Chapter 4

Numerical Model of Multi-Story Buildings Equipped with Friction Dissipators

4.1 Introduction

As a generalization of the previous chapter, a numerical model of the dynamic structural behavior of symmetric multi-story buildings equipped with friction dissipators (MSBFD) is proposed in this chapter. As the buildings are symmetric, their behavior is described by two lumped-mass models (the main frame and the bracing-dissipators combination) with a single degree-of-freedom per floor, which is the horizontal displacement in the considered direction (X or Y). In the following, only such 2D discrete MDOF models are considered.

First of all, the equations of motion of the models are formulated emphasizing that the number of 'active' degrees of freedom varies continuously between N and $2N$ (N is the number of floors) depending on the sticking-sliding conditions in the dissipators. Next, the proposed algorithm to solve these equations is described and its implementation in a software code (ALMA) is presented. Some numerical results obtained with ALMA are displayed and compared to those obtained with the commercial package ADINA. The agreement is satisfactory. Finally, some preliminary conclusions about the efficiency of friction dissipators to reduce the seismic response of buildings are derived.

4.2 Numerical Model of MSBFD

4.2.1 Frame with dissipators

Fig. 4.1 shows a typical 2D structure subjected to lateral forces and a seismic motion. This structure is known as *multi-story building* (MSB). If the building incorporates friction

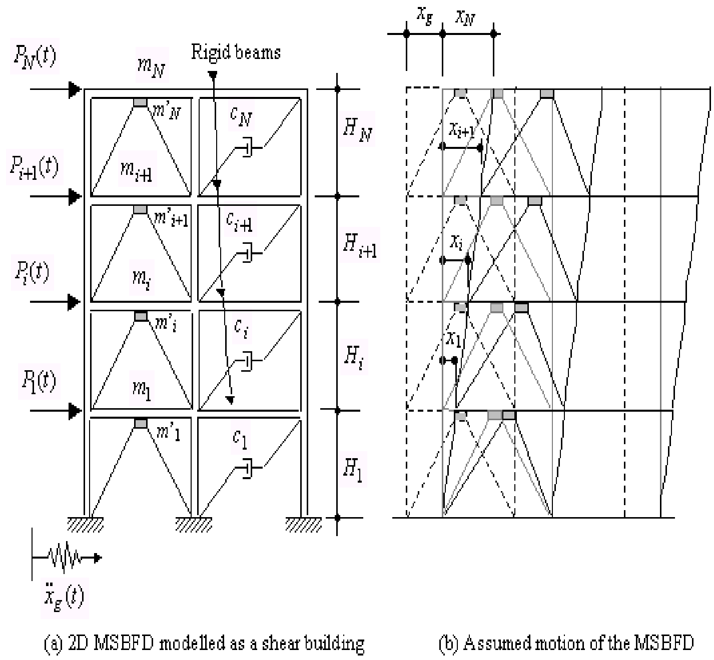


Figure 4.1 2D Multi-story building equipped with friction dissipators modelled as a shear builiding

dissipators at each floor, as in Fig. 4.1, this type of structure will be called *multi-story building with friction dissipators* (MSBFD).

4.2.2 Mechanical model

The mechanical model of the structure of Fig. 4.1 is shown in Fig. 4.2 for $N = 3$. The free-body diagram of the model of Fig. 4.2 is depicted in Fig. 4.3.

Despite the free-body diagram of Fig.4.3 belongs to a 3-degree-of-freedom system, the equations of motion will be developed for a generic N-story symmetric building.

4.2.3 Equations of motion

With the forces acting on the free-body diagrams of Fig. 4.3 it is possible to formulate the equations of motion of a MSBFD modelled as a 2D shear-building with N floors. For building structures that can not be considered as shear-buildings, a similar combination can be derived.

The total number of degrees of freedom will range between N (no sliding at any dissipator) and $2N$ (all dissipators slide simultaneously). Assuming that all $P_i(t) = 0$, the equations of motion of the $2N$ degrees of freedom for the system shown in Fig. 4.3 are:

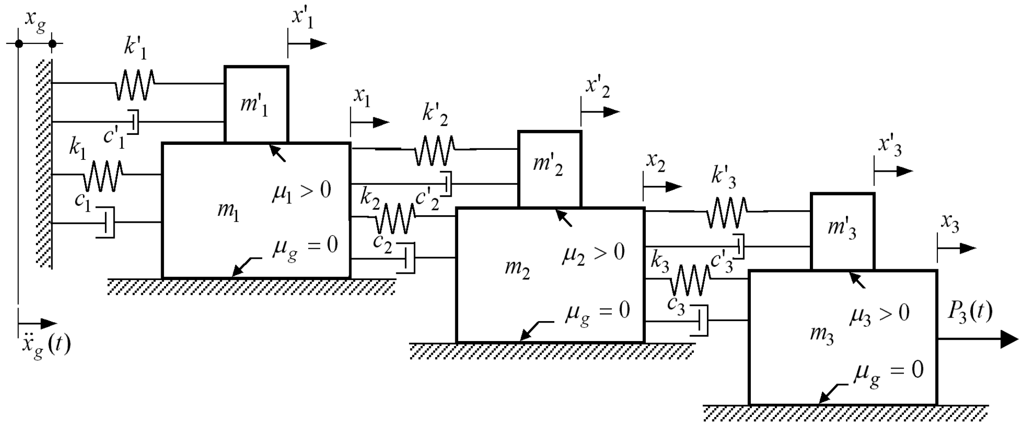


Figure 4.2 Mechanical model of a MSBFD (shear-building)

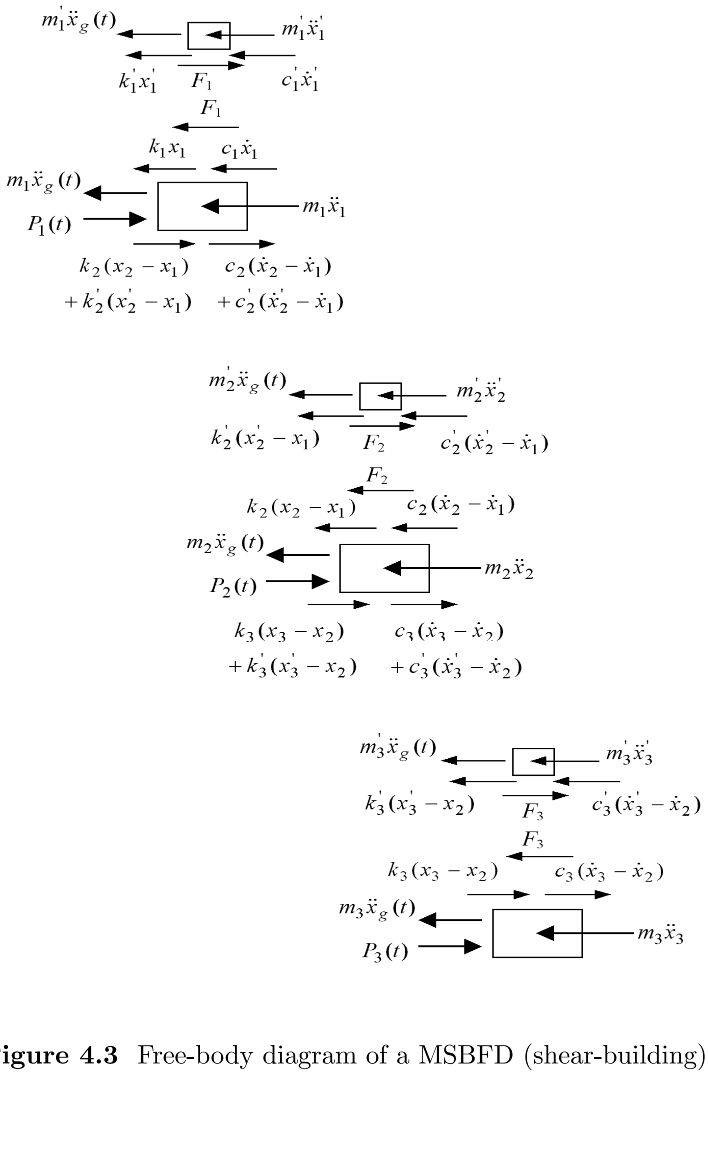


Figure 4.3 Free-body diagram of a MSBFD (shear-building)

$$
\begin{aligned}
m_1\left(\ddot{x}_1+\ddot{x}_g\right)+c_1\,\dot{x}_1+k_1\,x_1-c_2\left(\dot{x}_2-\dot{x}_1\right)-k_2\left(x_2-x_1\right)&\\
-c_2'\left(\dot{x}_2'-\dot{x}_1\right)-k_2'\left(x_2'-x_1\right)&=-F_1\\
m_1'\left(\ddot{x}_1'+\ddot{x}_g\right)+c_1'\,\dot{x}_1'+k_1'\,x_1'&=F_1
\end{aligned}
$$

$$
\begin{aligned}
m_2\left(\ddot{x}_2+\ddot{x}_g\right)+c_2\left(\dot{x}_2-\dot{x}_1\right)+k_2\left(x_2-x_1\right)-c_3\left(\dot{x}_3-\dot{x}_2\right)&\\
-k_3\left(x_3-x_2\right)-c_3'\left(\dot{x}_3'-\dot{x}_2\right)-k_3'\left(x_3'-x_2\right)&=-F_2\\
m_2'\left(\ddot{x}_2'+\ddot{x}_g\right)+c_2'\left(\dot{x}_2'-\dot{x}_1\right)+k_2'\left(x_2'-x_1\right)&=F_2
\end{aligned}
$$

$$\ldots$$

$$
\begin{aligned}
m_i\left(\ddot{x}_i+\ddot{x}_g\right)+c_i\left(\dot{x}_i-\dot{x}_{i-1}\right)+k_i\left(x_i-x_{i-1}\right)-c_{i+1}\left(\dot{x}_{i+1}-\dot{x}_i\right)&\\
-k_{i+1}\left(x_{i+1}-x_i\right)-c_{i+1}'\left(\dot{x}_{i+1}'-\dot{x}_i\right)-k_{i+1}'\left(x_{i+1}'-x_i\right)&=-F_i\\
m_i'\left(\ddot{x}_i'+\ddot{x}_g\right)+c_i'\left(\dot{x}_i'-\dot{x}_{i-1}\right)+k_i'\left(x_i'-x_{i-1}\right)&=F_i
\end{aligned}
$$

$$\ldots$$

$$
\begin{aligned}
m_N\left(\ddot{x}_N+\ddot{x}_g\right)+c_N\left(\dot{x}_N-\dot{x}_{N-1}\right)+k_N\left(x_N-x_{N-1}\right)&=-F_N\\
m_N'\left(\ddot{x}_N'+\ddot{x}_g\right)+c_N'\left(\dot{x}_N'-\dot{x}_{N-1}\right)+k_N'\left(x_N'-x_{N-1}\right)&=F_N
\end{aligned}
$$

where $\ddot{x}_g$ is the ground acceleration; m_i, c_i and k_i are, respectively, the mass, viscous damping and stiffness coefficients of the i-th floor and m_i', c_i' and k_i' are the corresponding values for the bracing-dissipator connection. F_i is the friction force between the dissipator and the structure (see Eqs. (A.9) and (A.10)). Values of F_i are limited by the corresponding friction coefficients μ_i and the prestressing forces N_i (see Chapter 2 and Appendix A):

$$|F_i|=\mu_i N_i \tag{4.1}$$

The block of $2\,N$ equations above can be grouped as:

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m_N \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_N \end{bmatrix} + \begin{bmatrix} c_1+c_2+c'_2 & -c_2 & 0 & 0 \\ -c_2 & c_2+c_3+c'_3 & \ddots & 0 \\ 0 & 0 & \ddots & -c_N \\ 0 & 0 & -c_N & c_N \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_N \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & -c'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \ddots & -c'_N \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}'_1 \\ \dot{x}'_2 \\ \vdots \\ \dot{x}'_N \end{bmatrix} + \begin{bmatrix} k_1+k_2+k'_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3+k'_3 & 0 & 0 \\ 0 & 0 & \ddots & -k_N \\ 0 & 0 & -k_N & k_N \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & -k'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \ddots & -k'_N \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_N \end{bmatrix} = - \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m_N \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \ddot{x}_g - \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{bmatrix}$$

and

$$\begin{bmatrix} m'_1 & 0 & 0 & 0 \\ 0 & m'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m'_N \end{bmatrix} \begin{bmatrix} \ddot{x}'_1 \\ \ddot{x}'_2 \\ \vdots \\ \ddot{x}'_N \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ -c'_2 & 0 & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & -c'_N & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_N \end{bmatrix}$$
$$+ \begin{bmatrix} c'_1 & 0 & 0 & 0 \\ 0 & c'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & c'_N \end{bmatrix} \begin{bmatrix} \dot{x}'_1 \\ \dot{x}'_2 \\ \vdots \\ \dot{x}'_N \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ -k'_2 & 0 & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & -k'_N & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$$
$$+ \begin{bmatrix} k'_1 & 0 & 0 & 0 \\ 0 & k'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & k'_N \end{bmatrix} \begin{bmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_N \end{bmatrix} = - \begin{bmatrix} m'_1 & 0 & 0 & 0 \\ 0 & m'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m'_N \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \ddot{x}_g + \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{bmatrix}$$

The first block deals with the motion of the main structure while the second one deals with the motion of dissipators. These two sets of equations can be written in matrix form as

$$\begin{aligned} \mathbf{M}^{ss}\,\ddot{\mathbf{x}}^s + (\mathbf{C}^{ss} + \mathbf{C}^{db})\,\dot{\mathbf{x}}^s + \mathbf{C}^{dc}\,\dot{\mathbf{x}}^d + (\mathbf{K}^{ss} + \mathbf{K}^{db})\,\mathbf{x}^s + \mathbf{K}^{dc}\,\mathbf{x}^d \\ = -\mathbf{M}^{ss}\,\mathbf{r}\,\ddot{x}_g - \mathbf{F} \end{aligned} \tag{4.2a}$$

$$\begin{aligned} \mathbf{M}^{dd}\,\ddot{\mathbf{x}}^d + \left(\mathbf{C}^{dc}\right)^T \dot{\mathbf{x}}^s + \mathbf{C}^{da}\,\dot{\mathbf{x}}^d + \left(\mathbf{K}^{dc}\right)^T \mathbf{x}^s + \mathbf{K}^{da}\,\mathbf{x}^d \\ = -\mathbf{M}^{dd}\,\mathbf{r}\,\ddot{x}_g + \mathbf{F} \end{aligned} \tag{4.2b}$$

or

$$\begin{bmatrix} \mathbf{M}^{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{dd} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}^s \\ \ddot{\mathbf{x}}^d \end{bmatrix} + \begin{bmatrix} \mathbf{C}^{ss} + \mathbf{C}^{db} & \mathbf{C}^{dc} \\ \left(\mathbf{C}^{dc}\right)^T & \mathbf{C}^{da} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}}^s \\ \dot{\mathbf{x}}^d \end{bmatrix}$$
$$+ \begin{bmatrix} \mathbf{K}^{ss} + \mathbf{K}^{db} & \mathbf{K}^{dc} \\ \left(\mathbf{K}^{dc}\right)^T & \mathbf{K}^{da} \end{bmatrix} \begin{bmatrix} \mathbf{x}^s \\ \mathbf{x}^d \end{bmatrix} = - \begin{bmatrix} \mathbf{M}^{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{dd} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{r} \end{bmatrix} \ddot{x}_g + \begin{bmatrix} -\mathbf{F} \\ +\mathbf{F} \end{bmatrix} \tag{4.3}$$

where

$$\mathbf{M}^{ss} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m_N \end{bmatrix} = \text{mass matrix of the main structure}$$

$$\mathbf{C}^{ss} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & 0 \\ -c_2 & c_2 + c_3 & \ddots & 0 \\ 0 & 0 & \ddots & -c_N \\ 0 & 0 & -c_N & c_N \end{bmatrix} = \text{damping matrix of the main structure}$$

$$\mathbf{K}^{ss} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & 0 & 0 \\ 0 & 0 & \ddots & -k_N \\ 0 & 0 & -k_N & k_N \end{bmatrix} = \text{stiffness matrix of the main structure}$$

$$\mathbf{x}^s = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \text{displacement vector of the main structure}$$

$$\mathbf{M}^{dd} = \begin{bmatrix} m'_1 & 0 & 0 & 0 \\ 0 & m'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m'_N \end{bmatrix} = \text{mass matrix of the braces + dissipators}$$

$$\mathbf{C}^{da} = \begin{bmatrix} c'_1 & 0 & \cdots & 0 \\ 0 & c'_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & c'_N \end{bmatrix}$$

$$\mathbf{C}^{db} = \begin{bmatrix} c'_2 & 0 & \cdots & 0 \\ 0 & c'_3 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\mathbf{C}^{dc} = \begin{bmatrix} 0 & -c'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ \vdots & 0 & \ddots & -c'_N \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \mathbf{C}^{da} + \mathbf{C}^{db} + \mathbf{C}^{dc} + \left(\mathbf{C}^{dc}\right)^T &= \mathbf{C}^{dd} = \begin{bmatrix} c'_1 + c'_2 & -c'_2 & 0 & 0 \\ -c'_2 & c'_2 + c'_3 & \ddots & 0 \\ 0 & 0 & \ddots & -c'_N \\ 0 & 0 & -c'_N & c'_N \end{bmatrix} \\ &= \text{damping matrix of the bracing system + dissipators} \end{aligned}$$

$$\mathbf{K}^{da} = \begin{bmatrix} k'_1 & 0 & \cdots & 0 \\ 0 & k'_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & k'_N \end{bmatrix}$$

$$\mathbf{K}^{db} = \begin{bmatrix} k'_2 & 0 & \cdots & 0 \\ 0 & k'_3 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\mathbf{K}^{dc} = \begin{bmatrix} 0 & -k'_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ \vdots & 0 & \ddots & -k'_N \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} \mathbf{K}^{da} + \mathbf{K}^{db} + \mathbf{K}^{dc} + \left(\mathbf{K}^{dc}\right)^T &= \mathbf{K}^{dd} = \begin{bmatrix} k'_1 + k'_2 & -k'_2 & 0 & 0 \\ -k'_2 & k'_2 + k'_3 & \ddots & 0 \\ 0 & 0 & \ddots & -k'_N \\ 0 & 0 & -k'_N & k'_N \end{bmatrix} \\ &= \text{stiffness matrix of the bracing system + dissipators} \end{aligned}$$

$$\mathbf{x}^d = \begin{bmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_N \end{bmatrix} = \text{displacement vector of the bracing system + dissipators}$$

$$\mathbf{r} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = \text{unit vector}$$

$$\mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{bmatrix} = \text{friction force vector}$$

Superindices s and d account for the structure and for the dissipators, respectively: $\mathbf{x}^s = [x_1, ..., x_N]^T$ and $\mathbf{x}^d = [x'_1, ..., x'_N]^T$. Dots above vectors $\mathbf{x}^s$ and $\mathbf{x}^d$ indicate time derivatives.

Eqs. (4.2a) and (4.2b) (or its alternative form (4.3)) above derived are also applicable to any time-varying load $\mathbf{P}(t)$. In this case, it is necessary to substitute the right terms of Eqs. (4.2a) and (4.2b), i.e., $-\mathbf{M}^{ss}\,\mathbf{r}\,\ddot{x}_g - \mathbf{F}$ and $-\mathbf{M}^{dd}\,\mathbf{r}\,\ddot{x}_g + \mathbf{F}$, by vectors $\mathbf{P}(t) - \mathbf{F}$ and $\mathbf{0} + \mathbf{F}$, respectively.

4.3 Proposed Solution of the Equations of Motion

4.3.1 Previous considerations

It is important to note that Eqs. (4.2a) and (4.2b) are coupled through interaction friction forces $\mathbf{F}$ and crossed matrices $\mathbf{C}^{db}$ (and $(\mathbf{C}^{db})^T$) and $\mathbf{K}^{db}$ (and $(\mathbf{K}^{db})^T$). This situation makes the solution process more difficult. However, the approach presented in this section is intended to make easy the numerical solution of Eqs. (4.2a) and (4.2b). Through the entire solution process, these equations will be split in two subsets termed with subindexes st (sticking condition) and sl (sliding condition). If the total number of sticking floors are called nst and the total number of sliding floors nsl, thus at any instant $N = nst + nsl$. Eqs. (4.2a) and (4.2b) can be written as

$$\begin{aligned} &\mathbf{M}^{ss}\,({}_{st+sl}\ddot{\mathbf{x}}^s) + (\mathbf{C}^{ss} + \mathbf{C}^{db})\,({}_{st+sl}\dot{\mathbf{x}}^s) + \mathbf{C}^{dc}\,({}_{st+sl}\dot{\mathbf{x}}^d) \\ &+(\mathbf{K}^{ss} + \mathbf{K}^{db})\,({}_{st+sl}\mathbf{x}^s) + \mathbf{K}^{dc}\,({}_{st+sl}\mathbf{x}^d) = -\mathbf{M}^{ss}\,\mathbf{r}\,\ddot{x}_g - ({}_{st+sl}\mathbf{F}) \end{aligned} \tag{4.4}$$

$$\mathbf{M}^{dd}\left({}_{st+sl}\ddot{\mathbf{x}}^d\right) + \left(\mathbf{C}^{dc}\right)^T\left({}_{st+sl}\dot{\mathbf{x}}^s\right) + \mathbf{C}^{da}\left({}_{st+sl}\dot{\mathbf{x}}^d\right)$$
$$+ \left(\mathbf{K}^{dc}\right)^T\left({}_{st+sl}\mathbf{x}^s\right) + \mathbf{K}^{da}\left({}_{st+sl}\mathbf{x}^d\right) = -\mathbf{M}^{dd}\,\mathbf{r}\,\ddot{x}_g + \left({}_{st+sl}\mathbf{F}\right) \tag{4.5}$$

where ${}_{st+sl}\mathbf{x}^s$ is the displacement vector of the main frame:

$${}_{st+sl}\mathbf{x}^s = \left[{}_{st}x_1, {}_{st}x_2, \ldots, {}_{st}x_i, \ldots, {}_{st}x_{nst}, {}_{sl}x_1, {}_{sl}x_2, \ldots, {}_{sl}x_i, \ldots, {}_{sl}x_{nsl}\right]^T$$

and ${}_{st+sl}\mathbf{x}^d$ is the displacement vector of the dissipators:

$${}_{st+sl}\mathbf{x}^d = \left[{}_{st}x'_1, {}_{st}x'_2, \ldots, {}_{st}x'_i, \ldots, {}_{st}x'_{nst}, {}_{sl}x'_1, {}_{sl}x'_2, \ldots, {}_{sl}x'_i, \ldots, {}_{sl}x'_{nsl}\right]^T$$

Displacements ${}_{st+sl}\mathbf{x}^s$ and ${}_{st+sl}\mathbf{x}^d$ can be split to yield the following vectors:

$$\begin{aligned} {}_{st}\mathbf{x}^s &= \left[{}_{st}x_1, {}_{st}x_2, \ldots, {}_{st}x_i, \ldots, {}_{st}x_{nst}\right]^T \\ {}_{st}\mathbf{x}^d &= \left[{}_{st}x'_1, {}_{st}x'_2, \ldots, {}_{st}x'_i, \ldots, {}_{st}x'_{nst}\right]^T \qquad \text{(sticking conditions)} \end{aligned}$$

and

$$\begin{aligned} {}_{sl}\mathbf{x}^s &= \left[{}_{sl}x_1, {}_{sl}x_2, \ldots, {}_{sl}x_i, \ldots, {}_{sl}x_{nsl}\right]^T \\ {}_{sl}\mathbf{x}^d &= \left[{}_{sl}x'_1, {}_{sl}x'_2, \ldots, {}_{sl}x'_i, \ldots, {}_{sl}x'_{nsl}\right]^T \qquad \text{(sliding conditions)} \end{aligned}$$

Velocities ${}_{st+sl}\dot{\mathbf{x}}^s$, ${}_{st+sl}\dot{\mathbf{x}}^d$ and accelerations ${}_{st+sl}\ddot{\mathbf{x}}^s$, ${}_{st+sl}\ddot{\mathbf{x}}^d$ are split in the same way.

It is important to point out that the degrees of freedom nst and nsl belonging to the sticking and sliding conditions at each floor, vary continuously through the entire time integration process.

4.3.2 Proposed algorithm

Eqs. (4.4) and (4.5) are solved numerically, step-by-step, using a modified version of the linear acceleration method (see Appendix B). This version is a generalization of the direct-time integration procedure presented in Chapter 3 for SSBFD.

Three nested iteration loops involving the coupling quantities ($\dot{\mathbf{x}}^d$, $\mathbf{x}^d$, $\dot{\mathbf{x}}^s$, $\mathbf{x}^s$ and $\mathbf{F}$) and the estimated accelerations ($\ddot{\mathbf{x}}^{s*}$ and $\ddot{\mathbf{x}}^{d*}$) at the step $k+1$ are performed. The procedure is described next:

INITIAL INSTANT $t_1 = 0$

At the first instant, $t_1 = 0$ displacements ${}_{st+sl}\mathbf{x}_0^s$, ${}_{st+sl}\mathbf{x}_0^d$ and velocities ${}_{st+sl}\dot{\mathbf{x}}_0^s$, ${}_{st+sl}\dot{\mathbf{x}}_0^d$ are known. For each floor i there are two possibilities:

1. If $\dot{x}_i(0) = \dot{x}_i'(0)$, there is sticking at floor i. The total number of floors under the sticking condition is nst. Hence, there will be nst equations out of Eq. (4.4) and nst equations out of (4.5).

2. If $\dot{x}_i(0) \neq \dot{x}_i'(0)$, there is sliding at floor i. The total number of floors under the sliding condition is nsl. Hence, there will be $nsl = N - nst$ equations out of Eq. (4.4) and nsl equations out of (4.5).

The unknown quantities of the nst equations in Eq. (4.4) are accelerations ${}_{st}\ddot{\mathbf{x}}_0^s$ and friction forces ${}_{st}\mathbf{F}_0$, while in the remaining nsl equations the unknown quantities are only the accelerations ${}_{sl}\ddot{\mathbf{x}}_0^s$. Friction forces ${}_{sl}\mathbf{F}_0$ can be computed using the following expression:

$$ {}_{sl}F_i(0) = \operatorname{sgn}\left[{}_{sl}\dot{x}_i(0) - {}_{sl}\dot{x}_i'(0)\right]\mu_i N_i \qquad (i = 1,\ 2,\ \ldots,\ nsl) \tag{4.6} $$

Accelerations ${}_{sl}\ddot{\mathbf{x}}_0^s$ can be calculated using the nsl equations of Eq. (4.5) at instant t_1:

$$ \begin{aligned} {}_{sl}\ddot{\mathbf{x}}_0^s \;=\;& {}_{sl}\left(\mathbf{M}^{ss}\right)^{-1}\left[-\,{}_{sl}(\mathbf{C}^{ss} + \mathbf{C}^{db})\,\left({}_{st+sl}\dot{\mathbf{x}}_0^s\right)\right. \\ & -\,{}_{sl}\mathbf{C}^{dc}\left({}_{st+sl}\dot{\mathbf{x}}_0^d\right) -\,{}_{sl}(\mathbf{K}^{ss} + \mathbf{K}^{db})\,\left({}_{st+sl}\mathbf{x}_0^s\right) \\ & \left. -\,{}_{sl}\mathbf{K}^{dc}\left({}_{st+sl}\mathbf{x}_0^d\right) -\,{}_{sl}\mathbf{M}^{ss}\,\mathbf{r}\,\ddot{x}_g(0) -\,{}_{sl}\mathbf{F}_0\right] \end{aligned} \tag{4.7} $$

where left subindices sl refer to the 1, 2, ... , nsl rows belonging to the nsl equations of the sliding condition. Hence, matrices ${}_{sl}\left(\mathbf{M}^{ss}\right)^{-1}$, ${}_{sl}(\mathbf{C}^{ss} + \mathbf{C}^{db})$, ${}_{sl}\mathbf{C}^{dc}$, ${}_{sl}(\mathbf{K}^{ss} + \mathbf{K}^{db})$, ${}_{sl}\mathbf{K}^{dc}$ and ${}_{sl}\mathbf{M}^{ss}$ are now of order $nsl \times N$.

On the other hand, the unknown variables of the nst equations in Eq. (4.5) are again friction forces ${}_{st}\mathbf{F}_0$ (accelerations ${}_{st}\ddot{\mathbf{x}}_0^d$ are equal to ${}_{st}\ddot{\mathbf{x}}_0^s$) and in the remaining nsl equations, the unknown quantities are accelerations ${}_{sl}\ddot{\mathbf{x}}_0^d$ (friction forces ${}_{sl}\mathbf{F}_0$ are obtained using Eq. (4.6)).

First loop. In order to determine accelerations ${}_{st}\ddot{\mathbf{x}}_0^s$ from Eq. (4.4) it is necessary to assume initial values of ${}_{st}\mathbf{F}_0$. In further visits to this loop, these values are no longer assumed, but rather updated. These assumed (or updated) values will be called ${}_{st}\mathbf{F}_0^*$. Thus, accelerations ${}_{st}\ddot{\mathbf{x}}_0^s$ can be calculated using the nst equations of Eq. (4.4) at instant t_1:

$$ \begin{aligned} {}_{st}\ddot{\mathbf{x}}_0^s = {}& {}_{st}\left(\mathbf{M}^{ss}\right)^{-1}\left[-\,{}_{st}(\mathbf{C}^{ss} + \mathbf{C}^{db})\,\left({}_{st+sl}\dot{\mathbf{x}}_0^s\right)\right. \\ & -\,{}_{st}\mathbf{C}^{dc}\left({}_{st+sl}\dot{\mathbf{x}}_0^d\right) -\,{}_{st}(\mathbf{K}^{ss} + \mathbf{K}^{db})\,\left({}_{st+sl}\mathbf{x}_0^s\right) \\ & \left. -\,{}_{st}\mathbf{K}^{dc}\left({}_{st+sl}\mathbf{x}_0^d\right) -\,{}_{st}\mathbf{M}^{ss}\,\mathbf{r}\,\ddot{x}_g(0) -\,{}_{st}\mathbf{F}_0^*\right] \end{aligned} \tag{4.8} $$

Go to the second loop.

Second iteration loop. After the values of accelerations ${}_{st}\ddot{\mathbf{x}}_0^s$ and ${}_{sl}\ddot{\mathbf{x}}_0^s$ have been calculated with Eqs. (4.8) and (4.7), friction forces ${}_{st}\mathbf{F}_0$ can be obtained using the corresponding nst equations of (4.5). Initial values of accelerations ${}_{sl}\ddot{\mathbf{x}}_0^d$, termed ${}_{sl}\ddot{\mathbf{x}}_0^{d*}$, are assumed (or updated).

$$\begin{aligned} {}_{st}\mathbf{F}_0 \;\; = \;\; & {}_{st}\mathbf{M}^{dd}\,({}_{st+sl}\ddot{\mathbf{x}}_0^{d*}) + {}_{st}(\mathbf{C}^{dc})^T\,({}_{st+sl}\dot{\mathbf{x}}_0^{s}) \\ & + {}_{st}\mathbf{C}^{da}\,({}_{st+sl}\dot{\mathbf{x}}_0^{d}) + {}_{st}(\mathbf{K}^{dc})^T\,({}_{st+sl}\mathbf{x}_0^{s}) \\ & + {}_{st}\mathbf{K}^{da}({}_{st+sl}\mathbf{x}_0^{d}) + {}_{st}\mathbf{M}^{dd}\,\mathbf{r}\,\ddot{x}_g(0) \end{aligned} \tag{4.9}$$

where ${}_{st+sl}\ddot{\mathbf{x}}_0^{d*} = \left[{}_{st}\ddot{\mathbf{x}}_0^{d}, {}_{sl}\ddot{\mathbf{x}}_0^{d*}\right]^T$. The left subindices st refer to the 1, 2, …, nst rows belonging to the nst equations of the sticking condition. Hence, matrices ${}_{st}\mathbf{M}^{dd}$, ${}_{st}(\mathbf{C}^{dc})^T$, ${}_{st}\mathbf{C}^{da}$, ${}_{st}(\mathbf{K}^{dc})^T$, and ${}_{st}\mathbf{K}^{da}$ are now of order $nst \times N$.

1. If ${}_{st}F_i(0) = {}_{st}F_i^*(0) \pm \varepsilon_f$ (where ε_f is the prescribed tolerance) it is necessary to check the following:
   (a) If $|{}_{st}F_i(0)| < \mu_i N_i$ go to the third loop.
   (b) If $|{}_{st}F_i(0)| \geq \mu_i N_i$ then ${}_{st}F_i(0)$ is set equal either to $\mu_i N_i$ if ${}_{st}F_i(0) \geq \mu_i N_i$, or to $-\mu_i N_i$ if ${}_{st}F_i(0) < -\mu_i N_i$. A sliding condition at floor i will be considered for next instant t_2. Go to the third loop
2. If ${}_{st}F_i(0) \neq {}_{st}F_i^*(0) \pm \varepsilon_f$, the new (updated) value of ${}_{st}F_i^*(0)$ is set equal to the ${}_{st}F_i(0)$ just calculated with Eq. (4.9). Go to the first loop.

Third iteration loop. After the values of accelerations ${}_{st}\ddot{\mathbf{x}}_0^s$, ${}_{sl}\ddot{\mathbf{x}}_0^s$ and friction forces ${}_{st}\mathbf{F}_0$ have been calculated, accelerations ${}_{sl}\ddot{\mathbf{x}}_0^d$ can be determined using the nsl equations of (4.5) at instant t_1:

$$\begin{aligned} {}_{sl}\ddot{\mathbf{x}}_0^{d} \;\; = \;\; & {}_{sl}(\mathbf{M}^{dd})^{-1}\,[-\,{}_{sl}(\mathbf{C}^{dc})^T\,({}_{st+sl}\dot{\mathbf{x}}_0^{s}) \\ & - {}_{sl}\mathbf{C}^{da}\,({}_{st+sl}\dot{\mathbf{x}}_0^{d}) + {}_{sl}(\mathbf{K}^{dc})^T\,({}_{st+sl}\mathbf{x}_0^{s}) \\ & - {}_{sl}\mathbf{K}^{da}({}_{st+sl}\mathbf{x}_0^{d}) - {}_{sl}\mathbf{M}^{dd}\,\mathbf{r}\,\ddot{x}_g(0) + {}_{sl}\mathbf{F}_0] \end{aligned} \tag{4.10}$$

where left subindices sl refer to the 1, 2, … , nsl rows belonging to the nsl equations of the sliding condition. Hence, matrices ${}_{sl}(\mathbf{M}^{dd})^{-1}$, ${}_{sl}(\mathbf{C}^{dc})^T$, ${}_{sl}\mathbf{C}^{da}$, ${}_{sl}(\mathbf{K}^{dc})^T$, ${}_{sl}\mathbf{K}^{da}$ and ${}_{sl}\mathbf{M}^{dd}$ are now of order $nsl \times N$.

1. If ${}_{sl}\ddot{x}_i'(0) = {}_{sl}\ddot{x}_i'^*(0) \pm \varepsilon_a$ (where ε_a is the prescribed tolerance) go to next instant t_2.

2. If ${}_{sl}\ddot{x}'_i(0) \neq {}_{sl}\ddot{x}'^{*}_i(0) \pm \varepsilon_a$, the new (updated) value of ${}_{sl}\ddot{x}'^{*}_i(0)$ is set equal to the ${}_{sl}\ddot{x}'_i(0)$ calculated with Eq. (4.10). Go to the second loop.

ANY INSTANT t_{k+1}

At each generic instant $k+1$ Eqs. (4.4) and (4.5) become, respectively

$$
\begin{aligned}
\mathbf{M}^{ss}\left({}_{st+sl}\ddot{\mathbf{x}}^{s}_{k+1}\right) + \left(\mathbf{C}^{ss} + \mathbf{C}^{db}\right)\left({}_{st+sl}\dot{\mathbf{x}}^{s}_{k+1}\right) \\
+\mathbf{C}^{dc}\left({}_{st+sl}\dot{\mathbf{x}}^{d}_{k+1}\right) + \left(\mathbf{K}^{ss} + \mathbf{K}^{db}\right)\left({}_{st+sl}\mathbf{x}^{s}_{k+1}\right) \\
+\mathbf{K}^{dc}\left({}_{st+sl}\mathbf{x}^{d}_{k+1}\right) = -\mathbf{M}^{ss}\,\mathbf{r}\,\ddot{x}_g(t_{k+1}) - \left({}_{st+sl}\mathbf{F}_{k+1}\right)
\end{aligned}
\tag{4.11}
$$

$$
\begin{aligned}
\mathbf{M}^{dd}\left({}_{st+sl}\ddot{\mathbf{x}}^{d}_{k+1}\right) + \left(\mathbf{C}^{dc}\right)^{T}\left({}_{st+sl}\dot{\mathbf{x}}^{s}_{k+1}\right) \\
+\,\mathbf{C}^{da}\left({}_{st+sl}\dot{\mathbf{x}}^{d}_{k+1}\right) + \left(\mathbf{K}^{dc}\right)^{T}\left({}_{st+sl}\mathbf{x}^{s}_{k+1}\right) \\
+\,\mathbf{K}^{da}\left({}_{st+sl}\mathbf{x}^{d}_{k+1}\right) = -\,\mathbf{M}^{dd}\,\mathbf{r}\,\ddot{x}_g(t_{k+1}) + \left({}_{st+sl}\mathbf{F}_{k+1}\right)
\end{aligned}
\tag{4.12}
$$

The response is computed from the one at previous instant k. It is initially assumed that the sticking-sliding conditions in the dissipators at instant k are the same at $k+1$. There are two possibilities:

1. If there was sticking at instant k at floor i, it is assumed that there is sticking at instant $k+1$ at floor i. The total number of floors under the sticking condition is nst. There will be nst equations out of Eq. (4.11) and nst equations out of (4.12).

2. If there was sliding at instant k at floor i, it is assumed that there is sliding at instant $k+1$ at floor i. The total number of floors under the sliding condition is nsl. There will be $nsl = N - nst$ equations out of Eq. (4.11) and nsl equations out of (4.12).

Initial conditions. Conversely to initial instant t_1, at each instant $k+1$ displacements ${}_{st+sl}\mathbf{x}^{s}_{k+1}$, ${}_{st+st}\mathbf{x}^{d}_{k+1}$ and velocities ${}_{st+sl}\dot{\mathbf{x}}^{s}_{k+1}$, ${}_{st+sl}\dot{\mathbf{x}}^{d}_{k+1}$ are unknown. Hence it is necessary to set some values for these quantities. Initial values of displacements and velocities are calculated using the interpolation criterion considered in the linear acceleration method [1, 61]:

$$
{}_{st}\mathbf{x}^{s}_{k+1} = {}_{st}\mathbf{x}^{s}_{k} + \Delta t\,{}_{st}\dot{\mathbf{x}}^{s}_{k} + \frac{(\Delta t)^2}{6}\left(2\,{}_{st}\ddot{\mathbf{x}}^{s}_{k} + {}_{st}\ddot{\mathbf{x}}^{s*}_{k+1}\right) \tag{4.13a}
$$

$$
{}_{sl}\mathbf{x}^{s}_{k+1} = {}_{sl}\mathbf{x}^{s}_{k} + \Delta t\,{}_{sl}\dot{\mathbf{x}}^{s}_{k} + \frac{(\Delta t)^2}{6}\left(2\,{}_{sl}\ddot{\mathbf{x}}^{s}_{k} + {}_{sl}\ddot{\mathbf{x}}^{s*}_{k+1}\right) \tag{4.13b}
$$

$$
{}_{st}\dot{\mathbf{x}}^{s}_{k+1} = {}_{st}\dot{\mathbf{x}}^{s}_{k} + \frac{\Delta t}{2}\left({}_{st}\ddot{\mathbf{x}}^{s}_{k} + {}_{st}\ddot{\mathbf{x}}^{s*}_{k+1}\right) \tag{4.13c}
$$

$$
{}_{sl}\dot{\mathbf{x}}^{s}_{k+1} = {}_{sl}\dot{\mathbf{x}}^{s}_{k} + \frac{\Delta t}{2}\left({}_{sl}\ddot{\mathbf{x}}^{s}_{k} + {}_{sl}\ddot{\mathbf{x}}^{s*}_{k+1}\right) \tag{4.13d}
$$

$$\begin{aligned}
{}_{st}\mathbf{x}^d_{k+1} &= {}_{st}\mathbf{x}^d_k + \left({}_{st}\mathbf{x}^s_{k+1} - {}_{st}\mathbf{x}^s_k\right) && \text{(4.14a)}\\
{}_{sl}\mathbf{x}^d_{k+1} &= {}_{sl}\mathbf{x}^d_k + \Delta t\, {}_{sl}\dot{\mathbf{x}}^d_k + \frac{(\Delta t)^2}{6}\left(2\, {}_{sl}\ddot{\mathbf{x}}^d_k + {}_{sl}\ddot{\mathbf{x}}^{d*}_{k+1}\right) && \text{(4.14b)}\\
{}_{st}\dot{\mathbf{x}}^d_{k+1} &= {}_{st}\dot{\mathbf{x}}^s_{k+1} && \text{(4.14c)}\\
{}_{sl}\dot{\mathbf{x}}^d_{k+1} &= {}_{sl}\dot{\mathbf{x}}^d_k + \frac{\Delta t}{2}\left({}_{sl}\ddot{\mathbf{x}}^d_k + {}_{sl}\ddot{\mathbf{x}}^{d*}_{k+1}\right) && \text{(4.14d)}
\end{aligned}$$

where ${}_{st}\ddot{\mathbf{x}}^{s*}$, ${}_{sl}\ddot{\mathbf{x}}^{s*}$ are the accelerations on each floor of the main structure, ${}_{st}\ddot{\mathbf{x}}^{s*}$, ${}_{sl}\ddot{\mathbf{x}}^{s*}$ are the accelerations of each dissipator (subindexes k and $k+1$ refer to instants k and $k+1$, respectively) and Δt is the time increment.

First iteration loop. With the calculated (or updated) values of ${}_{st+sl}\mathbf{x}^s_{k+1}$, ${}_{st+st}\mathbf{x}^d_{k+1}$ and ${}_{st+sl}\dot{\mathbf{x}}^s_{k+1}$, ${}_{st+sl}\dot{\mathbf{x}}^d_{k+1}$, accelerations ${}_{st}\ddot{\mathbf{x}}^s_{k+1}$ can be calculated from Eq. (4.11):

$$\begin{aligned}
{}_{st+sl}\ddot{\mathbf{x}}^s_{k+1} = {} & (\mathbf{M}^{ss})^{-1}\left[-\left(\mathbf{C}^{ss} + \mathbf{C}^{db}\right)\left({}_{st+sl}\dot{\mathbf{x}}^s_{k+1}\right) - \mathbf{C}^{dc}\left({}_{st+sl}\dot{\mathbf{x}}^d_{k+1}\right)\right.\\
& -\left(\mathbf{K}^{ss} + \mathbf{K}^{db}\right)\left({}_{st+sl}\mathbf{x}^s_{k+1}\right) - \mathbf{K}^{dc}\left({}_{st+sl}\mathbf{x}^d_{k+1}\right)\\
& \left.-\mathbf{M}^{ss}\,\mathbf{r}\,\ddot{x}_g(t_{k+1}) - \left({}_{st+sl}\mathbf{F}^*_{k+1}\right)\right] \qquad \text{(4.15)}
\end{aligned}$$

where ${}_{st+sl}\mathbf{F}^*_{k+1} = \left[{}_{st}\mathbf{F}^*_{k+1}, {}_{sl}\mathbf{F}_{k+1}\right]^T$. The friction force vector of the sticking condition, ${}_{st}\mathbf{F}^*_{k+1}$, is initially set equal to ${}_{st}\mathbf{F}_k$ and it is eventually updated (see second loop). ${}_{sl}\mathbf{F}_{k+1}$ is the friction force vector of the sliding condition. This vector is initially set equal to ${}_{sl}\mathbf{F}_k$.

1. If ${}_{st+sl}\left(\ddot{x}_i\right)_{k+1} = {}_{st+sl}\left(\ddot{x}^*_i\right)_{k+1} \pm \varepsilon_a$ (where ε_a is the prescribed tolerance) go to the second loop.

2. If ${}_{st+sl}\left(\ddot{x}_i\right)_{k+1} \neq {}_{st+sl}\left(\ddot{x}^*_i\right)_{k+1} \pm \varepsilon_a$, the new (updated) value of ${}_{st+sl}\left(\ddot{x}^*_i\right)_{k+1}$ is set equal to ${}_{st+sl}\left(\ddot{x}_i\right)_{k+1}$. Go to initial conditions.

Second iteration loop. With the calculated (or updated) values of displacements ${}_{st+sl}\mathbf{x}^s_{k+1}$, ${}_{st+st}\mathbf{x}^d_{k+1}$ and velocities ${}_{st+sl}\dot{\mathbf{x}}^s_{k+1}$, ${}_{st+sl}\dot{\mathbf{x}}^d_{k+1}$, friction forces ${}_{st}\mathbf{F}^*_{k+1}$ can be calculated using the nst equations of Eq. (4.12):

$$\begin{aligned}
{}_{st}\mathbf{F}_{k+1} = {} & {}_{st}\mathbf{M}^{dd}\left({}_{st+sl}\ddot{\mathbf{x}}^d_{k+1}\right) + {}_{st}\left(\mathbf{C}^{dc}\right)^T\left({}_{st+sl}\dot{\mathbf{x}}^s_{k+1}\right)\\
& - {}_{st}\mathbf{C}^{da}\left({}_{st+sl}\dot{\mathbf{x}}^d_{k+1}\right) + {}_{st}\left(\mathbf{K}^{dc}\right)^T\left({}_{st+sl}\mathbf{x}^s_{k+1}\right)\\
& + {}_{st}\mathbf{K}^{da}\left({}_{st+sl}\mathbf{x}^d_{k+1}\right) + {}_{st}\mathbf{M}^{dd}\,\mathbf{r}\,\ddot{x}_g(t_{k+1}) \qquad \text{(4.16)}
\end{aligned}$$

1. If ${}_{st}\left(F_i\right)_{k+1} = {}_{st}\left(F^*_i\right)_{k+1} \pm \varepsilon_f$ (where ε_f is the prescribed tolerance) it is necessary to check the following:

(a) If $\left|_{st}\left(F_i\right)_{k+1}\right| < \mu_i N_i$ go to the third loop.

(b) If $\left|_{st}\left(F_i\right)_{k+1}\right| \geq \mu_i N_i$ then $_{st}\left(F_i\right)_{k+1}$ is set equal either to $\mu_i N_i$ if $_{st}\left(F_i\right)_{k+1} \geq \mu_i N_i$, or to $-\mu_i N_i$ if $_{st}\left(F_i\right)_{k+1} < -\mu_i N_i$. Go to the third loop.

2. If $_{st}\left(F_i\right)_{k+1} \neq {}_{st}\left(F_i^*\right)_{k+1} \pm \varepsilon_f$, the new (updated) value of $_{st}\left(F_i^*\right)_{k+1}$ is set equal to the $_{st}\left(F_i\right)_{k+1}$ just calculated with Eq. (4.16). Go to the first loop.

Third iteration loop. With the calculated (or updated) values of displacements $_{st+sl}\mathbf{x}^s_{k+1}$, $_{st+st}\mathbf{x}^d_{k+1}$ and velocities $_{st+sl}\dot{\mathbf{x}}^s_{k+1}$, $_{st+sl}\dot{\mathbf{x}}^d_{k+1}$, accelerations $_{sl}\ddot{\mathbf{x}}^d_{k+1}$ can be determined using the nsl equations of Eq. (4.12):

$$\begin{aligned}
{}_{sl}\ddot{\mathbf{x}}^d_{k+1} \quad = \quad & {}_{sl}(\mathbf{M}^{dd})^{-1}[- {}_{sl}(\mathbf{C}^{dc})^T \left({}_{st+sl}\dot{\mathbf{x}}^s_{k+1}\right) \\
& - {}_{sl}\mathbf{C}^{da}\left({}_{st+sl}\dot{\mathbf{x}}^d_{k+1}\right) - {}_{sl}(\mathbf{K}^{dc})^T \left({}_{st+sl}\mathbf{x}^s_{k+1}\right) \\
& - {}_{sl}\mathbf{K}^{da}\left({}_{st+sl}\mathbf{x}^d_{k+1}\right) - {}_{sl}\mathbf{M}^{dd}\,\mathbf{r}\,\ddot{x}_g(t_{k+1}) + {}_{sl}\mathbf{F}_{k+1}]
\end{aligned} \tag{4.17}$$

1. If $_{sl}\left(\ddot{x}'_i\right)_{k+1} = {}_{sl}\left(\ddot{x}'^*_i\right)_{k+1} \pm \varepsilon_a$ (where ε_a is the prescribed tolerance) go to final considerations.

2. If $_{sl}\left(\ddot{x}'_i\right)_{k+1} \neq {}_{sl}\left(\ddot{x}'^*_i\right)_{k+1} \pm \varepsilon_a$, the new (updated) value of $_{sl}\left(\ddot{x}'^*_i\right)_{k+1}$ is set equal to the $_{sl}\left(\ddot{x}'_i\right)_{k+1}$ calculated with Eq. (4.17). Go to initial conditions.

Final considerations. As said before, at each sampling instant $k+1$ it is initially assumed that the condition of the previous instant k keeps for each i floor. However, during instant $k+2$ such a condition can change. Thus, after the fulfillment of the established conditions for the above iteration loops, the *sliding-sticking* condition at the i-th dissipator must be checked before going to next instant $k+2$. At each i-th floor, two possibilities must be considered:

1. If there was sticking, each computed value of the friction force, $_{st}\left(F_i\right)_{k+1}$, is compared to $\mu_i N_i$:

   (a) If $\left|_{st}\left(F_i\right)_{k+1}\right| < \mu_i N_i$ the sticking condition at the i-th floor keeps at instant $k+2$. Go to instant $k+2$.

   (b) If $\left|_{st}\left(F_i\right)_{k+1}\right| \geq \mu_i N_i$ there is sliding and therefore $_{st}\left(F_i\right)_{k+1}$ is either set equal to $\mu_i N_i$ if $_{st}\left(F_i\right)_{k+1} \geq \mu_i N_i$ or to $-\mu_i N_i$ if $_{st}\left(F_i\right)_{k+1} < -\mu_i N_i$. A sliding condition at the i-th floor must be considered at instant $k+2$. Go to instant $k+2$.

2. If there was sliding, each value of the relative velocity $\left({}_{sl}\dot{x}_i - {}_{sl}\dot{x}'_i\right)_{k+1}$ is computed;

   (a) if $\left({}_{sl}\dot{x}_i - {}_{sl}\dot{x}'_i\right)_k \left({}_{sl}\dot{x}_i - {}_{sl}\dot{x}'_i\right)_{k+1} > 0$, the sliding condition keeps at $k+2$ (i.e., there is no change in the direction of relative motion). Go to instant $k+2$.

(b) if $({}_{sl}\dot{x}_i - {}_{sl}\dot{x}'_i)_k \; ({}_{sl}\dot{x}_i - {}_{sl}\dot{x}'_i)_{k+1} \leq 0$ there is sticking. Moreover, the fulfillment of inequality $({}_{sl}\dot{x}_i - {}_{sl}\dot{x}'_i)_k \; ({}_{sl}\dot{x}_i - {}_{sl}\dot{x}'_i)_{k+1} \leq 0$ could mean that, instead of sticking, there has been a change in the direction of the relative motion and the sliding condition continues in the opposite direction. Despite this last possibility, at next instant $k+2$ a sticking condition will be considered. Go to instant $k+2$.

The above algorithm is displayed, in a simpler manner, in the flowchart shown in Fig. 4.4.

4.3.3 Stability and accuracy

The following values of time increment Δt and acceleration and force tolerances, ε_a and ε_f, were proved to be good to yield enough stability and acccuracy:

$$\begin{aligned}
\Delta t &= T_F/200 \\
\varepsilon_a &= PGA/100000 \\
\varepsilon_f &= \mu N/100000
\end{aligned}$$

where T_F is the fundamental period of the bare frame. The maximum absolute value of the ground acceleration, $|\ddot{x}_g|_{\max}$, could be used instead of the PGA (peak ground acceleration), if necessary.

4.3.4 ALMA program

The ALMA (**A**utomatic non**L**inear **M**atrix **A**nalysis) program holds the algorithm described above [62]. It is a FORTRAN 77 source code and is intended to solve the equations of motion of MSBFD. The input data have to be defined by the user in plain text files. In one file, for example, for a sinusoidal driving force, the values of P_0, $\bar{\omega}$, μN have to be defined for each level. In other file, the time increment Δt and the tolerances for accelerations ε_a and for sliding thresholds ε_f are set. In a third file, the property matrices of the main structure $\mathbf{M}^{ss}$, $\mathbf{C}^{ss}$, $\mathbf{K}^{ss}$ and those of the bracing and dissipators $\mathbf{M}^{dd}$, $\mathbf{C}^{dd}$, $\mathbf{K}^{dd}$ must be defined.

ALMA reads the input data files and determines the dynamic response for each floor. The output data are saved as text files which can be visualized later with a graphics package (i.e., EXCEL®). In fact, the time-history responses shown in this work were obtained in such a way. This procedure is illustrated in the block diagram of Fig. 4.5.

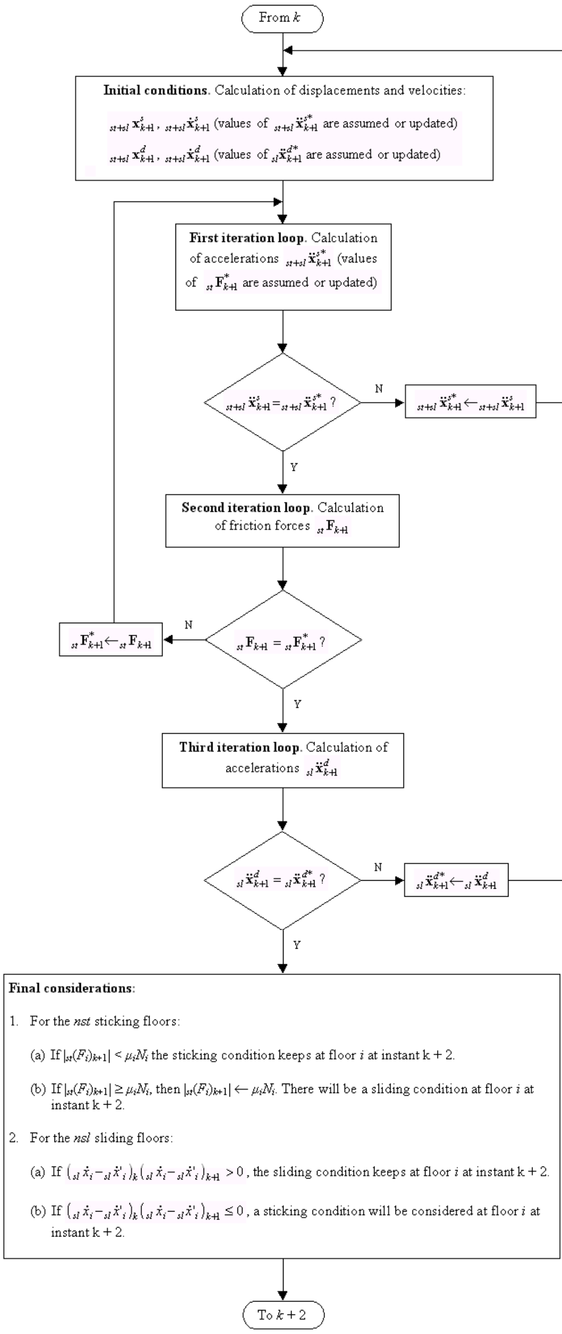


Figure 4.4 Flowchart of the proposed algorithm (ALMA program)

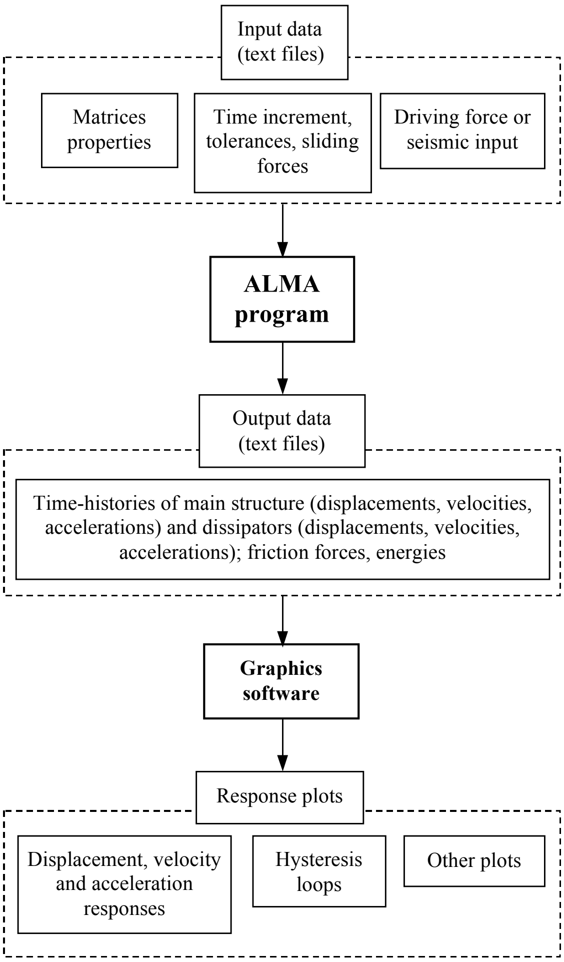


Figure 4.5 Procedure of data processing

Parameter	Main frame			Bracing + dissipators		
	1st floor	**2nd floor**	**3rd floor**	**1st floor**	**2nd floor**	**3rd floor**
Mass, lb·s/in^2	259.0	259.0	129.5	1.0	1.0	0.5
I_b, in^4	∞	∞	∞			
I_c, in^4	354.102	236.068	118.034			
A, in^2				0.5490	1.0979	1.6469
H, in	118.0	118.0	118.0			
L, in	196.8	196.8	196.8			

Table 4.1 Properties of the frame of Fig. 4.6

4.4 Practical Applications

4.4.1 Pulse loading on a 3-story building

As a first example of solution using ALMA, the dynamic response of the 3-story building of Fig. 4.6 affected for a pulse loading acting on the second floor, will be studied.

4.4.1.1 Description of the structure

The structure consists of three stories, each one holds a friction dissipator between the girder and the top of the braces, as shown in Fig. 4.6. The values registered in Table 4.1 were considered to determine the dynamic properties of the structure. The data belonging to the main frame were obtained from an example studied in [1]. The stiffness matrix of the main frame was determined assuming infinitely rigid beams ($E = 29 \times 10^6$ psi) and the matrix stiffness of the bracing system was calculated neglecting its compression resistance. The damping matrix of the main frame was calculated using the classical modal approach [1, 61]: $\xi_1 = 0.0275$, $\xi_2 = 0.0877$ and $\xi_3 = 0.1003$.

The data obtained with the values of Table 4.1 in matrix and vector forms are given next:

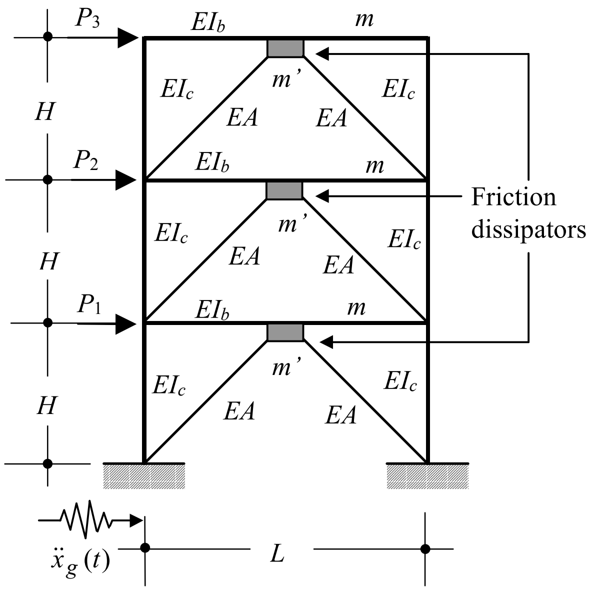


Figure 4.6 3-story building equipped with friction dissipators in each floor

$$\mathbf{M}^{ss} = \begin{bmatrix} 259.0 & 0 & 0 \\ 0 & 259.0 & 0 \\ 0 & 0 & 129.5 \end{bmatrix} \text{lb·in/s}^2$$

$$\mathbf{C}^{ss} = \begin{bmatrix} 1375.260 & -375.067 & -125.070 \\ -375.067 & 875.120 & -375.140 \\ -125.070 & -375.140 & 375.060 \end{bmatrix} \text{lb·in/s}$$

$$\mathbf{K}^{ss} = \begin{bmatrix} 250.0 & -100.0 & 0 \\ -100.0 & 150.0 & -50.0 \\ 0 & -50.0 & 50.0 \end{bmatrix} \times 10^3 \text{ lb/in}$$

$$\mathbf{T}^{ss} = \begin{bmatrix} 0.5708 \\ 0.2611 \\ 0.1791 \end{bmatrix} \text{s}$$

$$\mathbf{M}^{dd} = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \text{lb}\cdot\text{in/s}^2$$

$$\mathbf{C}^{da} = \mathbf{C}^{db} = \mathbf{C}^{dc} = \mathbf{C}^{dd} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{K}^{da} = \begin{bmatrix} 127.5 & 0 & 0 \\ 0 & 85.0 & 0 \\ 0 & 0 & 42.5 \end{bmatrix} \times 10^3 \text{ lb/in}$$

$$\mathbf{K}^{db} = \begin{bmatrix} 85.0 & 0 & 0 \\ 0 & 42.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 10^3 \text{ lb/in}$$

$$\mathbf{K}^{dc} = \begin{bmatrix} 0 & -85.0 & 0 \\ 0 & 0 & -42.5 \\ 0 & 0 & 0 \end{bmatrix} \times 10^3 \text{ lb/in}$$

In this case, the ground acceleration $\ddot{x}_g(t) = 0$ and $\mathbf{P}(t) = [0,\, P_2(t),\, 0]^T$. $P_2(t)$ is depicted in Fig. 4.7.

The sliding thresholds $\mu_1 N_1$, $\mu_2 N_2$ and $\mu_3 N_3$ are set equal respectively, to 20 kips, 25 kips and 10 kips. A time increment of 0.0005 s is used and the total time of analysis is 0.8 s. The prescribed tolerances are set equal to $\varepsilon_a = 0.01$ in/s^2 and $\varepsilon_f = 0.1$ lb.

4.4.1.2 Results

The third floor interstory drift response of the MSBFD for the main frame is shown in Fig. 4.8. In this figure the dissipator displacement is also plotted. The thresholds for each sliding force are defined by the two horizontal lines drawn in each case.

The hysteresis loops of the third floor are depicted in Fig. 4.9. As in the case of SSBFD considered in Chapter 3, these hysteresis loops are rectangular.

The energy time-histories of the entire structure are plotted in Fig. 4.10.

4.4.2 Ground acceleration on a benchmark building

4.4.2.1 Description of the structure

The next structure is classified as a *benchmark* problem. The dimensions and the properties are shown in Fig. 4.11 [63]. The cross sections of the braces are all equal to 118.71 cm^2. All braces work both in tension and compression.

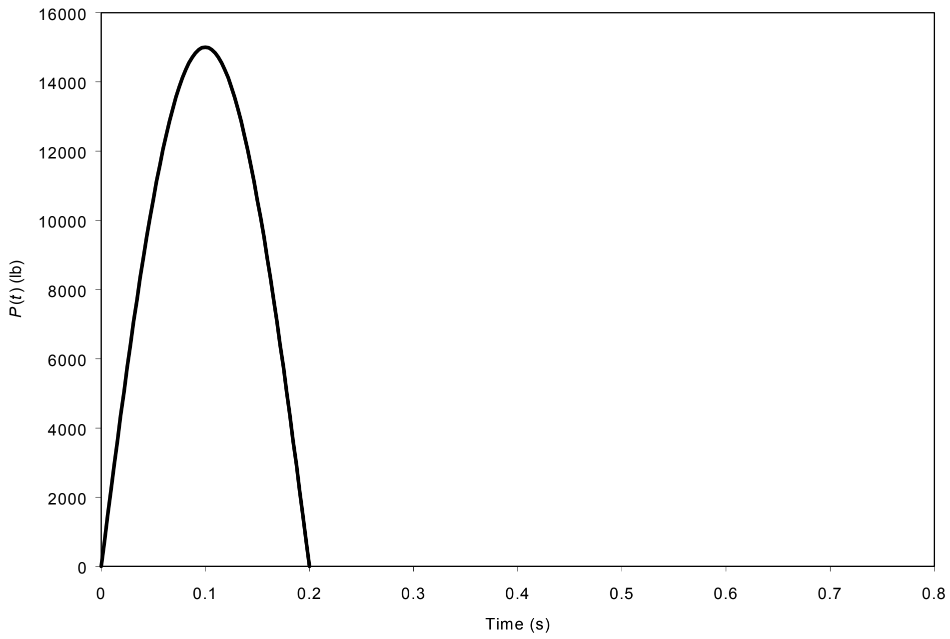


Figure 4.7 Sine pulse applied to the second floor of a 3-story building equipped with FD

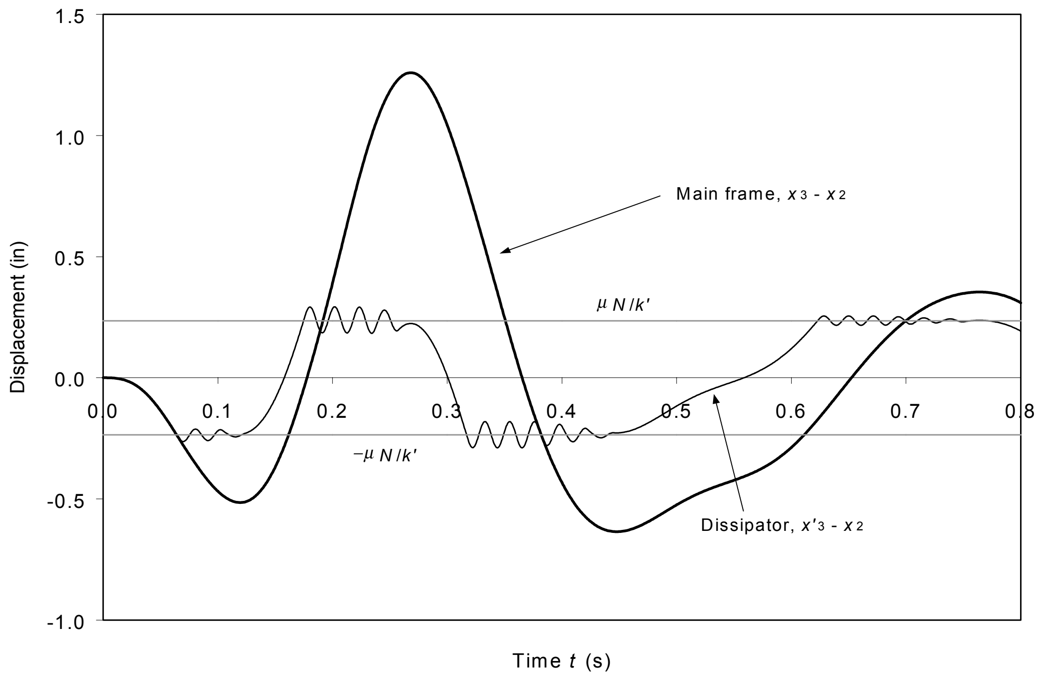


Figure 4.8 Relative displacement $x_3 - x_2$ of the MSBFD for the pulse loading

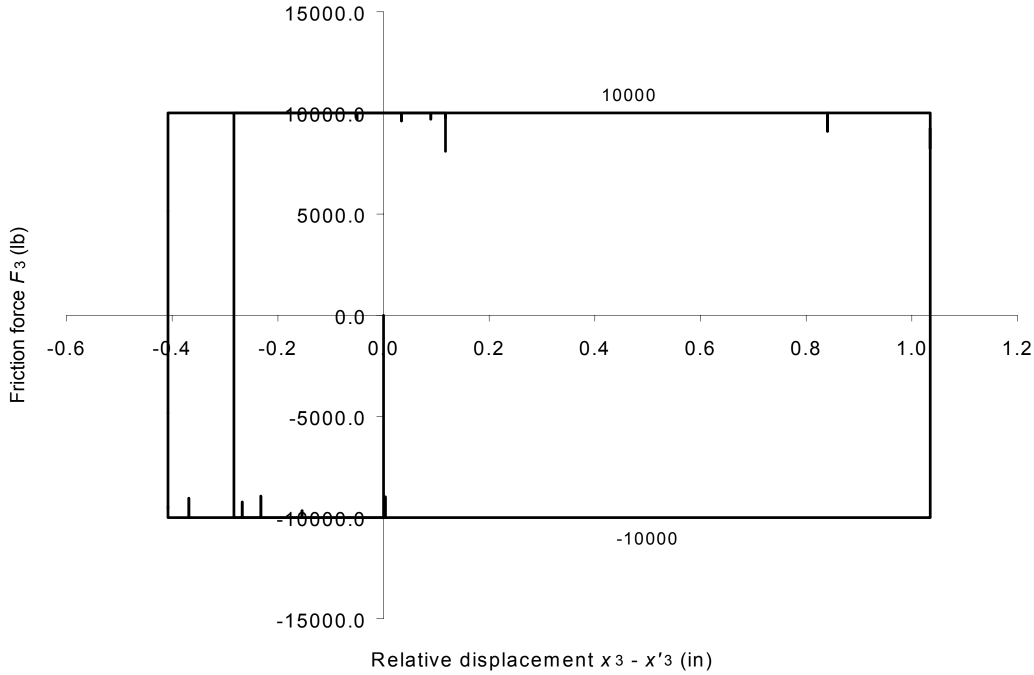


Figure 4.9 Third floor hysteresis loops of the MSBFD for the pulse loading

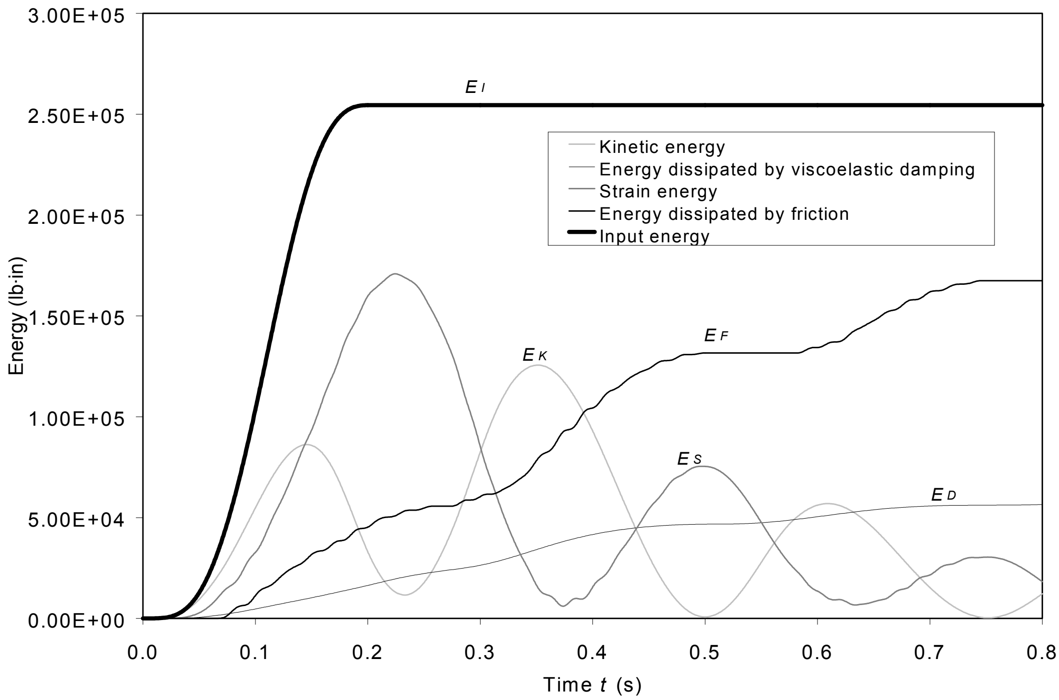


Figure 4.10 Energy response of the MSBFD of Fig. 4.6 for the pulse loading

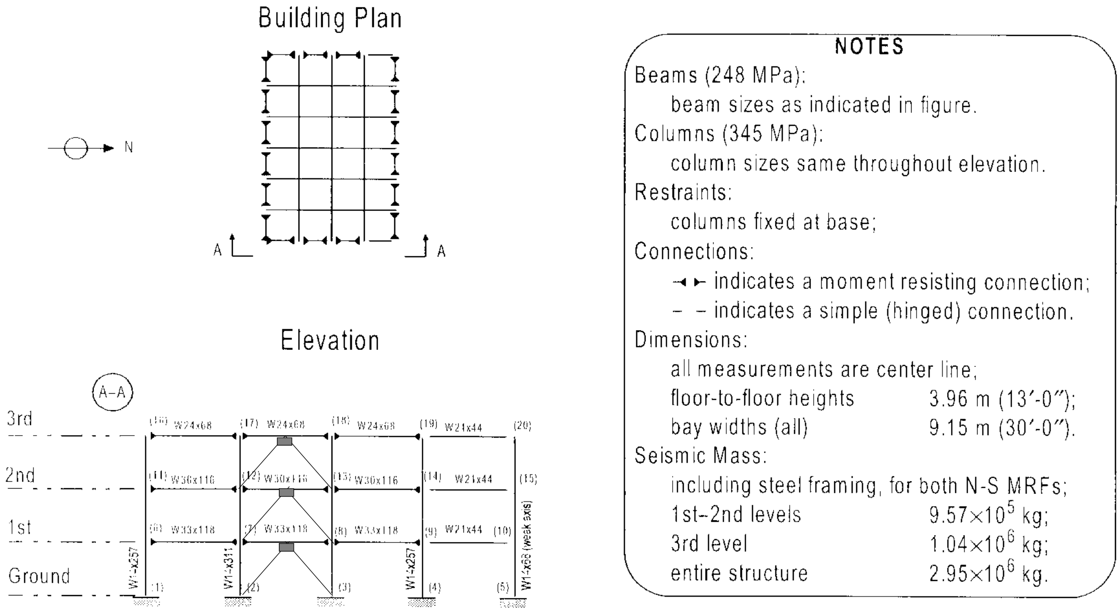


Figure 4.11 3-story building classified as a bench mark problem

The modal damping ratios of the bare frame are, respectively, $\xi_1 = 0.02$, $\xi_2 = 0.01451$ and $\xi_3 = 0.02$. For the bracing system and FDs, the damping ratios were considered zero.

The values of $\mathbf{M}^{ss}$, $\mathbf{C}^{ss}$, $\mathbf{K}^{ss}$, $\mathbf{M}^{dd}$, $\mathbf{C}^{da}$, $\mathbf{C}^{db}$, $\mathbf{C}^{dc}$, $\mathbf{K}^{da}$, $\mathbf{K}^{db}$, and $\mathbf{K}^{dc}$ are:

$$\mathbf{M}^{ss} = \begin{bmatrix} 4.78 & 0 & 0 \\ 0 & 4.78 & 0 \\ 0 & 0 & 5.18 \end{bmatrix} \times 10^5 \text{ kg}$$

$$\mathbf{C}^{ss} = \begin{bmatrix} 509.253 & -221.488 & 38.679 \\ -221.488 & 394.427 & 120.313 \\ 38.679 & 120.313 & 197.705 \end{bmatrix} \text{kN·s/m}$$

$$\mathbf{K}^{ss} = \begin{bmatrix} 436.575 & -237.345 & 41.445 \\ -237.345 & 313.526 & -128.930 \\ 41.445 & -128.930 & 93.585 \end{bmatrix} \text{MN/m}$$

$$\mathbf{T}^{ss} = \begin{bmatrix} 1.0101 \\ 0.3268 \\ 0.1715 \end{bmatrix} \text{s}$$

$$\mathbf{M}^{dd} = \begin{bmatrix} 350.11 & 0 & 0 \\ 0 & 350.11 & 0 \\ 0 & 0 & 350.11 \end{bmatrix} \text{kg}$$

$$\mathbf{C}^{da} = \mathbf{C}^{db} = \mathbf{C}^{dc} = \mathbf{C}^{dd} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{K}^{da} = \begin{bmatrix} 45.664 & 0 & 0 \\ 0 & 45.664 & 0 \\ 0 & 0 & 45.664 \end{bmatrix} \text{MN/m}$$

$$\mathbf{K}^{db} = \begin{bmatrix} 45.664 & 0 & 0 \\ 0 & 45.664 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MN/m}$$

$$\mathbf{K}^{dc} = \begin{bmatrix} 0 & -45.664 & 0 \\ 0 & 0 & -45.664 \\ 0 & 0 & 0 \end{bmatrix} \text{MN/m}$$

In this case, all $P_i(t) = 0$ and the values of $\ddot{x}_g(t)$ correspond to the accelerogram of the Northridge earthquake (Santa Monica station, Calif., 90° component, January 17, 1994), shown in Fig. 3.13. Since no amplification factor was considered, $PGA = 1.742g$. In this case a time increment of 0.00125 s was used and the total time of analysis was 10 s. The prescribed tolerances were $\varepsilon_a = 0.01$ cm/s^2 and $\varepsilon_f = 1.0$ N.

The sliding thresholds for the three dissipators are, respectively, $\mu_1 N_1 = 834.095$ kN, $\mu_2 N_2 = 740.831$ kN and $\mu_3 N_3 = 43.510$ kN.

4.4.2.2 Results

The hysteresis loops of the third floor is shown in Fig. 4.12. As expected, such loops are rectangular.

The energy time-histories of the entire structure are shown in Fig. 4.13.

4.4.3 Ground acceleration on a 10-story building

4.4.3.1 Description of the structure

This last analyzed structure consists of a 10-story building equipped with FDs in each floor. The values of matrices $\mathbf{M}^{ss}$, and $\mathbf{K}^{ss}$ can be found in [46]. Matrix $\mathbf{C}^{ss}$ was calculated considering all $\xi_i = 0.02$. Matrices $\mathbf{M}^{dd}$, $\mathbf{C}^{da}$, $\mathbf{C}^{db}$, $\mathbf{C}^{dc}$ were considered null; $\mathbf{K}^{da}$, $\mathbf{K}^{db}$, and $\mathbf{K}^{dc}$ were determined using the coefficients given in [46]. The values of the sliding thresholds,

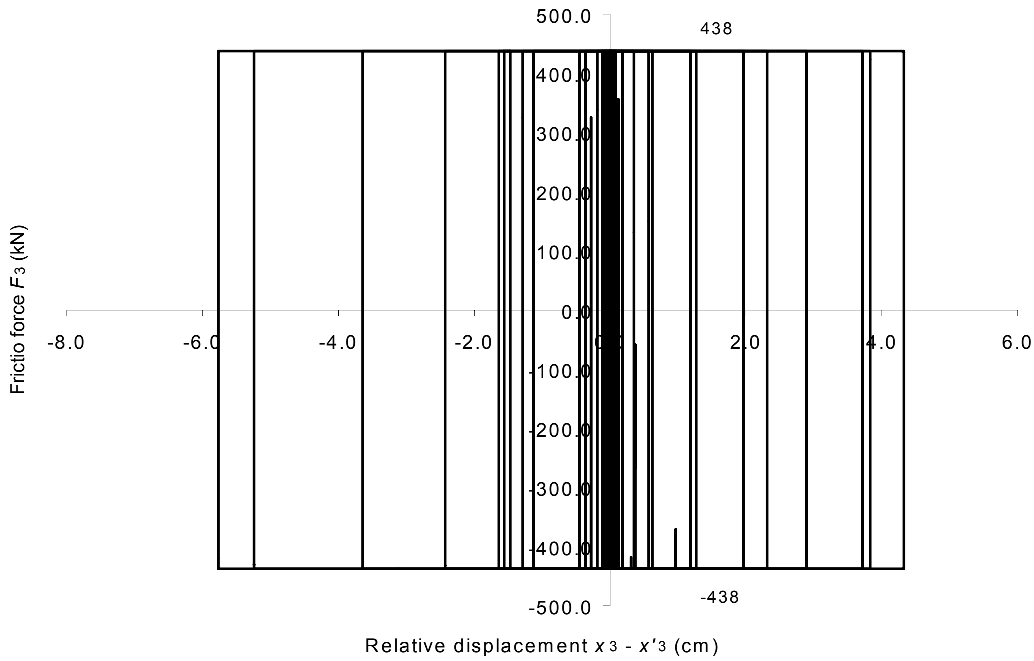


Figure 4.12 Third floor hysteresis loops of the benchmark building for a seismic input

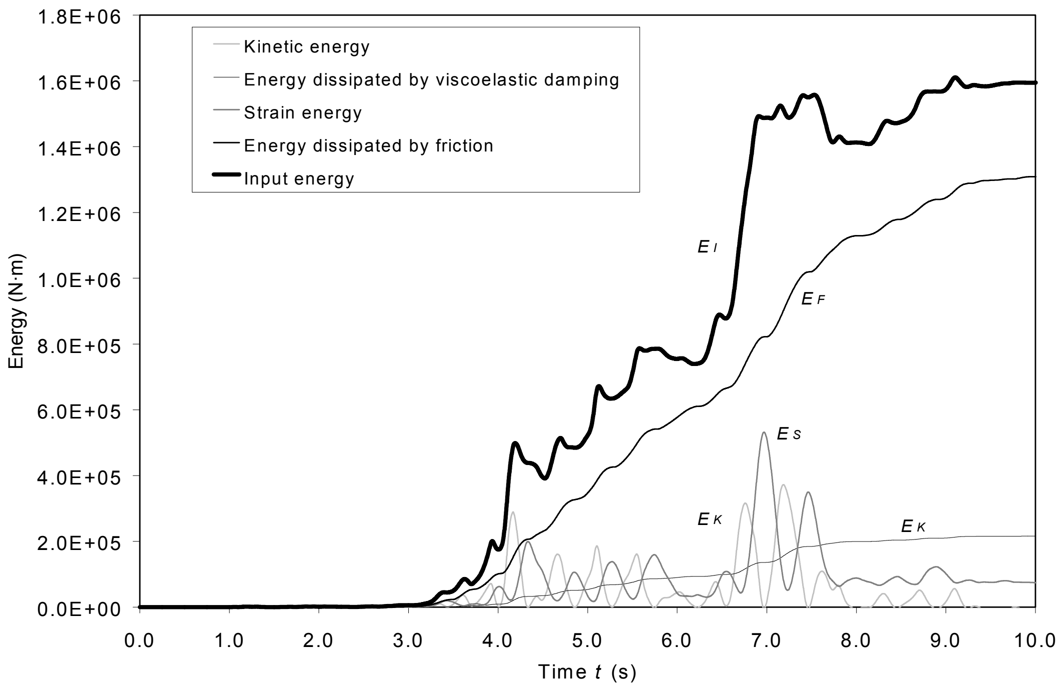


Figure 4.13 Energy response of the benchmark building for a seismic input

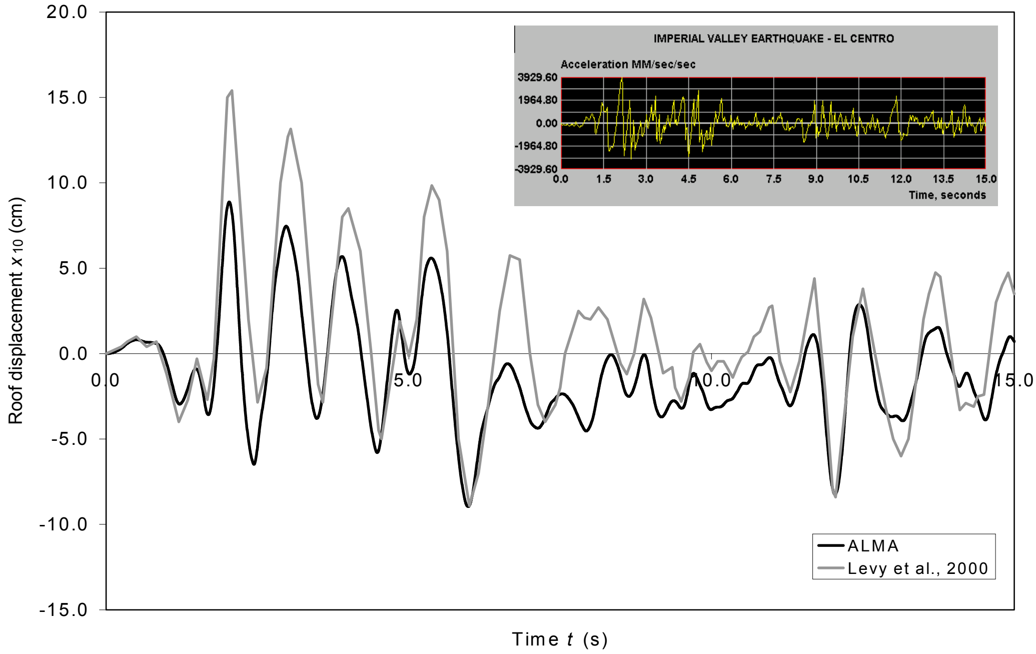


Figure 4.14 Displacement response of a 10-story building equipped with FDs and subjected to El Centro earthquake

$\mu_i N_i$, are also given in [46]. The tolerances used were, again, $\varepsilon_a = 0.01$ cm/s^2 and $\varepsilon_f = 1.0$ N.

The seismic input used in this case was the El Centro earthquake, N-S component, $PGA = 0.40g$. The time increment was 0.00125 s and the total time of analysis was 15 s.

4.4.3.2 Results

Fig. 4.14 shows a comparison between the top floor response obtained using ALMA and the one displayed in [46]. The difference between both plots might be due to the different models employed, since in [46] a elastic-perfectly plastic law was used. Nevertheless, some similarities can be observed.

4.5 Comparison between ALMA and ADINA

4.5.1 Agreement of results

As in the case of the SSBFD, in this section a comparison of results between the proposed algorithm (ALMA program) and ADINA is presented. This comparison is made with respect to the displacements of each floor relative to the ground.

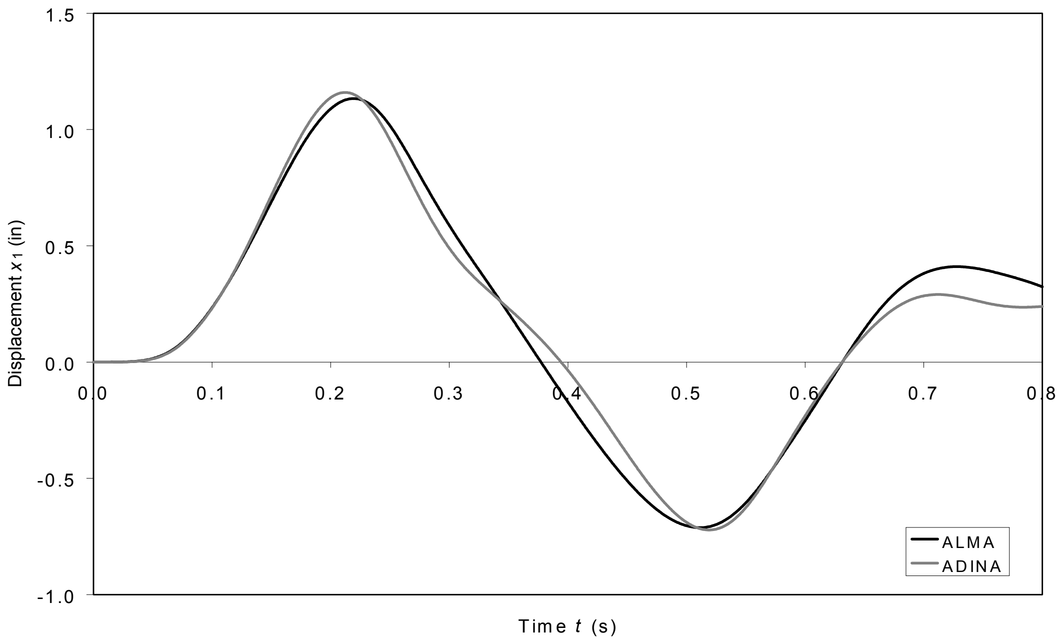


Figure 4.15 Comparison of first floor displacements between ALMA and ADINA for the pulse loading

In Figs. 4.15, 4.16 and 4.17 the interstory drift response for the pulse loading are presented, while and in Figs. 4.18, 4.19 and 4.20 the interstory drift response for the ground acceleration are shown. In each figure, the black line corresponds to the results obtained using ALMA, while the grey line shows the results obtained with ADINA. The agreement between both programs is good.

4.5.2 Computational efficiency

A comparison between the performances of ALMA and ADINA has been carried out. In virtually all of the cases significant differences have been found showing that ALMA is faster and requires less memory allocation. A description of an illustrative example is described next. The structure considered for the simulation is the benchmark building of Fig. 4.11. The building is subjected to a seismic input of 15 s of duration. The main features of the computer machine used in this test are: Pentium II processor at 233 MHz and 96 MB RAM. In order to get comparable results no additional software has been running at the same time. For both programs, the lengths of the discretization periods have been chosen as long as possible to obtain enough stability and accuracy. For ADINA the CPU time was 15 minutes and 500 MB HD memory was required, while for ALMA the CPU time was 3 minutes 20 seconds and no significant memory allocation was necessary. For further details, the system

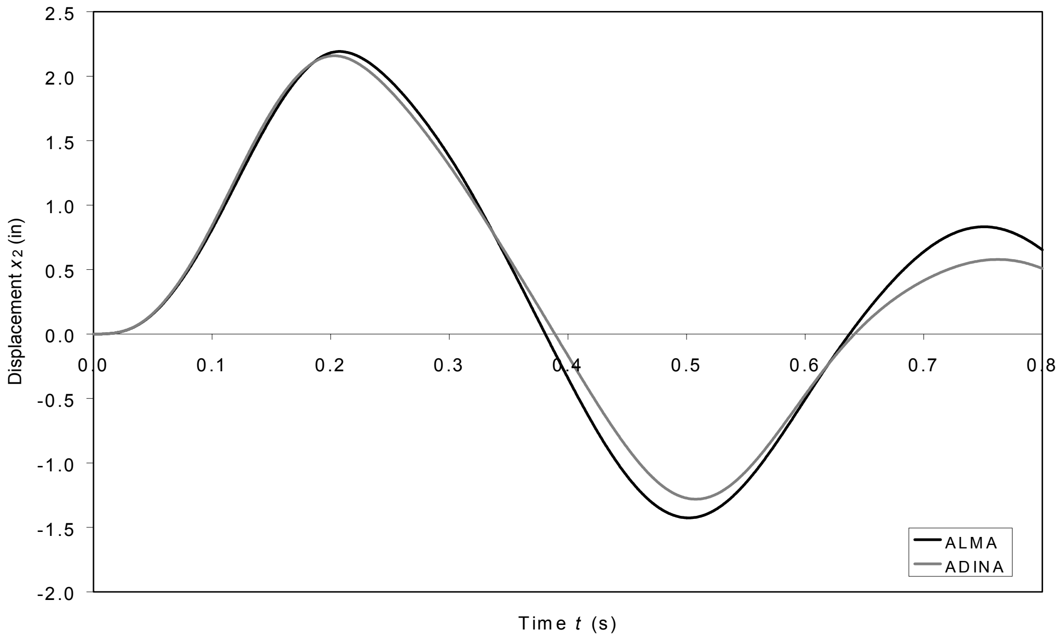


Figure 4.16 Comparison of second floor displacements between ALMA and ADINA for the pulse loading

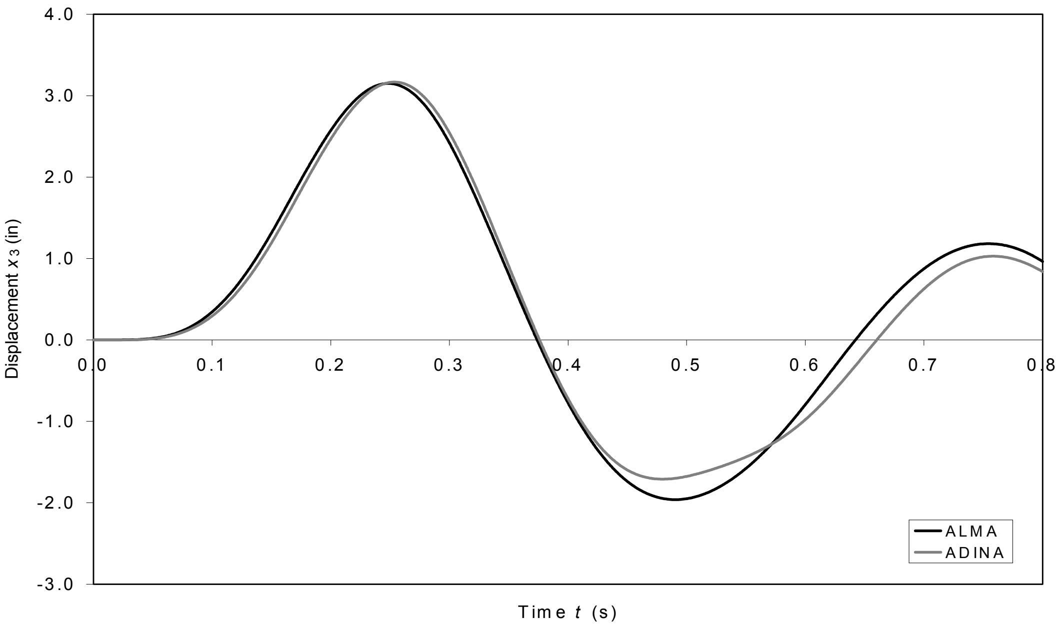


Figure 4.17 Comparison of third floor displacements between ALMA and ADINA for the pulse loading

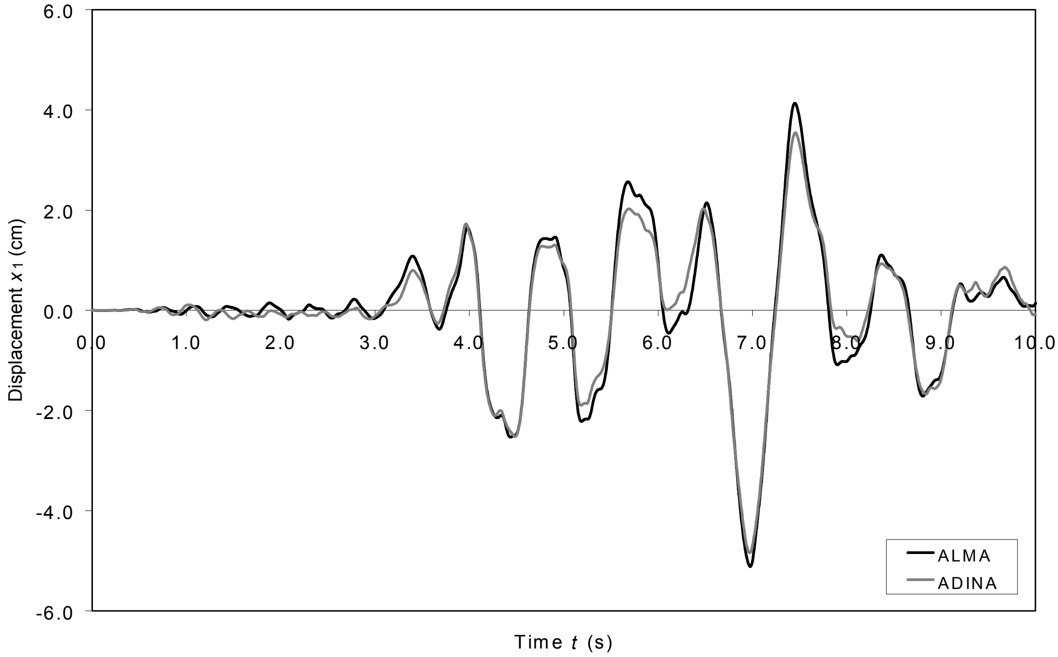


Figure 4.18 Comparison of first floor displacements between ALMA and ADINA for the Northridge earthquake

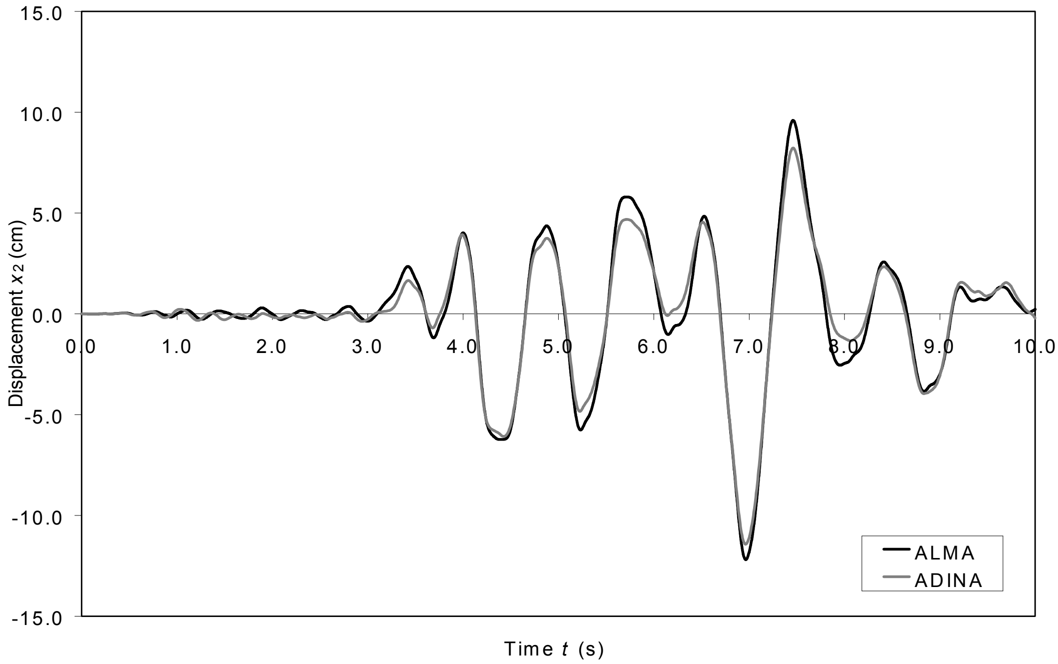


Figure 4.19 Comparison of second floor displacements between ALMA and ADINA for the Northridge earthquake

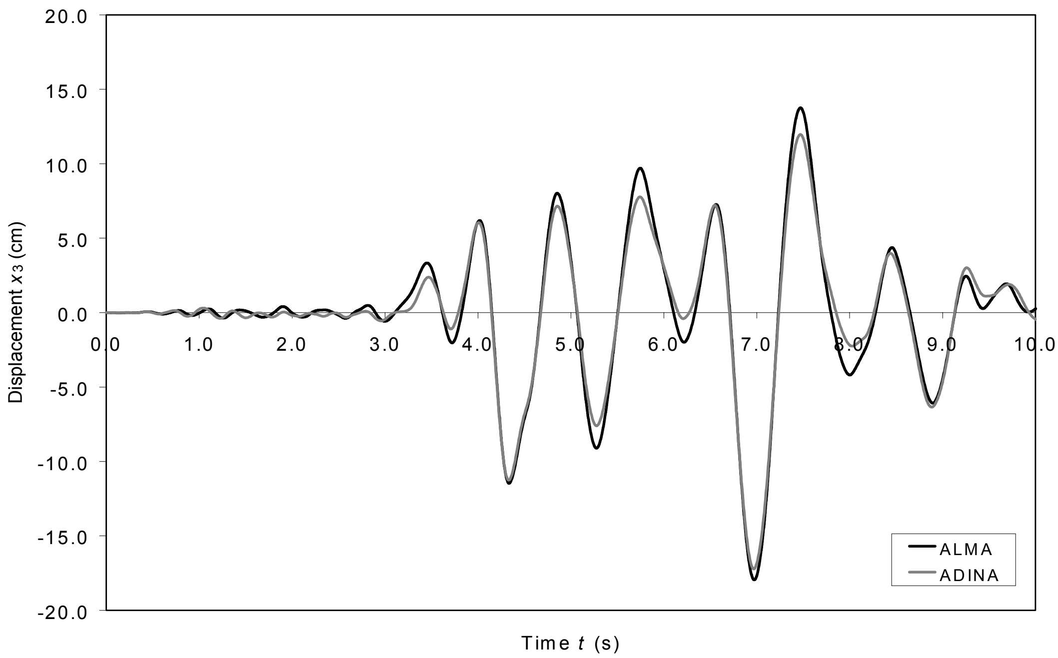


Figure 4.20 Comparison of third floor displacements between ALMA and ADINA for the Northridge earthquake

monitors for both programs are displayed in Fig. 4.21.

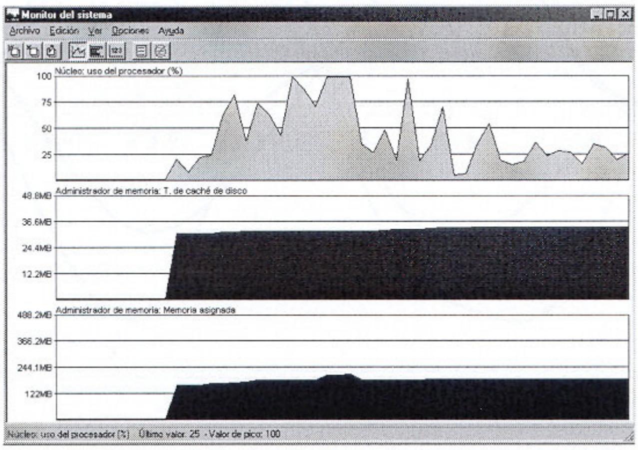
4.6 Efficiency of Friction Dissipators

As seen in Chapter 3, FDs are able to reduce the dynamic response of building structures subjected to lateral loads. For example, for the case of the pulse loading above studied, Figs. 4.22, 4.23 and 4.24 show comparisons between the displacements of the MSBFD and those of the bare frame (MSB with all $\mu_i N_i = 0$). The black lines correspond to the MSBFD (protected frame) inter-story drifts and the grey ones show the bare frame responses.

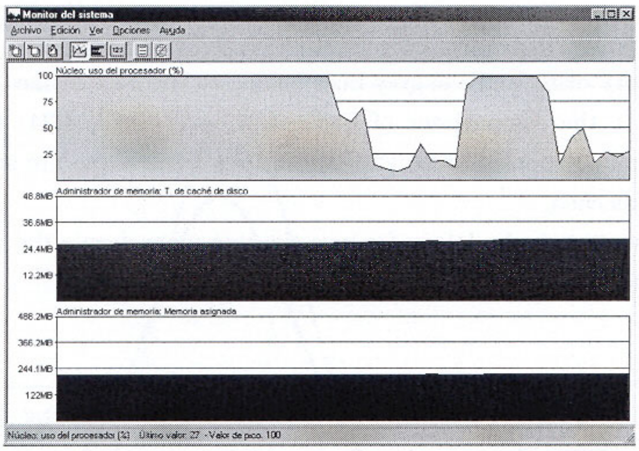
For the case of the seismic input, Figs. 4.25, 4.26 and 4.27 show comparisons between the displacements of the MSBFD and those of the bare frame. Again, the black lines belong to MSBFD (protected frame) interstory drifts and the grey ones belong to the bare frame responses.

As illustrated in the comparisons of Figs. 4.22, 4.23, 4.24, 4.25, 4.26 and 4.27, a significant reduction of the dynamic response of building structures subjected to lateral vibrations can be reached. More numerical analyses have been carried out, yielding similar conclusions.

Chapter 6 presents a methodology to investigate more deeply the capacity of friction dissipators to reduce the seismic response of buildings.



(a) System monitor for ALMA



(b) System monitor for ADINA version 7.5

Figure 4.21 Comparison between the system monitors for ALMA and ADINA

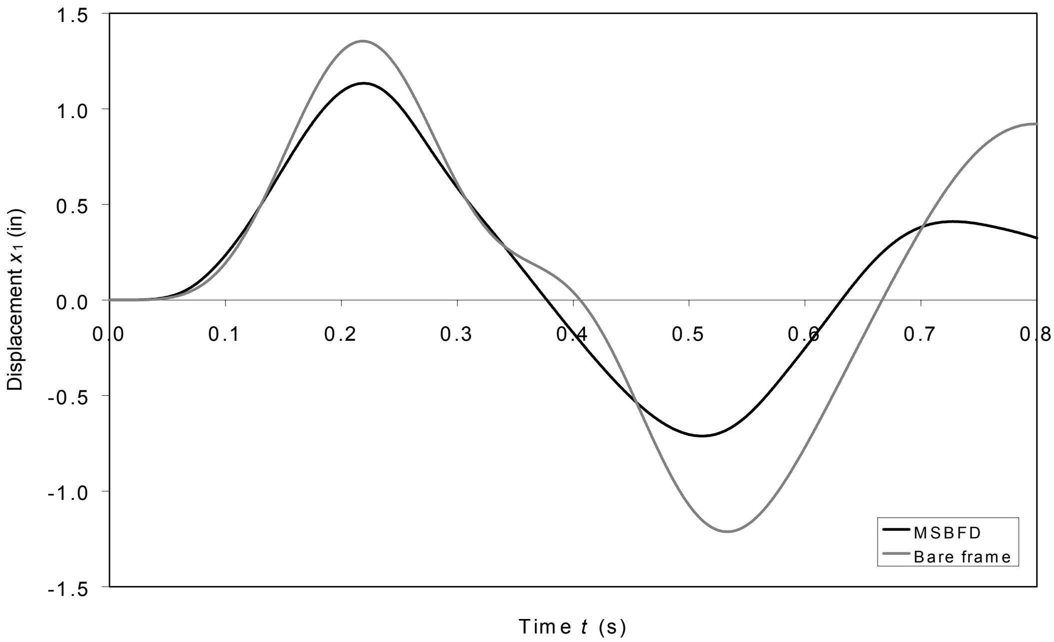


Figure 4.22 Comparison of x_1 displacements between the MSBFD and a bare frame for the pulse loading

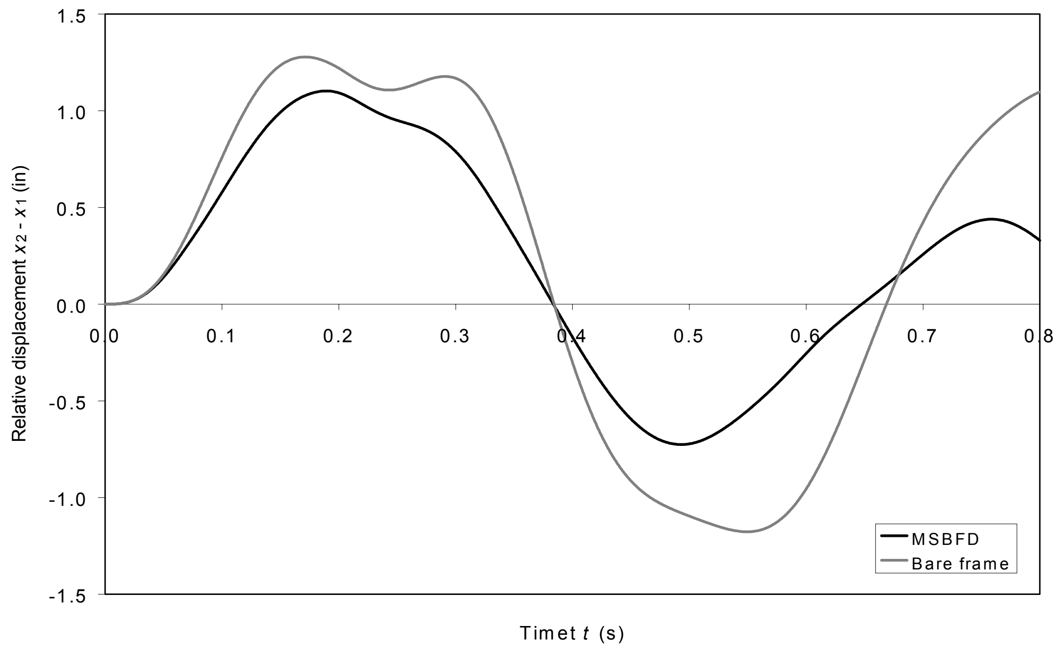


Figure 4.23 Comparison of relative displacements $x_2 - x_1$ between the MSBFD and a bare frame for the pulse loading

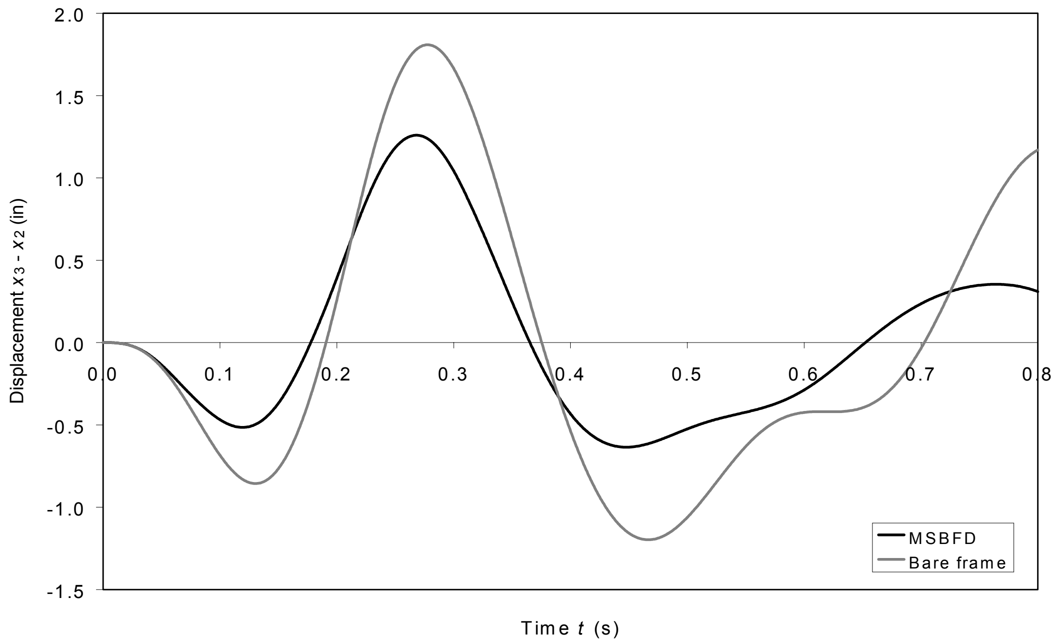


Figure 4.24 Comparison of relative displacements $x_3 - x_2$ between the MSBFD and a bare frame for the pulse loading

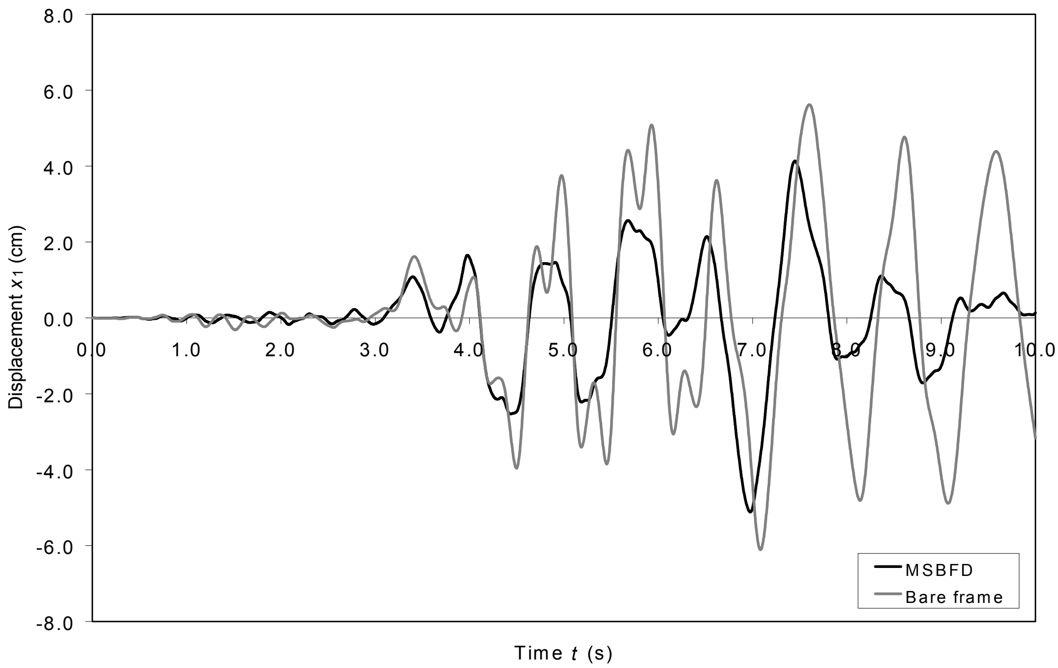


Figure 4.25 Comparison of displacements x_1 between the MSBFD and a bare frame for the Northridge earthquake

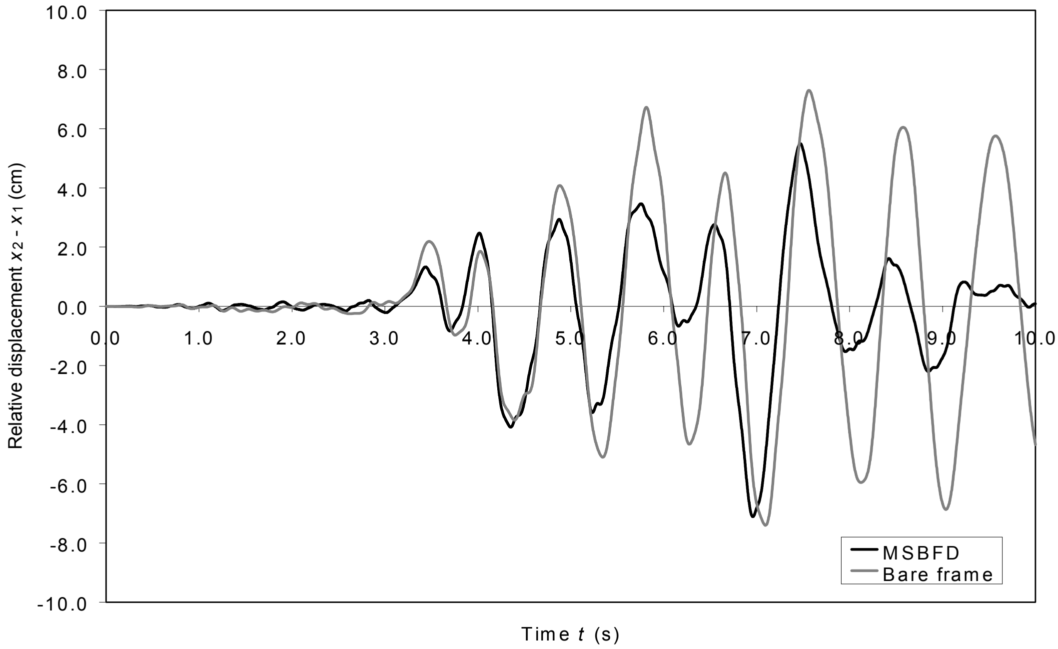


Figure 4.26 Comparison of relative displacements $x_2 - x_1$ between the MSBFD and a bare frame for the Northridge earthquake

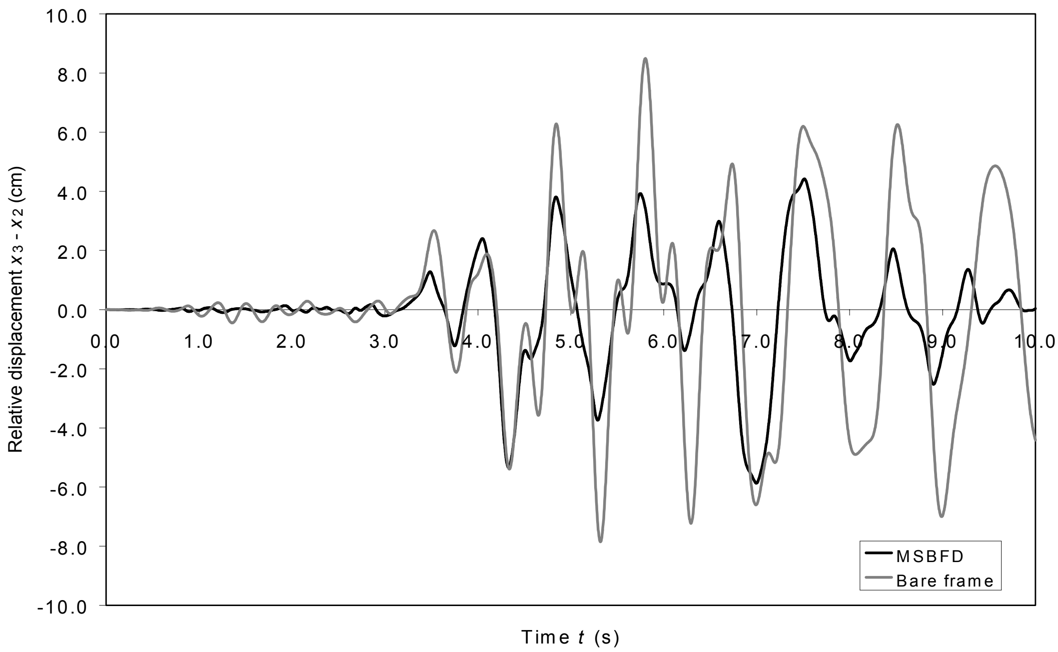


Figure 4.27 Comparison of relative displacements $x_3 - x_2$ between the MSBFD and a bare frame for the Northridge earthquake