Essays in Learning Uncertainty

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This dissertation is dedicated to Sadaf and my family.

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Abstract

Agents make decisions under uncertainty. They are not only uncertain about the true realizations of variables of interest, but also about their degree of uncertainty. Using survey of business uncertainty, I study firms' subjective uncertainty and show that firms' time varying subjective uncertainty can be driven by the learning channel. Next, I study implications of learning uncertainty for firms' investment patterns. To do so, I first build a simple model of firms' investment with learning uncertainty and show that learning uncertainty results in three main implications. Then, using Compustat data, I show that all three learning uncertainty's implications for firms' investment patterns are observable in the data. Since uncertainty about second moments affects agents' decisions, so it also has important implications for optimal conduct of policies such as monetary policy. I build a model with uncertainty about productivity's dispersion and study optimal response of monetary policy to the productivity dispersion shock.

Resum

Els agents prenen decisions sota incertesa. No només estan incerts sobre les realitzacions de les variables d'interès, sinó també sobre el seu grau d'incertesa. Utilitzant l'enquesta d'incertesa empresarial, estudio la incertesa subjectiva de les empreses i demostro que les variacions en la incertesa subjectiva de les empreses poden ser impulsades a través de l'aprenentatge. A continuació, estudio les implicacions de la incertesa en l'aprenentatge en els patrons d'inversió de les empreses. Per fer-ho, primer construeixo un senzill model d'inversió de les empreses amb incertesa en l'aprenentatge i demostro que aquesta té tres implicacions principals. Utilitzant les dades de Compustat, demostro que les tres implicacions de la incertesa d'aprenentatge per als patrons d'inversió de les empreses són observables a les dades. Atès que la incertesa sobre els segons moments afecta les decisions dels agents, també té implicacions importants per a la conducta òptima de polítiques com la política monetària. Finalment, construeixo un model amb incertesa sobre la dispersió de la productivitat i estudio la resposta òptima de la política monetària al xoc de dispersió de la productivitat.

Preface

Subjective uncertainty plays an important role in decision making process. Agents are uncertain about the realizations of many variables of interest such as sale revenue, exchange rates, income, productivity, etc. The degree of agents' uncertainty varies over the time, sometimes they become more certain and sometimes more uncertain. How does the degree of uncertainty vary over the time? Which factors affect and drive agents' uncertainty?

We can think of subjective uncertainty as the second moment of the subjective probability density function of the variables of interest. Agents by observing the realizations of the variables of interest can *learn* the whole distribution. Throughout three chapters of my thesis, I concentrate on variables that are assumed to be normally distributed with unknown second moments. By observing the history of realizations of those variables, agents can learn the second moments or variances of the data generating distributions. Learning second moments or in other words *learning uncertainty*, is a potential driver of agents' subjective uncertainty.

In the chapter 1, using firm level survey data, I study firms' subjective uncertainty. I show that the subjective uncertainty is time varying. Moreover, it responds significantly positively to the realized uncertainty and the conditional responsiveness to realized uncertainty decreases over the time. Given these findings, I propose learning uncertainty as a mechanism that maps realized uncertainty to subjective uncertainty.

After validating learning uncertainty as a possible driver of subjective uncertainty, in the chapter 2, I study the implications of learning uncertainty for firms' investment patterns. To do so, I first build a partial equilibrium model of firms investment with learning uncertainty. I show learning uncertainty results in 1-lower investment response to the idiosyncratic TFPR shocks for firms that experience more volatile productivity in their lifetime, 2-lower investment response to larger idiosyncratic TFPR shocks and 3-asymmetric responses to symmetric positive and negative idiosyncratic TFPR shocks (Asymmetric S shaped response). Next, using Compustat data, I estimate TFPR in firm level and study the dynamism of firms' investment rate response to idiosyncratic TFPR shocks. I show three mentioned implications of learning uncertainty for the investment patterns are observable phenomenon in the data. Finally, based on the finding from the survey of business uncertainty about drivers of subjective uncertainty, I assume Compustat firms are Bayesian learners and after building their time varying posteriors' uncertainty (variance of posterior beliefs) about idiosyncratic TFPR shocks, I study the impact of their posteriors' uncertainty on the investment response to TFPR shocks. I show that firms' posteriors' uncertainty about idiosyncratic TFPR shocks affects negatively their investment response to the shock which verifies learning uncer-

tainty as a potential driver of three mentioned investment patterns in the data.
In the chapter 3, I present a theoretical framework that features contractionary productivity dispersion shock which is a result of the interaction between substitutability of supplied labor and demanded goods. I introduce information friction as a source of nominal rigidity to study the impact of the dispersion shock on the conduct of monetary policy. In particular, I assume firms have incomplete information about the productivity dispersion when they set the price. I show that in the environment with nominal rigidity, replicating full-information flexible price equilibrium is always feasible and optimal. The optimal monetary policy is the policy which eliminates the dependence of the idiosyncratic nominal variables on the unknown productivity dispersion and as a result removes the information friction.

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Chapter 1

SUBJECTIVE UNCERTAINTY AND LEARNING

1.1 Introduction

Both micro-uncertainty and macro-uncertainty[1] play crucial roles in agents' decision making process. Originated by Bloom (2009), a growing branch of literature studies micro and macro-uncertainty shocks, their drivers and their implications. Berger and Vavra (2017), Jurado et al. (2015), Baker et al. (2016) and Fernandez-Villaverde et al. (2011) are among papers that show that uncertainty second moment shocks are counter-cyclical. Many theoretical frameworks rationalize counter-cyclicality of the uncertainty shocks through wait-and-see effects[2], risk premium effects[3] or precautionary motives[4]. In the modeling set-ups, they mostly concentrate on the *subjective uncertainty*; uncertainty about future realizations of variables of the interest. In this paper, I concentrate on the firm-level subjective micro-uncertainty and its drivers.

How is the subjective uncertainty formed? Which factors are drivers of subjective uncertainty? One potential driver is the *realized uncertainty*; an unexpected realization of a variable or in other words realized volatility. Many empirical studies, some of them already mentioned above, also used realized uncertainty as a proxy to verify the counter-cyclicality of the uncertainty shocks[5]. I show that firm-level

[1] By micro-uncertainty I refer to uncertainty about micro variables such as firm-level productivity, household income, etc. and by macro-uncertainty I refer to uncertainty about macro variables such as GDP, TFP, broad stock market indices, exchange rates, etc.

[2] see Bloom et al. (2018), Bachmann and Bayer (2013) .

[3] see Arellano et al. (2016) and Christiano et al. (2014).

[4] see Basu and Bundick (2017), Leduc and Liu (2016) and Ravn and Sterk (2017).

[5] For example, Berger et al. (2019) shows that realized volatilities are robustly followed by contractions.

idiosyncratic realized uncertainty and subjective uncertainty are significantly and positively correlated.

Using firm-level survey of business uncertainty, I study firms' subjective uncertainty about future sale growth rates. An interesting feature of this data is the fact that I do not only have access to subjective probability distributions about future realization of the sale growth, but also its previous realizations. Therefore, I can determine both micro subjective uncertainty and realized uncertainty and study their relation. I show that the subjective uncertainty is time varying, responds significantly positively to the realized uncertainty and the conditional responsiveness to realized uncertainty decreases over the time. These findings suggest that realized uncertainty can be a potential driver of subjective uncertainty. I am not the first who proposes realized uncertainty as a driver of subjective uncertainty. Altig et al. (2020) by using the same survey data, Bachmann et al. (2021) by using survey panel on German manufacturing firms and Boutros et al. (2020) by using survey on stock market predictions made by financial executives find the same result.

There are different possible mechanisms that can map realized uncertainty to subjective uncertainty such as GARCH, learning, etc.. However, the fact that the responsiveness of subjective uncertainty to realized uncertainty decreases over the time, suggests that learning is the best candidate. In this paper, I concentrate on the standard Bayesian Normal-Gamma learning. In particular, I assume that firms are uncertain about the variance of the underlying variable's data generating process, which is assumed to be normally distributed with a constant unknown variance, and learn the variance by observing the realizations of the mentioned variable over the time following Bayesian Normal-Gamma updating mechanism. I show that variance of the posterior beliefs about the underlying variable, which I will call *posterior beliefs' uncertainty* from now on, is a reliable predictor for forecasting subjective micro-uncertainty in the firm-level. I am not the first who proposes learning uncertainty as a possible driver of the subjective uncertainty. Boutros et al. (2020) by using 14,800 forecasts of one-year SP 500 returns made by Chief Financial Officers over a 12-year period, find that when return realizations fall outside of ex-ante confidence intervals, CFOs' subsequent confidence intervals widen considerably. They propose Bayesian learning as a possible driver. I do find the same result about idiosyncratic micro-uncertainty, in particular about idiosyncratic sale growth uncertainty.

I compare Bayesian learning with standard GARCH estimation. I find that estimated Bayesian posterior beliefs' uncertainty fits the data better than the time varying variance from GARCH(1,1) estimation to predict firm level subjective uncertainty. Of course, orders of the GARCH process and calibration of Bayesian prior's parameters can affect the comparison's result. Optimal model selection is out of the scope of this paper. I also find both Bayesian posteriors' uncertainty and the time varying variance obtained from GARCH approach overestimate firms' subjective uncertainty. To rephrase it, although subjective uncertainty is significantly and positively correlated with posterior beliefs' uncertainty and time varying uncertainty from GARCH estimation, in general it is lower than both. This finding is in line with overconfidence literature.

After validating learning as a reliable mechanism that map realized uncertainty to subjective uncertainty, in the next chapter I will study implications of learning uncertainty on firms' investment decision.

1.2 Subjective uncertainty and realized uncertainty

Firms' investment or hiring decisions are made under uncertainty and their belief about the future plays a crucial role in their decision making process. Firms' belief can be affected by various idiosyncratic or aggregate factors. In the following section using firm level survey data, I study firms' subjective probability distributions and in particular their subjective uncertainty in order to understand how the idiosyncratic subjective uncertainty is formed. The main data that I use for studying subjective uncertainty is *survey of business uncertainty* conducted by Federal Reserve Bank of Atlanta. The survey began in 2014 and I use the data until February 2022. It is a monthly survey and covers about 2,560 firms drawn from all 50 states, every major non-farm industry, and a range of firm sizes.

In the survey respondents provide information about their beliefs about next year sales growth, employment growth and capital expenditure growth and also report last year's realizations of the mentioned variables. The main innovation and advantage of this data is to let survey respondents freely select support points and probabilities in $N = 5$ point distributions over future sales growth, employment growth, and capital expenditure growth[6]. I drop all observations with negative assigned probability or probabilities that does not sum up to 100. For further information about details of the survey design and data cleaning please refer to Altig et al. (2020).

I define subjective uncertainty of firm i at time t for the variable x by $SDS^x_{i,t}$ as the standard deviation of subjective probability density function of firm i at time t about the variable x:

$$SDS^x_{i,t} = \left[\sum_{j=1}^{N=5} p^x_{i,t,j}\left(x_{i,t,j} - \overline{x}_{i,t}\right)^2\right]^{\frac{1}{2}} \quad , \quad \overline{x}_{i,t} = MS^x_{i,t} = \sum_{j=1}^{N=5} p^x_{i,t,j} x_{i,t,j}$$

$$x \in \{\textit{Sale Growth}, \textit{Capex Growth}, \textit{Employment Growth}\}$$

[6] Although raw survey responses about capital expenditure and employment are not in terms of the growth rate, following Altig et al. (2020) I re-express them in terms of growth rate.

where $x_{i,t,j}$ is the j-th support point chosen by firm i at the time t for the next year realization of the variable x with assigned probabilities $p^x_{i,t,j}$ and $\overline{x}_{i,t}$ is the corresponding subjective mean value.

The way that I define the subjective uncertainty clearly shows that the subjective uncertainty in the firm-level is time varying as it is a function of time varying variable x and the time varying subjective mean[7]. What factors shape and affect idiosyncratic subjective uncertainty? How does the previous realization of the variable affect the subjective uncertainty about its future realization? In the figure 1.1, I present the scatter plot with subjective uncertainty over next year sales growth rates on the vertical axis and percentiles of past sales growth rate over the last year on the horizontal axis. I present the same figure for subjective uncertainty of employment and capital expenditure growth rates in the appendix. Please note that here I do not control for any factor such as age, persistence or fixed effects.

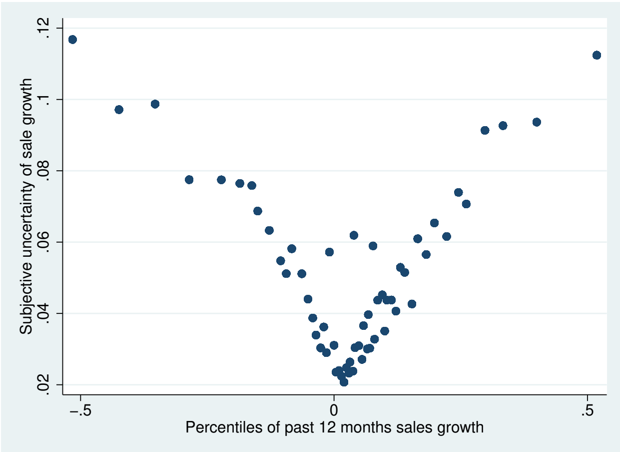


Figure 1.1: Subjective uncertainty about future sale growth rate versus previous realization.

As you notice subjective uncertainty has a V-shaped relationship to past sales growth. The V-shaped relationship between subjective uncertainty and the past growth rate suggests that there is a positive relationship between the *size* of the realization of the growth rates in the last year and the subjective uncertainty about their future realizations. To put it in another way, there is a positive relationship between realized volatility and subjective uncertainty. This result is in line with the finding in Altig et al. (2020) and the only difference is that I have access to the richer data from 2014 to February 2022. Bachmann et al. (2021) and Boutros et al. (2020) also reach the same finding that there is a positive relationship between the

[7] In the appendix, I also present the aggregate values of subjective uncertainty. As you can see there the aggregate values are also time varying with a large spike after Covid crisis.

realized volatility and the subjective uncertainty using different survey data.

In the next step, I go further and study the relationship between the subjective uncertainty and the size of the growth rates deeper. From now on, I only concentrate on the *sale revenue growth rate*, because capital expenditure growth and employment growth rates are functions of firms' endogenous decisions, however, sale growth rates can capture the beliefs about the firm's exogenous profit and productivity. Moreover, in the next chapters I am going to study the implications of uncertainty about revenue based measure of productivity which is closely related to the sale revenue growth. First, I define the *size shock* for the firm i at time t as:

$$SizeShock_{i,t} = |\boldsymbol{SaleG}_{i,t} - MS^{SaleG}_{i,t-1}|$$

$MS^{SaleG}_{i,t-1}$ is the lagged subjective mean for the firm i about the sale growth rate. The expression of the size shock implies that the larger is the distance between the realization of the sale growth rates and its lagged subjective mean, the larger is the size shock. We can think of the size shock as the *realized uncertainty.* This definition of the size shock is in line with Bayesian Normal-Gamma learning that you will see in the next subsection. To be specific, you will see that the variance of posterior beliefs about the sale growth is a function of the distance between the realization of the sale growth rates and the mean. Now, I study the impact of the size shock on the subjective uncertainty through the following regression:

$$\begin{aligned} SDS^{SaleG}_{i,t} = {} & \beta SizeShock_{i,t} + \rho SDS^{SaleG}_{i,t-1} \\ & + \lambda SizeShock_{i,t} Obs_{i,t} + \gamma Obs_{i,t} + F_i + G_{jt} + \epsilon_{i,t} \end{aligned} \tag{1.1}$$

$SDS^{SaleG}_{i,t}$ is the subjective uncertainty of firm i at the time t. $SDS^{SaleG}_{i,t-1}$ is the lagged subjective uncertainty and controls for its persistence. $Obs_{i,t}$ controls for the number of the observations of the sale growth rates by the firm i at the time t; in other words, it controls for the information set's size. F_i is the firms' fixed effect and G_{jt} is the sector-time fixed effect[8]. In the table 1.1, you can find the result of the regression.

The first row of the table 1.1 shows the impact of the size shock on the subjective uncertainty. As you see, the effect of the size shock on the subjective uncertainty is significantly positive. Time varying uncertainty models such as GARCH or learning uncertainty can result in this finding. We also observe significant positive persistence in the subjective uncertainty.

Is the impact of the realized uncertainty on subjective uncertainty constant over the time or does it change as firms observe more data? As you can see in the

[8]Because of the Nickell bias in the primary study, as I only have limited number of observations for each firm, in the appendix I will also study the same regression without firms' fixed effect and obtain the same result.

VARIABLES	$SDS_{i,t}^{SaleG}$
$SizeShock_{i,t}$	0.0714***
	(0.00511)
$SDS_{i,t-1}^{SaleG}$	0.0454***
	(0.0111)
$SizeShock_{i,t}Obs_{i,t}$	-0.00168***
	(0.000275)
$Obs_{i,t}$	0.000266
	(0.000397)
Observations	9,688
R-squared	0.619

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 1.1: Impact of the realized uncertainty on the subjective uncertainty.

third row of the table 1.1, the interaction between the size shock and the number of observations has a significant negative effect on the subjective uncertainty. To rephrase it, the larger is the information set's size the lower will be the responsiveness of the subjective uncertainty to the size shock. In order to study deeper the impact of the number of observations on the subjective uncertainty responsiveness to the size shock, I divide firms into two groups of young and old firms[9] and study the impact of the size shock on the subjective uncertainty for the both groups:

$$\begin{aligned} SDS_{i,t}^{SaleG} = & \left[\beta + \beta' d_{i,t}^{Old} + \lambda Obs_{i,t}\right] SizeShock_{i,t} \\ & + \rho SDS_{i,t-1}^{SaleG} + \gamma Obs_{i,t} + F_i + G_{jt} + \epsilon_{i,t} \end{aligned}$$

This regression is exactly same as (1.1) with the only difference that I introduce the interaction with the dummy to study the impact of the number of the observation on the responsiveness of the subjective uncertainty to the realized uncertainty. In the table 1.2 you can find the result of the regression[10].

Second row of the table 1.2 reconfirms that as the size of the information set increases, the responsiveness of the subjective uncertainty with respect to the re-

[9]To be more clear, I define $E_j\left(Obs_{i,t}^x\right)$ as the average number of observations of the variable x in the sector j over the whole sample time and I use this threshold to specify whether the firm i at the time t is young or old:

$$d_{i,t}^{Old} = 1 \quad , \quad if \quad Obs_{i,t}^x > E_j\left(Obs_{i,t}^x\right)$$

[10]In the appendix in order to control for the Nickell bias and as a robustness check, as it was discussed before, I also studied the same regression without firms' fixed effect.

VARIABLES	$SDS_{i,t}^{SaleG}$
$SizeShock_{i,t}$	0.0717***
	(0.00511)
$d_{i,t}^{Old}SizeShock_{i,t}$	-0.0198**
	(0.00847)
$SDS_{i,t-1}^{SaleG}$	0.0448***
	(0.0111)
$SizeShock_{i,t}Obs_{i,t}$	-0.000891**
	(0.000436)
$Obs_{i,t}$	0.000203
	(0.000398)
Observations	9,688
R-squared	0.619

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 1.2: Information set size and responsiveness of subjective uncertainty to realized uncertainty.

alized uncertainty decreases. This finding is a distinct feature of the learning mechanism.

1.3 Subjective uncertainty and learning uncertainty

As it was shown in the previous subsection, the subjective uncertainty of firms is time varying and has a significant positive correlation with the size shock. In other words, there is a positive relationship between the realized uncertainty and the subjective uncertainty. Models of time varying uncertainty such as GARCH can results in this finding.
However, the fact that the responsiveness of the subjective uncertainty to the realized uncertainty is decreasing in the number of observations (or the information set size) is the novel finding which is a distinct feature of learning models. Over the time and by observing more and more data, assuming that firms learn uncertainty, their responsiveness to the size shock will decrease as their prior become more and more precise. Firms by observing the data from the data generating process can learn the whole distribution of the data generating process. In this paper, I concentrate on the second moment and in particular on the parametric second moment (uncertainty) learning, specifically Bayesian Normal-Gamma learning. In this subsection, after presenting Bayesian updating mechanism as a possible driver of subjective uncertainty, I compare the subjective uncertainty of firms with

their posterior beliefs' uncertainty.
Let us assume the variable $x_{i,t}$ in each period conditional on the known μ_i^x and the unknown $\sigma_{x,i}^2$ is drawn from the distribution $N\left(\mu_i^x, \sigma_{x,i}^2\right)$ and the firm i is uncertain about the value of $\sigma_{x,i}^2$ while It has perfect information about μ_i^x. After observing realizations of $x_{i,t}$ over the time the firm can learn the unknown variance. By assuming Gamma prior for the precision $\theta_i^x = 1/\sigma_{x,i}^2$, we preserve the conjugacy of Bayesian learning. To rephrase it, by assuming that the precision θ_i^x is initially drawn from the $Gamma(\alpha_{i,0}^x, \beta_{i,0}^x)$ distribution which corresponds to the firm's prior[11], after observing the history of realizations of the shock which is denoted by the information set $I_{i,t}^x = \{x_{i,1}, x_{i,2}, ..., x_{i,t-1}\}$ the posterior of the firm about the precision will preserve Gamma distribution and in particular will be $Gamma(\alpha_{i,t}^x, \beta_{i,t}^x)$. Moreover, the conditional posterior distribution of the underlying variable $x_{i,t}$ will preserve normality. That is an interesting feature of the Normal-Gamma Bayesian learning which makes the learning process very tractable. The details of the beliefs updating in the Normal-Gamma Bayesian learning are provided below:

- Prior:

$$x_{i,t}|\theta_i^x \quad \overset{iid}{\sim} \quad N\left(\mu_i^x, \frac{1}{\theta_i^x}\right) \quad , \quad \theta_i^x \quad \sim \quad Gamma\left(\alpha_{i,0}^x, \beta_{i,0}^x\right)$$

- Posterior:

$$x_{i,t}|\theta_i^x, I_{i,t}^x \sim N\left(\mu_i^x, \frac{1}{\theta_i^x}\right) \quad , \quad \theta_i^x|I_{i,t}^x \sim Gamma(\alpha_{i,t}^x, \beta_{i,t}^x) \tag{1.2}$$

$$\alpha_{i,t}^x = \alpha_{i,t-1}^x + \frac{1}{2} \quad , \quad \beta_{i,t}^x = \beta_{i,t-1}^x + \frac{(x_{i,t-1} - \mu_i^x)^2}{2}$$

As it is indicated above both the prior and the posterior of $x_{i,t}$ are conditionally normal. The unconditional distribution will be Student's t-distribution which has a fatter tail:

$$x_{i,t}|I_{i,t}^x \quad \sim \quad t_{2\alpha_{i,t}^x}\left(\mu_i^x, \frac{\beta_{i,t}^x}{\alpha_{i,t}^x}\right)$$

From the equation (1.2) we can see that $\alpha_{i,t}^x$ evolve deterministically regardless of the realization of the shocks, however, $\beta_{i,t}^x$ is an increasing function in the distance between the realized shock and the mean. The larger is the distance, the larger will be $\beta_{i,t}^x$. The variance of the posterior beliefs in period t for the firm i about the underlying variable x can be expressed by:

$$\textit{Posterior beliefs' uncertainty:} \qquad \widetilde{\sigma}_{x,i,t}^2 = \frac{\beta_{i,t}^x}{\alpha_{i,t}^x - 1}$$

[11]Rational expectation assumption.

$$= \left[\frac{\beta^x_{i,t-1} + \frac{\left(x_{i,t-1}-\mu^x_i\right)^2}{2}}{\alpha^x_{i,t-1} - 1}\right]\left[\frac{\alpha^x_{i,t-1} - 1}{\alpha^x_{i,t-1} - \frac{1}{2}}\right]$$

$$= \left[\widetilde{\sigma}^2_{x,i,t-1} + \frac{\left(x_{i,t-1}-\mu^x_i\right)^2}{2(\alpha^x_{i,t-1} - 1)}\right]\left[\frac{\alpha^x_{i,t-1} - 1}{\alpha^x_{i,t-1} - \frac{1}{2}}\right]$$

$$= W^{Persist}_{x,i,t}\widetilde{\sigma}^2_{x,i,t-1} + W^{Shock}_{x,i,t}\left(x_{i,t-1}-\mu^x_i\right)^2$$

$W^{Persist}_{x,i,t}$ is the weight for the persistence term which is increasing over the sample size and $W^{Shock}_{x,i,t}$ is the weight for the size shock which is decreasing in the sample size; t. By assuming that the sample size is large enough, $t \to \infty$ then $\alpha^x_{i,t-1} \to t$, and we can approximately estimate posterior beliefs' uncertainty for the firm i at the time t about the variable x by $\widetilde{\sigma}^2_{x,i,t} \simeq \widetilde{\sigma}^2_{x,i,t-1} + \frac{\left(x_{i,t-1}-\mu^x_i\right)^2}{2t}$.
As you can see the variance of the unconditional posterior about the underlying variable is increasing in $\beta^x_{i,t}$. Moreover, the impact of the distance of the shock from the mean, or in other words realized uncertainty, on the posterior belief is decreasing over the time as firms receive more information and their priors become more precise. I also rewrote the estimated expression for the posterior beliefs' uncertainty for large values of t and $\alpha^x_{i,t}$. As you can see the posterior beliefs' uncertainty can approximately be expressed as the lagged posterior beliefs' uncertainty plus the new realization of the size shock divided by the number of observations. Please note that up to now I assumed the precision term is constant, is drawn once and is not time varying. However, the belief about the precision is time-varying and changes over the time by observing new data. Therefore, the mentioned learning process is ergodic. To put it in another way, by observing more and more data the true value will eventually be learnt. Given the limited number of observations that I have for each firm in the survey data and also to preserve the tractability of the learning process, I primarily concentrate on the ergodic learning. In the appendix, following Bakshi and Skoulakis (2010), Weitzman (2007) and Shephard (1994) I will introduce non-ergodic learning by applying Beta shocks to the precision term.
As it was mentioned before, I would like to compare subjective uncertainty of firms with their posterior beliefs' uncertainty and study their relationship. To do so, in the first step I need to build posterior beliefs about sale growth for all firms. In the survey of business uncertainty, I have data about the history of realized sale growth rates in the past. So by calibrating the initial priors' parameters and by applying the history of shocks to the Bayesian updating mechanism (1.2), I can obtain the time varying posterior parameters for each firm.
There are three parameters to be calibrated. I need to calibrate the mean values of realized shocks; μ^{SaleG}_i, and two initial prior parameters for each firm; $\alpha^{SaleG}_{i,0}$ and

$\beta_{i,0}^{SaleG}$. I target the average values of the realized sale growth rates for each firm i over the whole sample to calibrate μ_i^{SaleG} [12]. I target firm-level first and second moments of the subjective uncertainty for each firm i [13] to calibrate $\alpha_{i,0}^{SaleG}$ and $\beta_{i,0}^{SaleG}$.

After calibrating prior parameters' values and feeding Bayesian updating mechanism (1.2) with the history of the realized sale growth rates, I find posterior parameters' values $\alpha_{i,t}^{SaleG}$ and $\beta_{i,t}^{SaleG}$. Then, from the above mentioned formula, I easily find the posterior beliefs uncertainty about sale growth rates for each firm $\widetilde{\sigma}^2_{SaleG,i,t}$. In order to compare subjective uncertainty of each firm i with its posterior beliefs' uncertainty and study their relationship, I run the following regression[14]:

$$SDS_{i,t}^{SaleG} = \beta\widetilde{\sigma}_{SaleG,i,t} + \epsilon_{i,t} \tag{1.3}$$

Please note here I do not control for any firm or sector specific variables as I want to compare my learning based estimation and reported subjective uncertainty and see how well my estimation fit the data. The result of the regression is presented in the first column of the table 1.3. As you can see, there is a significant positive relationship between the subjective uncertainty and posterior beliefs' uncertainty. One may wonder why the estimated coefficient in the table 1.3 is less than 1? In order to answer this question I will study the relationship between subjective uncertainty and posteriors' uncertainty deeper by the end of this section.

Next, I compare Bayesian learning with the standard GARCH process and see which one fit the data better. To do so, I build time varying uncertainty that is derived from GARCH(1,1) estimation. The details of GARCH estimation is provided in the appendix. Next, I run the same regression as (1.3) with the only difference that I use time varying idiosyncratic uncertainty that is derived from GARCH estimation instead of Bayesian posteriors' uncertainty.

Does Bayesian learning perform better in predicting subjective uncertainty than GARCH? After comparing results in the first and second columns of the table 1.3 we notice that the responsiveness to the Bayesian posteriors' uncertainty is larger. Moreover, R-squared values for Bayesian estimation is larger than GARCH[15]. Therefore, in this simple comparison it seems that Bayesian learning fits data better than GARCH(1,1). Please note in this study I only compared Bayesian pos-

[12] $\mu_i^{SaleG} = \sum_t SaleG_{i,t}/T_i^x$ where T_i^{SaleG} corresponds to the total number of survey responses that I have for the firm i about the sale growth rates from 2014 to 2022.

[13] $E_i\left(SDS_{i,t}^{SaleG}\right)$ and $Var_i\left(SDS_{i,t}^{SaleG}\right)$ which corresponds to the firm-level mean and the variance of subjective uncertainty for each firm i over the whole sample T_i^{SaleG}.

[14] To make sure that the initial prior miscalibration does not affect the result, I drop all firms that I have less than 5 observations for them.

[15] BIC/AIC information criterion comparison also reconfirms that Bayesian learning performs better.

teriors' uncertainty with GARCH(1,1). Of course, orders of the GARCH process and calibration of Bayesian prior's parameters can affect the comparison's result. Optimal model selection is out of scope of this paper. It worth mentioning again that the fact that responsiveness of subjective uncertainty to the realized uncertainty is decreasing in the number of observations, is a unique feature of learning model that is absent in the GARCH estimation.

	Bayesian	GARCH
VARIABLES	$SDS_{i,t}^{SaleG}$	$SDS_{i,t}^{SaleG}$
$\widetilde{\sigma}_{SaleG,i,t}$	0.359***	0.210***
	(0.00387)	(0.00257)
Observations	9,829	9,829
R-squared	0.468	0.404

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 1.3: Driver of subjective uncertainty - Comparing posterior beliefs' uncertainty with time varying uncertainty estimated from GARCH(1,1) process.

In the figure 1.2, I plot subjective uncertainty and standard deviation of posterior beliefs about sale growth rates for two different firms from two different sectors[16], assuming that prior beliefs' uncertainty at each period t is equal to the lagged subjective uncertainty[17]. As you can see subjective uncertainty and posterior beliefs uncertainty follow each other very closely. Both firms have a spike in their uncertainty after the Covid crisis. The firm in the leisure and hospitality sector has a larger spike as they were more affected by the Covid crisis and the subsequent lockdown.

An interesting phenomenon that is observable in the data is the fact that posterior beliefs' uncertainty for most of the firms is above subjective uncertainty over the whole sample, even if we use lagged subjective uncertainty as the prior for learning. This finding that firms assign more weight to their prior when they experience large realized uncertainty and underreact to the realized uncertainty is in line with the overconfidence literature[18]. In the figure 1.3 you can see the histogram of the difference between the standard deviation of posterior beliefs and subjective uncertainties; $\widetilde{\sigma}_{SaleG,i,t} - SDS_{i,t}^{SaleG}$, for all firms over the whole sample[19]. As

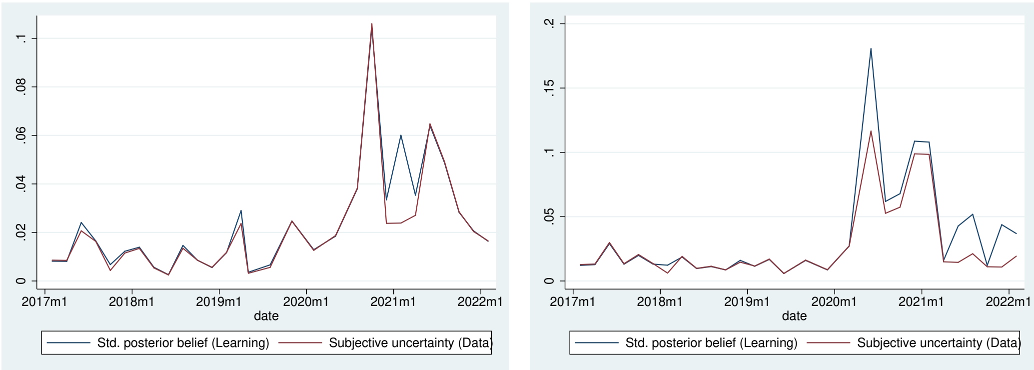
[16]Please note that due to the confidentiality of the survey of business uncertainty, firms' identity is antonymous.

[17]$\beta_{i,t-1}^{SaleG} = \left[SDS_{i,t-1}^{SaleG}\right]^2 \left(\alpha_{i,t-1}^{SaleG} - 1\right).$

[18]For example see Scheinkman and Xiong (2003).

[19]Here again I use lagged subjective uncertainty to pin down the prior for the Bayesian learning.

you can see the distribution is significantly right skewed with a large intensity at zero[20]. This finding justifies small values of coefficients in the table 1.3.



(a) A firm in finance and insurance sector . (b) A firm in leisure and hospitality sector.

Figure 1.2: Std. of posterior belief and subjective uncertainty.

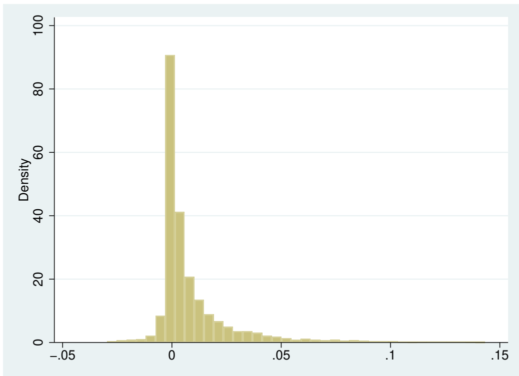


Figure 1.3: Histogram of the difference between std. of posterior beliefs and subjective uncertainties.

After validating Bayesian learning as a mechanism that can map idiosyncratic realized uncertainty to the idiosyncratic subjective uncertainty, in the next chapter I study what are the implications of learning uncertainty on the firms' decision making, in particular their investment decision.

1.4 Conclusion

In this chapter, I studied firms' idiosyncratic subjective uncertainty and its drivers. Using firm level survey of business uncertainty, I showed that the subjective un-

[20]Another reason for the right skewed distribution is miscalibration of prior parameters $\alpha_{i,0}$.

certainty about idiosyncratic sale revenue growth rate is time varying, responds significantly positively to the realized uncertainty and the conditional responsiveness to realized uncertainty decreases over the time. These findings suggest that realized uncertainty can be a potential driver of subjective uncertainty and the fact that the conditional responsiveness of subjective uncertainty to realized uncertainty decreases over the time validates *learning uncertainty* to be a potential mechanism that maps realized uncertainty to the subjective uncertainty. Using Normal-Gamma Bayesian learning and history of realized shocks, I built posterior beliefs for each firm. I showed that the variance of the posterior beliefs is a reliable predictor for subjective uncertainty. In the next chapter, I will study implications of learning uncertainty for firms' investment patterns.

Chapter 2

IMPLICATIONS OF LEARNING UNCERTAINTY

2.1 Introduction

In the previous chapter, I validated learning uncertainty as a potential driver of firms' subjective uncertainty. What are the implications of learning uncertainty for firms' behavior? Does learning uncertainty affect firms' investment patterns? In this chapter, I would like to answer theses questions. To do so, I first build a simple model of firms' investment with learning uncertainty and I show learning uncertainty results in three main implications. Next, I go to Compustat data to check whether we observe those implications in the data or not?

The model is an entrepreneurial partial equilibrium model in a rational expectation OLG environment. Agents are uncertain about the true realization of the variance of the idiosyncratic TFPR shocks data generating process[1]. However, over the time and by observing the history of TFPR shocks realizations they can learn the variance; or in other words learn the uncertainty. I use Bayesian Normal-Gamma learning. In order to keep the agents' utilities bounded and well-defined, following Weitzman (2007) and Bakshi and Skoulakis (2010), I use truncated inverse Gamma distribution for the variance of the log-TFPR.

In the model, risk averse agents have access to two different saving options for allocating their endowments; saving in a risk-less project or as an entrepreneur investing in a firm and obtain the risky profit. They would like to maximize the old age expected utility. How do agents allocate their wealth to different saving options? How does realized uncertainty affect the investment in this model? High realized uncertainty in firms' previous productivity, implies high subjective un-

[1] I assume idiosyncratic TFPR shocks conditional on a given variance follow log-Normal distribution with the known mean zero.

certainty about future productivity because of the learning channel. Due to the entrepreneur's risk aversion, high subjective uncertainty results in low investment in the risky project. After clarifying the main mechanism of the model, I study implications of learning uncertainty on firms' investment pattern.

In order to study implications of learning uncertainty on firms' investment pattern, I use impulse responses. Traditional impulse response approach in linear models does not work in my non-linear models with learning. In the model if I don't apply any shock except the shock of the interest, I will affect agents' beliefs because they are learning the exogenous process. To rephrase it, in my model of second moment learning, if I don't apply any other shock than the shock of the interest, as agents observe constant realizations of TFPR shocks, their posterior uncertainty eventually will converge to zero. In order to avoid this issue, following generalized impulse response approach, I redefine impulse responses and after that I study implications of learning uncertainty. I will show that learning uncertainty brings about three main implications.

The first implication of learning uncertainty is related to the environment that firms live in. Firms that live in a volatile environment have a lower investment response to TFPR shock of the same size compared to firms that live in a stable environment. The intuition is very simple. Living in a more volatile environment results in higher subjective uncertainty about the risky project. Therefore, the risky project is less attractive and risk-averse agents will allocate more their income to the safe project[2].

The second implication of learning uncertainty on the investment response concentrates on the size of the shock. Investment response to the large shocks are relatively lower than small shocks. The intuition for this finding is again related to the impact of the size of the shock on subjective uncertainty. Larger shocks implies higher subjective uncertainty because they are less probable or in other words they are closer to the tails. Higher subjective uncertainty results in lower investment response.

The third and the last implication of learning uncertainty on firms' investment pattern concentrates on the sign of the shock. Investment response to the positive shocks are relatively lower than negative shocks. When firms observe a positive shock, due to the persistence of the shock, their expectation about future realization of TFPR increases and at the same time their subjective uncertainty increases. These first and second moments effects of the positive shock have an opposing impact on the investment. To be more specific, higher expected TFPR increases the investment response while the higher subjective uncertainty decreases the invest-

[2]We can think of higher subjective uncertainty as a fatter tail posterior beliefs. A given large realization of the shock is less *black swan* for firms that lived in more volatile environment and as a result have higher subjective uncertainty. Lower investment response of firms with higher subjective uncertainty is in line with Nimark (2014) and Kozlowski et al. (2020).

ment response. On the other hand, when firms observe a negative shock, their expectation about future realization of TFPR decreases and again their subjective uncertainty increases. The first and second moments effects of the negative shock have the same direction negative impact on the investment. That is why the investment response to the negative shock is larger than the positive shock. The asymmetric response to symmetric shocks in my model is a result of the interaction between the first and the second moments' beliefs. Fajgelbaum et al. (2017) and Van Nieuwerburgh and Veldkamp (2006) obtain the similar form of asymmetry as a result of the endogenous uncertainty and procyclical precision about the economy's fundamentals.

Next, I study whether we observe three learning uncertainty's implications on investment pattern in the data or not? The main data that I use is the Compustat data. The first step is to estimate firm level productivity. In firm-level panels like Compustat, we have data about firms' revenues and expenditures instead of quantities and that is why I concentrate on the revenue based measure of total factor productivity (TFPR) instead of the quantity based measure of total factor productivity (TFPQ). I use cost share method to estimate production function's elasticities and then based on residual approach I estimate TFPR in the firm level. Next, following Castro et al. (2015), I extract TFPR shocks in the firm level which is the main explanatory variable in my study. The main dependent variable in my study is the firms' investment rate. It is defined as the ratio between the capital expenditure and the lagged book value of the tangible capital stock. Following Jeenas (2019), Ottonello and Winberry (2020) and Mongey and Williams (2017), I use the perpetual inventory method to construct a measure of the firm's capital stock.

In order to study implications of learning uncertainty on firms' investment pattern, I use local projection method to obtain impulse responses. I show that all three mentioned implications of learning uncertainty on firms' investment are observable phenomenon in the data. The second and third implications of learning uncertainty implies that firms' investment response to idiosyncratic TFPR shocks is asymmetric and S shaped. Ilut et al. (2018) find that employment growth rates responses to TFPR shocks is asymmetric and S shaped. I find the same pattern regarding the investment response.

Finally, assuming that Compustat firms are Bayesian learners, after building their time varying posterior uncertainty about idiosyncratic TPFR shocks using Normal-Gamma learning, I study the impact of their posterior uncertainty about TFPR shocks on the investment response to the TFPR shocks. I show that posterior uncertainty has a significant negative impact on the investment rate level which is in line with contractionary uncertainty shock literature. I also show that interaction with posterior uncertainty has a significant negative effect on firms' investment responsiveness to the idiosyncratic TFPR shocks. Moreover, controlling for posterior uncertainty affects three mentioned implications of learning uncertainty on investment patterns. To be specific, the difference between investment response of firms living in stable and volatile environment is less noticeable. The asymmetric S shape response is also less perceptible. All these findings reconfirm that learning uncertainty is a potential driver of three mentioned investment patterns.

2.2 Theory of learning uncertainty and investment

In the previous chapter, using firm level survey data, I showed that firms' subjective uncertainty is affected by the past realized uncertainty. I proposed Bayesian learning as a potential driver of subjective uncertainty and we saw that there is a significant positive correlation between Bayesian posterior beliefs' uncertainty and subjective uncertainty. After accepting learning uncertainty as a mechanism that drives subjective uncertainty, I would like to study its implications on firms' behavior and in particular their investment decision. To do so, I first build a simple entrepreneurial model of firm investment with learning uncertainty.

2.2.1 Model

The model is an entrepreneurial partial equilibrium model of firms' investment in a rational expectation OLG environment. In each period t two generations live; young and old. Young generation in period t allocates the endowment e between two different saving options; saving in a risk-free project or as an entrepreneur investment in a firm. Old generation consumes the returns from his young period's saving.

Firms:
The firm i at the time t using capital produces the good $Y_{i,t}$ through the Cobb-Douglas production function:

$$Y_{i,t} = \widetilde{A}_{i,t} K_{i,t}^{\alpha_K}$$

$\alpha_K < 1$ is capital share. The value of $\widetilde{A}_{i,t}$ measures the quantity based measure of total factor productivity of firm i at the time t or in other words it refers to TFPQ. Let us define revenue based measure of total factor productivity (TFPR) as[3]:

$$P_{i,t}\widetilde{A}_{i,t} = A_{i,t} = e^{z_{i,t}} \quad , \quad z_{i,t} = \rho_{i,0} + \rho_i z_{i,t-1} + \eta_{i,t} \quad , \quad \eta_{i,t}|\theta_i \overset{iid}{\sim} N\left(0, \frac{1}{\theta_i}\right)$$

[3] In my model I concentrate on TFPR instead of TFPQ because later on using Compustat data I estimate TFPR shocks in the firm-level. I will discuss in details why I use TFPR instead of TFPQ.

Young and old agents:

The young agent i of generation t is endowed with endowment e. He does not consume anything in the young age and only takes care of old age consumption. He has access to two saving options; saving in the risk-less project that yields the risk-free rate R in the period $t+1$ or as an entrepreneur investing in the firm i and obtain the risky profit $\pi_{i,t+1}$. The young agent allocates his endowment between these two saving technologies so his budget constraint is:

$$\underbrace{B_{i,t+1}}_{\textit{Investment in Risk-Free Project}} \quad + \quad \underbrace{K_{i,t+1}}_{\textit{Investment in Capital for Production at t+1}} \quad = \quad e$$

and his old age nominal budget constraint is:

$$C^{Old}_{i,t+1} = RB_{i,t+1} + \pi_{i,t+1} = RB_{i,t+1} + A_{i,t}K^{\alpha_K}_{i,t}$$

and he maximizes his old age utility:

$$\max_{C^{Old}_{i,t+1}} \quad E_{i,t}\left[U(C^{Old}_{i,t+1})\right], \quad st. \quad \textit{Budget Constraints}$$

$E_{i,t}$ refers to the expectation of the young agent i given the information set available to him at the time t which is denoted by $I_{i,t}$ [4]. I assume the standard CRRA utility function $U(C) = \frac{C^{1-\gamma}-1}{1-\gamma}$.

Information friction and learning:

Agents have perfect and complete information about the structure of the environment, except about the firms' TFPR shocks' precision term θ_i. Agents are uncertain about the true realization of the variance of the distribution of the TFPR shocks $\eta_{i,t}$. However, over the time and by observing the realizations of $\eta_{i,t}$ agents can learn its value. As it was mentioned before we already know:

$$\eta_{i,t}|\theta_i \quad \overset{iid}{\sim} \quad N\left(0, \frac{1}{\theta_i}\right) \tag{2.1}$$

The precision term $\theta_i = 1/\sigma_i^2$ is initially drawn from the following distribution:

$$\theta_i \quad \sim \quad Truncated - Gamma\left(\alpha_{i,0}, \beta_{i,0}, \underline{\vartheta}_i, \overline{\vartheta}_i\right)$$

$\alpha_{i,0}$ and $\beta_{i,0}$ are the shape and rate parameters of the standard Gamma distribution. $\underline{\vartheta}_i$ and $\overline{\vartheta}_i$ are lower and upper truncation thresholds. They define *the finite support* of the realized precision θ_i. Following Bakshi and Skoulakis (2010) and Weitzman (2007) I use truncated Gamma distribution, instead of standard Gamma

[4] Please note that $E_{i,t}[.]$ is the rational expectation operator.

distribution, in order to have well defined and bounded expected utility[5]. Because of the conjguacy of learning process, after observing the history of realized TFPR shocks $I_{i,t} = \{\eta_{i,1}, \eta_{i,2}, ..., \eta_{i,t-1}\}$ the posterior beliefs will preserve truncated Gamma distribution:

$$\theta_i|I_{i,t} \quad \sim \quad Truncated - Gamma\left(\alpha_{i,t}, \beta_{i,t}, \underline{\vartheta}_i, \overline{\vartheta}_i\right)$$

$$\alpha_{i,t} = \alpha_{i,t-1} + \frac{1}{2} \quad , \quad \beta_{i,t} = \beta_{i,t-1} + \frac{\left(\eta_{i,t-1} - \mu_i\right)^2}{2}$$

The conditional predictive distribution for $\eta_{i,t}$ preserves the above mentioned normal distribution and the unconditional distribution will follow:

$$\eta_{i,t}|I_{i,t} \quad \sim \quad Dampened - t\left(v_{i,t}, \underline{\xi}_{i,t}, \overline{\xi}_{i,t}\right)$$

$$v_{i,t} = 2\alpha_{i,t} \quad , \quad \underline{\xi}_{i,t} = 2\underline{\vartheta}_i\beta_{i,t} \quad , \quad \overline{\xi}_{i,t} = 2\overline{\vartheta}_i\beta_{i,t}$$

The unconditional predictive distribution is dampened Student's t-distribution, instead of the standard Student's t-distribution, because of using truncated Gamma distribution instead of the standard Gamma distribution. The probability density function of the mentioned dampened Student's t-distribution is [6]:

$$p^{DT}\left(y_{i,t}\right) = \frac{\gamma\left[\frac{v_{i,t}+1}{2}, \frac{\overline{\xi}_{i,t}}{2}\left(1 + \frac{y_{i,t}^2}{v_{i,t}}\right)\right] - \gamma\left[\frac{v_{i,t}+1}{2}, \frac{\underline{\xi}_{i,t}}{2}\left(1 + \frac{y_{i,t}^2}{v_{i,t}}\right)\right]}{\sqrt{\pi v_{i,t}}\left(\gamma\left[\frac{v_{i,t}}{2}, \frac{\overline{\xi}_{i,t}}{2}\right] - \gamma\left[\frac{v_{i,t}}{2}, \frac{\underline{\xi}_{i,t}}{2}\right]\right)\left(1 + \frac{y_{i,t}^2}{v_{i,t}}\right)^{\frac{v_{i,t}+1}{2}}}$$

Following the same discussion as in the previous chapter, in the baseline study I assume ergodic learning. The precision term is only drawn once and by observing data over the time its value will eventually be learnt. However, one can easily replace it with the non-ergodic learning process that is presented in the appendix[7].

[5] As it was discussed in the previous chapter, if the precision term θ_i is drawn from the standard Gamma distribution, the unconditional distribution of the variable $\eta_{i,t}$ will be Student's t-distribution. We know that the moment generating function for Student's t-distribution is undefined. Therefore, as TFPR in my model follows conditional log-normal distribution, the expectation term in the model will not exist if θ_i is drawn from the standard Gamma distribution.

[6] $y_{i,t} = \frac{\eta_{i,t}}{\sqrt{\beta_{i,t}/\alpha_{i,t}}}$ is the scaled TFPR shock and $\gamma[a, j] = \int_0^j u^{a-1}e^{-u}du$ is the lower incomplete Gamma function for $a > 0$ and $j > 0$.

[7] Please note there are available state of art models such as Gilchrist et al. (2014) that builds a quantitative general equilibrium model, featuring heterogeneous firms that face time-varying idiosyncratic uncertainty which generates contractionary uncertainty shocks effect on firms' investment. The reason that I avoid using those models is to preserve tractability of the model in an environment with learning uncertainty featuring truncated distributions.

Equilibrium:

The partial equilibrium is defined such that given prices and the information, young agents allocate their endowment optimally between the risk-free project and the investment in firms. The capital supply is pinned down by the following equilibrium condition:

$$RE_{i,t}\left[\left(C^{Old}_{i,t+1}\right)^{-\gamma}\right] = E_{i,t}\left[\left(C^{Old}_{i,t+1}\right)^{-\gamma} r_{i,t+1}\right]$$

$r_{i,t+1} = \alpha_K A_{i,t+1} K^{\alpha_K - 1}_{i,t+1}$ refers to the marginal product of capital of the firm i in the next period. High realized uncertainty in firms' previous productivity, implies high subjective uncertainty about future productivity because of the learning channel. Due to the entrepreneur's risk aversion, high subjective uncertainty results in low investment in the risky project. If agents were risk neutral; or in other words $\gamma = 0$, because of the well-known Oi-Hartman-Abel[8] effect and the complementarity channel between the capital and productivity, higher subjective uncertainty will result in higher allocation of endowment to the risky project or in other words higher investment. However, by introducing risk averse entrepreneurs, the risk aversion effect dominates the Oi-Hartman-Abel effect and higher subjective uncertainty results in lower investment.

2.2.2 Implications of learning uncertainty

In order to study the implications of second moment learning on firms' investment I use the *impulse response* approach. Impulse responses are useful tools for comparing the impact of different shocks of interest on the model. Traditional impulse response approach in linear models does not work in my non-linear model with learning. In linear models, we usually apply an exogenous shock of interest and study the impact of that shock on the variable of interest without applying any shock afterward. However, in non-linear models, especially those featuring learning, if we don't apply any shock except the shock of the interest, we are affecting agents' beliefs because they are learning the exogenous process. To put it in another way, in my model of second moment learning, if I don't apply any other shock than the shock of interest, as agents observe constant realizations of TFPR shock which are equal to the mean zero, their posterior's uncertainty eventually will converge to zero.

In order to solve this problem, following generalized impulse response method[9], I modify the traditional impulse response approach. In addition to the study case in which I would like to study the impact of a special shock of interest, I define

[8] see Oi (1961) , Hartman (1972) and Abel (1983).

[9] see Koop et al. (1996).

a new case as the *reference scenario*. In the reference scenario agents share the same prior as the study case. Moreover, they receive the exact same realizations of the TFPR shock from the conditional Normal distribution (2.1) as the study case, except at the given period t^* that I apply my shock of interest η^* in the study case. Instead, in the reference scenario, agents at the period t^* receive the shock which is equal to the mean of the distribution which is assumed to be zero. I define the impulse response of the investment to the shock η^* as the average values of $N = 1000$ times simulations of log-difference of $K_{i,t+1}$ between the study case and the reference scenario divided by the size of the shock itself [10] , [11]:

$$\%\Delta K_{i,t+1}^{\eta^*} = IRF^{\eta^*}(K_{i,t+1}) = \frac{E_N\left[log\left(K_{i,t+1}^{StudyCase}\right) - log\left(K_{i,t+1}^{Reference}\right)\right]}{|\eta^*|}$$

Three main implications of learning uncertainty that I study through impulse responses are investment responses of firms that live in stable versus volatile environments, investment responses to large versus small shocks and finally investment responses to positive versus negative shocks.

Calibration

Although the model is very simple and I only use it for qualitative illustrations and not the quantitative analysis, I choose parameter values that are in line with my empirical studies in the previous chapter and next section. I calibrate the model in the following way.

I choose values for elasticity $\alpha_K = 0.11$ which are equal to average values of estimated capital share following the cost share approach[12]. This value is also in line with average capital share found by De Loecker et al. (2020). I choose CRRA risk aversion $\gamma = 8$ which is a pretty high value. The reason for choosing this value is to obtain impulse responses that are visibly distinguishable from each other. In the appendix I study the effect of using lower values for the risk aversion. I choose quarterly risk-free rate $R = 1 + 0.5\%$. Regarding the TFPR process, I choose the constant term $\rho_{i,0} = 0.1$ and the persistence term $\rho_i = 0.7$ [13]. Regarding the

[10]Please note I divide the log-difference by the size of the shock because I would like to study the *responsiveness* of the investment to the shock regardless of its size.

[11]The operator $E_N[.]$ in the definition refers to the average values over $N = 1000$ times simulations. I take average values over simulation to make sure the result is not driven by the randomness of draws from the Normal distribution.

[12]The estimation of labor share and capital share is going to be discussed in details in the next section.

[13]Following the residual approach that is going to be explained in details in the next section, I estimate TFPR in firm level. I have fitted an AR(1) process and then I choose the average value for the constant term and the persistence term.

learning parameters, I choose $\alpha_{i,0} = 3.2$ and $\beta_{i,0} = 0.01$ which are equal to their corresponding average estimated values using sale growth subjective uncertainty from survey of business uncertainty. Targeting prior parameters is discussed in details in the previous chapter. I run simulations with the true realized variance value $\sigma^2_{i,0} = 1/\theta_{i,0} = \beta_{i,0}/\alpha_{i,0}$. Finally, I choose the variance truncation values $\underline{\vartheta}_i = 10^{-10}$ and $\overline{\vartheta}_i = 10^{+10}$ to make sure that I cover all possible positive support values for the variance.

Stable versus volatile environment

Learning uncertainty is a mechanism that maps realized uncertainty to subjective uncertainty and we know that firms' subjective uncertainty affects their investment decision. So the first implication of learning uncertainty on firms' investment that I study is the impact of living in a more volatile environment; or in other words experiencing more volatile TFPR shocks, on firms' investment.

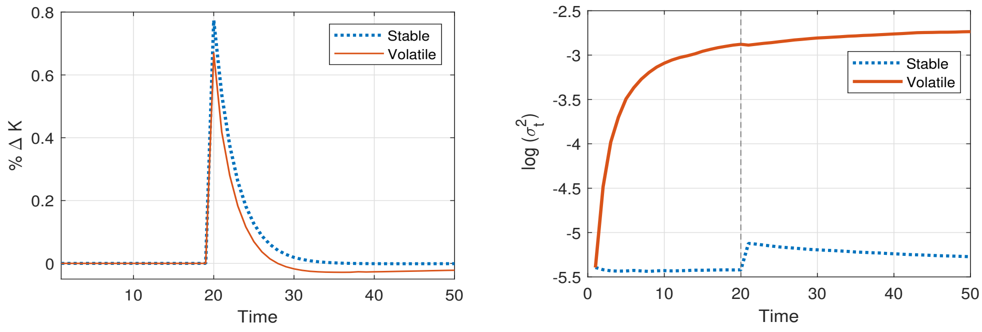
To do so, I define two different firms; a stable firm that receives TFPR shocks which are drawn from the distribution that has the standard deviation $\sigma_s = \sigma_{i,0}$ and a volatile firm that receives TFPR shocks which are drawn from the distribution that has the standard deviation $\sigma_v = 4\sigma_{i,0}$. Two firms share the similar prior initially. At a given period, both firms receive a similar large shock. To be more specific, at the period $t^* = 20$, I apply a shock that is 3 std. away from the mean; $\eta^* = 3\sigma_{i,0}$.

In the panel (a) of the figure A.3, I plot the investment impulse responses to the similar large shock for both stable and volatile firms. As you see, the conditional impulse responses of the stable and volatile firms are different[14]. In particular, the stable firm responds more to the given shock. In order to understand the intuition we need to study firms' posterior beliefs and in particular their posterior uncertainty. In the panel (b) of the figure A.3, I plot the logarithm of the posterior uncertainty for the both stable and volatile firms. As you see, living in a more volatile environment brings about higher posterior's uncertainty and as a result higher subjective uncertainty for the volatile firm. Higher subjective uncertainty makes the risky investment project less desirable for the risk averse agent and that is why he responds less to similar shock. Moreover, higher subjective uncertainty implies fatter tail posterior beliefs. As a result a large shock of the same size is less extreme, less informative and less *black swans* for the firm who lived in the more volatile environment.

In order to clarify that learning uncertainty results in mentioned different responses, in the figure 2.2, I plot the impulse response of investment in the case

[14] I obtain the same result for the negative shock. I will study the impact of the sign of the shock in details in the following subsections.

that both firms have perfect common information about the underlying uncertainty and they are not learning. As you see both firms respond identically to the given shock when they do not learn the uncertainty.



(a) Investment impulse responses.

(b) Posterior uncertainty.

Figure 2.1: Stable versus volatile environment - with learning uncertainty.

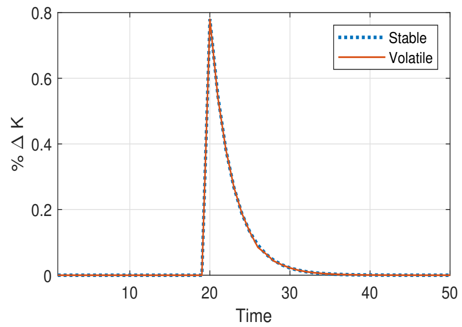
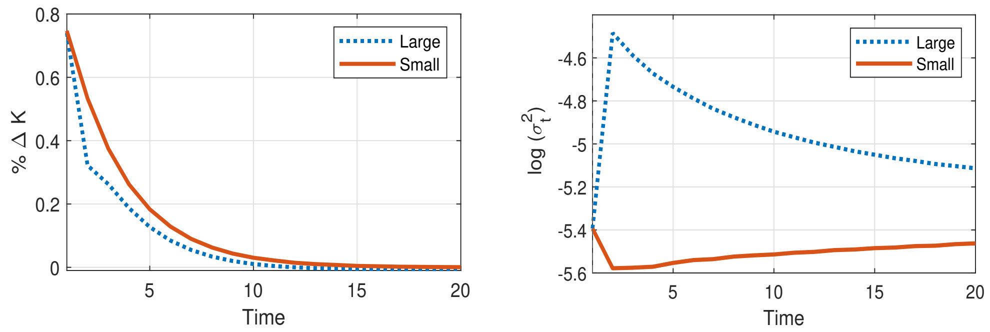


Figure 2.2: Stable versus volatile environment - without learning uncertainty.

Small versus large shocks

In the previous subsection, I studied the impact of the same shock on two different firms' investment that live in two different stable and volatile environments. In the following two subsections, however, I am going to study the impact of two different shocks on the same firm. Here, I concentrate on the size of the shock to study whether firms respond similarly to the small versus large shocks.
To do so, I run the simulation for a firm two times; once by applying a small shock $\eta^* = 0.3\sigma_{i,0}$ at the initial period $t^* = 1$ and another time by applying a large shock $\eta^* = 3\sigma_{i,0}$ initially, and then I study the impulse responses. You can find the impulse responses in the panel (a) of the figure A.5. As you see

the impulse responses of investment to the large shocks are lower than the small shocks. Again in order to understand the intuition, we need to study the posterior uncertainty. In the panel (b) of the figure A.5 I plot the logarithm of the posterior uncertainty for both simulations. As you see, the large initial shock has a significant positive impact on the subjective uncertainty. Higher subjective uncertainty results in lower investment afterward.
In order to clarify that learning uncertainty is the mechanism that brings about this result, in the figure 2.4, I plot the impulse response of investment in the case that there is no learning uncertainty. As you see impulse responses for small and large shocks are identical.



(a) Investment impulse responses.

(b) Posterior uncertainty.

Figure 2.3: Small versus large shocks - with learning uncertainty.

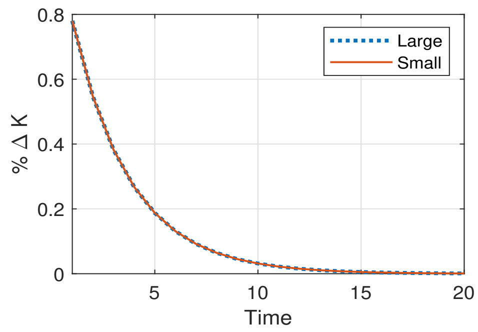


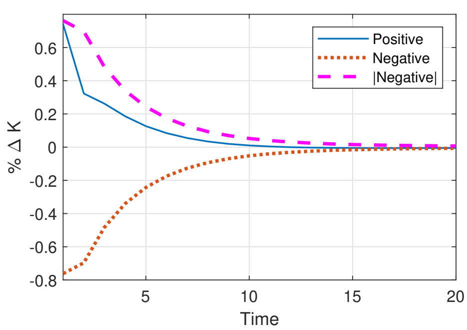
Figure 2.4: Small versus large shocks - without learning uncertainty.

Positive versus negative shocks

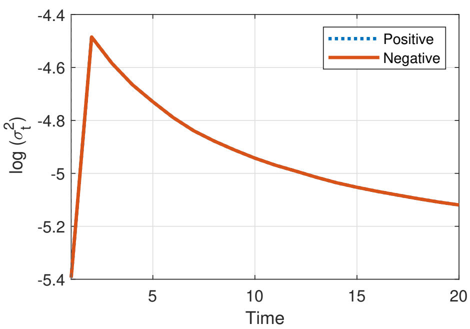
In the previous subsection, I studied the impact of the *size* of the shock on the investment response. In this subsection, however, I concentrate on the *sign* of the shock to study whether firms respond symmetrically to the symmetric positive versus negative shocks. To do so, I run the simulation for a firm two times; once by applying a positive shock $\eta^* = 3\sigma_{i,0}$ at the initial period $t^* = 1$ and another time by applying a negative shock $\eta^* = -3\sigma_{i,0}$ initially, and then I study the impulse responses. Impulse responses are depicted in the panel (a) of the figure A.7. As you notice, impulse responses to symmetric positive and negative shocks are asymmetric. The mentioned asymmetry is a result of the learning uncertainty. When firms observe a positive shock, due to the persistence of the shock, their expectation about future realization of TFPR increases and at the same time their subjective uncertainty increases as you can see in the panel (b) of the figure A.7. These first and second moments effects of the positive shock have an opposing impact on the investment. To be specific, higher expected TFPR increases the investment response while the higher subjective uncertainty decreases the investment response.

On the other hand, when firms observe a negative shock, their expectation about future realization of TFPR decreases and again their subjective uncertainty increases. The first and second moments effects of the negative shock have the same direction negative impact on the investment. That is why the investment response to the negative shock is larger than the positive shock.

In order to clarify that learning uncertainty is the mechanism that brings about the asymmetric responses, in the figure A.8, I plot the impulse response of investment in the case that there is no learning uncertainty. As you see there is no asymmetric responses anymore.



(a) Investment impulse responses.



(b) Posterior uncertainty.

Figure 2.5: Positive versus negative shocks - with learning uncertainty.

I conclude this section by summarizing three main implications of learning uncertainty for the firm investment:

1. Firms that have lived in a more volatile environment have a lower investment response to large TFPR shock of the same size.

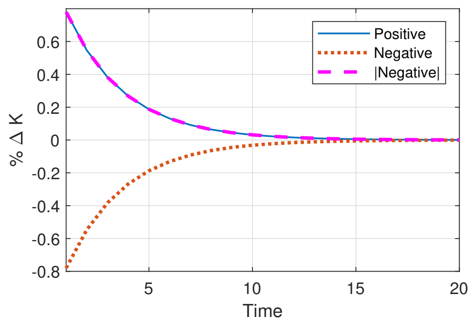


Figure 2.6: Positive versus negative shocks - without learning uncertainty.

2. Investment response to the large shocks are relatively lower than small shocks.

3. Investment response to the positive shocks are relatively lower than negative shocks[15].

In the next section, using Compustat data, I will empirically study firms investment response to TFPR shocks, and check whether we observe these three learning uncertainty implications in data or not.

2.3 Firms' investment response to the idiosyncratic TFPR shocks

In the previous chapter I showed that learning uncertainty in the micro-level is an observable phenomenon in data. Then, I introduced a simple model of firms' investment featuring learning uncertainty. I studied implications of learning uncertainty on firms' investment response to TFPR shocks. In this section using firm level data, I empirically study firms' investment response to the idiosyncratic TFPR shocks. Using local projections, I will show that all three mentioned implications of learning uncertainty for firms' investment are observable in the data. Before that, let us introduce the data that I use in addition to the main explanatory and dependent variables in my local projections.

Data:

I draw the firm-level dataset from the Compustat universe of publicly listed U.S. incorporated firms. I follow data cleaning procedure as Jeenas (2019) and Chiavari and Goraya (2021). More details about data cleaning is provided in the appendix.

[15]Points 2 and 3 imply that firms investment response to TFPR shocks are *asymmetric and S shaped.*

After cleaning, the panel has quarterly data from 1984 to 2020, including 22,256 firms in different sectors.

Dependent Variable:

The key dependent variable that I use in my study is the investment rate[16]. It is defined as the ratio between the capital expenditure[17] and the lagged book value of the tangible capital stock. Following Jeenas (2019), Ottonello and Winberry (2020) and Mongey and Williams (2017), I use the perpetual inventory method to construct a measure of the firm's fixed capital stock $K_{i,t}$. The details about the construction of the firms' capital stock is provided in the appendix.

Explanatory Variable:

The main explanatory variable that I consider is the revenue based measure of total factor productivity. In firm-level panels like Compustat, we have data about firms' revenues and expenditures instead of quantities and that is why I concentrate on the revenue based measure of total factor productivity (TFPR) instead of the quantity based measure of total factor productivity (TFPQ)[18]. I estimate TFPR following residual approach. Let us assume the firm i at the time t, produces the good $Y_{i,t}$ according to standard Cobb-Douglas production function:

$$Y_{i,t} = \widetilde{A}_{i,t} K_{i,t}^{\alpha_K^j} \left(LM_{i,t}\right)^{\alpha_L^j}$$

$\widetilde{A}_{i,t}$ is total factor productivity and $LM_{i,t}$ stands for the labor and materials used in the production and both have the same elasticity α_L^j. I assume that elasticities α_L^j and α_K^j are sector specific and same for all firms within the 2-digit SIC sector j. The firm's TFPR can easily be extracted from the following residual:

$$z_{i,t} = log\left(A_{i,t}\right) = log(Sale_{i,t}) - \alpha_L^j log(LM_{i,t}) - \alpha_K^j log(K_{i,t}) \tag{2.2}$$

$A_{i,t}$ stands for the firm i's revenue based measure of total factor productivity at the time t. $Sale_{i,t}$ is the firm i sale revenue at the time t. After deflating all variables by the price deflator $\texttt{IPDNBS}_{i,t}$ from FRED database, I use the item $\texttt{SALEQ}_{i,t}$ from Compustat for $Sale_{i,t}$, the item $\texttt{COGSQ}_{i,t}$ for $LM_{i,t}$ and the constructed capital stock for $K_{i,t}$. I estimate production elasticites α_K^j and α_L^j by using the cost share approach following Foster et al. (2008), De Loecker et al. (2020) and Chiavari and Goraya (2021)[19]. More details about determining elasticities are provided in the appendix.

[16]In order to avoid endogenity problem in my local projections, I use investment rates instead of widely used log-capital cumulative differences; because as you will see later I extract the explanatory variable TFPR shock using the residual approach.

[17]The item `CAPEX` in Compustat.

[18]Foster et al. (2008) discusses in details the difference between revenue based measure of total factor productivity and quantity based measure of total factor productivity in the firm level.

[19]Another standard approach for estimating elasticities is the production function estimation following Ackerberg et al. (2015). In my study two approaches result in very similar outcomes so I only take cost share approach into account.

Estimated TFPR values $A_{i,t}$ can be potentially serially auto correlated and also affected by many different firms' characteristics such as age, size, etc.. In order to extract the exogenous part of the TFPR process, following Castro et al. (2015), for each firm i at the time t in the sector j, I run the following regression:

$$z_{i,t} = \rho_j z_{i,t-1} + \gamma_j age_{i,t} + \lambda_j size_{i,t} + H_i + L_{j,t} + \eta_{i,t}$$

I control for the persistence in the TFPR process, age of the firm[20], size of the firm[21], firm's fixed effect H_i and sector-time fixed effect $L_{j,t}$. From now on I will call the residual $\eta_{i,t}$ the idiosyncratic TFPR shock. It is the main explanatory variable in my study. I drop all values of idiosycratic TFPR shocks below the first percentile and above the 99th percentile to control for outliers which might significantly affect the estimates. In the figure 2.7 I plot the histogram of idiosyncratic TFPR shocks. As you notice it is symmetrically distributed around zero.

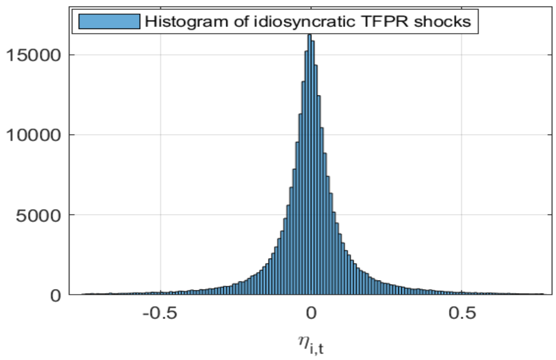


Figure 2.7: Idiosyncratic TFPR shocks $\eta_{i,t}$ distribution.

2.3.1 General investment response

In this subsection, I study general firms' investment response to the idiosyncratic TFPR shock. To confirm existence of implications of learning uncertainty for firms' investment in data, in next subsections, I divide firms into two groups of stable and volatile firms in order to study the impact of living in stable versus volatile environment. I also divide shocks into different quantiles in order to study the impact of the size of the shock and its sign on the investment responsiveness to the TFPR shock.

[20] I define age of the firm i as the number of quarters since the appearance of the firm in Compustat.

[21] I define size of the firm i in the sector j as $log(Sale_{i,t}) - log(Sale_{j,t})$ where $Sale_{j,t}$ is the total sum of sales in the sector j at the time t.

I follow Jorda (2005) local projection method to study firms' investment response to the TFPR shocks:

$$I_{i,t+h} = \beta_h \eta_{i,t} + \Gamma'_h W_{i,t-1} + \Lambda'_h W_{i,t-1}\eta_{i,t} + \sum_{m>0} \theta_h^m I_{i,t-m} + F_i + G_{j,t} + \epsilon_{i,t,h} \quad (2.3)$$

$I_{i,t+h}$ stands for the $h > 0$ periods ahead log-investment rate of the firm i; $I_{i,t+h} = log\,(\texttt{CAPEX}_{i,t+h}) - log\,(K_{i,t+h-1})$. $W_{i,t-1}$ is the vector of lagged control variables. I control for the size, age, liquid asset ratio, leverage ratio and also their interactions with the TFPR shock. $I_{i,t-m}$ controls for the serial correlation of dependent variable[22]. F_i is the firm fixed effect and $G_{j,t}$ is the sector-time fixed effect. The result of local projection for $h = 4$ is provided in the table 2.1. As you see TFPR shock $\eta_{i,t}$, lagged liquid asset ratio $liq_{i,t-1}$, lagged age and lagged size all have significant positive effects on the investment rate while lagged leverage ratio $lev_{i,t-1}$ has a significant negative impact on it[23]. In the figure 2.8, I plot general investment rate impulse response to the idiosyncratic TFPR shock for $h = 20$ periods ahead and corresponding 95$\%$ confidence intervals. Please note for impulse response figures in this section and upcoming ones, I eliminate the interaction of control variables with the shock, so that the local projection correctly illustrates the impulse response functions. Estimated coefficients of local projection and corresponding tables are provided in the appendix.

2.3.2 Implications of learning uncertainty

Stable versus volatile environment

In the previous section, after introducing a simple model of firms' investment with learning uncertainty, I studied implications of learning uncertainty for the firms' investment pattern. In the following subsections, I check if we empirically observe those implications in the data. The first implication of learning uncertainty on the investment pattern was firms that have lived in a more volatile environment have a lower investment response to TFPR shocks.

I divide firms into two different groups of stable firms and volatile firms. To do so, I first find the standard deviation of idiosyncratic TFPR shocks for each firm during the whole sample time $SD_i(\eta_{i,t})$. Then, I find the median standard deviation $Med\,[SD_i(\eta_{i,t})]$. In order to find the average values, in Compustat it

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[22] I choose $m = 6$ in all of my studies.

[23] After deflating variables, I define leverage ratio of the firm i as total debt divided by the total asset and the liquid asset ratio as cash and short term investments divided by the total asset: :

$$lev_{i,t} = log(\texttt{DLCQ}_{i,t} + \texttt{DLTTQ}_{i,t}) - log(\texttt{ATQ}_{i,t}) \quad , \quad liq_{i,t} = log(\texttt{CHEQ}_{i,t}) - log(\texttt{ATQ}_{i,t})$$

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.465***	0.537***	0.750***	0.713***
	(0.0904)	(0.0942)	(0.0974)	(0.102)
$liq_{i,t-1}$	0.0480***	0.0495***	0.0508***	0.0455***
	(0.00223)	(0.00232)	(0.00239)	(0.00249)
$lev_{i,t-1}$	-0.0436***	-0.0439***	-0.0415***	-0.0380***
	(0.00226)	(0.00235)	(0.00242)	(0.00252)
$age_{i,t-1}$	0.0228***	0.0263***	0.0322***	0.0353***
	(0.00314)	(0.00335)	(0.00357)	(0.00375)
$size_{i,t-1}$	0.148***	0.148***	0.149***	0.121***
	(0.00515)	(0.00540)	(0.00556)	(0.00586)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.566

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.1: General investment response to the TFPR shock.

is common to use median instead of mean because median is less responsive to extreme tail values. I define volatile firms as those firms whose idiosyncratic TFPR shocks' standard deviation is larger than the median value. Finally, I run the following local projection which is identical to (2.3) with the only difference that I introduce the interaction with the volatility dummy d_i^v:

$$d_i^v = 1 \quad if \quad SD_i(\eta_{i,t}) > Med\left[SD_i(\eta_{i,t})\right]$$

$$I_{i,t+h} = \beta_h \eta_{i,t} + \beta_h^v d_i^v \eta_{i,t} + \Gamma_h' W_{i,t-1} + \Lambda_h' W_{i,t-1} \eta_{i,t} + \sum_{m>0} \theta_h^m I_{i,t-m} + F_i + G_{jt} + \epsilon_{i,t,h}$$

The result of the local projection for $h = 4$ is provided in the table 2.2. As you can see, living in a more volatile environment results in a significant lower investment rate response to the TFPR shock[24]. This finding is in line with the first implication of learning uncertainty that was discussed in the previous section. In the figure 2.9, the impulse responses for $h = 20$ for the both stable and volatile firms are provided and as you see the impulse response for volatile firms is below stable firms. Please note for impulse response figures in this subsection and upcoming ones, same as before, I eliminate the interaction of control variables with

[24]Please note that the estimated coefficients for volatile firms (interacted with the dummy) is significant with p-values close to zero.

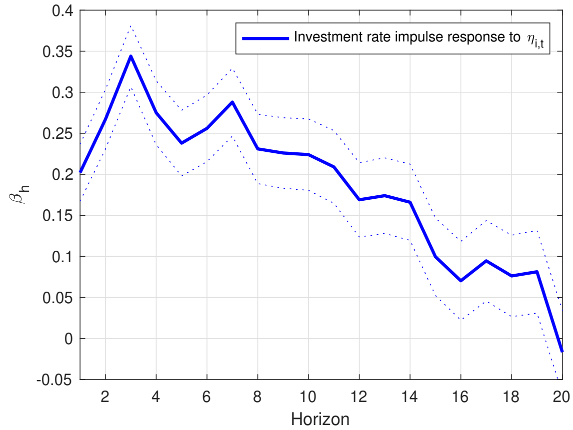


Figure 2.8: General investment response to the TFPR shock.

the shock, so that the local projection correctly illustrates the impulse response functions. Estimated coefficients of local projections and corresponding tables are provided in the appendix.

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.692***	0.936***	1.274***	1.247***
	(0.101)	(0.105)	(0.108)	(0.114)
$d_i^v \eta_{i,t}$	-0.285***	-0.499***	-0.656***	-0.671***
	(0.0553)	(0.0577)	(0.0592)	(0.0617)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.567

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.2: Investment response of firms who lived in stable versus volatile environment.

Small versus large shocks

The second implication of learning uncertainty for firms' investment pattern was investment response to the large shocks are relatively lower than small shocks. In order to see whether we observe this implication in data, I divide idiosyncratic TFPR shocks into two groups of small versus large shocks. Small shocks are

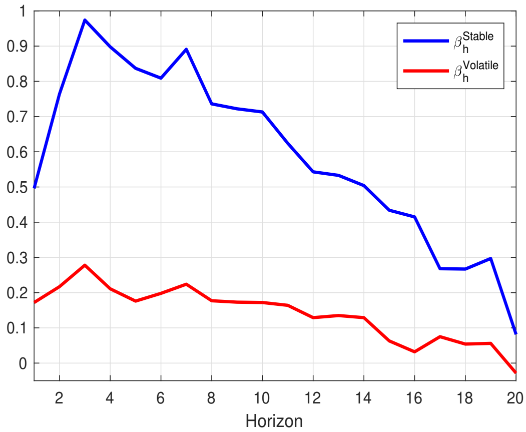


Figure 2.9: Investment response of firms who lived in stable versus volatile environment.

within 2 standard deviation of firms' idiosyncratic TFPR shocks $SD\left(\eta_{i,t}\right)$ [25] and large shocks are outside 2 standard deviation interval. Then I run the following local projection which is identical to (2.3) with the only difference that I introduce the interaction with the size dummy $d^l_{i,t}$:

$$d^l_{i,t} = 1 \quad if \quad |\eta_{i,t}| > 2SD\left(\eta_{i,t}\right)$$

$$I_{i,t+h} = \beta_h \eta_{i,t} + \beta^l_h d^l_{i,t} \eta_{i,t} + \Gamma'_h W_{i,t-1} + \Lambda'_h W_{i,t-1} \eta_{i,t} + \sum_{m>0} \theta^m_h I_{i,t-m} + F_i + G_{jt} + \epsilon_{i,t,h}$$

The result of the local projection for $h = 4$ is provided in the table 2.3. As you see, relative response to the large shocks are significantly lower than small shocks[26]. This is in line with the second implication of learning uncertainty. In the figure 2.10, the impulse responses for $h = 20$ for the both small and large shocks are depicted and as you notice the impulse response to small shocks are above large shocks[27].

Positive versus negative TFPR shocks

The third implication of learning uncertainty for firms' investment pattern was investment response to the positive shocks are relatively lower than negative shocks.

[25] Please note the standard deviation is among all firms over the whole sample.

[26] Please note that the estimated coefficients for large shocks (interacted with the dummy) is significant with p-values close to zero.

[27] Estimated coefficients of local projections and corresponding tables are provided in the appendix.

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.535***	0.622***	0.859***	0.870***
	(0.0916)	(0.0955)	(0.0987)	(0.104)
$d^l_{i,t}\eta_{i,t}$	-0.164***	-0.194***	-0.249***	-0.369***
	(0.0342)	(0.0357)	(0.0368)	(0.0384)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.566

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.3: Investment response to small versus large TFPR shocks.

In order to see whether we observe this implication in data, I divide idiosyncratic TFPR shocks into two groups of positive versus negative shocks and run the following local projection which is identical to (2.3) with the only difference that I introduce the interaction with the sign dummy $d^p_{i,t}$:

$$d^p_{i,t} = 1 \quad if \quad \eta_{i,t} > 0$$

$$I_{i,t+h} = \beta_h \eta_{i,t} + \beta^p_h d^p_{i,t} \eta_{i,t} + \Gamma'_h W_{i,t-1} + \Lambda'_h W_{i,t-1} \eta_{i,t} + \sum_{m>0} \theta^m_h I_{i,t-m} + F_i + G_{jt} + \epsilon_{i,t,h}$$

The result of the local projection for $h = 4$ is provided in the table 2.4. Relative response to the positive shocks are significantly lower than negative shocks[28]. This is in line with the third implication of learning uncertainty. In the figure 2.11, the impulse responses for $h = 20$ for the both positive and negative shocks are depicted and as you notice the impulse response to negative shocks are above positive shocks[29].

Forni et al. (2021) using a new econometric approach which combines quantile regressions and structural VARs decompose uncertainty shocks into positive (upside) uncertainty shocks and negative (downside) uncertainty shocks. They find that an increase in downside uncertainty generates significant negative effects on real economic activity while an increase in upside uncertainty has small positive effects on real economic activity. In my study, the asymmetric response of firms to negative and positive shocks is closely in line with their finding.

[28]Please note that the estimated coefficients for positive shocks (interacted with the dummy) is significant with p-values close to zero.

[29]Estimated coefficients of local projections and corresponding tables are provided in the appendix.

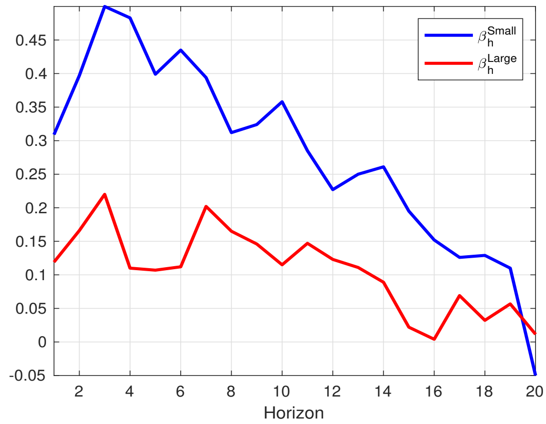


Figure 2.10: Investment response to small versus large TFPR shocks.

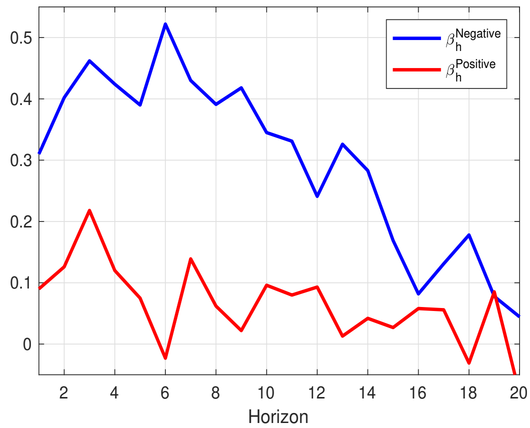


Figure 2.11: Investment response to positive versus negative TFPR shocks.

Asymmetric S shaped response

In the next step I combine the results of the previous two subsections. I divide the distribution of TFPR shocks into four groups of small-negative, small-positive, large-negative and large positive by interacting dummies $d^p_{i,t}$ and $d^l_{i,t}$. Then I run the following local projection:

$$I_{i,t+h} = \left[\beta^{sn}_h + \beta^{sp}_h(1 - d^l_{i,t})d^p_{i,t} + \beta^{ln}_h d^l_{i,t}(1 - d^p_{i,t}) + \beta^{lp}_h d^l_{i,t} d^p_{i,t}\right] \eta_{i,t}$$
$$+ \Gamma'_h W_{i,t-1} + \Lambda'_h W_{i,t-1}\eta_{i,t} + \sum_{m>0} \theta^m_h I_{i,t-m} + F_i + G_{jt} + \epsilon_{i,t,h}$$

The result of the local projection for $h = 4$ is provided in the table 2.5.

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.550***	0.647***	0.846***	0.841***
	(0.0933)	(0.0973)	(0.101)	(0.106)
$d^p_{i,t}\eta_{i,t}$	-0.203***	-0.256***	-0.223***	-0.292***
	(0.0547)	(0.0570)	(0.0587)	(0.0615)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.566

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.4: Investment response to positive versus negative TFPR shocks.

As you see the highest response is for the baseline small and negative shocks. Large-negative, small-positive and large-positive shocks all have lower responses compared to the baseline small-negative shocks. This finding suggests that firms' investment response to the idiosyncratic TFPR shocks is asymmetric and S shaped. In the figure 2.12, I plot the average values of impulse responses over $h = 10$ horizons for all four mentioned regions. My result is in line with Ilut et al. (2018). As you see in the figure 2.13, they find that employment growth rates response to TFPR shocks is asymmetric and S shaped. I find the same pattern regarding the investment response.

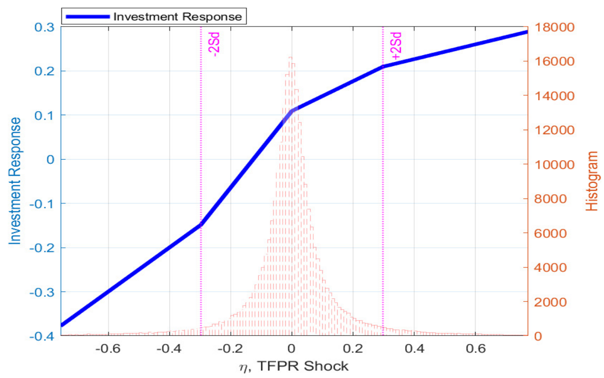


Figure 2.12: Asymmetric S shaped investment response to TFPR shocks.

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.736***	0.843***	1.057***	1.094***
	(0.0991)	(0.103)	(0.107)	(0.112)
$(1-d^l_{i,t})d^p_{i,t}\eta_{i,t}$	-0.433***	-0.476***	-0.427***	-0.484***
	(0.0812)	(0.0846)	(0.0869)	(0.0907)
$d^l_{i,t}(1-d^p_{i,t})\eta_{i,t}$	-0.311***	-0.336***	-0.380***	-0.490***
	(0.0515)	(0.0539)	(0.0554)	(0.0579)
$d^l_{i,t}d^p_{i,t}\eta_{i,t}$	-0.445***	-0.524***	-0.542***	-0.730***
	(0.0676)	(0.0705)	(0.0725)	(0.0759)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.567

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.5: Asymmetric S shaped investment response to TFPR shocks.

2.3.3 Investment response, subjective uncertainty and learning

In the previous subsection we saw that all three implications of learning uncertainty that were discussed in the section 2.2 and the subsection 2.2.2 are observable phenomenons in the data. In this subsection, I study the impact of idiosyncratic subjective uncertainty about TFPR shocks on firms' investment responsiveness.

In the chapter one, I validated learning uncertainty as a possible driver of subjective uncertainty. Generalizing that finding to Compustat firms and assuming Compustat firms are Bayesian learners who learn uncertainty by learning the distribution of the idiosyncratic TFPR shocks, I can obtain their time varying idiosyncratic posterior uncertainty by observing previous realizations of idiosyncratic TFPR shocks using the standard Normal-Gamma Bayesian learning that was discussed in the previous chapter. Let us assume idiosyncratic TFPR shocks $\eta_{i,t}$ are conditionally drawn from a Normal distribution with an unknown variance σ_i^2:

$$\eta_{i,t}|\sigma_i^2 \quad \overset{iid}{\sim} \quad N(0,\sigma_i^2) \qquad , \qquad \sigma_i^2 \quad \sim \quad Inverse-Gamma\left(\alpha_o^j,\beta_0^j\right)$$

α_0^j and β_0^j are the prior's shape and rate parameters and are assumed to be sector specific. I calibrate prior parameters in the following way. I find the variance of

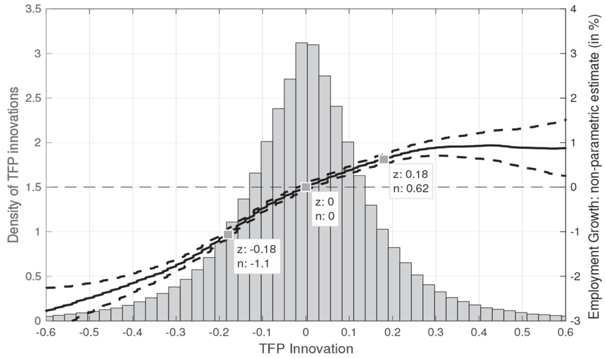


Figure 2.13: Asymmetric S shaped employment growth response to TFPR shocks. *Source: Ilut et al. (2018)*

idiosyncratic TFPR shocks for each firm over the whole sample time[30]. Then, by targeting the mean and the variance (first and second moments) of obtained idiosyncratic variances *within each sector*[31], I calibrate α_0^j and β_0^j.

Firms are uncertain about the true value of σ_i^2 but can learn it over the time by observing previous realizations of the idiosyncratic TFPR shocks ; $I_{i,t} = \{\eta_{i,1}, \eta_{i,2}, ..., \eta_{i,t-1}\}$. From the previous chapter and the equation (1.2) we know that after observing the information set $I_{i,t}$, the unconditional posterior distribution will be Student's t-distribution:

$$\eta_{i,t}|I_{i,t} \quad \sim \quad t_{2\alpha_{i,t}}\left(0, \frac{\beta_{i,t}}{\alpha_{i,t}}\right)$$

where $\alpha_{i,t}$ and $\beta_{i,t}$ are posterior beliefs' parameters obtained from the updating mechanism (1.2). I will call the variance of the above mentioned unconditional posterior distribution the posterior uncertainty; $\widetilde{\sigma}_{i,t}^2 = \frac{\beta_{i,t}}{\alpha_{i,t}-1}$.

After building the series of posterior uncertainty values for all firms in the sample, I study the impact of posterior uncertainty on the firms' investment response to the idiosyncratic TFPR shocks by running the following local projection[32]:

$$\begin{aligned} I_{i,t+h} = \beta_h \eta_{i,t} + \beta_h^{\sigma} \widetilde{\sigma}_{i,t}^2 + \beta_h^{\sigma'} \widetilde{\sigma}_{i,t}^2 \eta_{i,t} + \Gamma_h' W_{i,t-1} \\ + \Lambda_h' W_{i,t-1} \eta_{i,t} + \sum_{m>0} \theta_h^m I_{i,t-m} + F_i + G_{jt} + \epsilon_{i,t,h} \end{aligned}$$

which is identical to the local projection (2.3) with the only difference that I control for the posterior uncertainty $\widetilde{\sigma}_{i,t}^2$ and its interaction with the idiosyncratic

[30] Denoted by $Var_i\left(\eta_{i,t}\right)$.

[31] Denoted by $E_j\left[Var_i\left(\eta_{i,t}\right)\right]$ and $Var_j\left[Var_i\left(\eta_{i,t}\right)\right]$.

[32] I drop first 5 observations of each firm to make sure prior miscalibration does not affect the result.

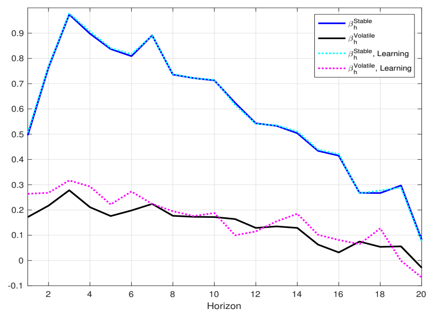
TFPR shock. The result of the regression is provided in the table 2.6. As you see in the second row, posterior uncertainty has a significant negative impact on the investment rate level. This is in line with contractionary uncertainty shock literature. Moreover, as you see in the third row, posterior uncertainty has a significant negative impact on the investment responsiveness to the idiosyncratic TFPR shock. This finding again confirms that learning uncertainty can affect investment response to idiosyncratic TFPR shocks and bring about three mentioned implications.

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.589***	0.662***	0.887***	0.915***
	(0.0935)	(0.0973)	(0.101)	(0.106)
$\widetilde{\sigma}^2_{i,t}$	-1.468***	-1.720***	-1.805***	-1.736***
	(0.300)	(0.312)	(0.321)	(0.333)
$\widetilde{\sigma}^2_{i,t}\eta_{i,t}$	-1.411***	-1.186***	-1.315***	-2.229***
	(0.297)	(0.309)	(0.316)	(0.329)
Observations	139,317	135,106	131,078	127,316
R-squared	0.612	0.593	0.585	0.566

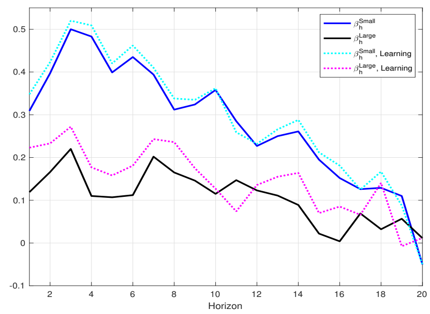
Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 2.6: Impact of posterior uncertainty on investment response to TFPR shocks.

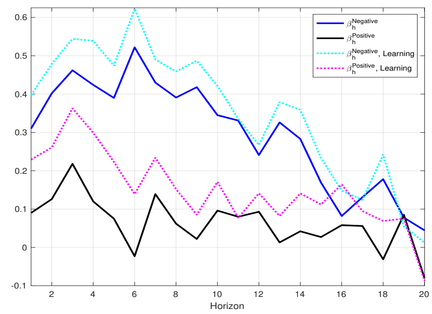
Next, I check whether controlling for posteriors' uncertainty affect 3 mentioned implications that were studied in the previous subsections or not. I run 3 local projections that were presented in the subsection 2.3.2, with the only difference that I control for the posterior uncertainty $\widetilde{\sigma}^2_{i,t}$. The results are provided in the figure 2.14. As you see in all three cases controlling for learning uncertainty, results in almost higher investment responses. Moreover, the difference between investment response of firms living in stable versus volatile environments, the difference between investment response to small versus large shocks and the difference between investment response to positive versus negative shocks is less noticeable.



(a) Stable versus volatile environment.



(b) Small versus large shocks.



(c) Positive versus negative shocks.

Figure 2.14: Investment response to TFPR shocks with and without controlling for learning uncertainty.

2.4 Conclusion

In the previous chapter I validated learning uncertainty as a possible driver of subjective uncertainty. In this chapter, I studied implications of learning uncertainty for firms' investment pattern. To do so, I first built a simple partial equilibrium model of firms investment with learning uncertainty. In my model, I used an OLG environment which is populated by risk averse young agents and old entrepreneurs who would like to maximize their old age utility by allocating their endowment into risk-free or risky saving options. Using impulse responses, I showed learning uncertainty results in three implications: 1-firms that have lived in a volatile environment have a lower investment response to idiosyncratic TFPR shocks compared to firms that have lived in a stable environment, 2-lower investment response to large idiosyncratic TFPR shocks compared to small shocks and 3-asymmetric responses to symmetric positive and negative idiosyncratic TFPR shocks. Implications 2 and 3 suggests that the firms' investment response to idiosyncratic TFPR shocks is asymmetric and S shaped.

Next, I went to Compustat data and studied firms' investment response to idiosyncratic TFPR shocks to prove the existence of three mentioed implications of learning uncertainty for firms' investment patterns in data. I first extracted TFPR shocks in firm level as the main explanatory variable using residual approach. I used cost-share method to estimate production function elasticities. Then, by means of local projection method I obtained impulse responses of investment rate to idiosyncratic TFPR shocks. I showed all three implications of learning uncertainty are observable phenomenon in the data.
Finally, based on the finding from the survey of business uncertainty about drivers of subjective uncertainty, I assumed that Compustat firms are Bayesian learners. After building their time varying posterior uncertainty about idiosyncratic TPFR shocks, I studied the impact of their posterior uncertainty about TFPR shocks on the investment response to the shocks. I show that posteriors' uncertainty has a significant *negative* effect on firms' investment responsiveness to the idiosyncratic TFPR shocks. Moreover, I showed that the difference between investment response of firms living in stable versus volatile environments, the difference between investment response to small versus large shocks and the difference between investment response to positive versus negative shocks is less noticeable when I control for the learning uncertainty. This finding reconfirms that learning uncertainty is a potential driver of three mentioned investment patterns.

Chapter 3

CONTRACTIONARY PRODUCTIVITY DISPERSION SHOCK AND OPTIMAL MONETARY POLICY

3.1 Introduction

In the previous chapters, I studied shocks to firms' subjective uncertainty, their drivers and their implications for the investment patterns. In this chapter, I concentrate on *productivity dispersion shock* as the main second moment shock of interest and its impact on the economy and the optimal conduct of the monetary policy.

Originated by Bloom (2009), a growing branch of literature studies the impact of the second moment shocks on business cycles. Berger and Vavra (2017), Jurado et al. (2015), Baker et al. (2016) and Fernandez-Villaverde et al. (2011) are among papers that show that second moment shocks are counter-cyclical. Many theoretical frameworks rationalize this evidence through wait-and-see effects[1], risk premium effects[2] or precautionary motives[3].

Dispersion shock is one form of the second moment shocks that plays an important role in business cycles. Kehrig (2015), Bachmann and Bayer (2014), Bachmann and Bayer (2013) and Bloom et al. (2018) provide evidence that dispersion shocks are counter-cyclical. Counter-cyclical productivity dispersion, which is a well established empirical fact, is absent in many friction-less models. According to the

[1] see Bloom (2009), Bachmann and Bayer (2013) .

[2] see Arellano et al. (2016) and Christiano et al. (2014).

[3] see Basu and Bundick (2017), Leduc and Liu (2016) and Ravn and Sterk (2017).

well known Oi-Hartman-Abel[4] effect second moment productivity shocks, due to the complementarity channel between the productivity and factors of production, are expansionary in models without friction. In this paper, in the first step, I build a friction-less model that departs from the Oi-Hartman-Abel effect and features contractionary productivity dispersion shock.

The proposed model is based on a simplified version of the framework that is introduced by Angeletos et al. (2020). In my static model there is a representative household consisting of a consumer and a continuum of workers who supplies labour to a continuum of firms that produce differentiated goods. I obtain the contractionary dispersion shock in the friction-less model by introducing taste for variety and substitutablity, not only for the consumed goods, but also for the supplied labor in the aggregate economy. For a small degree of substitutablity of either supplied labor or consumption good, the dispersion shock will be contractionary. The main intuition is that for small values of substitutablity, the model converges to the Leontief environment. As a result, the standard complementarity channel between factor of production and the productivity is broken and the second moment shock is not expansionary anymore.

In the next step, I study the impact of the dispersion shock on the conduct of the optimal monetary policy by using the information fiction as a source of nominal rigidity. Information friction refers to the scenario that at the time that firms set the price, they have incomplete (and *not* asymmetric) information about underlying aggregate productivity dispersion. Chosen prices are fixed and can not be updated after receiving more information. Following Angeletos et al. (2020) I will call the scenario in which the model features information driven nominal rigidity *sticky price* and the environment in which the price is set with complete information *flexible price*. Using information friction as a source of nominal rigidity is not the contribution of the paper, it has been widely used in the literature before [5]. The main contribution of the paper is the introduction of the uncertainty about the dispersion when prices are chosen.

In addition to the information friction, the model has another source of the distortion and that is the monopolistic competition. Please note I refer to the environment without information friction as friction-less because the welfare loss due to the monopolistic competition can simply be eliminated by implementing the standard optimal fiscal policy. Both monetary and fiscal policy makers are restricted and committed to follow pre-determined rules which are contingent on the realized states.

It is a well known fact that in the absence of information driven nominal rigidity,

[4] see Oi (1961) , Hartman (1972) and Abel (1983).

[5] see Mankiw and Reis (2002), Mackowiak and Wiederholt (2009), La'O and Tahbaz-Salehi (2022) and Angeletos and La'O (2020)

monetary policy is neutral. However, in the environment with information friction monetary policy has real effect. I show that the optimal monetary policy is the policy that replicates the full-information (flexible price) scenario. This policy is always feasible and basically eliminates the dependence of idiosyncratic nominal variables on the aggregate dispersion. To put it in another way, this policy is equivalent to the policy in the flexible price environment that makes the price setting function irrelevant of the aggregate dispersion. So if in the full information environment idiosyncratic marginal cost and price does not depend on the dispersion, in the environment with incomplete information, the unknown term does not play any role neither and the information friction is eliminated.
Reducing uncertainty in the market and eliminating information friction as an optimal policy is in contrast with "Paradox of Transparency" literature [6]. The optimal policy in my paper, that eliminates the information friction, is in line with Kohlhas (2022), however, it is *not* optimal because of the increase in the informativeness of prices nor reduction in the uncertainty of the central bank.
The structure of the chapter is as follows. In the section 3.2, I introduce the baseline model without any nominal rigidity and study the equilibrium. In the section 3.3, we see how we can obtain the contractionary dispersion shock in the frictionless model. In the section 3.4, I introduce information friction as a source of nominal rigidity and study the equilibrium. In the section 3.5, I study the optimal monetary policy in the environment with nominal rigidity and finally I conclude in the section 3.6.

3.2 Baseline model without nominal rigidity

In this section I present the baseline full information model in which there is not any source of nominal rigidity. I show how the interaction between substitutability of supplied labour and demanded goods can generate the contractionary productivity dispersion shock. The model is based on Angeletos et al. (2020).

3.2.1 Environment

Household:
The model is a one-period and static. There is a representative household consisting of a consumer and a continuum of workers who supplies labour to a continuum of firms, indexed by $i \in I = [0, 1]$. The household maximizes the utility:

$$U = \frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1-\epsilon} - 1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta} - 1}{1-\delta}$$

[6] see Morris and Shin (2005), Amador and Weill (2010), Ou et al. (2021) and Gaballo (2016).

where C is the aggregate consumption basket, N is the aggregate supplied labour and $\frac{M}{P}$ is the real money in the utility. $\gamma > 0$ parameterizes the income elasticity of labor supply and the risk aversion, $\epsilon < 0$ parameterizes the Frisch elasticity of labor supply [7] and $\delta > 0$ parameterizes the convexity of the utility with respect to the real balance. Aggregate consumption, labour and price are determined by the following CES aggregators.

$$C = \left[\int_I (c_i)^{\frac{\rho-1}{\rho}} di\right]^{\frac{\rho}{\rho-1}}, \quad N = \left[\int_I (n_i)^{\frac{\omega-1}{\omega}} di\right]^{\frac{\omega}{\omega-1}}$$

$$P = \left[\int_I (p_i)^{1-\rho} di\right]^{\frac{1}{1-\rho}} \quad , \quad W = (1-\tau)\left[\int_I (w_i)^{1-\omega} di\right]^{\frac{1}{1-\omega}}$$

where c_i is the consumed quantity of the commodity produced by the representative firm i at the price p_i and $\rho > 1$ is the elasticity of substitution between different consumed goods. n_i is the supplied labor for the production of the good that is produced by the firm i with wage w_i and $\omega < 0$ is the elasticity of substitution between different supplied labors.[8]

The representative household receives labor income and profits from all firms in the economy. Its nominal budget constraint is thus given by:

$$\int_I p_i c_i di + M = \int_I \Pi_i di + (1-\tau)\int_I w_i n_i di + T$$

where M is nominal demanded money, Π_i is the profit from the firm i.

Government:

There is a government which collects tax and redistribute it in a lump-sum fashion. In the household's budget constraint τ is the constant tax rate on the labour income and T denotes the lump-sum redistribution tax [9]. The government plays the role of the central bank at the same time and supply the nominal money M [10]. The government's budget constraint is:

$$T = \int_I \tau w_i n_i di + M$$

Firms:

The output of the representative firm in island i is given by:

$$y_i = A_i n_i$$

[7]Please note $\frac{-1}{\epsilon}$ is the Frisch elasticity of labor supply.

[8]I assume the household does not only have a taste for variety for the consumption good but also the supplied labor.

[9]τ is always chosen optimally such that the monopolistic competition distortion will be eliminated.

[10]The monetary policy rule will be discussed in the section of the model with nominal rigidity.

A_i is the productivity of the firm i and n_i, which is the only factor of production, is demanded labour for the production of the good i. Firms produce differentiated goods in monopolistic competitive fashion. The firm's realized profit is given by:

$$\pi_i = p_i y_i - w_i n_i$$

Markets clearing:
Labour supply in each firm i is equal to the labour demand at the market clearing wage w_i. Demand and supply for the produced goods of each firm i are equal at the market clearing price p_i.

Idiosyncratic productivity shocks:
As it was mentioned earlier A_i is the productivity of the firm i. It is log-normally distributed in the cross-section of firms:

$$a_i = log(A_i) \quad \sim \quad N\left(\overline{a} - \frac{\sigma^2}{2}, \sigma^2\right)$$

Idiosyncratic log-productivities are centered around $\overline{a} - \frac{\sigma^2}{2}$ and σ^2 captures the degree of dispersion in the productivity between different firms. This form of distribution guarantees that the second moment dispersion shock is only second moment shock and is not affecting average productivity $E(A_i) = exp(\overline{a})$. $\overline{a}$ is predetermined and known to everyone. $\sigma^2 \sim IG(\alpha_0, \beta_0)$ is drawn from the Inverse-Gamma distribution with the shape parameter $\alpha_0 \to \infty$ and the scale parameter $\beta_0 \to \infty$ such that $\frac{\beta_0}{\alpha_0} \to \sigma_0^2$ [11]. In the baseline model without information friction, firms perfectly observe the realizations of σ^2 before price setting [12].

3.2.2 Equilibrium

The equilibrium consists of the optimal allocations of labor, produced goods, money demand, prices and policy instruments such that:

- The representative household maximizes the utility subject to the budget constraint taking prices and wages as given.
- Firms maximize their profit subject to the demand constraint taking prices and wages as given.

[11]The assumptions about values of parameters α_0 and β_0 guarantees the existence of the moment generating functions. Moreover, with this parametric assumption firms will *not* have any form of dispersed or asymmetric information after observing realizations of a_i, in other words there is no learning after observing a_i.

[12]The household always observes σ^2.

- Prices and wages are set in a way that all markets clear.
- The government maximizes the ex-ante expected welfare given the optimal actions of firms and the representative household.

Now let us find the optimal actions by different agents to characterize the equilibrium.

Households:

The representative household maximizes the utility [13]

$$U = \frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1-\epsilon} - 1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta} - 1}{1-\delta}$$

subject to the budget constraint

$$\int_I p_i c_i di + M = \int_I \Pi_i di + \int_I (1-\tau) w_i n_i di + T.$$

Following the standard optimization problem that is presented in the appendix, we will obtain the consumption basket as:

$$\frac{p_i}{P} = \left(\frac{c_i}{C}\right)^{-\frac{1}{\rho}} \tag{3.1}$$

which is standard given the CES assumption. Moreover, the labour supply (labor basket) is given by:

$$\frac{(1-\tau)w_i}{W} = \left(\frac{n_i}{N}\right)^{-\frac{1}{\omega}} \tag{3.2}$$

The second order condition holds as long as $\omega < 0$ which implies that higher labour will be allocated to the firm with higher relative wage.

Finally the optimal money demand will be:

$$\left(\frac{M}{P}\right)^{-\delta} = C^{-\gamma} \tag{3.3}$$

Firms:

Firms maximize their profit π_i subject to the consumption basket (3.1). In the absence of information friction the optimal price setting will be:

$$p_i = \frac{\rho}{\rho - 1} \quad \frac{w_i}{A_i} \tag{3.4}$$

[13]Given the symmetry in the environment and the log-normal assumption, the welfare is well defined in the closed form. Please find its expression in the appendix.

which is standard price setting equation and implies the optimal price is the mark-up $\frac{\rho}{\rho-1}$ multiplied by the marginal cost $\frac{w_i}{A_i}$.

Government:
As it is shown in the appendix, in the baseline model without information friction, real variables are pinned down regardless of the conduct of the monetary policy so the monetary policy is neutral and does not have any real effect. Therefore, the government only chooses the optimal tax rate τ such that given the optimal decisions by households and firms, the expected welfare is maximized. The optimal fiscal policy eliminates monopolistic competition distortion.
Equilibrium conditions in closed form are provided in the appendix. As you see, given the log-normal distribution and symmetric assumptions, all real and nominal variable allocations are log-linear in terms of states.

3.3 Contractionary dispersion shock

As it was mentioned before the idiosyncratic productivity is log-normally distributed:

$$a_i = log(A_i) \quad \sim \quad N\left(\overline{a} - \frac{\sigma^2}{2}, \sigma^2\right).$$

Given $\overline{a}$ and σ^2 the expected value of the productivity will be:

$$E(A_i) = E\left(e^{a_i}\right) = e^{\overline{a}}$$

Therefore, $\overline{a}$ implies the average value of the productivity in the economy. Assumed distribution of productivities guarantees that σ^2 pins down the degree of productivity dispersion in the economy without affecting the average value.
I study the effect of an increase in the productivity dispersion on the economy. Does higher dispersion in the productivity results in higher output and employment or not?
As it is shown in the Appendix, the aggregate output can be expressed in terms of the average aggregate TFP $\overline{a}$ and the TFP dispersion σ^2:

$$log(Y) = log(C) = C_0 + C_A\overline{a} + C_\sigma\sigma^2$$

$$C_A = \frac{1-\epsilon}{\gamma-\epsilon} \quad , \quad C_\sigma = \frac{-(1-\epsilon)[1+\omega(\rho-2)]}{2(\rho-\omega)(\gamma-\epsilon)}$$

C_A is always positive so higher average TFP implies higher output. However, based on the model parameters C_σ can be positive (expansionary dispersion shock) or negative (contractionary dispersion shock).
For small values of the elasticity of substitution of labor (ω close to zero) or small

values of the elasticity of substitution of consumed goods (ρ close to 1), the coefficient C_σ will be negative and we will obtain contractionary dispersion shocks. The exact threshold values for these elasticities are provided in the appendix. Contractionary dispersion shock is a novel finding and is absent in standard friction-less models because of the well-known Oi-Hartman-Abel effect.

To understand the main intuition for this result let us study the relative labor supply in two different firms i and j:

$$log(n_i) - log(n_j) = \frac{-\omega(\rho - 1)}{\rho - \omega}(a_i - a_j)$$

Therefore, due to the complementarity effect, labor will be allocated to a more productive firm given the fact that $\omega < 0$ and $\rho > 1$. Let us assume $|\omega|$ converges to zero.[14] It means that we are converging to the Leontief environment in which the labour supply in all firms are constant and equal to the aggregate labor supply. By this assumption basically we are eliminating the complementarity effect between labor and productivity.

By assuming small values for elasticity of substitution of labor $|\omega|$ and large values for elasticity of substitution of consumption good ρ, the wage aggregator equation will converge to simple uniform integration while the price aggregator will be same as before:

$$P = \left[\int_I (p_i)^{1-\rho}\, di\right]^{\frac{1}{1-\rho}} \qquad , \qquad W = (1-\tau)\int_I w_i di$$

So given the mean preserving distribution of productivity and the law of large numbers, the responsiveness of aggregate wage to the dispersion shock is equal to the responsiveness of idiosyncratic wage to the dispersion shock. However, the responsiveness of aggregate price to the dispersion shock is *larger* than the responsiveness of idiosyncratic price to the dispersion shock. Moreover, the optimal price setting condition (3.4) implies that the responsiveness of idiosyncratic price to the dispersion shock is equal to the responsiveness of idiosyncratic wage to the dispersion shock. Therefore, after a positive dispersion shock, the aggregate real wage $\frac{W}{P}$ will decrease, which results in lower aggregate consumption and higher aggregate labor supply.

3.4 Model with nominal rigidity

3.4.1 Environment and equilibrium

Firms can set the price optimally if they have complete information about the aggregate states which affect their marginal cost of production. Now, I introduce

[14] The intuition for small values of ρ is similar.

the information friction as a source of nominal rigidity. To be specific, I assume firms have incomplete information about the productivity dispersion σ^2 when they set the price [15]. Prices are chosen based on firms' expectation about σ^2 and can not be updated afterward. In contrast to Angeletos et al. (2020) and Angeletos and La'O (2020) firms have complete information about the aggregate productivity $\overline{a}$ and the only source of uncertainty is the productivity dispersion σ^2. All firms share the same prior about the unknown dispersion. In particular, it is assumed that the dispersion is drawn from the following known distribution, but its true realization is unknown for firms:

$$\sigma^2 \sim Inverse - Gamma(\alpha_0, \beta_0) \quad s.t. \quad \begin{cases} \beta_0 \to \infty, \\ \alpha_0 \to \infty, \\ \frac{\beta_0}{\alpha_0} \to \sigma_0^2 \end{cases}$$

Parametric assumptions above guarantee the existence of moment generating function for the unconditional distribution of idiosyncratic productivities. Moreover, with this parametric assumption, firms will *not* have any form of asymmetric or dispersed information after observing the realization of idiosyncratic productivity a_i, in other words they will not learn from idiosyncratic productivity a_i.

So how do information friction and nominal rigidity affect the equilibrium conditions? Please note all assumptions are similar to the baseline model that is presented in the previous section and the only departure from the baseline model is the introduction of the information friction about productivity dispersion. Therefore, the only condition that is different from the baseline model is the price setting condition. As firms have incomplete information about the aggregate state, the objective of a firm is to maximize its *expectation* of the representative consumer's valuation of its profit subject to the consumption basket (3.1), namely:

$$\max_{p_i, n_i} \quad E\left[\frac{U'(C)}{P}\left(p_i y_i - w_i n_i\right)\right] \quad s.t. \quad \frac{p_i}{P} = \left(\frac{c_i}{C}\right)^{-\frac{1}{\rho}}$$

As it is shown in the appendix the profit maximization results in the following price setting condition:

$$\begin{aligned} p_i &= \frac{\rho}{A_i(\rho-1)} \quad \frac{E\left[\frac{U'(C)}{P} w_i n_i\right]}{E\left[\frac{U'(C)}{P} n_i\right]} \\ &= \frac{\rho}{A_i(\rho-1)} \quad \left(\frac{Cov\left[\frac{U'(C)}{P} n_i, w_i\right]}{E\left[\frac{U'(C)}{P} n_i\right]} + E(w_i)\right) \end{aligned} \tag{3.5}$$

[15] However, the household has complete information about the productivity dispersion.

This equation is equivalent to the equation (3.4) with the only difference that instead of the true realization of the marginal cost w_i, we have new expectation terms which is referring to the covariance channel between the marginal cost of production and the factor of production *(risk channel)* in addition to the expected marginal cost. You can easily see if the wage does not depend on the unknown productivity dispersion, equations (3.4) and (3.5) will be identical. We will come back to this in the next section when we study the optimal monetary policy.
In order to find the equilibrium conditions in the model with information friction, I use the standard guess and verify approach. Because of the symmetry in the environment and log-normal assumption, it is easy to show that all variables are log-linear in terms of the known states. I guess following policy functions for the household, firms and the government:

- Household's policy functions:
  - $log(n_i) = n_0 + n_a a_i + n_A \overline{a} + n_\sigma \sigma^2$. [16]
  - $log(N) = N_0 + N_A \overline{a} + N_\sigma \sigma^2$.
  - $log(C) = log(Y) = C_0 + C_A \overline{a} + C_\sigma \sigma^2$.
- Firms' policy functions:
  - $log(p_i) = \psi_0 + \psi_a a_i + \psi_A \overline{a}$.
  - $log(P) = P_0 + P_A \overline{a} + P_\sigma \sigma^2$.
- Fiscal Policy: $log(1 - \tau) = \tau_0$.
- Monetary Policy: $log(M) = m_\sigma \sigma^2$.

and using equilibrium conditions verify that my log-linear guess is valid. The values of 16 unknown agents' policy functions' coefficients (excluding fiscal and monetary policies) are determined and presented in the appendix. m_σ and τ_0 are policy tools in the control of the government. Same as before, τ_0 is chosen optimally such that the monopolistic competition distortion will be eliminated. Optimal monetary policy will be determined in the next section.
Please note that the household has complete information about aggregate dispersion when makes the decision about labor supply. That is why the dispersion appears in the labor supply policy function but not in the idiosyncratic price function. Moreover, based on the law of large numbers the aggregate values for labor, consumption and price are functions of the productivity dispersion in the market.

[16] I only consider the labor supply in each firm, because due to the market clearing labor supply and labor demand in each firm are equal.

Monetary policy's response to known states such as aggregate average productivity $\overline{a}$ does not have any real effect in the economy. To rephrase it, as it was discussed in the baseline model, the monetary policy response to known states is neutral. Therefore, I assume the monetary policy rule $log(M) = m_\sigma \sigma^2$ which only responds to the unknown dispersion. This policy has real effect on the economy and is not neutral anymore. Specifically, coefficients n_σ, C_σ and N_σ in policy functions depend on the monetary policy rule m_σ.

3.5 Optimal monetary policy

As it is discussed in previous sections, monetary policy in the environment without information friction is neutral and only affects nominal variables. However, in order to understand the optimal monetary policy in the environment with information friction, it is useful to study the impact of the monetary policy on nominal variables in the friction-less model without the information friction. In particular, let us study the effect of the monetary policy on firms' idiosyncratic nominal price and wage in the model without information friction:

$$log(p_i) = \psi'_0 + \psi'_a a_i + \underbrace{(m_\sigma - Q_s)}_{\psi'_\sigma} \sigma^2 + \psi'_A \overline{a} = log(w_i) - a_i + log\left(\frac{\rho}{\rho - 1}\right)$$

where ψ'_0, ψ'_a, ψ'_A and Q_s are constant functions of model parameters. You can find their values in the appendix and the equation (A.1).
Consider a specific monetary policy such that $m_\sigma = Q_s$ which results in $\psi_\sigma = 0$. In other words, for $m_\sigma = Q_s$ neither idiosyncratic nominal price nor idiosyncratic nominal wage respond to the productivity dispersion. This policy eliminates the dependence of idiosyncratic nominal variables on the productivity dispersion in the full information environment.
Now, let us go to the environment with information friction, in which the monetary policy is not neutral anymore. In order to determine the optimal conduct of monetary policy here, we need to understand what the source of friction in the market is and how the monetary policy can eliminate it. There are two sources of frictions in this environment; the first one is the monopolistic competition distortion and the second one is the information friction. The monopolistic competition distortion can be eliminated using the optimal fiscal policy. So we only have one source of friction left and that is the information driven nominal rigidity.
Monetary policy can easily remove this friction. As we saw before, the information friction only affects the firms' price setting problem. So by removing the dependence of idiosyncratic nominal variables on the productivity dispersion, the information friction becomes irrelevant. This policy is exactly equivalent to set

$m_\sigma = Q_s$. By doing so, the monetary policy eliminates the relevance of productivity dispersion both in the full information model and in the model without information friction. Please note that sign of Q_s is not predetermined and depends on many parameters that affects the cyclicality of the dispersion shock and convexity of the welfare. Therefore, we can not conclude that optimal monetary policy's response to the dispersion shock is necessarily expansionary or contractionary.
In the figure 3.1 you see the ex-ante expected welfare for a given parameterization of the model. As you notice, in the model with information friction for the monetary policy $m_\sigma = Q_s$, the expected ex-ante welfare is maximized and will be equal to the expected welfare in the full-information *flexible price* scenario.

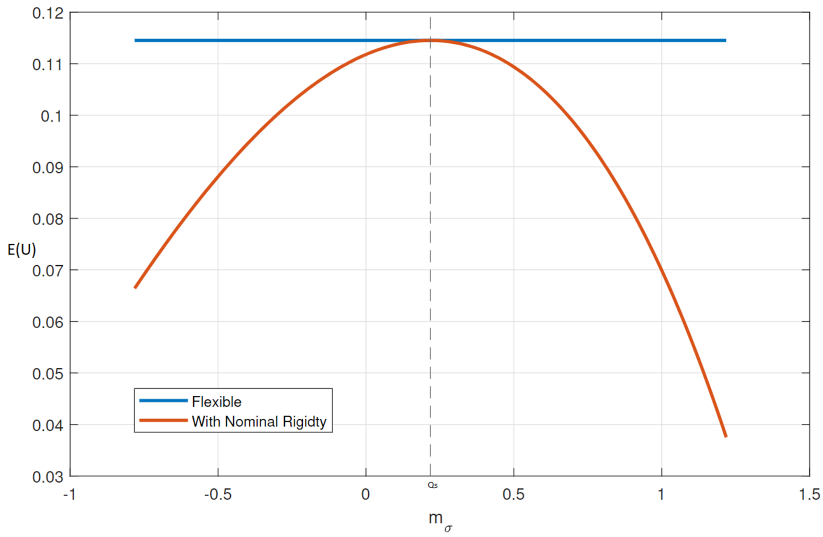


Figure 3.1: Expected welfare in response to the monetary policy in models with information friction and without information friction

3.6 Conclusion

In this chapter, I studied the impact of the productivity dispersion shock. By introducing taste for variety and substitutability, not only for the consumption good but also for the supplied labor, I managed to obtain the contractionary productivity dispersion shock in a fiction-less environment. The contractionary second moment shock, which is a departure from the well known Oi-Hartman-Abel effect, is a novel result and is derived when the elasticity of substitution either for the labor supply or consumption good is small enough. The intuition is very simple; by reducing the elasticity of substitution we converge to the Leontief environment

such that the complementarity channel between productivity and factors of production is broken.
In order to study the impact of the dispersion shock on the conduct of the monetary policy, I introduced information friction as a source of nominal rigidity following Angeletos and La'O (2020), Angeletos et al. (2020) and La'O and Tahbaz-Salehi (2022). In particular, I assumed firms have incomplete information about the productivity dispersion when they set prices of their goods.
I showed the optimal monetary policy is the policy that eliminates the reliance and dependence of the idiosyncratic nominal variables on the productivity dispersion. This policy basically replicates the flexible price full information equilibrium. My result is in contrast to the well-known "Paradox of Transparency" literature.

A

APPENDIX

A.1 Chapter 1 - Appendix

A.1.1 Subjective uncertainty and realized uncertainty

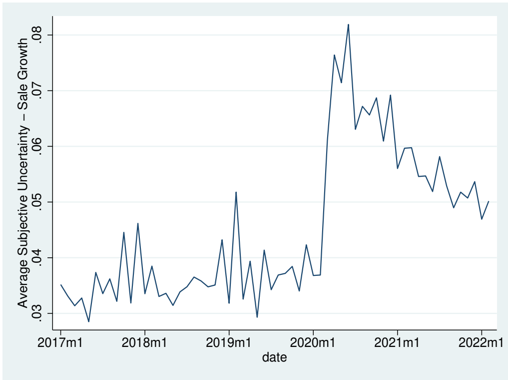
Aggregate subjective uncertainty

In the figure A.1, I plot the average value of subjective uncertainty about sale growth, employment growth and capital expenditure growth rates among firms between January 2017 and February 2022 [1].
As you can see the average subjective uncertainty about sale growth, employment growth and capital expenditure growth rates are time varying. Moreover, the figure exhibits a pronounced spike in uncertainty in March 2020 after the Covid crisis. This finding is in line with Baker et al. (2020).

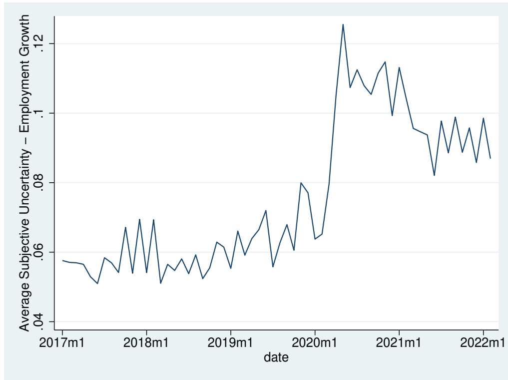
Subjective uncertainty and previous growth realizations

In the section 1.2 and in the figure 1.1, I presented the scatter plot with subjective uncertainty over next year sales growth rates on the vertical axis and 100 quantiles of past sales growth rate over the last year on the horizontal axis. Here, in the figure A.2, I present the same scatter plots for the capital expenditure growth and employment growth rates. There is a V-shaped relation between the subjective uncertainty and the past realization of the employment growth rate, same as what

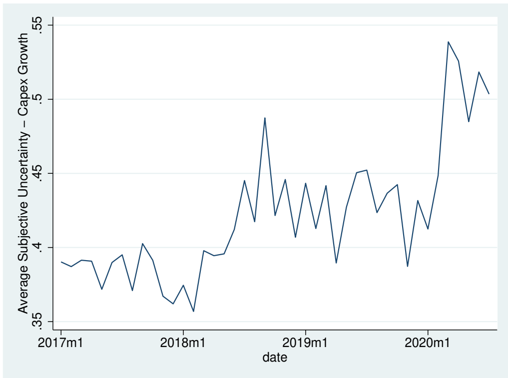
[1] Following Altig et al. (2020), I plotted subjective uncertainty after 2017 because of the modification in the panel rotation scheme in September 2016, which raised the number of respondents per topic from about 50 to 150. Moreover, the formulation of the sales question in September 2016 is revised, which significantly reduced response errors and the noisiness of average measures. The average values are noisier before this modification. In all other studies in the paper, as I am not concentrating on average subjective uncertainty values, I use the whole available sample from 2014 to 2022.



(a) Sale Growth



(b) Employment Growth



(c) Capex Growth

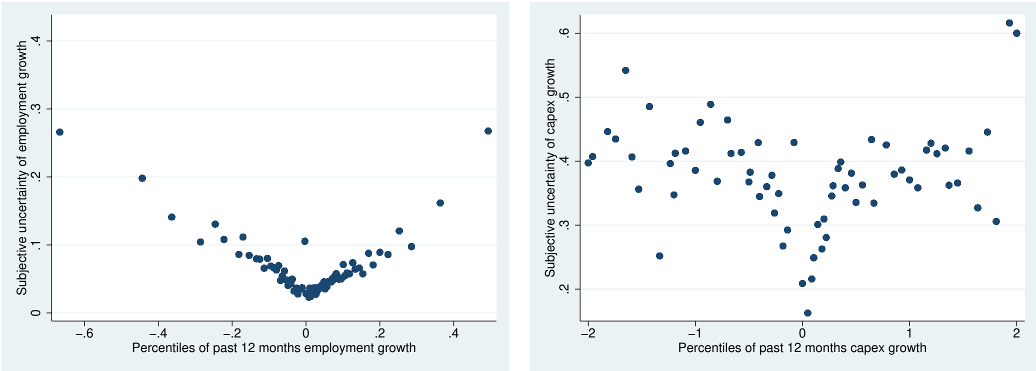
Figure A.1: Average values of subjective uncertainty.

we saw for the sale growth rate, however, for the capital expenditure the V-shape is not as evident as other two variables.

Impact of realized uncertainty on subjective uncertainty without firms' fixed effect

In the section 1.2 and in particular in the regression (1.1), I studied the impact of the size shock on the subjective uncertainty. In my primary study, I took firms' fixed effect into account. However, as I only have limited number of observations for each firm, my result might be affected by the Nickell bias. Here as a robustness check I will remove firms' fixed effect F_i and run the following regression:

$$\begin{aligned} SDS_{i,t}^{SaleG} &= \beta SizeShock_{i,t} + \rho SDS_{i,t-1}^{SaleG} \\ &\quad + \lambda SizeShock_{i,t} Obs_{i,t} + \gamma Obs_{i,t} + G_{jt} + \epsilon_{i,t} \end{aligned}$$



(a) Employment Growth Rate (b) Capex Growth Rate

Figure A.2: Subjective uncertainty about next year employment and capital expenditure growth rates versus past realizations.

In the table A.1 you can find the result. As you notice, there is still a significant positive relationship between the size shock (realized uncertainty) and subjective uncertainty. However, there is a significant decline in R-squared compared to the table 1.1.

VARIABLES	$SDS_{i,t}^{SaleG}$
$SizeShock_{i,t}$	0.0854***
	(0.00446)
$SDS_{i,t-1}^{SaleG}$	0.449***
	(0.00971)
$SizeShock_{i,t}Obs_{i,t}$	-0.00290***
	(0.000260)
$Obs_{i,t}$	0.000152*
	(8.90e-05)
Observations	9,908
R-squared	0.431

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A.1: Impact of the realized uncertainty on the subjective uncertainty.

Next, I will do the same study by introducing a dummy $d_{i,t}^{Old}$ in order to study the impact of the number of observations on the responsiveness of subjective uncertainty to the realized uncertainty[2]:

$$SDS_{i,t}^{SaleG} = \left[\beta + \beta' d_{i,t}^{Old}\right] SizeShock_{i,t} + \rho SDS_{i,t-1}^{SaleG} \\ + \lambda SizeShock_{i,t} Obs_{i,t} + \gamma Obs_{i,t} + G_{jt} + \epsilon_{i,t}$$

The result of the regression is provided in the table A.2. As you can see, again the responsiveness of the subjective uncertainty to realized uncertainty decreases as the number of observations increases. Although the result is not as significant as the case that I had firms' fixed effect.

VARIABLES	$SDS_{i,t}^{SaleG}$
$SizeShock_{i,t}$	0.0852***
	(0.00447)
$d_{i,t}^{Old} SizeShock_{i,t}$	-0.00626
	(0.00900)
$SDS_{i,t-1}^{SaleG}$	0.448***
	(0.00972)
$SizeShock_{i,t} Obs_{i,t}$	-0.00262***
	(0.000468)
$Obs_{i,t}$	0.000155*
	(8.90e-05)
Observations	9,908
R-squared	0.431

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A.2: Information set size and responsiveness of subjective uncertainty to realized uncertainty.

A.1.2 Subjective uncertainty and learning uncertainty

Non-ergodic Bayesian Learning

Previously in the section 1.3, I presented the Normal-Gamma Bayesian learning details and its updating formula in (1.2). The presented Bayesian learning is ergodic. To rephrase it, as the second moment term is drawn once without applying

[2]To be more clear, I define $E_j\left(Obs_{i,t}\right)$ as the average number of observations in the sector j over the whole sample time and I use this threshold to specify whether the firm i at the time t is young or old:

$$d_{i,t}^{Old} = 1 \quad , \quad if \quad Obs_{i,t} > E_j\left(Obs_{i,t}\right)$$

any shock afterward, the true value will eventually be learnt after observing more and more data.
Bakshi and Skoulakis (2010) and Weitzman (2007) based on Shephard (1994) present non-ergodic learning by applying Beta shocks to the precision term. Here I present details of non-ergodic Bayesian learning.
Let us assume the variable $x_{i,t}$ in each period conditional on μ_i^x and $\sigma^2_{x,i,t}$ is drawn from the distribution $N\left(\mu_i^x, \sigma^2_{x,i,t}\right)$ and the firm i is uncertain about the value of $\sigma^2_{x,i,t}$. By observing shocks $x_{i,t}$ over the time the firm can learn the unknown variance. Same as before, by assuming Gamma prior for the precision $\theta^x_{i,t} = 1/\sigma^2_{x,i,t}$, we preserve the conjugacy of Bayesian learning. The details of the beliefs updating in the Normal-Gamma Bayesian learning are provided below:

- Prior:

$$x_{i,t}|\theta^x_{i,t} \overset{iid}{\sim} N\left(\mu_i^x, \frac{1}{\theta^x_{i,t}}\right) \quad , \quad \theta^x_{i,0} \sim Gamma\left(\alpha^x_{i,0}, \beta^x_{i,0}\right)$$

$$\theta^x_{i,t+1} = \frac{1}{\omega_i^x}\eta^x_{i,t+1}\theta^x_{i,t} \quad , \quad \eta^x_{i,t+1} \sim Beta\left(\omega_i^x a^x_{i,t}, (1-\omega_i^x)a^x_{i,t}\right)$$

The constant $0 < \omega_i^x < 1$ controls the speed of the precision and usually takes values close to 1. Here as you can see, the precision term is not constant and changes in each period following above process. Normal-Gamma-Beta prior still preserves the conjugacy and after observing the history $I^x_{i,t}$ the posterior will be:

- Posterior:

$$x_{i,t}|\theta^x_{i,t}, I^x_{i,t} \sim N\left(\mu_i^x, \frac{1}{\theta^x_{i,t}}\right) \quad , \quad \theta^x_{i,t}|I^x_{i,t} \sim Gamma(\alpha^x_{i,t}, \beta^x_{i,t})$$

$$\alpha^x_{i,t} = \omega_i^x \alpha^x_{i,t-1} + \frac{1}{2} \quad , \quad \beta^x_{i,t} = \omega_i^x \beta^x_{i,t-1} + \frac{(x_{i,t-1} - \mu_i^x)^2}{2}$$

Please note the parameter $\alpha^x_{i,t}$ changes deterministically and regardless of the realizations of $x_{i,t}$, so the realizations of $x_{i,t}$ do not change the process of $\eta^x_{i,t}$. Given the limited number of observations that I have for each firm in the survey data and also to preserve the tractability of the learning process, I primarily concentrate on the ergodic learning in this paper.

Subjective uncertainty and GARCH

In the section 1.3, I compared Bayesian posterior's uncertainty with the subjective uncertainty and I showed that they are significantly and positively correlated. I also compared the learning model with GARCH(1,1) to see which one fits the data better. Here I provide details about GARCH(1,1) estimation. If the variable $x_{i,t}$ follows the GARCH(1,1) process we will have:

$$x_{i,t} = \mu_0 + \epsilon_{x,i,t} \quad , \quad \epsilon_{x,i,t} \sim N(0, \widetilde{\sigma}^2_{x,i,t})$$

$$\widetilde{\sigma}^2_{x,i,t} = \gamma_0 + \gamma_1 \epsilon^2_{x,i,t-1} + \delta_1 \widetilde{\sigma}^2_{x,i,t-1}$$

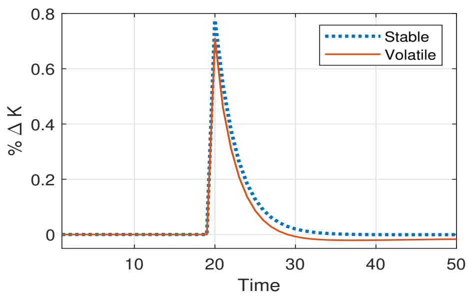
I estimate the process using likelihood maximization and after that find the time varying uncertainty in the firm level.

A.2 Chapter 2 - Appendix

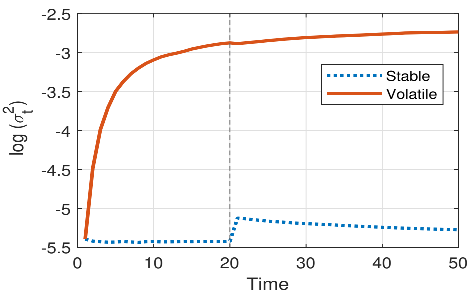
A.2.1 Implications of learning uncertainty, lower risk aversion calibration

In the chapter 2 and in the section 2.2.2 by using impulse responses I studied implications of learning uncertainty for firms' investment patterns. In my main study I chose the value of risk aversion for CRRA utility equal to $\gamma = 8$ which is a pretty high calibrated value. I chose this value to obtain impulse responses that are visibly distinguishable. Here, I do the same exercise and present 3 impulse responses that were discussed in the section 2.2.2 with the only difference that I choose the risk aversion parameter $\gamma = 5$.

- **Stable versus volatile environment:**



(a) Investment impulse responses.



(b) Posterior uncertainty.

Figure A.3: Stable versus volatile environment - with learning uncertainty.

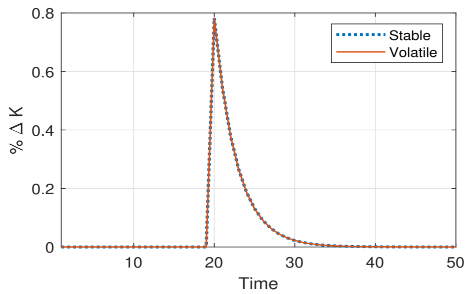
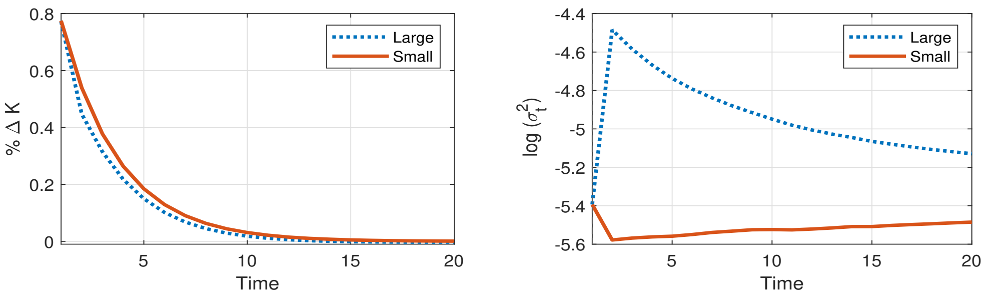


Figure A.4: Stable versus volatile environment - without learning uncertainty.

- **Small versus large shocks:**



(a) Investment impulse responses. (b) Posterior uncertainty.

Figure A.5: Small versus large shocks - with learning uncertainty.

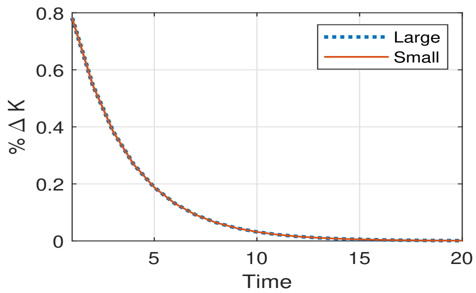
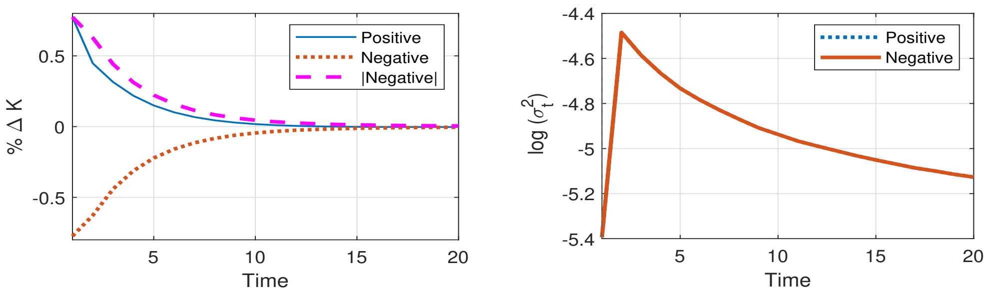


Figure A.6: Small versus large shocks - without learning uncertainty.

- **Positive versus negative shocks:**



(a) Investment impulse responses. (b) Posterior uncertainty.

Figure A.7: Positive versus negative shocks - with learning uncertainty.

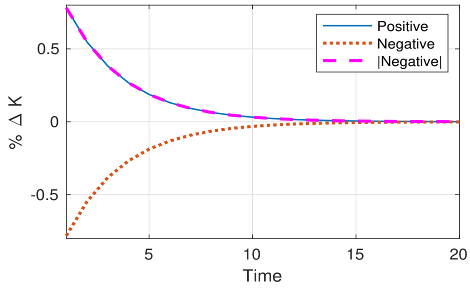


Figure A.8: Positive versus negative shocks - without learning uncertainty.

As you see all three implications of learning uncertainty are still valid with the only difference that in the impulse responses they are less visibly discernible. I did the same exercise with the risk aversion parameter $\gamma = 2$ and still all three implications hold.

A.2.2 Compustat data cleaning

I clean Compustat data following Jeenas (2019) and Chiavari and Goraya (2021). I drop financial firms with SIC codes between 6900-6999 and firms in utilities sector with SIC codes between 4900-4999. I drop all firms with missing or negative sales or cost of goods sold. I only concentrate on US firms so all firms with country code other than US are dropped. All firms without any industry information are also dropped.

A.2.3 Constructing capital stock

I need to construct capital stock for my study in the section 2.3, both for estimating the TFPR process and also the investment rate. I construct capital stock using the perpetual inventory method following Jeenas (2019), Ottonello and Winberry (2020) and Mongey and Williams (2017). In particular, I measure the initial value of firm i's capital stock as the earliest available entry of $PPEGTQ_{i,t}$, and then iteratively construct $K_{i,t}$ from $PPENTQ_{i,t}$ as:

$$K_{i,t} = K_{i,t-1} + PPENTQ_{i,t} - PPENTQ_{i,t-1}$$

A.2.4 Estimating production function's elasticities

Extracting TFPR from solow residual in the equation (2.2) requires estimation of elasticities α_L^j and α_K^j. Cost share approach and production function estimation approach are two standard approaches in the litreature to estimate elasticities. In

my study two approaches result in very similar outcomes so I only take cost share approach into account.
Consider a firm i in the sector j at the time t that produces the good $Y_{i,t}$ using labour and materials $LM_{i,t}$ and capital $K_{i,t}$ through the following Cobb-Douglas production function:

$$Y_{i,t} = \widetilde{A}_{i,t} K_{i,t}^{\alpha_K^j} (LM_{i,t})^{\alpha_L^j}$$

Static profit maximization of the firm results in following two optimal conditions:

$$\frac{\alpha_K^j Y_{i,t}}{K_{i,t}} = r_{i,t} \qquad , \qquad \frac{\alpha_L^j Y_{i,t}}{LM_{i,t}} = w_{i,t}$$

where $r_{i,t}$ is the rental cost of capital and $w_{i,t}$ is the cost of labour and materials. After rearrangement you can easily see:

$$\alpha_K^j = \frac{r_{i,t} K_{i,t}}{r_{i,t} K_{i,t} + w_{i,t} LM_{i,t}} \qquad , \qquad \alpha_L^j = \frac{w_{i,t} LM_{i,t}}{r_{i,t} K_{i,t} + w_{i,t} LM_{i,t}}$$

which basically implies that elasticities are equal to the relative cost shares.
I assume the quarterly cost of rental capital for all firms is constant and is equal to 3% [3]. I use the item $\mathtt{COGSQ}_{i,t}$ from Compustat for the labor and material cost share $w_{i,t} LM_{i,t}$ and constructed capital stock in the previous subsections for $K_{i,t}$. I calculate elasticities for each firm i at each time t. Finally, I take the median value in each sector j during *ten years rolling windows* as the targeted elasticity for all firms within the sector j.

A.2.5 Local projections' results

In the subsections 2.3.1 and 2.3.2, using local projection method, I studied general investment response to TFPR shocks and also implications of learning uncertainty in Compustat data to check whether we observe three mentioned investment patterns in the data or not. I also plotted impulse response functions. In order to obtain correct impulse responses, I eliminated the interaction of control variables with the shock. Here, I provide the details of estimation for each local projection.

[3] This is in line with average yearly deprecation rate of 10% and interest rate of 2%.

General investment response

$$I_{i,t+h} = \beta_h \eta_{i,t} + \Gamma_h' W_{i,t-1} + \sum_{m>0} \theta_h^m I_{i,t-m} + F_i + G_{j,t} + \epsilon_{i,t,h}$$

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.202***	0.267***	0.344***	0.275***
	(0.0177)	(0.0185)	(0.0190)	(0.0199)
$liq_{i,t-1}$	0.0482***	0.0498***	0.0511***	0.0455***
	(0.00223)	(0.00232)	(0.00239)	(0.00249)
$lev_{i,t-1}$	-0.0433***	-0.0437***	-0.0413***	-0.0381***
	(0.00225)	(0.00235)	(0.00241)	(0.00252)
$age_{i,t-1}$	0.0223***	0.0256***	0.0313***	0.0348***
	(0.00313)	(0.00335)	(0.00356)	(0.00375)
$size_{i,t-1}$	0.145***	0.144***	0.146***	0.118***
	(0.00511)	(0.00535)	(0.00552)	(0.00581)
$I_{i,t-1}$	0.241***	0.172***	0.207***	0.119***
	(0.00296)	(0.00309)	(0.00317)	(0.00331)
$I_{i,t-2}$	0.0901***	0.150***	0.0494***	0.0362***
	(0.00313)	(0.00327)	(0.00335)	(0.00351)
$I_{i,t-3}$	0.123***	0.0246***	0.00614*	0.00547
	(0.00313)	(0.00326)	(0.00335)	(0.00349)
$I_{i,t-4}$	0.0186***	0.00534	0.000432	0.0751***
	(0.00312)	(0.00326)	(0.00334)	(0.00349)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.566

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A.3: General investment response to the TFPR shock - without control interactions.

Stable versus volatile environment

$$d_i^v = 1 \quad if \quad SD_i(\eta_{i,t}) > Med\left[SD_i(\eta_{i,t})\right]$$

$$I_{i,t+h} = \beta_h \eta_{i,t} + \beta_h^v d_i^v \eta_{i,t} + \Gamma_h' W_{i,t-1} + \sum_{m>0} \theta_h^m I_{i,t-m} + F_i + G_{jt} + \epsilon_{i,t,h}$$

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.496***	0.764***	0.974***	0.898***
	(0.0521)	(0.0544)	(0.0558)	(0.0583)
$d_i^v \eta_{i,t}$	-0.324***	-0.547***	-0.696***	-0.687***
	(0.0541)	(0.0564)	(0.0579)	(0.0604)
$liq_{i,t-1}$	0.0478***	0.0491***	0.0503***	0.0447***
	(0.00223)	(0.00232)	(0.00239)	(0.00249)
$lev_{i,t-1}$	-0.0432***	-0.0435***	-0.0411***	-0.0379***
	(0.00225)	(0.00235)	(0.00241)	(0.00252)
$age_{i,t-1}$	0.0224***	0.0259***	0.0317***	0.0351***
	(0.00313)	(0.00335)	(0.00356)	(0.00375)
$size_{i,t-1}$	0.147***	0.147***	0.150***	0.122***
	(0.00512)	(0.00536)	(0.00553)	(0.00582)
$I_{i,t-1}$	0.241***	0.173***	0.207***	0.119***
	(0.00296)	(0.00309)	(0.00317)	(0.00330)
$I_{i,t-2}$	0.0902***	0.150***	0.0497***	0.0365***
	(0.00313)	(0.00327)	(0.00335)	(0.00350)
$I_{i,t-3}$	0.123***	0.0246***	0.00620*	0.00556
	(0.00313)	(0.00326)	(0.00335)	(0.00349)
$I_{i,t-4}$	0.0187***	0.00547*	0.000568	0.0753***
	(0.00312)	(0.00326)	(0.00334)	(0.00349)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.566

Standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table A.4: Investment response of firms who lived in stable versus volatile environment - without control interactions.

Small versus large shocks

$$d^l_{i,t} = 1 \quad if \quad |\eta_{i,t}| > 2SD\left(\eta_{i,t}\right)$$

$$I_{i,t+h} = \beta_h \eta_{i,t} + \beta^l_h d^l_{i,t} \eta_{i,t} + \Gamma'_h W_{i,t-1} + \sum_{m>0} \theta^m_h I_{i,t-m} + F_i + G_{jt} + \epsilon_{i,t,h}$$

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.309***	0.397***	0.500***	0.483***
	(0.0257)	(0.0268)	(0.0275)	(0.0287)
$d^l_{i,t}\eta_{i,t}$	-0.190***	-0.231***	-0.280***	-0.373***
	(0.0332)	(0.0346)	(0.0357)	(0.0373)
$liq_{i,t-1}$	0.0479***	0.0494***	0.0506***	0.0449***
	(0.00223)	(0.00232)	(0.00239)	(0.00249)
$lev_{i,t-1}$	-0.0432***	-0.0436***	-0.0412***	-0.0379***
	(0.00225)	(0.00235)	(0.00241)	(0.00252)
$age_{i,t-1}$	0.0224***	0.0257***	0.0315***	0.0349***
	(0.00313)	(0.00335)	(0.00356)	(0.00375)
$size_{i,t-1}$	0.148***	0.148***	0.151***	0.125***
	(0.00514)	(0.00539)	(0.00556)	(0.00585)
$I_{i,t-1}$	0.241***	0.172***	0.207***	0.118***
	(0.00296)	(0.00309)	(0.00317)	(0.00330)
$I_{i,t-2}$	0.0902***	0.150***	0.0495***	0.0365***
	(0.00313)	(0.00327)	(0.00335)	(0.00350)
$I_{i,t-3}$	0.123***	0.0246***	0.00621*	0.00553
	(0.00313)	(0.00326)	(0.00335)	(0.00349)
$I_{i,t-4}$	0.0187***	0.00550*	0.000590	0.0752***
	(0.00312)	(0.00326)	(0.00334)	(0.00349)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.566

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A.5: Investment response to small versus large TFPR shocks - without control interactions.

Positive versus negative shocks

$$d^p_{i,t} = 1 \quad if \quad \eta_{i,t} > 0$$

$$I_{i,t+h} = \beta_h \eta_{i,t} + \beta^p_h d^p_{i,t} \eta_{i,t} + \Gamma'_h W_{i,t-1} + \sum_{m>0} \theta^m_h I_{i,t-m} + F_i + G_{jt} + \epsilon_{i,t,h}$$

VARIABLES	$I_{i,t+1}$	$I_{i,t+2}$	$I_{i,t+3}$	$I_{i,t+4}$
$\eta_{i,t}$	0.310***	0.402***	0.462***	0.424***
	(0.0319)	(0.0334)	(0.0343)	(0.0359)
$d^p_{i,t}\eta_{i,t}$	-0.220***	-0.276***	-0.244***	-0.304***
	(0.0546)	(0.0569)	(0.0586)	(0.0614)
$liq_{i,t-1}$	0.0482***	0.0498***	0.0511***	0.0455***
	(0.00223)	(0.00232)	(0.00239)	(0.00249)
$lev_{i,t-1}$	-0.0432***	-0.0436***	-0.0413***	-0.0380***
	(0.00225)	(0.00235)	(0.00241)	(0.00252)
$age_{i,t-1}$	0.0224***	0.0257***	0.0315***	0.0349***
	(0.00313)	(0.00335)	(0.00356)	(0.00375)
$size_{i,t-1}$	0.144***	0.143***	0.145***	0.117***
	(0.00511)	(0.00536)	(0.00552)	(0.00581)
$I_{i,t-1}$	0.241***	0.172***	0.207***	0.119***
	(0.00296)	(0.00309)	(0.00317)	(0.00331)
$I_{i,t-2}$	0.0901***	0.150***	0.0494***	0.0363***
	(0.00313)	(0.00327)	(0.00335)	(0.00351)
$I_{i,t-3}$	0.123***	0.0245***	0.00612*	0.00542
	(0.00313)	(0.00326)	(0.00335)	(0.00349)
$I_{i,t-4}$	0.0186***	0.00542*	0.000485	0.0752***
	(0.00312)	(0.00326)	(0.00334)	(0.00349)
Observations	139,375	135,165	131,136	127,373
R-squared	0.612	0.593	0.585	0.566

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table A.6: Investment response to positive versus negative TFPR shocks - without control interactions.

A.3 Chapter 3 - Appendix

A.3.1 Baseline model without nominal rigidity

In this section after summarizing the model without nominal rigidity, the optimal conditions will be derived:

Environment:

Household:

- Utility:

$$U = \frac{C^{1-\gamma} - 1}{1-\gamma} - \frac{N^{1-\epsilon} - 1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta} - 1}{1-\delta}$$

- Aggregation:

$$C = \left[\int_I (c_i)^{\frac{\rho-1}{\rho}} \, di\right]^{\frac{\rho}{\rho-1}}, \quad P = \left[\int_I (p_i)^{1-\rho} \, di\right]^{\frac{1}{1-\rho}}$$

$$N = \left[\int_I (n_i)^{\frac{\omega-1}{\omega}} \, di\right]^{\frac{\omega}{\omega-1}}, \quad W = \left[\int_I [(1-\tau)w_i]^{1-\omega} di\right]^{\frac{1}{1-\omega}}$$

- Household's Budget Constraint:

$$\int_I p_i c_i di + M = \int_I \Pi_i di + \int_I (1-\tau) w_i n_i di + T$$

- Government's Budget Constraint:

$$T = \int_I \tau w_i n_i di + M$$

Firms:

- Monopolistic Competitive Firms producing differentiated goods:

$$y_i = A_i n_i$$

- Profit Maximization Problem:

$$\max_{p_i,n_i} \quad p_i y_i - w_i n_i \quad s.t \quad \frac{p_i}{P} = \left(\frac{\overbrace{A_i n_i}^{c_i = y_i}}{C} \right)^{-\frac{1}{\rho}}$$

$$\max_{p_i,n_i} \quad C\left(p_i\right)^{1-\rho} P^{\rho} - w_i n_i \quad s.t \quad \frac{p_i}{P} = \left(\frac{\overbrace{A_i n_i}^{c_i = y_i}}{C} \right)^{-\frac{1}{\rho}}$$

Optimal Conditions:

I summarize the main optimal conditions here. The details of the optimization are provided in the next section of the appendix.

- Consumption basket:

$$\frac{p_i}{P} = \left(\frac{c_i}{C}\right)^{\frac{-1}{\rho}}$$

- Labour supply:

$$\frac{(1-\tau)w_i}{W} = \left(\frac{n_i}{N}\right)^{\frac{-1}{\omega}} \quad or \quad \frac{P}{W} = \frac{C^{-\gamma}}{N^{-\epsilon}}$$

- Money demand:

$$\left(\frac{M}{P}\right)^{-\delta} = C^{-\gamma}$$

- Price setting :

$$p_i = \frac{\rho}{\rho - 1} \quad \frac{w_i}{A_i}$$

Equilibrium:

Using above equations we can easily find the equilibrium in a model without the nominal rigidity [4]. Start from the consumption basket equation. From the market

[4]Guess and verify is the standard approach in this literature to find the equilibrium. However, in the friction-less model for a better illustration, I find the equilibrium directly without using guess and verify method. I use this approach later in the model with nominal rigidity.

clearing and after some rearrangement we will have:

$$\frac{p_i^{1-\rho}}{P^{-\rho}}C = p_i A_i n_i \rightarrow \textit{Integration} \rightarrow PC = \int p_i A_i n_i di \rightarrow$$

$$\textit{Price Setting} \rightarrow PC = \frac{\rho}{\rho - 1}\int w_i n_i di$$

In the same way we will obtain the following condition from the labour basket:

$$NW = (1-\tau)\int w_i n_i di$$

So:

$$\frac{PC}{WN} = \frac{\rho}{(\rho-1)(1-\tau)} \quad \rightarrow \textit{Labour Supply} \rightarrow \quad \frac{C^{1-\gamma}}{N^{1-\epsilon}} = \frac{\rho}{(\rho-1)(1-\tau)}$$

Next, by dividing the consumption basket by labor supply we will have:

$$\frac{p_i}{w_i} \quad \frac{W}{P(1-\tau)} = \frac{\left(\frac{c_i}{C}\right)^{\frac{-1}{\rho}}}{\left(\frac{n_i}{N}\right)^{\frac{-1}{\omega}}}$$

$$\rightarrow \quad \frac{\rho}{(\rho-1)A_i}\frac{N^{1-\epsilon}}{C^{1-\gamma}(1-\tau)} = \frac{\left(\frac{c_i}{C}\right)^{\frac{-1}{\rho}}}{\left(\frac{n_i}{N}\right)^{\frac{-1}{\omega}}}\frac{N}{C} \quad \rightarrow$$

$$\frac{(A_i n_i)^{\frac{1}{\rho}}}{A_i n_i^{\frac{1}{\omega}}} = \frac{C^{\frac{1-\rho}{\rho}}}{N^{\frac{1-\omega}{\omega}}}$$

$$A_i^{\frac{1-\rho}{\rho}} n_i^{\frac{\omega-\rho}{\rho\omega}} = \frac{C^{\frac{1-\rho}{\rho}}}{N^{\frac{1-\omega}{\omega}}} \quad \rightarrow \quad n_i = \frac{C^{\frac{\omega(1-\rho)}{\omega-\rho}}}{N^{\frac{\rho(1-\omega)}{\omega-\rho}}} A_i^{\frac{\omega(\rho-1)}{\omega-\rho}}$$

We know $N = \left[\int_I (n_i)^{\frac{\omega-1}{\omega}} di\right]^{\frac{\omega}{\omega-1}}$ so by integrating n_i and after some rearrangement we will have:

$$\left[\frac{N}{C}\right]^{\frac{(\omega-1)(1-\rho)}{\omega-\rho}} = \int_I A_i^{\frac{(\omega-1)(\rho-1)}{\omega-\rho}} di$$

Using the law of large number and the mentioned distribution for A_i we will see:

$$\int_I A_i^{\frac{(\omega-1)(\rho-1)}{\omega-\rho}} di = e^{\frac{(\omega-1)(\rho-1)}{\omega-\rho}\left(\overline{a}-\frac{\sigma^2}{2}\right)+\frac{(\omega-1)^2(\rho-1)^2\sigma^2}{2(\omega-\rho)^2}}$$

So

$$\frac{C}{N} = e^{\overline{a}+\left[\frac{(\omega-1)(\rho-1)}{\omega-\rho}-1\right]\frac{\sigma^2}{2}}$$

We have already found:

$$\frac{PC}{WN} = \frac{C^{1-\gamma}}{N^{1-\epsilon}} = \frac{\rho}{(\rho-1)(1-\tau)}$$

So

$$e^{(1-\gamma)\overline{a}+\left[\frac{(\omega-1)(\rho-1)}{\omega-\rho}-1\right]\frac{(1-\gamma)\sigma^2}{2}} N^{\epsilon-\gamma} = \frac{\rho}{(\rho-1)(1-\tau)} \quad \rightarrow$$

$$log(N) = \underbrace{\frac{log\left(\frac{\rho}{(\rho-1)(1-\tau)}\right)}{\epsilon-\gamma}}_{N_0} \quad + \quad \underbrace{\frac{\gamma-1}{\epsilon-\gamma}}_{N_A}\overline{a} \quad + \quad \underbrace{\frac{(\gamma-1)[1+\omega(\rho-2)]}{2(\omega-\rho)(\epsilon-\gamma)}}_{N_\sigma}\sigma^2$$

And we can easily find $log(C)$:

$$log(C) = \underbrace{\frac{log\left(\frac{\rho}{(\rho-1)(1-\tau)}\right)}{\epsilon-\gamma}}_{C_0} \quad + \quad \underbrace{\frac{\epsilon-1}{\epsilon-\gamma}}_{C_A}\overline{a} \quad + \quad \underbrace{\frac{(\epsilon-1)[1+\omega(\rho-2)]}{2(\omega-\rho)(\epsilon-\gamma)}}_{C_\sigma}\sigma^2$$

After finding aggregate real variables we can easily find the idiosyncratic real variables:

$$n_i = \frac{C^{\frac{\omega(1-\rho)}{\omega-\rho}}}{N^{\frac{\rho(1-\omega)}{\omega-\rho}}} A_i^{\frac{\omega(\rho-1)}{\omega-\rho}} \quad \rightarrow$$

$$log(n_i) = \frac{\omega(1-\rho)}{\omega-\rho}C_0 - \frac{\rho(1-\omega)}{\omega-\rho}N_0 + \left(\frac{\omega(1-\rho)}{\omega-\rho}C_A - \frac{\rho(1-\omega)}{\omega-\rho}N_A\right)\overline{a} +$$

$$\left(\frac{\omega(1-\rho)}{\omega-\rho}C_\sigma - \frac{\rho(1-\omega)}{\omega-\rho}N_\sigma\right)\sigma^2 + \frac{\omega(\rho-1)}{\omega-\rho}a_i =$$

and $log(c_i) = a_i + log(n_i)$. As you can see all real variables are determined regardless of the monetary policy. Therefore, in the framework without nominal rigidity *the monetary policy is neutral.*

Regarding the nominal variables, from the money demand equation you can easily see that the aggregate price is pinned down by the monetary policy and aggregate output $P = \frac{M}{C^{\frac{\gamma}{\delta}}}$ and by replacing it the consumption basket we can easily find the relationship between nominal wages (both the idiosyncratic and the aggregate) and the monetary policy and real variables. Finally form the price setting equation, by replacing the idiosyncratic wage, you can easily find the relationship between idiosyncratic price and the monetary policy and real variables:

$$log(p_i) = \underbrace{\frac{-\gamma C_0}{\delta}}_{\psi'_0} + \underbrace{\frac{1-\omega}{\omega-\rho}}_{\psi'_a} a_i + \underbrace{\left(m_\sigma - \overbrace{\left[\frac{(1-\rho){\psi'_a}^2}{2} + \frac{\gamma C_\sigma}{\delta} - \frac{\psi'_a}{2} \right]}^{Q_s} \right)}_{\psi'_\sigma} \sigma^2 + \underbrace{\left(-\psi_a - \frac{\gamma C_A}{\delta} \right)}_{\psi'_A} \overline{a} \tag{A.1}$$

$$log(w_i) = log(p_i) + a_i - log\left(\frac{\rho}{\rho-1}\right)$$

I assume the monetary policy follows the rule $log(M) = m_\sigma \sigma^2$. The intuition for this policy rule is provided in the section of the model with nominal rigidity. As you can see ψ'_a is always negative which means more productive firms set lower prices.

Now let us go to *the optimal fiscal policy.* After finding the equilibrium allocations, we can express the ex-ante expected welfare of the government, before realization of the shock as follows:

$$E(U) = E\left(\frac{C^{1-\gamma}-1}{1-\gamma} - \frac{N^{1-\epsilon}-1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta}-1}{1-\delta} \right) =$$

$$E\left(\frac{e^{(1-\gamma)(C_0+C_A\overline{a}+C_\sigma\sigma^2)}-1}{1-\gamma} - \frac{e^{(1-\epsilon)(N_0+N_A\overline{a}+N_\sigma\sigma^2)}-1}{1-\epsilon} + \frac{e^{(1-\delta)[\frac{\gamma C_0}{\delta}+\frac{\gamma C_A}{\delta}\overline{a}+\frac{\gamma C_\sigma}{\delta}\sigma^2]}-1}{1-\delta} \right) = \tag{A.2}$$

$$\frac{e^{(1-\gamma)(C_0+C_A\overline{a}+C_\sigma\sigma_0^2)}-1}{1-\gamma} - \frac{e^{(1-\epsilon)(N_0+N_A\overline{a}+N_\sigma\sigma_0^2)}-1}{1-\epsilon} + \frac{e^{(1-\delta)[\frac{\gamma C_0}{\delta}+\frac{\gamma C_A}{\delta}\overline{a}+\frac{\gamma C_\sigma}{\delta}\sigma_0^2]}-1}{1-\delta}$$

I use the standard Normal-Gamma prior. It is well-known in the literature that if the conditional distribution of the variable x is normally distributed $x|\sigma^2 \sim N(\mu, \sigma^2)$ and $\sigma^2 \sim Inverse-Gamma(\alpha_0, \beta_0)$, then the unconditional distribution of the variable x is not normal anymore and will be distributed according to the fatter tail student's t distribution $x \sim t_{2\alpha_0}(\mu, \sigma_0^2)$. We know that the moment generating function for the student's t distribution does not exist, so in general the

expected welfare above is not well defined. However, by assuming the parametric assumption that $\alpha_0 \to \infty$ and $\beta_0 \to \infty$, the degree of freedom for the student's t distribution goes to infinity and we converge to the normal distribution. Moreover, as $\frac{\beta_0}{\alpha_0} \to \sigma_0^2$, the expected welfare converges to a finite value.
By assuming $log(1-\tau) = \tau_0$, we can express the optimal fiscal policy as:

$$\partial E(U)/\partial \tau_0 = 0 \quad \to$$

$$e^{(1-\gamma)(C_0+C_A\overline{a}+C_\sigma\sigma_0^2)} + e^{\frac{\gamma(1-\delta)}{\delta}(C_0+C_A\overline{a}+C_\sigma\sigma_0^2)}\frac{\gamma}{\delta} = e^{(1-\epsilon)(N_0+N_A\overline{a}+N_\sigma\sigma_0^2)}$$

And after replacing equilibrium values for policy functions' coefficients we can *numerically* find the optimal fiscal policy which eliminates the monopolistic competition distortion.

Contractionary Dispersion Shock

From the previous subsection we know $log(C) = log(Y) = C_0 + C_A\overline{a} + C_\sigma\sigma^2$. It is easy to see that C_A is always greater than zero which implies higher average TFP will increase the output. How about C_σ?

$$C_\sigma = -\frac{(1-\epsilon)[1+\omega(\rho-2)]}{2(\rho-\omega)(\gamma-\epsilon)}$$

We know $\epsilon < 0$, $\rho > 1 > 0 > \omega$ and $\gamma > 0 > \epsilon$. Therefore, you can easily see that there exists a value $\omega^* = \frac{-1}{\rho-2}$, by assuming $\rho > 2$, such that for $\omega^* < \omega < 0$ we will have $C_\sigma < 0$ or in other words, contractionary dispersion shock. In the same way, there exists a value $\rho^* = 2 - \frac{1}{\omega}$ such that for $1 < \rho < \rho^*$ the coefficient C_σ will be negative.
Either of these two conditions imply that we need a small degree of substitution for labor or consumed goods to be able to have a contractionary dispersion shock.

A.3.2 Model with nominal rigidity

Optimal Price Setting Condition

Let us study the profit maximization problem of a firm:

$$\max_{p_i,n_i} \quad E\left[\frac{U'(C)}{P}(p_i y_i - w_i n_i)\right] \quad s.t. \quad \frac{p_i}{P} = \left(\frac{c_i}{C}\right)^{-\frac{1}{\rho}}$$

After replacing the production function $c_i = y_i = A_i n_i$ in the consumption basket we will have:

$$n_i = \left(\frac{CP^\rho}{A_i}\right)(p_i)^{-\rho}$$

And then plugging it into the firms problem:

$$E\left[\frac{U'(C)}{P}\left(p_iA_i \quad \partial n_i/\partial p_i + A_in_i - w_i \quad \partial n_i/\partial p_i\right)\right] = 0$$

$$E\left[\frac{U'(C)}{P}\left(-\rho p_iA_i\frac{n_i}{p_i} + A_in_i + \rho w_i\frac{n_i}{p_i}\right)\right] = 0$$

$$p_i \quad = \quad \frac{\rho}{A_i(\rho-1)} \quad \frac{E\left[\frac{U'(C)}{P}w_in_i\right]}{E\left[\frac{U'(C)}{P}n_i\right]}$$

Equilibrium

I use guess and verify approach to determine equilibrium conditions. I guess following policy functions for agents and the government:

- Household's policy functions:
  - $log(n_i) = n_0 + n_aa_i + n_A\overline{a} + n_\sigma\sigma^2$.
  - $log(N) = N_0 + N_A\overline{a} + N_\sigma\sigma^2$.
  - $log(C) = log(Y) = C_0 + C_A\overline{a} + C_\sigma\sigma^2$.
- Firms' policy functions:
  - $log(p_i) = \psi_0 + \psi_aa_i + \psi_A\overline{a}$.
  - $log(P) = P_0 + P_A\overline{a} + P_\sigma\sigma^2$.
- Fiscal Policy: $log(1-\tau) = \tau_0$.
- Monetary Policy: $log(M) = m_\sigma\sigma^2$.

and plug these functions in the aggregation and optimal conditions using the market clearing in order to determine 16 unknown coefficients (excluding policies' coefficients):

- Aggregate consumption:

$$e^{C_0+C_A\overline{a}+C_\sigma\sigma^2} = \left[\int_I (y_i)^{\frac{\rho-1}{\rho}}\,di\right]^{\frac{\rho}{\rho-1}} \rightarrow \quad \textit{Market clearing} \quad \rightarrow \quad =$$

$$\left[\int_I \left(e^{a_i+(n_0+n_aa_i+n_A\overline{a}+n_\sigma\sigma^2)}\right)^{\frac{\rho-1}{\rho}}\,di\right]^{\frac{\rho}{\rho-1}} =$$

$$\left[\int_I \left(e^{\frac{\rho-1}{\rho}\left[a_i+(n_0+n_a a_i+n_A\overline{a}+n_\sigma\sigma^2)\right]}\right) di\right]^{\frac{\rho}{\rho-1}} =$$

$$e^{[n_0+n_A\overline{a}+n_\sigma\sigma^2]} \quad \left[\int_I \left(e^{\frac{(\rho-1)a_i}{\rho}[1+n_a]}\right) di\right]^{\frac{\rho}{\rho-1}}$$

$\rightarrow$ *Law of large numbers* $\rightarrow$ $=$

$$e^{[n_0+n_A\overline{a}+n_\sigma\sigma^2]}e^{[1+n_a]\overline{a}+\left[\frac{[1+n_a]^2(\rho-1)}{\rho}-1-n_a\right]\frac{\sigma^2}{2}}$$

After matching coefficients we will have:

$$C_A = n_a+n_A+1 \quad , \quad C_\sigma = n_\sigma+\frac{(\rho-1)[1+n_a]^2}{2\rho}-\frac{1+n_a}{2} \quad , \quad C_0 = n_0$$

- Aggregate labor:

$$e^{N_0+N_A\overline{a}+N_\sigma\sigma^2} = \left[\int_I (n_i)^{\frac{\omega-1}{\omega}}\, di\right]^{\frac{\omega}{\omega-1}} =$$

$$[\left[\int_I \left(e^{\frac{\omega-1}{\omega}\left[n_0+n_a a_i+n_A\overline{a}+n_\sigma\sigma^2\right]}\right) di\right]^{\frac{\omega}{\omega-1}} =$$

$$e^{n_0+n_A\overline{a}+n_\sigma\sigma^2} \quad \left[\int_I \left(e^{\frac{(\omega-1)n_a a_i}{\omega}}\right) di\right]^{\frac{\omega}{\omega-1}} = e^{n_0+n_A\overline{a}+n_\sigma\sigma^2}e^{n_a\overline{a}+\left[\frac{n_a^2(\omega-1)}{\omega}-n_a\right]\frac{\sigma^2}{2}}$$

After matching coefficients we will have:

$$N_A = n_a + n_A \quad , \quad N_\sigma = n_\sigma + \frac{(\omega-1)n_a^2}{2\omega} - \frac{n_a}{2} \quad , \quad N_0 = n_0$$

- Aggregate price:

$$e^{P_0+P_A\overline{a}+P_\sigma\sigma^2} = \left[\int_I (p_i)^{1-\rho} di\right]^{\frac{1}{1-\rho}} =$$

$$\left[\int_I e^{(1-\rho)(\psi_0+\psi_a a_i+\psi_A\overline{a})}di\right]^{\frac{1}{1-\rho}} = e^{\psi_0+\psi_A\overline{a}}\left[\int_I e^{(1-\rho)[\psi_a a_i]}di\right]^{\frac{1}{1-\rho}}$$

$$= e^{\psi_0+\psi_A\overline{a}} \quad e^{\psi_a\overline{a}+\left[(1-\rho)\psi_a^2-\psi_a\right]\frac{\sigma^2}{2}}$$

After matching coefficients we will have:

$$P_A = \psi_a + \psi_A \quad , \quad P_\sigma = \frac{(1-\rho)\psi_a^2}{2} - \frac{\psi_a}{2} \quad , \quad P_0 = \psi_0$$

- Money demand:

$$e^{\delta P_0+\delta P_A\overline{a}-\delta(m_\sigma-P_\sigma)\sigma^2}=e^{-\gamma(C_0+C_A\overline{a}+C_\sigma\sigma^2)}$$

After matching coefficients we will have:

$$-\delta P_A=\gamma C_A \quad , \quad \delta(m_\sigma-P_\sigma)=\gamma C_\sigma \quad , \quad -\delta P_0=\gamma C_0$$

- Consumption basket:

$$-\rho(\underbrace{log(p_i)-log(P)}_{\psi_a(a_i-\overline{a})-[(1-\rho)\psi_a^2-\psi_a]\frac{\sigma^2}{2}})=log(y_i)-log(Y)$$

$$\begin{aligned}&log(y_i)-log(Y)=log(A_i)+(log(n_i))-log(Y)=\\&a_i+[n_0+n_aa_i+n_A\overline{a}+n_\sigma^2]-(C_0+C_A\overline{a}+C_\sigma^2)=\\&[1+n_a]\,a_i+[n_A-C_A]\,\overline{a}+[n_\sigma-C_\sigma]\,\sigma^2+n_0-C_0\end{aligned}$$

Here after matching coefficients we will obtain only one new equation:

$$-\rho\psi_a=1+n_a$$

- Price Setting:
Using the consumption basket, labor basket and market clearing conditions after replacing the idiosyncratic wage w_i in the price setting condition you can easily see:

$$E\left(C^{1-\gamma}\quad\left(\frac{p_i}{P}\right)^{1-\rho}\right)=\frac{\rho}{\rho-1}\quad E\left(N^{-\epsilon+\frac{1}{\omega}}\quad\frac{n_i^{\frac{\omega-1}{\omega}}}{1-\tau}\right)=$$

$$\frac{\rho}{\rho-1}\quad E\left(N^{1-\epsilon}\quad\frac{\left(\frac{n_i}{N}\right)^{\frac{\omega-1}{\omega}}}{1-\tau}\right)$$

I first express the left hand side of the equation in terms of states:

$$E\left(C^{1-\gamma}\left(\frac{p_i}{P}\right)^{1-\rho}\right)=E\left(e^{(1-\gamma)(C_0+C_A\overline{a}+C_\sigma\sigma^2)+(1-\rho)\left(\psi_a(a_i-\overline{a})-\left[(1-\rho)\psi_a^2-\psi_a\right]\frac{\sigma^2}{2}\right)}\right)$$

$$=e^{(1-\gamma)C_0+(1-\rho)\psi_aa_i+[(1-\gamma)C_A-(1-\rho)\psi_a]\overline{a}}\quad E\left(e^{\left[(1-\gamma)C_\sigma-\frac{(1-\rho)^2\psi_a^2}{2}+\frac{(1-\rho)\psi_a}{2}\right]\sigma^2}\right)$$

Rearranging and taking log of the left hand side:

$$(1-\rho)\psi_aa_i+[(1-\gamma)C_A-(1-\rho)\psi_a]\overline{a}+(1-\gamma)C_0+$$

$$\left[(1-\gamma)C_\sigma - \frac{(1-\rho)^2\psi_a^2}{2} + \frac{(1-\rho)\psi_a}{2}\right]\sigma_0^2$$

And then let us go to the the right hand side

$$\frac{\rho}{\rho-1} \quad E\left(N^{1-\epsilon} \quad \frac{\left(\frac{n_i}{N}\right)^{\frac{\omega-1}{\omega}}}{1-\tau}\right) =$$

$$e^{log\left(\frac{\rho}{\rho-1}\right)} \quad E\left[e^{-\tau_0+(1-\epsilon)[N_0+N_A\overline{a}+N_\sigma\sigma^2]+\frac{\omega-1}{\omega}\left(n_a[a_i-\overline{a}]-\left[\frac{(\omega-1)n_a^2}{\omega}-n_a\right]\frac{\sigma^2}{2}\right)}\right] =$$

$$e^{log\left(\frac{\rho}{\rho-1}\right)} \quad e^{-\tau_0+(1-\epsilon)N_0+\frac{\omega-1}{\omega}n_a a_i+\left[(1-\epsilon)N_A-\frac{\omega-1}{\omega}n_a\right]\overline{a}} \quad E\left(e^{\left[(1-\epsilon)N_\sigma-\frac{(\omega-1)^2n_a^2}{2\omega^2}+\frac{(\omega-1)n_a}{2\omega}\right]\sigma^2}\right)$$

Rearranging and taking log of the right hand side:

$$\left[\frac{\omega-1}{\omega}n_a\right]a_i + \left[(1-\epsilon)N_A - \frac{\omega-1}{\omega}n_a\right]\overline{a}$$

$$+log\left(\frac{\rho}{\rho-1}\right)-\tau_0+(1-\epsilon)N_0+\left[(1-\epsilon)N_\sigma - \frac{(\omega-1)^2n_a^2}{2\omega^2} + \frac{(\omega-1)n_a}{2\omega}\right]\sigma_0^2$$

After matching coefficients we will have:

$$(1-\rho)\psi_a = \frac{\omega-1}{\omega}n_a \quad , \quad (1-\gamma)C_A-(1-\rho)\psi_a = (1-\epsilon)N_A - \frac{\omega-1}{\omega}n_a$$

$$, \quad (1-\gamma)C_0 + \left[(1-\gamma)C_\sigma - \frac{(1-\rho)^2\psi_a^2}{2} + \frac{(1-\rho)\psi_a}{2}\right]\sigma_0^2 =$$

$$log\left(\frac{\rho}{\rho-1}\right)-\tau_0+(1-\epsilon)N_0+\left[(1-\epsilon)N_\sigma - \frac{(\omega-1)^2n_a^2}{2\omega^2} + \frac{(\omega-1)n_a}{2\omega}\right]\sigma_0^2$$

So we obtained 16 linear equations to determine 16 unknown coefficients. Using basic algebra we can easily solve this system and find the coefficients. I first remind you the policy functions:

- Household's policy functions:
  - $log(N) = N_0 + N_A\overline{a} + N_\sigma\sigma^2$.
  - $log(C) = log(Y) = C_0 + C_A\overline{a} + C_\sigma\sigma^2$.
  - $log(n_i) = n_0 + n_a a_i + n_A\overline{a} + n_\sigma\sigma^2$.

- Firms' policy functions:
  - $log(p_i) = \psi_0 + \psi_a a_i + \psi_A \overline{a}$.
  - $log(P) = P_0 + P_A \overline{a} + P_\sigma \sigma^2$.

and then I present obtained coefficients:

- Aggregate consumption:

$$C_A = \frac{\epsilon - 1}{\epsilon - \gamma} \quad , \quad C_\sigma = \frac{\delta \left[m_\sigma - \frac{(1-\rho)\psi_a^2}{2} + \frac{\psi_a}{2} \right]}{\gamma}$$

$$C_0 = \frac{1}{(\epsilon - \gamma)} \left[log\left(\frac{\rho}{\rho - 1}\right) - \tau_0 + \left[(1-\epsilon) N_\sigma - \frac{(\omega - 1)^2 n_a^2}{2\omega^2} - (1-\gamma) C_\sigma + \frac{(1-\rho)^2 \psi_a^2}{2} \right] \sigma_0^2 \right]$$

- Aggregate labor:

$$N_A = C_A - 1 = \frac{\gamma - 1}{\epsilon - \gamma} \quad , \quad N_\sigma = C_\sigma - \frac{(\rho - 1)\rho\psi_a^2}{2} + \frac{\omega - 1}{2\omega} n_a^2 + \frac{1}{2}$$

$$N_0 = C_0$$

- Idiosyncratic labor:

$$n_a = \frac{\omega(\rho - 1)}{\omega - \rho} \quad , \quad n_A = N_A - n_a \quad , \quad n_\sigma = N_\sigma - \frac{(\omega - 1) n_a^2}{2\omega} + \frac{n_a}{2}$$

$$n_0 = N_0 = C_0$$

- Idiosyncratic price:

$$\psi_a = \frac{1 - \omega}{\omega - \rho} \quad , \quad \psi_A = -\psi_a - \frac{\gamma C_A}{\delta} \quad , \quad \psi_0 = P_0$$

- Aggregate price:

$$P_A = \psi_a + \psi_A \quad , \quad P_\sigma = \frac{(1-\rho)\psi_a^2}{2} - \frac{\psi_a}{2} \quad , \quad P_0 = \frac{-\gamma C_0}{\delta}$$

A.3.3 Optimization problems of agents

Household:
I use Lagrangian method to find the optimal decision for the household:

$$L = \frac{C^{1-\gamma}-1}{1-\gamma} - \frac{N^{1-\epsilon}-1}{1-\epsilon} + \frac{\left(\frac{M}{P}\right)^{1-\delta}-1}{1-\delta}$$
$$- \lambda\left[\int_I p_i c_i di + M - \int_I \Pi_i di - \int_I (1-\tau) w_i n_i di - T\right]$$

First order conditions with respect to different control variables imply:

- $\frac{\partial L}{\partial c_i}$:

$$C^{-\gamma} \quad \frac{\partial C}{\partial c_i} - \lambda p_i = C^{-\gamma} \quad \left(\frac{c_i}{C}\right)^{\frac{-1}{\rho}} - \lambda p_i = 0$$

$$\textit{Define:} \qquad \int_I p_i c_i di = X_c$$

$$C^{-\gamma+\frac{1}{\rho}} c_i^{1-\frac{1}{\rho}} = \lambda c_i p_i \Rightarrow C^{-\gamma+\frac{1}{\rho}} \int_I c_i^{\frac{\rho-1}{\rho}} di = \lambda X_c \Rightarrow C^{-\gamma+\frac{1}{\rho}} C^{\frac{\rho-1}{\rho}} = \lambda X_c$$

$$\Rightarrow \frac{C^{1-\gamma}}{X_c} = \lambda$$

$$C^{-\gamma+\frac{1}{\rho}} \quad c_i^{-\frac{1}{\rho}} = \frac{C^{1-\gamma}}{X_c} \quad p_i \quad \Rightarrow c_i^{-\frac{1}{\rho}} = \frac{C^{\frac{\rho-1}{\rho}} p_i}{X_c} \quad \Rightarrow p_i c_i = \frac{C^{1-\rho} p_i^{1-\rho}}{X_c^{-\rho}}$$

$$\Rightarrow X_c = \frac{C^{1-\rho}}{X_c^{-\rho}} \int_I p_i^{1-\rho} di$$

$$X_c = C\left[\int_I p_i^{1-\rho} di\right]^{\frac{1}{1-\rho}} = CP \quad , \quad \lambda = \frac{C^{1-\gamma}}{X_c} = \frac{C^{-\gamma}}{P}$$

$$c_i^{-\frac{1}{\rho}} = \frac{C^{\frac{\rho-1}{\rho}} p_i}{PC} \quad \Rightarrow\Rightarrow \quad \frac{p_i}{P} = \left(\frac{c_i}{C}\right)^{\frac{-1}{\rho}}$$

- $\frac{\partial L}{\partial n_i}$:

$$-N^{-\epsilon} \quad \frac{\partial N}{\partial n_i} + \lambda(1-\tau) w_i = -N^{-\epsilon} \quad \left(\frac{n_i}{N}\right)^{\frac{-1}{\omega}} + \lambda(1-\tau) w_i = 0$$ [5]

$$\textit{Define:} \qquad \int_I (1-\tau) w_i n_i di = X_n$$

[5] *It is easy to see that the second order condition holds as long as* $\omega < 0$.

$$N^{-\epsilon+\frac{1}{\omega}} \quad n_i^{1-\frac{1}{\omega}} = \lambda(1-\tau)w_i n_i \quad \Rightarrow N^{-\epsilon+\frac{1}{\omega}} N^{\frac{\omega-1}{\omega}} = \lambda X_n \quad \Rightarrow \lambda = \frac{N^{1-\epsilon}}{X_n}$$

$$N^{-\epsilon+\frac{1}{\omega}} \quad n_i^{\frac{-1}{\omega}} = \frac{N^{1-\epsilon}}{X_n}(1-\tau)w_i \quad \Rightarrow n_i^{\frac{-1}{\omega}} = \frac{N^{\frac{\omega-1}{\omega}}(1-\tau)w_i}{X_n} \quad \Rightarrow$$

$$(1-\tau)w_i n_i = \frac{N^{1-\omega}[(1-\tau)w_i]^{1-\omega}}{X_n^{-\omega}}$$

$$X_n^{1-\omega} = N^{1-\omega}\int_I [(1-\tau)w_i]^{1-\omega} di$$

$$\Rightarrow X_n = N \overbrace{\left(\int_I [(1-\tau)w_i]^{1-\omega} di\right)^{\frac{1}{1-\omega}}}^{W} = NW$$

$$\lambda = \frac{N^{-\epsilon}}{W}$$

$$n_i = \frac{N^{1-\omega}[(1-\tau)w_i]^{-\omega}}{(NW)^{-\omega}} \quad \Rightarrow\Rightarrow \quad \frac{(1-\tau)w_i}{W} = \left(\frac{n_i}{N}\right)^{\frac{-1}{\omega}}$$

$$N^{-\epsilon} \quad \left(\frac{n_i}{N}\right)^{\frac{-1}{\omega}} = \frac{C^{-\gamma}}{P}(1-\tau)w_i \quad \Rightarrow \quad w_i = \frac{N^{-\epsilon}P}{C^{-\gamma}(1-\tau)} \quad \left(\frac{n_i}{N}\right)^{\frac{-1}{\omega}}$$

$$N^{-\epsilon+\frac{1}{\omega}} \quad n_i^{-\frac{1}{\omega}} = \lambda(1-\tau)w_i = \frac{C^{-\gamma}}{P}(1-\tau)w_i$$

$$\Rightarrow \quad w_i = N^{-\epsilon+\frac{1}{\omega}} \quad n_i^{-\frac{1}{\omega}} \quad \frac{PC^{\gamma}}{1-\tau}$$

- $\frac{\partial L}{\partial M}$:

$$\frac{\left(\frac{M}{P}\right)^{-\delta}}{P} - \lambda = 0 \quad \Rightarrow \quad \left(\frac{M}{P}\right)^{-\delta} = C^{-\gamma}$$

Firms:
From the consumption basket we know:

$$n_i = \left(\frac{CP^{\rho}}{A_i}\right)(p_i)^{-\rho}$$

so maximizing the profit $\pi_i = p_i y_i - w_i n_i$ result in:
$\partial \pi_i / \partial p_i$:

$$p_i A_i \quad \partial n_i/\partial p_i + A_i n_i - w_i \partial n_i/\partial p_i = 0 \Rightarrow \quad -\rho p_i \left(\frac{A_i n_i}{p_i}\right) + A_i n_i + \rho w_i \left(\frac{n_i}{p_i}\right) = 0$$

$$A_i(1-\rho) + \rho\frac{w_i}{p_i} = 0 \quad \Rightarrow \quad p_i = \frac{\rho}{\rho-1} \quad \frac{w_i}{A_i}$$

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