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To My Teachers

Abstract

On Endogenous Market Incompleteness, Cycles, and Growth.

Alexandre Dmitriev

This doctoral thesis consists of three self-contained essays in Macroeconomics and Economic Growth.

Essay 1. "Technological Transfers, Limited Commitment and Growth"

Evidence shows that there are substantial rich-to-poor international capital flows although not as abundant as differences in rates of return would suggest. These flows are procylcical: abundant in good times and scarce in bad times. They have been reported to promote growth and stability in some countries, but merely to augment instability in the others. Conventional growth models face certain difficulties in accounting for this pattern. In this paper, we propose a dynamic model of capital flows to developing countries which is qualitatively consistent with these empirical regularities. The model is based on three main premises: i) international lending contracts are imperfectly enforceable; ii) access to the international financial markets results in technological transfers to a developing country from the rest of the world; iii) some of the productivity gains associated with the access to external financing are perishable. We solve for transitional dynamics of the model economy with endogenously incomplete markets and compare the results with the solutions obtained from the perfect risk-sharing and autarkic environments. In addition, we examine the implications of alternative assumptions about the severity of the repudiation punishment for growth, welfare and borrowing patterns. Our findings suggest that technological transfers may play a role of an important enforcement mechanism. In our framework, existence of substantial rich-to-poor capital flows is not inconsistent with the presence of default risk.

Essay 2. "A Note on Computing Partial Derivatives of the Value Function by Simulation"

The problems involving incentive compatibility constraints in the form of dynamic participation constrains have received wide attention in the literature due to the recent advances in dynamic optimization techniques. Often the optimality conditions for this class of problems involve partial derivatives of the value function with respect to some of the endogenous state variables. In this paper we suggest an algorithm for computing these partial derivatives by simulation. The attractive features of the algorithm include its rather wide scope of applicability and simplicity of implementation. Furthermore, the suggested method does not suffer from the curse of dimensionality and therefore it is particularly convenient for the models involving many state variables. (JEL C63)

Essay 3. "Institutions and Growth: Some Evidence from Estimation Methods Partially Robust to Weak Instruments"

This paper focuses on the empirical approach proposed by Hall and Jones (1999) to estimate the effect of what they call "social infrastructure" on productivity across countries. We attempt to address the criticism of Acemoglu et al (2001) directed towards this methodology for relying on the geographical instruments. To do so we consider the issue of weak identification in the linear instrumental variables model of Hall and Jones (1999). The evidence obtained from the partially robust estimators like the k-class and jackknife estimators is interpreted on the basis of the Monte Carlo studies. Our findings suggest that using some of the k-class estimators allows exclusive reliance on the linguistic variables to instrument for institutional quality despite their low correlation with the endogenous regressor in question. (JEL C15, O40)

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Introduction

This thesis consists of three self-contained essays. Although united under one title they differ in both the topics considered and approaches chosen. The first essay presents a dynamic model of capital flows to low- and middle income countries under endogenous market incompleteness. The second essay is a methodological contribution. It offers an algorithm which facilitates solving the models with dynamic participation constraints and many endogenous state variables. The third essay is an attempt to bring together the recent advances in the econometrics of weak instruments with the empirical methodology of the literature on deep determinants of economic growth. Despite these apparent differences, there is still a common theme relating the three essays. They attach special emphasis to a particular institutional aspect of the economy - imperfect enforcement of contracts. In the lines which follow we give an brief overview of the three essays included into this thesis.

Chapter 1, *Technological Transfers, Limited Commitment and Growth,* considers the relation between international capital flows, cycles, and growth. Our point of departure is a number of empirical regularities concerning levels and volatility of international capital flows that have been often considered in the literature as paradoxes. We start with the famous "Lucas paradox". Indeed, evidence suggests that there are substantial rich-to-poor international capital flows although not nearly as abundant as differences in rates of return would suggest. Another regularity we consider asserts that net capital inflows tend to be procylcical: abundant in good times and scarce in bad times. Kaminsky, Reinhart, and Vegh (2004) who report empirical evidence on this feature name the phenomenon "When it Rains, it Pours". Finally, access to external financial markets has been reported to promote growth and stability in some countries, but merely to augment instability in the others.

Conventional growth models have been facing certain difficulties in accounting for this pattern. In this essay, we propose a dynamic model of capital flows to developing countries which is qualitatively consistent with these empirical regularities. The model is based on three main premises. Our first premise is that international lending contracts are imperfectly enforceable. Following the sovereign debt literature, we assume that the available enforcement mechanisms are limited to a threat of permanent exclusion from the international markets. Hence, instead of exogenously limiting the amount of capital the developing countries may borrow, we incorporate a friction which allows a recipient country to borrow to the extent it will be willing honor its debts. Our second premise is that access to international financial markets is associated with increased efficiency of production in some sectors of the developing economy. This increase in productivity originates from the transfers of technologies from the industrialized world to the developing country which enjoys what Gerschenkron (1952) referred to as an "advantage of backwardness". Our final premise is that the recipient country will not be able to fully, if at all, enjoy the productivity benefits should it be excluded from the international markets. To some extent, this feature of the model can be motivated by an inherent property of technology - its partial excludability. Similar assumptions have been used by Cohen and Sachs (1986) and Eaton and Gersovitz (1984) who assume that foreign debt repudiation results in permanent loss of productive efficiency associated with foreign technology.

We solve for transitional dynamics of the model economy with endogenously incomplete markets and compare the results with the solutions obtained from the perfect enforcement and autarkic environments. In addition, we examine the implications of alternative assumptions about the severity of the repudiation punishment for growth, welfare and borrowing patterns. Our findings suggest that technological transfers may play a role of an important enforcement mechanism. In our framework, existence of substantial rich-to-poor capital flows is not inconsistent with the presence of default risk. This prediction of our model distinguishes itself from those of the existing international risk-sharing models with imperfect enforcement of lending contracts such as those Marcet and Marimon (1992) and Kehoe and Perri (2002). We overcome the difficulty that the models of sustained growth have in explaining the rich structure of observed capital flows and the "wide spectrum of borrowing patterns across low- and middle-income countries" (Marcet and Marimon 1992, p. 221). Our framework predicts that under limited commitment the pattern of capital flows depends heavily on the perishable productivity gains associated with the external financing opportunities. In addition, our model outperforms existing theories of economic growth in its ability to account for countercyclical behavior of net capital inflows to developing countries. Contrary to the implications of the models of perfect or exogenously restricted capital mobility our framework predicts is that the capital inflows to the emerging economies are acyclical.

Chapter 2, *A Note on Computing Partial Derivatives of the Value Function by Simulation,* contributes to the methodology for solving stochastic dynamic models with endogenously incomplete markets. In particular, we consider a class of problems where incompleteness of markets arises endogenously from the failure to perfectly enforce lending contracts. As is common in the limited commitment literature, instead of exogenously limiting the amount or type of assets the agents may trade, these models incorporate a friction which allows the agents to borrows to the extent they will be willing to repay later on. In this environment, the available enforcement mechanisms are limited to a threat of exclusion from the future intertemporal and interstate trade. Hence, the models we consider involve incentive compatibility constrains in the form of participation constrains. Often the optimality conditions for this class of problems involve partial derivatives with respect to some of the endogenous state variables of the optimal value function corresponding to the agent's outside option. Although many numerical methods can provide an approximation for the value function, in general, there is no reason to believe that a derivative of this approximation will be close in any sense to the actual value of the derivative. In this chapter we suggest a simple algorithm for computing these partial derivatives of the optimal value function by simulation.

The attractive features of the algorithm include its rather wide scope of applicability and simplicity of implementation. It can be used to study the questions of risk sharing under imperfect enforcement of contracts, as well as partnerships with limited commitment when several state variables appear in the model corresponding to the outside option. Such models may include habit formation preferences, several types of capital, or reputational co-state variables. The algorithm may still be applicable even though the default model fails to fit into a standard recursive framework. Furthermore, the suggested method is computationally inexpensive, it does not suffer from the curse of dimensionality and therefore it is particularly convenient for the models involving many state variables.

Chapter 3, *Institutions and Growth: Some Evidence from Estimation Methods Partially Robust to Weak Instruments,* belongs to the strand of literature which attempts to shed some light on the origins of the observed income disparities across countries. In recent years, the search for explanations has gone beyond economic variables to investigate "deeper" determinants of economic performance such as geography, integration and institutions. Designing an empirical strategy to access the importance of these factors in explaining variation in income levels is a formidable task. The challenge lies in disentangling the complex web of causality involving these deep determinants and the income levels.

The pioneering contributions of Hall and Jones (1999) as well as Acemoglu, Johnson and Robinson (2001) have focused on institutional quality as a potential determinant of comparative development. Both studies proposed empirical strategies and introduced new sets of instruments which allowed to demonstrate the causal effect of institutions in explaining income differences. Both approaches essentially share a common weakness which is anything but straightforward to overcome. To find a source of exogenous variation in institutions which would not have direct effect on current output levels the scholars had to go to the geographical and historical determinants of institutions. For instance, Hall and Jones (1999) rely on the distance from the equator while Acemoglu, Johnson, and Robinson (2001) utilize historical settler mortality to instrument for the institutional quality. Due to this it is natural to expect that the instruments proposed would be only weakly correlated with the endogenous variable of interest. The latter, however, often constitutes a source of severe problems for both estimation and inference purposes.

In Chapter 3 we focus on the empirical methodology proposed by Hall and Jones (1999) to estimate the effect of what they call "social infrastructure" on productivity across countries. We attempt to address the criticism of Acemoglu et al (2001) directed towards the Hall and Jones (1999) methodology for relying on the instruments with less than convincing theoretical justification. Hence, the central question we pose is whether the Hall and Jones (1999) results are driven by the use of "latitude" as well as Frankel and Romer (1999) predicted trade intensity to instrument for institutional quality. However, in attempt to accomplish this task we had to overcome the problem of weak identification.

The instruments proposed by Hall and Jones (1999) and in particular linguistic instruments are found to be only weakly correlated with their proxy for institutional quality. The questions we seek to address in the context their model are: Can one rely on two-stage least squares (TSLS) for estimation and inference purposes in view of the potential problem of weak identification? Can performance of TSLS improved upon by using the estimators partially robust to weak instruments? If so, which of the partially robust estimators estimators would be preferable?

To address these issues we conducted a Monte Carlo study and compared relative performance several partially robust estimators. We conclude that the linguistic instruments cannot be deemed as irrelevant. However, depending on the specification some of the examined estimators suffer from both bias and size distortions. To address the mentioned criticism of Acemoglu et al (2001) we reestimate the linear instrumental variable model of Hall and Jones (1999) across several specifications using methods partially robust to weak instruments. Furthermore, we interpret the evidence from the estimation on the basis of the Monte Carlo results. We conclude that using the partially robust estimators allows us to utilize the linguistic variables to instrument for institutional quality despite their low correlation with the endogenous regressor. Moreover, relying exclusively on the linguistic variables as instruments produces the results qualitatively consistent with the original findings of Hall and Jones (1999). In other words we tend to discard the argument that their results are driven by reliance on the "geographical" instruments.

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CHAPTER 1

Technological Transfers, Limited Commitment and Growth

1.1. Introduction

Several features concerning levels and volatility of international capital flows have been documented in the literature. First, international capital flows from the capital-rich to the capital-poor countries are too scarce in view of enormous differences in rates of return.[1] Second, there are substantial private capital flows to the developing countries.[2] Third, access of the capital-poor countries to the international financial markets has been reported to promote growth and stability in some cases but merely augment instability in the others.[3] Fourth, the net capital inflows are procyclical in most developing countries.[4] Conventional growth models have been reported to face certain difficulties in accounting for the observed pattern of capital flows from the industrialized to the low- and middle income countries.

In this paper we propose a dynamic model of capital flows to low- and middle income countries which is qualitatively consistent with these empirical regularities. Our benchmark is a stochastic growth model with two productive sectors one of which may enjoy productivity benefits associated with the access to external financing. We focus on the institutional aspects of the economy and consider environments which differ in the extent to which the international borrowing contracts are enforceable. To do so, we solve for transition dynamics of the model economy with endogenously incomplete markets and compare the results with

[1] The evidence on what Lucas (1990) argued to be a puzzle, has been reported by Reinhart and Rogoff (2004) and Lane (2004), among others.

[2] For instance, according to UNCTAD (1994, 2001) Foreign Direct Investment (FDI) inflows to developing countries increased from an annual average of $13.1 billion for 1981–1985 to $240.2 billion in 2000. Some researchers, as e.g. Albuquerque (2003, p. 354), tend to conclude that "International private capital flows represent a major sourse of financing economic activity in developing countries".

[3] The World Bank's *Global Development Finance* (2001, p. 71) report concludes that "although opening up domestic financial markets to international competition has attracted more capital to developing countries and has bolstered growth in some, the larger volume of capital market transactions has also contributed to a more volatile climate". An extensive review of the empirical evidence on the topic under a suggestive title: "Volatile International Capital Flows: A Blessing or a Curse?" is provided by Kaminsky (2004).

[4] Kaminsky, Reinhart, and Vegh (2004) report empirical evidence on this phenomenon which they name "When it Rains, it Pours".

the solutions obtained from the perfect risk-sharing and autarkic environments. In addition, we examine the role of alternative assumptions about the severity of the repudiation punishment and their implications for growth, welfare and borrowing patterns.

A number of explanation have been offered in the literature on "Lucas paradox" of why capital does not flow from rich to poor countries.[5] Yet, the evidence presented by Reinhart, Rogoff, and Savastano (2003) and Reinhart and Rogoff (2004, p. 53) tends to suggest that "some explanations may be more relevant than others". They argue that

> "...the key explanation to the "paradox" of why so little capital flows to poor countries may be quite simple: Countries that do not repay their debts have a relatively difficult time borrowing from the rest of the world" (Reinhart and Rogoff 2004, p. 56).

This is the avenue we will follow in this paper. Hence, our point of departure is that international lending contracts are imperfectly enforceable. In the absence of supranational authority, the available enforcement mechanisms are limited to a threat of exclusion from the international markets. Hence, instead of exogenously limiting the amount of capital the developing countries may borrow, we incorporate a friction which allows to a recipient country to borrow to the extent it will be willing to repay later on. Another rationale for relying on this assumption is that countercyclical capital inflows would be predicted both by theories of exogenously constrained access to the world credit markets and by theories of perfect capital mobility (Lane, 2004).

As argued by Albuquerque (2003) an open question which deserves attention in the context of the models with imperfect enforcement is the one concerning the levels of international capital flows. The reasons is that the models of international lending under limited commitment which allow for capital accumulation in the autarky such as those of Marcet and Marimon (1992) and Kehoe and Perri (2002) have very dramatic quantitative implications for international capital mobility. In words of Albuquerque (2003, p. 380) "these models provide an answer to Lucas' (1990) question, but an extreme one". They show that enforcement constrains result in negligible international capital flows both along the transition path and at the steady state distribution. The latter result is less than satisfying in view of the

[5]For instance, Barro, Mankiw, and Sala-i-Martin (1995) discussing international capital mobility in a neoclassical growth model exogenously limit the types of capital which can by financed by borrowing on the world market.

recent evidence on capital flows to developing countries. This is the issue we are going to address in this paper.

One of the reason for this failure is that the defaulter's punishment is not severe enough. This might stem from the failure of the existing theories of capital mobility under limited enforcement to model certain margins. The margin we argue to be important is presence of technological transfers a developing country will enjoy as a consequence of an access to the international markets. By the very nature of technology, that is its partial excludability, the recipient country will not be able to enjoy all the benefits associated with foreign technology should it switch to autarky. This feature makes the defaulter's punishment more severe, as compared to whose used by Marcet and Marimon (1992) or Kehoe and Perri (2002). In the context of our model, this default punishment will introduce a wedge between steady state distributions corresponding to the environment with imperfect enforcement of international lending contracts and the autarky. Whether this will generate non-negligible capital inflows to an economy during its transition from a low level of capital towards its ergodic distribution is the question which we will consider in this paper.

Hence, the second premise of our framework is that access to international financial markets is associated with increased efficiency of production in some sectors of the developing economy. This increase in productivity originates from the transmission of technologies from the industrialized world to the developing country which enjoys what Gerschenkron (1952) referred to as an "advantage of backwardness". A substantial amount of research has documented empirically the role of international capital flows for technological diffusion. Some studies emphasize the positive effect on productivity of openness and free capital movement *per se*. For example, Frankel and Romer (1999) argue that the benefits from integration for a developing country partially stem from the transfer of ideas from the rest of the world. In line with that the World Bank (2001, p. 59) *Global Development Finance* annual report states that there is ample evidence indicating towards the productivity benefits of the capital flows "through transfer of technology and management techniques". In a recent study, Alcalá and Ciccone (2004) provide empirical evidence indicating that openness promotes growth through its effect on TFP.

Other studies stress the importance of FDI as a mechanism of technological transfers to the developing countries from the rest of the world. For instance, according to World Bank (2001) FDI has been positively associated with the productivity of the foreign owned firms and with positive spillover to domestically owned firms.[6] Romer (1993) suggests that FDI has considerable potential to transfer ideas from the industrialized countries to the developing countries. FDI as a potential mechanism of technological transfers has been particularly emphasized due to its increasing role in the stream of international capital flows to low- and middle income countries. As documented by Thomas and Worrall (1994) already in the mid-eighties about a half of all capital flows to the developing countries took form of FDI. The fraction of FDI in the international capital flows kept increasing during the last two decades. Moreover, according to IMF (2003) it now constitutes the most important net flow for all regions.

Our final premise is that the recipient country will not be able to fully, if at all, enjoy the productivity benefits should it be excluded from the international markets. To some extent, this feature of the model can be motivated by an inherent property of technology - its partial excludability. Similar assumptions have been used by Cohen and Sachs (1986) and Eaton and Gersovitz (1984) who assume that foreign debt repudiation results in permanent loss of productive efficiency associated with foreign technology.

We consider a model with two agents, one risk-averse agent representing a developing country and the other risk neutral agent representing the rest of the world. We focus on the growth of the developing country which is assumed to have low initial level of capital. In this context, growth is understood as a transition from the initial low level of capital towards the steady state distribution. We analyze the model within three environments which differ in the extent to which the international lending contracts are being enforced. These are: (i) autarky; (ii) external financing with perfect enforcement of contracts; and (iii) external financing with limited enforcement of contracts. Under the latter regime, a developing country may at any moment appropriate the accumulated capital and refuse to honor its debt. In this case it will suffer a default punishment which will involve loss of any external financing opportunities in the future.

We assume that there are two productive sectors in the economy, which we refer to as domestic and foreign operated sector. Each of the sectors has Cobb-Douglas technology. The risk averse agent decides how much to invest in each of the sectors. The technology which converts investment into capital goods is non-linear and affected by the productivity

[6] Görg and Strobl (2001) provide a comprehensive review of the empirical literature on FDI and productivity spillovers. They also give account of other channels through which productivity spillovers occur such as movement of highly skilled personnel, the 'demonstration effect' or the 'competition effect'.

shocks. The foreign sector is assumed to be more productive due to technological transfers associated with external financing[7]. Failure to honor the external debt results in permanent loss of productivity benefits associated with foreign technology.

We consider two modifications of the model which differ in the default punishment a developing country will endure should it refuse to honor its contractual obligations. First, we analyze a model where in case of debt repudiation the country loses not only productivity benefits in the foreign operated sector but also accumulated capital in this sector. Furthermore, the country is deprived of the possibility to develop this sector on its own. Similar assumption has been used by Marcet and Marimon (1998), where they consider a partnership with limited commitment, and Albuquerque (2003), who studies composition of international capital flows. Under this assumption, the autarkic environment, which is hereafter referred to as one-sector autarky, is similar to the stochastic growth model of Brock and Mirman (1972) augmented with non-linear stochastic investment technology. Our key finding from this model is that perishable technological gains from external financing opportunities may eliminate the default risk even though they affect only some sectors of the economy.

The discussed above assumption of the punishment is case of deviation from the optimal plan may be judged as extremely severe. Indeed, the defaulting country loses not only all the productivity benefits and capital accumulated in the foreign operated sector but also a possibility to develop this sector on its own. Although, the latter cannot be ruled out as completely unrealistic[8], this feature is not especially attractive in our setting since our model economy consists of merely two productive sectors. Therefore, we consider a framework where in case of debt repudiation the developing country loses the technological advantage associated with access to external financing. However, the capital stock in all sectors of the economy remains productive with the TFP level of the domestically operated sector. Relying on this assumption we consider three representative cases which differ in the extent of the technological diffusion.

We overcome the difficulty that the models of sustained growth have in explaining the rich structure of observed capital flows and borrowing patterns across low- and middle-income countries. Our framework suggests that under limited enforcement the pattern of

[7]This assumption relies on the empirical evidence reviewed by Görg and Strobl (2001) who document that in the literature it is often argued that the positive spillovers only affect certain sectors of the economy.

[8]For instance, former soviet republics, after defaulting on the risk-sharing agreement with Russia known as USSR, might face serious difficulties should they intent to develop uranium enrichment and associated sectors.

capital movements depends heavily on the perishable productivity benefits associated with the external financing opportunities.

From a theoretical perspective, our findings allow to conclude that the existence of substantial capital flows from the developed to developing countries is not inconsistent with the presence of the default risk. We also conclude that technological transfers may play a role of an enforcement mechanism. In our framework even moderate technological benefits associated with external financing opportunities may substantially reduce the negative effect on the welfare of the failure to perfectly enforce contracts. Presence of technological diffusion in the environment with limited commitment induces a developing country to use foreign capital to both smooth consumption and invest more heavily in all the sectors of the economy including those directly unaffected by the technological transfers. The latter results in faster growth and significant welfare gains.

Our framework presents a case for capital controls. Contrary to Albuquerque (2003), the normative implications of our model do not advocate discouraging debt flows or encouraging FDI flows to the developing countries. Our claim is that lenders should encourage those capital inflows which are associated with perishable TFP benefits. These might include FDI in the sectors which depend on foreign blueprints or intangible assets, such as managerial skills.

Since we study models with dynamic participation constraints, which involve expected values of the future control variables, we are unable to use the results of standard dynamic programming. Our methodology relies on the contribution of Marcet and Marimon (1998) who have demonstrated that problems with incentive compatibility constraints fall into a general class of problems, which can be cast into an alternative recursive framework. Our numerical analysis utilizes the parameterized expectation approach (PEA) originally proposed by Marcet (1989). A particular version of simulation PEA which we use allows us to handle occasionally binding inequality constraints involving conditional expectations of the future choice variables.

Although PEA algorithm approximates the true equilibrium at the steady state distribution with arbitrary accuracy, the policy function obtained from the long-run simulations may not be a good approximation for the solution during the initial periods. This is of particular importance for our analysis since we consider an economy during the transition towards the steady state distribution. To overcome this problem we use a version of PEA featuring exogenous oversampling in order to find a distinct policy function for the initial periods.

Another non-standard feature of the problem we are solving is that the optimality condition in the limited enforcement environment involve partial derivatives of the value function associated with recursive formulation of the dynamic problem the agent faces in case of debt repudiation. In order to handle this issue, we utilize an algorithm proposed in Dmitriev (2006) to numerically compute partial derivatives of the value function with respect to several endogenous state variables.

The rest of the paper is organized as follows. Section 1.2 presents the baseline models corresponding to the three environments: one-sector autarky, external financing with full and limited enforcement. These models rely on the most stringent assumption about the defaulter's punishment. Section 1.3 describes the numerical algorithms for solving the models and analyzes the solutions for them. Section 1.4 introduces the main model with two-sector autarky which relies on a more moderate assumption concerning the debt repudiation punishment. Section 1.5 analyzes the numerical solutions corresponding to the models with two-sector autarky which differ in the magnitude of perishable productivity gains. Section 1.6 concludes.

1.2. The Baseline Model

The environments considered in the paper essentially share some features. There are two agents: agent 1 who is risk averse and can be interpreted as a developing country and agent 2 who is risk neutral and represents the industrialized countries. As in Marcet and Marimon (1992) the technologies that convert investment into capital are non-linear and are affected by a productivity shock.

1.2.1. Efficient growth mechanism under full commitment

It is assumed that there are two sectors in the economy which will be called domestic and foreign operated sector. In the case of external financing due to technological transfers the foreign operated sector will enjoy higher productivity as compared with the domestic sector.[9]

[9]The technological transfers partially originate from the fact that a part of capital inflows into a country will take form of FDI. It is often argued in the literature that the positive spillovers from FDI only affect certain firms in the domestic economy Görg and Strobl (2001).

The set of firms which are affected by the technological transfers from the rest of the world will be referred to as foreign operated sector.

In this environment, the efficient growth mechanism, Γ, represents a state-contingent investment and transfer plans $\Gamma = \{i_{1t}, i_{2t}, \tau_t\}$ which is obtained as a solution to a dynamic principal-agent problem for a given set of initial conditions and weights. The latter are comprised of the initial capital stocks k_{10}, k_{20}, the initial productivity shock θ_0, and the weight $\lambda \in \mathbb{R}_+$ assigned to the risk-averse agent in the planner's problem given by

Program 1.

$$\max_{\{c_{1t},\tau_t,i_{1t},i_{2t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[\lambda u(c_{1t}) + (-\tau_t) \right] \right]$$

subject to

$$c_{1t} - \tau_t + i_{1t} + i_{2t} = f(k_{1t}) + F(k_{2t}), \tag{1.1}$$

$$k_{1t+1} = (1-\delta)k_{1t} + g(i_{1t}, \theta_{t+1}), \tag{1.2}$$

$$k_{2t+1} = (1-\delta)k_{2t} + g(i_{2t}, \theta_{t+1}), \tag{1.3}$$

with $c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0$ *given.*

In this specification $u(\cdot)$ represents the instantaneous utility of the risk-averse agent. We denote as $f(\cdot)$ and $F(\cdot)$ the production functions corresponding to the domestic and foreign operated sectors of the economy. The function that transforms units of investment into units of capital is denoted as $g(\cdot)$. The consumption of the risk-averse agent is given by c_{1t}, the transfers from the risk-neutral agent to the risk averse one are denoted by τ_t. Investment in to the two sectors are given by i_{1t} and i_{2t}, and the corresponding capital stocks by k_{1t} and k_{2t}. The variable θ_{t+1} represents an exogenous stochastic shock, the realization of which is unknown at the time the investment decisions are made.

The following assumptions, relatively standard in the stochastic growth literature, will hold throughout the rest of the paper[10]: (i) the utility function $u(\cdot)$ of the agent 1 is strictly concave, twice differentiable and satisfies the Inada conditions: $\lim_{c\to 0} u'(c) = +\infty, \lim_{c\to\infty} u'(c) = 0$; (ii) the sectorial production functions $f(\cdot)$ and $F(\cdot)$ are concave and differentiable; (iii) the exogenous stochastic process θ_t is stationary and has bounded support; (iv) depreciation rate $\delta \in [0,1]$; (v) $g(\cdot,\theta)$ is differentiable and concave.

[10]Similar assumptions appear in Marcet and Marimon (1992), and Jones and Manuelli (1990), among others.

A note on the interpretation of this model should be made. As in the model of Acemoglu and Zilibotti (1997) the development takes the form of the capital accumulation in the existing sector considered as domestic as well as opening and subsequent accumulation in a new sector in the economy considered as foreign operated. The extent of the development in the domestically operated sector is summarized by the capital stock k_{1t}. Likewise the extent of the development in the foreign operated sector is summarized by the capital stock k_{2t} an initial value of which is lower than that of the domestic sector.

In addition to the equations (1.1), (1.2) and (1.3) the solution to the Program 1 must satisfy the following first order conditions[11]:

$$1 = \beta E_t \left[\frac{\partial g(i_{1t}, \theta_{t+1})}{\partial i_{1t}} \sum_{j=0}^{\infty} (\beta(1-\delta))^j f'(k_{1t+1+j}) \right], \tag{1.4}$$

$$1 = \beta E_t \left[\frac{\partial g(i_{2t}, \theta_{t+1})}{\partial i_{2t}} \sum_{j=0}^{\infty} (\beta(1-\delta))^j F'(k_{2t+1+j}) \right], \tag{1.5}$$

$$u'(c_{1t}) = \lambda^{-1}. \tag{1.6}$$

The model discussed above is based on the assumption that the planner can perfectly enforce both parties to follow the plan. In the remaining of the paper, this assumption will be relaxed and a number of assumptions regarding incentive compatibility will be considered. These assumptions will essentially differ in the extent of the punishment the risk-averse agent would have to endure should he deviate from the plan.

1.2.2. Efficient growth mechanisms under limited commitment

We begin with the most stringent assumption on the punishment in case of violation of the contract. We will assume that in case of default the developing country will appropriate the capital stock corresponding to the domestically operated sectors k_{1t}. The newly opened foreign sector will no longer be productive. This assumption can be justified on the grounds that the newly opened sector can be totally dependent on the technology and managerial skills transferred from the industrialized world.[12]

[11] See Appendix 1.7 for the derivation of the first order conditions.

[12] A similar assumption has been considered by Marcet and Marimon (1998) and Albuquerque (2003).

Hence, the failure to honor the contract will result in closing down the sector which cannot be operated using domestically available technologies. In case of debt repudiation, the country will switch to autarky and will remain excluded from the international markets forever. The problem the country would face in autarky takes the following form:

$$\max_{\{c_t, i_t\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

subject to

$$c_t + i_t = f(k_t),$$

$$k_{t+1} = (1-\delta)k_t + g(i_t, \theta_{t+1}),$$

where $c_t \geq 0, i_t, \geq 0$, and the initial values k_0, θ_0 are given by the corresponding values of capital stock of the domestically operated sector and the shock value at the time of deviation. Using the arguments of standard dynamic programming one can show[13] the existence of the time invariant policy functions $i(k,\theta)$, $c(k,\theta)$ and a value function $V^a(k,\theta)$. Hence, the reservation value for the risk-averse agent at time t is the utility of the autarkic solution $V^a(k_{1t}, \theta_t)$ given the capital stock k_{1t} and the productivity shock θ_t. The optimal allocations can be found by solving the following planner's problem with $\lambda \in \mathbb{R}_+$ and the participation constraint imposed on agent 1:

Program 2.

$$\max_{\{c_{1t}, \tau_t, i_{1t}, i_{2t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[\lambda u(c_{1t}) + (-\tau_t) \right] \right]$$

subject to

(1.7) $$c_{1t} - \tau_t + i_{1t} + i_{2t} = f(k_{1t}) + F(k_{2t}),$$

(1.8) $$k_{1t+1} = (1-\delta)k_{1t} + g(i_{1t}, \theta_{t+1}),$$

(1.9) $$k_{2t+1} = (1-\delta)k_{2t} + g(i_{2t}, \theta_{t+1}),$$

(1.10) $$E_t \left[\sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] \geq V^a(k_{1t}, \theta_t),$$

with $c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0$ *given.*

[13]See Appendix 1.7 for the solution to the dynamic programming problem under the autarky.

Since the constraint (1.10) involves expected values of the future variables, Program 2 is not a special case of the standard dynamic programming problems, and the Bellman equation will not be satisfied. However, as shown by Marcet and Marimon (1998) this problem falls into a general class of problems, which can be cast into alternative recursive framework.

The recursive saddle point problem associated with Program 2 will be given by

$$\max_{\{c_{1t},\tau_t,i_{1t},i_{2t}\}_{t=0}^{\infty}} \min_{\{\mu_t\}_{t=0}^{\infty}} \mathcal{H} = E_0 \sum_{t=0}^{\infty} \beta^t \{(\lambda + M_{t-1})\, u(c_{1t}) + (-\tau_t) + \mu_t\,(u(c_{1t}) - V^a(k_{1t},\theta_t))\} \tag{1.11}$$

subject to (1.7)-(1.9) and

$$M_t = M_{t-1} + \mu_t, \quad M_{-1} = 0, \qquad \mu_t \geq 0. \tag{1.12}$$

Indeed, the corresponding Lagrangian is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \lambda u(c_{1t}) + (-\tau_t) + \mu_t \left(E_t \left[\sum_{i=0}^{\infty} \beta^i u(c_{1t}) \right] - V^a(k_{1t},\theta_t) \right) \right\}$$

subject to (1.7)-(1.9), given $\mu_t \geq 0$, where $\beta^{-t}\mu_t$ is the Lagrange multiplier of (1.10) at t. The law of iterated expectations allows to imbed the conditional expectations E_t into E_0. Furthermore, reordering the terms and introducing the law of motion for M_t yields the above result.

As shown by Marcet and Marimon (1998), under certain assumptions[14] the solution to the recursive saddle point problem obeys a saddle point functional equation. Within our framework their result implies that there exists a unique value function,

$$W(k_1,k_2,M,\theta) = \min_{\mu\geq 0} \max_{\{c_1,\tau,i_1,i_2\}} \{(\lambda + M)\, u(c_1) + (-\tau) + \mu\,(u(c_1) - V^a(k_1,\theta)) + \beta E\,[W(k_1',k_2',M',\theta') \mid \theta]\}$$

subject to

$$c_1 - \tau + i_1 + i_2 = f(k_1) + F(k_2), \tag{1.13}$$

$$k_j' = (1-\delta)k_j + g(i_j,\theta'), \text{ for } j = 1,2 \tag{1.14}$$

[14]Marcet and Marimon (1998) state some interiority conditions needed for the existence of the saddle point problem. These are trivially satisfied in the famework considered here.

$$M' = M + \mu, \tag{1.15}$$

$$c_1, i_1, i_2 \geq 0, \tag{1.16}$$

for all (k_1, k_2, M, θ) and such that $W(k_{10}, k_{20}, M_{-1}, \theta_0)$ is the value of Program 2. The policy correspondence associated with the above saddle point functional equation is given by

$$\begin{aligned}\psi(k_1, k_2, M, \theta) \in \arg\min_{\mu \geq 0} \max_{\{c_1, \tau, i_1, i_2\}} & \{(\lambda + M)\, u(c_1) + (-\tau) + \mu\, (u(c_1) - V^a(k_1, \theta)) \\ & + \beta E\left[W(k_1', k_2', M', \theta') \mid \theta\right]\}\end{aligned}$$

subject to (1.13) - (1.16).

The key results demonstrated by Marcet and Marimon (1998) ensures that the optimal solution of Program 2 satisfies $(c_{1t}, \tau_t, i_{1t}, i_{2t}, \mu_t) = \psi\,(k_{1t}, k_{2t}, M_{t-1}, \theta_t)$ for all t with the initial conditions $(k_{10}, k_{20}, 0, \theta_0)$. That is there exist a time invariant policy correspondence ψ such that only the values of a small number of past variables $(k_{1t}, k_{2t}, M_{t-1}, \theta_t)$ matter. Hence, the problem is now in a recursive framework the solution to which can now be obtained from studying the saddle point functional equation.

Denoting γ_{1t} and γ_{2t} the Lagrange multipliers of the constraints (1.8) and (1.9), the first order conditions for this problem become:

$$(\lambda + M_t)\, u'(c_{1t}) = 1, \tag{1.17}$$

$$-1 - \beta E_t \left[\gamma_{jt+1} \frac{\partial g(i_{jt}, \theta_{t+1})}{\partial i_{jt}}\right] = 0, \text{ for } j = 1, 2 \tag{1.18}$$

$$f'(k_{1t}) - \mu_t \frac{\partial V^a}{\partial k_{1t}}(k_{1t}, \theta_t) + \gamma_{1t} - \beta(1 - \delta) E_t\left[\gamma_{1t+1}\right] = 0, \tag{1.19}$$

$$F'(k_{2t}) + \gamma_{2t} - \beta(1 - \delta) E_t\left[\gamma_{2t+1}\right] = 0, \tag{1.20}$$

$$E_t\left[\sum_{i=0}^{\infty} \beta^i u(c_{1t+i})\right] - V^a(k_{1t}, \theta_t) \geq 0, \tag{1.21}$$

$$\mu_t \left[E_t\left[\sum_{i=0}^{\infty} \beta^i u(c_{1t+i})\right] - V^a(k_{1t}, \theta_t)\right] = 0, \tag{1.22}$$

in addition to the technological constraints (1.7)-(1.9), the law of motion (1.12) for the co-state variable M_t, and non-negativity of the Lagrange multiplier $\mu_t \geq 0$.

1.3. Solutions to the Growth Models

In this section we will present the numerical solutions for various models of this paper as well as describe the algorithms for obtaining them. To obtain the numerical solution to the models we will rely on the parameterized expectation approach. With some exceptions, the functional forms utilized here are similar to those of Marcet and Marimon (1992). These are

$$f(k_{1t}) = Ak_{1t}^{\alpha} \text{ and } F(k_{2t}) = \widetilde{A}k_{2t}^{\alpha},$$

$$g(i_t, \theta_{t+1}) = a(\theta_{t+1} + s)\frac{i_t}{(1 + i_t)} + b,$$

$$u(c_{1t}) = c_{1t}^{\gamma+1}/(\gamma + 1),$$

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t,$$

where $\{\varepsilon_t\}$ are independent normally distributed random variables with zero mean and variance σ_{ε}^2.

1.3.1. Solving the problem with full enforcement

With the chosen functional forms the optimality conditions for the case of full enforcement are the following:

$$(1.23) \qquad (1 + i_{1t})^2 = \beta E_t \left[a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1 - \delta))^j A\alpha(k_{1t+1+j})^{\alpha-1} \right],$$

$$(1.24) \qquad (1 + i_{2t})^2 = \beta E_t \left[a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1 - \delta))^j \widetilde{A}\alpha(k_{2t+1+j})^{\alpha-1} \right],$$

$$(1.25) \qquad c_{1t}^{\gamma} = \lambda^{-1},$$

$$(1.26) \qquad c_{1t} - \tau_t + i_{1t} + i_{2t} = Ak_{1t}^{\alpha} + \widetilde{A}k_{2t}^{\alpha},$$

$$(1.27) \qquad k_{it+1} = (1 - \delta)k_{it} + a(\theta_{t+1} + s)\frac{i_{it}}{(1 + i_{it})} + b, \text{ for } i = 1, 2.$$

The first step of the PEA is to substitute the conditional expectations in (1.23) and (1.24) by the flexible functional forms that depend on the state variables and some coefficients[15]. Each of the parameterized expectations $i = 1, 2$ takes the form:

$$\psi(\omega^i; k_{1t}(\omega), k_{2t}(\omega), \theta_t) = \exp(\omega_1^i + \omega_2^i \log k_{1t}(\omega) + \omega_3^i \log k_{2t}(\omega) + \omega_4^i \log \theta_t),$$

where $\omega = (\omega^1; \omega^2)$. The use of the exponential polynomial guarantees that the left hand side of (1.23) and (1.24) would be positive. Increasing the degree of the polynomial would allow to approximate the solution with arbitrary accuracy[16].

The algorithm for solving the model takes the following steps:

(I) Fix the initial conditions and draw a series of $\{\theta_t\}_{t=1}^T$ that obeys the law of motion for the exogenous state variable. The number of periods T in the truncated series should be sufficiently large.

(II) For a given ω substitute the conditional expectations in (1.23) and (1.24) to yield:

$$(1.28) \qquad (1 + i_{it})^2 = \delta\psi(\omega^i; k_{1t}(\omega), k_{2t}(\omega), \theta_t) \text{ for } i = 1, 2$$

(III) Using the realizations of θ_t obtain recursively from (1.28) and (1.25)-(1.27) a series of the endogenous variables $\{c_{1t}(\omega), \tau_t(\omega), i_{1t}(\omega), i_{2t}(\omega), k_{1t}(\omega), k_{2t}(\omega)\}$ for this particular ω.

(IV) The next step involves running two separate non-linear regressions. The role of the dependent variables will be performed by the expressions inside the conditional expectation in the RHS of (1.23) and (1.24). Namely, the 'dependent variables' $Y_{1t}(\omega)$ and $Y_{2t}(\omega)$ would take form

$$\begin{aligned} Y_{1t}(\omega) &\equiv a(\theta_{t+1} + s)\sum_{j=0}^{\infty}(\beta(1-\delta))^j A\alpha(k_{1t+1+j}(\omega))^{\alpha-1}, \\ Y_{2t}(\omega) &\equiv a(\theta_{t+1} + s)\sum_{j=0}^{\infty}(\beta(1-\delta))^j \widetilde{A}\alpha(k_{2t+1+j}(\omega))^{\alpha-1}. \end{aligned}$$

[15] see Marcet and Lorenzoni (1998) for further details on the implementation of PEA.

[16] The fact that PEA can provide arbitrary accuracy if the approximation function is refined and a proof of convergence to the correct solution are given in Marcet and Marshall (1994). In practice the choice of degree of the exponential polynomial can be guided by the test for accuracy in simulations proposed by den Haan and Marcet (1994). Some practical issues on dealing with higher-order polynomials in the approximation function are discussed in den Haan and Marcet (1990).

Now, letting $S^i(\omega)$ be the result of the following regression:

$$Y_{it}(\omega) = \exp(\xi_1^i + \xi_2^i \log k_{1t}(\omega) + \xi_3^i \log k_{2t}(\omega) + \xi_4^i \log \theta_t) + \eta_{it},$$

for $i = 1, 2$, define $S(\omega) \equiv (S^1(\omega), S^2(\omega))$.

(V) The final step involves using an iterative algorithm to find the fixed point of S, and the set of coefficients $\omega_f = S(\omega_f)$ which would give the solution for the endogenous variables $\{c_{1t}(\omega_f), \tau_t(\omega_f), i_{1t}(\omega_f), i_{2t}(\omega_f), k_{1t}(\omega_f), k_{2t}(\omega_f)\}$.

1.3.2. Solving the problem with limited commitment

This section shows how to solve the model with limited enforcement using PEA adapted from Marcet and Marimon (1992). The main difference from the algorithm discussed above is that here the participation constraint might be binding in some periods and slack in the others. Furthermore, there is one more expectation to parameterize and an additional (co-)state variable M_{t-1} to include into the parameterization.

The following optimality conditions are to be satisfied:

$$\mu_t \left[u(c_{1t}) + E_t \left[\sum_{i=1}^{\infty} \beta^i u(c_{1t+i}) \right] - V^a(k_{1t}, \theta_t) \right] = 0, \tag{1.29}$$

$$E_t \left[\sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] - V^a(k_{1t}, \theta_t) \geq 0, \tag{1.30}$$

$$c_{1t}^{\gamma} = 1/\left(\lambda + \mu_t + M_{t-1}\right), \tag{1.31}$$

$$M_t = M_{t-1} + \mu_t, \tag{1.32}$$

$$\begin{aligned} (1 + i_{1t})^2 &= \beta E_t \left[a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1-\delta))^j \right. \\ &\quad \left. \times \left(A\alpha(k_{1t+1+j})^{\alpha-1} - \mu_{t+j+1} \frac{\partial V^a(k_{1t+j+1}, \theta_{t+j+1})}{\partial k_{1t+j+1}} \right) \right], \end{aligned} \tag{1.33}$$

$$(1 + i_{2t})^2 = \beta E_t \left[a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1-\delta))^j \widetilde{A}\alpha(k_{2t+1+j})^{\alpha-1} \right], \tag{1.34}$$

$$c_{1t} - \tau_t + i_{1t} + i_{2t} = Ak_{1t}^{\alpha} + \widetilde{A}k_{2t}^{\alpha}, \tag{1.35}$$

$$k_{jt+1} = (1-\delta)k_{jt} + a(\theta_{t+1} + s)i_{jt}/(1 + i_{jt}) + b, \text{ for } j = 1, 2, \tag{1.36}$$

in addition to the inequality constraint $\mu_t \geq 0$ and the initial conditions[17].

In order to solve this model with PEA the algorithm described for the case of full enforcement should be modified in the following way. First, in step II parameterize the conditional expectations in (1.29), (1.33) and (1.34) to yield

$$(1 + i_{it}(\omega))^2 = \delta\psi(\omega^i; k_{1t}(\omega), k_{2t}(\omega), M_{t-1}(\omega), \theta_t) \text{ for } i = 1, 2, \tag{1.37}$$

$$\mu_t \left[u(c_{1t}(\omega)) + \beta\psi(\omega^3; k_{1t}(\omega), k_{2t}(\omega), M_{t-1}(\omega), \theta_t) - V^a(k_{1t}(\omega), \theta_t) \right] = 0,$$

where $\omega = (\omega^1, \omega^2, \omega^3)$.

In step III the participation constraint should be taken into account. One way to proceed is to initially assume that the participation constraint is not binding, then $\mu_t(\omega) = 0$, $M_t(\omega) = M_{t-1}(\omega)$, and the solution for $c_{1t}(\omega)$ follows from (1.31). For this solution one has to check whether the constraint is indeed satisfied, that is if

$$u(c_{1t}(\omega)) + \beta\psi(\omega^3; k_{1t}(\omega), k_{2t}(\omega), M_{t-1}(\omega), \theta_t) \geq V^a(k_{1t}(\omega), \theta_t).$$

If that is the case one can proceed by solving for the rest of the endogenous variables from (1.37) and the feasibility constraints (1.35) - (1.36). Otherwise, the participation constraint must be binding, that is

$$u(c_{1t}(\omega)) + \beta\psi(\omega^3; k_{1t}(\omega), k_{2t}(\omega), M_{t-1}(\omega), \theta_t) = V^a(k_{1t}(\omega), \theta_t),$$

[17]From (1.19) and (1.20) using recursive substitution and the law of iterated expectations yields the following expressions for the lagrange multipliers γ_{1t} and γ_{2t}

$$\begin{aligned} \gamma_{1t} &= \beta E_t \left[a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1-\delta))^j \left(\mu_{t+j} \frac{\partial V^a}{\partial k_{1t+j}}(k_{1t+j}, \theta_{t+j}) - f'(k_{1t+j}) \right) \right], \\ \gamma_{2t} &= -\beta E_t \left[a(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1-\delta))^j F'(k_{2t+j}) \right]. \end{aligned}$$

Substituting the the above expressions into (1.18), and using again the law of iterated expectations and the functional forms for the production and investment functions yields the optimality conditions (1.33) and (1.34).

from which the solution for $c_{1t}(\omega)$ follows. The value of the multiplier $\mu_t(\omega)$ then follows from (1.31), the value of $M_t(\omega)$ from the law of motion (1.32), and the rest of the endogenous variables from (1.37) and (1.35) - (1.36).

Now, step IV will involve running three non-linear regressions for $i = 1, 2, 3$ of the form

$$Y_{it}(\omega) = \exp(\xi_1^i + \xi_2^i \log k_{1t}(\omega) + \xi_3^i \log k_{2t}(\omega) + \xi_4^i \log \theta_t + \xi_5^i M_{t-1}(\omega)) + \eta_{it},$$

where the 'dependent variables' are given by

$$\begin{aligned} Y_{1t}(\omega) &\equiv a\,(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1-\delta))^j \left[A\alpha(k_{1t+1+j}(\omega))^{\alpha-1} \right. \\ &\quad \left. -\mu_{t+j+1}(\omega) \frac{\partial V^a(k_{1t+j+1}(\omega), \theta_{t+j+1})}{\partial k_{1t+j+1}} \right], \\ Y_{2t}(\omega) &\equiv a\,(\theta_{t+1} + s) \sum_{j=0}^{\infty} (\beta(1-\delta))^j \widetilde{A}\alpha(k_{2t+1+j}(\omega))^{\alpha-1}, \\ Y_{3t}(\omega) &\equiv \sum_{i=1}^{\infty} \beta^i u(c_{1t+i}(\omega)). \end{aligned}$$

The last step is similar to the one in the the case of full enforcement.

A few notes on the algorithm should be made. First, in this algorithm μ_t will be positive by construction. Second, step IV involves calculation of the derivative of the value function in the autarky with the respect to its first argument. Marcet and Marimon (1992) provide derivation of this derivative which is convenient for computational purposes. The computation algorithm is given in Appendix 1.7.

1.3.3. Numerical solutions to the models

In this section we present the simulated series for the models discussed above. First, a short note should be made on the parameterization of the model. The values of the parameters used in the simulations except for the productivity parameters A and $\widetilde{A}$ are similar to those of Marcet and Marimon (1992). This concerns all the models considered throughout the paper. The choice of values for the depreciation rate of the capital (δ) and the discount factor (β) allows to interpret one period as a year. The values of the parameters are summarized in Table 1.1.

[insert Table 1.1 about here.]

A note on the weight λ in the planner's problem should be made. In all the reported simulations the value of λ is set to make expected discounted transfers at $t = 0$ equal to zero. This would ensure that the series reported corresponds to the equilibrium contract.

[insert Figure 1.1 about here.]

The simulation results for the environment with full enforcement are presented in Figure 1.1. These results will be compared with those obtained in the autarkic environment (see Figure 1.2 and 1.2). The initial value of capital stock in the domestic sector is set to one, while the foreign operated sector is initially assumed to be nonexistent.[18]

[insert Figure 1.2 and 1.2 about here.]

The results can be summarized in the following way. First, as expected, the consumption of the risk-averse agent in the PO environment is constant both in the steady state and along the transition. All the risk is born by the risk neutral agent, which is also reflected in the volatility of the transfers in the steady state.

Second, under full enforcement the developing country borrows heavily during the initial periods in order to boost investment in both sectors of the economy. Due to the access to external financing, the mean growth rate of output raises from 2.4% to 8.4% during the first 15 periods, and from 1.4% to 3.8% during the first 35 periods.

Third, during the initial periods the investment rates under PO environment are significantly higher that those in the autarky. Under full enforcement, as the capital accumulates in both sectors the investment rates decline. The opposite is observed in the autarkic environment. Higher investment level in the foreign sector than that of the domestic is due to the lower initial capital stock in the former. Remarkably, in the steady state the investment rates under PO environment are more volatile than those in the autarky.

Finally, under full enforcement access to external financial opportunities results in a welfare gain equivalent to a 92% "increase in consumption". By "increase in consumption" we refer to a permanent increase in consumption that would equate the present value under the autarky with the present values achieved under other environments

Remarkably, all of the results reported for the PO environment are also applicable to the model with limited commitment corresponding to Program 2. The implication of this

[18]This assumption is made to make Autarky directly comparable with other environments. In addition, as in Marcet and Marimon (1992) we assume that the initial capital stock in the Autarkic environment equals to one.

finding is that technological gains from external financing opportunities may eliminate the default risk.

A comment should be made on this finding according to which the solutions to the case of full enforcement and limited enforcement coincide. The fact that participation constraint turns out to be never binding can driven by the assumption of the punishment in case of deviation from the optimal plan, which is extremely severe. Should the country default it will lose not only the technological advantage and capital accumulated in the newly opened sector but also a possibility to develop this sector on its own. In the remaining of the paper we will address the issue of default punishment which might give some qualitatively different results.

1.4. The Main Model: Two-sector Autarky and Limited Enforcement

In this section, we will modify the assumption concerning the punishment incurred by the developing country in case of deviation from the optimal plan. It will be assumed that failure to follow the plan would result in the loss of the technological advantage in the newly opened sector[19]. However, the newly open sector will remain productive with the productivity level of the domestically operated sector. Furthermore, the country will preserve the accumulated capital in both sectors. In this formulation, the autarky would be given by

Program 3.

$$\max_{\{c_t, i_{1t}, i_{2t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

subject to

$$c_{1t} + i_{1t} + i_{2t} = f(k_{1t}) + f(k_{2t}), \tag{1.38}$$

$$k_{jt+1} = (1-\delta)k_{jt} + g(i_{jt}, \theta_{t+1}), \ \textit{for } j = 1, 2 \tag{1.39}$$

with $c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0$ *given.*

The arguments from the standard dynamic programming will ensure the existence of the time invariant policy functions $i_1(k_1, k_2, \theta), i_2(k_1, k_2, \theta), c(k_1, k_2, \theta)$ and a value function $V^{a2}(k_1, k_2, \theta)$. Hence, the reservation value for the agent 1 at time t is the utility of the autarkic solution $V^{a2}(k_{1t}, k_{2t}, \theta_t)$ given the capital stock accumulated in the domestically

[19]This assumption is close in spirit to those of Cohen and Sachs (1986) or Eaton and Gersovitz (1984) where foreign debt repudiation results in permanent loss of productive efficiency.

operated sector k_{1t}, the capital stock of the newly opened sector k_{2t} and the productivity shock θ_t.[20]

Under these less stringent assumptions on the default punishment, the optimal allocations can be found by solving the following planner's problem with $\lambda \in \mathbb{R}_+$ and the participation constraint imposed on agent 1.

Program 4.

$$\max_{\{c_{1t},\tau_t,i_{1t},i_{2t},k_{1t},k_{2t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[\lambda u(c_{1t}) + (-\tau_t)\right] \right]$$

subject to

$$c_{1t} - \tau_t + i_{1t} + i_{2t} = f(k_{1t}) + F(k_{2t}), \tag{1.40}$$

$$k_{jt+1} = (1-\delta)k_{jt} + g(i_{jt}, \theta_{t+1}), \ \textit{for } j = 1,2 \tag{1.41}$$

$$E_t \left[\sum_{i=0}^{\infty} \beta^i u(c_{1t+i}) \right] \geq V^{a2}(k_{1t}, k_{2t}, \theta_t), \tag{1.42}$$

with $c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0$ *given.*

Once again, in the above framework, the steady state distributions of capital will differ under full and limited enforcement due to the technology transfers. This feature would distinguish the present setup from the framework of Marcet and Marimon (1992) as far as the growth incentives for integration are concerned.

Similar to Program 2, the present problem can be cast into recursive framework the solution to which will be obtained from studying the saddle point functional equation. Denoting γ_{1t} and γ_{2t} the Lagrange multipliers of the constraints (1.41), the first order conditions for this problem become:

$$(\lambda + M_t)\, u'(c_{1t}) = 1,$$

$$-1 - \beta E_t \left[\gamma_{jt+1} \frac{\partial g(i_{jt}, \theta_{t+1})}{\partial i_{jt}} \right] = 0, \ \text{for } j = 1,2,$$

$$f'(k_{1t}) - \mu_t \frac{\partial V^{a2}}{\partial k_{1t}}(k_{1t}, k_{2t}, \theta_t) + \gamma_{1t} - \beta(1-\delta)E_t \left[\gamma_{1t+1}\right] = 0,$$

$$F'(k_{2t}) - \mu_t \frac{\partial V^{a2}}{\partial k_{2t}}(k_{1t}, k_{2t}, \theta_t) + \gamma_{2t} - \beta(1-\delta)E_t \left[\gamma_{2t+1}\right] = 0,$$

[20] See Appendix 1.7 for the optimality conditions corresponding to Program 3.

$$E_t\left[\sum_{i=0}^{\infty}\beta^i u(c_{1t+i})\right]-V^{a2}(k_{1t},k_{2t},\theta_t)\geq 0,$$

$$\mu_t\left[E_t\left[\sum_{i=0}^{\infty}\beta^i u(c_{1t+i})\right]-V^{a2}(k_{1t},k_{2t},\theta_t)\right]=0,$$

in addition to the technological constraints (1.40)-(1.42), the law of motion for the co-state variable M_t,

$$M_t = M_{t-1}+\mu_t,\quad M_{-1}=0$$

and non-negativity of the Lagrange multiplier $\mu_t \geq 0$.

Substituting the chosen functional forms and simplifying the first order conditions in a manner similar to the one described in footnote 6 yields the following optimality conditions:

$$\mu_t\left[u(c_{1t})+E_t\left[\sum_{i=1}^{\infty}\beta^i u(c_{1t+i})\right]-V^{a2}(k_{1t},k_{2t},\theta_t)\right]=0,$$

$$E_t\left[\sum_{i=0}^{\infty}\beta^i u(c_{1t+i})\right]-V^{a2}(k_{1t},k_{2t},\theta_t)\geq 0,$$

$$c_{1t}^{\gamma}=1/\left(\lambda+\mu_t+M_{t-1}\right),$$

$$M_t = M_{t-1}+\mu_t,$$

$$\begin{aligned}(1+i_{1t})^2 &= \beta E_t\left[a\left(\theta_{t+1}+s\right)\sum_{j=0}^{\infty}\beta^j(1-\delta)^j\left(A\alpha(k_{1t+1+j})^{\alpha-1}\right.\right.\\ &\quad\left.\left.-\mu_{t+j+1}\frac{\partial V^{a2}(k_{1t+j+1},k_{2t+j+1},\theta_{t+j+1})}{\partial k_{1t+j+1}}\right)\right],\end{aligned}$$

$$\begin{aligned}(1+i_{2t})^2 &= \beta E_t\left[a\left(\theta_{t+1}+s\right)\sum_{j=0}^{\infty}\beta^j(1-\delta)^j\left(\widetilde{A}\alpha(k_{2t+1+j})^{\alpha-1}\right.\right.\\ &\quad\left.\left.-\mu_{t+j+1}\frac{\partial V^{a2}(k_{1t+j+1},k_{2t+j+1},\theta_{t+j+1})}{\partial k_{2t+j+1}}\right)\right],\end{aligned}$$

$$c_{1t}-\tau_t+i_{1t}+i_{2t}=Ak_{1t}^{\alpha}+\widetilde{A}k_{2t}^{\alpha},$$

$$k_{jt+1}=(1-\delta)k_{jt}+a(\theta_{t+1}+s)i_{jt}/(1+i_{jt})+b,\ \text{for } j=1,2$$

in addition to non-negativity of the Lagrange multiplier $\mu_t \geq 0$ and the initial conditions.

1.5. Characterization of Equilibria

We solve the model in Program 4 with the PEA using an algorithm similar to the one described for the model in Program 2. As before, in all simulations the TFP parameter of the domestic sector (A) was set to one. When it comes to the TFP parameter of the foreign operated sector ($\widetilde{A}$), we consider three representative cases which differ in the magnitude of the technological transfers.

The simulation results are summarized in Figures 1.3-1.6 and Tables 1.3-1.5. We compare three institutional environments: the autarky equilibrium corresponding to Program 3 denoted as "au" in Figures 3-5, Pareto optimum allocation with perfect enforcement denoted as "po", and the equilibrium with limited enforcement corresponding to Program 4 denoted as "pc". For these figures we plot the first 50 periods as representative of the transition from the low level of capital to the steady state, and periods 100 to 200 as representative of the steady state distribution.

1.5.1. Equilibria with no technological transfers

First, we consider the case with no technological transfers whatsoever, which in terms of TFP's corresponds to $\widetilde{A} = A = 1$. Under lack of commitment, the behavior of the developing country is affected by the two opposing forces. On on hand, the country wants to default on its debt, something which would imply switching to autarky and staying there forever. Unlike the autarky assumption of the Program 2, Program 4 implies that the country would still be in a position to develop the foreign sector on its own with the expropriated capital to begin with. The opposing force is the threat of the punishment for defaulting. In this case, it is the loss of possibility to borrow in order either enhance growth or to smooth consumption against the unforeseen shocks or along the growth path. As before, the characterization of the capital accumulation and transfers during the transition can be obtained only from the numerical solutions, which are summarized in Figure 1.3 and Table 1.3.

[insert Figure 1.3 and Table 1.3 about here.]

An important feature of this case is that the steady state distributions of capital are quite similar across all the three environments, in both sectors. They are actually identical in the PC and PO environments as are the distributions of the corresponding investment rates. As reported in Table 1.3, the steady state capital stock in the autarky environment is slightly higher on average than in the other environments in either of the sectors. The reason for that is that in autarky the country has to self-insure against the cyclical fluctuations of output and the only source of self-insurance is the capital.

In each of the sectors, the investment is more volatile under full enforcement than under the autarky. This feature is similar to the one reported by Marcet and Marimon (1992), and represents an example where an increase in volatility of investment is desirable.

Despite absence of any technological spillovers, the positive effect of the access to external financing on growth is rather substantial under full enforcement. The growth rates go from 2.5 to 3% during the first 15 periods. Yet, this effect practically disappears once the assumption of perfect enforceability of contracts is relaxed. The overall gains, measured as permanent increase in consumption that would equate the present value under the autarky with the present values achieved under other regimes, differ significantly in the PO and PC environments. Failure to perfectly enforce contracts reduces the welfare gains by the factor of 25. In fact, during the transition the consumption paths under autarky and under limited enforcement are very similar. As can be seen from Figure 1.3, the key difference is that the consumption series under PC is smoother than that under the autarky during the transition. Furthermore, it is outright flat in the steady state while the consumption under autarky keeps fluctuating even in the steady state. Hence, with no technological transfers, the access to the external financing under limited enforcement allows to smooth out variation of output but not keep constant consumption along the transition. The possibility to smooth consumption through external financing results in the minor welfare gain under limited commitment. As in Marcet and Marimon (1992) enforcement constrains result in negligible transfers and severely reduce growth opportunities.

1.5.2. Equilibria with technological transfers of medium magnitude

The case with the technological transfers of medium magnitude is defined by two characteristic features. First, in the environments which grant access to the external financing, the foreign operated sector is more productive than the domestic one[21]. Second, the productivity differences between sectors are low enough to guarantee that the participation constraint is binding in some periods. The key feature of this case is that the productivity benefits introduce a gap between the average steady state capital stocks in the economy with and without external financing. The latter feature makes the punishment for default more severe that

[21] In terms of sectoral TFPs the case reported here corresponds to $A = 1$ and $\widetilde{A} = 1.1$.

in the previous case but not severe enough to eliminate risk of default. The characteristics of the efficient accumulation mechanisms under the three considered institutional setups are summarized in Figure 1.4 and Table 1.4.

[insert Figure 1.4 and Table 1.4 about here.]

The simulations demonstrate several distinctive features of the setup which encompasses both productivity benefits from external financing and risk of default. These can be summarized in the following way.

First, despite the presence of the default risk in the environment with limited commitment the capital movements from and to the developing country are no longer negligible. This result distinguishes the present setup from both the equilibrium with no technological transfers discussed in the previous section as well as the models of Marcet and Marimon (1992) or Kehoe and Perri (2002). This feature allows to conclude that presence of the default risk is not inconsistent with the capital flows of substantial magnitude.

Second, under limited enforcement the developing country borrows not only in order to smooth cyclical variation in consumption but also in order to invest heavier during the transition and hence foster growth. Remarkably, the borrower boosts investment in all productive sectors and not only those affected by the technological transfers. Once again, in this prediction the current case differs from the case with no technological diffusion, be it two-sector model discussed above or one-sector framework of Marcet and Marimon (1992). In other words, borrowing with an objective to promote growth can be an equilibrium outcome even in the environment with present risk of default.

Third, the behavior of the consumption path under limited enforcement is rather peculiar. During a few initial periods, the consumption path is flat. Although it is still lower that the consumption level under full enforcement, the series is well above the autarky consumption. In other words, in this environment consumption smoothing along the growth path is no longer absent. As the capital accumulates, the participation constraint starts binding at certain period. After that the consumption in the limited commitment environment rises every time the incentive compatibility constraint binds. As in the case with no technological transfers, the shape of the consumption series reminds that of the autarky. However, during the all the transition periods there is a diminishing wedge between the two series. This can be attributed to the diminishing difference in the accumulated capital stock in the environments with full and limited enforcement. As in the case with no technological diffusion, under limited enforcement the steady state distribution is characterized by a flat consumption schedule which can lie either above or below the autarky path.

Since the default risk is still present during the transition, under limited enforcement the paths of investment, transfers, and capital stock differ from those in the Pareto optimum. Transfers from abroad to the developing country are lower in this case relative to the full enforcement outcome. The investment rates inherit the same feature. In fact, in the sector unaffected by the productivity benefits the investment series falls rather quickly to the autarky level. However, due to the heavy investment during the initial periods, the capital stock under limited enforcement stays above the autarky capital stock during the transition. The latter result holds for all sectors including the domestic one.

Another regularity concerns the average capital stock of the economy in the steady state distribution. As shown by Marcet and Marimon (1992) the capital stock of a country in the environment with limited commitment is lower than that in the autarky. The driving force behind this result is the need to use capital as the only means of self-insurance in the autarkic environment. A similar result is obtained in our framework in the case when no technological diffusion takes place. When the technological transfers are present, however, this conclusion may no longer be true. Since the productivity of the foreign operated sector is higher under limited enforcement than in the autarky, so is the capital stock in the foreign sector. Hence, whether the overall capital stock will be higher in the autarky than under limited commitment depends on which of the two forces dominates. For instance, in the case with transfers of medium magnitude reported in Table 1.4, under limited enforcement the capital stock in the domestic sector is lower than that in the autarky. The converse is true for the foreign operated sector.

Some characteristic features of the solutions following from our framework are in line with the documented empirical regularities we began from in Section 1.1. For instance, Marcet and Marimon (1992) state that the observed cross-country differences in borrowing patterns and rich structure of capital flows find little explanation in the models of sustained growth. On the contrary, our framework predicts that under limited commitment, the extent to which a developing country will borrow depends on the magnitude of perishable productivity gains associated with external financing relative to the productivity in the autarky.

Another regularity is reported by Gertler and Rogoff (1990) and more recently Lane (2004) who document that the level of foreign debt in the developing countries is positively correlated with their income. This observation is in line with the predictions of our model as well. Indeed, countries which highly benefit from technological transfers in the foreign operated sector will be able not only to increase production due to the productivity gains but also due to the higher capital stock in all sectors. The latter stems from increased investment levels financed through transfers from abroad. Such countries will tend to have both higher income level and higher level of foreign debt.

Our model outperforms existing theories of economic growth in its ability to account for countercyclical behavior of capital inflows to developing countries. The quantitative predictions of our framework and cross-country empirical evidence documented by Kaminsky, Reinhart, and Vegh (2004) is summarized in Figure 1.5 . The upper histogram reports country correlations between the cyclical components of net capital inflows and real GDP for a sample of 80 developing countries for a period 1960-2003. The lower panel corresponds to the same statistics for the simulated solution of our model.[22] Contrary to the implications of the models of perfect or exogenously restricted capital mobility our framework predicts is that the capital inflows to the developing countries are acyclical. For example, for the same sequence of the exogenous shock our benchmark perfect risk-sharing model predicts the correlation of cyclical component in inflows and output to be -0.86 with the bootstrap standard error of 0.08, while the limited commitment model predicts this statistic to be not significantly different form zero.[23]

[insert Figure 1.5 about here.]

In our framework this cyclical behavior of capital flows is partly determined by endogenous incompleteness of the international lending markets. The basic intuition is the following. On one hand, a good realization of the shock increases the value of the autarkic alternative and therefore temptation of the borrower to default. Therefore, an incentive-compatible contract requires a once and for all increase in consumption of the recipient country. On the other hand, an expected increase in productivity of the investment technology incites the borrower to increase investment in every sector of the economy. This increase in consumption and investment is partially financed through an increase in output and partially

[22]We report the correlations from the simulated series affer removing the secular component with HP filter with the smothing parameter $\lambda = 400$ as suggested by Dolado, Sebastián, and Vallés (1993) for the annual data. Since, we are interested in the behavior of the economy along its transition path, making inference from simulating a long series is not a feasible option. Our strategy is therefore to rely on the logic of Bootstrap methods (see e.g. MacKinnon (2002)) to make better use of the information contained in the simulated series corresponding to the transition. The histogram of country correlations implied by our model is bases on 100.000 Bootstrap iterations.

[23]The value of the correlation we obtain is -.13 with the bootstrap standard error of 0.28.

through capital inflow from abroad. Hence, the cyclical behavior of our model economy is determined by the relative magnitude of these two opposing forces.

The reason that our models fails to predict the procyclical behavior of net capital inflows is that we abstract from a number factors which might matter. One of such factors emphasized in the empirical literature is that government policies tends to be procyclical.[24]

1.5.3. Equilibria with technological transfers of high magnitude

When the magnitude of technological transfers is high enough the defaulter's punishment becomes so severe that the participation constraint turns out to be never binding. Hence, the solution under limited commitment and that under perfect enforcement will coincide. This compels us to reiterate the conclusion obtained earlier from the model with one-sector autarky. Our results suggest that presence of perishable technological benefits associated with external financing may eliminate risk of default. The latter is true even though these benefits are enjoyed only by some sectors of the developing economy. The simulation results for the case with technological transfer of high magnitude are presented in Figure 1.6 and Table 1.5.

[insert Figure 1.6 and Table 1.5 about here.]

One final note will be made concerning the relation between the productivity benefits and the corresponding welfare gains. In the reported example the TFP level in the foreign operated sector ($\widetilde{A}$) is set to 1.35. This particular choice is motivated by the desire to find the lowest level of $\widetilde{A}$, which would ensure that the participation constraint does not bind. In this case, the welfare gain, measured as a permanent increase in consumption that would equate the present value of utility under the autarky with the present values achieved in the other environments, is large. It corresponds to the increase in consumption of 26%. Notice that these gains are driven by two forces. On one hand, it is higher productivity of the foreign operated sectors under PC than that under autarky which takes the credit. On the other hand, the spillovers increase the default punishment and by that facilitate borrowing during the initial periods in order to foster growth. The importance of the latter force for welfare improvement is more obvious in the case with no transfers reported in Table

[24]World Bank (2001, p.72) tentatively suggests that "... the procyclical nature of capital flows also reflects volatility induced by a country's own actions—and inactions—through uncertain government policies and, especially, the underdeveloped state of its own financial markets." Empirical evidence on the issue is documented in Kaminsky, Reinhart, and Vegh (2004).

1.3. In the absence of technological diffusion, the failure to enforce contracts results in a welfare loss corresponding to change in consumption of 3.4%. With introduction of moderate technological transfers, corresponding to the TFP level in the foreign operated sector $(\widetilde{A})$ of 1.1, the difference between welfare gains under full and limited enforcement falls by more than a half and becomes 1.6%. This reduction of relative welfare benefits can be attributed to an increase in the punishment for default.

To summarize, even moderate perishable technological benefits substantially reduce the negative effect on welfare of the failure to perfectly enforce lending contracts. In other words, in our framework technological transfers play a role of an important enforcement mechanism.

1.6. Conclusion

The objective of this study was to develop a model of international risk-sharing which would be qualitatively consistent with some features of capital flows to the low- and middle income countries documented in the literature. The model we developed is based on three main premises: i) international lending contracts are imperfectly enforceable; ii) access to the international financial markets results in technological transfers to a developing country from the rest of the world; iii) some of the productivity gains associated with the access to external financing are perishable.

We consider a two-sector stochastic growth model and compute optimal accumulation mechanisms in the environments which differ in the extent to which the borrowing contracts with the rest of the world are being enforced. Furthermore, we examine different assumptions concerning the defaulter's punishment and their implications for growth, welfare and borrowing patterns. The principal conclusions of this paper can be summarized in the following way:

First, we conclude that the existence of substantial capital flows from the developed to developing countries is not inconsistent with the presence of the default risk. This prediction of our model distinguishes itself from those of the existing international risk-sharing models with imperfect enforcement of lending contracts such as those Marcet and Marimon (1992) and Kehoe and Perri (2002).

Second, we overcome the difficulty that the models of sustained growth have in explaining the rich structure of observed capital flows and the "wide spectrum of borrowing patterns across low- and middle-income countries" (Marcet and Marimon 1992, p. 221). Our framework predicts that under limited commitment the pattern of capital flows depends heavily on the perishable productivity gains associated with the external financing opportunities. In our framework even moderate technological benefits associated with external financing opportunities may substantially reduce the negative effect on the welfare of the failure to perfectly enforce contracts. In this respect, we conclude that technological transfers may play a role of an important enforcement mechanism. Our model suggests that technological transfers to a developing country from the rest of the world may eliminate risk of default even though they affect only some sectors of the economy.

Third, our model outperforms existing theories of economic growth in its ability to account for countercyclical behavior of net capital inflows to developing countries. Contrary to the implications of the models of perfect or exogenously restricted capital mobility our framework predicts is that the capital inflows to the emerging economies are acyclical. A margin we abstract from in the present inquiry that might be responsible for our failure to predict procyclical inflows is that government policies tend to be procyclical. We leave modeling this feature as an avenue for future research.

Finally, we show that absence of technological diffusion in an environment with limited enforcement of contracts may result in scarce capital flows to developing countries, substantially reduce their growth opportunities and increase volatility of investment. On the over hand, presence of technological transfers in this environment may induce a developing country to use foreign capital to both smooth consumption against unforeseen shocks as well as along the growth path. Moreover, along the transition path the foreign capital will be used to invest more heavily in all the sectors of the economy including those directly unaffected by the technological diffusion. The latter will result in faster growth as well as more substantial welfare gains.

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1.7. Appendix: Derivations

1.7.1. Derivation of the necessary conditions in (1.4), (1.5) and (1.6).

Using the arguments of standard dynamic programming (see Stokey, Lucas, and Prescott (1989)) one can show the existence of the time invariant policy functions $i_1(k_1, k_2, \theta)$, $i_2(k_1, k_2, \theta)$ and a value function $V(k_1, k_2, \theta)$. The Lagrangian for the problem is given by

$$\begin{aligned} \mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ [\lambda u(c_{1t}) + (-\tau_t)] - \lambda_{1t} [c_{1t} - \tau_t + i_{1t} + i_{2t} - f(k_{1t}) - F(k_{2t})] \right. \\ & \left. -\mu_{1t}(k_{1t} - (1-\delta)k_{1t-1} - g(i_{1t-1}, \theta_t) - \mu_{2t}(k_{2t} - (1-\delta)k_{2t-1} - g(i_{2t-1}, \theta_t) \right\}, \end{aligned}$$

where λ_{1t}, μ_{1t} and μ_{2t} are the Lagrange multipliers associated with the constraints (1.1), (1.2) and (1.3). The corresponding f.o.c. are given by

$$\lambda u'(c_{1t}) = \lambda_{1t}, \tag{1.43}$$

$$1 = \lambda_{1t}, \tag{1.44}$$

$$-\lambda_{jt} + \beta E_t \left[\mu_{jt+1} \frac{\partial g(i_{jt}, \theta_{t+1})}{\partial i_{jt}} \right] = 0, \text{ for } j = 1, 2, \tag{1.45}$$

$$\lambda_{1t} f'(k_{1t}) - \mu_{1t} + (1-\delta)\beta E_t \left[\mu_{1t+1} \right] = 0, \tag{1.46}$$

$$\lambda_{1t} F'(k_{2t}) - \mu_{2t} + (1-\delta)\beta E_t \left[\mu_{2t+1} \right] = 0. \tag{1.47}$$

From the equation (1.46) using recursive substitution yields

$$\mu_{1t} = E_t \left[\sum_{j=0}^{\infty} (\beta(1-\delta))^j f'(k_{1t+j}) \lambda_{1t+j} \right].$$

Substituting the latter into (1.45) and using (1.44) as well as the law of iterated expectations yields

$$1 = \beta E_t \left[\frac{\partial g(i_{1t}, \theta_{t+1})}{\partial i_{1t}} \sum_{j=0}^{\infty} (\beta(1-\delta))^j f'(k_{1t+1+j}) \right].$$

The condition (1.5) is derived using the similar argument from (1.47),(1.45), and (1.44). The condition (1.6) follows directly from (1.43) and (1.44).

1.7.2. Approximating the value function and its derivative in the one-sector autarky.

The Bellman equation corresponding to the one-sector autarky is given by

$$V^a(k,\theta) = \max_{(c,i)\in A(k)} \left\{u(c) + \beta E\left[V^a(k',\theta') \mid \theta\right]\right\},$$

subject to

$$A(k) = \left\{(c,i) \in \mathbb{R}^2_+ : c + i = f(k)\right\},$$
$$k' = (1-\delta)k + g(i,\theta').$$

Denoting by $V'(k,\theta)$ the derivative of the value function with the respect to its first argument, the first order condition for the problem becomes

$$u'(c) = \beta E\left[V^{a\prime}(k',\theta')\frac{\partial g\left(i,\theta'\right)}{\partial i} \mid \theta\right]. \tag{1.48}$$

Applying the theorem of Benveniste - Scheinkman [25] yields the following condition for the derivative:

$$V^{a\prime}(k,\theta) = u'(c)f'(k) + \beta(1-\delta)E\left[V^{a\prime}(k',\theta') \mid \theta\right].$$

Rewriting the latter in the sequence form, using recursive substitution and the law of iterated expectations yields

$$V^{a\prime}(k_t,\theta_t) = E_t\left[\sum_{j=0}^{\infty}(\beta(1-\delta))^j u'(c_{t+j})f'(k_{t+j})\right]. \tag{1.49}$$

Now, rewriting (1.48) in the sequence form, using (1.49) and the law of iterated expectations yields the first order condition for the autarky

$$u'(c_t) = \beta E_t\left[\frac{\partial g(i_{1t},\theta_{t+1})}{\partial i_{1t}}\sum_{j=0}^{\infty}(\beta(1-\delta))^j u'(c_{t+1+j})f'(k_{1t+1+j})\right]. \tag{1.50}$$

In order to approximate the value function and its derivative the following algorithm can be used. First, parameterize the conditional expectation in (1.49) as

$$\psi(\omega,k_t,\theta_t) = \exp(P_n(\log\left(k_t\right),\log\left(\theta_t\right))),$$

[25] see Stokey, Lucas, and Prescott (1989) or Marcet and Marimon (1992) for details.

where P_n is a polynomial of degree n. Then, run a non-linear regression, which for $n = 2$ takes the form:

$$\begin{aligned} Y_t &= \exp(\omega_1 + \omega_2 \log(k_t) + \omega_3 \log(\theta_t) + \omega_4 (\log(k_t))^2 \\ &\quad + \omega_5 \log(k_t) \log(\theta_t) + \omega_6 (\log(\theta_t))^2) + \eta_t, \end{aligned}$$

where the dependent variable Y_t is given by the expression inside the conditional expectation in (1.49) evaluated the the autarky solution $\{c_t, k_t\}_{t=0}^{\infty}$.

A similar approach can be used to approximate the value function, except the parameterization of the conditional expectation should change to $\psi(\omega, k_t, \theta_t) = -\exp(P_n(\log(k_t), \log(\theta_t)))$ since utility of the agent 1 takes only negative values.

1.7.3. Derivation of the first order conditions for the two-sector autarky in Program 3.

The dynamic problem corresponding to the autarky with two open sectors is given by

$$\max_{\{c_t, i_{1t}, i_{2t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

subject to

$$c_{1t} + i_{1t} + i_{2t} = f(k_{1t}) + f(k_{2t}), \tag{1.51}$$

$$k_{jt+1} = (1-\delta)k_{jt} + g(i_{jt}, \theta_{t+1}), \quad \text{for} \quad j = 1, 2, \tag{1.52}$$

with $c_{1t} \geq 0, i_{1t}, i_{2t} \geq 0, k_{10}, k_{20}, \theta_0$ given.

The Lagrangian for the problem is given by

$$\begin{aligned} \mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \{ u(c_t) - \lambda \left[c_{1t} - \tau_t + i_{1t} + i_{2t} - f(k_{1t}) - f(k_{2t}) \right] \\ &\quad - \mu_{1t}(k_{1t} - (1-\delta)k_{1t-1} - g(i_{1t-1}, \theta_t) - \mu_{2t}(k_{2t} - (1-\delta)k_{2t-1} - g(i_{2t-1}, \theta_t) \}, \end{aligned}$$

where λ_{1t}, μ_{1t} and μ_{2t} are the Lagrange multipliers associated with the constraints (1.51) and (1.52). The corresponding f.o.c. are given by

$$u'(c_t) = \lambda_t, \tag{1.53}$$

$$-\lambda_t + \beta E_t \left[\mu_{jt+1} \frac{\partial g(i_{jt}, \theta_{t+1})}{\partial i_{jt}} \right] = 0, \text{ for } j = 1, 2, \tag{1.54}$$

$$\lambda_t f'(k_{jt}) - \mu_{jt} + (1-\delta)\beta E_t \left[\mu_{jt+1} \right] = 0, \text{ for } j = 1, 2. \tag{1.55}$$

Using recursive substitution and the law of iterated expectations (1.55) reduces to

$$\mu_{jt} = \beta E_t \left[\sum_{i=0}^{\infty} (\beta(1-\delta))^i f'(k_{jt+1+i}) \lambda_{t+i} \right], \text{ for } j = 1, 2,$$

which combined with (1.53) and (1.54) yields

$$u'(c_t) = \beta E_t \left[\frac{\partial g(i_{jt}, \theta_{t+1})}{\partial i_{jt}} \sum_{i=0}^{\infty} (\beta(1-\delta))^i f'(k_{jt+1+i}) u'(c_{t+i}) \right], \text{ for } j = 1, 2.$$

1.8. Figures and Tables

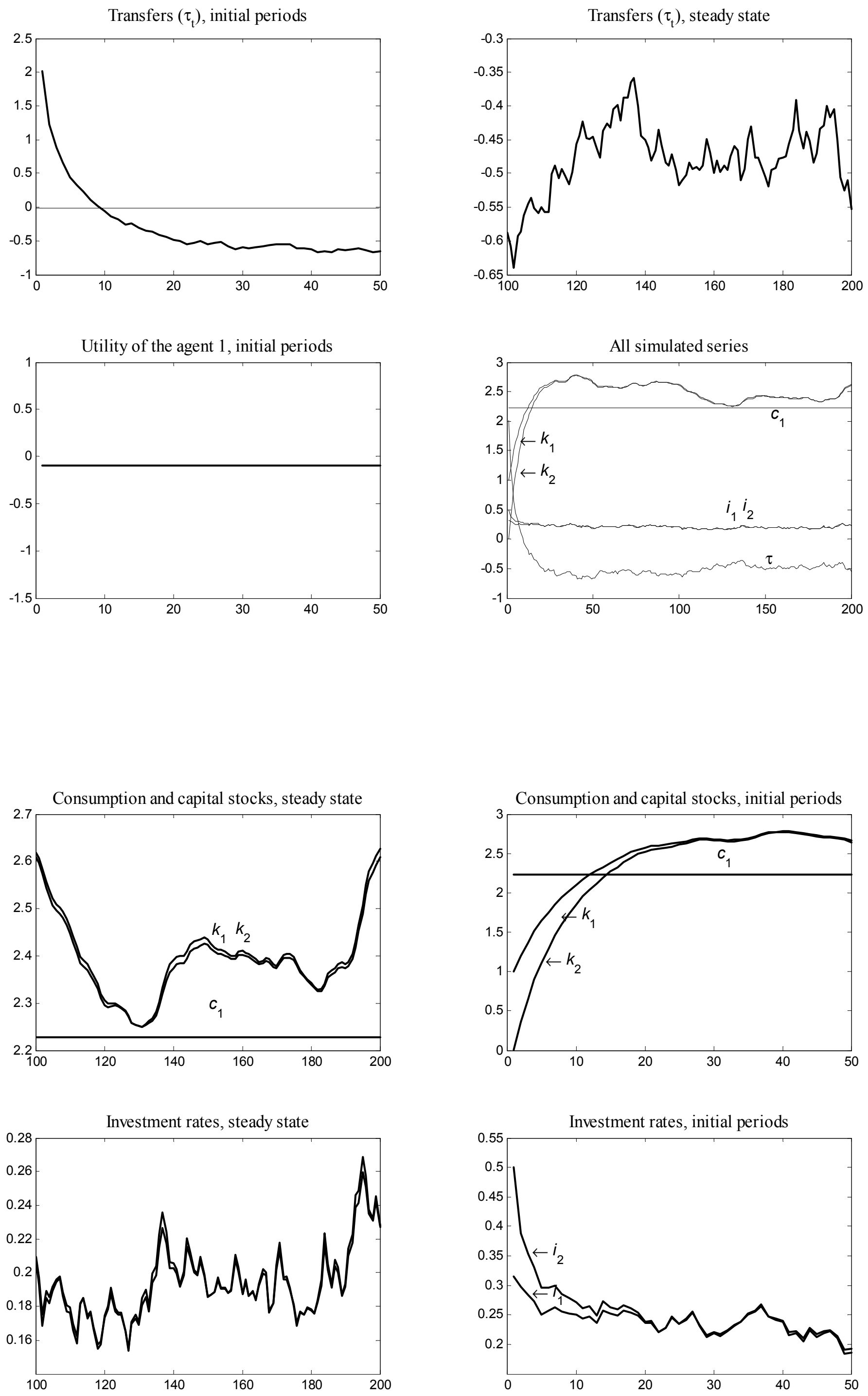

Figure 1.1. Efficient accumulation mechanism under full enforcement.

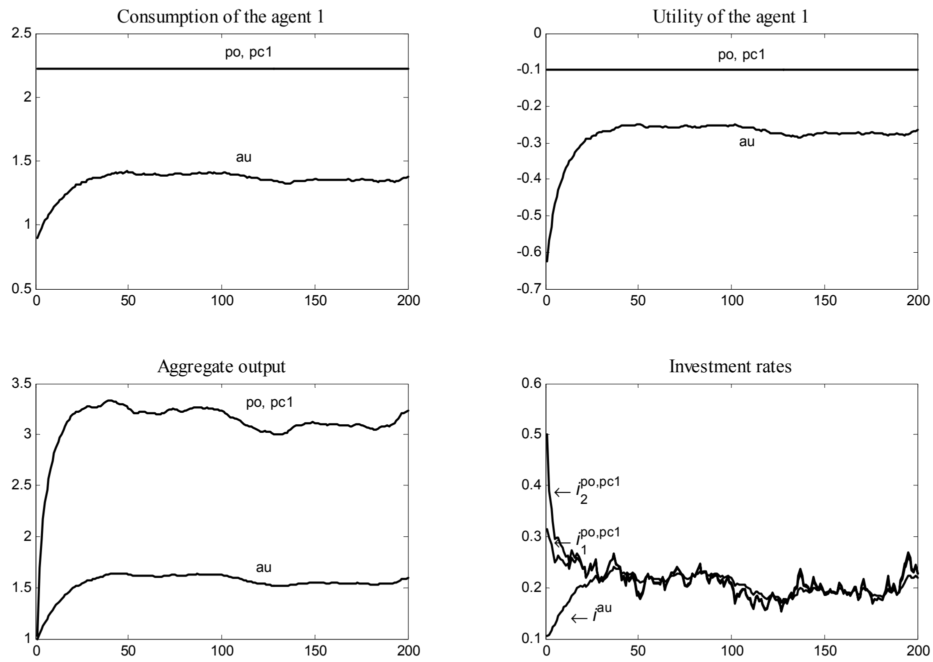

Figure 1.2. The model with one-sector autarky: efficient accumulation mechanisms

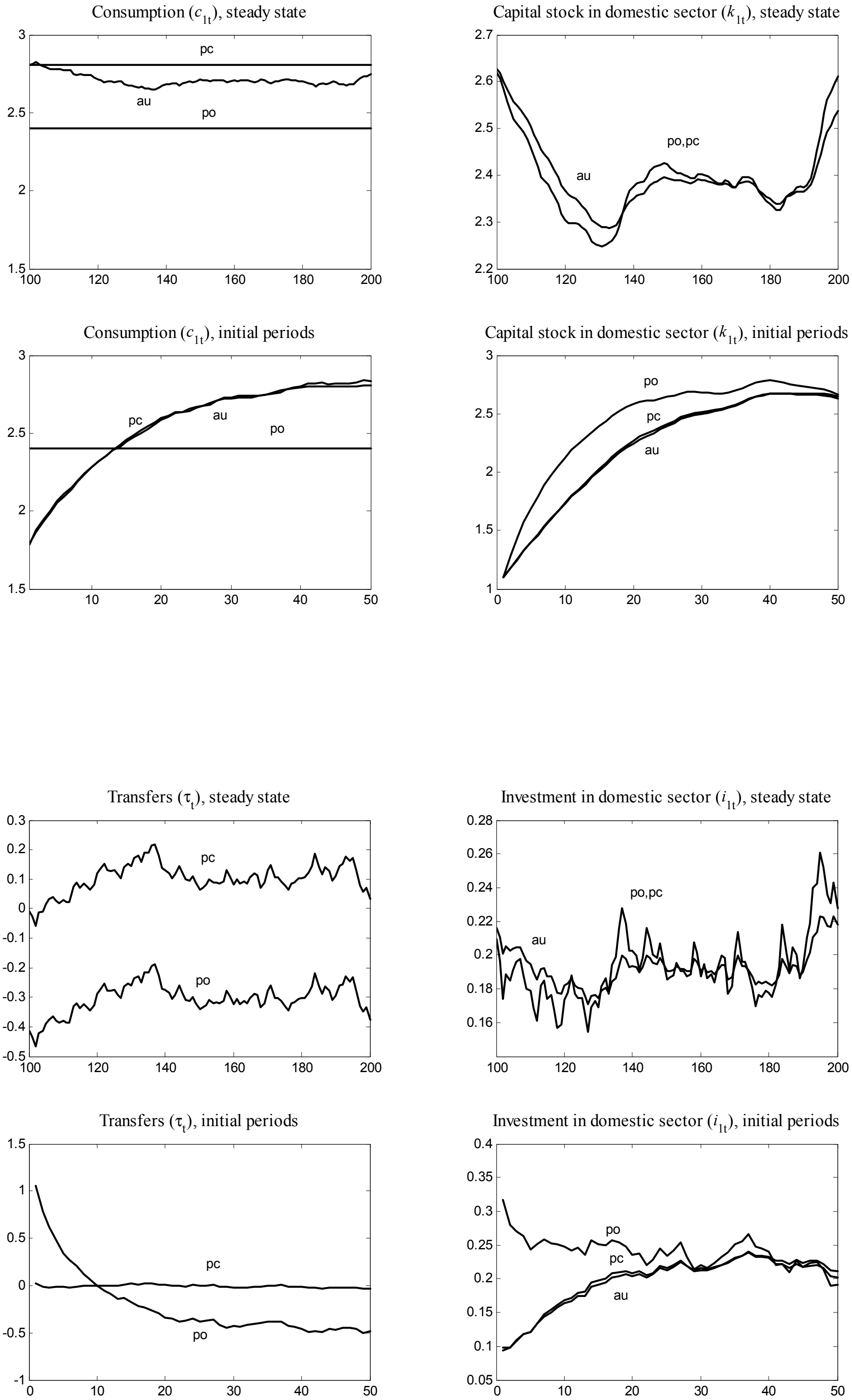

Figure 1.3. The model with two-sector autarky: no technological transfers.

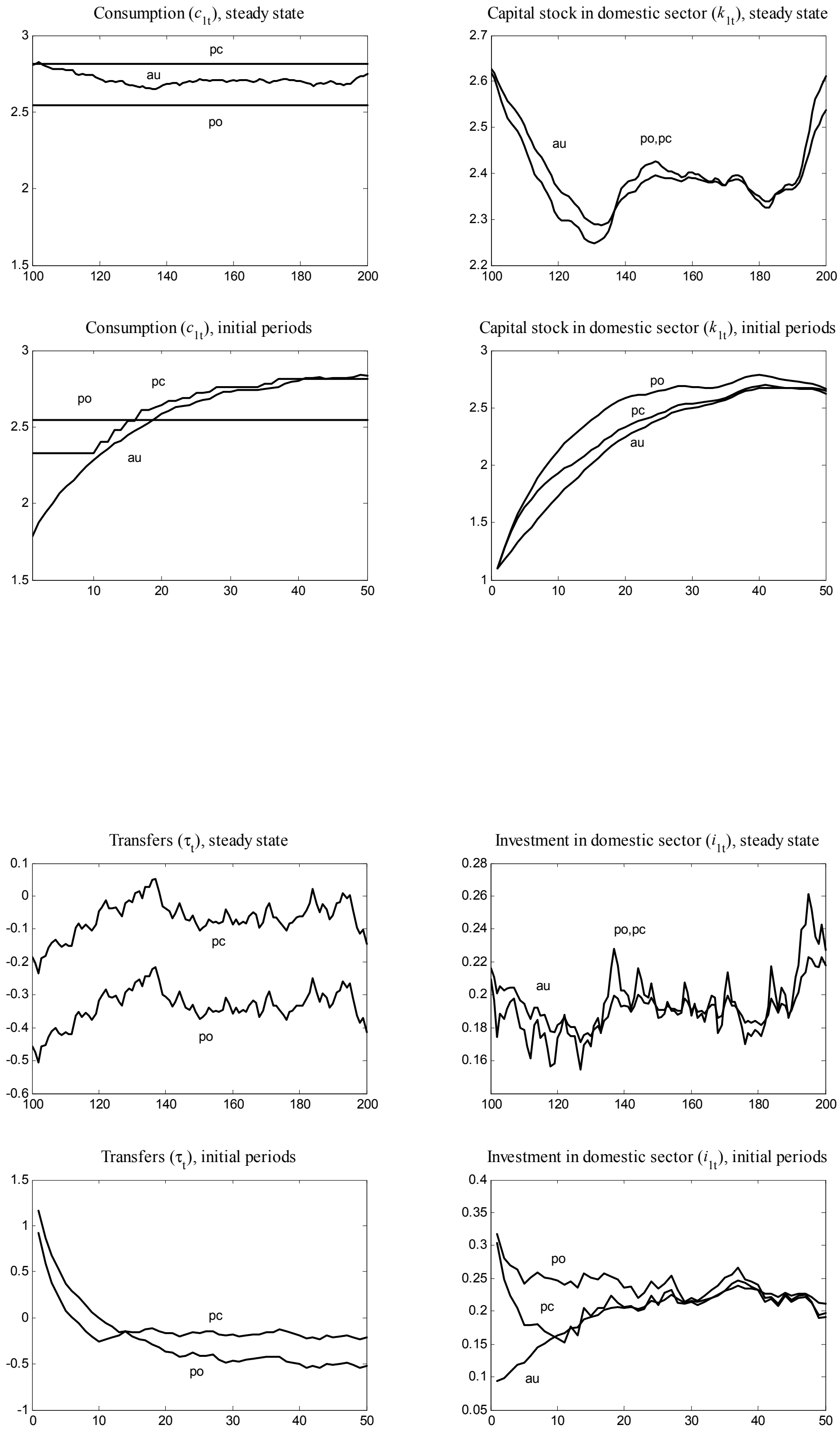

Figure 1.4. The model with two-sector autarky: technological transfers of medium magnitude.

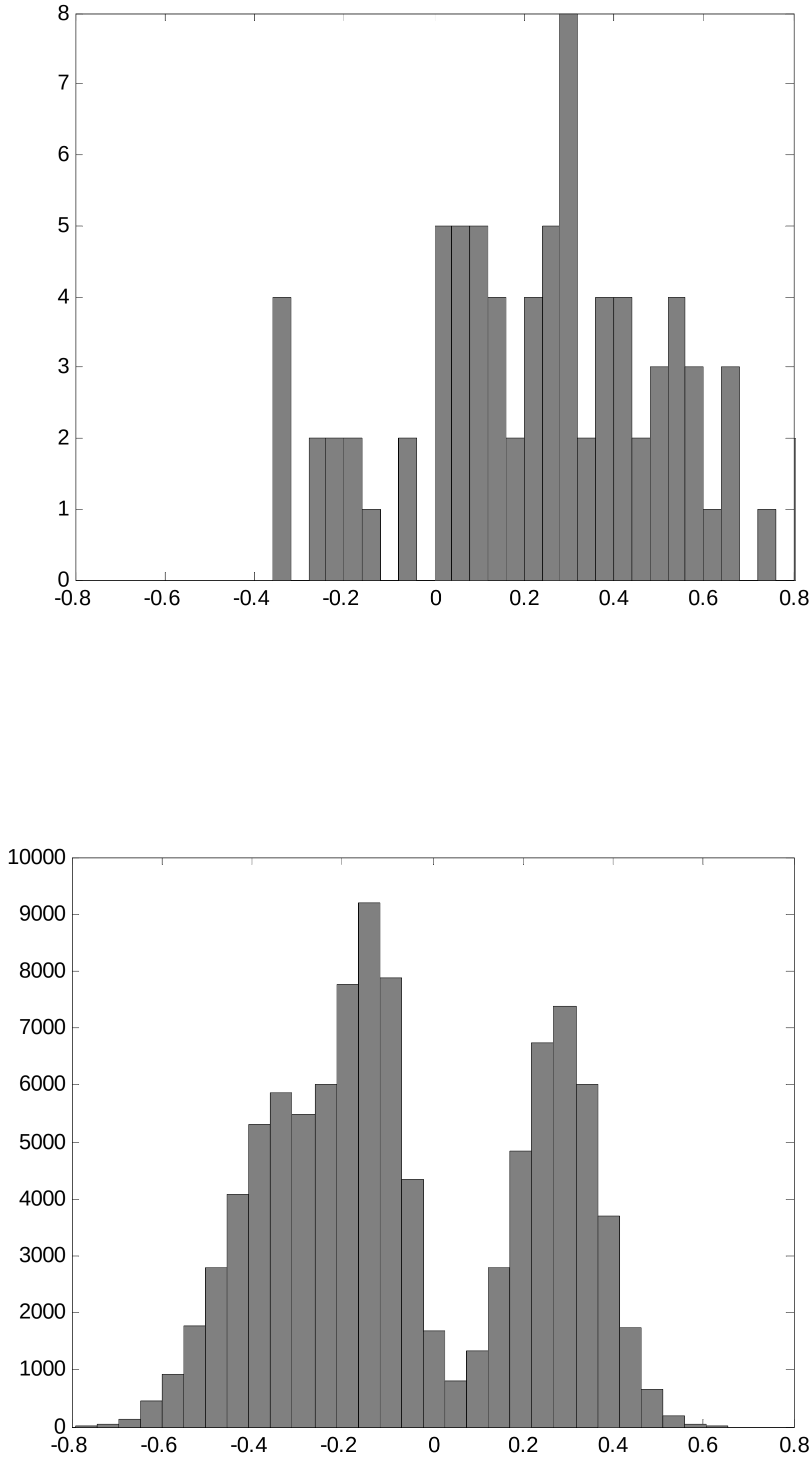

Figure 1.5. Histograms of country correlations between the cyclical components of net capital inflows and real GDP: data (upper panel) and model predictions.

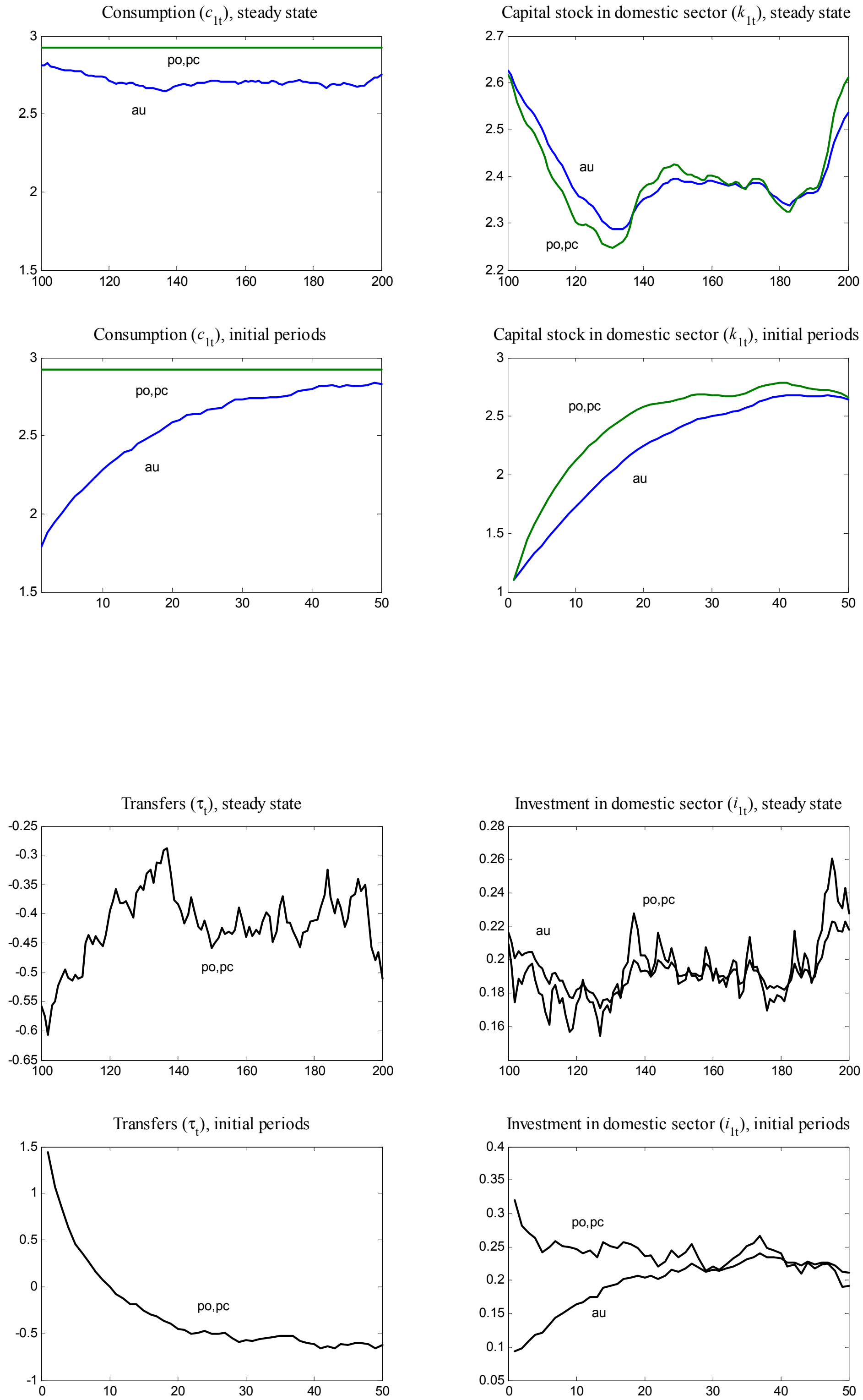

Figure 1.6. The model with two-sector autarky: technological transfers of high magnitude

Table 1.1. Parameterization of the models

Factor share of capital	$\alpha = 0.5$
Risk-aversion parameter of agent 1	$\gamma = -3$
Discount factor	$\beta = 0.95$
Autocorrelation parameter of $\log(\theta_t)$	$\rho = 0.95$
Standard deviation of innovations of $\log(\theta_t)$	$\sigma_\varepsilon = 0.03$
Depreciation rate	$\delta = 0.1$
Constants in the investment functions	$a = 0.6$; $s = 0.2$; $b = 0.13$

Note: Throughout the paper the values of the parameters used in the simulations except for the productivity parameters A and Ã are similar to those of Marcet and Marimon (1992).

Table 1.2. Simulation results: the models with one-sector autarky (Ã = 1.00)

Model	Utility of the agent 1	Mean of growth rate of output (15 periods)	Mean of growth rate of output (35 periods)	Mean of capital in domestic sector (steady state)	Increase in consumption
AU	-7.44	2.41%	1.38%	2.478	-
PO, PC1	-2.01	8.44%	3.80%	2.467	92.20%

Note: The institutional environments considered are the one-sector autarky corresponding to Program 2 (AU), the environment with perfect enforcement in Program 1 (PO), and the limited commitment environment in Program 2 (PC1). The productivity levels of domestic and foreign operated sectors are set to be identical. The utility of the agent 1 is measured at Time 0 using many independent replications of the model conditioning on $\theta_0 = 1$, and $k_{10} = 1$ in case of autarky and $k_{10} = 1$, $k_{20} = 0$ in case of the two sector models. "Mean of growth rate of output" refers to the mean across independent realizations during the first 15 and 35 periods respectively. The "Increase in consumptions" refers to the permanent increase in consumption that would equate the present value under the autarky with the present values achieved under other environments.

Table 1.3. Simulations: the case with no technological transfers (Ã = 1.00)

Model	Utility of the agent 1	Mean of growth rate of output (15 periods)	Mean of growth rate of output (35 periods)	Mean of capital in domestic/foreign sector (steady state)	Increase in consumption
AU3	-1.861	2.455%	1.381%	2.470 / 2.470	-
PO	-1.734	3.035%	1.463%	2.466 / 2.466	3.58%
PC	-1.856	2.470%	1.384%	2.466 / 2.466	0.14%

Note: The case with no technological transfers corresponds to the setup when the productivity levels of domestic and foreign operated sectors are identical. The institutional environments considered are the two-sector autarky in Program 3 (AU3), the environment with perfect enforcement in Program 1 (PO), and the limited commitment environment in Program 4 (PC). "Mean of growth rate of output" refers to the mean across independent realizations during the first 15 and 35 periods respectively. The utility of the agent 1 is measured at Time 0 using many independent replications of the model conditioning on $\theta_0 = 1$, and $k_{10} = 1.1$, $k_{20} = 0.9$. The "Increase in consumptions" refers to the permanent increase in consumption that would equate the present value under the autarky with the present values achieved under other environments.

Table 1.4. Simulations: the case of technological transfers of medium magnitude (Ã = 1.10)

Model	Utility of the agent 1	Mean of growth rate of output (15 periods)	Mean of growth rate of output (35 periods)	Mean of capital in domestic/foreign sector (steady state)	Increase in consumption
AU3	-1.861	2.455%	1.381%	2.470 / 2.470	-
PO	-1.542	3.161%	1.523%	2.467 / 2.641	9.87%
PC	-1.588	2.787%	1.462%	2.465 / 2.639	8.26%

Note: The case with no the case of technological transfers of medium magnitude corresponds to the setup when the productivity levels of foreign operated sectors is higher than that of the domestic sector. However, the productivity differences are not big enough to eliminate risk of default in the environment with limited commitment. The rest is similar to Table 1.3.

Table 1.5. Simulations: the case of technological transfers of high magnitude (Ã = 1.35)

Model	Utility of the agent 1	Mean of growth rate of output (15 periods)	Mean of growth rate of output (35 periods)	Mean of capital in domestic/foreign sector (steady state)	Increase in consumptio n
AU3	-1.861	2.455%	1.381%	2.470 / 2.470	-
PO	-1.168	3.450%		2.467 / 3.016	26.22%
PC	-1.168	3.450%		2.467 / 3.016	26.22%

Note: The case with no the case of technological transfers of high magnitude corresponds to the setup when the productivity levels of foreign operated sectors is higher than that of the domestic sector. Moreover, the productivity differences are big enough to eliminate risk of default in the environment with limited commitment. The rest is similar to Table 1.3.

CHAPTER 2

A Note on Computing Partial Derivatives of the Value Function by Simulation

2.1. Introduction

The purpose of this note is to propose a simple algorithm for computing partial derivatives of the optimal value function. The problems involving incentive compatibility constrains in the form of participation constrains have received wide attention in the literature due to the recent advances in the dynamic optimization techniques. Often the optimality conditions for this class of problems involve partial derivatives with respect to some of the endogenous state variables of the optimal value function corresponding to the dynamic programming formulation of an outside option. Although many numerical methods can provide an approximation for the value function, there is no reason to believe that a derivative of this approximation will be close in any sense to the actual value of the derivative. In this note we suggest an algorithm for computing these partial derivatives by simulation.

This issue has been previously considered by Marcet and Marimon (1992) who solved numerically a growth model with capital accumulation under one-sided lack of commitment. To circumvent the problem of finding the values of the derivatives Marcet and Marimon (1992) proposed a rather convenient method based on the ideas of Benveniste and Scheinkman (1979). Unfortunately their methods has limited applicability since it depends on the availability of the analytical solution for the derivatives as conditional expectations of the known functions of the model solution. In this note we propose a simple algorithm to fill this gap.

To demonstrate the main idea of the algorithm consider a typical example of a partnership with limited commitment[1]. Suppose that the enforcement technology available is such that if the agent deviates from the optimal plan he has no choice but to switch to autarky and stay there forever. In this case the reservation value for the agent would be the value function of the autarkic solution $V(x_t, s_t)$ evaluated at the values of the endogenous state variables

[1]Examples of such models include extentions of the economies in Kocherlakota (1996) and Alvarez and Jermann (2000) to a context with endogenous production as in Kehoe and Perri (2004, 2002) or Abraham and Carceles-Poveda (2006)

x_t, and exogenous shocks s_t at the moment of deviation. The optimality conditions for the planners problem will involve the values of the partial derivatives of the value function $V(x_t, s_t)$ with respect to some of the endogenous state variables. As is common in the endogenous incomplete markets literature, the intertemporal conditions (Euler equations) explicitly take into account the effect the changes in the values of state variables will have on the agent's incentives to deviate from the optimal plan. In the discussion that follows we will assume that these partial derivatives of the optimal value function actually exist[2].

In order to be able to use finite differences to approximate the values of the partial derivatives at a given point one would need to know the values of the optimal value function at a certain set of points. The numerical procedure we sketch below allows one to obtain approximations of these values with arbitrary precision. Moreover, achieving this accuracy is feasible for all points in the state-space which have economic relevance.

The initial step of the algorithm involves obtaining numerical solution to the autarkic problem using a procedure which satisfies three criteria. First, it approximates some unknown function with flexible functional forms of finite elements. Second, it can deliver an accurate solution as the number of the finite elements in the function goes to infinity. Finally, the resulting numerical solution must be such that it can be formulated as a set of policy functions approximated with flexible functional forms. In order to execute this step we will rely on a version of the parameterized expectation approach (PEA)[3]. Even though several procedures suit well for our purpose, the choice of PEA can be justified due to it's following inherent features. First, PEA is computationally efficient when there is a large number of state variables and stochastic shocks in the conditional expectations[4]. Furthermore, it does not impose a discrete grid on the endogenous state variables or the stochastic shocks. Second, Christiano and Fisher (2000, p. 1181) "describe PEAs ...[as] at least as accurate as all the other algorithms considered". Third, Monte Carlo integration in both evaluation of expected discounted returns and the numerical solution to the model with PEA are convenient

[2]In practical applications one would most often need to establish a stronger property than merely existence of the partials. The conditions which ensure differentiability of the value function or at least some of its restrictions are problem specific. Some of these conditions are considered by Benveniste and Scheinkman (1979) for the cases where the Bellman equation is satisfied and by Koeppl (2006) where it is not.

[3]The method was originally introduced by Marcet (1989). The modifications that followed differ along several dimensions. These include the manner the conditional expectations are parameterized, and the way the search for a fixed point is carried out. Which particular version would prove to be more suitable in terms of accuracy and computational efficiency for each class of problems is a question deserving further attention.

[4]This can be partially atributed to the fact that PEA incorporates both endogenous oversampling and Monte-Carlo integration.

for parallelizing.[5] Finally, the algorithm can be utilized to compute the transition towards the stationary distribution. Although the basic version of PEA discussed in den Haan and Marcet (1990) only suits to approximate the solution at the stationary distribution, the algorithm can be modified to include exogenous oversampling and solve for the transition (see Christiano and Fisher (2000) and Marcet and Marimon (1992) for further details).

The next step involves using Monte-Carlo integration in order to evaluate the conditional expectation of the discounted sum of future instantaneous utilities. This requires simulating a large number of realizations of the shocks using the law of motion for the exogenous state variables and their initial values. The initial values in question are simply the values of the state variables at which the value function is being approximated. The simulated series for the shocks along with the obtained approximated policy functions can subsequently be used to generate corresponding series of the endogenous variables consistent with the optimality and feasibility conditions. The obtained series will serve for calculating the discounted sums of the instantaneous returns, each of which would correspond to a particular realization of the stochastic process. An approximation of the value function at a given point is the outcome of averaging over the obtained discounted sums of the instantaneous returns.

The final step involves applying the method of finite differences to approximate the values of the partial derivatives at the point in question. Depending on the underlying assumptions about differentiability of the optimal value function one can choose to rely on several finite difference formulas to compute the derivatives.

The attractive features of the algorithm include its rather wide scope of applicability and simplicity of implementation. It can be used to study the questions of risk sharing under imperfect enforcement of contracts, as well as partnerships with limited commitment when several state variables appear in the model corresponding to the outside option. Such models may include habit formation preferences, several types of capital, or reputational co-state variables. Furthermore, the method may still be applicable even though the default model may fail to fall into a standard recursive framework. Furthermore, the suggested method is computationally inexpensive, it does not suffer from the curse of dimensionality and therefore it is particularly convenient for the models involving many state variables.

The rest of the paper is organized as follows. In Section 2.2 we sketch the idea behind the algorithm. In Section 2.3 we suggest some examples where the proposed algorithm will

[5]See, for instance, Creel (2005) for a discussion of parallelizing of Monte Carlo problems.

prove to be useful. In Section 2.4 we offer a worked out example of implementation of the algorithm and relate the algorithm with the available alternatives. Section 2.5 concludes.

2.2. The Algorithm

Typically, in the models with participation constraints the reservation value is the value function of the outside alternative evaluated at the current values of the endogenous state variables, $\overline{x}$, and exogenous shocks, $\overline{s}$. Suppose, that the value of the optimal value function at a point $(\overline{x}, \overline{s})$ is an outcome of a standard optimization problem for the outside alternative, which can be written as follows:

$$V(\overline{x}, \overline{s}) = \max_{\{a_t\}} E_0 \sum_{t=0}^{\infty} \beta^t r(x_t, a_t, s_t) \tag{2.1}$$

$$\begin{aligned} \text{subject to } x_{t+1} &= l(x_t, a_t, s_t), \quad a_t \in A(x_t, s_t), \\ x_0 &= \overline{x}, \; s_0 = \overline{s}, \end{aligned} \tag{2.2}$$

where r is an instantaneous utility function, $\beta \in (0, 1)$ the discount factor, $\{s_t\}$ an exogenous Markov stochastic process, x_t a vector of endogenous state variables, a_t a vector of control variables, A a feasibility correspondence and l the law of motion for the endogenous state variables. The functional equation to this problem can be derived using the standard dynamic programming techniques. It yields a time invariant policy function f such that optimal allocations satisfy $a_t = f(x_t, s_t)$.

The purpose of the suggested algorithm is to find an pointwise approximation to the partial derivatives $\frac{\partial}{\partial x_i} V(\overline{x}, \overline{s})$ of the value function with respect to its i-th argument. The algorithm takes the following three steps:

Step I. (Numerical Solution) Solve the model in (2.1) with a spectral method and formulate the solution in terms of approximated policy functions

$$a_t = \widehat{f}(\omega; x_t, s_t). \tag{2.3}$$

Step II. (Monte Carlo Integration). Simulate N sequences of the realizations of the stochastic process $\{s^n\}_{t=1}^{T}$ of size T with a starting value $s_0^n = \overline{s}$, for all $n = 1, ..., N$. For a each sequence $\{s^n\}_{t=0}^{T}$ simulate the series of the endogenous variables $\{x_t^n, a_t^n\}_{t=0}^{T}$ using approximated policy functions (2.3), the equations for motion for the state variables (2.2), and the initial values $x_0^n = \overline{x}$. Using the simulated series calculate the discounted sums of the instantaneous returns and average over N.

$$V(\overline{x}, \overline{s}) \simeq \frac{1}{N} \sum_{n=1}^{N} \sum_{t=0}^{T} \beta^t r(x_t^n, a_t^n, s_t^n).$$

Step III. (Numerical Differentiation) Repeat Step II to obtain approximations of the value function at two points, for instance $V(\overline{x} + \epsilon \boldsymbol{\iota}_i, \overline{s})$ and $V(\overline{x} - \epsilon \boldsymbol{\iota}_i, \overline{s})$, where $\boldsymbol{\iota}_i$ denotes a conformable vector of zeros with one on its i-th coordinate, and ϵ is a small positive number. Calculate the value of the partial derivative using, for example, Stirling's finite difference formula:

$$\frac{\partial}{\partial x_i} V(\overline{x}, \overline{s}) \simeq \frac{V(\overline{x} + \epsilon \boldsymbol{\iota}_i, \overline{s}) - V(\overline{x} - \epsilon \boldsymbol{\iota}_i, \overline{s})}{2\epsilon}.$$

The optimal choice of the method for calculating the derivatives in Step III it problem specific and its accuracy depends on the smoothness of the value function. The approaches available include a variety of difference formulas, Richardson Extrapolation, or curve fitting with cubic splines. These are described at length in the standard numerical methods texts such as Judd (1998), Mathews and Fink (2004), or Press, Teukolsky, Vetterling, and Flannery (1992).

A brief note should be made at this point on the accuracy of the algorithm. In principle, arbitrary accuracy of the approximation can be achieved, by simultaneously increasing the dimension of the approximating family of functions in Step I, increasing the size of Monte Carlo iterations in Steps I and II, and decreasing the denominator ϵ in Step III. However, in practical applications there are several sources of the approximation errors. First, in order to obtain the values of optimal value function at a point one relies on the approximations of the policy functions implied by the numerical solution to the model. Second, since we consider stochastic models, there is an additional error stemming from the evaluation of the integral in computation of expected discounted returns. Finally, numerical differentiation introduces two more sources of error: the truncation error and the roundoff error. The truncation error comes from omitting higher order terms in the Taylor series expansion. The roundoff error is associated with storing real numbers in computer's floating-point format. We will discuss some practical accuracy issues in the context of an example in section 2.4.

2.3. Some Examples of Applicability of the Algorithm

In this section we provide a number of examples where the proposed algorithm will proof to be useful. A common feature of all these examples, is that solving them boils down to designing an optimal social contract which takes into account not only technological but also incentive and legal constraints.

2.3.1. Risk-Sharing with Endogenous Market Incompleteness and Habit Formation Preferences

Here we present a model of international risk sharing which distinguishes itself from the celebrated model of Backus, Kehoe, and Kydland (1992) in two respects. First, following Kehoe and Perri (2002) we introduce a friction in the credit markets. We assume that the international loans are feasible only to the extent to which they can be enforced by the threat of exclusion from participating in any other international risk sharing arrangement. Second, we incorporate habit formation preference in the model. The latter can be motivated by the evidence presented by Fuhrer and Klein (2006, p. 722) who suggest that "habit formation characterizes consumption behavior amongst most of G-7 countries".

Consider the planner's problem of maximizing a weighted sum of utilities subject to individual participation constraints and feasibility constraints. In particular, the problem of interest is to choose allocations $\{c_{it}, i_{it}\}$ for $i = 1, ...I$ to solve

$$\max_{\{c_{it}, i_{it}\}} E_0 \sum_{i=1}^{I} \lambda_i \sum_{t=0}^{\infty} \beta^t u(c_{it}, h_{it})$$

subject to

$$\sum_{i=1}^{I} c_{it} + \sum_{i=1}^{I} i_{it} = \sum_{i=1}^{I} f\left(k_{it}, \theta_{it}\right), \tag{2.4}$$

$$k_{it+1} = (1-\delta)k_{it} + i_{it}, \tag{2.5}$$

$$h_{it+1} = h_{it} + \lambda(c_{it} - h_{it}), \tag{2.6}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{it+j}, h_{it+j}) \geq V_i^a\left(k_{it}, h_{it}, \theta_{it}\right), \tag{2.7}$$

where $c_{it}, i_{it} \geq 0$, $\boldsymbol{\theta}_t$ follows a first order vector autoregressive process, and the initial values for the state variables $k_{i0}, h_{i0}, \theta_{i0}$, and the initial non-negative weights are given. Here $V_i^a(k_{it}, h_{it}, \theta_{it})$ denotes the optimal value function corresponding to the autarkic environment

$$\max_{\{c_{it}, i_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, h_{it}) \tag{2.8}$$

subject to

$$c_{it} + i_{it} = f(k_{it}, \theta_{it}),$$

$$k_{it+1} = f(k_{it}, \theta_{it}) - c_{it} + (1-\delta)k_{it},$$

$$h_{it+1} = h_{it} + \lambda(c_{it} - h_{it}),$$

with the initial values being equal to the values of the state variables $k_{it}, h_{it}, \theta_{it}$ at the moment of deviation from the optimal plan.

In addition to the aggregate recourse constraint (2.4), participation constraint (2.7), and the equations of motion for the state variables (2.5)-(2.6) the constrained efficient allocations should satisfy the following risk sharing condition:

$$\frac{u_c(i,t) + \lambda\beta E_t \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j \left[\frac{\xi_{it+1+j}}{\xi_{it}} u_h(i, t{+}1{+}j) - \frac{\mu_{it+1+j}}{\xi_{it}} \frac{\partial V_i^a}{\partial h_{it+1+j}}(i, t{+}1{+}j)\right]}{u_c(s,t) + \lambda\beta E_t \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j \left[\frac{\xi_{st+1+j}}{\xi_{st}} u_h(s, t{+}1{+}j) - \frac{\mu_{st+1+j}}{\xi_{st}} \frac{\partial V_s^a}{\partial h_{st+1+j}}(s, t{+}1{+}j)\right]} = \frac{\xi_{st}}{\xi_{it}},$$

for $i, s = 1, ..., I$. Optimal allocation must also satisfy the intertemporal condition

$$\begin{aligned} &u_c(i,t) + \lambda\beta E_t \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j \left[\frac{\xi_{it+1+j}}{\xi_{it}} u_h(i, t{+}1{+}j) - \frac{\mu_{it+1+j}}{\xi_{it}} \frac{\partial V_i^a}{\partial h_{it+1+j}}(i, t{+}1{+}j)\right] \\ = \;& \beta E_t \left[\left(\frac{\xi_{it+1+j}}{\xi_{it}} u_c(i, t+1) + \lambda\beta \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j \left[\frac{\xi_{it+2+j}}{\xi_{it}} u_h(i, t{+}2{+}j)\right.\right.\right. \\ & \left.\left. - \frac{\mu_{it+2+j}}{\xi_{it}} \frac{\partial V_i^a}{\partial h_{it+2+j}}(i, t{+}2{+}j)\right]\right) (f_k(k_{it+1}, \theta_{it+1}) + 1 - \delta) - \frac{\mu_{it+1}}{\xi_{it}} \frac{\partial V_i^a}{\partial k_{it+1}}(i, t+1)\Bigg], \end{aligned} \tag{2.9}$$

along with the complementary slackness condition

$$\mu_{it}\left[E_t \sum_{j=0}^{\infty} \beta^j u(c_{it+j}, h_{it+j}) - V_i^a(k_{it}, h_{it}, \theta_{it})\right] = 0,$$

and the law of motion for the reputational co-state variables

$$M_{it+1} = M_{it} + \mu_{it},$$

where $\xi_{it} = \lambda_i + M_{it+1}, \mu_{it} \geq 0$, and $M_{i0} = 0$. In these first order conditions we have used the abbreviations $u_c(i,t)$ for $\frac{\partial u(c_{it},h_{it})}{\partial c_{it}}$, and we have used similar abbreviations for other terms. Notice, that the partial derivative of the optimal value function V_i^a enter both the intertemporal condition (Euler equation) and the risk-sharing condition.

2.3.2. Capital flows to developing countries under risk of debt repudiation.

Our next example is a dynamic principal-agent problem with one-sided lack of commitment similar in spirit to that of Marcet and Marimon (1992). In this setup, a capital-poor country, represented as a risk-averse agent, can borrow from the industrialized capital-rich countries, represented here as a risk neutral agent. While the club of the industrial countries is assumed to honor its contractual obligations, the developing country has an option to renege on its debt and suffer the consequences. In the absence of a supranational enforcement authority the punishment the borrower will incur in case of debt repudiation is the exclusion from any further intertemporal and inter-state trade with the industrialized countries. However, this will not preclude the poor country from being able to enter a risk sharing arrangement under two-sided lack of commitment with some other capital poor countries.

Constrained efficient allocations can be found as a solution to the following planner's problem:

$$\max_{\{c_t,\tau_t,i_t,\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[\lambda u(c_t) + (-\tau_t) \right] \right] \tag{2.10}$$

subject to

$$c_t - \tau_t + i_t = \theta_t f(k_t), \tag{2.11}$$

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{2.12}$$

$$E_t \left[\sum_{j=0}^{\infty} \beta^j u(c_{t+j}) \right] \geq V^d(k_t, \theta_t), \tag{2.13}$$

with initial conditions (k_0, θ_0) and non-negativity conditions $c_t \geq 0, i_t \geq 0$. In this specification c_t, i_t, and k_t are consumption, gross investment, and capital stock respectively; τ_t denotes transfers from the risk-neutral agent to the risk averse one; $\delta \in (0,1]$ is depreciation rate of capital; $\beta \in (0,1)$ is the discount factor. A random productivity shock θ_t follows a first order stationary Markov process with bounded support. The instantaneous utility $u(\cdot)$ of the risk-averse agent is strictly concave, twice differentiable and satisfies the Inada conditions. The production function $f(\cdot)$ is concave and differentiable.

The reservation value which appears in left hand side of (2.13) corresponds to the expected felicity of the agent's outside option: a partnership with two-sided lack of commitment. The allocation to the latter must satisfy the following planner's problem, with $\sum_{i=1}^{I} \lambda_i = 1$:

$$\max_{\{c_{it}, i_{it}\}} E_0 \sum_{i=1}^{I} \lambda_i \sum_{t=0}^{\infty} \beta^t u(c_{it}) \tag{2.14}$$

subject to

$$\sum_{i=1}^{I} c_{it} + \sum_{i=1}^{I} i_{it} = \sum_{i=1}^{I} \theta_{it} f(k_{it}),$$
$$k_{it+1} = (1-\delta) k_{it} + i_{it},$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{it+j}) \geq V_i^a (k_{it}, \theta_{it}). \tag{2.15}$$

with $c_{it} \geq 0, i_{it} \geq 0$. The initial values for k_{i0}, θ_{i0} are given by the corresponding values of the state variables in problem (2.10) at the moment of deviation. In the problem (2.14) above, the reservation value $V_i^a(k_{it}, \theta_{it})$ is given by

$$V_i^a (k_{it}, \theta_{it}) = \max_{\{c_{is}\}_{s=t}^{\infty}} E \left[\sum_{s=t}^{\infty} \beta^{s-t} u(c_{is}) \right] \tag{2.16}$$

subject to

$$k_{is+1} = \theta_{is} f(k_{is}) + (1-\delta) k_{is} - c_{is},$$

with $c_{it} \in [0, \theta_{is} f(k_{is})]$.

While the autarkic problem in (2.16) is the classical Brock and Mirman (1972) economy the solutions to the problem (2.14) will take the form of the following policy function:[6]

$$\Psi(k,\mu,\theta) = \arg\min_{\gamma\geq 0}\max_{c,i}\left\{\sum_{j=1}^{I}\left(\lambda_j+\mu_j+\gamma_j\right)u(c_j)+\beta E\left[W(k',\mu',\theta'))\mid\theta\right]\right\} \tag{2.17}$$

subject to

$$\mu_j' = \mu_j + \gamma_j,$$
$$\sum_{j=1}^{I} c_j + \sum_{j=1}^{I} i_j = \sum_{j=1}^{I}\theta_j f\left(k_j\right),$$
$$k_j' = (1-\delta)k_j + i_j,$$

in addition to the incentive compatibility constraint (2.15) and non-negativity constraints on the multipliers $\{\mu_j\}$.

The recursive contract methodology of Marcet and Marimon (1998) ensures that the optimal solution to the optimization problem (2.14) satisfies $(c_{it}, i_{it}, \gamma_{it}) = \Psi\left(k_{it}, \mu_{it}, \theta_{it}\right)$ for all t with the initial conditions $(k_{i0}, 0, \theta_{i0})$. In a particular case when symmetric treatment can be applied to the participants of the risk-sharing arrangement in (2.14) the reservation value on the right hand side of (2.13) is given by

$$V^d(k_t,\theta_t) = \frac{1}{\lambda_j}W(k_t,0,\theta_t).$$

Otherwise, this reservation value is simply the expected discounted sum of instantaneous utilities $\sum_{t=0}^{\infty}\beta^t u(c_{it})$ where $\{c_{it}\}$ are allocations obtained from from iterating on (2.17).

2.3.3. International borrowing under limited commitment and several types of capital.

Another example where the suggested algorithm will be useful is a framework with limited commitment with several types of capital. A version of this model is considered in Chapter 1 whereby the borrower which cannot fully commit to repay its debts accumulates capital in several productive sectors.

[6]Henceforth we adopt the dynamic programming convention by which primes denote the forwarded values of variables.

2.4. An Example of Implementation.

In this section we describe a practical computational strategy for implementing the algorithm. We will rely on the example in section 2.3.1 which considers international risk sharing problem of Backus, Kehoe, and Kydland (1992) augmented with the contract enforcement friction and habit formation preferences.

The intertemporal optimality condition to the planner's problem (2.9) includes partial derivatives of the value function corresponding to the dynamic programming formulation of the agents outside option. The functional equation for the autarkic problem is given below:

$$V(k,h,\theta) = \max_{(c,i)\in A(k,\theta)} \{u(c,h) + \beta E[V(k',h',\theta') \mid (k,h,\theta)]\}$$
$$h' = h + \lambda(c-h),$$
$$k' = (1-\delta)k + i$$
$$A(k,\theta) = \left\{(c,i) \in \mathbb{R}^2_+ : c + i = f(k,\theta)\right\},$$

The objective of the algorithm is to find the values of the partials in question $V_h(\cdot)$ and $V_k(\cdot)$ at a point $(\overline{k},\overline{h},\overline{\theta})$ which is likely to happen in equilibrium. Since the analytical expression for these derivatives is in general unavailable, we will have no choice but to rely on numerical differentiation. Another complication which arises here is that the closed form solution to the optimal value function is generally unavailable either. Hence, one needs to approximate value function at two points, e.g. $V(\overline{k}+\varepsilon,\overline{h},\overline{\theta})$ and $V(\overline{k}-\varepsilon,\overline{h},\overline{\theta})$ with arbitrary accuracy in order to be able to use finite differencing approach[7].

The first step of the algorithm involves solving the model with a spectral method which can approximate the policy functions with arbitrary accuracy. In this example we will utilize a version of PEA of Marcet (1989) which allows us to formulate the solution in terms of approximated policy functions.

The Euler equation for the problem is given by:

$$u_c(c,h) + \beta\lambda E_t\left[\sum_{j=0}^{\infty}\beta^j(1-\lambda)^j u_h(c_{t+j+1},h_{t+j+1})\right] \tag{2.18}$$

[7]The points of interest are $V(\overline{k}+\varepsilon,\overline{h},\overline{\theta})$ and $V(\overline{k},\overline{h},\overline{\theta})$ if we choose to rely on Newton's forward formula, $V(\overline{k}-\varepsilon,\overline{h},\overline{\theta})$ and $V(\overline{k},\overline{h},\overline{\theta})$ if we use Newton's backward formula, and $V(\overline{k}+\varepsilon,\overline{h},\overline{\theta})$ and $V(\overline{k}-\varepsilon,\overline{h},\overline{\theta})$ if we use Stirling's formula.

$$= \beta E_t \left[\left(u_c \left(c_{t+1}, h_{t+1} \right) + \beta \lambda \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j u_h \left(c_{t+j+2}, h_{t+j+2} \right) \right) \left(f_k(k_{t+1}, \theta_{t+1}) + 1 - \delta \right) \right].$$

To simplify the exposition we will consider the case of non-persistent habits which corresponds to $\lambda = 1$. In this particular case, the habit stock at $t+1$ is simply the level of consumption at t, and the Euler equation (2.18) reduces to:

$$\text{(2.19)} \quad u_c \left(c_t, h_t \right) + \beta E_t \left[u_h \left(c_{t+1}, h_{t+1} \right) \right] = \beta E_t \Big[\left(f_k(k_{t+1}, \theta_{t+1}) + 1 - \delta \right) \times \left(u_c \left(c_{t+1}, h_{t+1} \right) + \beta u_h \left(c_{t+2}, h_{t+2} \right) \right) \Big].$$

To solve the model numerically we will assume the functional forms relatively standard in the growth literature. The instantaneous utility function is given by

$$u(c_t, h_t) = \frac{\left(c_t - b h_t \right)^{1-\sigma}}{1-\sigma},$$

where $b \in (0,1)$ and $\sigma > 0$. One rationale for choosing additive functional form to introduce habits is to preserve the usual concavity properties of the utility function. The production function is Cobb-Douglas and is given by

$$f(k_t, \theta_t) = \theta_t k_t^{\alpha}.$$

The stochastic productivity follows a first-order autoregressive process in logs

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t,$$

where $\{\varepsilon_t\}$ are independent normally distributed random variables with zero mean and variance σ_ε^2. In this example, we restrict attention to one particular set of the parameters which are summarized in Table 2.1.

[Insert Table 2.1 about here.]

With the chosen functional forms the Euler equation becomes:

$$\left(c_t - b h_t \right)^{-\sigma} - \beta b E_t \left[\left(c_{t+1} - b h_{t+1} \right)^{-\sigma} \right] = \beta E_t \Big[\left(\alpha \theta_{t+1} k_{t+1}^{\alpha} + 1 - \delta \right) \times \left[\left(c_{t+1} - b h_{t+1} \right)^{-\sigma} - b \beta \left(c_{t+2} - b h_{t+2} \right)^{-\sigma} \right] \Big].$$

The sequences of optimal allocations $\{c_t, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ must satisfy the following system of stochastic difference equations:

$$\begin{aligned}(c_t - bh_t)^{-\sigma} \;&=\; \beta E_t \Bigg[b\,(c_{t+1} - bh_{t+1})^{-\sigma} \\ &\times \left(1 + \left(\alpha\theta_{t+1}k_{t+1}^{\alpha-1} + 1 - \delta\right)\left(\frac{1}{b} - \beta\left(\frac{c_{t+2} - bh_{t+2}}{c_{t+1} - bh_{t+1}}\right)^{-\sigma}\right)\right)\Bigg],\end{aligned} \tag{2.20}$$

$$k_{t+1} = \theta_t k_t^{\alpha} + (1-\delta)k_t - c_t, \tag{2.21}$$

$$h_{t+1} = c_t. \tag{2.22}$$

For the expositional purpose, we will rely on the version of PEA which is easiest to implement. The computational procedure takes the following steps:

- Fix the initial conditions and draw a series of $\{\theta_t\}_{t=1}^{T}$ that obeys the law of motion for the exogenous shocks with T sufficiently large.[8]
- Substitute the conditional expectations in (2.20) by the flexible functional forms that depend on the state variables k_t, h_t, θ_t and some coefficients to yield

  $$(c_t(\omega) - bh_t(\omega))^{-\sigma} = \beta\psi(\omega; k_t(\omega), h_t(\omega), \theta_t),$$

  where

  $$\psi(\omega; k_t(\omega), h_t(\omega), \theta_t) = \exp(P_n\,(\omega; \log k_t(\omega), \log h_t(\omega), \log\theta_t)),$$

  and P_n denotes polynomial of degree n. Reliance on the exponent of the logarithmic polynomial expansion guarantees that the left hand side of (2.20) would be positive. Given $c_t(\omega)$, the next period values for the capital and habit stocks follow directly from the corresponding laws of motion (2.21) and (2.22).
- Using the realizations of $\{\theta_t\}_{t=0}^{T}$ repeat the previous step in order to obtain recursively a series of the endogenous variables $\{c_t(\omega), k_{t+1}(\omega), h_{t+1}(\omega)\}_{t=0}^{T}$, for this particular parameterization ω.

[8]In order to ensure sufficient accuracy of the solution we chose $T = 50,000$ for all the numerical examples considered. The computational burden of this is still rather low since the model needs to be solved only once.

- Run the following non-linear regression

$$Y_t(\omega) = \exp(P_n\left(\xi; \log k_t(\omega), \log h_t(\omega), \log \theta_t\right)) + \eta_t,$$

where the role of the dependent variable $Y_t(\omega)$ is performed by the expression inside the conditional expectation in the RHS of (2.20).

- Letting $S(\omega)$ be the result of the regression in the previous step, use an iterative procedure to find the fixed point of S, and the set of coefficients $\omega_f = S\left(\omega_f\right)$. This would provide the solution for the endogenous variables $\{c_t(\omega_f), k_{t+1}(\omega_f), h_{t+1}(\omega_f)\}_{t=0}^{T}$ for this particular realization of the stochastic process $\{\theta_t\}_{t=1}^{T}$along with the approximated policy functions:

$$c_t(k_t, h_t, \theta_t) = bh_t + [\beta\psi(\omega_f; k_t, h_t, \theta_t)]^{-\frac{1}{\sigma}}, \tag{2.23}$$

$$k_{t+1}(k_t, h_t, \theta_t) = \theta_t k_t^{\alpha} + (1-\delta)k_t - bh_t - [\beta\psi(\omega_f; k_t, h_t, \theta_t)]_t^{-\frac{1}{\sigma}}, \tag{2.24}$$

$$h_{t+1}(k_t, h_t, \theta_t) = bh_t + [\beta\psi(\omega_f; k_t, h_t, \theta_t)]^{-\frac{1}{\sigma}}. \tag{2.25}$$

The simulated series consistent with optimality and feasibility conditions are reported in Figure 2.1. The initial values for capital, k_0, and habit stock, h_0 correspond to those of the deterministic steady state.

[Insert Figure 2.1 about here.]

Our objective is to find approximations of partials at a range of points. Supposing that the point of interest is $\left(\overline{k}, \overline{h}, \overline{\theta}\right)$ the algorithm proceeds as follows:

- Simulate N sequences of the realizations of the stochastic process $\{\theta^n\}_{t=0}^{\overline{T}}$ of size $\overline{T}$ with a starting value $\theta_0^n = \overline{\theta}$, for all $n = 1, ..., N$. For a each sequence $\{\theta^n\}_{t=0}^{T}$ simulate the series of the endogenous variables $\{k_t^n, h_t^n, c_t^n\}_{t=0}^{\overline{T}}$ using approximated policy functions and the laws of motion for the state variables (2.23)-(2.25), and the corresponding initial values $k_0^n = \overline{k}, h_0^n = \overline{h}$. Using the simulated series calculate the discounted sums of the instantaneous utilities and average over N.

$$V\left(\overline{k}, \overline{h}, \overline{\theta}\right) \simeq \frac{1}{N}\sum_{n=1}^{N}\sum_{t=0}^{\overline{T}}\beta^t\frac{\left(c_t^n - bh_t^n\right)^{1-\sigma}}{1-\sigma}.$$

- To obtain $V_k\left(\overline{k},\overline{h},\overline{\theta}\right)$ get approximations of the optimal value function at $V\left(k+\epsilon,h,\theta\right)$ and $V\left(k-\epsilon,h,\theta\right)$, where ϵ is a small positive number. Calculate the approximated value of the partial derivative using Stirling's finite difference formula:

$$\frac{\partial V\left(\overline{k},\overline{h},\overline{\theta}\right)}{\partial k}\simeq\frac{V\left(\overline{k}+\epsilon,\overline{h},\overline{\theta}\right)-V\left(\overline{k}-\epsilon,\overline{h},\overline{\theta}\right)}{2\epsilon}.$$

  The partial with respect to the habit stock is obtained in a similar way.

[Insert Figure 2.2 about here.]

Notice, that the length of the simulated series $\overline{T}$ can be very moderate due to discounting of the future utilities. The optimal value of ϵ is both computer and problem specific. The approximations to the derivatives at a range of points are reported in Figure 2.2.

2.4.1. Comparing performance of the algorithm with the available alternatives

In this section we will consider issue of accuracy of our algorithm in the context of an example. First, we will compare performance of our algorithm with the approach of Marcet and Marimon (1992) when such comparison is feasible. Furthermore, we present several special cases which allow us to isolate the contributions to the overall approximation error of the algorithm from different sources.

Once again, consider the risk-sharing problem with endogenous market incompleteness and habits discussed in section 2.3.1. Notice, that the optimality conditions for the autarkic problem (2.8) can be written in the sequence form as follows:

$$u_c\left(c,h\right)+\beta\lambda E_t\left[V_h(k_{t+1},h_{t+1},\theta_{t+1})\right]=\beta E_t\left[V_k(k_{t+1},h_{t+1},\theta_{t+1})\right], \tag{2.26}$$

$$V_k(k_t,h_t,\theta_t)=\beta E_t\left[V_k(k_{t+1},h_{t+1},\theta_{t+1})\right]\left(f_k(k_t,\theta_t)+1-\delta\right), \tag{2.27}$$

$$V_h(k_t,h_t,\theta_t)=u_h\left(c_t,h_t\right)+\beta(1-\lambda)E_t\left[V_h(k_{t+1},h_{t+1},\theta_{t+1})\right]. \tag{2.28}$$

The intertemporal condition (2.28) can be used to compare our algorithm with the Marcet and Marimon (1992) method. The latter requires solving the model numerically and expressing the derivatives of interest in terms of conditional expectations and functions of equilibrium path of the model. Starting from (2.28), using recursive substitution and the law of iterated expectations yields:

$$V_h(k_t, h_t, \theta_t) = u_h(c_t, h_t) + E_t\left[\sum_{j=1}^{\infty} \beta^j (1-\lambda)^j u_h(c_{t+j}, h_{t+j})\right].$$

Now, one can proceed by parameterizing the right hand side with flexible functional forms in the state variables (k_t, h_t, θ_t). An approximation of this derivative can be obtained by running one non-linear regression using the simulated series from the numerical solution of the model.

[Insert Figure 2.3 about here.]

The approximations of the derivative obtained using our algorithm and the approach of Marcet and Marimon (1992) are reported in Figure 2.3. In the graphs, we plot the approximated values of $V_h(k_t, h_t, \theta_t)$ for a range of one of the state variables while keeping the remaining ones fixed at their deterministic steady state values. The histograms plot the sample distributions of capital and habit stock from one long simulation of the model. Based on the graphs and histograms presented, a few observations can be made. First, the two algorithms produce indistinguishable results when the state variables take the values which often happen in equilibrium. Second, for the point which are unlikely to occur is equilibrium, the approximations differ significantly. To see this feature, consider the range of values of capital stock in excess of 6.5. The plots of the approximate derivatives reported in the upper panel of 2.3 do not coincide. Moreover, the upper tail of the histogram suggests that such values of k_t are not unlikely to happen in equilibrium. Notice, that while considering at a relatively high value of k_t we kept the remaining arguments of $V_h(k_t, h_t, \theta_t)$ at their deterministic steady state values. However, the points where capital is very high while consumption (and hence habit stock) are at the steady state level are rather unusual. This can be also noticed from the graph in Figure 2.1 which plots the evolution of the endogenous variables and exogenous shocks for 200 periods corresponding to the stationary distribution. This is an expected result, since we relied on a version of PEA which delivers good approximation to the policy function in the region of the state space which is frequently visited by the model in equilibrium. This by no means limits the applicability of the algorithm. If one happens to be interested in a different subset of the state space one can simply modify the oversampling scheme in Step I of the algorithm. We have followed this strategy in Chapter 1 of this thesis where the object of interest was the transition path towards the stationary distribution.

The framework we have chosen to serve as worked out example embeds several well known special cases. For instance, for $\lambda = 0$, it reduces to the stochastic growth model of Brock and Mirman (1972). In this case, the analytical form of the one-period return function r which maps the graph A of the feasibility correspondence Γ into the real numbers is known. Indeed, the correspondence describing the feasibility constraints is given by

$$\Gamma(k_t, \theta_t) = [f(1-\delta)k_t, f(k_t, \theta_t) + (1-\delta)k_t],$$

and the instantaneous return function becomes

$$r : A \to \mathbb{R} \text{ given by } r(k_t, k_{t+1}, \theta_t) = u(f(k_t, \theta_t) + (1-\delta)k_t - k_{t+1}),$$

where $A = \{(k_t, k_{t+1}, \theta_t) \in \mathbb{R}^3 : k_{t+1} \in \Gamma(k_t, \theta_t)\}$. Hence, by virtue of the Benveniste-Sheinkman theorem the derivative of interest can be expressed as

$$V_k(k_t, \theta_t) = u'(f(k_t, \theta_t) + (1-\delta)k_t - g(k_t, \theta_t))[f_k(k_t, \theta_t) + 1 - \delta],$$

where g is the optimal policy function for capital stock. This special case allows us to compare the simulation from our algorithm with the example where the only source of approximation errors is the approximation of the policy function g. This will allow us to isolate the contribution of the approximation errors in evaluation of the integrals and numerical differentiation to the overall approximation error of the algorithm. As can be seen from Figure 2.4 the approximations delivered by our algorithm are very close to the approximation which rely on the Benveniste-Sheinkman theorem. Once again, in the region of the state space which is often visited by the model in equilibrium the two approximations are virtually identical. This allows us to tentatively suggest that the main contribution to the approximation error of the algorithm comes from the approximation of the policy functions.

[Insert Figure 2.4 and Figure 2.5 about here.]

Our final special case allows us to compare the approximation of the derivative with its known exact solution. It is well known that for the functional forms $f(k_t, \theta_t) = \theta_t k_t^\alpha$, $u(c_t) = \log c_t$, and $\delta = 1$, the optimal policy function is defined by the simple law of motion $k_{t+1} = \alpha\beta\theta_t k_t^\alpha$. Moreover, the derivative of the value function has the following analytical solution:

$$V_k(k_t, \theta_t) = \frac{\alpha\theta_t k_t^{\alpha-1}}{(1-\alpha\beta)\theta_t k_t^\alpha} = \frac{\alpha}{(1-\alpha\beta)}\frac{1}{k_t}.$$

Notice that by replacing the approximated policy function with the known closed form solution, we can isolate the effect of the errors stemming from Monte Carlo integration and numerical differentiation on the accuracy of the approximation. In Figure 2.5 we compare the approximated derivatives obtained using exact policy function for k_t with the graph of the analytical derivative. The reported graphs are visually indistinguishable for the whole range considered, i.e. six standard deviations of k_t around its deterministic steady state value. The approximation errors stemming from Monte Carlo integration and numerical differentiation are of order of 10^{-9} of the value of the derivative. This suggests that obtaining accurate approximation of the policy functions in the region of state space of interest is crucial for the accuracy of the whole algorithm.

2.5. Concluding Remarks

The purpose of the note was to present an algorithm for computing the partial derivatives of the optimal value function by simulation. The procedure proposed is conceptually straightforward, computationally inexpensive, and simple to implement. Yet, it is flexible enough to handle dynamic model with a large number of state variables even when derivatives of interest cannot be expressed in terms of conditional expectations and functions of equilibrium path of the model. For our benchmark examples the algorithm has a performance comparable with approximation method introduced by Marcet and Marimon (1992). Furthermore, the algorithm can be applied to some problems which fail to fall into standard recursive framework and for which Bellman equation is not well defined.

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2.6. Appendix: Derivations

2.6.1. Optimality conditions for example in section (2.3.1).

Since the constraint (2.7) involves expected values of the future decision variables, this problem is not a special case of the standard dynamic programming problems, and the Bellman equation will not be satisfied. However, as shown by Marcet and Marimon (1998) it falls into a general class of problems, which can be cast into an alternative recursive framework. The recursive saddle point problem associated with dynamic problem above will be given by

$$\max_{\{c_{it},i_{it}\}} \min_{\{\mu_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{I} \{(\lambda_i + M_{it})\, u(c_{it}, h_{it}) + \mu_{it}\, (u(c_{it}, h_{it}) - V_i^a\,(k_{it}, h_{it}, \theta_{it}))\}$$

subject to (2.4)-(2.7) and

$$M_{it+1} = M_{it} + \mu_{it}, \quad M_{i0} = 0, \tag{2.29}$$

$$\mu_{it} \geq 0.$$

Indeed, the corresponding Lagrangian is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{i=1}^{I} \lambda_i u(c_{it}, h_{it}) + \sum_{i=1}^{I} \mu_{it} \left(E_t \sum_{j=0}^{\infty} \beta^j u(c_{it+j}, h_{it+j}) - V_i^a\,(k_{it}, h_{it}, \theta_{it}) \right) \right\}$$

subject to (2.4)-(2.6), given $\mu_{it} \geq 0$, where $\beta^{-t}\mu_{it}$ is the Lagrange multiplier of (2.7) at t. The law of iterated expectations allows to imbed the conditional expectations E_t into E_0. Furthermore, reordering the terms and introducing the law of motion for M_{it} yields the above result.

As shown by Marcet and Marimon (1998), under certain assumptions the solution to the recursive saddle point problem obeys a saddle point functional equation. Within our framework their result implies that there exists a unique value function,

$$\begin{aligned} W(k,h,M,\theta) \;\; = \;\; & \min_{\{\mu\}_{i=1}^{I}} \max_{\{c,i\}_{i=1}^{I}} \left\{ \sum_{i=1}^{I} [(\lambda_i + M_i)\, u(c_i, h_i) + \mu_i\, (u(c_i, h_i) - V_i^a\,(k_i, h_i, \theta_i))] \right. \\ & \left. + \beta E\left[W(k', h', M', \theta') \mid \theta\right] \right\} \end{aligned}$$

subject to

$$\sum_{i=1}^{I} c_i + \sum_{i=1}^{I} i_i = \sum_{i=1}^{I} f(k_i, \theta_i), \tag{2.30}$$

$$k_i' = (1-\delta)k_i + i_i, \tag{2.31}$$

$$h_i' = h_i + \lambda(c_i - h_i), \tag{2.32}$$

$$M_i' = M_i + \mu_i, \tag{2.33}$$

$$c_i, i_i, \mu_i \geq 0, \tag{2.34}$$

for all (k, h, M, θ) and such that $W(k_0, h_0, M_0, \theta_0)$ is the value of the optimization problem in question. The policy correspondence associated with the above saddle point functional equation is given by

$$\psi(k,h,M,\theta) \in \arg \min_{\{\mu\}_{i=1}^{I}} \max_{\{c,i\}_{i=1}^{I}} \Bigg\{ \sum_{i=1}^{I} \left[(\lambda_i + M_i)\, u(c_i,h_i) + \mu_i \left(u(c_i,h_i) - V_i^a(k_i,h_i,\theta_i) \right) \right] + \beta E\left[W(k',h',M',\theta') \mid \theta \right] \Bigg\}$$

subject to (2.30) - (2.34).

The key results demonstrated by Marcet and Marimon (1998) ensures that the optimal solution to the optimization problem we consider satisfies $(c_t, i_t, \mu_t) = \psi(k_t, h_t, M_t, \theta_t)$ for all t with the initial conditions $(k_0, h_0, 0, \theta_0)$. That is there exist a time invariant policy correspondence ψ such that only the values of a small number of past variables $(k_t, h_t, M_t, \theta_t)$ matter. Hence, the problem is now in a recursive framework the solution to which can now be obtained from studying the saddle point functional equation.

Denoting by m_{it}, n_{it} and γ_t the Lagrange multipliers of the constraints (2.5), (2.6) and (2.4), the first order conditions for this problem become:

$$(\lambda_i + M_{it} + \mu_{it}) \frac{\partial u(c_{it}, h_{it})}{\partial c_{it}} + \lambda n_{it} - \gamma_t = 0$$

$$m_{it} - \gamma_t = 0$$

$$\beta E_t \left[-\mu_{it+1} \frac{\partial V_i^a(k_{it+1}, h_{it+1}, \theta_{it+1})}{\partial k_{it+1}} + m_{it+1}(1-\delta) + \gamma_{t+1} \frac{\partial f(k_{it+1}, \theta_{it+1})}{\partial k_{it+1}} \right] = m_{it}$$

$$\beta E_t\left[\left(\lambda_i+M_{it+1}+\mu_{it+1}\right)\frac{\partial u(c_{it+1},h_{it+1})}{\partial h_{it+1}}-\mu_{it+1}\frac{\partial V_i^a\left(k_{it+1},h_{it+1},\theta_{it+1}\right)}{\partial h_{it+1}}+\left(1-\lambda\right)n_{it+1}\right]=n_{it}$$

$$\mu_{it}\left[E_t\sum_{j=0}^{\infty}\beta^j u(c_{it+j},h_{it+j})-V_i^a\left(k_{it},h_{it},\theta_{it}\right)\right]=0$$

$$E_t\sum_{j=0}^{\infty}\beta^j u(c_{it+j},h_{it+j})\geq V_i^a\left(k_{it},h_{it},\theta_{it}\right)$$

in addition to the aggregate resource constraint (2.4), the laws of motion (2.5)-(2.6) for state variables, the laws of motion (2.29) for the co-state variables M_{it}, and non-negativity of the Lagrange multipliers $\mu_{it}\geq 0$.

$$\left(\lambda_i+M_{it}+\mu_{it}\right)\frac{\partial u(c_{it},h_{it})}{\partial c_{it}}+\lambda n_{it}=\left(\lambda_j+M_{jt}+\mu_{jt}\right)\frac{\partial u(c_{jt},h_{jt})}{\partial c_{jt}}+\lambda n_{jt},\ \text{ for } i,j=1,...,I,$$

$$m_{it}=\beta E_t\left[m_{it+1}\left(\frac{\partial f\left(k_{it+1},\theta_{it+1}\right)}{\partial k_{it+1}}+1-\delta\right)-\mu_{it+1}\frac{\partial V_i^a\left(k_{it+1},h_{it+1},\theta_{it+1}\right)}{\partial k_{it+1}}\right],$$

$$n_{it}=\beta\left(1-\lambda\right)E_t\left[n_{it+1}\right]+\beta E_t\left[\left(\lambda_i+M_{it+1}+\mu_{it+1}\right)\frac{\partial u(c_{it+1},h_{it+1})}{\partial h_{it+1}}-\mu_{it+1}\frac{\partial V_i^a\left(k_{it+1},h_{it+1},\theta_{it+1}\right)}{\partial h_{it+1}}\right].$$

Using recursive substitution and the law of iterated projections yields

$$n_{it}=\beta E_t\sum_{j=0}^{\infty}\beta^j\left(1-\lambda\right)^j\left[\xi_{it+1+j}\frac{\partial u(c_{it+1+j},h_{it+1+j})}{\partial h_{it+1+j}}-\mu_{it+1+j}\frac{\partial V_i^a\left(k_{it+1+j},h_{it+1+j},\theta_{it+1+j}\right)}{\partial h_{it+1+j}}\right],$$

where the time varying planner weights are given by

$$\xi_{it}=\lambda_i+M_{it}+\mu_{it}.$$

Re-organizing the terms yields the following optimality conditions:

$$\frac{u_c\left(i,t\right)+\lambda\beta E_t\sum_{j=0}^{\infty}\beta^j\left(1-\lambda\right)^j\left[\frac{\xi_{it+1+j}}{\xi_{it}}u_h\left(i,t{+}1{+}j\right)-\frac{\mu_{it+1+j}}{\xi_{it}}\frac{\partial V_i^a}{\partial h_{it+1+j}}\left(i,t{+}1{+}j\right)\right]}{u_c\left(s,t\right)+\lambda\beta E_t\sum_{j=0}^{\infty}\beta^j\left(1-\lambda\right)^j\left[\frac{\xi_{st+1+j}}{\xi_{st}}u_h\left(s,t{+}1{+}j\right)-\frac{\mu_{st+1+j}}{\xi_{st}}\frac{\partial V_s^a}{\partial h_{st+1+j}}\left(s,t{+}1{+}j\right)\right]}=\frac{\xi_{st}}{\xi_{it}},$$

for $i,s=1,...,I$.

Optimal allocation must also satisfy the following intertemporal condition:

$$
\begin{aligned}
& u_c(i,t) + \lambda\beta E_t \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j \left[\frac{\xi_{it+1+j}}{\xi_{it}} u_h(i, t{+}1{+}j) - \frac{\mu_{it+1+j}}{\xi_{it}} \frac{\partial V_i^a}{\partial h_{it+1+j}} (i, t{+}1{+}j) \right] \\
= \; & \beta E_t \left[\left(\frac{\xi_{it+1+j}}{\xi_{it}} u_c(i, t+1) + \lambda\beta \sum_{j=0}^{\infty} \beta^j (1-\lambda)^j \left[\frac{\xi_{it+2+j}}{\xi_{it}} u_h(i, t{+}2{+}j) \right.\right.\right. \\
& \left.\left.\left. - \frac{\mu_{it+2+j}}{\xi_{it}} \frac{\partial V_i^a}{\partial h_{it+2+j}} (i, t{+}2{+}j) \right] \right) (f_k(k_{it+1}, \theta_{it+1}) + 1 - \delta) - \frac{\mu_{it+1}}{\xi_{it}} \frac{\partial V_i^a}{\partial k_{it+1}} (i, t+1) \right].
\end{aligned}
$$

In these first order conditions we have used the abbreviations $u_c(i,t)$ for $\frac{\partial u(c_{it}, h_{it})}{\partial c_{it}}$, and we have used similar abbreviations for other terms.

2.6.2. Optimality conditions for stochastic growth model with habits.

Consider the following dynamic optimization problem corresponding to the stochastic growth model with additive habit formation preferences:

$$\max_{\{c_t, i_t\}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

subject to

$$c_t + i_t = f(k_t, \theta_t), \tag{2.35}$$

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{2.36}$$

$$h_{t+1} = h_t + \lambda(c_t - h_t), \tag{2.37}$$

with the initial conditions k_0, h_0 given.

Using the arguments of standard dynamic programming (see Stokey, Lucas, and Prescott (1989)) one can show the existence of the time invariant policy functions $c(k,h,\theta), i(k,h,\theta)$ and a value function $V(k,h,\theta)$. The functional equation for the problem is given by

$$V(k,h,\theta) = \max_{(c,i)\in A(k)} \{u(c,h) + \beta E[V(k',h',\theta') \mid (k,h,\theta)]\}$$

$$h' = h + \lambda(c-h),$$

$$k' = f(k,\theta) + (1-\delta)k + i,$$

$$A(k,\theta) = \left\{(c,i) \in \mathbb{R}^2_+ : c + i = f(k,\theta)\right\},$$

The first order condition of the maximization problem in the right hand side of the functional equation is given by

$$u_c(c,h) = \beta E[V_k(k',h',\theta') - \lambda V_h(k',h',\theta') \mid (k,h,\theta)]. \tag{2.38}$$

Let $g(k,h,\theta)$ be the optimal policy function for investment. Then, the following identity must hold:

$$\begin{aligned} V(k,h,\theta) &= u(f(k,\theta) - g(k,h,\theta), h) \\ &\quad + \beta E[V((1-\delta)k + g(k,h,\theta), (1-\lambda)h + \lambda(f(k,\theta) - g(k,h,\theta)), \theta') \mid (k,h,\theta)]. \end{aligned}$$

Differentiating both sides of the equality above with respect to k yields

$$\begin{aligned} V_k(k,h,\theta) &= u_c(c,h)\left[f_k(k,\theta) - g_k(k,h,\theta)\right] \\ &\quad + \beta E\left[V_k(k',h',\theta')\left[(1-\delta) + g_k(k,h,\theta)\right]\right. \\ &\quad \left. + \lambda V_h(k',h',\theta')\left[f_k(k,\theta) - g_k(k,h,\theta)\right] \mid (k,h,\theta)\right], \end{aligned}$$

which using (2.38) reduces to

$$V_k(k,h,\theta) = \beta E\left[V_k(k',h',\theta')\left[f_k(k,\theta) + 1 - \delta\right] \mid (k,h,\theta)\right].$$

Differentiating both sides of the identity above with respect to h yields

$$\begin{aligned} V_h(k,h,\theta) &= u_h(c,h) - u_c(c,h)g_h(k,h,\theta) + \beta E\left[V_k(k',h',\theta')g_k(k,h,\theta)\right. \\ &\quad \left. + V_h(k',h',\theta')\left[(1-\lambda) - \lambda g_h(k,h,\theta)\right] \mid (k,h,\theta)\right], \end{aligned}$$

which using (2.38) reduces to

$$V_h(k,h,\theta) = u_h(c,h) + \beta(1-\lambda)E\left[V_h(k',h',\theta') \mid (k,h,\theta)\right].$$

The optimality conditions can be written in the sequence form in the following way:

$$u_c(c,h) + \beta\lambda E_t\left[V_h(k_{t+1},h_{t+1},\theta_{t+1})\right] = \beta E_t\left[V_k(k_{t+1},h_{t+1},\theta_{t+1})\right], \tag{2.39}$$

$$V_k(k_t,h_t,\theta_t) = \beta E_t\left[V_k(k_{t+1},h_{t+1},\theta_{t+1})\right]\left(f_k(k_t,\theta_t) + 1 - \delta\right), \tag{2.40}$$

$$V_h(k_t,h_t,\theta_t) = u_h(c_t,h_t) + \beta(1-\lambda)E_t\left[V_h(k_{t+1},h_{t+1},\theta_{t+1})\right]. \tag{2.41}$$

Combining (2.39) and (2.40) yields the following expression for derivative of the value function w.r.t. the capital stock:

$$V_k(k_t,h_t,\theta_t) = \left(u_c(c_t,h_t) + \beta\lambda E_t\left[V_h(k_{t+1},h_{t+1},\theta_{t+1})\right]\right)\left(f_k(k_t,\theta_t) + 1 - \delta\right)$$

Shifting the expression above one period ahead and substituting in into (2.40) gives

$$\begin{aligned} u_c(c,h) + \beta\lambda E_t\left[V_h(k_{t+1},h_{t+1},\theta_{t+1})\right] &= \beta E_t\left[\left(f_k(k_{t+1},\theta_{t+1}) + 1 - \delta\right)\right. \\ &\quad \left. \times\left(u_c(c_{t+1},h_{t+1}) + \beta\lambda V_h(k_{t+2},h_{t+2},\theta_{t+2})\right)\right], \end{aligned}$$

Using recursive substitution and the law of iterated expectations (2.41) provides an expression for the derivative of the value function w.r.t. the habit stock:

$$V_h(k_t, h_t, \theta_t) = u_h\left(c_t, h_t\right) + E_t\left[\sum_{j=1}^{\infty}\beta^j(1-\lambda)^j u_h\left(c_{t+j}, h_{t+j}\right)\right].$$

Finally, the Euler equation for the problem becomes:

$$u_c\left(c, h\right) + \beta\lambda E_t\left[\sum_{j=0}^{\infty}\beta^j(1-\lambda)^j u_h\left(c_{t+j+1}, h_{t+j+1}\right)\right] = \beta E_t\Bigg[\left(f_k(k_{t+1}, \theta_{t+1}) + 1 - \delta\right)$$

$$\times\left(u_c\left(c_{t+1}, h_{t+1}\right) + \beta\lambda\sum_{j=0}^{\infty}\beta^j(1-\lambda)^j u_h\left(c_{t+j+2}, h_{t+j+2}\right)\right)\Bigg].$$

2.7. Figures and Tables

Figure 2.1. Stochastic growth model with habits: a numerical solution with PEA

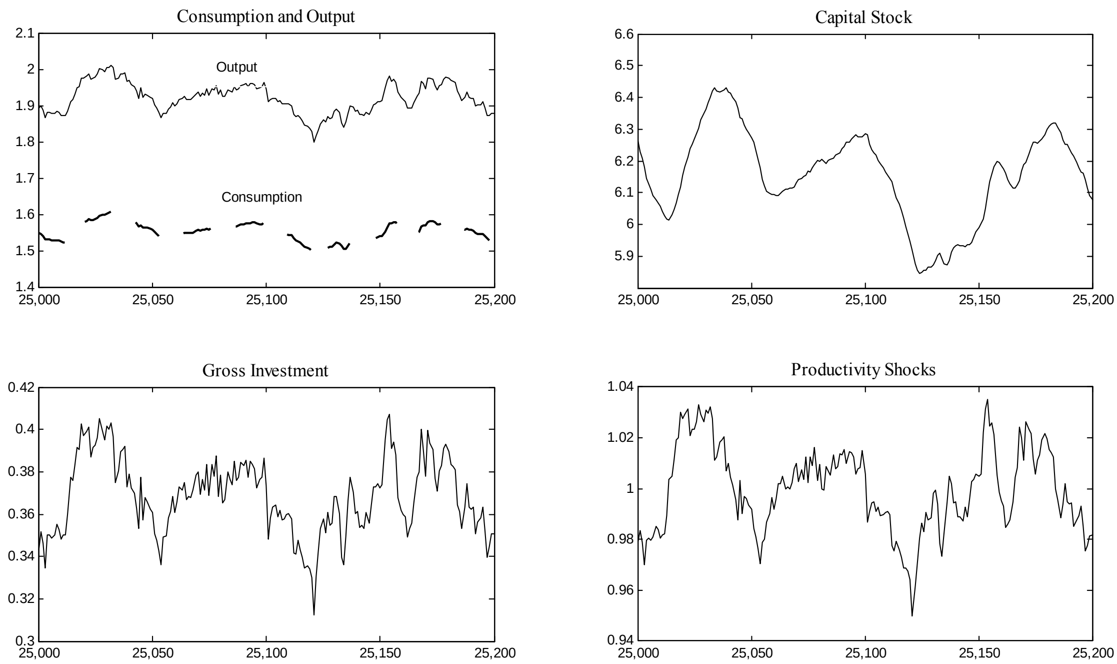

Figure 2.2. Growth model with habits: approximation of the value function and its partial derivatives

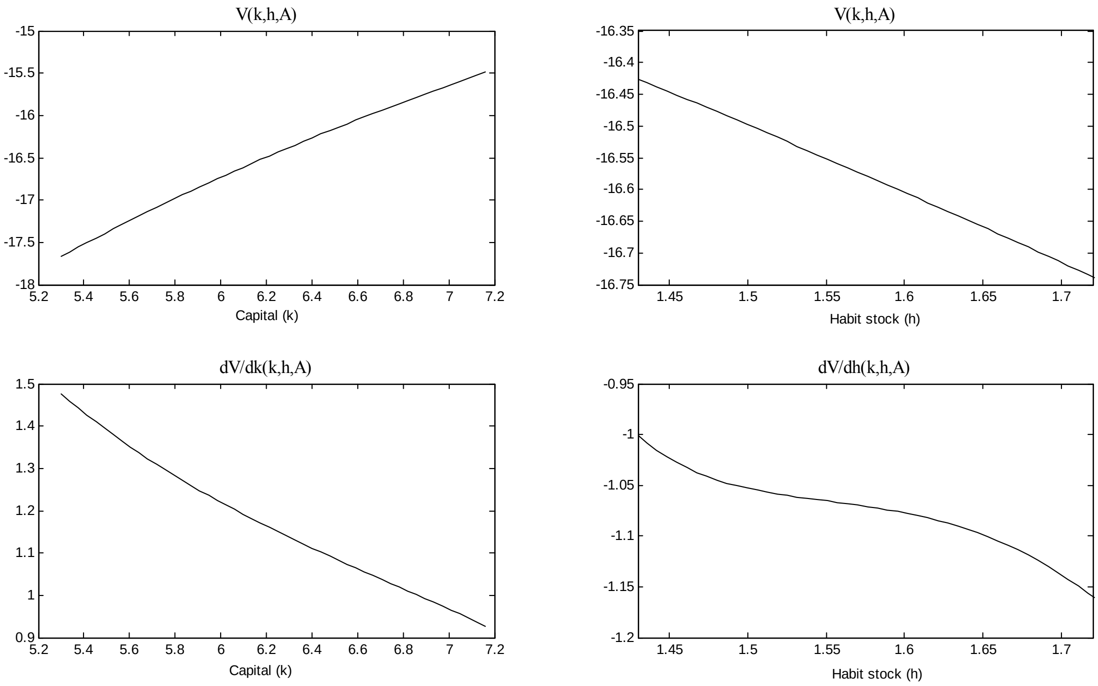

Figure 2.3. Alternative methods for the approximation of the derivatives

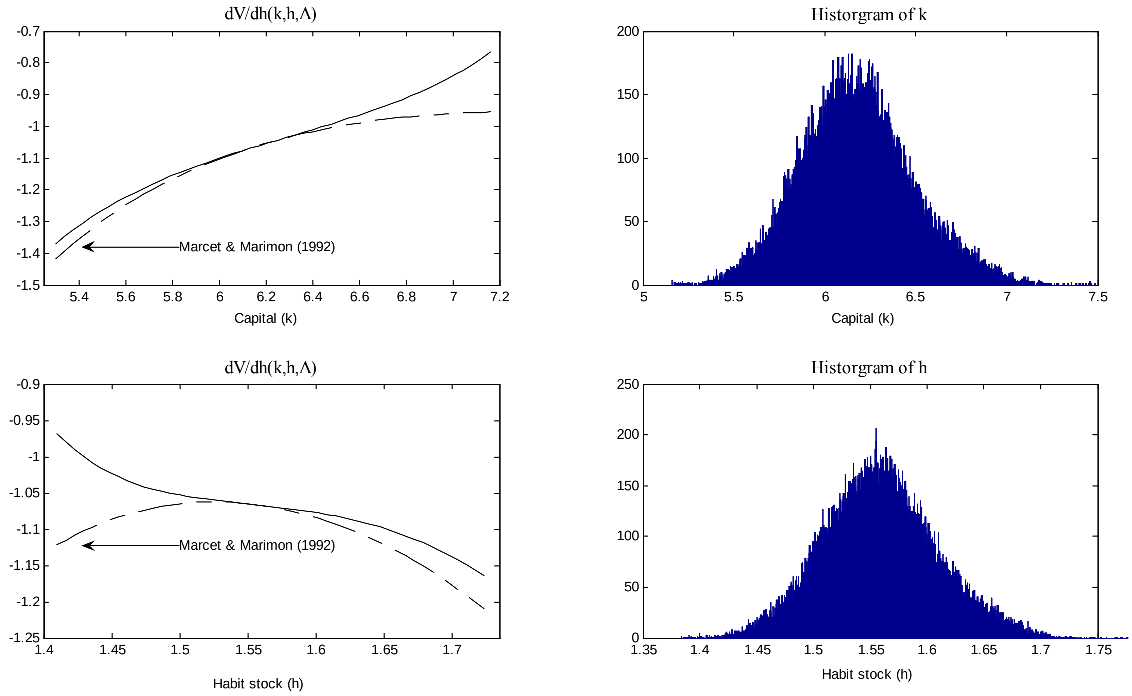

Figure 2.4. Stochastic growth model with separable preferences: simulations

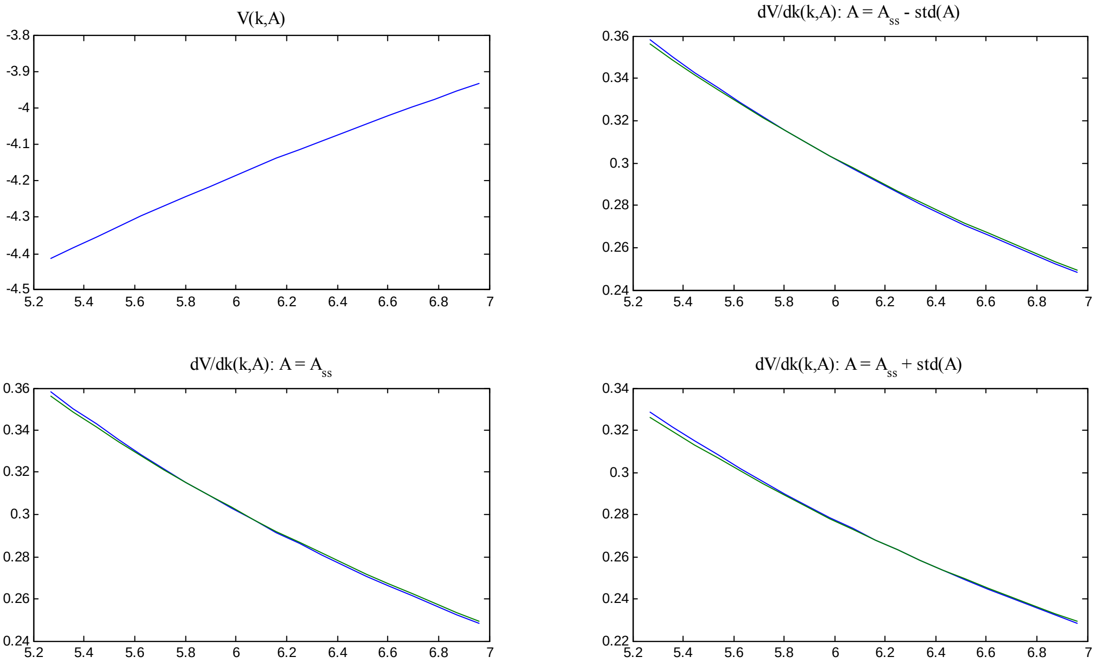

Figure 2.5. Comparison of numerical and analytical solutions

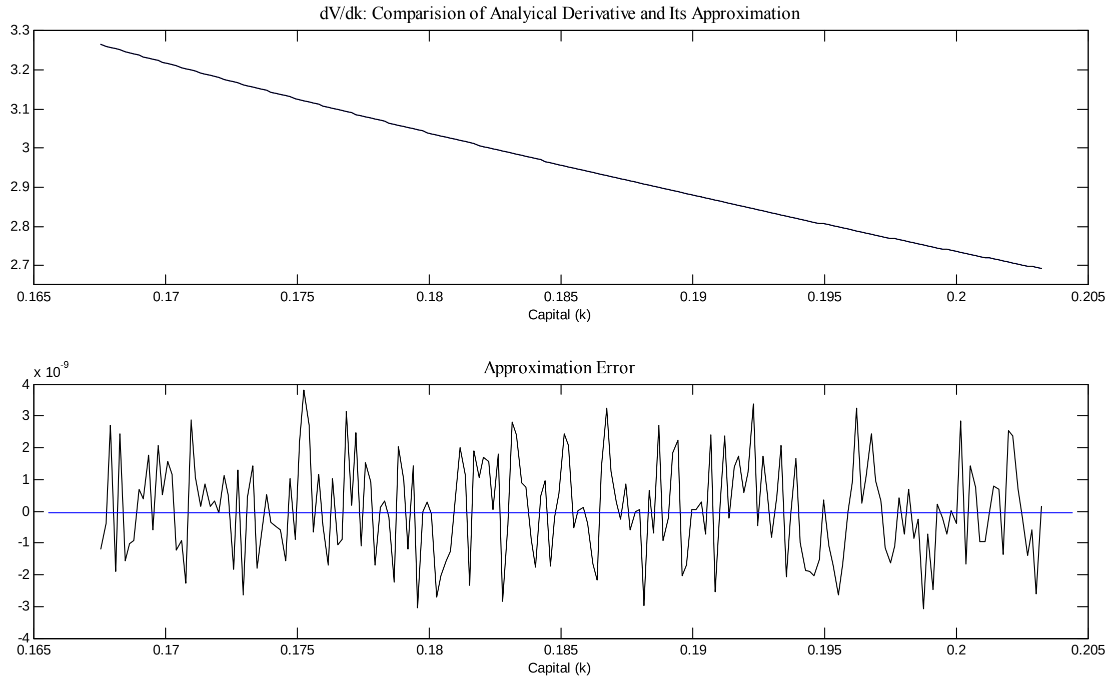

Table 2.1. Parameterization of the model

σ_ε	ρ	σ	β	δ	α	b
0.007	0.95	3	0.95	0.06	0.36	0.5

CHAPTER 3

Institutions and Growth: Some Evidence from Estimation Methods Partially Robust to Weak Instruments

3.1. Introduction

Explaining the vast disparities in average income between the richest and the poorest countries has always been one of the biggest challenges in economics. In recent years, the search for explanations has gone beyond economic variables to investigate "deeper" determinants of economic performance such as geography, integration and institutions[1]. Designing an empirical strategy to access the importance of these factors in explaining variation in income levels is a formidable task. The challenge lies in disentangling the complex web of causality involving these deep determinants and the income levels. This paper will narrow the focus on the view which asserts the primacy of institutions in particular the property rights and the rule of law in explaining the observed income differences.

The most influential contributions in this strand of literature have been the pioneering work of Robert Hall and Charles Jones (1999) as well as the study of Daron Acemoglu, Simon Johnson and James Robinson (2001). Both studies proposed empirical strategies and introduced new sets of instruments which allowed to shed some light on the causal effect of institutions in explaining income differences. Their approaches and instruments in particular have been both extensively utilized as well as criticized in the literature.

Both approaches essentially share a common weakness which is anything but straightforward to overcome. To find a source of exogenous variation in institutions which would not have direct effect on current output levels the scholars had to go to the geographical and historical determinants of institutions. For instance, Hall and Jones (1999) rely on the distance from the equator while Acemoglu, Johnson, and Robinson (2001) utilize historical settler mortality to instrument for the institutional quality. Due to this it is natural to expect

[1]We hereby rely on the taxonomy of deep determinants proposed by Rodrik and Subramanian (2003).

that the instruments proposed would be only weakly correlated with the endogenous variable of interest. The latter, however, often constitutes a source of severe problems for both estimation and inference purposes.[2] Even though, the latter point is strongly emphasized in the recent literature on weak identification in linear instrumental variables (IV) models it is often disregarded in many empirical applications[3]. Hence, one of the objectives of this study is to serve as an example which brings together the recent advances in the econometrics of weak instruments with the empirical methodology of the literature on deep determinants of economic growth.

This paper is concerned with the performance of the estimators partially robust to the presence of weak instruments in application to the Hall and Jones (1999) study (henceforth HJ99). HJ99 investigate whether differences in social infrastructure can explain observed cross-country variation of income per capita. To address the issues of endogeneity and measurement error in their proxy for institutional quality HJ99 estimate the effect of social infrastructure on per capita income using two-stage least squares (TSLS). The instruments of their choice reflect the extent of Western European influence which is suggested to be one of the forces behind the adoption of favorable infrastructure.

Two possible lines of criticism can be directed towards the HJ99 study. First, Acemoglu et al (2001) express their criticism towards the empirical approach of HJ99 for reliance on the two instruments with questionable relation to their theory, namely distance from the equator and Frankel-Romer predicted trade share[4]. Second, the instruments proposed by HJ99 and in particular linguistic instruments are found to be only weakly correlated with their proxy for institutional quality. The basic diagnostic tools proposed by Stock, Wright, and Yogo (2002) indicate that the weak instruments are potentially a problem in which case the performance of TSLS can be inferior. This study is an attempt to address both lines of criticism.

One of the objectives of the present study is address the Acemoglu et al (2001) critique and to assess whether the HJ99 results are driven by the use of "latitude" and Frankel-Romer predicted trade share as instruments for institutional quality. In order to address the issue

[2]See for example Hahn and Hausmann (2003).

[3]Two notable exceptions are the recent studies by Alcalá and Ciccone (2004) and Dollar and Kraay (2003) which try to identify partial effects of institutions and integration on income levels.

[4]Acemoglu et al (2001, p. 1373) express certain skepticism about HJ99 using "latitude" as an instrument for institutions noting that "...the theoretical reasoning behind these instruments is not entirely convincing". Some scholars e.g. Rodrik, Subramanian, and Trebbi (2004) go on to claim that the instruments proposed by Acemoglu et al (2001) are preferable to those of HJ99.

six different specifications are considered some of which utilize exclusively linguistic variables as instruments. However, in order to accomplish this task the problem of potentially weak identification is to be overcome. In this respect the question to be answered is whether one can rely on TSLS for both estimation and inference purposes given the potential problem of weak instruments. Furthermore, the question that arises is whether the performance of TSLS can be improved upon by using the estimators partially robust to weak instruments. Finally, given that the relative performance of the partially robust estimators heavily depends on characteristics of the model and data at hand, the question to be considered is which of the estimators one might rely on in the context of linear IV model of HJ99.

Furthermore, we seek to address the issue of whether linguistic variables proposed by HJ99 form valid and relevant instruments for their proxy of institutional quality. That is whether the instruments sufficiently identify the model and whether orthogonality condition is satisfied given the potential problem of weak instruments. This issue is of particularly importance for isolating partial effects of institutions and integration[5].

The relevance of HJ99 instruments from the econometric standpoint is addressed by evaluating performance of the estimators partially robust under weak instruments in a Monte Carlo experiment. Following the suggestions of Blomquist and Dahlberg (1999) and Stock, Wright, and Yogo (2002) the relative performance of k-class estimators and Jackknife estimators is examined under a data generating process which is believed to be applicable to the Hall and Jones (1999) study. To address the mentioned criticism of Acemoglu et al (2001) the linear IV model of HJ99 is reestimated across several specifications and the evidence from the partially robust estimators is interpreted on the basis of the Monte Carlo results.

3.2. Some Comments on the Hall and Jones (1999) Study

HJ99 estimate the effect of what they call "social infrastructure" on productivity across countries. They address the issue of reverse causality by using the fraction of population speaking English at birth, the fraction of population speaking one of the five major European languages at birth, the distance from the equator and the Frankel and Romer (1999) geography predicted trade intensity as instruments. The first three instruments are claimed

[5]As demonstrated by Dollar and Kraay (2003) the existing attempts to identify partial effects of institutions and trade using geography related instruments as "latitude" or settler mortality suffer from serious identification problems.

to reflect the extent of past Western European influence which is suggested to be one of the forces behind the adoption of favorable infrastructure.

Acemoglu et al (2001) have expressed certain dissatisfaction with the HJ99 instruments. One reason for that is that the overall results might heavily depend on the reliance on the "latitude" and Frankel-Romer predicted trade share as instruments. HJ99 do not provide any theoretical justification for the use of the Frankel-Romer predicted trade intensity. As for the "latitude", Hall and Jones (1999, p.101) argue that Europeans were "...more likely to settle in areas that were broadly similar in climate to West Europe". One possible problem with this argument is that climatic zones differ substantially across the regions with identical distance from the equator. Even though Acemoglu et al (2001) show that "latitude" has no independent effect on economic performance and hence forms a valid instrument, its relation to the HJ99 theory is still questionable. This partially constitutes a rationale to put more weight on the linguistic instruments[6].

The HJ99 theory stipulates that the extent to which primary languages of Western Europe are spoken as first language today should reflect the extent of positive Western European influence during 16-19th centuries. Some objections to this argument can be articulated as well. First, as noted by Acemoglu et al (2001) the 'Western influence' was not necessary positive in terms of institutional quality. An illustrative example they present is Belgian influence in the Congo and Western influence in the Gold coast. Second, timing could be an issue of concern. Indeed, suppose that the extent to which a European language is spoken in a certain country in the 18th century reflects the degree of European influence in the 18th century. The extent to which this language is spoken today does not have to be the same as it was a few centuries ago. This issue, however, might somewhat lose its importance in view of the Acemoglu et al (2001) theory. They argue that colonization often took form of either establishing "extractive posts" or creating "settler colonies". In the former case, colonizers exerted "negative" influence in terms of institutions and were likely to leave the colony. In the latter case, the colonizers had intentions to stay and hence established institutions resembling those of Western Europe. In light of this, one would expect smaller proportion of the population of former "extractive posts" speak a European language long after the colonizers are gone. On the contrary, in the former "settler colonies" like e.g. Neo Europes in which

[6]The linguistics instruments are of particular interest for identification purposes in bivariate specifications attempting to identify partial effects of e.g. institutions and trade. See Alcalá and Ciccone (2004), Dollar and Kraay (2003).

"life was modeled after home country" (Acemoglu et al 2001, p. 1376) one would expect large fraction of the inhabitants to speak the colonizer´s language today. Hence, relying on this argument one would expect an association between fraction of European languages spoken today with the "positive" European influence during colonization time. The latter is in line with the HJ99 hypothesis.

To assess whether the HJ99 results are driven by the two instruments with somewhat doubtful relation to their theory six specifications will be considered in this paper. The first three specifications include the full 127 country sample. The first one uses all four original instruments, namely the languages, the latitude, and Frankel-Romer predicted trade share. The second specification relies exclusively on the "latitude" and the trade share, while the last one utilizes only the language characteristics. The specifications iv) through vi) are identical to the first three except they rely on the 79 country sample with no imputed data.

Another potential problem in the HJ99 study is that the instruments might be only weakly correlated with the endogenous regressor. In the latter case the TSLS estimator will have non-normal sampling distribution. Whether the instruments should be deemed as weak or not depends on the purpose to which the instruments serve. The usual judgment criteria are the size distortion and the relative bias (Stock, Wright, and Yogo 2002).

Stock, Wright, and Yogo (2002) emphasize the point that the least applied researches should do is to apply the basic tools for detection of weak instruments as e.g. first-stage F-statistics. Following this suggestion we report the values of the first-stage F-statistics for the six basic specifications in Table 3.1. Based on Table 3.1 two observations can be made. First, all specifications free of imputed data exhibit low F-statistics[7].

[Insert Table 3.1 about here.]

The same can be said about both specifications which rely on language variables as instruments. Second, none of the reported F-statistics is far in excess of ten. These observations suggest that weak instruments are potentially a problem. Before proceeding to the next section a note should be made on the role of F-statistics as a diagnostic tool. The literature on weak instruments suggests that one should interpret the first stage F-statistics with certain care. As noted by Stock and Yogo (2001) the Staiger-Stock rule of thumb happens to be too conservative if limited information maximum likelihood of Fuller-k estimators are used but not conservative enough to ensure that TSLS Wald test does not suffer from size distortions.

[7] The rule of thumb proposed by Stock,Write and Yogo (2002) is that one should be concerned with the issue of weak instruments should the value of the first stage F-statistic fall short of ten.

In this respect Blomquist and Dahlberg (1999) express certain skepticism concerning excessive reliance on the F-statistics as a diagnostic tool. They state that "...it has become somewhat of a folklore that if the first stage F - statistics is large, the TSLS performs well. However the folklore is not correct". Their claim is supported by the simulations whereby average F-statistics of 29 with the sample size of 2408 does not preclude TSLS from having the average bias of -51.6%. In line with this argument Stock et al (2002) suggest using k-class or Jackknife estimators even if F-statistics are in excess of ten.

On the other hand, Staiger and Stock (1997) show that even if F-statistics are low (less that 5) the instruments do not have to be irrelevant. To overcome the potential problem they go on to suggest using alternative estimators more robust to the presence of weak instruments as for instance k-class estimators.

3.3. Monte-Carlo Simulations

As shown by Blomquist and Dahlberg (1999) in the Monte-Carlo experiments the ranking of the partially robust estimators heavily depends on the nature of the data generating process (DGP). Hence, they advocate complementing the estimates with a Monte-Carlo study for the relevant sample size and DGP believed to be applicable. Following this suggestion the relative performance of the five k-class estimators and two Jackknife estimators will be investigated in this section in the context of the linear IV model used by HJ99.

3.3.1. DGP and evaluation criteria

The linear IV regression model estimated by HJ99 is given by

$$\begin{aligned} log(Y/L) &= \alpha + \beta S + \varepsilon \\ S &= Z\delta + u \end{aligned}$$

where $log(Y/L)$ is an $N \times 1$ vector of log income per capita, Z is an $N \times K_2$ matrix of instruments, S is an $N \times 1$ vector of proxy for social infrastructure, α is a scalar.

The DGP is designed so that the generated data will replicate certain features of the actual observations. The dependent variable is generated as

$$y = a + bS + e$$

where $a = 7$, $b = 3$, and S is the proxy for social infrastructure from HJ99 study. Following the idea of Friedman (1984) the error term is generated by $e = k\widehat{u} + \xi$ where $\xi \sim N(0, \sigma_\xi^2)$ and $\widehat{u}$ is a projection of S to the space orthogonal to the space spanned by the instruments, i.e. $\widehat{u} = (I - Z(Z'Z)^{-1}Z')S$. The values of the parameters $\sigma_\xi^2 = 0.1$, and $k = 0.8$ are set so that first two empirical moments of generated y would be close (with 10% in absolute terms) to those of observed $log(Y/L)$ for a chosen extent of correlation between regressor and error.

The advantage of the proposed DGP is that the generated data resemble the original sample in terms of the sample size, the first two empirical moments and more importantly the extent of correlation of the instruments with the endogenous regressor. In fact, in each replication of the experiment the first-stage F-statistics are identical to those reported in Table 3.1.

The disadvantages of this formulation include rather restrictive assumptions of fixed instruments and Gaussian errors. The measurement error is not modelled hereby due it its unknown form. Hence, we implicitly assume that the measurement error will not change the ranking of the partially robust estimators. Finally, for the purpose of this exercise it is assumed that the instruments are orthogonal to the error term. The orthogonality assumption can be tested using e.g. the asymptotic methods suggested by Staiger and Stock (1997) or sample re-use methods. This strategy will be followed in the section 3.5.1 reexamining HJ99 results.

A brief note should be made concerning the choice of the measures to compare the estimators. Some of the estimators considered in this study (e.g. JIVE or UJIVE) do not have first or second finite sample moments. Due to this fact, Angrist, Imbens, and Krueger (1999) advocate use of median estimates and median absolute errors to evaluate the performance of the estimators. On the other hand, as pointed by Jerry A. Hausman and Kuersteiner (2001) the absence of finite sample moment is an issue of practical concern. As Jerry A. Hausman and Kuersteiner (2001) have show in the Monte-Carlo experiments the "moments problem" may cause certain estimators to have extremely high mean estimates or RMSE. On this ground, they advocate the use of mean estimates and RMSE to evaluate finite sample performance of the estimators.

Following Angrist, Imbens, and Krueger (1999) and Blomquist and Dahlberg (1999) the performance of the estimators in this study will be evaluated on the basis of five statistics. These are the percentage bias, the root mean square error (RMSE), the median absolute error and the quantiles around the true parameter value. Furthermore, we report the coverage rates which are computed as a fraction of replications when the calculated confidence interval covers the true parameter value. The confidence intervals are estimated using the following Bootstrap procedure with 1000 replications. First, for a sample size N we draw uniformly with replacement N observations. Second, we use instrumental variables on the generated data to get a new estimate $\widetilde{\beta}$. The bootstraped 95% confidence interval is calculated as $\left[\widetilde{\beta}_{0.025}, \widetilde{\beta}_{0.975}\right]$ where $\widetilde{\beta}_{0.025}$, and $\widetilde{\beta}_{0.975}$ are the $2,5\%$ and $97,5\%$ percentiles of the obtained sampling distribution.

3.3.2. Estimators

This study investigates the performance of the five k-class estimators and two Jackknife estimators in a linear IV model with potentially weak instruments. The Jackknife Instrumental Variable Estimator (JIVE) originally proposed by Angrist, Imbens, and Krueger (1995) is computed using the following algorithm:

(1) Using all the observations except for the i-th estimate the parameters of the first-stage regression: $S_{-i} = Z_{-i}\delta + u$.

(2) Use the estimated parameters $\widehat{\delta}$ with the instruments for the i-th observation Z_i to construct the fitted value for the i-th observation, $\widehat{S}_i$

(3) Repeat the steps 1) and 2) for all observations $i = 1...N$.

(4) Regress the dependent variable on the fitted values from step 1)-3) and exogenous regressors: $log(Y/L) = \alpha + \beta\widehat{S} + \widetilde{\varepsilon}$.

Hence, defining $\widehat{X}$ as a $N \times 2$ matrix with its i-th row given by $(1\ \widehat{S}_i)$ the JIVE estimator becomes

$$(\alpha \quad \beta)'_{jive} = \left(\widehat{X}\,'\widehat{X}\,\right)^{-1}\widehat{X}\,'log(Y/L).$$

The Unbiased Jackknife Instrumental Variable Estimator (UJIVE) suggested by Angrist, Imbens, and Krueger (1995) and will be defined as

$$(\alpha \quad \beta)'_{ujive} = \left(\widehat{X}\,'X\right)^{-1}\widehat{X}\,'log(Y/L),$$

where X as a $N \times 2$ matrix with its i-th row given by $(1\ S_i)$. That is the UJIVE estimator differs from JIVE in that instead of OLS at the second stage the IV is performed with $\widehat{X}$ used as instruments.

Following Stock and Yogo (2002, p. 7) this paper considers five k-class estimators: OLS, TSLS, the limited information maximum likelihood estimator (LIML), an estimator of the family the Fuller -k estimators, and bias-adjusted TSLS. Generally, this study relies on the definition of these estimators as given by Stock and Yogo (2002, p. 7)[8].

3.3.3. Simulation results

The simulations have been performed for the six chosen specifications.[9] The results are reported in Figure 3.1 as well as Tables 3.2 and 3.3. They can be summarized as follows.

First, the performance of TSLS estimator does not seem to be severely affected by the weak correlation of the instruments with the endogenous regressor. The results are robust across specifications.

Second, the Jackknife estimators are outranked by the k-class estimators across all specifications. In particular JIVE suffers from both severe average bias and size distortion. For instance, in the specification iv) which relies on 79 obs. sample and four instruments JIVE demonstrates a negative average bias of 24.8% while the calculated 95% confidence interval covers the true parameter only in 67% of cases.

Third, shifting from the four instruments to the linguistic instruments only makes the estimates much more imprecise. The RMSE almost doubles for the $k-$class estimators. The latter holds for both samples.

Finally, in the specification vi) the performance of OLS, LIML and Jackknife estimators is inferior both in terms of bias, RMSE, and median absolute error. Fuller - k estimator outranks the remaining estimators in terms of median square error, RMSE as well as the size distortion. Hence, Fuller - k estimator will be recommended for inference purposes. Furthermore, the performance of the latter remains stable across the specifications.

[Insert Figure 3.1, Table 3.2, and Table 3.3 about here.]

These results parallel the findings of Jerry A. Hausman and Kuersteiner (2001) who report reduction of bias and MSE using Fuller - k estimator relative to TSLS and LIML when the instruments are weak. Furthermore, using Monte-Carlo design of Hahn and Hausman (2002) the study of Hahn, Hausman, and Kuersteiner (2004) reached the conclusion that TSLS, jackknife TSLS (UJIVE) and Fuller estimators often perform better than for instance

[8] In the notation of Stock and Yogo (2002, p. 7) the Fuller - k estimator examined here corresponds to $c = 1$ which is the best unbiased estimator of second order among estimators with $k = 1 + a(\widehat{k}_{LIML}-1)-c/(T-K_1-K_2)$ for some constants a and c. Furthermore, for computational purposes we calculate $\widehat{k}_{LIML}$ as the minimum eigenvalue of $\left(\underline{Y}'M_{\underline{Z}}\underline{Y}\right)^{-1/2}\left(\underline{Y}'M_X\underline{Y}\right)\left(\underline{Y}'M_{\underline{Z}}\underline{Y}\right)^{-1/2}$.

[9] All the simulations and estimations in this paper were performed using MATLAB 6.5. The MATLAB code is available upon request.

JIVE or LIML. A potential explanation for this involves a notion of the "moments problem". Even though it has been long recognized that some partially robust estimators like LIML or JIVE do not have finite sample moments, it has been recently shown by Hahn, Hausman, and Kuersteiner (2004) that this feature can create problems in weak instrument situations. While both TSLS and Fuller estimators do not suffer from "moments problem", Jackknife TSLS (or UJIVE) has finite sample moments only up to the degree of overidentification. As it turns out, the simulation results obtained from the specification iv) relying on the two linguistic instruments are suggestive of the "moments problem" in this particular application. Both estimators with "no moments" perform poorly and so does UJIVE due to the low degree of overidentification. Fuller estimator and TSLS outrank the rest of the estimators which supports the claim of Hahn and Hausman (2003) that "instrument pessimism" is sometimes overstated for TSLS.

3.4. A Reassessment of the Hall and Jones (1999) Results

This section comments on the estimation results of the HJ99 linear IV regression model using partially robust estimators. The results across the six specification are reported in Table 3.4. They can be briefly summarized in the following way.

First, the point estimates obtained from TSLS, LIML, Fuller - k and BTSLS do not differ substantially within a particular specification. The exception is JIVE which gives somewhat lower point estimates than the rest of the partially robust estimators. This observation is consistent with the simulation results according to which JIVE is plagued by a negative bias while the k-class estimators are virtually free from it.

Second, in specifications i) through v) most of the estimates obtained from the partially robust estimators are found to be significant at any conventional level. The exceptions are the UJIVE estimates in specifications iii)-v) and LIML in v) which are found to be insignificant. The Monte-Carlo simulation results, however, suggest that in these cases UJIVE has much lower precision than the k-class estimators in terms of RMSE and median absolute error. Hence, for the inference purposes TSLS, BTSTS or Fuller - k are preferable.

Third, the specifications iii) and vi) that rely exclusively on the linguistic instruments produce somewhat higher point estimates than the rest of specifications. Furthermore, this becomes more apparent in the sample with no imputed data.

Finally, specification vi) which relies on the linguistic instruments and the sample with no imputed data is worth some special attention. In this specification all the estimates with notable exception of the Fuller -k are found to be insignificant[10]. To interpret this finding one should utilize the simulations results. They suggest that LIML and Jackknife estimators suffer from both bias and size distortion. Furthermore, as can be noted from Table 3.2 and Figure 3.1, these estimators are much more imprecise than TSLS or Fuller-k. The latter conclusion follows from both RMSE and median absolute error criteria. Relying on the Fuller-k yields a point estimate of 7.08 which is significant at any conventional level.

3.5. Robustness Checks

The criterion of the instrument relevance has been considered above in details. The issue which still requires some attention is instrument exogeneity. The problem which arises is that the exogeneity tests and overidentification test which are equivalent under conventional asymptotics are no longer equivalent with weak instruments. As shown by Staiger and Stock (1997) the tests may suffer from both size distortions and lower power against violations of the orthogonality condition. This constitutes a rationale to be sceptical about the overidentification tests reported by HJ99. To alleviate the problem, initially the guidelines provided by Staiger and Stock (1997) will be followed. Furthermore, a version of a bootstrap test of overidentifying restrictions will be considered.

3.5.1. Tests of overidentifying restrictions based on asymptotic approximations.

Following Staiger and Stock (1997) the test statistics utilized here are TR^2 from the regression of the IV residuals on the instruments and exogenous variables (ϕ_{reg}) and Basmann's test statistics (ϕ_{Bas}). Both tests are considered here for residuals of the $k-$ class regressions, in particular TSLS, LIML and Fuller - k. The results of the overidentification tests across specifications are reported in Table 3.5. Some comments on the results can be made. First, the null of instrument exogeneity cannot be rejected at any conventional level of significance across most specifications and tests proposed. Second, the notable exception is specification iv) (shorter sample, all four instruments) for which both Basmann test and TR^2 test reject the null at 10% level. For the purpose of this paper, however, the issue does not seem to be a major concern since the null is not rejected for any specification involving exclusively language instruments. Furthermore, according to Monte Carlo results of Staiger and Stock

[10]With t-statistics of 1.27 obtained from the TSTL and BTSLS (which are actually identical with only two instruments) some researchers like e.g. Frankel and Romer (1999) would call an estimate "marginally" significant.

(1997) the Basmann test should be preferred for inference. They go on to conclude that even though both test have size distortions, under the null the TSLS version tends to overreject while LIML version tends to underreject. Thus, Staiger and Stock (1997) advocate to rely on the Basmann-LIML test which in specification iv) fails to reject the null.

To summarize, the overidentifying restrictions were not rejected by the tests suggested by Staiger and Stock (1997) for the models with weak identification. This provides justification for the orthogonality assumptions made in the Monte Carlo experiments. Interpreting this result one should take into account that under certain parameter values, the Basmann -LIML test advocated by Staiger and Stock (1997) might have low power against small violations of orthogonality condition. This might suggest reexamining the results relying on sample re-use methods rather that asymptotic approximations.

3.5.2. Bootstrapping Basmann - LIML test

In order to overcome the potential problems associated with the Basmann-LIML test based on the asymptotic approximation the principle of the bootstrap can be applied. This claim can be justified on two grounds. First, as shown by Staiger and Stock (1997) asymptotic distribution of the Basmann-LIML test statistics does not depend on any unknown parameter and hence the test statistics is asymptotically pivotal. In this case the bootstrap method is likely to converge faster than the corresponding asymptotic approximation and have smaller size distortions (Horowitz 2001). Furthermore, the bootstrap tests might be particularly advantageous relative to the asymptotic ones under the conditions of weak identification. For instance, as argued by Wong (1996) who investigated the properties of the bootstrap Hausman test relative to the asymptotic approximation the advantage of using bootstrap increases as the correlation between the instruments and the endogenous regressors becomes low.

The bootstrap procedure utilized in this section is similar to the one of Wong (1996). However, unlike the latter the statistics bootstrapped here is LIML version of the Basmann test. Furthermore, the procedure makes use of the Friedman´s (1984) orthogonalization idea. In matrix notation the model considered is

$$y = Y\beta + X\gamma + u, \quad (3.1)$$

$$Y = Z\Pi + X\Phi + V, \quad (3.2)$$

where (3.1) is the structural equation of interest, y and Y are respectively $T \times 1$ vectors of T observations on the endogenous variables, (3.2) is a reduced form equation for Y. X is a $T \times K_1$ matrix of exogenous regressors, Z is a $T \times K_2$ matrix of instruments, u and V are $T \times 1$ vectors of error terms. The errors $\begin{pmatrix} u_t & V_t \end{pmatrix}'$, where u_t denotes the t-th observation on u, are assumed to have mean zero, to be serially uncorrelated and to be homoscedastic. Furthermore it is assumed that both exogenous variables and instruments are orthogonal to the error terms.

In the context of HJ99 study y is the $T \times 1$ vector of log income per capita, Z is a $T \times K_2$ matrix of instruments, Y is a $T \times 1$ vector of proxy for social infrastructure, X is a $T \times 1$ vector of ones, β is the main parameter of interest.

The bootstrap procedure follows a simple algorithm. First the model in (3.1) is estimated by a k-class estimator to obtain the residual vector $\widehat{u}(k) = y - Y\widehat{\beta}(k) - X\widehat{\gamma}(k)$. Those residuals will not be exactly orthogonal to the set of instruments, that is $\frac{1}{T}Z'\widehat{u}(k) \neq 0$. This, however, is an assumption imposed by the model. Following the idea of Friedman (1984) one might utilize the component of the residual vector orthogonal to the set of instruments given by $\widetilde{u}(k) = (I - Z'(Z'Z)^{-1}Z')\widehat{u}(k)$ where I denotes a $T-$ dimensional identity matrix.

Second, given a quadruple of (y, Y, Z, X) the bootstrap method is used to resample with replacement from the empirical distribution of $(\widetilde{u}(k), Y, Z, X)$. Denoting by $(\widetilde{u}(k)^*, Y^*, Z^*, X^*)$ the values drawn from the empirical distribution, for each bootstrap replication y^* is generated as $y^* = Y^*\widehat{\beta}(k) + X^*\widehat{\gamma}(k) + \widetilde{u}(k)^*$.

Next, using the bootstrap sample the model is estimated by a k-class estimator to obtain a residual vector $\widehat{u}^*(k) = y^* - Y^*\widehat{\beta}^*(k) - X^*\widehat{\gamma}^*(k)$. Then the Basmann´s statistic is calculated as

$$\phi^*_{Bas}(k) = \left[\widehat{u}^*(k)' P^*_{Z^\perp}\widehat{u}^*(k)\right] / \left[\widehat{u}^*(k)' M^*_{Z^\perp}\widehat{u}^*(k) / (T - K_1 - K_2)\right],$$

where $Z^{*\perp} = (I - X^*(X^{*\prime}X^*)^{-1}X^*)Z^*$, $P^*_{Z^\perp}$ and $M^*_{Z^\perp}$ are the corresponding projection and residualizing matrices given by $P^*_{Z^\perp} = Z^{*\perp\prime}(Z^{*\perp\prime}Z^{*\perp})^{-1}Z^{*\perp\prime}$ and $M^*_{Z^\perp} = I - P^*_{Z^\perp}$ respectively.

The Basmann´s statistic $\phi^*_{Bas}(k)$ is calculated for each of B bootstrap replications. The 90%, 95% and 99% percentiles from the obtained empirical distribution represent corresponding bootstrap critical values reported in Table 3.6.

For convenience the Basmann - LIML statistics $\phi_{Bas}(k_{LIML})$ for all six specifications are repeated in Table 3.6 along with the bootstrap critical values. The conclusion about the instrument orthogonality drawn from the bootstraped Basmann - LIML test coincides with those based on the asymptotic approximations. In each case the Basmann - LIML test statistic falls short of the tabulated critical values which implies failure to reject the overidentifying restrictions.

To summarize, the overidentifying restrictions have not been rejected by the tests robust to the presence of weak instruments for all specifications. The conclusion holds for both tests based on asymptotic approximations as well as the test relying on small sample properties. The evidence obtained provides some further support to the orthogonality assumption made in section 3.3.

3.6. Conclusion

This paper focuses on the empirical methodology proposed by HJ99 to estimate the effect of what they call "social infrastructure" on productivity across countries. In this study we attempt to address the criticism of Acemoglu et al (2001) directed towards the HJ99 methodology for relying on the instruments with less than convincing theoretical justification. Hence, one of the objectives of this study is to assess whether the HJ99 results are driven by the use of "latitude" and Frankel-Romer predicted trade share to instrument for institutional quality. In order to address the issue six different specifications have been considered some of which utilize exclusively linguistic variables as instruments. However, in attempt to accomplish this task one had to overcome the problem of weak identification.

The instruments proposed by HJ99 and in particular linguistic instruments are found to be only weakly correlated with their proxy for institutional quality. The basic diagnostic criteria for weak identification such as low values of the first stage F-statistics indicate that the performance of TSLS estimator might be inferior. Hence, the issue of concern is whether one can rely on TSLS for both estimation and inference purposes given the potential problem of weak identification. Furthermore, the questions that arise are whether the performance of TSLS can be improved upon by using the estimators partially robust to weak instruments and which of the estimators are preferable in the context of the HJ99 model.

To address these issues a Monte Carlo study comparing relative performance of TSLS and several partially robust estimators has been conducted. The results of the experiments can be summarized as follows. First, one may conclude that the instruments cannot be deemed as irrelevant. However, depending on the specification some of the examined estimators suffer from both bias and size distortions. For instance, JIVE´s performance is inferior compared to the other partially robust estimators across all specifications. Second, compared to the other examined partially robust estimators the TSLS estimator performs relatively well in terms of average bias, RMSE and median absolute error. Overall, the results of the Monte Carlo experiments suggest that among the estimators considered the preferred ones should be Fuller - k and TSLS. This finding constitutes further support for the claim of Hahn and Hausmann (2003) that "instrument pessimism" is sometimes overstated for TSLS.

To address the mentioned criticism of Acemoglu et al (2001) the linear IV model of HJ99 was reestimated across several specifications and the evidence from the estimation methods partially robust to weak instruments was interpreted on the basis of the Monte Carlo results. The main conclusion is that both specifications relying exclusively on the linguistic variables produce the results qualitatively consistent with HJ99. In the sample free from imputed data most of the estimators produce positive but insignificant estimates of the coefficient on the proxy for institutional quality. However, as the Monte Carlo experiments show LIML and Jackknife estimators demonstrate inferior performance both on the bias, median absolute error and RMSE criteria. This gives a rationale to rely on the evidence from TSLS and Fuller - k estimators. However, for the purpose of inference the Fuller - k might be preferable since it performs best on both the RMSE and median absolute error criteria and furthermore has lower size distortion. Based on the Fuller - k estimator the estimated coefficients on social infrastructure are significant at any conventional level with the point estimates of 5.95 and 7.08 in the large and small sample respectively. Hence, one can discard the argument that the HJ99 results are driven by reliance on the "geographical" instruments.

It stands to a reason to emphasize our finding that using the partially robust estimators allows to utilize the linguistic variables to instrument for institutional quality despite their low correlation with the endogenous regressor. This result is particularly important for identifying partial effects of institutions and trade in view of Dollar and Kraay (2003) argument who had shown that existing attempts to isolate partial effects using geography related instruments suffer from serious identification problems.

To summarize, the Monte Carlo experiments suggest that in the case of HJ99 study the TSLS and Fuller - k estimators are not plagued by either severe size distortion or bias despite low values of first-stage F-statistics. In the specifications that rely exclusively on the linguistic characteristics as instruments, the inference should be made on the basis of Fuller - k or TSLS estimators. We find the linguistic variables to be relevant instruments for the institutional quality. Relying on the latter estimators the coefficients on social infrastructure are found to be positive, significant and actually somewhat higher that the estimates reported by HJ99. Hence, we find the evidence contradicting the criticism of Acemoglu et al (2001) towards the HJ99 work.

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3.7. Tables and Figures

Table 3.1. First-stage F-statistics across specifications

	Specification	First-stage F statistic
i)	Full sample (127 countries), 4 instruments	12.843
ii)	Full sample (127 countries), latitude & FR tr.share	13.118
iii)	Full sample (127 countries), languages	6.4995
iv)	No imputed data (79 countries), 4 instruments	6.7206
v)	No imputed data (79 countries), latitude & FR tr.share	9.0664
vi)	No imputed data (79 countries), languages	2.1178

Table 3.2. Monte Carlo simulation results

	$k-$class estimators					Jackknife estimators	
	OLS	TSLS	LIML	Fuller$-k$	BTSLS	JIVE	UJIVE
Specification i.	(HJ99 full sample, 127 observations, 4 instruments); $\rho = 0.391$;						
Av. % Bias	0.18716	0.00038	0.01275	0.00718	0.01091	-0.1411	-0.0313
RMSE	0.57265	0.2048	0.21627	0.20966	0.21547	0.46973	0.24818
SIZE	0.001	0.974	0.98	0.976	0.976	0.79	0.98
Specification ii)	(HJ99 full sample, 127 observations, Distance/FR tr.share); $\rho = 0.452$;						
Av. % Bias	0.21938	0.00244	0.00563	0.00463	0.00244	-0.1698	-0.0369
RMSE	0.66804	0.27411	0.28069	0.26991	0.27411	0.57736	0.33411
SIZE	0	0.972	0.983	0.978	0.972	0.836	0.985
Specification iii)	(HJ99 full sample, 127 observations, languages); $\rho = 0.485$;						
Av. % Bias	0.23793	0.00215	0.01856	0.00289	0.00215	-0.3078	-0.083558
RMSE	0.72301	0.37207	0.39523	0.36123	0.37207	0.99496	0.55044
SIZE	0	0.974	0.97	0.972	0.974	0.75	0.996
Specification iv)	(Sample with no imputed data, 127 observations, 4 instruments); $\rho = 0.415$						
Av. % Bias	0.1956	0.00199	0.026224	0.01471	0.022731	-0.24868	-0.06136
RMSE	0.60356	0.26646	0.29361	0.2752	0.29515	0.79102	0.37653
SIZE	0.017	0.974	0.976	0.976	0.977	0.676	0.98
Specification v)	(Sample with no imputed data, 127 observations, 4 instruments); $\rho = 0.453$						
Av. % Bias	0.21561	0.00023	0.01105	0.00413	0.00023	-0.236	-0.05793
RMSE	0.6623	0.30874	0.32225	0.30314	0.30874	0.77153	0.41605
SIZE	0.08	0.981	0.984	0.977	0.981	0.814	0.989
Specification vi)	(Sample with no imputed data, 127 observations, 4 instruments); $\rho = 0.516$						
Av. % Bias	0.2511	-0.00968	-0.05535	0.01748	-0.00968	-0.88724	-0.61603
RMSE	0.7662	0.59901	0.91229	0.53488	0.59901	2.7237	2.6992
SIZE	0	0.998	1	0.989	0.998	0.543	1

Note: Average per cent bias (Av % Bias), RMSE and SIZE (coverage rate for a 95% confidence interval calculated as a proportion of replications when the confidence interval covers the true parameter value) for the estimates of β. Reported ρ indicates the average sample correlation coefficient between the endogenous regressor and the error term. The experiments rely on 1000 Monte Carlo replications and 1000 Bootstrap iterations.

Table 3.3. Quantiles around β and median absolute errors: six specifications

	Quantiles around β: *specification i.* (127 obs., 4 instruments)					Median abs. error
Estimator	0.10	0.25	0.50	0.75	0.90	
OLS	3.4193	3.48	3.562	3.6447	3.7085	0.562
TSLS	2.7319	2.8573	2.9906	3.1346	3.2583	0.13899
LIML	2.6858	2.8164	2.9531	3.106	3.24	0.14443
Fuller - *k*	2.71	2.8363	2.9705	3.1201	3.2517	0.13985
BTSLS	2.6896	2.8203	2.9627	3.1082	3.2385	0.14237
JIVE	2.3146	2.4337	2.573	2.7173	2.8351	0.42582
UJIVE	2.6103	2.7447	2.9018	3.0646	3.1974	0.16469
	Quantiles around β: *specification ii.* (127 obs., Dist/FR tr.s.)					Median abs. error
Estimator	0.10	0.25	0.50	0.75	0.90	
OLS	3.5099	3.579	3.6595	3.7346	3.8089	0.65953
TSLS	2.6616	2.8221	2.9962	3.1946	3.3695	0.18401
LIML	2.6309	2.7942	2.9726	3.1772	3.3553	0.19206
Fuller - *k*	2.6775	2.833	3.0038	3.2023	3.3696	0.18359
BTSLS	2.6616	2.8221	2.9962	3.1946	3.3695	0.18401
JIVE	2.1512	2.3055	2.4819	2.6725	2.8454	0.5172
UJIVE	2.4955	2.6745	2.8792	3.1003	3.3009	0.23703
	Quantiles around β: *specification iii.* (127 obs., languages)					Median abs. error
Estimator	0.10	0.25	0.50	0.75	0.90	
OLS	3.5657	3.6326	3.7164	3.7931	3.8609	0.7164
TSLS	2.5143	2.743	2.9957	3.2463	3.4448	0.25267
LIML	2.4401	2.681	2.9406	3.2171	3.4196	0.26497
Fuller - *k*	2.5409	2.7662	3.0036	3.2588	3.446	0.25442
BTSLS	2.5143	2.743	2.9957	3.2463	3.4448	0.25267
JIVE	1.5958	1.8203	2.0774	2.3214	2.5267	0.9225
UJIVE	2.113	2.4102	2.7506	3.0737	3.3454	0.35911
	Quantiles around β: *specification iv.* (79 obs., 4 instruments)					Median abs. error
Estimator	0.10	0.25	0.50	0.75	0.90	
OLS	3.4076	3.4921	3.5864	3.6807	3.7679	0.58644
TSLS	2.6475	2.823	2.9995	3.1749	3.3261	0.17646
LIML	2.5543	2.733	2.9279	3.1105	3.2695	0.1915
Fuller - *k*	2.6015	2.7716	2.9644	3.1409	3.2932	0.18238
BTSLS	2.5641	2.7496	2.9371	3.1262	3.2943	0.18966
JIVE	1.9199	2.0882	2.2619	2.4333	2.5895	0.73787
UJIVE	2.3986	2.6088	2.8259	3.04	3.2351	0.24815
	Quantiles around β: *specification v.* (79 obs., Dist/FR tr.s.)					Median abs. error
Estimator	0.10	0.25	0.50	0.75	0.90	
OLS	3.4611	3.5561	3.644	3.7398	3.8254	0.64401
TSLS	2.6126	2.7767	3.0009	3.2075	3.3982	0.21733
LIML	2.5687	2.7346	2.9644	3.1856	3.3727	0.23094
Fuller - *k*	2.6399	2.7919	3.0121	3.2138	3.4032	0.20939
BTSLS	2.6126	2.7767	3.0009	3.2075	3.3982	0.21733
JIVE	1.9145	2.0789	2.2868	2.5042	2.6813	0.71302
UJIVE	2.3607	2.5635	2.8198	3.0878	3.3063	0.28739
	Quantiles around β: *specification vi.* (79 obs., languages)					Median abs. error
Estimator	0.10	0.25	0.50	0.75	0.90	
OLS	3.5757	3.6575	3.7525	3.8431	3.928	0.75247
TSLS	2.2287	2.5641	2.9747	3.3728	3.7433	0.39954
LIML	1.9652	2.3395	2.8431	3.293	3.7234	0.47655
Fuller - *k*	2.3891	2.6725	3.0521	3.4074	3.7452	0.37327
BTSLS	2.2287	2.5641	2.9747	3.3728	3.7433	0.39954
JIVE	-0.40765	-0.032695	0.36074	0.71409	1.0726	2.639
UJIVE	-1.3881	-0.11133	1.2284	2.4316	3.6523	1.938

Note: The experiments are identical to those reported in Table 3.2.

Table 3.4. Estimation results, six specifications

		$k-$class estimators				Jackknife estimators	
	OLS	TSLS	LIML	Fuller$-k$	BTSLS	JIVE	UJIVE
i) 127 obs.; 4 inst	3.2891	5.0847	5.3003	5.2431	5.1855	4.7721	5.382
	(0.196)	(0.5079)	(0.6760)	(0.6332)	(0.5564)	(0.4554)	(0.7046)
ii) 127; Dist/FR		4.6698	4.7600	4.6919	4.6698	4.234	4.9118
		(0.6763)	(1.4348)	(0.7562)	(0.6763)	(0.6319)	(2.2469)
iii) 127; Languages		5.7981	6.2097	5.9513	5.7981	5.1492	6.8178
		(1.6004)	(3.2017)	(1.3260)	(1.6004)	(1.1287)	(16.162)
iv) 79; 4 instruments	3.0741	4.6612	5.2683	5.1423	4.8279	4.0898	5.1095
	(0.253)	(0.6121)	(3.7255)	(0.9958)	(0.7347)	(0.5586)	(3.0471)
v) 79; Dist./FR tr.sh		3.9388	3.9751	3.9154	3.9388	3.3444	4.1239
		(0.7048)	(7.9828)	(0.7330)	(0.7048)	(0.7700)	(9.6542)
vi) 79; Languages		6.538	8.6454	7.082	6.538	4.5276	15.417
		(5.1294)	(26.941)	(2.2586)	(5.1294)	(4.4353)	(27.715)

Note: The dependent variable is log of income per capita. The regressors are a constant and a proxy for social infrastructure. Depending on specification the instruments used include fraction of population speaking a English at birth, fraction of population speaking a European language, distance from the equator and Frankel and Romer (1999) predicted trade share. Standard errors are given in the parenthesis. The standard errors are computed using a Bootstrap procedure described in the text.

Table 3.5. Testing overidentifying restrictions, six specifications

	TR^2 test (ϕ_{reg})			Basmann test (ϕ_{Bas})		
	TSLS	LIML	Fuller$-k$	TSLS	LIML	Fuller$-k$
i) 127 obs.; 4 instruments	4.1767	4.0347	4.0442	4.1487	4.0031	4.0128
	(0.2430)	(0.2577)	(0.2567)	(0.2459)	(0.2611)	(0.2601)
ii) 127 obs.; Dist/FR tr.sh	1.377	1.3595	1.3694	1.3592	1.3418	1.3517
	(0.2406)	(0.2436)	(0.2419)	(0.2437)	(0.2467)	(0.245)
iii) 127 obs.; Languages	1.8229	1.6983	1.7448	1.8058	1.6807	1.72748
	(0.177)	(0.1925)	(0.1865)	(0.179)	(0.1948)	(0.1887)
iv) 79 obs.; 4 instruments	6.4307	5.8243	5.8462	6.5575	5.8899	5.9139
	(0.0924)	(0.1205)	(0.1193)	(0.0874)	(0.1171)	(0.1159)
v) 79 obs.; Dist/FR tr.sh	0.6153	0.61275	0.61975	0.5966	0.59409	0.60093
	(0.4328)	(0.4338)	(0.4311)	(0.4399)	(0.4408)	(0.4382)
vi) 79 obs.; Languages	2.1921	1.5775	1.8554	2.1691	1.5485	1.8279
	(0.1387)	(0.2091)	(0.1732)	(0.1408)	(0.2134)	(0.1764)

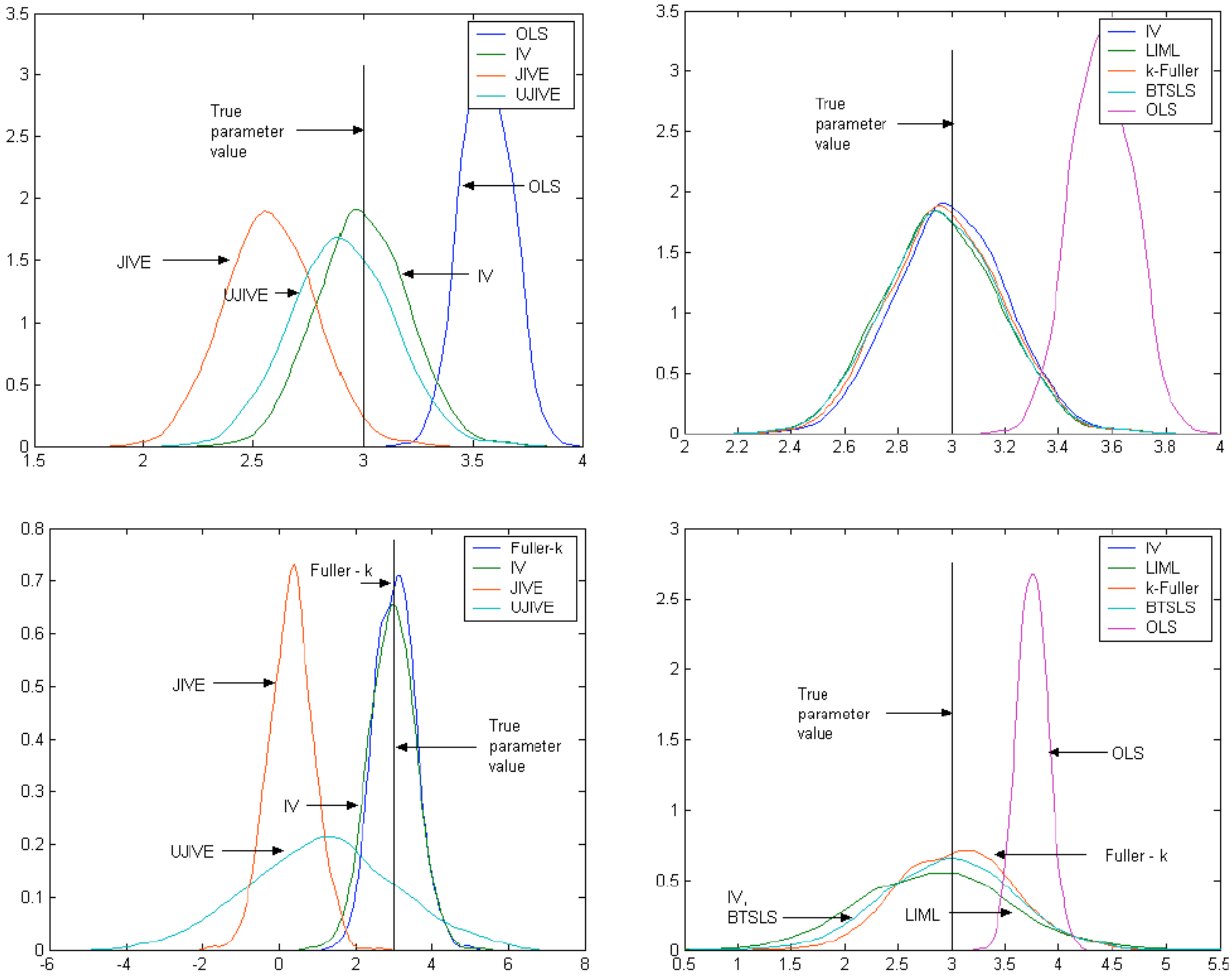
Note: The table reports TR^2 over-identification test statistics and Basmann's test statistics in the χ^2 form. The tests results are reported for the residuals of the k-class regressions namely TSLS, LIML and Fuller$-k$ respectively. The $p-$values reported in the parenthesis are based on asymptotic approximation. The dependent variable in the second stage IV regression is log of income per capita. The regressors are a constant and a proxy for social infrastructure. Depending on specification the instruments used include fraction of population speaking English at birth, fraction of population speaking a European language, distance from the equator, and Frankel and Romer (1999) predicted trade share.

Table 3.6. Bootstrap critical values for Basmann–LIML test of overidentifying restrictions

Specification	90% Crit. values	95% Crit. values	99% Crit. values	Basmann stat. $\phi_{Bas}(k_{LIML})$
i) Full sample, 4 instruments	7.3752	9.353	14.423	4.0031
ii) Full sample, latitude & FR tr.share	3.2166	4.564	7.7597	1.3418
iii) Full sample, languages	3.0171	4.3997	7.4694	1.6807
iv) No imputations, 4 instruments	6.2934	7.9244	11.602	5.8899
v) No imputations, latitude & FR tr.share	2.578	3.6441	6.4887	0.59409
vi) No imputations, languages	2.4323	3.5093	5.9893	1.5485

Note: The table repots a set of critical values for the LIML version of the Basmann's test of overidentifying restrictions and the corresponding test statistics for each of the six specifications considered. The bootstrap procedure is described in the text.

Figure 3.1. Kernel density estimates of beta: specifications i) and vi)



Note: The figure reports distributions for the estimators of the coefficient on institutional quality based on the kernel density estimation. The upper panel corresponds to the specification involving all four instruments and the larger sample (127 observations). The lower panel reports the results from the specification relying on the two linguistics instruments only and the sample free from imputed data (79 observations). Left panels serve for comparison of the jackknive estimators with some of the *k*-class estimators. The right panels present the distributions of all of the k-class estimators considered. The details of the DGP and the estimators used are described in the text.