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Theoretical and empirical issues on risk-sharing

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This thesis is dedicated to my family.

Usual disclaimer applies.

1 Introduction

Standard economic theory - together with common sense - suggests that, in an uncertain environment, risk-averse agents are better-off if they manage to insure the uncorrelated part of the fluctuations in their resources. In a frictionless environment, every agent's consumption should not respond to idiosyncratic shocks in his income. This condition can be interpreted as a cross-sectional version of the Permanent Income Hypothesis: individual consumption should not vary across agents in response to uncorrelated shocks in their individual income. Rational agents should insure one to the other against idiosyncratic movements in their incomes. They should achieve what is commonly called "full risk-sharing". However, casual observation of consumption data suggests that such condition is hardly met at all levels of aggregation. Individual consumption seems to be very correlated with both current and lagged individual income.

Here we address the issue of imperfect risk sharing from two standpoints.

In the first contribution, we address the problem of lack of optimal social insurance owing to the difficulties of eliciting socially optimal actions from individuals. A stream of research proposes to study the properties of constrained optimal allocations in presence of incentive problems using contract theory. Generally speaking, a contract is a mutual agreement between heterogenous agents, who jointly improve their welfare by redistributing their resources. Here, the contract also provides a theoretical benchmark: the optimal allocation.

An influential stream of literature assumes that the lack of risk insurance is generated by the technological problem of enforcement of contracts. Even though there is full and public information, perfect compliance may not be achieved because it is not legal, or because its cost would be prohibitive. There is no direct measure to force the agents in the contract. They can always step out, with no direct consequences. However, there is one indirect measure that can credibly make the life of the deviating agent less easy. The agent that refuses to comply will be excluded from the contract from the time of the deviation onwards. The utility of consuming in autarchy forever will be the exit option in the hands of each agent. An insurance contract, to be sustainable, should be self-enforcing, that is, it must deliver to each agent, at each point in time, at least the same utility he would get in autarchy. In practical terms, this amounts to tuning upon the trade-off between short-run benefits (enjoy the temporaneous good income realisation) and long-run costs of default (consume in autarchy forever).

In this thesis we extend the scope of this approach studying the consequences of collective default. In fact, we argue that, in several circumstances, restricting the alternative to autarchy consumption seems unrealistic. Generally speaking, borrowers may find access to credit through many different channels. Beside the official international financial markets, income fluctuation of sovereign countries, for instance, are smoothed away also within regional treaties, trade arrangements, and so on. Individuals tend to pool their resources through informal agreements within extended families, kinships, and so on. If coordination to punish jointly infractors is not sustainable, a borrower may take on a loan from one institution and at some point renege it to enter one of these other arrangements.

We provide a first characterisation of the dynamics of lending induced by the optimal contract robust to deviations of this sort. The equilibrium pattern of lending is shown to be complex and sensitive to the stochastic properties of the income process. More importantly, it displays very different features with respect to the case in which collective default is not permitted. First of all, less lending will be sustainable in equilibrium. Second, a borrower is going to be more likely to receive future lending the more he has been convinced not to default in the past and the less is the future utility from his best outside alternative (with individual default, the opposite would happen making the borrower with higher future utility be more likely to be promised more future lending). Third, lending dynamics will heavily depend on how individual incomes relate to each other (with individual default, only individual income characteristics count).

Risky assets and insurance demand. The second contribution of this thesis is empirical. Using panel data on wealth and income characteritics of Italian households, we aim to explain the scarce diffusion of health and property insurance products with low degree of risky asset held in their portfolios.

In the classical optimal insurance model a risk averse agent has to decide whether to buy an unfairly priced insurance policy, or increase savings to self insure. The main prediction is that the agent will buy insurance only if the marginal rate of substitution between endowments in the different states of nature exceeds the market premium rate. The more the consumer is able to transfer readily liquid resources to the future, the less the marginal rate of substitution is likely to be higher than the premium rate.

However, certain types of assets can be used as self insurance instruments more efficiently than others. Safe and liquid assets like transaction and saving accounts are certainly more suitable for precautionary saving than risky assets, such as stock or shares of mutual funds, or illiquid assets such as life insurance premiums or shares in pension funds. Following this reasoning, it is more likely that the marginal rate of substitution of an agent with high portfolio shares of risky or illiquid assets goes above the market premium rate, and thus more likely that the agent purchases private insurance.

We test this hypothesis using the data collected in the Survey of Household Income Wealth (SHIW) run by the Bank of Italy over the period 1989-2002. We estimate a censored regression model where the propensity to purchase health and property insurance is related to the degree of diversification the asset portfolios held by the households. The estimation's results confirm the existence of a positive relationship between the degree of portfolio diversification and the inclination to purchase property insurance. We obtain less mileage from the estimation of the health insurance equation. As a by-product, we present a detailed account of the diffusion of health and property insurance across Italian household grouped by socio-economic, demographic and territorial characteristics.

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Part II

Sustainable borrowing with local default

Sustainable borrowing with local default*

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Abstract

This paper studies the dynamic properties of self-enforced lending contracts between a credit institution and sovereign borrowers. In our set-up, borrowers can default collectively on the lending contract to share risk between them in alternative agreements. In these circumstances, the relevant outside options to the borrowers would be given by the utility attainable in these alternatives. The self-enforcing lending contract is obtained preventing borrowers' mutual improvement in any of the risk-sharing alternatives. We adapt the Lagrangian method developed by Marcet and Marimon (1998) to our environment and cast the problem for the efficient lending contract in a recursive framework. Depending on the different contractual conditions in the alternative, the dynamics of lending will display a variety of patterns in response to income shocks – in any case, quite different to the case with individual default only. We analytically solve for the self-enforcing credit contract in a simple one-shock environment to show the main properties of the lending contract.

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1 Introduction

Background and motivations. In the last 50 years, developing countries have had access to foreign capital mainly through international lending. In the 1970's, capital flows to less developed countries increased substantially, reaching in 1999 nearly half of their GDP.[1] Nevertheless, in spite of the several foreign debt default crises experienced over the last century, there is evidence that on average lenders have recovered their principal. Lending to sovereign borrowers has proved to be a profitable business. Even loans in default ended up being profitable ex-post. Eichengreen and Portes (1989) observed that, due to default, ex-post rates on British and U.S. loans to sovereign borrowers resulted lower than the rates established contractually. However, the same rates were still higher than those available on the domestic market.

An accurate analysis of the dynamics of international lending cannot overlook the problem of how repayments from sovereign entities are enforced. In effect, whilst domestic laws generally enforce contracts reallocating collateral from insolvent borrowers to lenders, a sovereign borrower cannot be obliged to repay its debt by virtually any international law. The absence of a third-party authority with recognised powers over the resources of sovereign debtors makes the enforcement of contracts with direct measures in general very difficult. However, even without the possibility of punishing defaulters with direct sanctions, some lending can still be sustained by threatening insolvent borrowers with the harshest indirect punishment: the perpetual foreclosure from any future lending. Sustainable repayment schemes should be designed so that borrowers receive at least the same utility as in their best outside alternatives. If the lender is capable to impose a complete embargo from future intertemporal trade, the best outside opportunity would be the utility of consuming in autarchy from the time of default onwards.

The lack of enforceability of contracts has been pointed out as a main cause of imperfect allocation of risk in more general environments. Marcet and Marimon (1992 and 1998) study the implications of imperfect participation and information on growth and consumption smoothing in a model with capital accumulation. Kocherlakota

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[1] World Development Indicators (2003), Table 4.16, External Debt.

(1996) addresses the issue more explicitly analysing a model of dynamic insurance between two risk-averse agents. Alvarez and Jermann (2000) study implications on asset price dynamics of the problem of enforceability with a decentralised version of the model of Kocherlakota. Eaton and Gersowitz (1981) stressed on the role of reputation for solvency to enforce lending to sovereign countries. In their setting, borrowers would repay debt only to the extent this gives them future opportunities to smooth income fluctuations through lending.[2]

In this paper, we depart from this scheme arguing that, in several circumstances, restricting the alternative to autarchy consumption seems unrealistic. Generally speaking, borrowers may find access to credit through many different channels. Beside the official international financial markets, income fluctuation of sovereign countries, for instance, are smoothed away also within regional treaties, trade arrangements, and so on. If coordination to punish jointly infractors is not sustainable, a sovereign borrower may take on a loan from one institution and at some point renege it to enter one of these other arrangements.

We assume that an alternative available to a borrower that defaults is to form a risk-sharing agreement with another defaulting borrower. This has a number of consequences. First, the lending contract may experience simultaneous default by several borrowers. Second, the desirability of the outside option is in general higher. Individual participation constraints may no longer be sufficient to ensure sustainability. The lender should avoid the forming of collective arrangements too.

In the formal set-up, there is a non-sovereign risk-neutral credit agency – the Lender – that borrows and lends to risk-averse sovereign agents – the Borrowers – endowed

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[2] Bulow and Rogoff (1989) showed that if a defaulter has access to a rich variety of cash-in-advance contracts, then no lending contract would be sustainable in equilibrium. The argument is quite strong, in that the cash-in-advance contracts they propose constitute a form of saving, though very sophisticated. It is easy to believe that a bad credit record can foreclose access to further borrowing. It is less likely that it keeps from *saving*. This result spurred an intense debate: Kletzer and Wright (2000) showed that some lending is indeed sustainable if the financial institution too cannot make binding commitments; Cole and Kehoe (1998) argue that a country has to maintain credibility in non-debt relationships (military, political etc.), outside the credit market, that would however be undermined by the default in credit markets. In this paper, we do not address this issue directly.

with stochastic income streams. Owing to the sovereign status of the borrowers, the lender is unable to elicit full participation from them. On the other hand, he is able to make perfectly binding commitments on future payments. After observing the income realisation, the borrowers can continue in the lender's contracts, or arrange some risk-sharing between them in a "local agreement" made up of the two borrowers in isolation. An important difference between the lender and the borrowers is given by the credit technology, to which only the lender has access. If the borrowers default on the lender, they can only trade on current income realisations.

Self-enforced contracts should be constructed so that no borrower finds any outside alternative more attractive, neither individually nor collectively. We propose a mechanism to avoid collective deviations that consists in breaking up consensus necessary to form any possible alternative agreements. Only one of the two borrowers threatening to default collectively is going to be promised at least the same treatment he would receive in the risk-sharing alternative. This operation will be performed efficiently, so that the lender's stream of profits are maximised, provided the borrowers are given some minimum initial utility.

We construct the set of contracts robust to collective default concentrating our analysis on the class of efficient contracts. A feature displayed by solutions satisfying these type of constraints is that they end up depending on an increasingly large number of variables, potentially infinite. A way to reduce this dimensionality would be setting out recursively the lender's optimal problem. As standard dynamic programming techniques cannot be employed when sustainability constraints include future choice variables, we use a version of the Lagrangian method proposed by Marcet and Marimon (1992 and 1998) to write the efficient contract problem as a function of a limited number of states variables.

We provide a first characterisation of the dynamics of lending induced by the optimal contract inspecting the first order conditions. The equilibrium pattern of lending is shown to be complex and sensitive to the stochastic properties of the income process. More importantly, it displays very different features with respect to the case in which collective default is not permitted. First of all, less lending will be sustainable in equilibrium. Second, a borrower is going to be more likely to receive future lending the more he has been convinced not to default in the past and the less is the future utility from his best outside alternative (with individual default, the opposite would happen making the borrower with higher future utility be more likely to be promised more future lending). Third, lending dynamics will heavily depend on how individual incomes relate to each other (with individual default, only individual income characteristics count). Fourth, some characterstics of the alternative agreements – namely the surplus' sharing rule and the degree of enforceability – are relevant to the way income shocks propagate to the pattern of lending.

Outline of the paper. The paper is organised in the following way. Section 2 describes the formal environment and briefly discuss the case of individual deviation. Section 3 introduces the possibility of collective deviation allowing the formation of local, mutually beneficial, alternative risk-sharing arrangements. After discussing the sustainability of borrowing and lending contracts in this modified environment, section 4 presents the optimal contract offered by the credit agency using a recursive formulation. In section 5, we look at the pattern of lending induced by the sustainable contract a simple one-shock economy with aggregate fluctuations proving the main features of the sustainable contract. The last section sets out our main conclusions and proposals for future research.

All results are proved in the Appendix.

2 Environment and hypotheses

Time is discrete and infinite.

There are two risk-averse sovereign agents – the **Borrowers** – indexed by $j = a, b$. Each one of them orders preferences on consumption (absorption) sequences $\{c_{jt}\}_{t\geq 0}$ of the only non-storable good of this economy by means of a Von-Neumann-Morgenstern expected utility representation.

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{jt})$$

The period utility function $u(.)$ is differentiable, increasing, strictly concave and satisfying standard Inada conditions; $\beta \in (0, 1)$ is the time preference factor.

Alongside, a risk-neutral credit agency, the **Lender,** has access to an exogenous credit market in which he can borrow and lend at the exogenous interest rate r. The lenders seeks to maximise the benefits proceeding from the credit arrangements reached with the borrowers.

$$\Pi = E_0 \sum_{t=0}^{\infty} R^{-t} \sum_{j=a,b} (y_{jt} - c_{jt})$$

where $R \equiv 1 + r$.

Uncertainty consists in the exogenous realisations of endowments y_{jt} to which each borrower is entitled. Borrowers are ex-ante identical: their incomes are drawn from the same compact support in R_{++}. Aggregate income of the borrowers $Y_t \equiv y_{at} + y_{bt}$ fluctuates over time. A state of this economy at time t is fully described by the list of agents' income realisations $\mathbf{y}_t = \{y_{at}, y_{bt}\}$. The conditional state history $\mathbf{y}^s/\mathbf{y}^t = \{\mathbf{y}^t_{,}\mathbf{y}_{t+1}, ...\mathbf{y}_s\}$ is the list of all the realisations of $\mathbf{y}$ up to $s \geq t$ after the unconditional history $\mathbf{y}^t = \{\mathbf{y}_0,\mathbf{y}_1, ...\mathbf{y}_t\}$, given $\mathbf{y}_0$. The probability of observing $\mathbf{y}^t$ is $\Pr(\mathbf{y}^t)$ and the conditional probability of observing $\mathbf{y}^s$ after history $\mathbf{y}^t$ is $\Pr(\mathbf{y}^s/\mathbf{y}^t)$. The expectation conditional to the information available up until t of the generic sequence $\{x_s\}_{s \geq t}$, $E_t \sum_{s=t}^{\infty} x_s$, is the average taken over all possible conditional histories $\mathbf{y}^{t+s}/\mathbf{y}^t$ after the node $\mathbf{y}^t$, $\sum_{s=t}^{\infty} \sum_{\mathbf{y}^{t+s}} \Pr(\mathbf{y}^{t+s}/\mathbf{y}^t) x(\mathbf{y}^s/\mathbf{y}^t)$.

Owing to their sovereign status, the borrowers are unable to commit ex-ante to agreements with the lender. On the other hand, the lender is able to bind himself to undertake future actions.

Upon realisations, incomes are publicly and costlessly observable.

2.1 Individual default

At time 0, the lender and the borrowers sign a contract consisting in a list of state-contingent transfers to be performed in the future. A useful way to interpret these transfers is to regard them as part of an implicit insurance contract resulting from periodical renegotiations of a standard lending contract. In any case, agreements of this sort would need an enforcement mechanism capable to induce the participants to undertake the transfers when they are called upon to do so.

With sovereign borrowers, obtaining compliance by direct measures may result too costly or even not feasible. However, if the agreement can credibly exclude borrowers from future participation, then compliance may be obtained indirectly by threatening borrowers with the worst possible indirect punishment. This amounts to excluding infractors perpetually from any future transfer scheme. Formally, this requires to bind the agreement to satisfy borrowers' individual rationality constraints to make sure that each participant obtains from the contract at least the same utility they would get outside.

To fix ideas we can think of a village in which there are two farmers working in a fertile land and a risk-neutral moneylender. At the end of every period t, they harvest a random production of crop y_{jt}, and then turn it over to the moneylender, who invests it outside the village at an exogenous interest rate r. The lender will offer contracts to a and b conditional to making non-negative expected profits at time 0,

$$E_0 \sum_{t=0}^{\infty} R^{-t} \left[(y_{at} + y_{bt}) - (c_{bt} + c_{bt}) \right] \geq 0$$

making sure that at time 0 borrowers get at least the intertemporal utilities U_a and U_b.

The lender can make binding promises, but farmers can't. At any time, farmers can step out of the agreement if the expected utility of consuming their own crop from then on is higher. To avoid deviations, the moneylender must design the transfer scheme so that there would be no ex-post benefit from going away. This amounts to constraining the contract to the following sequence of constraints.

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js}) \geq V_{jt}^A(\mathbf{y}^t) \tag{1}$$

for $j = a, b$ and all $t \geq 0$, where $V_{jt}^A(\mathbf{y}^t) \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} u(y_{js})$.

The right hand side of inequality 1 represents borrower j's time t best alternative to the contract offered by the lender: perpetual autarchy consumption. With limited participation at the individual level, borrowers will renege if the sequence of future transfers delivers less utility than their lifetime autarchy consumption. When a borrower is asked to repay part of the debt – for instance when experiencing a positive income realisation – he will face a trade-off between the short-term gain of not transferring resources to the lender, and the enduring gain of receiving transfers in states with low income in the future. He would not renege a contract capable to balance this intertemporal trade-off. [3] We will refer to the set of allocations robust to individual deviations with the notation $\Omega = \left\{c_{1t}^{I}, c_{2t}^{I}\right\}_{t\geq 0}$.

In the next section, we take the first step away from this framework making borrowers face more structured alternatives to the contract offered by the lender.

3 Collective default

Enforceability of the contract described above is obtained by trading off the enduring gains offered by the long term agreements from the lender with the one-period current gains and subsequent perpetual exclusion represented by autarchy consumption. The borrower's contractual choice reduces to only two possibilities: either deal with the credit agency or stay in autarchy.

This dichotomy appears to us somewhat radical. In many contexts, agents can smooth income fluctuations using different channels and institutions. Families, extended families and small communities constitute an instance of environments in which risk may be, and often is, shared. Different members may pool part of their incomes and diversify their idiosyncratic components. In some rural communities people tend to smooth individual income fluctuations within groups, often characterised by vicinity, kinship or family links.

Sovereign countries make no exception. The formation of international agreements may create a broader set of financial instruments available to the member countries. A trade agreement, for example, by increasing the volume of trade would foster the development of trade-associated financial instrument, such as short-term commercial credit, through which some income fluctuations could be smoothed away.

The alternatives for risk sharing and consumption smoothing may be more or less enforceable. In fact, enforcement by reputation seems to be a characteristic of agree-

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[3] Another interpretation of the participation constraints is that of "reputation of payments", as they keep agents from reneging in order to build a reputation of solvency.

ments between sovereign and non-sovereign entities. Within a restricted group of countries default could be credibly avoided by threatening infractors with direct sanctions, such as commercial embargoes, exclusion from regional treaties and the like. Even in the case of lack of commitment within the local agreement, the threat of default by one member would be partially balanced by the threat of default by the others.

We will see that the possibility of joining alternative risk-sharing agreements may induce collective default on the lending contract. Borrowers may simultaneously find it convenient to renege the credit contract and arrange something between them. In such an event, the lender will no longer avoid default offering contracts proof to individual deviations only. With collective deviations, the sustainable credit contract should make sure that the borrowers don't form any jointly improving alternative agreements too.

3.1 Local agreements

It is well known that with complete information self-enforcing contracts will never experience default. Nevertheless, to understand the characteristics of the self-enforcing contract, we need to know what happens if default indeed takes place. With individual deviations, this is straightforward. Borrowers in default can only consume their individual income from the time of deviation onwards. But now the borrowers can default on the lending contract jointly to arrange some alternative risk-sharing agreement. With collective default, we need to characterise the possible equilibria in all the possible risk-sharing agreements that could be reached by groups of defaulting borrowers.

We first describe the characteristics of the alternative agreement as if the borrowers had no access to the lender's services, and they could only smooth income fluctuations between them. We refer to the arrangements achievable without the lender as "Local Agreements", to convey the idea of exclusion from international lending and the suffering of aggregate fluctuations.

For the time being, we will consider the broad class of agreements satisfying the minimal rationality requirement of being more attractive than individual autarchy. We require that such agreements be *mutually improving* at the time of formation, to have all joiners be better-off with respect to the their autarchy.

Definition 1 (Local Agreements) *A Local Agreement is a risk-sharing arrangement formed at some* t *by the two borrowers inducing a consumption allocation* $\{c^L_{as;t}, c^L_{bs;t}\}_{s\geq t}$*, such that the following conditions are met.*

1. *It's **Feasible**. That is, it satisfies the sequence of period resource constraints.*

$$\sum_{j=a,b} c^L_{js;t}(\mathbf{y}^s) \leq \sum_{j=a,b} y_{jt}(\mathbf{y}^s)$$

2. *It is **Mutually Improving**. That is, its members would ex-ante improve upon the value of their autarchies. Letting* $V^L_{jt}(\mathbf{y}^t) \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c^L_{js;t})$ *be the expected utility to borrower* j *in the local agreement, this means the following.*

$$V^L_{jt}(\mathbf{y}^t) \geq E_t \sum_{s=t}^{\infty} \beta^{s-t} u(y_{js})$$

for $j = a, b$*.*

The (local) feasibility requirement is the first consequence of collective default. After default, borrowers would no longer have access to the credit technology managed by the lender. The property of mutual improvability calls for a degree of ex-ante desirability of local agreements. Absent the credit agency, mutual improvability ensures that the local agreement would indeed be formed.

Next, we will see the implications of the existence of these arrangements on the structure of incentives of the borrowers when deciding whether to comply or not to the credit contract offered by lender.

3.2 The sustainable lending contract

The credit agency will offer the borrowers enduring lending contracts guaranteeing them at least as the same conditions they would enjoy outside. For a deviating borrower, "outside" there are the following options: either remain in autarchy and consume his own endowments for good, or join a local agreement, provided the other borrower is willing to do the same. The members of a local agreement formed at some t would obtain the expected utilities $V^L_{at}(\mathbf{y}^t)$ and $V^L_{bt}(\mathbf{y}^t)$, which will constitute their best outside opportunities. By mutual improvability, these will always dominate the expected utility of consuming in individual autarchy. It is already clear that satisfying the sequence of participation constraints based on the utility of individual autarchy consumption will no longer be sufficient to make the credit contract self-enforcing.

Let's consider the local agreement available at time t. The trivial way to avoid collective default would be promising both borrowers the same utility they would obtain from the local agreement: at least utility $V_{at}^{L}(\mathbf{y}^t)$ to borrower a **and** at least utility $V_{bt}^{L}(\mathbf{y}^t)$ to borrower b. However, the lender has a subtler and surely more efficient way to prevent the formation of local agreements. A local agreement would arise only if the two borrowers consensually decide to default on the credit contract. The lender can disrupt consensus making sure that **either** a gets $V_{at}^{L}(\mathbf{y}^t)$ **or** b gets $V_{bt}^{L}(\mathbf{y}^t)$ – not necessarily both –, and then ensure that none of them deviate individually.[4] If this operation is performed at all t's, then the resulting credit contract will be proof to all deviations – individual and collective.

We are now ready to spell out rigorously the central concept of this paper. We will refer to the contract fulfilling these requirements as to the *Sustainable Lending Contract.*

Definition 2 (The Sustainable Lending Contract) *Let* $\mathbf{C} = \{c_{at}, c_{bt}\}_{t\geq 0} \in \mathbf{\Omega}$ *a consumption allocation proof to individual deviations. Let* $\left\{V_{at}^{L}(\mathbf{y}^t), V_{bt}^{L}(\mathbf{y}^t)\right\}_{t\geq 0}$ *be the sequence of intertemporal utilities obtainable joining local agreements. We define the set of sustainable credit contracts* $\mathbf{\Lambda}$ *as follows.*

$$\mathbf{\Lambda} = \left\{ \begin{array}{c} \mathbf{C} \in \Omega{:} E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{as}) \geq V_{at}^{L}(\mathbf{y}^t) \ \textit{\textbf{or}} \\ E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bs}) \geq V_{bt}^{L}(\mathbf{y}^t),\ \forall \mathbf{y}^t,\ t \geq 0 \end{array} \right\}$$

The lender is bound to offer contracts in the set $\mathbf{\Lambda}$. As for the case of individual deviations, at time 0 minimum profit requirement has to be satisfied.

$$E_0 \sum_{t=0}^{\infty} R^{-t} \left[y_{at} + y_{bt} - c_{bt} - c_{bt} \right] \geq 0$$

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[4] We write "**and**" and "**or**" in bold to stress their nature of logical operators.

4 The efficient contract

The lender will act efficiently. After promising the borrowers some initial future utility, he will solve an optimal contracting problem maximising the expected sum of benefits from the contract subject to the constraints defined in $\mathbf{\Lambda}$.

The solution to the lender's problem identifies the frontier of efficient contracts. Let's call it $F_{\mathbf{\Lambda}}$.

$$\begin{aligned} F_{\mathbf{\Lambda}} &= \sup_{C\in\mathbf{\Lambda}} E_0 \sum_{t=0}^{\infty} R^{-t} \sum_{j=a,b} (y_{jt} - c_{jt}) \qquad (2) \\ \text{s.t.} &: E_0 \sum_{t=0}^{\infty} \beta^t u(c_{jt}) \geq U_j \end{aligned}$$

Where $j = a, b$. U_a and U_b are time 0 promises of intertemporal utility that parametrise the frontier of efficient contracts.[5]

The set of sustainable credit contracts $\mathbf{\Lambda}$ displays a somewhat complex structure that impedes a straightforward characterisation of the frontier as it would be without commitment problems. Indeed, as uncertainty unfolds, the agents' collective outside opportunities $V_{at}^{L}(\mathbf{y}^t)$ and $V_{bt}^{L}(\mathbf{y}^t)$ may increase up to the point agents are tempted to renege the lender's agreement and join local agreements. The lender would then have to choose one of the two borrowers threatening default and promise more future utility to him. This would permanently modify the distribution of expected utility across borrowers in favour of the chosen borrowers.

This cannot be seen directly. Our definition of the set of sustainable lending contracts over which the lender should maximise lacks of analytical tractability. The next result addresses this issue.

Result 1 *The generic consumption allocation* $\mathbf{C}$ *will be proof to collective default if and only if it satisfies the following sequence of constraints.*

$$\max_{j=a,b} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js}) - V_{jt}^{L}(\mathbf{y}^t) \right] \geq 0, \text{ for all } t \qquad (3)$$

The sustainability requirement associated to collective default is expressed as in a more familiar way. We are now ready to restate the efficient sustainable lending

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[5] For instance, the expected value of their endowments at time 0.

contract maximisation problem in the more familiar way. The contract will maximise time 0 expected profits to the lender,

$$\max_{\{c_{at},c_{bt}\}_{t\geq 0}} E_0 \sum_{t=0}^{\infty} R^{-t} \sum_{j=a,b} (y_{jt} - c_{jt}) \tag{4}$$

so that the following sequence of sustainability constraints are satisfied for $j = a, b$ and $t \geq 0$,

$$\begin{aligned} \max_{j=a,b} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js}) - V_{jt}^{L}(\mathbf{y}^t) \right] &\geq 0 \\ E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js}) - V_{jt}^{A}(\mathbf{y}^t) &\geq 0 \end{aligned} \tag{5}$$

for $j = a, b$, making sure borrowers get U_a and U_b at time 0.

Alongside the individual participation constraints, the constraints associated to the utility attainable in a local agreement by the borrowers are to be satisfied. This is the key difference to the case of individual deviations. The structure of incentives of a borrower will also be influenced by the stochastic characteristics of the other in a direct manner. Not only will his outside opportunity depend on his income realisation, but also on its degree of ”compatibility” with the other borrower.

The solution to the efficient credit contract will then incorporate this peculiar asymmetry. The borrowers threat default as a group, but the relevant enforcement problem is still associated to individuals. The unity of the group determines the value of the outside option, but only individual agents are those who will eventually gather the long-term benefits for forgoing such option.

4.1 A Lagrangian formulation

Whenever the borrowers are tempted to deviate, either individually or collectively, they would be convinced to stay with a promise of permanently higher future utility. The efficient contract should keep track of all these promises made in the past to avoid default of borrowers. The mathematical solution of an optimisation problem with this backloading feature becomes quite complex, as the dimensionality of the control variables increases very rapidly with time. Every time the individual and sustainability constraints bind the future distribution of consumption between agents will be permanently affected. As a consequence, the contractual solution may have to account for the whole history of income realisations.[6]

In our environment, in addition, things are complicated by the asymmetry between the entity threatening default and that being awarded for not stepping out. The implications of this discrasy could not emerge so clearly from a solution that potentially involves an infinite number of variables. To reduce the number of the relevant variables, we need to construct a recursive version of the same contractual problem. Kydland and Prescott (1977) showed that these constraints cannot be reconducted to known law of motions of the current states variables and the policy function cannot take the usual form. The standard dynamic programming techniques that have been used extensively in the dynamic macroeconomics literature (see Stockey et al. 1989) are not suitable for problems with constraints involving expectations of future variables such as those in 1 and 5.

We develop a Lagrangian specification extending the techniques illustrated by Marcet and Marimon (1998, henceforth MM) to our environment. The strategy is to restate the optimisation problem including the constraints directly in a Lagrangian objective function and look for a saddle-point solution in the state-space enlarged with a co-state variable that keeps record of the whole history of binding constraints. Their set-up is quite general and it is suitable for a broad class of problems for which the standard Bellman equation doesn't apply. They present a number of practical examples for applications. Our main reference is going to be their "Example 1: A Partnership with Limited Commitment", in which they propose a recursive formulation of a problem of consumption risk-sharing with individual participation constraints.

Here we have to face a further complication that keeps us from applying MM's technique straight away. Apart from the sequence of individual participation constraints – which are still there to be fulfilled – there is also the sequence of sustainability constraints defined on the borrowers taken collectively. However, only individual agents should be receiving compensations for not stepping out of the contract. The co-state variables keeping record of these compensations need to be related to borrowers. We must find a way to link the collective sustainability constraint with individual utilities.

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[6] This is a typical feature of contracts based on reputation for solvency.

Henceforth, we will assume that the borrowers' individual participation constraints never bind and assume the interest rate r is equal to $\beta^{-1}-1$ to focus on risk-sharing and smoothing only. We will present and discuss the general case in the appendix.

Let $\{\overline{\gamma}_t(\mathbf{y}^t)\}_{t\geq 0}$ be the sequence of non-negative Lagrange multipliers associated to the collective sustainability constraints and $\alpha_j > 0$ that is associated to the initial promise of lifetime utility U_j to borrower j.

We may write the Lagrangian function as follows.

$$\begin{aligned} H &= E_0\left\{\sum_{t=0}^{\infty}\beta^t\left\{\sum_{j=a,b}(y_{jt}-c_{jt})+\right.\right. \\ &\quad \left.+\overline{\gamma}_t\max_{j=a,b}\left[E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{js})-V_{jt}^L(\mathbf{y}^t)\right]\right\}+ \\ &\quad \left.+\sum_{j=a,b}\alpha_j\left[\sum_{t=0}^{\infty}\beta^t u(c_{jt})-U_j\right]\right\} \qquad (6)\end{aligned}$$

For each binding constraint, the contract will optimally choose one agent to award to break unanimity in the deviating group.

The following result allows us to rewrite the planner's problem more suitably.

Proposition 1 *Let $\mathcal{I}_{jt}$ be an indicator function defined on borrower j such that, for all consumption allocations C,*

$$\mathcal{I}_{jt} = \begin{cases} 1 & \text{if } E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{js})-V_{jt}^L(\mathbf{y}^t) > E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{is})-V_{it}^L(\mathbf{y}^t),\ i\neq j \\ 0 & \text{Otherwise} \end{cases}$$

then, the collective participation constraint

$$\max_{j=a,b}\left[E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{js})-V_{jt}^L(\mathbf{y}^t)\right]\geq 0 \qquad (7)$$

is satisfied if and only if the following constraint is satisfied as well.

$$\sum_{j=a,b}\mathcal{I}_{jt}\left[E_t\sum_{s=t}^{\infty}\beta^{s-t}U(c_{js})-V_{jt}^L(\mathbf{y}^t)\right]\geq 0$$

The indicator $\mathcal{I}_t$ is a well defined function. Mapping from the set of consumption allocations, it selects which one of the borrower j's participation constraint is going to be the one to be taken into consideration in the efficient contract at each point time. Notice that $\mathcal{I}_{at} + \mathcal{I}_{bt} = 1$, which makes sure that at most one agent will get the compensation for not deviating and join a local agreement.

Let's incorporate this result in the Lagrangian, then maximise it with respect to the borrowers' consumption sequences $\mathbf{C} \equiv \{c_{at}, c_{bt}\}_{t\geq 0}$ and minimise it with respect to the corresponding sequence of Lagrange multipliers $\mathbf{\Gamma} \equiv \{\gamma_t\}_{t\geq 0}$ associated to the local agreements that could be formed out of the lending contract.

$$\begin{aligned}\min_{\mathbf{\Gamma}} \max_{\mathbf{C}} \mathbf{J} \equiv E_0 \Bigg\{ \sum_{t=0}^{\infty} \beta^t \Bigg\{ \sum_{j=a,b} (y_{jt} - c_{jt}) + \\ + \gamma_t \sum_{j=a,b} \mathcal{I}_{jt} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{jt}) - V_{jt}^{L}(\mathbf{y}^t) \right] \Bigg\} + \\ + \sum_{j=a,b} \alpha_j \left[\sum_{t=0}^{\infty} \beta^t u(c_{jt}) - U_j \right] \Bigg\}\end{aligned} \tag{8}$$

Proposition 2 *The problem for the efficient sustainable lending contract 8 can be written as follows.*

$$\begin{aligned}\min_{\mathbf{\Gamma}} \max_{\mathbf{C}} E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=a,b} \{(y_{jt} - c_{jt}) + \\ + \mu_t^j u(c_{jt}) + \mathcal{I}_{jt} \gamma_t \left[u(c_{jt}) - V_{jt}^{L}(\mathbf{y}^t) \right] \}\end{aligned}$$

Where $\mu_{jt+1} = \mu_{jt} + \mathcal{I}_{jt}\gamma_t$ (with its initial value μ_{j0} set equal to the Lagrange multiplier associated to the initial utility promise α_j) is the (co)state variable accounting for the history of past temptations of collective deviation up to time t and relative promises of future utility to avoid them.

Using the algebraic steps in MM and the law of iterated expectations, we can group terms more suitably to write the lender's objective function as a function of a reduced number of variables. The period return function (the function h in MM) now depends on current consumptions c_j, the state $\mathbf{y}$, the Lagrange multiplier γ, defined on the local agreement, two co-state variables μ_j and the relative indicator functions $\mathcal{I}_j$, which instead are associated to borrowers.

$$h(c, \mathcal{I}, \gamma, \mu, \mathbf{y}) = \sum_{j=a,b} \left\{ (y_j - c_j) + (\mu_j + \mathcal{I}_j \gamma)\, u(c_j) - \mathcal{I}_j \gamma V_j^{L}(\mathbf{y}^t) \right\}$$

We can interpret h as if the lender shifts every period the weights assigned to the borrowers. The lender will set the Lagrange multiplier γ positive whenever the collective constraint binds, that is, when the borrowers are tempted to form a local agreement. However, the lender will offer more intertemporal utility (renegotiate debt) only to one of them to refrain from doing so. This is captured by the indicator function $\mathcal{I}_{jt}$, which is set equal to one if j is the chosen borrower. The positive multiplier on the collective default constraint γ will increase this borrower's co-state variable μ_j accounting for the whole history of temptations and promises of future utility.

This formulation differs from the example with individual participation constraints from at least two viewpoints. First, the value function is obtained by maximising over agents' consumption streams, but it is minimised over constraints defined on the *group* formed by them. The lender chooses the utility weight to be assigned to agents, according to the evolution of the Lagrange multipliers relative to the local arrangement they may threat to form as uncertainty reveals. Second, whilst the emergence of a local agreement clearly depends on the degree of risk-sharing affinity between its members, their individual consumption streams will be affected according to how the same arrangements can be broken.

Whether the contract that solves the original problem in 4 is also a solution to the Lagrangian function in 6 is a technical matter. However, it becomes a substantive issue if we use the Lagrangian formulation to provide qualitative insights on the behaviour of consumption path induced by the efficient contract. To make sure that our problem can be solved using the results in MM, we should verify that the set of allocations satisfies some regularity conditions on the stochastic process and the return functions, and that the set of sustainable contracts has at least an interior point (assumption A1-A3 and A-5 in MM).[7] These conditions are satisfied in our environment.

The definition of the sustainable lending contract requires that **either** one **or** the other member of a binding local agreement should be promised more future utility to

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[7] These assumptions ensure that the set of sustainable allocations is closed, bounded and non-empty, and thus a solution to the original problem stated in 4 exists.

refrain from exiting. The set of sustainable lending contracts Λ is the intersection of the unions of two convex sets, which may be non-convex. Marcet and Marimon acknowledge that many economic problems are cast in non-convex environments and discuss extensively the limits of applying their framework to study them. However, convexity is not a necessary condition for the existence of a solution to the Lagrangian. Indeed, MM prove that if the Lagrangian has a saddle-point solution, then this is going to solve also the primal problem.[8]

4.2 A characterisation of the optimal contract

From now onwards, we shall assume that the saddle-point problem 6 admits a solution, call it $\{c^*_{at}, c^*_{bt}, \mathcal{I}^*_{at}, \mathcal{I}^*_{bt}, \gamma^*_t\}_{t\geq 0}$, so that $\{c^*_{at}, c^*_{bt}\}_{t\geq 0}$ also solves the lenders problem stated in 4.

Optimal consumption path of borrower j. At an optimum, the following first order conditions are satisfied.

$$u'(c_{jt}) = \frac{1}{\mu_{jt} + \mathcal{I}_{jt}\gamma_t} \tag{9}$$

For every date t and state history $\mathbf{y}^t$, $j = a, b$.

We see that j's consumption is increasing in the term $\mathcal{I}_{jt}\gamma_t$, which will be positive at an optimum if the constraint on the local agreement binds and at the same time the choice variable $\mathcal{I}_{jt}$ is set equal to one.

Complementary slackness condition.

$$\gamma_t \geq 0 \text{ and } \gamma_t \sum_{j=a,b} \mathcal{I}_{jt} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{jt}) - V^L_{jt}(\mathbf{y}^t) \right] = 0 \tag{10}$$

The first order condition for consumptions is obtained directly differentiating the Lagrangian in 9 with respect to c_{jt}. In an efficient contract c_{jt} is increasing with the Lagrange multiplier γ_t associated to the group, provided that the corresponding choice variable $\mathcal{I}_{jt}$ is set equal to one. Note that c_{jt} inherits the history of promises of future utility received in the past to avoid deviations represented by the co-state variable μ. Finally, the value of γ_t is set as usual: equal to zero if the sustainability constraint holds slack, or positive otherwise.

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[8] See Theorem 2, page 26 in MM.

Co-state variable. The law of motion of μ_{jt} is obtained using the complementary slackness condition and the condition on $\mathcal{I}_{at}$.

$$\mu_{jt+1} = \mu_{jt} + \mathcal{I}_{jt}\gamma_t$$

with $\mu_{a0} \equiv \alpha_a \geq 0$ and $\mu_{a0} \equiv \alpha_b \geq 0$.

The co-states μ_{at} and μ_{bt} represent the utility weight assigned to borrowers at time t and evolve according to the value of the local agreement and to what extent this makes his default credible.

4.3 Lending dynamics

Let $\mathcal{I}_t \equiv \mathcal{I}_{at}$. Then, we may combine the optimality conditions for the two borrowers to study how their utilities will evolve over time relative to each other.

$$\frac{u'(c_{bt})}{u'(c_{at})} = \frac{\mu_{at} + \mathcal{I}_t\gamma_t}{\mu_{bt} + (1 - \mathcal{I}_t)\gamma_t} \tag{11}$$

In the states in which the local agreement delivers relatively high utility to both agents – when the borrowers' aggregate income is high – the probability that the collective sustainability constraint binds increases. If this effectively happens, 10 implies that the current Lagrange multiplier γ should then take a positive value. The function $\mathcal{I}_t$ will determine which borrower is going to be convinced not to join the agreement with more future lending or with his debt payment recontracted. The lender will pick this borrower in an efficient manner.

From the definition of $\mathcal{I}_t$, this choice will depend on the difference between the term $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js})$ relatively to the utility of the local agreement at time t, $V_{jt}^L(\mathbf{y}^t)$.

The value of borrower's j outside option $V_{jt}^L(\mathbf{y}^t)$ is exogenously determined and likely to be influenced by a number of elements, such as the stochastic properties of the income process, the initial utility weight assigned in the agreement, the degree of local enforceability and the like. More importantly, it represents the relative portion of surplus he can extract from the local agreement. On the other hand, $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js})$ is determined endogenously. From condition 9, we see that it grows with μ_{jt}, that is, the more the agent has been tempted to default in the past, the more likely he is going to be chosen to break consensus today and in the future.

Let's regard all the possible cases.

1. Both borrowers are strictly better off in the lending contract. The collective participation constraint doesn't bind. No co-state variable is updated.

   $$\begin{array}{lcll} E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{at}) > V_{st}^L(\mathbf{y}^t) & \multirow{2}{*}{$\gamma = 0$} & \mathcal{I}_{at} = \text{ any} & \mu_{at+1} = \mu_{at} \\ E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bt}) > V_{bt}^L(\mathbf{y}^t) & & \mathcal{I}_{bt} = \text{ any} & \mu_{bt+1} = \mu_{bt} \end{array}$$

   The pattern of lending will not be modified.

2. Borrower b (a) would be tempted to default, but the other is strictly better off staying. Again, no co-state is updated.

   $$\begin{array}{lcll} E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{at}) > [<]\ V_{st}^L(\mathbf{y}^t) & \multirow{2}{*}{$\gamma = 0$} & \mathcal{I}_{at} = 1\ [= 0] & \mu_{at+1} = \mu_{at} \\ E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bt}) < [>]\ V_{bt}^L(\mathbf{y}^t) & & \mathcal{I}_{bt} = 0\ [= 1] & \mu_{bt+1} = \mu_{bt} \end{array}$$

   The pattern of lending will not be modified.

3. Both borrowers are tempted to default. The collective participation constraint binds. Borrower a (b) has the lowest incentive to leave and therefore is promised more intertemporal utility.

   $$\begin{array}{lcll} E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{at}) = \ [<]\, V_{st}^L(\mathbf{y}^t) & \multirow{2}{*}{$\gamma > 0$} & \mathcal{I}_{at} = 1\ [= 0] & \mu_{at+1} > [=]\, \mu_{at} \\ E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bt}) < \ [=]\, V_{bt}^L(\mathbf{y}^t) & & \mathcal{I}_{bt} = 0\ [= 1] & \mu_{bt+1} = [>]\, \mu_{bt} \end{array}$$

   The pattern of lending will be modified. The borrower with less incentives to default is the one chosen to break consensus in the local agreement. He will obtain a higher consumption profile (u is strictly concave). The lender will agree more future lending to him, rolling over the debt payment due at t.

The next remark summarises these considerations.

Remark 1 *If the local aggregate income is high, then it is more likely that the corresponding collective participation constraint binds. Moreover, the consumption profile of one borrower is more likely to grow above the consumption profile of the other borrower (the more likely he is going to have his repayment scheme renogotiated), the less the former will obtain in the local agreement and the more often he has been chosen to break-up local agreements in the past (the more likely he has renegotiated debt in the past).*

With this is mind, we can move on to the analysis of the evolution over time of the relative position of borrowers under the lending contract. The ratio $\xi_t \equiv \frac{u'(c_{bt})}{u'(c_{at})} = \frac{\mu_{at+1}}{\mu_{bt+1}}$ implied by condition 11 represents the enforceable allocation of future utility implementable by the contract after the realisation of uncertainty at time t. We normalise multipliers $m_{at} = \frac{\mathcal{I}_t}{\mu_{at}}\gamma_t$ and $m_{bt} = \frac{(1-\mathcal{I}_t)}{\mu_{at}}\gamma_t$ to write a law of motion for ξ_t.

$$\xi_t = \frac{1 + m_{at}}{1 + m_{bt}}\xi_{t-1} \tag{12}$$

As indicators have to sum up to 1, when $m_{at} > 0$ it must be the case that $m_{bt} = 0$, and vice-versa. Therefore, if the optimal contract chooses borrower a (b) to prevent default, then ξ_t must grow (fall) above (below) ξ_{t-1}. Borrower a (b) will enjoy better contractual conditions and the situation of agent b (a) will remain unchanged. As in other models with sovereign risk and full information, the threat of default introduces persistence in the responses of debt flows to income shocks.

Notice that, thus far, we have not specified how borrowers actually would interact in local agreements, apart from imposing the property of mutual improvability. The collective nature of default alone induces the peculiar mechanics of our sustainable lending contract. Indeed, in presence of any class of local agreements, the self-enforcing lending contract would display the same formal features.

However, the dynamics of lending might be linked to how local agreements happen to arise and how they are eventually broken up. The degree of local commitment and the initial utility weights assigned to the borrowers in the local agreement appear to be the main drivers behind these two events. In the appendix we illustrate two polar cases. In the first one, which we adopt for the numerical example in the next section, borrowers are able to fully commit to any mutually improving local agreement. Upon joint default, they would able to achieve the best local alternative: the "Local Pareto Optima" (LPO). This hypothesis may apply for alternative agreements between agents that are linked between each other by strong bonds like family or kin relationships. In the second case, we will relax the hypothesis of local full commitment allowing defaulting borrowers to join only self-enforced local agreements – the "Constrained Pareto Optima" (CPO). This assumption may suit, on the other hand, agreements between agents connected by weaker bonds, like geographical vicinity bewteen sovereing countries.

5 A one-shock example

We study the properties of the sustainable lending contract in a simple environment. A non-sovereign lender borrows and lends at the exogenous interest rate $r \equiv \beta^{-1} - 1$ to two ex-ante identical sovereign borrowers, a and b, with logarithmic utility $u(c(\mathbf{y}^t)) = \ln c(\mathbf{y}^t)$.

Aggregate fluctuations are deterministic. At time 1, aggregate endowment Y_t is "High". In all subsequent periods it is "Low".

$$Y_1 = H;\ Y_2 = L;\ Y_3 = L;\ ....$$

where $H > L > 0$.

At time 1, the world winds up in one of two possible states with equal probability. At time 1 aggregate income $Y_1 = H$ is split across agents according to the fixed proportion $q \in \left(\frac{1}{2}, 1\right)$ in the following way: either $y_{a1} = qH$ and $y_{b1} = (1-q)H$, or $y_{a1} = (1-q)H$ and $y_{b1} = qH$. At $t > 1$, borrowers' individual income is constant and equal to $\frac{1}{2}L$. All the uncertainty is resolved in the first period. Aggregate income experiences a high realisation that is unequally distributed according to the realisation of uncertainty and, ex-ante, agents are symmetrical.

As for the punishment technology, borrowers are free to default from the lending contract, though suffering the exclusion from future credit. Moreover, we assume that if borrowers are left in individual autarchy, they will lose fraction $\delta \in (0,1)$ of their current and future endowments.[9]

5.1 Sustainable lending

For the rest of this section, we shall assume that at time 1 the economy ends up in state in which $y_{a1} = qH$ and $y_{b1} = (1-q)H$.[10]

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[9] This assumption, although unnecessary in the general framework, is needed in this particular one to avoid trivial solutions.

[10] The alternativw case will display opposite but identical characteristics; for this clear symmetry, we drop the state contingent notation.

At $t=0$, the lender offers the borrowers a contract delivering them the expected utility U_j, which we assume to be equal to the expected utility of their income before uncertainty resolves. Borrowers can default collectively on the lender's contract to form a local risk-sharing agreement. Aggregate income is high only at time 1, therefore the collective sustainable constraint binds only at time 1. Letting $\mathcal{I} \equiv \mathcal{I}_a$and $(1-\mathcal{I}) \equiv \mathcal{I}_b$, the first order conditions for an optimum will be the same for all t's.

$$\frac{c_{at}}{c_{bt}} = \frac{\alpha_a + \mathcal{I}\gamma_1}{\alpha_b + (1-\mathcal{I})\,\gamma_1}$$

Where $\gamma_1 > 0$ is the Lagrange multiplier associated to the binding collective participation constraint at time 1.

Given the assumptions on the income process, as $y_{a1} > y_{b1}$ and $y_{at} = y_{bt}$ for all $t > 1$, agent's a utiliy from the local arrangement V_{a1}^{L} is going to be greater than agent b's, V_{b1}^{L}, the lender will set the indicator function $\mathcal{I}$ equal to 0, leading to the following ratio of equilibrium consumptions.

$$c_b = \frac{\alpha_b + \gamma}{\alpha_a} c_a$$

To obtain the consumption shares, we need to derive the utility of the collective outside options. We assume that in a local agreement borrowers are able to implement a local pareto optimal allocation, LPO, as described in the appendix.

Local Pareto Optimum. The intertemporal utilities the borrowers would get in the LPO are going to be the following.

$$\begin{aligned} V_{a1}^{LP} &= \frac{1}{1-\beta}\ln\left[(1-\beta)\,q + \beta\frac{1}{2}\right] H^{1-\beta}L^{\beta} \\ V_{b1}^{LP} &= \frac{1}{1-\beta}\ln\left[(1-\beta)\,(1-q) + \beta\frac{1}{2}\right] H^{1-\beta}L^{\beta} \end{aligned}$$

Note that $V_{j1}^{LP} \geq V_{j1}^{A}$ for $j = a, b$ meaning that the LPO is mutually improving.

In a steady state, evaluating the collective participation constraint at its value we get borrower's b consumption.

$$c_b^{LP} = \left[(1-\beta)\,(1-q) + \beta\frac{1}{2}\right] H^{1-\beta}L^{\beta}$$

We obtain borrower a's consumption equating his expected utility to the utility of consuming in autarchy.

$$c_a^{LP} = (1-\delta)q^{(1-\beta)}\frac{1}{2}^{\beta}H^{1-\beta}L^{\beta}$$

The contract implemented by the lender will display the following features.

1. Both borrowers are going to surrender some of the income of the first period to consume more in future low income periods. This is the consumption smoothing motive, which renders the contract with the lender more attractive than the local agreement. Clearly, the harsher are the aggregate fluctuations, the more strongly the smoothing incentive will work against collective default.

2. The cross-section income fluctuations will be redistributed across borrowers to avoid the formation of the local agreement in the first period (the only one that is relevant) promising just one potential defaulter more future lending. The lender minimises costs; the borrower with low income will need a promise of less lending to refrain from defaulting.

3. More importantly, when a receives the good realisation, the lender choses borrower b to break consensus in the local agreement, although his income realisation is lower, as this maximises the objective fuction of the lender. Differently to the individual default case, it is the borrower with the relatively lower realisation whose debt payment is forgiven and renegotiated.

The following remark proves this last property.

Remark 2 *If the good income realisation hits borrowers a, the lender will choose borrower b to break consensus in the local agreement. Setting* $\beta = 0.98$, $q = 0.55$, $\delta = 0.2$, $L = 2$ *and* $H = 10$, *the consumption shares will be the following.*

$$\begin{aligned} c_a^{LP} &= 4.1965 \\ c_b^{LP} &= 5.2252 \end{aligned}$$

On the other hand, if the lender had chosen borrower a to break the coalitions the consumption shares would have been the following.

$$\begin{aligned} c_a &= \left[(1-\beta)\,q + \beta\frac{1}{2}\right] H^{1-\beta}L^{\beta} = 5.2461 \\ c_b &= (1-\delta)(1-q)^{(1-\beta)}\frac{1}{2}^{\beta} H^{1-\beta}L^{\beta} = 4.1797 \end{aligned}$$

As $c_a^{LP} + c_b^{LP} < c_a + c_b$, the lender would maximise profits by choosing borrower b to break the coalition.

6 Concluding remarks and future developments

Conclusions. We analysed the interaction between default and alternative risk-sharing agreements focussing on the dynamics of lending to sovereign borrowers. We built a model of dynamic lending with limited commitment where borrowers have the option to default collectively and trade between them state contingent claims to their future incomes.

We defined a new class of sustainable lending contracts proof to collective default. We constructed the collective sustainability constraints to break up alternative agreements with the creation of a conflict of interest for only one of their potential members. We showed that the risk of collective deviations generates a peculiar asymmetry between the incentives to leave, which are connected to the unity of the group of borrowers, and the incentives to comply which are related to borrowers taken individually.

We provided a first characterisation of the lending dynamics implied in the optimal contract solving a version of the Lagrangian formulation proposed by Marcet and Marimon (1999). Dynamics are shown to be complex displaying substantially different features with respect to the case of individual default. In general terms, when collective default becomes a credible threat, just one borrower's repayment scheme will be renegotiated. The choice of which of the two borrowers will have his debt rolled over is made efficiently, minimising the cost to the lender. This cost will be less depending on two fundamentally different factors. The more the lender has renegotiated the payments with a borrower in the past and the lower is the utility the same borrower is going to get in the future alternative agreement, the less amount of resources to pay the lender will renegotiate to convince him not to step out. The second element is independent from the decisions of the lender, whereas the first are endogenously determined in the optimal lending contract. In contrast with the predictions of the individual default set-up a relatively low income realisation may increase the probability of receiving more lending in the future.

We analytically solved for the optimal lending contract in a simple one-shock economy with deterministic aggregate fluctuations, in which these conjectures are confirmed and some more insights are carried out.

Future research. The dynamics induced by the contract appear to be complex and heavily dependent on the underling stochastic properties of the income process. More mileage could be obtained solving the model numerically.

Another development would be the analysis of the market implications of breaking collective deviations. We would however need to cast our approach into a full-fledged general equilibrium environment, on the same lines of Kehoe and Perri (2002).

Finally, our equilibrium concept and the techniques developed to characterise it could be extended to analyse issues in other fields of research. The formulation could be adapted to include collective deviations to study the dynamics of cartel formation and stability, as well as the time pattern of merger & acquisition waves. Another fruitful extension would be analysing the dynamics of the formation of political majorities allowing the formation of coalitions of deviating parties.

References

[1] Abreu D. et. al (1990), "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring", *Econometrica*, 58, 1041-1064.

[2] Alvarez, F. and Jermann, U. (2000), "Efficiency, Equilibrium and Asset Pricing with Risk of Default", *Econometrica*, 68, 775-797.

[3] Bulow, G. and Rogoff, K. (1989), "Sovereign Debt: Is to Forgive to Forget?", *American Economic Review*, 79, 43-50.

[4] Cole, H.L., and Kehoe, P. J. (1998), ”Models of Sovereign Debt: Partial versus General Reputations”, *International Economic Review*, 39(1), 55-70.

[5] Eaton, J. and Gersovitz, M. (1981), ”Debt with Potential Repudiation”, *Review of Economic Studies*, 48, 289-309.

[6] Eichengreen, B. and Portes, R. (1989), ”Setting Defaults in the Era of Bond Finance”, *World Bank Economic Review*, 3(2), 211-239.

[7] *Global Development Finance. Financing the Poorest Countries,* (2002), World Bank Publication.

[8] Kydland F. E. and Prescott E. C. (1977), ”Rules Rather than Discretion: The Inconsistency of Optimal Plans”, *Journal of Political Economy*, vol. 85, no. 3.

[9] Kydland F. E. and Prescott E. C. (1979), ”Dynamic Optimal Taxation, Rational Expectations and Optimal Control”, *Journal of Economic Dynamics and Control*, vol. 2, pp. 79-91.

[10] Ljungqvist, L. and Sargent, T. (2000), ”Recursive Macroeconomic Theory”, MIT press, Cambirdge Mass.

[11] Kehoe P. and Perri F. (2002), ”International Business Cycles With Endogenous Incomplete Markets”, *Econometrica*, vol. 70, 907-928.

[12] Kletzer, K. M. and Wright, B. D. (2000), ”Sovereign Debt as Intertemporal Barter”, *American Economic Review*, 90, 621-639.

[13] Kocherlakota, N. (1996), ”Implication of Efficient Risk Sharing without Commitment”, *Review of Economic Studies*, 595-609.

[14] Marcet, A. and Marimon, R. (1992), ”Communication, Commitment, and Growth”, *Journal of Economic Theory*, 58, 219-249.

[15] Marcet, A. and Marimon, R. (1998), ”Recursive Contracts”, *mimeo.*

[16] Stokey *et al.* (1989), ”Recursive Methods in Economic Dynamics”, Harvard University Press, Cambridge, Mass.

7 Appendix

Proof of Result 1. The generic consumption allocation $\mathbf{C}$ will be proof to collective default if and only if it satisfies the following sequence of constraints.

It suffices to prove the statement for time t.

($\Rightarrow$) Consider a an allocation in Λ. At time t, there are three possibilities: either $E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(c_{as}) - V^L_{at}(\mathbf{y}^t) \geq 0$, **or** $E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(c_{bs}) - V^L_{bt}(\mathbf{y}^t) \geq 0$, **or** both. It cannot be the case that $E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(c_{as}) - V^L_{at}(\mathbf{y}^t) < 0$, **and** $E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(c_{bs}) - V^L_{bt}(\mathbf{y}^t) < 0$. It follows that the maximum between them must be non-negative.

($\Leftarrow$) Suppose not. Therefore there is an allocation $\widetilde{\mathbf{C}}_0 = \{\widetilde{c}_{1t}, \widetilde{c}_{2t}\}_{t\geq 0}$ that satisfies condition 3 and it is not a member of Λ. If $\widetilde{\mathbf{C}}_0$ is not in Λ, then it must be the case that **neither** $E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(\widetilde{c}_{as}) - V^L_{at}(\mathbf{y}^t) \geq 0$ **nor** $E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(\widetilde{c}_{bs}) - V_{bt}(\mathbf{y}^t) \geq 0$. It is only possible that $E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(\widetilde{c}_{as}) - V^L_{at}(\mathbf{y}^t) < 0$ **and** $E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(\widetilde{c}_{bs}) - V^L_{bt}(\mathbf{y}^t) < 0$. But this implies that $\max\limits_{j=a,b}\left[E_t\sum\limits_{s=t}^{\infty}\beta^{s-t}u(\widetilde{c}_{as}) - V^L_{at}(\mathbf{y}^t)\right] < 0$, which contradicts that $\widetilde{\mathbf{C}}_0$ satisfies 3. ■

Proof of Proposition 1. Let $\mathcal{I}_{jt}$ be an indicator function defined on borrower j such that, for all consumption allocations C,

$$\mathcal{I}_{jt} = \begin{cases} 1 & \text{if } E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{js}) - V^L_{jt}(\mathbf{y}^t) > E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{is}) - V^L_{it}(\mathbf{y}^t),\ i \neq j \\ 0 & \text{Otherwise} \end{cases}$$

then, the collective participation constraint

$$\max_{j=a,b}\left[E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{js}) - V^L_{jt}(\mathbf{y}^t)\right] \geq 0 \tag{13}$$

is satisfied if and only if the following constraint is satisfied as well.

$$\sum_{j=a,b}\mathcal{I}_{jt}\left[E_t\sum_{s=t}^{\infty}\beta^{s-t}U(c_{js}) - V^L_{jt}(\mathbf{y}^t)\right] \geq 0$$

($\Rightarrow$) Let the allocation C satisfy 7. We prove the statement for all the possible cases:

1. If $E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{as}) - V^L_{at}(\mathbf{y}^t) \geq 0$ and $E_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{bs}) - V^L_{bt}(\mathbf{y}^t) \geq 0$, then $\sum_{j=a,b}\mathcal{I}_{jt}\left[E_t\sum_{s=t}^{\infty}\beta^{s-t}U(c_{js}) - V^L_{jt}(\mathbf{y}^t)\right] \geq 0$ is trivially true.

2. If $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{as}) - V_{at}^L(\mathbf{y}^t) \geq 0$ and $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bs}) - V_{bt}^L(\mathbf{y}^t) < 0$,

   then $\mathcal{I}_t$ will command that $\mathcal{I}_{at} = 1$ and $\mathcal{I}_{bt} = 0$. It follows that

   $\sum_{j=a,b} \mathcal{I}_{jt} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_{js}) - V_{jt}^L(\mathbf{y}^t) \right] = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_{as}) - V_{at}^L(\mathbf{y}^t) \geq 0.$

3. Likewise, if $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{as}) - V_{at}^L(\mathbf{y}^t) < 0$ and $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bs}) - V_{bt}^L(\mathbf{y}^t) \geq 0$,

   then it will be $\mathcal{I}_{at} = 0$ and $\mathcal{I}_{bt} = 1$. It follows that

   $\sum_{j=a,b} \mathcal{I}_{jt} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_{js}) - V_{jt}^L(\mathbf{y}^t) \right] = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_{bs}) - V_{bt}^L(\mathbf{y}^t) \geq 0.$

($\Leftarrow$) By construction, $\mathcal{I}_t$ can be such that **either** $\mathcal{I}_{at} = 1$ and $\mathcal{I}_{bt} = 0$, **or** $\mathcal{I}_{at} = 0$ and $\mathcal{I}_{bt} = 1$ – **not** both. If in allocation C condition **??** is true for some t, then only one of the following cases is possible.

1. $\sum_{j=a,b} \mathcal{I}_{jt} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_{js}) - V_{jt}^L(\mathbf{y}^t) \right] = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_{as}) - V_{at}^L(\mathbf{y}^t) \geq 0$
2. $\sum_{j=a,b} \mathcal{I}_{jt} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_{js}) - V_{jt}^L(\mathbf{y}^t) \right] = E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_{bs}) - V_{bt}^L(\mathbf{y}^t) \geq 0.$

By result 1, this implies that condition 7 must be satisfied as well. The statement is true. ■

Proof of Proposition 2. The problem for the efficient sustainable lending contract 8 can be written as follows.

$$\min_{\mathbf{\Gamma}} \max_{\mathbf{C}} E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=a,b} \{(y_{jt} - c_{jt}) + \\ + \mu_t^j u(c_{jt}) + \mathcal{I}_{jt} \gamma_t \left[u(c_{jt}) - V_{jt}^L(\mathbf{y}^t) \right] \Big\}$$

Where $\mu_{jt+1} = \mu_{jt} + \mathcal{I}_{jt} \gamma_t$ (with its initial value μ_{j0} set equal to the Lagrange multiplier associated to the initial utility promise α_j) is the (co)state variable accounting for the history of past temptations of collective deviation up to time t and relative promises of future utility to avoid them.

A solution to 8 is found by maximising with respect to $\mathbf{C}$ and minimising with respect to $\mathbf{\Gamma}$ the function $\mathbf{J}(\mathbf{C}, \mathbf{\Gamma})$

$$E_0 \sum_{t=0}^{\infty} \beta^t \sum_{j=a,b} \left\{ (y_{jt} - c_{jt}) + \mathcal{I}_{jt} \gamma_t E_t \sum_{s=t}^{\infty} \left[\beta^{s-t} u(c_{js}) - V_{jt}^L(\mathbf{y}^t) \right] \right\}$$

Using the law of iterated allocations, $E_0\left[E_t\left(x_s\right)\right] = E_0\left(x_s\right)$, and

$$\sum_{t=0}^{\infty}\beta^t \sum_{j=a,b}\left[\mathcal{I}_{jt}\gamma_t\sum_{s=t}^{\infty}\beta^{s-t}u(c_{jt})\right] = \sum_{t=0}^{\infty}\beta^t\left[\sum_{j=a,b}\mu_{jt}u(c_{jt})\right]$$

where $\mu_{jt+1} = \mu_{jt} + \mathcal{I}_{jt}\gamma_t$, $\mu_{j0} = \alpha_j$, we get the result.[11] ■

7.1 Local commitment

In this section we discuss two hypotheses on contract enforceability in the local agreements.

Local Pareto Optimum.

In an LPO, the borrowers a and b will seek to smooth their income making transfers to each other subject to the sequence of period feasibility constraint. Amongst all the feasible allocations, a Local Pareto Optimum at time t would be an allocation solving the following programming problem.

$$\max_{\{c_{as},c_{bs}\}_{s\geq t}} E_t\sum_{s=t}^{\infty}\beta^{s-t}\left[\lambda^t u(c_{as}) + \left(1-\lambda^t\right)u(c_{bs})\right] \tag{14}$$

Subject to

$$c_{as} + c_{bs} \leqslant y_{as} + y_{bs}$$

for all $s \geq t$. $\lambda^t \in (0,1)$ is a parameter indicating the borrower's relative position in time t's local agreement. The solution of the programming problem in 14 does not provide a value for this parameter. Its determination is however crucial when we compute outside opportunity of the borrowers to be confronted with the utility attainable in the credit contract. We propose the value for λ^t implementing the allocation that would result in an complete markets equilibrium given income realisation $\{y_{at}, y_{bt}\}$.

Definition 3 (Local Pareto Optimum) ***Local Pareto Optimum*** *(LPO) is an intertemporal allocation* $\left\{c^{LP}_{as;t}, c^{LP}_{bs;t}\right\}_{s\geq t}$ *implemented at some* $t \geq 0$ *by the borrowers satisfying the following conditions.*

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[11] The result can be checked by direct substitution, see Sargent and Ljungqvist (2000).

(i) Risk-Sharing: $\frac{u'(c^{LP}_{bs;t})}{u'(c^{LP}_{as;t})} = \frac{\lambda^t}{1-\lambda^t}$

(ii) Resource Constraints: $c^{LP}_{as;t} + c^{LP}_{bs;t} \leq y_{as} + y_{bs}$

for all $s \geq t$ *and*

(iii) Intertemporal Budget Constraint: $E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{u'(c^{LP}_{js;t})}{u'(c^{LP}_{jt;t})} \left(c^{LP}_{js;t} - y_{js} \right) \leq 0,$ $j = a, b$.

Conditions (i) and (ii) have to be satisfied by a solution of the programming problem in 14. As required by Pareto optimality, the borrowers' marginal utilities will be in the constant proportion over time. Condition (iii) comes from our assumption of complete markets. The initial distribution of income, appropriately discounted by the future gains coming from mutual risk-sharing, will ultimately be what determines λ^t.

At time t, borrowers joining a LPO will be entitled to the intertemporal utilities $V^{LP}_{at}(\lambda^t) \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c^{LP}_{as;t})$ and $V^{LP}_{bt}(\lambda^t) \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c^{LP}_{bs;t})$.

In an LPO, the surplus would be shared between borrowers according to λ^t, which is linked to the income realisation observed at the time of default. With individual deviations, if income is *i.i.d.* or positively autocorrelated, borrowers with high income realisations will be dissuaded from defaulting with the promise of more future lending. With collective deviations, a high realisation to borrower a, for example, would induce a high λ, and hence a higher value of V^{LP}_{jt}. This would imply a lower probability of receiving future lending in the optimal lending contract. Therefore, a borrower experiencing a relatively low income realisation[12], is more likely to receive more future lending (or be given the chance to recontract its debt) than a borrower with a high income realisation.

Next, we see the opposite case allowing borrowers to default also on a local agreement.

Local Constrained Optimum.

With limited commitment, the local agreement has to be self-enforcing. An LPO starting at time t will maximise the following weighted sum of intertemporal utilities of the borrowers,

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[12] Relatively to the other borrower.

$$\max_{\{c_{as},c_{bs}\}_{s\geq t}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\lambda_C^t u(c_{as}) + \left(1-\lambda_C^t\right) u(c_{bs})\right]$$

making sure to satisfy, together with the period feasibility constraints, the following sequence of individual participation constraints.

$$E_s \sum_{n=0}^{\infty} \beta^{s-t} u(c_{js+n}) \geq E_t \sum_{s=t}^{\infty} \beta^{s-t} u(y_{js})$$

for all $s \geq t$.

Definition 4 (Local Constrained Optimum) *Let* $\left\{\phi_{as}^{LC}, \phi_{bs}^{LC}\right\}_{s\geq t}$ *be the sequence of Lagrange multipliers associated to the individual participation constraints, then a Local Constrained Optimum (LCO) is an allocation* $\left\{c_{as;t}^{LC}, c_{bs;t}^{LC}\right\}_{s\geq t}$ *implemented at some* $t \geq 0$ *by the borrowers satisfying the following conditions, for all* $s \geq t$.

(i) First Order Conditions: $\frac{u'(c_{bs;t}^{LC})}{u'(c_{as;t}^{LC})} = \frac{\sum_{i=1}^{s-t} \phi_{ai}^{LC}}{\sum_{i=1}^{s-t} \phi_{bi}^{LC}}$

(ii) Resource Constraints: $c_{as;t}^{LC} + c_{bs;t}^{LC} \leq y_{as} + y_{bs}$

The set of solutions to this programming problems has been studied in detail by Kocherlakota (1996). For simplicity, we set the initial Pareto weights equal across agents.[13]

At time t, borrowers joining a LPO will be entitled to the intertemporal utilities $V_{at}^{LC}(\mathbf{y}^t) \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{as;t}^{LC})$ and $V_{bt}^{LC}(\mathbf{y}^t) \equiv E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bs;t}^{LC})$.

With local limited commitment, the dynamics of sustainable lending are likely to be influenced by a larger number of variables. The division of local surplus would depend on the whole prospect of future threats of local defaults. Moreover, local limited commitment reduces the amount of utility attainable in an LCO. This increases the sustainability of lending, as the likelihood of threat of collective default would be reduced. Clearly, a lower level of individual autarchy utility makes the local agreement more enforceable. However, differently from the individual case, this would eventually

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[13] Ideally, the utility weights should reflect the expected future stream of utility obtainable in the agreement including the expected sequence of future temptations to default and corresponding utility promises. We are not pursuing this issue here.

harm the borrowers in term of lending. In effect, the harshest is autarchy, the more local surplus is generated, which in turn increases the chances that the borrowers decide to default collectively. But this means less lending is sustained in equilibrium. Interestingly, the original set-up in which only individual default is possible predicts exactly the opposite: the worse are the borrowers in individual autarchy, the more lending is sustainable in equilibrium.

7.2 Algebraic details of the one-shock example

Local Pareto Optima. According to the definition of LPO, an allocation implemented in any local Pareto optima should satisfy the risk-sharing condition and the period resource constraints

$$\frac{c_{at}^{LP}}{c_{bt}^{LP}} = \frac{\lambda^t}{1-\lambda^t}$$

For all $t \geq 1$, λ^t being the weight assigned to borrower a. Consumptions would be in constant proportion relative to each other, not constant over time.

Using the period resource constraint and the intertemporal budget constraint condition, we derive an expression for λ^t.

$$\lambda^t = (1-\beta)\, q + \beta \frac{1}{2}$$

The value of the initial weight assigned to borrower a is going to be a convex combination between the portion q and $\frac{1}{2}$ and it is easily interpretable. The more borrower a values future consumption (the higher β), the more of his (higher) initial realisation he is willing to forgo to smooth consumption across future states.

Given this weight, the borrower's consumptions will be fractions of local aggregate income

$$\begin{aligned} c_{at}^{LP} &= \left[(1-\beta)\, q + \beta \frac{1}{2}\right] Y_t \\ c_{bt}^{LP} &= \left[(1-\beta)\,(1-q) + \beta \frac{1}{2}\right] Y_t \end{aligned}$$

from which we obtain the outside options $V_{at}^{LP}$and V_{bt}^{LP}.

7.3 A general formulation

In this section we present the generalised version of the efficient lending contract. The lender's interest rate is general $(1+r) \equiv R < \beta^{-1}$ and individual participation constraints might bind.

The lender maximises the expected discounted sum of revenue from the contract.

$$\max_{\{c_{at},c_{bt}\}_{t\geq 0}} E_0 \sum_{t=0}^{\infty} R^{-t} \sum_{j=a,b} (y_{jt} - c_{jt})$$

so that the following sequence of sustainability constraints are satisfied for $j = a, b$ and $t \geq 0$,

$$\begin{aligned}
E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{as}) - V_{at}^{A}(\mathbf{y}^t) &\geq 0 \\
E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bs}) - V_{bt}^{A}(\mathbf{y}^t) &\geq 0 \\
\max_{j=a,b} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js}) - V_{jt}^{L}(\mathbf{y}^t) \right] &\geq 0 \\
E_0 \sum_{t=0}^{\infty} \beta^{s-t} u(c_{at}) - U_a &\geq 0 \\
E_0 \sum_{t=0}^{\infty} \beta^{s-t} u(c_{bt}) - U_b &\geq 0
\end{aligned}$$

By virtue of Proposition 1, we can construct a suitable indicator function $\widetilde{\mathcal{I}}_{jt}$ and look for an optimum for the following programming problem,

$$\begin{aligned}
&\min_{\mathbf{\Gamma},\mathbf{\Phi}} \max_{\mathbf{C}} E_0 \left\{ \sum_{t=0}^{\infty} R^{-t} \left\{ \left\{ \sum_{j=a,b} (y_{jt} - c_{jt}) + \right. \right. \right. \\
&\left. +\widetilde{\phi}_{jt} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js}) - V_{jt}^{A}(\mathbf{y}^t) \right] \right\} \\
&+\widetilde{\gamma}_t \left\{ \sum_{j=a,b} \widetilde{\mathcal{I}}_{jt} \left[E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{js}) - V_{jt}^{L}(\mathbf{y}^t) \right] \right\} + \\
&+\widetilde{\alpha}_a \left[E_0 \sum_{t=0}^{\infty} \beta^{s-t} u(c_{at}) - U_a \right] \\
&\left. +\widetilde{\alpha}_b \left[E_0 \sum_{t=0}^{\infty} \beta^{s-t} u(c_{bt}) - U_b \right] \right\}
\end{aligned}$$

where $\widetilde{\mathbf{\Phi}} \equiv \{\widetilde{\phi}_{at}, \widetilde{\phi}_{bt}\}_{t\geq 0}$ $\widetilde{\mathbf{\Gamma}} \equiv \{\widetilde{\gamma}_{at}, \widetilde{\gamma}_{bt}\}_{t\geq 0}$ are the sequences of Lagrange multipliers associated to the individual and collective participation constraints, respectively.

By an analogous argument as in proposition 2, we are allowed to write the problem for the efficient lending contract as follows.

$$\begin{aligned}&\min_{\mathbf{\Gamma},\mathbf{\Phi}} \max_{\mathbf{C}} E_0 \Bigg\{ \sum_{t=0}^{\infty} R^{-t} \sum_{j=a,b} \{(y_{jt} - c_{jt}) + \\ &+R\beta \left(\widetilde{\alpha}_j + \widetilde{\mu}_{jt}\right) u(c_{jt}) + \widetilde{\phi}_{jt} \left[u(c_{jt}) - V^A_{jt}(\mathbf{y}^t)\right] \\ &+\widetilde{\mathcal{I}}_{jt}\widetilde{\gamma}_t \left[u(c_{jt}) - V^L_{jt}(\mathbf{y}^t)\right]\Big\}\end{aligned}$$

Where,

$$\begin{aligned}\widetilde{\mu}_{jt+1} &= R\beta\widetilde{\mu}_{jt} + \widetilde{\phi}_{jt} + \widetilde{\mathcal{I}}_{jt}\gamma_t \\ \widetilde{\mu}_{j0} &= \alpha_j\end{aligned}$$

Optimal consumption path of borrower j. At an optimum, the following first order conditions are satisfied.

$$u'(c_{jt}) = \frac{1}{R\beta\widetilde{\mu}_{jt} + \widetilde{\phi}_{jt} + \widetilde{\mathcal{I}}_{jt}\gamma_t}$$

If $R\beta < 1$, the borrower will value current consumption more than the market. He would borrow more from the lender accept a contract that delivers him a consumption profile tilted backwards. Consumption will decrease monotonically until any of the participation constraints don't bind. From that moment on, the borrowers will only repay their debt.

Co-state variable. The law of motion of $\widetilde{\mu}_{jt}$ is obtained using the complementary slackness condition and the condition on $\mathcal{I}_t$

$$\begin{aligned}\widetilde{\mu}_{jt+1} &= R\beta\widetilde{\mu}_{jt} + \widetilde{\phi}_{jt} + \ \mathcal{I}_t\gamma_t \\ \widetilde{\mu}_{jt+1} &\geq R\beta\widetilde{\mu}_{jt}\end{aligned}$$

With strict equality if either $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{as}) > V^L_{at}(\mathbf{y}^t)$ or $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{as}) - V^L_{at} < E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{bs}) - V^L_{bt}(\mathbf{y}^t)$, and if $E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_{at}) > V^A_{at}(\mathbf{y}^t)$.

Part III

Portfolio risk and the demand for insurance by Italian households

Portfolio risk and the demand for health and property insurance in Italy[*]

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Abstract

Compared with other industrialised countries, the diffusion of health and property insurance policies across Italian households appears limited. It seems that Italian families still prefer to insure mainly through precautionary saving. Alongside, a very small fraction of families invest their savings in the financial markets. Pulling together these two observations, we argue that families that hold their financial wealth in a diversified portfolio find precautionary saving a less efficient insurance instrument with respect to private insurance policies. We estimate the relationship between health and property insurance diffusion and degree of portfolio diversification of households by means of censored regression analysis (tobit model) using the data collected in the Survey of Household Income Wealth (SHIW) run by the Bank of Italy over the period 1989-2002. The estimation's results confirm the existence of a positive relationship between the degree of portfolio diversification and the inclination to purchase property insurance. Less mileage is obtained from the estimation of the health insurance equation. As a by-product, we present a detailed account of the diffusion of health and property insurance across Italian household grouped by socio-economic, demographic and territorial characteristics.

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1 Background and motivations

It seems that the Italian market for private health and property insurance has not yet overcome its initial stage of development. In 2004 premiums gathered by Italian insurance companies operating in the non-life industry, excluding motor insurance, which is almost everywhere compulsory, accounted for as low as 1.1% of Italian GDP, amongst the lowest in Europe: 2.3% in the average country in the EU15, 2.8% in Germany and in the UK, 1.9 in Spain. Focussing on lines of business providing health insurance (sickness and injury), the gap displayed by the Italian sector is even larger. Only 0.33% of GDP is spent to purchase private health coverage, against 0.66% in the EU15, 1.48% in Germany, 0.49% in the UK, 0.61% in Spain.[1] Europe itself is considered underinsured if compared with the US, where the non-life non-motor lines of business accounted for 4.7% of GDP in 2003, and the health business for 2.4%.[2] The growth rate of non-life insurance market penetration during the last decade has been steady, but admittedly low.

Several analysts point out the role played by the free national health system hindering the growth of the private health insurance industry. However, aggregate statistics on the main sources of funding of health expenses in industrialised countries released in 2003 by the OECD do not seem to substantiate this explanation. Indeed, more than one fifth of the health expenses sustained in Italy are financed directly by the households, leading to the conclusion that against those outlays not covered (ex-post) by the national health system, most Italian families still prefer to self insure through precautionary saving rather than the private insurance market.

Italian households show little interest in financial assets markets as well. The degree of participation of Italian families in financial markets is remarkably low, if compared with other European countries. According to the computations carried out by Guiso and Jappelli (2002) using survey data collected in several European countries, only 15% of Italian households participate in the stock market, of which only 7% participate directly, i.e. without the intermediation of institutional investors, less than half the European average of 14.7%. Moreover, the structure of portfolio holdings of Italian

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[1] Source: CEA (Comite Europeen des Assurances), Associates Statistics, 2004.

[2] Source: OECD, Insurance Statistics Yearbook, 1994-2003.

families is very simple, as the largest share of assets held is in the form of transaction and saving accounts.

The objective of this paper is to build a bridge between these stylised facts discussing the impact of the relationship between precautionary saving and the patterns of households' financial market participation on the decision to purchase insurance coverage.

The reasoning draws from the classical optimal insurance model, where a risk averse agent has to decide whether to buy an unfairly priced insurance policy, or increase savings to self insure. The model predicts that the agent will buy insurance only if the marginal rate of substitution between endowments in the different states of nature exceeds the market premium rate. The more the consumer is able to transfer readily liquid resources to the future, the less the marginal rate of substitution is likely to be higher than the premium rate.

The basic point is that certain types of assets can be used as self insurance instruments more efficiently than others. Safe and liquid assets like transaction and saving accounts are certainly more suitable for precautionary saving than risky assets, such as stock or shares of mutual funds, or illiquid assets such as life insurance premiums or shares in pension funds. Following this reasoning, it is more likely that the marginal rate of substitution of an agent with high portfolio shares of risky or illiquid assets goes above the market premium rate, and thus more likely that the agent purchases private insurance.

The main contribution of this paper consists in testing this hypothesis using the data collected in the Survey of Household Income Wealth (SHIW) run by the Bank of Italy over the period 1989-2002. We estimate a censored regression model where the propensity to purchase health and property insurance is related to the degree of diversification the asset portfolios held by the households. The estimation's results confirm the existence of a positive relationship between the degree of portfolio diversification and the inclination to purchase property insurance. We obtain less mileage from the estimation of the health insurance equation. As a by-product, we present a detailed account of the diffusion of health and property insurance across Italian household grouped by socio-economic, demographic and territorial characteristics.

In the literature there are several contributions studying various aspects of private health insurance markets. Starr-McCluer (1996) evaluates the impact of private health insurance on American households' saving habits to verify the existence of substitutability between private insurance and self insurance. The author concludes that, in general, precautionary savings do not offset private insurance. Guariglia and Rossi (2001) test the same hypothesis on British household data reaching the same conclusions, although they find some degree of substitutability in areas with poor quality public health services. A similar argument is set out in the analysis of Jappelli, Pistaferri and Weber (2004) using Italian household data. Besley et al. (1998) for the United Kingdom and Costa and Garcia (2001) for the Catalan region in Spain find that the quality differential between public and private health services is an effective driver of private health insurance demand by wealthy households.

There appears to be a less rich variety of contributions on household property insurance, at least to our account. The analysis conducted by Guiso and Jappelli (1998) on the relationship between household income risk and the demand for property insurance appears to be the closest to our line of argument.

The rest of the paper is organised as follows. In section 2, we provide a detailed account on the diffusion of health and property insurance across Italian households grouped by demographic, socio-economic and financial market characteristics using panel data collected by the Bank of Italy. In section 3, we first briefly outline the main theoretical issues connected with the demand of insurance in presence of uninsurable risk and then we present the result of a broad regression analysis to validate our predictions. In section 4, we draw our conclusions.

All tables and pictures are in the appendix.

2 Descriptive analysis on health and property insurance diffusion by household characteristics

Our data source is a panel made up of the last seven waves of the Survey on Household Income and Wealth run periodically by the Bank of Italy on a geographically stratified sample of approximately 8.000 households over the period 1989-2002.[3] Households are assigned weights that are proportional to the probability of being extracted from the universe. Along with social and demographic characteristics, sampled households are asked to report family income and wealth, consumption and saving habits, portfolio decisions. In the section of our interest, households are asked whether they hold health (sickness and injury) and property insurance - excluding compulsory motor insurance - and how much money they have spent to purchase it.[4]

Table II in the appendix reports the evolution of market diffusion, measured by the percentage of families that were covered by some insurance policy at the moment of the survey over the time period 1989-2002. The percentages are referred to the population, as they are weighted according to the probability of the sample units to be included in the sample.

In 2002, according to the SHIW, 7.5% of Italian households benefited from sickness and injury risk coverage through private insurance; in 1989, at the beginning of the panel, the percentage was 4.4% peaking up to 12.7% in 1993. In 2002 17.1% of Italian households were covered by property damage insurance, from 11.4% in 1989 with a peak of 28.7% in 1995. Finally, 4.1% held both types of coverage in 2002, 1.6% in 1989, with a maximum of 5.4% in 1998 (table 1).

Next, we report a detailed account of the evolution of the fraction of insured households by class of homogeneous characteristics. We focus our analysis on three main household characteristics groups: socio-economic, demographic and territorial. Socio-economic characteristics include degree of financial portfolio diversification, income and wealth classes, schooling, profession, title of ownership of dwelling; demographic characteristics include gender, age and household size; territorial characteristics include region of residence and dimension of the urban centre. Statistics on age, education, main professional status and gender are referred to the head of the household, intended as the person in the household with highest income. The rest of the statistics are referred to the household in its unity.

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[3] The surveys were run in 1989, 1991, 1993, 1995, 1998, 2000 and 2002.

[4] Table I depicts the section of survey's questionnaire concerning the insurance status of the respondent.

For each characteristic, we report the degree and the evolution over the sample period of the diffusion of insurance coverage within groups, that is, the percentage of households with a given characteristic holding health insurance, property insurance or both health and property insurance.

The complete set of descriptive statistics is reported in the appendix.

2.1 Economic and financial characteristics

Income. The inclination of household to purchase both insurance policies increases with income in a fairly regular fashion. 14.8% of households in the 5th income quintile had purchased health insurance in 2002 (20.9% in 1995 and 16% in 1989), whereas 31.9% of them had property insurance (49.8% in 1995 and 31.8% in 1989). 8.6% of households in the highest income quintile had both coverages (13.1% in 1995 and 7.5% in 1989).

In 1995 the distribution of insured households shifted upward from the level in 1989, especially in correspondence of the third and fourth income quintiles, suggesting a change in the attitude towards risk by the middle class; in 2002 the distribution went back to the initial level, maintaining, though, the 1995 profile (table III).

Financial wealth. Again, propensity to buy insurance is increasing with financial wealth holdings. 14.1% of households in the highest income quintile had sickness and injury coverage in 2002 (22.1% in 1995 and 13.3% in 1989), 31.2%% of them had property insurance (51.4% in 1995 and 33.9% in 1989). 8.5% of households in the highest wealth quintile was covered against both types of risk (13.7% in 1995 and 7.7% in 1989).

The distribution of households with health insurance becomes convex in correspondence of the highest quintiles, whereas the relationship remains fairly linear for the households insured against property losses (table IV of the appendix).

Degree of portfolio diversification. As in Guiso and Jappelli (2001), we aggregate the different assets in three homogeneous risk classes: ”safe assets” which include transaction and saving accounts of various nature, ”bonds”, which includes government and corporate bonds with fairly safe returns and ”stock”, which includes shares, mutual funds and privately managed assets.

We then construct an index of portfolio diversification that increases with the number of assets held in the portfolio. We consider four configurations - only safe assets, safe assets and bonds, safe assets and stock and all assets - are ordered from totally safe to fully diversified. We ignore unbalanced portfolios, such as those composed by only stock, or only bonds.

In 2002, the highest percentages of insured households were in the groups with financial portfolios composed of safe assets and stocks and all assets. In 2002, 18.6% of the former group had health insurance (20.9% in 1995 and 12.3%), 36.9% had property insurance (52.6% in 1995 and 37.5% in 1989); 17% of the latter group had health insurance (20.9% in 1995 and 12.3% in 1989), 37% had property insurance in 2002 (51.9% in 1995 and 36% in 1989) (table V of the appendix).

2.2 Socio-demographic characteristics

Degree of schooling. As expected, the relationship between inclination to purchase private insurance and degree of schooling is increasing for both products. In 2002, more than 13% of households with head holding a college degree had health insurance (19.9% in 1995 and 9.5% in 1989); 24.5% had property insurance (42.7% in 1995 and 16.3% in 1989).

Higher education people, besides enjoying higher income levels, are likely to have better knowledge of insurance products that are available in the market (table VI).

Main professional status. The highest percentages of insured households are, for both insurance products, in the group of households with head member of the arts or professions, 23.8% and 28.3% in 2002, respectively (33.9% and 41.8% in 1995; 12.1% and 18.5% in 1989). Self employed people are likely to be more sensitive to future income losses, as they already face higher income uncertainty.

Also high percentages of managers are insured against sickness and injury, probably because they have access to company health insurance plans, apart from being in the higher income classes (table VII).

Title of home ownership. There is no apparent relationship between the percentage of household with health insurance and the title of home ownership.

On the other hand, the highest percentage of households insured against property damage is found, not surprisingly, within the group of home proprietors, 21.8% in 2002 (23.7% in 1995 and 14.3% in 1989) (table VIII).

Gender of household head. In 2002 the percentage of households with female head was still quite low, 36.6%, although the percentage grew steadily over the period under analysis (19.5% in 1989).

In 2002 the portion of households insured against sickness and injury was 4.6%% if the head of the family was a woman (6% in 1995 and 2.3 in 1989), 9.1 % if it was a man (12% in 1995 and 4.9% in 1989). 12.5% of households with female head had property insurance in 2002 (21.5% in 1995 and 8.4% in 1989) and 2.2% had both property and health insurance (3.0% in 1995 and 0.7% in 1989) (table IX).

Age of head. The age structure of household heads remained basically constant over the sample period. The age structure of insured households, on the contrary, changes over time and across insurance products. All age profiles are hump-shaped. For health insurance the maximum percentage of insured households moves from age class 41-50 years old in 1989 (6.6%) to age class 31-40 years old in 2002 (10.6%); for property insurance the hump moves from class 41-50 in 1989 (16.3%) to 51-65 in 2002 (21.9%) (table X).

Older people are more likely to own their homes and have easier access to the national health insurance system.

Family size. The percentage of households insured against sickness and injury is highest for families with 3 members in 2002, 4 in 1995 and 3 in 1989; it is the same size (3 members) over the sample period for households holding property insurance and for households holding both products (table XI).

2.3 Territorial characteristics

Geographical region. As expected, households living in southern Italy are the least likely to be covered by some private insurance policy. In 2002, 12% of households living in northern regions had health insurance (13% in 1995 and 6% in 1989), the highest percentage in Italy; property insurance too is more diffused in the North, 30% in 2002 (44% in 1995 and 20% in 1989). Geographical differences in income seem to be the main reason behind this distribution (table XII).

Size of urban centre. The relationship between the size of the urban center of residence and the inclination to purchase health insurance behaves somewhat erratically over the sample period; it is increasing with size in 1989, it is increasing in 1995 to end up being flat in 2002.

The behaviour of the share of households with property insurance shows more insights, being highest within households living in small communities, where most families own their homes (table XIII).

3 Insurance diffusion and portfolio risk

The financial portfolios of Italian households span rather few assets. A remarkably large portion of Italian families had no sort of financial assets. In 2002, this portion has decreased to about a fifth, from over 30% in 1989. More than half of the households (55% in 2002, from 49% in 1989) had their financial wealth invested in low or zero yield safe assets, such as transaction and saving accounts. A rather marginal fraction of households showed some degree of portfolio diversification in favour of less liquid higher yield assets. In 2002, 5% of the population had safe assets and bonds in their portfolio (20% in 1995 and 14% in 1989). Italian familes bear little financial risk in their portfolios as well. Risky assets appear in a minority of portfolios. Only of the families 17% safe assets and stocks (4% in 1995 and 2% in 1989) and 4% had a fully diversified portfolio between safe assets, bonds and stock (6% in 1995 and 4% in 1989) (table XIV).

We shall not go into further detail reporting cross tabulations of the various degrees of portfolio diversification of households as we did for insured households. There are present in the literature several excellent studies documenting demographic, social and economic dynamics of financial participation of Italian households, to which we redirect the interested reader (see for example Guiso and Jappelli, 2001 and Guiso et al. 2002).

We argue that this limited participation of households in financial markets is one important determinant of the scarce market diffusion of health and property insurance among Italian families. Standard insurance theory predicts that a risk averse agent will decide to purchase unfairly priced insurance instead of self insure if the marginal rate of substitution between wealth holdings in the different states of nature exceed the premium rate offered in the market. Otherwise, the agent will simply transfer more assets to the future for precautionary motives.

To fix ideas, let's consider a basic optimal insurance problem where an agent faces an income loss L, which occurs with probability p. The agent's income y is the summation of labour income w and financial income given by the rate of return r times assets A.

We start describing the features of the optimal solution assuming that the agent invests all her wealth in safe assets. The agent will then solve the following utility maximisation problem.

$$\max_{I\geq 0} pu\left[y-L+\left(\frac{1-\mu p}{\mu p}\right)I\right]+(1-p)u\left(y-I\right)$$

Where I is the actual amount of insurance bought and μ the load charged by the insurer. Depending on the size of the load factor μ, this problem may not display an internal solution (*i.e.* the agent will not purchase any insurance).

The optimal amount of insurance I^* will have to satisfy the following first order (necessary) conditions.

$$pu'\left[y-L+\left(\frac{1-\mu p}{\mu p}\right)I^*\right]\left(\frac{1-\mu p}{\mu p}\right)\leq(1-p)u'\left(y-I^*\right)$$

Let's now assume that the assets held in the portfolio of the same agent (or of someone else with same preferences) deliver an uncertain rate of return $\widetilde{r}=r+\varepsilon$, where ε is a random variable with 0 mean and variance σ_ε^2.[5]

Letting $v(y)=u(y+\varepsilon A)$, we can write the first order conditions for the optimal choice of I as a function of the non stochastic part of income y.

$$pv'\left[y-L+\left(\frac{1-\mu p}{\mu p}\right)I^{**}\right]\left(\frac{1-\mu p}{\mu p}\right)\leq(1-p)v'\left(y-I^{**}\right)$$

Eeckoudt and Kimball (1992) show that if $u(.)$ exhibits decreasing absolute risk aversion and increasing prudence, then $v(.)$ will be strictly more risk averse than $u(.)$, implying that $I^{**}>I^*$, that is, the optimal amount of insurance will be higher than in absence of non insurable risk.

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[5] In this simple set-up the rate of return includes capital gains (or losses).

Under plausible assumptions on the utility function, it may also be the case that at some values of the load factor μ an agent holding her wealth in a low-risk portfolio will not purchase insurance, whilst the same agent holding risky assets will do.

The results described above lead to the important implication that not all types of assets are suitable for precautionary saving. An asset, to serve efficiently as a self insurance instrument, should be, firstly, reasonably safe. If this wasn't so, the asset would at best replace its uncertainty with that of the event to be insured. Secondly, it has to be fairly liquid, to be readily available to be converted into cash to finance the losses if nature winds up into bad states.

In line with this reasoning, transaction and saving accounts are certainly more suitable for precautionary saving than risky assets such as stock or shares of mutual funds, or illiquid assets such as life insurance premiums or shares in pension funds. A higher proportion of risky or illiquid assets in wealth holdings, by increasing the marginal rate of substitution between states of nature, makes private insurance coverage more attractive.

3.1 Regression analysis

In this section we test the conjecture on the database constructed upon the information found in the SHIW by means of discrete choice and censored regression analysis.

We proceeded with the following specification strategy.

Preliminary estimations. The descriptive analysis reported above on insurance diffusion and the literature on financial markets participation suggested that the positive correlation between insurance diffusion and degree of portfolio diversification might very likely be driven spuriously by common factors.

In order to identify such factors, we run a series of preliminary regressions: an ordered probit regression for the index of portfolio diversification of households, plus separate and joint probit regressions for the diffusion of health and property insurance. All equations contained constant terms and year dummies.

In the ordered probit equation for the degree of financial diversification the coefficients associated with age, education size of household are significantly positive; being male, resident in the North and in the Center, self-employed and home owner increases the likelihood to have a more diversified portfolio; being a tenant and to reside in a urban center with more than 500,000 inhabitants decrease it (table B1 in appendix B).

In the individual probit equation for health insurance, the significant coefficients are the same and they show the same signs. So happens to the estimated coefficients in the individual probit equation for property insurance, apart from that associated to being a tenant, which is negative and significant. The coefficients estimated in the binomial probit equation are in line with those of the individual equations, confirming though the existence of a strong aggregate idiosyncratic component (table B2 and B3 in appendix B).

Main regressions. We use these variables likely to be common drivers as controls when estimating the censored models. We separately regress the natural logarithm of the premiums paid by households for health and property insurance against four dummy variables constructed upon possessing the following portfolio configurations: ”only safe assets”, ”safe assets and bonds”, ”safe assets and stock” and ”safe assets, bond and stock”, controlling for all the variables found statistically significant in the preliminary estimations. According to the arguments set out above, the coefficients and their signs should be interpreted as follows. A large and positive coefficient associated to a portfolio configuration with risky assets would mean that households tend to compensate portfolio uncertainty by purchasing more insurance against health and property risk. A negative coefficient on the less risky portfolio configurations should be interpreted as the inclination of household to use their wealth as a self insurance device.

The estimation of both the censored models yielded coefficients associated to the portfolio configuration with a higher degree of diversification, that is, ”safe assets and bond”, ”safe assets and stock” and ”safe assets, bond and stock” are positive, statistically significant and in line with our predictions. The coefficients on the safe portfolio configuration were positive too, indicating that safe assets holdings do not offset completely the demand for private coverage against health and property risk.

In the property insurance equation the coefficients increase with share of non risk-free assets in the portfolios. The coefficient associated to the most diversified portfolio configuration (safe assets, bonds and stocks) is lower than those associated to the other configurations. This suggests that the income uncertainty brought about by holding risky assets is already diversified within the same portfolio.

The interpretation of the coefficients of the health insurance equation is less straightforward. The coefficient on safe assets is larger than that associated to the portfolio configuration with safe assets and bonds.

We included amongst the regressors the possession of life insurance and the participation in a defined pension plan. Both coefficient were positive and significant, suggesting the existence of complementarity between different insurance products.

We believe that this result relates to the particular morphology of most Italian households' financial portfolios. As we said, financial markets participation in Italy is still at a very early stage of development. Percentages of households with wealth allocated in bonds and stock are very low, slightly above 25% for all three diversified portfolio typologies, as opposed to the more than 50% for the safe configuration alone. It is likely that there is still variability across people with this portfolio configuration which was not captured by the control variables included in our estimations.

4 Summing up

The diffusion of insurance coverage against future monetary losses associated to sickness, injuries and property damage in Italy is limited. The existence of a free national health system does not suffice to account for the limited development of health insurance, as a consistent portion of health expenses are directly financed by household individual savings. Alongside, Italian households don't seem to appreciate financial markets as a valid instrument to diversify their wealth holdings, at least comparatively with other industrialised countries. We propose to link these observations arguing that high portfolio diversification, generally associated to significant holding of risky and illiquid assets, may constitute an incentive to purchase actuarially unfair insurance coverage, instead of self insure through precautionary saving.

To verify this main conjecture on the data, we used the following specification strategy.

Using a panel dataset made up of seven waves of the Survey of Household Income and Wealth run bi-annually by the Bank of Italy, we ran a series of preliminary regressions to identify the common drivers behind the decision to purchase insurance and the choice of the degree of portfolio diversification. We then proceeded to estimate the relationship between the likelihood to hold private coverage and the degree of portfolio diversification, controlling for the censorship and previously identified common drivers.

As regards property insurance the results indicate a clearly stronger inclination to purchase insurance for those households holding risky and illiquid assets. The outcome of the estimation of the health insurance equation is less sharp. The coefficient associated to the safest portfolio configuration was positive and in several specifications strongly significant. We think this was due to the yet early stage of development of financial markets.

We provided a detail account of the insurance habits of Italian households holding constant by a rich variety of demographic, territorial and socio-economic family characteristics.

References

[1] Banca d'Italia (2004), "Archivio Storico dell'Indagine sui Bilanci delle Famiglie Italiane, 1977-2002", versione 3.0.

[2] Besley, Tim et al. (1998), "Private and Public Health Insurance in the UK", European Economic Review, vol. 42, 491-497.

[3] Costa, Joan and Jaume Garcia (2001), "Demand for Private Health Insurance: Is there a Quality Gap?", Universitat Pompeu Fabra, Working Paper Series 531.

[4] Eekhoudt Louis, and Miles Kimball (1992), "Background Risk, Prudence and the Demand for Insurance", in Contributions to Insurance Economics, G. Dionne ed. London: Kluwer Academic Press.

[5] Fabbri, Daniele and Monfardino, Chiara (2002), "Public vs. Private Healthcare Service Demand in Italy", Università di Bologna, Working Paper no. 457.

[6] Guariglia Alessandra and Rossi Maria Cristina (2004), "Private Medical Insurance and Saving: Evidence from the British Household Panel Survey", Journal of Health Economics, 23, 761-83.

[7] Guiso, Luigi and Jappelli, Tullio (1998), "Background Uncertainty and the Demand for Insurance against Insurable Risks", Centro Studi in Economia e Finanza, Working Paper no. 2, Dipartimento di Scienze Economiche - Università degli Studi di Salerno.

[8] Guiso, luigi and Jappelli, Tullio (2002), "Household portfolio in Italy", in Guiso, L., Haliassos, M. and Jappelli, T. (eds), Household Portfolios, Cambridge MA.: MIT Press.

[9] Guiso, luigi and Jappelli, Tullio (2002), "Household stockholding in Europe: where do we stand and where do we go?", CEPR discussion paper no. 3694.

[10] Jappelli, Tullio; Pistaferri, Luigi and Weber, Guglielmo (2004), "Health Care Quality and Economic Inequality", Centro Studi in Economia e Finanza, Working Paper no. 119, Dipartimento di Scienze Economiche - Università degli Studi di Salerno.

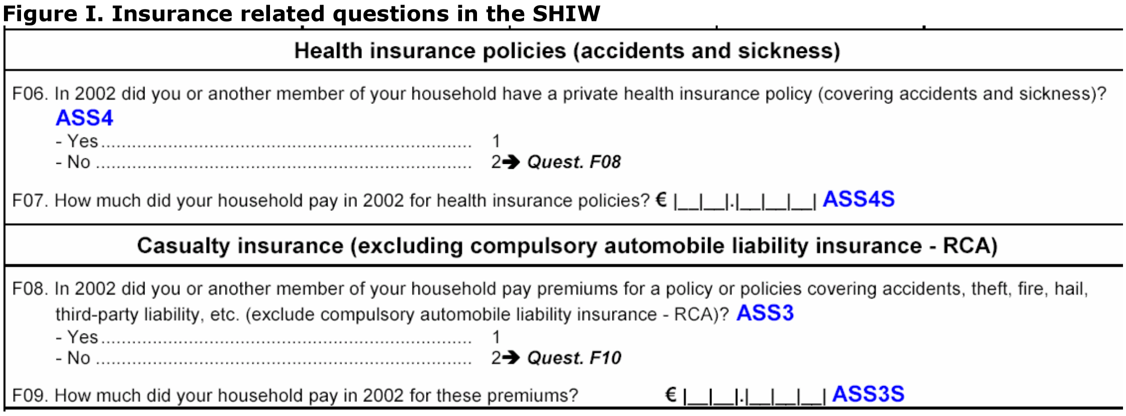
[11] OECD (2003), "Private Health Insurance in OECD Countries", OECD Publications, France.

[12] OECD (2004), "Insurance Statistics Yearbook 1994-2003", OECD Publications, France.

[13] Starr-McCluer, Martha (1996), "Health Insurance and Precautionary Savings", American Economic Review, Volume 86, no. 1, 285 - 295.

Appendix A

Figure I. Insurance related questions in the SHIW



Health insurance policies (accidents and sickness)

F06. In 2002 did you or another member of your household have a private health insurance policy (covering accidents and sickness)? **ASS4**

- Yes 1
- No .... 2➔ ***Quest. F08***

F07. How much did your household pay in 2002 for health insurance policies? € |__|__|.|__|__|__| **ASS4S**

Casualty insurance (excluding compulsory automobile liability insurance - RCA)

F08. In 2002 did you or another member of your household pay premiums for a policy or policies covering accidents, theft, fire, hail, third-party liability, etc. (exclude compulsory automobile liability insurance - RCA)? **ASS3**

- Yes 1
- No .... 2➔ ***Quest. F10***

F09. How much did your household pay in 2002 for these premiums? € |__|__|.|__|__|__| **ASS3S**

Table II. Insurance diffusion across Italian households

Type of coverage	**1989**	**1991**	**1993**	**1995**	**1998**	**2000**	**2002**
Health	4.4%	4.6%	12.7%	10.3%	10.1%	8.7%	7.5%
Property	11.4%	14.4%	12.9%	28.6%	24.7%	21.1%	17.1%
Health & Property	1.6%	2.3%	4.2%	5.1%	5.4%	4.7%	4.1%

Cross descriptive statistics

A1. Economic and financial characteristics of households

Table III. Distribution of insured people by income quintiles

	1989				**1991**			
	Population	**Health**	**Property**	***Health & Property***	**Population**	**Health**	**Property**	***Health & Property***
1st quintile	32.0%	1.7%	3.6%	0.1%	24.5%	0.9%	4.4%	0.1%
2nd quintile	25.0%	3.3%	9.5%	0.5%	23.1%	2.5%	9.3%	0.7%
3rd quintile	20.1%	3.9%	13.2%	2.2%	21.0%	3.8%	14.7%	1.1%
4th quintile	14.7%	6.1%	18.0%	2.6%	19.5%	6.8%	21.5%	4.0%
5th quintile	8.2%	16.0%	31.8%	7.5%	11.8%	14.2%	32.6%	9.1%

	1993				**1995**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
1st quintile	25.7%	2.6%	4.2%	0.5%	22.2%	3.5%	9.4%	1.0%
2nd quintile	21.5%	7.6%	6.9%	1.3%	19.5%	5.5%	17.2%	2.1%
3rd quintile	18.4%	13.2%	12.2%	2.7%	20.3%	10.8%	29.5%	4.1%
4th quintile	18.7%	19.3%	18.1%	5.7%	19.5%	12.2%	40.7%	6.2%
5th quintile	15.8%	27.9%	30.1%	14.1%	18.5%	20.9%	49.8%	13.1%

	1998				**2000**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
1st quintile	18.4%	2.1%	8.1%	0.9%	15.0%	1.5%	4.2%	0.3%
2nd quintile	17.6%	5.4%	10.7%	2.2%	17.1%	1.5%	7.8%	0.2%
3rd quintile	19.6%	8.3%	21.6%	3.6%	20.4%	5.9%	16.7%	2.0%
4th quintile	20.6%	10.4%	31.2%	5.3%	20.9%	9.2%	24.3%	4.2%
5th quintile	23.8%	21.1%	44.6%	12.8%	26.6%	19.1%	39.8%	12.6%

	2002			
	Population	***Health***	***Property***	***Health & Property***
1st quintile	12.9%	0.6%	1.1%	0.4%
2nd quintile	16.5%	1.2%	5.8%	0.4%
3rd quintile	18.3%	4.5%	11.3%	1.8%
4th quintile	21.4%	8.3%	18.9%	4.4%
5th quintile	31.0%	14.8%	31.9%	8.6%

Table IV. Distribution of insured people by wealth quintiles

	1989				**1991**			
	Population	**Health**	**Property**	***Health & Property***	**Population**	**Health**	**Property**	***Health & Property***
1st quintile	22.0%	1.6%	3.0%	0.5%	19.8%	1.6%	4.0%	0.1%
2nd quintile	32.0%	3.0%	7.4%	0.8%	25.1%	2.2%	6.7%	0.3%
3rd quintile	24.2%	4.3%	11.9%	1.2%	22.6%	3.7%	13.2%	1.6%
4th quintile	13.9%	6.7%	20.2%	2.6%	18.8%	5.2%	22.4%	2.4%
5th quintile	8.0%	13.3%	33.9%	7.7%	13.8%	13.9%	34.3%	9.8%

	1993				**1995**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
1st quintile	21.9%	5.5%	1.8%	0.4%	19.8%	3.6%	8.5%	0.6%
2nd quintile	20.5%	8.5%	7.9%	2.2%	19.2%	5.3%	17.1%	1.4%
3rd quintile	19.2%	9.2%	10.1%	1.4%	19.2%	6.6%	26.0%	2.3%
4th quintile	19.6%	17.0%	16.5%	5.2%	20.8%	12.7%	37.5%	6.6%
5th quintile	18.8%	24.8%	30.5%	12.5%	21.1%	22.1%	51.4%	13.7%

	1998				**2000**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
1st quintile	17.7%	3.5%	8.4%	1.2%	17.5%	2.1%	6.1%	0.2%
2nd quintile	19.8%	6.3%	14.2%	2.6%	16.5%	4.9%	10.9%	1.6%
3rd quintile	17.3%	7.3%	19.0%	2.9%	18.6%	4.4%	16.9%	1.8%
4th quintile	22.3%	10.1%	30.7%	5.0%	22.6%	8.1%	21.6%	3.8%
5th quintile	22.9%	20.6%	44.7%	13.4%	24.9%	19.4%	40.9%	13.0%

	2002			
	Population	***Health***	***Property***	***Health & Property***
1st quintile	17.1%	2.6%	3.0%	0.8%
2nd quintile	15.7%	4.8%	7.5%	1.6%
3rd quintile	16.3%	3.5%	12.3%	1.4%
4th quintile	20.5%	6.9%	19.1%	4.1%
5th quintile	30.4%	14.1%	31.2%	8.5%

Table V. Distribution of insured households by degree of portfolio diversification

	1989				**1991**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
No assets	31.1%	3.3%	7.0%	1.5%	19.2%	1.5%	5.3%	0.4%
Only safe	49.1%	3.4%	8.8%	1.1%	54.5%	3.8%	12.4%	1.4%
Safe assets and bonds	13.8%	6.2%	19.9%	2.5%	18.9%	4.6%	20.4%	2.1%
Safe assets and stock	1.8%	12.3%	37.5%	3.0%	3.3%	12.6%	40.5%	8.7%
All assets	4.2%	14.5%	36.0%	5.7%	4.1%	23.5%	34.9%	18.2%

	1993				**1995**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
No assets	17.4%	4.7%	3.5%	1.3%	16.6%	2.2%	6.3%	1.3%
Only safe	55.4%	11.7%	9.9%	2.8%	53.3%	10.0%	27.5%	4.7%
Safe assets and bonds	17.5%	15.5%	21.6%	6.8%	19.8%	12.1%	38.1%	4.8%
Safe assets and stock	5.0%	28.0%	31.8%	13.7%	4.1%	20.9%	52.6%	13.1%
All assets	4.6%	28.2%	31.6%	11.4%	6.2%	22.0%	51.9%	13.9%

	1998				**2000**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
No assets	14.0%	0.3%	3.2%	0.1%	19.6%	2.2%	6.9%	0.5%
Only safe	61.3%	8.2%	21.7%	3.8%	53.3%	6.3%	16.8%	2.9%
Safe assets and bonds	6.7%	14.6%	35.9%	8.6%	6.4%	6.1%	26.8%	4.0%
Safe assets and stock	12.9%	23.2%	46.1%	14.3%	15.5%	21.5%	44.4%	13.6%
All assets	5.0%	20.6%	50.6%	12.6%	5.2%	22.5%	41.9%	13.9%

	2002			
	Population	***Health***	***Property***	***Health & Property***
No assets	19.3%	1.2%	3.2%	0.6%
Only safe	54.5%	5.5%	13.6%	2.4%
Safe assets and bonds	5.1%	7.7%	24.8%	5.7%
Safe assets and stock	16.9%	18.6%	36.9%	11.7%
All assets	4.2%	17.0%	37.0%	9.1%

A2. Socio-demographic characteristics of households

Table VI. Distribution of insured households by educational qualification

	1989				1991			
	Population	Health	Property	Health & Property	Population	Health	Property	Health & Property
None	9.5%	0.5%	1.2%	0.0%	8.9%	0.5%	1.8%	0.0%
Elementary	38.3%	2.4%	9.8%	0.5%	37.8%	2.8%	12.2%	1.5%
Middle	23.9%	4.6%	12.7%	2.0%	24.3%	4.2%	16.5%	1.8%
High	21.3%	7.5%	15.5%	3.0%	22.4%	7.3%	19.1%	3.9%
College	6.7%	9.5%	16.3%	4.5%	6.3%	12.9%	20.2%	5.8%
Graduate	0.4%	20.0%	45.2%	0.0%	0.2%	19.0%	17.5%	1.5%

	1993				1995			
	Population	*Health*	*Property*	*Health & Property*	*Population*	*Health*	*Property*	*Health & Property*
None	10.7%	1.6%	3.8%	0.5%	10.2%	1.2%	8.3%	0.4%
Elementary	34.8%	7.3%	8.9%	2.3%	33.4%	5.1%	24.6%	2.6%
Middle	27.6%	13.8%	13.8%	3.8%	26.6%	11.3%	29.3%	5.2%
High	20.8%	22.7%	19.2%	7.9%	23.4%	17.5%	38.4%	8.9%
College	5.9%	23.3%	26.0%	10.7%	6.2%	19.9%	42.7%	10.7%
Graduate	0.2%	35.9%	26.0%	9.3%	0.2%	65.9%	37.2%	18.1%

	1998				2000			
	Population	*Health*	*Property*	*Health & Property*	*Population*	*Health*	*Property*	*Health & Property*
None	9.2%	0.4%	7.1%	0.3%	9.1%	1.5%	4.1%	0.3%
Elementary	29.3%	5.6%	19.4%	2.6%	29.2%	3.8%	18.3%	2.2%
Middle	26.5%	9.9%	26.5%	4.8%	26.0%	7.4%	18.0%	3.0%
High	27.2%	15.4%	31.5%	9.1%	27.5%	15.0%	29.5%	8.5%
College	7.6%	21.1%	35.3%	11.5%	8.0%	16.5%	29.9%	11.1%
Graduate	0.2%	6.5%	25.8%	4.6%	0.2%	23.0%	67.3%	23.0%

	2002			
	Population	*Health*	*Property*	*Health & Property*
None	7.7%	1.2%	1.7%	1.2%
Elementary	28.5%	3.6%	12.5%	2.1%
Middle	27.2%	6.5%	17.3%	3.6%
High	28.6%	12.3%	23.6%	5.8%
College	7.8%	13.4%	24.5%	9.2%
Graduate	0.2%	7.8%	28.4%	5.7%

Table VII. Distribution of insured households by main employment status

	1989				1991			
	Population	**Health**	**Property**	***Health & Property***	**Population**	**Health**	**Property**	***Health & Property***
Blue collar	19.4%	3.3%	7.2%	0.8%	19.0%	3.2%	9.9%	1.1%
White collar	15.3%	4.3%	10.2%	2.1%	13.7%	3.3%	14.4%	1.2%
Junior manager	4.7%	7.3%	23.9%	1.7%	5.4%	9.7%	17.1%	3.4%
Manager	1.5%	18.7%	36.9%	10.1%	1.4%	6.5%	25.3%	3.5%
Member of the arts or professions	3.9%	12.1%	18.5%	5.9%	3.3%	20.8%	33.9%	13.1%
Sole proprietor	15.8%	8.6%	20.4%	3.4%	15.0%	8.5%	21.7%	4.7%
Other (includes pensioners)	39.3%	1.5%	7.2%	0.4%	42.2%	2.2%	11.6%	1.3%

	1993				1995			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
Blue collar	16.8%	11.5%	7.8%	2.2%	16.4%	8.2%	25.2%	3.2%
White collar	15.7%	18.0%	16.6%	5.4%	13.0%	11.6%	35.5%	5.1%
Junior manager	2.2%	19.0%	27.7%	8.7%	3.3%	26.2%	40.0%	14.5%
Manager	1.3%	40.2%	29.5%	13.3%	1.3%	30.0%	51.5%	13.3%
Member of the arts or professions	2.2%	34.7%	32.7%	23.2%	2.6%	33.9%	41.8%	17.3%
Sole proprietor	15.4%	26.7%	20.1%	9.8%	13.7%	23.3%	39.9%	12.6%
Other (includes pensioners)	46.5%	4.7%	9.1%	1.3%	49.7%	4.3%	22.7%	2.2%

	1998				2000			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
Blue collar	14.9%	6.9%	22.1%	3.1%	14.7%	5.5%	15.0%	0.9%
White collar	14.2%	10.7%	24.1%	4.7%	13.2%	10.0%	23.5%	4.2%
Junior manager	2.8%	18.0%	36.3%	9.2%	2.9%	14.0%	30.1%	9.1%
Manager	1.1%	29.7%	38.3%	14.5%	1.3%	21.5%	33.8%	15.4%
Member of the arts or professions	3.8%	27.2%	42.0%	19.4%	3.2%	32.5%	45.6%	22.2%
Sole proprietor	13.1%	25.5%	38.2%	14.6%	11.8%	18.0%	29.6%	11.2%
Other (includes pensioners)	50.1%	4.7%	19.8%	2.4%	52.9%	5.1%	17.9%	2.9%

	2002			
	Population	***Health***	***Property***	***Health & Property***
Blue collar	15.4%	4.8%	13.7%	1.8%
White collar	14.6%	8.6%	19.7%	4.6%
Junior manager	2.2%	15.0%	24.7%	8.0%
Manager	1.3%	14.0%	19.5%	4.2%
Member of the arts or professions	3.8%	23.8%	28.3%	15.8%
Sole proprietor	11.7%	16.2%	26.2%	8.8%
Not employed	50.9%	4.2%	14.1%	2.4%

Table VIII. Distribution of insured households by title of property of dwelling

	1989				1991			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
Home owner	62.7%	4.6%	14.3%	1.8%	64.2%	5.3%	18.2%	2.9%
Tenant	27.4%	4.5%	6.2%	1.5%	24.1%	3.6%	6.4%	0.8%
With right of redemption	1.5%	4.2%	5.8%	0.0%	1.4%	3.0%	10.4%	3.0%
Usufructuary	8.5%	1.9%	8.1%	0.5%	10.4%	3.0%	9.6%	1.8%

	1993				1995			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
Home owner	62.4%	14.0%	17.2%	5.3%	64.9%	11.7%	35.6%	6.5%
Tenant	24.8%	10.1%	5.1%	2.1%	23.4%	7.9%	14.1%	2.3%
With right of redemption	0.9%	5.9%	7.5%	0.6%	0.8%	2.9%	23.7%	1.6%
Usufructuary	11.9%	11.7%	7.1%	2.9%	10.9%	7.6%	18.1%	2.8%

	1998				2000			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
Home owner	65.9%	10.8%	29.8%	6.2%	68.7%	10.1%	25.2%	6.0%
Tenant	22.6%	9.0%	14.0%	4.0%	20.5%	5.8%	9.6%	1.9%
With right of redemption	0.6%	3.4%	14.5%	2.0%	0.7%	0.6%	8.7%	0.6%
Usufructuary	10.8%	8.7%	16.6%	3.9%	10.1%	5.7%	16.8%	1.7%

	2002			
	Population	***Health***	***Property***	***Health & Property***
Home owner	68.8%	7.9%	21.8%	4.8%
Tenant	20.6%	6.0%	6.1%	2.2%
With right of redemption	0.5%	9.6%	3.8%	0.7%
Usufructuary	10.1%	7.1%	8.4%	2.8%

Table IX. Distribution of insured househould by gender of head

	1989				*1991*			
	Population	**Health**	**Property**	**Health & property**	**Population**	**Health**	**Property**	**Health & property**
Male	80.5%	4.9%	12.2%	1.8%	78.8%	5.0%	15.3%	2.5%
Female	19.5%	2.3%	8.4%	0.7%	21.2%	3.0%	11.1%	1.4%

	1993				*1995*			
	Population	**Health**	**Property**	**Health & property**	**Population**	**Health**	**Property**	**Health & property**
Male	71.9%	15.5%	14.9%	5.3%	71.7%	12.0%	31.4%	5.9%
Female	28.1%	5.5%	7.8%	1.5%	28.3%	6.0%	21.5%	3.0%

	1998				*2000*			
	Population	**Health**	**Property**	**Health & property**	**Population**	**Health**	**Property**	**Health & property**
Male	71.9%	11.7%	27.3%	6.5%	64.6%	10.1%	23.2%	5.7%
Female	28.1%	5.9%	17.9%	2.5%	35.4%	6.1%	17.1%	2.8%

	2002			
	Population	**Health**	**Property**	**Health & property**
Male	63.4%	9.1%	19.9%	5.1%
Female	36.6%	4.6%	12.2%	2.2%

Table X. Distribution of insured household by age of head

	1989				**1991**			
	Population	**Health**	**Property**	***Health & Property***	**Population**	**Health**	**Property**	***Health & Property***
<30	7.4%	5.2%	7.3%	1.4%	6.5%	4.3%	14.2%	2.7%
31-40	16.8%	5.2%	12.4%	1.8%	16.3%	5.9%	13.9%	1.6%
41-50	21.0%	6.6%	16.3%	3.5%	20.2%	6.1%	17.8%	3.3%
51-65	28.6%	4.7%	12.5%	1.2%	29.5%	5.2%	14.9%	2.9%
>65	26.2%	1.3%	6.9%	0.5%	27.6%	2.1%	11.7%	1.0%

	1993				**1995**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
<30	6.5%	14.6%	9.6%	2.7%	5.4%	10.8%	24.5%	4.4%
31-40	18.3%	19.6%	15.6%	6.2%	17.9%	16.6%	31.0%	7.6%
41-50	20.0%	21.0%	16.6%	7.6%	19.4%	15.5%	34.6%	7.7%
51-65	27.3%	11.4%	13.9%	4.0%	28.2%	10.0%	33.7%	5.5%
>65	27.8%	3.1%	8.4%	0.9%	29.2%	3.1%	18.9%	1.6%

	1998				**2000**			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
<30	4.9%	15.2%	24.8%	7.3%	4.9%	7.8%	14.3%	1.8%
31-40	17.5%	14.4%	26.3%	7.3%	18.5%	11.6%	23.0%	5.8%
41-50	19.8%	14.6%	29.5%	8.0%	19.1%	12.2%	23.3%	6.7%
51-65	26.9%	10.4%	29.0%	5.8%	27.0%	10.3%	24.6%	5.9%
>65	30.9%	3.7%	16.8%	2.0%	30.6%	3.4%	16.4%	2.3%

	2002			
	Population	***Health***	***Property***	***Health & Property***
<30	4.0%	5.8%	6.5%	0.3%
31-40	18.4%	10.6%	19.2%	5.0%
41-50	20.2%	10.1%	20.1%	6.4%
51-65	26.3%	8.3%	21.9%	4.6%
>65	31.1%	3.4%	11.2%	2.0%

Table XI. Distribution of insured household by number of members of household

	1989				*1991*			
	Population	**Health**	**Property**	**Health & property**	**Population**	**Health**	**Property**	**Health & property**
1	17.3%	2.7%	6.3%	0.5%	18.2%	3.4%	10.0%	1.1%
2	24.8%	4.0%	11.0%	1.0%	23.7%	4.5%	14.2%	1.9%
3	23.7%	5.5%	13.6%	2.4%	23.9%	4.2%	17.5%	2.3%
4	23.1%	5.3%	13.2%	2.4%	23.6%	6.7%	15.8%	3.9%
5	7.5%	2.9%	12.6%	0.9%	7.4%	3.1%	13.2%	0.9%
6	2.8%	4.2%	10.5%	2.9%	2.4%	3.6%	12.3%	3.3%
7	0.5%	1.6%	17.1%	0.0%	0.7%	1.6%	3.1%	0.0%
8	0.2%	0.0%	6.3%	0.0%	0.1%	0.0%	3.7%	0.0%

	1993				*1995*			
	Population	**Health**	**Property**	**Health & property**	**Population**	**Health**	**Property**	**Health & property**
1	17.5%	5.4%	8.5%	1.5%	18.3%	4.7%	*17.1%*	1.3%
2	24.6%	8.6%	11.5%	3.1%	25.4%	9.0%	27.8%	5.1%
3	23.5%	14.8%	16.6%	5.9%	23.5%	12.9%	35.5%	6.2%
4	23.6%	18.3%	15.6%	5.7%	22.9%	13.9%	32.7%	7.0%
5	7.6%	17.5%	10.5%	4.4%	7.4%	10.0%	26.3%	4.8%
6	2.2%	16.7%	8.3%	3.1%	1.8%	8.6%	29.5%	7.3%
7	0.8%	14.3%	5.6%	2.8%	0.5%	6.0%	9.9%	0.0%
8	0.1%	15.5%	0.0%	0.0%	0.1%	9.5%	34.4%	9.5%

	1998				*2000*			
	Population	**Health**	**Property**	**Health & property**	**Population**	**Health**	**Property**	**Health & property**
1	20.7%	4.6%	15.9%	1.5%	20.9%	4.3%	12.0%	2.1%
2	26.8%	10.6%	25.2%	6.0%	28.0%	7.2%	21.6%	4.3%
3	23.1%	12.6%	30.1%	7.6%	22.5%	10.8%	25.9%	6.0%
4	21.2%	10.9%	28.7%	6.1%	20.8%	12.7%	26.3%	7.0%
5	6.2%	13.9%	20.6%	5.2%	5.8%	9.1%	15.2%	3.3%
6	1.6%	8.6%	18.6%	4.7%	1.5%	9.7%	17.9%	5.2%
7	0.4%	26.6%	6.1%	6.1%	0.2%	1.7%	8.3%	1.7%
8	0.1%	15.4%	15.4%	15.4%	0.1%	6.9%	10.9%	5.4%

	2002			
	Population	**Health**	**Property**	**Health & property**
1	23.3%	3.3%	9.5%	1.1%
2	26.6%	7.0%	17.6%	3.5%
3	21.7%	10.0%	23.1%	5.6%
4	20.9%	9.5%	18.7%	5.6%
5	5.8%	7.7%	15.4%	4.4%
6	1.3%	12.5%	19.6%	11.9%
7	0.3%	18.4%	31.4%	18.4%
8	0.1%	0.0%	51.4%	0.0%

A3. Territorial characteristics of households

Table XII. Distribution of insured households by geographical region

	1989				1991			
	Population	**Health**	**Property**	***Health & Property***	**Population**	**Health**	**Property**	***Health & Property***
North	47.9%	6.3%	19.6%	2.7%	47.6%	6.4%	22.6%	3.7%
Center	19.3%	4.7%	7.1%	1.3%	20.1%	4.8%	12.2%	1.8%
South& Islands	32.9%	1.4%	2.0%	0.2%	32.3%	1.8%	3.6%	0.4%

	1993				1995			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
North	49.1%	15.4%	19.7%	6.1%	48.5%	12.8%	43.6%	7.8%
Center	18.7%	16.9%	11.4%	4.7%	18.2%	15.2%	23.5%	5.5%
South& Islands	32.1%	6.3%	3.4%	1.0%	33.2%	3.9%	9.5%	0.8%

	1998				2000			
	Population	***Health***	***Property***	***Health & Property***	***Population***	***Health***	***Property***	***Health & Property***
North	48.0%	14.6%	38.4%	8.8%	46.9%	13.8%	35.7%	8.3%
Center	19.1%	12.1%	23.2%	5.3%	19.5%	7.5%	14.7%	3.7%
South& Islands	32.9%	2.3%	5.4%	0.5%	33.6%	2.2%	4.3%	0.4%

	2002			
	Population	***Health***	***Property***	***Health & Property***
North	46.5%	12.2%	29.7%	7.7%
Center	19.9%	6.8%	13.2%	2.1%
South& Islands	33.5%	1.3%	1.9%	0.1%

Table XIII. Distribution of insured households by dimension of urban center

	1989				1991			
	Population	Health	Property	*Health & Property*	Population	Health	Property	*Health & Property*
<20,000	47.2%	4.5%	13.1%	1.9%	46.6%	4.1%	16.7%	2.4%
20-40,000	11.2%	2.5%	11.5%	0.7%	13.5%	2.9%	10.4%	1.4%
40-500,000	29.1%	3.9%	9.8%	1.3%	25.9%	5.8%	15.4%	2.8%
>500,000	12.5%	6.6%	8.9%	2.2%	13.9%	5.6%	8.7%	1.6%

	1993				1995			
	Population	*Health*	*Property*	*Health & Property*	*Population*	*Health*	*Property*	*Health & Property*
<20,000	48.3%	11.7%	12.7%	3.8%	48.6%	8.6%	29.3%	4.6%
20-40,000	12.9%	15.9%	16.2%	5.0%	13.0%	11.1%	25.6%	5.4%
40-500,000	26.1%	12.8%	13.9%	4.6%	25.6%	12.0%	30.4%	5.9%
>500,000	12.8%	13.0%	8.5%	3.9%	12.8%	12.5%	25.4%	5.0%

	1998				2000			
	Population	*Health*	*Property*	*Health & Property*	*Population*	*Health*	*Property*	*Health & Property*
<20,000	48.6%	9.8%	25.5%	5.6%	47.4%	8.7%	23.6%	4.7%
20-40,000	12.6%	9.3%	23.0%	4.8%	13.2%	9.2%	19.1%	5.1%
40-500,000	25.0%	10.8%	22.4%	5.1%	26.3%	8.6%	19.3%	4.6%
>500,000	13.8%	10.6%	27.5%	5.6%	13.1%	8.4%	17.1%	4.6%

	2002			
	Population	*Health*	*Property*	*Health & Property*
<20,000	46.3%	7.9%	20.3%	4.8%
20-40,000	13.4%	7.7%	14.3%	3.4%
40-500,000	26.5%	7.3%	15.7%	3.8%
>500,000	13.8%	6.1%	11.7%	2.5%

Table XIV. Type of assets held by Italian households

	1989	**1991**	**1993**	**1995**	**1998**	**2000**	**2002**
Combinations of assets							
Only safe assets	49%	55%	55%	53%	61%	53%	55%
Safe assets and bonds	14%	19%	18%	20%	7%	6%	5%
Safe assets and stock	2%	3%	5%	4%	13%	16%	17%
All assets	4%	4%	5%	6%	5%	5%	4%
No assets or other	31%	19%	17%	17%	14%	20%	19%

Appendix B

Regression analysis

B1. Preliminary estimations: ordered probit model

	Dependent variable: financial participation index			
Regressor	***Coefficient***	***Std. err.***	***z-stat***	***p-value***
Age	0.04072	0.00331	12.29	0.00
(Age^2)/1000	-0.32150	0.03108	-10.34	0.00
Education	0.34640	0.03423	10.12	0.00
(Education^2)/1000	-0.38384	5.33153	-0.07	0.94
Male	0.16641	0.01728	9.63	0.00
Size of household	0.02423	0.00626	3.87	0.00
Resident in the North	0.92745	0.01651	56.17	0.00
Resident in the Center	0.57587	0.01863	30.92	0.00
<20,000	-0.02421	0.01576	-1.54	0.12
20-40,000	0.02255	0.01708	1.32	0.19
>500,000	-0.08051	0.02043	-3.94	0.00
Employee	0.03415	0.02085	1.64	0.10
Self employed	0.11054	0.02626	4.21	0.00
Home owner	0.16723	0.02404	6.96	0.00
Tenant	-0.19987	0.02632	-7.59	0.00

B2. Preliminary estimations: separate probit models

	Dependent variables							
	Property insurance possession				***Health insurance possession***			
Regressor	***coefficient***	***std. err.***	***z-stat***	***p-value***	***Coefficient***	***std. err.***	***z-stat***	***p-value***
Age	0.039677	0.005090	7.79	0.00	0.044371	0.007352	6.04	0.00
(Age^2)/1000	-0.371394	0.047983	-7.74	0.00	-0.471139	0.073427	-6.42	0.00
Education	0.309475	0.054728	5.65	0.00	0.240488	0.072404	3.32	0.00
(Education^2)/1000	-19.126610	8.246258	-2.32	0.02	-2.318873	10.457340	-0.22	0.83
Male	0.089812	0.025815	3.48	0.00	0.136122	0.032515	4.19	0.00
Size of household	0.061818	0.009396	6.58	0.00	0.049050	0.012115	4.05	0.00
Resident in the North	1.309430	0.026792	48.87	0.00	0.783883	0.032547	24.08	0.00
Resident in the Center	0.752801	0.030580	24.62	0.00	0.640905	0.035312	18.15	0.00
<20,000	0.032049	0.022398	1.43	0.15	-0.007763	0.027887	-0.28	0.78
20-40,000	-0.007282	0.024577	-0.30	0.77	0.019430	0.029451	0.66	0.51
>500,000	-0.164290	0.031887	-5.15	0.00	-0.088865	0.035311	-2.52	0.01
Employee	-0.074743	0.030593	-2.44	0.02	0.025463	0.038684	0.66	0.51
Self employed	0.267516	0.034116	7.84	0.00	0.543333	0.039254	13.84	0.00
Home owner	0.368234	0.039616	9.30	0.00	0.113092	0.043415	2.60	0.01
Tenant	-0.312003	0.045240	-6.90	0.00	-0.046674	0.048555	-0.96	0.34

B3. Preliminary estimations: binomial probit model

	Dependent variables							
	Property insurance possession				***Health insurance possession***			
Regressor	***coefficient***	***std. err.***	***z-stat***	***p-value***	***Coefficient***	***std. err.***	***z-stat***	***p-value***
Age	0.036768	0.003466	10.61	0.00	0.055580	0.004868	11.42	0.00
(Age^2)/1000	-0.347907	0.032155	-10.82	0.00	-0.576668	0.047873	-12.05	0.00
Education	0.303104	0.038669	7.84	0.00	0.307437	0.050619	6.07	0.00
(Education^2)/1000	-18.183030	5.900312	-3.08	0.00	-12.473940	7.402244	-1.69	0.09
Male	0.091650	0.018110	5.06	0.00	0.115955	0.023029	5.04	0.00
Size of household	0.059051	0.006376	9.26	0.00	0.031042	0.007847	3.96	0.00
Resident in the North	1.196860	0.019556	61.20	0.00	0.714162	0.023290	30.66	0.00
Resident in the Center	0.662920	0.022362	29.64	0.00	0.646271	0.026055	24.80	0.00
<20,000	0.050721	0.017230	2.94	0.00	0.045236	0.021704	2.08	0.04
20-40,000	0.026316	0.018704	1.41	0.16	0.080773	0.022604	3.57	0.00
>500,000	-0.180019	0.025458	-7.07	0.00	-0.046864	0.028602	-1.64	0.10
Employee	-0.008739	0.021528	-0.41	0.69	0.057736	0.026729	2.16	0.03
Self employed	0.291220	0.024160	12.05	0.00	0.569586	0.027835	20.46	0.00
Home owner	0.408789	0.026468	15.44	0.00	0.041758	0.029890	1.40	0.16
Tenant	-0.275135	0.030519	-9.02	0.00	-0.141426	0.033532	-4.22	0.00
Wald test of $\rho=0$: χ (1) = 664.207; Prob > χ = 0.0000								

B4. Main estimations: censored tobit model

	Dependent variables							
	ln of property insurance premiums				***ln of health insurance premiums***			
Regressor	***coefficient***	***std. err.***	***z-stat***	***p-value***	***Coefficient***	***std. err.***	***z-stat***	***p-value***
Safe assets	1.560293	0.177712	8.78	0.00	1.889224	0.326733	5.78	0.00
Safe assets & bonds	1.663240	0.208075	7.99	0.00	1.187840	0.381845	3.11	0.00
Safe assets & stock	2.552402	0.225928	11.30	0.00	3.322492	0.398848	8.33	0.00
Safe assets, bonds & stock	2.166623	0.248290	8.73	0.00	3.030124	0.433068	7.00	0.00
Wealth	-0.831402	0.082713	-10.05	0.00	-0.666053	0.136639	-4.87	0.00
(Wealth^2)/1000	85.862430	5.609639	15.31	0.00	77.581550	9.322630	8.32	0.00
Income	0.000049	0.000005	10.19	0.00	0.000063	0.000008	8.16	0.00
(Income^2)/1000	0.000000	0.000000	-7.83	0.00	0.000000	0.000000	-5.12	0.00
Age	0.114266	0.023091	4.95	0.00	0.365389	0.045192	8.09	0.00
(Age^2)/1000	-1.264671	0.214525	-5.90	0.00	-4.098484	0.444272	-9.23	0.00
Education	1.743446	0.261846	6.66	0.00	2.517774	0.483673	5.21	0.00
(Education^2)/1000	-209.669100	39.481660	-5.31	0.00	-229.066500	70.877670	-3.23	0.00
Life insurance	0.118790	0.180820	0.66	0.51	-0.342985	0.288233	-1.19	0.23
Defined pension plan	1.601657	0.148033	10.82	0.00	3.057332	0.236591	12.92	0.00
Male	0.345303	0.120662	2.86	0.00	0.757173	0.219553	3.45	0.00
Size of household	0.037971	0.045393	0.84	0.40	-0.144451	0.076851	-1.88	0.06
Resident in the North	6.534281	0.143333	45.59	0.00	4.636963	0.239992	19.32	0.00
Resident in the Center	3.283820	0.154873	21.20	0.00	4.642682	0.257276	18.05	0.00
<20,000	0.377942	0.117594	3.21	0.00	0.599097	0.208222	2.88	0.00
20-40,000	0.090731	0.125565	0.72	0.47	0.697145	0.216253	3.22	0.00
>500,000	-1.127239	0.163862	-6.88	0.00	-0.408395	0.276038	-1.48	0.14
Employee	-0.200235	0.145187	-1.38	0.17	0.258280	0.258332	1.00	0.32
Self employed	0.647412	0.163747	3.95	0.00	3.606563	0.279126	12.92	0.00
Home owner	0.796964	0.196758	4.05	0.00	-1.604130	0.318656	-5.03	0.00
Tenant	-1.060264	0.205657	-5.16	0.00	-0.213725	0.326520	-0.65	0.51
	46226 left-censored observations at ln(property insurance premiums)<=0 9619 uncensored observations				51401 left-censored observations at ln(health insurance premiums)<=0 4444 uncensored observations			