UNIVERSITAT POLITÈCNICA DE CATALUNYA

Department of Chemical Engineering

ENERGY OPTIMISATION AND CONTROLLABILITY IN COMPLEX DISTILLATION COLUMNS

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CHAPTER 7. CONTROLLABILITY OF THE DIVIDED WALL COLUMN DEPENDING ON DESIGN

7.1 Abstract

In this chapter, the influence of the DWC design over its controllability is studied. The different designs contemplated are designs with different number of trays in the column sections. Firstly, two designs with stages optimally distributed between sections and different total number of stages are compared. Secondly, based on an optimal design, two non-optimal designs are studied to analyse the effect of transferring separation stages from the upper part of the main column to the lower part of the main column and inversely. In the same way, a third non-optimal design is studied to analyse the effect of transferring separation stages from the prefractionator to the main column. The study is done for a specific distillation example but the reasons of the process behaviour are discussed, which are useful to understand the general DWC behaviour.

7.2 Introduction

A plant is controllable if there is a controller that yields acceptable performance for all expected plant variations. Therefore, controllability is independent of the controller and is a property of the plant. Controllability can only be affected by changing the plant itself, that is, by design modifications (Skogestad et al., 1996).

For the DWC, some controllability works have been published (see references in chapter four). However, controllability changes derived from design modifications have not been analysed. This is the objective of this chapter.

In section 2.6, an optimisation procedure for the DWC design was described. One of the main objectives of this chapter is to know whether or not the optimal DWC design has good controllability properties compared to other designs.

In chapter four, the DWC controllability was studied. In this chapter, the same tools described and used in chapter four for controllability analysis are used.

7.3 Comparison between two optimal designs

The optimisation procedure described in 2.6.2 is based on minimum reflux conditions. Different designs are obtained depending on the used *RR/MRR*. In chapter three it was concluded that, relative to other distillation arrangements, the DWC is more economically favourable when long columns (obtained using small *RR/MRR*) are considered. In this section, the controllability of two DWC designs solving the same distillation problem is compared. Both designs are optimised using the design method described in section 2.6.2. One of the designs has been obtained using *RR/MRR*=2 and the other has been obtained using *RR/MRR*=1.23. The design with *RR/MRR*=2 has fewer trays than the design with *RR/MRR*=1.23.

The distillation problem to be solved is the separation (described in 4.11) of a mixture with relative volatilities α=(4.65:2.15:1) into 0.99 pure products. Feed is 1 kmol/min of equimolar saturated liquid. Products are saturated liquid streams, too. The design with fewer trays (*RR/MRR*=2) has *NT*=46, *NP*=13, *NM*=33, *NS*=17, *NCB*=8, *NCD*=26, and *NF*=7. The design with more trays (*RR/MRR*=1.23) has *NT*=58, *NP*=18, *NM*=40, *NS*=21, *NCB*=10, *NCD*=31 and *NF*=9.

The nominal operation of both separation processes has been optimised. Steady state nominal operations are described in Table 7.1. In the column with more trays, *L/D*=3.98 and *V/B*=4.98. In the column with fewer trays, *L/D*=6.93 and *V/B*=7.92. Energy savings of the column with more trays are of 42%.

Table 7.1:.Nominal operation of the compared processes

	Design with 58 trays	Design with 46 trays
Reflux rate (kmol/min)	1.335	2.322
Boilup (kmol/min)	1.670	2.657
Distillate flowrate (kmol/min)	0.335	0.335
Bottoms flowrate (kmol/min)	0.335	0.335
Side stream flowrate (kmol/min)	0.330	0.330
SPLITD	0.634	0.634
SPLITB	0.424	0.500
Holdup in reboiler and reflux drum (kmol)	10	10
Holdup in the rest of trays (kmol)	0.5	0.5

For the design with fewer trays, the maximum eigenvalue of the state matrix A is -0.0042 rad/min, and for the design with more trays, the maximum eigenvalue of the state matrix A is -0.0040 rad/min. These values indicate similar main open-loop time constants. However, the main open-loop time constant for the column with more trays (1/0.0040=150 min) is a bit larger than the main open-loop time constant for the column with fewer trays (1/0.0042=138 min).

7.3.1 Inventory control "DB"

In this section, the two DWC designs with "DB" inventory control are compared. "DB" is the typical stabilisation for isolated columns and the stabilisation considered in most of the literature. According to the results in 4.7.4.1, with "DB" stabilisation, the tuning of the inventory control loops does not influence the controllability of the system.

The possible manipulated variables for the composition control are *L*, *V*, *S*, *SPLITD*, and *SPLITB*. According to *MRI*, *CN*, and *II*, the two designs have the same preferred control structure, which is *L V S*. The dependence of this result on the frequency at what the singular value decomposition is analysed is small. For the design with more trays, *MRI* and *CN* indicate *L V S* as the best structure at least from s=0.001 to s=0.05. At higher frequencies, structures implicating *SPLITD* or *SPLITB* have better controllability indexes. For the design with fewer trays, *L V S* is the best structure from s=0.001 to s=0.1. At higher frequencies, structures implicating *SPLITD* or *SPLITB* have better controllability indexes.

For the column with more trays, the preferred structure indicated by the controllability indexes at w<0.05 rad/min is *L V S*, and the preferred structure at w>0.05 rad/min is *L S SPLITD*. Evaluation at the bandwidth frequency (w_B) is the interesting one. Therefore, if w_B<0.05 rad/min, *L V S* is preferred and if w_B>0.05 rad/min, *L S SPLITD* is preferred. The attainable bandwidth depends on the control structure and the tuning. It has been seen that for control structures, and different tunings, w_B<0.05. Therefore, *L V S* is the preferred structure. Similarly, the column with fewer trays has bandwidths lower than 0.1 rad/min, independently of the control structure and tuning, and therefore, *L V S* is the preferred control structure.

At w=0.04 rad/min, and for *L V S* control structure, the design with more trays has better controllability indexes than the design with fewer trays. The design with more trays has *MRI*=0.54 and *CN*=25. The design with fewer trays has *MRI*=0.26 and *CN*=91. But *MRI* and *CN* values depend a lot on the frequency at what the singular value decomposition is analysed. It is important to analyse these indexes at the bandwidth frequency and the bandwidth of the columns being compared may differ.

As was already seen in chapter four, for "DB" inventory control, at low frequencies, *CN* is very high. However, *CN* decreases at higher frequencies.

PI controllers are assumed in each composition control loop. Since $\boldsymbol{S}$ depends on the tuning, the bandwidth frequency can be modified changing the tuning. Although in MIMO domain it can not be assured that the bandwidth frequency will increase increasing any of the proportional constants K_c of the PI controllers and/or decreasing any of the integral time constants τ_c of the PI controllers, through the analysis of a large set of tunings, it has been observed that this is the bandwidth behaviour.

Consider that *CN* at the bandwidth has to be smaller than 100 to be acceptable. For the column with fewer trays, the *CN* is lower than 100 only for frequencies larger than 0.04 rad/min. No tuning has been found such that a bandwidth of 0.04 rad/min is obtained and the control system is stable for the rejection of a disturbance in the feed flowrate. In the case of the column with more trays, *CN* lower than 100 is obtained for frequencies larger than 0.0085 rad/min, and tunings giving this bandwidth frequency and stable control for the rejection of different disturbances have been found. This result indicates a superior controllability of the column with more trays.

With the same scaled tuning, shown in Table 7.2, the column with more trays has a bandwidth of 0.0035 rad/min and the column with fewer trays has a bandwidth of 0.0014 rad/min. This would indicate a natural ability of the column with more trays to have a larger bandwidth, which is a good property. The controllability indexes of the two columns at their bandwidth frequencies are compared and the ones of the column with fewer trays are worse, due basically to a very high *CN*.

A preliminary analysis of stability through the maximum singular value of $\boldsymbol{T}$ indicates that both columns have similar stability margins. With the tuning in Table 7.2, both columns have a peak of 2.3. However, through the maximum singular value of $wi^*\boldsymbol{Ti}$, the column with more trays is found to be more robust because of a lower peak. With the tuning in Table 7.2, the peak for the column with more trays is 0.29 and the peak for the column with fewer trays is 0.9. This confirms that the column with more trays permits naturally higher bandwidths and smaller *CN* because this is so for similar (or better) robustness. Therefore, the controllability of the column with more trays is superior to the controllability of the column with fewer trays.

Table 7.2: Scaled tuning for the L S V control structure

	K_c	τ_c
Loop 1: $(L\text{-}x_{AD})$	0.518	83 min
Loop 2: $(S\text{-}x_{BS})$	-1.999	83 min
Loop 3: $(V\text{-}x_{CB})$	0.5	81 min

For the column with more trays with the tuning in Table 7.2, simulation results for the setpoint tracking of +0.001 in purity of A and C are shown in Figures 7.1 and 7.2.

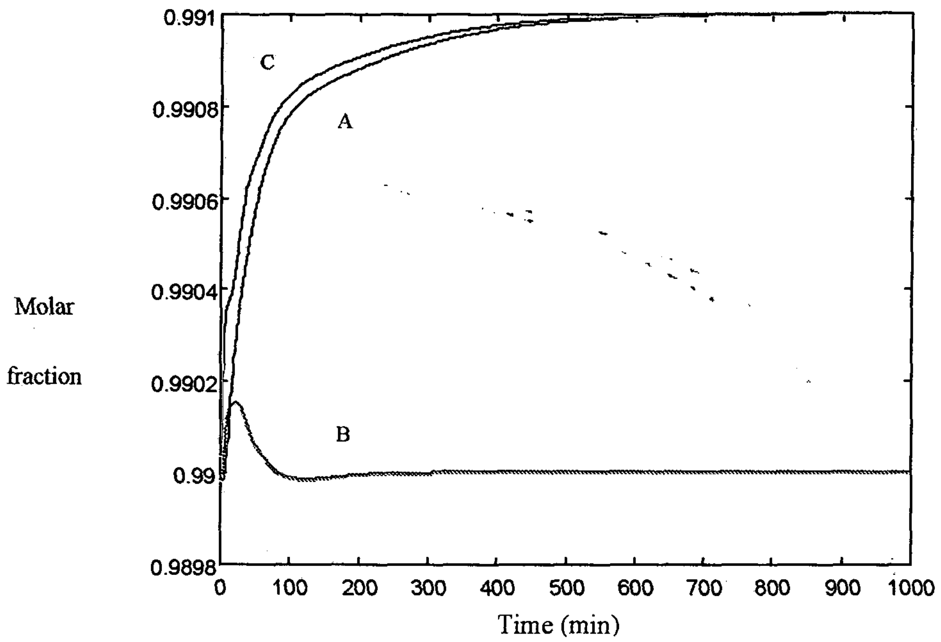

Figure 7.1: Output profiles for a disturbance in F (column with more trays)

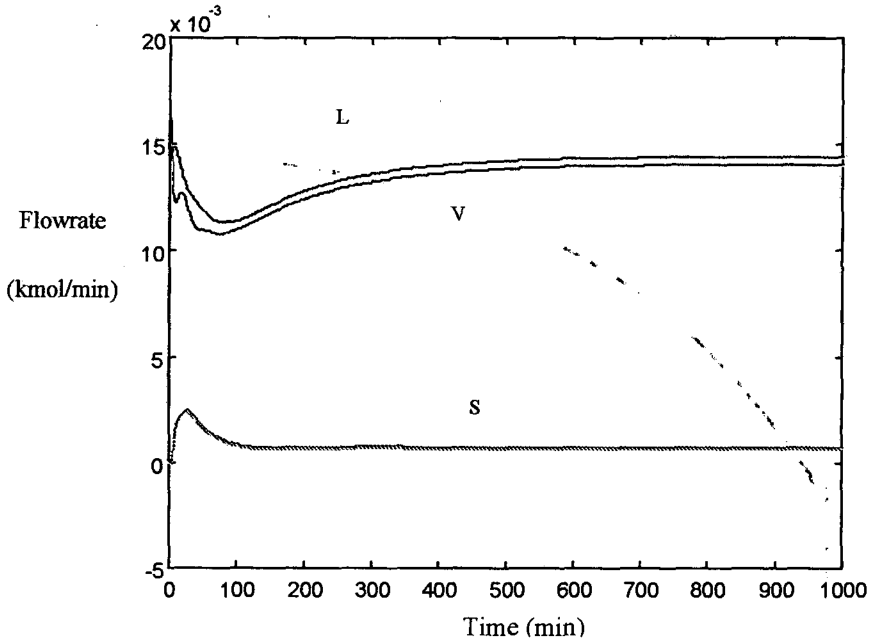

Figure 7.2: Input profiles for a disturbance in F (column with more trays)

For the column with fewer trays with the tuning in Table 7.2, simulation results for the setpoint tracking of +0.001 in purity of A and C are shown in Figures 7.3 and 7.4. Comparing the output profiles for both designs (Figures 7.1 and 7.3), it is seen that the column with more trays achieves the setpoints much faster. Comparing the input profiles, it is seen that the column with fewer trays needs much longer increases in *L* and *V* input variables.

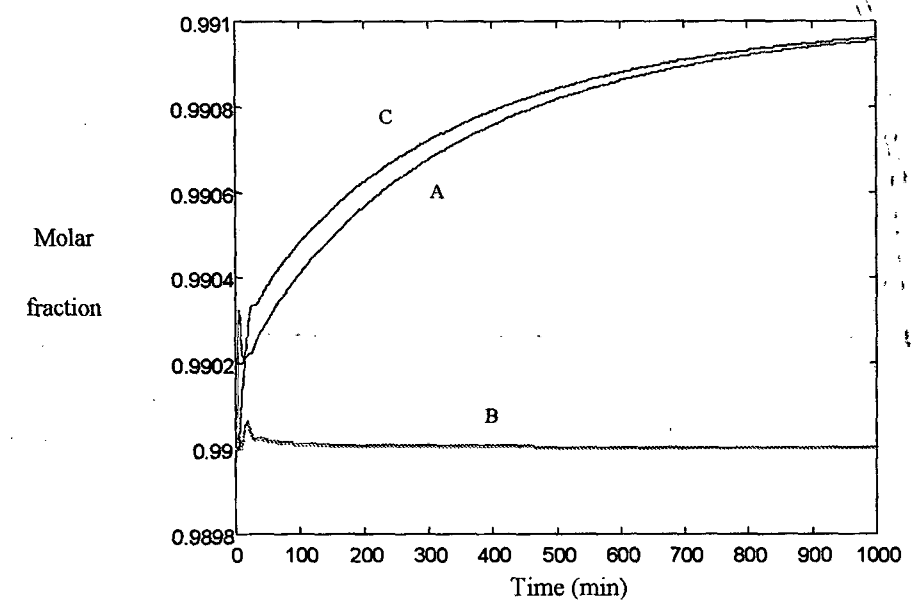

Figure 7.3: Output profiles for a disturbance in *F* (column with fewer trays)

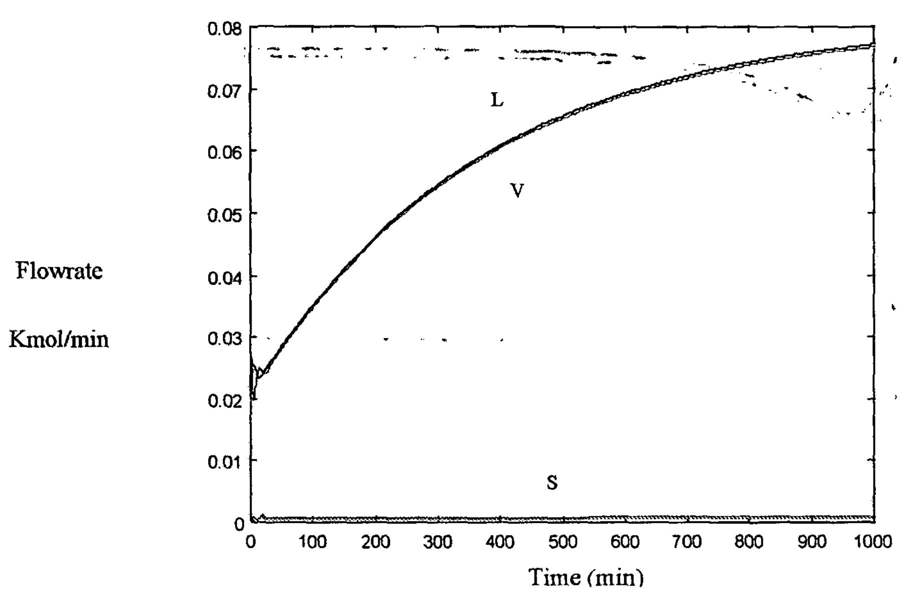

Figure 7.4: Input profiles for a disturbance in *F* (column with fewer trays)

In chapter four, it was already seen that high *CN* in the DWC is a problem associated to "DB" inventory control. High *CN* means that gain in the direction of maximum gain and gain in the direction of minimum gain are very different. The directions of maximum and minimum gains are obtained with the singular value decomposition. The singular value decomposition of $\boldsymbol{G}$ can be seen in equation 7.1. At steady state, $\boldsymbol{u}$ and $\boldsymbol{v}$ matrixes are real and its columns can be interpreted as directions in the output and input spaces (Skogestad et al., 1996).

$$\boldsymbol{G}=\boldsymbol{u}\sigma\boldsymbol{v}^H \qquad (7.1)$$

For the design with more trays and *L S V* control structure,

$$\boldsymbol{u}=\begin{pmatrix} 0.74 & -0.66 & 0.15 \\ 0.07 & 0.30 & 0.95 \\ -0.67 & -0.69 & 0.27 \end{pmatrix} \quad \sigma=\begin{pmatrix} 144 & 0 & 0 \\ 0 & 11.7 & 0 \\ 0 & 0 & 0.69 \end{pmatrix} \quad \boldsymbol{v}=\begin{pmatrix} 0.62 & -0.06 & 0.78 \\ -0.07 & -0.99 & -0.02 \\ -0.78 & 0.04 & 0.62 \end{pmatrix} \qquad (7.2)$$

As indicated by the singular value decomposition in 7.2, the input direction with higher gain is (0.62, -0.07, -0.78), with a gain of 144 and an output direction of (0.74, 0.07, -0.67). The input direction with lower gain is (0.78, –0.02, 0.62), with a gain of 0.69 and an output direction of (0.15, 0.95, 0.27). These vectors are scaled. The corresponding non-scaled ones are: input direction with higher gain, (0.16 -0.004 -0.26), and input direction with lower gain, (0.20 -0.001 0.20). In the direction of lower gain, external flows (*D*, *S*, and *B*) do not practically change.

For the design with fewer trays, the input and output directions at steady state are similar to the ones for the design with more trays. However, there are some differences. The singular value decomposition is:

$$\boldsymbol{u}=\begin{pmatrix} 0.72 & -0.65 & 0.24 \\ 0.04 & 0.38 & 0.92 \\ -0.67 & -0.66 & 0.30 \end{pmatrix} \quad \boldsymbol{s}=\begin{pmatrix} 219 & 0 & 0 \\ 0 & 11.6 & 0 \\ 0 & 0 & 0.27 \end{pmatrix} \quad \boldsymbol{v}=\begin{pmatrix} 0.65 & -0.03 & 0.75 \\ -0.04 & -0.99 & -0.005 \\ -0.75 & 0.03 & 0.66 \end{pmatrix} \qquad (7.3)$$

The non-scaled input direction with higher gain is (0.30 -0.003 -0.39), and the non-scaled input direction with lower gain is (0.34 0.0003 0.35). Again, in the direction of lower gain, external flows do not practically change. This means that in the direction of lower gain, the gain is given by the reflux ratios. A larger gain for the column with more trays (0.69 compared to 0.27) indicates that in the column with more trays, reflux ratios influence more the compositions.

CN are lower at higher frequencies because initial responses do not respond to the external flow changes, which are the main responsible of directionality. Initial responses obey the changes in internal streams (reflux ratios) without the effect of the external streams (mass balance), and the gains due to internal streams are smaller to the ones due to the external streams in the direction of higher gain, and larger in the direction of lower gain.

For the column with more trays, *MRI* and *CN* of the 2x2 control subsystems are calculated at w=0.04 rad/min. The 2x2 control subsystems are L and V controlling x_{AD} and x_{CB}, L and S controlling x_{AD} and x_{BS}, and V and S controlling x_{BS} and x_{CB}. For the first pair of loops, *MRI*=0.78 and *CN*=17. For the second one, *MRI*=0.45 and *CN*=13. And for the third one, *MRI*=0.39 and *CN*=21. Directionality of all three pairs of loops is equally important, what indicates that the directionality problem in the DWC is not comparable to that of a simple distillation column. Specifically, *CN* of the third pair of loops is the highest one. In the lower part of the main column, a strong directionality exists. The gain is high when the external flow B change, and low when it does not change.

Gains due to L, S, and V increments are calculated for the column with more trays and the column with less trays with steps of 1%, 0.1% and 0.01% in the input variables. Different gain matrixes are obtained. However, it has been observed that the column with more trays has in general larger gains than the column with fewer trays.

The ***RGA*** analysis indicates that L-x_{AD}, S-x_{BS}, V-x_{CB} is the best pairing. For the column with more trays, the ***RGA*** diagonal elements are plotted in Figure 7.5. For the column with fewer trays, the ***RGA*** diagonal elements are plotted in Figure 7.6. Steady state ***RGA*** for the column with more trays and the column with fewer trays are respectively indicated in equations 7.4 and 7.5. It can be observed that the column with fewer trays has larger ***RGA*** values, what indicates more control difficulties. It can also be observed that ***RGA***(1,1) and ***RGA***(3,3) decrease with frequency, what indicates that the coupling between loops L-x_{AD} and V-x_{CB} is reduced when the frequency increases. In other words, coupling is less apparent in initial responses. This is due to the fact that initially, when L or V change, only the effect of changes in reflux ratios affects the crossed outputs (purity of C and A), and this effect is weaker than the effect of changes in the external flows, which appears later in time.

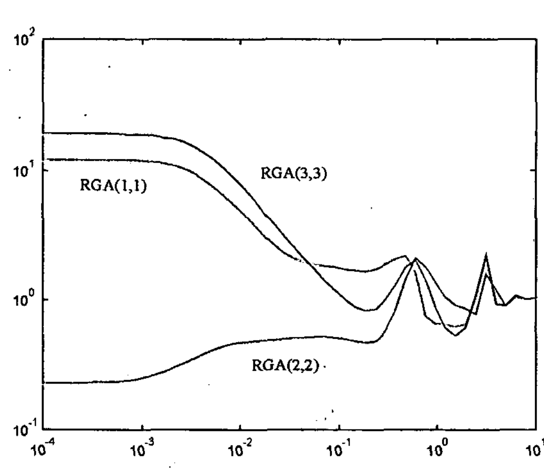

Figure 7.5: ***RGA*** of L S V for the column with more trays

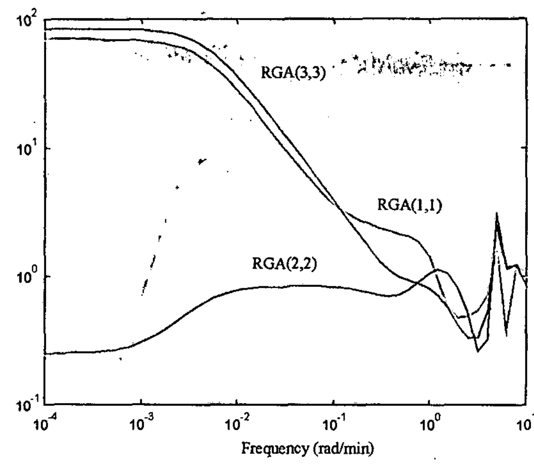

Figure 7.6: ***RGA*** of *L S V* for the column with fewer trays

$$\boldsymbol{RGA}(0)=\begin{pmatrix} 11.80 & 0.004 & -10.80 \\ 7.55 & 0.22 & -6.77 \\ -18.35 & 0.77 & 18.58 \end{pmatrix} \quad (7.4), \quad \boldsymbol{RGA}(0)=\begin{pmatrix} 67.64 & 0.010 & -66.65 \\ 15.16 & 0.24 & -14.40 \\ -81.80 & 0.74 & 82.06 \end{pmatrix} \quad (7.5)$$

At steady state, external flow changes are the main cause of interaction. Smaller ***RGA*** values for the column with more trays indicate less interaction. The reason is that, in the column with more trays, the relative importance of internal flow changes to external flow changes is larger. Dynamically, long columns favour decoupling. Introduction of trays between the products makes the travel of external flows effect longer. This retains the initial decoupling and improves the ***RGA***.

Notice that preferred structures and pairings are in accordance to the conclusions in chapter four for DWC operated at optimal conditions (see 4.11.1).

To know the control requirement of disturbances and setpoint changes, the ***CLDG*** and the ***PRGA*** for the two studied columns are analysed. In Figures 7.7 and 7.8, ***CLDG*** and ***PRGA*** elements of the column with more trays for output x_{AD} and all inputs are plotted. It can be observed that the ***CLDG*** for *F* disturbance is the largest ***CLDG*** element and the ***PRGA*** for *S* input is the largest ***PRGA*** element. These two curves are almost equal at low frequencies. Since the ***PRGA*** values at high frequencies are not important if tracking setpoints at high frequencies is not important, *F* is the most limiting variable. If $|Li|$ is larger than the ***CLDG*** of *F*, acceptable disturbance rejection and setpoint tracking is expected.

In Figures 7.9 and 7.10, ***CLDG*** and ***PRGA*** elements of the column with more trays for output x_{BS} and all inputs are plotted. The highest of all ***PRGA*** and ***CLDG*** elements is the ***CLDG*** for *F* disturbance. Therefore, if $|Li|$ is larger than the ***CLDG*** of *F*, acceptable disturbance rejection and setpoint tracking is expected. Smaller ***PRGA*** values in Figure 7.10 than in Figure 7.8 would indicate that setpoint tracking of output x_{BS} is easier than setpoint tracking of output x_{AD}.

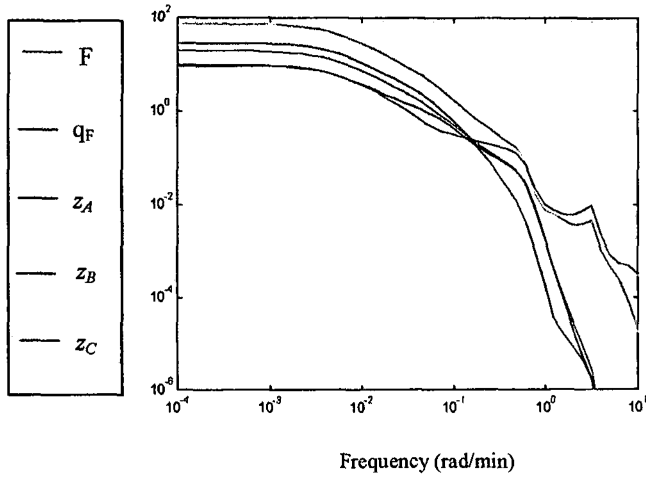

Figure 7.7: ***CLDG*** for output x_{AD} in the column with more trays

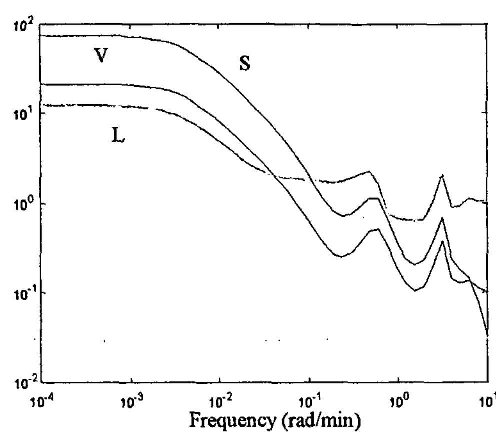

Figure 7.8: ***PRGA*** for output x_{AD} in the column with more trays

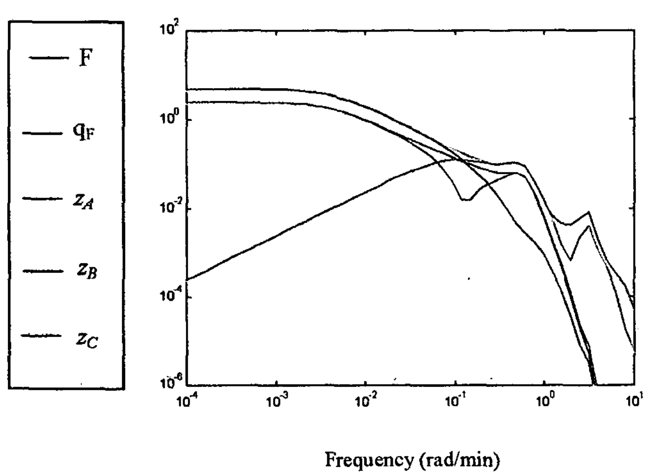

Figure 7.9: ***CLDG*** for output x_{BS} in the column with more trays

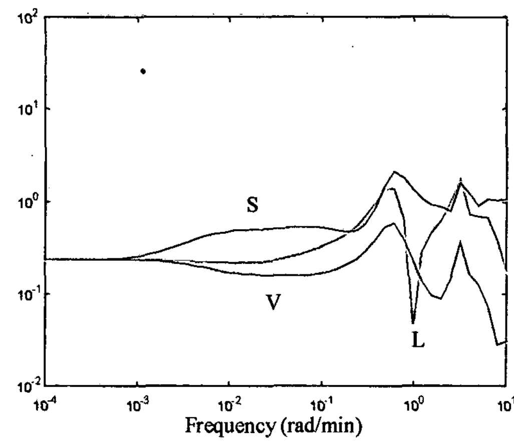

Figure 7.10: ***PRGA*** for output x_{BS} in the column with more trays

In Figures 7.11 and 7.12, ***CLDG*** and ***PRGA*** elements of the column with more trays for output x_{CB} and all inputs are plotted. F disturbance has again the highest ***CLDG***. Therefore, with a tuning tight enough to control F disturbances, the other disturbances and setpoint tracking should be controlled. Comparing ***PRGA*** of input S in Figures 7.8, 7.10, and 7.12, it is observed that it is lower for output x_{BS} (Figure 7.10). This would indicate that setpoint tracking of output x_{BS} is easier.

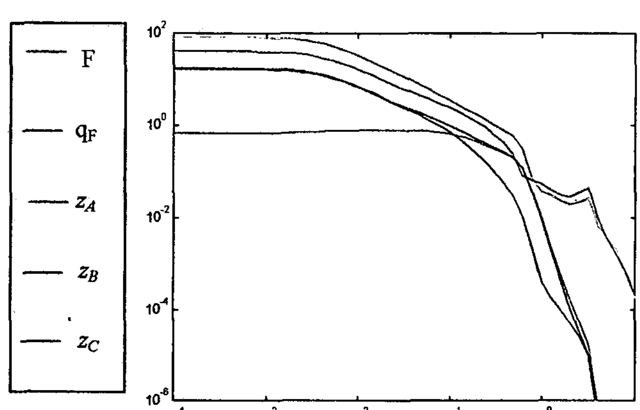

Figure 7.11: ***CLDG*** for output x_{CB} in the column with more trays

In Figures 7.13 and 7.14, ***CLDG*** and ***PRGA*** elements of the column with fewer trays for output x_{AD} and all inputs are plotted. Of all the disturbances, F has the highest ***CLDG***. However, the ***PRGA*** of input S is still higher and a tuning such that $|Li|$ is higher than it is required for x_{AD} setpoint tracking.

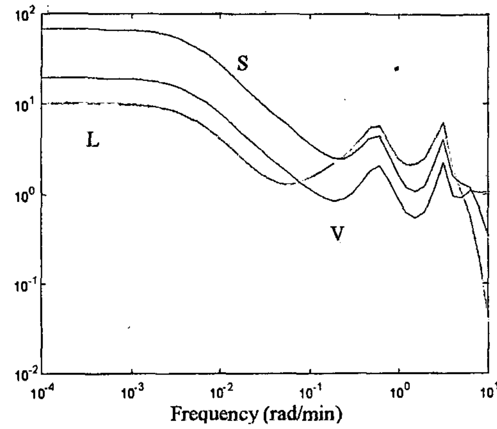

Figure 7.12: ***PRGA*** for output x_{CB} in the column with more trays

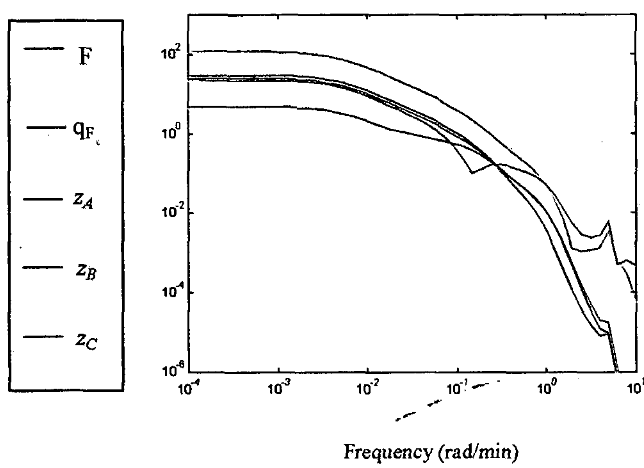

Figure 7.13: ***CLDG*** for output x_{AD} in the column with fewer trays

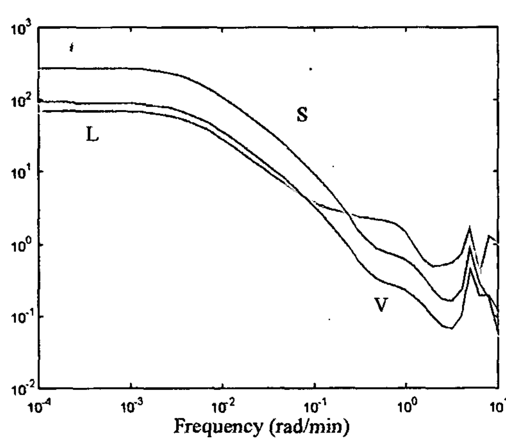

Figure 7.14: ***PRGA*** for output x_{AD} in the column with fewer trays

In Figures 7.15 and 7.16, ***CLDG*** and ***PRGA*** elements of the column with fewer trays for output x_{BS} and all inputs are plotted. F and z_B are the highest of all ***PRGA*** and ***CLDG*** curves. As found for the column with more trays, in Figures 7.14 and 7.16 it is seen that ***PRGA*** elements for output x_{BS} are lower than ***PRGA*** elements for output x_{AD}.

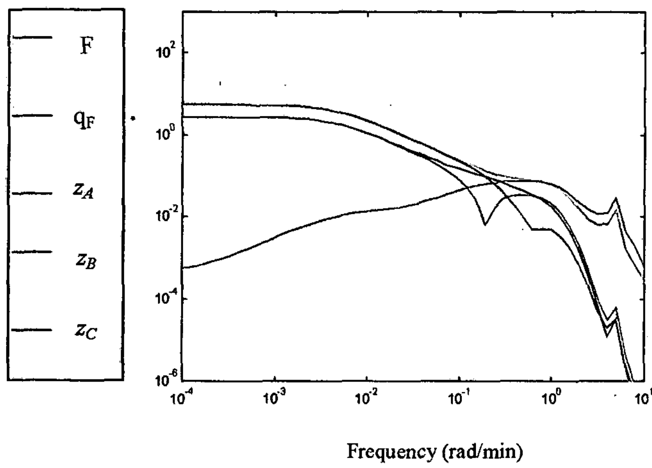

Figure 7.15: ***CLDG*** for output x_{BS} in the column with fewer trays

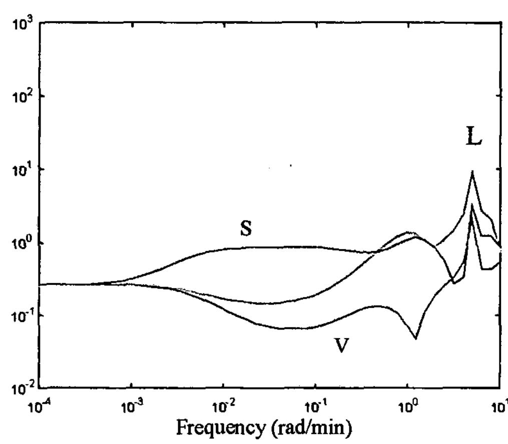

Figure 7.16: ***PRGA*** for output x_{BS} in the column with fewer trays

In Figures 7.17 and 7.18, ***CLDG*** and ***PRGA*** elements of the column with fewer trays for output x_{CB} and all inputs are plotted. As for output x_A, setpoint tracking demands the most aggressive tuning. As found for the column with more trays, comparing ***PRGA*** of input S in Figures 7.14, 7.16, and 7.18, it is observed that it is lower for output x_{BS} (Figure 7.16). This would indicate that setpoint tracking of output x_{BS} is easier.

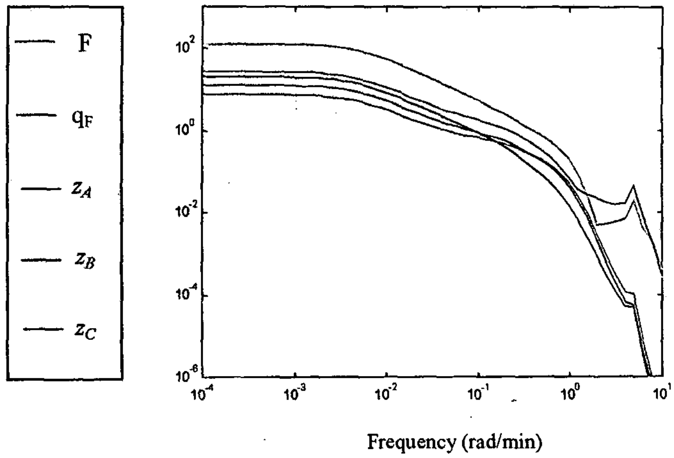

Figure 7.17: ***CLDG*** for output x_{CB} in the column with fewer trays

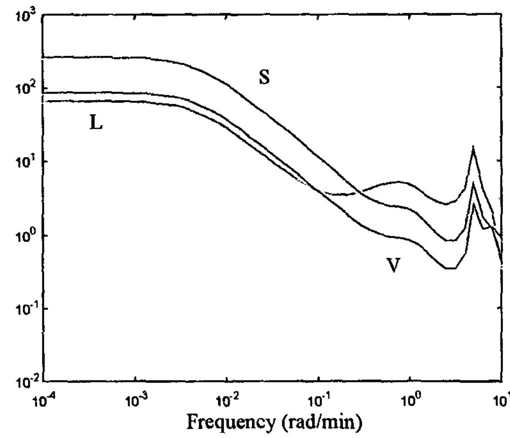

Figure 7.18: ***PRGA*** for output x_{CB} in the column with fewer trays

Comparing the ***CLDG*** of the two columns, it can be seen that ***CLDG*** elements of disturbance F are higher for the column with fewer trays (they have higher values at low frequencies and cross 1 magnitude at higher frequencies). Comparing the ***PRGA*** of the two columns, it can be seen that ***PRGA*** elements of outputs x_{AD} and x_{CB} are higher for the column with fewer trays. This would indicate that control is easier in the column with more trays.

As explained in 4.7.3, for acceptable control of output i, at all frequencies where ***CLDG*** elements are larger than one, ***L(i,i)=G(i,i)*K(i,i)*** should be larger than all ***CLDG*** elements corresponding to output i. With the tuning shown in Table 7.2, for the column with more trays, ***L(1,1)***, ***L(2,2)***, and ***L(3,3)*** cross 1 magnitude at w=0.088, 0.038, and 0.15 rad/min, respectively. At low frequencies, for all outputs, ***L(i,i)*** is larger than all ***CLDG*** of output i. However, ***L(1,1)*** is not larger than ***CLDG*** of F disturbance at all frequencies. Therefore, a tighter tuning is required by the column with more trays to control disturbances of 20% in F (remember that the tuning is done for maximum expected disturbances of 20%). On the other hand, with the same tuning (shown in Table 7.2) for the column with fewer trays, ***L(1,1)*** crosses the unity magnitude at 0.15 rad/min, ***L(2,2)*** at 0.044 rad/min, and ***L(3,3)*** at 0.22 rad/min. In this case, for all outputs, ***L(i,i)*** is larger than all ***CLDG*** of output i at all frequencies. Therefore, although ***CLDG*** and ***PRGA*** are higher for the column with fewer trays, the tuning in Table 7.2 is sufficient for the control of the column with fewer trays and it is not sufficient for the column with more trays. However, the tuning required for control of F in the column with more trays may be achieved without damaging stability.

In a similar way, although it seemed that the control of output x_{BS} could be easier because of smaller ***PRGA*** values, since ***L(2,2)*** is also smaller than ***L(1,1)*** and ***L(3,3)***, this can not be said.

The addition of trays makes the time constants of the individual loops slower, what is indicated by the fact of giving smaller SISO bandwidths. (Notice that, having both columns the same scaled tuning, non-scaled K_c in the column with more trays are smaller because nominal flowrates are smaller).

The fact that the column with more trays has higher MIMO bandwidth and lower SISO bandwidths than the column with fewer trays is due to the smaller directionality and interaction. The difference between the maximum and the minimum singular values of ***S*** is larger for the column with fewer trays. Having the column with fewer trays all three loops quicker, the MIMO bandwidth, which corresponds to the worst direction, is smaller.

It has been seen that all the indexes derived from the singular value decomposition, as well as the ***RGA*** values, indicate that the column with more trays will present better controllability. Incrementing the number of trays of the DWC controlled by *L S V* paired structure, *CN* becomes smaller. Since for this structure high *CN* is a major problem, the addition of trays plays an important role to improve the controllability. The addition of trays helps the decoupling and reduces interaction, two controllability aspects of the MIMO dimension. However, regarding individual loops, addition of trays makes them slower, requiring higher controller gains.

7.3.2 Inventory control "LV"

In this section, the controllability of the two DWC designs is compared considering "LV" inventory control. As explained in 4.6, for the design with fewer trays, for which reflux ratios are larger than 4, "LV" inventory control structure is preferred to "DB". As found in 4.7.4.4, the tuning of "LV" inventory control has a large influence on the composition control. The same tuning is chosen for both columns. It consists in K_c=0.266 (non-scaled) at both inventory control loops.

According to *MRI* and *CN*, the preferred composition control structure for both designs is *B D S*. For the column with more trays, this is the best structure for frequencies lower than 0.14 rad/min.

For the column with fewer trays, this is the best structure for frequencies lower than 0.08 rad/min. Since the bandwidth is expected to be lower than these frequencies, *D B S* is the preferred control structure.

At the same frequency, w=0.04 rad/min, the column with more trays has better controllability indexes, as is indicated in the first row of Table 7.3. With a scaled tuning of K_c=–1.2 and τ_c=80 min for the three loops, the column with more trays has a bandwidth frequency of 0.023 rad/min. With the same tuning, the column with fewer trays has a bandwidth frequency of 0.0095 rad/min. At these bandwidth frequencies, the column with more trays has better controllability indexes, as is indicated in the second row of Table 7.3. With a tuning of K_c=–0.18 and τ_c=82 min for the three loops, the column with more trays has a bandwidth frequency of 0.006 rad/min. With the same tuning, the column with fewer trays has a bandwidth frequency of 0.003 rad/min. At these bandwidth frequencies, the column with more trays has again better controllability indexes, as is indicated in the third row of Table 7.3. Therefore, it seems that the column with more trays has in general a higher bandwidth and better controllability indexes. However, for none of the columns the *CN* values are too high, especially if values are compared to the ones obtained with "DB" inventory control.

With "LV" inventory control, external flows are controlling A and C products composition. External flows will only change if compositions are not at the setpoints. This makes interaction and directionality smaller, compared to the "DB" inventory control. Imagine for example the effect of *D* on the purity of C. It will be given trough a change in *V*, but not in *B*, what will give a smaller interaction. *CN* frequency dependence is also different from that of "DB" inventory control. *CN* has not those large values at low frequencies and it is quite constant in a wide range of frequency. For "LV" inventory control, internal flows are much more responsible of the directionality than for "DB" inventory control.

Table 7.3: Controllability indexes for the column with more trays and the column with fewer trays.

	Column with more trays		Column with fewer trays	
	MRI	*CN*	*MRI*	*CN*
w=0.04 rad/min	0.78	2.0	0.23	6.8
Bandwidth frequency K_c=-1.2, τ_c=80 min	1.4	2.0	0.98	6.3
Bandwidth frequency K_c=-0.18, τ_c=82 min	4.8	1.9	2.9	4.1

RGA of the two columns has been analysed. For the column with more trays, ***RGA*** diagonal elements are plotted in Figure 7.19. For the column with fewer trays, ***RGA*** diagonal elements are plotted in Figure 7.20. ***RGA*** absolute values at the frequency of 0.04 rad/min for the column with more trays and the column with fewer trays are respectively indicated in equation 7.6. *D S B* paired control structure has been found to give the lower interactions. The column with more trays has a ***RGA*** closer to identity, but both ***RGA*** indicate weak interactions.

Therefore, according to ***RGA***, *MRI* and *CN*, the column with more trays has better controllability, but both columns have acceptable values of the controllability indexes.

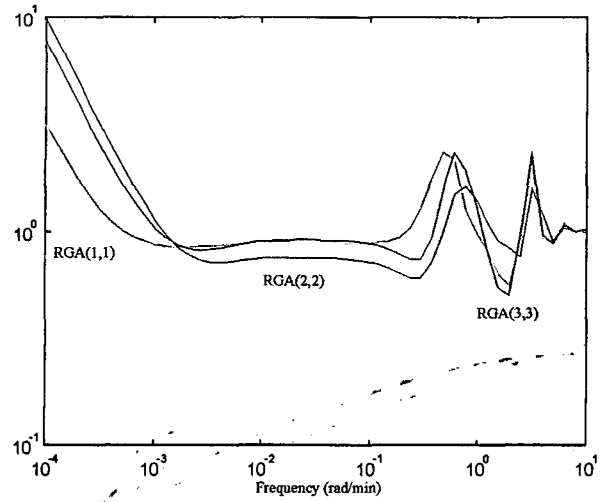

Figure 7.19: Diagonal elements of ***RGA*** matrix for the column with more trays

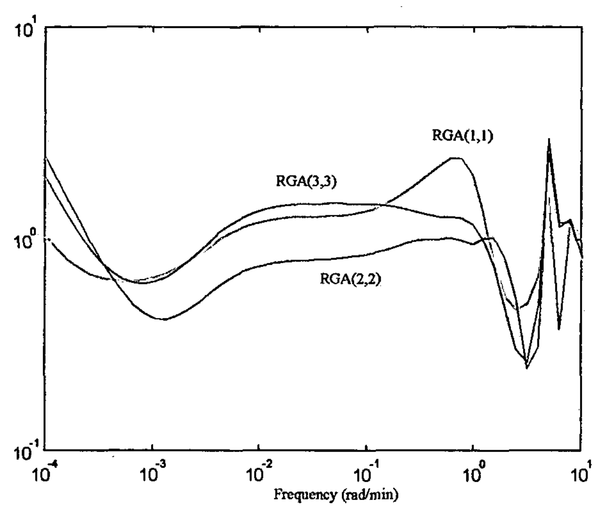

Figure 7.20: Diagonal elements of ***RGA*** matrix for the column with fewer trays

$$RGA(w=0.04)=\begin{pmatrix}0.87 & 0.13 & 0.02\\ 0.15 & 0.74 & 0.12\\ 0.04 & 0.14 & 0.89\end{pmatrix} \qquad RGA(w=0.04)=\begin{pmatrix}1.26 & 0.15 & 0.39\\ 0.29 & 0.79 & 0.06\\ 0.52 & 0.07 & 1.44\end{pmatrix} \quad (6)$$

Turning the attention to the ***CLDG***, it is seen that for both columns, the disturbance with higher ***CLDG*** is *F*. In Figure 7.21, the ***CLDG*** for all loops and disturbance *F* are plotted. It can be observed that for outputs x_{AD} and x_{CB}, ***CLDG*** of the column with fewer trays (dotted lines) crosses the unity magnitude at higher frequencies than ***CLDG*** of the column with more trays. Contrarily, for output x_{BS}, ***CLDG*** of the column with more trays crosses the unity magnitude at a higher frequency than ***CLDG*** of the column with fewer trays. This fact could indicate that outputs x_{AD} and x_{CB} are more difficult to be controlled with the column with fewer trays, while output x_{BS} is more difficult to be controlled with the column with more trays.

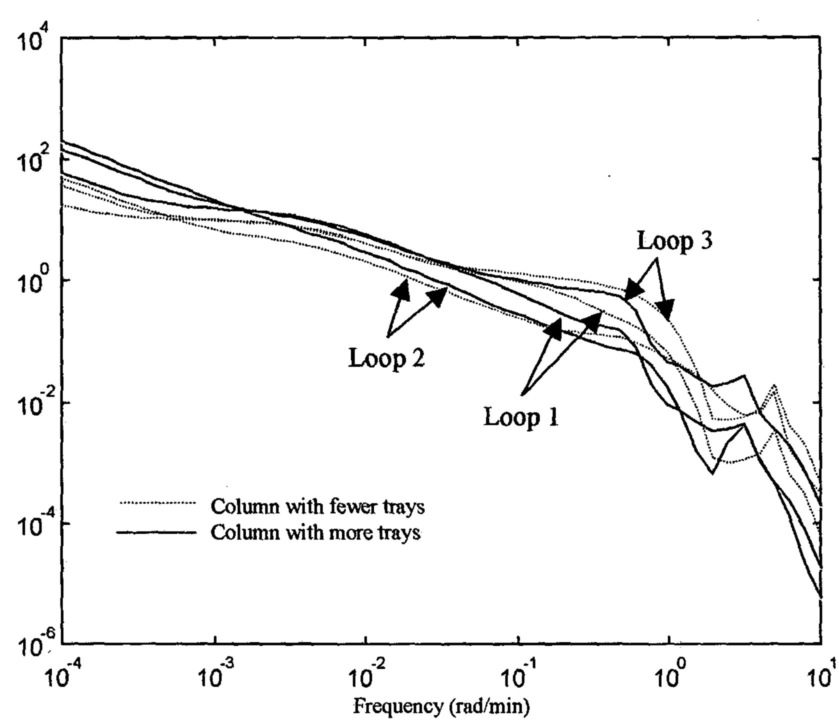

Figure 7.21: ***CLDG*** of all output and disturbance *F*

With a scaled tuning of K_c=–1.2 and τ_c=80 min in all loops, for the column with fewer trays, ***L(1,1)*** crosses the unity magnitude at 0.045 rad/min, ***L(2,2)*** at 0.026 rad/min, and ***L(3,3)*** at 0.048 rad/min. ***L(1,1)*** and ***L(3,3)*** are not larger than ***CLDG*** of *F* at all frequencies. Therefore, the tuning should be tighter for the control of *F*. With the same tuning, for the column with more trays, ***L(1,1)*** crosses the unity magnitude at 0.053 rad/min, ***L(2,2)*** at 0.034 rad/min, and ***L(3,3)*** at 0.055 rad/min. In this case, all three ***L(i,i)*** are larger than ***CLDG*** of output *i*, which indicates acceptable disturbance rejection. Contrarily at what happened with "DB" inventory control, with

"LV" inventory control, with the same tuning for both columns, the individual loops are quicker for the column with more trays.

Until now, all indicates that the control of the column with more trays is easier. For the K_c=–1.2, τ_c=80 min tuning, a preliminary stability analysis through the plot of the complementary sensitivity function ***T*** indicates smaller stability margins for the column with more trays. ***T*** plots can be seen in Figure 7.22. However, through the plot of *wi*Ti*, robust stability is found for both columns and a smaller peak is found for the column with more trays (0.6) than for the column with fewer trays (0.75).

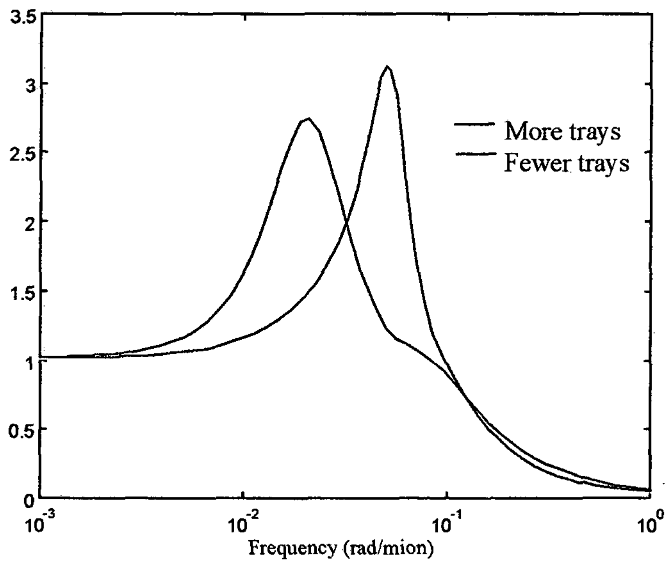

Figure 7.22: Maximum singular value of ***T***

In Figures 7.23 and 7.24, simulation results for the rejection of a disturbance in F of –10% during 5 min are shown for the column with more trays. The tuning is K_c=–1.2 and τ_c=80 min.

In Figures 7.25 and 7.26, simulation results for the rejection of a disturbance in F of –10% during 5 min are shown for the column with fewer trays. The tuning is K_c=–1.2 and τ_c=80 min. In spite of having better controllability indexes, simulation results do not show an improved performance of the columns with more trays.

For "LV " inventory control, addition of trays improves controllability indexes. However, the improvement is not as large as the improvement found for "DB" inventory control. In the case of "DB" inventory control, the main advantage of adding trays is the reduction of *CN* but for "LV" inventory control, high *CN* is not a problem.

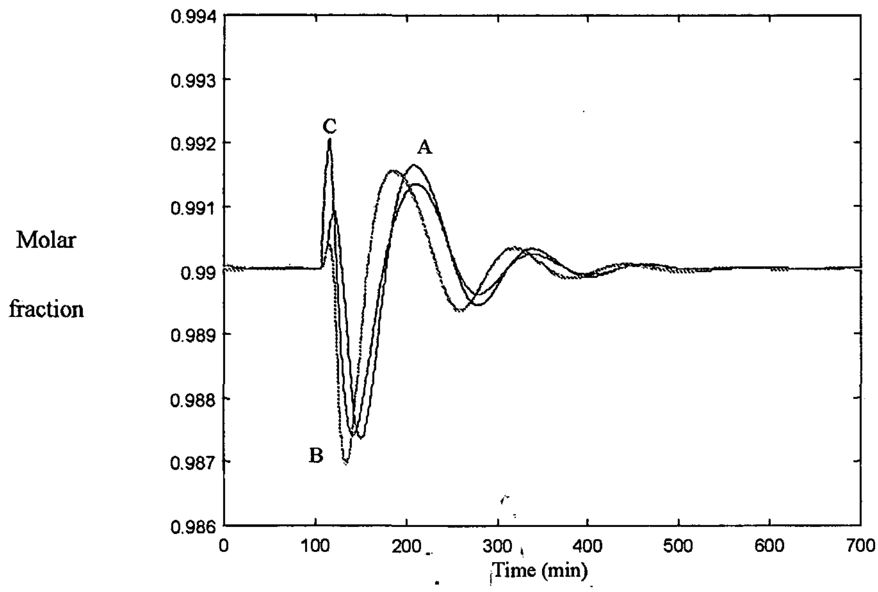

Figure 7.23: Output profiles for a disturbance in F

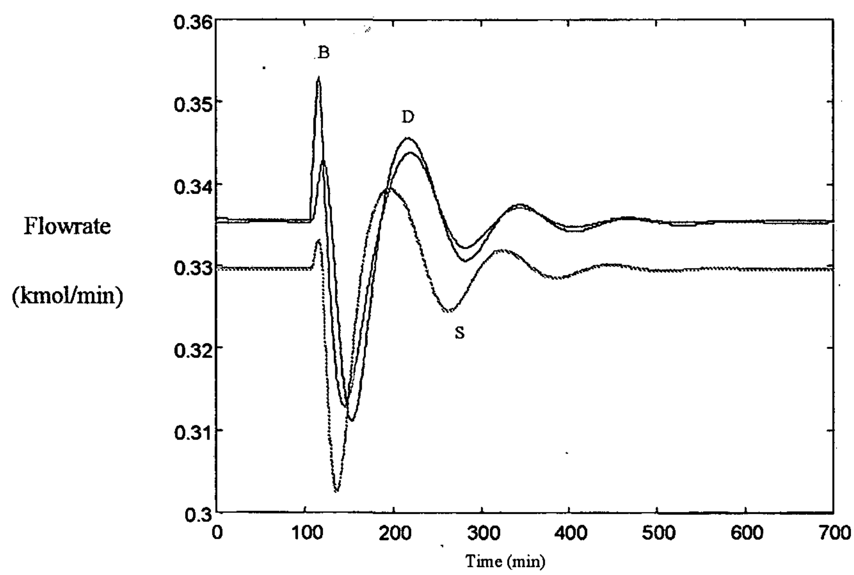

Figure 7.24: Input profiles for a disturbance in F

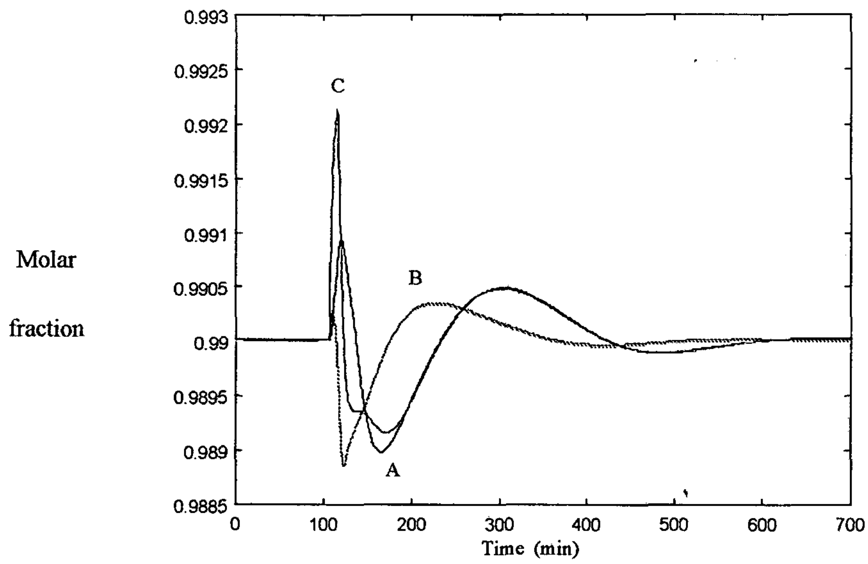

Figure 7.25: Output profiles for a disturbance in F

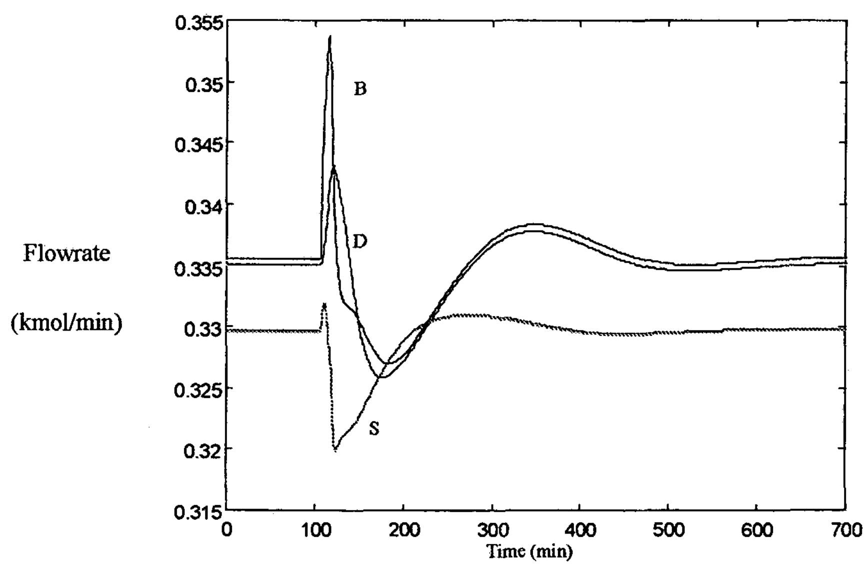

Figure 7.26: Input profiles for a disturbance in F

7.4 Study of non-optimal designs

In this section, the possibility to obtain better controllability using non-optimal designs is explored. The objective is to study the controllability of a column in which the distillation effort is transferred from the lower part of the main column to the upper part of the main column, based on the design with more trays defined in section 7.3. This is done with a design called D1, obtained decreasing the number of trays of the upper part of the main column and increasing the number of trays in the lower part of the main column. For D1, *NT*=58, *NP*=18, *NM*=40, *NS*=23, *NCB*=11, *NCD*=32 and *NF*=9. Inversely, to study the controllability of a column in which the distillation effort is transferred from the upper part of the main column to the lower part of the main column, design D2, for which the number of trays of the upper part of the main column has been increased and the number of trays in the lower part of the main column has been decreased, is considered. For D2, *NT*=58, *NP*=18, *NM*=40, *NS*=19, *NCB*=9, *NCD*=30 and *NF*=9. Finally, to study the controllability when distillation effort is transferred from the prefractionator to the main column, design D3 is considered. It is obtained increasing the number of trays in the prefractionator and decreasing the number of trays in the main column. For D3, *NT*=58, *NP*=20, *NM*=38, *NS*=20, *NCB*=10, *NCD*=29 and *NF*=10. The same separation problem studied in section 7.3 is considered.

Operation is optimised in the three cases. Nominal optimal operation can be seen in Table 7.4. The energy loss due to a non-optimal design for D1 is 3%, for D2, it is 8%, and for D3, it is 3%.

Table 7.4: Optimal operation for the separation processes with designs D1, D2, and D3

	Design D1	Design D2	Design D3
Reflux rate (kmol/min)	1.394	1.469	1.383
Boilup (kmol/min)	1.728	1.803	1.718
Distillate flowrate (kmol/min)	0.336	0.335	0.335
Bottoms flowrate (kmol/min)	0.334	0.335	0.335
Side stream flowrate (kmol/min)	1.330	1.330	1.330
SPLITD	0.591	0.534	0.641
SPLITB	0.400	0.310	0.441
Holdup in reboiler and reflux drum (kmol)	10	10	10
Holdup in the rest of trays (kmol)	10	10	10

7.4.1 Comparison between the optimal design, D1 and D2

7.4.1.1 "DB" inventory control

Comparing the three designs with *L S V* control structure, it is found that controllability indexes of the optimal design are better. However, *L S V* is not the preferred structure for the non-optimal designs. Specifically, for design D2, *L S SPLITD* control structure has been found to be the preferred, and its controllability indexes are considerably better than those of the optimal design with *L S V* structure. Therefore, according to the singular value decomposition analysis, using a non-optimal design, control structures implicating the split variables become preferred, and better than *L S V* structure for the optimal design. These results indicate the possibility of a new trade-off between cost minimisation and controllability in a DWC. (In chapter four it was already seen that a trade-off between energy optimality and controllability might be obtained taking operation away from optimal conditions). In Table 7.5, the singular value decomposition results at *w*=0.04 rad/min are shown. For each design, the three best structures are indicated.

Table 7.5: Controllability indexes at *w*=0.04 rad/min for the base design, D1, and D2

Design	Control structure	*MRI*	*CN*	*II*
Optimal	*L S V*	0.54	25.7	0.021
	V S SPLITD	0.41	26.8	0.015
	L S SPLITD	0.36	24.2	0.015
D1	*L S SPLITB*	0.53	18.4	0.028
	L S SPLITD	0.53	18.5	0.028
	L V SPLITB	0.62	25.3	0.024
D2	*L S SPLITD*	0.91	10.3	0.088
	L S SPLITB	0.82	11.5	0.072
	V S SPLITD	0.91	12.8	0.071

Comparison of the optimal design with L S V control structure and D2 with L SPLITD S control structure

From the results shown in Table 7.5, and comparing the ***RGA*** diagonal elements of the two designs, shown in Figures 7.5 and 7.27, it is seen that, acording to the controllaiblity indexes, D2 with *L SPLITD S* paired structure has better controllability than the optimal design with *L S D* paired structure. Comapring the ***RGA*** values, a matrix closer to identity is found for the D2 design. The same is observed comparing steady state ***RGA*** of the optimal design in equation 7.4 with steady state ***RGA*** of D2 in equation 7.7.

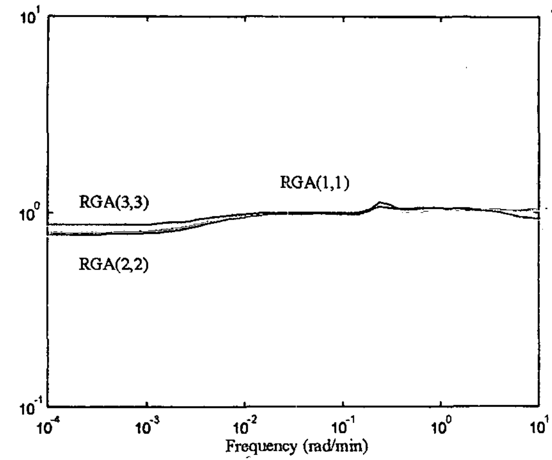

Figure 7.27: ***RGA*** diagonal elements of D2 with *L SPLITD S* structure

$$\boldsymbol{RGA}(w=0)=\begin{pmatrix}0.77 & 0.23 & 0.002\\ 0.09 & 0.75 & 0.16\\ 0.14 & 0.02 & 0.84\end{pmatrix}\qquad \boldsymbol{RGA}(w=0.04)=\begin{pmatrix}0.98 & 0.04 & 0.001\\ 0.01 & 0.96 & 0.04\\ 0.02 & 0.02 & 0.97\end{pmatrix}\qquad (7.7)$$

Through simulations, it is seen that the tuning for D2 and *L SPLITD S* control structure consisting in K_c=0.14, τ_c=70 min (L-x_{AD}), K_c=-0.75, τ_c=83 min ($SPLITD$- x_{BS}), and K_c=0.15, τ_c=75 min (S- x_{CB}), gives accepatable control. In Figures 7.28 and 7.29, the control action for the rejection of a disturbnce +10% in F during 5 min is shown. For this tuning, the bandwidth is 0.0067 rad/min, *MRI*(s=0.0067)=1.13, *CN*(s=0.0067)=45, the peak of the maximum singular value of ***T*** is smaller than 2 and the peak of the maximum singular value of *wi**Ti** is smaller than 1. Therefore, the control structure has good controllability and robustness.

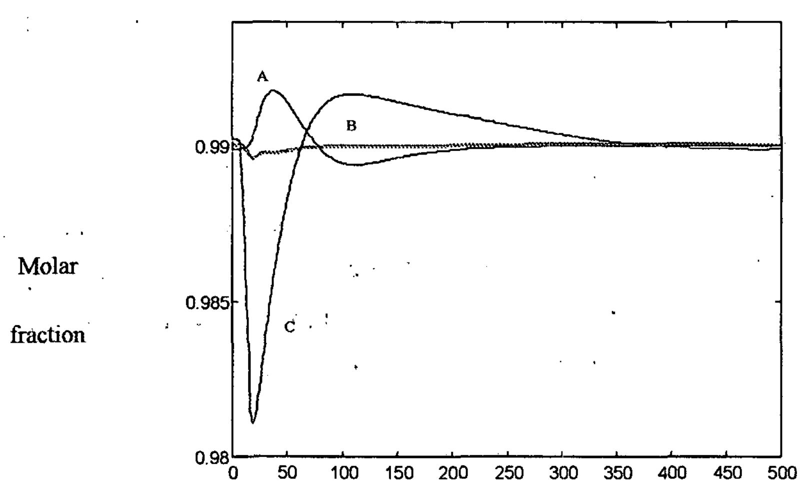

Figure 7.28: Output profiles for a disturbance in F

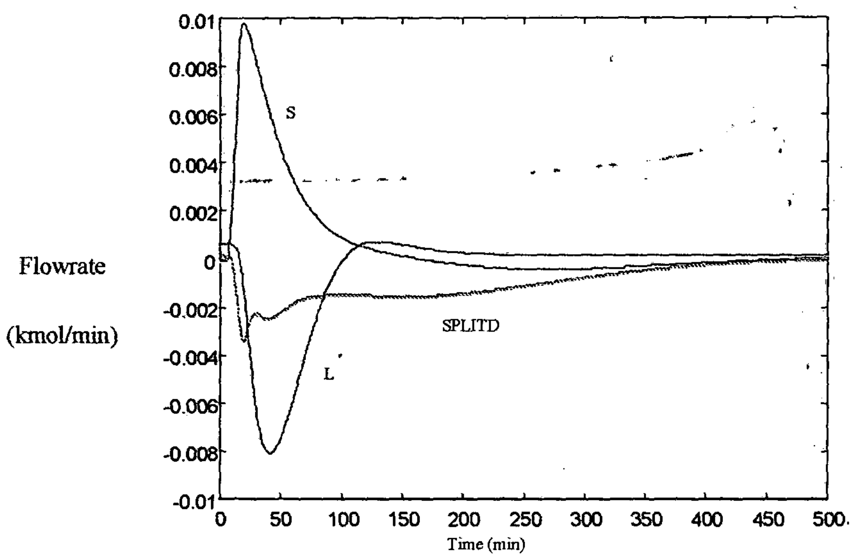

Figure 7.29: Input profiles for a disturbance in F

With the same tuning K_c=0.14, τ_c=70 min (L-x_{AD}), K_c=-0.75, τ_c=83 min (S-x_{BS}), and K_c=0.15, τ_c=75 min (V-x_{CB}), the optimal column with L S V control structure has CN=202. In order to have a smaller CN at the bandwidth frequency, a tighter tuning is needed. The tuning is chosen the less tight for CN<100. This tuning is K_c=1, τ_c=83 min (L-x_{AD}), K_c=-4, τ_c=83 min (S-x_{BS}), and K_c=1, τ_c=83 min (V-x_{CB}). For this tuning, the bandwidth frequency is 0.0075, MRI(s=0.0075)=0.7, CN(s=0.0075)=102, the peak of the maximum singular value of T is 2.1, and the peak of the maximum singular value of $wi*Ti$ is 1. Therefore, controllability indexes are worse than the ones for the non-optimal column for similar stability margins, what confirms the superiority of the non-optimal column. In Figures 7.30 and 7.31, the simulated profiles of the optimal design control action are shown. The same disturbance loaded in the previous example (10% in F) is loaded. Since the tuning has been tighted to have a smaller CN, profiles corresponding to a tighter tuning are observed: smaller variation in the inputs and larger variation in the outputs.

From this section it can be concluded that the use of a non-optimal design can be a good option to improve the controllability of a DWC with "DB" inventory control. The main reason is that the non-optimal design puts the split variables in the set of preferred manipulated variables and the resulting control structures do not have the problem of directionality that control structures with L and V have. For the example examined, differences in controllability are notable for an energy loss of 8%.

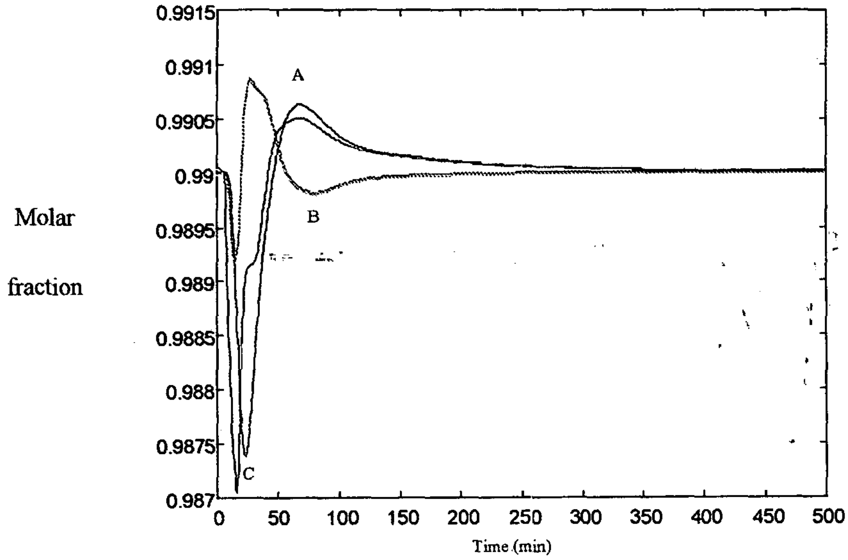

Figure 7.30: Output variables for a disturbance in *F*

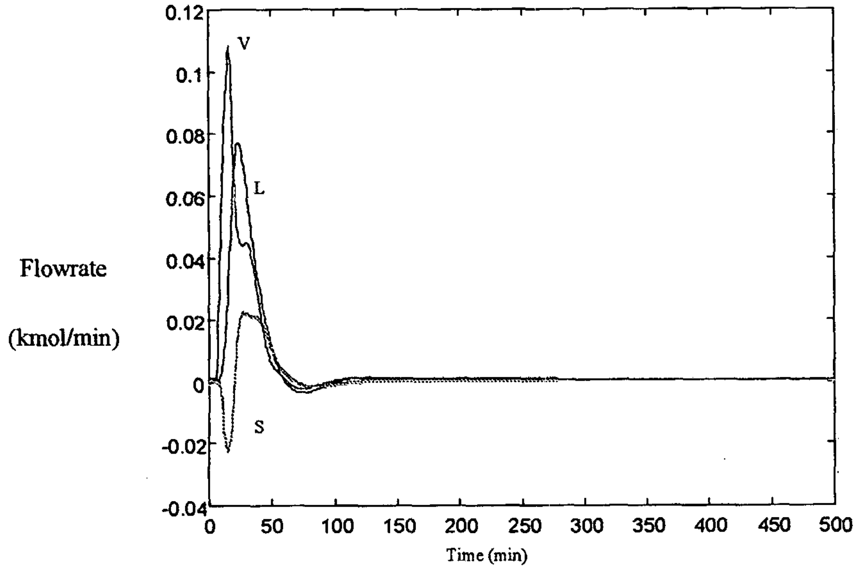

Figure 7.31: Manipulated variables for a disturbance in *F*

7.4.1.2 "LV" inventory control

Similarly at what happens for "DB" inventory control (seen in 7.4.1.1), for "LV" inventory control, the preferred structure for the optimal design does not include the split variables as manipulated variables while the preferred structures for designs D1 and D2 does. Specifically, the preferred structure for the optimal design is *D B S*, the preferred structure for D1 is *D B SPLITB*, and the preferred structure for D2 is also *D B SPLITB*. However, for "LV" inventory control, the non-optimal designs with their preferred control structures have worse controllability indexes than the optimal design with *D S B* control structure. In Table 7.6, the singular value decomposition of the three designs is shown. Results indicate that in the case of "LV" inventory control, the change to non-optimal designs has not improved the controllability of the system.

Table 7.6: Controllability indexes for the base design, D1 and D2

Design	Control structure	*MRI*	*CN*	*II*
Optimal	*DBS*	0.78	2.05	0.38
w=0.04	*B S SPLITB*	0.45	3.14	0.14
	B S SPLITD	0.38	3.74	0.10
D1	*D B SPLITB*	0.59	2.75	0.21
w=0.043	*D B SPLITD*	0.58	2.78	0.21
	D S SPLITD	0.43	3.01	0.14
D2	*D B SPLITB*	0.72	2.17	0.33
w=0.039	*D B SPLITD*	0.72	2.20	0.32
	D S SPLITD	0.54	2.34	0.23

7.4.2 Comparison between the optimal design and design D3

Comparing D3 with the optimal design, the following results are found:

- For "DB" inventory control, *L S V* (preferred structure for the optimal design) is not the preferred structure for D3. *L S SPLITD* is the preferred structure for D3. Besides, the controllability indexes of D3 with *L S SPLITD* are better than the controllability indexes of the optimal design with *L S V* control structure. The case is similar to that of D2 design. The change of design makes the split variables appear in the set of preferred manipulated variables and reduce the problem of high *CN*.

- With "LV" inventory control, the same preferred control structure is found for D3 and for the optimal design, which is *D S B*. Controllability indexes are slightly better for D3 design.

Therefore, as it was found analysing designs D1 and D2, analysing D3 it is seen that a non-optimal design can be used to improve the controllability of the DWC. In this case, the non-optimal design transfers distillation effort from the prefractionator to the main column.

7.5 Controllability of different operating conditions

In section 7.3 it has been seen that the addition of trays in a DWC may improve the controllability of the column. In chapter four, it was seen that moving from optimal operation to another operation, controllability was improved. In this section, the combination of these two effects is analysed. To do that, the designs studied in section 7.3 are considered.

For the column with more trays, controllability at three different nominal operating conditions is analysed. Specifically, the optimal operation and two non-optimal operations (called operation 1 and operation 2) are compared. Nominal split variables for the optimal operation are *SPLITD*=0.634 and *SPLITB*=0.424. Operation 1 is defined fixing *SPLITD* at 0.600 and *SPLITB* at 0.424. The boilup increases by 4.5%. Operation 2 is defined fixing *SPLITD* at 0.650 and *SPLITB* at 0.424. Boilup increases by 2.6%.

Table 7.7: Controllability indexes for operation 1 and the column with more trays

s=0.04 rad/min	"DB"	"LB"	"DV"	"LV"
First preferred structure	L S SPLITB MRI=1.14 CN=8.4	D S SPLITD MRI=1.01 CN=2.7	B S SPLITB MRI=0.98 CN=2.8	D B S MRI=0.95 CN=1.8
Another structure	L S V MRI=0.80 CN=19.0	V D S MRI=0.94 CN=3.7	L B S MRI=0.86 CN=4.1	D S SPLITB MRI=0.67 CN=2.4

Table 7.8: Controllability indexes for optimal operation and the column with more trays

s=0.04 rad/min	"DB"	"LB"	"DV"	"LV"
First preferred structure	L V S MRI=0.54 CN=25	V D S MRI=0.68 CN=4.5	L B S MRI=0.62 CN=4.7	D B S MRI=0.78 CN=2.05
Second preferred structure	V S SPLITD MRI=0.41 CN=26	V S SPLITB MRI=0.44 CN=5.7	B S SPLITD MRI=0.40 CN=6.1	B S SPLITB MRI=0.45 CN=3.1

Table 7.9: Controllability indexes for operation 2 and the column with more trays

S=0.04 rad/min	"DB"	"LB"	"DV"	"LV"
First preferred structure	L S SPLITD MRI=1.12 CN=8.3	D S SPLITD MRI=0.92 CN=2.9	B S SPLITD MRI=1.02 CN=2.5	D B SPLITD* MRI=0.75 CN=2.5
Another structure	L S V MRI=0.21 CN=67.3	V D S MRI=0.27 CN=13	L B S MRI=0.24 CN=12.9	D B S MRI=0.32 CN=4.9

In Tables 7.7, 7.8 and 7.9, the *MRI* and *CN* of the preferred sets of manipulated variables for the three operating conditions and the column with more trays are shown. Values for the different inventory control structures are separately indicated. The following can be observed:

- At optimal operation, the preferred sets of manipulated variables do not include the split variables. On the contrary, at non-optimal operations, the preferred sets of manipulated variables include the split variables.
- For inventory control "DB", "LB", and "DV", the controllability indexes indicate different preferred sets of manipulated variables for optimal and non-optimal operations, and preferred sets of manipulated variables for non-optimal operations present better controllability indexes.
- With "DB" inventory control, preferred structures at non-optimal operations, including split variables, give an important reduction of the *CN*. With "LV" inventory control, preferred structures at non-optimal operations and preferred structures at optimal operation have similar controllability indexes.

According to these results, for the column with more trays with "DB" inventory control, a trade-off is possible between energy optimality and controllability. For the column with fewer trays, the same controllability analysis at different operating conditions was studied in section 4.11.2, and the same conclusions were obtained. Comparing results in section 4.11.2 with results in this section, it is seen that the combination of column with more trays at non-optimal operation effectively gives the best controllability indexes (specially, the lowest *CN*).

7.6 Conclusions

Comparing DWC designs with stages optimally distributed between sections but with different total number of stages, it is seen that high *CN* is a problem associated to "DB" inventory control, which can be solved increasing the number of trays of the DWC. For "DB" inventory control, high *CN* is a major problem. Thus, the addition of trays plays an important role in the controllability improvement. For "LV " inventory control, addition of trays is also found to improve controllability. However, in this case, the improvement is less important because acceptable controllability indexes are found in all cases. The addition of trays within design optimality does not change the preferred set of manipulated variables.

Comparing the controllability of optimal and non-optimal DWC designs, it is found that the use of non-optimal designs may improve controllability when inventory control is "DB". The main reason is that for non-optimal designs, split variables (*SPLITD* and *SPLITB*) appear in the set of preferred manipulated variables, and are able to reduce the large directionality of the *L S V* control structure. In the case of "LV" inventory control, the change to non-optimal designs has not been found to improve the controllability of the system. These results indicate a trade-off between design optimality and controllability in a DWC with "DB" inventory control.

Finally, for "DB" inventory DWC, it is found that the effect of controllability improvement obtained moving out from optimal operation and the effect of controllability improvement obtained with the addition of trays are added.