Essays on Political and Public Economics

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Introduction

This thesis deals with three issues related to public and political economics: (1) The effect of income distribution on conflict; (2) the effect of decentralization on corruption in the presence of powerful local elites; and (3) the effect of the number of parties on the negotiation outcomes in a legislature. To study these relationships is useful to understand several situations that involve collective actions, elite behaviors, and bargaining processes. Moreover, these subjects are relevant for any type of democratic economy. However, we must recognize that these essays were motivated by stylized facts that seem to be more common in developing than in developed countries, like the existence of high income inequalities and the weakness of institutional rules.

In chapter I, we study the relationship between conflict and income distribution. Commonly, the theoretical studies that have analyzed this relationship have assumed that the conflicts in a society are directly over wealth. When this is the case, it is unsurprising that income redistribution might generate a decrease in the level of conflict. Nevertheless, several conflicts in a society are not directly over wealth but over group interests or social choices. When this is the case, it is not clear how income redistribution may affect the conflict intensity.

A priori, one can think that a bad distribution of wealth might have two opposite effects on the level of conflict in a society. First, a high level of inequality could motivate the poorest to get into a conflict in order to capture, via their social preferences, some resources from the others. If this is the case, the relationship between conflict and inequality is expected to be positive. Second, conflicts consume resources, and generally winning probabilities depend on the quantity of resources allocated by a group in supporting its cause. Hence, since the poorest have little chances to win they do not have incentives to get into a conflict. If this is the case, the relationship between conflict and inequality is expected to be negative.

In order to understand this relationship, we develop a contest model for social choices among groups with different wealth. In this context, we study how the interaction among group-size, wealth, and its distribution affects both conflict intensity and group success probabilities. Our most surprising result is that, under certain circumstances, more between-group income equality does not necessarily imply less conflict intensity. Thus, opposite to the common wisdom, it is not always true that improvements in income distribution reduce the level of conflict in a society. We end this chapter by presenting some empirical evidence on political campaigns that supports our theoretical findings.

Chapter II essentially focuses on developing countries. Here, we care on the relationship among decentralization, corruption and political accountability in this type of economies. The main question to this respect is whether or not decentralization promotes good governance and persuades politicians against corruption in the presence of powerful local elites. Like some authors have argued, the existence of these elites is an idiosyncratic characteristic of these countries.
We motivate our discussion with some suggestive evidence about the relationship between decentralization and corruption. In particular, we show that the negative effect that fiscal decentralization has on corruption in developed countries cannot be confirmed in developing economies. In the rest of the chapter, we build an imperfect information model of corruption and political accountability to study if the influence of local elites on the allocation of public resources can explain this outcome.

We find that the power of the elites can explain the lack of success of decentralization in combating corruption in developing countries. However, we identify other unexpected factors that play an important role in this relationship. The first is the existence of many regions with a relative weak accountability sector. When this occurs, political accountability does not work appropriately, the local elites are able to demand corruption at a low cost, and the incumbents can accept these demands by facing a low probability of detection. The second element is the design of grants. Usually, this instrument is used intensively in developing countries in order to reduce the between-region income inequalities. We show that if a part of these grants is not invested in improving the accountability systems, then the incumbents can allocate these resources discretionarily in corruption at no cost. Finally, the decentralization design also matters. The assignment of many tasks to small jurisdictions, in which the spoils of the incumbents are low, reduces the cost of corruption for the local elites and increases its demand.

In the last chapter, we deal with another institutional issue that also affects the allocation of resources and the promoted public policies through the political process. More precisely, we study the effect of both the number of parties and the level of ideological polarization on the bargaining outcomes of a legislature. To understand this relationship is of special interest for many democracies in which either the institutional rules or the cultural characteristics or both have allowed for a large number of parties in the legislatures. This is the case of many established democracies in Latin America, some large democratic economies like India, and some new east and middle-east democracies.

The chapter is motivated for the confusion of some authors about the role of both number of parties and polarization in a bargaining process. These scholars have used these two concepts indistinctly, and by doing so, have concluded that a large number of parties complicates the policymaking process. However, the evidence they present is far to be statistically robust. We show that in order to get strong conclusions to this respect, it is crucial to distinguish between these two dimensions.

In order to understand the effect of these two elements on the negotiation outcomes, we use a bargaining approach. We show that when the government party is negotiating with another party, and the level of ideological polarization between these is high enough, the former party can be better if the latter is split in several parties. This illustrates the existence of a trade-off between these two issues.
Chapter I

Conflict and Wealth

Abstract

We study how the interaction among group-size, wealth, and its distribution affects both conflict intensity and group success probabilities in a society. Here conflict is due to differences in preferences for social outcomes which are not necessarily related to individual wealth. By using a contest model and considering prizes with different characteristics (public, mix private-public), we show that less between-group income inequality sometimes generates more conflict. We also prove that a sufficiently high income inequality can explain the group-size paradox. We present some evidence that support our findings by using information on U.S. House campaign race.

There is a common belief that income inequality increases social unrest and so the level of conflict in a society. Moreover, some scholars (e.g. Sen, 1972) have stressed the strong connection that exists between these elements and have claimed that inequality might be the source of social revolutions. However, few theoretical studies have analyzed this relationship, and only some recent empirical studies have explored if income distribution can explain the likelihood of conflict, mainly of civil wars.

Commonly, the theoretical studies that have analyzed this relationship have assumed that inequality is the direct cause of conflict. In other words, these models assume that conflict is directly over wealth (e.g. Grossman, 1994; Horowitz, 1993). When this is the case, it is unsurprising that income redistribution might generate a decrease in the level of conflict.

Nevertheless, several conflicts in a society are not directly over wealth but over group interests. In this context, conflict is understood as a between-group contest for social choices in which no collective decision rule is necessarily established. When this is the case, it is not clear how income redistribution may affect the conflict intensity.

In this paper we deal with this notion of conflict, i.e. conflict on group interest. Thus, in contrast to the traditional studies on this topic, here conflict is due to differences in preferences for social outcomes that are not necessarily related to individual wealth. Some examples of this type of conflict are cities or neighbourhoods competing for different locations of a public facility (hospital, park, library, etc.) or a public project, industries struggling for government support, people contributing to political campaigns, cities competing to celebrate some international sport event (e.g. Olympic Games, Football World Cup), and even a civil war over political power.
In this context, we are going to study the effect of wealth and its distribution on the level of conflict. Additionally, group-size will also matter because both number of people in a group and their wealth affect between and within-group income inequality. Thus, our purpose is to study how the interaction among group-size, wealth, and its distribution affects both conflict intensity and group success probabilities in a society.

Following previous studies in this area we will use a framework of pure contest, i.e. where the utility derived from the people engaged in the conflict comes only from their most preferred choice. The model assumes that the society is divided in groups whose members share the same preferences for the social outcomes, but they do not necessarily have the same level of wealth. We also assume that the success probability of each group depends on the resources spent by its members in supporting their preferred outcome. Under these conditions each person in each group has to decide how much to contribute to the cause in order to maximize her expected utility. The total resources spent in the conflict by the groups will measure the level of conflict in the society.

We start by studying the case in which the contest’s prize is a pure public good. In this case, our results show that most of the time very poor people are not willing to engage in any conflict, i.e. they prefer to take their total wealth for themselves instead of spending money in the contest. Then, if only very poor people form a group, this group might be marginalized from any social choice.

With regard to the equilibrium winning probabilities, we find the following results. First, wealthier (in terms of average wealth) and larger groups are more successful than poorer and smaller groups. Second, when the groups have the same average wealth, larger groups spend more on conflict than smaller groups and then attain a higher winning probability. Third, it is not necessarily true that larger groups are more successful or that wealthier groups are more successful, it depends on the interaction between group-size and average wealth. Thus, even though the contest’s prize is totally public, it is possible to see smaller groups in a society being quite successful because of a higher average wealth. We show that in order to observe this outcome, the total wealth of the smaller groups needs not to be higher than the total wealth of the larger groups.

We explore the effect of between-group income redistribution over the level of conflict by transferring wealth from a richer to a poorer group (progressive transfer of income). By doing so, we find that income equality does not necessarily imply less conflict intensity; it depends on the relative size (number of people) of the implied groups and its winning probabilities. We identify three cases. First, when the poorer group is smaller than the richer or equal to it (and so the winning probability of the former is smaller than the respective probability of the latter), a progressive transfer of income increases the level of conflict. Second, when the poorer group is larger and its winning probability is higher than that of the richer group, a progressive transfer of income decreases the level of conflict. Finally, there is an ambiguous effect when even being larger the poorer group its winning probability is smaller than that of the richer group.

The intuition behind these results goes as it follows. As it was mentioned above, these findings depend on two factors, the relative group-size and the relative winning probabilities. In our framework, the winning probabilities are concave in the group-
average wealth, i.e. that the marginal probability decreases as the group-average wealth increases. Then, if there is an exogenous rise in the average wealth of any group, this group will have more incentive to increase the optimal contribution when its winning probability is low. On the other hand, the relative group-size will define the individual relative transfer. For instance, when the poorer group is smaller than the richer, a progressive transfer implies that the increase in the individual wealth of a person who belongs to the poorer group is relatively higher than the decrease in the individual wealth of a person who belongs to the richer group. In this case, we say that there is a high relative transfer. The opposite occurs when the poorer group is larger than the richer one. In such case, we say that there is a low relative transfer.

Now let us combine the two effects. When the poorer group is smaller than the richer, the marginal probability of the former is large. Moreover, any between-group progressive transfer implies a high relative transfer. Then, each individual in the poorer group will allocate a higher fraction of the transfer to the conflict than the fraction that was allocated by each individual in the richer group from this transfer. The final result is an increase in the level of conflict. The contrary occurs when the poorer group is larger (low relative transfer) and its winning probability higher (low marginal probability). The ambiguity appears when the two effects go in the opposite direction, i.e. when the poorer group even though being larger (low relative transfer) has a smaller probability (high marginal probability) than the richer group.

We also explore the effect of a within-group progressive transfer of income on the level of conflict. We find that within-group income inequality usually does not affect conflict intensity. Actually, this result is a corollary of the Neutrality theorem for private provision of public goods (Warr, 1983). The novelty of our result is that this neutrality still remains when there is an interest conflict with other groups for the provision of the good. However, in the same direction of Bergstrom et al. (1986), we show that in the presence of corner solutions this neutrality does not necessarily hold.

We also study the case in which the prize has a varying mix of public and private characteristics. Since under these conditions part of the prize decreases as the group-size increases, then it is not necessarily true that the winning probability of a group (and so the level of conflict) increases as its group-size increases. When this result still holds, the effects of a between-group progressive transfer of income over the level of conflict are similar to those found for the pure public prize case. Conversely, when this is not the state of affairs (i.e. the winning probabilities decrease with the group-size), in most of the situations the effect of a between-group redistribution of income on the conflict intensity is ambiguous. Nevertheless, we prove that independently on the degree of publicness of the prize, when the poorer group is smaller than the richer group a low between-group income inequality always increases the conflict intensity.

We present some empirical support to our theoretical findings by using information of US state campaign spending in House race. Under the assumption that people in each city compete with people in other cities of the same state in order to get their preferred set of candidates elected, we find that states with a higher between-city income inequality spend less in House campaigns that those with a lower inequality.
There are three groups of theoretical papers closest in spirit to ours. In the first set are those studies in which conflict is directly over wealth, e.g. Grossman (1991), Horowitz (1993), and Harms and Zink (2002). When this is the case, a redistribution of income always generates a decrease in the level of conflict. In the second group are those that share our same notion of conflict, e.g. Hirshleifer (1991), Skaperdas (1992; 1998), Esteban and Ray (1999; 2001). These studies have not cared about the role of the individual wealth over conflict intensity. In our paper, we combine the notion of conflict of the latter with the inequality issues of the former studies. Finally, the paper is also related with the rent-seeking and the collective action literature (e.g. Katz, Nitzan and Rosenberg, 1990; Nitzan, 1991). These papers have mainly concentrated on the effect of the free-riding behaviour on the group winning probabilities but have not paid attention to any income inequality issue.

There are some empirical studies which are also related to ours, e.g. Collier and Hoeffler (2001); Collier, Hoeffler and Söderbom (2001); Hegre, Gissinger and Gleditsch (2002). These studies have found no effect of income inequality (Nationwide Gini coefficients) on (armed) conflict. From the point of view of our findings, this evidence might be biased because of some measurement errors. As we shall see in detail later on, the key point is that both between-group income inequality and within-group inequality may affect the level of conflict in a different way. Nationwide Gini coefficients measure the inequality in a society as a whole, and they do not separate these two issues.

The remainder of the chapter is as follows. In section 1 the model and its characteristics are described, and section 2 presents and discusses the main results. Section 3 makes a brief discussion on the relationship between wealth and group-success, and section 4 extends the model to the case in which the contest’s prize has a mix of public and private characteristics. Section 5 presents some empirical evidence that supports our theoretical findings. Conclusions are presented in the last section.

1. The Model

Suppose that a society composed by \( n \) individuals with different wealth must choose an outcome from a finite set of issues \( G \). Think of these options as different locations of a public facility or a public project (hospital, park, library, etc.), political candidates receiving contributions, a law that might favour an economic sector, the selection of a city to celebrate some international event, etc.

Individuals not only differ in wealth but also in their valuation of these outcomes. Assume that each person derives utility only from her most preferred outcome. We fix this gain to one. Thus, if an individual prefers outcome \( g \in G \) over all other outcomes, and this is chosen by the society, this player gets an extra unit of utility, otherwise she does not receive anything. Furthermore, all those who rank a certain option \( g \) first form a group. We identify this group also by \( g \). The number of people in a group is denoted by \( n_g \), where \( \sum_{g \in G} n_g = n \).

We assume that preferences for the outcomes are distributed randomly among individuals, not necessarily correlated with their wealth. In other words, we allow the
existence of any within-group income distribution. Notice that a particular case is that in which everybody with the same wealth has the same favourite option. This could be the case of different neighbourhoods where the people have the same level of wealth competing for the location of a public facility.

Let us denote by \( i \) (and some times by \( j \)) individuals. Each individual \( i \) has an exogenous wealth \( w_i \) and spends a nonnegative amount of resources \( r_i \) in the contest in order to maximise her expected utility. We assume that individuals cannot borrow, and that individual wealth is public information. With the required normalization we define the individual wealth net of conflict expenditure by \( c_i = w_i - r_i \). Assuming that utility is separable between net wealth and the contest prize, the expected utility of an individual who belongs to group \( g \) is given by:

\[
EU_i = p_g + f(c_i) \tag{1}
\]

where \( p_g \) is the success probability of group \( g \), and \( f(\cdot) \) is a function that is assumed to be continuous, thrice differentiable, with \( f'(\cdot) > 0 \), \( \lim_{c_i \to 0} f'(c_i) = \infty \), \( f''(\cdot) < 0 \), and \( f'''(\cdot) > 0 \).

We assume that the winning probability of a group depends on the effort contributed by its members in support to their preferred outcome.\(^1\) Denoting by \( R_g \) the total amount of resources contributed by group \( g \) in the conflict (i.e. \( R_g = \sum_{i \in g} r_i \)), and by \( R \) the total amount of resources expended by the society in the conflict (i.e. \( R = \sum_g R_g \)), this probability is defined as follows:

\[
p_g = \frac{R_g}{R} \tag{2}
\]

for all \( g = 1, \ldots, G \), provided that \( R > 0 \). If \( R = 0 \) then the winning probabilities are given by an arbitrary vector \( \{ \tilde{p}_1, \ldots, \tilde{p}_G \} \). We assume that this vector is such that \( R_{g'} > 0 \) for some \( g' \neq g \).

Observe that \( R \) can be interpreted as an indicator of the conflict scale or conflict level. Let us define \( R_i = R - r_i \) and \( R_g = R - R_g \). Summarizing, each individual in each group takes as given the efforts contributed by everyone else in the society and chooses \( r_i \geq 0 \) to maximize equation 1 subject to 2. The resources expended by an individual \( i \) who belongs to group \( g \) is described by the following conditions (see the appendix):

\[
\frac{1}{R}(1 - p_g) = f'(c_i) \quad \text{if} \quad f'(w_i) < \frac{R_g}{R^2} \tag{3a}
\]

\[
r_i = 0 \quad \text{if} \quad f'(w_i) \geq \frac{R_g}{R^2} \tag{3b}
\]

---

\(^1\) Contest success probabilities have been axiomatized by Skaperdas (1996). Here we assume a simple form for the success probabilities.
Under interior solution equation 3a describes the usual equilibrium condition according to which the marginal utility of the contribution must be equal to its marginal disutility. In this framework, a Nash equilibrium is a vector of individual contributions such that equation 3a is satisfied for every individual in every group. Sometimes we shall refer to the people whose best response is given by \( r_i=0 \) as inactive people, whereas we shall call active people those whose best response implies \( r_i>0 \). Using the same criteria we will differentiate between inactive groups, those with \( r_i=0 \ \forall \ i \in g \), and active groups, those with \( r_i>0 \) for at least one \( i \in g \).

It is also possible to define the equilibrium in terms of the success probabilities and \( R \), rather than in terms of the personal contributions. Given that \( f'(\cdot) \) decreases monotonically, from equation 3a the individual best response can be written as:

\[
 r_i = \text{Max}\left\{0, w_i - f^{-1}\left(\frac{1}{R}(1 - p_g)\right)\right\}
\] (4)

Combining equation 2 and 4 we get:

\[
 p_g = \frac{1}{R} \sum_{i \in g} \text{Max}\left\{0, w_i - f^{-1}\left(\frac{1}{R}(1 - p_g)\right)\right\}
\] (5)

Equilibrium can now be interpreted as a vector \( p \ (G \times 1) \) of success probabilities (such that \( p_g \geq 0 \ \forall \ g \), and \( \Pi = \sum_g p_g = 1 \)) and a positive scalar \( R \), such that equation 5 is satisfied for every group. Notice that 5 implicitly defines \( p_g \) as a function of \( R \). With the system of \( G \) equations given in 5 plus the condition that \( \Pi = 1 \), we can solve for the equilibrium vector \( \langle p, R \rangle \).

**Proposition 1.** There always exists an equilibrium vector \( \langle p, R \rangle \) such that equation 5 is satisfied for each group \( g \), \( p_g \geq 0 \ \forall \ g \), and \( \Pi = \sum_g p_g = 1 \). Moreover, this equilibrium is unique.

The proofs of proposition 1 and the rest of propositions are in the appendix. From equation 5 (and also from equation 3a) it can be seen that the equilibrium winning probabilities, and so the level of conflict, are functions of both the individual wealth and the group size. In what follows we care about the role played by these two exogenous variables in the model. We shall start by studying the simplest case in which everybody with the same wealth has the same favourite outcome and so belongs to the same group. We shall refer to that as the within-group income equality case. Later we shall extend our findings to the within-group income inequality case, i.e., where groups consist of people with different wealth. From now on, we are going to assume that there exists an interior solution for every individual. Only in some special cases are we going to relax this assumption.
2. Analysis

Within-Group Income Equality Case

For the moment let us assume that everybody with the same wealth has the same favourite outcome and so belongs to the same group. In this case, for any group $g$, $w_i = w_g \forall i \in g$, where $w_g$ is the common individual wealth of group $g$. It is also the case in which $w_g = \overline{w}_g$, where $\overline{w}_g$ denotes the average wealth of group $g$. We denote by $W_g$ the total wealth of group $g$ (i.e. $W_g = \sum_{i \in g} w_i$).

Replacing $w_i$ by $\overline{w}_g$ in equation 3a it follows that $r_i = r_g \forall i \in g$. Thus, the winning probability for group $g$ can be rewritten as $p_g = \frac{n_g r_g}{R}$ and equation 3a can be represented as follows:

$$\frac{1}{R} (1 - p_g) = f'\left(\frac{\overline{w}_g - p_g R}{n_g}\right) \quad \text{if} \quad f'(w_g) < \frac{1}{R - g} \quad (6)$$

From equation 6 it can be inferred that $p_g$ and $R$ are completely defined by $\overline{w}_g$ and $n_g$.

We start our analysis by stating the effect that these variables have over the equilibrium. Our strategy consists of examining how success probabilities change over the cross-section of groups (i.e. how $\Pi = \sum_g p_g (\overline{w}_g, n_g, R)$ change) when either $\overline{w}_g$ or $n_g$ change, keeping constant the level of conflict. Since $\Pi$ decreases as $R$ increases (see proof of proposition 1), once we know how $\Pi$ changes, it can be inferred how $R$ must move in order to recover a new equilibrium (i.e. in order to recover $\Pi = 1$).

Proposition 2: Assume that people with the same wealth share the same favourite outcome in the society (i.e. there is within-group income equality) and that there is an interior solution for everybody, then:

(a) Both the level of conflict and the winning probability of group $g$ are strictly increasing in the average wealth of group $g$.

(b) Both the level of conflict and the winning probability of group $g$ are strictly increasing in the group-size of $g$.

It is possible to extract more conclusions from proposition 2. First, wealthier (in terms of average wealth) and larger groups are more successful than poorer and smaller groups. Second, when the groups have the same average wealth, larger groups spend more on conflict than smaller groups and then attain a higher winning probability. This means that Olson’s paradox does not necessarily hold under our framework. Actually,
it replicates the result found by Katz et al. (1990) for rent-seeking activities over public goods. Third, it is not necessarily true that either larger groups or wealthier groups are more successful.

The key point in the conclusions above is that group-success depends on the interaction between group-size and average wealth. Thus, at the end of the day it is possible to see smaller groups being quite successful because of a higher average wealth. This could be an explanation, alternative to the free-rider effect, to explain the aforementioned paradox. In section 3 we explore this interaction in more detail.

Consider now corner solutions. When for a certain group, say group $g$, $f'(w_g) = f'(\overline{w}_g) \geq 1/R_{-g}$, people in this group are not going to take part in the conflict. The condition implies that groups with a small enough average wealth (given its size) might be out of the social conflict. This issue can explain why in some societies there are very poor groups which are marginalized from the social choices even when these are large in size.

Let us come back to the case in which there is an interior solution for every group. Now we are going to analyze the effect of income inequality over the equilibrium. Since, for the moment, we are interested in keeping the within-group income equality, in this section we are going to analyze only the effect of a between-group progressive transfer of income over the level of conflict. By such a transfer we refer to the case in which a richer group (call it group $h$) transfers part of its total wealth to a poorer group (call it group $l$) keeping constant both the total wealth in the society and the within-group income distribution. Within-group transfers of income will be studied in the next section.

Similarly as before, the effect of a between-group transfer can be analyzed by looking how the success probabilities change over the cross-section of groups when the transfer is done and $R$ is kept constant. Notice that by taking one unit of money from $\overline{w}_h$ (richer group average wealth) and transferring it to group $l$, the poorer group average wealth ($\overline{w}_l$) will increase by $n_h/n_l$. Taking this into account, the change in $\Pi$ when there is a progressive transfer can be computed as:

$$
\Delta\Pi|_R = \frac{\partial p_l}{\partial \overline{w}_l} \frac{n_h}{n_l} - \frac{\partial p_h}{\partial \overline{w}_h} \bigg|_R
$$

From proposition 2, we already know the derivatives implied in 7. Then, replacing those and manipulating algebraically we obtain:

\[ \text{However, at the end of the day the contribution of the new member compensates the decreases in the contribution of the current members.} \]

\[ ^3 \text{Notice that when } \overline{w}_g \text{ is small, } f'(\overline{w}_g) \text{ is high (in fact when } \overline{w}_g \text{ goes to zero, } f'(\overline{w}_g) \text{ goes to infinity). However, it is not enough to have a corner solution whenever } R_g \text{ also matters. Ceteris paribus from proposition 2, we can infer that } R_g \text{ decreases with } n_g. \text{ Thus, if } n_g \text{ is high, in order to have a corner solution for group } g \text{ it is required a smaller average wealth. In fact, this average wealth must satisfy } \overline{w}_g \leq f^{-1}(1/R_g). \]
$$\Delta \Pi \bigg|_R = R n_h \left( \frac{1}{R^2 - \Omega_i} - \frac{1}{R^2 - \Omega_h} \right)$$  \hspace{1cm} (8)$$

where $\Omega_g = \frac{n_g}{f''(\bar{w}_g - p_g R/n_g)} < 0$ for $g=h,l$. Thus, when $\Omega_l < \Omega_h$ ($\Omega_l > \Omega_h$), the transfer makes $\Pi$ smaller (higher) than one, and in order to recover the equilibrium conditions, $R$ must decrease (increase) whenever $p_g$ and $R$ are negatively related. Notice that $\Omega_l < \Omega_h$ if and only if $\frac{n_l}{n_h} > \frac{f''(\bar{w}_i - p_i R/n_i)}{f''(\bar{w}_h - p_h R/n_h)}$. The opposite is true when $\Omega_l > \Omega_h$. These results are stated in the following proposition.

**Proposition 3**: Assume that people with the same wealth share the same favourite outcome in the society (i.e. there is within-group income equality), and that there is an interior solution for everybody. Then, a progressive transfer of income generates a decrease in the level of conflict if $\frac{n_l}{n_h} > \frac{f''(\bar{w}_i - p_i R/n_i)}{f''(\bar{w}_h - p_h R/n_h)}$. When this inequality is reversed, the transfer generates an increase in the level of conflict. If both terms are equal, the transfer does not affect the level of conflict.

Whether $\Omega_l$ is smaller or larger than $\Omega_h$ depends critically on the implied parameters and, some times, on the concavity of $f(.)$. Notice that there are two forces involved in these inequalities, the relative group-size ($n_l/n_h$) and the relation between $f(.)$'s second derivatives. The relative group-size will define the individual relative transfer. For instance, when the poorer group is smaller than the richer, a progressive transfer implies that the increase in the individual wealth of a person who belongs to the poorer group is relatively higher to the decrease in the individual wealth of a person who belongs to the richer group. If this is the case, we will say that there is a high relative transfer. The opposite will happen when the poorer group is larger than the richer one; if so, we will say that there is a low relative transfer. When $n_l/n_h = 1$, then there is an equivalent relative transfer. With regard to the ratio of second derivatives, we will give some intuition later on.

We want to see under which conditions income redistributions might increase or decrease the level of conflict. In order to classify our result in a systematic way, it is important to remember the relationship among wealth, group-size, and winning probabilities. Using the result of proposition 2, when $n_l = n_h$ then $p_l < p_h$. Similarly, when $n_l < n_h$ then $p_l < p_h$. However, when $n_l > n_h$ the relationship between the equilibrium probabilities is not clear. It might be that $p_l > p_h$ if the number of members in the poorer group is high enough to offset the negative effect coming from its smaller average wealth. If this is not the case, then it must be again that $p_l < p_h$. Keeping in mind these facts, the effects of a between-group transfer on the conflict intensity are stated in the following proposition.
Proposition 4: Assume that people with the same wealth share the same favorite outcome in the society (i.e. there is within-group income equality) and that there is interior solution for everybody.

a) If either \( n_l = n_h \) or \( n_l < n_h \), then a progressive transfer of income increases the level of conflict.

b) If \( n_l > n_h \) and \( p_l < p_h \) (i.e. number of members in the poorer group is not enough to compensate for the smaller average wealth), then the effect of a progressive transfer of income over the level of conflict is ambiguous.

c) If \( n_l > n_h \) and \( p_l \geq p_h \) (i.e. number of members in the poorer group is enough to compensate for the smaller average wealth), then a progressive transfer of income reduces the conflict intensity.

Proposition 4 shows that in our framework income equality does not necessarily generate a decrease in the conflict intensity. Only when the poorer group has a higher winning probability (i.e. group-size compensates its small average wealth), income redistribution reduces the conflict intensity. The only ambiguity found occurs when \( n_l > n_h \) and \( p_l < p_h \). Note that if \( n_l \) is high enough compared to \( n_h \) such that the probabilities are not too different (keeping \( p_l \) still smaller than \( p_h \)) then the population ratio will tend to be greater than the ratio of second derivatives.4 If this is the case, then the level of conflict will decrease. Nevertheless, even when \( n_l \) is high but the probabilities are further from each other, the final result will depend not only on the implied parameters but also on the concavity of \( f(.) \). If \( f''(.) \) increases quite fast, then the opposite result might be found.

As it was mentioned above, these findings depend on two forces, the relative group-size and the relation between \( f(.) \)’s second derivatives. We already know that the relative group-size will define the individual relative transfer. Now let us consider the ratio of second derivatives \( f''(c_i) / f''(c_h) \). We have shown that there is an inverse relation between this ratio and the ratio of probabilities \( (p_l/p_h) \) (see the proof of proposition 4), so we can relate this force to the relative winning probabilities. It is easy to show that the winning probabilities are concave in the group-average wealth, i.e. that the marginal probability decreases as the group-average wealth increases. Then, if there is an exogenous increase in the average wealth of any group, this group will have more incentive to increase the optimal contribution when its winning probability is low.

Now let us combine the two effects. When \( n_l = n_h \) there is both an equivalent relative transfer – i.e. the relative group-size effect is absent – and a high marginal probability in the poorest group. Thus, when there is a progressive transfer of income, the poorer group will spend a higher proportion of the transfer in the conflict than the proportion that was spent by the richer group from the same amount of wealth. In the new equilibrium the intensity of conflict will increase. We conclude that the “pure” effect of income redistribution over the level of conflict is positive.

When the poorer group is smaller than the richer, there is both a high relative transfer and a high marginal probability. In this case, the relative winning probabilities effect is

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4 Notice that if \( p_l \) and \( p_h \) are close, equation 6 implies that \( \vec{w} - p_l R/n_l \) is also close to \( \vec{w} - p_h R/n_h \).
reinforced by the relative group-size effect. Similar as before, the final result is an increase in the level of conflict. The contrary occurs when the poorer group is larger (low relative transfer) and its winning probability higher (low marginal probability). The ambiguity appears when the two effects go in the opposite direction, i.e. when the poorer group in spite of being larger (low relative transfer) has a smaller probability (high marginal probability) than the richer group.

To conclude this section let us consider again groups with corner solution. Notice that if it is the case, a progressive transfer of income done from an active to an inactive group that is not sufficiently high to turn active the poorest group after the transfer decreases the level of conflict. In this case the persons in the poorer group will find more profitable to take the money coming from the transfer for them and still keep away from the conflict. In the new equilibrium the poorer group does not reinvest the total proportion of the transfer that was spent in the conflict by the richer group, and so the conflict intensity will decrease. If the poorest group turns active, the level of conflict may increase or decrease depending on how both the number of active people and their average wealth changes.

**Within-Group Income Inequality Case**

Now, we concentrate on the more general case in which people with different wealth form groups. In this case it can be shown that equation 6 also characterises the equilibrium (See appendix). Then it is possible to generalise proposition 2 though 5 for this case.

Under these circumstances, it also makes sense to study the effect of a within-group progressive transfer. By such a kind of transfer we refer to the case in which the richer people in a group transfer part of their total wealth to the poorer people in the same group, keeping constant the total wealth of that group. Since the equilibrium condition (equation 6) depends on the group average wealth, and it does not change when there is a within-group progressive transfer, then neither the winning probabilities nor the conflict intensity are affected by such a kind of transfers.

Nevertheless, within-group distribution might affect the equilibrium when there are some inactive people in a group (the equilibrium condition in this situation is stated in the appendix). If this is the case, any within-group redistribution from the inactive to the active people increases the average wealth of the latter, and thus increases both the group winning probability and the level of conflict. Notice that in this case the number of active people keeps constant. When the redistribution goes the opposite way, and the number of active people changes, it is hard to make a prediction; the final effect shall depend on how both the number of active people and their average wealth change. When these two variables increase after the redistribution, both the group winning

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5 Actually, this result comes directly from equation 3a. We already know that at equilibrium $w_i - r_i = k \forall i \in g$, where $k$ is a positive constant. Solving for $r_i$ and summing up over $i$, we get $R = W - n_k$. Thus, within-group income distribution does not matter. The important fact of the previous analysis is that we know how $k$ looks, and so we can generalise the propositions in the previous sections.
probability and the level of conflict increase. On the other hand, when these two variables decrease the opposite result comes about.\(^6\)

Actually, this result is a corollary of the Neutrality theorem for private provision of public goods (Warr, 1983). Such theorem says that regardless of the differences in individual preferences the private provision of a public good is unaffected by the redistribution of income.\(^7\) Notice that in our case the winning probability is a public good for the group and it is provided privately. The novelty of our result is that this neutrality still remains when there is an interest conflict with other groups over the provision of the good. However, in the same direction of Bergstrom et al. (1986), we also show that in the presence of corner solutions this neutrality not necessarily holds.

Some political scientists have argued that group heterogeneity (for instance, in wealth) matters for the success of collective action (e.g. Marwell and Oliver, 1993), and then there should be something missing in the Neutrality theorem. In this line, some authors have shown that there are others assumptions, apart from the absences of corner solutions, that may change this result. For instance, linearity in the production function of the public goods, the “pureness” of the public good, and the existence of perfect markets (e.g. Cornes and Sandler, 1994, 1996 (pp. 184-190); Bardhan, et al., 2002).

3. Wealth and Group-Success

The relationship between group-success and group-size has received special attention in the collective action theory. As we saw in section 2, the explicit inclusion of wealth in the analysis opens a new and, to our knowledge, unexplored perspective in which this relationship can be affected. In this section we shall study more carefully how the interaction between wealth and group-size may affect the success of a group involved into a contest.

The most representative thesis in this respect is due to Olson (1965). In his theory on collective action, he concedes that because of the free-riding effect and because pay-offs are not always pure public goods, larger groups are less successful than smaller groups in looking for their interests. This result is known as the “group-size paradox”. Using

\(^6\) Notice that when there is a transfer of income from active to inactive people four cases may come about: The active people and their average wealth decrease; the active people and their average wealth increase, the active people increase, but their average wealth decreases; and the active people decrease, but their average wealth increases. In the last two cases, the final effect will depend on the specific values that these endogenous variables (\(\overline{w}_g^4\) and \(n_g^4\)) take at the new equilibrium.

\(^7\) Our neutrality result assumes that preferences are the same for everybody in the group. It is easy to extend this result when this is not the case. Assume that each individual \(i\) in each group values the prize at \(x_i \in (0,1]\). So her expected utility is given by \(EU_i = x_i p_g + f(c_i)\), and the interior solution requires \(x_i (1 - p_g)/R = f'(w_i - r_i)\). This condition implies that for each pair of active members of \(g\), say \(i\) and \(j\), there exists a \(\theta_{ij} \in (0, \infty)\) such that \(\theta_{ij}(w_i - r_i) = w_j - r_j\). Following the same steps that we used to get equation 5A in the appendix, the equilibrium condition can be written as \(\sum_{i \in g} \frac{1}{R} (1 - p_g) = \frac{1}{x_i} f' \left( \frac{\sum_{i \in g} \theta_{ij}(w_i - p_g R)}{\Theta_g} \right)\), where \(\Theta_g = \sum_{i \in g} \theta_{ij}\). From this condition it follows that neither the level of conflict nor the winning probability are affected by a within-group redistribution.
our framework, in this section we shall explore if wealth also has this effect over group success.

From proposition 2 we know that when two groups have the same average wealth, the larger group will spend more on conflict than the smaller group and thus will attain a higher winning probability. It is also true that smaller and poorer groups (in terms of average wealth) are less successful than larger and richer groups. These facts imply that the group-size paradox does not necessarily hold in our framework.

Nevertheless, proposition 2 also suggests that it may be possible to observe a smaller group being more successful than a larger group if the former has enough wealth to compensate by its size. At that time, we are interested in knowing under which conditions this outcome might come about. Plainly, a necessary condition to get this result is that the average wealth of the smaller group must be higher than the average wealth of the larger group. However, as example 1 illustrates, it is not a sufficient condition.

**Example 1**: Assume that there are two groups (group $s$ and $b$) in contest and $f(c_i) = \ln(c_i)$. Let $n_s=3$, and $n_b=30$. For any pair $(\bar{w}_s, \bar{w}_b)$ we can solve for the equilibrium vector $(p_s, p_b, R)$. Table 1 shows some computations. Notice that with $\bar{w}_s$ enough high then $p_s > p_b$. However, although $\bar{w}_s > \bar{w}_b$, this result can be reverted. That is the case in which $(\bar{w}_s, \bar{w}_b) = (103, 100)$.

<table>
<thead>
<tr>
<th>$\bar{w}_s$</th>
<th>$\bar{w}_b$</th>
<th>$p_s$</th>
<th>$p_b$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>161</td>
<td>100</td>
<td>0.60</td>
<td>0.40</td>
<td>59.5</td>
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<tr>
<td>112</td>
<td>100</td>
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<tr>
<td>103</td>
<td>100</td>
<td>0.49</td>
<td>0.51</td>
<td>48.6</td>
</tr>
<tr>
<td>71</td>
<td>100</td>
<td>0.40</td>
<td>0.60</td>
<td>39.7</td>
</tr>
</tbody>
</table>

Therefore, in order to observe a smaller group with a higher winning probability, we must impose some extra conditions on either its average wealth or its total wealth. Example 1 brings an additional clue to this respect. Notice that in the two first cases in which $p_s > p_b$, the total wealth of the smaller group is smaller than the total wealth of the larger group. Thus, $W_s > W_b$ is not a necessary condition in order to get $p_s > p_b$.

We start by analysing the case in which there are only two groups in contest ($G=2$). Call these groups $s$ and $b$, and assume that the former is smaller in size than the later, i.e. $n_s<n_b$. Then, in this case we have $n = n_s + n_b$, and $W = W_s + W_b$.

**Proposition 5**: Assume that $G=2$, $R$ is the equilibrium level of conflict, and both $s$ and $b$ are active groups.

(a) The smaller group ($s$) will be more successful than the larger group ($b$) if:
\( W_s - \frac{n_s}{n}W \geq \left( 1 - \frac{n_s}{n} \right)R \)  

(9)

(b) Even though \( \bar{w}_s > \bar{w}_b \), the smaller group \( (s) \) will be less successful than the larger group \( (b) \) if:

\[
0 < W_s - \frac{n_s}{n}W \leq \left( \frac{1}{2} - \frac{n_s}{n} \right)R
\]

(10)

Condition 9 is a sufficient requirement to have the smaller group be more successful than the larger group. Notice that the second term in the left-hand side of this inequality \( (n_s, W/n) \) can be interpreted as the wealth that group \( s \) would have if the total wealth in the society were distributed equally among all its individuals. Then, part (a) of proposition 5 says how large should the income inequality between the two groups be in order to have the smaller group be more successful than the larger group. The required inequality is a fraction \( (1-n_s)/n \) of the equilibrium level of conflict. Thus, for a level of conflict \( R \) this inequality must be higher as smaller is \( n_s \). On the other hand, part (b) says that when the between-group income inequality is not too high, this outcome can be reverted. It can be checked that these conditions are satisfied in example 1.

Notice that condition 9 can be written as \( W_s \geq n_s/W_b + R \), which not necessarily implies \( W_s > W_b \). Then, for \( R \) and \( n_s \) small enough it can happen, as in example 1, that 9 holds but \( W_b > W_s \). Additionally, notice that 9 can also be written in terms of \( s \) and \( b \)'s average wealth as follows: \( \bar{w}_s - \bar{w}_b \geq \frac{1}{n_s}R \). This condition shows directly the minimum income inequality required between \( s \) and \( b \) in order to have \( p_s > p_b \).

In the appendix we generalize previous conditions for the case in which there are more than two groups \( (G>2) \). Different to 9 and 10, the new conditions include the sum of equilibrium probabilities of the other groups (\( \Pi^- \)). In this case, for a given level of \( R \), the income inequality required between \( s \) and \( b \) to get \( p_s > p_b \) is higher as \( \Pi^- \) goes to one.

From this discussion we can extract two conclusions. First, a sufficiently high income inequality between a small and a large group can explain the group-size paradox when the contest’s prize has pure public characteristics. Second, to observe this outcome, the total wealth of the smaller groups must not be necessarily higher than the total wealth of the larger groups.

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\( ^8 \) Condition 10 can be written in terms of average wealth as follows: \( 0 < \bar{w}_s - \bar{w}_b \leq \left( \frac{1}{2n_s} \right) \frac{1}{n_s}R \).
4. Extension: Mix Private-Public Prize

So far we have studied the effect of income distribution over both the level of conflict and the winning probabilities when the contest’s prize is a pure public good. In this section we consider the case of a prize with a varying mix of public and private characteristics. To this end we assume the prize has a public component $P$, which is equally enjoyed by all the groups’ members irrespective of the groups size (i.e. does not have any congestion); and a private component $M$ (say money), which to simplify we assume is equally divided among the group’s members.

One could assume another type of distributive rule for the private part of the prize. For instance, $P$ might be distributed accordingly to the individual contributions. Actually, this rule makes sense when the group members have different levels of wealth and so different contributions. However, with such a kind of rule it is not possible to extract general analytical results from our framework. In what follows, we are going to restrict ourselves to the equality distributive rule.

Following Esteban et al. (2001), we call $\lambda \in [0,1]$ the share of publicness of the prize. Thus, if the group $g$ wins the contest, it will receive a prize $z_g$ given by:

$$z_g = z(\lambda, n_g) = \lambda P + (1 - \lambda) \frac{M}{n_g}$$

(11)

Therefore, the expected utility of an individual who belongs to group $g$ is given now by:

$$E U_i = p_g z_g + f(c_i)$$

(12)

Equation 12 replaces equation 1. The rest of the framework keeps the same. Thus, taking as given the contribution of everybody else each individual $i$ maximises 12 subject to equation 2. Assuming interior solution for every individual, similar to section 3 it can be shown that the unique equilibrium vector $(p, R)$ must satisfy (see the appendix):

$$\frac{I}{R} (1 - p_g) z_g = f' \left( \bar{w}_g - \frac{p_g R}{n_g} \right)$$

(13)

with $p_g \geq 0 \ \forall g$;

and $I I = \sum_g p_g = I$

Since part of the prize decreases as the group-size increases, then it is not necessarily true that the winning probability of group $g$ (and so the level of conflict) increases as $n_g$ increases. Actually, Esteban et al. (2001) have already studied the group-size effect when there is a mix public-private prize. Here we restate their main results to this respect in term of our framework and use it in order to study how between-group income distribution affects both the winning probabilities and the conflict intensity.
To do so, it is useful to define two new variables. First, call $\theta_g$ the share of publicness as perceived for an individual of group $g$ as:

$$\theta_g = \frac{\lambda P}{\lambda P + (1 - \lambda)M/n_g} \quad (14)$$

Additionally, from the utility function $u_g = z_g + f(c_g)$ with $c_g = w_g - r_g$, we define for each group $g$ the average elasticity of the marginal rate of substitution (MRS) with respect to the effort ($\eta_g$) as follows:\(^9\)

$$\eta_g(\bar{w}_g, \bar{r}_g) = \frac{\partial \text{MRS}/\partial r_g}{\partial r_g/\partial \bar{r}_g} = -f''(\bar{w}_g - \bar{r}_g) \frac{\bar{r}_g}{f'(\bar{w}_g - \bar{r}_g)} > 0 \quad (15)$$

With these two variables we can state the following results.

**Proposition 6**: Consider the prize $z_g$ and assume that there is an interior solution for everybody in the game described above, then:

(a) Both the level of conflict and the winning probability of group $g$ are strictly increasing in the average wealth of group $g$.

(b) Both the level of conflict and the winning probability of group $g$ are strictly increasing in the group-size of $g$ if and only if $\eta_g(.) > (1 - \theta_g)$.

**Corollary**: The level of conflict and the winning probability of group $g$ are strictly increasing in the group-size of $g$ if either: (i) For any $\lambda \in [0,1]$, $\eta_g(.) > 1$; or (ii) the prize is totally public ($\lambda = 1$).

The effect of wealth on both the winning probabilities and the conflict intensity is similar to that found for the pure public prize case. However, the effect of the group-size on these variables can differ from that found in section 2. Now it depends on whether the average elasticity of the marginal rate of substitution is higher, equal, or smaller than the share of privateness of the prize.

Let us study the income inequality effect. Notice that under interior solution the within-group income distribution neutrality still holds. Thus, in what follows we care on the between-group income inequality. We employ the same strategy used in section 3 to study the effect of a between-group progressive transfer. We obtain the following results.

---

\(^9\) When group $g$ wins the contest, an individual $i$ who belongs to this group gets utility $u_i = z_g + f(w_i - r_i)$. Evaluating this utility on the average individual (i.e. that with wealth equals $\bar{w}_g$, and contribution $\bar{r}_g$), we get $\bar{u}_g$. For this individual, the marginal rate of substitution equals to $MRS = \frac{\partial \bar{u}_g/\partial \bar{r}_g}{\partial \bar{u}_g/\partial \bar{z}_g}$.  

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Proposition 7. Consider the prize $z_g$ and assume that there is an interior solution for everybody in the game described above, then:

(a) If $\eta_s(\cdot) > (1 - \theta_s)$, the results in propositions 4 and 5 apply.

(b) If $\eta_s(\cdot) < (1 - \theta_s)$: (i) If $n_l < n_h$ and $p_l < p_h$, then a progressive transfer of income increases the conflict intensity; (ii) If either $n_l < n_h$ and $p_l > p_h$ or $n_l > n_h$, then the effect of a progressive transfer of income over the level of conflict is ambiguous.

Proposition 7 says that when the winning probabilities increase with the group-size, the results of a between-group progressive transfer of income over the level of conflict are similar to those found for the pure public prize case. However, when this is not the state of affairs (i.e. the winning probabilities decrease with the group-size), the possibilities and the results differ. For instance, now it is possible to have the poorer group being more successful than the richer because of its smaller (and not its larger) size. On the other hand, when the poorer group is larger it can never be more successful. In these two cases the effect of a between-group redistribution of income over the conflict intensity is ambiguous.

We end this discussion by remarking an important result. Notice that no matter what the degree of publicness of the prize is, when the poorer group is smaller than the richer group a low between-group income inequality always increases the conflict intensity.

5. Empirical Evidence: Political Campaigns

The model developed in previous sections predicts that income inequality affects the level of conflict positive or negatively depending on the relative group-size. When the prize is totally public, from Proposition 4 we know that if poorer groups in the society are smaller or equal in size to richer groups, a higher between-group income inequality implies a smaller level of conflict. This is also true when the prize is a mix of public and private characteristics. Nevertheless, the opposite result may come about when poorer groups are larger (enough) in size than richer groups. If this is the case, a higher between-group income inequality might imply a higher level of conflict.

In this section we present some empirical evidence that support our findings about the relationship between income inequality and conflict intensity. To do so, we are going to use information on US campaign contributions in the House race. We have chosen the political campaign example for two reasons. First, it fits well our theoretical framework; second, there is a comprehensive data set available for US’ campaigns during the last decades.

In the literature there are two theoretical approaches to explain campaign contributions or expenditure. The “political man” theory, which assumes that contributors are passive consumers of the position selected by a candidate; and the “economic man” theory, which assumes that contributors are investors who buy the position of the candidates in order to seek for some rents. In the former case, candidates pre-select their political positions, and people or interest groups (like Political Action Committees (PACs) in the US case) contribute to the candidate whose position is closest to their interest. In the
latter case, people or interest groups contribute to the campaign of the candidate whose position has been bought.\(^{10}\)

Regardless the assumption about the individual behaviour (consumer or rent-seeker), campaigns contributors choose a candidate or a set of candidates on the basis of their preferences. In terms of our model, conflict in the political campaigns case is due to these differences in the individual (or the interest group) preferences for candidates. In this context, people invest resources in their preferred candidate (or candidates) in order to get her elected. Thus, the political campaigns example is a good case to test for the empirical predictions of our model.

As we said before, it is not easy to collect information in order to contrast empirically our theoretical findings. For instance, although there are some interesting data sets on internal conflicts around the world, there is not information on wealth for the groups in conflict. The studies on this topic have included nationwide income inequality measures to explain either civil war initiation or duration. Nevertheless, this kind of measure accounts for the inequality in a society as a whole, and it does not separate the between-group from the within-group income inequality. In previous sections, we have shown that these two inequalities may affect the level of conflict in a different way. Thus, by exploiting the available information on political campaigns, and by making some rational assumptions on the people behaviour, we are able to overcome these information restrictions.

We concentrate on the state campaign spending in House race. We have collected information for the three political cycles during the period 1991-1996. Actually, the information corresponds to a panel data with three periods, but since there are not important variations over time, we work with the time-average for each variable (in other words, we present between-group estimations). The information about the financing of the campaigns is from the U.S. Federal Election Commission (FEC).\(^{11}\) Our measure of conflict will be the expenditure in House campaigns at the state level.

Before defining a good proxy for the between-group income inequality, we must identify the groups in conflict. Once it is done, we can look for a measure of group wealth. Actually, we do not have any direct information that allows us to identify groups. A priori, people or interest groups are joined by their preference for a candidate or a set of candidates, but this information is not available. A reasonable assumption is that groups are defined by geographical characteristics, in particular by the city where their members live. This implies that in each state, people from a city compete with people from others cities in order to get their set of preferred policies applied. From now on we identify each city as a group, and we take the per capita income in each of them as the measure of group average wealth. Information of per capita personal income for each Metropolitan Statistical Area (MSA) in US is from the Bureau of Economics

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\(^{10}\) A survey on these theories can be found in Mueller (2003), chapter 20.

\(^{11}\) This information is available at: [http://www.fec.gov](http://www.fec.gov). The file for each electoral cycle (time for electing Representatives to House is every even numbered year) contains information on the campaigns of all individuals who have registered under the Federal Election Campaign. We excluded campaigns that have not received contributions or made expenditures aggregating in excess of $5,000 (i.e. candidates who are not statutory candidates under the 1979 Amendments to the FEC).
Analysis. With this information we are able to compute different measures of between-city income inequality in each state.

At this point, our purpose is to estimate a reduced equation that explains the campaign spending in House race in each state in terms of its respective between-cities income inequality. We also shall include other control variables that have been usually included in previous analysis of political campaigns: the state per capita personal income, the state per capita government spending, the state population, and the number of campaigns in race. 12 Basic statistics are reported in table 2 for the 40 states with complete information. 13

<table>
<thead>
<tr>
<th>Variable</th>
<th>No. Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>State per capita campaign spending in House Race, cycle 1995/96 ($)</td>
<td>40</td>
<td>1.77</td>
<td>0.41</td>
<td>0.72</td>
<td>2.66</td>
</tr>
<tr>
<td>State population 1996 (thousands)</td>
<td>40</td>
<td>6,271</td>
<td>5,878</td>
<td>1,120</td>
<td>31,230</td>
</tr>
<tr>
<td>State per capita personal income 1996 ($)</td>
<td>40</td>
<td>22,727</td>
<td>3,327</td>
<td>17,171</td>
<td>32,135</td>
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<tr>
<td>State per capita government spending 1996 ($)</td>
<td>40</td>
<td>3,078</td>
<td>535</td>
<td>2,318</td>
<td>4,514</td>
</tr>
<tr>
<td>Within state std. deviation of log of MSA per capita income 1996.</td>
<td>40</td>
<td>0.14</td>
<td>0.05</td>
<td>0.08</td>
<td>0.28</td>
</tr>
<tr>
<td>Number of campaigns</td>
<td>40</td>
<td>34</td>
<td>32</td>
<td>6</td>
<td>182</td>
</tr>
<tr>
<td>Within-state Correlation: MSA population and personal per capita income</td>
<td>40</td>
<td>0.66</td>
<td>0.36</td>
<td>-1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Average of population ratios between richer and poorer cities</td>
<td>40</td>
<td>7.26</td>
<td>4.93</td>
<td>0.26</td>
<td>18.84</td>
</tr>
<tr>
<td>Percentage of population ratios between richer and poorer cities that are equal or higher than 1</td>
<td>40</td>
<td>0.75</td>
<td>0.18</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 2
Basic Statistics

12 Information on state’s government spending and population is from U.S. Census Bureau, and number of campaigns from FEC.
13 15 of the 55 states were excluded from the analysis because of missing value observations (specially in campaign expenditure). The states excluded were: Alaska, American Samoa, Delaware, District of Columbia, Guam, Hawaii, Montana, New Hampshire, North and South Dakota, Puerto Rico, Rhode Island, Vermont, Virginia Island, and Wyoming.
What is our expected relationship between campaign spending and between-group income inequality? We already know that the expected effect of the between-group income inequality on the level of conflict depends on the relative group-size (and also on the prize characteristics). Thus, we need to check the relationship between group-size and average wealth among cities from the same state.

From our sample the mean of the MSA correlation between per capita personal income and population is 0.66. This pattern is similar in almost all the states, although the values of the correlations run from 0.1 to 1.\(^4\) Thus, we can say that in almost all the states those cities with a higher per capita income have in average a higher population as well. In order to explore more this relationship, we computed for each state all the possible population ratios between pairs of MSA that have in the numerator the population of a richer city and in the denominator the population of a poorer one. From now on, we refer to these relative measures as the population ratios. Table 2 also reports the average of these ratios and the percentage of cases in which they are equal or higher than one. From this statistics we can also infer that, in general, the populations of the richer cities are higher than the populations of the poorer ones.

In terms of our model, this information suggests that we are in the case in which poorer groups in the society are smaller in size than richer groups. Thus, independently on whether people are able to extract some private benefits from elections (i.e. a mix private-public prize), we expect that those states with a higher between-city income inequality spend less money in the House race. In other words, we expect a negative parameter for this relationship.

The estimation results are reported in table 3. As measure of between-group income inequality it is used the standard deviation of the log of the MSAs’ per capita income.\(^5\) The columns differ in the control variables included. Standard deviations are robustly estimated. In the line of some recent studies (e.g. Ansolabehere, et al., 2002), there is evidence that campaign contributions are not a form of policy-buying, but rather a form of political consumption. This conclusion comes from the fact that the government spending is not relevant in explaining campaign spending whereas personal income is. The income elasticity is quite near to that found in previous studies.

Concentrate now in the between-group inequality. Column 2 in table 3 presents the results when the campaign spending is controlled by this variable. As we expected, the sign of the respective parameter is negative, i.e. a higher between-group income inequality implies a lower level of conflict. Moreover, this parameter is significantly different from zero. This evidence supports the predictions of our theoretical model.

Since both the correlation between population and income and the percentage of population ratios equal or higher than one are low in some states, the sign of our relevant parameter might be the opposite for some states. Columns 3 through 6 in table 3 report some results that exploit explicitly the relationship between population and income. We do it by using the three measures mentioned above.

\(^4\) The only exception is Nevada, where this correlation is negative.

\(^5\) The results are quite similar when we use alternative inequality measures as the variance of MSAs’ per capita income, or Gini coefficients.
Table 3  
U.S. State Campaign Spending in House Race  
(Cycles 1991/92 to 1995/96)


<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tbody>
<tr>
<td>Between-group income inequality</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(BGII)</td>
<td>(a)</td>
<td>(1,00)**</td>
<td>(1,25)*</td>
<td>(1,00)**</td>
<td>(1,29)**</td>
<td>(0,91)**</td>
</tr>
<tr>
<td>BGII * Dummy Corr(population,income)</td>
<td>(b)</td>
<td>(0,68)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>BGII * Corr(population,income)</td>
<td></td>
<td></td>
<td>0.53</td>
<td></td>
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<td></td>
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<tr>
<td>BGII * Dummy proportion of population ratios &gt;1</td>
<td>(c)</td>
<td>(1,00)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>BGII * Log(average of population ratios)</td>
<td>(d)</td>
<td></td>
<td></td>
<td>-0.27</td>
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<td></td>
</tr>
<tr>
<td>Log of per capita personal income</td>
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<tr>
<td></td>
<td>0.75</td>
<td>0.74</td>
<td>0.79</td>
<td>0.81</td>
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<td>(0,40)*</td>
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<td>(0,34)**</td>
<td>(0,34)**</td>
<td>(0,36)**</td>
<td>(0,38)**</td>
</tr>
<tr>
<td>Log of per capita government spending</td>
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<td>-0.17</td>
<td>-0.13</td>
<td>-0.16</td>
<td>-0.18</td>
<td>-0.16</td>
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<td></td>
<td>(0,32)</td>
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<tr>
<td>Log of population</td>
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<td>(0,07)**</td>
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<td>(0,09)**</td>
<td>(0,10)**</td>
<td>(0,10)**</td>
<td>(0,11)**</td>
</tr>
<tr>
<td>Number of campaigns in race</td>
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<tr>
<td></td>
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<td>(0,002)**</td>
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<td></td>
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<td></td>
<td>-3.04</td>
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<td>-2.91</td>
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<td>(2,95)</td>
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<td>0.33</td>
<td>0.34</td>
</tr>
<tr>
<td>No. Obs.</td>
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<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

Standard deviations are robustly estimated. *** = Significant at the .01 level; ** = .05 level; and * = .1 level.  
(a) Between-group income inequality: Corresponds to the within state standard deviation of log of MSA per capita personal income; (b) Dummy Corr(population,income): 1 if the MSA correlation between population and per capita income is smaller than 0.23; (c) Dummy proportion of population ratios >1: 1 if the percentage of population ratios between richer and poorer cities higher that one is smaller that 50%; (d) Log(Average of population ratios): Correspond to the log of the average of population ratios between richer and poorer cities. The population ratios between richer and poorer cities correspond to all the possible population ratios between pair of MSA that have in the numerator the population of a richer city and in the denominator the population of a poorer one.

The first one is the correlation between population and the per capita wealth. We create dummy variables for different intervals of the correlation and estimate different specifications with the interaction between these binary variables and the between-group inequality. We do not find any significant effect. Column 3 reports the regression with the best fit, where the dummy variable takes the value of one if the correlation is smaller than 0.2 and 0 otherwise. Alternatively, we include an interaction between the correlation and the between-group inequality. This variable is also not significant (Columns 4).
The second variable is the percentage of cases in which the population ratios are equal or higher than 1. Once more, we create dummy variables for different intervals of this percentage and introduce interaction between that and the between-group inequality. There is not any significant effect. Column 5 reports the regression with the best fit, where the dummy variable takes the value of one if the percentage is smaller than 50% and 0 otherwise.

Finally, we use the log of the average of population ratios. We introduce an interaction between this variable and the between-group inequality, which may allow us to obtain not only a different magnitude for the inequality effect in each state but also a different sign for those states where the average of the ratios is smaller than one. The parameter related to this interaction is negative, as we expect, but it is not significantly different from zero.

As our theoretical model predicts, the results for the political campaign spending case support the idea that the between-group income inequality affects the level of conflict in a society, and that, this effect depends on the relationship between group size and income. For this particular case, we have found that on average a higher between-city income inequality implies a lower level of campaign spending.

6. Conclusions

This paper studies how the interaction among group-size, wealth, and its distribution affects both conflict intensity and group success probabilities in a society when there is a contest for either a pure public prize or a mix private-public prize. Different to the traditional studies on this topic, in this paper we assumed that conflict is due to differences in preferences for social outcomes that are not necessarily related to the individual wealth, and in particular is not generated by income inequality.

Using a contest model between interest groups that introduces explicitly the individual wealth, we find some interesting results. First, poorest people generally are not willing to engage in any conflict. Second, less inequality does not imply less conflict intensity. In fact, the “pure” effect that income redistribution has over the level of conflict is positive. Only under some especial conditions (when the poorer groups have a higher winning probability than the richer ones), income redistribution reduces the conflict intensity.

Third, neither winning probabilities nor conflict intensity are affected by the within-group income inequality in the absence of corner solutions. However, when there are inactive people in a group, this result does not hold any more, and the final effect on the level of conflict depends on how both the number of active people and their average wealth changes. We consider important to explore others assumptions, apart from the absences of corner solutions, that may change this neutrality result. In particular, introducing non-perfect substitutibility in the winning probability function may be an appealing variation to take into account in our framework. We leave it in the open agenda.
Finally, the interaction between group-size and wealth can explain why very small groups with high average wealth are more successful than larger groups with smaller average wealth when groups are competing for public goods. We show that to observe this outcome, the total wealth of the smaller group must not be necessarily higher than the total wealth of the larger group.

Since many of the internal armed conflicts around the world are not directly over wealth, previous findings can partially explain why income inequality (measured by nationwide Gini’s coefficients in a society) has been irrelevant in explaining civil wars likelihood in a country. Nationwide Gini coefficients measure the inequality in a society as a whole but, as we have seen, the between-group and the within-group inequality may affect in a different way the level of conflict. Information in a nationwide Gini mixes these two issues. Unfortunately, there is no available information on group wealth to test this issue for the case of internal conflicts. We presented some evidence for the case of House campaigns in US that support the hypothesis that between-group income inequality in fact affects the level of conflict.

Appendix

Individual Optimal Contributions with a Pure Public Prize. Plugging equation 2 in 1 and taking as given the contribution of the rest of people, each individual \( i \) who belongs to group \( g \) maximizes \( EU_i = \frac{R_g}{R} + f(c_i) \) over \( r_i \). It can be verified that \( EU_i \) is strictly concave in \( r_i \). From the first order condition we get \( \frac{R - R_g}{R^2} = f'(c_i) \).

Reorganizing terms and using the winning probability function we get equation 3a. Since \( \lim_{r_i \to 0} f'(c_i) = \infty \) then the individual contribution will be always smaller than the individual wealth. On the other hand note that
\[
\frac{\partial EU_i}{\partial r_i} \bigg|_{r_i=0} = \frac{R_g}{R^2} - f'(w_i). \]
This marginal utility is positive if and only if \( f'(w_i) < \frac{R_g}{R^2} \). Thus, when this inequality holds, the total amount of resources spent by individual \( i \) in the conflict is strictly positive and described by equation 3a. On the other hand, when \( f'(w_i) \geq \frac{R_g}{R^2} \), the marginal utility is not positive, and the best response of agent \( i \) is \( r_i=0 \).

Proof of proposition 1. To prove it we use equation 5. This equation implicitly defines \( p_g \) as a function of \( R \). It can be readily verified that \( p_g \) is a continuous function of \( R \). Using the implicit function theorem it can be shown that for \( p_g>0 \), \( p_g \) is strictly decreasing in \( R \). From equation 5:

\[
\frac{\partial p_g}{\partial R} = \frac{\frac{1}{R^2} \sum_{i \in g} \left( w_i - f' \left( \frac{|l - p_g|}{R} \right) \right) - \frac{1}{R} \sum_{i \in g} \left( f'\left( \frac{|l - p_g|}{R} \right) \right) \frac{1}{R^2} \left( 1 - p_g \right) - \frac{1}{R} \sum_{i \in g} \left( f'' \left( \frac{|l - p_g|}{R} \right) \frac{1}{R} \right)}{\frac{1}{R} \sum_{i \in g} \left( f'\left( \frac{|l - p_g|}{R} \right) \right) \frac{1}{R}} \tag{A1}
\]
Since \( f''(.) < 0 \) and \( \sum_{i \in g} \left( w_i - f'(l - p_g) / R \right) > 0 \) under interior solution, both the numerator and the denominator in A1 are positive. Then it follows that \( \frac{\partial p_g}{\partial R} < 0 \) \( \forall g \).

Further, for every \( g \) there exists a positive constant \( K_g \) such that \( p_g > 0 \) if and only if \( R \leq K_g \).

Consider the function \( \Pi = \frac{1}{R} \sum_{g} \sum_{i \in g} \text{Max}\left\{ 0, w_i - f^{-1}\left( \frac{1}{R} (1 - p_g) \right) \right\} \). At equilibrium, \( R \) must be such that \( \Pi = 1 \) with \( p_g > 0 \) \( \forall g \). Note that \( \Pi \) is strictly decreasing in \( R \), and tends to zero as \( R \) goes to infinity. On the other hand, when \( R \) goes to zero then, \( p_g > 0 \), and \( \Pi \) goes to infinity. It follows that there must be some \( R \) for which \( \Pi = 1 \). Further, it is unique.

**Proof of proposition 2.** Assume that we are at equilibrium.
(a) Applying the implicit function theorem to equation 6 and keeping \( R \) constant we get
\[
\frac{\partial p_g}{\partial w_i} = \frac{n_g R f''(.)}{R^2 f''(.) - n_g} > 0 .
\]
Since \( \frac{\partial p_g}{\partial R} = \frac{\partial \Pi}{\partial w_i} \mid_R \), an increase in \( p_g \) makes \( \Pi(.) > 1 \).

Thus, since \( p_g \) and \( R \) are negatively related, \( R \) must increase to recover the equilibrium conditions implied in 6. It proves that the level of conflict increases as the average wealth of group \( g \) increases. Now, we concentrate on the final effect on \( p_g \). Until now, the winning probabilities of the other groups (\( p_{g'} \)) have not changed and \( p_g \) has gone up. Since \( R \) increased, all the probabilities must go down to recover the equilibrium condition \( \Pi(.) = 1 \). Then, at the new equilibrium the final \( p_g \) must be larger that the initial \( p_g \) to assure it. It proves that the winning probability of group \( g \) increases as the average wealth of this group increases.

(b) Same as before, keeping constant \( R \) (and \( w_g \)) in equation 6, we get
\[
\frac{\partial p_g}{\partial n_g} = \frac{f''(.) R^2 p_g}{n_g (f''(.) R^2 - n_g)} > 0 .
\]
Following the same arguments used above, we can prove statement b.

**Proof of proposition 3.** See the proof in the text.

**Proof of proposition 4.** Assume that we are at equilibrium. First of all, in order to prove this proposition we claim that, for a given \( R \): (i) If \( p_i < p_h \), then \( \frac{f''(w_i - p_i R / n_i)}{f''(w_h - p_h R / n_h)} > 1 \); (ii) if \( p_i > p_h \), then \( \frac{f''(w_i - p_i R / n_i)}{f''(w_h - p_h R / n_h)} \leq 1 \). Let us prove claim (i).

When \( p_h > p_i \), equilibrium condition 6 implies \( f'(w_h - p_h R / n_h) < f'(w_i - p_i R / n_i) \).

From the characteristics of \( f(.) \), we have that \( \frac{f''(w_i - p_i R / n_i)}{f''(w_h - p_h R / n_h)} > 1 \). The same steps can be used to prove claim (ii). Using these claims and previous results we have that:
(a) When either \( n_l = n_h \) or \( n_l < n_h \) it must be that \( p_l < p_h \). Then it follows that
\[
\frac{n_l}{n_h} < \frac{f''(\bar{w}_l - p_lR/n_l)}{f''(\bar{w}_h - p_hR/n_h)}
\]
and from proposition 3 the level of conflict increases.

(b) In this case both \( \frac{n_l}{n_h} \) and \( \frac{f''(\bar{w}_l - p_lR/n_l)}{f''(\bar{w}_h - p_hR/n_h)} \) are higher than one, thus the effect on the level of conflict is ambiguous.

(c) In this case \( \frac{n_l}{n_h} > \frac{f''(\bar{w}_l - p_lR/n_l)}{f''(\bar{w}_h - p_hR/n_h)} \), thus the level of conflict decreases.

**Between-Group Income Inequality Case.** Equations 3a and 3b define the equilibrium conditions in this case. Assume there is interior solution for everybody. Since at equilibrium \( \frac{I}{R} (1 - p_g) \) is the same for any pair of persons in the same group, say \( i \) and \( j \), from 3a it must be that \( f'(w_l - r_i) = f'(w_j - r_j) \). Given the monotonicity of \( f(.) \), this equality holds if and only if:
\[
w_i - r_i = w_j - r_j \tag{A2}
\]

Equation A2 implies that, at equilibrium, the wealth net of conflict expenditure of all active individuals who belong to the same group must be equal to the same constant. It is possible to derive this constant in terms of the equilibrium variables \( R \) and \( p_g \). In order to do so, take any group \( g \), fix any active individual in this group, say person \( i \), and use A2 to sum up the equilibrium contributions of the other active individuals in the group, then:
\[
\sum_{j \neq i} r_j = \sum_{j \neq i} (w_j - w_i + r_i) = \sum_{j \neq i} w_j - (n_g - 1)w_i + (n_g - 1)r_i \tag{A3}
\]

Reorganizing terms in A3, we can write down the total amount of resources spent on the conflict by the group \( g \) in terms on the personal contribution of any active member, say person \( i \), and her wealth:
\[
R_g = (W_g - n_g w_i) + n_g r_i \tag{A4}
\]

Notice that if \( w_i = \bar{w}_g \), A4 implies \( R_g = n_g r_i \) \( \forall i \), and then equation 3a can be written as \( \frac{I}{R} (1 - p_g) = f'(\bar{w}_g - \frac{p_gR}{n_g}) \). We shall prove with equation A6 that it is true not only for this case but in general for every \( w_i \). Combining equation A4 and 2 and solving for \( r_i \) we get:
\[
r_i = \frac{p_g R - (W_g - n_g w_i)}{n_g} \tag{A5}
\]

Notice that the equilibrium contribution can also be written as \( r_i = \bar{r}_g + w_i - \bar{w}_g \), where \( \bar{r}_g = R_g / n_g \). Thus, richer people spend resources on the group’s average contribution. Finally, using A5 we can rewrite 3a as follows:
\[
\frac{1}{R} (1 - p_g) = f^\prime \left( \frac{w_g}{n_g} - \frac{p_g R}{n_g} \right)
\]  

(A6)

which actually is the same equilibrium condition obtained for the within-group income equality case. If there are inactive people in \( g \), equation A6 must be written as

\[
\frac{1}{R} (1 - p_g) = f^\prime \left( \frac{w_g^a - p_g R}{n_g^a} \right),
\]

where an \( A \) has been added as superscript in \( n_g \) and \( w_g \) to denote the active number of people in a group and their respective average wealth. This is so because when adding contributions in A3 only active people matter. Notice that if it is the case, it cannot be done any comparative statistics because \( n_g^a \) and \( w_g^a \) are endogenous.

**Proof of proposition 5.** Assume that \( G=2 \) and \( R \) is the equilibrium level of conflict.

(a) From equation 9 we have:

\[
W_s - \frac{n_s}{n} W \geq \left( 1 - \frac{n_s}{n} \right) R \quad \Rightarrow \quad W_s - \frac{n_s}{n} W > \left( p_s - \frac{n_s}{n} \right) R
\]

\[
\Leftrightarrow \quad nW_s - n_s (W_s + W_b) > (n_s + n_b) p_s - n_s R
\]

\[
\Leftrightarrow \quad (n - n_s) W_s - n_s W_b > (n_b p_s - n_s \{1 - p_s\}) R
\]

\[
\Leftrightarrow \quad \frac{n_s W_s - n_s W_b}{n_s n_b} > \left( \frac{n_b p_s - n_s p_b}{n_s n_b} \right) R
\]

\[
\Leftrightarrow \quad \bar{w}_s - \bar{w}_b > \left( \frac{p_s}{n_s} - \frac{p_b}{n_b} \right) R
\]

\[
\Leftrightarrow \quad \bar{w}_s - \frac{p_s}{n_s} R > \bar{w}_b - \frac{p_b}{n_b} R
\]

\[
\Leftrightarrow \quad f^\prime \left( \bar{w}_s - \frac{p_s}{n_s} R \right) < f^\prime \left( \bar{w}_b - \frac{p_b}{n_b} R \right)
\]

\[
\Leftrightarrow \quad p_s > p_b
\]

The last line comes from the equilibrium conditions (equation 6).

(b) Assume \( W_s \) is such that 10 holds (which implies \( \bar{w}_s > \bar{w}_b \)), but \( p_s > p_b \). From the equilibrium conditions (equation 6) it follows that

\[
f^\prime \left( \frac{\bar{w}_s - p_s}{n_s} R \right) < f^\prime \left( \frac{\bar{w}_b - p_b}{n_b} R \right).
\]

From the proof of part a, this inequality implies that \( W_s - \frac{n_s}{n} W > \left( p_s - \frac{n_s}{n} \right) R \). This last inequality contradicts the initial assumption because in this case \( p_s > 1/2 \).
Generalization of proposition 5 for \(G>2\). Consider groups \(s\) and \(b\), where \(n_s<n_b\). Call \(\Pi^- = \Pi - p_s - p_b\), and assume \(G>2\), \(R\) is the equilibrium level of conflict, and both \(s\) and \(b\) are active groups.

a) The smaller group \((s)\) will be more successful than the larger group \((b)\) if:

\[
W_s - \frac{n_s}{n_s + n_b}(W_s + W_b) \geq \left(1 - \frac{n_s}{n_s + n_b}\right)\left(1 - \Pi^-\right)R
\]

\(\text{(A7)}\)

b) Even though \(\bar{w}_s > \bar{w}_b\), the smaller group \((s)\) will be less successful than the larger group \((b)\) if:

\[
0 < W_s - \frac{n_s}{n}(W_s + W_b) \leq \left(1 - \frac{n_s}{n_s + n_b}\right)\left(1 - \Pi^-\right)R
\]

\(\text{(A8)}\)

The proof is similar to the proof of proposition 6.

Individual Optimal Contributions with a Mix Public-Private Prize. Each individual \(i\) who belongs to group \(g\) maximizes \(EU_i = \frac{R_g}{R} z_g + f(c_i)\) over \(r_i\). Assuming interior solution for everybody, the first order condition can be written as:

\[
\frac{1}{R} (1 - p_g) z_g = f'(c_i)
\]

\(\text{(A9)}\)

Following the same steps and arguments used in the pure public prize case, from A9 we can redefine the equilibrium in terms of \(p\) and \(R\), and prove that the equilibrium vector \((p,R)\) always exists and is unique. Moreover, following the same steps used to obtain equation A6 we get that at equilibrium:

\[
\frac{1}{R} (1 - p_g) z_g = f'\left(\frac{p_g R}{n_g}\right) \quad \forall g
\]

\(\text{(A10)}\)

This system of \(G\) equations plus the conditions \(\Pi = \sum_g p_g = 1\), and \(p_g \geq 0 \quad \forall \quad g\), complete the equilibrium description.

Proof of proposition 6. Assume that we are at equilibrium.  
(a) Applying the implicit function theorem to equation 13 and keeping \(R\) constant we get:

\[
\frac{\partial p_g}{\partial \bar{w}_g} = \left.\frac{n_g R f''(.)}{R^2 f''(.) - n_g z_g}\right| > 0.
\]

Following the same arguments used in proposition 2(a) we prove statement 6(a).
(b) Keeping constant \( R \) (and \( \bar{m}_g \)) in equation 13 by the implicit function theorem we get

\[
\frac{\partial p_g}{\partial n_g} = \frac{-R/n_g \left( (1-p_g)(1-\lambda)M + f''(\cdot)Rp_g \right)}{n_g z_g - f''(\cdot)R^2}.
\]

Since \( f''(\cdot) < 0 \), then the sign of this derivative depends on the sign of the term in parenthesis in the numerator. Thus, \( \frac{\partial p_g}{\partial n_g} > 0 \) if and only if:

\[
\frac{1}{R} (1-p_g)(1-\lambda)M < f''(\cdot)Rp_g
\]

\[\iff\]

\[
\frac{1}{R} (1-p_g) z_g (1-\theta_g) < f''(\cdot) \frac{Rp_g}{n_g}
\]

\[\iff\]

\[
f'(\cdot) (1-\theta_g) < f''(\cdot) \tilde{R}_g
\]

\[\iff\]

\[
(1-\theta_g) < \eta_g
\]

Since \( \eta_g > 0 \) and \( \theta_g \in [0,1] \), the corollary follows immediately.

**Proof of proposition 7.** Consider again two groups \( h \) and \( l \) with \( \bar{m}_h > \bar{m}_l \). We can apply the same strategy used in section 2 to study any between-group progressive transfer. By Using equation 7 and replacing the respective derivative or each group (see proposition 7a) we get that a progressive transfer of income generates a decrease in the level of conflict if

\[
\frac{n_h z_h}{n_l z_l} > \frac{f'''(\bar{m}_l - p_l R/n_l)}{f'''(\bar{m}_h - p_h R/n_h)}.
\]

When this inequality is reversed, the transfer generates an increase in the level of conflict. The difference with the condition in proposition 3 is that the right-hand term includes the ratio \( z_l/z_h \). Notice that, since the term \( n_h z_h \) is strictly increasing in \( n_g \), then \( n_l z_l/n_h z_h > 1 \) if \( n_l > n_h \).

First, consider the case in which \( \eta_g(\cdot) > (1-\theta_g) \), i.e. that in which the winning probability is strictly increasing in the group-size. In this case, we have the same possibilities studied in proposition 4. Following the same arguments used to prove it, we can get the same results. The only difference has to do with the line of reasoning in proposition 4b. In this new condition the ambiguity arises because the ratio \( f'''(c_l)/f'''(c_h) \) can be higher, equal, or smaller than 1. This proves part (a) of the proposition.

Now, consider the case in which \( \eta_g(\cdot) < (1-\theta_g) \), i.e. that in which the winning probability strictly decreases in group-size. When this occurs, there are three alternatives: (1) \( n_l < n_h \) and \( p_l < p_h \), (2) \( n_l < n_h \) and \( p_l > p_h \), and (3) \( n_l > n_h \) and \( p_l < p_h \). Notice that, from the results in proposition 7(b), the possibility \( n_l > n_h \) and \( p_l > p_h \) never happens. We apply the same arguments used in the proof of propositions 4. Under the alternative 1 it follows that, \( n_l z_l/n_h z_h < 1 \), and \( f'''(c_l)/f'''(c_h) > 1 \), thus the result in proposition 7(b-i) arises immediately. Under alternatives 2 and 3, it is not possible to infer whether the ratio \( f'''(c_l)/f'''(c_h) \) is above or below the unit. This is so because we do not
know if $z_i(1-p_i)$ is higher, equal, or smaller than $z_i(1-p_i)$. Thus, in these cases the effect of a between-group progressive transfer on the level of conflict is ambiguous.
Chapter II

Decentralization, Corruption, and Political Accountability in Developing Countries.

Abstract

Does decentralization reduce the level of corruption in the presence of powerful local elites? This is a relevant question for developing countries. We motivate this paper with some empirical evidence. Using cross-country information we find that the negative average effect of decentralization on corruption documented in the literature is absent for developing countries. We build an imperfect information model of corruption and political accountability to study if the influence of local elites on the allocation of public resources can explain this outcome. We find that not only the power of the elites but also other unexpected factors matter. In particular, both the existence of relative poor and rich regions with a weak accountability sector and the design of decentralization and grants can also explain the lack of success of decentralization in combating corruption in these economies.

An important part of the literature on fiscal federalism has cared on the potential benefits of decentralization on corruption. The main question to this respect is whether or not decentralization promotes good governance and persuades politicians against corruption. There is a partial agreement that decentralization reduces the level of corruption. This conclusion is based on both some well-known theoretical results and some of the empirical evidence available.

Nevertheless, this is not the common perception in developing countries. Some authors have informally claimed that some idiosyncratic characteristics of these economies, like the existence of powerful local elites, have not allowed decentralization to reduce the level of corruption. In this paper we study how the existence of these local elites affects the relationship between decentralization and corruption.

The arguments that support the idea that decentralization reduces the level of corruption are based on at least two theories. First, jurisdictional competition discourages local governments from establishing distortionary policies that might drive away factors of production to less interventionist jurisdictions (Brenna and Buchanan 1980; Shleifer and Vishny, 1993). Second, decentralization improves political accountability (Seabright, 1996). The idea behind this thesis is that decentralization grants the citizens of each region with the power to decide directly whether to re-elect a government or not, whereas centralization ensures that regions no longer have the same power in the re-election decision. It allows decentralization to encourage good governance.
Other authors have claimed that decentralization may bring about more corruption in developing countries. The reason to think so is simple: those factors that allow decentralization to reduce corruption fail systematically in these economies. For instance, jurisdictional competition requires the existence of well-behaved common markets and that is not the rule in developing countries (Litvack, Ahmad, and Bird, 1998). Furthermore, although in most of these countries popular election systems are established, powerful elites make difficult a broadly based local participation in elections (Prud’homme, 1995; Tanzi, 1995). This issue obscures political accountability through elections and makes developing countries more vulnerable to corrupt bureaucracies.

The empirical evidence about the relationship between decentralization and corruption also exhibit different results. The most representative study in this field is due to Fisman and Gatti (2002), who work with a cross-section of 55 (developing and developed) countries. They use the sub-national share of total government spending as measure of decentralization and the International Country Risk Guide corruption index as measure of corruption. Their results show that more decentralization implies less corruption. However, Treisman (2000, 2002) finds the opposite result by using different measures of decentralization and quality of government. Regardless of that, whether or not the effect of decentralization on corruption is negative in both developed and developing countries is still an open issue.

To motivate our discussion, we present some suggestive evidence about the relationship between decentralization and corruption in developing countries. In order to be consistent with the available evidence, we use the same sample, data set, decentralization definition, corruption index and econometric specification used by Fisman and Gatti (2002). We show that the negative effect that fiscal decentralization has on corruption in developed countries can not be confirmed in developing economies. In the rest of the paper we formalize the idea that the lack of success of decentralization in combating corruption in developing countries can be explained by the existence of powerful local elites. In doing so, we find out new elements that are relevant to understand this relationship.

We start by developing and analyzing an incomplete information model of corruption and political accountability in a decentralized system. We understand corruption as the use of public resources for private gains. By political accountability, we mean the capacity of citizens to detect a corrupt incumbent and to remove him from office\(^\text{16}\). The game involves the voters of the jurisdiction, the respective incumbent, and a local elite that demands corruption from the office. The asymmetry in the model arises from the incumbent’s type (corrupt or non-corrupt). At election time, citizens cannot observe this type but only a signal about it. This signal is produced and sent by a local accountability sector.

The accountability sector is an organized local group interested in good governance. This sector can be understood as a technology that invests all its resources in

\(^{16}\) It is important to note that this concept differs from the Seabright’s definition of accountability, which refers to the probability that the welfare of a region can determine the re-election of the government.
supervising the incumbent’s performance. These resources depend positively on the per capita income of the jurisdiction. We assume that the probability of detecting the incumbent in corruption increases as the resources of the accountability sector increases. In our framework, not only the accountability sector but also the elite can influence the political process by affecting the probability of detecting the incumbent in corruption. It can do it through two mechanisms. First, it can invest some resources in order to hold up the task of the accountability sector. In this way the elite reduces the probability of detection. Second, we assume the elite has economic control over a proportion of citizens. As this proportion increases, it is more difficult to detect the incumbent in corruption activities.

If the incumbent is non-corrupt, then at equilibrium there is not corruption. The interesting case is that in which the incumbent is corrupt. If this is the case, we show that at equilibrium both the level of political accountability and the level of corruption are simultaneously determined by the power of the local elite, the per capita income of the jurisdiction, the incumbent’s office spoils, and the incumbent’s share in corruption – i.e. the proportion that incumbent reserves to himself from the resources allocated in corruption.

To model the centralized case, we use the same framework described above. The novelty is that, under centralization, there is a local elite in each jurisdiction demanding corruption not to a local incumbent but to a central bureaucrat. This extension does not affect the equilibrium representation of the model. Thus, both the level of corruption and political accountability under centralization depend on the total power of the local elites at the national (federal) level, the national per capita income of the federation, the federal incumbent’s office spoils, and the federal incumbent’s share in corruption.

Our aim is to study how corruption and political accountability change when a federation moves from a centralized to a decentralized system. We do it by analysing how the parameters of the model change between the federal (national) level and the jurisdictional level. We start by comparing the power of the elites at the national level against the power of the elite at the jurisdictional level. Our model predicts that, if the latter is larger than the former, then decentralization increases (reduces) the level of corruption (political accountability). The opposite happens if the latter is smaller than the former. The final effect of decentralization (via the elites' power) on national corruption and accountability is difficult to predict. It depends on the distribution of these powers across the jurisdictions and the initial level of corruption.

The second relevant comparison is between the national and the jurisdictional per capita income. We show that, if the jurisdictional per capita income is larger than the federal per capita income, decentralization reduces (increases) the level of corruption (political accountability) if and only if the resources of the accountability sector grow above the locally generated taxes. Otherwise, decentralization increases (reduces) the level of corruption (political accountability). An analogous result is obtained if the jurisdictional per capita income is smaller than the national per capita income. Once again, the final effect of decentralization (via per capita income) on corruption and political accountability is ambiguous. It depends on the dispersion of income across the jurisdictions. Nevertheless, it is not hard to think that corruption (political
accountability) will increase (decrease) in many jurisdictions in developing countries, which usually are characterized by a relative weak accountability sector.

An important corollary of this result has to do with the use of grants or transfers from the central government to the jurisdictions. Grants affect the amount of resources that an incumbent can allocate in his jurisdiction positively, but do not affect the amount of resources that the accountability sector has to invest on political accountability. When this happens, our model predicts an increment (reduction) in the level of corruption (political accountability). This is an important issue for developing countries where the central governments use transfers intensively in order to reduce the high between-jurisdiction income inequality. In order to avoid corruption while reducing inequalities, the design of these transfers must include some grants to the accountability sectors.

The third relevant element is the offices spoils. National office spoils are expected to be larger than jurisdictional office spoils, independently of the level of development. Our model predicts that under these circumstances, decentralization increases the level of corruption. Nevertheless, the effect on political accountability is ambiguous. Thus, the decentralization design also affects the level of corruption. For instance, the office spoils in small municipalities are farther from the national ones than the respective spoils in states. Therefore, when a federation is decentralized, corruption will increase more if it focuses on small jurisdictions than if it focuses on states. Decentralization in developing countries has allocated many tasks to small municipalities.

The last key element is the incumbent’s share in corruption. Some authors (e.g. Tanzi, 1995) have claimed that rewards to local politicians are relatively smaller than those received by central bureaucrats. Our model predicts that when this is the case and this share is not too high, decentralization reduces the level of corruption. However, the effect on accountability is ambiguous.

The final effect of decentralization on the nationwide level of corruption and political accountability depends on the combination of all the factors mentioned above. Thus, it is difficult to make a clear prediction about this effect. Nevertheless, our empirical evidence suggests that decentralization has not effected the level of corruption in developing countries. This outcome can be explained by the interaction of the parameters in our model.

How local elites affect the relationship between corruption and decentralization has not been formally studied in the literature. The paper by Bardhan and Mookherjee (2000) is quite close to this issue. They investigate the determinants of relative capture of local and national governments. However, in their model capture is produced on the political position of the government with respect to a public policy. Our model is more specific in terms of corruption. Here, elites do not influence explicitly the position of the government in a public policy but influence the allocation of public resources between public goods and private goods.

The rest of the chapter is organized as follows. Section 1 presents the empirical motivation. Section 2 develops the decentralized framework, and section 3 analyses its comparative statics. Section 4 extents the model to the case of centralization, and
section 5 studies how both the level of corruption and political accountability change when a federation is decentralized. Section 6 concludes. The appendix contains all proofs.

1. Empirical Motivation

As we have already discussed, most of the empirical evidence has supported the hypothesis that decentralization reduces corruption. For the purpose of this paper, there is still an open issue that has not been studied in the literature. It has to do with whether or not the dissuasive effect of decentralization on corruption is systematically present in both developing and developed countries. In this section, we present some empirical evidence on that.

In order to be consistent with the available evidence that supports the existence of a negative relationship between decentralization and corruption, we are going to use the same sample, data set, corruption indicator, and definition of decentralization used by Fisman and Gatti (2002) (hereafter F&G). The decentralization index corresponds to the ratio between the total expenditure of subnational (state and local) governments and the total spending by all government levels (state, local, and central). Correspondingly, the measure of corruption is the International Country Risk Guide (ICRG)’s corruption index. This index has been rescaled such that it lies between 0 and 1, where 0 indicates least corruption. The other variables included in the analysis are: per capita income, population, government size, and civil liberties. All the variables are averages for the period 1980-1995, except population which corresponds to a geometric average. The exact definition of the complete set of variables is given in the appendix.17

We work with the same basic econometric specification used by F&G, which assumes that corruption is a function of fiscal decentralization, per capita income, population, public sector’s size, and civil liberties. In order to test our hypothesis, we allow for a different effect of decentralization in developed and developing countries. The results are reported in table 1. All the standard deviations of the parameters are robustly estimated.

Columns 1 and 2 present the F&G’s estimation and our replica respectively. The discrepancies should be due to the data differences discussed in footnote 17. The rest of columns introduce the interactions between the dummy for developing countries and the decentralization index. Columns 3 through 5 differ in the GDP level taken into account to define a developing country. From the estimations, it follows that decentralization reduces corruption significantly in developing countries, but that effect is totally reversed in countries with low income. In other words, we cannot reject in any regression the hypothesis that the effect of decentralization over corruption is null in

17 There are three differences between the F&G’s data set and the one used here: (1) Population is taken from Heston, Summers and Aten, Penn World Table Version 6.1, whereas F&G’s source is World Development Bank Indicators. (2) For government size (total government expenditure divided by GDP) F&G use Barro (1991)’s information. When we use this source the country sample is reduced in a high proportion, and it does not coincide with the F&G’s sample. Thus, we use government size from Heston et al., which additionally includes information for the whole period 1980-1995. (3) The GDP information used by F&G is in 1985 price, and the one used here is 1996 price.
developing countries. This result supports the idea that decentralization has not been decisive in reducing corruption in these economies. It is also interesting to note that, when we allow for differences between developing and developed countries, the decentralization effect becomes stronger in the former set of countries. Depending on the developing country definition, it increases (in absolute terms) between 0.10 and 0.17 points. 18

Table 1
Corruption and Decentralization (LS)

<table>
<thead>
<tr>
<th></th>
<th>(1) F&amp;G estimation (a)</th>
<th>(2) replica</th>
<th>(3) (2) plus effect for developing countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decentralization Index (local and state share of total expenditure)</td>
<td>-0.42 (-2.97)***</td>
<td>-0.52 (-3.65)***</td>
<td>-0.67 (-3.51)***</td>
</tr>
<tr>
<td>(Developing country dummy) x (Decentralization Index) (b)</td>
<td>0.58 (2.45)***</td>
<td>0.36 (1.44) **</td>
<td>0.45 (2.26)***</td>
</tr>
<tr>
<td>Log of GDP</td>
<td>-0.08 (-2.38)***</td>
<td>-0.13 (-3.13)***</td>
<td>-0.09 (-2.05)***</td>
</tr>
<tr>
<td>Civil Liberties</td>
<td>0.02 (1.47)</td>
<td>0.02 (1.08)</td>
<td>0.02 (1.17)</td>
</tr>
<tr>
<td>Log of population</td>
<td>0.011 (0.85)</td>
<td>0.03 (2.06)**</td>
<td>0.02 (1.72)*</td>
</tr>
<tr>
<td>Government size</td>
<td>-1.07 (-3.33)***</td>
<td>-0.48 (-2.08)***</td>
<td>-0.46 (-1.95)***</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.69</td>
<td>0.66</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Test statistics for decentralization effect in developing countries equal to zero (P-value)

<table>
<thead>
<tr>
<th></th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of obs.</td>
<td>56</td>
<td>56</td>
</tr>
</tbody>
</table>

18 Regressions in table 1 do not include the dummy for developing countries without interaction. Notice, we are controlling by GDP. When we introduce this dummy instead of GDP, we get the same conclusions. Since there is a high correlation between GDP and this dummy, when we introduce both in the regressions, the effect of decentralization disappears in both developed and developing countries.

Estimations in table 1 may present some endogeneity problems. As F&G observe, corrupt central governments can affect the composition of public spending. Thus, by
keeping more rents in the centre, they can expand their rent extraction potential. As in F&G, we employ the legal origin of the country to instrument for the decentralization index. The idea is that Civil legal codes (like the French) encourage government centralization, whereas Common systems (like the British) have the opposite effect. Thus, our instrument is directly correlated with the centralization index, and it is expected to affect corruption only through this effect.

Table 2
Corruption and Decentralization (IV)

<table>
<thead>
<tr>
<th>ubo</th>
<th>-1,10</th>
<th>-1,03</th>
<th>-0,96</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decentralization Index (local and state share of total expenditure)</td>
<td>(-3,73)***</td>
<td>(-3,75)***</td>
<td>(-3,86)***</td>
</tr>
<tr>
<td>Developing country dummy x Decentralization Index</td>
<td>0,82</td>
<td>0,54</td>
<td>0,58</td>
</tr>
<tr>
<td>(Decentralization Index)</td>
<td>(3,31)***</td>
<td>(2,25)**</td>
<td>(2,98)***</td>
</tr>
<tr>
<td>Log of GDP</td>
<td>-0,04</td>
<td>-0,09</td>
<td>-0,09</td>
</tr>
<tr>
<td>(a)</td>
<td>(-0,99)</td>
<td>(-2,12)**</td>
<td>(-2,43)**</td>
</tr>
<tr>
<td>Civil Liberties</td>
<td>0,01</td>
<td>0,00</td>
<td>0,00</td>
</tr>
<tr>
<td>Log of population</td>
<td>0,04</td>
<td>0,03</td>
<td>0,03</td>
</tr>
<tr>
<td>(a)</td>
<td>(2,38)**</td>
<td>(2,08)**</td>
<td>(1,95)*</td>
</tr>
<tr>
<td>Government size</td>
<td>-0,41</td>
<td>-0,55</td>
<td>-0,53</td>
</tr>
<tr>
<td>(a)</td>
<td>(-1,59)</td>
<td>(-2,22)**</td>
<td>(-2,20)**</td>
</tr>
<tr>
<td>R-squared</td>
<td>0,66</td>
<td>0,65</td>
<td>0,68</td>
</tr>
<tr>
<td>P-value: Test statistics for decentralization effect in developing countries equals to zero</td>
<td>0,24</td>
<td>0,09</td>
<td>0,11</td>
</tr>
<tr>
<td>P-value: F-test statistics for joint significance of instruments in first stage regressions</td>
<td>0,00</td>
<td>0,00</td>
<td>0,00</td>
</tr>
<tr>
<td>P-value: Hausman test for consistency</td>
<td>0,71</td>
<td>0,97</td>
<td>0,99</td>
</tr>
<tr>
<td>Number of obs.</td>
<td>56</td>
<td>56</td>
<td>56</td>
</tr>
</tbody>
</table>

t-statistics are in parentheses. Standard errors are robustly estimated. Corruption index is rescaled to take values between 0 and 1, where 0 = least corruption. All regressions are estimated with a constant term. 

The IV estimations are reported in table 2. As before, we cannot reject the hypothesis that decentralization does not affect corruption in developing countries. Additionally, even though we cannot reject the hypothesis that the LS estimator is consistent, the effect of decentralization on corruption in developed countries estimated by IV is larger.

19 There are five classifications: (1) English common Law; (2) Socialist laws; (3) French Commercial Code; (4) German Commercial Code; (5) Scandinavian Commercial Code (See the appendix).
20 For an extended discussion about the validity of this instrument see F&G (2002) pp. 337.
than the respective effect estimated by LS. After correcting for endogeneity, our main conclusion remains the same, i.e. decentralization has an important effect in reducing corruption in developed countries, but this effect is not observed in less developed economies. How can this outcome be explained? We care on this issue in the rest of the paper.

2. The Decentralized Federation

We start by analysing an incomplete information model of political accountability and corruption in a single jurisdiction, i.e. when the federation is totally decentralized. The game is played by the jurisdiction’s voters, their respective incumbent and one local elite. There is also an organized local group interested in good governance that is called the accountability sector. This sector is not a formal player in the game, but just an information technology. In the game, the local elite demands corruption from the incumbent (in form of public resources) in order to obtain private gains. The resources allocated by the incumbent in this activity are identified as corruption.

**Incumbent**

At the beginning of the game, there is an incumbent who is (exogenously) in office. This incumbent has an amount of resources $\tau(y)$ that should be invested in a public good $z$ but might go to corruption $r$. $\tau$ are the locally generated taxes, which are assumed to be a positive function of the regional income $y$ (i.e. $\tau' > 0$). The unit price of the public good is normalized to be one. Thus, the incumbent budget constraint is $\tau = z + r$. All this variables are measure in per capita terms.

The incumbent can be of two types $t \in \{c, n\}$, where $c$ stands for “corrupt” and $n$ for “non-corrupt”, with $Pr(t = n) = 1$. An incumbent of type $n$ receives an infinitely negative utility from corruption; thus, he will always reject any corruption demand. An incumbent of type $c$ receives a linear positive utility from corruption. For any unit of resources that he allocates to corruption to serve the elite’s demand, he will ask for himself an exogenous share $\beta \in (0,1)$. We shall refer to $\beta$ as the incumbent’s share. It can be understood as the incumbent’s share arising from a bargaining game between the incumbent and the elite. Thus, when incumbent accepts a level of corruption $r$, he will receive $\beta r$ units of utility. The remaining $(1-\beta)r$ will go to the elite. No matters his type, an incumbent gets spoils (“ego-rents”) $S > 0$ if he stays in office.

**Accountability sector**

In our framework, political accountability is understood as the capacity of citizens to detect the incumbent in corruption and to remove him from the office. In the model there is an accountability sector that cares on improving political accountability in order to encourage good governance. You can think this sector is formed by civic associations, independent (non-influenced) media, and central government’s control offices. This sector is endowed with an amount of resources $A$ that will be totally invested in supervising the incumbent’s performance. These resources are also measure
in per capita terms. We allow these resources to depend positively on the per capita jurisdiction’s income \((y)\), then \(A=A(y)\) with \(A’ \> 0\).

The main task of the accountability sector is to send a signal to the citizens announcing whether the incumbent is corrupt or not. Thus, this sector can be understood as a technology that invests all its resources into accountability and makes an announcement about the incumbent’s type. In particular, it is not a formal player. We assume this sector is not influenced by any player in the game; thus, it will only transmit true information to the voters.

**Voters**

Let \(u(z)\) be the utility that voters receive from the public good supplied by the incumbent, with \(u\) strictly increasing. An incumbent of type \(n\) will provide a utility \(u(\tau)\) to the voters, whereas an incumbent of type \(c\) will deliver \(u(\tau - r)\). After observing the outcome, voters must decide whether they re-elect the incumbent or randomly elect a candidate from the opposition whose type will be \(n\) with probability \(\gamma\).

Nevertheless, we assume that voters are not able to observe their payoff directly at the time of elections but only a signal from the accountability sector. If the incumbent’s type is \(n\), the accountability group will receive and send a signal \(s=n\). However, if the incumbent is corrupt, they will receive and send a signal \(s=c\) with probability \(\delta \in [0,1]\), and \(s=n\) with probability \(1-\delta\). With this information, the citizens vote in order to maximize their expected utility. The probability of detecting the incumbent in corruption \((\delta)\) will be established endogenously in the model. We will define it formally later on.

**Elite**

The elite demands corruption \(r\) from the jurisdiction’s incumbent in order to produce some personal benefits. One can think on some specific project that affects directly and positively the elite’s benefits: licenses, public contracts, market interventions, etc.\(^{21}\) When the incumbent accepts the corruption demand, the elite receives the fraction \(1-\beta\) of \(r\). With this amount of resources, it is going to produce \(Q((1-\beta)r)\) benefits, where \(Q’ > 0\).

We assume that the elite can influence the political process by affecting the probability of detecting the incumbent in corruption \((\delta)\). It can do it through two mechanisms. First, it can invest some resources \(H\) in order to hold up the task of the accountability sector. In this way the elite reduces \(\delta\). For instance, these resources may be spent on bribing other involved public workers, falsifying some documents, altering the account books, and so on. Second, the elite has economic control over a proportion \(\theta \in [0,1/2]\) of citizens, which makes it more difficult to detect the incumbent in corruption activities. One can think that the elite has some monopsonistic power in the jurisdiction’s labor

\(^{21}\) Notice that in some of these cases corruption may also affect the citizens’ welfare positively. However, since our analysis is not about welfare but about corruption, we do not care on these external effects.
market and so it can induce these people to cover any signal of corruption. If this is the case, the resources invested by the accountability sector will be less productive as $\theta$ increases. We refer to $\theta$ as the elite’s power.

To simplify, we assume the elite does not face any cost when it demands corruption to the incumbent. This implies that, if the incumbent’s type is $n$, the elite will not face any penalty if it insinuates a corruption agreement to the former. Assuming a linear $Q(.)$, the elite’s expected payoff will be $\pi = (1 - \gamma)(1 - \beta)r - H$.

Detection probability and accountability level

Up to now there are three variables affecting the detection probability ($\delta$): $A$, which is a function of $y$ and has a positive effect on it; and $H$ and $\theta$, which affect $\delta$ negatively. In addition to these three effects, we shall allow for a moral hazard component. This component takes into account the fact that the more rent is allocated to corruption as a proportion of the local taxes, the easier it is for the accountability sector to find out about corruption. To simplify the algebra, while preserving sufficient richness of structure, we will assume:

$$\delta = (1 - \theta)\frac{A}{A + H} \Psi(r/\tau)$$

where $\Psi(0) = 0$, $\Psi(1) = 1$, $\Psi'(.) > 0$, $\Psi''(.) > 0$, $\Psi'(0) < \infty$, $\Psi''(0) = 0$, and $\lim_{r \to \tau} \Psi'(.) = \infty$. The four first assumptions ensure $\Psi$ belongs to the interval $[0,1]$, and both the moral hazard probability and its marginal rate strictly increase in $r/\tau$. The remaining are technical assumptions. Keep in mind that $\tau$ is a function of $y$, so ultimately $\Psi(.)$ is also a function of $y$. We call this component the moral hazard probability.\(^{22}\)

As we mentioned earlier, political accountability in our framework is understood as the ability of citizens to detect the incumbent in corruption and remove her from office. One can be tempted to relate this concept directly to the detection probability. However, $\delta$ may not represent this concept accurately because of the moral hazard component. Consider the following situation. Imagine there is a variable that affects the level of corruption negatively and so $\Psi(.)$, but, at the same time, affects $(1 - \theta)(A/(A + H))$ (the other part of $\delta$) positively. When the first effect dominates the second effect, the final result is a reduction in $\delta$.\(^{23}\) If we do not make any distinction between the level of accountability and the detection probability, we conclude that the former also decreases. However, since in the new situation either elite has less influence on $\delta$ (via $H$ or $\theta$) or

\(^{22}\) Notice that, if $r = \tau$, $\delta = (1 - \theta)A/(A + H) < 1$. However, as we shall see below, although $\delta$ is not equal to one when the incumbent spends the whole taxes in corruption, with this form of the detection probability we are able to obtain interior solutions for corruption. We could keep equation 1 for $r < \tau$ and redefine $\delta = 1$ if $r = \tau$. It does not add any new to our results.

\(^{23}\) Later on, we shall formally see that the situation described in this example always holds.
the accountability sector is more effective or both (it is so because \((1-\theta)(A/(A+H))\) has increased), this conclusion is not right at all.

Thus, in order to get what we call the degree of political accountability \((\delta_m)\), we remove the moral hazard probability from \(\delta\):

\[
\delta_m = (1-\theta) \frac{A}{A+H} \tag{2}
\]

Game and Equilibrium

In order to keep the framework as simple as possible, we assume that an incumbent type \(c\) does not extract rents without the elite participation. This allows us to concentrate on the corruption generated from the elite intervention. With the accountability group investing \(A\) in accountability, the timing of the game is as follows:

1. Given \(\theta\), the elite offers a contract \((H,r)\) to the incumbent.
2. The incumbent decides whether to accept \((Y)\) or reject \((N)\) the contract.
3. The citizens observe the accountability sector signal and vote for the candidate (the incumbent or another candidate of unknown type) that maximizes their expected utility.

The equilibrium of the game has two components. The first one is the game between the elite and the incumbent, which determines the levels of both corruption and political accountability. The second is the equilibrium in the election game, which establishes whether the incumbent is re-elected or not. To model the equilibrium in the corruption market, we focus on perfect Bayesian equilibrium restricted to pure-strategy equilibria in which citizens always vote for their preferred candidate.

The complete description of the equilibrium strategies and proofs of the following propositions can be found in the appendix. Here we state the equilibrium conditions when there is a positive level of corruption. We now introduce subscript \(j\) to denote jurisdictions and superscript \(d\) to denote outcomes and parameters under decentralization.

**Proposition 1.** When incumbent is of type \(c\), at equilibrium the incumbent always accepts the contract (with positive corruption) offered by the elite. The equilibrium contract \(\{\hat{H}^d_j, \hat{r}^d_j\}\) satisfies the following conditions:

\[
(1-\beta^d) = \frac{(1-\theta^d)}{\beta^d} A(\cdot) S^d \left( \frac{\Psi(\cdot) \hat{r}^d_j}{\tau(\cdot) - \Psi(\cdot)} \right) \tag{3}
\]

\[
\hat{H}^d_j = A(\cdot) \frac{(1-\theta^d)}{\beta^d} \left( \frac{\Psi(\cdot)}{\hat{r}^d_j} S^d - I \right) \tag{4}
\]
where $A(\cdot)$ and $\tau(\cdot)$ depend on $y_j$, and $\Psi(\cdot)$ is evaluated at $r_j^d/\tau(\cdot)$. Equation 3 implicitly sets the equilibrium level of corruption in jurisdiction $j$ ($r_j^d$) under decentralization. This happens at the point where the elite’s marginal income of corruption $1 - \beta^d$ equals the elite’s marginal cost of corruption. Equation 4 sets the minimum level of $H_j^d$ required by the incumbent to accept $r_j^d$. At equilibrium, the public good supply, the accountability level, and the detection probability are given respectively by:

$$\hat{z}_j = \tau(\cdot) - \hat{r}_j^d$$  
$$\hat{\delta}_j^d = \left(1 - \theta^d\right)\frac{A(\cdot)}{A(\cdot) + H_j^d}$$  
$$\hat{\delta}_j^d = \hat{\delta}_j^d \Psi\left(\frac{r_j^d}{\tau(\cdot)}\right)$$

### 3. Analysis

From now on, we will focus on the sort of equilibria with positive corruption (described in proposition 1), i.e. those in which the incumbent is of type $c$. This way we will be able to analyze how the level of corruption, the detection probability, and the level of accountability are affected when the parameters of the model change. At equilibrium, each of these three outcomes depends simultaneously on the jurisdiction income ($y_j$), the offices spoils ($S^d$), the elite’s power ($\theta^d$), and the incumbent’s share ($\beta^d$). To save notation, we drop again the subscript $j$ and superscript $d$.

**Corruption**

First of all, we analyze the level of corruption. Notice that equation 3 sets an implicit function of corruption in terms of $y$, $S$, $\theta$, and $\beta$. Proposition 2 states the effect that each of these factors has on the level of corruption.

**Proposition 2.** Assume there is positive corruption in the jurisdiction (i.e. equilibrium is described by proposition 1), then the level of corruption ($r$):

- a) Decreases as the jurisdiction income ($y$) increases if and only if 
  $$\left(1/\eta_1 \right) r^*/\tau < A'/A,$$
  where $\eta_1 = \tau^2 \frac{(1 - \beta)\beta}{\left(1 - \theta\right) A S \Psi''''(\cdot)} \in (0,1)$. Otherwise, corruption increases.

- b) Decreases as the office spoils increase ($S$).

- c) Increases as the elite’s power increases ($\theta$).

- d) Increases as the incumbent’s share ($\beta$) increases if $\beta < \frac{1}{2}$, and decreases as $\beta$ increases if $\beta > \frac{1}{2}$.

The jurisdiction’s income affects the level of corruption through two channels. On the one hand, the amount of resources invested by the accountability sector in its task also goes up as $y$ increases. This influences corruption negatively via the increase in the
elite’s marginal cost of corruption. On the other hand, the locally generated taxes ($\tau$) grow as $y$ increases. Thus, for the same level of corruption, it produces a decrease in the ratio $\tau/\tau$, which reduces the probability of detection (via the moral hazard probability) and encourages the demand for corruption. Since both things occur whereas the elite’s marginal income keeps constant, the final effect on the level of corruption will depend on which of the two effects on the marginal cost dominates the other. When the marginal accountability rate ($A'/A$) increases proportionally more than the marginal tax rate ($\tau'/\tau$) - exactly in proportion $1/\eta_1$ - the corruption marginal cost increases, and so the level of corruption decreases.

The result in proposition 2(a) has an important implication for the accountability sector success. This says that if $y$ increases, the resources invested in accountability must grow relatively faster than the generated taxes in order to get a reduction in the level of corruption. This result can also be used to interpret the effect of any central (or between-jurisdiction) transfer in our framework. When there are transfers (or grants) from the national to the jurisdictional level, the incumbent’s budget is positively affected whereas the accountability sector’s resources remain the same. In terms of our framework, this implies $\tau'/\tau > 0$, and $A'/A = 0$. From proposition 2(a), it follows that, under these circumstances, the level of corruption increases. Thus, in order to assure a better allocation of these transfers, central government should use part of these resources to invest directly in improving accountability (i.e. investing in $A$) in order to assure a better allocation of these resources.

Some authors have previously claimed that a high level of central transferences incentives corruption and affects the fiscal performance in jurisdictions negatively. The explanation they have given to this effect is that local voters and local politicians receive fiscal or political benefits from grant programs without internalizing their full cost (Rodden (2002)). Our model exposes an alternative explanation for this phenomenon, i.e. since transfers only increase the potential resources to be invested in corruption but do not affect the resources invested in accountability, they encourage corruption.

Results (b) and (c) in proposition 1 are quite intuitive. In both cases, the marginal benefit of corruption keeps constant, but the marginal cost changes. When the office spoils $S$ go up, the marginal cost of corruption increases, and then, corruption decreases. Alternatively, a rise in the elite’s power ($\theta$) makes the accountability sector less efficient, reduces the marginal cost of corruption, and, as a result, corruption increases.

Statement (d) says that if $\beta$ is small enough and it increases, then corruption goes up. It is direct that the elite’s marginal income of corruption decreases as $\beta$ increases. However, there is also a reduction in the marginal cost of corruption because the elite must now invest less resources in affecting the level of accountability in order to incentive the incumbent’s participation. If $\beta$ is smaller than $\frac{1}{2}$, then it will still be
profitable for the elite to demand more corruption. Actually, this case is the most interesting, as it may properly reflect what occurs in the real world\textsuperscript{24}.

It is also interesting to see how the public good supply is affected in all these cases. From equation 5, it immediately follows that the public good supply increases whenever corruption decreases. The opposite is true when corruption increases as a result of a change in $\theta$, $S$, or/and $\beta$. Nonetheless, when the increment in corruption is due to an increase in the jurisdictions income, the public good supply may go up or down depending on how much both the accountability marginal rate and the taxes marginal rate change.

Detection probability

Now consider the detection probability. Proposition 3 states the results for the comparative statics.

\textbf{Proposition 3}. Assume there is positive corruption in the jurisdiction (i.e. equilibrium is described by proposition 1). The probability of detecting the incumbent in corruption ($\delta$):

a) Increases as the jurisdiction income ($y$) increases if and only if $(1/\eta_1)\tau'/\tau > A'/A$ (i.e. as the level of corruption increases); otherwise, it decreases.

b) Increases as the office spoils ($S$) decrease (i.e. as the level of corruption increases).

c) Increases as elite’s power ($\theta$) increases (i.e. as the level of corruption increases).

d) Increases as the incumbent’s share ($\beta$) increases if and only if $\Phi > -\beta(1-2\beta)$,

\[\Phi = (1-\theta) \frac{AS}{\tau^2} \Psi''(\cdot) - 2(1-\beta)\beta > 0.\]

Notice that if $\beta<\frac{1}{2}$, this condition always holds (i.e. it increases as the level of corruption increases).

We must be careful in the interpretation of results in proposition 3. Essentially, all these results say that the detection probability increases whenever the level of corruption increases and vice versa. This is so because the moral hazard component always dominates the total effect over the detection probability. Thus, when $r$ increases the moral hazard component goes up and so the detection probability\textsuperscript{25}.

A direct way to see that the moral hazard probability dominates the final effect on $\delta$ is through the incumbent’s participation constraint. At equilibrium, this constraint implies $\hat{\delta} = \beta \hat{r}/S$ (See proof of proposition 3). Hence, keeping constant $\beta$ and $S$, the detection probability increases whenever the level of corruption increases. When the changes in corruption stem from a variation in $S$, the final effect is strengthened by it. When it

\textsuperscript{24} Computations for Latin America show that the rate that officials ask for public contracts runs from 8% to 25%.

\textsuperscript{25} This assertion is true when the rise in $r$ is not due to an increment in $y$ (if so, unambiguously $r/\tau$ increases). However, when $r$ increases as a result of an increment in $y$, the ratio $r/\tau$ does not necessarily increase, and the final effect on the moral hazard probability is ambiguous. As proposition 3 shows, even in this case, the total effect on $\delta$ is also dominated by the change in $r$. 

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stems from a variation in β, the final effect will depend, among other things, on the
type of β (the way it is described in proposition 3(d)).

Thus, as we have already discussed in section 2, it is better if we focus on an
appropriate measure of political accountability. By doing so, we can see whether or not
the voters are able to detect a corrupt incumbent not via the level of corruption but via
the efficiency of the accountability sector.

**Political Accountability**

Equation 2 shows that beside the direct effect of θ and y, the accountability level
depends crucially on the elite’s investment $H$. The comparative statics’ results are stated
in proposition 4.

**Proposition 4.** Assume there is positive corruption in the jurisdiction (i.e. equilibrium is
described by proposition 1). The level of political accountability ($\delta_a$):

a) Increases as the jurisdiction income ($y$) increases if and only if

$$
(1 / \eta_1 - \eta_2) \tau / \tau < A' / A,
$$

where

$$
\eta_2 = \frac{\Phi \tau}{\Psi''(1 - \beta) \beta \eta_1} > 0.
$$

Otherwise, it decreases.

b) Can increase or decrease as the office spoils ($S$) increase. Only when the effect of $S
over $r$ is large enough it increases.

c) Decreases as the elite’s power ($\theta$) increases.

d) Increases as the incumbent’s share ($\beta$) increases if $\beta > \frac{1}{2}$. If $\beta < \frac{1}{2}$, it increases only if

the effect of $\beta$ over $r$ is small enough.

Statement (a) says that when the jurisdiction’s income increases, the marginal
accountability rate must grow at least $1 / \eta_1 - \eta_2$ times the marginal tax rate in order to
observe an increment in accountability. We cannot infer the sign of this rate of growth
(see appendix), but since $\eta_2 > 0$, it follows that $1 / \eta_1 > (1 / \eta_1 - \eta_2)$. Then, the condition
in proposition 4(a) is less demanding than the required condition to have a decrease in
the corruption level (proposition 2(a)). Thus, an increment in accountability is not
enough to observe a reduction in corruption.

For a clearer intuition of the remaining results, let us analyze the elite’s contribution to
decrease the efficiency of the accountability sector ($H$). From equation 4, there are two
forces affecting $H$ as either $S$, or $\theta$, or $\beta$ change. One is the direct effect observed in 4,
and the other is the effect through $r$ - more specifically through the term $\Psi(\cdot)/r$. Notice
that this ratio can be interpreted as the moral hazard probability per unit of corruption. It
is easy to show that, keeping constant $y$ and so $\tau$, $\Psi(\cdot)/r$ strictly increases in $r$. Thus,
when this ratio goes up (i.e. $r$ increases) the elite will be willing to raise $H$ in order to
compensate the increment in the detection probability. Thus, the final effect over $H$ will
depend on the combination of the direct effect and the effect through $\Psi(\cdot)/r$.

---

For instance, when the level of accountability increases in a small proportion and the detection
probability is still dominated by the moral hazard component, we can observe more corruption.
Through the direct effect $H$ increases as $S$ increases. In other words, when the office spoils are large, the elite must invest more in affecting accountability in order to get the same level of corruption. However, since office spoils affect the level of corruption negatively, then $\Psi(.)/r$, and so $H$, decrease as $S$ goes up. To observe a reduction in $H$, and so an increment in the accountability level, it is necessary that the latter effect dominates the former. This implies a large enough impact of $S$ over $r$ (the appendix states the formal condition).

Even though the adjustment in $H$ is ambiguous when the elite’s power ($\theta$) goes up ($H$ decreases via the direct effect but increases since $r$ increases), the direct effect is enough to reduce the accountability level. This result depends crucially on the assumption that the accountability level depends directly and negatively on $\theta$.

Finally, consider the effect of the incumbent’s share ($\beta$) on $\delta_a$. When $\beta$ affects corruption negatively (i.e. $\beta>\frac{1}{2}$), the two forces reduce $H$, and thus, the level of accountability increases. However, when $\beta<\frac{1}{2}$, its effect over $H$ is ambiguous ($H$ decreases via the direct effect but increases since $r$ increases). Hence, in order to have an improvement in accountability, it is required that corruption does not increase excessively (the appendix states the formal condition).

Summing up, we have found the following results. First, when the jurisdiction’s income increases, the level of corruption goes down and the accountability level increases if the accountability sector grows sufficiently above the locally generated taxes. Second, the office spoils affect the level of corruption negatively, but in order to affect the level of accountability positively a high enough impact over it is required. Third, the elite’s power affects corruption positively and the accountability level negatively. Finally, when $\beta<\frac{1}{2}$ - which actually is the most interesting case - an increment in the incumbent’s share increases the level of corruption but has an ambiguous effect on political accountability.

4. The Centralized Federation

So far, the model presented in section 2 describes how both corruption and political accountability are determined in each jurisdiction in a decentralized federation. In this section we consider the case in which the federation is totally centralized. In order to do so, we use exactly the same framework we introduced in section 2. The main difference is that under centralization there is only one central incumbent in the federation who receives corruption demands from $J>1$ elites, one in each jurisdiction (where $J$ is the number of jurisdictions). From now on, we use superscript $c$ to denote parameters and outcomes under centralization.

There are some issues we must take into account in this new framework. First, we are characterizing the national per capita level of corruption $r^c$, i.e. the amount of resources allocated in corruption as proportion of the total population in the federation. In particular, $r^c = \sum_{j=1}^{J} r^c_j$, where $r^c_j$ is the amount of resources allocated in corruption in
each jurisdiction $j$ under centralization as proportion of the total population in the federation. $H^c = \sum_{j=1}^{J} H^c_j$, is defined in a similar way.

Second, under centralization the relevant parameters are those at the federal level. For instance, the power of the elites is their total power at the federal level. We define $\theta^c_j$ as the percentage of people that elite $j$ controls in its jurisdiction as proportion of the federal population. Thus, the total power of the elites at the federal level is $\theta^c = \sum_{j=1}^{J} \theta^c_j$.

The other relevant parameters are the national (federal) per capita income $y^c$, the central (federal) office spoils $S^c$, and the central incumbent’s share in corruption $\beta^c$. Keeping in mind these changes, we define the probability of detecting the central incumbent in a corruption agreement with the elites in the following way:

$$\delta^c = \left(1 - \theta^c\right) \frac{A(.)}{A(.) + H^c} \Phi\left(r^c / \tau(.)\right)$$

where $A(.)$ and $\tau(.)$ depend on $y^c$.

The last issue has to do with the accountability sector. When the system moves from centralization to decentralization, we are implicitly assuming that the accountability sector is decentralized at the same time. In other words, we are imposing that under centralization there is one national accountability sector which supervises the central incumbent, whereas under decentralization there is one group in each jurisdiction carrying out the same task. In order to avoid any extra effect, we keep the characteristics of the accountability sector unchanged at the two levels, i.e. both the jurisdictional and the national sector will use the same technology.$^{27}$

The timing of the game is similar to the decentralization case:

1. Each elite $j$ simultaneously offers a contract $\left(H^c_j, r^c_j\right)$ to the central incumbent.
2. The central incumbent decides if she accepts ($Y$) or rejects ($N$) each contract.
3. The citizens observe the accountability sector’s signal and vote for the incumbent or for another candidate of unknown type.

The equilibrium of this game is presented in the appendix. We show that the equilibrium characterization of the centralized case is exactly the same as the decentralized case. In particular, when incumbent is of type $c$, the national level of per capita corruption is completely defined by:

---

$^{27}$ This implies that the centralized and decentralized accountability group do not differ in its productivity. In our framework, one can easily introduce a parameter to take into account differences in the accountability sector’s productivity between the centralized and the decentralized system. In addition, one may introduce some across-jurisdictions positive externalities between the accountability sectors. However, it complicates the analysis and does not add any interesting result for our purpose.
\[ (1 - \beta^c) = \frac{(1 - \theta^c)AS^c}{\beta^c} \left( \frac{\Psi(.)\hat{\tau}^c}{\hat{\tau}^c} - \frac{\Psi(.)}{\hat{\tau}^c} \right) \]  

where \( \Psi(.) \) is evaluated at \( \hat{\tau}^c \). Similarly, \( \hat{H}^d \) and \( \hat{\delta}^d \) can be written using equations 4 and 6 (see the appendix).

It is important to notice that in our decentralized game, the elites neither compete among them for the public resources nor collude in a single national elite. We do it in order to keep our framework as simple as possible. A way to introduce this kind of behaviors is through the bargaining power of the incumbent (\( \beta \)). If the elites compete for the public resources, one can expect an increment in \( \beta \) as the system moves from centralization to decentralization. If they collude in a single elite, one can expect a reduction in this parameter. Actually, we are going to discuss the effect of these changes latter on.

5. Centralization versus Decentralization

The aim of this section is to evaluate how both corruption and political accountability change when a federation moves from a centralized to a decentralized system. From the discussion in the previous section, the only difference between the two systems is the respective set of parameters \( \{y, S, \theta, \beta\} \). Thus, we can use the results in propositions 2 and 4 to analyze the expected change in the level of corruption and accountability when a federation is decentralized. Our analysis compares the national outcomes under centralization against the jurisdiction \( j \)'s outcomes under decentralization. Recovering national outcomes under decentralization is a question of average.

Elite’s power

When a federation moves from a centralized to a decentralized system, the relevant parameter of the elite’s power is not the total power of the elites at the federal level (\( \theta^c \)) but the power of each elite at the jurisdictional level (\( \theta^d \)). Notice that \( \theta^d \) can be higher, equal, or smaller than \( \theta^c \). Thus, if \( \theta^d > \theta^c \), the level of per capita corruption (political accountability) in jurisdiction \( j \) under decentralization will be larger (smaller) than the national level of per capita corruption (political accountability) under centralization. The opposite will happen if \( \theta^d < \theta^c \).

The final effect of decentralization on the national level of corruption and political accountability depends critically on both the distribution of powers (\( \theta^d \)) across the jurisdictions and the initial level of corruption. Table 3 presents an example for a federation formed by 3 jurisdictions, each of them with population equals 10. In each of the three cases, the respective elite of the jurisdiction controls a different proportion of
people, whereas at the national level they always control the same percentage of citizens (30%).

In case I, as the federation moves from centralization to decentralization, corruption increases in jurisdictions 1 and 2 (0.4>0.3) and decreases in jurisdiction 3 (the opposite happens with the level of accountability). In case II, corruption increases in 1, decreases in 3 and keeps the same in 2. Finally, in case III the national level of corruption does not change when the federation is decentralized. This illustrates the fact that the distribution of powers across jurisdictions matters. The second issue has to do with the aggregation of these changes. Since the effect of the elite’s power on both corruption and accountability is not constant and depends on the initial level of corruption, it is difficult to predict the final effect of decentralization on the national level of corruption and political accountability in cases I and II.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Example: Distribution of Elites’ Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controlled Pop.</td>
<td>Elite’s power</td>
</tr>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>National</td>
<td>30</td>
</tr>
<tr>
<td>j=1</td>
<td>10</td>
</tr>
<tr>
<td>j=2</td>
<td>10</td>
</tr>
<tr>
<td>j=3</td>
<td>10</td>
</tr>
</tbody>
</table>

Thus, we conclude that the final effect of decentralization – via the elites’ power - on corruption is ambiguous. Nevertheless, since the elites play an important role in many jurisdictions in developing countries, one can expect a significant increment in the level of corruption in these municipalities after decentralization.

**Per capita income**

To see the effect of decentralization - via per capita income - on corruption, we must compare the national per capita income (yc) against the per capita income of each jurisdiction j. This analysis makes sense if the federation has an important dispersion of income across jurisdictions. Such is the case in most developing economies.

The per capita income in jurisdiction j (yj) may be higher, equal, or smaller than the national per capita income (yc). Additionally, the change in per capita income when a federation moves from centralization to decentralization may affect both the marginal accountability rate (A′/A) and the marginal tax rate (τ′/τ) in a different proportion. Thus, in order to understand the effect of decentralization on both corruption and accountability, we need to consider all the possible situations that can arise. Table 4

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28 See the appendix. Assuming Ψ''''(.) > 0, then \( \frac{\partial^2 A}{\partial \theta^2} > 0 \). However, the sign of \( \frac{\partial^2 \delta}{\partial \theta^2} \) is ambiguous.

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summarizes all these situations for the corruption outcome. For obvious reasons, the case in which \( y_j = y^c \) is not reported.

### Table 4

**Effect of Decentralization -Via Per Capita Income- on Corruption**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Parameter Conditions</th>
<th>( r ) Effect</th>
<th>Accountability Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_j &gt; y^c )</td>
<td>( A'/A &gt; I/\eta, \tau'/\tau )</td>
<td>Decreases</td>
<td>Rich regions</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>with relative strong</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>accountability sector</td>
</tr>
<tr>
<td>( y_j &lt; y^c )</td>
<td>( A'/A &lt; I/\eta, \tau'/\tau )</td>
<td>Increases</td>
<td>Poor regions</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>with relative weak</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>accountability sector</td>
</tr>
<tr>
<td></td>
<td>c) ( A'/A &gt; I/\eta, \tau'/\tau )</td>
<td>Decreases</td>
<td>Poor regions</td>
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<td></td>
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<td>with relative strong</td>
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<td>accountability sector</td>
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<tr>
<td></td>
<td>d) ( A'/A &lt; I/\eta, \tau'/\tau )</td>
<td>Increases</td>
<td>Poor regions</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>with relative weak</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>accountability sector</td>
</tr>
</tbody>
</table>

When the per capita income in jurisdiction \( j \) is larger than the national per capita income, then \( A(y_j) > A(y^c) \) and \( r(y_j) > r(y^c) \). Nevertheless, both the resources of the accountability sector and the locally generated taxes can be affected in different proportions. If \( A'/A > I/\eta, \tau'/\tau \), i.e. the jurisdiction \( j \) is a rich region with a relative strong accountability sector (case (a) in table 4), the level of corruption in this jurisdiction under decentralization will be smaller than the national level of corruption under centralization. The opposite will happen if \( A'/A < I/\eta, \tau'/\tau \), i.e. the jurisdiction \( j \) is a rich region with a relative weak accountability sector (case (b) in table 4). Like table 4 shows, a similar analysis can be done when \( y_j < y^c \) (cases (c) and (d)). Similarly, using the result in proposition 4(c) one can analyses the effect of decentralization on political accountability.

Once again, the final effect of decentralization (via per capita income) on the national level of corruption is ambiguous. It depends on how the jurisdictions are distributed among the four cases characterized in table 4. However, it is not difficult to think that most of the jurisdictions in developing countries can be classified in cases (b) and (d) i.e. regions with weak accountability sectors. Thus, our prediction is that the level of corruption (accountability) will increase (decrease) in an important proportion of jurisdictions and only decrease (increase) in a few rich jurisdictions with relative strong accountability sector.

Another issue to take into account is the use of grants or transfers under decentralization. In the presence of significant between-jurisdiction income inequalities, the design of central or between-region transfers plays an important role. As we mentioned in section 2, transfers affect the level of corruption positively. Since these inequalities across regions are relatively larger in developing countries than they are in developed countries, and because most of the developing countries use these transfers intensively to finance the poorest (the majority of) jurisdictions, the final effect of decentralization on the overall corruption may be positive. Thus, to avoid a re-escalation
of corruption through the transfer system, its design must involve transfers to the accountability sector.

Office Spoils

Now consider the office spoils $S$. Since the effect of spoils on political accountability is ambiguous, we concentrate only in their effect on corruption. Office spoils in a jurisdiction ($S^d$) are surely smaller than national ones ($S^c$) everywhere around the world. From our results, it implies that, when the system moves from centralization to decentralization, there must be an increment in the level of corruption in every jurisdiction and so in the national level of corruption.

A priory, there is not a significant difference between a developing and a developed country in this effect. Nevertheless, it is important to note that most of the developing countries have moved from a centralized to a decentralized system by assigning an important amount of decisions to small municipalities. It is different to the case of most developed countries, where states play an important role in the policymaking process. Since the office spoils in small municipalities are farther from the national ones than the respective spoils in states, corruption is expected to increase more in those countries in which decentralization focuses in small jurisdictions than in those in which it focuses in states. Thus, the decentralization design is an issue that should be taken into account.

Incumbent’s share

Since the effect of $\beta$ on the level of political accountability is ambiguous, we concentrate on its effect on corruption. We do not have any explicit expectation about the change of $\beta$ as the economy moves from a centralized to a decentralized system. Some authors (e.g. Tanzi, 1995) have claimed that rewards to local politicians are relatively smaller than those received by central bureaucrats, i.e. $\beta^c > \beta^d$. If this is the case, the common perception is that corruption under decentralization must be larger than corruption under centralization. This should be so because the local governments are cheaper than the central government.

Assume $\beta^c > \beta^d$. We have shown that if $\beta$ has a rational value (i.e. $\beta<\frac{1}{2}$) and it decreases, corruption is expected to be smaller in every jurisdiction under decentralization. In other words, if $\beta^c > \beta^d$, then decentralization reduces the level of corruption in the federation. This result is opposite to the informal prediction mentioned above. The reason is that we are taking into the account the strategic behaviour of the elite. As the incumbent’s share decreases, the elite’s marginal income of corruption ($1-\beta$) increases. If $\beta$ is small enough, the elite has to invest less resources ($H$) to persuade the incumbent, and then the level of corruption decreases to recover the equilibrium condition in equation 4.

Therefore, we must take into account the complete set of changes described above in order to understand the final effect of decentralization on nationwide corruption. As we have shown, it is quite difficult to make a clear prediction about it. Nevertheless, the evidence presented in section 1 suggests that decentralization has not been decisive in
reducing the level of corruption in developing countries. This outcome can be explained by the opposite effects that our model predicts.

6. Conclusions

There is a partial agreement in both theoretical and empirical literature that decentralization reduces the level of corruption. We have shown in this paper that this is the case in developed countries where the mechanisms that allow decentralization to incentive good governance work properly. However, because such mechanisms usually fail in developing countries, it is not more the case for these economies.

The power of local elites in these countries is one of the aspects that reduces political accountability and encourages bad governance. Thus, the implementation of policies that affect this power negatively can be useful in order to reduce corruption. For instance, if there is an important degree of monopsony in the labor market, it may be required to promote industrial or agricultural competition and to foster between-jurisdiction migration.

Although we emphasize the negative impact that local elites have on both the degree of political accountability and the level of corruption, there are other factors which have not allowed decentralization to work appropriately in developing countries. For instance, the existence of relative poor and rich regions with a weak accountability sector can explain this issue. Another important aspect is the high between jurisdiction income inequality in these countries, which intensify the use of transfers in order to finance the poorest regions. We have shown that grants affect corruption positively if the transfer system does not involve any improvement in the productivity of the accountability sector. Our theoretical results suggest that, in order to avoid corruption, any increase in the amount of transfers must be accompanied by a rise - at least as large as the rise in the transfers - in the amount of resources allocated to political accountability.

Finally, most developing countries have moved from a centralized system to a decentralized system that assigns an important amount of decisions to small municipalities. In terms of our model, this implies a dramatic reduction in the office spoils which encourages corruption. In order to take advantage of the potential benefits of decentralization while persuading politicians against corruption, it may be useful to empower states’ governments in which the office spoils are not too far from the central ones.

Appendix

In propositions 1 through 4 we omit the subscript $j$ and the superscript $c$.

Proof of proposition 1. The equilibrium strategies are:

1. The elite offers the incumbent a contract $(\hat{H}, \hat{r})$ to that satisfies the following conditions:
\[(1 - \beta) = \frac{(1 - \theta) AS}{\beta} \left( \frac{\Psi'(r/\tau) - \Psi(\cdot)}{r^2} \right) \quad (A1)\]

\[\hat{H} = A \left( \frac{(1 - \theta) \Psi(\cdot)}{\beta} \frac{S - 1}{r} \right) \quad (A2)\]

2. An incumbent of type \( n \) rejects the contract, and an incumbent of type \( c \) accepts it.

3. Voters re-elect the incumbent if \( s = n \); otherwise they do not re-elect the incumbent and vote for a challenger who is non-corrupt with probability \( \gamma \).

Now, we prove that the previous strategies characterize any pure-strategy perfect Bayesian equilibrium of the game. First, consider the voters’ behaviour whose strategies are conditioned to the signal \( s \). The voters’ beliefs are given by:

\[
\Pr(t = n) = \begin{cases} 
\gamma & \text{if } s = n \\
\gamma + (1 - \gamma)(1 - \delta) & \text{if } s = c \\
0 & \text{Otherwise}
\end{cases}
\quad (A3)
\]

To remove the incumbent from the office when \( s = c \) is a strictly dominant strategy. Now let’s assume \( s = n \). If in this case voters do not re-elect the incumbent and choose a challenger, the latter will be non-corrupt with probability \( \gamma \). Nevertheless, since \( \gamma/(\gamma + (1 - \gamma)(1 - \delta)) \geq \gamma \) for any \( \delta \in [0,1] \), then to re-elect the incumbent is a strictly dominant strategy.

Now consider the incumbent’s strategy. Since an incumbent of type \( n \) receives an infinitively negative utility from corruption she will always reject (N) any offer of the elite with positive corruption. Incumbent’s type \( c \) payoffs are \( V(c,N) = S \) if she rejects the elite’s contract, and \( V(c,Y) = (1 - \delta)(S + \beta r) + \delta \beta r \) if she accepts it. Thus, she will accept any contract in which \( V(c,Y) \geq V(c,N) \). This implies \( \delta \leq \beta r/S \), which actually is the incumbent’s participation constraint.

The elite maximizes its payoff \( \pi = (1 - \gamma)(1 - \beta)r - H \), subject to \( \delta \leq \beta r/S \) (Incumbent participation constraint), \( H \geq 0 \), and \( 0 \leq r \leq \tau \). The first constraint implies \( H \geq A \left( \frac{(1 - \theta) \Psi(r/\tau)}{\beta} \frac{S - 1}{r} \right) \). Since \( \pi \) strictly decreases in \( H \), then the incumbent will choose \( H = A \left( \frac{(1 - \theta) \Psi(r/\tau)}{\beta} \frac{S - 1}{r} \right) \). Notice this implies that at equilibrium the incumbent’s participation constraint holds with equality. We assume the parameters of the model are such that \( H > 0 \), for that it is required that \( \beta r < \Psi(\cdot)(1 - \theta)S \). This reduces the problem to the following programme:
\[
\begin{align*}
\text{Max} & \quad \pi = (1 - \gamma) \left[ (1 - \beta) r - A \left( \frac{(1 - \theta)}{\beta} \frac{\Psi(r/\tau)}{r} S - 1 \right) \right] \\
\text{s.t.} & \quad 0 \leq r \leq \tau
\end{align*}
\]

Equation A1 characterizes the first order condition (FOC) of this programme. Notice that \( \lim_{r \to a} \frac{\partial \pi}{\partial r} = (1 - \beta) - \frac{(1 - \theta)}{\beta} \frac{\Psi'(r) - \Psi(r)}{r^2} \), where \( a \) is any constant. Using L'Hopital, and from the properties of \( \Psi \), \( \lim_{r \to 0} \frac{\Psi'(r) - \Psi(r)}{r^2} = \frac{1}{2 \tau^2} \lim_{r \to 0} \Psi''(r) = 0 \). Thus, \( \lim_{r \to 0} \frac{\partial \pi}{\partial r} = (1 - \beta) > 0 \). From the characteristics of \( \Psi \), it also follows that \( \lim_{r \to \tau} \frac{\partial \pi}{\partial r} = -\infty \). Then, there is at least one interior solution for \( r \). The second order condition (SOC) of the programme is given by:

\[
\frac{\partial^3 \pi}{\partial r^2} = \frac{1}{r} \left( \frac{(1 - \theta)}{\beta} \frac{\Psi'(\cdot)}{\Psi''(\cdot)} \right) \]

(A4)

From the FOC, the first term in the parenthesis of equation A4 equals to \( 2(1 - \beta) \). It follows that at any maximum \( 2(1 - \beta) \beta < (1 - \theta) \Psi''(\cdot) \frac{AS}{\tau^2} \).

Plugging the optimal corruption in the incumbent’s participation constraint, we get \( \tilde{H} = A \left( \frac{(1 - \theta)}{\beta} \frac{\Psi'(\tilde{r}/\tau)}{\tilde{r}} S - 1 \right) \). At equilibrium, the elite offers the contract \( \{\tilde{H}, \tilde{r}\} \) to the incumbent independently on its type, and an incumbent of type \( c \) always accepts it.

**Proof of proposition 2.** Equation 3 sets an implicit function of corruption in terms of the parameters of the model. Call \( L = (1 - \beta) \beta r^2 - (1 - \theta) AS \frac{\Psi''(\cdot)}{r} - \Psi(\cdot) \) = 0.

Using the implicit theorem function, \( \frac{\partial r}{\partial l} = -\frac{\partial L/\partial l}{\partial L/\partial r} \), where \( l = \{y, \theta, \beta, S\} \). Notice \( \partial L/\partial r = -r \left( (1 - \theta)(AS/\tau^2) \Psi''(\cdot) - (1 - \beta) \beta \right) = -r \Phi \), where \( \Phi = (1 - \theta) \frac{AS}{\tau^2} \Psi''(\cdot) - 2(1 - \beta) \beta \). From the SOC (see proof of proposition 1) it follows that \( \Phi > 0 \), thus \( \partial L/\partial r < 0 \). We use it for the following computations.

**Jurisdiction’s income:** Deriving \( L \) with respect to \( y \), applying the implicit function theorem, and manipulating algebraically the expression we get:

\[
\frac{\partial r}{\partial y} = \frac{(1 - \theta) S}{r \Phi} \left( A(r^2/\tau^3) \Psi''(\cdot) \tau' - A'(\Psi'(\cdot)r/\tau - \Psi(\cdot)) \right)
\]

Using equation 3 and reorganizing terms, we can write this derivative as:
\[
\frac{\partial r}{\partial y} = \frac{r}{\Phi} \left( (1 - \theta)(1/\tau^2) AS \Psi'''(\tau')/\tau - (1 - \beta) \beta A' A \right) \quad (A5)
\]

Calling \( \eta_i = \frac{\tau^2 (1 - \beta) \beta}{(1 - \theta) AS \Psi'''(\tau)} \), statement (a) follows. Notice that from the SOC the denominator in \( \eta_i \) is higher than its numerator, thus \( \eta_i \in (0,1) \).

**Office Spoils:** Deriving \( L \) with respect to \( S \), applying the implicit function theorem, using equation 3, and reorganizing terms we get
\[
\frac{\partial r}{\partial S} = -\frac{(1 - \beta) \beta r}{\Phi S} < 0.
\]

**Elite’s power:** Deriving \( L \) with respect to \( \theta \), applying the implicit function theorem, using equation 3, and reorganizing terms we get
\[
\frac{\partial r}{\partial \theta} = \frac{(1 - \beta) \beta r}{\Phi (1 - \theta)} > 0.
\]
It can be also shown that
\[
\frac{\partial^2 r}{\partial \theta^2} = \frac{(1 - \beta) \beta}{\Phi (1 - \theta)^2} \left( \frac{\partial r}{\partial \theta} \Phi (1 - \theta) - r \left( \frac{\partial \Phi}{\partial \theta} (1 - \theta) - \Phi \right) \right),
\]
with
\[
\frac{\partial \Phi}{\partial \theta} = \frac{AS \left( (1 - \theta) \Psi'''(\tau') \frac{\partial r}{\partial \theta} \right)}{\tau^2} \frac{r}{\tau}.
\]
If we assume \( \Psi''' < 0 \), then \( \frac{\partial^2 r}{\partial \theta^2} > 0 \).

**Incumbent’s share:** Deriving \( L \) with respect to \( \beta \), applying the implicit function theorem, and reorganizing terms we get
\[
\frac{\partial r}{\partial \beta} = \frac{(1 - 2 \beta) r}{\Phi}.
\]
This derivative is positive if and only if \( \beta < \frac{1}{2} \), and negative if and only if \( \beta > \frac{1}{2} \).

**Proof of proposition 3.** There are two possibilities to analyse the effect of \( y, S, \theta, \) and \( \beta \) on the detection probability. The first one is to analyze what occurs to \( H \) when any of these exogenous change by using equation 4. With this information and the results in proposition 2 we can get the final effect on \( \delta \). However, there is a simpler way to do it. Since at equilibrium the incumbent’s participation constraint holds with equality, we can use the fact that \( \hat{\delta} = \beta (\hat{r}/S) \). From here, we get the following results.

\[
\text{Jurisdiction’s income: } \frac{\partial \delta}{\partial y} = \frac{\beta}{S} \frac{\partial r}{\partial y}, \text{ then } \text{sign}(\partial \delta/\partial y) = \text{sign}(\partial r/\partial y).
\]

\[
\text{Office Spoils: } \frac{\partial \delta}{\partial S} = \frac{\beta r}{S^2} \left( \frac{\partial r}{\partial S} S - r \right) < 0, \text{ since } \partial r/\partial S < 0.
\]

\[
\text{Elite’s power: } \frac{\partial \delta}{\partial \theta} = \frac{\beta}{S} \frac{\partial r}{\partial \theta} > 0, \text{ since } \partial r/\partial \theta > 0.
\]
Incumbent’s share: $$\frac{\partial \delta}{\partial \beta} = \frac{1}{S} \left( r + \beta \frac{\partial r}{\partial \beta} \right) = \frac{r}{S} \left( 1 + \frac{\beta (1-2\beta)}{\Phi} \right)$$. Thus, $$\frac{\partial \delta}{\partial \delta} > 0$$ if and only if $$\Phi > -\beta(1-2\beta)$$ (keep in mind that $$\Phi$$ depends also on $$\beta$$). Notice that when $$\beta<\frac{1}{2}$$ this condition holds.

**Proof of Proposition 4.** We apply the same strategy used in proof of Proposition 3. From equation 6 and the incumbent’s participation constraint we get $$\hat{\delta}_a = \frac{\beta \, \hat{r}}{S \, \Psi(.)}$$. Using it we get the following results.

**Jurisdiction’s income:** Computing $$\frac{\partial \delta_a}{\partial y}$$, using equation 3, and after some manipulation we get:

$$\frac{\partial \delta_a}{\partial y} = \frac{\beta}{\Psi(.)^2 S} \left( \frac{\partial r (1-\beta) \beta r^2}{\partial y (1-\theta) AS} + \Psi'(r/(\tau)^2 r' \right) \tag{A6}$$

The sign of this derivative depends on the sign of the term in parenthesis. Notice that if $$\partial r/\partial y < 0$$ then A6 is positive. However, when $$\partial r/\partial y > 0$$ its sign is ambiguous. The sufficient condition to have $$\partial \delta_a/\partial y > 0$$ is $$\frac{\partial r}{\partial y} < \left( \frac{1-\theta}{\beta} \right) AS \frac{\Psi'(r)}{r' \beta \tau^2 \Psi(.)}$$. Using equation A5 and reorganizing terms we can rewrite this condition as $$\left( \frac{l}{\eta_1} - \eta_2 \right) \frac{\tau'}{\tau} < \frac{A'}{A}$$, where $$\eta_2 = \frac{\Phi}{\Psi''(.) (1-\beta) \beta r \eta_1} > 0$$.

**Office Spoils:** Computing $$\frac{\partial \delta_a}{\partial S}$$, using equation 3, and after some manipulation we get:

$$\frac{\partial \delta_a}{\partial S} = \frac{\beta r}{S \Psi(.)} \left( \frac{1}{S} + \frac{\partial r}{\partial S} \eta_3 \right) \tag{A7}$$

where $$\eta_3 = \frac{(1-\beta) \beta r}{(1-\theta) AS \Psi(.)}$$. Notice that $$\eta_3 \in (0,1)$$. It is so because at equilibrium $$H>0$$, which implies $$\beta r < \Psi(.) (1-\theta) S < \Psi(.) (1-\theta) AS$$ (see proof or proposition 1). Since $$\partial r/\partial S < 0$$, the sign of A7 depends on the sign of the term in parenthesis. It follows that, $$\partial \delta_a/\partial S > 0$$ if and only if $$|\partial r/\partial S| > 1/(\eta_3 S)$$.

**Elite’s power:** Computing $$\frac{\partial \delta_a}{\partial \theta}$$, using equation 3, and after some manipulation we get $$\frac{\partial \delta_a}{\partial \theta} = -\eta_3 \frac{\beta}{\Psi(.) S} \frac{\partial r}{\partial \theta} < 0$$.
Incumbent’s share: Computing $\frac{\partial \delta_a}{\partial \beta}$, using equation 3, and after some manipulation we get:

$$\frac{\partial \delta_a}{\partial \beta} = \frac{r}{\Psi(\cdot)S} \left( 1 - \frac{\partial r}{\partial \beta} \beta \eta_3 \right) \quad (A8)$$

It is direct that if $\beta > \frac{1}{2}$, then $\partial r/\partial \beta < 0$ and so $\partial \delta_a/\partial \beta > 0$. However, if $\beta < \frac{1}{2}$ the sign of A8 is ambiguous. The sufficient condition to have $\partial \delta_a/\partial \beta > 0$ is $\partial r/\partial \beta < l/\beta \eta_2$, otherwise expression in A8 is negative.

Centralized Model. Assume there are $J > 1$ elites demanding corruption to the central incumbent (one in each of the $J$ jurisdictions) indexed by $j$ and endowed with economic power $\theta$. At equilibrium, the citizens’ strategies are exactly the same we described in the decentralized game (see proof of proposition 1). Thus, let us study the behaviour of the rest of the players.

Elite $j$ maximizes its payoff $\pi_j^c = (1 - \beta) r_j^c - H_j^c$, subject to $\delta^c \leq \beta^c r^c / S^c$ (with $r^c = \sum_{j=1}^J r_j^c$), $H_j^c \geq 0$, $0 \leq r_j^c \leq \tau$, $r^c \leq \tau$, and taking as given $r_i^c \ \forall i \neq j$. The first constraint implies $H_j^c \geq A \left( \frac{(1 - \theta^c) \Psi(r^c/\tau)}{\beta^c r^c S^c - \tau} \right) - \sum_{i \neq j} H_i^c$. Since $\pi_j^c$ strictly decreases in $H_j^c$, then the elite chooses $H_j^c = A \left( \frac{(1 - \theta^c) \Psi(r^c/\tau)}{\beta^c r^c S^c - \tau} \right) - \sum_{i \neq j} H_i^c$.

It implies that at equilibrium $H^c = A \left( \frac{(1 - \theta^c) \Psi(r^c/\tau)}{\beta^c r^c S^c - \tau} \right)$, where $H^c = \sum_{j=1}^J H_j^c$. We assume the parameters of the model are such that $H_j^c > 0$. This reduces the elite $j$ problem to the following programme:

$$\text{Max } \pi_j^c = (1 - \beta^c) r_j^c - A \left( \frac{(1 - \theta^c) \Psi(r^c/\tau)}{\beta^c r^c S^c - \tau} \right) + \sum_{i \neq j} H_i^c$$

s.t. $0 \leq r_j^c \leq \tau$ and $r^c \leq \tau$

At equilibrium, the level of corruption in region $j$ must satisfy:

$$(1 - \beta^c) = \frac{(1 - \theta^c) A S^c}{\beta^c} \left( \frac{\Psi'(\cdot) \hat{c}^c / \tau - \Psi(\cdot)}{\hat{c}^c} \right) \quad (A9)$$

Equation A9 represents the corruption reaction curve of elite $j$. With the $J$ system of equation, we can solve for the level of corruption in each jurisdiction. However,
equation A9 also defines implicitly the national per capita level of corruption $r^c$. Notice that the equilibrium representation of national corruption under centralization is exactly the same we obtained for the level of corruption under decentralization in jurisdiction $j$. The rest of equilibrium conditions are getting by following the same steps we used in proposition 1.

**Data Set Description**

**Corruption Index**: Originally ranking from 0 to 6, with 6 indicating lower corruption. Rescaled from 0 to 1, with 0 indicating lower corruption. Source: International Country Risk Guide. Taken from Fisman and Gatti (2002).

**Decentralization**: Total expenditure of subnational (state, and local) government over total spending by all levels (state, local, and central) of government. Source: *Government Finance Statistics*, International Monetary Found. Taken from Fisman and Gatti (2002).

**GDP**: Real GDP per capita in constant dollars, chain series, expressed in international price, base 1996. Source: Heston, Summers, and Aten, Penn World Table Version 6.1.

**Civil Liberties**: Gastil index of civil liberties. It takes values from 1 to 7, where 7 refers to the highest level of freedom. Source: Freedom House.

**Population**: Source: Heston, Summers, and Aten, Penn World Table Version 6.1.

**Government Size**: Total government expenditure divided by DGP. Source: Heston, Summers, and Aten, Penn World Table Version 6.1.

**Legal Origin**: Origin of a country’s legal system. These dummy variables classify the legal origin in five groups: (1) English common Law; (2) Socialist laws; (3) French Commercial Code; (4) German Commercial Code; (5) Scandinavian Commercial Code. Source: La Porta, Lopez, Shleifer, Vishny (1999).
Chapter III

Bargaining in Legislature: Does the Number of Parties Matter?

Abstract
How the number of parties affects the policymaking process in a legislature is an unexplored issue in the political economics literature. We use a bargaining model to study whether a government party prefers to negotiate with another compact party (2-parties legislature) or with several \( (m) \) parties (\( m+1 \)-parties legislature). In the model, parties negotiate on both a public (ideological) and a distributive (private) policy. Those legislators who belong to the same party share the same ideological position. We show that there is a maximum level of ideological polarization in the 2-parties legislature for which, above that, the government party always prefers to negotiate in an \( m+1 \)-parties legislature. This threshold increases as the number of non-government parties \( (m) \) in the \( m+1 \)-parties legislature increases. This demonstrates the existence of a trade-off between number of parties and polarization in the negotiation process in a legislature.

Consider the following situation: A policy maker wants to promote a public policy in a legislature. For instance, to introduce more flexible rules in the labor market, to modernize the pension system, to reform the taxation system, etc. This policy maker has the support of the government party, but he still requires some extra votes in order to get the sanction of the reform in the legislature. Does the policy maker prefer to bargain with one compact party or with several parties in order to promote the desired policy in the legislature? In which of the two cases it is more likely to obtain the required legislative support? This paper deals with these questions.

The scholars in political science have not agreed on this issue. Some authors consider that it is more difficult to form stable coalitions or to get policy agreements when there are many parties in a legislature. Other authors in the same area disagree with this position. They assert that, high levels of fragmentation are not conducive to effective policymaking due to the ability of parties to form these coalitions or to get policy agreements through a bargaining process. Nevertheless, almost everybody recognizes that these coalitions or agreements are more difficult to get when there is a high level of ideological polarization among those parties (See Jones, 2005).

An informal comparison of the percentage of executive’s legislative initiatives approved by the legislature (executive’s success in legislature) among some Latin American countries suggests that, not only the number of parties but also the ideological
polarization matter. For instance, Uruguay and Paraguay have the same number of effective parties (2.73) but the executive is much more successful in the latter (83%) than in the former (57%) democracy. A possible explanation for this difference is that the ideological polarization among political parties in Paraguay is considerably smaller than the respective polarization in Uruguay (the indexes are 0.58 and 4.05, respectively).

Argentina and Ecuador are countries with similar ideological polarization (1.71 and 1.78, respectively). However, the effective number of parties is twice in Ecuador (3.18 versus 6.71). In this case, the percentage of executive’s legislative initiatives approved by the legislatures in Argentina is significantly bigger than that approved in Ecuador (64% and 42%, respectively). The last example is Brazil and Chile. Brazil has more parties but is less polarized than Chile (7.81 versus 2.02, and 4.63 versus 5.82 respectively). In this case, the success of the executive in both countries is almost the same, even though Brazil does a bit better (72% in Brazil and 69% in Chile).

The discussion presented above suggests that any theoretical model trying to explain the role of the number of parties in the policymaking process must also take into account the level of polarization. However, no study has incorporated formally these two issues. Furthermore, as far as we know, there are not formal theoretical papers about the effect of the number of parties on the policymaking process.

In this paper we use a bargaining approach to study the effect of the number of parties on the policymaking process. One of the seminal papers about strategic bargaining in legislatures is due to Baron and Ferejohn (1989). In their framework, the legislators must decide how to distribute some private benefits in a unicameral, majority rule legislature. They use an alternating-offer bargaining model in which each of the legislators has a probability of being the proponent. The recognized legislator makes an offer on how the benefits should be distributed. Under a closed rule, the proposal is voted against the no allocation of benefits. If the proposal is accepted by a majority, the game ends and the payments are done. If it does not obtain majority, the legislature moves to the next session in the same fashion with a member recognized to make another proposal, and so on. In any equilibrium, the first proposal is passed, and benefits are distributed to a minimal majority.

The Baron et al. (1989)’s model has been extended in different ways by other scholars in both political and economics science (for instance, Jackson and Moselle, 2002). Nevertheless, there is not any formal analysis about the effect of the number of parties on the legislative bargaining process. Perhaps, the lack of attention to this issue is due to the technical complications that emerge when comparing the multiplicity of stationary sub-perfect equilibria in the multi-person alternating-offer bargaining models with the unique stationary sub-perfect equilibrium in the bilateral framework. To avoid these issues...

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29 The effective number of parties is an average for the most recent election and corresponds to the number of parties in lower/single house adjusted by its number of seats (Source: Jones, 2005). Polarization is measured by the Taylor and Herman 1971’s index. It uses information on the legislators’ perception about the distance in the ideological position between their party and the rest of parties (Latinobarometro 2002-04; source: Jones, 2005). Executive’s success in legislature is taken from Saiegh (2005).
technical problems, we restrict ourselves to the use of a take-it-or-leave-it bargaining procedure. Despite this simplification, our results show that we can learn a lot about the effect of the number of parties on the policymaking process.

More precisely, we use a variation of Baron et al. (1989)’s framework. Similar to them, we model a bargaining process in a unicameral legislature but with a closed agenda and only one session. Additionally, following Jackson et al. (2002), we introduce two dimensions in the bargaining process, a public decision (ideological) and a distributive decision (private). Following some of the main papers on party formation (Baron, 1993; Levy, 2004), we assume that those legislators who form a party commit on the same ideological position.

In our model, the government party (party $A$) makes an offer to the legislature which consists of both a public policy proposal and a distributive policy vector on a private good $X$. The legislators vote this proposal against the status quo. If the proposal passes, both the distributive and the public policies are implemented. If it is rejected, the status quo is assigned.

To study the effect of the number of parties on the legislative outcomes, we consider two extreme cases. In the first case, we assume each of the $m$ legislators who do not belong to the government party represents a different political party and takes his decisions individually. We refer to this situation as the $m+1$-parties legislature. In the second case, we assume those legislators who do not belong to the government party form a unique political party $B$. We call this situation the 2-parties legislature. In this case, the members of party $B$ commit to a unique position in the public policy (ideological) dimension.

Our model is able to replicate the observation that both the number of parties and the ideological polarization matter when negotiating in legislatures. More precisely, we find that there is a threshold in the level of polarization in the 2-parties legislature for which, beyond that, party $A$ always prefers to negotiate in an $m+1$-parties legislature. We call this threshold the maximum level of polarization party $A$ holds in the 2-parties legislature. Furthermore, we show this threshold is situated somewhere between the ideological position of the most polarized legislator and the respective position of the less polarized legislator in the $m+1$-parties legislature. We also prove that this maximum level of polarization that party $A$ is willing to hold in the 2-parties legislature increases as the number of non-government parties ($m$) in the $m+1$-parties legislature increases. This illustrates clearly the trade-off between number of parties and polarization.

The results described above are found when two circumstances take place. First, the government party is willing to compensate to everybody in the legislature in order to promote its most preferred public policy. Second, the amount of resources this party has is enough to promote this policy. When either the resources of the government party are not enough to achieve its most preferred public policy or this party is not willing to promote it, some similar results are got. However, we show there is an extreme situation in which the government party always prefers the 2-parties to the $m+1$-parties legislature. This occurs when either there is not distributive issue or the willingness of
the government party to promote its most preferred policy in the $m+1$-party legislature is too low.

The paper by Weingast, Shepsle and Johnsen (1981) has been associated with the effect of the number of parties on the policymaking process. Strictly speaking, they analyse the efficiency of the budget distribution among the districts of a federation in a legislature. They find that this distribution turns out to be more inefficient as the number of districts increases. Assuming a positive correlation between number of districts and number of parties, some scholars have claimed that the policymaking process becomes more complicated as the number of political parties increases. Our paper differentiates from this in at least two ways. First, we not only take into account legislative decisions on a private distribution but also on a public issue. Second, our analysis is not about efficiency. It cares on how both the number of parties and the level of polarization in a legislature affect the total utility of the government party and deviate it implemented policies from its initial ideological position.

The paper by Tabellini and Alesina (1990) has been also related to this topic. They study the effect of polarization in the preferences of the voters on the budget deficit. They find that this deficit increases as polarization goes up. If this polarization is transmitted to the parties in the legislature, then one can anticipate that it is more difficult for a government party to promote fiscal reforms when the legislature has a high level of polarization. Some authors (see below) have employed the terms polarization and number of parties indistinctly. By doing so and using the Tabellini and Alesina’s result, they have concluded that the budget deficits increase as the number of parties increases. However, as we are going to show polarization and number of parties are two different dimensions in the political process.

Some studies have explored empirically the relationship between number of parties and budget deficits by using cross-country information (for instance, Stein, Talvi and Grisanti, 1998; Mulas-Granados, 2003). They have found a no robust negative relationship between these two variables. The main assumption in these analyses is that the number of parties is a good proxy for both the level of polarization and the number of districts. However, although the number of parties might have a no perfect positive correlation with the number of districts, its correlation with the level of polarization is not clear. Actually, like in the examples mentioned above, one can observe legislatures with two parties and a high level of polarization, and legislatures with a large number of parties and a low level of polarization. We are going to show that it is crucial to separate the number of parties and the level of polarization in order to capture the real effect of the former on the negotiation outcomes in a legislature.

The rest of the chapter is organized as follows. Section 1 describes the framework for both the 2-parties and the $m+1$-parties legislature and presents their respective equilibrium representation. Section 2 analyses the equilibrium in each type of legislature, and section 3 studies whether the government party always prefers the 2-parties to the $m+1$-parties legislature or not. Section 4 concludes. The appendix contains all proofs.
1. Model

A policy maker or the government party, henceforth party $A$, wants to promote a public policy in a legislature. It needs $n$ votes to promote the policy, but it only has $n_A < n$ seats. Thus, party $A$ still requires $m = n - n_A$ votes.

The utility of each legislator depends on two arguments. A public policy $y$ (ideological decision), with feasible set $Y \in [0,1]$; and a distributive policy $(x_1,\ldots,x_n)$, with $x_i \geq 0 \ \forall i$, and $\sum_{i=1}^{n} x_i \leq X$. These rewards may include local investment projects, budgetary transfers, public employments, governmental contracts, etc.

Thus, $u_i(y,x_i):[0,1] \times \mathbb{R} \to \mathbb{R}$, where $u_i(.)$ is continuous and strictly increasing in $x_i$ for every $y \in Y$. We make two extra assumptions on the preferences of the legislators. First, the ideological decision is separable from the distributive decision. More precise, for any $(y,x_1,\ldots,x_n)$ and $(y',x'_1,\ldots,x'_n)$, $u_i(y,x_i: \hat{y}_i) > u_i(y',x_i: \hat{y}_i)$ if and only if $u_i(y,x_i: \hat{y}_i) > u_i(y',x_i: \hat{y}_i)$. Second, we assume $u_i$ is single peaked in $y$ for every $x_i$.

We denote the peak of $u_i$ by $\hat{y}_i$, and from now on we write the utility of legislator $i$ as $u_i(y_,x_i: \hat{y}_i)$.

We assume that $u_i$ has a unique local maximum or peaked at $\hat{y}_A \ \forall i \in A$. In other words, those legislators who belong to party $A$ share the same preferred ideological position. Without loss of generality, only those $i = 1,2,\ldots,n_A$ belong to party $A$. We fix ideological positions in such a way that $\hat{y}_1 < \hat{y}_A < \ldots < \hat{y}_n$.

In the legislature there is only one session. In this session, party $A$ makes an offer $(y,x_1,\ldots,x_n)$ to the legislators in order to maximize its total utility $\sum_{i=1}^{n} u_i(y,x_i: \hat{y}_A)$, subject to $\sum_{i=1}^{n} x_i \leq X$ and $x_i \geq 0 \ \forall i$. The legislators vote the proposal against the status quo $(0,0,\ldots,0)$. We set the status quo of the distributive policy at zero because at the beginning of the session no private resources have been allocated. If the $n$ legislators accept the offer of party $A$, then both the distributive and the public policies are implemented. Otherwise, the status quo is assigned.

The $m+1$-parties legislature game

In the $m+1$-parties legislature, each legislator who does not belong to party $A$ represents a single political party with a different ideological position. In other words, each of these legislators has a different peak for the public policy in the utility function. Furthermore, there is not any possibility of commitment among these legislators. We assume that the peaks of those $i's \not\in A$ are such that $\hat{y}_{n_A+i} < \hat{y}_{n_A+2} < \ldots < \hat{y}_n$. Figure 1 depicts the ideological position of the legislators in this game.
An equilibrium offer in the $m+1$-parties legislature is a vector $(y_{(m+1)}^*, x_{(m+1)}^*, \ldots, x_{n_{(m+1)}}^*)$, such that the following conditions are satisfied:

\[ (y_{(m+1)}^*, x_{(m+1)}^*, \ldots, x_{n_{(m+1)}}^*) \in \arg \max \left\{ \sum_{i \in A} u_i (\cdot) \right\} \]

\[ u_i (y_{(m+1)}^*, x_{(m+1)}^*, \ldots, x_{n_{(m+1)}}^*) \geq u_i (y^*, \theta : \hat{y}_i) \quad \forall i \notin A \]  

\[ u_i (y_{(m+1)}^*, x_{(m+1)}^*, \ldots, x_{n_{(m+1)}}^*) \geq u_i (y^*, \theta : \hat{y}_A) \quad \forall i \in A \]

\[ \sum_{i=1}^n x_i \leq X, \text{ and } x_i \geq 0 \quad \forall i \]

where the subscript $(m+1)$ refers to the type of legislature. Condition 2 assures those $i'$s $\notin A$ always accept the offer of party $A$ (Participation Constraint $i'$s $\notin A$). Condition 3 assures the offer of party $A$ is at least as good as the status quo for those $i'$s $\in A$ (Rationality Constraint $i'$s $\in A$). Condition 4 implies feasibility and no-negative transfers. Notice that neither condition 2 nor 3 care on how the private good (if any) is distributed among those members of party $A$.

**The 2-parties legislature game**

To model party formation and platform setting is beyond the scope of this paper. This issue has been already studied by different authors, for instance Baron (1993), Jackson et al. (2002), Levy (2004), and Morelli (2004). In these models, legislators who form a party commit on a single ideological platform which depends on the distribution of the preferences of voters, the set of potential candidates, and the institutional rules.

Following the studies in this area, in our model we assume parties are policy-oriented in its ideological position, and thus its members commit to a unique position in the public policy dimension $y$. Hence, in the 2-parties legislature those legislators who do not belong to party $A$ form a single political party $B$, with a unique ideological position $\hat{y}_B$. Notice that we have already imposed this assumption on party $A$.

Once again, we do not care on how those $i'$s $\notin A$ agree on $\hat{y}_B$. However, to make interesting our comparison with the $m+1$-parties legislature, we shall study the case in which $\hat{y}_B \in (\hat{y}_{n_{(m+1)}}), \hat{y}_n)$. In other words, we shall assume $\hat{y}_B$ is a linear combination of the ideological position of the legislators in the $m+1$-parties legislature, i.e. $\hat{y}_B = \sum_{i \notin A} \theta_i \hat{y}_i$, with $\sum_{i \notin A} \theta_i = 1$, and $0 < \theta_i < 1$. Figure 2 depicts the ideological position of the parties in this game.
An equilibrium offer in the 2-parties legislature is a vector \((y_{(2)}, x_{(2)}, \ldots, x_{(n)})\), such that:

\[
\left\{ \sum_{i \in A_i} x_i \leq X, \text{ and } x_i \geq 0 \quad \forall i \right. \]

Conditions 5 through 8 play the same role of conditions 1 through 4 in the \(m+1\)-parties legislature. Thus, these conditions assure optimality, participation of those \(i's \not\in A\), rationality of those \(i's \in A\), feasibility, and no-negative transfers. Formally, the only equilibrium condition that changes with respect to the \(m+1\)-parties legislature is the participation constrain. Now, in equation 6, those \(i's \not\in A\) do not care on \(y_i\) but on \(Y_A\).

2. Analysis

From now on, we restrict ourselves to the case in which \(u_i(y, x_i) = -b_i |y - \hat{y}_i| + x_i\). This utility function satisfies the assumptions made above. The parameter \(b_i\) measures the valuation of legislator \(i\) for the public policy relative to the distributive policy. We understand this parameter as how difficult is to convince legislator \(i\) to vote for a given public policy.

Accordingly to our definition of equilibrium, the only parameter that differs between the 2 and the \(m+1\)-parties legislature is the peak of \(u_i\). It implies that the vector of \(b_i's\) will be the same in the two types of legislatures. In other words, we are assuming that the \(b_i's\) are not part of the ideology of the parties but are a personal characteristic that keep unchanged regardless of the party composition.

For the analysis, we fix the status quo of the public policy to zero, i.e. \(y^* = 0\). To make things interesting, we also impose \(\hat{y}_A > 2\hat{y}_n\). This implies that if party \(A\) wants to promote its preferred public policy \(\hat{y}_A\), it has to offer \(x_i > 0 \quad \forall i \not\in A\).

The equilibrium representation in each type of legislature depends on both the willingness of party \(A\) to promote its preferred public policy \(\hat{y}_A\) and the feasibility of
this public policy. Notice that, under some circumstances, party \( A \) may have incentives to take for itself the complete amount of private good \( X \) and does not promote any public policy that implies positive transfers to the other parties. Furthermore, although party \( A \) can be willing to promote \( \hat{y}_A \), the amount of \( X \) will constraint its decisions. We start by analysing the first issue.

**Party \( A \)'s willingness to promote public policy**

First, consider the \( m+1 \)-parties legislature. Notice that the public policy that \( A \) can promote free of transfers to those \( i's \in A \) in this type of legislature is \( 2\hat{y}_{n_{A+1}} \). When this is the level of \( y \), legislator \( n_A + 1 \) receives his status quo utility, and the other legislators who do not belong to \( A \) get a utility above their respective reservation utility. This implies that those \( i's \notin A \) will not require any private transfer to accept this public policy.

**Proposition 1.** Consider a legislature with \( m+1 \)-parties, and assume \( X \) is large enough to promote any public policy \( y \in Y \).

i) Consider the pair of public policies \( y \) and \( \bar{y} \in Y \), with \( y < \bar{y} \). Let \( r \) and \( q \), with \( r \geq q \), such that \( y \) implies \( x_{i(m+1)}(\bar{y}) > 0 \) for those \( i = n_A + 1,...,q \), and \( \bar{y} \) implies \( x_{i(m+1)}(\bar{y}) > 0 \) for those \( i = n_A + 1,...,r \). If \( q < r \), party \( A \) prefers \( \bar{y} \) to \( y \) if and only if

\[
\sum_{i=n_A+1}^r b_i \geq \sum_{i=n_A+1}^q b_i + \sum_{i=q+1}^r b_i \frac{\bar{y} - 2\hat{y}_i}{\bar{y} - y} .
\]

If \( q = r \), previous condition reduces to

\[
\sum_{i=n_A+1}^r b_i \geq \frac{\sum_{i=n_A+1}^q b_i}{y - \bar{y}} \sum_{i=q+1}^r b_i.
\]

ii) Party \( A \) does not allocate private resources in promoting any level of public policy \( y \in Y \) if and only if

\[
\sum_{i=n_A+1}^r b_i < \sum_{i=n_A+1}^q b_i \frac{y - 2\hat{y}_i}{y - 2\hat{y}_{n_A+1}} \quad \forall q = n_A + 1,...,n .
\]

The conditions in proposition 1 reduce to compare the relative valuation of the public policy between those legislators who belong to party \( A \) and those who not. Thus, party \( A \) is willing to promote a level of public policy \( y \) if the sum of the relative valuation for the public policy of its members (\( \sum_{i\in A} b_i \)) is larger than the respective sum over those legislators who do not belong to \( A \) and have an ideological position below \( y \). In other words, if a public policy \( y \) implies that party \( A \) must offer a positive distributive policy to those legislators \( i = n_A + 1,...,r \), then \( A \) promotes \( y \) if \( \sum_{i\in A} b_i \) is large enough with respect to \( \sum_{i=n_A+1}^r b_i \).

When \( \sum_{i\in A} b_i \) is low enough, \( A \) does not promote any public policy that implies a positive distributive offer to those legislators who belong to another party. In this case, \( A \) promotes the level of \( y \) free of transfers to those \( i's \notin A \), i.e. \( 2\hat{y}_{n_A+1} \). From proposition 1, we get the following corollary:
Corollary 1: In the \( m+1 \)-parties legislature:
(i) If \[ \sum_{i \in A} b_i \geq \sum_{i \notin A} b_i, \]
then party \( A \) is willing to promote the closest feasible public policy to \( \hat{y}_A \).
(ii) If \[ \sum_{i \in A} b_i < b_{n+1}, \]
party \( A \) does not have incentives to allocate private resources in promoting any level of public policy.
(iii) If \[ b_{n+1} < \sum_{i \in A} b_i < \sum_{i=n+1} b_i, \]
then there is a public policy \( y \in (2\hat{y}_{n+1}, \hat{y}_A) \) which is the closest public policy to \( \hat{y}_A \) that party \( A \) is willing to promote.

Consider now the 2-parties legislature. Same as above, it is important to observe that the public policy that \( A \) can promote free of transfers to those \( i's \notin A \) in this type of legislature is \( 2\hat{y}_B \). Since we have imposed \( \hat{y}_B \in (\hat{y}_{n+1}, \hat{y}_A) \), it follows that \( 2\hat{y}_B > 2\hat{y}_{n+1} \). The respective willingness conditions of party \( A \) to promote the public policy in the 2-parties legislature are stated in proposition 2.

Proposition 2. Consider a legislature with 2-parties. Party \( A \) will promote the closest feasible public policy to \( \hat{y}_A \) if and only if \[ \sum_{i \in A} b_i \geq \sum_{i=n+1} b_i = \sum_{i \notin A} b_i. \] Otherwise, it will not allocate private resources in promoting any level of public policy.

Notice that the required condition in the 2-parties legislature to observe the government party promoting a public policy with positive redistribution offers to those \( i's \notin A \) is more demanding than the respective condition in the \( m+1 \)-parties legislature. It occurs because, different to the \( m+1 \)-parties legislature, if party \( A \) wants to promote a public policy above that level of \( y \) free of transfers in the 2-parties legislature (i.e. above \( 2\hat{y}_B \)), it has to offer a positive distributive policy to all those legislators who do not belong to its party (i.e. \( x_{i(2)}^* > 0 \forall i's \notin A \)). It does not happen in the \( m+1 \)-parties legislature.

Regardless of the type of legislature, propositions 1 and 2 also tell us that the number of seats that the government party has in the legislature plays an important role. This is so, because the conditions to observe the government party promoting its preferred public policy are more likely satisfied when \( n_A \) is high. It gives theoretical support to the empirical observation that those governments with more seats in the legislature are more successful in promoting its executive initiatives.

From now on, we call \( w_p \) to the largest level of public policy that party \( A \) is willing to promote in the \( p \)-parties legislature, \( p = \{2,m+1\} \). In table 1, we report the respective level of \( w_p \) for each of the cases we have studied above. Notice that when \( w_{m+1} \in (2\hat{y}_{n+1}, \hat{y}_A) \), \( w_2 = 2\hat{y}_B \). Using the results in previous propositions, we are already able to write down the equilibrium representation in each type of legislature depending on both the incentives to promote the public policy and its feasibility.
Table 1  
Party A’s willingness to promote public policy

<table>
<thead>
<tr>
<th>Required condition</th>
<th>Largest level of public policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_{i \in A} b_i \geq \sum_{i \in A} b_i$</td>
<td>$w_{m+1} = \hat{y}_A$</td>
</tr>
<tr>
<td>$b_{n+1} &lt; \sum_{i \in A} b_i &lt; \sum_{i \in A} b_i$</td>
<td>$w_{m+1} \in (\hat{y}_{n,1}, \hat{y}_A)$</td>
</tr>
<tr>
<td>$\sum_{i \in A} b_i &lt; b_{n+1}$</td>
<td>$w_{m+1} = 2\hat{y}_{n+1}$</td>
</tr>
</tbody>
</table>

The $m+1$-parties legislature equilibrium

In this part, we describe the equilibrium offer for both the case in which party A is willing to promote its most preferred policy (i.e. $w_{m+1} = \hat{y}_A$), and the case in which it does not have incentives to spend resources in promoting any public policy (i.e. $w_{m+1} = 2\hat{y}_{n+1}$). Since the equilibrium offer for the case in which $w_{m+1} \in (2\hat{y}_{n+1}, \hat{y}_A)$ does not differ too much from the former case, we present it in the appendix. Using equations 1 through 4 and proposition 1, the optimal offer of party A in the $m+1$-parties legislature can be written as follows (See the appendix):

If $w_{m+1} = \hat{y}_A$ (i.e. $\sum_{i \in A} b_i \geq \sum_{i \in A} b_i$):

$$y^{*}_{(m+1)} = \begin{cases} 
\frac{X}{b_{n+1}} + 2\hat{y}_{n+1} & \text{if } 0 \leq X < 2\sum_{i=n+1}^{j} b_i (\hat{y}_{n+1} - \hat{y}_{n+1}) \\
\frac{X + 2\sum_{i=n+1}^{j} b_i \hat{y}_i}{\sum_{i=n+1}^{j} b_i} & \text{if } 2\sum_{i=n+1}^{j} b_i (\hat{y}_{j+1} - \hat{y}_i) \leq X < 2\sum_{i=n+1}^{j} b_i (\hat{y}_{j+1} - \hat{y}_i) + n_A + 2 \leq j \leq n-1 \\
\hat{y}_A & \text{if } X \geq \sum_{i=n+1}^{n} b_i (\hat{y}_A - 2\hat{y}_i) \\
0 & \text{if } y^{*}_{(m+1)} \leq 2\hat{y}_i \quad \forall i \notin A \\
b_i (y^{*}_{(m+1)} - 2\hat{y}_i) & \text{if } 2\hat{y}_i < y^{*}_{(m+1)} \leq \hat{y}_A 
\end{cases} \quad (9)$$

$$x^{*}_{i(m+1)} = \begin{cases} 
X \geq 0 & \text{if } y^{*}_{(m+1)} \leq 2\hat{y}_i \\
\forall i \notin A & \text{if } 2\hat{y}_i < y^{*}_{(m+1)} \leq \hat{y}_A 
\end{cases} \quad (10)$$

If $w_{m+1} = 2\hat{y}_{n+1}$ (i.e. $\sum_{i \in A} b_i < b_{n+1}$):

$$y^{*}_{(m+1)} = 2\hat{y}_{n+1} \quad (11)$$

$$x^{*}_{i(m+1)} = 0 \quad \forall i \notin A \quad (12)$$
Equation 9 and 10 describe the optimal policy offer for the case in which party $A$ has incentives to promote the closest public policy to $\hat{y}_A$. Equation 9 shows how the public policy offer changes as $X$ increases. If there is not distributive issue (i.e. $X = 0$), party $A$ offers that level of $y$ free of transfers to those $i's \notin A$, i.e. $2\hat{y}_{n+1}$. The optimal public policy offer increases as $X$ increases. Only when $X$ is high enough, party $A$ is able to promote $\hat{y}_A$, and its members (or some of them) get a positive distributive policy. If $X$ is such that $y_{(m+1)}^* \leq \hat{y}_A$, then those $i's \in A$ do not receive any distributive policy, i.e. $x_i = 0 \ \forall i \in A$.

Equation 10 describes the distributive policy that party $A$ offers to those $i's \notin A$ as a function of the optimal public policy. Those legislators who do not belong to $A$ and have an ideological position below enough the optimal public policy offer $y_{(m+1)}^*$ receive a positive distributive policy. If $y_{(m+1)}^* > 2\hat{y}_n$, then all those legislators who do not belong to $A$ receive a positive distributive policy offer.

Equations 11 and 12 represent the equilibrium offer when party $A$ does not have incentives to allocate private resources in promoting any level of public policy. Regardless of $X$, $A$ always offers the level of public policy free of transfers ($2\hat{y}_{n+1}$). In this case, the complete amount of private resources, if any, is distributed among those legislators who belong to party $A$.

Like we say above, the equilibrium offer for the case in which $w_{m+1} \in (2\hat{y}_{n+1}, \hat{y}_A)$ is similar to that represented in equations 9 and 10 (see equations A11 and A12 in the appendix). In this case, the equilibrium public policy offer increases as $X$ increases in the same fashion described by equation 9. However, once $w_{m+1}$ is achieved, this offer keeps at this level regardless of the amount of $X$.

Figure 3 illustrates the equilibrium public policy offer for a 4-parties legislature (i.e. a legislature with a government party and other three parties). The up-gross line describes the public policy offer when $w_{m+1} = \hat{y}_A$. Notice that the slope of the optimal public policy decreases as the number of legislators who receive a positive distributive offer increases. It occurs because the changes in the private resources are less productive in promoting the public policy as the number of legislators who demand a positive distributive policy goes up.

The down-tiny line describes the public policy offer for the case in which $w_{m+1} = 2\hat{y}_{n+1}$. Here, $y_{(4)}^* = 2\hat{y}_{n+1}$ for any $X \geq 0$. The intermediate line describes the public policy offer for the case in which $w_{m+1} = \bar{y} \in (2\hat{y}_{n+1}, \hat{y}_A)$. Up to $\bar{y}$, the policy offer is exactly equal to the offer that party $A$ proposes in the case in which $w_{m+1} = \hat{y}_A$. Once $\bar{y}$ is achieved, $y_{(4)}^* = \bar{y}$ regardless of $X$. 

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The 2-parties legislature equilibrium

Using equations 5 through 8 and proposition 2, the optimal offer of party \( A \) in the 2-parties legislature \( \{x^*_{(2)}, y^*_{(2)}\} \) can be written as follows:

If \( w_2 = \hat{y}_A \) (i.e. \( \sum_{i \in A} b_i \geq \sum_{i \in B} b_i \)):

\[
y^*_{(2)} = \begin{cases} 
\frac{X}{\sum_{i \in B} b_i} + 2\hat{y}_B & \text{if} \quad 0 \leq X < (\hat{y}_A - 2\hat{y}_B)\sum_{i \in B} b_i \\
\hat{y}_A & \text{if} \quad X \geq (\hat{y}_A - 2\hat{y}_B)\sum_{i \in B} b_i 
\end{cases}
\]  
(13)

\[
x^*_{i(2)} = \begin{cases} 
0 & \text{if} \quad y^*_{(2)} = 2\hat{y}_B \\
\hat{y}_A - \frac{X}{\sum_{i \in B} b_i} & \text{if} \quad 2\hat{y}_B < y^*_{(2)} \leq \hat{y}_A \quad \forall i \not\in A 
\end{cases}
\]  
(14)

If \( w_2 = 2\hat{y}_B \) (i.e. \( \sum_{i \in A} b_i < \sum_{i \in B} b_i \)):

\[
y^*_{(2)} = 2\hat{y}_B \quad X \geq 0
\]  
(15)

\[
x^*_{i(2)} = 0 \quad \forall i \not\in A
\]  
(16)

Equations 13 and 14 describe the optimal policy offer for the case in which \( w_2 = \hat{y}_A \). Equation 13 shows how the public policy offer changes as \( X \) increases. If there is not distributive issue (i.e. \( X = 0 \)), party \( A \) offers that level of \( y \) free of transfers to those \( i's \not\in A \), i.e. \( 2\hat{y}_B \). The optimal public policy offer increases at a constant rate as \( X \)
increases. Once $X$ is high enough, and $\hat{y}_A$ is achieved, the public policy offer keeps constant at this level regardless of $X$.

As we have already commented, different to the $m+1$-parties legislature, in this case all those legislators who do not belong to $A$ receive a positive distributive policy offer if the public policy offer is bigger than $2\hat{y}_B$. This offer is described by equation 14. Equation 15 and 16 describe the policy offer for the case in which $w_2 = 2\hat{y}_B$. In this case, no matters the level $X$, $2\hat{y}_B$ is the equilibrium public policy offer. Furthermore, party $A$ offers no distributive policy to those legislators who do not belong to its party.

Figure 4 illustrates the equilibrium public policy offer for the situation in which the three legislators who do not belong to $A$ in figure 3 form a unique political party $B$ (i.e. a 2-parties legislature). The up-gross line describes the public policy offer for the case in which $w_2 = \hat{y}_A$. In this case, the slope of the public policy offer is constant. Moreover, this slope is equal to the slope in the $m+1$-parties legislature when $A$ is offering a positive distributive policy to all those legislators who do not belong to its party. The down-tiny line describes the public policy offer for the case in which $w_2 = 2\hat{y}_B$.

**Figure 4**

Equilibrium public policy offer for a 2-parties legislature

3. **The $m+1$-parties versus the 2-parties legislature**

We are already able to study whether party $A$ always prefers the 2-parties to the $m+1$-parties legislature or not. In order to do it, we compare the total utility that party $A$ attains in each type of legislature. Additionally, we are going to study in which type of legislature party $A$ is more successful in promoting its ideal public policy. For the comparison, we shall focus on the case in which party $A$ has incentives to promote $\hat{y}_A$.
in both types of legislatures (i.e. \( w_{m+I} = w_2 = \hat{y}_A \)). However, we start by analysing two cases in which party \( A \) is always better off in the 2-parties legislature.

**Proposition 3.** If either \( X = 0 \) or \( 2\hat{y}_{n_{A+I}} \leq w_{m+I} \leq 2\hat{y}_B \), then party \( A \) always prefers the 2-parties to the \( m+I \)-parties legislature.

The result in proposition 3 follows from the restriction we have imposed on \( \hat{y}_B \). When there is not distributive policy (\( X = 0 \)), the utility of the legislators only depends on the promoted public policy. In this situation, the best party \( A \) can do is to promote that public policy level free of transfer in each type of legislature (\( 2\hat{y}_{n_{A+I}} \) in the \( m+I \)-parties legislature, and \( 2\hat{y}_B \) in the 2-parties legislature). Since we have restricted ourselves to the case in which \( \hat{y}_B \in (\hat{y}_{n_{A+I}}, \hat{y}_A) \), party \( A \) is always better in the 2-parties than in the \( m+I \)-parties legislature.

Consider the case in which \( 2\hat{y}_{n_{A+I}} \leq w_{m+I} \leq 2\hat{y}_B \). We already know that it implies \( \sum_{i \in A} b_i < \sum_{i \in A} b_i \), which at the same time implies \( w_2 = 2\hat{y}_B \) (See propositions 1 and 2). Under these circumstances, in the 2-parties legislature \( y^*_{(2)} = 2\hat{y}_B \) and the distributive policy equals zero for those \( i's \notin A \). Since in the \( m+I \)-parties legislature, party \( A \) must offer a positive distributive policy to all those \( i's \notin A \) with \( 2\hat{y}_i < w_{m+I} \), and the equilibrium public policy is always smaller than \( 2\hat{y}_B \), then \( A \) prefers the 2-parties legislature.

With regards to the success of party \( A \) in promoting its preferred public policy, notice that in the two cases mentioned in proposition 3, \( y^*_{(m+I)} < y^*_{(2)} \). Thus, even though \( \hat{y}_A \) is not achieved, we conclude that party \( A \) is more successful in promoting its most preferred public policy in the 2-parties than in the \( m+I \)-parties legislature.

Concentrate in the case in which party \( A \) has incentives to promote \( \hat{y}_A \) in both types of legislatures (i.e. \( w_{m+I} = w_2 = \hat{y}_A \)). Later on, we shall discuss how these results can be extended to the case in which party \( A \) only has incentives to promote a public policy below \( \hat{y}_A \). We divide our analysis according to three sub-cases that can arise in the \( m+I \)-parties legislature: (i) \( \hat{y}_A \) is feasible; (ii) \( \hat{y}_A \) is unfeasible but \( x_i > 0 \ \forall i \notin A \); (iii) \( \hat{y}_A \) is unfeasible, and \( x_i > 0 \) for only some \( i's \notin A \). Notice that the sub-case in which \( X = 0 \) has been already studied in proposition 3.

**Proposition 4.** Assume \( w_{m+I} = w_2 = \hat{y}_A \) and feasibility of \( \hat{y}_A \) in the \( m+I \)-parties legislature (i.e. \( X \geq \sum_{i=0}^n b_i (\hat{y}_A - 2\hat{y}_j) \)), then:

(i) If \( \hat{y}_A \) is also feasible in the 2-parties legislature (i.e. \( X \geq (\hat{y}_A - 2\hat{y}_B) \sum_{i \in B} b_i \)), party \( A \) prefers the 2-parties to the \( m+I \)-parties legislature if and only if
\[ \hat{y}_b \geq \frac{\sum_{i \not \in A} b_i \hat{y}_i}{\sum_{i \not \in A} b_i} = \bar{y}_i, \text{ where } \bar{y}_i \in \left( \hat{y}_{n+1}, \hat{y}_n \right). \] Otherwise, party A prefers the \( m+1 \)-parties legislature.

(ii) If \( \hat{y}_A \) is unfeasible in the 2-parties legislature (i.e. \( X < (\hat{y}_A - 2\hat{y}_B) \sum_{i \in B} b_i \)), party A always prefers the \( m+1 \)-parties to the 2-parties legislature. Nevertheless, if \( \hat{y}_A \) is feasible in the \( m+1 \)-parties legislature and unfeasible in the 2-parties legislature, then \( \hat{y}_b < \bar{y}_i \).

Proposition 4(i) says that if \( \hat{y}_A \) is feasible in both types of legislatures, party A prefers the \( m+1 \)-parties to the 2-parties legislature if there is a high enough ideological polarization between party A and party B in the latter case. Whether the ideological polarization is high or low depends on the threshold \( \bar{y}_i \). Moreover, this threshold belongs to our interval of interest \( \left( \hat{y}_{n+1}, \hat{y}_n \right) \). Nevertheless, because of the feasibility assumption of \( \hat{y}_A \), party A is able to promote its preferred public policy in both types of legislatures.

The statement in (ii) follows from the fact that \( \hat{y}_A \) is only feasible in the \( m+1 \)-parties legislature. However, both the unfeasibility of \( \hat{y}_A \) in the 2-parties legislature and its feasibility in the \( m+1 \)-parties legislature implies \( \hat{y}_b < \bar{y}_i \), i.e. a high enough ideological polarization between parties A and B in the former type of legislature\(^{30} \). It also implies that party A is more successful in promoting its preferred public policy in the 2-parties than in the \( m+1 \)-parties legislature.

The difference between \( \hat{y}_A \) and \( \bar{y}_i \) can be understood as the maximum level of ideological polarization in the 2-parties legislature for which party A prefers this type of legislature to the \( m+1 \). We call to this difference the maximum level of polarization party A holds in the 2-parties legislature. Thus, if \( \hat{y}_A - \bar{y}_i \) is smaller than \( \bar{y}_i - \hat{y}_B \) (i.e. \( \bar{y}_i < \hat{y}_B \)), A always prefers the 2-parties to the \( m+1 \)-parties legislature. In proposition 5, we study the comparative static of \( \bar{y}_i \).

Notice that if \( b_i = 1 \ \forall i \not \in A \), then \( \bar{y}_i = \frac{\sum_{i \not \in A} \hat{y}_i}{m} \), which is the mean of the ideological position of those legislators who do not belong to party A. However, when this is not the case, \( \bar{y}_i \) can be above or below this mean depending on the combination of the implicated parameters (\( \hat{y}_i \)'s, \( b_i \)'s and \( m \)). In proposition 5, we study the comparative static of \( \bar{y}_i \).

**Proposition 5.** The threshold \( \bar{y}_i \):

(i) Increases as \( \hat{y}_i \) increases, \( i \not \in A \).

(ii) Decreases as \( b_i \) increases, \( i \not \in A \), if \( \hat{y}_i < \bar{y}_i \). Otherwise, it increases.

\(^{30}\) Notice that \( \hat{y}_b < \bar{y}_i \) does not imply unfeasibility of \( \hat{y}_A \) in the 2-parties legislature.
(iii) Decreases as \( m \) increases.

The results (i) and (ii) in proposition 5 are quite intuitive. Statement (i) says that, if the ideological position of a legislator who does not belong to party \( A \) becomes closer to the ideological position of the government party, then party \( A \) is willing to hold less polarization in the 2-parties legislature. It occurs because to bargain with this legislator in the \( m+1 \)-parties legislature is cheaper after the change than before the change.

Statement (ii) tells us that, if the relative valuation for the public policy of a legislator who has an ideological polarization far enough from \( \hat{y}_A \) (i.e. \( \hat{y}_i < \hat{y}_j \)) increases, then party \( A \) is willing to hold more polarization in the 2-parties legislature. The opposite is true if the relative valuation that increases corresponds to a less polarized legislator (i.e. \( \hat{y}_i > \hat{y}_j \)). In the former case, to negotiate in the \( m+1 \)-parties legislature with the involved legislator becomes more expensive after the change. The opposite happens in the latter case. It explains these results.

The most important result is in statement (iii). It says that the level of polarization that party \( A \) is willing to hold in the 2-parties legislature increases as the number of non-government parties (\( m \)) in the \( m+1 \)-parties legislature increases. This illustrates clearly the trade-off between number of parties and polarization in the negotiation process in a legislature.

Consider the second sub-case, i.e. that in which \( X \) is not enough to promote \( \hat{y}_A \), but all those legislators who do not belong to \( A \) receive a positive distributive policy offer \((\gamma^*_{(m+1)} \in (2\hat{y}_n, \hat{y}_A))\). Proposition 6 states the results for this situation.

**Proposition 6.** Assume \( w_{m+1} = w_2 = \hat{y}_A \), unfeasibility of \( \hat{y}_A \) in the \( m+1 \)-parties legislature, and \( 2\hat{y}_n < \gamma^*_{(m+1)} < \hat{y}_A \) (i.e. \( 2\sum_{i=a+1}^{n} b_i (\hat{y}_n - \hat{y}_i) \leq X < \sum_{i=a+1}^{n} b_i (\hat{y}_A - 2\hat{y}_i) \)), and so \( x_i > 0 \ \forall i \notin A \), then:

(i) If \( \hat{y}_A \) is also unfeasible in the 2-parties legislature (i.e. \( X < (\hat{y}_A - 2\hat{y}_B) \sum_{i \in B} b_i \)),

party \( A \) prefers the 2-parties to the \( m+1 \)-parties legislature if and only if \( \hat{y}_B \geq \hat{y}_j \).

Otherwise, party \( A \) prefers the \( m+1 \)-parties legislature.

(ii) If \( \hat{y}_A \) is feasible in the 2-parties legislature (i.e. \( X \geq (\hat{y}_A - 2\hat{y}_B) \sum_{i \in B} b_i \)), party \( A \)

always prefers the 2-parties to the \( m+1 \)-parties legislature. Nevertheless, if \( \hat{y}_A \) is unfeasible in the \( m+1 \)-parties legislature and feasible in the 2-parties legislature, then \( \hat{y}_B > \hat{y}_j \).

In this case, \( \hat{y}_A \) is unfeasible in the \( m+1 \)-parties but it can be either feasible or unfeasible in the 2-parties legislature. This implies that in the former legislature, party \( A \) spends the total amount of \( X \) in promoting the closest public policy to \( \hat{y}_A \), although it is not necessarily true in the latter case. Furthermore, regardless of the type of legislature, both \( x^*_{(2)} \) and \( x^*_{(m+1)} \) are strictly positive \( \forall i \notin A \).
Result (i) says that if \( \hat{y}_A \) is not feasible in both types of legislatures, party \( A \) prefers the \( m+1 \)-parties legislature if there is a high enough ideological polarization between party \( A \) and party \( B \) in the 2-parties legislature. If this is the case, then \( A \) can promote a closer policy to \( \hat{y}_A \) in the \( m+1 \)-parties than in the 2-parties legislature, i.e. \( y^*_j > y^*_i \).

Statement in (ii) follows from the fact that \( \hat{y}_A \) is only feasible in the 2-parties legislature. However, in this case the feasibility in the 2-parties legislature implies \( \hat{y}_B > \tilde{y}_j \). We have already study the threshold \( \tilde{y}_j \) in proposition 5.

Let us consider the last sub-case, i.e. that in which \( \hat{y}_A \) is unfeasible, and \( x_i > 0 \) for only some \( i’ \)’s \( \not\in A \).

**Proposition 7.** Assume \( w_{m+1} = w_2 = \hat{y}_A \), unfeasibility of \( \hat{y}_A \) in the \( m+1 \)-parties legislature, and \( 2\hat{y}_n < y^*_j \) (i.e. \( 0 < X = 2\sum_{i=n+1}^{n-l} b_i (\hat{y}_A - 2\hat{y}_j) \), and so \( x_i > 0 \) for only some \( i’ \)’s \( \not\in A \)), then:

(i) If \( \hat{y}_A \) is also unfeasible in the 2-parties legislature (i.e. \( X < (\hat{y}_A - 2\hat{y}_B) \sum_{i\in B} b_i \)), party \( A \) prefers the 2-parties to the \( m+1 \)-parties legislature if and only if

\[
\hat{y}_B \geq \frac{1}{2} \left( X + 2 \sum_{i=n+1}^{n-l} b_i \hat{y}_i \right) - \frac{X}{\sum_{i=n+1}^{n-l} b_i} = \tilde{y}_2.
\]

Otherwise, party \( A \) prefers the \( m+1 \)-parties legislature.

(ii) If \( \hat{y}_A \) is feasible in the 2-parties legislature (i.e. \( X \geq (\hat{y}_A - 2\hat{y}_B) \sum_{i\in B} b_i \)), party \( A \) always prefers the 2-parties to the \( m+1 \)-parties legislature. Nevertheless, if \( \hat{y}_A \) is unfeasible in the \( m+1 \)-parties legislature and feasible in the 2-parties legislature, then \( \hat{y}_B > \tilde{y}_j \).

The intuition behind the statements in proposition 7 is similar to that given for propositions 4 and 6. Nevertheless, there is a crucial difference with respect to our previous results. Unlike to the threshold \( \tilde{y}_j \), \( \tilde{y}_2 \) is not necessarily bounded by \( \hat{y}_n \) and \( \hat{y}_n \). It opens the possibility to observe either \( \tilde{y}_j < \hat{y}_n \) or \( \tilde{y}_2 > \hat{y}_n \). In the former case, party \( A \) always prefers the \( m+1 \)-parties legislature (because the condition in the statement (i) does not hold). In the latter case, party \( A \) always prefers the 2-parties legislature.

The comparative statics of \( \tilde{y}_2 \) is similar to that stated in proposition 5 for \( \tilde{y}_j \). We present it in the appendix. However, different to \( \tilde{y}_j \), \( \tilde{y}_2 \) also depends on \( X \). We show that \( \tilde{y}_2 \) increases as \( X \) increases. In other words, if \( X \) increases party \( A \) will hold less polarization in the 2-parties legislature. This occurs because of with the same increment
in $X$, party $A$ can increase $y^*_X$ more than $y^*_{X_j}$, i.e. a change in $X$ is more productive in the $m+1$-parties than in the 2-parties legislature.\textsuperscript{31}

The findings in propositions 4, 6 and 7, are summarized in table 2.

### Table 2
**Summary: $m+1$-parties versus 2-parties legislature when $w_{m+1} = w_2 = \hat{y}_A$**

<table>
<thead>
<tr>
<th>Condition</th>
<th>$\hat{y}_A$ unfeasible in the $m+1$-parties legislature</th>
<th>$\hat{y}_A$ feasible in the $m+1$-parties legislature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^*<em>{(m+1)} \in {2\hat{y}</em>{m+1}, 2\hat{y}_A}$</td>
<td>$A$ prefers the 2-parties legislature iff $\hat{y}_B \geq \hat{y}_I$</td>
<td>$A$ prefers the 2-parties legislature iff $\hat{y}_B \geq \hat{y}_I$</td>
</tr>
<tr>
<td>$y^*_{(2)} = \hat{y}_A$</td>
<td>$A$ always prefers the 2-parties legislature</td>
<td>$A$ always prefers the 2-parties legislature</td>
</tr>
<tr>
<td>If so $y^<em>_{(2)} \geq y^</em>_{(m+1)}$</td>
<td>It implies $\hat{y}_B \geq \hat{y}_I$</td>
<td>It implies $\hat{y}_B \geq \hat{y}_I$</td>
</tr>
<tr>
<td>$\hat{y}_B &lt; \hat{y}_I$</td>
<td>Here, $y^<em>_{(2)} \leq y^</em>_{(m+1)}$</td>
<td>Here, $y^<em>_{(2)} \leq y^</em>_{(m+1)}$</td>
</tr>
</tbody>
</table>

To end our analysis, let us briefly discuss how these results can be extended to the case in which $w_{m+1} \in \{2\hat{y}_{m+1}, \hat{y}_A\}$. When this is the case, we already know from proposition 2 that party $A$ is only willing to promote $2\hat{y}_B$ in the 2-parties legislature. Actually, we have already studied part of this case in proposition 3, for the situation in which $2\hat{y}_{m+1} \leq y^*_{(2)} < 2\hat{y}_B$ (See proposition 3). When $2\hat{y}_B < w_{m+1} < \hat{y}_A$, it is possible to find a threshold in the level of polarization of the 2-parties legislature for which party $A$ always prefers the $m+1$-parties legislature. However, since it does not add any new to our discussion, we do not present these results here.

### 4. Conclusions

In this paper we have analysed how both the number of parties and the level of polarization affect the policymaking process in a legislature. To study this relationship, we built a bargaining model for a unicameral legislature with closed agenda and only one session. In this session, legislators must decide on both a public policy and a distributive policy.

To study the effect of the number of parties on the legislative outcomes, we have considered two extreme cases: The $m+1$-parties legislature, in which the government

\textsuperscript{31} From 9 and 13, it follows that: $\delta y^*_{(m+1)}/\delta X = \int \sum_{m+1}^* b_i$ and $\delta y^*_{(2)}/\delta X = \int \sum_{m+1}^* b_i$ respectively. Actually, these are the slopes we depict in figures 3 and 4.
party has to negotiate with \( m \) sovereign legislators; and the 2-parties legislature, in which those legislators who do not belong to the government party form a unique political party \( B \). In this case, the members of party \( B \) commit to a unique position in the public policy dimension.

Our model is able to replicate the observation that not only the number of parties but also the ideological polarization matter when negotiating in legislatures. First, we find that there is a threshold in the level of polarization of the 2-parties legislature for which, beyond that, the government party always prefers to negotiate in an \( m+1 \)-parties legislature. Moreover, this threshold increases as the number of parties (\( m \)) in the \( m+1 \)-parties legislature increases. This illustrates clearly the trade-off between number of parties and polarization.

These results show how important is to take into account both the number of parties and the level of ideological polarization in any analysis about the effect of the former on the policymaking process. To explore the empirical implications of these findings is still an open issue. We leave that in the agenda for further research.

**Appendix**

**Proof of proposition 1.** First of all, notice that the level of \( y \) that \( A \) can promote free of transfers to those \( i \)'s \( \notin A \) is \( 2\hat{y}_{ni} \). Using condition 2 with equality, we are able to get \( x_{(m+1)}^* \), for those \( i \)'s \( \notin A \), as a function of the optimal public policy offer \( y_{(m+1)}^* \):

\[
x_{(m+1)}^* = b_i \left[ y_{(m+1)}^* - \hat{y}_i \right] - b_i \hat{y}_i.
\]

It is easy to prove that \( x_{(m+1)}^* > 0 \) if and only if \( y_{(m+1)}^* > 2\hat{y}_i \). Thus, we can write:

\[
x_{(m+1)}^* = \begin{cases} 
0 & \text{if } y_{(m+1)}^* \leq 2\hat{y}_i \\
\left[ b_i \left( y_{(m+1)}^* - \hat{y}_i \right) \right] & \text{if } 2\hat{y}_i < y_{(m+1)}^* \leq \hat{y}_d,
\end{cases} \quad \forall i \notin A
\]

We start by proving statement (i). Consider the pair of public policies \( \overline{y} \) and \( \underline{y} \in Y \), with \( \overline{y} > \underline{y} \). Without loss of generality, assume \( \overline{y} \) and \( \underline{y} \) are such that:

a) \( 2\hat{y}_q < y \leq 2\hat{y}_{q+1} \) for \( q = n_A + 1, \ldots, n-1 \), and \( q + 1 = A \) if \( q = n \) (i.e. in order to promote \( \underline{y} \), party \( A \) must offer a positive distributive policy to those legislators \( i \)'s \( \notin A \), such that \( i \leq q \)).

b) \( 2\hat{y}_r < \overline{y} \leq 2\hat{y}_{r+1} \), \( r \geq q \) (i.e. in order to promote \( \overline{y} \), party \( A \) must offer a positive distributive policy to those legislators \( i \)'s \( \notin A \), such that \( i \leq r \)).

Party \( A \) prefers \( \overline{y} \) to \( \underline{y} \) if and only if:

\[
\sum_{d \in \Delta} u_d \left( \overline{y}, x_{(m+1)}^* (\overline{y}) \right) \geq \sum_{d \in \Delta} u_d \left( \underline{y}, x_{(m+1)}^* (\underline{y}) \right) \tag{A2}
\]
Replacing the utility function, assuming feasibility, and using the fact that 
\[\sum_{i \in A} x_i^*(y) = X - \sum_{i \notin A} x_i^*(y);\] after some algebraic manipulation, condition A2 can be written as follows:

\[\left(\overline{y} - y\right)\sum_{i \in A} b_i \geq \sum_{i = n+1}^{q} x_i^*(\overline{y}) - \sum_{i = n+1}^{q} x_i^*(\overline{y}) \]  
(A3)

Using A1 and after some manipulation, condition A3 reduces to:

\[\sum_{i \in A} b_i \geq \sum_{i = n+1}^{q} b_i \left(\frac{y - 2\hat{y}_i}{\overline{y} - y}\right) \quad \text{for} \quad q < r; \quad \text{(A4)}\]

\[\sum_{i \in A} b_i \geq \sum_{i = n+1}^{q} b_i \quad \text{for} \quad q = r \quad \text{(A4')}

A4 and A4’ are the conditions stated in (i). Now, we prove statement (ii). Since the level of \(y\) that \(A\) can promote free of transfers to those \(i's \notin A\) is \(2\hat{y}_{n+1}\), then party \(A\) does not allocate resources in promoting any public policy if and only if:

\[\sum_{i \in A} u_i (2\hat{y}_{n+1}, x_i^*(\hat{y}_{n+1}); (y)) > \sum_{i \in A} u_i (y, x_i^*(\hat{y}_{n+1}); (y)), \quad \forall y \in (2\hat{y}_{n+1}, \hat{y}_A) \]  
(A5)

Without loss of generality, assume \(2\hat{y}_q < y \leq 2\hat{y}_{q+1}\) for \(q = n+1,...,n-1\); and if \(q = n\), then \(q + 1 = A\). Following the same steps we used above, condition A5 reduces to:

\[\sum_{i \in A} b_i < \sum_{i = n+1}^{q} b_i \left(\frac{y - 2\hat{y}_i}{\overline{y} - y}\right) \quad \text{(A6)}\]

**Proof of proposition 2.** In the 2-parties legislature, the level of \(y\) that \(A\) can promote free of transfers to those \(i's \notin A\) is \(2\hat{y}_B\). Using condition 6 with equality, we get

\[x_i^*(\hat{y}_{i(2)}) = b_i (y_i^*(\hat{y}_{i(2)}) - \hat{y}_B) \quad \text{if} \quad y_i^*(\hat{y}_{i(2)}) > \hat{y}_B, \quad \forall i \notin A \]  
(A7)

Then, we can write \(x_i^*(\hat{y}_{i(2)})\) for those \(i's \notin A\) as follows:

\[x_i^*(\hat{y}_{i(2)}) = \begin{cases} 
0 & \text{if} \quad y_i^*(\hat{y}_{i(2)}) \leq \hat{y}_B \\
 b_i (y_i^*(\hat{y}_{i(2)}) - \hat{y}_B) & \text{if} \quad 2\hat{y}_B < y_i^*(\hat{y}_{i(2)}) \leq \hat{y}_A, \quad \forall i \notin A \end{cases} \]

Thus, for any \(\overline{y} \in (2\hat{y}_B, \hat{y}_A)\) and \(y < \overline{y}\), party \(A\) prefers \(\overline{y}\) to \(y\) if and only if:

\[\sum_{i \in A} u_i (\overline{y}, x_i^*(\overline{y}) (y)) \geq \sum_{i \in A} u_i (y, x_i^*(y)) \]  
(A8)

In order to promote any \(y > 2\hat{y}_B\) in this legislature, party \(A\) has to offer \(x_i^*(y) > 0\) to all those \(i's \notin A\). Following the same steps we used before, this condition can be written as:
\[(\bar{y} - y)\sum_{i \in A} b_i \geq \sum_{i=1}^{n_A} x^*_i(\bar{y}) - \sum_{i=1}^{n_B} x^*_i(y) \]  

(A9)

Using A7 and after some manipulation, condition A9 reduces to:

\[\sum_{i \in A} b_i \geq \sum_{i=1}^{n_A} b_i = \sum_{i \in B} b_i \]  

(A10)

**Equilibrium characterization when** \(u_i(y, x_i) = -b_i|y - \hat{y}_i| + x_i\)

*The m+1-parties legislature:* First, assume \(w_{m+1} = \hat{y}_A\) (i.e. \(\sum_{i \in A} b_i \geq \sum_{i \in B} b_i\)). In this case, equation A1 represents the party \(A\)'s optimal distributive offer to those \(i \notin A\). From here, equation 10 follows. Summing up \(x^*_{i(m+1)}\) over those \(i \notin A\), one finds that party \(A\) requires \(X = \sum_{i=1}^{n_{n+1}} b_i (\hat{y}_A - 2\hat{y}_i)\) in order to promote \(\hat{y}_A\). For any \(0 < X < \sum_{i=1}^{n_{n+1}} b_i (\hat{y}_A - 2\hat{y}_i)\), the feasibility condition implies \(\sum_{i=1}^{n_{n+1}} x^*_{i(m+1)} = X\), where \(j\) is the legislator with the closest ideological position to \(\hat{y}_A\) who is receiving a positive distributive offer. Using this feasibility condition, equation 10 and the different intervals of \(X\), we get equation 9.

Following the same strategy, one can get the equilibrium offer when \(w_{m+1} = \bar{y} \in (2\hat{y}_{n+1}^*, \hat{y}_n)\). The optimal offer in this case can be written as:

\[y^*_{(m+1)} = \begin{cases} 
\frac{X}{b_{n_{n+1}}} + 2\hat{y}_{n_{n+1}} & \text{if } 0 \leq X < 2b_{n_{n+1}}(\hat{y}_{n_{n+2}} - \hat{y}_{n_{n+1}}) \\
X + 2\sum_{i=1}^{j} b_i \hat{y}_i & \text{if } 2\sum_{i=1}^{j} b_i (\hat{y}_j - \hat{y}_i) \leq X < 2\sum_{i=1}^{j} b_i (\hat{y}_{j+1} - \hat{y}_i) \\
\bar{y} & \text{if } X \geq 2\sum_{i=1}^{q-l} b_i (\hat{y}_q - \hat{y}_i) 
\end{cases} \]  

(A11)

\[x^*_{i(m+1)} = \begin{cases} 
0 & \text{if } y^*_{(m+1)} \leq 2\hat{y}_i \\
\frac{b_i (y^*_{(m+1)} - 2\hat{y}_i)}{\bar{y}} & \text{if } 2\hat{y}_i < y^*_{(m+1)} \leq \bar{y}, \forall i \notin A 
\end{cases} \]  

(A12)

Finally, when \(\sum_{i \in A} b_i < b_{n_{n+1}}\), from proposition 1 and its corollary, we already know that \(A\) promotes that level of \(y\) free of transfers to those \(i's \notin A\), i.e. \(w_{m+1} = 2\hat{y}_{n_{n+1}}\).

*The 2-parties legislature:* We proceed same as above. First, assume \(w_2 = \hat{y}_A\) (i.e. \(\sum_{i \in A} b_i \geq \sum_{i \in B} b_i\)). In this case, equation A7 represents the party \(A\)'s optimal distributive offer to those \(i's \notin A\). From here, equation 14 follows. Using it, and
summing up \( x^*_{i(2)} \) over those \( i' \)'s \( \not\in A \), one finds that party \( A \) requires
\[
X \geq (\hat{y}_A - 2\hat{y}_B) \sum_{i \in A} b_i
\]
in order to promote \( \hat{y}_A \). For any
\[
0 < X < (\hat{y}_A - 2\hat{y}_B) \sum_{i \in A} b_i,
\]
the feasibility condition implies \( \sum_{i \in A} x^*_{i(2)} = X \). Using
this feasibility condition and equation 14, we get equation 13. When \( \sum_{i \in A} b_i < \sum_{i \in A} b_i \),
from proposition 2 we already know that \( A \) promotes that level of \( y \) free of transfers to those \( i' \)'s \( \not\in A \), i.e. \( w_2 = 2\hat{y}_B \).

**Proof of proposition 3.** Assume \( X = 0 \). From equation 9, 11, 13 and 15, we know that
\( A \) promotes \( 2\hat{y}_{n+1} \) and \( 2\hat{y}_B \) in the 2-parties and the \( m+1 \)-parties legislature
respectively. Thus, party \( A \) prefers the 2-parties to the \( m+1 \)-parties legislature if and
only if
\[
\sum_{i \in A} b_i [2\hat{y}_B - \hat{y}_A] \geq \sum_{i \in A} b_i [2\hat{y}_{n+1} - \hat{y}_A].
\]
This inequality holds if and only if \( \hat{y}_B > \hat{y}_{n+1} \). Since \( \hat{y}_B = \sum_{i \in B} \theta_i \hat{y}_i \), with \( 0 < \theta_i < 1 \) \( \forall i \in B \), this inequality always holds.

Consider the case in which \( 2\hat{y}_{n+1} < w_{m+1} < 2\hat{y}_B \). From proposition 1, we already know
that if \( w_{m+1} < \hat{y}_A \), then \( \sum_{i \in A} b_i < \sum_{i \in A} b_i \). We also know that under these circumstances,
\( w_2 = 2\hat{y}_B = \hat{y}_{(2)}^* \). Thus, the promoted public policy in the \( m+1 \)-parties legislature is
smaller than the respective public policy in the 2-parties legislature. Moreover, the
optimal public policy in the 2-parties legislature does not implies a positive
distributive policy to those \( i' \)'s \( \not\in A \). Since in the \( m+1 \)-parties legislature party \( A \) must offer a
positive distributive policy to those \( i' \)'s \( \not\in A \) with \( 2\hat{y}_i < w_{m+1} \), then \( A \) always prefers the
2-parties to the \( m+1 \)-parties legislature.

**Proof of proposition 4.** Assume \( w_{m+1} = \hat{y}_A \) (i.e. \( \sum_{i \in A} b_i > \sum_{i \in A} b_i \)), and \( \hat{y}_A \) is feasible
in the \( m+1 \)-parties legislature (i.e. \( X \geq \sum_{i \in A} b_i (\hat{y}_A - 2\hat{y}_1) \)).

(i) Assume \( \hat{y}_A \) is feasible in the 2-parties legislatures, i.e. \( X \geq (\hat{y}_A - 2\hat{y}_B) \sum_{i \in A} b_i \).
Thus, party \( A \) prefers the 2-parties to the \( m+1 \)-parties legislature if and only if
\[
\sum_{i \in A} b_i \leq \sum_{i \in A} x^*_{i(2)} \leq \sum_{i \in A} x^*_{i(m+1)}.
\]
Using equations 10 and 14, and after some algebraic
manipulation, this condition holds if and only if \( \hat{y}_B \geq \sum_{i \in A} b_i \hat{y}_i / \sum_{i \in A} b_i = \hat{y}_i \). Now
we prove \( \hat{y}_i \in (\hat{y}_{n+1}, \hat{y}_n) \). Assume \( \hat{y}_i \geq \hat{y}_n \). This is true if and only if
\[
\sum_{i \in A} b_i \hat{y}_i \geq \sum_{i \in A} b_i \hat{y}_n.
\]
However, this inequality is never satisfied because \( \hat{y}_A \geq \hat{y}_i \)
\( \forall i \in A \), and \( \hat{y}_n > \hat{y}_{n+1} \). Now assume \( \hat{y}_i \leq \hat{y}_{n+1} \). This is true if and only if
\[
\sum_{i \in A} b_i \hat{y}_i \leq \sum_{i \in A} b_i \hat{y}_{n+1}.
\]
Once again, this inequality is never satisfied because
\( \hat{y}_{n+1} \geq \hat{y}_i \) \( \forall i \not\in A \), and \( \hat{y}_{n+1} > \hat{y}_n \).

(ii) Assume \( \hat{y}_A \) is unfeasible in the 2-parties legislatures, i.e. \( X < (\hat{y}_A - 2\hat{y}_B) \sum_{i \in A} b_i \).
Since \( y^*_{i(m+1)} = \hat{y}_A > y^*_{i(2)} \) and \( x^*_{i(m+1)} > x^*_{i(2)} = 0 \) \( \forall i \in A \), it follows that party \( A \)
always prefers the \( m+1 \)-parties legislatures. Both unfeasibility of \( \hat{y}_A \) in the 2-
parties legislatures and feasibility of \( \hat{y}_A \) in the \( m+1 \)-parties legislatures (the current case) imply \( \sum_{i=n+1}^n b_i(\hat{y}_A - 2\hat{y}_i) < (\hat{y}_A - 2\hat{y}_B)\sum_{i=n+1}^n b_i \). Simple algebraic manipulation shows that this inequality reduces to \( \hat{y}_B < \hat{y}_j \).

**Proof of proposition 5.** Consider \( \tilde{y}_j = \sum_{i \notin A} b_{j,i} / \sum_{i \notin A} b_i \), then:

(i) \( \frac{\partial \tilde{y}_j}{\partial \hat{y}_h} = \frac{b_h}{\sum_{i \notin A} b_i} > 0 \), \( h \notin A \)

(ii) \( \frac{\partial \tilde{y}_j}{\partial b_i} = \frac{\hat{y}_h \sum_{i \notin A} b_{i,h} - \sum_{i \notin A} b_{i} \hat{y}_i}{\left( \sum_{i \notin A} b_i \right)^2} \), \( \forall h \notin A \). Then, \( \frac{\partial \tilde{y}_j}{\partial b_i} < 0 \) if and only if \( \hat{y}_i < \frac{\sum_{i \notin A} b_{i,h} \hat{y}_h}{\sum_{i \notin A} b_i} = \tilde{y}_j \). If this inequality holds in the opposite way, then \( \frac{\partial \tilde{y}_j}{\partial b_i} > 0 \).

(iii) When \( m \) increases, both the nominator and the denominator of \( \tilde{y}_j \) increases. However, since \( \hat{y}_i \in (0,1) \) \( \forall i \), the latter will increase more than the former. Thus, \( \tilde{y}_j \) decreases.

**Proof of proposition 6.** Assume \( w_{m+1} = \hat{y}_A \) (i.e. \( \sum_{i \notin A} b_i \geq \sum_{i \notin A} b_i \)), and \( \hat{y}_A \) is unfeasible in the \( m+1 \)-parties legislature, but \( x^*_{i(m+1)} > 0 \) \( \forall i \notin A \), (i.e. \( 2\sum_{i=n+1}^{n-l} b_i(\hat{y}_n - \hat{y}_i) < X < \sum_{i=n+1}^n b_i(\hat{y}_A - 2\hat{y}_i) \)).

(i) Assume \( \hat{y}_A \) is unfeasible in the 2-parties legislature, i.e. \( X < (\hat{y}_A - 2\hat{y}_B)\sum_{i \notin A} b_i \). If it happens, then \( x^* = 0 \) \( \forall i \in A \) in both types of legislatures. Thus, party \( A \) prefers the 2-parties to the \( m+1 \)-parties legislature if and only if \( \sum_{i \notin A} b_i |y^*_{i(2)} - \hat{y}_A| \geq \sum_{i \notin A} b_i |y^*_{i(m+1)} - \hat{y}_A| \). It is easy to see that this condition holds if and only if \( y^*_{i(2)} \geq y^*_{i(m+1)} \). Using equations 9 and 13, this inequality can be written as \( \frac{X + 2\hat{y}_B \sum_{i=n+1}^n b_i}{\sum_{i=n+1}^n b_i} \geq \frac{X + 2\sum_{i=n+1}^n b_i \hat{y}_i}{\sum_{i=n+1}^n b_i} \). Simple algebraic manipulation shows that this inequality reduces to \( \hat{y}_B > \tilde{y}_j \).

(ii) Assume \( \hat{y}_A \) is feasible in the 2-parties legislature. Since \( y^*_{i(2)} = \hat{y}_A > y^*_{i(m+1)} \) and \( x^*_{i(2)} \geq x^*_{i(m+1)} = 0 \) \( \forall i \in A \), then it follows that party \( A \) always prefers the 2-parties legislature. Both feasibility of \( \hat{y}_A \) in the 2-parties and unfeasibility of \( \hat{y}_A \) in the \( m+1 \)-parties legislature imply \( \sum_{i=n+1}^n b_i(\hat{y}_A - 2\hat{y}_i) > (\hat{y}_A - 2\hat{y}_B)\sum_{i=n+1}^n b_i \). Simple algebraic manipulation shows that this inequality reduces to \( \hat{y}_B > \tilde{y}_j \).
Proof of proposition 7. Assume \( w_{m+1} = \hat{y}_A \) (i.e. \( \sum_{i \in A} b_i \geq \sum_{i \in A} b_i \)), \( \hat{y}_A \) is unfeasible in the \( m+1 \)-parties legislatures, and \( x_{i(m+1)}^* = 0 \) for at least one \( i \notin A \), i.e. \( 0 < X \leq \sum_{i=n+1}^{n-l} b_i (\hat{y}_A - 2 \hat{y}) \).

(i) If \( \hat{y}_A \) is also unfeasible in the 2-parties legislature, i.e. \( X < (\hat{y}_A - 2 \hat{y}_B) \sum_{i=1}^{l} b_i \), then \( x_{i(2)} = x_{i(m+1)} = 0 \) \( \forall i \in A \). Thus, party \( A \) prefers the 2-parties to \( m+1 \)-parties legislature if and only if \( \sum_{i \in A} b_i |\hat{y}_A - 2 \hat{y}_B| \geq \sum_{i \in A} b_i |y^*_{i(m+1)} - \hat{y}_A| \). This inequality is satisfied if and only if \( y^*_i \geq y^*_{i(m+1)} \). Using equations 9 and 13, this inequality can be written as \( X + 2 \hat{y}_B \sum_{i=n+1}^{m} b_i \geq X + 2 \sum_{i=n+1}^{m} b_i \hat{y}_i \), \( n_a + l \leq j \leq n - 1 \). Simple algebraic manipulation shows that this inequality holds if and only if \( \hat{y}_B > \frac{1}{2} \left( \frac{X + 2 \sum_{i=n+1}^{m} b_i \hat{y}_i}{\sum_{i=n+1}^{m} b_i} - \frac{X}{\sum_{i=n+1}^{m} b_i} \right) = \bar{\bar{y}}_2 \). Consider the boundaries of \( \bar{\bar{y}}_2 \). Notice that \( \bar{\bar{y}}_2 \geq \hat{y}_{n+1} \) if and only if \( \frac{X + 2 \sum_{i=n+1}^{m} b_i \hat{y}_i}{X + 2 \sum_{i=n+1}^{m} b_i \hat{y}_{n+1}} \geq \frac{\sum_{i=n+1}^{m} b_i}{\sum_{i=n+1}^{m} b_i} \). We cannot assure this condition always holds. Similar, \( \bar{\bar{y}}_2 \leq \hat{y}_n \) if and only if \( \frac{X + 2 \sum_{i=n+1}^{m} b_i \hat{y}_i}{X + 2 \sum_{i=n+1}^{m} b_i \hat{y}_n} \leq \frac{\sum_{i=n+1}^{m} b_i}{\sum_{i=n+1}^{m} b_i} \). Once again, we cannot assure this condition always holds.

(ii) The proof follows exactly the same steps we used in the proof of proposition 4(ii).

Comparative statics of \( \bar{\bar{y}}_2 \). Consider \( \bar{\bar{y}}_2 = \frac{1}{2} \left( \frac{X + 2 \sum_{i=n+1}^{m} b_i \hat{y}_i}{\sum_{i=n+1}^{m} b_i} - \frac{X}{\sum_{i=n+1}^{m} b_i} \right) \). Since the upper-limit of the sum in the first term of \( \bar{\bar{y}}_2 \) (\( j \)) is endogenous, we must be careful when analyzing how \( \bar{\bar{y}}_2 \) changes as the exogenous variables that affect it change. Thus, assume \( y^*_{i(m+1)} \in (2 \hat{y}_j, 2 \hat{y}_{j+1}) \) is far enough from its boundaries (i.e. the changes in the exogenous variables do not affect \( j \)), then \( \bar{\bar{y}}_2 \):

(i) Increases as \( \hat{y}_i \) increases, \( i \notin A \). Notice that \( \frac{\partial \bar{\bar{y}}_2}{\partial \bar{\bar{y}}_h} = \frac{b_h}{\sum_{i=n+1}^{m} b_i} > 0 \), \( h \notin A \)

(ii) Decreases as \( b_i \) increases, for \( i \notin A \) and \( i \leq j \), if \( \hat{y}_i < \frac{1}{2} \left( \frac{X + 2 \sum_{i=n+1}^{m} b_i \hat{y}_i}{\sum_{i=n+1}^{m} b_i} - \frac{X}{\sum_{i=n+1}^{m} b_i} \right) = \theta \), with \( \theta > \bar{\bar{y}}_2 \). Otherwise, it
increases. Computing the derivative we get:
\[
\frac{\partial \hat{y}_2}{\partial b_h} = \frac{1}{2} \left( 2 \hat{y}_h \sum_{i=2}^{j} b_i - X - 2 \sum_{i=2}^{j} b_i \hat{y}_i + \frac{X}{\left( \sum_{i=2}^{j} b_i \right)^2} \right), \quad \forall h \not\in A \text{ and } h \leq j.
\]

Simple algebraic manipulation shows that \( \frac{\partial \hat{y}_2}{\partial b_h} < 0 \) if and only if
\[
\hat{y}_h < \frac{1}{2} \left( \frac{X + 2 \sum_{i=2}^{j} b_i \hat{y}_i}{\sum_{i=2}^{j} b_i} - \frac{X}{\sum_{i=2}^{j} b_i \left( \sum_{i=2}^{j} b_i \right)} \right) = \theta, \quad \text{with } \theta > \bar{y}_2.
\]
If this inequality holds in the opposite direction, then \( \frac{\partial \hat{y}_2}{\partial b_h} > 0 \).

(iii) Increases as \( b_i \) increases, for \( i \not\in A \) and \( i > j \). Notice that \( \frac{\partial \hat{y}_2}{\partial b_h} = \frac{1}{2} \left( \frac{X}{\sum_{i=2}^{j} b_i \left( \sum_{i=2}^{j} b_i \right)} \right) > 0, \quad \forall h \not\in A \text{ and } h > j \).

(iv) Increases as \( X \) increases. Notice that \( \frac{\partial \hat{y}_2}{\partial X} = \frac{1}{2} \left( \frac{1}{\sum_{i=2}^{j} b_i} - \frac{1}{\sum_{i=2}^{j} b_i} \right) > 0. \)

These results are similar to those presented in proposition 5. In this case, the difference \( \hat{y}_A - \bar{y}_2 \) is understood as the maximum level of ideological polarization in the 2-parties legislature which for party \( A \) prefers the 2-parties to the \( m+1 \)-parties legislature. Thus, the effect of \( \hat{y}_i \) \((i \not\in A)\) on \( \bar{y}_2 \) has the same interpretation we gave to the statement (i) in proposition 5.

Apparently, the effect of \( b_i \) \((i \not\in A)\) on \( \bar{y}_2 \) is different to the effect of \( b_i \) \((i \not\in A)\) on \( \bar{y}_1 \) but it is not. It still says that, when the relative valuation for the public policy of a legislator who does not belong to party \( A \) and has a close enough ideological position to this \((i.e. \text{either } \hat{y}_i > \theta \text{ or } i > j)\) increases, then \( A \) is willing to hold less polarization in the 2-parties legislature. It is also important to notice that \( \theta \) is close to \( \bar{y}_2 \), although the former is always larger than the latter.

The only one difference with respect to proposition 5 is that, in the current case, the threshold \( \bar{y}_2 \) also depends on \( X \). If \( X \) increases, party \( A \) will hold less polarization in the 2-parties legislature. This occurs because of with the same increment in \( X \), party \( A \) can increase \( y_{(m+1)}^* \) more than \( y_{(2)}^* \). In other words, \( X \) is more productive in the \( m+1 \)-parties than in the 2-parties legislature.
References


