Expectations and Macroeconomics: Learning and Loose Commitment

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Abstract

The three chapters of this thesis analyze different issues regarding the role of expectations in macroeconomics. The first two chapters consider that private economic agents can either be rational and forward looking or may actually need to use past data to learn how to form expectations. The first two chapters try to describe and estimate the different dynamics that these two types of expectation mechanisms induce. The third chapter analyzes the interaction between policy makers and forward-looking rational agents. The policy is time-inconsistent in such setting, leaving room for the debate between discretion and commitment. The third chapter analyzes optimal policy when the policy maker only defaults on past promises under certain conditions. I now turn to describe each chapter in detail.

Chapter 1

The New Keynesian model with rational expectations unrealistically predicts that unanticipated credible disinflations can be achieved instantaneously and with no output costs. Learning techniques are quite suitable to analyze regime changes such as a disinflation. We set up a theoretical model where some agents are rational but an empirically plausible small proportion of private agents need to learn the behaviour of the economy. In this context, a permanent change in the inflation target leads inflation to respond sluggishly while the output gap is temporarily affected. The calibrated model explains quite well transition dynamics during the Volker disinflation.

Chapter 2

The modelling of expectations and its degree of backward looking behaviour are issues of main concern in the inflation dynamics research agenda. One approach in the literature is to use expectations from surveys as an approximation to actual expectations. We estimate the Phillips curve allowing for a simultaneous role of rational and survey expectations. We formulate both a generalization of the usual Phillips curve and a structural version. We find that marginal cost determines inflation while the role of detrended output appears to be problematic. We also find that survey expectations are an important component of inflation dynamics but they do not fully reflect economic agents' rationality. A significant level of rationality is found even when controlling for survey expectations. Our results suggest that taking survey expectations as representative of actual expectations is a doubtful methodology.

Chapter 3 (joint with Davide Debortoli)

Due to time-inconsistency or political turnover, policymakers' promises are not always fulfilled. We analyze policy problems combining commitment and discretion. We consider three settings where the planner occasionally defaults on past promises. In the first setting, a default may occur in any period with a given probability. In the second, a planner does not default during a finite tenure but disregards the promises of previous planners. In the third, we make the likelihood of default a function of endogenous variables. We formulate these problems recursively, and provide techniques that can be applied to a general class of models. Our method can be used to analyze the plausibility and the importance of commitment. In a fiscal policy application, we find that average allocations become closer to the discretion solution. We also discuss how welfare changes with the degree of commitment.

Contents

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Dedicated to my family and friends,
Specially dedicated to Irina,
my father Jaime,
my mother Rute,
my brother Pedro,
my grandmother Helena,
and my friends Ana Maria and Manuela Harthley.

Chapter 1

Learning the inflation target

1.1 Introduction

The New Keynesian (NK) model with Rational Expectations (RE) is widely used in modern monetary economics. This model allows for a short-run role of monetary policy since firms do not freely update prices every period. However, the NK model predicts that an unanticipated fully credible disinflation is accomplished with no output cost and that inflation jumps immediately to its new target. These results are clearly at odds with conventional wisdom. Ball (1994b) analyzes a panel of countries and shows that disinflations very frequently cause recessions, moreover inflation adjusts slowly to target.

We maintain the widely used NK model as our benchmark and we examine how the model's predictions change when a proportion of the private sector follows a learning algorithm. Learning, as in Marcet and Sargent (1989) and Evans and Honkapohja (2001), assumes that economic agents behave as econometricians because they have limited information about the underlying economic model. The learning literature is mainly devoted to analyze convergence properties of learning algorithms. Learning is not commonly employed as an expectations mechanism *per se*, instead it is seen as a refinement criteria for RE equilibria.[1] This paper uses learning techniques to explain transition dynamics during a disinflation. Assuming that the private sector learns is specially suited for regime changes since RE unrealistically assume that the private sector expectations catch up immediately.[2] However, we show that if all agents are *learners* then the economy takes too much time to converge to its new steady state. We believe that this fact has inhibited the use of learning techniques during regime changes. We show that if besides *learners*, rational agents are also present then convergence is faster and in line with the data. Other papers introduce the issue of heterogenous expectations, for instance Giannitsarou (2003) studies a variety of heterogenous forms in a learning framework. Coupling rational or forward looking agents with *learners* or backward looking agents in theoretical models is uncommon, an exception is Evans et al. (1993).

There are other arguments that motivate a departure from RE. Firstly, Estrella and Fuhrer (2002) show that RE induce counterfactual observations in a general class of macroeconomic models, which include the NK framework. Secondly, the econo-

[1]Some exceptions are Sargent (1999), Marcet and Nicolini (2003), Ferrero (2004), Primiceri (2005), Marcet and Nicolini (2005), Giannitsarou (2005).

[2]For instance Richard Clarida, Jordi Galí and Mark Gertler (1999), p.1703. expressed that: *"Finally, with a few exceptions, virtually all the literature ignores the issue of transition to a new policy regime. In particular, the rational expectations assumption is usually employed. Policy simulations thus implicitly presume that the private sector catches on immediately to any regime change. In reality, however, there may be a period of transition where the private sector learns about the regime change. This kind of scenario may be highly relevant to a central bank that has accommodated inflation for a sustained period of time but is intent on embarking on a disinflation. Modelling private sector learning is a challenging but nonetheless important task."*

metric evidence from the Phillips curve suggests that a backward looking component is found to be present. Modern macroeconomic models have been reluctant to assume departures from RE. The common arguments are that too many degrees of freedom are introduced, agent's expectations are inconsistent with the model and that expectations formation does not change with policy. We address the previous criticisms by considering a learning algorithm where no degrees of freedom are introduced, and the learning algorithm is endogenous to the model and to policy.

Galí and Gertler (1999) consider the NK model when the firms that update prices may compute optimal prices or simply use a rule of thumb. This formulation makes last period inflation to appear in the model's reduced form. The authors estimate the parameters in the model concluding that backward looking behavior is statistically significant but quite small. Indeed, the previous model implies that disinflations are too fast and almost costless. In our model the backward looking mechanism is a learning algorithm which is suitable for regime changes in general and for a disinflation episode in particular. The learning algorithm uses the same structural form of RE, being theoretically more defensible and providing a more robust justification of backward looking behavior. In addition, our learning algorithm satisfies a consistency criteria proposed in Marcet and Nicolini (2003), while Galí and Gertler (1999) fail to pass this requirement. On the empirical side Nunes (2005b) presents evidence that supports our approach.

In our model when a disinflation is pursued, a recession will take place while inflation is reduced gradually to target. If the inflation target is raised, the economy will experience a temporary boom while inflation rises to its new level. Besides explaining disinflation stylized facts we set the initial and final inflation target to match the Volcker disinflation, making the model dynamics comparable with the data.

The paper is structured as follows: section 2 describes the model, section 3 makes a preliminary evaluation, section 4 presents and evaluates the heterogenous expectations model, and section 5 concludes.

1.2 Description of the model

We use the NK model in reduced form as derived in Woodford (2003) and first suggested by Calvo (1983). Since the derivation of the model is available in the previous references we will just describe the reduced form equations. The Aggregate Supply (AS) curve is given by

$$\pi_t = \kappa z_t + \beta \tilde{E}_t \pi_{t+1} \tag{1.1}$$

where π_t denotes inflation, z_t denotes the output gap and $\tilde{E}_t$ denotes expectations, which may not be rational. It is usually introduced a shock term in Eq. (1.1) denoted by cost-push shock. The Investment-Saving (IS) curve is described as

$$z_t = \tilde{E}_t z_{t+1} - \sigma^{-1}(r_t - r_t^n - \tilde{E}_t \pi_{t+1}) \tag{1.2}$$

where r_t is the interest rate set by the central bank and r_t^n is the natural interest rate. Usually it is assumed that the natural interest rate follows an AR(1) process

$$r_t^n = \rho r_{t-1}^n + \varepsilon_t \tag{1.3}$$

The model is closed with an equation for the interest rate. We will assume a Taylor rule[3] of the type

$$r_t = \pi^* + \varphi_\pi(\pi_t - \pi^*) + \varphi_z z_t \tag{1.4}$$

where the constant term in the interest rate rule is set to make the inflation target π^* consistent in equilibrium. Plugging the interest rate rule in the IS equation and rearranging the system one obtains

$$y_t = a + b\tilde{E}_t y_{t+1} + \varkappa r_t^n \tag{1.5}$$

where $y_t = [z_t, \pi_t]'$, $a = \frac{1}{\sigma+\varphi_z+\kappa\varphi_\pi}[\pi^*(\varphi_\pi - 1), \kappa\pi^*(\varphi_\pi - 1)]'$, $\varkappa = \frac{1}{\sigma+\varphi_z+\kappa\varphi_\pi}[1, \kappa]'$ and

$$b = \frac{1}{\sigma + \varphi_z + \kappa\varphi_\pi}\begin{bmatrix} \sigma & 1 - \beta\varphi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \varphi_z) \end{bmatrix} \tag{1.6}$$

Taking the results of Blanchard and Kahn (1980), Bullard and Mitra (2002) find that if $\kappa(\varphi_\pi - 1) + (1 - \beta)\varphi_z > 0$ then there is a unique RE solution to the model described in Eq. (1.5). Note that Bullard and Mitra (2002) consider a model where $a = 0$. If a is a constant matrix, condition (1c) in Blanchard and Kahn (1980) must be satisfied. Since this condition is trivially met, uniqueness conditions are equivalent.

The Minimal State Variable (MSV) solution takes the form of $y_t = A + Cr_t^n$.[4] In the learning literature it is assumed that economic agents do not know the RE solution but do know the functional form of the MSV solution. At each moment in time, the private sector will use the available data to estimate A and C with a learning algorithm; we denote the estimates obtained at time t of A by A_t and of C by C_t. Therefore, at time t the private sector will think that the economy behaves as

[3] For details see Taylor (1993).

[4] For further details on MSV solutions see McCallum (1983).

$y_t = A_t + C_t r_t^n$, this equation is known as the Perceived Law of Motion (PLM). Given the estimates of the private sector, expectations are formed as $\tilde{E}_t y_{t+1} = A_t + C_t \rho r_t^n$. Inserting expectations in Eq. (1.5) and rearranging the terms to fit the functional form of the MSV solution yields $y_t = a + bA_t + (bC_t\rho + \varkappa) r_t^n$. The later equation describes the Actual Law of Motion (ALM) of the economy. The mapping from the PLM to the ALM is called the T-map, and in this model it is given by

$$T(A, C) = (a + bA, bC\rho + \chi) \tag{1.7}$$

where we dropped the time subscripts for convenience. The fixed point in the T-map is the RE solution, where the PLM and ALM are equal. So, we assume that agents can not directly solve for RE but know the functional form of the MSV and using a learning algorithm update A_t and C_t.

It is common in the learning literature to analyze if a given solution is E-stable. The E-stability concept means that at a given fixed point of the T-map the following equation is locally stable

$$\frac{d}{d\tau}(A, C) = T(A, C) - (A, C) \tag{1.8}$$

Frequently, if a solution is E-stable then the learning equilibrium converges to the RE solution. Bullard and Mitra (2002) compute the E-stability conditions for this model. It turns out that the condition for E-stability is equal to the condition for uniqueness.

We still did not describe the way that *learners* parameters are updated, we assume that private agents use a Recursive Least Squares (RLS) formula given by

$$\phi_t = \phi_{t-1} + R_t^{-1} x_{t-1}(y_{t-1} - \phi'_{t-1} x_{t-1})' * \alpha_{M,t} \tag{1.9}$$

$$R_t = R_{t-1} + \alpha_{R,t}(x_{t-1}x'_{t-1} - R_{t-1}) \tag{1.10}$$

$$\text{where } \phi_t = \begin{bmatrix} A_{z,t} & A_{\pi,t} \\ C_{z,t} & C_{\pi,t} \end{bmatrix} = \begin{bmatrix} A'_t \\ C'_t \end{bmatrix}, x_{t-1} = \begin{bmatrix} 1 \\ r^n_{t-1} \end{bmatrix}, \alpha_{M,t} = \begin{bmatrix} \alpha_{out,t} & 0 \\ 0 & \alpha_{inf,t} \end{bmatrix}.$$

If $\alpha_{R,t} = \alpha_{out,t} = \alpha_{inf,t} = t^{-1}$ RLS is equivalent to Ordinary Least Squares. It is also common in the literature to assume $\alpha_{R,t} = \alpha_{out,t} = \alpha_{inf,t} = p$, where $p \in (0,1)$. Assuming a constant tracking parameter means that recent observations are given more weight, such rules may be optimal under regime changes. We are considering a more general RLS algorithm than the ones considered in Evans and Honkapohja (2001). Our formulation allows different tracking parameters for updating the parameters related to the output and inflation equation. If there is a regime shift it will be optimal for *learners* to use higher tracking parameters, nevertheless the regime shift may not have equal consequences on the two regressions that *learners* perform. We will determine the optimal tracking parameters that *learners* use during the disinflation with the Internal Consistency (IC) analysis of Marcet and Nicolini (2003). Therefore, we do not want to restrict *learners* to use the same tracking parameter in different regressions.

1.2.1 The heterogeneous framework

Now we assume that aggregate expectations are a weighted average between expectations of *Learners* and *Rationals*

$$\tilde{E}_t y_{t+1} = \psi E_t^L y_{t+1} + (1-\psi) E_t^R y_{t+1} \tag{1.11}$$

Plugging Eq. (1.11) in Eq. (1.5) we obtain

$$y_t = a + b\psi E_t^L y_{t+1} + b(1-\psi) E_t^R y_{t+1} + \varkappa r_t^n \tag{1.12}$$

A fixed proportion of agents $1-\psi$ are rational and therefore do know how *learners* form expectations and the proportion of *learners* and *rationals* in the economy. Rational agents are aware that not everybody is rational and predict inflation and the output gap accordingly. If $\psi = 0$ then rational agents predict that inflation can jump immediately to its new target and that no recession occurs. On the other hand, if the proportion of *learners* is nearly one then *rationals* should predict as *learners* do and disinflations will be more costly.

The literature has proposed models, on empirical grounds, where the private sector has both backward and forward looking behavior.[5] In the model that we present in this section, part of the private sector uses a learning algorithm. In fact, this introduces a foundation for backward looking behavior, i.e. if agents do not have sufficient knowledge to compute RE, they can still use available data optimally to form expectations. Note that the tracking parameters will be determined using the IC requirement making our *learners* not to be purely backward looking.

[5]For instance see Fuhrer and Moore (1995a), Fuhrer (1997), Roberts (1997, 1998), Clarida et al. (1999) and Galí and Gertler (1999).

In practical terms it is the AS equation that is subject to empirical tests. Galí and Gertler (1999) argue correctly that the NK Phillips curve already assumes a relationship between marginal cost and the output gap. When deriving the NK Phillips curve an intermediate step yields an equation with marginal costs

$$\pi_t = \zeta mc_t + \beta E_t \pi_{t+1} \tag{1.13}$$

Using this equation it is shown that known problems in the literature with the NK Phillips curve disappear. The authors address the issue that some firms may have adaptive expectations and estimate the following equation

$$\pi_t = \zeta mc_t + \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} \tag{1.14}$$

where $\gamma_b + \gamma_f = 1$ if $\beta = 1$. The authors conclude that the fraction of firms with backward looking behavior is smaller than the ones with forward looking behavior, but statistically significant. According to Galí and Gertler (1999) the weight on backward looking behavior is estimated to be between 0.2 and 0.4.

Nunes (2005b) examines an equation similar to Galí and Gertler (1999) but where inflation expectations are an average between RE and expectations from the Survey of Professional Forecasters. Several authors such as Roberts (1997, 1998) and Carroll (2003a) pointed out to the importance of survey expectations to determine the dynamics of inflation. Nunes (2005b) reports the following results[6]

$$\pi_t = \underset{(0.0065)}{0.0260} mc_t + \underset{(0.0706)}{0.3432} S_t \pi_{t+1} + (1 - 0.34329) E_t \pi_{t+1} \tag{1.15}$$

[6] It has been imposed that the coefficients on survey and rational expectations sum to one. The results are very similar for the unrestricted estimation. The sample period goes from 1968 fourth quarter to 2004 second quarter. The equation is estimated with Generalized Method of Moments, the J-test is 0.6181 and there are no signs of weak instruments.

where $S_t\pi_{t+1}$ is the average inflation expectation from the Survey of Professional Forecasters. The results of Nunes (2005b) and Galí and Gertler (1999) are similar, the difference relies on the modelling of non rational firms. Figure 1.1 computes one quarter ahead expectations from the Survey of Professional Forecasters versus one quarter ahead GDP inflation. For most of the disinflation period, expectations systematically overpredict inflation. Moreover, figure 1.1 also shows that, in periods of rising inflation, expectations underpredict inflation; hence the overall picture is consistent with learning behavior. Also Branch and Evans (2005) argue that surveys can be accurately described by a learning algorithm. Hence, there is empirical evidence that expectations are heterogenous and that $\psi = 0.3$.

1.3 Model evaluation

In the eighties, inflation was considerably reduced as can be seen in figure 1.2. There is not a clear methodology to identify the beginning and the end of a disinflation. Ball (1994b), using a different detrending method from ours, reports that trough inflation occurred 16 quarters after peak inflation.[7] The author considers that the effects in the output gap lasted for 20 quarters. During this period the economy went into recession, as can be seen by observing the output gap on figure 1.3. Figure 1.4 computes the Federal Funds interest rate, which rose significantly in this period to reduce inflation. The sacrifice ratio is a common measure reported for disinflations, it is computed as the sum of the output gap divided by the change in inflation. The computation of the sacrifice ratio is quite sensitive to the methodology used. Ball

[7] The data appendix contains the description of the data series used throughout the paper.

(1994b) assumes that natural output would grow log-linearly from the start of the disinflation to the end. This methodology serves the authors purposes to identify the relative costs of different episodes in a systematic way. The sacrifice ratio computed with Ball (1994b) methodology and considering that the disinflation lasts for 16 quarters and 20 quarters is 0.89% and 1.38% respectively.[8] For the data series presented in this paper, and treating as zero the observations where the output gap is positive, the sacrifice ratio is 0.56%. In line with the literature we obtain natural output by using a band pass filter. Our sacrifice ratio is lower than what has been previously considered but it seems to us that our detrending method is closer to what we expect to obtain with a theoretical model.

We will analyze how the model behaves during the Volcker disinflation. We model the disinflation as a change in the central bank's inflation target. Primiceri (2005) explains why inflation rose in the seventies and fell in the eighties due to central bank learning. We do not have such goal in mind. Primiceri model can account for costly disinflations because a completely backward looking Phillips curve is assumed. Instead we focus on the learning side of firms and how such setup can induce empirically plausible disinflation patterns.

We will use the values of $\kappa = 0.1$ and $\sigma = 0.64$. These values are consistent with Nunes (2005b) and Woodford (2003).[9] We set $\rho = 0.35$ and $\beta = 1.0$ in accordance with the literature. For the interest rate rule we assume $\varphi_\pi = 1.5$ and $\varphi_z = 0.5$ as in "the" Taylor rule. For these parameters the solution is unique and E-stable.

[8]We are considering that output grows log-linearly during 16 quarters or 20 quarters and the change in inflation is 11.6%.

[9]Note that the value of $\kappa = 0.026$ in Eq. (1.15) has to be multiplied by four if we consider annualized inflation as we do. A similar adjustment was also performed for σ.

We assume that at the beginning of the disinflation the economy is at steady state and inflation is 15.3%, then the inflation target is lowered to 3.7%. These values correspond to filtered inflation 1980 first quarter and 1984 fourth quarter as can be observed in figure 1.2, using other values does not change the results. The NK model is linearized around a zero steady state value of inflation. If one would assume a disinflation from 11.6% to 0%, i.e. keeping the magnitude of the disinflation, then the simulated series would only differ by a constant. In accordance with the previous observation, in Ball (1994b) the initial level of inflation has no clear effect on the sacrifice ratio. We keep the option of a disinflation to a non-zero inflation target to make the model dynamics comparable with the Volcker disinflation.

1.3.1 The limiting cases

The RE model can be obtained by setting $\psi = 0$. Such model can not account for disinflation behavior. As Mankiw and Reis (2002) emphasize, in the NK model under RE an unanticipated credible disinflation results in an immediate reduction of inflation and there are no output costs. It is usually inferred that price stickiness is translated to inflation stickiness but such inference is not correct.

The other extreme case refers to $\psi = 1$, this is the model where the economy is solely inhabited by *learners*. To simulate this model we need to obtain the values for $\alpha_{inf}, \alpha_{out}$.[10] The IC criteria for this model implies that $(\alpha_{inf} = 0.5, \alpha_{out} = 0.5)$. We will examine this criteria in more detail in the model with heterogenous expec-

[10] For the estimation of the variance-covariance matrix the private sector is assumed to use a low tracking parameter of 0.05. It is optimal for *learners* to do so because even with the regime change the steady state values of this matrix are not altered. Assuming other values for the variance-covariance matrix tracking parameter does not change the results.

tations.[11] The parameters found in our analysis are higher than the values that the literature usually assumes, the main reason is that the large bulk of the literature does not focus on regime changes.

In figure 1.5, we plot the average inflation and output gap generated by 5000 simulations of the model. Qualitatively the model accounts well for what one would expect, inflation is reduced sluggishly converging to the new target and in the short run the economy experiences a recession. Doing the symmetric experience, i.e. raising the inflation target, inflation rises sluggishly to the target while the economy experiences a boom. The drawbacks of this first model is that convergence is too slow and that expectations are completely backward looking.

In order to obtain a more precise notion of convergence we will report some statistics. We will run the model 5000 times and compute the mean of all realizations. Firstly we will report the first time period when mean inflation and mean output gap differ by less than 0.005 from their steady state levels. Secondly, we will also report the following measure that takes into account volatility,

$$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-x^*)^2}{N}} = \sqrt[2]{\frac{\sum_{n=1}^{N}[(x_{t,n}-\bar{x}_t)^2+(\bar{x}_t-x^*)^2]}{N}} \tag{1.16}$$

where N is the total number of simulations, $x_{t,n}$ is the n-th realization at a chosen t and x^* is steady state value of x, $\bar{x}_t$ is the mean value across realizations at time t. For large enough t one expects the second term to be very small. We will report the computations for $t = 16$ and $t = 20$. For the previous model and for $N = 5000$, the following table summarizes convergence statistics

[11] For further details see the appendix on the IC criteria.

	out		inf	
	$t = 16$	$t = 20$	$t = 16$	$t = 20$
$\bar{x}_t$	-0.0615	-0.0535	0.1060	0.0963
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-\bar{x}_t)^2}{N}}$	0.0227	0.0232	0.0161	0.0167
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-x^*)^2}{N}}$	0.0655	0.0583	0.0709	0.0616
$t : \bar{x}_t - x_t < 0.005$	84		85	

Table 1

Both at $t = 16$ and $t = 20$ average inflation and average output are still far from steady state. The economy takes 85 quarters to reach its new steady state. These results broadly confirm that convergence is slow when compared with the Volcker disinflation. The limiting case of $\psi = 1$ can explain disinflations qualitatively but fails on a quantitative dimension.

1.4 Solving the heterogenous expectation model

The previous section showed that the limiting cases of pure rational expectations or pure learning do not provide satisfactory transition dynamics for the Volcker disinfaltion. The heterogeneous expectation model carries a non trivial solution, which we will explain in detail.

1.4.1 The i.i.d. case

We will start with a more simple case where the natural interest rate in the heterogenous expectation model is i.i.d.. For pure expositional purposes we also assume that learners use a unique tracking parameter. In this case the learning algorithm is simplified to

$$E_t^L y_{t+1} = E_{t-1}^L y_t + \alpha(y_{t-1} - E_{t-1}^L y_t) \tag{1.17}$$

In order to solve the rational agents problem we conjecture that the MSV solution takes the form

$$y_t = A + B E_t^L y_{t+1} \tag{1.18}$$

Hence

$$y_{t+1} = A + B E_{t+1}^L y_{t+2} \tag{1.19}$$

$$y_{t+1} = A + B(E_t^L y_{t+1}(1-\alpha) + \alpha y_t) \tag{1.20}$$

Since $E_t^R y_{t+1} = A + B(E_t^L y_{t+1}(1-\alpha) + \alpha y_t)$ the ALM will be

$$y_t = a + b\psi E_t^L y_{t+1} + b(1-\psi)A + b(1-\psi)B(E_t^L y_{t+1}(1-\alpha) + \alpha y_t) + \varkappa r_t^n \tag{1.21}$$

Rearranging terms

$$\begin{aligned} y_t = {} & (I - b(1-\psi)B\alpha)^{-1}[(a + b(1-\psi)A)] + \\ & + (I - b(1-\psi)B\alpha)^{-1}[(b\psi + b(1-\psi)(1-\alpha)B)E_t^L y_{t+1}] \end{aligned} \tag{1.22}$$

So to solve for rational agents we need to consider the following two equations

$$A = (I - b(1-\psi)B\alpha)^{-1}(a + b(1-\psi)A) \tag{1.23}$$

$$B = (I - b(1-\psi)B\alpha)^{-1}(b\psi + b(1-\psi)(1-\alpha)B) \tag{1.24}$$

The second equation is a quadratic matrix equation on B that can be solved using the generalized eigenvalues method.[12] After computing B solving for A is a trivial problem.

[12]For a discussion see Uhlig (1999).

The MSV solution changed when we introduced rational agents. Adaptive *learners* could then realize that their expectations are taken into account by *rationals*. This would make *learners* to estimate a different MSV and in its turn *rationals* would estimate again another MSV and ... this problem would be taken to infinity. To show this complication is not necessary we will argue that if *learners* behave as if *rationals* did not exist then their expectations do converge to an equilibrium. For this economy, convergence conditions under tracking can be computed by examining directly the learning algorithm.[13] The learning algorithm is

$$E_t^L y_{t+1} = E_{t-1}^L y_t + \alpha(y_{t-1} - E_{t-1}^L y_t) \tag{1.25}$$

Using Eq. (1.22)-(1.24) the equation just described can be written as

$$E_t^L y_{t+1} = (I(1-\alpha) + \alpha B)E_{t-1}^L y_t + \alpha A + \alpha(I - b(1-\psi)B\alpha)^{-1}\varkappa r_{t-1}^n \tag{1.26}$$

The previous system will be stable as long as the matrix $(I(1-\alpha)+\alpha B)$ has eigenvalues with absolute value smaller than one. The asymptotic mean of expectations is $Ey = (I-B)^{-1}A$ which corresponds to the RE equilibrium $[0,\pi^*]'$. The asymptotic variance is $vec\sum = (I-(I(1-\alpha)+\alpha B)\otimes(I(1-\alpha)+\alpha B))^{-1}vec(\alpha(I-b(1-\psi)B\alpha)^{-1}\varkappa\sigma(\alpha(I-b(1-\psi)B\alpha)^{-1}\varkappa)')$. Using the normality assumption for r_t^n one concludes $E_t^L y_{t+1} \sim N([0,\pi^*]', \sum)$.

[13]This approach closely follows Evans and Honkapohja (2001) section 3.3.

1.4.2 The autocorrelated shocks case - Approximate Linear Solution

In the last section, the fact that agents were only predicting a constant for output and inflation made the analysis of coupling *learners* and *rationals* easier. The technical difficulty with fully rational agents is that when predicting future variables, rational agents must take into account the learners' expectations formation process. Once *learners* estimate more than an average an explicit MSV where a fixed point exists is not so easily obtained. When the natural interest rate is autocorrelated as it is usually assumed in the NK model *learners* follow the algorithm Eqs. (1.9) - (1.10) which is no longer linear.

Let's first introduce a simplifying assumption that will enable an explicit solution. We make the assumption that *rationals* do not realize that learners parameters will change in the following period. For future reference we will denote these agents as *near rationals*. *Near rationals* will still be aware of regime shifts and if these agents would be the sole inhabitants of our NK economy disinflations would still be costless.

Near rationals will solve the following equation

$$y_t = A^R + B^R A_t^L + C^R r_t^n + D^R C_t^L r_t^n \tag{1.27}$$

where variables with R upper script are variables for *near rationals* and variables with L upper script are variables for learners MSV, $y_t = A_t^L + C_t^L r_t^n$. Expectations for *near rationals* are formed as $E_t^R y_{t+1} = A^R + B^R A_t^L + C^R \rho r_t^n + D^R C_t^L \rho r_t^n$, i.e. learners parameters A_t and C_t are not taken to evolve. So plugging these expectations back in the ALM yields

$$y_t = a + b(1-\psi)A^R + b(I\psi + (1-\psi)B^R)A_t^L + \\ + (b(1-\psi)C^R\rho + \varkappa)r_t^n + b(I\psi + (1-\psi)D^R)\rho C_t^L r_t^n \tag{1.28}$$

The solution must satisfy $A^R = (I - b(1-\psi))^{-1}a$, $B^R = (I - b(1-\psi))^{-1}b\psi$, $C^R = (I - b(1-\psi)\rho)^{-1}\varkappa$, $D^R = (I - b(1-\psi)\rho)^{-1}b\psi\rho$. The way that this problem was solved ensures that once learners converge to RE equilibrium so do *near rationals*.[14] Hence, the relevant question to be posed is whether *learner's* expectations will converge to equilibrium. The relevant T-map is

$$T(A^L, C^L) = (a + b(1-\psi)A^R + b(I\psi + (1-\psi)B^R)A^L, \\ b(1-\psi)C^R\rho + b(I\psi + (1-\psi)D^R)C^L\rho + \chi) \tag{1.29}$$

The fixed point in the T-map is

$$A^L = (I - b(I\psi + (1-\psi)B^R))^{-1}(a + b(1-\psi)A^R) \tag{1.30}$$

$$C^L = (I - b(I\psi + (1-\psi)D^R)\rho)^{-1}(\varkappa + b(1-\psi)C^R\rho)) \tag{1.31}$$

The fixed point for A^L and C^L corresponds to the RE equilibrium, namely $A^L = [0, \pi^*]'$. This result is not surprising, it means that the presence of *near rationals* does not alter the long run behavior of the economy.

E-stability is obtained if the matrices $b(I\psi + (1-\psi)B^R) - I$ and $b(I\psi + (1-\psi)D^R)\rho - I$ have all eigenvalues with negative real parts. The previous conditions can be written as $B^R - I$, $D^R - I$.

[14]Molnar (2004) used the solution method derived here and considered that the proportion of *near rationals* depends on their forecasting performance. Confirming our results, the author concluded that agents with more rationality make convergence faster.

For the parametrization previously considered one obtains

$$B^R = \begin{bmatrix} 0.2003 & -0.5216 \\ 0.0668 & 0.8261 \end{bmatrix}, D^R = \begin{bmatrix} 0.0585 & -0.0605 \\ 0.0077 & 0.1311 \end{bmatrix} \quad (1.32)$$

The previous matrices have all eigenvalues with real parts smaller than one, hence the fixed point is E-stable.[15] Our approximate solution is quite useful in three aspects. First, one can analyze the E-Stability conditions analytically. Second, when the economy converges *learners* parameters will not change and then our solution is exact. Thirdly, our analysis helped identify the form of a solution which we may carry for a non linear case.

1.4.3 The autocorrelated shocks case - Full non Linear Solution

The solution for *near rationals* is accurate in steady state but may perform poorly during a transition period. We will now get a solution for rational agents allowing for a different solution during the transition period. This is a standard procedure in models where the RE solution during a transition period may differ from the steady state solution, for instance see Marcet and Marimon (1992). We will apply a method of parameterized expectations described in Lorenzoni and Marcet (1999). We first choose as an initial guess for RE the *near rationals* solution. In a first step we use

[15] Evans and Honkapohja (1998) showed that E-stability implies local convergence of the learning algorithm in a class of models that contain the NK framework. The difference between this economy and the NK framework are the matrices that constitute the T-map. Evans and Honkapohja (1998) results can be applied to the model presented in this section. Also note that when the economy converges, as it is assumed in the E-Stability concept, *near rationals* do not commit mistakes, being completely rational. Convergence conditions under recursive least squares and tracking are not always the same; simulation based results suggest that if agents use a tracking algorithm the economy also converges to equilibrium.

the guess and carry out 1000 simulations of the disinflation period for 20 quarters. In a second step, we regress y_{t+1} on the regressors of the *near rationals* solution, $\{1, A_t^L, \rho r_t^n, C_t^L \rho r_t^n\}$. In the third step we form a new guess for the solution. The new guess is an weighted average between the previous guess and the parameters estimated in step 2. We repeat the previous steps until the guess and the estimated parameters are equal. The RE solution is a fixed point in the previously described algorithm.[16]

1.4.4 The tracking parameters

We will now proceed on explaining how we make the vector of the tracking parameters $\alpha = (\alpha_{inf}, \alpha_{out})$ endogenous, this avoids introducing additional degrees of freedom in the model and having agents who make big mistakes. We will employ the concept of Internal Consistency (IC) first introduced by Marcet and Nicolini (2003). Before introducing the formal concept we need some notation. Let $y_t(\alpha)$ denote the values generated in the economy when *learners* use α as tracking parameters, $E_t^{\bar{\alpha}} y_{t+1}(\alpha)$ denote *learners* prediction at t of y_{t+1} when *learners* in the economy use α as tracking parameters and the predictions are made using $\bar{\alpha}$. So, for a given time horizon T and a number close to zero $\varepsilon > 0$ the vector α is consistent if

$$\left(\frac{1}{T} \sum_{t=1}^{T} (y_{t+1}(\alpha) - E_t^{\alpha} y_{t+1}(\alpha))^2 \right) \leq \min_{\bar{\alpha}} E \left(\frac{1}{T} \sum_{t=1}^{T} (y_{t+1}(\alpha) - E_t^{\bar{\alpha}} y_{t+1}(\alpha))^2 \right) + \varepsilon \tag{1.33}$$

[16] We also tested for different functional forms of the solution for rational agents, being the results robust. In particular, we checked how the mistakes of *near rationals* could be improved by the regressors $1/t, 1/t^2$. This solution undoubtedly imposes continuity between the transition period solution and the *near rationals* solution.

That is to say, α is internally consistent if all *learners* use it and predictions made by these tracking parameters are good when compared with predictions made by other tracking parameters $\bar{\alpha}$. Note that different vectors of tracking parameters α are associated with a different solution for rational agents during the transition period. We chose the time horizon to be 16 or 20 quarters, which is the duration of the Volcker disinflation episode. We computed results for $\varepsilon = 0.00001$ which approximately corresponds to 1.5% and 1% of the MSE for output and inflation when the tracking parameters are $(\alpha_{inf} = 0.5, \alpha_{out} = 0.3)$. Robustness analysis can be found in the appendix. Expectations on Eq. (1.33) are computed by Monte Carlo integration using 1000 simulations. Even though it is common to assume a unique tracking parameter, during the disinflation the private sector may realize that the output gap is more stable than the level of inflation, hence the private sector may choose different tracking parameters for output and inflation. We present results for inflation in figure 1.6 and for output in figure 1.7. The horizontal axis represents the tracking parameters $(\alpha_{inf}, \alpha_{out})$, and the vertical axis represents $(\bar{\alpha_{inf}}, \bar{\alpha_{out}})$. Figure 1.6 and 1.7 show the results for $T = 20$ and $\varepsilon = 0.00001$. In these figures a value of 1 means that condition (1.33) is met, a value of 0 means that $(\bar{\alpha_{inf}}, \bar{\alpha_{out}})$ is inefficient given that all *learners* use $(\alpha_{inf}, \alpha_{out})$. If a 1 occurs in the diagonal it means that $(\alpha_{inf}, \alpha_{out})$ is internally consistent.

When predicting inflation, one can see on figure 1.6 that $\alpha_{inf} = 0.5$ is always optimal. On figure 1.7 we see that when α_{inf} is low $\alpha_{out} = 0.5$ predicts well and when α_{inf} is high $\alpha_{out} = 0.1$, being $\alpha_{out} = 0.3$ better for intermediate values of α_{inf}. The pair $(\alpha_{inf} = 0.5, \alpha_{out} = 0.3)$ is internally consistent. The robustness analysis showed that the pair ($\alpha_{inf} = 0.5, \alpha_{out} = 0.1$) can also be internally consistent. The first pair is more robust and the results almost do not change if we consider the second pair.

We also would like to know if the tracking parameters that we found in the IC analysis reflect the actual learning behavior during the Volcker disinflation. The IC is a theoretical concept that may induce different tracking parameters than the ones used in reality. In Eq. (1.15), Nunes (2005b) suggests that 0.3 of agents forecast as in the SPF. Branch and Evans (2005) and this paper suggest that surveys can reflect learning behavior. Using surveys we will obtain an indicative measure of the tracking parameter. When we regress inflation on inflation expectations, we obtain[17]

$$\pi_t = -0.005 + 1.113 S_{t-1}\pi_t \tag{1.34}$$

$$(0.003) \quad (0.08)$$

We can not reject unbiasedness over the full sample, we accept the hypothesis that the constant is zero and the coefficient on inflation expectations is one. The significance level of this test is 0.23. To obtain the tracking parameter we estimate equation $S_t\pi_{t+1} = (1-\alpha)S_{t-1}\pi_t + \alpha\pi_t$ and we obtain:[18]

$$S_t\pi_{t+1} = -0.000 + (1 - 0.175)S_{t-1}\pi_t + 0.175\pi_t \tag{1.35}$$

$$(0.000) \quad (0.043)$$

The previous regression suggests that private agents used an average tracking parameter of 0.175 over the full sample. However, it is likely that in periods of structural change the tracking parameter is higher. Since we are focusing on the Volcker dis-

[17]We are considering a Newey West correction of 12 lags.

[18]To check for robustness, we also estimated $S_t\pi_{t+1} = (1-\alpha)S_{t-2}\pi_{t-1} + \alpha\pi_{t-1}$ and the relevant results do not change.

inflation we repeat the analysis for the subsample 1980 first quarter to 1984 fourth quarter. This corresponds to 20 periods, as we considered in the IC analysis.

$$S_t\pi_{t+1} = 0.001 + (1 - 0.442)S_{t-1}\pi_t + 0.442\pi_t \qquad (1.36)$$

$$(0.001) \qquad (0.077)$$

Now the value of the tracking parameter is 0.442.[19]. This value is very close to 0.5 which is the value that we obtained with the IC criteria. It is not our claim that one can model survey expectations in such a simple way as we did here. Nevertheless, it is a comforting result that the estimated and the internally consistent tracking parameter are similar, drawing further evidence that our model is a good description of the Volcker disinflation.

Evaluating the heterogenous model

Figure 1.8 plots the average paths of inflation and output gap and figure 1.9 plots the interest rate, considering $\alpha_{out} = 0.3$ and $\alpha_{inf} = 0.5$. The sacrifice ratio assuming the disinflation lasts for 16 is 0.56%, a value quite similar to the computations in this paper but lower than the estimates of Ball (1994b). The economy where *learners* and *rationals* coexist converges in line with the data. There is still a temporary recession while inflation gradually moves to target, more importantly convergence is not too slow. The following table supports the previous claim, the economy is very near its new steady state in 15 quarters.

[19]The R^2 of this regression is 0.49 and the Durbin-Watson statistic is 2.3.

	out		inf	
	$t = 16$	$t = 20$	$t = 16$	$t = 20$
$\bar{x}_t$	-0.0035	-0.0011	0.0401	0.0382
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-\bar{x}_t)^2}{N}}$	0.0192	0.0194	0.0111	0.0107
$\sqrt[2]{\frac{\sum_{n=1}^{N}(x_{t,n}-x^*)^2}{N}}$	0.0195	0.0194	0.0115	0.0108
$t : \bar{x}_t - x_t < 0.005$	15		15	

Table 2

The learning literature has usually considered that *learners* use OLS or very low tracking parameters because the focus has not been on regime changes. If *learners* are only one third of the population and these agents use OLS or very low tracking parameters then our simulations show that the economy does not reach the new steady state in a plausible amount of time. This observation casts doubts on some recent papers in the literature that have taken OLS learning as a good description of reality.

1.4.5 Comparison with the literature

As already mentioned Galí and Gertler (1999) consider the hybrid formulation of the NK model. The main difference to our model relies on the formulation of backward looking behavior. We determine the tracking parameters with the IC criteria while using past inflation is not internally consistent. Moreover the empirical analysis of survey expectations supports our framework of a tracking parameter around 0.5. Nevertheless, we simulated the hybrid formulation of Galí and Gertler (1999) and concluded that their model can not explain disinflation. Output and inflation are back to target in 4 and 5 quarters respectively and the sacrifice ratio is 0.09%, an extremely low value when compared with the data.

Schorfheide (2005) assumes that the inflation target is subject to regime changes and estimates a NK model with Bayesian methods assuming that private agents have either full or partial information.[20] When private agents do not have full information they are assumed to use Bayesian learning. The author estimates the model under both specifications and concludes that the Bayesian posterior favors the full-information version of the model but the dynamics in the 80's are better captured by the delayed response of the learning specification. Schorfeide's model can not properly account for the Volcker disinflation because the option under learning can capture the sluggish decay of inflation in the 80's but predicts a positive output growth failing to account for the growth rates in this time period.

Erceg and Levin (2003) build a model to explain disinflation dynamics. The authors assume that wages and prices are determined by staggered four-quarter nominal contracts, capital is subject to quadratic adjustment costs, and the inflation target is both subject to transitory and persistence shocks. In addition, the private sector makes use of the Kalman filter to infer the value of the unobservable inflation target. The authors analyze the Volcker disinflation concluding that the model can account for the empirical observations in the data. Our paper maintains the widely used NK benchmark avoiding building a new model to explain disinflations. Erceg and Levin (2003) assumptions introduce additional degrees of freedom making it easier to match the model with the data. We do not introduce more degrees of freedom since the learning algorithm lies in the class with the same functional form of RE and the only free parameters are made endogenous to the model and to policy.

[20]For another related application of Bayesian learning see Andolfatto and Gomme (2003).

Ball (1995) explains disinflation dynamics by modeling the central bank to be non-credible. The credibility approach has only achieved limited success. Under RE the private sector can not make systematic mistakes, so on average the private sector will have correct beliefs about the central bank objectives. Consequently, during disinflations, recessions are as likely as booms, which is a counterfactual observation as Ball (1994b) describes. Combining imperfect credibility with staggered price adjustments yields the prediction that if credibility is sufficiently low a recession will always occur. However, even quite credible central banks (e.g. Germany) did not manage to avoid recessions when pursuing a disinflation. The previous observation casts doubt that credibility alone can explain disinflation dynamics.[21]

Mankiw and Reis (2002) consider a flexible price model where agents form expectations rationally but only revise them periodically. In such a setting, disinflations are costly but the assumption of flexible prices is crucial. The authors consider that economic agents face a fixed probability of being able to update their information set. In the NK model, firms face a fixed probability of being able to update prices and when doing so firms always have the most recent information set. If in Mankiw and Reis (2002) setting one would assume sticky prices then there would be no difference from the NK framework where disinflations are costless. Ball (1994a) considered a model where prices are not fixed and firms could choose a predetermined time-varying path for prices until the next adjustment, as in Mankiw and Reis (2002). Ball (1994a) showed that this feature would be an improvement upon previous specifications in his paper but remarked that "...time-varying prices are not a convincing explanation because they are uncommon in the real world" and that "Economists should aim for

[21] See Clarida and Gertler (1997).

a theory of disinflation that is consistent with the prevalence of fixed prices.”. The learning algorithm presented in this paper implies costly disinflations with sticky and flexible prices.

Sargent (1982) reports four cases of big (hyper)inflations that were stopped suddenly and argues that this experiences are consistent with RE. In accordance with the previous author, Ball (1994b) reports that when the inflation change is bigger the sacrifice ratio is smaller. Our model can account for the previous observations. For bigger changes in inflation the IC requirement will imply bigger tracking parameters reducing the cost of the disinflation. That is to say, in the Volcker disinflation *learners* may have adapted slowly while in big disinflations *learners* may have adapted very fast. In addition, the experiences reported in Sargent (1982) involved drastic policy changes in the exchange rate regime and fiscal stance, such notorious changes may also have led *learners* for a fast adaptation.

1.5 Conclusions

This paper analyzes the NK model under a disinflation when part of the private sector forms expectations using a learning algorithm. The NK model under RE is not able to account for the observed inflation persistence when an unanticipated credible disinflation is under way. This paper shows that when learning is introduced in the NK model, transition dynamics during an unanticipated disinflation become consistent with the data. Assuming that the private sector learns is specially suited for regime changes since RE unrealistically assume that the private sector expectations catch up immediately. Moreover, the learning mechanism uses the same functional form of RE providing a foundation for backward looking behavior. Using the internal consistency requirement makes tracking parameters to be endogenous, leaving no free parameters in the expectations formation process.

The literature has often concluded that inflation expectations are neither purely backward looking nor purely forward looking. To incorporate the previous observation we assumed that, an empirically plausible, small proportion of the private sector is not forward looking during the regime change. Aside from the empirical appeal of our formulation we show that the advantages also spill over to a theoretical formulation. Even though learning is suitable to analyze regime changes, convergence is usually too slow under this assumption. Our empirically consistent heterogenous framework solves this problem since a part of the private sector still learns but convergence is not too slow. Our model generates persistence in inflation and a recession, which are patterns observed during the Volcker disinflation, and in addition convergence to the new steady state is plausible.

We make a contribution to the literature by showing that learning techniques can be useful to describe transition dynamics. We managed to explain disinflation dynamics with the widely used NK model as our benchmark, we thus avoid introducing arbitrary features into the model to achieve our goal. Moreover, our results do not hinge on specific assumptions; we used the NK framework and our results also carry over to a less appealing flexible price model. Hence, this paper suggests a robust explanation for disinflation dynamics in general and for the Volcker disinflation in particular.

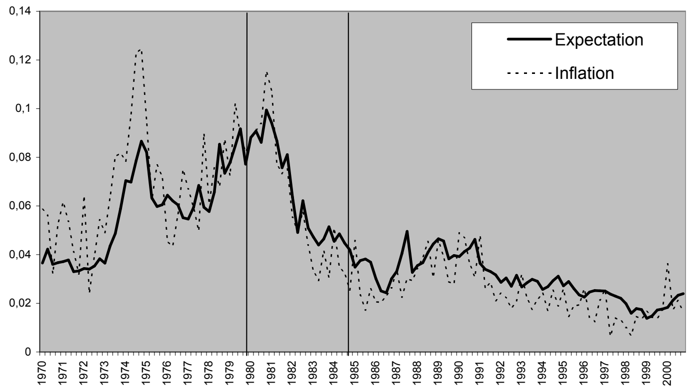


Figure 1.1: Survey of Professional Forecasters GDP deflator expectations

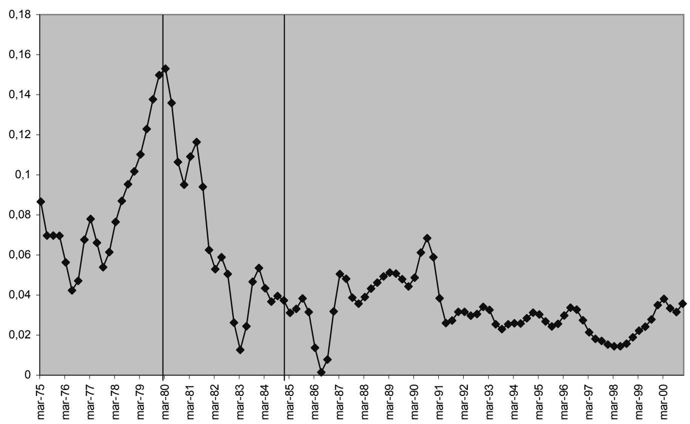


Figure 1.2: Inflation

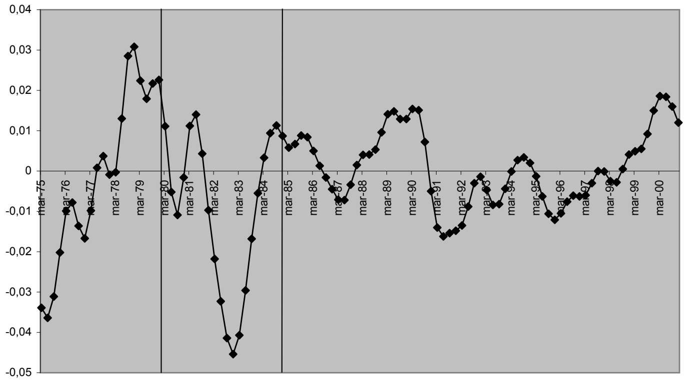


Figure 1.3: Output Gap

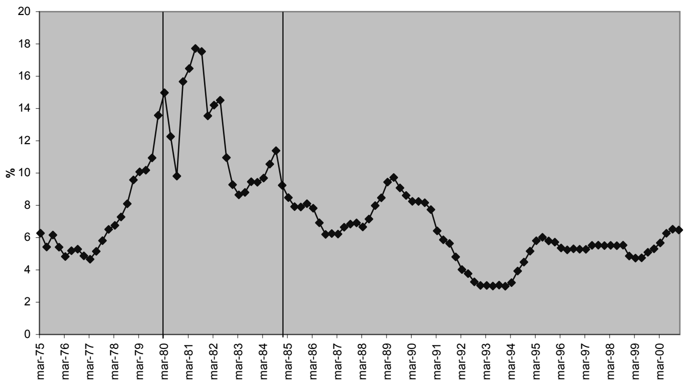


Figure 1.4: Federal Funds interest rate

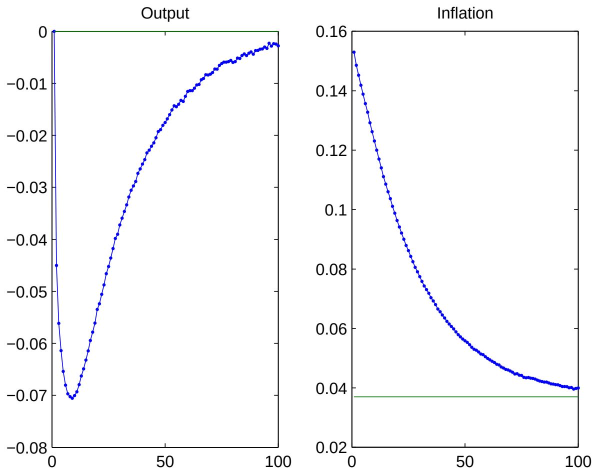


Figure 1.5: *Learners* economy - output and inflation.

out	inf																										sum
0,9	0,9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,9	0,7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,9	0,5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	25
0,9	0,3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,9	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	25
0,7	0,3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,5	0,9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,5	0,7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,5	0,5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	25
0,5	0,3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,5	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,3	0,9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,3	0,7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,3	0,5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	25
0,3	0,3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,3	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,1	0,9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,1	0,7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,1	0,5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	25
0,1	0,3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,1	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0,1	0,1	0,1	0,1	0,1	0,3	0,3	0,3	0,3	0,3	0,5	0,5	0,5	0,5	0,5	0,7	0,7	0,7	0,7	0,7	0,9	0,9	0,9	0,9	0,9	out
		0,1	0,3	0,5	0,7	0,9	0,1	0,3	0,5	0,7	0,9	0,1	0,3	0,5	0,7	0,9	0,1	0,3	0,5	0,7	0,9	0,1	0,3	0,5	0,7	0,9	inf

Figure 1.6: Internal Consistency Table for inflation.

out	inf																										sum
0,9	0,9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,9	0,7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,9	0,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,9	0,3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,9	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,7	0,1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0,5	0,9	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	0	0	0	0	8
0,5	0,7	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	0	0	0	0	8
0,5	0,5	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	0	0	0	0	8
0,5	0,3	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	0	0	0	0	8
0,5	0,1	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	0	0	0	0	8
0,3	0,9	0	1	1	0	0	0	1	1	1	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	11
0,3	0,7	0	1	1	0	0	0	1	1	1	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	11
0,3	0,5	0	1	1	0	0	0	1	1	1	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	11
0,3	0,3	0	1	1	0	0	0	1	1	1	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	11
0,3	0,1	0	1	1	0	0	0	1	1	1	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	11
0,1	0,9	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	10
0,1	0,7	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	10
0,1	0,5	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	10
0,1	0,3	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	10
0,1	0,1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	10
		0,1	0,1	0,1	0,1	0,1	0,3	0,3	0,3	0,3	0,3	0,5	0,5	0,5	0,5	0,5	0,7	0,7	0,7	0,7	0,7	0,9	0,9	0,9	0,9	0,9	out
		0,1	0,3	0,5	0,7	0,9	0,1	0,3	0,5	0,7	0,9	0,1	0,3	0,5	0,7	0,9	0,1	0,3	0,5	0,7	0,9	0,1	0,3	0,5	0,7	0,9	inf

Figure 1.7: Internal Consistency Table for output.

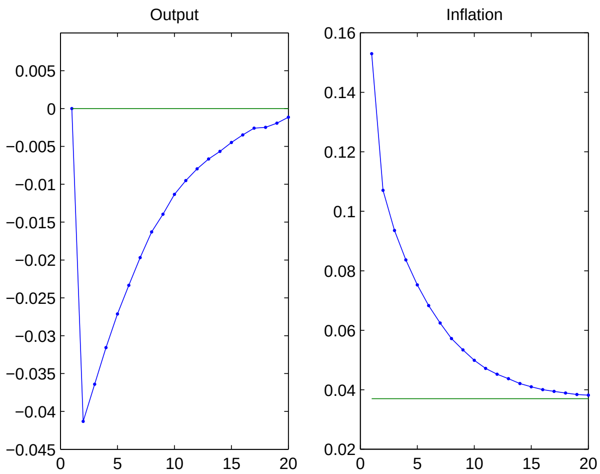


Figure 1.8: Learners and rationals economy

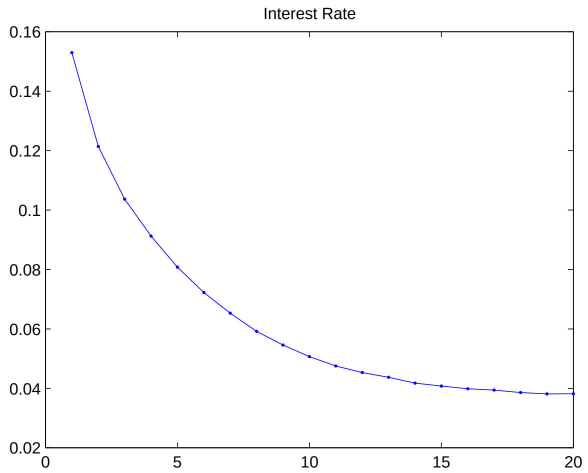


Figure 1.9: Learners and rationals economy

Chapter 2

Inflation Dynamics: The Role of Expectations

2.1 Introduction

The Phillips curve has strong implications for the design of monetary policy and the nature of business cycles. Not surprisingly it has received considerable attention from researchers. Despite the effort devoted to the understanding of inflation dynamics, the debate remains unsettled. Several research lines have been proposed but none has reached the success required to gather consensus.[1] The New Keynesian Phillips Curve (NKPC) is claimed to explain inflation dynamics more accurately than prior theories. Taylor (1980), Calvo (1983) and others stressed the role of staggered wage and price setting by forward looking individuals and firms. Changes in nominal variables have persistent effects on real variables, matching the empirical results of

[1]Goodfriend and King (1997) and Woodford (2003) provide a survey on the Phillips curve research.

VAR analysis.

At an empirical level the NKPC has been heavily tested.[2] In earlier studies, the equation relating inflation, expected future inflation and detrended output did not appear to be robust. The coefficient on detrended output was usually negative, which contradicts the theory. In addition, it was pointed out that the NKPC does not generate enough inflation persistence. The data shows that inflation only changes gradually; this observation is particularly evident during disinflations where inflation is reduced sluggishly. In the NKPC price stickiness is not translated into inflation stickiness, hence the inflation level can be changed instantaneously in sharp contrast with the empirical patterns.

The work of Galí and Gertler (1999) (henceforth GG) made an important contribution to reconciling the NKPC with the data. The authors emphasized that the New Keynesian setup considers marginal cost to be the relevant determinant of inflation. Under certain theoretical assumptions there is a direct link between marginal cost and the output gap. However, at an empirical level the link between marginal cost and the output gap is weak. Most studies that estimated the NKPC used some measure of detrended output. Therefore, the empirical failures of the NKPC could be due to the misuse of detrended output and not in the theory itself. The authors reestimated the NKPC using marginal cost and obtained quite satisfactory results. In addition, GG formulated a hybrid Phillips curve. Some firms were assumed to use a rule of thumb pricing decision while others were completely rational. The hybrid formulation was able to generate more inflation persistence than the usual NKPC.[3]

[2]Authors that estimate the NKPC include Fuhrer and Moore (1995b), Fuhrer (1997), Roberts (1995, 1997, 1998), Galí and Gertler (1999), Galí et al. (2001, 2003b).

[3]GG suggested that the proportion of backward looking firms was too small to explain disinflation

The proportion of backward looking firms was estimated to be small but statistically significant.

The outlook of the inflation dynamics literature suggests that certain deviations from RE may be desirable. The direct analysis of survey expectations shows that these are based on an intermediate degree of rationality. Combining the previous two observations, some authors were led to use survey expectations as representative of actual expectations.[4] When the Phillips curve is estimated with survey expectations the coefficient on output gap has the correct sign and is statistically significant, an outcome that is not easily obtained with RE. On one hand, the use of survey expectations is appealing since one does not need to take a stand on the exact rationality content of expectations or the process of their formation. On the other hand, the fact that expectations from surveys are not fully rational also inhibited their use since they go against the RE hypothesis.

This paper estimates the NKPC allowing for a simultaneous role of rational and survey expectations. We do not make an a priori judgement neither on the usefulness of surveys nor on the plausibility of RE, and instead we let the data speak. We estimate a generalization of the usual Phillips curve and a structural version in the spirit of GG. It is important to stress that our methodology is not a horse race on the forecasting performance between RE and surveys. By estimating a Phillips curve with two kinds of expectations our methodology estimates which type of expectations

behaviour. Nunes (2005a) uses the tools of Marcet and Sargent (1989) to introduce learning in the New Keynesian model during a disinflation. The author assumes that a small and empirically plausible proportion of agents are learners while others are rational. The model is shown to match the Volker disinflation dynamics quite well.

[4]Roberts (1995, 1997, 1998), Kozicki and Tinsley (2001), Erceg and Levin (2003), Carroll (2003b), Adam and Padula (2003), Mankiw et al. (2003) among others took this option.

reflect the pricing behavior of firms. Surveys may forecast very poorly but if surveys reflect firms expectations then our methodology should point out that surveys are a relevant variable in the pricing decisions. Even though our methodology indicates a less important role of RE than what GG had suggested, for most specifications RE appear to be more important than surveys. Nevertheless, we find that the weight of surveys is statistically significant, inducing inflation persistence in the NKPC. Our last finding is that the marginal cost appears to be a robust measure of economic activity while the output gap does not, confirming the results of GG.

Our results indicate that dismissing RE in favor of survey expectations is a flawed methodology. Despite the presence of survey expectations, the weight on RE is large and significant. We reach the important conclusion that true expectations have a degree of rationality that is not contained in surveys. Our results give empirical grounds to the claim that economic agents can have a high degree of rationality but face little or no incentives to accurately report their forecasts in surveys. There are other explanations that may be equally discouraging for surveys. The Phillips curve is based on the pricing decisions of individual firms. Survey respondents are asked to predict the overall change in prices. It may be easier for a firm to do its pricing decisions rationally than for a survey respondent to predict inflation. Each firm usually has very detailed information about its input and output markets making the pricing decision easier. Survey respondents face a much harder task since they must predict the aggregate evolution of a basket of prices without a specific knowledge of each price determinants. Since the Phillips curve is based on the pricing decisions of firms it can reflect high levels of rationality while surveys may not.

The paper is structured as follows: section 2 describes the NKPC and earlier empirical results, section 3 introduces surveys, section 4 estimates a generalization of the usual Phillips curve, section 5 describes the hybrid structural formulation, section 6 provides the robustness analysis, and section 7 concludes.

2.2 The New Keynesian Phillips Curve

The NKPC assumes an environment populated by monopolistically competitive firms that produce a differentiated product with a constant price elasticity. To induce frictions in the model it is assumed that firms face a constraint on price adjustment. Calvo (1983) framework avoids a cumbersome aggregation problem where it would be necessary to keep track of the price history of firms. It is assumed that in any given period each firm has a fixed probability $1 - \theta$ to adjust its price. This probability is independent of the last adjustment and other economic conditions. While this assumption is clearly a simplification to actual price setting behaviour, it allows for an explicit and tractable formulation. An alternative option is to consider that firms only set prices when the optimal and actual price differ by a certain amount. The latter formulation does not yield an explicit solution and has therefore not received as much attention as the Calvo setup.

It can be shown that under Calvo pricing, the aggregate (log) price level p_t is a combination of lagged price p_{t-1} and the optimal reset price p_t^* as the following equation describes:

$$p_t = \theta p_{t-1} + (1 - \theta)p_t^* \tag{2.1}$$

Denote the nominal marginal cost at t by mc_t^n and the subjective discount factor by β. Then for a profit maximizing firm facing the Calvo pricing rules, the optimal reset price may be expressed as:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{mc_{t+k}^n\right\} \tag{2.2}$$

In resetting price at time t, the firm takes into account the path of future nominal marginal costs, given the likelihood that its price may remain fixed. Combining equations 2.1 and 2.2 yields an equation for inflation of the form:

$$\pi_t = \lambda mc_t + \beta E_t\left\{\pi_{t+1}\right\} \tag{2.3}$$

where $\pi_t \equiv p_t - p_{t-1}$ and $\lambda \equiv (1-\theta)(1-\beta\theta)\theta^{-1}$. Under certain conditions there is a direct relationship between real marginal cost and the output gap[5]:

$$mc_t = \kappa x_t \tag{2.4}$$

where the output gap is the difference between output and the output level that would arise if prices were completely flexible. Using the previous relationship one can rewrite the NKPC as:

$$\pi_t = \lambda\kappa x_t + \beta E_t\left\{\pi_{t+1}\right\} \tag{2.5}$$

The previous equation performs poorly in the data, the estimated coefficient on the output gap is negative, which is in direct contradiction with the theory. According

[5]See Woodford (2003) for details.

to equation 2.4, the real marginal cost and the output gap should have a strong contemporaneous correlation. GG show that the marginal cost lags the output gap, casting doubt on equation 2.4 and hence on the relevance of detrended output in the NKPC. Figure 2 plots marginal cost and quadratically detrended GDP showing that the relation between the two variables is not simply captured by equation 2.4. It is also visible in the figure that marginal cost lags output. GG and Sbordone (2002) consider equation 2.3 and show that the NKPC performs quite well, once one takes into account that marginal cost is the relevant real activity measure as the theory suggests.

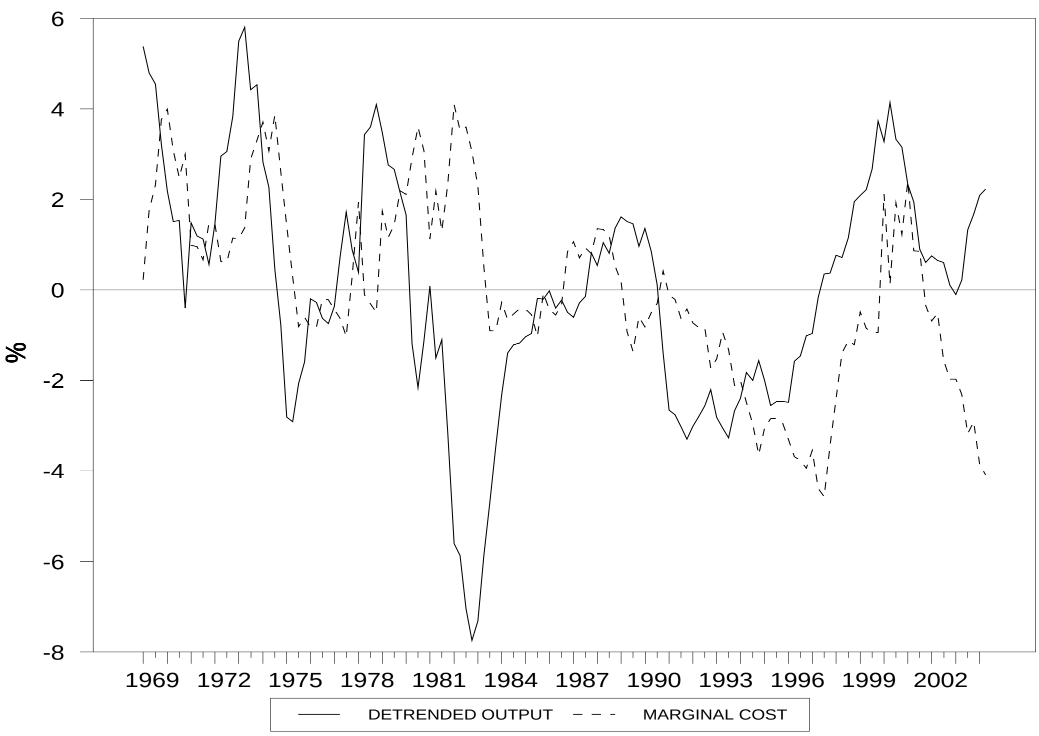


Figure 2.1: Detrended output and marginal cost

One undesirable feature of the NKPC is that the inflation level can change instantaneously without affecting economic activity. Any credible disinflation could be achieved without any output loss, which is a counterfactual observation as Ball (1994b) describes. Since even credible central banks experienced costly disinflations, the NKPC was heavily criticized in its inability to generate inflation persistence. The previous criticisms led researchers to propose the hybrid NKPC of the following kind:

$$\pi_t = \delta x_t + \gamma_f E_t \{\pi_{t+1}\} + \gamma_a \pi_{t-1} \tag{2.6}$$

where γ_f is the weight on forward looking expectations and γ_a is the weight on adaptive expectations. An implication of the hybrid formulation is that disinflations always cause recessions. The hybrid Phillips curve had not obtained large empirical success since it was estimated with the output gap as the relevant measure of economic activity. GG also estimate a hybrid version of the Phillips curve using the marginal cost instead of detrended output and obtain good results. The coefficient on marginal cost is positive and significant, the coefficient on lagged inflation is small but statistically significant.

2.2.1 Reviewing the Empirical Evidence

Before jumping to our econometric methodology and results we will describe how we obtain the marginal cost and we will provide a description of the Survey of Professional Forecasters.

The measure of marginal cost

The marginal cost is not directly observable, but using economic theory we can obtain an observable measure of marginal cost. A firm that minimizes cost will equalize the real marginal cost to the real wage divided by the marginal product of labor. Let $MC_{t,t+k}$ be the real marginal cost in $t+k$ for a firm that optimally resets price in t. Given the Cobb-Douglas technology the previous relation is described by:

$$MC_{t,t+k} = \frac{W_{t+k}/P_{t+k}}{(1-\alpha)(Y_{t,t+k}/N_{t,t+k})} \tag{2.7}$$

where $Y_{t,t+k}$ and $N_{t,t+k}$ are output and employment for a firm that optimally reset price in period t, and $1-\alpha$ is the exponent of labour in the Cobb Douglas production function. Since firm level data is not available it is not possible to compute the previous measure of marginal cost. Define the observable aggregate marginal cost in the following manner:

$$MC_{t+k} = \frac{W_t/P_t}{(1-\alpha)(Y_t/N_t)} \tag{2.8}$$

Under the assumptions of a Cobb-Douglas production technology and an isoelastic demand curve Sbordone (2002) obtains the following log-linear relation between $MC_{t,t+k}$ and MC_{t+k}:

$$mc_{t,t+k} = mc_{t+k} - \frac{\varepsilon\alpha}{1-\alpha}(p_t^* - p_{t+k}) \tag{2.9}$$

where $mc_{t,t+k}$ and mc_{t+k} are the log deviations of $MC_{t,t+k}$ and MC_{t+k} respectively. Following GG and Adam and Padula (2003) we will set $mc_{t,t+k} = mc_{t+k}$, and in a later section we will use alternative measures of marginal cost. In this case, the (log) marginal cost is simply the labour income share or equivalently the real unit labour costs. Following previous studies, we will use the labour income share in the non-farm business sector.

Survey Expectations

Survey expectations are computed using the data from the Survey of Professional Forecasters (SPF). As the title suggests, respondents of this survey are professional forecasters, those who make regular economic forecasts as part of their jobs. Around 80 economic institutions report their predictions for a wide range of economic variables on a quarterly basis. Croushore (1993) provides a detailed description of the survey. Carroll (2003b) reports that households' inflation forecasts use information from the SPF. His results suggest that the SPF influences the beliefs of a wide spectrum of economic agents'. Therefore, the survey is relevant to the overall expectations in the economy.

Survey participants report their GDP deflator forecast for the current and the following quarter. We compute survey expectations from the mean forecast for current and next quarter. GDP inflation is our measure of inflation throughout the paper since this is the variable that survey expectations try to predict. Survey expected inflation and current inflation are plotted in figure 2.

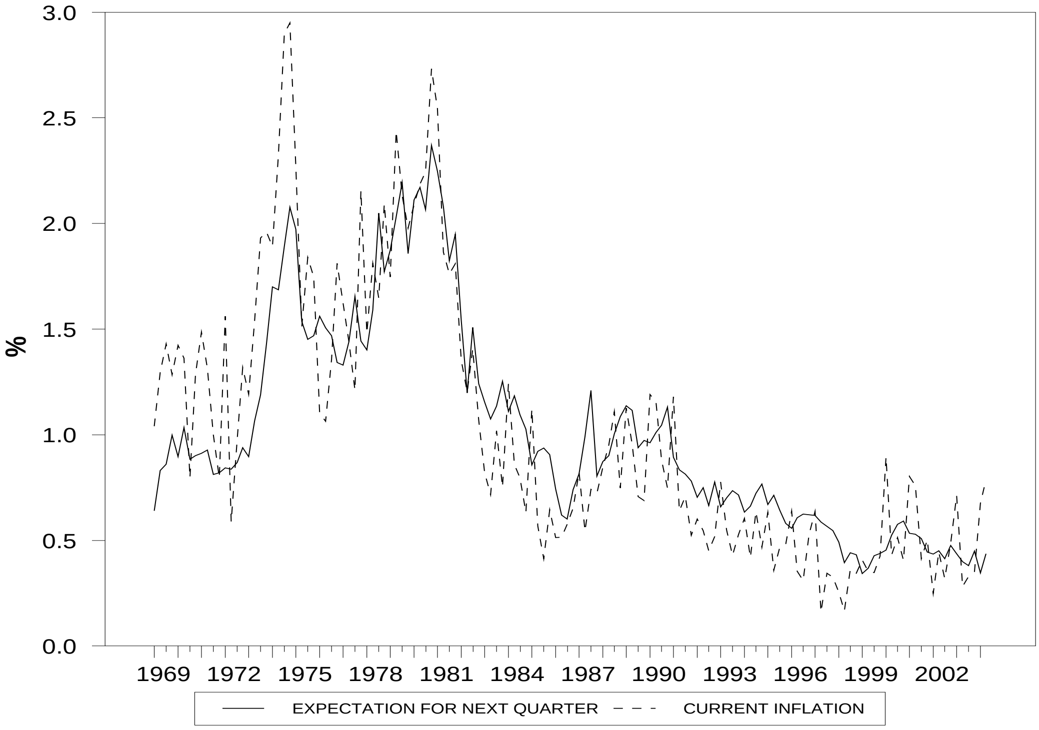


Figure 2.2: Inflation and Expected Inflation

Estimation Results and Methodology

Following earlier studies, we will estimate the forward looking and hybrid Phillips curve with detrended output and marginal cost by linear generalized method of moments (GMM). Using the law of iterated expectations in equation 2.5 and 2.6 we can write the orthogonality conditions that underlie GMM as:

$$E_t \left\{ (\pi_t - \delta x_t - \beta \pi_{t+1}) z_t \right\} = 0 \tag{2.10}$$

$$E_t \left\{ (\pi_t - \delta x_t - \gamma_f \pi_{t+1} - \gamma_a \pi_{t-1}) z_t \right\} = 0 \tag{2.11}$$

where z_t denotes the vector of instruments. Any variable dated t and earlier is a valid instrument, in the sense that it belongs to the information set of rational agents. Nevertheless, to allow for publication lags we will only include variables dated $t-1$ and earlier. Our set of instruments contains four lags of inflation and two lags of marginal cost, wage inflation, output gap and expected inflation.[6] As GG we use a Newey-West correction for the covariance matrix with a bandwidth of 12 lags. The output gap is computed by fitting a quadratic trend to GDP.[7] Our data sample goes from 1968:4 to 2004:02 where the starting date is determined by the availability of survey data.

Table 1 shows the GMM estimations of equation 2.5 and 2.6 using our described methodology. Equation 2.5 excludes a priori lagged inflation while equation 2.6 does not. In the first two rows, detrended GDP is included as a regressor while in the last two rows the marginal cost is used instead. As in earlier studies the first two rows in table 1 show the difficulties in obtaining a positive coefficient on the output gap. Accounting for a hybrid curve instead of a completely forward looking one does not change the disappointing negative sign on the output gap. The last column reports the p-value for the Hansen's J statistic of overidentifying restrictions, which confirms the validity of the regressions. An important contribution of GG was to point out that the marginal cost is the relevant measure of activity. Indeed, once we replace the output gap x_t by the marginal cost s_t the results improve.

[6]We also used two other alternative instrument sets in all regressions in this paper, being the results robust. The second instrument set includes four lags of inflation and two lags of marginal cost, wage inflation, output gap, expected inflation, interest rate spread and commodity price inflation. The third instrument set includes four lags of all the variables in the second instrument set. Our instrument sets are based on the paper of GG and Galí et al. (2001, 2003b).

[7]We also used the output gap obtained with the Hodrick-Prescott filter. The main results are unchanged.

Table 1

	$E_t\{\pi_{t+1}\}$	π_{t-1}	x_t	s_t	J-test
Detrended GDP					
	1.0296	-	-0.0161	-	0.7493
	(0.0197)		(0.0048)		
	0.7824	0.2322	-0.0078	-	0.6448
	(0.0929)	(0.091)	(0.0039)		
Marginal Cost					
	0.9927	-	-	0.0065	0.6926
	(0.0252)			(0.0090)	
	0.6620	0.3167	-	0.0098	0.6106
	(0.0552)	(0.0526)		(0.0060)	

2.3 Alternatives to Full Rationality

The NKPC with marginal cost may not be seen as completely satisfactory, since a plausible theory relating marginal cost and the output gap is necessary. In view of the empirical failures of the NKPC in both the forward looking and the hybrid form led researchers to question the RE hypothesis. The belief that economic agents do not make systematic mistakes has supported RE as a crucial assumption in modern macroeconomics. However, RE have been criticized and intermediate formulations of rationality have been proposed. In fact, the literature on inflation dynamics has quite frequently adopted some form of deviation from full rationality. Roberts (1995) estimates the Phillips curve using rational and survey expectations as alternative measures. When survey expectations are employed the quadratically detrended output gap is positive and significant, while under RE the output gap is insignificant. Fuhrer and Moore (1995b) use the relative real wage contracting hypothesis to formulate a sticky inflation Phillips curve, which is shown to perform well. As Roberts (1998) discusses, the sticky inflation equation is observationally equivalent to a hybrid Phillips curve.[8] Roberts (1997) uses a formulation that nests both the sticky prices and the sticky inflation hypothesis. The model is estimated taking survey expectations as actual expectations. The author brings evidence in favor of sticky prices. Roberts (1997) results suggest that the NKPC is valid but the RE assumption is too strong and that considering some form of sticky expectations is a possible solution. Fuhrer (1997) also adopts a hybrid formulation to explain short-run inflation dynamics. The author argues that the estimates of the output gap are not significant and the role of backward looking behaviour is found to be dominant. Roberts (1998) shows that survey expectations have rational and backward looking components. In addition, taking survey expectations as the true expectations in the economy enhances the fit of the Phillips curve. Kozicki and Tinsley (2001) estimate several competing Phillips curves using survey expectations. Mankiw and Reis (2002) consider a flexible price model where agents form expectations rationally but revise them only periodically. Erceg and Levin (2003) build a model to explain disinflation behaviour where economic agents use the Kalman filter to infer the unobservable inflation target. Survey expectations are used to calibrate the signal to noise ratio. Carroll (2003b) considers that households form expectations by absorbing information from professional forecasters. Survey expectations are also treated as the actual expectations and the author states that his formulation addresses the inflation persistence issue. Adam and Padula (2003) also follow the strategy of estimating the NKPC using survey expectations. The authors find that both the marginal cost and detrended output gap

[8]Holden and Driscoll (2003) show that Fuhrer and Moore (1995b) framework is not microfounded and once one takes the standard optimizing assumptions Fuhrer and Moore (1995b) formulation collapses to the usual Phillips curve.

are correctly signed and statistically significant. Estrella and Fuhrer (2002) show that RE induce counterfactual observations in a general class of macroeconomic models. The authors propose several solutions that induce some form of backward looking behaviour.

2.3.1 Using Survey Expectations

Estimations under RE show that lagged inflation is significant. This empirical finding could be present because actual expectations would be a mixture between rational and adaptive expectations. Expectations of inflation collected from surveys are found to be partially backward and partially forward looking, in agreement with the previous hypothesis.[9] Hence, if survey expectations represent actual inflation expectations then the Phillips curve would exhibit the desired inflation persistence. Moreover, Adam and Padula (2003) make reference to the known result in the literature that expectations from surveys are usually unbiased in large samples but are biased in small samples, i.e. survey expectations do not make use of all available information. The previous result makes the authors criticize GG results on the grounds that the usual orthogonality conditions of generalized method of moments under RE are no longer valid. The suggested equation to be estimated is:

$$\pi_t = \delta x_t + \beta S_t \{\pi_{t+1}\} \tag{2.12}$$

where $S_t \{\pi_{t+1}\}$ denote survey expectations for π_{t+1} formed at time t. The empirical studies that estimated the NKPC with surveys also found a role for lagged

[9] Examples of work in this spirit include Roberts (1998), Carroll (2003b), Mankiw et al. (2003), Branch (2004).

inflation that was not entirely captured in survey expectations. Taking into account this fact some authors estimate the following equation:

$$\pi_t = \delta x_t + \gamma_s S_t \{\pi_{t+1}\} + \gamma_a \pi_{t-1} \tag{2.13}$$

The estimation of equations like 2.12 and 2.13 met more empirical success than their RE counterparts. Table 2 reviews the estimates for both the marginal cost and the output gap. Overall, the results are quite good. Either considering the output gap or the marginal cost, the coefficient is positive and statistically significant as the theory implies.[10] It seems that the empirical failure of the NKPC would not simply lie in the link between the marginal cost and the output gap but in the assumption of RE.

[10]Even though some authors use instrumental variables or GMM to estimate the previous equations on the ground of measurement error, it is also theoretically correct to use ordinary least squares in the above equations. The results and the conclusions remain unaltered.

Table 2

	$S_t\{\pi_{t+1}\}$	π_{t-1}	x_t	s_t	J-test
Detrended GDP					
	1.1515	-	0.0505	-	0.5973
	(0.032)		(0.0052)		
	0.6461	0.4202	0.0386	-	0.5071
	(0.0752)	(0.0688)	(0.0046)		
Marginal Cost					
	0.9486	-	-	0.0678	0.5073
	(0.0355)			(0.0135)	
	0.5218	0.4039	-	0.0493	0.4932
	(0.0790)	(0.0756)		(0.0118)	

The content of Survey Expectations

We obtained very different results depending on the measure of expectations used. Up to now our results suggest that survey expectations do not correspond closely to RE. Since the respondents of this survey are professional forecasters one could expect that this survey would correspond very closely to RE. However, surveys are usually criticized on the grounds that respondents have no incentives to accurately report their forecasts. To have an idea of the content of surveys we will estimate an equation of the following type:

$$S_t\{\pi_{t+1}\} = (1-\psi)E_t\{\pi_{t+1}\} + \psi(\textit{learning mechanism}) \tag{2.14}$$

We assume that the SPF inflation forecast is an average of RE predictions and predictions based on a learning mechanism. If ψ is close to zero we would conclude that survey respondents are close to full rationality. We will estimate equation 2.14 with GMM. The orthogonality condition is given by:

$$E_t \{(S_t \{\pi_{t+1}\} - (1-\psi)\pi_{t+1} - \psi(\textit{learning mechanism}))z_t\} = 0 \tag{2.15}$$

Equation 2.14 is still very general because we did not specify the *learning mechanism* that we will employ. The learning literature as in Marcet and Sargent (1989) would suggest a learning scheme of the type:

$$S_t \{\pi_{t+1}\} = S_{t-1} \{\pi_t\} + \alpha(\pi_t - S_{t-1} \{\pi_t\}) \tag{2.16}$$

where α is a constant between zero and one. This learning scheme, also known as tracking, considers that expectations are updated using the last prediction error. RE or learning schemes induce different transition dynamics, such differences are very noticeable during regime changes.[11] If one does not want to include variables from period t then we can consider a learning scheme as:

$$S_t \{\pi_{t+1}\} = S_{t-2} \{\pi_{t-1}\} + \alpha(\pi_{t-1} - S_{t-2} \{\pi_{t-1}\}) \tag{2.17}$$

We estimated equation 2.14 using different specifications. In the learning component of survey expectations we first included the variables suggested in equations 2.16 or 2.17. We also considered different specifications by including more variables in the learning component. For all specifications and all instrument sets ψ is very frequently bigger than 0.75. This means that surveys are quite different from RE. Confirming Nunes (2005a) and Branch and Evans (2005) our results also suggest that expectations can be properly modelled as a learning algorithm.[12]

[11] For a discussion of these issues with an application to the Volker disinflation see Nunes (2005a)

[12] This evidence is contrary to Roberts (1998). For a further discussion on this issue and an examination of the Michigan survey see Nunes (2005c).

2.4 Rational or survey expectations?

Using expectations from surveys seems rather encouraging, nevertheless the choice of surveys over RE remains undiscussed. The RE hypothesis has been one of the bedrocks of modern economics and dismissing such a widely employed assumption should not be done trivially. The discussion about the proper modelling of expectations is unsettled. On the one hand, RE supporters dismiss survey expectations on the grounds that these frequently fail to pass rationality tests and that there are no incentives for participants to reveal their true beliefs. On the other hand, survey supporters claim that inflation dynamics can not be properly modelled under RE and that estimations with surveys solved some crucial problems in the NKPC. To solve these disputes we let the data speak and allow for a simultaneous role of rational and survey expectations. Using linear GMM we will estimate the following specification:

$$\pi_t = \delta x_t + \gamma_f E_t \left\{\pi_{t+1}\right\} + \gamma_s S_t \left\{\pi_{t+1}\right\} \tag{2.18}$$

Equation 2.18 is not a horse race on the forecasting performance of surveys and RE. By considering a Phillips curve we try to understand which type of expectations are representative of the firms pricing decisions. In table 3, the first and the second rows use the quadratically detrended output gap while the third and fourth rows use the marginal cost. The second and the fourth rows are estimated with the restriction $\gamma_f + \gamma_s = 1$. The weight on RE is dominant while the weight on survey expectations is small but statistically significant. The estimated coefficient on the output gap is negative while the estimated coefficient on the marginal cost is positive. These results are in accordance with GG but are contradictory to the papers that used survey expectations. Our results indicate that estimating the Phillips curve using only survey expectations can be misleading.

Table 3

	$E_t\{\pi_{t+1}\}$	$S_t\{\pi_{t+1}\}$	x_t	s_t	J-test
Detrended GDP					
	0.8639	0.1955	-0.0044	-	0.6633
	(0.0981)	(0.1156)	(0.0072)		
$\gamma_f + \gamma_s = 1$	0.9828	0.0172	-0.0145	-	0.7101
	(0.0768)	(0.0768)	(0.0055)		
Marginal Cost					
	0.6885	0.3317	-	0.0205	0.6181
	(0.077)	(0.0678)		(0.0098)	
$\gamma_f + \gamma_s = 1$	0.6568	0.3432	-	0.0260	0.6875
	(0.0706)	(0.0706)		(0.0065)	

Note: for the first-stage regression the F-statistic is 45 with a p-value of 0.00. There are no signs of weak instruments since the F-statistic is well above values proposed in Stock et al. (2002).

GG report that lagged inflation appears in the Phillips curve with a small but statistically significant coefficient. To check the robustness of our findings we will also allow for a role of lagged inflation. To do so we propose the following specification:

$$\pi_t = \delta x_t + \gamma_f E_t\{\pi_{t+1}\} + \gamma_s S_t\{\pi_{t+1}\} + \gamma_a \pi_{t-1} \tag{2.19}$$

Table 4 shows that the main conclusions remain unchanged. The weight on RE is dominant and the marginal cost is a robust measure of economic activity while the output gap is not.[13]

[13]When we use detrended output by employing the Hodrick-Prescott filter the coefficient is negative in the unrestricted and restricted specification.

Table 4

	$E_t\{\pi_{t+1}\}$	$S_t\{\pi_{t+1}\}$	π_{t-1}	x_t	s_t	J-test
Detrended GDP						
	0.6624	0.145	0.2299	0.0011	-	0.536
	(0.0932)	(0.0980)	(0.0925)	(0.0058)		
$\gamma_f+\gamma_s+\gamma_a=1$	0.7122	0.031	0.2567	-0.0042	-	0.601
	(0.0872)	(0.0583)	(0.0895)	(0.0046)		
Marginal cost						
	0.5593	0.1998	0.2382	-	0.0182	0.5024
	(0.0766)	(0.0691)	(0.0823)		(0.0078)	
$\gamma_f+\gamma_s+\gamma_a=1$	0.5568	0.2051	0.2381	-	0.0185	0.5946
	(0.0664)	(0.0693)	(0.0794)		(0.0053)	

2.5 Structural Formulation of the Hybrid Phillips Curve

The regressions considered in the previous section where rational and survey expectations are present constitute a generalization of the usual Phillips curve. Nevertheless, our methodology did not enable us to estimate the deep parameters of the economy. This section will follow the methodology of GG in deriving a structural hybrid Phillips curve.

The aggregate price level evolves according to:

$$p_t = \theta p_{t-1} + (1-\theta)\bar{p}_t^* \tag{2.20}$$

where $\bar{p}_t^*$ is an index of prices reset at t. A proportion $(1-\omega)$ of firms are forward looking and set prices p_t^f, while the remaining ones are backward looking and set prices p_t^b. Hence the index of reset prices is given by:

$$\bar{p}_t^* = (1-\omega)p_t^f + \omega p_t^b \tag{2.21}$$

Forward looking firms behave as before and set prices as:

$$p_t^f = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{mc_{t+k}^n\right\} \tag{2.22}$$

In GG backward looking firms follow a rule of thumb where prices set in period t only depend on information from period $t-1$ and earlier. These firms are assumed to be able to observe average reset prices from last period. The observed reset prices should then be adjusted for inflation. Since only information from period $t-1$ and earlier is available, lagged inflation is used as a proxy for current inflation. Therefore, prices set by backward looking firms obey the following rule:

$$p_t^b = \bar{p}_{t-1}^* + \pi_{t-1} \tag{2.23}$$

By combining equations 2.20 - 2.23 one can obtain a Phillips curve of the form:

$$\pi_t = \lambda mc_t + \gamma_f E_t\left\{\pi_{t+1}\right\} + \gamma_b \pi_{t-1} \tag{2.24}$$

where

$$\lambda \equiv (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1} \tag{2.25}$$

$$\gamma_f \equiv \beta\theta\phi^{-1}$$

$$\gamma_b \equiv \omega\phi^{-1}$$

$$\phi \equiv \theta + \omega\left[1-\theta(1-\beta)\right]$$

Equation 2.24 can be estimated by nonlinear GMM. Nonlinear GMM is sensitive to the normalization and hence GG use two methods, in method 1 the inflation coefficient is not normalized while in method 2 it is. Equations 2.26 and 2.27 correspond to method 1 and 2 respectively.

$$E_t \{(\phi\pi_t - (1-\omega)(1-\theta)(1-\beta\theta)mc_t - \beta\theta\pi_{t+1} - \omega\pi_{t-1})z_t\} = 0 \qquad (2.26)$$

$$E_t \left\{(\pi_t - (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1}mc_t - \beta\theta\phi^{-1}\pi_{t+1} - \omega\phi^{-1}\pi_{t-1})z_t\right\} = 0 \qquad (2.27)$$

In order to have a benchmark, we reestimated the hybrid Phillips curve of GG using our instrument set and our data sample. In table 5, the first and the second rows use methods 1 and 2 leaving β unrestricted. The third and the fourth rows impose the restriction $\beta = 1$. Table 5 broadly confirms the results of GG and Galí et al (2001). The proportion of backward looking firms is estimated to be relatively small but statistically significant. The coefficient on marginal cost is positive in all specifications. The degree of price rigidity is reasonable but somewhat higher than what Taylor (1999) suggests.

Table 5

	ω	θ	β	$\lambda : s_t$	J-test
Unrestricted β					
(1)	0.1957	0.861	0.9471	0.0197	0.5543
	(0.0963)	(0.0256)	(0.0326)	(0.0086)	
(2)	0.3985	0.8773	0.9494	0.0098	0.6106
	(0.0884)	(0.0327)	(0.0436)	(0.006)	
Restricted β					
(1)	0.1538	0.8645	-	0.0153	0.6523
	(0.0964)	(0.0295)		(0.007)	
(2)	0.3838	0.8909	-	0.0058	0.7023
	(0.0919)	(0.0381)		(0.0041)	

2.5.1 Structural Phillips curve with rational and survey expectations

In this section we suggest a modification of the hybrid Phillips curve that serves our purposes. We will assume that some firms are forward looking while others use survey expectations to reset prices. We change the backward looking rule to:

$$p_t^b = \bar{p}_{t-1}^* + S_{t-1}\{\pi_t\} \tag{2.28}$$

This equation keeps the backward looking spirit of equation 2.23 in the sense that only information dated $t-1$ and earlier is used. It is now assumed that current inflation is forecasted as in surveys and that lagged inflation is no longer used. In our opinion, this formulation is more realistic because prices should be updated with π_t. If one considers that π_t is not available then it seems more natural to us to include a forecast of π_t and not π_{t-1}. There are two interpretations to justify the inclusion of survey expectations in our equation. The first interpretation is that survey expectations may correspond more accurately to actual forecasting behaviour of firms. The second interpretation is that survey forecasts are publicly available allowing firms to observe and use them, as Carroll (2003b) suggests.

Combining equations 2.20 - 2.22 and 2.28 one can derive the following equation:

$$\pi_t \varphi = (1-\omega)(1-\theta)(1-\beta\theta)mc_t + \beta\theta E_t\left\{\pi_{t+1}\right\} \\ + \theta\omega\pi_{t-1} - (1-\theta)\omega\beta\theta S_t\left\{\pi_{t+1}\right\} + (1-\theta)\omega S_{t-1}\left\{\pi_t\right\} \tag{2.29}$$

where

$$\varphi = (\theta + \omega\left[1 - \theta(1-\beta\theta)\right]) \tag{2.30}$$

Note that if one substitutes $S_t\left\{\pi_{t+1}\right\}$ by π_t and $S_{t-1}\left\{\pi_t\right\}$ by π_{t-1} then equation 2.29 becomes equivalent to the usual hybrid Phillips curve in equation 2.24.

We follow the same estimation strategy of GG and use nonlinear GMM in the following two normalizations:

$$E_t\{(\varphi\pi_t - (1-\omega)(1-\theta)(1-\beta\theta)mc_t - \beta\theta\pi_{t+1} - \theta\omega\pi_{t-1} \\ + (1-\theta)\omega\beta\theta S_t\left\{\pi_{t+1}\right\} - (1-\theta)\omega S_{t-1}\left\{\pi_t\right\})z_t\} = 0 \tag{2.31}$$

$$E_t\{(\pi_t - (1-\omega)(1-\theta)(1-\beta\theta)\varphi^{-1}mc_t - \beta\theta\varphi^{-1}\pi_{t+1} - \theta\omega\varphi^{-1}\pi_{t-1} \\ + (1-\theta)\omega\beta\theta\varphi^{-1}S_t\left\{\pi_{t+1}\right\} - (1-\theta)\omega\varphi^{-1}S_{t-1}\left\{\pi_t\right\})z_t\} = 0 \tag{2.32}$$

If expectations of firms are better captured by surveys than by lagged inflation then we expect our estimates of ω to be larger than in GG specification. The overall results are quite satisfactory (see table 6), for all specifications the parameters are estimated in a reasonable range. The proportion of firms forecasting as in surveys is higher than the proportion of backward looking firms in GG, as one would expect. Under the first normalization the proportion of forward looking firms is dominant while in the second specification both types of firms are estimated to have the same importance. The degree of price rigidity θ is slightly lower than under GG specification. The coefficient on the marginal cost is always correctly signed and statistically significant. This is also an improvement relative to GG, since in table 5 the coefficient on marginal cost is always positive but it is not always significant. In this section, the estimations of a microfounded Phillips curve confirm the results obtained in earlier sections. Some papers in the literature have taken survey expectations as representative, for instance see Roberts (1995, 1997, 1998), Kozicki and Tinsley (2001), Erceg and Levin (2003), Carroll (2003b), Adam and Padula (2003), Mankiw et al. (2003). We would like to stress that our results suggest that considering survey expectations as representative is a doubtful methodology.

Table 6

	ω	θ	β	$\lambda : s_t$	J-test
Unrestricted β					
(1)	0.2908	0.8447	0.9268	0.0221	0.5552
	(0.1038)	(0.0257)	(0.0374)	(0.0085)	
(2)	0.5142	0.8362	0.9163	0.0149	0.6066
	(0.0919)	(0.0318)	(0.0543)	(0.0064)	
Restricted β					
(1)	0.2283	0.8509	-	0.0164	0.6587
	(0.1036)	(0.0293)		(0.0068)	
(2)	0.5079	0.8385	-	0.0101	0.6995
	(0.094)	(0.0348)		(0.0043)	

2.6 *Robustness Analysis*

We will proceed with four robustness exercises. The first one explores sub-sample stability. The second considers the inclusion of extra lags of inflation. The third exercise uses an alternative measure of marginal cost. Finally, we will consider the non-farm business deflator instead of the GDP deflator.

The appendix shows the baseline estimation and the results presented in the robustness analysis when the instruments include the contemporaneous survey expectations. There are two ways to interpret the inclusion of surveys in the Phillips curve. One is that some firms observe the inflation prediction of the SPF and use this information to update prices. Then, there are reasons to exclude the current SPF forecast from the information set, since this forecast is only available in the middle of the quarter. Alternatively, we may think that the forecast of the SPF is similar to the actual forecasting behaviour of firms. In this case, there is no reason to exclude the current SPF prediction. Our results are more robust if the current SPF forecast is included but the main conclusions do not change with the information set. For direct comparability with previous results in the literature we present the regressions without the current SPF forecast in the main part of the paper.

2.6.1 Sub-sample Stability

To some extent we are constrained in the sub-sample stability analysis because GMM can be somewhat inefficient in small samples. Intuitively, as the number of free parameters decreases the first stage regression predicted values will be closer to actual values making OLS (or non linear Least Squares) results to manifest. We will use two overlapping sub-samples, the first one goes from 1968:04 to 1989:4 and the second one goes from 1980:01 to 2004:02. Table 7 presents the results for the first and the second sub-samples.

The main results remain unchanged. Marginal cost has a significant impact on short-run inflation dynamics. In the first specification the importance of surveys is not dominant in both sub-samples and is quite small in the second period. In the second specification, the role of backward looking firms is as important as that of forward looking firms in both sub-samples. The degree of price rigidity is lower for the first sub-sample, indicating that in a period of high inflation prices are reset more frequently. This would be in accordance with the implications of a state pricing model.

The sub-sample results when the current survey forecast is included as an instrument is presented in table II of the appendix. In this case RE are clearly dominant in both specifications and both sub-samples.

Table 7

	ω	θ	β	$\lambda : s_t$	J-test
1968:04 to 1989:4					
Unrestricted β					
(1)	0.3664	0.7377	0.8942	0.0559	0.792
	(0.0648)	(0.0323)	(0.0485)	(0.0131)	
(2)	0.5224	0.7221	0.925	0.0394	0.8073
	(0.0614)	(0.0419)	(0.0638)	(0.0114)	
Restricted β					
(1)	0.3713	0.717	-	0.0497	0.8543
	(0.0683)	(0.0357)		(0.0116)	
(2)	0.6881	0.6047	-	0.0432	0.875
	(0.0483)	(0.0611)		(0.0115)	
1980:01 to 2004:02					
Unrestricted β					
(1)	0.0793	0.86	1.0033	0.019	0.6731
	(0.0778)	(0.024)	(0.0439)	(0.0094)	
(2)	0.5242	0.8476	0.8936	0.0139	0.7027
	(0.0941)	(0.025)	(0.0747)	(0.0052)	
Restricted β					
(1)	0.0715	0.8609	-	0.0195	0.7491
	(0.0767)	(0.0239)		(0.008)	
(2)	0.4954	0.8535	-	0.0084	0.7799
	(0.0991)	(0.0304)		(0.0038)	

2.6.2 Inclusion of further lags of inflation

If further lags of inflation are omitted from the Phillips curve it may happen that our estimates of ω are biased downwards. To test this hypothesis we include three additional lags of inflation and test whether their sum is significant. The results presented in table 8 show that the sum of coefficients Ψ is never statistically different from zero. The inclusion of the additional lags deteriorates the regression estimates, note that the standard deviations of all estimators are now bigger. Importantly, the proportion of forward looking firms does not decrease. This section suggests that the high estimate of forward looking firms is not a consequence of omitted inflation lags. Table III in the appendix shows the results for the alternative instrument set. The main results are unchanged.

Table 8

	ω	θ	β	Ψ	$\lambda : s_t$	J-test
Unrestricted β						
(1)	0.1728	0.9146	0.9519	0.0468	0.0086	0.5155
	(0.1671)	(0.0542)	(0.0717)	(0.0482)	(0.0084)	
(2)	0.3025	0.9049	0.9822	0.0286	0.0063	0.5248
	(0.1871)	(0.0772)	(0.0948)	(0.0544)	(0.0077)	
Restricted β						
(1)	0.1382	0.9019	-	0.0173	0.0081	0.6193
	(0.1471)	(0.0516)		(0.0223)	(0.0081)	
(2)	0.246	0.9152	-	0.023	0.0048	0.5997
	(0.1536)	(0.0705)		(0.0243)	(0.0076)	

2.6.3 Alternative Measures of Marginal Cost

We have thus far presented results under the assumption that $mc_{t,t+k} = mc_{t+k}$, this amounts to set $\alpha = 0$ in equation 2.9. Galí et al. (2001) also use an alternative parameterization.[14] By definition, the average markup equals the inverse of average real marginal cost, therefore it follows that:

$$\alpha = 1 - S_t \mu_t \tag{2.33}$$

where $S_t \equiv W_t N_t / P_t Y_t$ denotes the labour income share. Also given our assumptions, the steady state markup has the following relationship with ε:

[14]Bear in mind the correction made in Galí et al. (2003a) to the paper Galí et al. (2001).

$$\varepsilon = \frac{\mu}{\mu - 1} \tag{2.34}$$

Galí et al (2001) report the labour income share and the markup in the U.S. to be approximately 2/3 and 1.1 respectively. Inserting these values in equation 2.33 and 2.34 we obtain our values for α and ε. It is possible to show that combining equations 2.20 - 2.22, 2.28 and 2.9 the following Phillips curve is obtained:

$$\begin{aligned} \pi_t \varphi = {} & (1-\omega)(1-\theta)(1-\beta\theta)\xi mc_t + \beta\theta E_t \left\{ \pi_{t+1} \right\} \\ & + \theta\omega\pi_{t-1} - (1-\theta)\omega\beta\theta S_t \left\{ \pi_{t+1} \right\} + (1-\theta)\omega S_{t-1} \left\{ \pi_t \right\} \end{aligned} \tag{2.35}$$

where $\varphi \equiv (\theta + \omega \left[1 - \theta(1-\beta\theta) \right])$ and $\xi \equiv (1-\alpha)/(1+\alpha(\varepsilon-1))$. The orthogonality conditions under the two normalizations are:

$$\begin{aligned} & E_t\{(\varphi\pi_t - (1-\omega)(1-\theta)(1-\beta\theta)\xi mc_t - \beta\theta\pi_{t+1} - \theta\omega\pi_{t-1} \\ & + (1-\theta)\omega\beta\theta S_t \left\{ \pi_{t+1} \right\} - (1-\theta)\omega S_{t-1} \left\{ \pi_t \right\}) z_t\} = 0 \end{aligned} \tag{2.36}$$

$$\begin{aligned} & E_t\{(\pi_t - (1-\omega)(1-\theta)(1-\beta\theta)\varphi^{-1}\xi mc_t - \beta\theta\varphi^{-1}\pi_{t+1} - \theta\omega\varphi^{-1}\pi_{t-1} \\ & + (1-\theta)\omega\beta\theta\varphi^{-1} S_t \left\{ \pi_{t+1} \right\} - (1-\theta)\omega\varphi^{-1} S_{t-1} \left\{ \pi_t \right\}) z_t\} = 0 \end{aligned} \tag{2.37}$$

The results are presented in table 9, where our calibration implies that $\xi = 0.2$. The proportion of firms making use of survey expectations is now bigger than a half. The results do confirm that the proportion of rational firms is economically large and statistically significant. The coefficient on marginal cost always has the expected sign and is statistically significant.[15] With the new measure of marginal cost the degree of price rigidity is lower. Galí et al. (2003a) also report that considering $\xi = 0.2$ implies a lower estimate for the degree of price rigidity. Table IV in the appendix shows that if the contemporaneous forecast of surveys is an instrument then the importance of RE is always dominant. Despite the fact that RE are usually dominant, we stress that the main argument in this paper is not that RE are always dominant in every specification. Our main point is that taking survey expectations as representative seems to be a flawed methodology.

Table 9

	ω	θ	β	$\lambda : s_t$	J-test
Unrestricted β					
(1)	0.5061	0.4542	0.8257	0.0413	0.5100
	(0.0678)	(0.0680)	(0.0750)	(0.0115)	
(2)	0.5904	0.5030	0.9326	0.0231	0.5884
	(0.0599)	(0.0670)	(0.0777)	(0.0079)	
Restricted β					
(1)	0.5823	0.4044	-	0.0350	0.6247
	(0.0565)	(0.0690)		(0.0098)	
(2)	0.6267	0.4835	-	0.0209	0.6792
	(0.0487)	(0.0679)		(0.0063)	

2.6.4 Alternative Measures of Inflation

We have so far focused on the GDP deflator/inflation because this is the variable predicted in the SPF. Nevertheless, we want to examine whether our results change when we consider the non-farm business deflator instead. Table 10 shows the results. Now, the role of surveys is even more limited. For the first specification, ω is negative and indistinguishable from zero. For the second specification, ω is positive but small.

[15]Note that now $\lambda \equiv (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1}\xi$.

Table 10

	ω	θ	β	$\lambda : s_t$	J-test
Unrestricted β					
(1)	-0.0881	0.8736	0.9902	0.0233	0.6280
	(0.0650)	(0.0352)	(0.0343)	(0.0136)	
(2)	0.2332	0.9015	0.9656	0.0088	0.5836
	(0.0877)	(0.0434)	(0.0435)	(0.0075)	
Restricted β					
(1)	-0.1009	0.8724	-	0.0229	0.6533
	(0.0609)	(0.0368)		(0.0133)	
(2)	0.2122	0.9219	-	0.0043	0.6798
	(0.0887)	(0.0626)		(0.0069)	

2.7 Conclusion

Allowing for a simultaneous role of rational and survey expectations we have estimated a generalization of the usual NKPC and its hybrid structural version. Both specifications led to similar results. We find evidence that the marginal cost is a robust measure of economic activity in the NKPC as GG and Sbordone (2002) had suggested. We also confirm that detrended output is not robust. More importantly, we find that the proportion of firms using surveys is higher than the proportion of simply backward looking firms of GG. Surveys may be used by firms because they reflect the actual forecasting behaviour of economic agents or because some firms observe these forecasts and use them in their pricing decisions. Despite the importance of survey expectations, RE play a significant role in inflation dynamics. For most specifications the proportion of rational firms is estimated to be dominant.

We present strong evidence that the actual pricing decision of firms entails significantly more rationality than what survey expectations suggest. Survey expectations may also be a mixture of rational and backward looking expectations, but besides the rationality embedded in surveys there is still a large number of firms forming expectations in accordance with RE. This result suggests that survey participants do not have the correct incentive to provide an accurate answer. Another possible explanation that is worthwhile pursuing in future research is the following. Individual firms may be efficiently performing their pricing decisions making markets to reflect high levels of rationality. Firms have day to day knowledge of their specialized input and output markets, having all the necessary information to set prices. However, a survey respondent or an economist trying to predict inflation may have a much harder job. Survey respondents have to predict aggregate inflation without having a day to day and specialized knowledge of every market that determines each price that is included in the inflation basket. This explanation could make the Phillips curve to reveal levels of rationality that are not reflected in surveys. The two explanations of our results are equally discomforting for the assumption that survey expectations reflect the true expectations in the economy. Even though, several papers in the literature had used this assumption, we find evidence that it is flawed.

An important avenue for research that remains open relates to the cyclical behaviour of the real marginal cost. Even though, the marginal cost seems to be a robust measure shaping inflation dynamics it would be desirable to find a direct link between inflation and the output gap. For future research, we will take the approach of this paper to estimate the Euler equation for output. We expect that different groups in the population may have different degrees of rationality. Fuhrer and Rudebusch (2004) address the issue of backward and forward looking behaviour and obtain contradictory results depending on the specification and the estimation method. It seems highly desirable to examine the role that survey expectations may play in such setting.

Chapter 3

Loose Commitment

3.1 Introduction

3.1.1 Motivation and Contribution

In a general class of macroeconomic models, households' behavior depends on expectations of future variables. Characterizing optimal policy in such circumstances is intricate. A planner influences households' expectations through its actions, and households' expectations influence the actions of the planner. Following the seminal papers by Kydland and Prescott (1977) and Barro and Gordon (1983a), the literature has taken two different approaches to tackle this problem - commitment and discretion. Under commitment, it is assumed that the planner will never default on its past promises. Under discretion, a planner can never make and fulfil a promise. These two settings are clearly extreme. In addition, models' predictions under commitment or discretion can differ significantly. It seems more reasonable to assume that institutions and planners sometimes fulfill their promises and sometimes do not.

This paper proposes several frameworks combining commitment and discretion. We first consider a setting where current promises will be fulfilled with a given probability. In another setting, promises are only kept during a finite tenure. Lastly, we make the likelihood of default a function of endogenous variables. There may be several interpretations for the *loose commitment* settings just described. A political economy interpretation is that governments fulfil their own promises but it is possible that another government is elected and today's promises will not be kept. Another interpretation is that a government commits to future plans, but if particular events arise, such as wars or political instability, defaulting becomes inevitable. As is common in the discretion literature, we consider that a default on past promises occurs whenever a reoptimization takes place. For the purposes of this paper it is indifferent whether the reoptimization is undertaken by the same planner or by a newly appointed one.

The contribution of this paper is in part methodological. We considerably generalize and extend the work of Roberds (1987) and Schaumburg and Tambalotti (2005). The methods that these authors propose can only be applied to linear-quadratic models. However, linear quadratic approximations are only valid under full commitment and the timeless perspective. Since *loose commitment* is a clear departure from full commitment, the methods previously proposed in the literature are extremely restrictive. We provide a methodology that can be applied to a large class of microfounded models, and we prove that the solution of these problems is recursive.

It is not possible to tell a priori whether allocations and welfare under *loose com-*

mitment will be closer to the full commitment or the full discretion cases. Since such results are model dependent, we believe it is important to apply our methodology to any particular model. As an illustration of what can be learnt, we provide an application to fiscal policy. In our application, we find that average allocations are substantially closer to discretion. When the probability of keeping promises is decreased from 1 to 0.75, most variables move more than half of the distance towards discretion. The true probability of keeping promises in the real world is an interesting issue open to debate. A value of 0.75 means that governments fulfil on average 75% of their promises. When political turnover is concerned, this value means that governments stay in power during 4 years on average. Our findings are robust across several setups that we consider. When the planner commits for 4 years, most allocations are still closer to discretion. We also discuss how the welfare gains change as a function of the probability to commit or the implied average time period before a default. Finally, when the probability of commitment/reelection depends on state variables, the planner actively manipulates the state variables in order to enhance commitment.

3.1.2 Other applications

The main goal of this paper is to present a methodology allowing to combine commitment and discretion. To make our results clear and illustrate the issues that we can address we also provide an application to a fiscal policy model. There are many other interesting applications of the methods presented in this paper.

Alesina and Tabellini (1990) considered that policymakers with different preferences alternate in office. The authors conclude that each government leaves an inefficiently low amount of resources to its successor. The authors kept most of their analysis in a two period model and always considered discretionary policy. We are currently extending their model to a dynamic infinite horizon setup, where each government can still commit taking into account the possibility of reelection. This modeling strategy allows us to obtain predictions regarding the steady state level of debt and isolate the effects of political disagreement from those of lack of commitment.

For the sake of simplicity we kept the probability of commitment, or the regime duration as parameters. In the third setup, we make the model richer by considering that the commitment probability is a function of state variables, but we do not explicitly model the decision to default. Our setups can be interpreted as simplifications of more complex models where the decision to default is modeled explicitly. We think that the methods presented here are a useful step for combining commitment and discretion in a tractable way, preserving important features of macro models such as infinite horizon, endogenous dynamic state variables and non-linear utility. Endogenizing the probability of default or the regime duration is a desirable extension of this paper that we are currently pursuing.

In this paper, we only analyze fiscal policy. The literature has analyzed extensively monetary policy in the Barro-Gordon setup. However, as we explain in the methodology section, under reasonable assumptions the Barro-Gordon setup is only a valid approximation to a microfounded model if one assumes a timeless perspective of full commitment. The timeless perspective excludes the *loose commitment* setups proposed here. It is interesting to analyze monetary policy and its interactions with fiscal policy under *loose commitment* in a microfounded model.

3.1.3 Methodology

In a very specific model, Roberds (1987) considers that promises may not always be kept. The author's model and assumptions are very specific, and his method is not generalizable to other applications. Schaumburg and Tambalotti (2005) propose a setup equal to one of the three settings described here, and apply it to a monetary model without state variables. Nevertheless, the authors follow a restrictive linear quadratic approach that was criticized by Klein et al. (2004). Moreover, there is an additional drawback of applying the linear quadratic approach in these types of problems. As shown by Debortoli and Nunes (2006), a correct linear-quadratic approximation can in general be derived if one imposes the timeless perspective assumption. The timeless perspective assumes that the problem is initialized at the full commitment steady state and that default never occurs. The *loose commitment* framework clearly requires a departure from the timeless perspective. As a consequence, using the linear-quadratic approach with *loose commitment* is inappropriate not only because solutions may be inaccurate, but also because the specification of the original model is violated.

The tools for the analysis of time-inconsistent and time-consistent policy are recent. The key reference for solving time-inconsistent models is Marcet and Marimon (1998). Klein and Rios-Rull (2003) show how to solve for the time-consistent policy with linear quadratic techniques. Klein et al. (2004) recognize that the techniques proposed in Klein and Rios-Rull (2003) do not deliver controlled accuracy and propose a technique based on generalized Euler equations and a steady state local analysis. Judd (2004) proposes global approximation methods instead of steady state local analysis.

We prove the recursivity of the solution using the tools of Marcet and Marimon (1998). In the solution procedure, we use a global method and generalized Euler equations taking the recent contributions of Judd (2004) and Klein et al. (2004). We show how to solve for linear and non-linear models, without and with state variables relying only on one fixed point. As a by-product, our methodology can be used as a homotopy method to obtain the time-consistent solution.

3.1.4 Literature Review

Reputational equilibria, as in Backus and Driffill (1985), is a recurrent topic in the time-consistency literature. Unlike the reputational equilibria literature we are not aiming at building setups where a planner of a certain type resembles another type. We aim at characterizing the solution of planners that can make credible promises, but may be out of charge when it is time to fulfill them. Our results hold in a more plausible and standard infinite horizon framework and we are not limited to models without state variables, as is often the case in reputation models.

Another related topic is the trigger strategies as in Barro and Gordon (1983b). Our paper is not aimed at building equilibria where private agents try to enforce a given equilibrium. To enforce a given equilibrium atomistic private agents need to develop and coordinate on highly sophisticated expectations mechanisms. Even if such strategies are possible, they are very hard to implement and may not be enforced every period. Hence, the planner may not always be forced to fulfil its promises, as in the *loose commitment* setting.

Flood and Isard (1989) consider a central bank commitment to a rule with escape clauses. The rule does not incorporate some important shocks affecting the economy. When such shocks hit the economy, it may be better to abandon the rule. One can interpret that our probability of default is their probability of anomalous shocks. Another interpretation is that we consider policymakers who are more rational, and do not leave important shocks outside the commitment rule. In such interpretation, the rule is always better and the planner only defaults if the commitment technology becomes inoperative. An important difference is that our setting can have endogenous state variables.

Persson et al. (2006), elaborating on an earlier proposal of Lucas and Stokey (1983), suggest a mechanism that makes the commitment solution to be time-consistent. Each government should leave its successor with a carefully chosen maturity of nominal and indexed debt for each contingent state of nature and at all maturities. Even though such strategies do eliminate the time-consistency problem, this structure of debt is not observed in reality. Our view is that at certain points in time the commitment solution may be enforced, but in some contingencies discretion is unavoidable.

The paper is organized as follows: section 3.2 introduces the probabilistic model, section 3.3 describes the T-periods model, section 3.4 provides an application to optimal taxation, section 3.5 considers an extension with endogenous probabilities and section 3.6 concludes.

3.2 The probabilistic model

We will consider a general model where a planner is not sure whether its promises will be kept or not. As we had explained, this uncertainty can be due to several factors. For simplicity, we assume that these events are exogenous and that in any period the economy will experience default or commitment with given exogenous probabilities. In Section 3.5, we will relax this assumption. Since it is indifferent whether it is the same or a new planner who defaults and reoptimizes, we use the terms "reelection", "new planner" and "default" interchangeably.

To make matters simple, we abstract from any shock other than the random variable s_t describing default (D) or commitment (ND) in period t. It is a straightforward generalization to include other sources of uncertainty, but the notation would be harder to follow. More formally, suppose the occurrence of Default or No Default is driven by a Markov stochastic process $\{s_t\}_{t=1}^{\infty}$ with possible realizations $\bar{s}_t \in \Phi \equiv \{D, ND\}$, and let Ω^t be the set of possible histories up to time t:

$$\Omega^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : \bar{s}_j \in \Phi, \forall j = 1, ..., t\} \tag{3.1}$$

We only consider the histories $\omega^t = \{D, \bar{s}_1, \bar{s}_2, ..., \bar{s}_t\}$ that start with default. This is because in the initial period there are no promises to be fulfilled or equivalently the current government has just been settled. Before turning to the planner we describe the problem of individual agents.

3.2.1 Individual agents and constraints

The economy is populated by individual agents such as rational utility maximizing households and profit maximizing firms. As is standard to assume, economic agents maximize their objectives taking as given the actions of the government. We describe a very general setting where the first order conditions (FOCs) of households and firms fit the following functional form:

$$b_1(c_t(\omega^t), k_t(\omega^t)) + \beta E_t b_2(c_{t+1}(\omega^{t+1}), k_{t+1}(\omega^{t+1})) = 0 \tag{3.2}$$

where b_1 and b_2 are vectors of functions, β is the discount factor, E_t denotes rational (mathematical) expectations using available information. The vectors k and c denote the set of states and controls from the perspective of the government.

Given our institutional setting, consumers will believe the promises of the current planner, but will consider that if a different planner comes into play, then different policies will be implemented and past promises will not be kept. As it is common in the time-consistency literature, economic agents will take future controls that can not be committed upon as functions of the state, i.e. $c_{t+1}(\{\omega^t, D\}) = \Psi\{k_{t+1}(\{\omega^t, D\})\}$ where we use the short notation $\{\omega^t, D\}$ to denote $\{\omega^t, \bar{s}_{t+1} = D\}$. $\Psi(.)$ denotes the policy function that rational agents anticipate to be implemented in future periods.[1] The constraint therefore becomes:

$$\begin{aligned} b_1(c_t(\omega^t), k_t(\omega^t)) + \beta Prob(\{\omega^t, ND\}|\omega^t) b_2(c_{t+1}(\{\omega^t, ND\}), k_{t+1}(\{\omega^t, ND\})) \\ + \beta Prob(\{\omega^t, D\}|\omega^t) b_2(\Psi\{k_{t+1}(\{\omega^t, D\})\}, k_{t+1}(\{\omega^t, D\})) = 0 \end{aligned} \tag{3.3}$$

where we use the short notation $Prob(\{\omega^t, ND\}|\omega^t)$ to denote $Prob(\{s_j\}_{j=0}^{t+1} = \{\omega^t, ND\}|\{s_j\}_{j=0}^{t}$

[1]For further discussions on this issue see Klein et al. (2004).

ω^t). The planner will then take as given the FOCs of economic agents. In addition, the planner will have other constraints such as feasibility and its own budget constraint, which either fit the functional form of Eq. (3.3) or the following functional form:

$$k_{t+1}(\omega^{t+1}) = \ell(c_t(\omega^t), k_t(\omega^t)) \tag{3.4}$$

Eq. (3.4) describes the evolution of the states, being ℓ a vector of functions and where it is understood that $k_{t+1}(\{\omega^t, ND\}) = k_{t+1}(\{\omega^t, D\})$, $\forall \omega^t$.[2]

3.2.2 The planner

When default occurs, a new planner is appointed and it will be taking decisions from that point onwards. Therefore, it is convenient to separate all histories ω^t with respect to the first time when default occurs. This is because we want to know which histories correspond to which planner. We now define the subset of Ω^t of histories where only commitment as occurred up to time t as:

$$\Omega^t_{ND} \equiv \{\omega^t = \{D, \{\bar{s}_j\}^t_{j=1}\} : \bar{s}_j = ND, \forall j = 1, ..., t\} \tag{3.5}$$

and the subsets of histories where the first default occurs in period i,

$$\Omega^t_{D,i} \equiv \{\omega^t = \{D, \{\bar{s}_j\}^t_{j=1}\} : (\bar{s}_i = D) \wedge (\bar{s}_j = ND), \forall j = 1, ..., i-1\}, \text{ if } i \leq t \tag{3.6}$$

$$\Omega^t_{D,i} = \oslash, \text{ if } i > t$$

By construction note that $\{\Omega^t_{ND}, \Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ is a partition of the set Ω^t. Moreover, it can be seen that the sets Ω^t_{ND} and $\Omega^i_{D,i}$ are singletons.[3] Therefore, in order to

[2] We consider this formulation for notational convenience. In the presence of additional sources of uncertainty one should consider the more general form $k_{t+1}(\omega^{t+1}) = \ell(c_t(\omega^t), k_t(\omega^t), \omega^{t+1})$.

[3] Ω^t_{ND} only contains the history $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, ..., \bar{s}_t = ND\}$ and similarly the set $\Omega^i_{D,i}$ only contains the history $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, ..., \bar{s}_{i-1} = ND, \bar{s}_i = D\}$.

avoid confusion between histories and sets of histories, we will refer to these singleton sets as ω_{ND}^t and $\omega_{D,i}^i$ respectively.

In figure 3.1 we show a more intuitive representation of the particular partition of histories specified above, where we use the name of the unique history ending in a given node to denote the node itself. White nodes indicate when a new planner is settled (default has occurred), while black nodes indicate the cases where the first planner is still in power (no default has occurred). We can see that in any period t there is only one history ω_{ND}^t such that commitment has always occurred in the past, or in other words the planner settled in period 0 is still in charge. Moreover, there is also only one history $\omega_{D,i}^i = \left\{\omega_{ND}^{i-1}, D\right\}$, meaning that the first default occurred in period i. In our institutional setting, a new planner is then settled from the node $\omega_{D,i}^i$ onward and it will make its choices over all the possible histories passing through the node $\omega_{D,i}^i$, that is the sets $\Omega_{D,i}^t, \forall t \geq i$.

We will now write the problem of the current planner where to simplify notation, and without loss of generality, we abstract from the presence of constraints in the maximization problem:

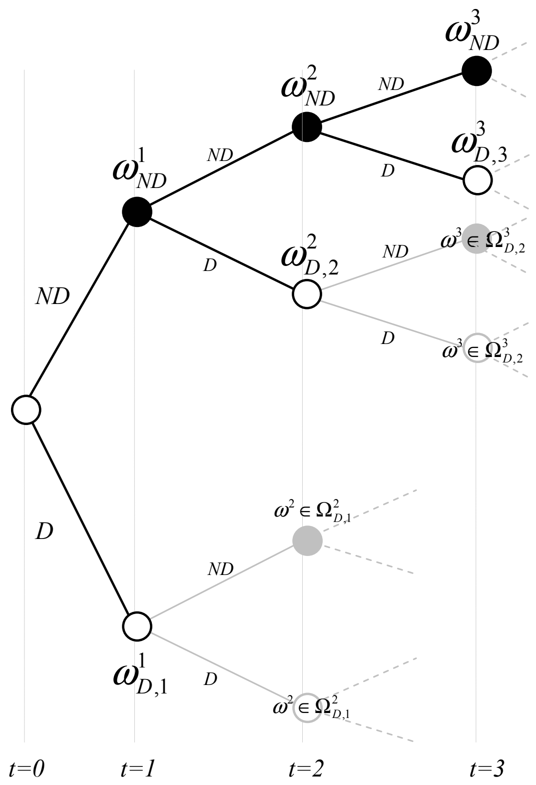


Figure 3.1: Diagram of the possible histories

$$
\begin{aligned}
W(k_0) = \max_{\substack{\{c_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t \in \Omega^t}} [\sum_{t=0}^{\infty} \sum_{\omega^t_{ND}} \beta^t \{Prob(\omega^t) u(c_t(\omega^t), k_t(\omega^t))\} \quad (3.7) \\
+ \max_{\substack{\{c_t(\omega^t)\}_{t=1}^{\infty} \\ \omega^t \in \Omega^t_{D,1}}} \left\{ \sum_{t=1}^{\infty} \sum_{\omega^t \in \Omega^t_{D,1}} \beta^t \{Prob(\omega^t) u(c_t(\omega^t), k_t(\omega^t))\} \right\} \\
+ \max_{\substack{\{c_t(\omega^t)\}_{t=2}^{\infty} \\ \omega^t \in \Omega^t_{D,2}}} \left\{ \sum_{t=2}^{\infty} \sum_{\omega^t \in \Omega^t_{D,2}} \beta^t \{Prob(\omega^t) u(c_t(\omega^t), k_t(\omega^t))\} \right\} \\
+ ...]
\end{aligned}
$$

where we are using the short notation $Prob(\omega^t) = Prob(\{s_j\}_{j=0}^t = \omega^t)$. Eq. (3.7) makes it explicit that inside the maximization problem of the current government there are other planners maximizing welfare during their tenures. Given that $\{\Omega^t_{ND}, \Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ is a partition of the set Ω^t, all the histories are contemplated in our formulation. Since $\forall\ t > i$, $\Omega^t_{D,i} = \{\omega^i_{D,i}, \{\bar{s}_j\}^t_{j=i}\}$, we can rewrite the probabilities for $\omega^t \in \Omega^t_{D,i}$ in the following way:

$$Prob(\omega^t) = Prob(\omega^i_{D,i} \wedge \omega^t) = Prob(\omega^t|\omega^i_{D,i})Prob(\omega^i_{D,i}), \forall\ \omega^t \in \Omega^t_{D,i},\ t \geq i. \quad (3.8)$$

Substituting for these expressions into Eq. (3.7) and collecting the common term in the summation, we obtain:

$$W(k_0) = \max_{\substack{\{c_t(\omega^t)\}^\infty_{t=0} \\ \omega^t \in \Omega^t}} \left\{ \sum_{t=0}^{\infty} \sum_{\omega^t_{ND}} \beta^t \{Prob(\omega^t) u(c_t(\omega^t), k_t(\omega^t))\} \right. \quad (3.9)$$

$$\left. + \sum_{i=1}^{\infty} \beta^i Prob(\omega^i_{D,i}) \left[\max_{\substack{\{c_t(\omega^t)\}^\infty_{t=i} \\ \omega^t \in \Omega^t_{D,i}}} \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega^t_{D,i}} \beta^{t-i} \{Prob(\omega^t|\omega^i_{D,i}) u(c_t(\omega^t), k_t(\omega^t))\} \right] \right\}$$

Since we are assuming that any future planner is also maximizing we can define the value functions:

$$\xi_i(k_i(\omega^i_{D,i})) \equiv \max_{\substack{\{c_t(\omega^t)\}^\infty_{t=i} \\ \omega^t \in \Omega^t_{D,i}}} \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega^t_{D,i}} \beta^{t-i} \{Prob(\omega^t|\omega^i_{D,i}) u(c_t(\omega^t), k_t(\omega^t))\} \quad (3.10)$$

where it was made explicit that each planner assigns probability one to its initial node. The value functions $\xi_i(k_i)$ summarize the happenings after the node $\omega^i_{D,i}$. Since $\Omega^t_{D,i} \cap \Omega^t_{D,j} = \oslash$ for $i \neq j$, the choices of future planners are independent between themselves. This formulation is very general since one can assume several institutional settings that the future planners will face. For example, one can assume that some future planners have full commitment while others do not. For simplicity we will assume that all future planners face the same institutional settings which at this stage we do not specify, thus we assume that $\xi(k_i) = \xi_i(k_i)$ $\forall i$.[4] Since all the histories $\{\Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ are already being maximized by other planners, it is equivalent to consider that the initial planner maximizes over the single history $\{\omega^t : \omega^t \in \Omega^t_{ND}\} \equiv \omega^t_{ND}$ instead of $\omega^t \in \Omega^t$. We can therefore rewrite the problem at period $t = 0$ as:

$$W(k_0) = \max_{\{c_t(\omega^t_{ND})\}^{\infty}_{t=0}} \sum_{t=0}^{\infty} \Big\{\beta^t \{Prob(\omega^t_{ND}) u(c_t(\omega^t_{ND}), k_t(\omega^t_{ND}))\} + \sum_{i=1}^{\infty} \beta^i Prob(\omega^i_{D,i}) \xi(k_i(\omega^i_{D,i}))\Big\} \quad (3.11)$$

We will now assume that the random variable s_t is i.i.d. to further simplify the problem. It is straightforward to generalize our formulation to Markov processes. Also to simplify notation denote $Prob(\{\omega^t, ND\}|\omega^t) = \pi$ and $Prob(\{\omega^t, D\}|\omega^t) = 1 - \pi$, which implies that:

$$Prob(\omega^t_{ND}) = \pi^t \quad (3.12)$$

$$Prob(\omega^t_{D,t}) = \pi^{t-1}(1 - \pi). \quad (3.13)$$

With this formulation at hand we are ready to show that our problem can be written as a saddle point functional equation (SPFE), and that the optimal policy functions of the planner are time-invariant and depend on a finite set of states.

[4]In a companion paper we relax this assumption, focusing on political disagreement issues.

The recursive formulation

Collecting results from the previous section, the problem of the current planner is:

$$\max_{\{c_t(\omega^t_{ND})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}(\beta\pi)^t\{u(c_t(\omega^t_{ND}),k_t(\omega^t_{ND}))+\beta(1-\pi)\xi(k_{t+1}(\omega^{t+1}_{D,t+1}))\} \tag{3.14}$$

$$\begin{aligned} s.t: \quad & k_{t+1}(\omega^{t+1}_{ND}) = k_{t+1}(\omega^{t+1}_{D,t+1}) = \ell(c_t(\omega^t_{ND}),k_t(\omega^t_{ND})) \\ & b_1(c_t(\omega^t_{ND}),k_t(\omega^t_{ND})) + \beta(1-\pi)b_2(\Psi\{k_{t+1}(\{\omega^t_{ND},D\})\},k_{t+1}(\{\omega^t_{ND},D\})) \\ & \qquad + \beta\pi b_2(c_{t+1}(\omega^{t+1}_{ND}),k_{t+1}(\omega^{t+1}_{ND})) = 0 \end{aligned}$$

Due to the fact that we do have future controls in the constraints through the term $\beta\pi b_2(c_{t+1}(\omega^{t+1}_{ND}),k_{t+1}(\omega^{t+1}_{ND}))$, the usual Bellman equation is not satisfied.[5] Building on the results of Marcet and Marimon (1998), we show that problems of this type can be rewritten as a SPFE that generalizes the usual Bellman equation. This result is summarized in proposition 1.

Proposition 1 *Problem (3.14) can be written as saddle point functional equation as:*

$$W(k,\gamma) = \min_{\lambda\geq 0}\max_{c}\{h^m(c,k,\lambda,\gamma)+\beta(1-\pi)\xi(k')+\beta\pi W(k',\gamma')\} \tag{3.15}$$

$$s.t: k' = \ell(c,k)$$

$$\gamma' = \lambda, \;\; \gamma_0 = 0$$

[5] For details see Stokey et al. (1989).

where

$$h^m(c,k,\lambda,\gamma) = u(c,k) + \lambda g_1(c,k) + \gamma g_2(c,k) \tag{3.16}$$

$$g_1(c,k) = b_1(c,k) + \beta(1-\pi)b_2(\Psi\{l(c,k)\}, l(c,k)) \tag{3.17}$$

$$g_2(c,k) = b_2(c,k) \tag{3.18}$$

Proposition 1 makes it clear that the current planner maximizes utility of the representative agent subject to the constraints $k' = \ell(c,k)$ and $g_1(c,k) + \beta\pi g_2(c',k') = 0$, where the latter is incorporated in h^m. If there is no commitment, the continuation of the problem is $\xi(k')$. If the current promises will be fulfilled, then the continuation of the problem is $W(k',\gamma')$, and promises are summarized in the co-state variable γ'. The optimal policy functions of such problem are time invariant and depend on a finite number of states, as proposition 2 describes.[6]

Proposition 2 *The solution of problem (3.14) is a time invariant function with state variables (k_t, γ_t), that is to say:*

$$\psi(k,\gamma) \in \arg\min_{\lambda\geq 0}\max_{c}\{h^m(c,k,\lambda,\gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k',\gamma')\} \tag{3.19}$$

$$s.t : k' = \ell(c,k)$$

$$\gamma' = \lambda,\ \gamma_0 = 0$$

3.2.3 Equilibrium

In the institutional setting built in Eq. (3.14), we only assume that all planners from period 1 onward will face the same problems. From now on, we also assume that

[6] As it is common in the time-consistent literature we do not prove that the optimal policy function is unique. Nevertheless, we found no evidence of multiple solutions.

all future planners face the same institutional setting as we specify in period 0. In other words, we specify their problems in the same way as the problem of the planner in period 0. Thus we can use the following definition of equilibrium.

Definition 3 *A Markov Perfect Equilibrium where each planner faces the same institutional setting must satisfy the following conditions.*

1. Given $\Psi(k)$ and $\xi(k)$, the sequence $\{c_t\}$ solves problem (3.14);

2. The value function $W(k, \gamma)$ is such that $\xi(k) = W(k, 0) \equiv W(k)$;

3. The policy functions $\psi(k, \gamma)$ solving problem (3.14) are such that $\Psi(k) = \psi(k, 0)$.

The second part of the definition imposes directly that the problem of the initial and future planners must be equal. In a companion paper, we relax this assumption and focus on political disagreement issues. The third part of the definition imposes a consistency requirement in the constraints. More precisely, we require the policy function $\Psi(k)$ that agents expect to be implemented under default to be consistent with the optimal policy function. We refer to the notion of Markov Perfect Equilibrium because the function Ψ only depends on the natural state variables k. Also, in this equilibrium neither the planner nor individual agents desire to change behavior. Individual agents are maximizing and their beliefs are correct. The planner, taking as given Ψ and $\xi = W$, is also maximizing.

3.2.4 Solution strategy

There are different ways to solve our problem. One approach would be to prove that iterating on the SPFE is a contraction. By doing so, we could solve our problem in a very similar way to the usual value function iteration. We will follow a different approach, we will solve our problem using FOCs to the lagrangian. Our generic problem is:

$$W(k_0) = \underset{\{c_t,k_{t+1}\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} (\beta\pi)^t \left[u\left(c_t,k_t\right) + \beta\left(1-\pi\right)\xi\left(k_{t+1}\right)\right] \tag{3.20}$$

$$s.t. \quad k_{t+1} = \ell\left(c_t,k_t\right)$$

$$g_1\left(c_t,k_t\right) + \beta\pi g_2\left(c_{t+1},k_{t+1}\right) = 0$$

$$\forall t = 0,...,\infty$$

where g_1 and g_2 are defined by Eqs. (3.17, 3.18) respectively.

Details on the FOCs can be found in the appendix. It is important to mention that the term $\xi_{k,t+1}$ appears in the FOCs. As we had anticipated, the current planner will try to influence future planners. The value function $\xi\left(k_{t+1}\right)$ summarizes the welfare that agents will achieve with a planner appointed at $t+1$. From the perspective of the planner appointed at $t+1$, the state variables k_{t+1} can not be changed. Nevertheless, from the perspective of the current planner, who is in charge at period t, k_{t+1} can be manipulated.[7] The FOC with respect to k_{t+1}, thus considers both the possibility that the current planner stays in power and the possibility that a new planner is appointed. In the case that a new planner is appointed the current planner can only affect future decisions through the states k_{t+1}, which in turn influence the value function $\xi\left(k_{t+1}\right)$.

[7] Note that, when default occurs, the lagrange multiplier is set to zero and cannot be used to influence incoming planners.

The FOCs expressed in Eqs. (C.6-C.9) allows us to solve for the optimal policy. If the problems of the current and future planners differs, we could proceed in the following way. We could first obtain the value functions $\xi\left(k_{t+1}\right)$ and the optimal policy functions $\Psi\left(k_{t+1}\right)$ corresponding to future planners. Given $\xi\left(k_{t+1}\right)$ and $\Psi\left(k_{t+1}\right)$, we could then solve for the policy functions of the current planner.

As described in Definition 3, we are particularly interested in the formulation where future planners face the same problem as the current planner, i.e. where $\xi\left(k_t\right) = W(k_t)$ and hence $\xi_{k,t+1} = W_{k,t+1}$. In this case, one possible solution strategy relies on the solution of two fixed points. In a first step, we could guess the functions $W_{k,t+1}$. We could then solve for the optimal policy $c_t = \psi(k_t, \lambda_{t-1})$. This second step would involve solving a fixed point problem, because according to our equilibrium definition $\Psi(k) = \psi(k, 0), \forall k$. Once obtained the policy function, we could update our guess of W and $W_{k,t+1}$ and repeat the procedure until convergence.

We will show a solution method that only relies on solving one fixed point. To obtain the derivative $W_{k,t+1}$ we can use envelope results, which are summarized in result 1.

Result 1 *Using envelope results it follows that:*

$$\frac{\partial W\left(k_t\right)}{\partial k_t} = \frac{\partial u\left[c_t\left(k_t\right), k_t\right]}{\partial k_t} + v_t \ell_{k,t} + \lambda_t g_{1,k,t} \tag{3.21}$$

where all variables are evaluated using the optimal policy of a planner appointed in period t, given the state k_t.

Result 1 uses the fact that the planners are maximizing a function, which allows the use of envelope principles.[8] It is important to note that in Eq. (3.21) all the

[8] A proof of this envelope result is available upon request.

variables are evaluated with the optimal policy that the government elected at t implements. For instance, one has to bear in mind that the policy function at time t of a planner appointed at t does not depend on the lagrange multiplier.

By Definition 3, the policy functions that the current and future planners implement are equal. If we use the envelope result to substitute $\xi_{k,t+1} = W_{k,t+1}$, the FOCs only depend on the functions $\psi(k_t, \lambda_{t-1})$ and $\Psi(k)$, where $\Psi(k) = \psi(k, 0), \forall k$. We can use a collocation method to solve for the optimal policy functions. This solution method is simpler, because it relies on one fixed point instead of two. As a side product, our methodology can be used as a homotopy to obtain the time-consistent solution. Starting from the time-inconsistent solution, one can gradually reduce the probability of commitment to zero in order to obtain the time-consistent solution.

We want to stress that in our framework global solution methods proposed in Judd (1992) and Judd (2004) are much more appropriate. The linear quadratic approximation proposed in Benigno and Woodford (2004) or Benigno and Woodford (2006) is only valid in a timeless perspective. The timeless perspective assumes that initial commitments are equal to the steady-state commitment, not being useful to analyze transition dynamics. There are several reasons that make the linear quadratic approach inappropriate in our framework. Firstly, we consider that commitments may be broken and consequently we need to focus on transition dynamics. Secondly, our model does not have a steady state point around which one can take an approximation. Thirdly, under discretion the allocations can be very far from the commitment steady-state. Our method is more suitable and it is also simpler. Even for an exactly linear quadratic model Schaumburg and Tambalotti (2005) need to solve three fixed points to get their solution using a less reliable method.

Beside these numerical considerations, there is an important drawback of applying the linear-quadratic approach to study problems with *loose commitment* settings. Indeed, as shown by Debortoli and Nunes (2006), a correct linear-quadratic approximation of a general model can only be derived by imposing the timeless perspective approach. However, allowing for the occurrence of a default explicitly violates the timeless perspective assumption. Therefore, applying the linear-quadratic approach to study problems characterized by *loose commitment* contradicts the microfoundations of the original model.

3.3 T-periods model

We will now consider another institutional setting, where a planner knows that it will be in charge during T periods. After that a new planner is appointed. As in previous sections, we will assume that the future planner faces the same institutional settings as the initial planner. Using the same notation as in section 3.2, we can write the problem as:

$$W(k_0) = \max_{\{c_t\}_{t=0}^{T-1}} \sum_{t=0}^{T-1} (\beta)^t \{u(c_t, k_t)\} + \beta^T W(k_T) \tag{3.22}$$

$$s.t: k_{t+1} = \ell(c_t, k_t), \qquad t = 0, 1, ..., T-1.$$

$$b_1(c_t, k_t) + \beta b_2(c_{t+1}, k_{t+1}) = 0, \qquad t = 0, 1, ..., T-2.$$

$$b_1(c_t, k_t) + \beta b_2(\Psi_0\{k_{t+1}\}, k_{t+1}) = 0, \quad t = T-1.$$

The objective function includes the instantaneous utility of all the periods during the tenure and the value function of the future planner. The constraints that the planner faces reflect the institutional setting just described. Up to the last period, the current planner can credibly commit. In the last period of the tenure, private agents know that in the next period another planner will be appointed and no credible promises can be made. Therefore, private agents expect that in period T a new planner implements the policy function Ψ_0.

By appealing to standard dynamic programming techniques, it is clear that the policy functions of the planner appointed at $t = 0$ are equal to the policy functions of the planner appointed at $t = T$. The proof of such result is simple and only requires to consider the tenure of each planner as one big period and use infinite horizon dynamic programming results.[9]

Proposition 4 *Denote $\Psi_{j,i}$ as the optimal policy function of a planner appointed at $t = j$ in the time period $t = j + i$. That is to say,*
$\{\Psi_{j*T,0}(k_{j*T}), \Psi_{j*T,1}(k_{j*T}), ..., \Psi_{j*T,T-1}(k_{j*T})\} = \psi(k)$ *where*

$$\psi(k) \in \arg\max_{c} \textit{Problem (3.22)}$$

Then $\Psi_{j*T,i}(k) = \Psi_{(j+1)*T,i}(k)$, $\forall k, j, 0 \leq i \leq T - 1$.

The previous proposition states that the solution of problem (3.22) is a tenure invariant function with state variables (k). It is important to stress that we are not claiming that the policy functions are time-invariant. Indeed, the policy function that a planner implements in one period is different from the policy function that the same

[9] Another proof follows from applying finite horizon dynamic programming results and using the definition that the problems of different planners are equal.

planner implements in another period. Another important remark in Proposition 4 is that we are only considering the state variables when the tenure begins. A planner appointed in $t = 0$ will implement policy functions for all the periods $t = 0, 1, ...T - 1$ that only depend on the initial state k_0. If the model would have some sources of shocks, such as productivity, the state-space would be huge. Since we just kept shocks away for notational convenience, we want to use techniques that can easily incorporate exogenous shocks. To do so we will solve for policy functions that depend on the past lagrange multiplier and the past state variable.[10] These policy functions are still time variant but tenure invariant. As before, we can apply envelope results, which allow us to simplify our problem to a single fixed point. The FOCs of this problem are easily obtained and for brevity we will not state them here.

3.4 An optimal taxation problem

In the previous sections we have formulated optimal policy problems in a general form. We will now refer to a specific optimal taxation problem mentioned in Marcet and Marimon (1998) and Klein et al. (2004). We chose this model because it is a benchmark in the literature, where both the commitment and discretion solutions have been analyzed. A representative household derives utility both from private $\{c_t\}$ and public consumption $\{g_t\}$. The representative agent rents capital $\{k_t\}$ to a firm and inelastically supplies one unit of labor. Capital and labor markets are competitive, but financial markets are not available to the government. Thus, the government collects taxes $\{\tau_t\}$ and provides the public good under a balanced budget

[10]We can do so by using the results of Marcet and Marimon (1998) in a finite horizon economy.

constraint. The household problem is:

$$\max_{\{k_{t+1},c_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{3.23}$$

$$s.t : c_t + k_{t+1} = k_t + (1 - \tau_t)[w_t + (r_t - \delta)k_t]$$

where r, w, β and δ refer to the interest rate, wage, the discount factor and the depreciation rate respectively. There is uncertainty in this economy because it is not know in advance whether the planner will default or not. Wages and interest rates are determined in perfectly competitive markets:

$$r_t = f_k(k_t) \tag{3.24}$$

$$w_t = f(k_t) - f_k(k_t)k_t \tag{3.25}$$

where $y_t = f(k_t)$ is production. The FOCs of the households are:

$$u_c(C(k_t, k_{t+1}, g_t), g_t) = \beta E_t u_c(C(k_{t+1}, k_{t+2}, g_{t+1}), g_{t+1})\{1 + [1 - T((k_{t+1}, g_{t+1})][f_k(k_{t+1}) - \delta]\} \tag{3.26}$$

where we have already substituted the interest rate, the resource constraint and the balanced budget condition, which are described by:

$$c_t \equiv C(k_t, k_{t+1}, g_t) = f(k_t) + (1 - \delta)k_t - k_{t+1} - g_t \tag{3.27}$$

$$\tau_t \equiv T(k_t, g_t) = g_t/(f(k_t) - \delta k_t) \tag{3.28}$$

As in Klein et al. (2004), we also consider the possibility that the government only taxes capital income and therefore $\tau_t \equiv T(k_t, g_t) = g_t/((r_t - \delta)k_t)$.

In this model, the government would like to manipulate expectations. If the government commits to a given level of taxes tomorrow, then private agents will accumulate a certain amount of capital. Since we have a balanced budget, committing to lower taxes may imply less public consumption and may not always be optimal. Note that to provide an efficient level of public consumption the government under commitment may promise higher taxes than the government under discretion. This result may not seem intuitive to the reader. In Chamley (1986), government expenditure is exogenous, capital and labor can be taxed at different rates, and the government can accumulate assets making households highly indebted. In that model, there is a big incentive to tax capital very highly in earlier periods to obtain large amounts of assets and eliminate distortionary taxation in later periods. Our model is different in many respects. We are assuming endogenous public expenditure, no labor taxation (capital income model) or equal tax rates on capital and labor (total income model), and balanced budget. Here, the government's main task is to implement a good mix between private and public consumption. Unlike a model where the government can accumulate assets, taxing at very high or very low rates has an immediate effect on the ratio of private to public consumption. Hence, the government acting with discretion may have a strong incentive to reduce taxes in order to increase private consumption. But since the government under discretion ignores the effects of taxes on capital accumulation, the ratio of private to public consumption implemented in equilibrium will not be optimal. The models that we present confirm this intuition. When only capital income is available, the tax base is very low and the government under discretion tries to provide more public consumption and implements inefficiently high taxes. When total income is available for taxation, the tax base is higher and the discretion government tries to reduce taxes. However, ignoring dynamic effects, it still implements an inefficient equilibrium. For further discussions of this issue see

Klein et al. (2004).

In order to proceed to the numerical solution, we specify a per-period utility function:

$$u(c_t, g_t) = \log(c_t) + \gamma_g \log(g_t) \tag{3.29}$$

and a standard production function:

$$y_t = k_t^{\alpha} \tag{3.30}$$

We use a standard calibration for an annual model of the US economy. Table 3.1 summarizes the values used for the parameters.

Table 3.1: Parameter values

β	δ	α	γ_g
0.96	0.08	0.36	0.50

3.4.1 Probabilistic Model

We now consider the probabilistic model introduced previously. In case of default, households believe that k_{t+2} and g_{t+1} will be given by the functions $h(k_{t+1})$ and $\Psi(k_{t+1})$ respectively. Households anticipate the changes in power and therefore Eq. (3.26) is written as:

$$\begin{aligned} & u_c(C(k_t, k_{t+1}, g_t), g_t) = \qquad (3.31) \\ & = \beta\pi u_c(C(k_{t+1}, k_{t+2}, g_{t+1}), g_{t+1})\{1 + [1 - T((k_{t+1}, g_{t+1})][f_k(k_{t+1}) - \delta]\} \\ & + \beta(1-\pi)u_c(C(k_{t+1}, h(k_{t+1}), \Psi(k_{t+1})), \Psi(k_{t+1}))\{1 + [1 - T((k_{t+1}, \Psi(k_{t+1}))][f_k(k_{t+1}) - \delta]\} \end{aligned}$$

If one is tempted to match political cycles with commitment cycles, then the value of 0.75 is realistic, since it corresponds to a planner being in office for 4 years on average. A calibration based on the political history of the US implies a value of 0.8, while the political history of Italy would imply a calibration around 0. We will first examine the model where only capital income is taxed. Table 3.2 shows average allocations in the economy. In this case, discretion implies higher taxes, which induce lower capital accumulation and consumption but more provision of the public good. The first feature that one should stress is that average allocations in the economy seem to be closer to the discretion solution rather than to the commitment one. It may be expected that decreasing the probability of commitment by 25% would make allocations move by 25% of the difference between commitment and discretion. Nevertheless, a decrease in the probability of commitment from 1 to 0.75 leads to a bigger change in the average allocations of the economy towards the discretion steady state. For example, in the capital income tax model the absolute drop in capital is already 59% of the difference between full commitment and discretion.

Figure 3.2 plots the average path during the first 25 quarters, if one starts at the steady state value of capital under default and no promises have been made. The picture confirms the results of table 3.2, since the path for $\pi = 0.75$ is relatively closer to the discretion path. Also notice that in the commitment solution, as credibility starts to build, taxes and public consumption start to be lowered gradually. In figure 3.3 we plot the paths followed for a given history. We consider the history where by chance a new planner is reappointed every four years. There are noticeable differences in the accumulation path of capital. It is not just the realization of default or

Table 3.2: Capital Income Tax - Average Values

	0,00	**0,25**	**0,50**	**0,75**	**1,00**
k	1,602	1,663	1,755	1,918	2,366
g	0,232	0,230	0,227	0,222	0,203
y	1,185	1,201	1,224	1,264	1,364
c	0,825	0,838	0,857	0,889	0,971
τ	0,776	0,768	0,756	0,735	0,673
λ	0,000	-0,205	-0,541	-1,227	-3,644

commitment that makes allocations to change. Even considering the same history, the policies that the government implements lead to different allocations. The figure also shows that, when a new planner is reappointed, taxes and public consumption jump to high levels. Only after this initial increase do these variables start to be decreased gradually.

We now turn attention to the case where total income is taxed. Average allocations are shown in table 3.3. Again the result is that average allocations get away from the commitment steady state quite quickly. Figure 3.4 plots average paths and figure 3.5 plots the paths for a specific realization, where a new government is reappointed every four years. The conclusions are the same as before. The fact that a new planner is reappointed creates a change in policy. Nevertheless, for the same history different policies induce non-negligible differences in capital accumulation and output.

We finally focus on implications for welfare, which we measure as compensating variation in (private and public) consumption. The improvement from discretion to commitment is 2,19% and 0,13% in the capital and total income model respectively. Table 3.4 shows the welfare gains for different probabilities of commitment. We have normalized the welfare gain of moving from complete discretion to full commitment to

Table 3.3: Total Income Tax - Average Values

	0,00	**0,25**	**0,50**	**0,75**	**1,00**
k	4,391	4,387	4,379	4,363	4,259
g	0,408	0,409	0,411	0,416	0,447
y	1,703	1,703	1,702	1,700	1,685
c	0,944	0,943	0,940	0,934	0,897
τ	0,302	0,303	0,304	0,308	0,333
λ	0,000	-0,048	-0,135	-0,334	-1,453

1. The first line refers to the situation where only capital income is taxed. When the probability of default increases from 0 to 0.25, we see that only 15% of the benefits of commitment are achieved. In the total income model, even if the probability of keeping past promises is 0.75, only 39% of the gains are achieved. We plot the relative welfare gains as a function of π in figure 3.6. The function is convex suggesting that increasing π from low to intermediate levels results in relative small welfare gains. Most of the gains from enhancing commitment can only be achieved when π is already high. In figure 3.7 we plot the relative welfare gains as a function of the expected time before a default occurs ($1/(1-\pi)$). In this metric the welfare gains function is concave. The welfare gains per unit of time of moving from 1 to 2 years are higher than the gains of moving from 1 to 4 years.

In a related work on optimal monetary policy, Schaumburg and Tambalotti (2005) found qualitatively different results. First, allocations move linearly in the probability π.[11] For instance, when π moves from 1 to 0.75 inflation goes 25% of the distance towards discretion. Secondly, most of the welfare gains are achieved at low levels of commitment. In other words, the welfare is always concave regardless of the metric

[11] Besides the original calibration, we also tried an annual calibration of their model. The results do not change qualitatively, and are available upon request.

used.[12] When π is 0.75, about 90% of the welfare gains from commitment are obtained. Comparing absolute welfare measures in our model and theirs is unclear, but the welfare gains of moving from discretion to commitment are also much higher in their model.[13]

It is interesting to test whether the differences just mentioned between monetary and fiscal policy are also found in other models. In any case, these results explain why economists and policy makers have devoted considerably more attention to increase credibility in monetary policy than in fiscal policy. The institutional changes aimed at building central bank credibility in the 80's were justified by the potential welfare gains when credibility is low. Nowadays, central banks have more credibility but are not fully committed to any future action or rule. The reason seems to be that the benefits of increasing commitment even further are small. Arguably, it may be difficult to establish a fiscal authority with full commitment, because such an institution would interfere with democratic choices taken at different points in time. If one believes that fiscal policy commitment is unlikely to be high, then the welfare gains that can be obtained at intermediate levels of commitment are also unlikely to be high.

[12]The authors did not compute welfare as compensating variations but as life-time utility. In our model, the relative gains (values reported in the tables) are virtually unchanged if we use life time utility, the absolute values do change to 3,5% and 0,4% in the capital and total income model respectively.

[13]In Barro-Gordon models the welfare loss penalizes quadratically deviations of inflation from zero and deviations of the output gap from a target level. The inflation and output gap under commitment are nearly zero. Under discretion the inflation is quite high and the output gap is still zero. Since standard calibrations give a much higher weight to inflation deviations in the loss function, the gains from commitment are substantial.

Table 3.4: Welfare Gain

	0	**0,25**	**0,5**	**0,75**	**1**
Capital Income Tax	0,000	0,150	0,344	0,608	1,000
Total Income Tax	0,000	0,066	0,175	0,392	1,000

3.4.2 T-Periods Model

In this section we will apply the T-periods setting to the fiscal policy model described previously. For brevity considerations we skip the FOCs and we proceed directly with the analysis. This model displays political cycles. Figure 3.8 and figure 3.9 plot the paths for planners facing different tenure lengths. During each tenure, allocations move towards commitment values. When past promises are broken there is a sudden movement towards the discretion value. Longer tenures allow allocation to be closer to the full commitment solution.

We compare average allocations when the planner knows with certainty that it will be in charge during 1,2,4,8 and infinitely many periods. Obviously, the extreme values considered correspond to the default and commitment cases. Given the political history of the US, tenures of 8 periods can be considered an upper bound, while tenures of 4 years have been the norm. Tables 3.5 and 3.6 present the average allocations for the capital income and total income taxation model respectively. In the capital income model, for the benchmark tenure of 4 periods, capital moves towards discretion by 69% of the total difference between discretion and commitment. In the total income model, capital moves by 88% of the difference between commitment and discretion. Hence, results show that average values are still close to the discretion case both for the capital and the total income taxation model. Table 3.7 shows the

Table 3.5: T-period model - Capital Income Tax - Average Allocations

	DEF	**2**	**4**	**8**	**COM**
k	1,602	1,700	1,837	1,995	2,366
g	0,232	0,229	0,225	0,219	0,203
y	1,185	1,211	1,245	1,282	1,364
c	0,825	0,846	0,873	0,904	0,971
τ	0,776	0,764	0,746	0,725	0,673
λ	0,000	-0,315	-0,825	-1,538	-3,644

Table 3.6: T-period model - Total Income Tax - Average Allocations

	DEF	**2**	**4**	**8**	**COM**
k	4,391	4,385	4,375	4,357	4,259
g	0,408	0,410	0,413	0,418	0,447
y	1,703	1,703	1,701	1,699	1,685
c	0,944	0,942	0,938	0,932	0,897
τ	0,302	0,303	0,306	0,310	0,333
λ	0,000	-0,071	-0,199	-0,406	-1,453

results for welfare. In the more realistic total income taxation model, when the planner knows that it will stay in power during 4 years, only 27% of the welfare gain is achieved. Figure 3.10 plots relative welfare as a function time.

3.5 Extension - endogenous probabilities.

We are finally going to consider an extension where the probability of defaulting depends on the states of the economy. Since capital is the only natural state variable in the economy and all allocations depend on capital, we will consider that

Table 3.7: Welfare Gain T-period model

	DEF	**2**	**4**	**8**	**COM**
Capital Income Tax	0,000	0,256	0,528	0,745	1,000
Total Income Tax	0,000	0,102	0,266	0,492	1,000

the probability of defaulting today depends on the current capital stock. We think it is plausible to assume that when capital is higher there is a higher probability of reelection. We will consider the following probability function:

$$F(k_t) = 1 - \frac{1}{(\frac{k_t}{\tilde{k}})^\rho + 1} \tag{3.32}$$

where $\tilde{k}$ and ρ are parameters to be defined. Note that $\tilde{k}$ is a normalization such that $F(\tilde{k}) = 0.5$ and that the higher is ρ, the easier it is for the planner to influence its reelection probability. In the case of $\rho = 0$ the probability is always constant. The planner and households will consider that the probability of commitment in the next period is $F(k_{t+1})$ instead of π. For instance, the objective function of the planner is:

$$\sum_{t=0}^{\infty} \beta^t \frac{\Pi_{j=0}^{t}(F(k_t))}{F(k_0)} \{u(c_t, k_t) + \beta(1 - F(k_{t+1}))W(k_{t+1})\} \tag{3.33}$$

All the proofs considered previously also apply in this setting using minor modifications.[14] We can use a homotopy from the model in section 3.2 to this model by changing ρ from 0 to the desired value. This model raises an extra difficulty, because both the derivative and the level of the value function appear in the FOCs, hence one needs to approximate the value function. We choose $\rho = 5$ and $\tilde{k}$ to be equal to the average capital allocation when $\pi = 0.5$. Our normalization of $\tilde{k}$ allows us to directly compare the results with the probabilistic model when $\pi = 0.5$.

Results are presented in table 3.8 both for the capital income and the total income model. We see that capital is now higher. Since the probability of commitment is

[14]It is useful to redefine the objective of the planner using the definition $\theta_{t+1} = \theta_t F(k_{t+1})$, with $\theta_0 = 1$. Note that the special term on $F(k_0)$ in the objective function does not induce any time-inconsistency problem because k_0 is predetermined.

increasing in capital the planner has a further motive to accumulate capital. In the capital income model this effect is quite visible, while in the total income model this effect is more subtle.[15] Similarly, if we turn our attention to welfare, we notice that in the capital income model with endogenous probability the gain is 54.3% of the the total gain from commitment. This value is much higher than the welfare gain of 34.5% obtained in the benchmark case of $\pi = 0.5$ (as reported in table 3.4). In the total income model, welfare with endogenous probability is almost identical to the benchmark case of $\pi = 0.5$. There are two reasons for this discrepancy. First, in the capital income model higher probability of commitment leads to higher capital, which in turn increases the probability of commitment. This self reinforcing mechanism is not present in the total income model. The second reason is that the pure discretion and pure commitment solutions in the first model are very different, while this is not so in the second one. Overall, our results suggest that governments accumulate more capital to be reelected, and this is a good policy since it reduces political turnover increasing the commitment probability.

Table 3.8: Endogenous Probability - Average Values

	Capital Income Tax		**Total Income Tax**	
	$\pi = 0.5$	**End. Prob.**	$\pi = 0.5$	**End. Prob.**
k	1.755	1.896	4.379	4.381
g	0.227	0.222	0.411	0.411
y	1.224	1.259	1.702	1.702
c	0.857	0.885	0.940	0.941
τ	0.756	0.738	0.304	0.304
λ	-0.541	-0.864	-0.135	-0.134

[15] If we increase ρ in the total income taxation model then capital starts to be visibly higher.

3.6 Conclusions

The time-consistent and time-inconsistent solutions can differ dramatically. It is not clear which assumption about the planners' commitment technology is more plausible. It seems more realistic to consider that planners only face a *loose commitment* technology. We have considered different formulations of *loose commitment* and applied these tools to optimal fiscal policy. We examined a setup where the fiscal authority is reappointed every period with a given probability, another setup where the fiscal authority stays in power for T periods and finally a setup where the probability of reelection is endogenous. Combining these polar cases is a straightforward extension. Even though our settings may be naturally interpreted in the spirit of political turnover, one can also consider that the same planner may default on its own plans and reoptimize.

From the methodological point of view, our contribution is to show a solution technique for problems of limited commitment with the following main features. First, it can be applied to a wide class of non-linear models, with or without state-variables keeping the model's micro-foundations structure intact. Second, building on the results of Marcet and Marimon (1998), we proved that the solution to our problem is recursive. Third, we implemented an algorithm which is relatively inexpensive, because it only requires the solution of one fixed-point, and makes use of global approximation techniques which are pointed out in the literature as more reliable. Finally, as a by-product, our procedure can be used as a homotopy method to find the time-consistent solution.

We show that in the optimal taxation model under *loose commitment*, average allocations seem to be closer to the time-consistent solution. Regarding welfare, we find that for a probability of commitment around 0.75 or a tenure of 4 years, most of the gains from commitment are not achieved. While the welfare gains are a concave function of the expected time before a default, they are a convex function of the probability of commitment. These results are very different from those obtained in the literature regarding monetary policy. We believe that our results give support to the low interest in building independent or credible fiscal authorities.

The methodology that we have proposed here can be applied to many interesting economic setups. In a companion paper, following the insights of Alesina and Tabellini (1990), we consider the case where different planners have different objectives. We examine how political disagreement and political turnover influence the long-run level of debt. We manage to explain why governments do accumulate debt, and how this level depends on the degree of political disagreement and lack of commitment. In another paper, we analyze the interactions between fiscal and monetary policy. Finally, we are considering an extension that explicitly models the costs of defaulting.

Figure 3.2: Capital Income: Average Allocations

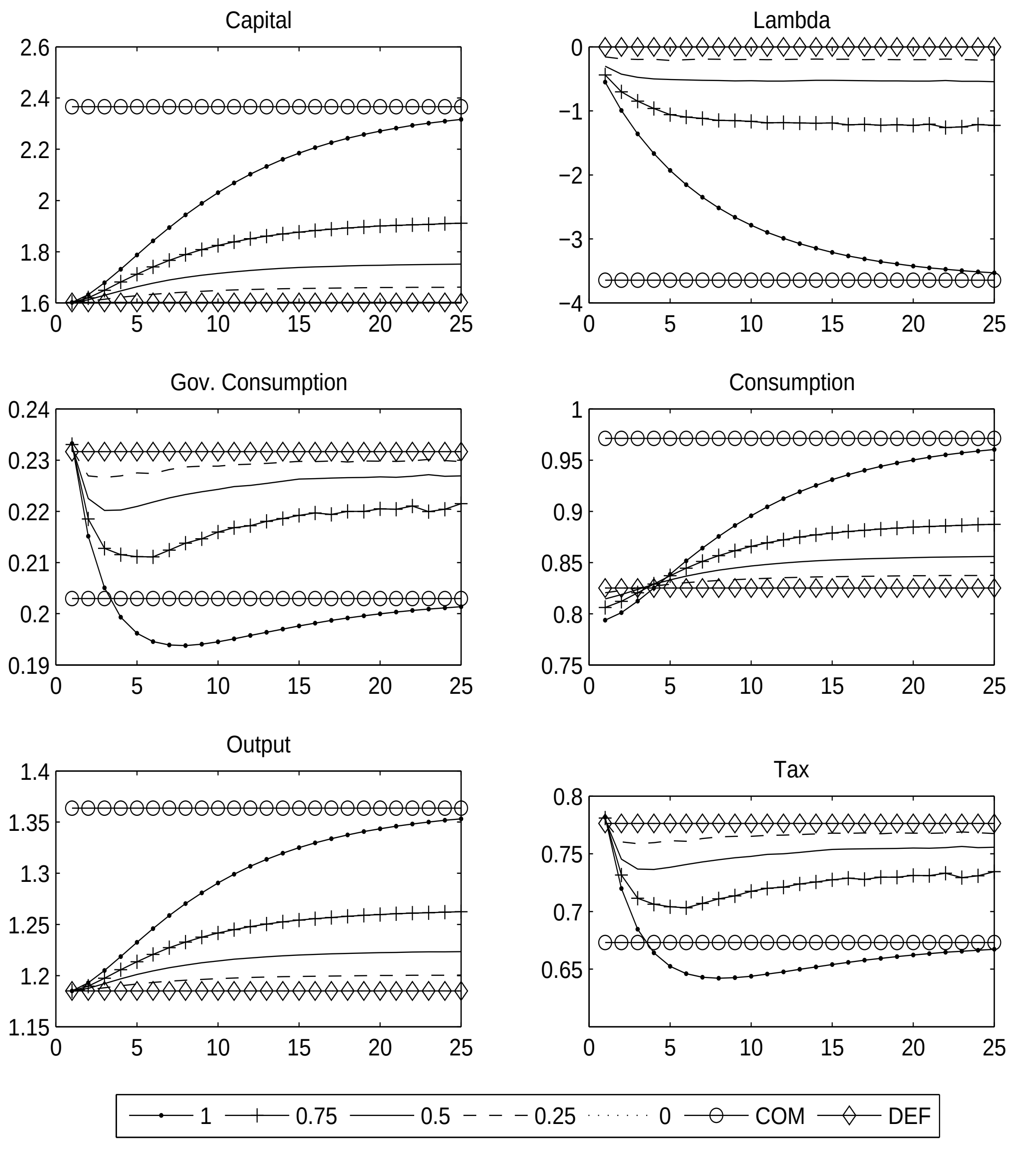


Figure 3.3: Capital Income: Default every 4 periods

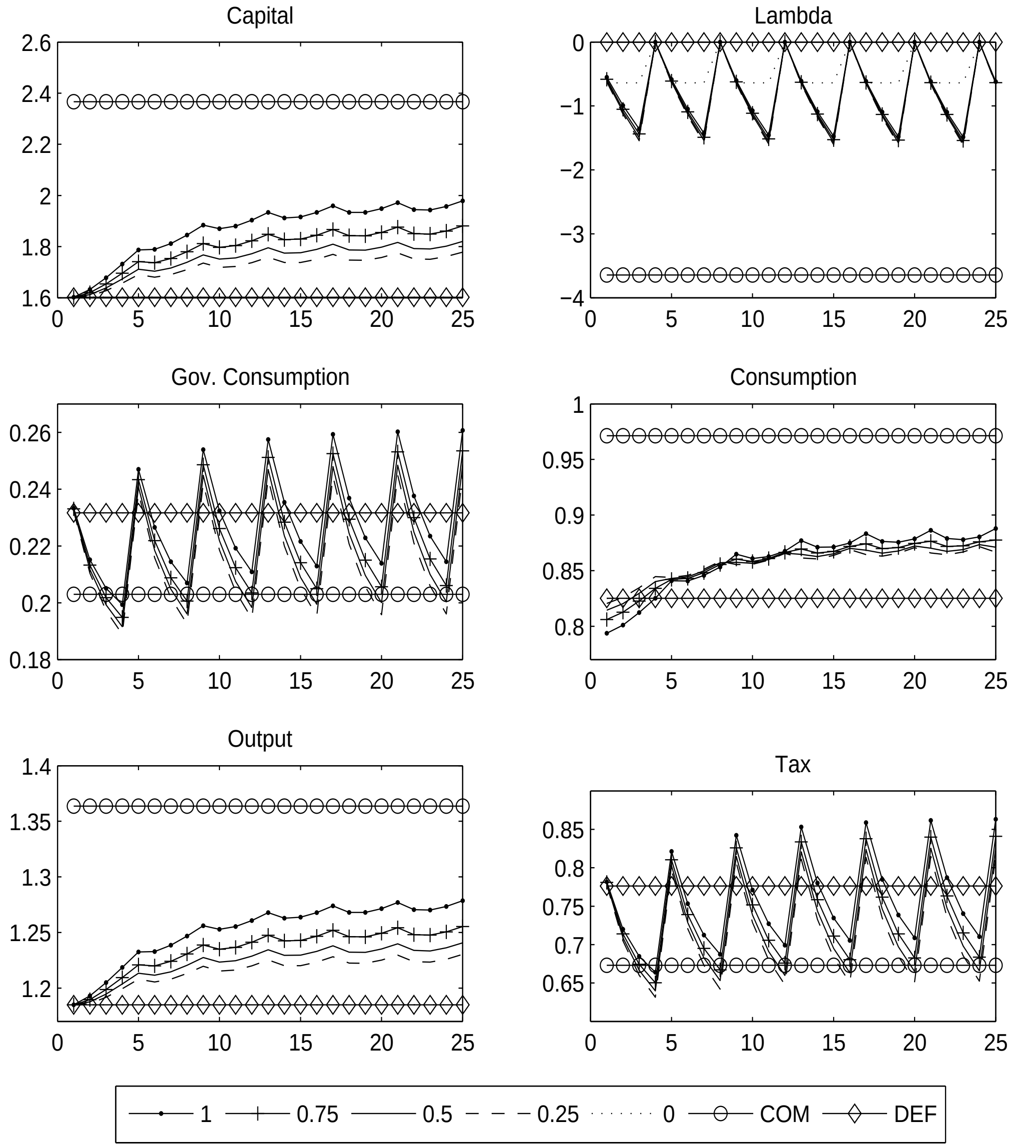


Figure 3.4: Total Income: Average Allocations

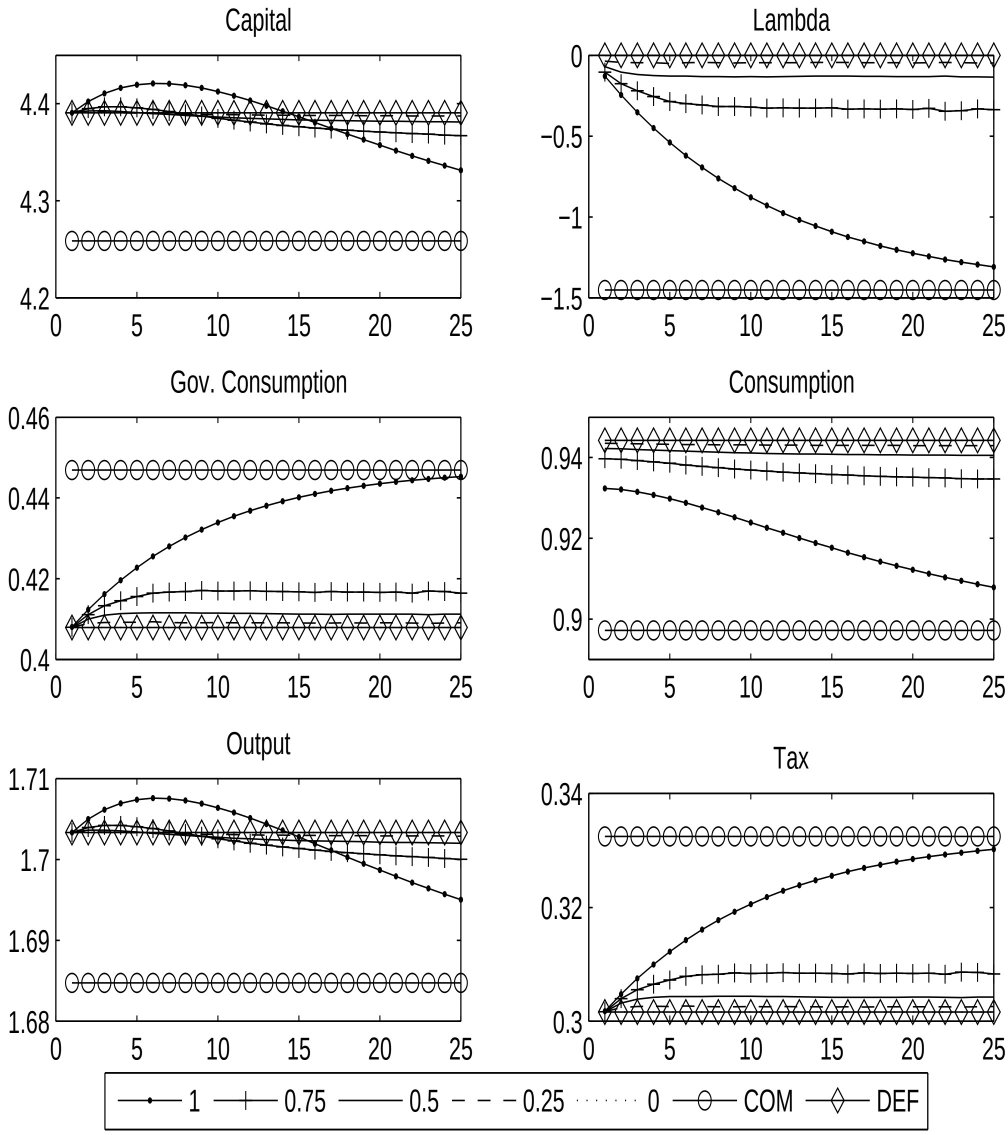


Figure 3.5: Total Income: Default every 4 periods

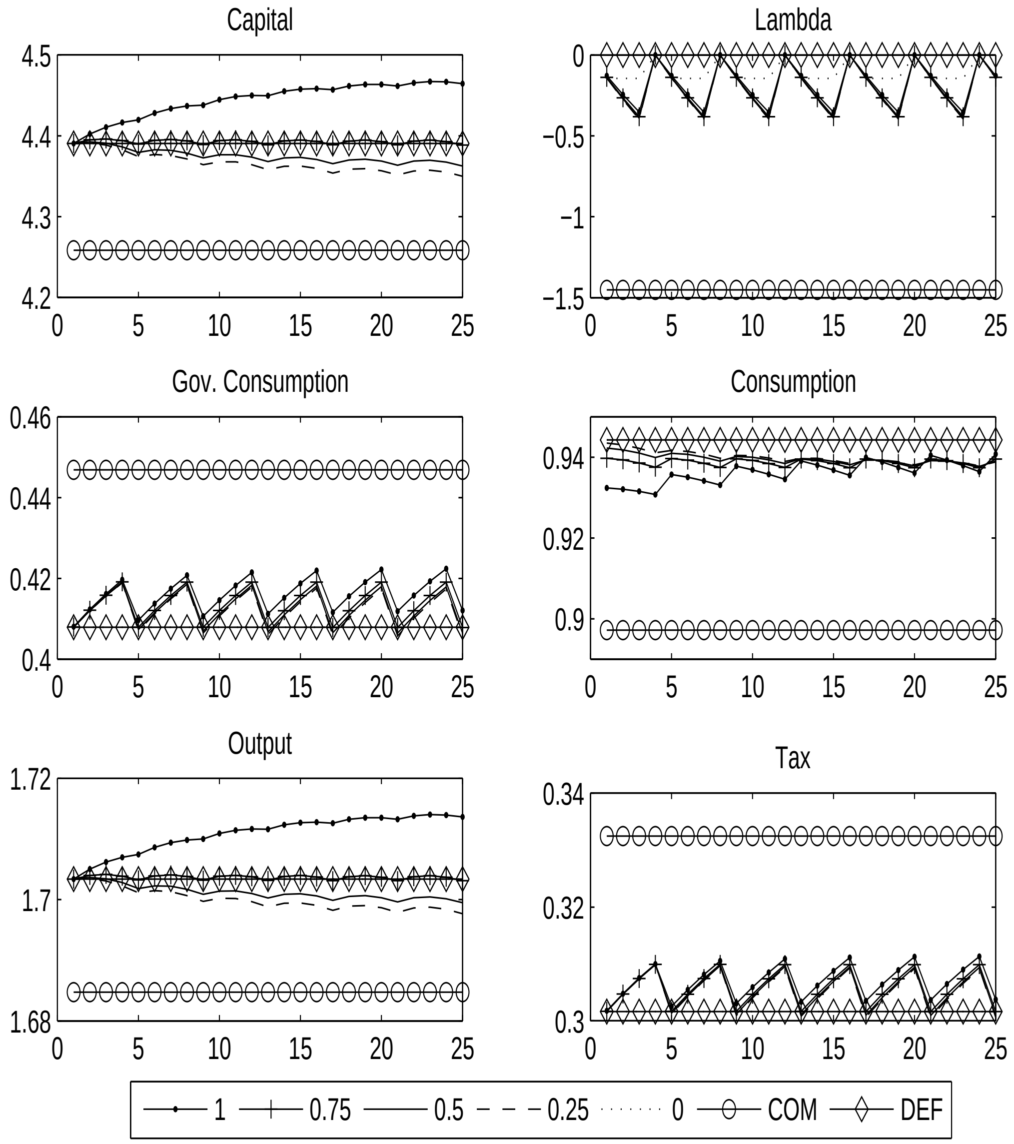


Figure 3.6: Welfare Gains on π axis: Probabilistic Model

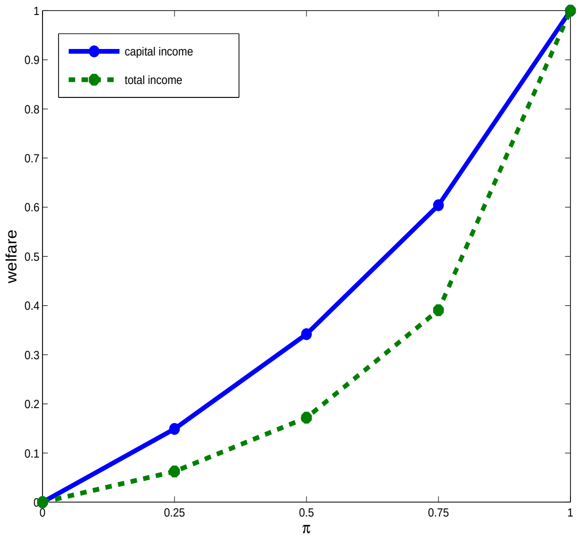


Figure 3.7: Welfare Gains on expected time axis: Probabilistic Model

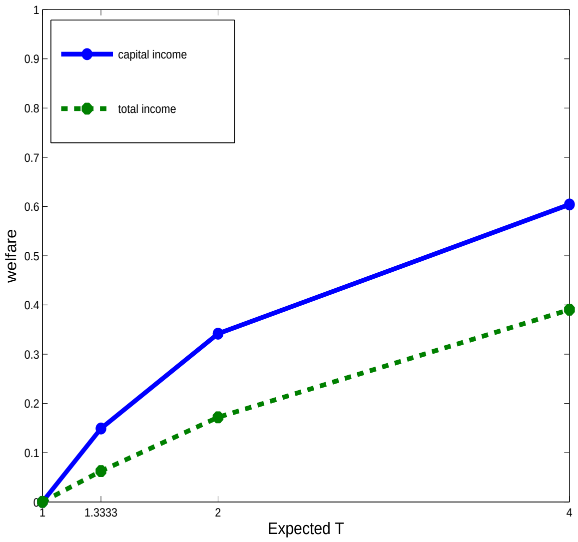


Figure 3.8: Capital Income: T-periods model

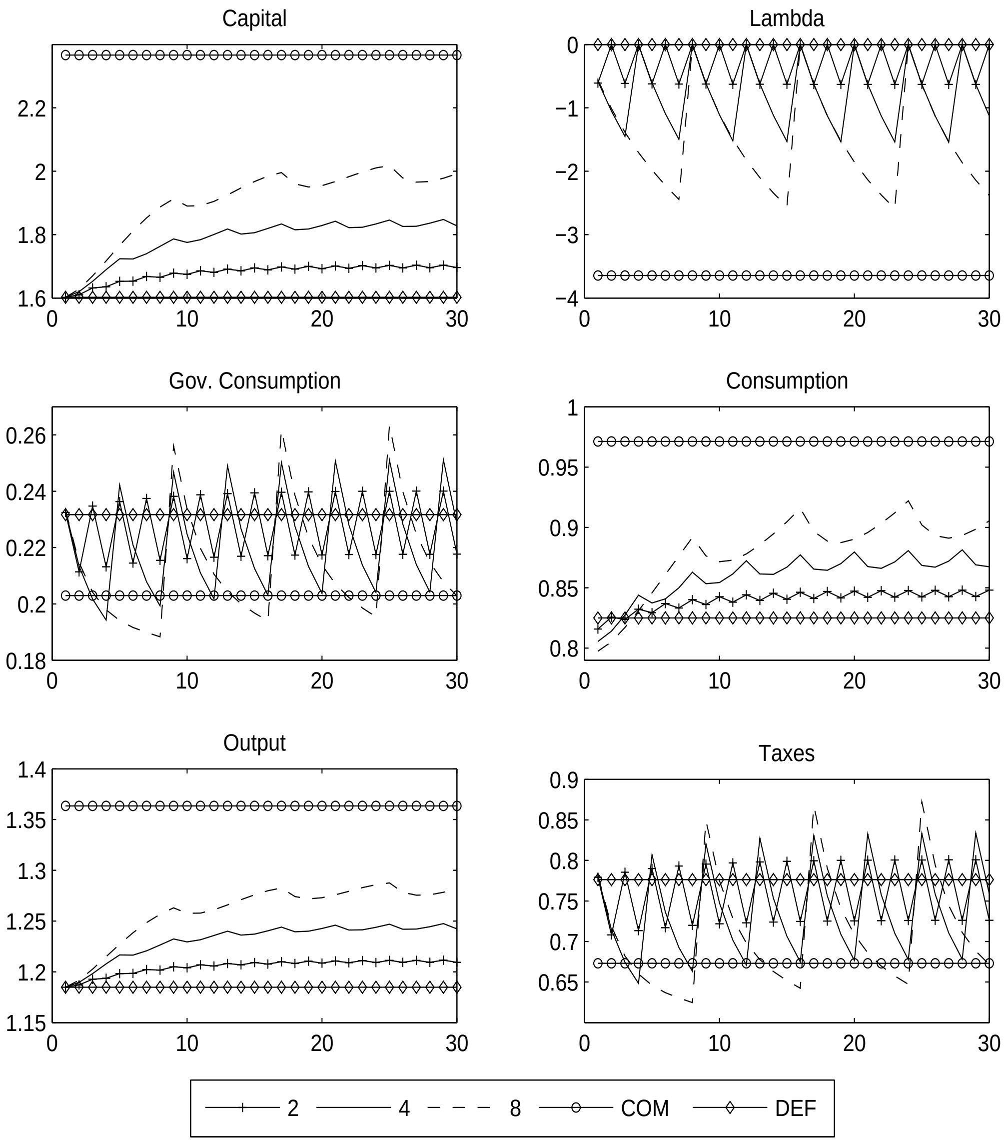


Figure 3.9: Total Income: T-periods model

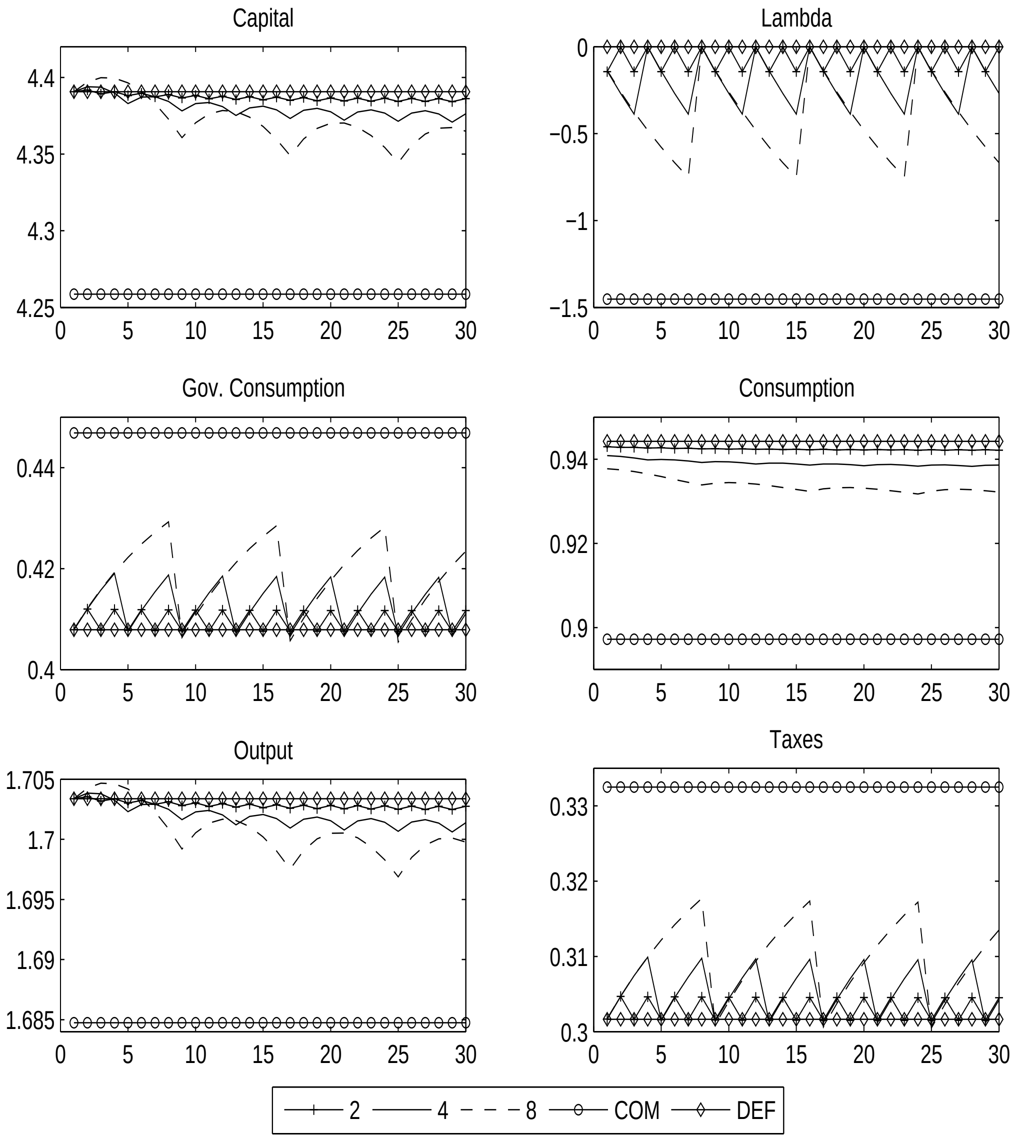
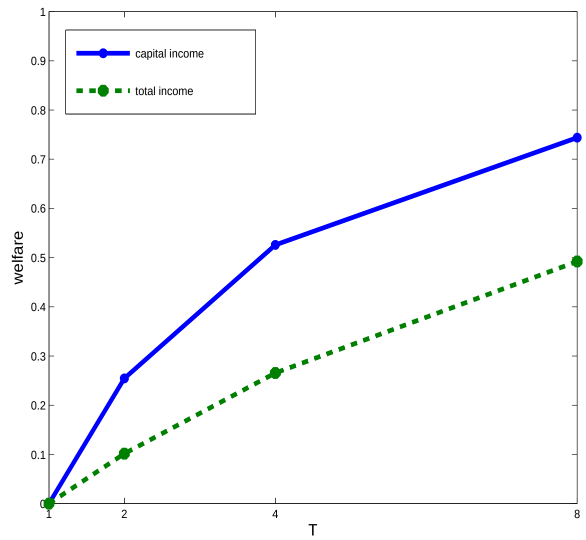


Figure 3.10: Welfare Gains on time axis: T-periods Model



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Appendix A

Appendix: Learning the Inflation Target

A.1 Internal Consistency Appendix

The magnitude of shocks influences the prediction performance of tracking parameters. Insofar as there is a regime change it is optimal to give more importance to recent observations, this is precisely what high tracking parameters do. On the other hand, shocks create noise in the economy, and if one gives more importance to recent observations then predictions will be harmfully influenced by recent shocks. We first considered that besides the autocorrelated natural interest rate there is a non-autocorrelated cost-push shock. We also did sensitivity analysis by considering the presence of other shocks, being the results robust. In the alternative specification we considered two non-autocorrelated shocks that influence Eq. (1.12) directly. We do not assume that the shocks are correlated because the correlations between the output gap and inflation are influenced in this time period by the disinflationary episode.

We estimated the data standard deviation of the output gap and inflation from 1980 to 1984 to be 0.02 and 0.04 respectively. We set the magnitude of shocks so that for the internal consistent tracking parameter the model yields plausible variances when compared with the data during the Volcker disinflation. Note that this implies too high variances for the period after the disinflation.

A.1.1 *Learners'* model

We considered the standard deviations for the natural interest rate innovation and the cost-push shock to be 0.015 and 0.01 respectively. The internally consistent parameters correspond to $\alpha_{out} = 0.5$ and $\alpha_{out} = 0.7$ and α_{inf} ranging from 0.5 to 0.9. For any of these parameters the simulated disinflation is slower than the Volcker disinflation.

A.1.2 Main model

We first set the standard deviation for the innovation in the natural interest rate and the cost-push shock to be 0.015 and 0.01 respectively. If the tracking parameter for output is 0.3 and for inflation is 0.5, for an average of 5000 simulations and for a range of 20 periods, the average standard deviation for output is 0.022 and for inflation is 0.031. In this specification the internally consistent parameters were $\alpha_{inf} = 0.5, \alpha_{out} = 0.3$ as can be seen in figures 1.6 and 1.7. We conducted sensitivity analysis considering an alternative value of T=16, and $\varepsilon = 0.00002$, being the results robust. The parameters $\alpha_{inf} = 0.5, \alpha_{out} = 0.1$ are also internally consistent for $\varepsilon = 0.00002$ but the simulated series are very similar under the two sets of parameters.

We tried different specifications. We also set the standard deviation for the cost-push shock and the innovation in the natural interest rate to be 0.02 being the results robust. We also assumed that the natural interest rate is autocorrelated and there are two i.i.d. shocks affecting Eq. (1.12) directly. We set the standard deviation in the innovation of the natural interest rate to be 0.02, and of the shock affecting output and inflation to be 0.01 and 0.02 respectively. Once again the results are robust.

A.2 Data Appendix

All variables refer to the USA. Inflation was computed using the seasonally adjusted monthly Consumer Price Index (CPI) for all urban consumers and all items. The source is the United States Department of Labor: Bureau of Labor Statistics and the series code is CPIAUCSL. Quarter inflation is computed as the sum of the months in the quarter divided by the sum of the CPI of the months of the previous quarter. The reported series were filtered using the band pass filter eliminating components with periodicity smaller than 4 quarters.[1]

The series for Gross Domestic Product at constant prices and seasonally adjusted is the official series of the Federal Reserve System. A first series was computed by eliminating the components with periodicity smaller than 32 quarters. A second series was computed by eliminating components smaller than 4 quarters. The output gap is computed by the log of the second series divided by the first series.

[1] For a description of the band pass filter see Baxter and King (1999).

The federal funds interest rate source is the Federal Reserve System. The monthly annualized rates were transformed to quarterly annualized rates using a geometric average.

The Survey of Professional Forecasters reports the mean values for the GDP deflator prediction for the current quarter and the following quarter. Expected inflation is computed as the annualized change from the current quarter prediction to the next quarter prediction. The GDP deflator source is the Federal Reserve System.

Appendix B

Appendix: Inflation Dynamics: The Role of Expectations

B.1 Structural Phillips curve with contemporaneous survey in the instrument set

Table I - Full Sample Results

	ω	θ	β	$\lambda : s_t$	J-test
Unrestricted β					
(1)	0.2091	0.8553	0.9548	0.0203	0.6508
	(0.0763)	(0.0267)	(0.033)	(0.0087)	
(2)	0.3768	0.8726	0.9736	0.01	0.6997
	(0.0744)	(0.0338)	(0.0413)	(0.0066)	
Restricted β					
(1)	0.1839	0.8560	-	0.0166	0.7325
	(0.0767)	(0.0291)		(0.0069)	
(2)	0.3678	0.876	-	0.0080	0.7695
	(0.0759)	(0.0338)		(0.0045)	

Table II - Sub-sample Analysis

	ω	θ	β	$\lambda : s_t$	J-test
1968:04 to 1989:4					
Unrestricted β					
(1)	0.2353	0.7662	0.9374	0.0530	0.8464
	(0.0622)	(0.0273)	(0.0363)	(0.0118)	
(2)	0.3771	0.7815	0.9568	0.0317	0.8555
	(0.0629)	(0.0338)	(0.0452)	(0.0108)	
Restricted β					
(1)	0.2350	0.7578	-	0.0472	0.8953
	(0.0639)	(0.029)		(0.0112)	
(2)	0.3867	0.7754	-	0.0283	0.9030
	(0.0646)	(0.0365)		(0.0098)	
1980:01 to 2004:02					
Unrestricted β					
(1)	0.1460	0.854	0.9702	0.0219	0.7457
	(0.0593)	(0.0210)	(0.0471)	(0.0088)	
(2)	0.4010	0.8669	0.9607	0.0110	0.7601
	(0.0679)	(0.0247)	(0.0571)	(0.0053)	
Restricted β					
(1)	0.1336	0.8536	-	0.0191	0.8191
	(0.0588)	(0.0231)		(0.0072)	
(2)	0.4038	0.8705	-	0.0081	0.8282
	(0.0692)	(0.0289)		(0.0041)	

Table III - Including further lags of inflation

	ω	θ	β	Ψ	$\lambda : s_t$	J-test
Unrestricted β						
(1)	0.1981	0.906	0.9779	0.0326	0.0079	0.6185
	(0.1278)	(0.0587)	(0.0779)	(0.0485)	(0.0083)	
(2)	0.2760	0.9089	1.0192	0.0155	0.0042	0.6546
	(0.1407)	(0.0857)	(0.1003)	(0.0541)	(0.0078)	
Restricted β						
(1)	0.1693	0.9004	-	0.0188	0.0078	0.7026
	(0.0962)	(0.0532)		(0.0221)	(0.0080)	
(2)	0.2197	0.924	-	0.0257	0.0040	0.7097
	(0.0954)	(0.0705)		(0.0247)	(0.0076)	

Table IV - Alternative Measures of Marginal Cost

	ω	θ	β	$\lambda : s_t$	J-test
Unrestricted β					
(1)	0.2848	0.5752	0.8984	0.0376	0.6190
	(0.0653)	(0.0487)	(0.0412)	(0,0106)	
(2)	0.385	0.6645	0.9679	0.0153	0.7015
	(0.0647)	(0.0533)	(0.0409)	(0,0069)	
Restricted β					
(1)	0.2986	0.5780	-	0.0311	0.7290
	(0.0662)	(0.0475)		(0,0088)	
(2)	0.3865	0.6720	-	0.0136	0.7731
	(0.0640)	(0.0476)		(0.0048)	

Table V - Non-farm Business Deflator

	ω	θ	β	$\lambda : s_t$	J-test
Unrestricted β					
(1)	-0.0506	0.8680	0.9764	0.0256	0.7088
	(0.0671)	(0.0332)	(0.034)	(0.0132)	
(2)	0.2172	0.9063	0.9744	0.0078	0.6777
	(0.0859)	(0.0459)	(0.0388)	(0.0074)	
Restricted β					
(1)	-0.0587	0.864	-	0.0241	0.726
	(0.0624)	(0.0358)		(0.0127)	
(2)	0.2084	0.9222	-	0.0043	0.7506
	(0.0867)	(0.0623)		(0.0069)	

Appendix C

Appendix: Loose Commitment

C.1 Proofs

Proof. of Proposition 1

Drop history dependence and define:

$$r(c_t, k_t) \equiv u(c_t, k_t) + \beta(1-\pi)\xi(l(c_t, k_t))$$

$$g_1(c_t, k_t) \equiv b_1(c_t, k_t) + \beta(1-\pi)b_2(\Psi\{l(c_t, k_t)\}, l(c_t, k_t))$$

$$g_2(c_{t+1}, k_{t+1}) \equiv b_2(c_{t+1}, k_{t+1})$$

Our problem is thus:

$$\max_{\substack{\{c_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t=\omega^t_{ND}}} \sum_{t=0}^{\infty} (\beta\pi)^t \{r(c_t, k_t)\} \tag{C.1}$$

$$s.t: \quad k_{t+1} = \ell(c_t, k_t)$$

$$g_1(c_t, k_t) + \beta\pi g_2(c_{t+1}, k_{t+1}) = 0$$

which fits the definition of Program 1 in Marcet and Marimon (1998). To see this more clearly note that our discount factor is $\beta\pi$ and we have no uncertainty. Since ω_{ND}^t is a singleton, we have previously transformed our stochastic problem into a non-stochastic problem. Therefore, we can write the problem as a saddle point functional equation in the sense that there exists a unique function satisfying

$$\begin{aligned} W(k,\gamma) &= \min_{\lambda\geq 0}\max_c\{h(c,k,\gamma,\lambda)+\beta\pi W(k',\gamma')\} \\ s.t: \ \ k' &= \ell(c,k) \\ \gamma' &= \lambda, \ \gamma_0 = 0 \end{aligned} \tag{C.2}$$

where

$$h(c,k,\lambda,\gamma) = r(c,k) + \lambda g_1(c,k) + \gamma g_2(c,k) \tag{C.3}$$

or in a more intuitive formulation define:

$$h^m(c,k,\lambda,\gamma) = u(c,k) + \lambda g_1(c,k) + \gamma g_2(c,k) \tag{C.4}$$

and the saddle point functional equation is:

$$\begin{aligned} W(k,\gamma) &= \min_{\lambda\geq 0}\max_c\{h^m(c,k,\lambda,\gamma)+\beta(1-\pi)\xi(k')+\beta\pi W(k',\gamma')\} \\ s.t: \ \ k' &= \ell(c,k) \\ \gamma' &= \lambda, \ \gamma_0 = 0 \end{aligned} \tag{C.5}$$

■

Proof. of Proposition 2: Using Proposition 1, this proof follows trivially from the results of Marcet and Marimon (1998). ■

C.2 First Order Conditions - Probabilistic Model

To solve the problem first set up the Lagrangian, using ν_t and λ_t as Lagrange multipliers for the two constraints. Thus, we need to find the FOCs of the following problem:

$$\underset{\{\nu_t,\lambda_t\}_{t=0}^{\infty}}{Min}\ \underset{\{c_t,k_{t+1}\}_{t=0}^{\infty}}{Max}\ \mathscr{L} = \sum_{t=0}^{\infty} (\beta\pi)^t\, u\left(c_t, k_t\right) + \beta\left(1-\pi\right)\xi\left(k_{t+1}\right)$$

$$+\nu_t\left(\ell\left(c_t,k_t\right) - k_{t+1}\right) + \lambda_t\left(g_1\left(c_t,k_t\right) + \beta\pi g_2\left(c_{t+1},k_{t+1}\right)\right)$$

The FOCs are[1]:

$$\frac{\partial \mathscr{L}}{\partial c_t} : u_{c,t} + v_t \ell_{c,t} + \lambda_t g_{1,c,t} + \lambda_{t-1} g_{2,c,t} = 0 \tag{C.6}$$

$$\frac{\partial \mathscr{L}}{\partial k_{t+1}} : \beta\left(1-\pi\right)\xi_{k,t+1} - v_t + \beta\pi\left(\lambda_t g_{2,k,t+1} + u_{k,t+1} + \lambda_{t+1} g_{1,k,t+1} + v_{t+1}\ell_{k,t+1}\right) = 0 \tag{C.7}$$

$$\frac{\partial \mathscr{L}}{\partial \nu_t} : k_{t+1} = \ell\left(c_t, k_t\right) \tag{C.8}$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_t} : g_1\left(c_t,k_t\right) + \beta\pi g_2\left(c_{t+1},k_{t+1}\right) = 0 \tag{C.9}$$

$$\forall t = 0,...,\infty \qquad \lambda_{-1} = 0$$

where, using Eqs. (3.17,3.18) it follows that:

$$g_{1,c,t} = b_{1,c,t} + \beta\left(1-\pi\right)\left[\ell_{c,t}\left(b_{2,c,t+1}\Psi_{k.t+1} + b_{2,k,t+1}\right)\right]$$

$$g_{2,c,t} = b_{2,c,t}$$

$$g_{1,k,t} = b_{1,k,t} + \beta\left(1-\pi\right)\left[\ell_{k,t}\left(b_{2,c,t+1}\Psi_{k.t+1} + b_{2,k,t+1}\right)\right]$$

$$g_{2,k,t} = b_{2,k,t}$$

[1]For notational simplicity we treat k and c as scalars instead of vectors. The symbol $f_{x,t}$ indicates the partial derivative of the function $f(x_t)$ with respect to x_t. We suppressed the arguments of the functions for readability purposes.