Introduction

Nowadays managing companies requires making many decisions of qualitative nature related to the design and activities of their logistic systems. These decisions concern, among others, the location of facilities, the structure of the fleets and their management, as well as the policies for the storage, transportation and delivery of raw materials, products or information. Indeed, an appropriate planning of such activities can result in significant economical improvements.

The role of *Operations Research* (OR) in this field is to provide the decision makers with quantitative tools, based on which they can base their decisions. It is within this framework that in this thesis we will study some of the decisions appearing in the design of logistic systems, and, in particular, those related to the location of facilities from which clients will be serviced, as well as the policies for managing the fleets that will actually provide service to the clients. Methodologically, we will study these problems from the mathematical programming point of view.

Location-Routing and Related Problems

One of the most important strategic decisions to be taken in the design of logistic systems concerns the location of facilities. Potential locations of the facilities must fulfill a series of requirements such as physical and economical accessibility, and particular conditions inherent to the specific application (gouvernmental restrictions, company policies, etc.). Finding such potential locations is not always an easy task, and often constitutes the objective of a preliminary study. Once the set of candidate locations is determined, decision makers must choose a subset of them that satisfies the system specifications and that is optimal with respect to a given objective. The objective can have rather different natures; we can seek cost minimization, coverage maximization, optimization of some equity measure, or a combination of the above. Moreover, in the general case, it can even be desired to consider several different objectives simultaneously.

The efficiency of a system, however, will also depend on many other factors, the structure and management of the fleets being among the most important ones. Thus, apart of the location of facilities, decision makers must commonly face the choice on the number or types of vehicles assigned to each located facility and the design of the routes followed by each vehicle, which also implies the allocation of clients to facilities. The family of problems addressing these decisions is commonly referred to as *Vehicle Routing Problems* (VRP).

Operations researchers have been attracted by both location and routing problems for many decades. In fact, these two areas are still nowadays very active from a research point of view. This is due to the fact that these problems give raise to a wide variety of mathematical programming models, most of which are really challenging. Both location and routing problems fall in most cases in the field of *Combinatorial Optimization* (CO) and it is within this area that we will study the problems appearing in this thesis. In the vast OR literature devoted to these subjects heuristic as well as exact algorithms can be found for an extensive collection of problems.

Most often, even if the two kinds of problems, namely location of facilities and vehicle routing, appear together in a given scenario, they are studied and solved separately. This tendency to tackle location and routing problems in successive steps is commonly justified by the great difficulty of the combined problem, inherited from its components. Since both, location and routing problems are already $\mathcal{NP}$–hard under most scenarios, their combination leads undoubtedly to a really hard problem. However, it is well-known that studying vehicle routing only when locations have already been fixed, leads, in general, to suboptimal solutions even if allocation of clients to facilities is done simultaneously with the location, using some estimation of the travel costs.

Despite the difficulty of the problems that we have just mentioned, computational facilities available nowadays and the success already achieved in solving location as well as routing problems provide us with a reasonable framework to deal with the combined problems without splitting them into different levels. In fact, the literature addressing combined problems has been increasing in the last years, as well as the number of different variants of the problem considered within this family, generally corresponding to real world needs. However, most of this existing literature addresses *Location-Routing Problems* (LRP) by means of heuristic approaches, since in general, even medium-size problems are not yet solvable with exact algorithms, within reasonable amounts of time.

Diversity in LRPs is introduced from four main sources; the structure of the system, the scope of the decision, the nature of the data, and the type of the planning horizon. A complete classification of LRPs can be found in Laporte (1988).

- While, in the simplest systems, only clients and plants (or primary facilities) are considered, an extensive literature can be found where systems with three layers are studied. The extra layer in such systems is the set of warehouses (or secondary facilities). Also, different policies for the routes of the vehicles can be considered. Most often, vehicles perform return trips starting at a facility (primary or secondary) and visiting points in a lower layer, but this policy is not generalized. Another constraint that appears frequently in the literature is that routes be disjoint. Moreover, some side constraints are commonly included on the problem, such as length constraints on the vehicle routes, time windows, or capacity constraints on some resources consumed by the facilities or the vehicles for providing a service. Problems considering this latter type of constraints are referred to as capacitated problems.
- The number of facilities to open and/or the number of vehicles associated with them, as well as the values of their capacities in the case of capacitated problems, can be fixed parameters, or they can be taken as decision variables. On the other hand, in problems with three layers, locations of facilities can be all decision variables, or we can have fixed locations in one of the layers.
- At the same time, problems can be classified with respect to the nature of the data. In most real applications there is some uncertainty in part of the data characterizing the problem, that has been classically tackled by means of estimators. In some applications, however, models obtained this way do not give an appropriate representation of the real problem, and stochastic models are preferable, since the explicit consideration of stochasticity in the model gives raise to more reliable solutions.
- Finally, the distinction between static and dynamic models has to be made. Although models considering a single period are most popular, it is rare that a system can be built from scratch or modified at a single stage, while its environment remains unaltered. On the contrary, frequently the system and the environment evolve successively in order to adapt to each other. Modelization of this phenomenon is achieved through dynamic (or multi stage) models.

This thesis focuses on the study of a particular static LRP. It is assumed that facilities are capacitated and there is exactly one vehicle at each open facility that is uncapacitated. This situation occurs, for instance, when repairmen have to be assigned to machines and in the logistics design of security trucks for money collection. Both, a deterministic and a stochastic version are studied. Mathematical programming formulations of the considered models are presented. Also, algorithmic approaches have been developed and tested in both cases.

Some other problems appear either as subproblems of the LRP or as auxiliary problems during its resolution. This is the case of the *Generalized Assignment Problem* (GAP), its stochastic version (SGAP), and the *Elementary Shortest Path Problem with Resource Constraints* (ESPPRC). While the GAP is a well-known problem in CO and many tools have already been developed to deal with it, not much is known about the other two. Therefore, they have been studied in more detail in this work.

Bounds for a Deterministic Location-Routing Problem

First, we have formulated the deterministic LRP by means of an integer programming model that considers location of facilities, allocation of clients to facilities and vehicle routing decisions at the same level. The solution of a reinforced linear relaxation of this model is the basis of a rounding procedure that provides the starting solution for a *Tabu Search* (TS) heuristic. Also, we have developed a lower bound based on the structure of the problem.

Solutions of the LRP considered in this work can be seen as combinations of routes associated with each facility (or plant). This structure points out *Column Generation* (CG) as a promising approach for this LRP, given the good results it has already obtained in several pure routing problems. To explore the applicability of this approach to our problem, we have first modeled it as a partition problem with side constraints, and then developed a CG algorithm that seeks good feasible routes for each plant, and tries to combine them appropriately. In the search for promising routes within this scheme, solutions to an ESPPRC or to a *Traveling Salesman Problem* (TSP) with profits have to be identified. This has motivated the deeper study of the former of these problems for which we have developed a simple heuristic.

Stochastic Versions of Generalized Assignment and Location-Routing Problems

As mentioned before, real world needs cannot always be satisfied by means of deterministic models. Uncertainties appear in most contexts, and deterministic models based on estimations are not always suitable. This fact has stimulated the evolution of Stochastic Programming since the first attempt to explicitly address uncertainty in optimizations problems carried out by G. B. Dantzig in 1955.

In the case of logistic systems design, uncertainty in the data can be due to different sources, and it can have different natures. For instance, the demands of the clients are not likely to be known in advance, the travel times are often scenario dependent (weather conditions, congestion..), and even the availability of resources is sometimes only known up to a point. Thus, most often it is much more realistic to model some of these measures by means of random variables. Moreover, it can be the case that uncertainty is not about the value of the demand of a customer, but refers to its presence as an actual point of demand. Uncertainties considered in this paper, are of this latter type.

Whereas for the deterministic case a rich literature exists on any of the subproblems of the LRP, namely pure Location, Vehicle Routing, and also Assignment Problems, literature on stochastic versions of this problems is rather poor. To the best of our knowledge, there are relatively few publications addressing Stochastic Vehicle Routing problems, and one single publication on a SGAP (Mine, Fukushima, Ishikawa, and Sawa 1983) independent of our work.

In this thesis, before tackling a stochastic version of the LRP defined above, we study a stochastic GAP, where the presence of each job is stochastic, and can be modeled with a Bernoulli distributed random variable. The reason is that the subproblem of the considered LRP that allocates clients within the set of open plants has this structure. We present three heuristics as well as three versions of an exact method developed to solve it. We also present a lower bound, based on Stochastic Linear Programming techniques, which is much tighter than the linear relaxation, and plays an important role in the efficiency of the exact algorithms.

Finally, we study a stochastic LRP, where the presence of each client is uncertain and, as in the case of the SGAP, it is modeled as a Bernoulli random variable. The capacity of a facility is defined as the maximum number of clients it can service. The problem is formulated with a recourse model, where the recourse term has two parts, penalties are paid for unserviced customers, and savings in route costs are derived from skipped customers. A heuristic as well as a lower bound for the problem are presented, together with the results of some computational experiences.

Structure of the Thesis

In the following chapters we describe in detail the problems and approaches studied in this thesis. The structure of this work is as follows; in Chapter 1 we present a review on the existing literature related to this work. Next, we introduce the deterministic LRP under study in Chapter 2, where a compact model is proposed and a TS heuristic is presented together with our lower bound. In Chapter 3 we present the CG approach. As a part of this approach we study the ESSPRC and propose a simple heuristic for it. Chapter 4 includes all our work on the stochastic GAP; three heuristics, three versions of an exact algorithm, and a lower bound. Finally, in Chapter 5 we describe the heuristic and the lower bound we have developed for the stochastic version of the LRP. We conclude this work with some remarks and future research proposals in Chapter 6.