Self control and debt: theory and evidence

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To my family
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Preface

When buying on credit, anecdotal and empirical evidence suggest that the consumers focus more on the size of the monthly payments than on the interest charged on the loan or the total amount they pay. The lenders typically use low monthly payments and longer loan durations as the main advertising messages and interest rates usually appear only in small prints. An anonymous writer in an internet blog says \(^1\)

...Geez, when I sold my last house, someone asked me what the monthly payments would be...they didn’t ask what the asking price for the house was! I’ve heard some people shop for cars based only on monthly payments, too. I thought educated programmers were supposed to be above this! I mean focusing on monthly payments is like the gutter of financial stability!

The tendency of consumers to base their loan decisions almost solely on monthly payments also seems to be a cause for concern for regulators and consumer protection agencies. A booklet published by the Washington State Attorney General’s office advises the borrowers on their automobile purchases as follows:

...Your automobile purchase or lease is a series of negotiations: price, trade-in value, optional products or services, and financing terms. Treat each negotiation as a separate transaction. Don’t focus on monthly payment alone! ...Beware of the dealer who tries to focus on monthly payments...

It is hardly surprising that the consumers end up financially overextended when they base loan decisions on how much they are going to pay each month and ignore the size of the debt burden. The recent U.S. sub-prime mortgage crisis demonstrates the importance of such financial decisions and their impact on the economy. The crisis eventually led Federal reserve to propose new regulations on sub-prime mortgage lending. According to the Financial Times, “the proposed new rules go beyond

\(^1\)http://blogs.sun.com/jimgris/entry/open_letter_to_john_berger
disclosure requirements and industry guidance to ban some controversial lending and advertising practices outright and restrict others.”  

A victim of the sub-prime mortgage meltdown in Ohio, one of the worst affected areas in terms of mortgage foreclosures and evictions, is reported saying

“I do blame myself a little bit,” Mrs. X acknowledges. “I feel dumb.” She explains that she was focused on the monthly payment when she borrowed from Countrywide, not the interest rate or taxes due. “Once we got the loan documents at the closing, I just came home and stuck them in a drawer.”

The first two chapters of this thesis aim to explore the role of self-control problems in explaining this puzzling behavior. In particular, we analyze the loan demand, choice of maturity as well as the allocation of periodic payments for the consumers who exhibit self-control problems. Our analysis relies on the assumption of the “quasi-hyperbolic discounting” (Laibson [21]) to model self-control. This approach is based on the idea that the self-control problems occur as a result of a preference for immediate gratification. In order to model the preference for immediate gratification, it is assumed that people have hyperbolic -rather than exponential- discount functions (Strotz [37]; Phelps and Pollack [28]; Laibson [21]; O’Donoghue and Rabin [25], [26]). The quasi-hyperbolic discount function, when evaluated at period $t$, is equal to 1 for $s = t$ and equal to $\beta^{s-t}$ for $s = t + 1, t + 2, ...$ with $\beta \leq 1$. The case of $\beta = 1$ corresponds to the exponential discounting. When $\beta < 1$, the consumer has time-inconsistent preferences. This type of discounting implies that the discount rate decreases with the passage of time. Allowing for discount rates to vary over time results in time-inconsistency because plans that are optimal from the perspective of today are not necessarily optimal from the perspective of tomorrow. Time inconsistency can lead to lack of commitment because “intentions and actions are not aligned” (Laibson et al. [22]). In this thesis, we are going to use the terms “self-control problems”, “quasi-hyperbolic discounting” and “dynamically inconsistent preferences” interchangeably.

In the first chapter of the thesis, we consider a partial equilibrium setting in which the loan maturity and the interest rates are fixed and the consumers can choose how much to borrow and how to allocate the repayments between different periods. We take the loan demand and the repayment behavior of the exponential consumer as benchmark and compare them with the behavior of consumers who exhibit dynamically inconsistent preferences. Time-inconsistency requires us to make assumptions

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3Source: newbricks.blogspot.com/2007/09/ground-causes-accidents-claim-pilots.html

4See Gul and Pesendorfer [18] for a different approach
on the consumers’ analysis of their own future behavior. More specifically, we distinguish two types of consumers with dynamically inconsistent preferences a sophisticated consumer who is fully aware of her self-control problems and a naive consumer who is completely unaware of her self-control problems. We also look at the effect of having either full or no commitment power. We show that having self-control problems implies higher levels of borrowing whilst awareness of self-control problems or a lack of commitment puts downward pressure on that level. We also demonstrate that having self-control problems and being naive about them can induce a preference for back-loaded repayment plans which require higher repayments towards the end of the loan term. In other words, naivety can lead to a preference for repayment schedules that emphasize lower monthly payments early in the loan term.

The second chapter looks at the interaction between a monopolistic lender and a consumer with quasi-hyperbolic discounting in a setting where we assume equal monthly payments and allow the consumer to choose between different loan maturities. In particular, we consider an economy in which a monopolistic lender offers either a short-term or a long-term loan to a consumer with time-inconsistent preferences who wishes to finance the purchase of a single durable good. We establish the following properties of the equilibrium contract of installment credit. First, as long as the interest rate on loans are higher than the market risk free rate, in an equilibrium contract, the consumers with dynamically inconsistent preferences choose the loan with longer maturity (hence lower monthly payments). In contrast, the exponential consumer is indifferent between the short term and long term loans at the interest rates offered. Next, we identify a threshold value for the degree of time inconsistency above which the lender is able to extract from the naive consumer more than her willingness to pay in the first period. Finally, if, in addition, the degree of naivety exceeds a certain threshold, the lender is able to extract more profits from the naive consumer relative to the sophisticated consumer.

The results of the first two chapters provide some support to our conjecture that self-control problems are linked to consumers’ tendency to focus more on monthly payments than on the interest rate or on the total cost of the loan.

Third chapter of this thesis, which is a joint work with Alena Bicakova, provides empirical evidence on the impact of self-control problems on consumers’ debt repayment behavior. The analysis is motivated by the survey evidence which shows that a significant percentage of borrowers blame the debt mismanagement as the reason for running into financial troubles. This contrasts with the standard economic argument that default on debt is caused by the unexpected negative shocks to income or to expenditure. Our conjecture is that the debt mismanagement and lack of self-control are linked, and they have adverse effect on the debt repayment performance of the borrowers. In order to test this conjecture, we use a unique administrative data set of a major consumer credit counselling charity in the UK. Credit counselling
assists heavily indebted borrowers by setting up and administering repayment plans, so called debt management plans or DMPs. We first construct an extremely simple theoretical model of a debt management plan to show that borrowers who have self control problems and are naive about it (all else equal) are more likely to fail to complete the DMP. We then identify individuals with self-control problems using two different indicators: self-reported reasons for running into financial troubles, and smoking and estimate and compare the DMP drop out rates for the group of individuals identified as those with self-control problems with the rest of the CCCS clientele. Our results indicate that self-control problems increase the probability of dropping out from a debt management plan by 12% and 31% when the self-reported reasons for becoming overindebted and smoking are used as indicators, respectively. To the extent that these indicators can be used as a valid proxy for self-control problems, the results indicate a potential link between default rates and lack of self-control. We believe that this link needs to be explored further in future studies.
Chapter 1

Why do consumers focus more on the size of monthly payments than on interest rates?

1.1 Introduction

A reader named Tom writes in to ask:

"Youre always talking about minimizing interest rates when it comes to reducing debt. Shouldnt you really focus on whatever methods you can use to minimize the total amount that you have to pay each month?"

If you look exclusively at your monthly budget and at nothing beyond, it makes sense on some level to merely minimize the total amount you have to pay in bills each month. However, once you start carrying that perspective out to its logical conclusion, it stops making much sense at all. ¹

Previous research on loan demand provides empirical evidence which suggest that the demand is more sensitive to maturity than it is to the interest rate. Attanasio et al. [2] analyze the elasticity of the loan demand with respect to the interest rate and the maturity of the loan using micro data from the Consumer Expenditure Survey (1984-1995) on auto loan contracts. They find that the aggregate loan demand is highly sensitive to loan maturity, but unresponsive to changes in the interest rate, which they interpret as evidence of liquidity constraints. Karlan and Zinman [19] estimate the elasticities of demand using randomized trials (random loan offers distributed via direct mail to over 50,000 individuals) implemented by a major South African

micro-finance lender. They also find that loan size decisions are far more responsive to changes in loan maturity than to changes in interest rate. They, however, note that “an alternative (or complementary) explanation to liquidity constraints” ... would be that “income and age proxy for financial sophistication, i.e. perhaps the poor and inexperienced use a decision rule that lead them to focus on monthly payments rather than the interest rates”.

Wertenbroch [38] reviews empirical evidence of “how the consumers with self-control problems cope with choice situations in which it is difficult to assess their global resource constraints” (e.g., a life-time budget constraint). Wertenbroch [38] argues that the consumers with dynamically inconsistent preferences base their consumption decisions on the immediately available resources. Credit may lead to excessive consumption or overextending debt since “perceived liquidity” (boosted by available credit) increases current spending by temporarily relaxing the current budget constraint (Ausubel [3]; Laibson [21]) as predicted by “the behavioral life-cycle hypothesis” of Shefrin and Thaler [33].

Stango and Zinman [35] argue that the lenders emphasize monthly payments because the consumers exhibit what they call “payment/interest bias-a systematic tendency to underestimate interest rates associated with a loan principal and payment stream”. They empirically quantify the bias and show that the households who display such bias are more likely to hold loans with higher interest rates. This relationship applies to loans obtained from finance companies but not to loans from banks. They attribute their findings to a “cognitive bias in how the consumers perceive interest rates, namely, exponential growth bias, the well-documented tendency for individuals to dramatically underestimate the growth or decline of exponential series”.

Can self-control problems modeled by dynamically inconsistent preferences provide additional insights into understanding why consumers focus more the on monthly payments than on the interest rate on the loan or the total price they pay? In this chapter, we try to provide an answer.

We consider an economy that lasts for three periods where the income process is exogenous and there is no uncertainty. The assumption of quasi-hyperbolic discounting allows us to distinguish four different types of consumers: an ‘exponential consumer’, a sophisticated consumer who is fully aware of her self-control problem but has no commitment power, a consumer with dynamically inconsistent preferences but have full commitment power, and a naive consumer who is fully unaware of her self-control problem. Each type of consumer wishes to finance the purchase of a durable good and can do so by taking a loan with a maturity of two periods. We allow consumers to choose how much to repay in each period. We compare the loan demand and the allocation of periodic payments across different types. We demonstrate that self-control problems induced by quasi-hyperbolic discounting im-
ply higher levels of borrowing. However, awareness of self-control problems or a lack of commitment puts downward pressure on that level. These results are linked to the previous findings in the literature. Wertenbroch [38] argues that some consumers are capable of employing “self-rationing strategies to compensate for their inability to stick to global constraints”. For example, they may choose not to finance “hedonic” consumption or alternatively “self-impose stricter payment terms” (Wertenbroch and Soman [39], Dhar and Wertenbroch [13], O’Curry and Strahilevitz [24], Puri [30], Prelec and Lowenstein [29]). Ameriks et al. [1] provide survey evidence which suggest that self-control problems are not always associated with overconsumption. In some cases, self-control problems may lead to underconsumption and higher levels of wealth. Meier and Sprenger [23] present empirical evidence that individuals with present bias preferences have higher levels of credit card borrowing. Our results also suggest that naive consumers prefer back-loaded repayment plans, namely, plans which require higher repayments towards the end of the loan term. In other words, self-control problems coupled with naivete can induce a preference for repayment schedules that emphasize lower monthly payments early in the loan term.

This chapter is organized as follows. Section 1.1 provides an introduction as well as an overview of the anecdotal and empirical evidence on the monthly payment bias. Section 1.2 contains the model and the results. Section 1.3 concludes. All the proofs are provided in the Appendix.

1.2 The model

Consider an economy with three periods, where the income process is exogenous and there is no uncertainty. We closely follow the model in Attanasio [2] but we allow the consumer to have quasi-hyperbolic discounting. We assume that the consumer wishes to finance the purchase of a durable good, whose value is fixed to $K$. One may think that the manufacturer of the durable sets this value.

The consumer faces a deterministic sequence of wages and she can finance a fraction $\Phi \in [0, 1]$ of the purchase with a loan of maturity of two periods. The borrowing and the lending rates are denoted by $r^b$ and $r^l$, respectively. We assume that the consumer takes them as given and that $r^b - r^l > 0$.

We make a series of simplifying assumptions. First, we assume that the decision to buy the durable has already been made and the consumer simply chooses how much to borrow and how to allocate the repayments between periods. This assumption allows us to abstract from the buying decision and focus on loan demand and repayment dynamics. Second, we assume that the borrowing can only be made in the first period which rules out the possibility of transferring the third period resources to the second period. This assumption is indeed a strong one, and relaxing it has the advantage
of studying the repayment dynamics in a more realistic setting. Nevertheless, we believe it is a reasonable assumption to make given that our focus is on installment credit which typically is tied to the purchase of a durable good. Third, we assume that the amount borrowed can not exceed the value $K$ of the durable. Fourth, we assume that the wages in the first period fall short of the value $K$ of the durable. This assumption rules out a cash purchase. Fifth, we assume that the consumer can not sell the durable in the second or third periods which allows us to abstract from the impact of the secondary markets. Finally, in our economy, default is not allowed so that the loan has to be repaid in full by the end of the last period. One can assume that the punishment should the borrower default is prohibitively severe. This assumption is also made for convenience, as it allows us to focus solely on the loan demand and the allocation of periodic payments. Nevertheless, the obvious extension to the analysis in this chapter would be to study the default behavior and the lender-borrower interaction when there is income uncertainty.

We assume that the utility from consuming the durable is constant and is the same for all consumer types. The utility over the non-durable consumption is defined as follows. We assume that the consumer has quasi-hyperbolic preferences over the consumption stream $(c_1, c_2, c_3)$. The quasi-hyperbolic discount function, when evaluated at period $t$, is equal to 1 for $s = t$ and equal to $\beta \delta^{s-t}$ for $s = t + 1, t + 2, \ldots$ with $\beta \leq 1$. The case of $\beta = 1$ corresponds to the exponential discounting. When $\beta < 1$, the consumer has time-inconsistent preferences. Therefore, period one preferences are

$$u(c_1) + \beta \delta u(c_2) + \beta \delta^2 u(c_3),$$

while period two preferences are

$$u(c_2) + \beta \delta u(c_3).$$

This type of discounting implies that the consumer will systematically be more impatient in the near future. The consumer always employs the discount rate $\beta \delta$ between today and tomorrow and the rate $\delta$ between all consecutive periods. With dynamically inconsistent preferences, an optimal decision from the perspective of today concerning the tomorrow and the day after will not be consistent with the optimal decision from the perspective of tomorrow concerning the same periods.

We want to analyze the the effect of self-control on loan demand (i.e. fraction financed) and on the allocation of monthly payments over periods. Our benchmark case is the exponential discounting. Salanié and Treich [32] argue that comparing the consumers who has exponential and quasi-hyperbolic discount functions by looking at the effect of changing $\beta$ is not an appropriate comparative statics exercise. As

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<sup>2</sup>This is typically the case for real-life installment loan arrangements.
\( \beta \) changes, both the degree of time inconsistency as well as the discount factor applied to future periods change. They suggest that in order to isolate the effect of self-control, one needs to compare a consumer with no-commitment power with the one with full commitment power. Their criticism applies to our comparative statics exercise of comparing results under the quasi-hyperbolic discounting and the exponential discounting. Therefore, we consider four different types of consumers. The first is the \textit{sophisticated consumer} who is fully aware of her dynamically inconsistent preferences and takes this into account when making choices today. However, she is not able to commit her choices made in the first period. The second is the \textit{naive consumer} who is fully unaware of her future preferences. Her decisions are based on her current preferences only with the perception that these preferences will stay the same in the future. The third is the \textit{exponential consumer} with time-consistent preferences. The last is the \textit{consumer with full commitment power} who exhibits dynamically inconsistent preferences but has the ability to commit her choices in the future periods.

The generic problem for each type of the consumer is to choose \( c_1, c_2, c_3, \Phi \) (fraction financed), and \( P \) and \( P' \) (repayments in the second and last periods) to maximize the associated preferences as described above. In the next subsection, we derive the first order conditions for the maximization problem of each type consumer. In what follows, indices \( \{S, N, FC, Exp\} \) refer to the sophisticated consumer, the naive consumer, the consumer with full commitment power and the exponential consumer, respectively.

\subsection{The sophisticated consumer}

The sophisticated consumer chooses \( c_1^S, c_2^S, c_3^S, \Phi_S \) (fraction financed), and \( P_S \) and \( P'_S \) (repayments in the second and last periods) to maximize the associated preferences as described before. The maximization problem is subject to the following constraints:

\begin{align*}
  c_1^S + K(1 - \Phi_S) & \leq w_1, \quad (1) \\
  c_2^S + P_S & \leq w_2 + A_1^S(1 + r^I), \quad (2) \\
  c_3^S + P'_S & \leq w_3 + A_2^S(1 + r^I), \quad (3) \\
  \Phi_S & \leq 1, \quad (4) \\
  -\Phi_S & \leq 0, \quad (5) \\
  -P_S & \leq 0, \quad (6) \\
  P_S - \Phi_S K(1 + r^b) & \leq 0, \quad (7)
\end{align*}

where \( c_j^S, j = 1..3 \) is the \textit{jth} period non-durable consumption, \( K \) is the (fixed) value of the durable, \( w_j, j = 1..3 \) is the \textit{jth} period income with \( w_1 < K \). The savings at the end of the period \( j \) are denoted by \( A_j^S \) where \( A_1^S = w_1 - c_1 - K(1 - \Phi) \) and
\( A_2^S = w_2 + A_1^S(1 + r^l) - c_2^S - P_S \). By definition, we have \( P_S' = [\Phi_S K (1 + r^b) - P_S](1 + r^b) \). Finally, \( \lambda_k^S \) denotes the Kuhn-Tucker multiplier associated with the constraint \( k \) for \( k = 1, \ldots, 7 \). The constraints (1) to (3) are the period-by-period inter-temporal budget restrictions. In the first period, the sum of the consumption and the down payment can not exceed the income and the difference between the two constitutes the first period savings. In the second period, the sum of the consumption and the first repayment can not exceed the amount of available resources in that period, namely, the sum of the income and the savings from the first period. Any amount that is not consumed or spent is the savings in the second period and so on. The constraints (4) to (7) are there to ensure interior values for the finance share and positive repayments in each period. Later in the chapter, we will analyze the implications of relaxing the assumption of positive repayments in each period.

The sophisticated consumer’s maximization problem is solved by backwards induction. We start with the last period.

**The Last Period:** In the last period the consumer chooses \( c_3^S \) to maximize \( u(c_3^S) \). She takes as given the values chosen for \((\Phi_S, P_S, c_1^S, c_2^S)\) in the early periods. As this is the last period, she sets \( A_3^S = 0 \). The optimal choice of \( c_3^S \) from the perspective of the last period is given as

\[
c_3^S = w_3 + A_2^S(1 + r^l) - P_S'.
\]

**The Second Period:** \( c_3^S \) from the last period serves as a restriction for the choice of \( c_2^S \). The consumer again takes \( \Phi_S \) and \( c_1^S \) as given and chooses \( c_2^S \) and \( P_S \) to maximize

(P2) \[ u(c_2^S) + \beta \delta u(w_3 + A_2^S(1 + r^l) - P_S') \]

subject to

\[
\begin{align*}
c_2^S + P_S & \leq w_2 + A_1^S(1 + r^l), \\
-P_S & \leq 0, \\
P_S & \leq \Phi_S K (1 + r^b).
\end{align*}
\]

Assuming positive repayments in each period \((0 < P_S < \Phi_S K (1 + r^b))\), the first order conditions (FOCs) are given as

\[
\begin{align*}
u'(c_2^S) + \beta \delta u'(c_3^S) \frac{\partial c_3^S}{\partial c_2^S} - \lambda_2^S &= 0, \\
\beta \delta u'(c_3^S) \frac{\partial c_3^S}{\partial P_S} - \lambda_2^S &= 0, \\
\lambda_2^S A_2^S &= 0, \quad \lambda_2^S \geq 0.
\end{align*}
\]

Using \( c_3^S = w_3 + A_2^S(1 + r^l) - P_S' \), we have \( \frac{\partial c_3^S}{\partial c_2^S} = -(1 + r^l) \) and \( \frac{\partial c_3^S}{\partial P_S} = r^b - r^l \).
Rearranging the first order conditions, we get
\[ u'(c^S_2) = \beta \delta u'(c^S_3) \left[ \frac{\partial c^S_3}{\partial P} - \frac{\partial c^S_3}{\partial c^S_2} \right] = \beta \delta u'(c^S_3)(1 + r^b). \]

By assumption we have \( r^b - r^l > 0 \) and \( u' > 0 \) which implies that \( \lambda^S_2 > 0 \). Therefore we have \( A^S_2 = 0 \) and
\[ c^S_2 = w_2 + \left[ w_1 - c^S_1 - K(1 - \Phi_S) \right] (1 + r^l) - P_S. \]

This relationship serves as a restriction in the first period problem together with the first order conditions above.

**The First Period:** In the first period the consumer chooses \( c^S_1 \) and \( \Phi_S \) to maximize

\[
(P1) \quad u(c^S_1) + \beta \delta u \left( w_2 + [w_1 - c^S_1] (1 + r^l) - P_S \right) + \beta \delta^2 u \left( w_3 + A^S_2 (1 + r^l) - P_S' \right)
\]
subject to
\[
c_1^S + K(1 - \Phi_S) \leq w_1, \\
-\Phi_S \leq 0, \\
\Phi_S \leq 1.
\]

Assuming an interior value for the finance share \((0 < \Phi_S < 1)\), the first order conditions can be derived as
\[
\begin{align*}
    u'(c^S_1) + \beta \delta u'(c^S_2) \frac{\partial c^S_2}{\partial c^S_1} + \beta \delta^2 u'(c^S_3) \frac{\partial c^S_3}{\partial c^S_1} - \lambda^S_1 &= 0, \\
    \beta \delta u'(c^S_2) \frac{\partial c^S_2}{\partial \Phi_S} + \beta \delta^2 u'(c^S_3) \frac{\partial c^S_3}{\partial \Phi_S} + \lambda^S_1 K &= 0, \\
    \lambda^S_1 A^S_1 &= 0, \quad \lambda^S_1 \geq 0.
\end{align*}
\]

The sophisticated consumer, by definition, knows that she will set \( A^S_1 = 0 \). This can be used to compute
\[
\begin{align*}
    \frac{\partial c^S_2}{\partial c^S_1} &= -(1 + r^l), \quad \frac{\partial c^S_3}{\partial c^S_1} = (1 + r^l) K, \\
    \frac{\partial c^S_2}{\partial \Phi_S} &= 0, \quad \frac{\partial c^S_3}{\partial \Phi_S} = (1 + r^b)^2 K.
\end{align*}
\]

We will now show that, given \( r^b > r^l \), \( 0 < \Phi_S < 1 \) and \( 0 < P_S < \Phi_S K(1 + r^b) \), the sophisticated consumer sets \( A^S_1 = 0 \). To demonstrate this, assume that \( A^S_1 > 0 \) which implies \( \lambda^S_1 = 0 \). We rearrange the first order conditions to get
\[
\beta \delta^2 u'(c^S_3)(1 + r^b)[\beta(1 + r^l) - (1 + r^b)] = -\lambda^S_1.
\]
This is a contradiction since the assumptions \( u'(c) > 0, 0 < \beta < 1 \) and \( r^b > r^l \) dictate that the left hand side of the equation above is strictly negative. This proves that \( A^S_1 = 0 \). In summary, the first order conditions for the sophisticated consumer’s problem imply the following:

\[
\begin{align*}
  u'(c^S_1) &= \delta(1 + r^b)u'(c^S_2), \\
  u'(c^S_2) &= \beta \delta(1 + r^b)u'(c^S_3), \\
  A^S_1 &= A^S_2 = A^S_3 = 0.
\end{align*}
\]

The equations above clearly demonstrate the time-inconsistency implied by the assumption of the quasi-hyperbolic discounting. Time-inconsistency arises because the marginal rate of substitution between the first and second period consumption is different than that between the second and the third period consumption.

1.2.2 The consumer with full commitment power

The consumer with full commitment power chooses \( (c^FC_1, c^FC_2, c^FC_3, \Phi_{FC}, P_{FC}) \) in the first period to maximize

\[
\begin{align*}
  u(c^FC_1) + \beta \delta u(c^FC_2) + \beta^2 \delta^2 u(c^FC_3)
  \end{align*}
\]

subject to

\[
\begin{align*}
  c^FC_1 + K(1 - \Phi) &\leq w_1, \quad (1b) \\
  c^FC_2 + P_{FC} &\leq (1 + r^l)A^FC_1 + w_2, \quad (2b) \\
  c^FC_3 + P'_{FC} &\leq (1 + r^l)A^FC_2 + w_3, \quad (3b) \\
  \Phi_{FC} &\leq 1, \quad (4b) \\
  - \Phi_{FC} &\leq 0, \quad (5b) \\
  - P_{FC} &\leq 0, \quad (6b) \\
  P_{FC} - \Phi_{FC}K(1 + r^b) &\leq 0, \quad (7b)
\end{align*}
\]

and commits to these choices in the next periods. In the above, \( c^FC_j, j = 1..3 \) denotes the \( jth \) period non-durable consumption, \( K \) stands for the (fixed) value of the durable, and \( w_j, j = 1..3 \) is the \( jth \) period income with \( w_1 < K \). The savings at the end of the period \( j \) are denoted by \( A^FC_j \) where

\[
\begin{align*}
  A^FC_1 &= w_1 - c^FC_1 - K(1 - \Phi_{FC}) \\
  A^FC_2 &= w_2 + A^FC_1(1 + r^l) - c^FC_2 - P_{FC}.
\end{align*}
\]

The repayment in the last period is defined as \( P'_{FC} = \Phi_{FC}K(1 + r^b) - P_{FC} \) and the lagrange multiplier associated with the constraint \( ib, i = 1..7 \), is denoted by \( \lambda_{ib} \). The first order conditions for this problem
are

\[ u'(c_1^{FC}) - \lambda_{1b} - \lambda_{2b}(1 + r^i) - \lambda_{3b}(1 + r^i)^2 = 0, \quad (1.1) \]
\[ \beta \delta u'(c_2^{FC}) - \lambda_{2b} - \lambda_{3b}(1 + r^i) = 0, \quad (1.2) \]
\[ \beta \delta^2 u'(c_3^{FC}) = \lambda_{3b}, \quad (1.3) \]
\[ \lambda_{1b} K + \lambda_{2b} K(1 + r^i) - \lambda_{3b} K(1 + r^b)^2 + \lambda_{3b} K(1 + r^i)^2 - \lambda_{4b} + \lambda_{5b} + \lambda_{7b} K(1 + r_b) = 0, \quad (1.4) \]
\[ -\lambda_{2b} - \lambda_{3b}(1 + r^i) + \lambda_{3b}(1 + r^b) + \lambda_{5b} - \lambda_{7b} = 0, \quad (1.5) \]
\[ \lambda_{1b} A_1^{FC} = 0, \quad \lambda_{2b} A_2^{FC} = 0, \quad \lambda_{3b} A_3^{FC} = 0, \quad \lambda_{1b}, \lambda_{2b}, \lambda_{3b} \geq 0, \quad (1.6) \]
\[ \lambda_{4b} \Phi_{FC} = 0, \quad \lambda_{5b}(1 - \Phi_{FC}) = 0, \quad \lambda_{6b} P_{FC} = 0, \quad (1.7) \]

We will assume again an interior value for the finance share \((0 < \Phi_{FC} < 1)\) and positive repayments in each period \((0 < P_{FC} < \Phi_{FC} K(1 + r^b))\). This leaves us with \(\lambda_{4b} = \lambda_{5b} = \lambda_{6b} = \lambda_{7b} = 0\). Therefore, the equation (1.5) implies that \(\lambda_{3b}[r^b - r^i] = \lambda_{2b}\). Using the assumption \(r^b > r^i\) and the equation (1.3), we have \(\lambda_{3b} > 0\) and therefore, \(\lambda_{2b} > 0\). Thus, \(A_2^{FC} = A_3^{FC} = 0\).

To demonstrate that \(A_1^{FC} = 0\), let’s proceed by assuming \(A_1^{FC} > 0\), which implies \(\lambda_{1b} = 0\). Using \(\lambda_{1b} = 0\) in the equations (1.1) and (1.2), we obtain

\[ u'(c_1^{FC}) = \lambda_{2b} + \lambda_{3b}(1 + r^i)(1 + r^i) = \beta \delta u'(c_2^{FC})(1 + r_i). \]

Moreover, combining the equations (1.1), (1.2), (1.4) and (1.5), we get

\[ u'(c_1^{FC}) = \lambda_{2b} + \lambda_{3b}(1 + r^i)(1 + r^i) = \lambda_{3b}(1 + r^b)(1 + r^b) = \lambda_{2b} + \lambda_{3b}(1 + r^i)(1 + r^b) = \beta \delta u'(c_2^{FC})(1 + r_b). \]

This clearly is a contradiction since we assumed \(r^b > r^i\). Thus, we have \(A_1^{FC} = 0\). In conclusion, first order conditions for the maximization problem of the consumer with full commitment power imply the following:

\[ u'(c_1^{FC}) = \beta \delta (1 + r^b) u'(c_2^{FC}), \]
\[ u'(c_2^{FC}) = \delta u'(c_3^{FC})(1 + r^b), \]
\[ A_1^{FC} = A_2^{FC} = A_3^{FC} = 0. \]

The equations above show that the marginal rate of substitution between \(c_1^{FC}\) and \(c_2^{FC}\) is different than that between \(c_2^{FC}\) and \(c_3^{FC}\). That is, the marginal rate of substitution is time-varying. This is due to the assumption of quasi-hyperbolic discounting.

### 1.2.3 The naive consumer

The naive consumer solves the same first period problem as the consumer with full commitment power. They exhibit the same amount of consumption in the first period,
$c_1$, and the same fraction $\Phi$ to be financed. In the second period, the naive consumer chooses $(c_2^N, c_3^N, P_N)$ to maximize

$$u(c_2^N) + \beta \delta u(c_3^N)$$

subject to

$$- P_N \leq 0, \quad (1c)$$
$$c_2^N + P_N \leq w_2, \quad (2c)$$
$$c_3^N \leq (1 + r^l) A_2^N + w_3 - P', \quad (3c)$$
$$P_N \leq \Phi N (1 + r_b), \quad (4c)$$

where $c_j^N, j = 1..3$ stands for the $j$th period non-durable consumption, $K$ is the (fixed) value of the durable, and $w_j, j = 1..3$ is the $j$th period income with $w_1 < K$. The savings at the end of the period $j$ are denoted by $A_j^N$ where $A_1^N = w_1 - c_1^N - K(1 - \Phi_N)$ and $A_2^N = w_2 + A_1^N (1 + r^l) - c_2^N - P_N$. The repayment in the last period is defined as $P_N' = [\Phi_N K (1 + r_b) - P_N] (1 + r^b)$ and the lagrange multiplier associated with the constraint $ic, i = 1..7$, is denoted by $\lambda_{ic}$. This problem admits the following first order conditions:

$$u'(c_2^N) = \lambda_{2c} + \lambda_{3c} (1 + r^l),$$
$$\beta \delta u'(c_3) = \lambda_{3c},$$
$$- \lambda_{2c} - \lambda_{3c} (1 + r^l) + \lambda_{3c} (1 + r^b) = 0,$$
$$\lambda_{2c} A_2^N = 0, \quad \lambda_{3c} A_3^N = 0, \quad \lambda_{2c}, \lambda_{3c} \geq 0.$$

Assuming an interior value for the finance share and positive repayments in each period, the naive consumer’s first order conditions can be used to derive

$$u'(c_2^N) = \beta \delta (1 + r^b) u'(c_3^N), \quad (1.8)$$
$$A_1^N = A_2^N = 0.$$

Given the second period’s choice as above, the last period’s optimal choice is to set $A_3^N = 0$ and consume $w_3 - P_N'$.

### 1.2.4 Optimal finance share and repayment amounts

In order to explicitly calculate the finance share and the periodic payments implied by the first order conditions in each problem, we will assume that per-period utility for each type of consumer is logarithmic, $u(c) = \ln(c)$.

We begin by the case of the sophisticated consumer. First, we substitute for $c_2 = w_3 - P_S$ and $c_2 = w_2 - P_S$ in $u'(c_2^S) = \beta \delta (1 + r^b) u'(c_3^S)$ and solve for the
repayment in the second period \( (P_S) \) as a function of the optimal finance share \( (\Phi_S) \) as follows:

\[
P_S = \frac{\beta \delta w_2 (1 + r^b) + \Phi_S K (1 + r^b)^2 - w_3}{(1 + r^b)(1 + \beta \delta)}.
\]  

(1.9)

Next, we substitute for \( c_1^S = w_1 - K(1 - \Phi) \) and \( c_2^S = w_2 - P_S \) in the equation \( u'(c_1^S) = \delta (1 + r^b) u'(c_2^S) \) to get

\[
P_S = w_2 - \delta (1 + r^b) w_1 + \delta (1 + r^b) K - \delta (1 + r^b) K \Phi_S.
\]  

(1.10)

Finally, we use the equations (1.9) and (1.10) to solve for \( \Phi_S \) as follows:

\[
\Phi_S = \frac{1}{K (1 + r^b) (1 + \delta (1 + \beta \delta))} \left[ w_2 + \frac{w_3}{1 + r^b} \right] 
\]

\[
... + \frac{\delta (1 + \beta \delta)}{(1 + \delta (1 + \beta \delta))} \left[ 1 - \frac{w_1}{K} \right].
\]  

(1.11)

The first period payment, \( P_S \), can be calculated by substituting back for \( \Phi_S \) in either equation (1.9) or equation (1.10). The repayment in the last period, \( P'_S \), can be found by substituting for \( P_S \) in \( P'_S = [\Phi K (1 + r^b) - P_S] (1 + r^b) \).

As for the consumer with full commitment power, \( \Phi_{FC} \) and \( P_{FC} \), finance share and the first repayment can be solved following exactly the same steps as before. First, we derive the two equations for \( P_{FC} \) as follows:

\[
P_{FC} = \frac{\delta w_2 (1 + r^b) + \Phi_{FC} K (1 + r^b)^2 - w_3}{(1 + r^b)(1 + \delta)}, \]

(1.12)

\[
P_{FC} = w_2 - \beta \delta (1 + r^b) w_1 + \beta \delta (1 + r^b) K - \beta \delta (1 + r^b) K \Phi_{FC}.
\]  

(1.13)

Then, we solve for \( \Phi_{FC} \) as

\[
\Phi_{FC} = \frac{1}{K (1 + r^b) (1 + \beta \delta (1 + \delta))} \left[ w_2 + \frac{w_3}{1 + r^b} \right] 
\]

\[
... + \frac{\beta \delta (1 + \delta)}{(1 + \beta \delta (1 + \delta))} \left[ 1 - \frac{w_1}{K} \right].
\]  

(1.14)

The naive consumer solves the same first period problem as the consumer with full commitment power. Therefore, their choice of the fraction financed must be the same. They differ with respect to the optimal decisions made in the second period. As the second period arrives, the consumer with full commitment power sticks to the choices she made in the first period but the the naive consumer revises her plans. She chooses a different \( P \) than she originally planned. In conclusion, we have

\[
\Phi_N = \Phi_{FC},
\]
and the equation (1.8) can be used to obtain the naive consumer’s second period repayment as follows:

\[ P_N = \frac{\beta \delta w_2 (1 + r^b) + \Phi_N K (1 + r^b)^2 - w_3}{(1 + r^b)(1 + \beta \delta)}. \]  
(1.15)

1.2.5 Comparison with the exponential consumer

In this section, we will compare each type of consumer’s choice of the fraction financed and the repayments in the second and third periods. Recall that we consider four different types; the sophisticated consumer, the consumer with full commitment power, the naive consumer and the exponential consumer.

Let us begin by our comparison with the sophisticated and the exponential types. Exponential discounting is a special case of hyperbolic discounting with \( \beta = 1 \). Therefore, the relevant comparative statics exercise is to look at how \( \Phi_S \) and \( P_S \) change with \( \beta \). The total derivative of \( P_S \) can be expressed as follows:

\[ \frac{dP_S}{d\beta} = \frac{\partial P_S}{\partial \Phi_S} \frac{d\Phi_S}{d\beta} + \frac{\partial P_S}{\partial \Phi_S} \frac{d\Phi_S}{d\beta}. \]

The Lemmas 1.1, 1.2 and 1.3 below summarize our results.

**Lemma 1.1** In an interior solution for consumption in each period, i.e. for \( c_i > 0, i = 1, 3 \), we have

\[ \frac{\partial P_S}{\partial \Phi_S} > 0, \frac{\partial \Phi_S}{\partial \beta} < 0 \text{ and } \frac{\partial P_S}{\partial \beta} > 0. \]

An immediate consequence of this lemma is that \( \Phi_S > \Phi_{Exp} \). In other words, the sophisticated consumer finances a higher fraction of the loan compared to the exponential consumer.

**Lemma 1.2** In an interior solution for consumption in each period, we have

\[ \frac{dP_S}{d\beta} > 0 \]

The previous lemma showed that as \( \beta \) increases, the fraction financed increases. The larger the fraction financed, the higher the debt burden is in the next periods. Lemma 1.2 argues that in the case of the sophisticated consumer, the effect of higher \( \Phi \) on \( P \) is offset by the effect of higher \( \beta \) on \( P \), leaving \( \frac{dP_S}{d\beta} > 0 \), provided that the consumption in each period is strictly positive.

**Lemma 1.3** Assume an interior solution for consumption in each period. We have \( P_S < P_{Exp} \) iff \( \Phi_{Exp} < \Phi_S \).
In summary, when compared with the exponential consumer, the sophisticated consumer borrows more. Hence, she puts less money down. Moreover, she chooses to repay less in the second period.

In order to fully understand the effect of self-control on the loan demand as well as the repayment pattern, we need to compare the sophisticated consumer with a consumer that has the same preferences but also has the full commitment power. The following proposition makes that comparison.

**Lemma 1.4** \( \Phi_S < \Phi_{FC} \) and therefore \( P_S < P_{FC} \).

That is, when compared with a consumer with the same preferences but with full commitment power, the sophisticated consumer borrows less and repays less in the next period. This proposition shows the implications of the conflict that the sophisticated consumer faces as a result of her lack of self-control. The sophisticated consumer knows that, when the next period comes, she will end up paying less than she thinks she should. This results in her borrowing less than she would if she had a way to commit to her choices.

Finally, in order to analyze the consequences of not being aware of self-control problems, we look at how the naive consumer’s choice compares with other types. The following lemma summarizes our findings.

**Lemma 1.5** \( \Phi_S < \Phi_N = \Phi_{FC} \) and \( P_S < P_N < P_{FC} \).

The naive consumer and the consumer with full commitment power borrows the same amount. However, the naive consumer repays less debt in the next period relative to the consumer with full commitment power. More interestingly, the sophisticated consumer exercises self-control by restricting her borrowing, and ends up repaying less in the next period than both the naive consumer and the consumer with full commitment power.

Finally, combining Lemmas 1.1-1.5, we summarize our results in the following proposition.

**Proposition 1.6** \( \Phi_{Exp} < \Phi_S < \Phi_N = \Phi_{FC} \), \( P_{Exp} > P_S \) and \( P_S < P_N < P_{FC} \).

The exponential consumer borrows less than the other types. The sophisticated consumer borrows more than the exponential consumer because she is more impatient and wishes to consume more now. For the same reason, even though she has borrowed more, the sophisticated consumer repays less in the second period compared to the exponential consumer.

If the sophisticated consumer had the commitment power, she would borrow even more. In fact, she would borrow at the same level as the consumer with the full commitment power. This is exactly what the naive consumer does. Both the naive and
the sophisticated consumer’s current preferences imply the same optimal allocation of repayments over the following two periods. These are the allocations that the consumer with full commitment power chooses. The sophisticated consumer knows that she can not commit to these allocations. As a result, she does not borrow as much as she wishes. She knows that her next self is as impatient as she is and would choose to consume more at the expense of repaying less towards the debt. She restricts her borrowing because she knows she can not commit herself. The naive consumer, not being aware of her own self-control problems, borrows as much as the consumer with the full commitment power but ends up paying less in the second period.

We imposed interior solutions for the fraction financed, consumption in each period, and positive repayments in the second and third periods. We will now analyze under which conditions on the parameters \( w_i, \beta, \delta, K \) of the model those interior solutions occur. The lemmas below summarizes our results.

**Proposition 1.7** The sufficient condition on the parameters \( \{ w_i, \beta, \delta, K, r^b \} \) of the model under which we have \( P > 0, P' > 0, 0 < \Phi < 1 \) and \( c_i > 0, i = 1, 2, 3 \) for all types of the consumers can be written as follows:

\[
K \in \left( \max[K_1, K_2, K_3], \overline{K} \right),
\]

where \( K_1, K_2, K_3 \) and \( \overline{K} \) are given as

\[
\overline{K} = w_1 + \frac{w_2}{1 + r^b} + \frac{w_3}{(1 + r^b)^2},
\]

\[
K_1 = w_1 + \frac{w_2}{1 + r^b} - (1 + \delta) \frac{w_3}{(1 + r^b)^2},
\]

\[
K_2 = w_1 - (1 + \beta \delta) \frac{w_2}{1 + r^b} + \frac{w_3}{(1 + r^b)^2},
\]

\[
K_3 = \beta \delta (1 + \delta) w_1 + \frac{w_2}{1 + r^b} + \frac{w_3}{(1 + r^b)^2}.
\]

The requirement that the value of the durable must not exceed the present value of future wages is necessary to obtain interior solution for consumption in each period. Note that the condition in Proposition 1.7 neither requires the wage pattern to be increasing nor it is satisfied for all increasing wage patterns. For instance, if the third period wage is sufficiently high, the consumer would like to borrow more than the value of the durable in the first period, violating \( \Phi < 1 \). Similarly, this can result in the consumer wanting to increase her borrowing in the next period violating \( P > 0 \). In the next section, we provide a numerical example with parameters values which are chosen to satisfy the sufficient condition described in the proposition above.
A numerical example with logarithmic utility

In order to demonstrate the findings in Lemmas 1.1-1.5 and in Proposition 1.6, we give here a numerical example with \( u(c_i) = \ln(c_i), i = 1, 2, 3 \). The values we assume for the parameters are as follows: \( K = 100, w_2 = 30, w_3 = 60, \delta = 0.9, r_b = 5\% \) and \( r_l = 4\% \). Given these values, consumption in each period is zero at around \( w_1 = 17 \). For illustration purposes, we assume that \( w_1 = 20 \), which makes \( c_i > 0, i = 1..3, \) for all types. In figures 1.1 and 1.2 we graph \( \Phi, P \) and \( P' \) against \( \beta \) for each consumer type, as \( \beta \) varies between 0 and 1.

Figure 1.3 looks at how consumption in each period changes with \( \beta \) for each type of consumer. Recall that \( \beta = 1 \) corresponds to the case of the exponential consumer. In this particular numerical example, for all consumer types, we have \( c_1 > c_2 > c_3 \). In the case of sophisticated consumer, \( c_1 \) and \( c_2 \) decreases whilst \( c_3 \) increases with \( \beta \). As for the naive consumer and the consumer with full commitment power, \( c_1 \) decreases while \( c_2 \) and \( c_3 \) increase with \( \beta \). In the case of the naive consumer the difference between the second and third period consumption first increases and then decreases with \( \beta \), as \( \beta \to 1 \).

Figure 1.4 shows the allocation of repayments for each consumer type when \( \beta = 0.9 \). The exponential consumer puts more money down, pays more in the second period and less in the third period as opposed to the naive consumer.

Figure 1.5 graphs the loan demand and the total price paid for the durable across types, again for \( \beta = 0.9 \). We see that the exponential consumer borrows less and pays less (in terms of the total price) than all other types do. The naive consumer borrows the most and pays the highest total price. The sophisticated consumer would borrow more if she could commit to her choices. The consumer with full commitment power pays less for the durable in terms of total price than the naive consumer even though they have borrowed the same amount.

1.2.6 Corner solutions for \( P \) and \( P' \)

Analysis of the previous section assumes strictly positive repayment amounts in each period, that is we impose that \( P \in (0, \Phi K(1 + r^b)) \) for each type. In this section, we will discuss the implications of relaxing this assumption for the repayment behavior for each type of consumer. The proposition below summarizes the results.

**Proposition 1.8** If consumers are allowed to postpone the payment of the debt until the last period, first period savings can be zero or positive. If they can pay their debt in full in the second period, the second period savings can be zero or positive. In all cases, we have \( \Phi_{Exp} < \Phi_S = \Phi_{FC} = \Phi_N \).

If we allow consumers to pay nothing in the second period, that is, if we let consumers choose \( P = 0 \), there are two different solutions (for each type): one with
$A_1 > 0$ and another with $A_1 = 0$. The savings $A_2$ and $A_3$ are both equal to zero in each of these solutions. Note that, when $P = 0$, we have $P' = \Phi K(1 + r^b)$. That is, raising $\Phi$ raises $P'$. In either of these solutions all consumers, except the exponential consumer, choose to borrow exactly the same amount. They all borrow more, and consequently pay more in the last period, than the exponential consumer. The intuition behind this result is as follows. When $P = 0$, the difference between having full or having no commitment power vanishes. The sophisticated consumer and the naive consumer choose the same repayment amount ($P = 0$) next period. As a result, the sophisticated consumer chooses the same fraction financed as the consumer with full commitment power does. The exponential consumer borrows less because she is more patient compared to the others.

Let us now look at the case where consumers can pay their debt in full in the second period, that is when $P = \Phi K(1 + r^b)$ and $P' = 0$. In this case, there are again two different solutions (for each type): one with $A_2 > 0$ and another with $A_2 = 0$. In both cases, we have $A_1 = 0$ and $A_3 = 0$. Both solutions imply the same result: (i) the exponential consumer borrows less than the sophisticated consumer (ii) the sophisticated consumer borrows the same amount as the naive consumer and the consumer with full commitment power. The case of $P = \Phi K(1 + r^b)$ simply represents the situation where the loan maturity is one period. Moreover, consumers do not save in the first period ($A_1 = 0$). Interestingly, whilst dynamically inconsistent preferences affect the allocation of consumption between the second and third periods, they do not impact the choice of the fraction financed. We attribute this to the peculiarity of the logarithmic utility. To see this, consider the first order conditions for the maximization problem of the sophisticated consumer and the consumer with full commitment power when $A_2 > 0$, $0 < \Phi < 1$, and $P = \Phi K(1 + r^b)$. The sophisticated consumer’s FOC implies

$$u'(c^S_2) = \beta \delta u'(c^S_3)(1 + r^l),$$

$$\beta \delta^2 (1 - \beta) u'(c^S_3) \frac{\partial c^S_3}{\partial \Phi} (1 + r^l) + \beta \delta^2 u'(c^S_3)(1 + r^b)(1 + r^l) = u'(c^S_3).$$

Similarly, the FOC for the consumer with full commitment power gives

$$u'(c^{FC}_2) = \delta u'(c^{FC}_3)(1 + r^l),$$

$$u'(c^{FC}_1) = \beta \delta^2 u'(c^{FC}_3)(1 + r^l)(1 + r^b).$$

Note that, when $\beta = 1$ FOC for both the sophisticated and the naive consumers reduce to the set of equations that describe the exponential consumer’s optimal choice for $\Phi$ and $c_i$, $i = 1...3$. When $\beta < 1$ and $u(c) = \ln(c)$, these equations imply the same optimal fraction financed for the sophisticated consumer and the consumer with full commitment power.$^3$ The conflict between selves does not impact the choice

$^3$The proof can be found in the Appendix.
of the sophisticated consumer when the loan has a maturity of one period. Therefore, the payment in the next period also does not differ for the sophisticated and the naive types. However, the sophisticated consumer saves less and consumes more in the second period as opposed to the consumer with full commitment power. The effect of dynamic inconsistency manifests itself only via the allocation of consumption between the second and third periods. The same argument applies to the case of \( A_1 = A_2 = A_3 = 0. \)

### 1.3 Conclusion

We have demonstrated that dynamically inconsistent preferences imply higher levels of borrowing. However, the level of borrowing is negatively effected by the degree of awareness of self-control problems and positively affected by the ability to commit future choices. We proved these by showing that (i) the sophisticated consumer’s awareness of her own self-control problems results in her restricting her level of borrowing (ii) the sophisticated consumer would borrow more if she had the means to commit her choices.

Our findings also suggest that, self-control problems can induce a preference for repayment schedules with higher repayments towards the end of the loan term. There are retail installment loans as well as mortgages that offer such back-loaded repayment plans. These type of loans are typically marketed to subprime borrowers. As the recent US subprime mortgage crisis testifies, these are the loans with the worst level of delinquency and default rates.
Figure 1.1: Finance rates $\Phi_S$, $\Phi_{FC}$ and $\Phi_N$ against $\beta$
Figure 1.2: Payments in second and third periods against $\beta$
Figure 1.3: Consumption in each period against $\beta$

(a) Sophisticated consumer

(b) Naive consumer

(c) Consumer with full commitment power
Figure 1.4: Allocation of payments across periods, $\beta = 0.9$
Figure 1.5: Loan demand and the total price paid across types, $\beta = 0.9$
Appendices
.A Proofs

Proof of Lemma 1.1

First, we take the derivative of $\Phi_S$ with respect to $\beta$ in equation (1.11) to obtain

$$\frac{\partial \Phi_S}{\partial \beta} = \frac{\delta^2}{K[1 + \delta(1 + \beta \delta)]^2} \left[ K - w_1 - \frac{w_2}{1 + r^b} - \frac{w_3}{(1 + r^b)^2} \right].$$

If $c_i^S > 0$ for $i = 1, 3$, then $K < w_1 + \frac{w_2}{1 + r^b} + \frac{w_3}{(1 + r^b)^2}$. It follows that, in an interior solution for consumption in each period, we have $\frac{\partial \Phi_S}{\partial \beta} < 0$.

Next, we compute $\frac{\partial P_S}{\partial \beta}$ using (1.10) as follows:

$$\frac{\partial P_S}{\partial \beta} = \frac{\delta}{(1 + \beta \delta)^2} \left[ w_2 - \Phi_S K(1 + r^b) + \frac{w_3}{1 + r_b} \right].$$

When $c_i^S > 0$ for $i = 1, 3$, we have $w_2 + \frac{w_3}{1 + r_b} > \Phi_S K(1 + r^b)$. Thus, in an interior solution for consumption, we have $\frac{\partial P_S}{\partial \beta} > 0$.

Finally, $\frac{\partial P_S}{\partial \Phi_S}$ can be computed as

$$\frac{\partial P_S}{\partial \Phi_S} = \frac{K(1 + r^b)}{1 + \beta \delta} > 0.$$

This completes the proof. $\square$

Proof of Lemma 1.2

As shown in Lemma 1.1, we have

$$\frac{dP_S}{d\beta} = \frac{\delta(1 + r^b)}{1 + \beta \delta} \left[ \frac{\delta[K - w_1]}{[1 + \delta(1 + \beta \delta)]^2} - \frac{\Phi_S K}{1 + \beta \delta} \right].$$

This implies that

$$\frac{dP_S}{d\beta} > 0 \text{ iff } \Phi_S < \frac{\delta(1 + \beta \delta)}{[1 + \delta(1 + \beta \delta)]^2} \left[ 1 - \frac{w_1}{K} \right].$$

Using equation 1.11, one can show that

$$\Phi_S = \frac{1}{K(1 + r^b)(1 + \delta(1 + \beta \delta))} \left[ w_2 + \frac{w_3}{1 + r^b} \right] + \frac{\delta(1 + \beta \delta)}{(1 + \delta(1 + \beta \delta))} \left[ 1 - \frac{w_1}{K} \right] < \frac{\delta(1 + \beta \delta)}{[1 + \delta(1 + \beta \delta)]^2} \left[ 1 - \frac{w_1}{K} \right].$$

This completes the proof. $\square$
Proof of Lemma 1.3
Evaluating the equations (1.9) and (1.10) at $\beta = 1$, it is easy to show that the following two equations must simultaneously hold:

\[ P_S - P_{Exp} = \delta(1 + r^b)[\Phi_{Exp} - \Phi_S], \] 
\[ .16 \]
\[ P_S - P_{Exp} = \frac{K(1 + r^b)}{(1 + \beta\delta)(1 + \delta)} \left[ (1 + \beta\delta)\Phi_S - (1 + \delta)\Phi_{Exp} \right]. \] 
\[ .17 \]

The equation (.16) implies that $P_S < P_{Exp}$ iff $\Phi_{Exp} < \Phi_S$. The equation (.17) implies that $P_S < P_{Exp}$ iff $(1 + \beta\delta)\Phi_S < (1 + \delta)\Phi_{Exp}$. This completes the proof. □

Proof of Lemma 1.4
We are going to prove that $\Phi_S < \Phi_{FC}$ and $P_S < P_{FC}$. Using equations (1.11) and (1.14), it is straightforward to show that $\Phi_S < \Phi_{FC}$. Moreover, equations (1.9), (1.10), (1.12) and (1.13) imply that the following must simultaneously hold:

\[ P_{FC} - P_S = \delta(1 + r^b)(1 - \beta)[w_1 - K] + \delta(1 + r^b)K[\Phi_S - \beta\Phi_{FC}] \] 
\[ .18 \]
\[ P_{FC} - P_S = \frac{1}{(1 + \delta)(1 + \beta\delta)} \left[ \delta(1 - \beta)[w_2 + \frac{w_3}{1 + r^b}] \right. \]
\[ + K(1 + r^b)[(1 + \beta\delta)\Phi_{FC} - (1 + \delta)\Phi_S] \] 
\[ .19 \]

We know that $\Phi_{FC} > \Phi_S$. Assume that $\beta\Phi_{FC} > \Phi_S$ and therefore, $(1 + \beta\delta)\Phi_{FC} - (1 + \delta)\Phi_S > 0$. Equation (.19), implies that $P_{FC} - P_S > 0$ and $\Phi_S > \beta\Phi_{FC}$, which is a contradiction. This completes the proof. □

Proof of Lemma 1.5
Using equations (1.12) and (1.15), we get

\[ P_{FC} = P_N + \frac{\delta(1 - \beta)}{(1 + \delta)(1 + \beta\delta)} \left[ w_2 + \frac{w_3}{1 + r^b} - \Phi_{FC}K(1 + r^b) \right]. \]

Assuming an interior solution for the consumption in each period implies that we have $(w_2 + \frac{w_3}{1 + r^b}) > \Phi_{FC}K(1 + r^b)$. Thus, $P_{FC} > P_N$ for $\beta < 1$. Note that, for $\beta = 1$, $P_{FC} = P_{NC}$ as expected.

As for the naive and the sophisticated consumer’s repayments in the second period, one can use equations (1.9) and (1.15) to obtain

\[ P_S - P_N = \frac{K(1 + r^b)}{1 + \beta\delta} \left[ \Phi_S - \Phi_N \right]. \]
Since the naive consumer and the consumer with full commitment power solve the same first period problem, they chose the same fraction financed. That is, we have $\Phi_N = \Phi_{FC} > \Phi_S$ which implies that $P_S < P_N$. □

Proof of Proposition 1.6

The proof follows from the Lemmas 1.1-1.5. □

Proof of Proposition 1.7

Proof relies on the Proposition 1.6. First note that if $\Phi_{FC} < 1$, then

$$\Phi_{Exp} < \Phi_S < \Phi_N = \Phi_{FC} < 1.$$ 

Using equation (1.14), $\Phi_{FC} < 1$ iff

$$K > \beta\delta(1 + \delta)w_1 + \frac{w_2}{1 + rb} + \frac{w_3}{(1 + rb)^2}.$$ 

Second, $P_S > 0$ implies $0 < P_S < P_{Exp}$ and $0 < P_S < P_N < P_{FC}$. Moreover, we have

$$P_S > 0 \iff \Phi_S < 1 - \frac{w_1}{K} + \frac{w_2}{\delta K(1 + rb)^2}.$$ 

The equation (1.11) can be used to prove that $\Phi_S$ satisfies the condition above iff the following holds:

$$K > w_1 - (1 + \beta\delta) \frac{w_2}{1 + rb} + \frac{w_3}{(1 + rb)^2}.$$ 

Third, it is easy to show that the following is true:

$$P_{Exp}' > 0 \iff \Phi_{Exp} > \frac{w_2}{K(1 + rb)} - \frac{w_3}{\delta K(1 + rb)^2},$$

$$P_{FC}' > 0 \iff \Phi_{FC} > \frac{w_2}{K(1 + rb)} - \frac{w_3}{\delta K(1 + rb)^2},$$

$$P_N' > 0 \iff \Phi_N > \frac{w_2}{K(1 + rb)} - \frac{w_3}{\beta\delta K(1 + rb)^2},$$

$$P_S' > 0 \iff \Phi_S > \frac{w_2}{K(1 + rb)} - \frac{w_3}{\beta\delta K(1 + rb)^2},$$

where, we have

$$\frac{w_2}{K(1 + rb)} - \frac{w_3}{\delta K(1 + rb)^2} < \frac{w_2}{K(1 + rb)} - \frac{w_3}{\beta\delta K(1 + rb)^2}.$$
The inequality above and the Proposition 1.6 imply that, if the condition for \( P_{\text{Exp}}' > 0 \) is satisfied, then the conditions \( P_{S}' > 0, P_{N}' > 0 \) and \( P_{F C}' > 0 \) are also satisfied. The necessary and sufficient condition for \( P_{\text{Exp}}' > 0 \) is

\[
K > w_1 + \frac{w_2}{1 + r^b} - (1 + \delta) \frac{w_3}{(1 + r^b)^2}.
\]

Fourth, the following is true for all types:

\[
c_1, c_2 > 0 \iff \Phi > 1 - \frac{w_1}{K},
\]

\[
c_3 > 0 \iff \Phi < \frac{w_3}{K(1 + r^b)^2} + \frac{w_2}{K(1 + r^b)}.
\]

Again, using the Proposition 1.6, this implies that the sufficient condition for \( c_1, c_2, c_3 > 0 \) (for all types) are

\[
\Phi_{\text{Exp}} > 1 - \frac{w_1}{K} \quad \text{and} \quad \Phi_{\text{FC}} < \frac{w_3}{K(1 + r^b)^2} + \frac{w_2}{K(1 + r^b)}
\]

The two conditions above are satisfied iff

\[
K_1 < w_1 + \frac{w_2}{1 + r^b} + \frac{w_3}{(1 + r^b)^2}.
\]

Finally, the assumption \( w_1 < K \) guarantees that \( 1 - \frac{w_1}{K} > 0 \). Therefore, in an interior solution for consumption, we have \( \Phi > 0 \) for all consumer types. This concludes the proof. \( \square \)

**Proof of Proposition 1.8**

Consider first the case of \( P = 0 \) with \( 0 < \Phi < 1 \) (for all types). Sophisticated consumer uses backward induction to solve her optimization problem. It follows from \( P_{S} = 0 \) that we have \( P_{S} < \Phi_{S}K(1 + r^b) \) and therefore \( \lambda^S_{1} = 0 \). We also have \( \lambda^S_{4} = \lambda^S_{5} = 0 \) by the assumption that \( 0 < \Phi_{S} < 1 \). The sophisticated consumer’s maximization problem in the second period, (P2), give rise to following first order conditions:

\[
\beta \delta u'(c^S_3)(r^b - r^l) = \lambda^S_2 - \lambda^S_6.
\]

Assume that \( A^S_2 > 0 \), which implies \( \lambda^S_2 = 0 \). Substituting for \( \lambda^S_2 = 0 \) in equation (.20), we obtain \( \beta \delta u'(c^S_3)(r^b - r^l) = -\lambda^S_6 \). This is a contradiction since \( r^b > r^l \) and \( \lambda^S_6 \geq 0 \). Thus we must have \( A^S_2 = 0 \) in any solution. Assume that \( A^S_1 = 0 \). The sophisticated consumer’s maximization problem in the first period, (P1), imply the following first order condition:

\[
u'(c^S_1) = \beta \delta^2 (1 + r^b)^2 u'(c^S_3).
\]

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Having $A_1^S = 0$ implies $c_1^S = w_1 - K(1 - \Phi_S)$ and $c_3^S = w_3 - \Phi_S K(1 + r^b)$. Assuming $u(c) = \ln(c)$, and substituting for $c_1^S$ and $c_3^S$ in equation (.21), we can solve for $\Phi_S$ as follows:

$$\Phi_S = \frac{w_3 - \beta \delta^2 (1 + r^b)^2 [w_1 - K]}{K (1 + r^b)^2 (1 + \beta \delta^2)}.$$  

The derivative of $\Phi_S$ with respect to $\beta$, given below, is negative for $c_i > 0$, $i = 1..3$.

$$\frac{\partial \Phi_S}{\partial \beta} = -\frac{\delta^2 [(1 + r^b)^2 (w_1 - K) + w_3]}{K (1 + r^b)^2 (1 + \beta \delta^2)^2}.$$  

This implies that $\Phi_S > \Phi_{Exp}$. Assume now that $A_1^S > 0$, and therefore $\lambda_1^S = 0$. The first and the second period maximization problems, (P1) and (P2), together imply the following first order conditions:

$$u'(c_1^S) = (1 + r^l) \beta \delta u'(c_2^S),$$  

$$u'(c_2^S)(1 + r^l) = \delta u'(c_3^S)(1 + r^b)^2.$$  

With $u(c) = \ln(c)$ and $A_2^S = A_3^S = 0$, consumption in each period can be solved as

$$c_1^S = \frac{c_2^S}{\beta \delta (1 + r^l)};$$  

$$c_2^S = \frac{\beta \delta}{1 + \beta \delta} \left( w_2 + (w_1 - K)(1 - \Phi_S)(1 + r^l) \right);$$  

$$c_3^S = w_3 - \Phi_S K(1 + r^b)^2.$$  

Substituting for $c_i^S$, $i = 1, 2, 3$ in equations (.22) and (.23) yields:

$$\Phi_S = \frac{1}{K (1 + \beta \delta + \beta^2 \delta^2)} \left[ w_3 \frac{1 + \beta \delta}{(1 + r^b)^2} - w_2 \frac{\beta \delta^2}{1 + r^l} - [w_1 - K] \beta \delta^2 \right].$$  

Taking the derivative of $\Phi_S$ with respect to $\beta$, we get

$$\frac{\partial \Phi_S}{\partial \beta} = \frac{\left( \beta^2 (1 - \delta) - 1 \right) \left[ w_2 (1 + r^b)^2 + (1 + r^l)[w_3 + (w_1 - K)(1 + r^b)^2] \right]}{(1 + \beta \delta + \beta^2 \delta^2) K (1 + r^l)(1 + r^b)^2}.$$  

It is easy to show that $c_i^S > 0$, $i = 1..3$ implies $\frac{\partial \Phi_S}{\partial \beta} < 0$. Therefore, we have $\Phi_S > \Phi_{Exp}$. First order conditions for the consumer with full commitment power, namely the equations (1.1)-(1.7), imply

$$u'(c_1^{FC}) = \beta \delta^2 (1 + r^b)^2 u'(c_3^{FC}) \text{ when } A_1^{FC} = 0,$$

$$u'(c_2^{FC})(1 + r_i) = \delta u'(c_3^{FC})(1 + r^b)^2 \text{ when } A_1^{FC} > 0,$$

$$u'(c_1^{FC}) = \beta \delta (1 + r^l) u'(c_2^{FC}) \text{ when } A_1^{FC} \geq 0.$$  

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Comparing the equations above with equations (.21), (.22) and (.23), we conclude that $\Phi_S = \Phi_F C$. Finally, by definition, the naive consumer chooses $\Phi_N = \Phi_S$ both for $A_N^1 = 0$ and $A_N^1 > 0$.

Consider now the case of $P = \Phi_K (1 + r^b)$ with $0 < \Phi < 1$. The fact that we have $\Phi_S > 0$ implies $P_S > 0$ and therefore $\lambda_0^S = 0$. Moreover, we have $\lambda_1^S = \lambda_0^S = 0$ by the assumption $0 < \Phi_S < 1$. The sophisticated consumer’s first period problem, (P1), yield the following first order conditions:

$$u'(c_1^S) = \beta \delta u'(c_2^S) = \lambda_1^S,$$  \hspace{1cm} (.24)

$$\beta \delta u'(c_2^S)(r^l - r^b) + \lambda_1^S K = 0.$$  \hspace{1cm} (.25)

Assume that $A_1^S > 0$ and therefore $\lambda_1^S = 0$. Substituting this in (.25), we get $\beta \delta u'(c_2^S)(r^l - r^b) = 0$. This is a contradiction since $r^b > r^l$. Thus, we must have $A_1^S = 0$ in any solution. The equations (.24) and (.25) together imply

$$u'(c_1^S) = \beta \delta u'(c_2^S)(1 + r^b).$$  \hspace{1cm} (.26)

Assume first that $A_2^S = 0$. This and the fact that $A_1^S = 0$ yields $c_1^S = w_1 - K(1 - \Phi_S)$ and $c_2^S = w_2 - \Phi_S K(1 + r^b)$. Assuming $u(c) = \ln(c)$, and substituting for $c_1^S$ and $c_2^S$ in the equation (.26), $\Phi_S$ can be solved as follows:

$$\Phi_S = \frac{w_2}{K (1 + r^b)(1 + \beta \delta)} + [1 - \frac{w_1}{K}] \frac{\beta \delta}{1 + \beta \delta}.$$  \hspace{1cm} (29)

The derivative of $\Phi_S$ with respect to $\beta$ can be calculated as

$$\frac{\partial \Phi_S}{\partial \beta} = \frac{\delta}{(1 + \beta \delta)^2 K} [-\frac{w_2}{1 + r^b} + K - w_1].$$  \hspace{1cm} (30)

Assuming $c_1^S > 0$ and $c_2^S > 0$ imply $\frac{w_2}{1 + r^b} > \Phi_S K > K - w_1$. Thus, we have $\frac{\partial \Phi_S}{\partial \beta} < 0$ which implies $\Phi_S > \Phi_{Exp}$. It is easy to show that, when $A_2^{FC} = 0$, the first order conditions for the consumer with full commitment power, namely the equations (1.1)-(1.7), imply

$$u'(c_1^{FC}) = \beta \delta u'(c_2^{FC})(1 + r_b).$$

Comparing the equation above with the equation (.26), we conclude that $\Phi_S = \Phi_{FC}$. Finally, by definition, we have $\Phi_{FC} = \Phi_N$. Assume now that $A_2^S > 0$. The sophisticated consumers maximization problem in the first period, (P1), imply the following first order conditions:

$$u'(c_2^S) = \beta \delta u'(c_3^S)(1 + r^l),$$  \hspace{1cm} (.27)

$$\beta \delta^2 (1 - \beta) u'(c_3^S) \frac{\partial c_3^S}{\partial \Phi_S}(1 + r^l) + \beta \delta^2 u'(c_3^S)(1 + r^b)(1 + r^l) = u'(c_1^S).$$  \hspace{1cm} (.28)
With \( u(c) = \ln(c) \) and \( A_1^S = A_3^S = 0 \), we have
\[
\begin{align*}
c_1^S &= w_1 - K(1 - \Phi_S), \\
c_3^S &= c_2^S\beta\delta(1 + r^l), \\
c_3^S &= \frac{\beta\delta}{1 + \beta\delta}[w_3 + [2 - \Phi_S K(1 + r^b)](1 + r^l)].
\end{align*}
\]
Substituting for \( c_2^S \) and \( c_3^S \) in the equations (.27) and (.28), we can solve for \( \Phi_S \) as
\[
\Phi_S = \frac{1}{K(1 + \beta\delta(1 + \delta))} \left[ \frac{w_3}{(1 + r^b)(1 + r^l)} + \frac{w_2}{1 + r^b} - \frac{[w_1 - K]\beta\delta(1 + \delta)}{1 + r^b} \right].
\]
The derivative of \( \Phi_S \) with respect to \( \beta \) can be calculated as
\[
\frac{\partial \Phi_S}{\partial \beta} = \frac{-\delta(1 + \delta)}{(1 + \beta\delta(1 + \delta))^2} \left[ \frac{w_3}{K(1 + r^b)(1 + r^l)} + \frac{w_2}{K(1 + r^b)} - \frac{[w_1 - K]}{K(1 + r^b)} \right] < 0.
\]
The first order conditions (1.1)-(1.7) for the consumer with full commitment power give
\[
\begin{align*}
u'(c_{FC}^1) &= \delta u'(c_{FC}^3)(1 + r^l), \quad (.29) \\
u'(c_{FC}^1) &= \beta\delta^2 u'(c_{FC}^3)(1 + r^l)(1 + r^b). \quad (.30)
\end{align*}
\]
Assuming \( u(c) = \ln(c) \), we get
\[
\begin{align*}
c_{FC}^3 &= (1 + r^b)(1 + r^l)\beta\delta^2 c_{FC}^1, \\
c_{FC}^3 &= c_{FC}^2\delta(1 + r^l), \\
c_{FC}^2 &= \frac{w_3}{(1 + r^l)(1 + \delta)} + \frac{w_2}{1 + \delta} - \frac{\Phi_{FC} K(1 + r^b)}{1 + \delta}.
\end{align*}
\]
Substituting for \( c_{FC}^1 \) and \( c_{FC}^3 \) in the equations (.29) and (.30), we can solve for \( \Phi_{FC} \) as follows:
\[
\Phi_{FC} = \frac{1}{K(1 + \beta\delta(1 + \delta))} \left[ \frac{w_3}{(1 + r^b)(1 + r^l)} + \frac{w_2}{1 + r^b} - \frac{[w_1 - K]\beta\delta(1 + \delta)}{1 + r^b} \right].
\]
This implies that \( \Phi_{FC} = \Phi_S \). Finally, by definition, we have \( \Phi_{FC} = \Phi_N \). This completes the proof. \( \square \)
Chapter 2

Closed-end credit under quasi-hyperbolic discounting

“Unfortunately, most people don’t consider the total cost when they borrow money. If they can make the minimum payments on their debt, they figure they’re doing OK. They don’t see the cash that’s seeping out of their lives every day.”

“When you shift the dialogue from total payments to monthly payments, you shift from a discussion of $3,000 to $90 per month. People buy based on cash flow, not total price. This is one of the reasons leasing is so popular. Think about it. How do most car dealers present their prices? They sell payments, not price. So should you.”

2.1 Introduction

The 2001 Survey of Consumers (U.S.) reveals that about forty percent of the households regard self-control as a general problem, and they believe that availability of credit cards might trigger overspending and overborrowing. Interestingly, a significant percentage of people think that this is a problem for others but not for themselves (Durkin [14]).

In this chapter we analyze the interplay between the self-control problems and consumers’ choice of the loan terms (i.e. the maturity, the down payment amount and the interest rate). We model self-control problems by assuming that the consumers have quasi-hyperbolic discounting.

Literature on self-control and credit behaviour focused mainly on revolving credit (e.g. credit cards). Shui and Ausubel [34], using US data on credit cards, observe

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1Liz Pulliam Weston; a personal finance columnist for MSN Money
2Matt Michel, article on “equipment pricing” http://www.phccweb.org
that consumers over-respond to teaser rates because they underestimate their future borrowing. They also find that consumers are reluctant to switch credit cards despite the fact that there are better offers elsewhere. Using the 1995 Survey of Consumer Finances, Gross and Souleles [17] identified the following “credit card portfolio puzzle”: many consumers hold credit card debt although 1) they have substantial amount of illiquid assets for retirement and 2) they have liquid assets with interest rates much lower than the interest paid on their credit card debt. Laibson et al. [22] focuses on the first puzzle. They show that consumers with time-inconsistent preferences (hyperbolic discounting) “borrow actively in revolving credit card market and accumulate large stocks of illiquid wealth” as in observed data. Bertaut and Haliassos [5] focus on the second puzzle. They show that consumers do not pay credit balances in full because doing so serves as a self-control mechanism as it reduces the credit limit available to them. They conclude that “debt revolvers are motivated primarily by self-control considerations rather than consumption smoothing”. Bertaut and Haliassos [6], using data from several waves of the Survey of Consumer Finances, document the following belief of the card holders; debit card serves as the instrument of self-control (i.e. commitment device) whereas credit card create self-control problems since it stimulates overspending.

In this chapter, we focus on closed-end credit. Most common examples of closed-end credit include mortgages, auto loans loans, and personal loans. The consumer borrows a specific amount of money and repays it in full over a stipulated period of time in fixed installment payments. According to the 2004 Survey of Consumer Finances, 46 percent of the US households had installment debt (Bucks et al. [8]).

Anecdotal and empirical evidence suggest that, when shopping for a loan, consumers focus more on low monthly payments and longer loan durations rather than the interest rate. Attanasio et al. [2] analyze the elasticity of the loan demand with respect to the interest rate and the maturity of the loan using micro data from the Consumer Expenditure Survey (1984-1995) on auto loan contracts. They find that the aggregate loan demand is highly sensitive to loan maturity, but unresponsive to changes in the interest rate, which they interpret as evidence of liquidity constraints. Karlan and Zinman [19] estimate the same elasticities of demand using randomized trials (random loan offers distributed via direct mail to over 50,000 individuals) implemented by a major South African micro-finance lender. They also find that loan size decisions are far more responsive to changes in loan maturity than to changes in interest rate. They, however, note that “an alternative (or complementary) explanation to liquidity constraints is that income and age proxy for financial sophistication, i.e. perhaps the poor and inexperienced use a decision rule that lead them to focus on monthly payments rather than the interest rates”. In line with this interpretation,
“mortgage professor” Jack Guttentag ³ advises the mortgage borrowers as follows:

“The worst mistake by far is making decisions based solely on the affordability of monthly payments, without considering how the decisions will affect the equity in their homes. I call this malady “payment myopia”... Payment myopia is not one specific mistake but a general approach to financial decisions that can lead to a lifetime of mistakes. It arises out of a lifestyle focus on the present, and an unwillingness to defer gratification. Those afflicted want what they want now.”

According to Jack Guttentag, payment myopic consumers also choose the longest term available because it results in the lowest payment and they usually prefer the loans for which no down payment is required. Appendix B provides more examples on the lending practices that potentially target borrowers who exhibit the so called ”payment myopia”.

Can we attribute the presence of the “payment myopia” to the dynamically inconsistent preferences? How do self-control problems effect the consumers’ choice of loan terms? In order to answer these questions we consider a simple model of installment borrowing which helps us to look at the interaction between a monopolistic lender and a consumer who has time-inconsistent preferences. The consumer wishes to finance the purchase of a single durable good. The lender offers either of the two contracts: a short term loan with time to maturity of one period and a long term loan with time to maturity of two periods. We assume equal monthly installments. The lender has exponential discounting whereas the consumer can have either exponential discounting (benchmark) or quasi-hyperbolic discounting. We consider the following different types of time-inconsistent consumers: a sophisticated consumer who is aware of her self-control problem and a naive consumer who is fully or partially naive about it. In each case, we compare the equilibrium contract to the benchmark. In particular we look at look at how the choice of credit terms (i.e. maturity, down payment and interest rate) differ for each type.

We find that under benchmark case exponential discounting, both the lender and the consumer are indifferent between the contract with the long-term loan and the contract with the short-term loan. Under quasi-hyperbolic discounting, however, the equilibrium contract is the long term loan provided that the market interest rate exceeds the long run discount rate of the consumer. We also analyze whether self-control problems affect the sophisticated consumer’s choice and how the rational, monopolistic lender reacts to naive consumer’s underestimation of her own time-inconsistency. We find that the sophisticated consumer uses “consistent planning”

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based on backward induction and therefore she is able to correct for the effects of self-control. Therefore, self-control problems affect naives in a different way than it affects the sophisticates. We identify a threshold value for the finance rate (fraction of the durable purchase that is financed), above which the lender is able to extract from the naive consumer more than her willingness to pay in the first period. The condition for the equilibrium finance rate can also be written as a condition on the degree of time-inconsistency. If the finance rate is above the threshold value, equivalently, if the time-inconsistency is strong enough, the lender is able to extract more profits from the naive consumer relative to the sophisticated consumer even though they have the same time-preference parameters.

Our analysis in this chapter is closest in spirit to those of Eliaz and Spiegler [15] and Della Vigna and Malmendier [12]. In Eliaz and Spiegler [15], a principal contracts with agents whose preferences change from one period to the other. Agents have diverse abilities to forecast their future preference changes. Preferences can change due to any reason, (including the possibility of present bias) but there is no discounting in their model. Della Vigna and Malmendier [12] analyze the profit maximizing contract of a monopolist facing consumers with dynamically inconsistent preferences in the context of health club industry. The nature of the contract studied in Della Vigna and Malmendier [12] differ from the contract we analyze in this chapter. The profit maximizing monopolist in Della Vigna and Malmendier [12] offers a two part tariff, an initial payment followed by a price paid per visit. Therefore, the consumer still can choose how much to consume of the services in the second period. In Eliaz and Spiegler [15], the principal lends the agent a sum of money in period 1, and “in the next period” the agent chooses to reallocate the repayment between that period and period 3. In the model of this chapter, as it is typically the case for installment loans, once the contract is signed in the first period, the loan terms are fixed (including the size of the periodic payments). Both papers consider the case of sophisticated, naive and partially naive consumers. In our analysis as well as in theirs, naivete of the hyperbolic agent allows the monopolist to exploit the naive consumer’s misperception of her own preferences.

There are also experimental papers by Wertenbrouch et al. [38] and Ranyard and Craig [31] which analyze the implications of self-control problems on consumer credit. Wertenbrouch et al. [38] introduce a multi-period model of “mental budgeting” and test the implications of the model by means of four different experiments in order to study consumer credit in the presence of self-control problems. The results of their experiments suggest that people with self-control problems, if they are aware of it, use debt aversion as an internal commitment mechanism by avoiding debt or by self imposing stricter payment terms. Ranyard and Craig [31] use a survey to study the role of installment credit in personal budgeting and the way people evaluate it.
However, they focus on the notion of mental accounting. They conclude that mental accounts help consumers in personal budgeting and their use of installment credit.

The remainder of the chapter is organized as follows. In section 2.2, we present a simple model of installment credit. In section 2.3, we characterize the equilibrium contract for the cases of exponential discounting and quasi-hyperbolic discounting. In section 2.4, we conclude. In Appendix A, we provide the proofs of all the results of this chapter. Appendix B documents some anecdotal evidence, which suggests that the self-control problems might be playing a role in the consumer’s choice of installment credit terms.

### 2.2 A simple model of installment borrowing

In order to analyze the effect of time-inconsistent preferences on credit choice, we consider a simple model of installment borrowing. The part of the model regarding the consumer’s problem closely follows the model in Attanasio et al. [2]. In the model, a monopolistic lender faces a single borrower (consumer) with time-inconsistent preferences. The consumer wishes to finance the purchase of a single durable good, whose price is fixed to $K > 0$. There are three periods, periods 0, 1 and 2. The borrowing and the purchase of the durable can only take place in period 0 and the consumer can not sell the durable at the subsequent periods. At each period, the consumer receives a deterministic income $w > 0$.

Let $\Phi$ be the fraction of the required expenditure $K$ financed. Then, $\Phi K$ is the amount financed (loan size) and $K(1 - \Phi)$ is the required amount of down payment. We assume that the consumer can not borrow more than $K$, collateral, and that $w < K$. Therefore we have $\Phi \in [1 - w/K, 1]$. The assumption $w < K$ rules out the cash purchase and it provides a motive for the consumer to move the resources from the future periods to the present period (i.e. borrow) in order to consume the durable good in the present. The durable good provides to its owner a flow of (per-period) benefits $c$, including the period of the purchase.

Since the income process $\{w_t = w \text{ for } t = 0, 1, 2\}$ is deterministic there is no risk. Market interest rate is exogenous and is denoted by $r_f$.

**Discounting.** We assume that the consumer has quasi-hyperbolic preferences (Phelps and Pollack [28]; Laibson [21]; O’Donoughe and Rabin [25], [26]). The quasi-hyperbolic discount function for time $s$ evaluated at period $t$, equals to 1 for $s = t$ and is equal to $\beta^s \delta^{s-t}$ for $s = t+1, t+2, \ldots$ where $\beta \in (0, 1]$ and $\delta \in (0, 1]$. The case of $\beta = 1$ corresponds to time-consistent exponential discounting. When $\beta < 1$, the consumer has time-varying, stationary discounting. This type of discounting generates time-inconsistency due to the difference between the short run and the long run discount rates. We refer to consumer’s selves in periods 0, 1, and 2 as *self-0*, *self-1* and *self-2*.
respectively. We assume that self-2 is passive, in the sense that she obeys the wishes of previous selves by following whatever decision has been taken by them.

As it is explained above, with quasi-hyperbolic discount function, self-0 time preferences are given as \(\{1, \beta \delta, \beta \delta^2, \ldots\}\). Suppose that self-0 thinks that the future selves’ time preferences are \(\{1, \hat{\beta} \delta, \hat{\beta} \delta^2, \ldots\}\). A consumer with preference parameters \((\beta, \hat{\beta}, \delta)\) is said to have time-consistent exponential discounting if \(\beta = \hat{\beta} = 1\). We will consider the following cases of time-inconsistency: if \(\beta = \hat{\beta} < 1\), the consumer is said to be sophisticated and if \(\beta < 1\) and \(\hat{\beta} = 1\), the consumer is (fully) naive (O’Donoghue and Rabin [27]). That is, sophisticated consumer is fully aware of her hyperbolic preferences and correctly anticipates that her future selves will have hyperbolic preferences. In contrast, naive consumer is completely unaware of her time-inconsistency and she thinks that her future selves will discount exponentially. Note that these are the two extreme cases. If consumer’s self-0 believes that her future selves’ preferences are \(\{1, \hat{\beta} \delta, \hat{\beta} \delta^2, \ldots\}\) with \(\beta < \hat{\beta}\), then she is said to be partially naive (O’Donoghue and Rabin [27]). Note that the difference \(\hat{\beta} - \beta\) can be interpreted as the degree of naivete. Similarly, the difference \(\beta - 1\) reflects the degree of time-inconsistency. We assume that the lender knows consumer’s preferences.

![Figure 2.1: The timing of the model](image)
2.2.1 The timing of the model

The lender has all the bargaining power. At the beginning of period 0, the lender offers the contract $i$ with $(r_i, \Phi_i)$, where $r_i$ denotes the interest rate on a loan amount of $\Phi_i K$ that has a maturity of $i = 1, 2$ periods and $\Phi_i$ refers to the upper bound on $\Phi_i$, that is, an upper bound on the fraction of $K$ financed. Hence, $K(1 - \Phi_i)$ is the minimum down payment requirement set by the lender. Therefore, the consumer’s choice of $\Phi_i$ is restricted to the set $[1 - w/K, \Phi_i]$. We refer to the loan as a short-term loan if $i = 1$ and, a long-term loan if $i = 2$. The consumer’s first period self (self-0) either accepts the offer and sets $\Phi_i^* \equiv \Phi_i^*(r_i, \Phi_i)$ or rejects it.

If the consumer’s self-0 is offered the short term loan and accepts the offer, for a loan amount of $\Phi_1 K$, she puts $K (1 - \Phi_1)$ down in the first period. This contract requires a payment

$$P_1 = \Phi_1 K (1 + r_1)$$

in the next period. If self-0 is offered the long term loan and accepts the offer, for a loan amount of $\Phi_2 K$, she puts $K (1 - \Phi_2)$ down in the first period. This contract requires two equal installment payments of $P_2$ in the second and third periods where

$$P_2 = \Phi_2 K (1 + r_2)^2/(2 + r_2).$$

The installment payment $P_2$ can be calculated using the following formula

$$[\Phi_2 K (1 + r_2) - P_2](1 + r_2) = P_2$$

where the left hand side is the debt due in the third period after having made a payment of $P_2$ in the second period. Since the third period is the final period, the consumer has to pay all of the remaining debt.

**Leaving the contract.** At the beginning of period 1, consumer’s next self (self-1) can choose to stay in the contract or leave it. If the consumer walks out of the contract, the lender seizes the durable. We assume that the durable does not depreciate.

The third period is a passive period in the sense that the consumer’s self-2 follows whatever decision has been taken in previous periods.

**2.2.2 Consumer Payoffs.**

The consumer’s self-0 begins period 0 with wage $w$. After observing the offer of the lender - the contract $i$ with $(r_i, \Phi_i)$, $i = 1, 2$ - she decides whether to accept and buy the durable or reject the offer. Conditional on buying, she also decides on the fraction of amount financed $\Phi_i^*(r_i, \Phi_i)$, which in turn determines the down payment to
be made. Since there is no non-durable good consumption, in each period, whatever remains from the wage $w$, after making the periodic loan payments, is saved at rate $r_f$. Therefore the first period savings $A^1_1$ is equal to $w - K(1 - \Phi^*_i)$. Consumer receives a per-period benefit $c$ from consumption of the durable.

In the second period, if \textit{self-0} has accepted an offer, \textit{self-1} decides whether to stay in the contract chosen or leave. If the accepted offer was a short term loan and, if \textit{self-1} decides to stay, she makes the periodic payment $P_i$ to lender out of that period’s wage $w$ and saves the rest at $r_f$. She enjoys the benefit $c$ from the durable. If she decides to leave, she saves $w$ at $r_f$. If \textit{self-0} was offered a long term loan and accepted, then (i) if \textit{self-1} decides to stay, she pays $P_2 < P_1$ out of her wage $w$ and saves the rest at $r_f$. She enjoys benefits $c$ from the consumption of the durable (ii) if \textit{self-1} decides to leave the contract, she saves all of her wage at $r_f$. Therefore the second period savings is equal to $A_i^2 = w + A_1^1(1 + r_f) - P_i$.

In the last period there is no decision to be taken (passive period). If consumer’s \textit{self-0} has been offered and accepted the short term loan and if \textit{self-1} decides to stay in the contract, she has paid the loan in full at the previous period. Therefore consumer’s \textit{self-2} just saves her wage and consumes $c$. If \textit{self-0} has been offered the long term loan and accepted, and if \textit{self-1} decides to stay, \textit{self-2} makes the last installment payment $P_2$, saves the rest at $r_f$ and consumes $c$.

If consumer’s \textit{self-0} refuses either of the contracts, she saves her wages $w$ at the rate $r_f$ until the last period. We can now state the payoffs from the perspective of \textit{self-0} and \textit{self-1}.

**Payoffs from the perspective of \textit{self-0}.** If the consumer’s \textit{self-0} rejects either of the two offers, her time-0 payoff is given by

$$U_0 = \beta \delta^2[w + (w + w(1 + r_f))(1 + r_f)].$$

If consumer’s \textit{self-0} is offered and accepts the short term loan and, if \textit{self-1} decides to stay in the contract, the time-0 payoff attached to this option is

$$U_1 = \beta \delta^2[w + A_1^1(1 + r_f)] + c(1 + \beta \delta + \beta \delta^2).$$

If \textit{self-0} is offered and accepts the long term loan, and if \textit{self-1} decides to stay, the payoff attached to this choice from the perspective of \textit{self-0} is

$$U_2 = \beta \delta^2[w + A_2^2(1 + r_f) - P_2] + c(1 + \beta \delta + \beta \delta^2).$$

Finally if \textit{self-1} leaves the contract with maturity $i$, the relevant payoff from the perspective of \textit{self-0} is

$$U_i^d = \beta \delta^2[w + (w - K(1 - \Phi_i))(1 + r_f))(1 + r_f)] + c,$$
where \( A_i^2 = w + A_1(1 + r_f) - P_i \) and \( A_i^1 = w - K(1 - \Phi_i) \) and \( i = 1, 2 \).

**Payoffs from the perspective of self-1.** If self-1 decides to stay, given that the self-0 has accepted the short term loan, her time-1 payoff is given as

\[
V_1 = \beta \delta w + \beta \delta A_1^1(1 + r_f)^2 + \beta \delta[w - P_1](1 + r_f) + c(1 + \beta \delta). \tag{2.5}
\]

If self-1 has decided to stay given that self-0 has accepted the long term loan, then her time-1 payoff is

\[
V_2 = \beta \delta w + \beta \delta A_2^1(1 + r_f)^2 + \beta \delta[w - P_2](1 + r_f) - \beta \delta P_2 + c(1 + \beta \delta). \tag{2.6}
\]

If self-1 decides to leave the contract, her time-1 payoff is

\[
V_d^i = \beta \delta A_i^1(1 + r_f)^2 + \beta \delta w(1 + r_f) + \beta \delta w. \tag{2.7}
\]

### 2.2.3 Lender profits

We assume that the loan provision is costly and that the lender incurs the same cost \( \alpha > 0 \) for both types of loans. The lender’s profit from providing the loan with maturity of \( i \) periods is denoted by \( \Pi_i(r_i, \Phi_i^*(r_i, r_f)) \). They are given as

\[
\Pi_1 = \Phi_1^*K[\delta(1 + r_1) - 1] - \alpha, \\
\Pi_2 = \Phi_2^*K[\frac{(1 + r_2)^2}{(2 + r_2)^2(1 + \delta)} - 1] - \alpha. \tag{2.8}
\]

### 2.3 Equilibrium contract

In this section we will analyze the equilibrium contract that a monopolistic lender offers to a single consumer with preference parameters \((\beta, \hat{\beta}, \delta)\). The proofs of the results are provided in Appendix A. We begin by defining a feasible contract.

**Definition 2.1 Feasible contract.** A contract \((r_i, \Phi_i)\) is feasible if the profits from loan provision, net of the opportunity cost of the funds, is nonnegative.

**Remark 2.2** The market interest rate \( r_f \) is the opportunity cost of the funds. By definition a feasible contract has the property that \( r_i > r_f \geq 1 - 1/\delta \) for \( i = 1, 2 \). This is a necessary condition for the lender to to provide the loan contract with maturity \( i \). The first inequality is strict due to the assumption that the loan provision has a positive cost, \( \alpha \).
We now proceed to characterize the equilibrium contract for each type of the consumer. We begin by describing the lender’s maximization problem. The lender first chooses, given the consumer’s choice of $Φ^*$, $(r_1, \Phi_1)$ to maximize $Π_1$ and $(r_2, \Phi_2)$ to maximize $Π_2$ respectively. $Π_1$ and $Π_2$ are given as in equation (2.8). Each maximization problem is subject to the relevant participation constraint of the consumer as follows;

$$U(r_1, \Phi_1^*(r_1, \Phi)) \geq \max\{U_d(\Phi), U_0\} \text{ for } i = 1 \text{ or } i = 2$$

where $U_0, U_1, U_2$ and $U_d(\Phi)$ are given by equations (2.1) to (2.4). The lender then selects the contract with maturity $i$ that gives the highest profit $Π_i$ and offers $(r_i, \Phi_i)$ to the consumer. Therefore, the lender’s maximization problem, given the consumer’s choice of $Φ^*$, can be written as follows:

$$\max_i\{\max_i (r_1, \Phi_1)(Π_1), \max_i (r_2, \Phi_2)(Π_2)\}$$

where each maximization problem inside the big parenthesis is subject to the relevant participation constraint of the consumer as described above.

Given the lender’s problem, we can proceed to define the equilibrium contract.

**Definition 2.3 Equilibrium contract.** An equilibrium contract is a feasible contract such that it solves lender’s maximization problem.

**Lemma 2.4 (Consumer’s choice of $Φ_1^*$) A consumer with preference parameters $(β, \hat{β}, δ)$ chooses $Φ_1^* = Φ^* = 1 - w/K$ (maximum down payment) if she accepts a contract and commits to her choice, and chooses $Φ_1^* = \Phi$ (minimum down payment) if she accepts a contract with the intention to leave it at the next period.

**Lemma 2.5 (Lender’s choice of $Φ_1$) In an equilibrium contract, the lender sets $Φ_1 = \Phi = 1 - w/K$ for any consumer with preference parameters $(β, \hat{β}, δ)$.

Therefore, in an equilibrium contract, the down payment requirement and the finance rate on the short term and long term loans are equal and, they are the same for exponentials, naives and sophisticates. As a result we have $A_i^1 = A_1 = 0$ for all $i$. That is, the consumer with $(β, \hat{β}, δ)$ preferences does not save in period 0 if she takes a loan. This is due to the difference between the market interest rate $r_f$ and the borrowing rates $r_i, i = 1, 2$.

We use the case of exponential discounting as a benchmark of our analysis. The proposition below provides the characterization of the equilibrium contract under exponential discounting, namely, the equilibrium contract offered to a consumer with preference parameters $(β, \hat{β}, δ)$ such that $\hat{β} = β = 1$.  

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Proposition 2.6 (Characterization of the equilibrium contract under exponential discounting) Under exponential discounting, the lender offers \((r_1, 1 - w/K)\) or \((r_2, 1 - w/K)\) and, both the consumer and the lender are indifferent between the two offers where \(r_i, i = 1, 2\) are determined by the following two equations:

\[
\begin{align*}
-\delta P_1(r_1, 1 - w/K) + c(1 + \delta + \delta^2) &= w, \\
-\delta(\delta + 1)P_2(r_2, 1 - w/K) + c(1 + \delta + \delta^2) &= w.
\end{align*}
\]

Notice that the installment payments \(P_1\) and \(P_2\) are functions of \((r_1, \Phi)\) and \((r_1, \Phi)\) respectively, as defined in subsection 2.2.1.

When the following condition is met, the consumer prefers signing the contract first and then leaving it next period, to rejecting it in the first place. Since our results use this condition, we will explain it in detail below.

**Condition 2.7** \(U_d(\Phi) \leq U_0\)

The Condition 2.7 can be expressed in several ways. First, it implies that \(\beta\delta^2K(1 - \Phi)(1 + r_f)^2 \geq c\). That is, the consumer prefers signing a contract today and then leaving it next period, to rejecting it, whenever the benefit of using the durable for one period exceeds the associated opportunity cost. Opportunity cost is the down payment \(K(1 - \Phi)\) which is saved at \(r_f\) for two periods. Second, given \(\Phi^* = \Phi = 1 - w/K\), it can be expressed as a condition on the equilibrium finance rate:

\[\Phi^* \leq \tilde{\Phi}\]

where \(\tilde{\Phi} = 1 - \frac{c}{\beta\delta^2K(1 + r_f)^2}\). If the finance rate \(\Phi^*\) is high enough such that it exceeds the threshold value \(\Phi\) (equivalently if the down payment requirement is low enough), then the consumer prefers signing the contract at period 0 and then leaving it next period, to rejecting it. Finally, given \(\Phi^* = 1 - w/K\), it implies a condition on \(\beta\):

\[\beta \geq \tilde{\beta}\]

where \(\tilde{\beta} = \frac{c}{w\delta^2(1 + r_f)^2}\). Recall that the difference, \(1 - \beta\), measures the degree of time inconsistency, or the degree of the conflict between the consumers’ self-0 and self-1.

We can now proceed to the characterization of the equilibrium contract for the case of quasi-hyperbolic discounting. We begin by the following remark:
Remark 2.8 In this model, if $\Phi^* > \Phi$, then the lender may prefer that the consumer leaves the contract at the second period. Let us denote the lender profit when consumer leaves the contract at the second period by $\Pi_l$ which is given by

$$\Pi_l = -\Phi K - \alpha + \delta K|_{\Phi = 1 - w/K} = w - [K(1 - \delta) + \alpha].$$

Note that given $\Phi = 1 - w/K$, the profit is constant (i.e. independent of the interest rate). If the $\Pi_l$ is sufficiently high to exceed the profits from the loan provision, the lender would simply offer an interest rate which is high enough to induce the consumer to accept the offer and then leave the contract next period. In this case the lender is able to maintain the time-inconsistent consumer at the payoff level $U_d(1 - w/K)$.

We can now state the propositions that characterize the equilibrium contract under quasi-hyperbolic discounting. Consider a consumer with preference parameters $(\beta, \hat{\beta}, \delta)$ where $\beta < 1$ and $\hat{\beta} \in [\beta, 1]$ (quasi-hyperbolic discounting). Consider the interest rates $r_2^*, r_2'$ such that:

$$U_1(r_1^*) = U_0, \quad U_2(r_2^*) = U_0 \quad \text{and} \quad V_1(r_1') = V_d, \quad V_2(r_2') = V_d.$$

Proposition 2.9 (Characterization of the equilibrium contract under quasi-hyperbolic discounting: $\Phi^* \leq \Phi \text{ and } r_f > \frac{1}{\delta} - 1$). The lender offers $(r_2^*, 1 - w/K)$ and, both the naive and the sophisticated consumers accept and stay.

Proposition 2.10 (Characterization of the equilibrium contract under quasi-hyperbolic discounting: $\Phi^* \leq \Phi \text{ and } r_f = \frac{1}{\delta} - 1$). The lender offers $(r_2^*, 1 - w/K)$ or $(r_2', 1 - w/K)$ and, the lender, the naive consumer and the sophisticated consumer are all indifferent between the two offers.

Proposition 2.11 (Characterization of the equilibrium contract under quasi-hyperbolic discounting: $\Phi^* > \Phi \text{ and } r_f > \frac{1}{\delta} - 1$). If the lender does not favor liquidation, then the lender offers $(\min(r_2^*, r_2'), 1 - w/K)$ to the sophisticated consumer, and she accepts and stays. The naive consumer is offered $(r_2', 1 - w/K)$, and she accepts and stays. If the lender favors liquidation, then the lender offers any $r > r_2'$ and, both the naive and the sophisticated consumers accept and leave the contract at the second period.

Remark 2.12 In Proposition 2.11, the lender does not favor liquidation if we have $\Pi_2(\beta, \hat{\beta}, \delta)(\min(r_2^*, r_2'), 1 - w/K) > \Pi_l$.

Proposition 2.13 (Characterization of the equilibrium contract under quasi-hyperbolic discounting: $\Phi^* > \Phi \text{ and } r_f = \frac{1}{\delta} - 1$). If the lender favors liquidation, then the lender offers any $r > r_i'$, $i = 1, 2$, and both the naive and the sophisticated consumers leave the contract at the second period.
consumers accept and leave the contract at the second period. If the lender does not favor liquidation, then the lender offers \((r'_1, 1 - w/K)\) or \((r'_2, 1 - w/K)\) to the naive consumer and, both the lender and the naive consumer are indifferent between the offers. If \(r^*_i \leq r'_i\), then the lender offers \((r^*_1, 1 - w/K)\) or \((r^*_2, 1 - w/K)\) to the sophisticated consumer and, both the lender and the sophisticated consumer are indifferent between the two offers. If \(r^*_i > r'_i\), then the lender offers \((r'_1, 1 - w/K)\) or \((r'_2, 1 - w/K)\) to the sophisticated consumer and, both the lender and the sophisticated consumer are indifferent between the two offers.

**Remark 2.14** In Proposition 2.13, the conditions for the lender not to favor liquidation when facing the naive consumer and the sophisticated consumer are given as

\[
\Pi_2^\beta(\beta, \hat{\beta}, \delta)(r'_2, 1 - w/K) > \Pi_l \quad \text{and} \quad \Pi_2^\beta(\beta, \hat{\beta}, \delta)(\min(r^*_2, r'_2), 1 - w/K) > \Pi_l,
\]

**Corollary 2.15** The following relationship holds between \(r^*_i\) and \(r'_i\):

\[
r^*_i \leq r'_i \iff [c - \beta \delta^2 w(1 + r_f)^2 \leq c \delta (1 - \beta)] \quad \text{for} \quad i = 1, 2.
\]

The expression \(c(1 + \delta(\beta - 1)) - \beta \delta^2 w(1 + r_f)^2\) measures the discounted payoff at \(t = 0\) from consuming the durable today and tomorrow, in the presence of the conflict of interest between self-0 and self-1. The term \(c \delta (\beta - 1)\) < 0 reflects the difference in the valuation of self-0 and self-1. The difference in the valuations - hence a potential conflict of interest - is due to the time inconsistent preferences. Absent any commitment mechanism, sophisticated consumer corrects for this conflict by agreeing to pay an interest rate higher than \(r^*_0\), which is her first period willingness to pay if there was a way to commit staying, i.e. making self-1 stay.\(^4\) The conditions that \(r^*_i \leq r'_i\) and \(\Phi^* > \hat{\Phi}\) together impose conditions on \(\beta\) and \(\delta\) as follows:

\[
\Phi^* > \hat{\Phi} \quad \text{and} \quad r^*_i \leq r'_i \iff \beta < \tilde{\beta} \quad \text{and} \quad \delta \geq \tilde{\delta}
\]

where \(\tilde{\delta} = \frac{c}{w(1 + r_f)^2}\).

**The consumer surplus and the lender profits**

Let us define the consumer surplus for a consumer with \((\beta, \hat{\beta}, \delta)\) preferences as the difference between the payoff level that the lender maintains the consumer in the equilibrium contract and the perceived reservation payoff - both from the perspective of self 0. The proposition 2.16 summarizes the consumer surpluses in an equilibrium contract.

\(^4\)See the proof of Lemma 2.5 in Appendix A.
Proposition 2.16 (Consumer surplus) In an equilibrium contract the exponential consumer surplus is zero. If $\Phi^* \leq \Bar{\Phi}$, then the sophisticated and the naive consumer surpluses are zero and if $\Phi^* > \Bar{\Phi}$, then the sophisticated consumer surplus is zero whereas the naive consumer surplus is $c\delta(\beta - 1) < 0$.

For the sophisticated consumer the perceived payoff is equal to the actual payoff obtained since sophisticates, by definition, forecast their future preferences correctly. For the naive consumer, the perceived payoff at period 0, $U_d(\Phi)|_{\Phi=1-w/K}$, is different than the actual payoff obtained, $U_2(r'_2)$, again discounted to period 0, as long as the finance rate $\Phi^*$ is above the threshold $\Bar{\Phi}$. In this case, the naive surplus is negative and the lender is able to extract from the naive consumer more than her first period willingness to pay. Interestingly, when this is the case, the naive consumer’s surplus does not depend on the degree of naivete. Even the smallest degree of naivete has the same effect as being completely naive, the consumer with $\hat{\beta} \in (\beta, 1)$ ends up paying $r'_i$, just as the consumer with $\hat{\beta} = 1$.

Let $\Pi_1^{(\beta, \hat{\beta}, \delta)}$ denote the maximum profit the lender can obtain by providing a loan of maturity $i$ to a consumer with preference parameters $(\beta, \hat{\beta}, \delta)$, provided that the consumer stays in the contract to the end. The Proposition 2.17 and the Corollary 2.18 describe the lender profits in an equilibrium contract.

Proposition 2.17 (Lender profits). In an equilibrium contract we have

(a) $\Pi_1^{(\beta, \hat{\beta}, \delta)} = \Pi_2^{(\beta, \hat{\beta}, \delta)}$ for $\beta = \hat{\beta} = 1$,
(b) $\Pi_2^{(\beta, \hat{\beta}, \delta)} \geq \Pi_1^{(\beta, \hat{\beta}, \delta)}$ for all $\hat{\beta} \in [\beta, 1]$ with $\beta < 1$ iff $r_f \geq \frac{1}{\delta} - 1$

Namely, when the consumer discounts exponentially, both the lender and the consumer are indifferent between a contract with a long-term loan and a contract with a short-term loan. Under quasi-hyperbolic discounting, the lender prefers the long term loan and therefore proposes it, only when the market interest rate exceeds the long term discount rate ($\delta$). In other words, if $r_f > \frac{1}{\delta} - 1$, the equilibrium contract is the one with a long term loan and, if $r_f = \frac{1}{\delta} - 1$, both the lender and the naive and the sophisticated consumers are indifferent between the contract with a long-term loan and a contract with a short-term loan. Note that under exponential discounting we have $r_f = 1/\delta - 1$. We now proceed to compare the lender profits for the cases of naive and sophisticated consumers.

Corollary 2.18 Assume that $\Phi^* > \Bar{\Phi}$ and $r^*_i < r'_i$, then we have

$$\Pi_i^{(\beta, \hat{\beta}, \delta)}|_{\hat{\beta} \in (\beta, 1]} - \Pi_i^{(\beta, \hat{\beta}, \delta)}|_{\hat{\beta} = \beta} > 0$$
Recall that $\Phi^* > \tilde{\Phi}$ and $r_i^* < r_i'$ together imply that $\beta < \tilde{\beta}$ and $\delta > \tilde{\delta}$. Therefore, the corollary says that whether the lender is able to extract more profits from the naive consumer relative to the sophisticated consumer depends on the structure of individual time preferences. Recall that the naive surplus is given as $c\delta(1 - \beta)$ which can be written as $c(\hat{\beta}\delta - \beta\delta)$ with $\hat{\beta} = 1$. Since the effect of the smallest degree of naivete is the same as full naivete, this expression measures the difference between the perceived and the true discount rates by the naive consumer between first and second periods. This is intuitive as the difference in the lender profits stems from the fact that (i) preferences are time-inconsistent ($\beta < 1$) (ii) the naive consumer over-estimates the true $\beta$ as $\hat{\beta}$. For the lender to extract more surplus from the naives relative to sophisticates, the conflict must be strong enough (measured by the degree of time inconsistency, or $1 - \beta$) and the impact of the conflict (i.e the difference between actual and perceived discount rates) must be high enough where the latter is reflected in the size of $\delta$ when $\hat{\beta} = 1$. Therefore a direct implication of the Corollary 2.18 is that the lender profits are not increasing with the degree of naivete. Note also that the difference between naive and sophisticated profits is preserved even when $r_f = 1/\delta - 1$.

### 2.4 Conclusion

Empirical evidence suggest that loan demand is more sensitive to changes in the maturity than the changes in the interest rate. Anecdotal evidence points to the consumer’s tendency to focus more on lower monthly payments and longer loan durations rather than the interest rate or the total price of the loan. Can we explain these evidence by assuming that the consumers have self-control problems resulting from time-inconsistent preferences? With the aim of providing an answer to this question, we analyze the interaction between a monopolistic lender and a consumer with time-inconsistent preferences in a simple model of installment borrowing. The consumer wishes to finance the purchase of a single durable good. The lender offers either a short term or a long term loan. The consumer faces a limit on the amount of loan, the limit being the value of the durable good. Each loan offer specifies the interest rate, the maturity of the loan as well as the down payment requirement. We consider three cases for the consumer’s time preferences: time-consistent exponential, time-inconsistent naive and, time-inconsistent sophisticated. We characterize the equilibrium contract in each case and compare the time-inconsistent consumer’s choice of the installment credit terms with the benchmark case of exponential discounting.

In the benchmark case of exponential discounting, the lender offers interest rates and sets the down payment requirements on the two contracts in such a way that
both the lender and the consumer are indifferent between a long-term and a short-term loan. We identify the condition under which this indifference no longer holds for the case of quasi-hyperbolic discounting, namely, the condition that the market interest rate exceeds the long-term discount rate. In addition, we identify conditions on the degree of time-inconsistency and degree of naivete under which the lender extracts more from the naive consumer than her first period willingness to pay and the lender extracts more profits from the naive consumer relative to the sophisticated consumer.

In our economy, it is possible to leave the contract in the second period which makes it difficult for the sophisticated consumer’s current self to constrain the choice of her future selves. Considering that lease contracts are more flexible concerning the decision to terminate the contract, relative to buying on credit, an interesting direction for future research emerges; namely, using the framework of the analysis in this chapter to study a time-inconsistent consumer’s choice between leasing and buying on credit. Another interesting direction for future research would be allowing for renegotiation of the contract terms in future periods and comparing the behaviour of naive and sophisticated agents under two scenarios (renegotiation vs. no renegotiation).
Appendices
.A Proofs

Throughout the proofs we assume the following tie breaking rules: (i) if the discounted benefit attached to accepting an offer is equal to that of rejecting it we assume that the consumer accepts (ii) if the discounted benefit attached to staying in the contract is equal to that of leaving we assume that consumer stays.

In the following we will consider the scenario where monopolistic lender faces a consumer with preference parameters $(\beta, \hat{\beta}, \delta)$. The benchmark case (exponential discounting) is obtained by setting $\beta = 1$.

Proof of lemma 2.4 (Consumer’s choice of $\Phi^*_i$.)

From the perspective of the consumer, we have the following different cases regarding the choice of $\Phi^*_i$:

Case 1

If the consumer is offered the short term loan and accepts the offer with the intention of staying in the contract, she chooses the value of $\Phi_1$ in order to maximize

$$
\max \beta \delta^2 [w + [w + [w - K(1 - \Phi_1)](1 + r_f) - P](1 + r_f)]
\tag{48}
$$

subject to $\Phi_1 \in [1 - w/K, \Phi_1]$

The first order condition gives

$$
\beta \delta^2 K [1 + r_f][r_f - r_1] = \lambda_2 - \lambda_1
$$

where $\lambda_1$ and $\lambda_2$ are the Lagrange multipliers associated with the inequality constraints $\Phi_1 \geq 1 - w/K$ and $\Phi_1 \leq \Phi_1$ respectively.

A feasible contract has to satisfy $r_f < r_1$. Therefore we have $\lambda_1 > 0$ and $\lambda_2 = 0$ and the consumer would like to set $\Phi_1$ as low as possible. The solution to the maximization problem is thus $\Phi^*_1 = 1 - w/K$. Therefore she puts all of her wage as down payment in the first period and she sets $A^1_1 = 0$.

Case 2

If the long term loan is offered and, if the consumer accepts the offer with the intention of staying in the contract, she solves the following maximization problem for $\Phi_2$

$$
\max \beta \delta^2 [w - P' + [w + [w - K(1 - \Phi_2)](1 + r_f) - P'](1 + r_f)]
\tag{49}
$$

over $\Phi_2$

subject to $\Phi_2 \in [1 - w/K, \Phi_2]$
where \( P' = \Phi_2 K \frac{(1 + r_2)^2}{2 + r_2} \). The first order condition gives

\[
\beta \delta^2 [(1 + r)^2 - \frac{(1 + r_2)^2}{2 + r_2} (2 + r_f)] = \lambda_2 - \lambda_1
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multipliers associated with the inequality constraints \( \Phi_2 \geq 1 - w/K \) and \( \Phi_2 \leq \Phi_2 \) respectively.

It is easy to show that

\[
(1 + r_f)^2 - \frac{(1 + r_2)^2}{2 + r_2} (2 + r_f) \geq 0 \quad \text{iff} \quad r_f \geq r_2
\]

Since a contract exists only if \( r_f < r_2 \), consumer sets \( \Phi_2 \) to the minimum. Therefore the solution to the maximization problem is \( \phi_2^* = 1 - w/K \), which implies \( A_2^* = 0 \).

**Case 3**

If the consumer’s self-0 signs any of the two contracts with the intention of leaving it in the next period, her choice of \( \Phi_i \) is the solution to the maximization problem(s) below:

\[
\max \quad \beta \delta^2 [w + [w + [w - K(1 - \Phi_i)](1 + r_f)](1 + r_f)]
\]

subject to \( \Phi_i \in [1 - w/K, \Phi_i] \)

Using the first order condition, we have

\[
\beta \delta^2 K [(1 + r_f)^2 = \lambda_2 - \lambda_1]
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrange multipliers associated with the inequality constraints \( \Phi_i \geq 1 - w/K \) and \( \Phi_i \leq \Phi_i \) respectively. The function that is maximized is increasing in \( \Phi_i \). Therefore, the consumer wishes to set \( \Phi_i \) as high as possible, i.e., at \( \Phi_i^* = \Phi_i \).

Thus, we have proved the following:

(a) The consumer, if she accepts a contract and commits to her choice, wishes to set the finance rate optimally at \( \Phi_i^* = \Phi^* = 1 - w/K \) (maximum down payment).

(b) The consumer, if she accepts a contract with the intention of leaving it in the next period, wishes to set the finance rate optimally at \( \Phi_i^* = \Phi_i \) (minimum down payment).

This concludes the proof of Lemma 2.4. \( \square \)
Proof of lemma 2.5 (Lender’s choice of $\Phi$)

We will show that, for any consumer with $(\beta, \hat{\beta}, \delta)$ preferences, the lender sets the down payment requirement optimally at $\Phi_i = \Phi = 1 - w/K$. Depending on the parameters $(w, r_f, K, c, \beta, \delta)$ of the model, we either have $U^i_d(\Phi_1) \leq U_0$ or $U^i_d(\Phi_1) > U_0$.

CASE 1: $U^i_d(\Phi_1) \leq U_0$

Consider the interest rates $r^*_1$, $r^*_2$, $r'_1$ and $r'_2$ such that $U_1(r^*_1) = U_0$, $U_2(r^*_2) = U_0$, $V_1(r'_1) = V_d$, and $V_2(r'_2) = V_d$.

Claim 1.a. If $U^i_d(\Phi_1) \leq U_0$, $i = 1, 2$, then we have $\Phi_i = 1 - w/K$ and

$$r^*_1 \leq r'_1 \quad \text{and} \quad r^*_2 \leq r'_2 \quad \text{for all } \beta \leq 1.$$ 

Proof of the Claim 1.a: Given $\Phi_i^* = 1 - w/K$, we have $A_i^* = 0$ and $r^*_1$ and $r^*_2$ are determined by equating equation (2.1) to equation (2.2) and equation (2.3) respectively:

$$-\beta \delta^2 P_1(r^*_1)(1 + r_f) + c(1 + \beta \delta + \beta \delta^2) = \beta \delta^2 w(1 + r_f)^2,$$ \hspace{1cm} (9)

$$-\beta \delta^2 P_2(r^*_2)(2 + r_f) + c(1 + \beta \delta + \beta \delta^2) = \beta \delta^2 w(1 + r_f)^2.$$ \hspace{1cm} (10)

Similarly, $r'_1$ and $r'_2$ are determined by equating the equation (2.7) to (2.5) and (2.6) respectively:

$$-\beta \delta P_1(r'_1)(1 + r_f) + c(1 + \beta \delta) = 0,$$ \hspace{1cm} (11)

$$-\beta \delta P_2(r'_2)(2 + r_f) + c(1 + \beta \delta) = 0.$$ \hspace{1cm} (12)

Consider first the case of the short term loan $(i = 1)$. Combining the equations (9) and (11), we get

$$P_1(r^*_1) - P_1(r'_1) = \frac{c[1 + \delta(\beta - 1)] - \beta \delta^2 w(1 + r_f)^2}{\beta \delta^2(1 + r_f)}.$$ 

The condition $U^i_d(\Phi_1) \leq U_0$ can also be written as $\beta \delta^2 K(1 - \Phi_1)(1 + r_f)^2 \geq c$. Since $\Phi_1 \in [1 - w/K, 1]$ and $c[1 + \delta(\beta - 1)] < c$, we have

$$0 \leq \beta \delta^2 K(1 - \Phi_1)(1 + r_f)^2 - c \leq \beta \delta^2 w(1 + r_f)^2 - c[1 + \delta(\beta - 1)].$$
This implies that $P_1(r_1^*) - P_1(r_1') \leq 0$. $P_i(r)$ is increasing in $r$ and therefore $r_1^* \leq r_1'$. The proof of the assertions for the long term loan follows the same lines of argument.

Given $r_1^* \leq r_1'$, if the consumer’s self-0 accepts an offer at $r_1^*$ or at $r_2^*$, self-1 would agree with her and stay in the contract of her choice (i.e. there is no conflict between current and the future selves at these rates). Therefore, there the lender can not exploit the conflict by setting the down payment requirement to zero (i.e. $\Phi_i = 1$) for either of the contracts with maturity $i = 1, 2$. We conclude that the lender sets $\Phi_i = \Phi = 1 - \frac{w}{K}$.

Remark .19 Note that self-1 is willing to pay a higher rate in any of the two contracts, however self-0, if she is offered a higher rate, rejects and therefore effectively constraints the choice of self-1. This is important because it implies that the naive and sophisticated consumer behaviour coincide when $U_{d1}(\Phi_i) \leq U_0$, $i = 1, 2$.

Remark .20 When $U_{d1}(\Phi) \leq U_0$, the lender is able to maintain the time-inconsistent consumer at the payoff level $U_0$ by offering $(r_1^*, \Phi)$ on the short term loan and by offering $(r_2^*, \Phi)$ on the long term loan. Exponential consumer is also maintained at the payoff level $U_0$. The lender offers the exponential agent $(r_i, 1 - \frac{w}{K})$ where $r_i$ is given by the following equations:

\[-\delta P_1(r_1) + c(1 + \delta + \delta^2) = w,\]
\[-\delta(\delta + 1)P_2(r_2) + c(1 + \delta + \delta^2) = w.\]

CASE 2: $U_{d1}(\Phi_1) > U_0$

Consider the interest rates $r_1^0$, $r_2^0$, $r_1'$ and $r_2'$ such that $U_1(r_1^0) = U_{d1}^1, U_2(r_2^0) = U_{d2}^2$, $V_1(r_1') = V_d$, and, $V_2(r_2') = V_d$.

Claim 2.a. If $U_{d1}(\Phi_1) > U_0$, then the following relationship holds:

\[r_1^0 \leq r_1' \text{ and } r_2^0 \leq r_2' \text{ for } \beta < 1,\]
\[r_1^0 = r_1' \text{ and } r_2^0 = r_2' \text{ for } \beta = 1.\]

Proof of the Claim 2.a: Recall that, if the consumer accepts an offer with the intention to stay in the contract, she sets $\Phi_i^* = 1 - \frac{w}{K}$, and $A_i^1 = 0$ and if she wishes to leave, she sets $\Phi_i^* = \Phi_i$. The interest rates $r_i^0$ and $r_i^0$ are determined by
equating the equation (2.4) to (2.2) and (2.3) respectively:

\[-\beta \delta^2 P_1(r_1^0)(1 + r_f) + c(\beta \delta + \beta \delta^2) = \beta \delta^2[w - K(1 - \Phi_1)](1 + r_f)^2,\]

\[.13\]

\[-\beta \delta^2 P_2(r_2^0)(2 + r_f) + c(\beta \delta + \beta \delta^2) = \beta \delta^2[w - K(1 - \Phi_2)](1 + r_f)^2.\]

\[.14\]

Self-1 takes as given the values of \(\Phi^*\) and \(\Phi_i\). Therefore \(r'_1\) and \(r'_2\) are determined in exactly the same way as before, by equating the equation (2.7) with (2.5) and (2.6), respectively:

\[-\beta \delta P_1(r'_1)(1 + r_f) + c(1 + \beta \delta) = 0,\]

\[-\beta \delta P_2(r'_2)(2 + r_f) + c(1 + \beta \delta) = 0.\]

After some algebra we get

\[P_1(r_1^0) - P_1(r'_1) = \frac{c\delta(\beta - 1) - \beta \delta^2[w - K(1 - \Phi_1)](1 + r_f)^2}{\beta \delta^2(1 + r_f)}.\]

Since \(c\delta(\beta - 1) \leq 0\) and \(w \geq K(1 - \Phi_1)\), we have \(P_1(r_1^0) - P_1(r'_1) \leq 0\). \(P_i(r)\) is increasing in \(r\) which implies that \(r_1^0 \leq r'_1\). If \(\beta = 1\), the consumer has exponential discounting and the lender sets \(\Phi_i = 1 - w/K\). As a result we have \(r_1^0 = r'_1\). This concludes the proof \(\Box\).

We now need to analyze the conflict between the selves regarding the decision of leaving the contract. Let us begin by describing the nature of the conflict. Consider first the contract with a short term loan \((i = 1)\). For any \(r > r_1^0\), self-0’s optimal response is to accept the offer with the intention of leaving it next period. Self-1, however, is as impatient as self-0 regarding the consumption of durable and is willing to pay a higher interest rate \(r'_1\). Therefore for any \(r \in [r_1^0, r'_1]\), self-1 does not agree with the choice of self-0 (see figure .2).

Insert Figure .2 Here

The sophisticated consumer knows that the true discount rate employed by self-1 between periods 1 and 2 is \(\beta \delta\). Therefore, she correctly predicts that the Claim 2.a holds.

Naive consumer’s self-0 thinks that self-1’s discount rate between periods 1 and 2 is \(\hat{\beta} \delta\) and therefore, she mistakenly predicts that interest rates that make self-1 indifferent between staying or leaving, \(r_{i}^{\beta}, i = 1, 2\), are determined as follows:

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\[ -\hat{\beta}\delta P_1(r_{1}^{0})(1 + r_f) + c(1 + \hat{\beta}\delta) = 0, \]
\[ -\hat{\beta}\delta P_2(r_{1}^{0})(2 + r_f) + c(1 + \hat{\beta}\delta) = 0. \]

In other words, naive self-0 incorrectly believes that the following is true:

\[ P_1(r_{1}^{0}) - P_1(r_{1}^{\hat{\beta}}) = \frac{e^{\hat{\beta}(\beta-1)} - \beta\delta^2[w - K(1 - \overline{\Phi}_1)](1 + r_f)^2}{\beta\delta^2(1 + r_f)}. \]

The equation above implies that \( r_{1}^{0} \leq r_{1}^{\hat{\beta}}. \) Note that, when \( \hat{\beta} = 1 \) and \( \overline{\Phi}_1 = 1 - w/K, \) we have \( r_{1}^{0} = r_{1}^{\hat{\beta}}. \) In addition, we have \( r_{1}^{\hat{\beta}} = r_{1}^{1} \) when \( \hat{\beta} = \beta. \) The same argument applies to the case of the long term loan.

**Claim 2.b.** If \( U_d^i(\overline{\Phi}_i) > U_0, \) then \( r_{i}^{*} > r_{i}^{0} \) with \( i = 1, 2. \)

**Proof of the Claim 2.b:** Consider first the short term loan \( (i = 1). \) The interest rates \( r_{1}^{*} \) and \( r_{1}^{0} \) are determined by the following set of equations:

\[ -\beta\delta^2 P_1(r_{1}^{*})(1 + r_f) + c(1 + \beta\delta + \beta\delta^2) = \beta\delta^2 w(1 + r_f)^2, \]
\[ -\beta\delta^2 P_1(r_{1}^{0})(1 + r_f) + c(\beta\delta + \beta\delta^2) = \beta\delta^2[w - K(1 - \overline{\Phi}_1)](1 + r_f)^2. \]
The two equations can be combined to obtain
\[
P_1(r_0) - P_1(r_1^*) = \frac{\beta \delta^2 K (1 - \Phi_1)(1 + r_f)^2 - c}{\beta \delta^2 (1 + r_f)}.
\]
The condition \(U_1^d(\Phi_1) > U_0\) can also be expressed as \(\Leftrightarrow c > \beta \delta^2 K (1 - \Phi_1)(1 + r_f)^2\) and \(P_1\) is increasing in \(r\). This proves that \(r_1^0 < r_1^*\). The same argument applies to the case of the long term loan. \(\square\)

**Claim 2.c.** If \(U_i^d(\Phi_i) > U_0, i = 1, 2\), then we have
\[
r_i^* \leq r_i' \Leftrightarrow c - \beta \delta^2 w (1 + r_f)^2 \leq c \delta (1 - \beta).
\]

**Proof of the Claim 2.c:** The proof follows by observing that
\[
P_1(r_1^*) - P_1(r_1') = \frac{c [1 + \delta (\beta - 1)] - \beta \delta^2 w (1 + r_f)^2}{\beta \delta^2 (1 + r_f)}, \Box
\]

**Claim 2.d.** If \(U_i^d(\Phi_i) > U_0\), then the lender sets \(\Phi_i = \Phi = 1 - w/K\).

**Proof of the Claim 2.d:** Assume first that the lender sets \(\Phi_i = 1 - w/K\). Note that the lender profit, when consumer leaves the contract at the second period, is given as
\[
\Pi_l = -\Phi K - \alpha + \delta K |_{\Phi=1-w/K} = w - [K (1 - \delta) + \alpha].
\]
Given \(\Phi_i\), the profit is constant (i.e. independent of the interest rate). If the \(\Pi_l\) is sufficiently high to exceed the profits from the loan provision, the lender simply offers an interest rate which is high enough to induce the consumer to accept the offer and then leave the contract next period. In this case, the lender is able to maintain the time-inconsistent consumer at the payoff level \(U_d(1 - w/K)\).

Consider the sophisticated consumer. Recall that \(r_i^\beta = r_i'\) when \(\hat{\beta} = \beta\). Suppose that the lender offers a short term loan with an interest rate \(r \in (r_1^0, r_1')\). In this case, leaving the contract at time 1 is not feasible for self-0 because she knows that self-1 would choose to stay in the contract, violating her plans. Therefore, the conflict lowers self-0’s outside option from \(U_d(1 - w/K)\) to \(U_0\) (for this range of interest rates only). Assume first that \(r_1^* \leq r_1'\). For any \(r \in [r_1^0, r_1^*]\) we have \(U_1(r) > U_0\), and therefore self-0 accepts the offer and stays in the contract. For any \(r \in (r_1^*, r_1')\), we have \(U_1(r) < U_0\) and therefore self-0 rejects the offer. Thus, provided that the profits from the loan provision at \(r_1^*\) exceeds \(\Pi_l\), the profit maximizing rate of interest is \(r_1^*\).
At $r_1^*$, self-0 accepts the offer, self-1 stays and the lender is able to maintain self-0 at the payoff level $U_0$. Assume now that $r_1^* > r_1'$. Then for any interest rate $r \in [r_1^0, r_1']$ we have $U_1(r) > U_0$ and therefore the consumer would accept the offer and stay in the contract. And for any $r > r_1'$, the consumer would accept the offer and then leave the contract at the next period because we have $U_d(\Phi) > U_0$. This implies that for $r_1^* > r_1'$, the profit maximizing level of interest rate is $r_1'$ provided that the profits from the loan provision at $r_1'$ exceeds $\Pi_l$. At $r_1'$, self-0 accepts, self-1 stays in the contract and the lender is able to maintain the consumer at the payoff level $U(r_1')$. Same arguments apply to the case of the long term loan.

Consider now the naive consumer. Recall that if $\Phi = 1 - w/K$ then $\beta \in (\beta, 1]$ implies $r_i^\beta \in [r_i^0, r_i']$. Consider first the case where $r_1^* \leq r_1'$ and assume that $r_1^\beta \leq r_1^*$. For any $r \in (r_1^0, r_1')$, self-0 accepts the offer, self-1 stays and for any $r \in (r_1^\beta, r_1')$, self-0 accepts the offer thinking that self-1 would leave but self-1 stays. Therefore the profit maximizing interest rate is $r_1'$. Note that, this is true for any $\beta \in (\beta, 1]$. At $r_1'$, there is a difference in the payoff perceived by self-0, $U_d(\Phi)$, and the actual payoff she is able to obtain, $U_1(r_1')$. Assume now that $r_1^\beta > r_1^*$. For any $r \in (r_1^0, r_1^\beta]$, self-0 correctly predicts that self-1 would stay and, since $U(r) > U_0$, she accepts the offer. For any $r \in (r_1^\beta, r_1')$, self-0 again correctly predicts that self-1 would stay but since $U(r) < U_0$, she rejects the offer. For any $r \in (r_1^\beta, r_1')$, self-0 accepts the offer mistakenly believing that self-1 would leave but self-1 stays. Finally, for any $r > r_1'$, self-0 accepts the offer thinking that self-1 would leave and self-1 leaves. Hence, if profits from the loan provision at $r_1'$ exceeds $\Pi_l$, then the profit maximizing rate of interest is $r_1'$. At this rate, consumer’s self-0 accepts the offer, self-1 stays. payoff perceived by self-0, $U_d(\Phi)$, is different than the payoff obtained, $U_1(r_1')$. Consider now the case where $r_1^* > r_1'$. Recall that $r_1^\beta \leq r_1'$. For any interest rate $r \in (r_1^0, r_1^\beta]$, the first best option for the naive consumer’s self-0 is to accept the offer and then leave the contract next period. But, she correctly anticipates that self-1 would stay. Since we have $U(r) > U_0$, the next best choice for her is to accept the offer. For any $r \in (r_1^\beta, r_1')$, self-0 mistakenly believes that self-1 would leave and therefore accepts the offer but self-1 stays. Hence, if profits from the providing the loan at $r_1'$ exceeds $\Pi_l$, then the profit maximizing rate of interest is $r_1'$. Therefore, again the payoff that is perceived by self-0, $U_d(\Phi)$, is different then the payoff that is obtained, $U_1(r_1')$.

Finally, in all of the cases above, if the lender favors liquidation, that is, if $\Pi_l > \Pi_2(r_2^2, 1 - w/K) \geq \Pi_1(r_1^*, 1 - w/K)$, then he is able to maintain the naive consumer at the payoff level $U_d(1 - w/K)$ by offering any $r > r_1'$.

Assume now that the lender sets $\Phi_l = 1$. If $\Phi_l = 1$, we have $\Pi_l < \Pi(r_1, 1)$ and $U_d(1) > U_0$. That is, the lender does not favor liquidation and the consumer strictly prefers signing the contract first and then leaving it to rejecting it in the first place. Since for the sophisticated consumer intentions and actions are aligned, we
would never have the situation where the consumer accepts an offer thinking that she would leave the contract at the next period, but she ends up staying. Therefore, the lender, when facing the sophisticated consumer, would be strictly better off by setting \( \Phi_i = 1 - w/K \) instead of setting \( \Phi_i = 1 \).

Let us now check if relaxing the down payment requirement for naive consumers, by setting \( \Phi_i = 1 \), does make the lender better off. Consider again the case of an offer with a short term loan. Recall that the payoff maximizing choice for the consumer’s self-0 is \( \Phi_i^* = 1 - w/K \) if self-0 accepts the offer thinking that self-1 would stay, and \( \Phi_1^* = \Phi_1 \) whenever self-0 accepts the offer with the intention of leaving it. Following the same lines of the argument as in CASE-1, the profit maximizing choice of interest rate is again \( r'_1 \). At this rate, the naive consumer’s self-0 accepts the offer with the intention of leaving it at the next period. She mistakenly believes that self-1 would agree with her, thus she sets \( \Phi^* = \Phi_1 = 1 \), but self-1 stays. Note that, the value of \( r'_1 \) depends on the value of \( \Phi^* \) and hence \( \Phi_1 \). Therefore, the lender would prefer setting \( \Phi_1 = 1 \) to setting \( \Phi_1 = 1 - w/K \) iff

\[
\Pi_1(r'_1(\Phi^*), \Phi^*)|_{\Phi_1=1-w/K} < \Pi_1(r'_1(\Phi^*), \Phi^*)|_{\Phi_1=1}
\]

where

\[
\Pi_1 = \delta P_1(r'_1(\Phi^*), \Phi^*) - \Phi^* K - \alpha \quad \text{and}
\]

\[
P_1(r'_1(\Phi^*), \Phi^*) = \frac{c(1 + \beta \delta)}{\beta \delta (1 + r_f)}.
\]

As \( \Phi^* \) increases, the interest that makes the self-1 indifferent between staying and leaving, \( r'_1(\Phi^*) \), decreases, leaving \( P_1 \) constant. This implies that the lender profits at \( \Phi_1 = 1 \) is lower. Therefore we conclude that the lender sets \( \Phi_1 = 1 - w/K \), also for the naive consumer. The same argument applies to the case of the long term loan. This concludes the proof of Claim 2-d. \( \square \)

We showed that the lender’s profit maximizing choice of the down payment requirement is \( \Phi = 1 - w/K \) for any consumer with preference parameters \( (\beta, \tilde{\beta}, \delta) \). This concludes the proof of Lemma 2.5. \( \square \)

**Proofs of Propositions 2.9, 2.10, 2.11, 2.13 and the Corollary 2.15**

The proofs follow from the Lemmas 2.4 and 2.5 and noting that (i) the exponential discounting is a special case of quasi hyperbolic discounting with \( \beta = 1 \) and, (ii) when the consumer discount exponentially, \( r_f = 1 - 1/\delta \). \( \square \)
Proof of Proposition 2.16

Using the proofs of Lemmas 2.4 and 2.5, we know that if $U_d(\Phi)|_{\Phi=1-w/K} \leq U_0$, the lender is able to maintain exponential, naive and sophisticated consumers at the payoff level $U_0$ and the consumer surplus is zero for all types. When $U_d(\Phi)|_{\Phi=1-w/K} > U_0$, the lender is able to maintain the sophisticated consumer at the payoff level $U_0$ if $r^*_i \leq r'_i$ and at the payoff level $U_d(\Phi)|_{\Phi=1-w/K}$ if $r^*_i > r'_i$. By definition the exponential and the sophisticated consumer surpluses are equal to zero. The naive consumer surplus is given by

$$U_1(r'_i) - U_1(r^0_i) = U_2(r'_2) - U_2(r^0_2) = c\delta(\beta - 1) - \beta\delta^2[w - K(1 - \Phi)](1 + r_f)^2.$$  

Since in an equilibrium contract the lender sets $\Phi = 1 - w/K$, this reduces to $c\delta(\beta - 1) < 0$. The naive consumer’s surplus does not depend on the degree of naivete which is measured by $\hat{\beta} - \beta$ for fixed $\beta$. □

Proof of Proposition 2.17

For any $(r_i, \Phi), i = 1, 2$ we have

$$\Pi_1(r_1, \Phi) = \delta P_1(r_1, \Phi) - \Phi K - \alpha,$$

$$\Pi_2(r_2, \Phi) = \delta(1 + \delta)P_2(r_2, \Phi) - \Phi K - \alpha.$$  

Given $\Phi = 1 - w/K$, this implies that

$$\Pi_2(r_1^*, 1 - w/K) \geq \Pi_1(r_1^*, 1 - w/K) \iff (1 + \delta)P_2(r_2, 1 - w/K) \geq P_1(r_1, 1 - w/K).$$

Using equations (.9) and (.10) to substitute for $P_1(r_1^*, 1 - w/K)$ and $P_2(r_2^*, 1 - w/K)$, we get

$$\Pi_2(r_1^*, 1 - w/K) \geq \Pi_1(r_2^*, 1 - w/K) \iff \frac{1 + r_f}{2 + r_f} \geq \frac{1}{1 + \delta}. $$

Similarly, using equations (.13) and (.14) we have

$$\Pi_2(r_1^0, 1 - w/K) \geq \Pi_1(r_2^0, 1 - w/K) \iff \frac{1 + r_f}{2 + r_f} \geq \frac{1}{1 + \delta}, $$

and using equations (.11) and (.12) we have

$$\Pi_2(r_1', 1 - w/K) \geq \Pi_1(r_2', 1 - w/K) \iff \frac{1 + r_f}{2 + r_f} \geq \frac{1}{1 + \delta}. $$
This shows that
\[ \Pi_2^{(\hat{\beta},\beta,\delta)} \geq \Pi_1^{(\hat{\beta},\beta,\delta)} \iff r_f \geq \frac{1}{\delta} - 1 \text{ for any } \beta \leq 1, \delta \leq 1, \beta \leq \hat{\beta}. \]

Under exponential discounting \((\beta = \hat{\beta} = 1)\) we have \(r_f = \frac{1}{\delta} - 1\), which implies
\[ \Pi_2^{(\beta,\beta,\delta)} = \Pi_1^{(\beta,\beta,\delta)}. \]

**Proof of Corollary 2.18**

Using the proofs of Lemma 2.4, Lemma 2.5 and the Proposition 2.16, the lender profits are given as follows:

If \( U_d(\Phi)_{|\Phi=1} - w/K \leq U_0 \), then
\[
\Pi_1^\beta(r_1^*, 1 - \frac{w}{K}) = \Pi_1^\hat{\beta}(r_1^*, 1 - \frac{w}{K}) = \delta P_1(r_1^*, 1 - \frac{w}{K}) + w - K - \alpha \text{ and,} \\
\Pi_2^\beta(r_2^*, 1 - \frac{w}{K}) = \Pi_2^\hat{\beta}(r_2^*, 1 - \frac{w}{K}) = \delta(1 + \delta)P_2(r_2^*, 1 - \frac{w}{K}) + w - K - \alpha.
\]

If \( U_d(\Phi)_{|\Phi=1-w/K} > U_0 \), then the naive profits are given as
\[
\Pi_1^{(\beta,\hat{\beta},\delta)}(r_1^*, 1 - \frac{w}{K})_{|\beta \in (\beta,1]} = \frac{c(1+\beta \delta)}{\beta(1+r_f)} + w - K - \alpha, \\
\Pi_2^{(\beta,\hat{\beta},\delta)}(r_2^*, 1 - \frac{w}{K})_{|\beta \in (\beta,1]} = (1 + \delta)\frac{c(1+\beta \delta)}{\beta(2+r_f)} + w - K - \alpha
\]
and, the sophisticated profits are given as:

If \( r_1^* \leq r_1' \), then
\[
\Pi_1^{(\beta,\hat{\beta},\delta)}(r_1^*, 1 - \frac{w}{K})_{|\beta = \beta} = \\
\frac{c(1+\beta \delta + \beta^2 \delta^2) - \beta \delta^2 w(1 + r_f)^2}{\beta \delta(1 + r_f)} + w - K - \alpha \text{ and,} \\
\Pi_2^{(\beta,\hat{\beta},\delta)}(r_2^*, 1 - \frac{w}{K})_{|\beta = \beta} = (1 + \delta)\frac{c(1+\beta \delta + \beta^2 \delta^2) - \beta \delta w(1 + r_f)^2}{\beta \delta(2 + r_f)} + w - K - \alpha.
\]

If \( r_1^* > r_1' \), then
\[
\Pi_1^{(\beta,\hat{\beta},\delta)}(r_1', 1 - \frac{w}{K})_{|\beta = \beta} = \frac{c(1+\beta \delta)}{\beta(1+r_f)} + w - K - \alpha \text{ and,} \\
\Pi_2^{(\beta,\hat{\beta},\delta)}(r_2', 1 - \frac{w}{K})_{|\beta = \beta} = (1 + \delta)\frac{c(1+\beta \delta)}{\beta(2 + r_f)} + w - K - \alpha.
\]
Therefore, if \( U_d(\Phi)_{\Phi=1-w/K} > U_0 \) and if \( r_i^* > r_i' \), naive and sophisticated consumers pay the same level of interest, \( r_i' \). That is, we have

\[
\Pi_i^{(\beta,\hat{\beta},\delta)}|_{\hat{\beta} \in (\beta,1]} - \Pi_i^{(\beta,\hat{\beta},\delta)}|_{\hat{\beta} = \beta} = 0 \quad \text{for} \quad r_i^* > r_i'.
\]

When \( r_i^* \leq r_i' \), naive consumer pays \( r_i' \) and the sophisticated consumer pays \( r_i^* \). The difference between the lender profits at \( r_i' \) and \( r_i^* \) can be calculated as:

\[
\Pi_1^{(\beta,\hat{\beta},\delta)}|_{\hat{\beta} \in (\beta,1]} - \Pi_1^{(\beta,\hat{\beta},\delta)}|_{\hat{\beta} = \beta} = \\
... = -\frac{1}{\beta \delta (1 + r_f)} [c(1 + \delta(\beta - 1)) - \beta \delta^2 w (1 + r_f)] > 0,
\]

\[
\Pi_2^{(\beta,\hat{\beta},\delta)}|_{\hat{\beta} \in (\beta,1]} - \Pi_2^{(\beta,\hat{\beta},\delta)}|_{\hat{\beta} = \beta} = \\
... = -\frac{1 + \delta}{\beta \delta (2 + r_f)} [c(1 + \delta(\beta - 1)) - \beta \delta^2 w (1 + r_f)] > 0. \square
\]
.B Anecdotal evidence on lending practices

In this appendix we give more anecdotal evidence on consumers’ tendency to focus more on monthly payments than on interest rates or the total cost of the loan.

**Focusing only on monthly payments**

Consumer protection agencies underline particularly on one aspect of consumer attitude towards buying on credit: It seems that people generally focus on the amount of monthly payments rather than the total price of the loan when comparing different loans or when consolidating their debt. Consumers seem to apply a simple rule of thumb “the lower the monthly payment, the more attractive the loan is”. As an example, consider the following quote from one of the publications of Ohio Department of Commerce-Consumer Affairs. The publication is mainly about predatory lending tricks and how to recognize them. They warn the consumers about the lending practices such as “selling the monthly payment” or “flipping by repeated financing” (debt consolidation). It is written

“When dealing with loans it is a mistake to focus exclusively on the monthly payment...of equal or greater significance are what is the loan’s interest rate and other finance charges?..How long will it last and how much will it cost in total payments?”

The following two examples are typical dealer scams targeting consumers who choose to finance their automobiles at the car dealer.

**Selling the monthly payment**

“Bonus Scam: “We lowered your payments! Come back and re-sign!”

About a week after you drive home with your new car, the car dealer calls you up and says “Great news! we were actually able to get you into a better loan with a lower monthly payment and less APR! Just stop by our dealership when you get a minute, and resign the papers to get your lower payment.”

You go down to the dealer to re-sign the paperwork, and this is where a huge red flag should shoot up for you: You notice that they changed your loan from 48 months to 60 months, or even 72 months. What they did in reality was INCREASE the APR on you not lower it like they lied. Then

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5The publication can be found at http://www.com.state.oh.us/dfi/app/Tricks-Final2.pdf
to mask their crime, they spread out the loan over more months, which lowers the monthly payment, tricking you into thinking you are saving money when you’re getting ripped off. Yes there are people out there who believe the dealer and think they are actually paying lower APR. This does not even pass the common sense test either and here is the crucial point: With most lenders, if you increase the number of months in the loan, the APR goes UP, not down, just look at their rate card! Make sure you point that out to them. The salespeople who pull this scam sometimes make buyers re-sign the paperwork and don’t give them copies, and the buyer finds out later on the dealer lied about the “reduced APR.

How to avoid the scam: If they call you later, say “No thanks, I like my payment the way it is.” Tell them to fax or email it to you in writing first to review it for unexpected changes. Better yet don’t answer the phone. Want a lower rate? Refinance your car loan, or send extra principle monthly, it has the same effect”.

**Negotiate for monthly payments**

“A car salesman will often try to negotiate with you over the monthly payment, not the car price. This is a costly trick. By focusing on monthly payments, he can obscure the price of the car ...Therefore, you should negotiate for price. And you must know the price you can afford BEFORE you go into the dealer. Once you know the price you want, THEN negotiate for the monthly payment. If you got your loan through your bank or credit union, they will have already worked out a payment schedule anyway.

Why do many service members negotiate for monthly payment? Because you usually figure out your monthly budget to figure out if you can afford to purchase a car. That is an important step. But when you are actually at the dealership, negotiate for price. If you don’t, the car dealer can have you paying a lot more”.

Below is an example of loan arrangement, that potentially targets people with self-control problems.

**Temptation of buying now, paying later**

Watch Out For... BALLOON LOANS

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6source: http://www.military.com/Finance/
“From time to time you may find a lender offering a particularly attractive loan with very low monthly payments. This could be a “balloon loan,” one which has a very large final payment (a “balloon” payment is generally thought of as any payment that is more than twice the amount of any other payment). With these types of loans, little or none of the monthly payment goes to reduce the principal (the amount borrowed); only the interest is repaid during the term of the loan. The principal is mostly or entirely paid off in the final payment.

Balloon loans can be difficult for consumers who are unprepared for the large final payment. The information about the final balloon payment must be given to the borrower during the loan application process. Balloon loans are prohibited by federal law when the length of the loan is less than 5 years.”

**Triggering terms in loan advertisements**

Is it possible to change the consumer’s perception of the price by adjusting the payment scheme to consumer’s psychological tendencies? Anecdotal evidence suggests that consumers are more sensitive to the size of the monthly payments than the total amount they pay. That is, potential buyers are more sensitive to an increase in the monthly payment amount than they are to an increase in the total price of the loan. From the perspective of lenders, it is possible to offer any amount of monthly payment by increasing the duration of the loan and the interest rate simultaneously. Therefore, an automobile finance company can make a car look more affordable by using monthly payments and longer loan durations as main advertising messages.

Perhaps the financing companies are also aware of the tendency of the consumers to focus more on how much they are going to pay each month rather than total price of the loan. If one looks at a typical loan advertisement for automobiles, one sees that while the monthly payment is the main advertising tool, most of the time the total price of the loan appears in small prints.

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Chapter 3

Self control and debt: evidence from data on credit counselling

3.1 Introduction

People run into financial difficulties for various reasons. Is overindebtedness always caused only by unexpected adverse events such as loss of job, drop in income, illness or divorce? Can it also be merely a result of lack of financial planning? Chakravarty and Rhee [10] using PSID data for the U.S. between 1984-95, for example, reported that among people who filed for bankruptcy, around 40 percent stated credit misuse as the reason. We reproduce Panel B of Table 1 from their paper below:

<table>
<thead>
<tr>
<th>Reason for filing (chapter 7)</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job Loss</td>
<td>12.17</td>
</tr>
<tr>
<td>Marital distress</td>
<td>14.28</td>
</tr>
<tr>
<td><strong>Credit Misuse</strong></td>
<td>41.26</td>
</tr>
<tr>
<td>Health care bills</td>
<td>16.40</td>
</tr>
<tr>
<td>Lawsuits and Harassments</td>
<td>15.87</td>
</tr>
</tbody>
</table>

Source: Table 1, Panel B from Chakravarty and Rhee [10]. U.S. between 1984-95, PSID.

One possible reason for credit misuse may-be the lack of financial literacy. If debt mismanagement problems are due to lack of financial literacy, then the financial education should play a key role in helping people take “better” (from the economic theory point of view, “optimal”) decisions. However, as reported in Durkin [14] and discussed in Bertaut and Haliassos [6], there is survey evidence suggesting that

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1This chapter is a joint work with Alena Bicakova
people are generally familiar with basic financial concepts, i.e. they are aware of APR (annual percentage rate) and other terms of the credit card contracts. If we are to describe “unaware” as not knowing the APR on a bank type credit contract, the level of “awareness” is around 80 percent among the people surveyed.

**Self control problems** represent an alternative and entirely different reason for debt mismanagement. The 2001 Survey of Consumers (U.S.) reveals that about forty percent of the households regard self-control as a general problem, and they believe that availability of credit cards might trigger overspending and overborrowing. Interestingly, a significant percentage of people think that this is a problem for others but not for themselves (Durkin [14], Bertaut and Haliassos [6]).

Credit counselling assists heavily indebted borrowers by setting up and administering repayment plans, so called debt management plans (DMPs). In this chapter, we use a unique administrative data set of a major consumer credit counselling charity in the UK to explore the effect of self-control problems as well as the effect of various other individual characteristics on the debt repayment behavior of borrowers in financial difficulties.

Despite the fact that the credit counselling industry has been growing rapidly, the economic research in this area is still scarce. To our knowledge, there are only two papers on credit counselling that provide quantitative analysis, each with a rather different focus: Staten et. al. [36] use detailed credit bureau information from the U.S. to explore the effect of credit counselling on borrowers’ credit scores and future repayment behavior. They find that credit counselling has a significant positive effect on borrowers’ credit worthiness, in particular for those borrowers who have low initial credit scores. They also document improvement in many aspects of borrowing behavior of people who received counselling. Beltz [4] on the other hand analyzes the relationship between counselling agencies and creditors. He concludes that a closer co-operation between creditors and the counselling agencies would be preferable to current status where competition is used to lower the fair-share contribution rates.

Our analysis focuses on the counselling process itself. Namely, we ask who are the borrowers that are more likely to succeed in a DMP and therefore benefit from the counselling most, and who are the ones who do not.

We begin by developing a simple model of repayment behavior of a borrower in a debt management plan. Our theoretical model rely on the quasi-hyperbolic discounting ((Strotz [37]; Phelps and Pollack [28]; Laibson [21]; O’Donoghue and Rabin [25], [26]) to model self-control. However, our empirical analysis is neutral to this modelling choice (i.e not restricted on the time preference interpretation of self-control). The quasi-hyperbolic discount function for time $s$ evaluated at period $t$, equals to 1 for $s = t$ and is equal to $\beta \delta^{s-t}$ for $s = t + 1, t + 2, \ldots$ where $\beta \in (0, 1]$ and $\delta \in (0, 1]$. The case of $\beta = 1$ corresponds to time-consistent, exponential discounting. Allowing for the discount rates to be time-varying results in time-inconsistency because.
plans that are optimal from the perspective of today may not be optimal from the perspective of tomorrow. Time-inconsistency of preferences requires us to make assumptions on the consumers’ analysis of their own future behavior. More specifically, we distinguish two types of consumers with dynamically inconsistent preferences: a sophisticated consumer who is fully aware of her self-control problems and a naive consumer who is partially or completely unaware of her self-control problems. We compare the results to the exponential discounting benchmark. The simple model predicts that sophisticated hyperbolic discounters would have the same DMP dropout rates as exponentials whereas naive hyperbolic discounters would drop out from the DMP more often.

We use the dataset of clients of a major consumer credit counselling charity in the UK to test the predictions of our model. Our estimation sample consists of all the borrowers who started a DMP between January 2003 and November 2006 (60,495 individuals). We observe their debt repayment behavior (DMP performance) as well as various individual characteristics, budget information (income and expenditures), and DMP terms (amount of debt, duration, monthly payment). We construct two indicators of self-control problems using two types of information: self-reported reasons for running into financial troubles and smoking. Individuals who reported difficulties in managing their finances (poor shopping habits, lack of financial planning, no budget etc.) as the reason for running into financial troubles, and smokers are both regarded as suffering from self-control problems. We then compare the debt repayment behavior of individuals with and without self-control problems. We estimate a Cox proportional hazard model of the probability of dropping out from a DMP as a function of various individual characteristics including the two self-control indicators.

Preliminary results show that self-control problems increase the drop-out probability at any time by 12% and 31% respectively, depending on the self-control indicator used. We also find that the drop-out probability decreases with age and that women are substantially more likely to stay on DMP than men. A single woman is almost 40% less likely to drop than a single man. A couple is more likely to stay on the DMP if it is a woman who contacts CCCS. Having a mortgage decreases the probability of the DMP drop-out. While being self-employed increases the probability of dropping out, working as a full-time employee reduces it.

The remainder of the chapter is organized as follows. In the next section we introduce a very simple theoretical model of a DMP and analyze the behavior of the exponential, naive and sophisticated borrowers. Section 3.3 describes the data and the estimation method. In Section 3.4 we present the estimation results, and in Section 3.5 we conclude. Appendix A describes the variables that are used in the estimation. In Appendix B we discuss the role of credit counselling as a commitment mechanism to help borrowers overcome their self-control problems.
3.2 A simple model of a DMP

We construct a simple model of a DMP repayment behavior with income uncertainty that serves to illustrate the repayment performance of different types of borrowers while on a DMP. We consider borrowers that differ only with respect to their time preferences as explained below.

Discounting. We assume that borrowers have quasi-hyperbolic discount function (Strotz [37]; Phelps and Pollack [28]; Laibson [21]; O’Donoghue and Rabin [25], [26]). The quasi-hyperbolic discount function for time \( s \) evaluated at period \( t \), equals to 1 for \( s = t \) and is equal to \( \beta \delta^{s-t} \) for \( s = t + 1, t + 2, ... \) where \( \beta \in (0, 1] \) and \( \delta \in (0, 1] \). The case of \( \beta = 1 \) corresponds to time-consistent, exponential discounting. When \( \beta < 1 \), the consumer has time-inconsistent discounting. Consumers with time inconsistent preferences are like different selves at different times (preferences change over time). We refer to consumer’s self in period 0 as self-0, and that of period two and period three as self-1 and self-2 respectively. With quasi-hyperbolic discount function, self-0 time preferences are given as \( \{1, \beta \delta, \beta \delta^2, ...\} \). Suppose that self-0 thinks that the future selves’ time preferences are \( \{1, \hat{\beta} \delta, \hat{\beta} \delta^2, ...\} \). A consumer with preference parameters \((\beta, \hat{\beta}, \delta)\) is said to have time-consistent exponential discounting if \( \beta = 1 \) with \( \hat{\beta} = 1 \). We will consider the following cases of time-inconsistency: if \( \beta < 1 \) with \( \hat{\beta} = \beta \), the consumer is said to be sophisticated and if \( \beta < 1 \) with \( \beta < \hat{\beta} \), the consumer is naive. That is, the sophisticated consumer is fully aware of her hyperbolic preferences and correctly anticipates that her future selves will have hyperbolic preferences. In contrast, the naive consumer is partially or completely unaware of her time-inconsistency. She thinks that her future selves will either discount exponentially (fully naive) or she overestimates her true \( \beta \) (partially naive). The difference \( \hat{\beta} - \beta \) can be interpreted as the degree of naivete. The difference \( \beta - 1 \) reflects the degree of time-inconsistency.

3.2.1 Borrower behavior in a DMP

We assume that the time is discrete and there are three periods (0,1,2). We consider a borrower with preference parameters \((\beta, \hat{\beta}, \delta)\) and a given debt level \( P \) at \( t=0 \). We define the DMP as an agreement between the debt counselor and the borrower such that the borrower promises to pay \( P \) at \( t=1 \) upon accepting the DMP at \( t=0 \). We assume for simplicity that the consumption takes place only in period 1 and the per period utility \( u_t(c_t) = c_t \) is linear in \( c_t \). The income at \( t = 1 \) is a random variable that takes the value \( y_h \geq P \) with probability \( \alpha \) and the value \( y_l < P \) with probability \( 1 - \alpha \). There is no saving or borrowing while on a DMP.

At time \( t = 0 \) the borrower decides whether to accept a DMP (debt management plan) or reject it and default on his debts. If the borrower rejects the DMP and
defaults, he incurs a cost \( d_0 \) at \( t = 1 \). If the borrower accepts the DMP and if the realization of the income \( y \) at \( t = 1 \) is \( y_t \) then the borrower cannot meet his payment obligation and drops out. In this case the borrower gets the utility \( u(y_t) = y_t \) from consuming \( y_t \) at \( t = 1 \) and incurs the cost \( d_1 \) at \( t = 2 \). If the realization of income is \( y_h \) then the borrower decides whether to stay in DMP or drop out. If he stays, he consumes \( u(y_h - P) = y_h - P \) at \( t = 0 \). If he drops out, he incurs the cost \( d_1 \) at \( t = 2 \).

We compute the conditional drop-out probability, namely the probability that a borrower with \( (\beta, \hat{\beta}, \delta) \) preferences drops out from a DMP at \( t = 1 \) conditional on accepting a DMP at \( t = 0 \) under the following assumptions:

**Assumption 3.1** At \( t = 0 \), expected benefit of rejecting the DMP is higher than the expected benefit of accepting and then dropping-out next period if \( y = y_h \).

**Assumption 3.2** At \( t = 0 \), paying the debt when \( y = y_h \) is preferred to dropping-out when \( y = y_h \).

The Assumptions 1 and 2 imply that the no borrower has strategic default motives at \( t = 0 \) when signing the DMP. That is, dropping out from a DMP when there is a positive shock to income is never ex-ante optimal. We believe this assumption is justifiable in the context of credit counselling, since DMPs are voluntary agreements.
We begin by deriving the condition under which the Assumption 1 holds. In other words, we will derive the condition that any borrower prefers rejecting the DMP to accepting it at \( t = 0 \) with the intention of dropping out at \( t = 1 \) if \( y = y_h \):

\[
-\beta \delta d_0 + \beta \delta [(1 - \alpha) y_l + \alpha y_h] > \beta \delta \left[ \alpha (y_l - \hat{\beta} \delta d_1) + (1 - \alpha) (y_h - \hat{\beta} \delta d_1) \right],
\]

which implies that

\[
\hat{\beta} > \frac{d_0}{\delta d_1 \delta}. \tag{3.1}
\]

We proceed to derive the condition for accepting a DMP, namely, the condition that the expected benefit of agreeing a DMP at \( t = 0 \) with the intention of staying at \( t = 1 \) when \( y = y_h \), exceeds that of rejecting the DMP, both evaluated from the perspective of \( t = 0 \):

\[
-\beta \delta d_0 + \beta \delta [(1 - \alpha) y_l + \alpha y_h] \leq \beta \delta \left[ \alpha (y_l - \hat{\beta} \delta d_1) + (1 - \alpha) (y_h - P) \right]
\]

After some algebra (3.2) simplifies to a condition on \( \hat{\beta} \):

\[
\hat{\beta} \leq \frac{d_0 - (1 - \alpha) P}{\alpha d_1 \delta}. \tag{3.2}
\]

Note that, if (3.1) and (3.2) both hold, then the Assumption 2 is automatically satisfied.

When \( t = 1 \) arrives, the condition for staying in DMP becomes,

\[
y_h - P \geq y_h - \beta \delta d_1
\]

which after some algebra simplifies to

\[
\beta \geq \frac{P}{\delta d_1}. \tag{3.3}
\]

Recall that the exponential borrower has \( \beta = \hat{\beta} = 1 \). It follows from (3.1), (3.2) and (3.3) that the exponential borrower accepts the DMP iff \( \delta > \frac{d_0}{\delta d_1} \equiv \delta \) and \( \delta \leq \frac{d_0 - (1 - \alpha) P}{\alpha d_1} \equiv \overline{\delta} \), and drops out only when \( y = y_l \). Note that \( \delta \in (\underline{\delta}, \overline{\delta}) \) implies \( d_0 > P \). Given \( \delta \in (\underline{\delta}, \overline{\delta}) \), it follows from (3.1), (3.2) and (3.3) that: (i) the sophisticated borrower (\( \beta = \hat{\beta} \)) with \( \beta \in (\frac{d_0}{\delta d_1}, 1) \) accepts the DMP and drops out with probability
α (ii) the naive borrower who shares the same β, accepts the DMP if \( \hat{\beta} \in (\frac{d_0}{d_1}, 1) \) and drops out with probability α (iii) the sophisticated borrower with \( \beta < \frac{d_0}{d_1} \) rejects the DMP and (iv) the naive borrower who shares the same \( \beta \) accepts the DMP if \( \hat{\beta} \in (\frac{d_0}{d_1}, 1) \) and drops out with probability \( 1 > \alpha \) if \( \beta < \frac{P}{d_1} \). The following proposition summarizes our results.

**Proposition 3.3** Assume that Assumptions 1 and 2 hold, and that \( \delta \in (\tilde{\delta}, \bar{\delta}) \). The exponential borrower accepts the DMP and drops out with probability α. The sophisticated borrower accepts the DMP if \( \beta > \frac{d_0}{d_1} \) and drops out with probability α. The naive borrower, who shares the same \( \beta \), accepts the DMP if \( \hat{\beta} > \frac{d_0}{d_1} \) and drops out with probability α. The sophisticated borrower rejects the DMP when \( \beta < \frac{d_0}{d_1} \), and the naive borrower who shares the same \( \beta \) accepts the DMP if \( \hat{\beta} > \frac{d_0}{d_1} \), and drops out with probability \( 1 \) if \( \beta < \frac{P}{d_1} \).

The proposition above demonstrates that, under the Assumptions 1 and 2, the exponential and the sophisticated borrowers drop out of the DMP only if \( y = y_l \) whereas it is possible to have the naive borrower to drop out when \( y = y_h \).

The following example illustrates the proposition by assuming a particular joint distribution on \( \beta \) and \( \hat{\beta} \).

**Example Distribution for \( \beta \) and \( \hat{\beta} \)**

Consider the population of borrowers with their \((\beta, \hat{\beta})\) jointly uniformly distributed as follows

\[
f_{\beta,\hat{\beta}}(\beta, \hat{\beta}) = \begin{cases} 2 & \text{if } 0 \leq \beta \leq \hat{\beta} \leq 1, \\ 0 & \text{otherwise.} \end{cases}
\]

The marginal distribution of \( \hat{\beta} \) can be calculated as

\[
f_{\hat{\beta}}(\hat{\beta}) = \int_{\beta}^{1} f_{\beta,\hat{\beta}}(\beta, \hat{\beta}) \, d\beta = 2\hat{\beta}
\]

if \( 0 \leq \hat{\beta} \leq 1 \) and \( f_{\beta}(\hat{\beta}) = 0 \) otherwise.

Figure 3.2 illustrates the behaviour of a borrower with \((\beta, \hat{\beta}, \delta)\) preferences as \( \beta \) and \( \hat{\beta} \) vary.

The point \((1,1)\) on the upper corner represents the exponential borrowers, if \( \delta \in (\frac{d_0}{d_1}, \frac{d_0-(1-\alpha)P}{d_1}) \), they accept the DMP and stay if \( y = y_h \). The line segment
Figure 3.2: DMP outcome for different combinations of $\beta$ and $\hat{\beta}$

joining the points $\left(\frac{d_0}{\delta d_1}, \frac{d_0}{\delta d_1}\right)$ and $(1, 1)$ represents the sophisticated borrowers who accept and stay if $y = y_n$. The area with coordinates $\left(\frac{d_0}{\delta d_1}, 0\right)$, $\left(\frac{d_0}{\delta d_1}, \frac{P}{\delta d_1}\right)$, $(1, \frac{P}{\delta d_1})$ and $(1, 0)$ represent the naive borrowers who accept the DMP thinking they would stay but end up dropping at $t = 1$ when $y = y_n$.

The sophisticated hyperbolic discounters and exponentials do not drop from the DMP at $t = 1$ once they have accepted a DMP at $t = 0$, unless the realization of $y$ is $y_l$ (negative shock). Therefore for this type of borrowers the drop out probability is given by $\alpha$.

The probability (for a naive borrower) of dropping out at $t = 1$ conditional on accepting the DMP at $t = 0$ can be calculated as follows:

$$\alpha + (1 - \alpha)Pr(\beta \leq \frac{P}{\delta d} \mid \hat{\beta} > \frac{d_0}{\delta d_1}) = \alpha + (1 - \alpha)\frac{\int_{0}^{P/\delta d} \int_{d_0/\delta d_1}^{1} 2d\hat{\beta}d\beta}{\int_{d_0/\delta d_1}^{1} 2\hat{\beta}d\beta}$$

$$= \alpha + (1 - \alpha)\frac{2P}{d_0 + \delta d_1}.$$
3.3 Empirical analysis

3.3.1 About CCCS

We use the client database of the Consumer Credit Counselling Service (CCCS), the leading provider of the free debt counselling service in the UK.\(^2\) CCCS acts as a mediator between borrower/debtor and typically “many lenders” by negotiating a debt consolidation plan known as debt management plan (DMP) with lenders on behalf of consumers.

People in financial difficulties reach CCCS through their free phone number or their website.\(^3\) An assessment of the situation is performed over the phone and consumers are offered one of the following: financial and budget advice (providing self-help material) or a counselling interview.\(^4\) If the client is a candidate for a debt management plan (DMP), CCCS negotiates a repayment plan with creditors on behalf of the consumer (creditors are asked to freeze interest, stop penalties, and accept a longer repayment period and sometimes a reduced sum). Therefore DMP is essentially a debt consolidation plan. CCCS only deals with unsecured debt (debt accumulated on credit cards, store cards, catalogue orders etc.).\(^5\)

CCCS was introduced into the UK in 1993.\(^6\) Table below shows the number of people who started a DMP and the total number of people who were on a DMP at the end of each year between 2003-2006. Note that, these numbers take into account that there were borrowers who successfully completed their DMPs or who found other means of paying their debt and therefore left CCCS.

<table>
<thead>
<tr>
<th>Year</th>
<th># people started a DMP</th>
<th># people on DMP at year’s end</th>
<th>average debt per client</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>13,448</td>
<td>30,399</td>
<td>£27,565</td>
</tr>
<tr>
<td>2004</td>
<td>16,984</td>
<td>37,825</td>
<td>£29,341</td>
</tr>
<tr>
<td>2005</td>
<td>24,444</td>
<td>51,133</td>
<td>£30,763</td>
</tr>
<tr>
<td>2006</td>
<td>35,135</td>
<td>73,655</td>
<td>£31,370</td>
</tr>
</tbody>
</table>


\(^2\)CCCS is a registered debt charity and therefore has a non profit status. It is funded by fair share contributions from credit industry.

\(^3\)Almost half of CCCS clients are recommended to CCCS by their lenders.

\(^4\)The interview can take place over the phone or face to face in one of the centres and it consists of a full review of the credit and debt situation followed by a recommendation.

\(^5\)Secured debts (mortgages) are considered as “priority debts” and monthly payment obligations on such debts are taken into account when agreeing a monthly budget with the client.

\(^6\)It started with a pilot scheme in Leeds. During 1996 new centres opened in Nottingham, Birmingham and Cardiff, Galsgow, Newcastle, Chester, London and Limavady in Northern Ireland. Last year the charity opened new centers in Eastbourne, Sussex and more recently a center in Halifax.
In 2006, a typical CCCS client on a DMP is in mid-30’s, married with children with an average debt of just over 31,000 pounds.

3.3.2 Data Description and Key Variables

Our raw data contains information on all CCCS clients who have started a DMP between January 2003 and January 2007 (approximately 75,000 individuals) when the data was extracted. For the reasons we explain in section 3.3.3, our estimation sample uses 60,495 of these individuals.

Besides individual characteristics (such as age, gender, marital status, no of dependants, smoking, employment status, type of housing etc.), information about the debt (total amount owed to creditors, no of debts, information on each debt, debt type, creditor name etc.), and the agreed DMP budget (income, expenditures) and terms (amount of debt on which the DMP is set up, duration, monthly payments), we also have a self-reported reason of why the individuals ran into financial difficulties (i.e. why they have built up an amount of debt they cannot repay or handle without CCCS help).

Reason for Debt Indicator

Corresponding to the theoretical model, we are in particular interested in the repayment behavior of borrowers who may be classified as the ones with self-control problems, and how they compare to the rest of the CCCS clients. The fact that the self-control problems are rather difficult to measure is an obstacle to this objective. We use smoking as a potential indicator of general self control problems, as risky behaviors (such as substance abuse including smoking and alcohol consumption) are traditionally associated with low self control in psychology literature. We also consider some of other individual characteristics which may be correlated with self control problems such as age and gender, since being older is usually associated with increased financial discipline whereas being a single female or a single mother are usually associated with increased likelihood of financial troubles.\textsuperscript{7}

However, we also use a new indicator of self-control problems, which is based on a unique information contained in our data - (self-reported) reasons for running into financial troubles. We propose and construct an indicator that identifies individuals who have self-control problems as the ones who ran into financial troubles purely due mismanagement of their debts, rather than for any exogenous reason. Tables below summarize the variable “reason for debt” for our estimation sample. Although multiple reasons can be stated, around 74 percent of the clients state only a single reason for their debt.

\textsuperscript{7}See Bertaut and Haliassos [5] for a discussion.
Insert Tables 3.1 and 3.2 here.

<table>
<thead>
<tr>
<th>Category</th>
<th>Reason for debt</th>
<th>Freq.</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC_prob</td>
<td>No Budget</td>
<td>1663</td>
<td>2.09</td>
</tr>
<tr>
<td></td>
<td>Lack Of Money Education</td>
<td>182</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>Poor Shopping Habits</td>
<td>23</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Bank Account Problems</td>
<td>286</td>
<td>0.36</td>
</tr>
<tr>
<td>NS_job</td>
<td>Unemployment</td>
<td>4238</td>
<td>5.34</td>
</tr>
<tr>
<td></td>
<td>Change In Employment</td>
<td>1848</td>
<td>2.33</td>
</tr>
<tr>
<td></td>
<td>Failed Business</td>
<td>706</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>Spouse/Partner Not Working</td>
<td>238</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Temporary Layoff/Strike</td>
<td>100</td>
<td>0.13</td>
</tr>
<tr>
<td>NS_ill</td>
<td>Injury/Illness</td>
<td>5175</td>
<td>6.52</td>
</tr>
<tr>
<td></td>
<td>Caring For Relatives/Friends</td>
<td>755</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>Death In The Family</td>
<td>813</td>
<td>1.02</td>
</tr>
<tr>
<td>NS_preg</td>
<td>Pregnancy/Childbirth</td>
<td>1172</td>
<td>1.48</td>
</tr>
<tr>
<td>NS_incshock</td>
<td>Reduced Income</td>
<td>13840</td>
<td>17.43</td>
</tr>
<tr>
<td></td>
<td>Lost Part Time Income</td>
<td>151</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>Reduced Benefits</td>
<td>510</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>Reduction In Hours/Overtime</td>
<td>854</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>Salary Fluctuates/Commission</td>
<td>707</td>
<td>0.89</td>
</tr>
<tr>
<td>NS_sep</td>
<td>Separation/Divorce</td>
<td>6455</td>
<td>8.13</td>
</tr>
</tbody>
</table>

Table continues at the other page as Table 3.2.

Unfortunately, the reason for debt - although revealed during the over-the-phone counselling session and immediately reported (entered to the computer) by the counselor - is subject to two types of distortions/biases and a considerable change during 2005. In addition, the indicator also includes several categories that do not have clear interpretation, such as “overcommitted on credit”. The “reason for debt” information is automatically added in the letters that are sent to creditors along with a proposed repayment plan (DMP). During our discussion with the counsellors regarding the interpretation of each reason for debt, we discovered that there was a change in the trend of its reporting at the beginning of year 2005. The first bias come from the fact that the clients are reluctant to admit that they could not manage their finances well, and the counsellors are less likely to report these reasons unless the client specifically mentions it. The reason for the later is that the reason for debt is part of the
<table>
<thead>
<tr>
<th>Category</th>
<th>Reason for debt</th>
<th>Freq.</th>
<th>Percent.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other</td>
<td>Over Committed On Credit</td>
<td>25008</td>
<td>31.5</td>
</tr>
<tr>
<td></td>
<td>Used Credit For Living Exp.</td>
<td>9209</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>Used Credit For Business Exp.</td>
<td>273</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>Expenditure Excessive</td>
<td>499</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>Utility Arrears</td>
<td>62</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>School Expenses</td>
<td>123</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>High Mortgage/Rent</td>
<td>273</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>High Vehicle Costs</td>
<td>136</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Home Repair Expenses</td>
<td>264</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>Housing Cost Arrears</td>
<td>175</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>Increased Housing Payments</td>
<td>726</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>Insurance Problems</td>
<td>8</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Legal Expenses</td>
<td>56</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>Child Support Problems</td>
<td>343</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>Missed Work-Bad Weather</td>
<td>26</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Moving/Relocation Expenses</td>
<td>883</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>Low Paid</td>
<td>294</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>Part Time Work Only</td>
<td>451</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>Vehicle Accident</td>
<td>81</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Vehicle Repairs</td>
<td>101</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>Substance Abuse</td>
<td>24</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Unknown</td>
<td>667</td>
<td>0.84</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>79,398</td>
<td>100.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of reasons</th>
<th>Freq.</th>
<th>Percent.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45,063</td>
<td>74.49</td>
</tr>
<tr>
<td>2</td>
<td>12,474</td>
<td>20.62</td>
</tr>
<tr>
<td>3</td>
<td>2,509</td>
<td>4.15</td>
</tr>
<tr>
<td>4</td>
<td>391</td>
<td>0.65</td>
</tr>
<tr>
<td>5</td>
<td>53</td>
<td>0.09</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0.02</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>Total</td>
<td>60,495</td>
<td>100.00</td>
</tr>
</tbody>
</table>

information pack each client receives as the summary of the counselling session and clients may not be happy to be advised of their financial illiteracy. This means that
financial-mismanagement-type reasons for debt are likely to be underreported in our data. The second bias and the change in 2005 have the following reason. As CCCS discovered that creditors were willing to accept the negotiated DMP terms more readily when a specific reason for debt, and in particular, clear negative shock (such as loss of job, drop in income etc.) was stated as the justification for the borrower’s inability to repay, in 2005 CCCS started encouraging its counsellors to try to track down and report more specific reason for debt, and avoid using less clear categories. For example, if the client said I was over committed to credit, the counsellors would ask for a more specific reason.

Despite these limitations, we believe we can still make use of the indicator in the following “conservative” way. First, this indicator allows us to distinguish individuals who ran into financial difficulties for a particular exogenous negative shock. The key categories of these shocks are: unexpected illness, divorce, pregnancy, loss of job, and reduced income. These are clear facts that took place and are less likely to be subject to the above mentioned biases. Second, we use the categories, although less frequently populated, that suggest that the main reason for the financial troubles was mismanagement of the debt, as a proxy for the self-control problems. These are the reasons that suggest that the clients did not manage their finances (had no budget), lack financial education etc. which led them to have financial difficulties in the absence of a negative shock. We keep in mind the fact that this indicator is underreported and interpret the estimated effect of self-control problems on the debt repayment behavior as the lower bound. We use reasons which suggest a clear negative shock to construct indicators for specific negative-shock sub-groups and the debt mismanagement indicator and compare the DMP performances of these sub-groups to the rest of the population (the mix of the CCCS clients that are on a DMP and stated a reason for debt that has a less clear interpretation). We interpret the effect of the specific negative shocks (prior to starting a DMP) on DMP performance to capture either their permanent nature or correlation overtime. We interpret the coefficient of the debt mismanagement indicator in the model of the DMP performance as the effect of the self-control problems on the debt repayment behavior.

In addition, in order to capture the potential changes in the trend of the reason-for-debt reporting, we include a set of dummy variables that identify the year-month when the DMP started in our estimation.

8Variables such as “no-budget” are rather infrequently populated.
9We observe that if the client does not report a clear negative shock, “reduced income” is chosen more often as the reason for debt.
DMPs and DMP performance measures

During the counselling appointment, a budget is agreed and the monthly surplus is calculated. The monthly surplus equals monthly disposable income minus the regular monthly expenditures and the payments for secured debts. The DMP is based on the monthly surplus, i.e. it constitutes the required monthly payment of the borrower towards the DMP. Given the monthly surplus and the total amount owed to creditors, the DMP length is determined. Individuals’ debt-repayment behavior (their performance on the DMP) is captured by the DMP status of the client at the date the data was extracted. 10 The DMP status of a client may be active (still on DMP), self administration (an individual found other means of paying the debt and left CCCS), successful completion (debt is paid off under the DMP) or non payer (dropped from DMP). As the amount of debt owned is usually high and the available monthly surplus is only limited, the typical DMP length is over 10 years and most of the clients who have not dropped yet or exited in another way are still on a DMP. A non-payer (an individual dropped from the DMP) is a client that either missed 2 consecutive payments, or 4 payments in 12 months.

3.3.3 Estimation Sample

The estimation sample consists of individuals started a DMP between January 2003 and November 2006. We consider only individuals who were on a DMP for at least 3 months. There are two reasons to do that. First, given the definition of the drop out (two consecutively missed payments which can be observed only in the third month information), for these individuals - unless they explicitly tell CCCS that they discontinue the DMP - it cannot be determined whether they dropped or not. Also, intuitively, it is hard to assess the DMP performance during such a short period. Even more importantly, the three month rule excludes the borrowers who have agreed a DMP but never started paying. We consider this to be a special case, which may not be comparable with the DMP drop out after some repayments have been made. Further, we consider and compare only the regular DMP repayment versus default. We therefore exclude all other alternative ways of exiting DMP before the planned ending date, i.e. successful self-administration.11 However, individuals who chose (had to chose due to no other alternative) self-administration due to inability of paying the monthly payments required by the DMP are left in the sample and classified as DMP drop-outs. Finally, we drop individuals with missing information on any of the key variables and obvious outliers such as negative or zero income, age exceeding 120

10 The DMP information in CCCS database is updated on a continuous bases as new clients (new DMPs) are added every day. The data we have has monthly frequency. We are able to keep track of whether the client has made the payments since the start of the DMP.

11 The most typical case being an earlier repayment of the debt via selling of one’s property.
This leaves us with 60,495 individuals who have started or have been observed on the DMP during the analyzed period.

**Summary Statistics**

The following tables provide summary statistics for the variables that are used in the estimation.

Insert Tables 3.4, 3.5 here

<table>
<thead>
<tr>
<th>Stats</th>
<th>Mean</th>
<th>Var</th>
</tr>
</thead>
<tbody>
<tr>
<td>smoker</td>
<td>0.221</td>
<td>0.172</td>
</tr>
<tr>
<td>woman</td>
<td>0.555</td>
<td>0.247</td>
</tr>
<tr>
<td>couple</td>
<td>0.476</td>
<td>0.249</td>
</tr>
<tr>
<td>#dependants</td>
<td>0.476</td>
<td>0.249</td>
</tr>
<tr>
<td>mortgage</td>
<td>0.214</td>
<td>0.168</td>
</tr>
<tr>
<td>selfempl</td>
<td>0.030</td>
<td>0.029</td>
</tr>
<tr>
<td>fulltime</td>
<td>0.605</td>
<td>0.239</td>
</tr>
<tr>
<td>age</td>
<td>38.941</td>
<td>147.887</td>
</tr>
</tbody>
</table>

* In British Pounds.

About 55% of the individuals who contacted CCCS are women and the average age is 39.\(^{13}\) There is about 48% of couples and the average number of dependants is 0.48 in the sample. 6% of the contact individuals are self-employed, while 61% work full-time. There are 22% of households with smokers and 21% of households who have mortgage. A typical household in our sample has a monthly income of about 1,336 pounds per month and spends about 1,127 pounds a month. Table 3.6 below summarizes the specific-reason-for-debt variables that we use in the estimation. While 25.5% of the CCCS clients state reduction in income as the reason why they built up debt which they have troubles repaying, for 11.5% of the clients it is loss

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\(^{12}\) We didn’t find any systematic pattern in the observations that were dropped form the data.  
\(^{13}\)Some of them, however, represent a bigger household.
of job, for 10.9% it is illness or death in the family, for 10.7% it is separation or divorce, and for about 1.9% it is pregnancy. We classify 3.5% of clients as the ones who have self-control problems, based on the reported reasons that suggest debt mismanagement.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Freq.</th>
<th>Percent.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS_incshock</td>
<td>15,425</td>
<td>25.50</td>
</tr>
<tr>
<td>NS_job</td>
<td>6,933</td>
<td>11.46</td>
</tr>
<tr>
<td>NS_ill</td>
<td>6,582</td>
<td>10.88</td>
</tr>
<tr>
<td>NS_sep</td>
<td>6,455</td>
<td>10.67</td>
</tr>
<tr>
<td>NS_preg</td>
<td>1,172</td>
<td>1.94</td>
</tr>
<tr>
<td>SC_prob</td>
<td>2,140</td>
<td>3.54</td>
</tr>
</tbody>
</table>

As will be explain in the next section, typical DMP is over 10 years long and in our sample there are only very few regular successful completions of the DMP. To simplify the Cox proportional hazard model (described in the next session) that we use to analyze the DMP performance (duration and the probability of dropping from the DMP) we classify the individuals who has already successfully paid their debt off (309 individuals) as “active”.

Given our classification of the active and the non-payers, the distribution of the DMP outcomes (performance) in our estimation sample is as follows: There are 48,075 (about 80%) of active DMPs and 12,420 (about 20%) of non-payers, i.e. drop outs. See table below:

<table>
<thead>
<tr>
<th>dropDMP</th>
<th>Freq.</th>
<th>Percent.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active</td>
<td>48075</td>
<td>79.47</td>
</tr>
<tr>
<td>Nonpayer</td>
<td>12420</td>
<td>20.53</td>
</tr>
<tr>
<td>Total</td>
<td>60495</td>
<td>100.00</td>
</tr>
</tbody>
</table>

### 3.3.4 Empirical methodology: survival analysis

We use the conjectures of the theoretical model presented in section 3.2.1 to analyze the DMP performance of different types of CCCS clients (borrowers). Namely, we explore who successfully completes his or her DMP and repay their debts and who is not being able to make use of the DMP and drops out.

If we observed only the successfully completed DMPs or the drop outs, we could use a binary variable model for the estimation of the probability of dropping from a
DMP versus successful completion, as done in the theoretical model. However, the structure of the data and the time interval in which we observe the DMPs make this approach infeasible. As the average DMP is designed to last over 10 years and our data start only in 2003, vast majority of the DMP plans in our data are still on-going. We observe only very little instances of success - when the DMPs were completed, and certain proportion of failures - plans that stopped because the client dropped out. In most cases, the plans are still on and it is not yet known whether they will become a success or failure. However, even in these instances, it is possible to make meaningful comparisons of how long have the DMPs been on and compare them to the failures.

The best way to estimate the probability of DMP failure with this type of data is the duration analysis. It is based on the estimation of the hazard rate - the rate at which a failure happens given that it hadn’t happened yet. This methodology takes into account the duration of the DMPs as well as whether they failed or were censored. The censoring date in our data is January 2007, when the data was extracted. We observe all the DMPs of the CCCS clients who started a DMP after 2003 up to either their failure or the censoring date. The DMP duration corresponds to what is named as "spells" in the duration analysis. We observe the starting date for all the DMPs, but the ending date is observed only for the few successful completions\textsuperscript{14} and for the DMPs that failed. In the duration analysis terminology, we have the so-called flow data, in which we observe complete spells for the individuals who dropped from a DMP prior to January 2007, while the rest of the spells are right-censored in January 2007.

There is a range of models available for the estimation of the hazard rate (or the corresponding survival rate), both parametric and non-parametric, and with continuous or discrete time. Given the theoretical model, the empirical analysis has the following preferences: 1. We are interested only in the overall effect of various time-invariant individual characteristics and debt circumstances. 2. We do not want to impose any particular shape (functional form) on the hazard function.

\subsection*{3.3.5 Cox’s proportional hazard model}

We choose a simple proportional hazard model estimated by semiparametric estimator proposed by Cox\textsuperscript{11}, which doesn’t impose any restrictions on the baseline hazard function of the model. Cox suggested a partial likelihood estimator for a model with the following specification\textsuperscript{15}: the hazard rate of an individual $i$ with the spell length $t_i$, i.e. the probability that a spell ends at a particular time, given it lasted until that

\textsuperscript{14} To keep things simple, we classify the few successful completions as being still on a DMP for a very long time, so that in the limit their drop out probability approaches zero.

\textsuperscript{15} The exposition of the model here is based on Greene\textsuperscript{16}. 

79
\[
\lambda(t_i) = \lim_{\Delta s \to 0} \frac{\text{Prob}(s \leq t \leq s + \Delta s / t \geq s)}{\Delta s}
\]
is defined in the Cox’s proportional hazard model as
\[
\lambda(t_i) = \exp(-x t_i \beta) \lambda_0(t_i).
\]

where \(x\) are individual or spell-specific characteristics and \(\lambda_0\) is the baseline hazard function. Cox’s partial likelihood estimator allows to estimate \(\beta\) (the effect of the covariates \(x\) on the hazard rate) without estimating the baseline hazard \(\lambda_0(t_i)\).

The partial likelihood estimator is based on the expression for a probability that a particular spell ends among the corresponding risk set (set of spells that have not ended yet). It therefore estimates the probability that a given individual drops from the DMP divided by the sum of the probabilities that any of the individuals that are still on the plan drop from the DMP. It is this conditioning on being one of the individuals from the risk set that allows \(\lambda_0(t_i)\) be unconstrained, as it drops out from the expression that is estimated. The censored observations are included in the traditional way, as described for example in Cameron and Trivedi [9]. As we know the exact date when a spell starts, ends, or is censored, we measure the spells in days. This allow us to regard the model as the one with a continuous time and to avoid the presence of spells with exactly the same length. However, if ties still occur, they are treated by the method suggested in Breslow [7].

### 3.4 Estimation Results

Table 3.8 presents the results from the estimation of the Cox proportional hazard model. The three pairs of columns correspond to three different specifications that we estimate. In the first one, we explain the failure to stay on the DMP plan only with the set of the variables describing the various reasons why individuals ran into financial difficulties, so that they had to seek CCCS’s help. In the second, we also add various demographic characteristics, indicator for smoking, having a mortgage, self-employment and full-time employment status as well as logarithm of monthly income and household expenditures. In the third specification, we also add dummy variables that control for the starting point (year and month) of the DMP in calendar time.

We focus on the impact of the various reasons for debt on the probability of dropping out of the DMP. The key indicator that describes the individuals who ran into financial troubles because of the mismanagement of their debt (no budget, lack of financial education etc.) is presented in the first row of the table. Its coefficient is
always positive and significant for all the three specifications, ranging from 0.186 in the first specification to 0.122 in the third one. The effect somewhat declines when we control for additional variables (moving from the first to the second specification), but the magnitude is fairly robust across all the three specifications. The preferred third specification suggests that individuals, who reported mismanagement as the reason for debt, drop out from the DMP with 12\% higher probability than the rest of the CCCS clients. We interpret the debt-mismanagement indicator as a sign of self-control problems. If our interpretation is correct, the result we find supports the prediction of our theoretical model that the naive hyperbolic discounters are more likely to drop from the DMP.

The estimates of the other reasons for debt are more sensitive to the inclusion of additional variables. Namely, the effect of experiencing illness changes sign from negative to positive and the effect of pregnancy and of the reduction in income cease to be significant when we move from the first to the second and third specification. The occurrence of the specific shocks is highly correlated with age, gender, marital status, number of dependence and possibly also smoking. It is likely that in the first specification these shocks capture also the effect of these other factors that are not present in the estimation. The results in the preferred third specification suggest that individuals who experienced a particular negative shock, namely loss of job, illness in the family or divorce, that led them into debt, are more likely to drop from the DMP than the rest of the CCCS clients.\footnote{Insignificance of the reduced income is not surprising, knowing that it is often chosen -given the relatively vague definition - to explain the reason to the creditor in the absence of a clear negative shock.} This outcome is likely to be driven either by the permanent nature of these shocks or by the high probability of subsequent shocks, when the shocks are correlated overtime. In terms of our theoretical model, we interpret this finding as follows: the individuals who were hit by this kind of shock may have, as a consequence or even a priori, a higher probability of the occurrence of a low income state (the bad state of the world) than the rest.

As mentioned earlier, for some individuals, we observe multiple reasons for debt, meaning that the same person may be observed to have mismanaged their debt and experienced a negative shock at the same time. We also try to interact the debt mismanagement indicator with the negative shock indicators to be able to see, whether individuals with self-control problems handle negative shocks worse or better than the rest of the population. However, neither of these interactions has been found significant.\footnote{The results, not presented here, are available form the authors on request.}

Another result that we focus on in relation to the predictions of our model, is smoking, as it may be an alternative measure of self-control problems. The effect of smoking is highly significant and more than twice as big as the debt mismanagement
indicator. It suggests that smoking increases the probability of dropping out from the DMP by 31%. Again, provided that individuals who smoke have self-control problems, the finding is consistent with our expectations as well as with the predictions of our model.

However, when looking at debt mismanagement and smoking together, we find that their interaction of the two variables is not significant in the model, suggesting that they are two alternative measures of self-control problems, rather than two complementary or substitutable indicators. In addition to measuring self-control problems, smoking is also likely to have a direct income effect: given the high and still rising tobacco prices, smoking expenditures substantially increase the monthly “indispensable” spending, leaving smaller amount of income available to repay one’s debt and, in general, making households more vulnerable to income fluctuations.

We next focus on the demographic characteristics. We observe that the probability of the DMP drop-out decreases with age, although at a decreasing speed. Result that we find in particular interesting, is that women are much more likely to stay on DMP than men. As sometimes the CCCS client represents (in terms of debts, income and expenditures) the whole family, we interact the female indicator with marital status (couple) to see if the results are not driven only by the fact that women are more (or less) likely to be representing the couple and dealing with CCCS. The results however show that women indeed don’t drop from the DMP as often as men. A single woman is 28% less likely to drop than a single man, and a married man is 12% less likely to drop than a single man. It is also clear that the family is more likely to stay on the DMP if it is a woman who contacts CCCS. Married women are about 14% (female indicator plus the interaction term) less likely to drop out than married men.

Having a mortgage decreases the probability of the DMP drop-out by 14%, suggesting that the cost of default (namely loss of home) has a positive effect on repayment.

While being self-employed increases the probability of dropping out by 17%, working as a full-time employee reduces it by 12% when compared to the rest of the population. The high volatility and uncertainty of the income of the self-employed is likely to increase their sensitivity to various shocks that may lead to inability to repay. Steady stream of income of the full-time employees, on the other hand, makes the regular repayment easier.

The last two indicators, the logarithm of monthly income and the logarithm of monthly expenditure of the financial unit (whether it is an individual or a household), describe the regular financial inflow and outflow. As mentioned before, the DMP monthly payment is determined at the initial counselling session by deducting

\[18\] The results, not presented here, are available from the authors on request.
the regular expenditures\textsuperscript{19} from the regular monthly income. The effect of the two variables is in line with our prior expectations: while the amount of regular expenditure increases the probability of dropping out of the DMP, the amount of regular income makes individuals more likely to stay.

Finally, in the third specification, we also include dummy variables that indicate the year and month of the beginning of the DMP, to capture any changes that take place overtime, including the changes in the total state of the economy, the changes in the consumer credit availability, price changes as well as the potential changes in the composition of the CCCS’s clientele. Comparing the second and the third specification suggest that while improving the fit of the model, the starting date indicators do not change the results substantially.

We have so far discussed the effect of the various factors on the DMP drop-out rate but haven’t explored how the probability of dropping out evolves overtime. This is captured by the baseline hazard function, evaluated at the means of the variables, presented in Figure 3.3. The duration on the y-axis is measured in days.

The Figure 3.3 shows that overall the hazard rate of dropping out of the DMP is not monotonic, although it is predominantly declining with the time spent on the DMP. After peaking at about 220 days (7 months) since the starting date, the hazard rate steadily declines up to about 650 days (around 2 years), when the decline somewhat slows down. The very short durations (less than 60 days) are omitted in the model. The reason is the institutional features of the DMP and the conditions for dropping out: unless clients contact CCCS themselves, telling them that they want to drop out from the DMP, they are not dropped until one of the two conditions is met: they either miss 4 individual payments in 12 months or they miss two consecutive payments. It is the latter which basically implies that nobody (unless they purposefully do so) can drop from the DMP before the two month period of the two consecutive missed payments elapse. We believe that this is also the reason that drives the initial increase in the hazard rate, so we attribute it a purely institutional interpretation.

The predominately declining hazard rate may imply the presence of negative duration dependence, i.e. negative effect of time spent on DMP on the probability to drop. However, the observed shape of the hazard rate may be equally likely driven by unobserved heterogeneity (at the absence of any duration dependence): if the non-repaying people drop early on, the remaining pool of individuals who are at risk of dropping may, as a result, be less likely to drop. In our analysis, we do not try to distinguish which of the two interpretation is correct.

The shape of the hazard rate also confirms our choice of the estimation model with a fully flexible baseline hazard rate, as we find that it is neither constant, nor

\textsuperscript{19}Including necessary payments such as repayment of secured debt.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef. (SE)</th>
<th>Coef. (SE)</th>
<th>Coef. (SE)</th>
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<td>0.122**</td>
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<td>0.101**</td>
<td>0.103**</td>
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<td>NS_ill</td>
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<td>0.096**</td>
<td>0.100**</td>
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<td>NS_preg</td>
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<td>-0.013</td>
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<tr>
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<td>0.075*</td>
<td>0.077*</td>
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<tr>
<td>smoker</td>
<td>0.322**</td>
<td>0.311**</td>
<td></td>
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<td>-0.279**</td>
<td></td>
</tr>
<tr>
<td>couple (C)</td>
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<td>-0.123**</td>
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<td>0.047**</td>
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<td>0.036**</td>
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<td>-0.429**</td>
<td></td>
</tr>
<tr>
<td>Start. Month</td>
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<td>no</td>
<td>yes</td>
</tr>
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</table>

| N              | 60495      | 60495      | 60495      |
| Log-likelihood | -128427.722 | -127329.08 | -127147.51 |
| $\chi^2_{(\cdot)}$ | 80.193     | 2288.971   | 2578.277   |

Significance levels: †: 10%  *: 5%  **: 1%

The degrees of freedom of the $\chi^2_{(\cdot)}$ distribution for the three models are 6, 18, and 64.

Robust standard errors are calculated according to Lin and Wei (1989).
monotonic. As this is a proportional hazard model, the baseline hazard function does not vary across individuals and the effect of the respective covariates only shifts the hazard rate up and down. The effects of the RHS variables therefore hold at any point in time during the DMP. Figures 3.4 and 3.5 further illustrate how the hazard rate of the individuals who have self-control problems (as measured by the debt-mismanagement indicator and smoking indicator respectively) differ from the hazard rate of the rest of the CCCS clients.

### 3.5 Conclusion

Consumer credit counselling assists heavily indebted borrowers by providing financial advice and by setting up and administering repayment plans, so called debt management plans (DMPs). In this chapter we explore the effect of self-control on the debt repayment performance of borrowers who have been enrolled in a DMP. We develop a simple model of a DMP and analyze the repayment behaviour of borrowers with and without self-control problems. We model self-control by hyperbolic discounting and allow the borrower to be fully aware or partially naive about her self-control problems. We compare the results to the exponential discounting benchmark. The model predicts that sophisticated hyperbolic discounters would have the same DMP drop-out rates as exponentials whereas naive hyperbolic discounters would drop out from the DMP more often.

We use administrative data from a major consumer credit counselling charity in the UK to test the predictions of our model. We identify the individuals with self-control problems using two different self-control indicators: self-reported reasons for running into financial troubles and smoking. We explore the debt repayment behavior of individuals who are on a DMP, comparing the group of individuals we identified as the ones that have self-control problems to the rest of the CCCS clientele. Specifically, we use Cox proportional hazard model to estimate the probability of staying on a DMP vs. the probability of a DMP drop out. We control for the occurrence of specific negative shocks as reasons for financial difficulties prior to entering a DMP, as well as other household-specific characteristics.

Preliminary results show that self-control problems increase the drop-out probability at any stage of the DMP by 12% and 31% when reason for debt and smoking are used as indicators respectively. Our results also indicate that the drop-out probability decreases with age and that women are substantially more likely to stay on a DMP than men. Having a mortgage as well as working as a full-time employee decreases the probability of the DMP drop-out, whereas being self-employed increases it.

We also find that the interaction of the two indicators of self-control problems,
smoking and debt mismanagement, is not significant. This suggest that there may be individual (qualitative) variations in self-control and that each of these indicators might be capturing a different aspect of self-control problems. In a related paper, Khwaja et al. [20] find that individual differences in smoking behaviour are not captured by the “subjective rates of time discount revealed through committed choice scenarios” but they are related to other measures of self-control such as impulsivity and the length of the financial planning horizon. They conclude that there may be problems of self-control which are not reflected in time-varying discount functions.

To conclude, we find that when we control for the permanent negative shocks, borrowers who admit that they cannot manage their finances well, (i.e. reported reasons for debt suggesting debt misuse) and borrowers who smoke, are more likely to drop from a DMP. To the extend that these indicators can be used as a valid proxy for self-control problems, our results suggest that self-control problems have adverse effect on the debt repayment behavior.

Figure 3.3: Hazard Rate from the Cox’s Model - at Means of the Variables
Figure 3.4: Hazard Rates for Individuals with Self-control Problems (Who Mismanaged their Debt) and for Individuals without Self-control Problems
Figure 3.5: Hazard Rates for Smokers and Non-Smokers
Appendices
A List of Variables used in the estimation

**SC_prob** Indicator for self control problems, Dummy, 1 if the reason for debt stated is: no budget or lack of money education or poor shopping habits or bank account problems

**NS_job** Indicator for job loss, Dummy, 1 if the reason for debt stated is: unemployment or change in employment or failed business or spouse/partner not working or temporary layoff/strike

**NS_ill** Indicator for illness, Dummy, 1 if the reason for debt stated is: injury/illness or caring for relatives/friends or death in the family

**NS_preg** Indicator for pregnancy, Dummy, 1 if the reason for debt stated is pregnancy/childbirth

**NS_incshock** Indicator for shocks to income, Dummy, 1 if the reason for debt stated is: reduced income or lost part time income or reduced benefits or reduction in hours/overtime or salary fluctuates/commission

**NS_sep** Indicator for separation or divorce, Dummy, 1 if the reason for debt stated is separation

**smoker** Dummy, 1 if the CCCS client smokes

**woman** Dummy, 1 if the client is female

**couple** Dummy, 1 if the client is married or has a partner

**age** Indicator for the client age

**# dependants** Indicator for the no of dependants the client has

**mortgage** Dummy, 1 if the client has mortgage

**selfempl** Dummy, 1 if the client is self-employed

**fulltime** Dummy, 1 if the client is employed full time

**lnexpend** Indicator for monthly expenditures, in British Pounds, in logarithm.

**lnincome** Indicator for monthly income, in British Pounds, in logarithm.

**Start. Month and Year In** Set of binary indicators of the year and the month client has started the DMP.
Counselling as a commitment mechanism

If the counselling agencies are in fact able to renegotiate better repayment terms with lenders than the borrowers themselves, the DMPs can in principle help people hit by a negative shock. As for the people who are financially illiterate, the role of credit counselling should be providing financial education and advice. Can credit counselling help borrowers with self control problems?

A DMP is essentially a debt consolidation plan. It transfers for example, credit card debt- which is a line of credit- to a closed end credit paid over a stipulated amount of time in equal installments. That is, it changes the type of the loan agreement and the repayment scheme. One might argue that this particular repayment arrangement as well as the counselling provide the borrower with some spending and repayment discipline, if the borrower manages to stop borrowing on credit cards completely. In principle, being in DMP limits the chances of people borrowing further from the current lenders who agreed to re-negotiate the original debt contract. There is still a chance to borrow from other lenders who lack information about borrower’s repayment capabilities. This depends on the availability information sharing among these other lenders and counselling agency. In the UK the DMPs are not registered in a credit report, however borrowing further while on a DMP, if detected, results in the termination of the DMP agreement by the credit counselling agency.

If the credit counselling does help borrowers with self-control problems, there are two interesting questions one can ask: to what extent does being in a DMP serve as a commitment mechanism for borrowers who suffer from self control problems or to what extend it is possible to teach self-control? In this section, given our choice of modelling self control problems by hyperbolic discounting, we will discuss the limitations associated with testing these questions. Let us begin by describing the implications of different scenarios regarding the contractual environment.

By definition, sophisticated hyperbolic consumers (rational but time-inconsistent consumers) are aware of their time inconsistency and therefore they would like to “commit” whenever possible. However, commitment may not always be possible since the capability to constrain the future selves and to commit depends on the availability of commitment mechanisms and the contracting environment.

As for the contracting environment, two scenarios are possible. The first scenario is to assume that there are commitment devices, including credit counselling, that would help sophisticates to control the effect of their self-control problems (i.e. control spending, commit to debt obligations etc), which would prevent them from running into financial difficulties purely due to self control problems. That is, the CCCS clientele would consist of exponential and sophisticates who were hit by a negative shock and we would not be able to distinguish exponentials from sophisticates. However, by definition we would not expect naives to make use of the counselling services as a
commitment mechanism. Assuming that the negative shocks occur randomly across different types of people, naives would have higher drop out rates.

The second scenario is to assume that there are no commitment devices available except for the counselling which plays the role of the imperfect commitment device possibly through controlling spending and further borrowing and helping to figure out a better forecast of repayment capabilities. If this is the case, then sophisticates, using the commitment mechanism, would have similar drop out rates as exponentials whereas naives would drop out more often due to same reason explained above.

If the post-DMP data on individuals (who either successfully completed or failed a debt management plan) were available we could be able to test whether the effect of counselling on the future financial performance is temporary or permanent. If the affect of being on a DMP is helping borrowers commit their budget and pay their debt, by acting as an imperfect commitment device, the improvement on self-control problems would be temporary. If the effect is through teaching (learning) self-control then the effect would be permanent. Unfortunately, we do not have post-DMP data on individuals and therefore we can not test whether the effect of counselling is temporary or permanent.

\[^{20}\text{As long as the counselling is voluntary, it can best be an ‘imperfect” commitment mechanism.}\]
Bibliography


