Peeling failure in beams strengthened by plate bonding. A design proposal

doctoral thesis by Eva Oller Ibars

directed by:
Diego Cobo del Arco

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Universitat Politècnica de Catalunya
Departament d'Enginyeria de la Construcció

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ABSTRACT

The strengthening of aging infrastructures is in most cases required because of the necessity for increased levels of service loads or because of the degradation of structural materials. The technique of strengthening by externally bonding steel plates has been in practice as an alternative to other traditional methods since the late 1960's. However, steel plates present some disadvantages in terms of weight and corrosion that can be solved by replacing them with fiber reinforced polymer (FRP) laminates. FRP laminates provide benefits such as high strength-to-weight and stiffness-to-weight ratios, corrosion resistance as well as reduced installation costs due to their easy-handling. As a result, FRP laminates have been used extensively to strengthen reinforced concrete structures in many parts of the world since the 1990's.

Existing experimental work has shown that the application of externally bonded laminates can result in a catastrophic brittle failure in the form of a premature debonding of the laminate before reaching the design load. Since the weakest point in the bond between the concrete and the external reinforcement is the concrete layer near the surface, the most common laminate debonding occurs in the concrete cover along a weakened layer or along the embedded steel reinforcement. Laminate debonding can initiate in areas far from the anchorage zone due to the effects of flexural or shear cracks, or can initiate at the laminate end due to a high stress concentration at the cut-off point.

The main aim of this research has been the development of a simple effective method to design and verify the strengthening of an existing structure with an externally bonded plate while preventing the premature peeling failure that causes the laminate to debond. Special attention has been drawn on to transfer of stresses from laminate to concrete through the interface, which is the main key in the correct performance of externally reinforced concrete structures.

After a historical overview of the existing experimental and theoretical lines of research, the suitability of using existing theoretical models to forecast and prevent peeling failures is evaluated in Chapter 2 by means of an experimental bending test database. This experimental database includes results from the existing literature and results from an experimental program conducted by the author at the Structural Technology Laboratory of the Department of Construction Engineering at the Technical University of Catalonia.

To solve the weaknesses of the existing theoretical models, in Chapter 3, Non-Linear Fracture Mechanics theory is applied in a pure shear case to model the behavior of the interface and its premature failures. By assuming a bilinear bond-slip relationship, the evolution of the debonding process is studied in a pure shear specimen as the applied load is increased. The laminate tensile stress and interfacial shear stress distributions, together with the maximum transferred force are obtained as a function of three model

parameters (the fracture energy, the maximum shear stress and the sliding associated to this stress). Expressions are suggested for the evaluation of these parameters.

The formulae of a pure shear specimen are then extended to a general case of a beam under transverse loads in Chapter 4. For this purpose, the evolution of the debonding process is studied for two specific cases: a beam element between two cracks, and a beam element between the laminate end and the nearest crack. The laminate tensile stress and interfacial shear stress distributions are obtained for the different stages observed in the debonding process. A specific highlight observed was that the transferred force between cracks is at maximum when the maximum shear stress reaches the less-loaded crack. In this instance, depending on the crack spacing, a macrocrack may or may not have already initiated. Another point observed is related to the beam element between the laminate end and the nearest crack, which is similar to the pure shear specimen previously studied in Chapter 3 when the strain distribution in the concrete support is not considered. This formulation has been applied to one beam of the experimental program to find the different stress distributions.

The different laminate and interface stress distributions derived in Chapter 4 allow us to understand the behavior of an externally reinforced element, but are awkward for design purposes. Chapter 5 describes both a new design and verification method based on a maximum shear force-bending moment relationship associated to the theoretical maximum transferred force between two consecutive cracks before peeling occurs. After calculating the predicted value for the maximum shear force from the peeling relationship, the developed method verifies the debonding at the laminate end by checking the transferred force between the laminate end and the first crack in the laminate. The reliability of this proposal is verified by means of the assembled bending test database.

Finally, the main conclusions drawn from the work presented in this dissertation are summarized in Chapter 6. Future work and research lines are suggested as well.

RESUMEN

La necesidad de refuerzo estructural en una infraestructura existente puede venir motivada por la aparición de nuevos condicionantes de uso o por la degradación de los materiales. Desde finales de los años sesenta, la técnica del refuerzo mediante la adhesión de platabandas metálicas se ha llevado a la práctica como alternativa a otros métodos de refuerzo tradicionales. Sin embargo, las platabandas metálicas presentan algunas desventajas, como son su peso y su posible corrosión por agentes atmosféricos, que pueden solventarse sustituyéndolas por laminados de polímeros reforzados con fibras (FRP). Estos materiales poseen relaciones resistencia/peso y rigidez/peso mayores que el acero, facilitando su colocación, reduciendo costes y plazos de ejecución. Por ello, desde su aparición en los años noventa, se ha observado un uso creciente de los laminados FRP en el campo del refuerzo estructural.

En numerosos estudios empíricos se observa como la aplicación de laminados encolados puede resultar en una rotura frágil que conduce al desprendimiento prematuro del refuerzo antes de alcanzar la carga última. Este desprendimiento del laminado suele iniciarse en el recubrimiento, es decir, en el punto más débil de la interfase entre hormigón y laminado, bien sea por tensiones excesivas en sus extremos, bien sea por el efecto de las fisuras de flexión o cortante en zonas alejadas al anclaje.

El principal objetivo de este trabajo es el desarrollo de un método simple y efectivo para dimensionar y comprobar el refuerzo de estructuras existentes con laminados adheridos de tal forma que se eviten los modos prematuros de rotura que conducen al desprendimiento del laminado. Se ha dedicado especial atención a la transferencia de tensiones de laminado a hormigón que resulta el punto clave del correcto comportamiento de este tipo de refuerzo.

En el Capítulo 2, tras una revisión histórica de las líneas de investigación existentes, experimentales y teóricas, se ha evaluado mediante una base de datos experimental la fiabilidad de los modelos teóricos existentes para pronosticar y prevenir los modos de rotura prematuros antes mencionados. Esta base de datos experimental incluye resultados de la literatura existente y de una campaña experimental llevada a cabo por el autor en el Laboratorio de Tecnología de Estructuras de la Universidad Politécnica de Cataluña.

Para resolver las deficiencias de los modelos teóricos existentes, en el Capítulo 3, se ha aplicado la teoría de la Mecánica de Fractura No Lineal a un caso de corte puro para modelizar el comportamiento de la interfase y sus roturas prematuras. Estableciendo una relación bilineal entre la tensión tangencial y el deslizamiento entre adherentes, se ha estudiado la evolución del proceso de desprendimiento del refuerzo con la carga aplicada en un espécimen sometido a un estado de corte puro. Se han obtenido las distribuciones de tensiones tangenciales de la interfase y de tensiones normales en el laminado junto a la fuerza máxima transferida en función de tres parámetros (energía de fractura, máxima tensión tangencial y deslizamiento asociado a dicha tensión). Además, se sugieren expresiones para evaluar estos parámetros.

La formulación de un caso de corte puro se ha extendido a un caso general de una viga bajo cargas transversales en el Capítulo 4. Con este objetivo, se ha estudiado la evolución del desprendimiento del laminado en dos casos específicos: un elemento entre dos fisuras contiguas, y un elemento entre el extremo del laminado y la siguiente fisura. Se han obtenido las distribuciones de tensiones tangenciales en la interfase y tensiones de tracción en el laminado para las distintas fases del proceso. Cabe mencionar que la fuerza transferida entre dos fisuras alcanza su máximo valor cuando la tensión tangencial máxima llega a la fisura menos cargada. En este instante, dependiendo de la distancia entre fisuras, ya se puede haber iniciado o no la formación de una macrofisura. El elemento entre el extremo del laminado y la siguiente fisura es similar al caso de corte puro estudiado en el Capítulo 3 siempre que se desprecie la contribución del hormigón traccionado en la zona del recubrimiento. La formulación desarrollada en este capítulo se ha aplicado a una de las vigas de la campaña experimental para obtener la evolución de las diferentes distribuciones de tensiones con la carga aplicada.

Las distribuciones de tensiones en el laminado e interfase presentadas en el Capítulo 4 nos ayudan a comprender el comportamiento de un elemento reforzado con laminados adheridos en su cara traccionada, sin embargo, resultan complejas en la práctica. En el Capítulo 5 se describe un nuevo método de dimensionamiento y verificación basado en la obtención de una relación entre el máximo cortante antes de que se produzca el desprendimiento prematuro del refuerzo y el momento aplicado. Esta relación está asociada a la fuerza máxima transferida entre fisuras. A partir de la predicción del valor máximo de cortante, se verifica el desprendimiento del extremo del laminado evaluando la fuerza transferida entre dicho punto y la siguiente fisura. Se ha verificado la fiabilidad de esta propuesta mediante la base de datos de ensayos a flexión.

Finalmente, en el Capítulo 6 se resumen las principales conclusiones del trabajo presentado en esta tesis y se sugieren futuras líneas de investigación.

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NOTATION

Notations and abreviations are explained in the main text when they first occur. A list of them is presented here with the corresponding SI-units in brackets.

Roman capital letters

$A_{tr,c}^{*}$	area of the strengthened section transformed into concrete (mm^2)
A_e	area of concrete in tension (mm^2)
A_L	cross-sectional area of externally bonded reinforcement (mm^2)
A_s	cross-sectional area of internal steel reinforcement in tension (mm^2)
A_s'	cross-sectional area of internal steel reinforcement in compression (mm^2)
A_w	cross-sectional area of steel shear reinforcement (mm^2)
C_c	compressive force in the concrete (N)
C_F	empirical constant in the fracture energy definition
C_i	integration constant
COV	coefficient of variation
E_a	modulus of elasticity for the adhesive layer (MPa)
E_c	modulus of elasticity for concrete (MPa)
E_L	modulus of elasticity for the external reinforcement (MPa)
E_s	modulus of elasticity for steel (MPa)
F	applied load (kN)
F_{cr}	cracking load (kN)
F_s	service load (kN)
F_u	failure load (kN)
$F_{u,exp}$	experimental failure load (kN)
F_y	yield load (kN)
ΔF	increment of the external applied load (kN)
G	energy of the system ($MPa\ mm$)
G_a	shear modulus of the adhesive layer (MPa)
G_c	shear modulus of the concrete (MPa)
G_C	energy required to grow an existing crack ($MPa\ mm$)
G_F	fracture energy by unit bonded area ($MPa\ mm$)
G_F^{I}	fracture energy of Zone I of the bond - slip relationship ($MPa\ mm$)
G_F^{II}	fracture energy of Zone II of the bond - slip relationship ($MPa\ mm$)
G_r	shear modulus of the resin between composite layers (MPa)
I_c	second moment of inertia of the concrete cross-section (mm^4)
I_L	second moment of inertia of the laminate cross-section (mm^4)
$I_{tr,c}^{*}$	second moment of inertia of the strengthened section transformed to concrete (mm^4)
$I_{tr,L}^{*}$	second moment of inertia of the strengthened section transformed to laminate (mm^4)
K	constant in the existing definitions of the maximum transferred force

L	laminate bonded length (*mm*)
L_b	remaining bonded length (*mm*)
$L_{b,end}$	bonded length at the laminate end (*mm*)
$L_{crack,crit}$	critical diagonal crack location (*mm*)
$L_{L,1}$	laminate length calculated for the concrete tooth model (*mm*)
$L_{L,eff}$	effective length of the steel plate in the shear span (*mm*)
L_{mcrack}	macrocrack length (*mm*)
$L_{mcrack,left}$	macrocrack length starting from crack I (*mm*)
$L_{mcrack,right}$	macrocrack length starting from crack J (*mm*)
L_{lim}	limit between a short and long bonded length
L_{shear}	shear span (*mm*)
$L_{shear\ crack}$	critical shear crack location of a conventionally reinforced concrete beam (*mm*)
$L_{shear\ \mathrm{mod}}$	fictitious shear span (*mm*)
$M(x)$	bending moment acting on the x coordinate (*Nmm*)
M_{cr}	cracking moment (*kNm*)
M_c	bending moment acting on the concrete section (*Nmm*)
M_d	design bending moment of the strengthened section (*Nmm*)
$M_{d,0}$	design bending moment of the unstrengthened section (*Nmm*)
M_I	bending moment acting on crack I (*Nmm*)
M_J	bending moment acting on crack J (*Nmm*)
$M_{J,\mathrm{max}}^{(i)}$	bending moment associated to the maximum force of point (i) of the peeling limit relationship (*Nmm*)
M_H	bending moment acting on crack H (*Nmm*)
M_{lim}	limit bending moment (*kNm*)
M_{peel}	bending moment that causes laminate peeling-off (*kNm*)
$M_{peel,\ \mathrm{min}}$	lower bound of the peeling bending moment (*kNm*)
$M_{peel,\ \mathrm{max}}$	upper bound of the peeling bending moment (*kNm*)
$M_{peel,\ pure\ flexure}$	peeling bending moment in a pure flexure case (*Nmm*)
$M_{peel,V=0}$	peeling bending moment at the laminate end when the shear force is zero (*kNm*)
M_y	yield bending moment of the strengthened section (*Nmm*)
$M_{y,0}$	yield bending moment of the unstrengthened section (*Nmm*)
M_u	ultimate bending moment of the strengthened section (*Nmm*)
$M_{u,0}$	ultimate bending moment of the unstrengthened section (*Nmm*)
$M_{u,exp}$	experimental bending moment at failure (*kNm*)
ΔM_{IJ}	bending moment increment between crack I and crack J (*Nmm*)
N_c	axial force acting on the concrete section (*N*)
P	transferred force (*N*)
P_{exp}	experimental maximum transferred force in a pure shear specimen (*N*)
P_{max}	maximum transferred force (*N*)
$P_{\mathrm{max},L=Lb}$	maximum transferred force for a pure shear specimen whose length is the bonded length L_b (*N*)
$P_{\mathrm{max},L=Lb,end}$	maximum transferred force for a pure shear specimen whose length is the bonded length at the laminate end $L_{b,end}$ (*N*)
$P_{\mathrm{max},L=scr}$	maximum transferred force for a pure shear specimen whose length is the crack distance (*N*)
$P_{\mathrm{max},L=scr\ \mathrm{lim}}$	maximum transferred force for a pure shear specimen whose length is the crack distance limit (*N*)

$P_{\max,L=scr-Lmcrack}$	maximum transferred force for a pure shear specimen whose length is the remaining bonded length along crack distance (*N*)
ΔP_{scr}	transferred force between cracks I and J (*N*)
$\Delta P_{scr,end}$	transferred force between the laminate end and the nearest crack (*N*)
$\Delta P_{\max,scr}$	maximum transferred force between two adjacent cracks, I and J (*N*)
$\Delta P_{\max,K-I}$	maximum transferred force between crack I and the zero shear stress location (*N*)
T_L	tensile force in the laminate (*N*)
T_s	tensile force in the internal steel rebars (*N*)
U	internal strain energy (*MPa mm*)
U_L	stored energy in the laminate (*MPa mm*)
U_y	bond strength according to Colotti and Spadea (2001) (*Nmm*)
$V(x)$	shear force acting at the *x* coordinate (*N*)
V_{cr}	shear force that causes diagonal cracking (*kN*)
ΔV_{cr}	shear component resisted by the plate (*kN*)
V_{cracks}	shear force that prevents peeling failure near cracks (*kN*)
V_{cu}	shear capacity of the concrete in the RC beam alone without the contribution of the stirrups
V_{end}	shear force that prevents peeling failure at the laminate end (*kN*)
V_I	shear force acting on crack I (*N*)
V_J	shear force acting on crack J (*N*)
$V_{J,\max}$	maximum shear force acting on crack J (*N*)
$V_{J,\max}^{(i)}$	maximum shear force associated to point (i) of the peeling limit relationship (*N*)
V_{peel}	shear force that causes laminate peeling-off (*kN*)
V_{pred}	predicted shear force that causes laminate peeling-off (*kN*)
V_u	shear force that causes failure along the cracked section (*kN*)
$V_{u,exp}$	experimental shear force at failure (*kN*)
$V_{y,0}$	shear force associated to the yield bending moment of the unstrengthened section (*N*)
W	released energy when the fracture increases in length (*MPa mm*)
X_{exp}	experimental value for a certain parameter
X_u	ultimate theoretical value expected for a certain parameter

Roman lower case letters

a	unplated length between the support and the laminate end (*mm*)
a	crack length (*mm*)
b	concrete section width (*mm*)
b_L	laminate width (*mm*)
b_{L0}	reference width according to Brosens (2001) (*mm*)
c_1	constant of the fracture energy or maximum shear stress equation defined by FIB Task Group 9.3 FRP (2001)
c_2	constant of the maximum slip definition defined by FIB Task Group 9.3 FRP (2001)
c_F	constant determined by linear regression analysis of shear test results
d	effective depth of the concrete section (*mm*)
da	differential crack length (*mm*)
dx	differential length (*mm*)

f_c	compressive strength of concrete (*MPa*)
f_{cd}	design value of concrete compressive strength (*MPa*)
f_{ck}	characteristic cylinder compressive strength in concrete (*MPa*)
f_{cm}	mean value of concrete compressive strength (cylinder) (*MPa*)
$f_{cm,cylinder}$	mean value of concrete cylinder compressive strength (*MPa*)
$f_{cm,cube}$	mean value of concrete cube compressive strength (*MPa*)
f_{ctm}	mean value of axial tensile strength in concrete (*MPa*)
$f_{ctm,eff}$	effective tensile strength in concrete given by Zhang (1997) (*MPa*)
f_{Ly}	yield strength of the externally bonded reinforcement (*MPa*)
f_v	concrete formal shear stress (*MPa*)
f_y	yield strength of internal steel reinforcement (*MPa*)
f_{yd}	design yield strength of internal steel reinforcement (*MPa*)
f_{ym}	mean value of yield strength of internal steel reinforcement (*MPa*)
h	total depth of the concrete section (*mm*)
k	stress intensity factor
k_e	empirical constant in the width influence factor
k_b	width influence factor
k_c	concrete surface influence factor
n	layers of the composite laminate
q	transverse distributed load (*N*/*mm*)
r	concrete cover (*mm*)
s	relative displacement between concrete and laminate (*mm*)
s_{LM}	slip associated to the maximum shear stress (*mm*)
s_{L0}	maximum slip before laminate debonding (*mm*)
s_{cr}	distance between cracks I and J (*mm*)
$s_{cr,\lim}$	limit between a short and long crack distance (*mm*)
$s_{cr,\lim}^{(i)}$	limit between a short and long crack distance at point (i) of the peeling limit relationship (*mm*)
$s_{cr,\lim\ end}$	limit between a short and long distance at the laminate end (*mm*)
$s_{cr,\min}$	minimum stabilized crack spacing (*mm*)
$s_{cr,\min\ Stage\ 3c}$	minimum crack spacing for the development of Stage 3c (*mm*)
$s_{cr,\max}$	maximum stabilized crack spacing (*mm*)
$(s_{cr} - L_{mcrack})_{,\lim}$	limit crack distance for the remaining bonded length between cracks (*mm*)
$(s_{cr} - L_{mcrack})_{,\lim\ end}$	limit distance for the remaining bonded length at the laminate end (*mm*)
s_w	distance between stirrups (*mm*)
t_a	thickness of the adhesive layer (*mm*)
$t_{c,ref}$	concrete thickness where stresses are influenced by the external reinforcement
t_L	laminate thickness (*mm*)
t_r	resin thickness (*mm*)
u	displacement on the longitudinal direction (*mm*)
u_s	internal steel to concrete average bond strength (*MPa*)
u_L	internal steel plate to concrete average bond strength (*MPa*)
u_L	laminate displacement on the longitudinal direction (*mm*)
v	displacement on the vertical direction (*mm*)
w_k	crack width (*mm*)
x	longitudinal coordinate (*mm*)
x	neutral axis depth (*mm*)

x_0	neutral axis depth of the unstrengthened section (*mm*)
x_I	location of crack I (*mm*)
x_J	location of crack J (*mm*)
x_K	zero shear stress location (*mm*)
x_{LM}	maximum shear stress location (*mm*)
$x_{LM,left}$	maximum shear stress location closest to crack I (*mm*)
$x_{LM,\max}$	maximum value for the length of Zone II in a pure shear specimen (*mm*)
$x_{LM,P}$	maximum shear stress location for the maximum transferred force (*mm*)
$x_{LM,right}$	maximum shear stress location closest to crack J (*mm*)
x_{L0}	macrocrack tip location (*mm*)
$x_{L0,left}$	location of the tip of the macrocrack initiated in crack I (*mm*)
$x_{L0,right}$	location of the tip of the macrocrack initiated in crack J (*mm*)
y_0	height of the compression block in the unstrengthened section (*mm*)
y_c	position of the center of gravity in the concrete cross-section (*mm*)
y_{G*}	distance from the bottom concrete fiber to the gravity center of the strengthened section (*mm*)
y_L	position of the center of gravity in the laminate cross-section from its top fiber (*mm*)
Δy	increase on the compression height block (*mm*)
Δy_u	ultimate increase on the compression height block (*mm*)
z_L	laminate lever arm (*mm*)
$z_{L,I}$	laminate lever arm in crack I (*mm*)
$z_{L,J}$	laminate lever arm in crack J (*mm*)
z_s	steel lever arm (*mm*)
$z_{s,I}$	steel lever arms in crack I (*mm*)
$z_{s,J}$	steel lever arms in crack J (*mm*)

Greek capital letters

ϑ	ratio of bond strength to stirrup tensile strength
Ω_1	constant defined to solve the differential equation governing the laminate tensile stresses in Zone I of the bond-slip relationship (1/*mm*)
Ω_2	constant defined to solve the differential equation governing the laminate tensile stresses in Zone II of the bond-slip relationship (1/*mm*)
Ω_1^*	constant defined in the simplified linear approach to solve the differential equation governing the laminate tensile stresses in Zone I of the bond-slip relationship (1/*mm*)

Greek lower case letters

α	factor defined as the first static moment of the plate divided by both the plate width and the homogeneous moment of inertia of the section ($1/mm^2$)
β	fraction of the concrete tensile strength

ε_{cu}	concrete ultimate strain ($\mu\varepsilon$)
ε_{L}	laminate strain ($\mu\varepsilon$)
$\varepsilon_{L,\max}$	maximum experimental laminate strain ($\mu\varepsilon$)
$\varepsilon_{L,pure\ flexure}$	laminate strain associated to peeling in a pure flexure case ($\mu\varepsilon$)
ε_{Lu}	ultimate strain of the externally bonded laminate ($\mu\varepsilon$)
$\varepsilon_{Lu,k}$	characteristic value of the ultimate strain of the laminate according to the manufacturer ($\mu\varepsilon$)
ε_{sy}	yielding strain of the internal steel reinforcement ($\mu\varepsilon$)
ϕ_{s}	diameter of longitudinal tensile steel rebars (*mm*)
ϕ_{w}	diameter of shear steel rebars (*mm*)
γ	shear deformation
η	experimental constant of the shear capacity based models of Ali et al. (2001) that depends on the load type and equals to 1.6 for point loads
κ	constant given by the product of μ by the square of the crack distance (*MPa*)
λ	parameter on the resolution of the differential equation for shear stresses in linear elastic models given by equation (2.9) (1/*mm*)
μ	constant depending on the β concrete fraction and the crack distance (MPa/mm^2)
θ	arc tangent of the quotient between bending moment and shear force (*rad*)
$\theta^{i)}$	arc tangent of the quotient between bending moment and shear force for point (i) of the peeling limit relationship (*rad*)
θ_{exp}	θ associated to the experimental failure load (*rad*)
ρ_{L}	externally bonded reinforcement ratio
ρ_{s}	longitudinal internal steel reinforcement ratio in tension
$\rho_{s'}$	longitudinal internal steel reinforcement ratio in compression
ρ_{w}	shear steel reinforcement ratio
σ	normal stress (*MPa*)
σ_{c}	concrete stress on the longitudinal direction (*MPa*)
$\sigma_{c,b}$	concrete tensile stress on the bottom fiber of the section (*MPa*)
σ_{I}	maximum principal stress at the laminate end (*MPa*)
$\sigma_{I,\max}$	maximum experimental principal stress at the laminate end (*MPa*)
σ_{Iu}	maximum principal stress at the laminate end at failure (*MPa*)
σ_{II}	minimum principal stress at the laminate end (*MPa*)
$\sigma_{II,\max}$	minimum experimental principal stress at the laminate end (*MPa*)
σ_{IIu}	minimum principal stress at the laminate end at failure (*MPa*)
σ_{L}	laminate tensile stress (*MPa*)
σ_{L}^{I}	laminate tensile stress in Zone I of the bond-slip relationship (*MPa*)
σ_{L}^{II}	laminate tensile stress in Zone II of the bond-slip relationship (*MPa*)
$\sigma_{L,I}$	laminate tensile stress in crack I (*MPa*)
$\Delta\sigma_{L,IJ}$	laminate tensile stress increment between cracks I and J (*MPa*)
$\Delta\sigma_{L,IJ,\max}$	maximum laminate tensile stress increment between cracks I and J (*MPa*)
$\sigma_{L,J}$	laminate tensile stress in crack J (*MPa*)
$\sigma_{L,J\ end}$	laminate tensile stress in the nearest crack (crack J) to the laminate end (*MPa*)

$\sigma_{L,H\ (H\text{-}J)}$	laminate tensile stress in crack H, the second crack nearest to the laminate end (*MPa*)
$\sigma_{L,\min}$	minimum tensile stress in the laminate (*MPa*)
$\sigma_{L,\max}$	maximum tensile stress in the laminate (*MPa*)
σ_s	internal steel tensile stress (*MPa*)
$\sigma_{s,I}$	internal steel tensile stress in crack I (*MPa*)
$\sigma_{s,J}$	internal steel tensile stress in crack J (*MPa*)
$\Delta\sigma_{s,IJ}$	steel tensile stress increment between cracks I and J (*MPa*)
σ_x	tensile stress in the bottom concrete layer at the laminate end (*MPa*)
σ_y	interfacial normal stress (*MPa*)
$\sigma_{y,\max}$	maximum analytical normal stress at the laminate end under failure load (*MPa*)
$\sigma_{y,u}$	ultimate interfacial normal stress at the laminate end (*MPa*)
τ	interfacial shear stress (*MPa*)
τ_{FES}	fiber end shear stress (*MPa*)
τ_{LM}	interfacial maximum shear stress (*MPa*)
$\tau_{\min}$	minimum shear stress (*MPa*)
$\tau_{\max}$	maximum analytical shear stress at the laminate end under failure load (*MPa*)
$\tau_{\max,exp}$	maximum experimental shear stress at the laminate end under failure load (*MPa*)
$\tau_{\max,FEM}$	maximum numerical shear stress at the laminate end under failure load calculated by the Finite Element Methods (*MPa*)
τ_{PES}	plate end shear stress that causes plate end shear failure (*MPa*)
τ_u	ultimate shear stress at the laminate end according to Mohr-Coulomb criterion (*MPa*)
$\Delta\tau_{mod}$	modification factor (*MPa*)
υ	relationship between laminate tensile stresses in both cracks I and J
ξ	parameter given by equation (2.5)
ξ_b	bond parameter given by equation (2.49)
ξ_1	constant defined to solve the differential equation governing the laminate tensile stresses between cracks in Zone I of the bond-slip relationship when assuming Navier-Bernouilli's law (1/*mm*)
ξ_2	constant defined to solve the differential equation governing the laminate tensile stresses between cracks in Zone II of the bond-slip relationship when assuming Navier-Bernouilli's law (1/*mm*)
ψ	constant defined to solve the differential equation governing the laminate tensile stresses between cracks in both Zones I and II of the bond-slip relationship when assuming Navier-Bernouilli's law (1/*mm*)
ζ	parameter on the resolution of the differential equation for interfacial normal stresses in linear elastic models given by equation (2.20) (1/*mm*)

Abreviations

AFRP	Aramid Fiber Reinforced Polymer
CC	Concrete crushing
CFRP	Carbon Fiber Reinforced Polymer
GFRP	Glass Fiber Reinforced Polymer
F	Fabrication procedure
FRP	Fiber Reinforced Polymer
LEFM	Linear Elastic Fracture Mechanics
M	Material
NLFM	Non-Linear Fracture Mechanics
NR	Not clearly reported
O	Other modes of failure
Pr	Preloaded prior to laminate bonding
P	Pultruded laminates
P	Premature peeling failure
PC	Peeling initiated near cracks
PED	Plate end debonding
PES	Plate end shear failure
R	Plate rupture
RC	Reinforced concrete
S	Shear failure
Th	Theoretical
W	Wet lay-up laminates