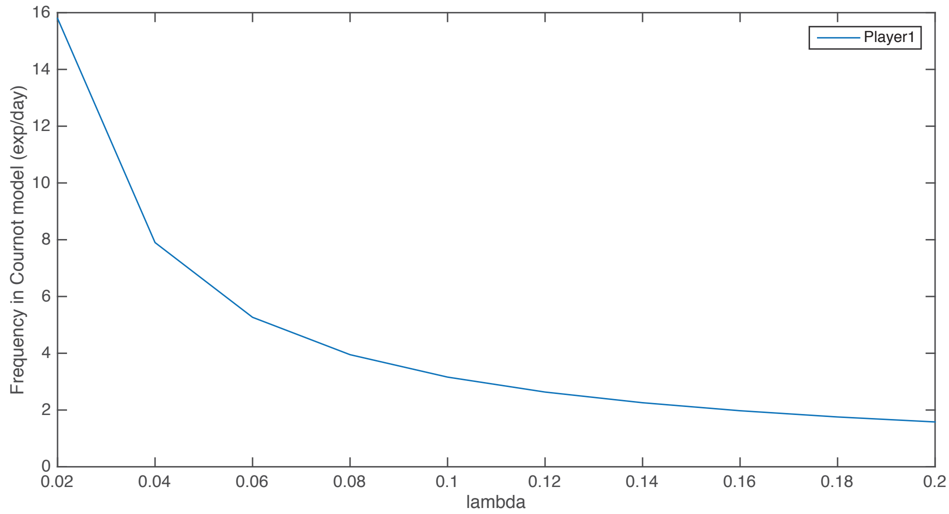


Figure 5.8. Evolution of frequencies with variation of parameter λ with Cournot's game.

5.4.5 Sensitivity analysis for Cournot's model with S-Shape function

In general, S-shape function is not linear, as it has been presented in analytical developments. If parameter a is bigger than one (Eq. 5.12), then marketshare function beneficiates the airline that increments its frequencies. This is, if customers are very sensitive to utility, then improvements in characteristics of supply result in better competitive position.

This experiment is carried out with Cournot's game (Eq. 5.19), which is not linear. For this reason, the problem is solved with algorithm that finds the equilibrium in a matrix of strategies. Figure 5.9 shows a sensitive analysis of frequencies for both players and for different values of parameter a. Also, in this experiment, costs are evaluated with Equations 5.7 and 5.8 (refered to Appendix 1). Finally, the parameters considered for this experiment are summarized in Table 5.4.

Table 5.4. Set of data for sensitivity analysis of parameter a and S-Shape function.

Variable / parameter	Value
P (EUR)	200.00
λ (EUR/PAX)	0.20
α	0.95
t_F (h)	1.00
t_G (h)	0.50
q (seats)	180
c_1 (cEUR per flight)	6,014.70
c_1 (cEUR per flight)	6,014.70
a	[1.00 … 1.50]

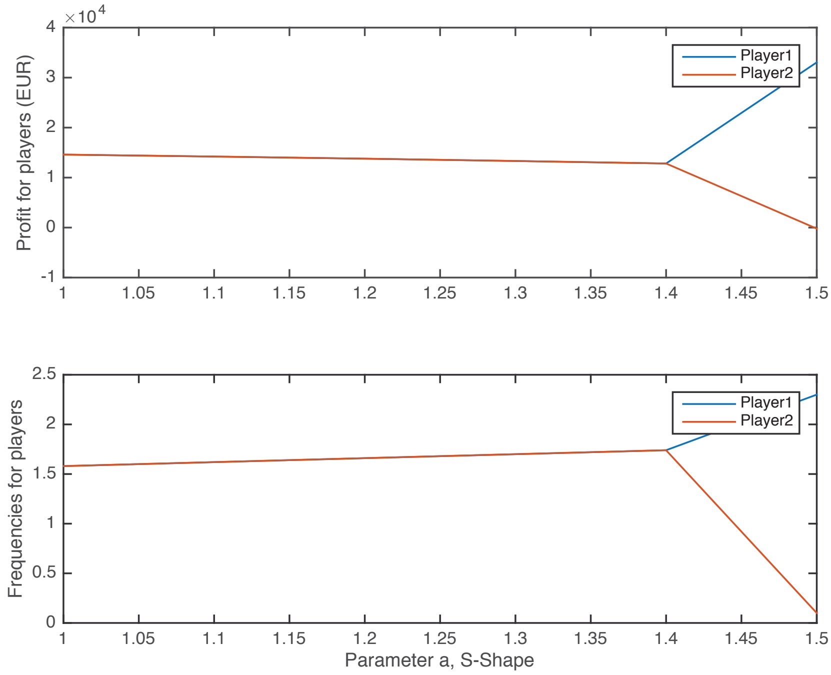


Figure 5.9. Evolution of profits for both players with S-Shape function and Cournot’s game.

If costs are diferent, the sensitivity analysis for parameter a gives in a diferent figure (Figure 5.10). Costs for player 2 are 80% than costs for player 1. It is very interesting how this set of parameters creates good conditions to incentivate one of two players to increase frequencies to capturate demand (this is sensitive to utility) and this strategy gives good results for player 2. This simple experiment explains one of most agressive mechanisms in airline industry.

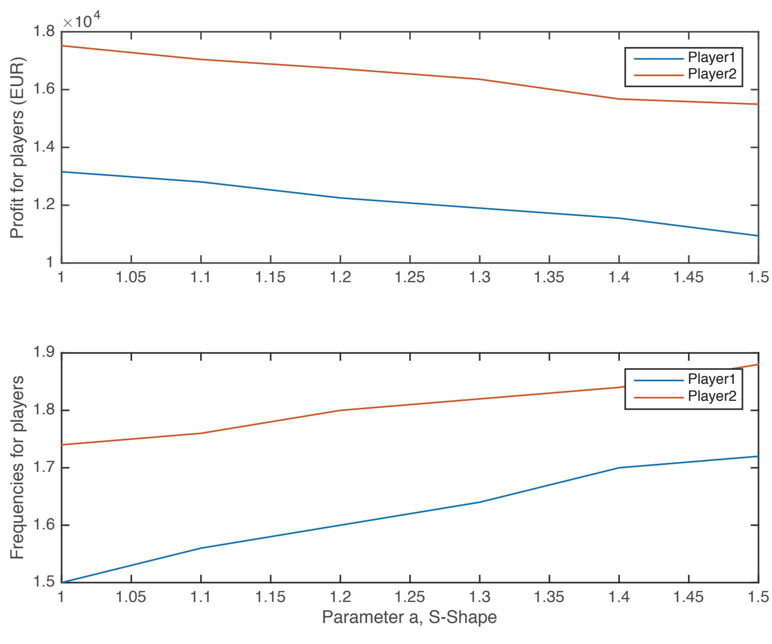


Figure 5.10. Evolution of profits for two players, S-Shape function, different costs and Cournot’s game.

If the experiment is carried out with a variation of difference in costs, then the interaction of S-Shape function is critical for improvement of efficiency. Table 5.5 and Figure 5.11 show system performance.

Table 5.5. Set of data for sensitivity analysis of costs and S-Shape function.

Variable / parameter	**Value**
P (EUR)	200.00
λ (EUR/PAX)	0.20
α	0.95
t_F (h)	1.00
t_G (h)	0.50
q (seats)	180
c_1 (cEUR per flight)	6,014.70 … 3,007.35
c_1 (cEUR per flight)	3,007.35 … 6,014.70
a	1.25

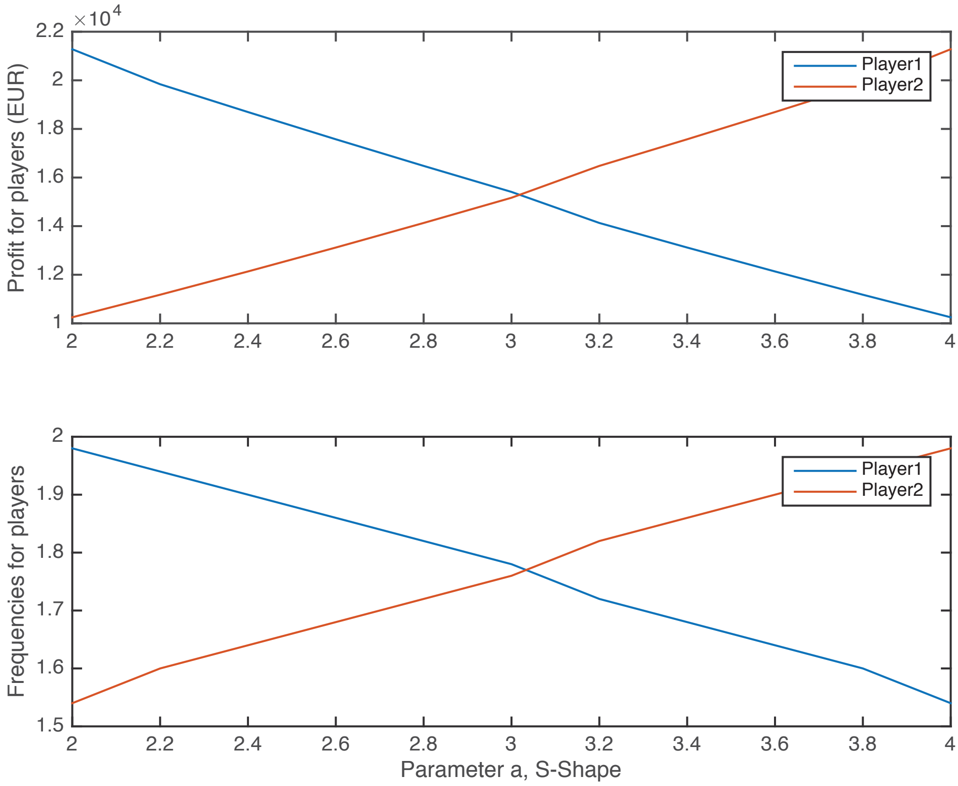


Figure 5.11. Evolution of profits for both players with S-Shape function and Cournot's game.

5.4.6 Cournot's game with charges by utilization

This is the last experiment for this section and is focused on evaluating impact of charges by utilization.

Table 5.6. Set of data for sensitivity analysis of navigation fee.

Variable / parameter	**Value**
P (EUR)	200.00
λ (EUR/PAX)	0.20
α	0.95
t_F (h)	1.00
t_G (h)	0.50
q (seats)	180
c_1 (cEUR per flight)	6,014.70
c_1 (cEUR per flight)	4,811.76
a	1.25
Charges by utilization (EUR)	[100 … 500]
Capacity of system (expeditions)	10

The experiment considers high charges for utilization and low capacity because the goal is to observe the effects on frequency allocation for both players. The results are very explicit, frequencies are more insensitive to this strategy but profits show the effect of this mechanism. In this game, demand is not affected by this charge (price model or demand model does not depend on charge), however it could be a good approach for further developments. What is more, in practice, airline managers validate this output and take into consideration that they flight where demand wants and they transfer these kind of concepts to final price ticket as much as they can.

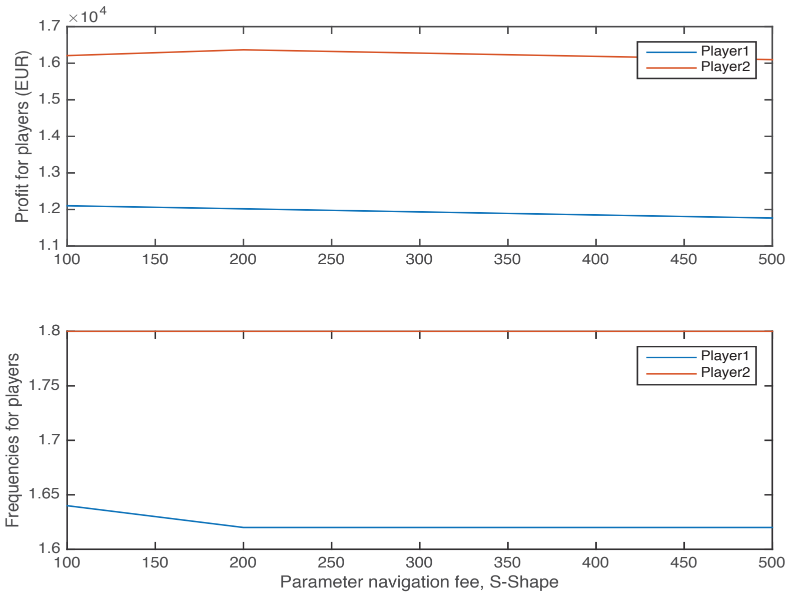


Figure 5.12. Evolution of profits for both players if charges by use are considered.

5.5 Conclusions

This chapter develops a set of ideas regarding airline competition and game theory models. Basically, models are based on Cournot and Stackelberg statement. Therefore, price variation is not considered, only competition in frequencies and its impacts on demand performance (quantity, price and marketshare).

Stackelberg's model highlights some interesting results. First, new entrants have opportunities in the market if they run lower costs than the current supply. This idea is consistent with practices in industry, LCC achieve their share because low cost model (based on point to point services) is less costly than legacy carriers or network carriers. If the current operator has less costs is very difficult for new entrant to achieve enough marketshare to survive. Obviously, this model does not show dynamics of bankrupt. Furthermore, Stackelberg's model provides high levels of frequencies and lower prices, which is better for customers.

Cournot's model shows that equilibrium at long term is possible if both competitors defence their business. The competition achieves this equilibrium in terms of frequency (or capacity in general term) but not in price (Bertrand's game). Previous authors have considered that this situation is more realistic to model current practices, and then Cournot's is good choice. The quantity of frequencies is fewer than optimal supply with Stackelberg's model, which is consistent with behavior of competitors in a long game.

For both models, Cournot and Stackelberg, costs are critical to define differences between two players. However, the performance achieves high gradients with evolution of elasticity of demand against utility provided by airlines. Sensitivity analysis for parameter a of S-Shape function demonstrates the high interaction of marketshare distribution on tendency to increment supply. Further developments can introduce logit model at numerical level. S-Shape function allows larger analyitical developments that result in compact equations.

Finally, charges for utilization near of capacity constraints are interesting and soft results are provided in this analysis. However, numerically, the additional term proposed is light compared with airline's profits. What is more, this charge has not translation to demand in these models and this point is critical to understand globally the performance of this kind of mechanisms for regulation. Despite of these arguments, charges for inefficient practices are consistent and further research is desirable.

6 Conclusions and future research

6.1 Conclusions of this thesis

Airline network configuration is a critical aspect of strategy for operators. The value proposition for passengers has a strong dependency on this configuration because this allows direct services or low fares. For this reason, airlines have to balance well different aspects as routes, frequencies or aircraft size. Analytical models are very useful to understand the strategy and the strenghtness of business model which are central aspects to survive in a competitive environment. They provide interesting causal-models with simple equations and few parameters. Models presented in this work are relevant in the way that provide compact formulations to evaluate the cost for operator and for passengers.

The developed models are capable to evaluate the impact of fixed costs in the network design. Fixed costs are a key issue because leasing or ownership of aircraft is the main asset that is weighting the profit and loss account or the balance sheet. Furthermore, human labor is other key aspect running business model. The experience of legacies is a good example to take into consideration the interest to review and actualize the network frequently.

One of the most interesting highlights is the good performance of point-to-point configuration when the demand is large enough to achieve high load factors. If operating costs are low, these load factors can be lower and this configuration beats the hub-and-spoke configuration.

However, if frequencies are high or demand is not very high, then the consolidation of operations at one hub is the best solution. Furthermore, if aircraft becomes larger, fares results lower and attractiveness for passengers increase. In this case, coordination at hubs is a key to minimize connection times. This parameter penalizes passenger travel time and it is associated with quality of service. For large networks and with equal aircraft operating cost, hub and spoke can reduce point to point network costs about 40%.

Stopover configuration is old fashioned in industry. It is related to the begginings of aviation when aircrafts have to stop due technical reasons. The technological evolution of aircrafts and the high cost of stops have impacted negatively in this performance. However, for low levels of demand, few routes but long stage lengths, it could be an interesting solution. There are two factors that penalize this network: density of airports and demand. The problem is due to logistics issues (many to many distribution) because if many routes are necessary the fleet rises dramatically. In the future, if manufacturers continue increasing aircraft capacity, it could be possible to study long haul systems of stopovers (with open skies agreements and a minimum space in the market).

In addition, from the point of view of policy makers, if air transport continues growing and load factors do not perform better, then new structural measures can be necessary. Analytical models are suitable for policy planning because outputs are valuable and models are less costly.

Regarding practicioners of airline planning, the main conclusion is that these models suit fine to optimize costs. Saving costs related to aircraft or crew staff is mandatory. Analytical models presented have improved previous formulation to be more accurated, however this fine tunning at level of flight scheduling only is possible with OR tools. Compact formulations have had to assume some mean values and this point makes that in practice some resource could be non-well utilized. Furthermore, the precision of planning is mandatory to run the business because aircrafts and employes have to be localized and assigned to flights. Flight schedule, routings and pairings are the basic information to develop control in the operation.

This work presented two algorithms to solve assignments in airlines. First, a Complete Enumeration Algorithm with Exhaustive Search Algorithm provides always the optimal solution for a problem, if the size's problem is not very large. One of the restrictions is time of computation, but the other is the impossibility to enumerate all the space of feasible solutions (it could be done in research areas but airlines have fewer computational tools). For this reason, a Tabu Search Algorithm provides a good trade-off of quasi-optimal solutions and time of computation. The mechanisms that have been designed to variate solutions are simple but very robust. The main evidence is the good performance of this algorithm in the test proposed. There are several lines of improvements that are explained in the proposed future lines of research.

One of the main conclusions related to metaheuristics is the interest to transfer knowledge to industry with these techniques but oriented to small and medium airlines. They could be potential users of simple tools because currently they are using simple sheets of calculation. However, metaheuristics or this Tabu Search Algorithm works with some parameters that are sophisticated for many practicioners in the industry. Some additional efforts of development could be necessary to improve this aspect and provide close tools for industry.

Solving the integrated problem has interest from scientific point of view and it could be a plus for large networks. However, daily operation requires update aircraft routings and crew pairings independently. For this reason, it is more interesting to advance with separated models.

About complexity in airline networks, this short chapter provides some indices to understand the topology of airline networks. At the end, most of them are very similar. However degree distribution, cluster coefficient and Gini index are the most interesting indices to measure levels of concentration. All airlines concentrate their operations in main bases, it is related to cost efficiency. Airline managers have to balance well the advantages of economies of concentration and robustness of decentralization. There is a trade-off of costs which impact is perceived by demand.

In addition, two real cases studies have been presented. They are two low cost airlines with different size and years of operation. The clusterization of point to point networks improves their grow if demand grows in parallel. These airlines do not achieve levels of concentration of hub and spoke networks but they are relatively concentrated. Aircrafts flight where people want. Then, the reliatibility is better in airlines that run point-to-point network. Airlines that hold high degree indices at main airports and provide connection services have poor performance of reliability. In this way, analytical model constates that evolution of delays and costs are worse for hub and spoke networks. In practice, low cost airlines manage better indices of punctuality because they avoid connection services. This is considered a problem of coordination with high associated cost and problem of investment for airlines (related to resources needed).

Finally, related to competivity models. The main conclusion is that models developed with game theory confirm that airline industry has to be focused on cost control. This is because airlines preserve their market share with more frequencies or cutting cost to reduce fares. Both actions are good for passengers but if demand does not buy tickets at the same rate, load factors go down and efficienty is penalized. In this way, simple models highlights that equilibrium point at very competitive markets could be nearest of inefficiency if air transport systems are congested. Then, some additional regulations are necessary and charges associated with indicators of efficiency could be good for airline industry.

To conclude, network configuration is a strategical issue for managing airlines. It is the main asset to build the value proposition that passengers perceive and it gives sustainability to business model. This industry is costly oriented and an efficient network configuration allows reducing costs, better planning, resources utilization factors and offer better service to passengers. However, this cost reduction could be applied to maintain inefficient networks and preserve competitive positions. This process is not correct and liberalization allows it. Some regulations and charging structures are necessary to ensure that airline networks are developed in interest of social and efficient transport system.

6.2 Future lines of research

Analytical models are very interesting to introduce new paradigms of network configuration in the research. There are some findings in this thesis that encourage proposing further lines of research. First, propose analytical models for mixed network configurations. Pure strategies are interesting to clarify some tendencies but main airlines combine both networks. Previous works exist but they do not provide compact formulation because it is easier to evaluate with algorithms. Second, introduce in equations for analytical models the effect of stochastic demand and variation on stage length configurations.

Third, analyse some trending scenarios like long haul routes with stopover between main hubs in the world, operated by only one alliance. The goal is to understand if regulated service could improve efficiency and promote global mobility (in similarity with round-the-world ships). Some critics with aggressive liberalized markets argue that current policies improve performance of transport systems. Efficient long haul routes could be supported by two findings of this thesis: network design analysis indicates that some configurations only can be achieved by global operators (not monopolies) but it could be interesting to provide good services at low cost levels, and competition destroy this developments because wars in terms of frequency or fares dissuades operators.

Fourth, the models proposed in this thesis have not considered the problem of congestion as a cost. It could be a good early step to understand main problems at hubs. Futhermore, coordination in a hub is complex, and analytical models can integrate probabilistic approaches to consider dynamic effects on performance. In addition, pricing schemes for efficiency practices are welcome in the industry, which is focused in environmental issues, and game theory (Cournot’s model) provides an interesting line of developments for the future.

Fifth, related to airline network planning with metaheuristics, the main line of research for the future is to improve the Tabu Search to design routes starting from flight schedule and designing three movements: swap sub-routes, insert sub-routes, cancel flights. Furthermore, a more sophisticated Reactive Tabu Search can improve the performance of the algorithm.

Sixth, there is a specific question that appears and it is interesting, the influence of higher speed in air transport performance. New business models can be driven. If speed was higher at the same costs, different resources allocation could be proposed and different performance could be achieved. This point is interesting because fuel consumption is related to speed, and fixed costs are related to time of flight. There is a trade-off that never has been studied, but manufacturers are showing new aircrafts models because people who travel in first classes of conventional airplanes could be potential customers of this concept of business (plus, long haul airlines are going to propose high density configurations for international markets).

Seventh, related to complexity, this is the main area of interest for the author of this thesis. Complex science allows considering a lot of topics and perspectives (including game theory). The first step that this thesis allows doing is to develop software with the aim to solve real situations of disruptions where time of computation has to be a key of success. Then, a variation of TSA to design sub-routes from flight schedule can be a great improvement. The analysis of topology of networks is a first step, now it is interesting to include a dynamic perspective. On the other hand, the airport has been considered as passive agent but it is not real, airports can play a key role to improve realibility (addressing customized operations). Then, metaheuristics for airport operations improvement is a line of research for the future. This is to allocate resources fine to increase reliability of the system.

Finally, game theory has been developed in many papers, always thinking how airlines can improve their performance. This thesis shows that there is a large way to provide further researchs. There are some simple games that are future lines of research: Bertrand model, Bayes-Nash, etc. In addition, include demand models for market share estimation based on real data is a main topic. However, it is the moment to apply game theory to recover some regulations in the air transport industry based on the airport role. Also, airport pricing (landing and navigation charges) considering the good use of common and constrained resources is a trending line of research for the future. For this issue, the collaboration of European Commission and Eurocontrol is desirable.

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APPENDICES

APPENDIX 1. Aircraft operating cost breakdown

Data about aircraft operating cost has been adopted from Cook (2011) and Belobaba (2007). Also, additional data has been found at main aircraft manufacturer webs (i.e. Airbus, 2015).

Table A1.1. Aircraft operating cost breakdown.

	B733	B734	B735	B738	B752	B763	B744	A319	A320	A321	AT43	AT42	A380	A350
MTOW (tn)	60.4	65.6	55.2	72.6	107.1	180.7	392.5	66.6	73.6	86.4	16.8	22.1	590	268
Rotation	4.5	4.1	5.1	4	2.7	1.8	1.5	4.9	4.4	4.2	4.7	5	-	-
Flight (min)	90	100	80	120	200	390	510	90	110	120	60	60	-	-
Taxi (h/day)	1.4	1.5	1.5	1.3	0.9	0.8	0.7	1.5	1.6	1.5	1.3	1.2	-	-
Turnaround time (min)	80	90	70	70	90	120	180	60	60	60	100	90	-	-
SH/ day (est.)	14.2	14.5	14.3	14.0	14.0	16.1	18.0	13.8	14.1	14.1	13.8	13.7	-	-
BH/day (est.)	8.2	8.3	8.3	9.3	9.9	12.5	13.5	8.9	9.7	9.9	6.0	6.2	-	-
BH / day	8.2	8.2	8.2	9.3	9.8	12.3	13.9	8.8	9.7	9.8	6	6.3	-	-
Seats	127	145	113	161	200	246	356	133	153	187	43	60	538	325
Fuel (kg/min)	40.6	40.2	37.1	42.9	55.2	78.8	163.5	38.4	39.3	46.5	7.2	10.5	226.8	140.5
Maintenance (€/BH)	740	790	680	610	890	1140	1610	790	720	850	380	470	-	-
Fleet (€/SH)	320	380	360	540	560	710	1090	510	610	730	160	230	-	-
Crew (€/BH)	330	320	310	400	420	680	870	310	360	370	160	180	-	-
Fleet (€/day)	4528	5504	5130	7542	7812	11431	19566	7013	8581	10293	2213	3151	-	-
Crew (€/day)	2706	2624	2542	3720	4116	8364	12093	2728	3492	3626	960	1134	-	-
Maintenance (€/day)	6068	6478	5576	5673	8722	14022	22379	6952	6984	8330	2280	2961	-	-

Note: block-hour (BH), service-hour (SH).

The relationship between MTOW and capacity (seats) of the plane is functional and depends on technology. ATR aircrafts have a differents performance respect to jets. Also, new aircrafts like A350 or A380 are lighter than classic A3xx of B7xx (see figure A1.1). In particular, B747 are very heavy related to capacity and it is compared with other aircrafts, this is the reason because of it is out of passenger routes for many airlines.

However, in this work and for simplicity, only only jets are considered (see figure A1.2). Polynomial relationship can be estimated between MTOW and seat capacity.

Mainly, the critical question is the relationship between fuel consumption and seat supply. Really, consumption is close related with MTOW and flight performance. For this work this assumptions are simplified and only linear relation between fuel and seats are considered for jets (see figure A1.3).

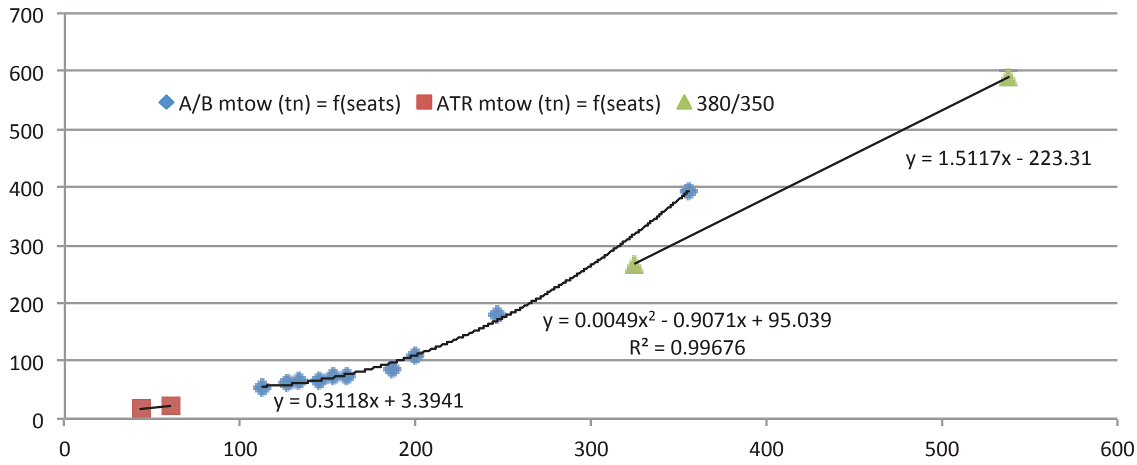

Figure A1.1. Functional relationship between seats and MTOW by type of aircraft.

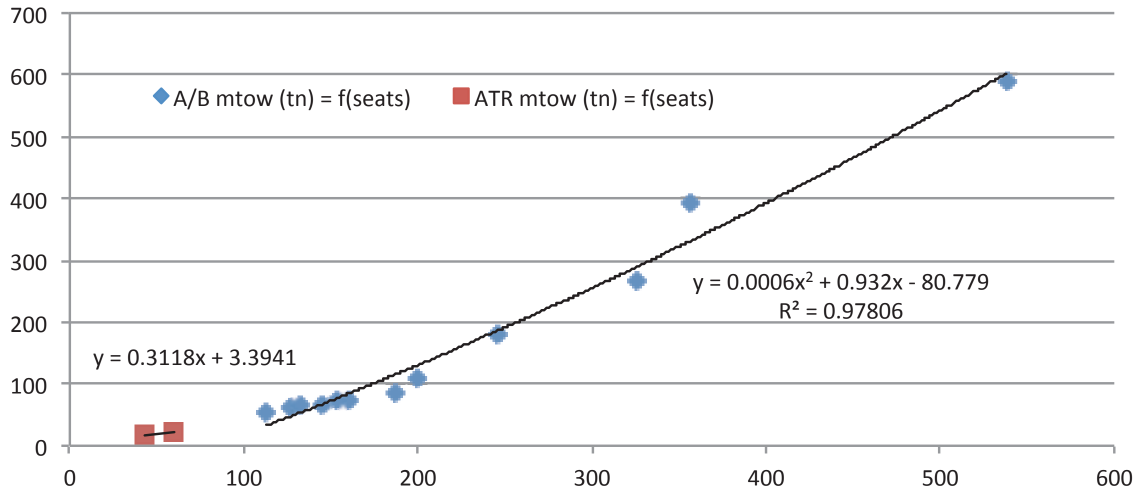

Figure A1.2. Polynomial relationship between seats and MTOW.

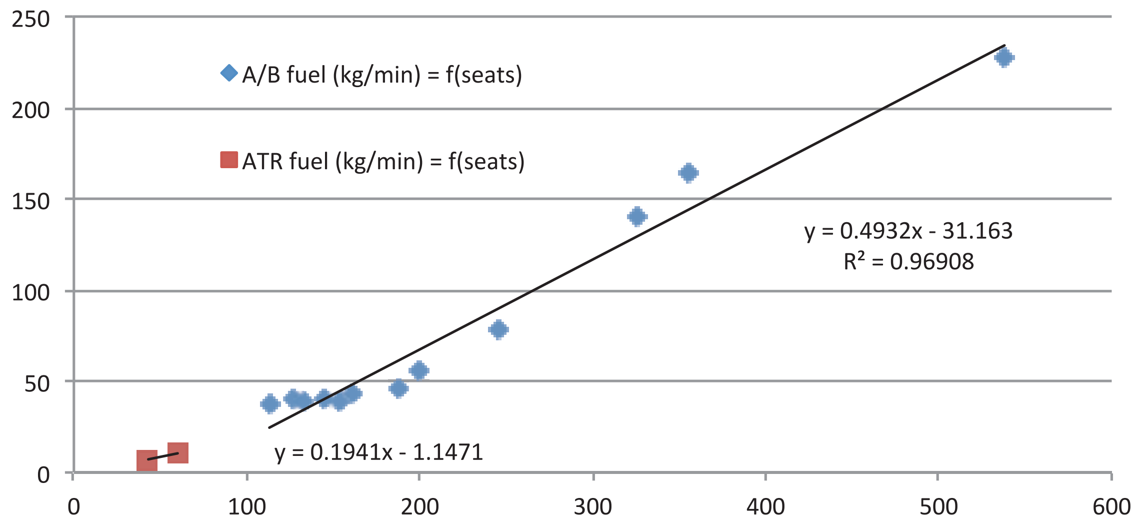

Figure A1.3. Linear relationship between seats and fuel consumption (kg/min).

Furthermore, total fuel cost for a flight depends on distance (unitary cost per minut can be calculated with figures above). Distance and time of flight can be estimated with a linear regression (explained in chapter 2). But, relationship between fuel consumption and distance is linear (Horton, 2010). At zero distance, minimum fuel is required for climbing phase (associated with 20 minutes aprox.).

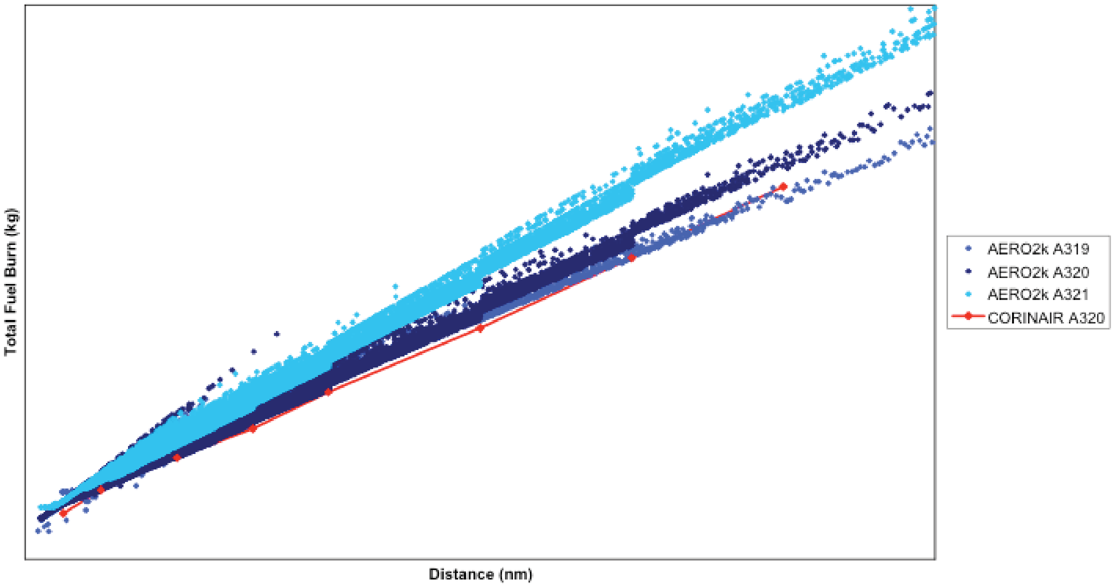

Figure A1.4. Comparison against fuel burned and distance for A319 and A32x.

Source: Horton, 2010.

Maintenance is an important and complicated activity for airlines industry. There are a lot of factors that managers have to consider to plan and budget. Mainly, some maintenance activities have to be carried out independently of flight hours because they depend only on aircraft age (Ackert, 2010). However, other activites depend on block-hours. Furthermore, financial analysts and airline managers tend to consider maintenance as a variable cost or fixed imputable cost. Definitively, they don't want to have fixed cost in PLA because it difficult to analyse profitability of routes. In practice, if the utilization of aircraft is high the error by assuming maintenance dependency on block-hours is small.

For this work, linear relationship between maintenance and seats capacity of aircraft is considered as variable cost and it is related to block-hours (see figure A1.5).

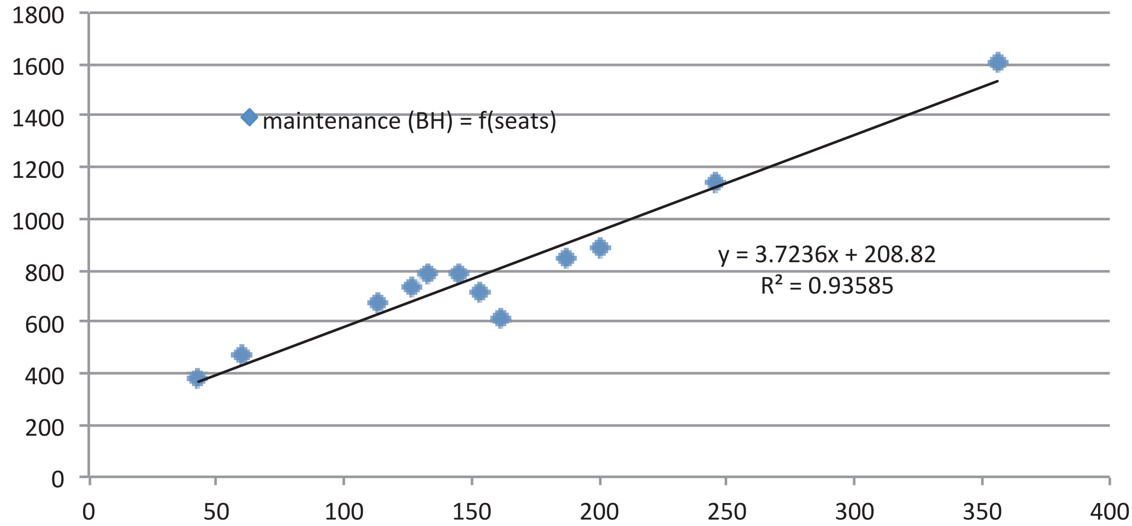

Figure A1.5. Linear relationship between maintenance cost (euro per block-hour) and seats.

Regarding fleet cost, a functional relationship between it and seats is determined by regression. Furthermore, some analyses have been carried finding a lot of disparity between data and it is because a lot of parameters take part in this question (MTOW, degree of technology, efficiency, etc.). Figure A1.6 shows the regression between fleet cost (euros per day) and capacity of aircraft measured in seats. Fleet cost is a fixed cost and depreciation is according to life of asset. This focus is more realistic than others that consider aircraft as variable or imputable fixed cost.

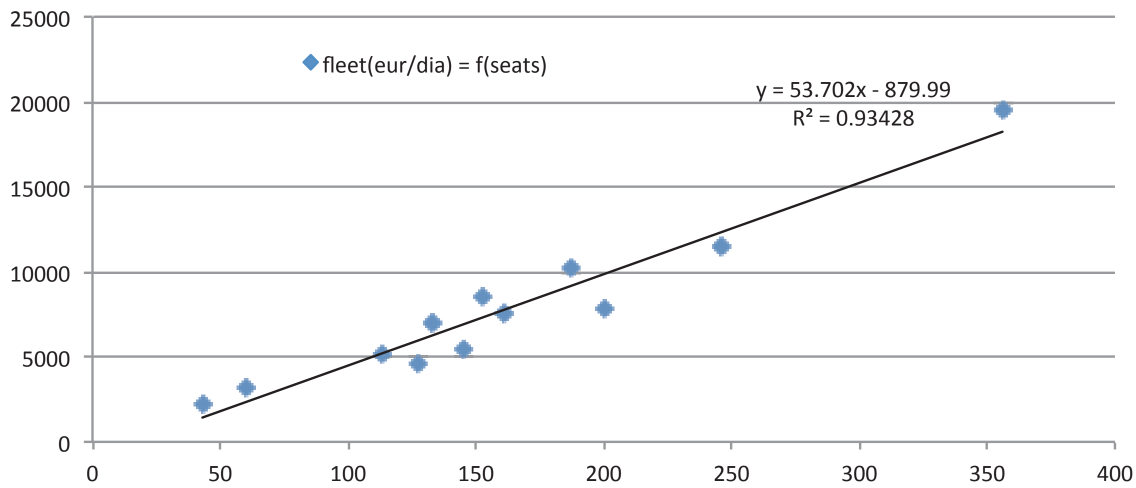

Figure A1.6. Linear relationship between fleet cost (euro per day) and seats.

Finally, crew costs varies with aircraft capacity measured in seats (i.e. Airbus estimates that extra crew should be added for any 50 seats of increased capacity) and time of flight. Distance and time of flight have impact on crew size because of the rotation. When crew cost is estimated from PLA and operating metrics this issue is not well analysed. However, figure A1.7 shows crew costs for different aircrafts. It is possible to see how big planes (typical for long haul) require extra crew, much more crew than extrapolation from smaller indicates. Here there is an impact of long haul routes and it is mandatory to calculate crew cost per day and not per block-hour. If the airline network has good utilitzation factors and network is large, then it is possible to calculate extra crew cost by block-hours. For new networks this method can raise important errors estimating profit and loss account. In this work, first regression is the definitive to calculate costs.

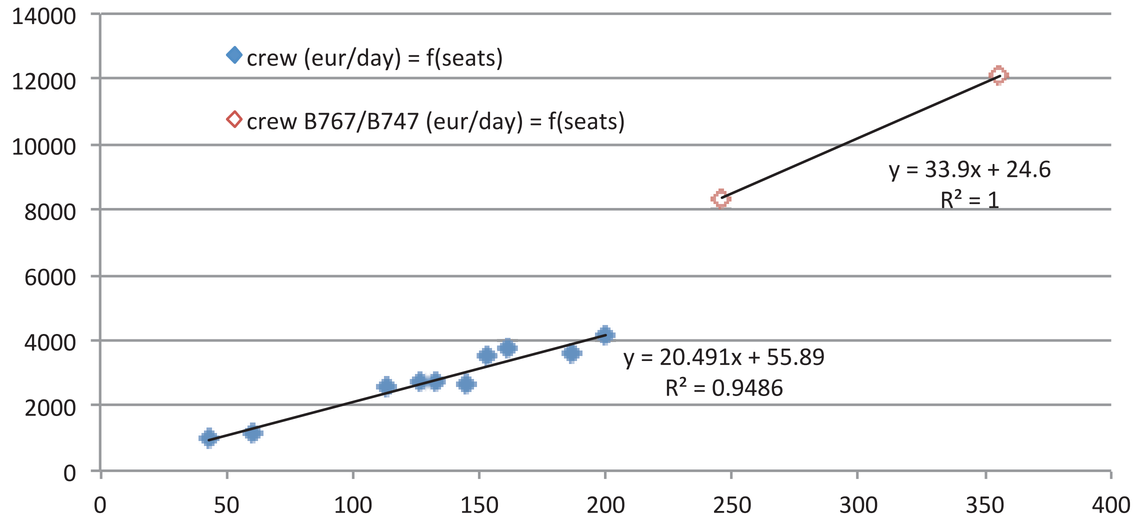

Figure A1.7. Linear relationship between crew cost (euro per day) and seats.

Estimating charges for landing and en-route can be complicated and it depends on each country or airport. With the aim of simplicity and to ensure understandable model, this work assumes averages values that IATA (2009) had estimated and they have been updated to actual cost. Then cost of landing and en-route control are related to navigation and infrastructure services, this cost is $c_N = 2{,}144€/flight$ and are equal for all flights (really, this cost depends on length, weight, etc.).

Finally, operator cost can be calculated by expressions contained in table A1.2 and it is possible to apply the following expression.

$$C_O = \sum_i \left(c_{K,i} + c_{M,i} + c_{N,i}\right) + t_C \left(\sum_j c_{W,j} + \sum_r \xi(\mathrm{r}) c_{E,r} \right)$$

Where, C_O is the operator cost for a set of I flights, R routes and J aircrafts (sum of aircraft operating cost for I flights); $c_{K,i}$, $c_{M,i}$, $c_{N,i}$ are fuel, maintenance and navigation cost per flight i (i=1,…, I) described in Table A1.2. The cycle time is t_C and multiplies fixed cost that are estimated per day of operation: $c_{W,j}$ is the ownership cost for aircraft j, $c_{E,r}$ is the crew cost for route r, being ξ(r) a function to increase crew sets depending on stage length.

Finally, total aircraft operating cost can be calculated by expressions contained in this table.

Table A1.2. Expressions to calculate aircraft operating cost breakdown.

Concept	Units	Expression	Coeficients	Variables
Fuel consumption, cost per flight of one aircraft.	€/flight	$c_K = \begin{cases} p_K(k_0 + k_1 q)t_F & q \geq 100 \\ p_K(k_0 + k_1 100)t_F & 100 > q \geq 0 \end{cases}$	$k_0 = -1868.16$ kg/h $k_1 = 29.59$ kg/h·seat	q, capacity (seats). t_F, time flight (h)
Maintenance, cost per flight of one aircraft	€/flight	$c_M = (m_0 + m_1 q)t_F$	$m_0 = 208.82$ €/h $m_1 = 3.7236$ €/h·seat	q, capacity (seats) t_F, time flight (h)
Ownership, cost per day of one aircraft	€/day·aircraft	$c_W = \begin{cases} w_0 + w_1 q & q \geq 50 \\ w_0 + w_1 50 & 50 > q \geq 0 \end{cases}$	$w_0 = -879.99$ €/day.acft $w_1 = 53.702$ €/day·acft.seat	q, capacity (seats) acft = aircraft
Crew, cost per day of one crew	€/day·crew	$c_E = (e_0 + e_1 q)$	$e_0 = 55.89$ €/day·crew $e_1 = 20.491$ €/day·crew·seat	q, capacity (seats)
Time of flight	h	$t_F = (b_0 + b_1 l)$	$b_0 = 0.338802$ h $b_1 = 0.001111$ h/km	l, stage length of flight (km)
Price of fuel	€/kg	$p_K = 0.6$	-	Constant

Table A1.3. Aircraft operating cost breakdown – average values.

Costs €	Average
MTOW (tn)	100.0
Rotation	3.9
Fight (min)	160.8
Taxi (h/day)	1.3
Turnaround time (min)	89.2
Service-hours per day (est.)	14.5
Bock-hours per day (est.)	9.2
Seats	160.3
Fuel (kg/min)	50.0
Maintenance cost (per block-hour)	805.8
Fleet cost (SH)	516.7
Crew cost (SH)	276.0
Fleet cost (day)	7,730.2
Crew cost (day)	4,008.8
Maintenance cost (day)	8,035.4

Note: Derivated from Table A1.1.

APPENDIX 2. Algorithm. Pseudocodes

1. Routing and pairing generation

```
function [output] = fgen(A,tg,tgm,f,B,I,J)
%FGEN routes generator
%   takes FS and tg and generates all routes feasibles for
aircraft
%   f is the current flight
%   tg is time around
%   B is route matrix
%   I,J position of current route
%   A (FS) (1c)Number_Flight (2c)D_air (3c)SDT (4c)A_air (5c)SAT
if nargin==3
    opc=1;
else
    if f>0
        opc=2;
    elseif f==0
        opc=3;
    end
end

if opc==1
    [n,m]=size(A);
    v=A(:,1);
    B=v;
    I=1;
    J=1;
    for i=1:n
        B=f(A,tg,tgm,v(i),B,I,J);
        I=size(B,1)-(n-i)+1;
    end
    output=B;
elseif opc==2
    j=find(A(:,1)==f);
    a=A(j,4);
    sat=A(j,5);
    t=sat+tg;
    t2=sat+tgm;
    x=find(A(:,2)==a & A(:,3)>=t & A(:,3)<t2); %vector indice
candidatos
    if isempty(x)==0
        %disp('lleno')
        v=A(x,1); %vuelos candidatos
        auxV=[0];
        k=1;
        l=1;
        C=v;%primera generada
        for i=1:length(v)
            C=f(A,tg,tgm,v(i),C,k,l);
            k=size(C,1)-(length(v)-i)+1;
        end
        [dfC,dcC]=size(C);
```

```
        auxV=[zeros(1,dcC) ;C];
    else
        auxV=[0];
    end
    [dfauxV,dcauxV]=size(auxV);
    if I==0 && J==0 %primera ruta
        B=[f*ones(dfauxV,1) auxV];
        output=B;
    else
        [Fil,Col]=size(B);
        if J==Col
            %al final
            aB=[B(1:I-1,:) zeros(I-1,dcauxV);
ones(dfauxV,1)*B(I,1:J) auxV; B(I+1:end,:) zeros(Fil-I,dcauxV)];
            B=aB;
        elseif J<Col && Col-J<dcauxV
            s=dcauxV-(Col-J);
            %mat=ones(dfauxV,1)*B(I,:)
            aB=[B(1:I-1,:) zeros(I-1,s); ones(dfauxV,1)*B(I) auxV;
B(I+1:end,:) zeros(Fil-I,s)];
            B=aB;
        elseif J<Col && Col-J>dcauxV
            s=(Col-J)-dcauxV;
            aB=[B(1:I-1,:); ones(dfauxV,1)*B(I,1:J) auxV
zeros(dfauxV,s); B(I+1,:)];
            B=aB;
        elseif J<Col && Col-J==dcauxV
            aB=[B(1:I-1,:); ones(dfauxV,1)*B(I) auxV;
B(I+1:end,:)];
            B=aB;
        end
        output=B;
    end
elseif opc==3
    output=B;
end
end
```

2. Complete Enumeration Algorithm

```
function [C,df] = fpcr(s)
%FVR Summary of this function goes here
%   s = seed [1 1 ... 1 0 .. 0]

n=length(s);
v=sum(s);
v=fix(v);
w=n-v;
C=[];
df=0;
if v>1 && w>=1
    s1=s(1:v);
    s2=s(v+1:w+v);
```

```
    %j=2 y mas
    for j=v:-1:2
        z=[s1(1:j-1) s2(1)];
        [x,k]=fun([s1(j:v) s2(2:w)]);
        y=[ones(size(x,1),1)*z x];
        C=[y];
        df=df+k;
    end
    %j=1
    [x,k]=fun([s1 s2(2:w)]);
    y=[ones(size(x,1),1)*s2(1) x];
    C=[y];
    df=df+k;
elseif v==1 && w>1
    s1=s(1:v);
    s2=s(v+1:w+v);
    [x,k]=fun([s1 s2(2:w)]);
    y=[s2(1)*ones(size(x,1),1) x];
    C=[y];
    df=df+k;
elseif v==1 && w==1
    s1=s(1);
    s2=s(2);
    y=[s2 s1];
    C=[y];
    df=df+1;
end
C=[s];

if nargout==2
    df=df+1;
end

end
```

3. Exhaustive Search Algorithm

```
function [routes,TC, cadena,C,U,Prod,dC] =
fselroutes(mr,mc,mf,fleet)
%FSELECT Summary of this function goes here
%   Detailed explanation goes here

%nflights
n=size(mf,1);
%m routes
m=size(mr,1);

U=funU(mr,mf);

if fleet>m
    %all routes are possible
    %then
    TC=sum(mc);
    routes=1:m;
```

```
    cadena='ok';
    C=[];
    U=[];
    Prod=[];
    dC=0;
elseif fleet<=m
    seed=ones(1,fleet);
    seed=[seed zeros(1,m-fleet)];
    [C,dC]=fpcr(seed);
    [c0,c1,c2]=fcobertura(C,U);
    Prod=C*mc;
    if isempty(c1)==0
        %satisface coberturas
        [TC,k]=min(Prod(c1));
        k=c1(k);
        routes=[1:m].*C(k,:);
        routes=routes(routes>0);
        cadena='ok';
    elseif isempty(c1)==1
        %o falta flota para cubrir o se cubre re-recorriendo
vuelos
        if isempty(c2)==0
            [TC,k]=min(Prod(c2));
            k=c2(k);
            routes=[1:m].*C(k,:);
            routes=routes(routes>0);
            cadena= 'constraints';
        else
            cadena='more fleet is required';
            routes=[];
            TC=0;
        end
    end

end

end
```

4. Tabu Search Algorithm

```
function [routes,TC, cadena, tabu_list,cycle_list,Cob] =
fstabu(mr,mc,mf,fleet,K)
%FSTABU Summary of this function goes here
%   Detailed explanation goes here

%parameters tabu
if nargin==4
    K=1000; %iter
end
%K=5; %iter
tenure=500; %tabu tenure
```

```
max_cycle=2; %maxima repeticion de una solucion
exito=0;
exopt=0;
Cseedopt=1e10;
Lcl=10;
cycle_list=[];
tabu_list=[];
Cob=[];
%nflights
n=size(mf,1);
%m routes
m=size(mr,1);

%matrix covering
U=funU(mr,mf);

if fleet>m
    %all routes are possible
    %then
    TC=sum(mc);
    routes=1:m;
    cadena='ok';
elseif fleet<=m
    seed=ones(1,fleet);
    seedo=[seed zeros(1,m-fleet)];
    seed=seedo;
    seede=[seedo 0]; %ayuda a reducir flotas
    %a partir de aqui tabu
    if prod(seed*U)==1
        Cseed=seed*mc;
        Cseedopt=Cseed;
        cycle_list=[seed Cseed 1+max_cycle];
    else
        Cseed=1e20;
        Cseedopt=Cseed;
        cycle_list=[];
    end

    tabu_list=[0 0 1];
    r=[];
    for i=1:K
        %2 aleatorios entre 1 y m
        %permuto seed
        exito=0;
        while exito==0
            nonul=fpos(seede==1);
            nul=fpos(seede==0);
            c_nonul=length(nonul);
            c_nul=length(nul);
            c1=randi([1 c_nonul],1,1);
            c2=randi([1 c_nul],1,1);
            r=[nonul(c1) nul(c2)];

            %r=randi([1 m+1],1,2)
```

```
            %busco r en la lista tabu_list
            [x,y]=ismember(r,tabu_list,'rows');
            s=[r(2) r(1)];
            [v,w]=ismember(s,tabu_list,'rows');
            %if r pertenece a la lista y esta tenure porque iter
bloque es
            %superior a i
%             if i==K
%                 seed=zeros(1,m);
%                 seed(2)=1;
%                 seed(10)=1;
%                 seed(11)=1;
%                 Cseed=seed*mc;
%                 exito=5;
%                 r=[22 22];
%                 x=5;
%             end
            tabu_list;
            if isempty(tabu_list)
                exito=1;
                tabu_list=[r i+tenure];
            elseif (x==1 && tabu_list(y,3)>=i) && (v==1 &&
tabu_list(w,3)>=i)
                if Cseed<Cseedopt -> aspiration criterion
                exito=1;
                else
                exito=0;%esta tenure
            elseif (x==1 && tabu_list(y,3)<i) %&& (v==1 &&
tabu_list(w,3)>=i)
                exito=1;%no esta tenure
                tabu_list;
            elseif (v==1 && tabu_list(w,3)<i) && (x==1 &&
tabu_list(y,3)>=i)
                exito=2;%no esta tenure
                tabu_list;
            elseif x==0
                exito=1;
                tabu_list;
            elseif x==1 && v==0
                exito=2;
                tabu_list;

            end
        end
        if exito==1
            aux=seede(r(1));
            seede(r(1))=seede(r(2));
            seede(r(2))=aux;
            seed=seede(1:m);
            Cseed=seed*mc;
            Cob(i)=prod(seed*U);
            if Cseed<Cseedopt && Cob(i)==1;
                exopt=1;
            else
```

```
                %[x,y]=ismember(r,tabu_list(:,1:2),'rows');
                %tabu_list(y,3)=i+K;
                exopt=0;
            end
        elseif exito==2
            aux=seede(s(1));
            seede(s(1))=seede(s(2));
            seede(s(2))=aux;
            seed=seede(1:m);
            Cseed=seed*mc;
            Cob(i)=prod(seed*U);
            if Cseed<Cseedopt && Cob(i)==1
                exopt=1;
            else
                %[v,w]=ismember(s,tabu_list(:,1:2),'rows');
                %tabu_list(w,3)=i+K;
                exopt=0;
            end
        elseif exito==5
            seed;
            Cob(i)=prod(seed*U);
            if Cseed<Cseedopt && Cob(i)==1
                exopt=1;
            else
                exopt=0;
            end
        end
%         Cob
        %calculo el coste asociado de esta seed (Cseed)

        if exopt==1
            if isempty(cycle_list)
%                 disp 'uno'
                cycle_list=[cycle_list;seed Cseed i+max_cycle];
                Cseedopt=Cseed;
            else
                [vseed,
wseed]=ismember(seed,cycle_list(:,1:m),'rows');
                if isempty(vseed)
%                     disp 'dos'
                    %no hay coincidencia
                    %la acumulo
                    cycle_list=[cycle_list;seed Cseed
i+max_cycle];
                    Cseedopt=Cseed;
                else
%                     disp 'tres'
                    %hay coincidencia y esta en wseed
                    if cycle_list(wseed,m+2)<i %esta y la almaceno
                        cycle_list(wseed,m+2)= i+max_cycle;
                        Cseedopt=Cseed;
                        % if wseed>=i esta bloqueada por
reincidencia
                    end
```

```
                end
%                 %quito todos cuya m+2 sea inferior a k,
liberados
%                 for j=1:size(cycle_list,1)
%                     if cycle_list(j,m+2)<i
%                         cycle_list(j,:)=[];
%                     end
%                 end
                if size(cycle_list,1)>Lcl
                    %quito los que superan la lista
                    [vo1,vo2]=sort(cycle_list(:,m+1));
                    cycle_list=cycle_list(vo2(1:Lcl),:)
                end
            end
            i
        end

    end

    if isempty(cycle_list)
        TC=0;
        routes=[];
        cadena='ko';
    else
        [val,ival]=min(cycle_list(:,m+1));
        C=cycle_list(ival,1:m);
        %tiene que cumplir cobertura
        TC=C*mc;
        routes=[1:m].*C;
        routes=routes(routes>0);
        cadena='ok';
    end

end

cycle_list;
tabu_list;

end
```

APPENDIX 3. Flight schedule for cases studies

This appendix shows airline data for case studies of Chapter 4. Due an agreement of confidentiality the set of data has been manipulated to delete all of data that could reveal the identity of both airlines.

Table A3.1. Airline 1. Flight frequencies of airline 1 for one week.

Origin	Destination	Week	Day 1	d.2	d.3	d.4	d.5	d.6	d.7
15	14	2	1	0	0	0	1	0	0
14	15	2	1	0	0	0	1	0	0
15	6	2	0	1	0	0	0	1	0
6	15	2	0	1	0	0	0	1	0
15	9	2	1	0	0	1	0	0	0
9	15	2	1	0	0	1	0	0	0
12	10	2	0	0	0	1	0	0	1
10	12	2	0	0	0	1	0	0	1
8	10	4	1	0	0	1	1	0	1
10	8	4	1	0	0	1	1	0	1
8	18	4	1	0	0	1	1	0	1
18	8	4	1	0	0	1	1	0	1
8	24	2	0	0	1	0	0	1	0
24	8	2	0	0	1	0	0	1	0
1	10	2	0	1	0	0	1	0	0
10	1	2	0	1	0	0	1	0	0
1	59	2	0	1	0	0	1	0	0
59	1	2	0	1	0	0	1	0	0
14	10	3	1	0	1	0	1	0	0
10	14	3	1	0	1	0	1	0	0
14	18	2	1	0	0	0	1	0	0
18	14	2	1	0	0	0	1	0	0
14	24	3	1	0	1	0	1	0	0
24	14	3	1	0	1	0	1	0	0
6	10	3	0	1	0	1	0	1	0
10	6	3	0	1	0	1	0	1	0
6	11	2	0	1	0	0	1	0	0
11	6	2	0	1	0	0	1	0	0
6	16	3	1	0	0	1	0	1	0
16	6	3	1	0	0	1	0	1	0
6	18	2	1	0	0	1	0	0	0
18	6	2	1	0	0	1	0	0	0
6	24	1	1	0	0	0	0	0	0
24	6	1	1	0	0	0	0	0	0
6	55	1	0	0	0	0	0	1	0
55	6	1	0	0	0	0	0	1	0
6	44	1	1	0	0	0	0	0	0
44	6	1	1	0	0	0	0	0	0

9	10	3	1	0	1	0	0	1	0
10	9	3	1	0	1	0	0	1	0
9	16	2	0	0	0	1	0	0	1
16	9	2	0	0	0	1	0	0	1
9	28	2	0	0	1	0	0	1	0
28	9	2	0	0	1	0	0	1	0
9	18	4	0	1	0	1	0	1	1
18	9	4	0	1	0	1	0	1	1
9	24	7	1	1	1	1	1	1	1
24	9	7	1	1	1	1	1	1	1
9	58	1	0	0	0	0	0	0	1
58	9	1	0	0	0	0	0	0	1
9	59	2	0	0	1	0	0	1	0
59	9	2	0	0	1	0	0	1	0
9	44	1	0	0	1	0	0	0	0
44	9	1	0	0	1	0	0	0	0
9	55	0	0	0	0	0	0	0	0
55	9	0	0	0	0	0	0	0	0
7	16	2	0	0	1	0	0	0	1
16	7	2	0	0	1	0	0	0	1
7	10	2	0	0	1	0	0	0	1
10	7	2	0	0	1	0	0	0	1
11	59	2	0	1	0	0	1	0	0
59	11	2	0	1	0	0	1	0	0
3	59	2	1	0	0	1	0	0	0
59	3	2	1	0	0	1	0	0	0
2	21	2	1	0	0	1	0	0	0
21	2	2	1	0	0	1	0	0	0
2	20	2	1	0	0	1	0	0	0
20	2	2	1	0	0	1	0	0	0
13	18	1	0	0	0	0	0	1	0
18	13	1	0	0	0	0	0	1	0
13	24	1	0	0	0	0	0	1	0
24	13	1	0	0	0	0	0	1	0
13	28	0	0	0	0	0	0	0	0
28	13	0	0	0	0	0	0	0	0
5	24	1	0	0	0	0	0	1	0
24	5	1	0	0	0	0	0	1	0
4	18	0	0	0	0	0	0	0	0
18	4	0	0	0	0	0	0	0	0
4	24	0	0	0	0	0	0	0	0
24	4	0	0	0	0	0	0	0	0
25	24	3	1	0	1	0	1	0	0
24	26	3	1	0	1	0	1	0	0
28	55	2	0	1	0	0	1	0	0
55	28	2	0	1	0	0	1	0	0
28	49	2	0	0	0	0	0	1	1
49	28	2	0	0	0	0	0	1	1
28	41	3	0	0	1	0	0	1	1

41	28	3	0	0	1	0	0	1	1
28	45	2	0	0	0	0	0	1	1
45	28	2	0	0	0	0	0	1	1
28	59	2	1	0	0	0	1	0	0
59	28	2	1	0	0	0	1	0	0
18	17	2	1	0	0	0	1	0	0
17	18	2	1	0	0	0	1	0	0
18	55	3	1	0	1	0	1	0	0
55	18	3	1	0	1	0	1	0	0
18	54	2	0	1	0	0	1	0	0
54	18	2	0	1	0	0	1	0	0
18	49	3	0	0	1	0	0	1	1
49	18	3	0	0	1	0	0	1	1
18	41	7	1	1	1	1	1	1	1
18	41	7	1	1	1	1	1	1	1
41	18	7	1	1	1	1	1	1	1
41	18	7	1	1	1	1	1	1	1
18	27	2	1	0	0	1	0	0	0
27	18	2	1	0	0	1	0	0	0
18	53	2	0	1	0	0	1	0	0
53	18	2	0	1	0	0	1	0	0
18	45	4	0	1	0	1	0	1	1
45	18	4	0	1	0	1	0	1	1
18	30	2	0	0	1	0	0	0	1
30	18	2	0	0	1	0	0	0	1
18	57	2	1	0	0	0	1	0	0
57	18	2	1	0	0	0	1	0	0
18	66	2	0	0	1	0	0	1	0
66	18	2	0	0	1	0	0	1	0
18	59	2	0	0	0	1	0	1	0
59	18	2	0	0	0	1	0	1	0
18	61	2	1	0	0	1	0	0	0
61	18	2	1	0	0	1	0	0	0
18	65	2	1	0	0	0	1	0	0
65	18	2	1	0	0	0	1	0	0
18	20	6	1	1	1	1	1	0	1
20	18	6	1	1	1	1	1	0	1
24	23	6	1	1	1	1	1	0	1
23	24	5	1	1	1	1	0	0	1
24	55	2	0	1	0	0	1	0	0
55	24	2	0	1	0	0	1	0	0
24	54	2	0	1	0	0	1	0	0
54	24	2	0	1	0	0	1	0	0
24	49	4	0	1	0	1	0	1	1
49	24	4	0	1	0	1	0	1	1
24	41	7	1	1	1	1	1	1	1
24	41	7	1	1	1	1	1	1	1
41	24	7	1	1	1	1	1	1	1
41	24	7	1	1	1	1	1	1	1

24	45	7	1	1	1	1	1	1	1
24	45	7	1	1	1	1	1	1	1
45	24	7	1	1	1	1	1	1	1
45	24	7	1	1	1	1	1	1	1
24	53	2	0	1	0	0	1	0	0
53	24	2	0	1	0	0	1	0	0
24	57	2	1	0	0	0	1	0	0
57	24	2	1	0	0	0	1	0	0
24	67	2	0	0	1	0	0	0	1
67	24	2	0	0	1	0	0	0	1
24	59	2	0	0	0	1	0	0	1
59	24	2	0	0	0	1	0	0	1
24	61	2	1	0	0	1	0	0	0
61	24	2	1	0	0	1	0	0	0
24	20	4	1	0	0	1	1	0	1
20	24	4	1	0	0	1	1	0	1
24	65	2	1	0	0	1	0	0	0
65	24	2	1	0	0	1	0	0	0
17	23	2	0	1	0	0	1	0	0
23	17	2	0	1	0	0	1	0	0
17	41	2	0	0	0	0	0	1	1
41	17	2	0	0	0	0	0	1	1
17	45	2	0	0	0	0	0	1	1
45	17	2	0	0	0	0	0	1	1
19	41	3	0	0	1	0	0	1	1
41	19	3	0	0	1	0	0	1	1
19	45	2	0	0	1	0	0	0	1
45	19	2	0	0	1	0	0	0	1
21	41	5	0	1	1	1	0	1	1
41	21	5	0	1	1	1	0	1	1
21	49	3	0	0	1	0	0	1	1
49	21	3	0	0	1	0	0	1	1
21	45	4	0	1	0	1	0	1	1
45	21	4	0	1	0	1	0	1	1
23	41	3	0	0	1	0	0	1	1
41	23	3	0	0	1	0	0	1	1
23	20	4	0	1	0	1	0	1	1
20	23	4	0	1	0	1	0	1	1
22	59	2	1	0	0	0	1	0	0
59	22	2	1	0	0	0	1	0	0
22	20	4	1	0	0	1	1	0	1
20	22	4	1	0	0	1	1	0	1
41	20	3	0	0	1	0	0	1	1
20	41	3	0	0	1	0	0	1	1
45	20	3	0	0	1	0	0	1	1
20	45	3	0	0	1	0	0	1	1
49	20	3	0	0	1	0	0	1	1
20	49	3	0	0	1	0	0	1	1
20	54	2	0	1	0	0	1	0	0

54	20	2	0	1	0	0	1	0	0
20	55	2	0	1	0	0	1	0	0
55	20	2	0	1	0	0	1	0	0
20	59	2	1	0	0	1	0	0	0
59	20	2	1	0	0	1	0	0	0
65	59	2	1	0	0	0	1	0	0
59	65	2	1	0	0	0	1	0	0
52	56	3	0	0	1	0	0	1	1
56	52	3	0	0	1	0	0	1	1
52	51	3	0	0	1	0	0	1	1
51	52	3	0	0	1	0	0	1	1
58	54	4	1	0	0	1	1	1	0
54	58	4	1	0	0	1	1	1	0
58	55	7	1	1	1	1	1	1	1
55	58	7	1	1	1	1	1	1	1
50	54	5	1	0	1	0	1	1	1
54	50	5	1	0	1	0	1	1	1
50	55	3	0	1	0	0	1	0	1
55	50	3	0	1	0	0	1	0	1
50	48	7	1	1	1	1	1	1	1
48	50	7	1	1	1	1	1	1	1
50	53	4	1	0	0	1	1	0	1
53	50	4	1	0	0	1	1	0	1
42	60	2	0	0	1	0	0	0	1
60	42	2	0	0	1	0	0	0	1
54	55	2	0	1	0	0	1	0	0
55	54	2	0	1	0	0	1	0	0
54	53	3	0	1	0	1	1	0	0
53	54	3	0	1	0	1	1	0	0
54	59	7	1	1	1	1	1	1	1
54	59	7	1	1	1	1	1	1	1
54	59	0	0	0	0	0	0	0	0
59	54	7	1	1	1	1	1	1	1
59	54	7	1	1	1	1	1	1	1
59	54	0	0	0	0	0	0	0	0
47	48	2	1	0	0	1	0	0	0
48	47	2	1	0	0	1	0	0	0
47	43	1	0	0	0	0	0	0	1
43	47	1	0	0	0	0	0	0	1
47	60	0	0	0	0	0	0	0	0
60	47	0	0	0	0	0	0	0	0
47	59	7	1	1	1	1	1	1	1
47	59	7	1	1	1	1	1	1	1
59	47	7	1	1	1	1	1	1	1
59	47	7	1	1	1	1	1	1	1
57	29	2	1	0	0	0	1	0	0
29	57	2	1	0	0	0	1	0	0
43	48	4	1	0	1	0	1	0	1
48	43	4	1	0	1	0	1	0	1

60	55	7	1	1	1	1	1	1	1
60	55	7	1	1	1	1	1	1	1
55	60	7	1	1	1	1	1	1	1
55	60	7	1	1	1	1	1	1	1
60	48	7	1	1	1	1	1	1	1
60	48	7	1	1	1	1	1	1	1
48	60	7	1	1	1	1	1	1	1
48	60	7	1	1	1	1	1	1	1
60	53	0	0	0	0	0	0	0	0
53	60	0	0	0	0	0	0	0	0
60	44	7	1	1	1	1	1	1	1
44	60	7	1	1	1	1	1	1	1
60	46	2	0	1	0	0	0	1	0
46	60	2	0	1	0	0	0	1	0
60	64	0	0	0	0	0	0	0	0
64	60	0	0	0	0	0	0	0	0
60	63	0	0	0	0	0	0	0	0
63	60	0	0	0	0	0	0	0	0
55	59	7	1	1	1	1	1	1	1
59	55	7	1	1	1	1	1	1	1
55	53	7	1	1	1	1	1	1	1
55	53	7	1	1	1	1	1	1	1
53	55	7	1	1	1	1	1	1	1
53	55	7	1	1	1	1	1	1	1
55	44	4	1	0	1	0	1	0	1
44	55	4	1	0	1	0	1	0	1
55	33	1	0	1	0	0	0	0	0
33	55	1	0	1	0	0	0	0	0
55	29	2	0	1	0	0	1	0	0
29	55	2	0	1	0	0	1	0	0
55	38	2	1	0	0	1	0	0	0
38	55	2	1	0	0	1	0	0	0
55	62	1	0	0	1	0	0	0	0
62	55	1	0	0	1	0	0	0	0
56	59	3	0	0	1	0	0	1	1
59	56	3	0	0	1	0	0	1	1
51	59	3	0	0	1	0	0	1	1
59	51	3	0	0	1	0	0	1	1
48	59	7	1	1	1	1	1	1	1
48	59	7	1	1	1	1	1	1	1
48	59	7	1	1	1	1	1	1	1
59	48	7	1	1	1	1	1	1	1
59	48	7	1	1	1	1	1	1	1
59	48	7	1	1	1	1	1	1	1
48	44	3	0	1	0	1	0	1	0
44	48	3	0	1	0	1	0	1	0
53	39	2	0	1	0	1	0	0	0
39	53	2	0	1	0	1	0	0	0
53	33	3	1	0	1	0	1	0	0

33	53	3	1	0	1	0	1	0	0
53	38	4	0	1	1	1	0	1	0
38	53	4	0	1	1	1	0	1	0
53	31	1	0	1	0	0	0	0	0
31	53	1	0	1	0	0	0	0	0
44	59	7	1	1	1	1	1	1	1
59	44	7	1	1	1	1	1	1	1
44	39	1	0	1	0	0	0	0	0
39	44	1	0	1	0	0	0	0	0
44	33	2	0	1	0	1	0	0	0
33	44	2	0	1	0	1	0	0	0
44	29	2	0	1	0	0	1	0	0
29	44	2	0	1	0	0	1	0	0
44	38	2	0	1	0	1	0	0	0
38	44	2	0	1	0	1	0	0	0
44	31	1	1	0	0	0	0	0	0
31	44	1	1	0	0	0	0	0	0
46	59	3	0	1	0	1	0	1	0
59	46	3	0	1	0	1	0	1	0
67	59	2	1	0	0	1	0	0	0
59	67	2	1	0	0	1	0	0	0
30	59	2	1	0	0	0	1	0	0
59	30	2	1	0	0	0	1	0	0
34	59	1	0	0	0	1	0	0	0
59	34	1	0	0	0	1	0	0	0
40	59	1	0	0	0	1	0	0	0
59	40	1	0	0	0	1	0	0	0
36	59	0	0	0	0	0	0	0	0
59	36	0	0	0	0	0	0	0	0
39	59	2	0	1	0	1	0	0	0
59	39	2	0	1	0	1	0	0	0
29	59	3	1	0	1	0	1	0	0
59	29	3	1	0	1	0	1	0	0
29	38	4	1	0	1	0	1	0	1
38	29	4	1	0	1	0	1	0	1
38	59	5	1	1	0	1	1	0	1
59	38	5	1	1	0	1	1	0	1
33	59	5	1	1	1	1	0	1	0
59	33	5	1	1	1	1	0	1	0
37	59	1	0	1	0	0	0	0	0
59	37	1	0	1	0	0	0	0	0
32	59	1	0	0	0	1	0	0	0
59	32	1	0	0	0	1	0	0	0
35	59	1	0	0	0	1	0	0	0
59	35	1	0	0	0	1	0	0	0
31	59	1	0	1	0	0	0	0	0
59	31	1	0	1	0	0	0	0	0
62	59	1	0	0	1	0	0	0	0
59	62	1	0	0	1	0	0	0	0

Table A3.2. Airline 2. Flight frequencies of airline 2 for one week.

Origin	Destination	Freq/week
116	96	56
96	116	56
99	108	42
126	139	42
139	126	42
108	99	42
116	97	32
97	116	32
30	96	28
132	96	28
133	139	28
138	96	28
139	30	28
116	94	28
139	133	28
94	116	28
96	30	28
96	37	28
37	96	28
96	132	28
96	138	28
30	139	28
122	132	26
132	122	26
132	125	26
125	132	26
124	132	24
132	133	24
132	124	24
133	132	24
132	131	23
131	132	23
41	37	21
12	96	21
137	96	21
138	37	21
126	132	21
30	41	21

114	96	21
132	30	21
161	96	21
172	96	21
173	96	21
175	176	21
175	96	21
176	175	21
115	96	21
95	116	21
96	12	21
37	41	21
116	95	21
96	114	21
96	115	21
132	126	21
37	138	21
96	137	21
30	132	21
96	161	21
96	172	21
96	173	21
96	175	21
41	30	21
116	83	20
101	99	20
99	101	20
83	116	20
38	116	19
116	38	19
116	174	19
174	116	19
37	34	18
116	89	18
28	88	18
30	34	18
34	30	18
34	37	18
88	28	18
89	116	18
85	116	17
25	96	17
96	25	17

116	90	17
89	96	17
90	116	17
90	96	17
96	89	17
96	90	17
116	85	17
176	75	16
116	75	16
4	116	16
75	116	16
119	96	16
75	176	16
3	37	16
116	4	16
37	3	16
96	119	16
140	132	15
132	140	15
41	88	15
88	41	15
133	123	14
28	96	14
133	137	14
37	44	14
137	133	14
116	30	14
138	3	14
116	37	14
138	75	14
138	99	14
138	116	14
138	125	14
37	116	14
139	4	14
37	132	14
139	122	14
139	124	14
116	132	14
139	131	14
116	138	14
30	4	14
143	96	14

147	96	14
37	176	14
164	96	14
172	173	14
4	30	14
173	172	14
17	96	14
38	96	14
175	4	14
116	92	14
116	93	14
176	37	14
30	45	14
30	75	14

Note: more than 2000 relations are considered in one week, here there are only the first two hundred.