



**Departament de Teoria  
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UNIVERSITAT POLITÈCNICA DE CATALUNYA

# **MULTIDIMENSIONAL SPECKLE NOISE, MODELLING AND FILTERING RELATED TO SAR DATA**

by

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**Ph.D. Dissertation**

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*A mis padres, José y María del Pilar  
y hermanos, Sergio y Eva.*



-Podrías decirme, por favor, qué camino he de tomar para salir de aquí?  
-Depende mucho del punto adonde quieras ir -contestó el Gato.  
-Me da casi igual dónde -dijo Alicia.  
-Entonces no importa qué camino sigas -dijo el Gato.  
-...siempre que llegue a alguna parte -añadió Alicia, a modo de explicación.  
-Ah!, seguro que lo consigues -dijo el Gato-, si andas lo suficiente.

*-Would you tell me, please, which way I ought to go from here?  
-That depends a good deal on where you want to get to -said the Cat.  
-I don't much care where -said Alice.  
-Then it doesn't matter which way you go -said the Cat.  
-...so long as I get somewhere -Alice added as an explanation.  
-Oh!, you're sure to do that -said the Cat-, if you only walk long enough.*

Alicia en el país de las maravillas,  
*Alice's Adventures in Wonderland,*  
CARROLL, Lewis (Charles Lutwidge Dodgson) (1832-1898)



# Preface

This thesis represents the work carried out during the last four years in the Electromagnetics and Photonics Engineering Group of the Signal Theory and Communications Department at the Technical University of Catalonia (UPC), Barcelona (Spain) and the Institute of High Frequency and Radar Systems of the German Aerospace Center (DLR), Oberpfaffenhofen (Germany). Four years... is a long period of time in which one learns numerous things. But perhaps, the most important lesson for me does not have anything to do with science or technology, but with what one learns about life from the relation with other people. It is for this reason that I want to dedicate the first lines of this thesis to all those people, who, in a greater or smaller degree, have made it possible.

Without doubt, this thesis would not have seen the light without the considerable support of my thesis advisor, Xavier Fàbregas Cànovas. His aid and advice, as well as his critical spirit and detailed vision about radar polarimetry have been fundamental in the development of this work. With him, I have had the possibility to maintain multitude of scientific and human discussions, from which I have learned to see things from a more calmed and rational point of view.

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During these four years, I have had diverse office mates, with which I have shared many and good moments, and whose friendship is one of the most important fruits of this period of my life. From my first period at the UPC, I keep very pleasing memories of Emilio and Alfonso. Special mention deserves Eduard, since although we shared office during a pair of months, since then we maintain a good friendship. During my stay in Germany, I shared office with Vito, who introduced me to the DLR Italian community. During the last part of this thesis, I have shared office with Xavier Fàbregas Cànovas, which has caused him to transform from my thesis advisor to a work colleague and a friend.

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Also, I would like to mention the *Contubernio*, since all of them have participated in and suffered this thesis.

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# Abstract

Synthetic Aperture Radar (SAR) systems have emerged, during the last decades, as a useful tool to gather and to analyze information from the Earth's surface. Owing to its coherent nature, this type of systems can collect electromagnetic scattering information with a high spatial resolution, but, on the other hand, it yields also speckle. Despite speckle is a true electromagnetic measurement, it can be only analyzed as a noise component due to the complexity associated with the scattering process. A noise model for speckle exists only for single channel SAR systems. Consequently, the work presented in this thesis concerns the definition and the comprehensive validation of a novel series of multidimensional speckle noise models, together with their application to optimal speckle noise reduction and information extraction.

First, a speckle noise model for the Hermitian product complex phase component is derived in the spatial domain and translated, subsequently, to the wavelet domain. This analysis is especially relevant to interferometric SAR data. This model demonstrates, on the one hand, that the wavelet transform itself is an interferometric phase noise filter that maintains spatial resolution. On the other hand, it makes possible a high spatial resolution coherence estimation. In a second part, a speckle noise model for the complete Hermitian product is proposed. It is proved that speckle is due to two noise components, with multiplicative and additive natures, respectively. The multidimensional speckle model, relevant for polarimetric SAR data, is finally derived by extending the Hermitian product noise model.

From a multidimensional speckle noise reduction point of view, this noise model allows to prove that the covariance matrix entries can be processed differently without corrupting the signal properties. On the other hand, it allows to redefine, and to extend, the principles under which an optimum multidimensional speckle noise model has to be set out. On the basis of these principles, a novel polarimetric speckle noise reduction algorithm is proposed.

## KEYWORDS

Synthetic Aperture Radar (SAR), Multidimensional SAR imagery, SAR Interferometry, SAR Polarimetry, Polarimetric SAR Interferometry, Speckle Noise, Speckle Noise Modelling, Speckle Noise Filtering, Coherence Estimation, Wavelet Transform



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