Financial Intermediation in Europe

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Chapter 1

Introduction

The thesis studies several aspects of banking activities in a European context. The first two chapters analyze the structure of large-value settlement systems as well as inter-bank markets after the introduction of the Single Currency in the European Monetary Union. The third chapter is a model of corporate control where banks are large shareholders with monitoring powers.

Chapter 2, which is joint with Thomas Rønde, studies international large value payment systems. The last decades have witnessed a substantial increase in the volume of transactions that are processed via large-value payment systems. Because of this development, payment systems have grown to be one of the most likely channels through which financial crises could propagate. The regulation of payment systems is therefore of increasing importance for financial stability. We analyze the regulation of international large value payment systems where, as in the EU, banking supervision is a national task. We build a model of a cross-border payment system, and focus on the choice between gross and net settlement. Gross settlement is safer than net settlement in terms of systemic risk, but more costly in operation because of high reserve requirements. In the model, the communication between the national supervisors is endogenized. It is shown that the national supervisors' incentives are not perfectly aligned when deciding upon
the mode of settlement. This creates a moral hazard problem in the communication between the supervisors. As a result, a net settlement system is implemented too often. We show that if the private banks have the same information as the regulators, the decision on the settlement system should be taken by the regulators. However, if the private banks have superior information about the risk of the foreign banks, it can be optimal to leave this decision in the hands of the private sector. Privately organized settlement systems are shown to be particularly efficient when the systemic impact of a bank’s failure is low.

In chapter 3, I analyze the effects of the Single Currency on the structure of inter-bank markets for liquidity. The model is based on a Diamond/Dybvig economy where banks are subject to solvency and liquidity shocks. Market participants receive noisy signals about each other’s solvency. Banks can cope with liquidity shocks either by liquidating parts of a profitable investment technology, or by borrowing funds on the inter-bank market. In a two-country setting, it is assumed that the signal received by foreign banks is less accurate than the one received by domestic banks. I find that an international inter-bank market can develop only when differences in liquidity needs across countries are small relative to the difference in signal quality. It is shown that when an equilibrium with integrated market exists, then there are multiple equilibria, some of which are characterized by an inefficient level of cross-border lending. In particular, a segmented inter-bank market is always an equilibrium. An equilibrium with an integrated market is shown to dominate one with segmented markets if the difference in signal quality is not too large.

In chapter 4 of this thesis, I develop a model of corporate finance where banks own equity in non-financial firms. Determinants of ownership structure in bank-based economies like Germany have so far been neglected in the literature. I analyze the decision of a company owner to sell stakes in his firm to outsiders, when among the potential shareholders there are banks. It
is assumed that ownership generates private benefits of control. Firms differ inversely in their expected performance and in the level of private benefits. Contrary to other studies, I take benefits to be divisible among blockholders. Owners signal the quality of their company to potential investors by varying the number of blocks sold. The model is consistent with empirical evidence showing that large blocks are sold at a discount to banks but otherwise trade at a premium. Furthermore, it is shown that firms with a high potential performance will choose a higher degree of dispersion.
Chapter 2

Regulation of International Large Value Payment Systems

Joint with Thomas Rønde

2.1 Introduction

The last decades have witnessed a substantial increase in the volume of transactions that are processed via large-value payment systems. In the US, for example, the combined volume processed via CHIPS and Fedwire more than quadrupled during the 1980s.\footnote{see Horii and Summers (1994).} This trend is both a result of technological change and of increased financial activity. Because of this development, payment systems have grown to be one of the most likely channels through which financial crises could propagate. Furthermore, the growing integration of financial markets has led to a rapid increase in cross-border transactions, and raises fears that crises in different parts of the world could affect financial stability in Europe. The design of large value payment systems, both on a domestic and on an international level, is thus of growing concern for system participants and financial regulators.\footnote{The G-10 countries, for example, established working committees on the risks of interbank payment systems. Results of this work are the Angell Report (1989) and the}
In the European Union, the cross-border settlement system TARGET has been implemented recently. TARGET is a Real-Time-Gross-Settlement system (RTGS). In these type of systems, all transfers made between participating banks are cleared and settled immediately and irrevocably. The advantage of RTGS systems is the reduction in systemic risk: the failure of a participating bank can not trigger the consecutive failure of other banks via the payment system, because all payments have already been cleared. The alternative to a RTGS system is a net settlement system. Here, payments are cleared only at pre-specified settlement times. Furthermore, only the net amounts of liabilities are actually transferred. Net systems are cheaper in operation, because the amount of reserves needed to settle is far lower than in RTGS systems. However, if a bank is unable to settle at the end of the day, its failure can have severe consequences for the other participants, as large amounts of expected payments will not be received. Clearly, there is a trade-off in efficiency between the two systems: if the failure of banks is likely, and if the amounts transferred are large, then a RTGS system is the better choice. On the other hand, if the banks are quite safe, and the opportunity cost of holding reserves is high, a netting system is more attractive. This basic trade-off between net and gross settlement systems has been analyzed by Freixas and Parigi (1998). An overview on the functioning of large value payment systems and a discussion of the risks involved can be found in Summers (1991) as well as Schoenmaker (1995a).

The EU member states have designed TARGET to be a RTGS system in order to reduce systemic risk as much as possible. However, parallel to TARGET, privately organized net settlement systems are operational (such as Euro-Clearing and CEDEL.) It is commonly argued that private banks prefer to use netting systems even if systemic risk is high, because they do not bear the full cost of a systemic crisis. Limited liability induces the private banks to implement too risky a payment system, as some of the


3see, e.g., Padoa-Schioppa (1995), and European Monetary Institute (1996).
increased risk is shifted to the banks' depositors and to the counterparties in the settlement system. The coexistence of privately organized netting systems might therefore undermine the European System of Central Banks' (ESCB) objectives of reducing systemic risk (see, for instance, Giannini and Monticelli (1995)). The possibility of coexistence of several payment systems is discussed in Rochet and Tirole (1996).

In this paper, we raise the question why the ESCB has restricted itself to providing a RTGS system. Given the trade-offs between net and gross settlement, it might be efficient to have a dual system, where safe banks settle net, while the riskier banks settle gross. Still, why let the net settlement systems be privately organized? After all, a public regulator is more likely to pursue the overall interest of the country than a private system operator. Finally, as a related question, we ask whether a privately organized system can be more efficient than a public one. Notice that we do not attempt to find the optimal regulatory framework. Instead, we want to assess the efficiency of the payment system currently implemented in the European Union. For a general analysis of optimal regulation in an international environment, see Caillaud et al. (1996).

We argue that the division of banking supervision in the ESCB creates an incentive problem for supervisory authorities. Supervision remains on the national level, and national regulators are to decide about access to the payment system. In a netting system, some of the costs of a local bank's failure are borne by the foreign economies. If supervisors are concerned rather with their country's welfare than with welfare on a community level, they might allow a high-risk bank to participate in the netting system. As a result, systemic risk would be too high.

In this paper, we analyze an economy with two countries, where consumers make and receive cross-border transfers. In each country there is one commercial bank. The existence of banks is justified on two grounds: firstly, they are able to invest in profitable, long-run technologies, which short-lived consumers could not do. Secondly, banks are participating in an international
payment system, enabling customers to make transfers abroad. Banking supervision is the task of the local central bank. Banks seek to maximize profits, and the central banks maximize their own country's welfare. We assume that the local central bank can observe the risk of the local bank at no cost, but it cannot observe the risk of the foreign bank. Concerning the private bank's information, we study two different scenarios. In the first, we assume that private banks have the same information as the central banks, while in the second one, private banks can perfectly observe each other's type. We have in mind different stages of financial integration in Europe. Growing interactions of banks in different countries are to be expected. In the process, banks are likely to acquire better information about each other's risk. This information might however not be accessible in the same way to foreign regulators.

The incentives of both private and central banks to set up gross or net settlement systems are studied. Due to the division of supervisory powers, the central banks have to rely on each other's information when designing a cross-border public settlement system. We analyze a game where the central banks can exchange information about the local bank's risk before deciding whether the banks can settle on a net basis. The local supervisors have incentives to understate the risk of the local bank, because the foreign economy carries some of the costs of failure in a net settlement system. Therefore, the national regulators allow too risky banks into the netting system. The private banks face limited liability, which induces them to choose net settlement too often. Systemic risk is therefore higher than desirable both when the public and the private sector decide upon access to the netting system.

We find that if the private banks have the same information as the public authorities, the decision about the mode of settlement should be made by the regulators. However, if the private banks possess superior information about the foreign banks' risks, it can be efficient to leave this decision to the private banks.

Other authors (e.g., Giovannini (1992) and Schoenmaker (1995b)) have studied problems concerning the division of banking supervisory powers within
the European Union. To our knowledge, however, this is the first paper that provides a formal analysis of the regulation of international payment systems. As a novel feature, we take the division of supervisory powers explicitly into account and endogenize the communication between the sovereign national regulators.

The paper is organized as follows. In section 2, the model is described. Section 3 discusses the effects of gross and net settlement on the banks’ portfolio choice and on systemic stability. Sections 4 and 5 are dedicated to the case of 'Partial Contagion' where a bank's failure does not force the other bank into bankruptcy. In Section 4, we study a benchmark case where local supervisors are informed about the other bank’s risk. Section 5 then proceeds to analyze the efficiency of public and private payment systems. In section 6, we discuss the results for the case of 'Full Contagion', where the failure by one bank leads to the failure of the other bank. Finally, section 7 concludes. All proofs are in the appendix.

2.2 The Model

The Economy We consider an OLG model with two countries and three periods. In the first two periods, a continuum of consumers are born in each country, with one unit of endowments. Consumers live for two periods, and wish to consume only in the second period of their lives. All in all, there are two generations of consumers, born either at time 0 ("consumers A") or at time 1 ("consumers B"). Consumers are risk neutral.

In each country, there is one bank. At time 0, the banks collect deposits from the local consumers of generation A. These deposits can be invested both in central bank reserves, yielding zero interest, and in a risky country-specific technology. At time 1, consumers A withdraw their money, and consumers B deposit. All consumers demand a non-negative expected return on their deposits.

At time 2, the risky technology yields a return of $R$, if successful, and 0
otherwise. The probability that Country $i$'s risky technology fails is denoted $\tilde{q}_i$. In order to have asymmetric information about the riskiness of the local bank, we assume that $\tilde{q}_i$ is a random variable. For simplicity, it is assumed that $\tilde{q}_i$ is uniformly distributed between 0 and 1. The realization of $\tilde{q}_i$ is denoted $q_i$ ($q_i$ is also called the bank's "type"). At time 1, the risky technology can be liquidated prematurely. Liquidation is costly, and the risky technology only pays $L$, $0 < L < 1$, if successful, and 0 if unsuccessful.

**Payment Systems** Consumers invest their endowments in the bank in order to make transfers to consumers in the other country. In particular, for each unit deposited, the consumers transfer $t$ to the other economy. Transfers between banks are made via a payment system.

Payments can be settled either on a gross or on a net basis. In a gross settlement system, payments are settled immediately and irrevocably (as in RTGS systems). Gross settlement requires the banks to hold central bank reserves equal to the amount that will be transferred to the other bank.\(^4\) Since the total transfers made every period are constant and equal to $t$, banks have to hold $t$ reserves in each period.\(^5\)

In a net settlement system, incoming and outgoing payments are cleared at the end of the day, and only the net liabilities are transferred. The banks send and receive transfers of $t$. As long as there is no failure, no reserves are needed in order to settle.

We assume that a gross settlement system is always in operation. The private banks can, however, settle on a net basis, if it is approved by the central banks. We abstract from any costs of setting up a net settlement system.

\(^4\)We assume that there are no overdraft facilities.

\(^5\)Alternatively, we could interpret this set-up as a gross system with collateralized daylight overdrafts. $t$ would then be equivalent to the amount of eligible assets that need to be held as collateral.
2.2. **THE MODEL**

**The Bankruptcy Rule** We need to define a bankruptcy rule that determines the payment obligations of the two netting partners in the case that one of them defaults. We assume that banks are always obliged to fulfill their settlement obligations to the other bank, regardless of whether the counterpart has declared itself bankrupt or not. We furthermore assume that depositors' claims on a bank's assets are senior to claims from the other bank. If a bank is not able to fulfill its payment obligations to either the depositors or to the other bank, it has to declare itself bankrupt.

These rules imply: (1) a failure of one bank's risky technology leads to bankruptcy of this bank, and (2) the other bank (if it itself is not bankrupt) is obliged to transfer $t$ to the failing bank, which that bank then will use to pay to its consumers. Because the transfer $t$ needs to be made only if one of the parties default, but otherwise nets out against incoming payments, we refer to it as the *Additional Settlement Obligation* (ASO). In a gross settlement system, there is no ASO because settlement always occurs immediately.

**Supervision and Regulation** In each country, the local central bank is in charge of supervision, and has the formal authority to decide whether the private bank can settle net with the foreign bank. At time 0, the central banks can observe perfectly the local bank's type, $q_i$, but they receive no information about the foreign bank's type, $q_j$. The central banks design the payment system as to maximize local welfare. The optimal mode of settlement depends on the risk of both the local and the foreign bank. The central banks have therefore incentives to exchange information about the risk of the private banks.

We model the information exchange in the following way: Before the game starts (e.g. at time $T = -1$), the central banks agree upon a scheme that decides for which $(q_1, q_2)$ net settlement should be allowed. Afterwards, the regulatory game is as follows: (1) The private banks apply to the central banks to settle on a net basis, (2) The central banks report the local bank's type. If the types are such that netting should be allowed according to the
pre-negotiated scheme, the permission is granted. Otherwise, the banks use the gross system. Notice that a netting system can be established only if all parties (i.e. all banks and central banks) agree to do so. We discuss the scheme and the regulatory game in more detail later.

Information  At time zero, the local bank and local central bank can observe the local bank's type. We assume, except as a benchmark, that the central banks cannot observe the risk of the foreign bank. We consider two different scenarios regarding the private banks' information: in the first, the local bank does not receive any information about the foreign bank. In the second, it can perfectly observe the foreign bank's type.

At time 1, central banks, private banks, and consumers receive a perfect signal about the success of the risky projects in both countries. For simplicity, we assume that the consumers observe the type of payment system only at the time when the signal about the return of the risky asset is received. The distribution of types is common knowledge.

The timing is summarized in the picture:

\footnote{Suppose the consumers could observe the payment system before making transfers. The choice of payment system contains information about the risk of the banks. The interest rate would therefore have to be contingent on the type of settlement system chosen to avoid that the consumers withdraw their deposits. A contingent interest rate would make the choice of payment system more efficient. It would not, however, change our results qualitatively.}
2.3 GROSS AND NET SETTLEMENT

\[ T = -1 \quad T = 0 \quad T = 1/2 \]

- The central banks negotiate the scheme
- Consumers A deposit 1
- \( q_t \) realized and observed by banks and local central bank
- Banks apply to settle net
- Central banks signal the type
- Decision on access to net system
- Banks invest

\[ T = 1 \quad T = 1 + 1/2 \quad T = 2 \]

- Shock realized and observed
- Net: transfers are settled
- Consumers B deposit 1
  - or bank declares bankruptcy
- Consumers A withdraw

- Transfers \( t \) are made
- Gross: transfers settled
- Returns realized
- Net: transfers are settled
- Consumers B withdraw

2.3 Gross and Net Settlement

2.3.1 The Consumers

At time 0, when the deposit contract with the consumers A is signed, neither the banks' types nor the settlement system are known. However, the risk incurred by the consumers A when depositing in the bank depends on both these factors. To find the interest rate \( r \) demanded by the consumers A, we solve the game backwards: The banks take the interest rate as given when deciding which type of settlement system to implement. Therefore, we first determine the choice of settlement system as a function of the banks' types and the interest rate. This is done in section 5. Afterwards, we find the interest rate that ensures consumers A a non-negative expected return on their deposits. The interest rate is derived in appendix B.

The OLG structure of the model implies that the banks use consumers B's deposits to return the deposits to consumers A. At time 2, the consumers B are then paid back their deposits out of the returns from the risky technology. Consumers B observe both the type of settlement system and the success of the risky technology in the two countries before depositing. The consumers
B do not deposit in a bank with an unsuccessful technology, as it will not be able to return the deposits at time 2. In the following sections, it is discussed under which conditions the consumers B invest in a bank with a successful technology. For now, notice: Consumers B face no uncertainty about the value of the bank’s assets. Hence, if the consumers B deposit in the bank, they will demand zero interest rate, as they are certain to be repaid.

### 2.3.2 Gross Settlement

When settling on a gross basis, the banks need to hold \( t \) reserves every period to be able to settle all outgoing transfers. The banks use the deposits of consumers B to pay consumers A. The consumers B deposit only 1, but the banks have to pay \( 1 + r \) to consumers A. Hence, in the first period, the banks hold additional \( r \) reserves on top of the settlement requirements. At time 0, banks invest \( t + r \) into reserves, and the remaining \( 1 - t - r \) into the risky technology.

At time 1, if consumers B deposit, consumers A withdraw \( 1 + r \). Each bank has then \( t \) reserves, and \( 1 - t - r \) of the risky asset. If the project is successful, the banks have \( (1 - t - r)R + t \) at time 2. Therefore, Consumers B deposit if and only if the technology is successful, and

\[
(1 - t - r)R + t \geq 1
\]  

(2.1)

We will assume that (2.1) holds.\(^7\)

When the risky technology is unsuccessful, consumers B do not deposit. The bank is forced into bankruptcy at time 1, because it cannot pay \( 1 + r \) to consumers A. Since the liquidation value of an unsuccessful technology is zero, the consumers A only receive a payment of \( t + r \).

For known \( q_i \), bank i’s expected profits in the gross system are

\[
\pi^G_i(q_i) = (1 - q_i)[(1 - r - t)R - (1 - t)].
\]  

(2.2)

\(^7\)If (2.1) does not hold, the banks cannot invest in the risky technology when settlement is done on a gross basis. Welfare and profits are then trivially equal to zero.
2.3. **GROSS AND NET SETTLEMENT**

Here, \( t \) can be interpreted as the average reserve holdings of a bank due to the settlement system used. The higher these reserve holdings, the less efficient is gross settlement, since less can be invested in the more profitable, risky technology. Similarly, a high interest rate results in lower profits because of the additional reserves that the banks needs to hold.

The expected welfare is

\[
W^*_i(q_i) = (1 - t - r) [(1 - q_i)R - 1].
\]

Both welfare and profits are increasing in the return on the risky technology. The difference between expected profits and expected welfare is a result of the limited liability, since the bank only pays \( r + t \) to the consumers \( A \) when it fails.

### 2.3.3 Net Settlement

In a net system, banks don’t need to hold reserves for settlement purposes. But, they still have to hold \( r \) to pay the consumers \( A \) the promised interest rate. Suppose that the risky technology of one of the banks fails. The bank with the low return goes bankrupt because the consumers \( B \) do not deposit. The profits of the other bank are adversely affected as well because of the Additional Settlement Obligation (ASO.) Following our assumptions, ASO is equal to \( t \). The bank holds only \( r \) reserves, which it has to pay to the consumers \( A \) at time 1. To pay (parts of) ASO, it is therefore necessary to liquidate \( \text{Min}\{1 - r, \frac{t}{r}\} \) of the risky technology. Our analysis depends crucially on whether the bank can repay the consumers \( B \) after liquidating some of the risky technology. If the bank cannot repay the consumers \( B \), these do not deposit, and all of the risky technology is liquidated to pay the consumers \( A \). This is the case of "**Full Contagion**". On the other hand, if the bank is able to repay the consumers \( B \), these will deposit, and the bank survives. We refer to this case as "**Partial Contagion**". Partial Contagion
occurs if and only if\(^8\)
\[
(1 - r - \frac{t}{L})R \geq 1. \tag{2.4}
\]

Under Partial Contagion, a bank will make positive profits if its own risky technology is successful (and zero profits if it isn’t.) Profits are highest when the other bank also succeeds. For given \(q_i, q_j\), the expected profits of Bank \(i\) trading with Bank \(j\) are given as:

\[
\pi_N^i(q_i, q_j) = (1 - q_i) [(1 - r)R - 1] - (1 - q_i)q_j \frac{R}{L}. \tag{2.5}
\]

We see that with net settlement, the foreign bank’s success rate \(q_j\) is crucial for expected profits: A high counterparty risk reduces profits of a netting system because it increases the probability that the bank needs to pay the Additional Settlement Obligation to the failing bank. Furthermore, the amount of transfers made every period, \(t\), enters the profits through ASO. Last, note that profits are increasing in the liquidation value of a successful project as it reduces the fraction of the risky technology that needs to be liquidated to pay the ASO.

The expected welfare of country \(i\) is:

\[
W_N^i(q_i, q_j) = (1 - r) [R(1 - q_i) - 1] - t \left[ (1 - q_i)q_j \frac{R}{L} - q_i(1 - q_j) \right]. \tag{2.6}
\]

The consumers receive only \(r\) from the local bank when it fails. In addition to this, the consumers receive \(t\) (ASO) from the foreign bank whenever it survives.

Under Full Contagion, the banks only earn positive profits if they both are successful. A bank with a successful project will be forced into bankruptcy if its trading partner fails. If a net settlement system is implemented, profits and welfare are therefore given as:

\[
W_N^i(q_i, q_j) = (1 - r) \left[ (1 - q_i)(1 - q_j)R + (1 - q_i)q_j L - 1 \right] \tag{2.7}
\]

\(^8\)Notice that condition 2.4 also implies that the consumers \(B\) deposit in the gross system when the return is high.
2.4 Benchmark: Symmetric Information

2.4.1 Global supervision

We first study which type of settlement system the local central bank would choose if it observed both banks’ types (global supervision).\(^9\) The outcome

\[ \pi^*_N(q_i, q_j) = (1 - q_i)(1 - q_j)(1 - r)R - 1. \]  

(2.8)

Notice that under Full Contagion, the size of ASO, \(t\), affects neither profits nor welfare. The banks always go bankrupt if one of them fails, and since the own consumers’ claims on the bank’s assets are senior to those of the other bank, the additional settlement is never be made.

Figure 2.1 shows the regions for partial and full contagion for different \(R\) and \(L\). The region of partial liquidation is obtained for a high liquidation value and high return on the risky technology.

For expositional reasons, we focus first on the case of Partial Contagion. The case of Full Contagion is discussed in Section 6.

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\(^9\)Note that this is not the solution that maximizes total ex-ante expected welfare, because we continue assuming that central banks maximize local welfare, not global.
will be used as a benchmark to analyze the effectiveness of the payment system when the central banks only observe the local bank’s type (local supervision).

Define $\Delta W^i(q_t, q_j)$ as the welfare gain of Country $i$ by settling net instead of gross with Country $j$:

$$\Delta W^i(q_t, q_j) \equiv W^i_N(q_t, q_j) - W^i_G(q_t)$$

$$= t(R - 1)(1 - q_t) - q_t \left( q_1 + (1 - q_1) \frac{R}{L} \right)$$

The lines representing $\Delta W^1(q_1, q_2) = 0$ and $\Delta W^2(q_2, q_1) = 0$ are displayed in Figure 2.2. $\Delta W^i(q_t, q_j) = 0$ divides the $(q_t, q_j)$-space into two regions where the welfare of Country $i$ is maximized either with gross or with net settlement. In the area below the curve, $\Delta W^1(q_t, q_j) > 0$, and netting is the welfare maximizing settlement mode. Above the curve, $\Delta W^1(q_t, q_j) < 0$, such that gross settlement leads to a higher welfare. Transfers are settled on a net basis only if it is beneficial for both parties; that is, when $\Delta W^1, \Delta W^2 \geq 0$. Under global supervision, the banks will only settle net in the lower left corner of Figure 2.2 (region $A$).

The central banks allow net settlement only if the risk of the foreign bank is not too high. This is due to the basic trade-off between gross and net settlement systems: If the foreign bank is risky, gross settlement is the preferable option, since it eliminates the risk of contagion. On the other hand, if the foreign bank is relatively safe, net settlement maximizes welfare because of lower reserve holdings.

$\Delta W^i = 0$ is downward sloping because of the differences in reserve requirements in both systems. The investment into the risky technology is smaller in a gross than in a net system. Therefore, as the probability of failure of this technology increases, welfare in the net system is decreasing more than in the gross system. In order to keep $\Delta W^i = 0$, the quality of the population of foreign banks in the net system must improve.
Suppose now that the foreign bank was known to be riskless, i.e. $q_j = 0$. From figure 2.2, we see that then, the local supervisor would admit all local banks into the net system, even if their risk of failure was very high. The reason for this is again that under net settlement, the bank invests more into the risky technology than it would under gross settlement. If the local bank succeeds, its profits are therefore higher in the net system. On the other hand, it has $t$ fewer reserves to pay to its consumers if it fails. But in case of failure, it will receive the ASO from the foreign bank, which is exactly $t$. The payments made to consumers are thus the same in both types of settlement systems, while profits are higher in the net system. Hence, if the foreign bank paid ASO with certainty, welfare would always be higher in a net system.

Figure 2.2 illustrates how the interests of the two countries never coincide completely. In the regions $A$ and $D$ both countries prefer a net and a gross settlement system, respectively. In region $B$ ($C$), however, Country 1 (2), which has the lower risk, prefers gross settlement, while Country 2 (1) prefers to net. The interpretation of this graphical result is straightforward: The country with the lower risk is more reluctant to settle net, as it is relatively...
more likely to pay ASO than to receive it. We will say that the interests of countries become more aligned as the regions $B$ and $C$ get smaller.

For future reference, let us briefly discuss how $\Delta W^1 = 0$ and $\Delta W^2 = 0$ change as functions of $R$ and $L$. An increase in the liquidation value, $L$, makes a failure by the foreign bank less costly, since less liquidation is needed to pay the Additional Settlement Obligation. Net settlement is therefore more attractive for higher $L$. A higher $L$ also makes countries’ interests more aligned, because a failure leads to smaller losses for the counterparty. An increase in $L$ will therefore move $\Delta W^1 = 0$ and $\Delta W^2 = 0$ out and closer together.

Comparative statics in $R$ is a bit more tedious, as it affects the profits both under net and gross settlement. For a given set of parameters, the consumers’ utility is lower under net settlement since they receive less in the event that the two banks fail. Thus, the welfare under both types of settlement can only be the same (i.e., $\Delta W^i = 0$) if the bank’s expected profits are higher with net than with gross settlement. This implies that the expected value of the risky technology that the bank holds until time 2, must be larger under a net than under a gross settlement system. Hence, net settlement becomes more attractive as $R$ increases. In Figure 1, $\Delta W^1 = 0$ and $\Delta W^2 = 0$ move out, so the region with net settlement increases.

2.4.2 The Private Proposal

The private banks move first in the regulatory game and propose which mode of settlement should be used. Here, we study which type of system bank $i$ prefers if it can observe the foreign bank’s type, $q_j$.

Bank $i$ prefers to settle in a net system whenever $\pi^i_N(q_i, q_j) \geq \pi^i_G(q_i)$. From (2.2) and (2.5), we derive a threshold $q^{PR}$ for bank $j$’s risk such that bank $i$ is indifferent between the two systems:

$$q^{PR} = \frac{R - 1}{R} L$$
2.4. BENCHMARK: SYMMETRIC INFORMATION

The banks will propose net settlement if and only if \( q_1, q_2 \leq q^{PR} \). In figure 2.3, this region corresponds to the lower left square. A bank prefers to net out transfers only when its counterpart’s risk of failure is small; that is, when the probability of contagion is low. The threshold \( q^{PR} \) does not depend on the bank’s own risk because of limited liability: The bank earns zero profits whenever it fails itself. Consequently, the bank’s own risk does not influence the choice between net and gross settlement.

As long as the local bank is successful, it carries the full cost of the foreign bank’s failure because it pays \( ASO \). The bank does not, however, take fully into account the loss experienced by consumers if both banks fail. It therefore only internalizes the full cost of a net settlement when it is sure not to fail. Figure 2.3 illustrates this point: \( q^{PR} \) is above \( \Delta W^i = 0 \) except for \( q_i = 0 \), where they coincide. We can summarize:

**Remark 1** Limited liability induces the private banks to propose too risky a settlement system

\( q^{PR} \) increases (decreases) with changes in \( \Delta W^i = 0 \). The comparative statics for \( q^{PR} \) are similar to those described above.
2.5 Local Supervision

Now, we turn to the choice of settlement system when the central banks only have information about the local bank's type. First, the information exchange among the central banks is described and the equilibrium characterized. Afterwards, we turn to the full regulatory game. We will consider two different assumptions about the private banks' information. We first assume that the private banks have the same information as the central banks. Under this assumption, the public sector should always regulate the access to the payment system. This case is of some independent interest, but we mainly consider it in order to study the efficiency of public regulation. We then assume that the private banks observe the foreign bank's risk. Here, it might be optimal to let the private banks decide upon access to the payment system, since they have superior information.

2.5.1 The Information Exchange

Both the local and the foreign bank's type matter when the central banks have to decide between gross and net settlement. The central banks have no information about the foreign bank's risk because banking supervision is local. To regulate the international payment system more efficiently, the central banks might want to exchange information about the banks' risks. The central banks are sovereign regulators, and are not subject to any international authority. To capture this situation, we model the information exchange in the spirit of "cheap-talk": The central banks can costlessly signal the risk of the banks through oral or written communication. There is, however, no formal mechanism (or institution) that can align the incentives of the central banks through side payments.

In cheap-talk models, signalling is both costless and non-binding. The signals sent by a player do not affect his pay-off directly\(^\text{10}\). In this type of

\(^{10}\text{Signalling matters therefore only to the extent in which it affects the players' beliefs about the other players' types and actions.}\)
2.5. LOCAL SUPERVISION

models, the results are sensitive to the exact formulation of the communication process. To sidestep this problem, we assume instead that signals are binding.\footnote{Notice, the binding scheme can implement everything that can be obtained with a non-binding scheme - and sometimes more. The information exchange, as we model it, therefore gives the upper bound on how much information can be exchanged in any cheap-talk game.}

The information exchange is modelled the following way: At $T = -1$, before the banks’ types are realized, the central banks agree upon a binding scheme that maps the signals sent by the central banks into a net or a gross settlement system. At this stage there is no conflict of interest as the countries are identical before the risks are realized. The central banks therefore choose the scheme that maximizes the expected welfare of the countries. At $T = 0$, after the risks have been realized, the central banks send a signal about the local bank’s risk. Once the signals have been sent, the settlement system is given by the scheme.

There exists, of course, a incentive compatible scheme where the central banks agree always either to accept or to reject the private proposal. In this case, the information exchange does not play a role. Proposition 1 characterizes the equilibrium scheme in the more interesting case where the information exchange matters for the choice of settlement system. Invoking the revelation principle, we restrict attention to incentive compatible schemes. We consider only piecewise continuous schemes.

**Proposition 1.** Suppose the foreign bank’s risk is uniformly distributed between 0 and $\bar{q}$. Then an incentive compatible scheme, $\Phi^n(\cdot)$, consists of $n$ intervals. Let interval $z$, $1 \leq z < n$, be defined as $I_z \equiv [q_{z-1}^n, q_z^n)$ with $q_{z-1}^n < q_z^n$ and $q_0^n = 0$. Interval $n$ is defined as $I_n \equiv [q_{n-1}^n, \bar{q}]$. If the bank in Country $i$ belongs to interval $z$, it will settle net with the bank in Country $j$ iff $q_j \leq q_{z-1}^n$. \$ \{q_1^n, q_2^n, \ldots, q_{n-1}^n\} \$ are given as the solution to the following
system of equations:

\[
\begin{align*}
\Delta W^i \left( q^i_{n-1}, \frac{1}{2} q^n_1 \right) &= 0 \\
\Delta W^i \left( q^n_{n-2}, \frac{1}{2} (q^n_1 + q^n_2) \right) &= 0 \\
\Delta W^i \left( q^n_{n-3}, \frac{1}{2} (q^n_2 + q^n_3) \right) &= 0 \\
&\quad \vdots \\
\Delta W^i \left( q^n_1, \frac{1}{2} (q^n_{n-2} + q^n_{n-1}) \right) &= 0
\end{align*}
\]

Proof. See appendix.

The reason why the equilibrium scheme has "jumps" is essentially the same as in the cheap-talk models with one sender and receiver\textsuperscript{12}: The interests of the central banks are not totally aligned. The central banks would therefore (for some types) like to induce a belief that is different from the bank's true type. On the other hand, as the incentives of the central banks are still somewhat aligned, they do not want to induce beliefs that are too different from the true type. Paraphrasing Stein (1989): They would like to tell small lies but not big lies. If the scheme was continuous, the central banks would have the possibility of telling small lies, and the scheme would not be incentive compatible. When the scheme consists of constant parts with jumps, only big lies make a difference. Incentive compatibility can thus be achieved. Here, we have two central banks that both send and receive signals. This is the reason why the equilibrium scheme is characterized by a system of equations instead of the (simpler) difference equation characterizing the equilibrium in the before mentioned cheap-talk games.\textsuperscript{13}


\textsuperscript{13}Melumad and Shibano (1991) compare binding and non-binding schemes when there is only one sender and receiver. They show that when schemes are binding, an incentive compatibility scheme can consist of both constant parts, and parts that coincide with
2.5. LOCAL SUPERVISION

To see how the equilibrium works, consider the scheme with two intervals, $\Phi^2(-)$. Suppose the central bank sends a signal $q > q_1^2$. A gross settlement system is then implemented independently of the signal sent by foreign central bank. Instead, the central bank could send a signal $q \leq q_1^2$. Then, banks would settle net whenever $q < q_1^2$, and settle gross otherwise. The own signal sent thus only matters whenever $q > q_1^2$. The central bank sends the signal that maximizes the local welfare. The expected risk of the foreign bank (given that $q < q_1^2$) is $\frac{1}{2}q_1^2$. Hence, the central bank sends a signal $q \leq q_1^2$ if and only if $\Delta W^\ast(q, \frac{1}{2}q_1^2) \geq 0$. The scheme has to be constructed such that the central banks reveal the type of the local bank truthfully for all $q \in [0, \bar{q}]$. This requires that $\Delta W^\ast(q, \frac{1}{2}q_1^2) \geq 0$ for $q \leq q_1^2$, and $\Delta W^\ast(q, \frac{1}{2}q_1^2) < 0$ for $q > q_1^2$. It follows that $\Delta W^\ast(q_1^2, \frac{1}{2}q_1^2) = 0$. This can also be seen from Figure 2.4 where $\Phi^2(-)$ is symmetric around $\Delta W^\ast = 0$ for $q = q_1^2$.

When the central banks decide which signal to send, they consider the average risk of the foreign bank, $\frac{1}{2}q_1^2$. To induce truth-telling, the region for which the banks settle net has to be larger than under symmetric information. The banks will therefore sometimes settle on a net basis even if the welfare of one of the countries (or both) would be higher under gross settlement.

2.5.2 Public Regulation

In this section we consider the case where neither the private banks nor the central banks know the risk of the foreign bank. In principle, the private banks could be allowed to exchange information before making suggestions to the central banks about the payment system. However, because the banks' interests are independent of their type, a bank will always want to settle net as long as the risk of the foreign bank is lower than $q^{PR} = (R - 1)L/R$. This implies that the banks cannot exchange information: All types will claim to the sender's most preferred action. In our model, we have two parties that both send (and receive). As the interests of the two parties sending the signals do not coincide, the incentive compatible scheme can only consist of constant parts, even if we have binding schemes.
be of low risk in order to settle net with the foreign bank if it is of low risk. Hence, there are two reasons why the central banks should regulate the access to the payment system. First, the private banks maximize profits instead of welfare, and second, they cannot exchange any information. We will analyze the case where the private and central banks have the same information to study the efficiency of public regulation rather than to discuss who should regulate the access to the payment system.

As a first result, we show that the information exchange between the central banks allows them to regulate more efficiently. Formally, the proof consists of showing that the welfare under the two interval scheme is higher than if the banks always settled either on a gross or on a net basis.

**Remark 2** The information exchange increases the efficiency of public regulation

**Proof.** See appendix.

The private banks foresee that the central banks will exchange information before accepting or rejecting a net settlement system. Proposition 2 shows that foreseeing the public regulation, the private banks always propose to implement a net settlement system. The central banks therefore have the authority to decide which type of settlement system should be used.

**Proposition 2.** In equilibrium, the private banks always propose to implement a net settlement system. The central banks therefore decide whether the private banks should settle on a net and or on a gross basis.

**Proof.** See appendix.

Because the private banks always propose net settlement, the banks’ proposal does not reveal any additional information about the foreign bank’s type. The central banks do not update the belief that the risk of foreign bank is uniformly distributed on the interval $[0,1]$. Hence, the optimal scheme is characterized by Proposition 1 for $\bar{q} = 1$. 
2.5. LOCAL SUPERVISION

Figure 2.4: The Public Scheme with Two Intervals \((R = 5, L = 0.6)\)

Figure 2.4 illustrates the equilibrium scheme for \(n = 2\) and \(\bar{q} = 1\). For two intervals, one single constraint determines the endpoint of the first interval, \(q_1^2\). The solution is given by \(\Delta W^i(q_1^2, \frac{1}{2}q_1^2) = 0\). The banks settle on a net basis whenever they both are of a type lower than \(q_1^2\). Schemes with more than two intervals are discussed in appendix C.

The relative efficiency of the equilibrium scheme with respect to the benchmark case of Global Supervision depends on the parameters. Efficiency is increasing in \(L\), because the central bank’s incentives are more aligned for high \(L\). The welfare loss of the public scheme with respect to the benchmark is depicted in figure 2.5 in the following subsection.

2.5.3 Access to the Net Settlement System

Now consider the case that bank \(i\) can observe the other bank’s type, \(q_j\). This assumption is supposed to capture internationally integrated financial markets. As markets integrate, cross-country relationships between banks will be enhanced. and banks will acquire more and more information about each other.
In section 4.2. we saw that the private banks propose to use a net settlement system whenever \( q_i, q_j < q_{PR} \). If the banks have proposed a net settlement system, the central bank in country \( i \) then updates its beliefs to that the risk of the foreign bank, \( q_j \), is uniformly distributed between 0 and \( q_{PR} \). Proposition 3 shows that in this case, the central banks always accept the private banks' proposal.

**Proposition 3.** Under Partial Contagion and symmetric information between the private banks, public regulation cannot improve upon the private proposal.

**Proof.** See appendix.

The proof has two parts. First, it is shown that for all schemes with two intervals or more, at least one interval boundary is larger than \( q_{PR} \). The only possible incentive compatible scheme has therefore one interval; that is, the central banks decide at \( T = -1 \) either always to accept or to reject the private proposal. In the second part of the proof it is shown that given the updated belief about the foreign bank's type, it is optimal to accept the private banks' proposal of a net settlement system.

Proposition 3 implies that if the private banks can observe each other's type, they will in fact decide the settlement method. The private banks propose to settle on a net basis for too high risks. However, the moral hazard problem faced by the central banks is so severe that the equilibrium scheme would involve netting with an even riskier population. The central banks maximize welfare by accepting the private proposal.

Figure 2.5 illustrates the relative efficiency both if the private proposal is accepted, and also of the public two-interval scheme of subsection 2.5.2. The lines mark the percentage deviation of expected welfare with respect to expected welfare in the benchmark of Global Supervision. Both the private
2.5. LOCAL SUPERVISION

Figure 2.5: Welfare Loss with respect to Global Supervision for the schemes of sections 5.2 and 5.3. ($R = 4, t = 0.2$)

proposal and the public scheme perform worse than the benchmark. The efficiency loss is especially high in the public scheme: for the parameters chosen, welfare is up to 5% smaller than in the benchmark case. We also see that the public scheme performs better for high $L$, i.e. when the incentives of the regulators are more aligned.

On the other hand is the welfare loss under a private system particularly large for high $L$. This result is driven by the interest rate: When $L$ is high, netting is attractive, since little of the risky technology needs to be liquidated to pay ASO. The banks will therefore settle on a net basis even for relatively high risks. The interest rate increases rapidly with the maximal risk for which the banks settle net (see appendix B). Hence, for high $L$, the interest rate is also high. This induces a welfare loss because the banks have to hold a large amount of reserves in the first period to pay consumers $A$.

2.5.4 Summary

Let us briefly summarize the results obtained under partial contagion. The central banks' incentives are only partially aligned, because the country with the higher risk benefits more from net settlement than the other country. The central banks do not observe the foreign bank's type, so to choose between a
net and gross settlement system they exchange information about the banks' types. This creates a moral hazard problem. The central banks might not reveal that the local bank is of high risk to be able to establish a net settlement system. We showed that to induce truth-telling, the central banks have to allow net settlement under circumstances where gross settlement would be preferable in terms of welfare.

The private banks propose net settlement for too a high risk of failure, since they do not consider the consumers' welfare loss when both banks fail simultaneously. The settlement system is therefore too risky (in expected terms) both when the private banks and the central banks decide upon the access to it.

If the private bank has no information about the foreign bank, the local supervisor should decide about access to the netting system. We showed, however, that if the local bank observe the foreign bank's type (e.g., because the bank is active in the foreign market), the local bank's proposal should be followed by the central bank.

2.6 Full Contagion

In the preceding section, we analyzed the model for parameter values for which the failure of one bank did not lead to bankruptcy of the other bank (Partial Contagion (PC)). In this section, we turn to the other case of Full Contagion (FC). Under full contagion, it is not possible to solve the model in closed form. Instead, we solve it numerically. The following contains a discussion of the results obtained, and points out the most important differences between the two cases.

For a gross system, neither expected profits nor expected welfare differ from those in the Partial Contagion case. But suppose that banks settle on a net basis. The bankruptcy rules for netting imply that a bank needs to pay the Additional Settlement Obligation when the counterparty fails. In this
section we assume that the banks cannot pay both the deposits of consumers $B$ and $ASO$. As a consequence, if the foreign bank fails, but the local one doesn't, consumers $B$ do not deposit at time 1. This in turn implies that consumers $A$'s claims can not be fulfilled. The local bank needs to liquidate its entire investment, and declare bankruptcy - even though its own project was successful. Furthermore, since the claims of consumers $A$ are senior to those of the other bank, the value of the liquidation goes to the consumers only, and the additional settlement to the other bank is never made.

Hence, both countries are worse off under $FC$ compared to $PC$: the country of the failure's origin because it does not receive $ASO$, and the other country because the bank goes bankrupt. Net settlement is therefore less attractive under $FC$.

### 2.6.1 Benchmark: Global Supervision

We first look at the central banks' choice of the settlement system when they also observe the foreign bank's type. Comparing expected welfare under gross and net settlement, we define the difference between the two as\(^{14}\)

$$
\Delta W_{FC}(q_1, q_2) \equiv W_N^*(q_1, q_2) - W_C^*(q_1, q_2)
= t \left[ R(1-q_1) - 1 \right] - (1-r)(1-q_2)q_3(R-L)
$$

The first term in this expression indicates the welfare loss in a gross system because of higher reserve holdings, while the second term refers to the costs of contagion in a netting system. $\Delta W^*(q_1, q_2) = 0$ and $\Delta W^*(q_3, q_4) = 0$ are shown in Figure 2.6. The incentives of the two countries are still not completely aligned. Under net settlement, the country with the lowest risk incurs a higher cost of failure relative to gross settlement.

\(^{14}\)We use the subscripts $FC$ and $PC$ to indicate Full/Partial Contagion only when necessary to avoid confusion.
Under PC the local bank was always allowed to settle when the foreign bank's portfolio bore no risk. From Figure 2.6 we see that this not the case under FC. If the local bank is a of high risk type, it is not allowed to settle net, even if the foreign bank never fails. This leads to the following remark:

**Remark 3** The access of local high-risk banks to the netting system is more restricted in Full Contagion than in Partial Contagion.

Behind this result lies the fundamental difference between Partial and Full Contagion. Under Partial Contagion, the local supervisor always allowed the local bank to settle net as long as the foreign bank was sufficiently safe (see Figure 2.2). The reason was that in case of the local bank's failure, fewer reserve holdings in the net system were offset by the incoming ASO from the other bank. Under Full Contagion, the foreign bank is not able to pay ASO. The local bank is therefore only allowed to settle net if the technology succeeds with sufficiently high probability.

To illustrate how the switch from Partial to Full Contagion affects the choice between net and gross settlement, let us focus on those parameter values that divide the two cases, i.e. \((1 - r - \frac{1}{L})R = 1\).
Remark 4 Suppose $(1 - r - \frac{1}{L})R = 1$. If the central bank could observe the foreign bank's type, the access criteria for participation in the net system would be more restrictive in Full Contagion than in Partial Contagion.

Net settlement is less attractive under $FC$ because the welfare is lower when only one of the banks fails. Therefore, as we go from $PC$ to $FC$, the curve $\Delta W'(q_0, q_1) = 0$ makes a discrete jump inwards, and the region with net settlement decreases. Figure 2.6 illustrates this discontinuity.

An increase in the liquidation value makes net settlement more attractive and aligns the incentives of the central banks. The region with net settlement is thus increasing with $L$. In Full Contagion, the increase in welfare under net settlement is due to a higher expected utility for the consumers rather than higher profits. The transfers made abroad, $t$, do not influence welfare in the netting system (as ASO is never paid). More transfers decrease efficiency in gross settlement because of higher reserve holdings. Accordingly, a higher $t$ results in a larger region with net settlement. The effect of an increasing return on the risky technology, $R$, is essentially the same as in the Partial Contagion case: Netting becomes more attractive and $\Delta W^* = 0$ moves out.\textsuperscript{15}

2.6.2 The Public System

We start by analyzing the publicly implemented payment system when the central banks cannot extract any information from the private banks. The central banks rely on the same kind of information exchange that was described in the previous sections: first, central banks agree upon a binding scheme that determines when net or gross settlement is used. Then, the central banks send signals about the local banks' types. Because the structure of the game is unchanged, we establish as a first result

Remark 5 The incentive compatible schemes in the Full Contagion Case are given by Proposition 1.

\textsuperscript{15}In the discussion of the comparative statics we have left out just the indirect effect that works through the interest rate. Numerical simulations show that this effect is small.
Figure 2.7: The Public Two-Interval Scheme: the switch from Partial to Full Contagion

Proof. Analogous to the proof of Proposition 1.

For the remainder of this section, we will analyze the public scheme with 2 intervals. \( q_2 \) is the endpoint of the first interval, and is given as the solution to \( \Delta W^i(q_1^2, q_1^2/2) = 0 \). Our model parameters influence \( q_2 \) in the same way in which they affect the line \( \Delta W^* = 0 \): \( q_2 \) is increasing in \( R, L \) and \( t \).\(^{16}\)

As we switch from Partial to Full Contagion, the line \( \Delta W^i = 0 \) jumps inwards. Figure 2.7 shows how the change in \( \Delta W^i = 0 \) lowers \( q_2^2 \), and illustrates the effects this has on total expected welfare.

Despite of a large change in \( q_1^2 \) (for the parameters chosen in the figure, \( q_1^2 \) decreases by 23%), the effect on welfare is not so strong (here, the loss in welfare is around 7%). The expected welfare is affected negatively because welfare in a netting system goes down. However, the larger area with gross settlement compensates partly for this loss.\(^{17}\)

\(^{16}\)Foreseeing public regulation, the private banks will also here propose a net settlement system if they have no information about the foreign bank's type.

\(^{17}\)Note that the area for net settlement decreases quadratically since both countries lower the threshold.
2.6. FULL CONTAGION

2.6.3 The Private Proposal

When the private bank has perfect information about the foreign bank's type, it weighs expected profits in gross and net settlement. Comparing equations (2.2) and (2.8), we find that bank \( i \) wants to use net settlement with the other banks if and only if \( q_j < q_{PR} \) where

\[
q_{PR} = \frac{t(R - 1)}{(1 - r)R - 1}.
\]

(2.10)

In a \( q_i/q_j \)-graph, the threshold is again a horizontal line, because the banks only consider the profit in the states of the world where their own technology succeeds.

In the analysis of Partial Contagion, we established that the local bank chooses too risky a settlement system because it does not consider the lower payments to consumers in the event that both banks fail. Under Full Contagion, however, the problem of limited liability is more serious. Here, the consumers' utility is lower in a netting system if either bank fails. Since this is not taken into account by the banks, they are willing to accept a higher risk of contagion than what is socially optimal. Figure 2.8 illustrates this point. Under FC, \( q_{PR} \) lies strictly above the curve \( \Delta W^i(q_i, q_j) = 0 \). The area for which the private bank prefers netting is thus larger than the welfare-maximizing one.

Remark 6 The possibility of full contagion encourages the private banks to engage in more risk-taking.

To study the implications of Partial and Full Contagion for the choice of the settlement mode, we look again at those parameters that mark the border between the two, i.e. \( (1 - r - t/L)R = 1 \). Here, there is an important difference between public and private incentives. Starting in the region of PC and approaching the border to FC, the profits of the local bank go to zero in the state of the world where the bank has to pay ASO. On the border, and
in the region $FC$, the profits are zero in this state of the world. As there is no discontinuity in the profits between $PC$ and $FC$, there is no discontinuity in $q^{PR}$. The welfare in the net settlement system decreases discretely as we switch from $PC$ to $FC$, but this is not taken into account by the private banks.

An increase in the cross-border transfers lowers the profits in the gross system, because the reserve requirements go up, but it does not affect the net system. The region where the private banks propose a net settlement system thus increases in $t$.

Under full contagion, the effect of an increase in $R$ is surprising. Here, the private banks will propose to settle net even if the risk of the foreign bank is so high that the total expected return from the risky technology is lower than in a gross settlement system. The profits in the net system are still at least as high as in the gross system because the expected payment to the consumers is lower. The part of the profits stemming from the technology is highest in the gross system, so an increase in $R$ diminishes the region with net settlement.
2.6. FULL CONTAGION

2.6.4 Access to the Net Settlement System

We can now compare efficiency of the private proposal with the public scheme with 2 intervals. Compared to the benchmark, the private banks as well as the central banks allow too many banks to settle their payments in the net system. This results in a loss of expected welfare.

As we enter the region of full contagion, net settlement becomes less efficient because the cost of a failure is larger. The central banks take this into account, and the access to net settlement becomes more restrictive. The private banks, on the other hand, do not consider the additional cost of failure under full contagion, since it is carried by the consumers. The private banks' proposal does not change from partial to full contagion. It follows that as we switch to full contagion, public regulation becomes more efficient, compared to the private proposal.

Under full contagion, it can thus be that \( q_1^2 < q_{PR} \). Under these circumstances, the central banks can exchange additional information after observing the private banks' proposal. The central banks sometimes overrule the private proposal, and they have the real authority to decide which type of

Figure 2.9: The Private Proposal versus Public Regulation in Full Contagion
\( (L = 0.2, t = 0.2) \)
settlement system to implement. Figure 2.9 shows the public and private threshold as well as the efficiency losses with respect to the benchmark case.

We see that $q^{PR}$ decreases in $R$, while $q^P$ decreases. For sufficiently high return on the risky technology, the private threshold is lower than the public one. Then, the central banks always accept the private proposal as it was the case under partial contagion.

2.7 Conclusion

In this paper, we have analyzed the regulation of international large-value payment systems when supervision of the banking industry is a national task. We modeled the national regulators' decision to provide access to gross and net settlement systems. As a novel feature, the communication between the regulators about the private banks' risk was endogenized. Furthermore, we studied the outcome that private banks would choose if they were not subject to regulation, and compared the efficiency of publicly and privately implemented systems.

Both the public and private solutions are shown to be inefficient, since too risky banks are allowed into the netting systems. Systemic risk is therefore higher than desirable. Private systems are too risky because banks face limited liability. Banks do therefore not take into account the full cost of bankruptcy. The inefficiency of the publicly implemented system stems from national supervision. The national supervisors' incentives are not perfectly aligned, because the foreign economy carries some of the costs of failure in a net settlement system. Therefore, the local supervisors have incentives to understate the risk of the local bank to induce net settlement in cases where the foreign economy would prefer gross settlement.

We find that if the private banks have the same information as the public authorities, the decision about the mode of settlement should be made by the regulators. If the private banks possess superior information about the foreign bank's risk, it might under some circumstances be optimal to leave
the decision to the private banks. The efficiency of a privately implemented payment system depends crucially on the systemic impact of a failure. If the systemic impact is relatively low, so the failure by one bank does not trigger the failure of the other, the banks internalize most of the costs of net settlement. A privately implemented payment system does therefore relatively well in terms of welfare. Within our model, we show that public regulation cannot improve upon a privately organized payment system under these circumstances. On the other hand, if failures propagate through the payment system, the consumers carry most of the costs of net settlement. Therefore, a privately implemented payment system performs worse, and the case for public regulation is stronger.

Our model shows that there exist circumstances under which the national regulators should follow a "hands-off" policy with regard to the payment system. Accordingly, the model gives some justification to the decision of leaving the implementation of netting systems in the hands of the private sector. Here, however, a warning note is in place. Privately organized payment system are shown to perform particularly well when the banks possess superior information about their foreign counterparts, and when transfer volumes are low such that the impact of a failure is small. As of now, banking is essentially a national industry. Financial integration is likely change this as more banks become European players. While this might lead to better information about the risk of foreign banks, it will also increase the transfer volumes. It is therefore possible that the two conditions favoring privately implemented payment systems cannot coexist.
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Appendix A

Proofs of Propositions and Remarks

A.1 Proposition 1

The equilibrium consists of an incentive compatible scheme such that if a central bank signals that the type of the private bank is \( q_i \), it commits to settle on a net basis whenever the foreign central bank signals \( q_j \leq \Phi^n(q_i) \). For this scheme to be feasible, in the sense that all types settle net whenever the foreign bank has a risk lower than \( \Phi^n(\cdot) \), \( \Phi^n(\cdot) \) has to be symmetric: \( \Phi^n(\cdot) = (\Phi^n)^{-1}(\cdot) \) for all \( q \in [0, \bar{q}] \). Define \( f_1(q_1) \) such that \( W^1(q_1, f_1(q_1)) = 0 \). It follows from (2.9):

\[
\begin{equation}
    f_1(q_1) = \frac{(1 - q_1)(R - 1)L}{q_1L + (1 - q_1)R} \tag{A.1}
\end{equation}
\]

Step 1: \( f_1(\cdot) \) is not symmetric.

Proof: Calculations show that the only solution to \( f_1(q_1) = (f_1)^{-1}(q_1) \) is

\[
\bar{q}_1 = \frac{R + (R - 1)L - \sqrt{(R + (R - 1)L)^2 - 4(R - L)(R - 1)L}}{2(R - L)}
\]

\( f_1(\cdot) \) is therefore not symmetric.

Step 2: \( \Phi^n(q) \) cannot be continuously increasing or decreasing on an open...
Proof: Suppose there exists some \( q \) st. \( \Phi^n(q) < f_1(q) \). If \( \Phi^n(\cdot) \) is strictly increasing, there exists some \( \varepsilon \) st. \( \Phi^n(q) < \Phi^n(q + \varepsilon) \leq f_1(q) \). A central bank with a private bank of type \( q \) will therefore have incentives to deviate and signal the type \( q + \varepsilon \). On the other hand, if \( \Phi^n(\cdot) \) is strictly decreasing, there exist some \( \varepsilon \) st. \( \Phi^n(q) < \Phi^n(q - \varepsilon) \leq f_1(q) \). A central bank with a private bank of type \( q \) will therefore have incentives to deviate. A similar argument applies to \( \Phi^n(q) > f_1(q) \). If \( \Phi^n(\cdot) \) is strictly or decreasing on an open set, it has to be that \( \Phi^n(\cdot) = f_1(\cdot) \). However, as \( f_1(\cdot) \) is not symmetric, this not a feasible scheme.

Step 3: Suppose there is a discontinuity at \( q^n_i \) s.t. \( \lim_{q \to (q^n_i)^-} \Phi^n(q) = q^n_{j+1} \) and \( \lim_{q \to (q^n_i)^+} \Phi^n(q) = q^n_j \) and \( q^n_j \neq q^n_{j+1} \). Then it has to hold:

\[
\Delta W^n \left( q^n_i, \frac{1}{2} (q^n_j + q^n_{j+1}) \right) = 0
\]  
(A.2)

Proof: Incentive compatibility implies that all \( q \in [q^n_{i-1}, q^n_i) \) prefer to settle net with all \( q_j \leq q^n_{j+1} \), instead of settling net only with \( q_j \leq q^n_j \), while the opposite is true for \( q \in [q^n_i, q^n_{i+1}) \). Because of continuity of \( W^n_G \) and \( W^n_N \), a necessary condition for this to hold is:

\[
(q - q^n_j)W^n_G(q^n_i) + q^n_jW^n_N \left( q^n_i, \frac{1}{2} q^n_j \right) = 0
\]  
(A.3)

\[
(q - q^n_{j+1})W^n_G(q^n_i) + q^n_{j+1}W^n_N \left( q^n_i, \frac{1}{2} q^n_{j+1} \right) = 0
\]  

(A.3) can be rewritten as (A.2), and proof follows.

Step 4: \( \Delta W^n(q_i, q_j) \geq 0 \) implies that \( \frac{\partial \Delta W^n(q_i, q_j)}{\partial q_i} < 0 \).

Proof: \( \Delta W^n(q_i, q_j) \geq 0 \) implies \( q_j \leq (R - 1)L/R \). Furthermore,

\[
\frac{\partial \Delta W^n(q_i, q_j)}{\partial q_i} = t \left( - (R - 1) + q_j (R/L - 1) \right) < 0 \iff q_j < (R-1)L/(R-L)
\]

As \( L > 0 \), \( \Delta W^n(q_i, q_j) \geq 0 \) therefore implies \( \frac{\partial \Delta W^n(q_i, q_j)}{\partial q_i} < 0 \).
A.2. PROPOSITION 2

Step 5: Consider the discontinuity at $q^n_i$ analyzed in step 3. It has to hold that $q^n_j < q^n_{j+1}$.

Proof: Incentive compatibility requires that

$$\frac{\partial}{\partial q} \left[ q^n_j \Delta W^n(q, \frac{1}{2} q^n_j) - q^n_{j+1} \Delta W^n(q, \frac{1}{2} q^n_{j+1}) \right]_{q=q^n_i} \geq 0 \Leftrightarrow$$

$$\left( q^n_j - q^n_{j+1} \right) \frac{\partial \Delta W^n(q, \frac{1}{2} (q^n_j + q^n_{j+1}))}{\partial q} \big|_{q=q^n_i} \geq 0$$

From Step 3 and 4 it therefore follows that $q^n_{j+1} > q^n_j$.

Step 6: The system of equations in Proposition 1 characterizing $\{q^n_1, q^n_2, ..., q^n_{n-1}\}$ follows from symmetry of $\Phi^n(\cdot)$, (A.2), and $q^n_1 < q^n_2 < ... < q^n_{n-1}$.

A.2 Proposition 2

Step 1: $\Phi^2(\cdot)$ does not exist.

Proof: The private sector proposes to settle on a net basis whenever $q_1, q_2 \leq (R - 1)L/R$. After the private banks have proposed a net settlement system, the central banks believe that the risk of foreign bank is uniformly distributed between 0 and $(R - 1)L/R$.

With 2 intervals, $q^n_2$ is given as the solution to $\Delta W^n(q^n_2, \frac{1}{2} q^n_2) = 0$. This implies that

$$\frac{1}{2} \left( R - L \right) \left( \frac{1}{2} q^n_2 \right)^2 + \left( \frac{1}{2} R + (R - 1)L \right) q^n_2 - (R - 1)L = 0$$

It can be shown that there is only one relevant solution to this equation. Moreover, we find

$$-\frac{1}{2} \left( R - L \right) \left( \frac{(R - 1)L}{R} \right)^2 + \left( \frac{1}{2} R + (R - 1)L \right) \left( \frac{(R - 1)L}{R} \right) - (R - 1)L < 0$$

which implies that $q^n_1 > (R - 1)L/R$. Therefore, knowing that the private banks prefer to settle net, there does not exist an incentive compatible
scheme with two intervals.

**Step 2:** When an equilibrium with 2 intervals does not exist, there cannot be an equilibrium with \( n > 2 \) intervals either.

**Proof:** We prove that in any scheme with \( n > 2 \) intervals, \( \Phi^n(\cdot) \) does not exist. First note that for any \( q_t, q_j \) for which \( \Delta W^1(q_t, q_j) = 0 \), we have

\[
\frac{dq_j}{dq_t} \bigg|_{\Delta W=0} = -\frac{\partial \Delta W/\partial q_t}{\partial \Delta W/\partial q_j} = \frac{q_j(R/L - 1) - (R-1)}{q_t + (1-q_t)R/L} < 0. \tag{A.4}
\]

This follows from \( \frac{dq_j}{dq_t} \bigg|_{\Delta W=0} < 0 \iff q_j \leq (R-1)L/(R-L) \), which is satisfied as \( q_t, q_j < (R-1)L/R \).

The proof is by contradiction. Suppose, \( \Phi^n(\cdot), n > 2 \), exists. Then it has to hold that \( \Delta W^1(q^n_{n-1}, \frac{1}{2}q^n_1) = 0 \) and \( q_1^2 > (R-1)L/R \geq q^n_{n-1} \). From (A.4) it follows then that \( q_1^2 < q^n_1 \). But as \( q_1^2 < q^n_{n-1} \), we arrive at a contradiction. Hence, \( q^n_{n-1} > q_1^2 \), and there exists no \( \Phi(n) \) with \( n > 2 \).

**Step 3:** It is optimal for the public sector to accept the private banks’ proposal of net settlement system.

**Proof:** Step 1 and 2 showed that the public sector cannot exchange any additional information. The only remaining question is therefore whether the central banks should accept a net settlement given the belief that the risk of the foreign bank is uniformly distributed between 0 and \( (R-1)L/R \). Calculations show that \( \Delta W^1 \left(q_t, \frac{1}{2} \frac{(R-1)L}{R} \right) \geq 0 \) for all \( q_t \leq \frac{R}{R+L} \). As \( q_t \leq \frac{(R-1)L}{R < \frac{R}{R+L}} \), it is optimal for the central banks to accept the private banks’ proposal of a net settlement system.

### A.3 Proposition 3

In all the possible schemes characterized by Proposition 1, the central banks obtain a gross settlement system by signalling \( q_t = 1 \). In equilibrium, the central banks reveal the type truthfully, so it has to hold that \( \Delta W^1 \left(q_t, \frac{1}{2} \Phi^n(q_t) \right) \geq \)
A.4. REMARK 2

0. This implies that \( \frac{1}{2} \Phi^n(q_i) \leq \frac{(R-1)(1-q_i)L}{q_i L + (1-q_i)R} \). It follows that \( \frac{1}{2} \Phi^n(q_i) \leq \frac{(R-1)L}{R} = q^{PR} \). Whenever the central banks decide to accept a net settlement system, it is also the private banks' preferred mode of settlement. Foreseeing this, the private banks always propose a net settlement system.

A.4 Remark 2

We need to show that the two-interval scheme leads to a higher expected welfare than would always choosing gross or net settlement. If the central banks implement the scheme with two intervals, they can always obtain a gross settlement system by signalling \( q = 1 \). In equilibrium, however, the central banks signal the banks' true types. It follows therefore from a revealed preference argument that the scheme with two intervals gives higher expected welfare than always implementing a gross settlement system.

Next, we show the scheme with two intervals also dominates a net settlement system for all \((q_1, q_2)\). Let \( \bar{q} \) be the maximal risk for which the banks' settle net under a scheme with two intervals; that is, the banks settle net iff. \( q_1, q_2 \leq \bar{q} \). The expected welfare as a function of the welfare as function of \( \bar{q} \) is given as

\[
E(W) = \int_0^\bar{q} \int_0^\bar{q} W_N dq_1 dq_2 + \int_0^\bar{q} \int_0^1 W_G dq_2 dq_1 + \int_0^1 \int_0^1 W_G dq_2 dq_1,
\]

where the interest rate is given by (B.1) with \( \bar{q} = q^{PR} \). Integration and maximization yields the first-order condition:

\[
\bar{q}^2 R(1 - \frac{L}{2}) - \frac{3}{2} \bar{q}(RL + R - L) + 2L(R - 1) = 0
\]

Let \( \bar{q}^* \) be the solution to the first order condition. Analysis of the first order derivative shows that welfare increases up to \( \bar{q} = \bar{q}^* \), and decreases afterwards. Calculations show that \( q_1^2 > \bar{q}^* \). Netting for all \((q_1, q_2)\) will therefore give a lower expected welfare than the scheme with two intervals, as \( 1 > q_1^2 > \bar{q}^* \).
Appendix B

The Interest Rate

Consider first partial contagion. In a net settlement system, the consumers receive $1+r$ when the local bank does not fail. When it does fail, the payment received by consumers depends on whether the foreign bank fails or not. If the foreign bank succeeds, the consumers receive the reserves the bank holds plus ASO, $r + t$. If the foreign bank also fails, the consumers are only paid $r$. In a net settlement system, the consumer in Country $i$ will therefore have the following expected payments on their deposits

$$P^N_{PC}(q_i, q_j) = (1 - q_i)(1 + r) + q_i(1 - q_j)(r + t) + q_i q_j r.$$  

Under full contagion, consumers receive $1 + r$ only if both banks succeed. If the own bank is forced into liquidation due to the foreign bank's failure, they receive all reserves $r$ plus the full liquidation value of the technology, $(1 - r)L$. If both banks fail, only the reserves $r$ are received. The expected payments are

$$P^F_{PC}(q_i, q_j) = (1 - q_i)(1 - q_j)(1 + r) + (1 - q_i)q_j [r + (1 - r)L] + q_i r.$$  

Similarly, in a gross settlement system, the consumers receive $1 + r$ if the bank succeeds, and $t + r$ if it fails. Hence, the expected payments are

$$P^G(q_i, q_j) = (1 - q_i)(1 + r) + q_i(t + r).$$
APPENDIX B. THE INTEREST RATE

The consumers foresee for which \((q_1, q_2)\) net settlement will be chosen in equilibrium. They also know if partial or full contagion applies. Consider first a private system. The interest rate that ensures the consumers exactly zero return on the deposits is then the solution to the following equation:

\[
\begin{align*}
\int_0^{q^{PR}} \int_0^{q^{PR}} P_N(q_1, q_2) dq_2 dq_1 + \int_0^{q^{PR}} \int_{q^{PR}}^{1} P_G(q_1, q_2) dq_2 dq_1 + \int_{q^{PR}}^{1} P_G(q_1, q_2) dq_2 = 1
\end{align*}
\]

Solving for \(r\), we obtain

\[
\frac{1}{2} - \frac{t}{2} \left( 1 - \frac{(q^{PR})^4}{2} \right)
\]

In a public system with \(n\) intervals, the interest rate is given by

\[
\sum_{i=0}^{n-1} \int_{q_i}^{q_{i+1}} \int_{q_{n-i-1}}^{q_{n-i}} P_N(q_1, q_2) dq_2 dq_1 + \sum_{i=0}^{n-1} \int_{q_i}^{q_{i+1}} \int_{q_{n-i-1}}^{1} P_G(q_1, q_2) dq_2 dq_1 = 1
\]

Finally, we calculate the interest rate in the case of global supervision. It is given as the solution to the following equation:

\[
\begin{align*}
\int_0^{q^*} \int_0^{f_1(q_1)} P_N(q_1, q_2) dq_2 dq_1 + \int_0^{q^*} \int_{f_2(q_1)}^{(R-1)L} P_N(q_1, q_2) dq_2 dq_1 + \int_{q^*}^{1} \int_0^{f_1(q_1)} P_G(q_1, q_2) dq_2 dq_1 + \\
\int_{q^*}^{1} \int_{f_2(q_1)}^{(R-1)L} P_G(q_1, q_2) dq_2 dq_1 + \int_0^{(R-1)L} \int_0^{1} P_G(q_1, q_2) dq_2 dq_1 = 1
\end{align*}
\]

where \(f_1(q_i)\) is given by (A.1), \(f_2(q_i) = \frac{(R-1)L-q_i R}{(R-1)L-q_i(R-L)}\) and \(q^*\) is the solution to \(f_2(q_i) = f_1(q_i)\).
Appendix C

Schemes with more than 2 Intervals

In the equilibrium scheme described in section 5.1., the number of incentive constraints increases with the number of intervals considered. Because the problem is not a recursive one, calculations quickly become very complex, especially in the full contagion case, where the interest rate is relevant for the calculations. This section contains some numerical results on the public schemes in partial contagion.

**Result 1:** Schemes with three intervals exist for high values of $L$.

**Result 2:** No scheme with four intervals exists.

**Result 3:** The scheme with three intervals, $\Phi^3(\cdot)$, dominates the scheme with two intervals, $\Phi^2(\cdot)$ only for a very limited range of parameter values, for high $R$, $L$, and $t$.

All results are derived numerically.

Result 1 shows that three interval schemes can exist only if the central banks' incentives are relatively aligned. Moreover, from result 2 we see that the incentives of the central banks are so disaligned that the maximal number of intervals the scheme can have is either two or three. Result 3 is more surprising. It shows that expected welfare if the countries commit to a two-interval scheme is usually higher than if they commit to three-
interval scheme. Unlike cheap-talk models, equilibria with more intervals are therefore not necessarily more efficient. The reason is that the general shape of the equilibrium scheme changes from an even to an odd number of intervals. This fact is illustrated by Figure C.1.
Chapter 3

Inter-bank Liquidity Channels and the Creation of a Unique Market in the EU

3.1 Introduction

One of the main objectives of the introduction of a single currency in the European Union has been to reduce market frictions and imperfections in financial markets. In order to reach this goal, the setting of an integrated interbank market is particularly important, at least for two main reasons: first, because interbank markets are instrumental in allowing for a smooth working of the payment systems (so that a bank that is lacking liquidity in the payment system is able to borrow from another bank), and second, because the efficient allocation of funds requires channeling liquidity to the banks and countries that need them most.

The objective of this paper is to understand how an interbank market may emerge after introducing the Single Currency in the European Monetary Union. The motivation for doing so is that interbank markets, as any credit market, cannot be simply analyzed with the standard supply and demand instruments that are at work for the markets of consumption goods. So, any
rigorous analysis of the effect of a single currency on the interbank market has to depart from the Modigliani-Miller setting of perfect capital markets and use an imperfect information setting where financial intermediaries have a role to play.

The theoretical analysis of interbank markets (Bhattacharya and Gale (1987), Bhattacharya and Fulghieri (1994), Rochet and Tirole (1996), Holmström and Tirole (1997), Algher (1999), and Freixas, Parigi and Rochet (1999)) has already yielded some important insights on the effect of the interbank market on resource allocation and the incentive effects it generates. Still, the area seems to be at an early stage, so that there is not a unique paradigm of an interbank model.

Our model uses a Diamond and Dybvig type of framework, where consumers are uncertain about the time they need to consume. This generates liquidity shocks, which we assume are present both at the individual and at the aggregate level. To be able to cope with these shocks, banks can invest in a storage technology. Because this technology has a lower return than alternative investment opportunities, it is efficient for banks to use the interbank market. In order to introduce credit risk, the model assumes that banks have some risk of failure. As in Rochet and Tirole (1996), they monitor each other in the interbank market, thus obtaining a signal on the solvency probability of each of their peers. In a two-country setting we assume that cross border information about banks is less precise than home country information. Hence, when a bank tries to borrow from a foreign bank, it does so either because it belongs to a liquidity short country or else because it has generated a "bad" signal at a domestic level and is therefore unable to borrow in his home country. As is intuitive, depending of the proportion of the two types of motivations for borrowing abroad, an integrated interbank market may exist or not.

Using this framework allows us to derive the following results:

1. We show that having a single currency is no guarantee for having a sin-
3.2. THE MODEL

We regard an economy with two countries.

Consumers In each country, there is a continuum of consumers of a total measure of one, who possess one unit of endowment each at time 0. Consumers are risk neutral and face liquidity shocks as in a Diamond and Dybvig (1988) type of model: they need to consume either at time 1 or at time 2. At time 0, consumers deposit their endowments in a bank, and can withdraw funds at the time they need to consume. Deposits are fully insured by a deposit insurance, so no bank runs occur.\footnote{The Deposit Insurance Company is assumed to raise funds at unit cost.} We assume that the demand-deposit contract promises them a consumption of 1 in either period, $C_1 = 1$ or $C_2 = 1$.
Banks' Investment  Banks receive the consumers' endowments at time 0, and can invest them in a risky technology or in reserves. Denote the investment in the risky technology $I$ and the one in reserves $1 - I$.

Each unit invested in the risky technology yields an uncertain payoff $\bar{R}$ at time 2, where $\bar{R} = R$ with probability $p$, and $\bar{R} = 0$ with probability $1 - p$. Investment in the technology is assumed to be ex-ante efficient in that $pR > 1$.

This risky asset can also be (partially) liquidated at time 1, with the following technology: liquidation of $\Delta I$ units gives a liquidation value of $l(\Delta I)$, where $l(\Delta I)$ is increasing and concave. Specifically, we use a logarithmic liquidation function $l(\Delta I) = \ln(\Delta I + 1)$. Using this function, we have $l(0) = 0$, $l'(\Delta I) > 0$ and $l''(\Delta I) < 0$ with $l'(0) = 1$.

If the risky technology pays a positive return $R$, banks are called solvent, otherwise they are insolvent. Banks have limited liability.

Liquidity Shocks  Banks are uncertain about the liquidity demand they face at time 1. For a fraction $q$ of all banks, a high fraction of consumers $\pi_H$ is impatient and wishes to withdraw at time 1. A fraction $1 - q$, on the other hand, faces a low liquidity demand $\pi_L$, $\pi_L < \pi_H$. The remaining consumers are impatient and withdraw at time 2.

The variable $q$ reflects the aggregate demand for liquidity and is uncertain as well. With probability $1/2$, $q = q_H$. Then, a country is in a state of high liquidity demand, because many banks face high time-1 withdrawals ($\pi_H$). Similarly, with probability $1/2$ the country faces a low liquidity demand with $q = q_L < q_H$, so that fewer banks face a high amount of withdrawals.

The probability of solvency and liquidity are uncorrelated. We assume that the liquidity shocks in both countries are not perfectly correlated.

Information  At time 0, the ex-ante probability of being solvent, $p$, is common knowledge. At time 1, all banks receive a common, non-verifiable signal $s_D$ about the solvency of domestic banks. The signal can either be good ($\bar{s}$)
3.2. THE MODEL

Solvency

or bad (s) and is defined as

\[ \text{prob}(s_D = s | \text{solvent}) = \text{prob}(s_D = s | \text{insolvent}) = \alpha_D. \]

Denote \( \theta \equiv p\alpha_D + (1-p)(1-\alpha_D) \) the ex-ante probability that the good signal is received about a bank.\(^2\) We assume that the signal is informative, i.e., \( \alpha_D \in (\frac{1}{2}, 1] \).

The signals received in the foreign country can only be observed with some noise. We assume that

\[ p(s_F = s | s_D = s) = 1 - \beta \]

regardless of the bank’s solvency, where \( \beta \in (0, \frac{1}{2}) \). The lower \( \beta \), the better is the information flow between countries. Defining \( \epsilon \equiv \beta(2\alpha_D - 1) \), we can write the quality of the foreign signal \( \alpha_F \) as \( \alpha_F = \alpha_D - \epsilon \).

Each bank is then characterized by a pair \( \{s_D, s_F\} \), denoting the signals that have been received by domestic and foreign banks about this particular bank.

Note that we assume that a bank cannot observe its own solvency, but only the signal. Therefore, it has no informational advantage over the other market participants with respect to himself. This assumption allows us to leave aside problems of moral hazard.

\(^2\)Because we are dealing with a continuum of banks, the ex-ante probability is equal to the ex-post fraction of banks of this type.
Efficiency  We assume that $\text{prob}(\text{solv} | \bar{s}) R \leq l'(I)$. That is, when the bad signal has been received about a bank, then the marginal value of liquidation of the last unit of the investment technology is higher than the continuation value of the risky technology. Since $l'(\Delta I)$ is decreasing in the amount liquidated, this implies that it is efficient to liquidate the entire risky technology of a low signal bank.

On the other hand, the assumption $p R > 1$ implies that $\text{prob}(\text{solv} | \bar{s}) R > 1$, such that (partial) liquidation of a high-signal bank is never efficient.

Timing  We summarize the timing of the model:

<table>
<thead>
<tr>
<th>t = 0</th>
<th>t = 1</th>
<th>t = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumers deposit</td>
<td>$q$ and $\pi$ observed</td>
<td>returns $\bar{R}$ realized</td>
</tr>
<tr>
<td>banks invest:</td>
<td>signals $s$ received</td>
<td>patient consumers withdraw</td>
</tr>
<tr>
<td>$I$ in risky asset</td>
<td>liquidation takes place</td>
<td>interbank loans repaid</td>
</tr>
<tr>
<td>$1 - I$ in reserves</td>
<td>interbank market</td>
<td>impatient consumers withdraw</td>
</tr>
</tbody>
</table>

3.3 One Country

We analyze the banks' problem at time 1, taking the level of investment in the risky asset as given. At time 1, banks can find themselves in different situations, depending on the time 1 liquidity demand by consumers, and on the signal received about solvency. A bank's basic liquidity demand is the difference between consumers' demand and the supply of reserves $\pi - (1 - I) \geq 0$ with $\pi \in \{\pi_H, \pi_L\}$. Banks with excess liquidity at time 1 can either store it until time 2, or lend it to other banks at an interest rate. All banks with a low demand for liquidity, $\pi = \pi_L$, are called lenders.

Banks with a high liquidity demand $\pi_H$ have to obtain the liquidity needed either by liquidating their risky technology, or by borrowing funds from lenders. If they cannot meet the consumers' demand, they are forced
to declare bankruptcy.

A bank's ability to repay and thus to obtain a loan depends on the signal about this bank. After receiving the signal $\bar{s}$ or $\bar{s}$, the lenders update their beliefs about the borrower's probability of solvency

$$p(solv|\bar{s}) = \frac{\text{prob}(solv \text{ and } \bar{s})}{\text{prob}(\bar{s})} = \frac{p\alpha_D}{\theta}$$

$$p(solv|\bar{s}) = \frac{\text{prob}(solv \text{ and } \bar{s})}{\text{prob}(\bar{s})} = \frac{p(1 - \alpha_D)}{1 - \theta}.$$ 

Since $p(solv|\bar{s}) > p(solv|\bar{s})$, the interest rate charged to $\bar{s}$-banks is lower than the one that $\bar{s}$-banks need to pay. We assume that for a $\bar{s}$-bank, the interest rate would have to be so high that the payments could not be met at time 2, while the interest rate charged to a $\bar{s}$-bank is sufficiently low, or

**Assumption 1:** The expected net present value of a loan to a bank is positive if $s_D = \bar{s}$ and negative if $s_D = \bar{s}$.

Because of assumption 1, only banks with $s = \bar{s}$ are able to borrow funds - these banks are called *borrowers* - while those with $s = \bar{s}$ do not have any access to outside financing.

Let us first solve the lenders' problem. By assumption, they only lend to $\bar{s}$-banks, charging an interest rate $r$. If a lender lends proportionally to all borrowers, then for each unit lent at time 1, he will receive a certain payment of $\bar{P}_D(1 + r)$ at time 2, where $\bar{P}_D \equiv p(solv|s_D = \bar{s})$ denotes the updated probability of facing a solvent bank when receiving the signal $\bar{s}$. Moreover, the bank can liquidate some amount $\Delta I_L$ of its risky technology, obtaining $l(\Delta I_L)$, where $0 \leq \Delta I_L \leq I$. Last, the lender has the possibility of storing reserves.

Lenders can liquidate in order to lend the proceeds to borrowers. This is profitable as long as the cost of liquidation does not exceed the return from lending. Since the cost of liquidation is increasing in the amount liquidated,
more liquidation will take place for a higher interest rate. The optimum is reached when the marginal cost of liquidation \( \frac{R}{\bar{p}_D(\Delta I_L)} \) is equal to the return from lending, \( \bar{p}_D(1 + r) \). Since \( l'(\Delta I) \leq 1 \), lenders want to liquidate only if \( 1 + r > \frac{R}{\bar{p}_D} \).

Borrowers need to obtain liquidity to satisfy their consumers' demands. They have the choice between borrowing at rate \( 1 + r \) and liquidating \( \Delta I_B \) with \( 0 \leq \Delta I_B \leq L \). Again, the optimal amount of liquidation is determined by the interest rate, because the borrowing bank will liquidate as long as the cost of liquidation is cheaper than the cost of borrowing. Equality of the marginal cost of both sources of liquidity determines the optimum, or \( 1 + r = \frac{R}{\bar{p}_D(\Delta I_L)} \). Thus, liquidation will occur only if \( 1 + r > R \). In appendix A.1, the optimal levels of liquidation for both lenders and borrowers are calculated to be

\[
\begin{align*}
l(\Delta I_L)(1 + r) &= \max \left\{ 0, \ln \left( \frac{\bar{p}_D(1 + r)}{R} \right) \right\} \\
l(\Delta I_B)(1 + r) &= \max \left\{ 0, \ln \left( \frac{1 + r}{R} \right) \right\}.
\end{align*}
\]

Note that the borrowers' and lenders' decisions to liquidate differ: borrowers are willing to liquidate larger amounts, and for a lower interest rate than lenders do. This reflects the fact that the cost of borrowing at interest rate \( r \) is higher than the returns from lending at the same rate: borrowers have to pay back whenever they survive, while lenders receive a positive profit from lending only when both the borrower and they themselves survive. Consequently, liquidation is more attractive for borrowers. Denote

\[
\Lambda(1 + r) \equiv (1 - q)l(\Delta I_L)(1 + r) + q\theta l(\Delta I_B)(1 + r)
\]

the supply of liquidity by lenders and borrowers. It follows that

**Lemma 1** The supply of liquidity \( \Lambda(1 + r) \) is a non-decreasing function of the interest rate.
3.3. ONE COUNTRY

Finally, those banks who need liquidity, but about whom the bad signal has been received, are not able to borrow on the inter-bank market. They need to obtain all liquidity through liquidation; independently of the cost of doing so. Note that we have assumed that it is more efficient to liquidate all assets of a bank with the bad domestic signal. However, because of limited liability, this type of bank has no incentives to liquidate all its assets.

Lemma 2 Banks with \(s_D = s\) liquidate no more than necessary to satisfy consumers' demand.

3.3.1 The Interbank Market

The equilibrium interest rate \(1 + r\) clears the interbank market (IBM) at time 2. The agents active in this market are a measure of \((1 - q)\) lenders and \(q\theta\) borrowers. Denote the aggregate liquidity shortage in the market at time 1 by \(\Omega\), given by

\[
\Omega \equiv (1 - q) \left[ \pi_L - (1 - I) \right] + q\theta \left[ \pi_H - (1 - I) \right].
\]

For \(\Omega \leq 0\), there is excess liquidity in the market. Lenders compete for giving loans, and as a result, the interest rate will be so low that lenders are

\(^3\)The banks' maximization problem is set up in appendix A.1.
Figure 3.3: For $\Omega < 0$ (i.e. excess liquidity), the interest rate is constant in $\Omega$. At $\Omega = 0$ it exhibits a jump from $1 + r = 1/p$ to $1 + r = R$. The kink occurs at the point from which on lenders also wish to liquidate.

just indifferent between storing and lending. Hence,

$$1 + r = \frac{1}{p_D}$$  \hspace{1cm} (3.2)

Since $pDR > 1$ implies $1 + r < R$, it follows that neither borrowers nor lenders want to liquidate, i.e. $\Delta I_L = \Delta I_B = 0$.

On the other hand, if $\Omega > 0$, the economy faces a liquidity shortage. Now, liquidation by borrowers and/or lenders is necessary to generate the liquidity needed. The total amount liquidated should equalize demand and supply for liquidity, i.e.

$$\Lambda (1 + r) = \Omega.$$  \hspace{1cm} (3.3)

The higher $\Omega$, the more liquidation is needed. To induce a higher level of liquidation, the returns from lending (or the cost of borrowing) need to increase, i.e. the interest rate has to rise. Thus, the equilibrium interest rate is increasing in $\Omega$. The interest rate is derived in appendix B and illustrated in figure 3.3. Note that the functional form of the interest rate changes at the point from which on lenders start to liquidate.
In what follows, we assume that for \( q = q_L \), there is excess liquidity (point \( A \) in figure 3.3), while for \( q = q_H \), the liquidity shortage is so high that both lenders and borrowers liquidate (point \( B \)).

### 3.4 Two countries

In this section, we regard two countries with different aggregate liquidity demands. Let us denote the country with excess liquidity country \( L \) (low liquidity shock \( q_L \)), and the one with a liquidity deficit country \( H \) (high liquidity shock \( q_H \)).

We allow for the establishment of an international inter-bank market (IBM), where banks with liquidity needs can obtain liquidity by borrowing from banks in the other country. For the remainder of this section, we make the following assumption:

**Assumption 2:** (No Correspondent Banking) A bank cannot borrow and lend at the same time.

This assumption rules out that banks can profit from interest differentials and act in different markets at the same time. It will be relaxed in section 3.5.1.

Lending decisions are again made on the basis of the signals about solvency. Because the quality of the signal received by the foreign lender is lower than the one received by the domestic lender, the signals received about one bank in different countries are not perfectly correlated. Banks with liquidity needs can therefore find themselves with either one of the signal pairs \( \{s_D, s_F\} = \{\bar{s}, \bar{s}\}, \{\bar{s}, \bar{s}\}, \{s, \bar{s}\}, \) or \( \{s, \bar{s}\} \). The first two types are able to borrow in their home country. Furthermore, banks with \( \{\bar{s}, \bar{s}\}, \{s, \bar{s}\} \) are in principle able to obtain a loan in the foreign country, while \( \{s, s\} \) are not able to obtain any funds.
Lemma 3  Suppose that banks can infer the signal received in the other country. Then, they update their beliefs about a bank’s solvency in the following way:

\[ \text{prob}(\text{solv}|\{\bar{s}, s_i\}) = \bar{p}_D \quad \text{prob}(\text{solv}|\{s, s_i\}) = p_D \quad \text{for } s_i \in \{\bar{s}, \bar{s}\}. \]

The proofs follow directly from the assumptions about the signal structure: the foreign signal only adds noise to the domestic signal. The quality of the combined signal is therefore never higher than the one of the domestic one, and banks who can observe both signals disregard the one that has been received by the foreign banks.\(^4\)

If the other country’s signal cannot be inferred, then the only banks that can obtain a loan in both countries are those with the signal pair \{\bar{s}, \bar{s}\}. Denote \(\psi_i\) the fraction of \{\bar{s}, \bar{s}\}-banks of country \(i\), \(i \in \{L, H\}\) that choose to borrow abroad. The fractions will be determined endogenously.

Lemma 4  Borrowers from country \(L\) borrow in country \(L\) (\(\psi_L = 0\))

We obtain this result because banks choose to borrow in the country where the interest rates are lowest. For borrowers in the liquid country (country \(L\)), borrowing in country \(H\) is more expensive than in country \(L\) for two reasons: firstly, liquidity is scarcer in \(H\) than in \(L\), and secondly, \(L\)-borrowers have to pay a premium reflecting the worse quality of information. Therefore, for those banks who can choose where to borrow (\{\bar{s}, \bar{s}\}-banks), borrowing at home is always cheaper. On the other hand, those banks with the signal pair \{\bar{s}, \bar{s}\} would always like to borrow abroad, since they cannot borrow in their home country because of assumption 1. However, because they would be the only type of banks doing so, the foreign lenders infer that \(s_D = \bar{s}\), and by Lemma 3 and assumption 1 find that lending to these banks is not profitable. Hence, \(\psi_L = 0\), and only borrowers from country \(H\) borrow abroad.

\(^4\)A bank might be able to infer the signal received in the other country from the borrowers’ strategic behavior.
3.4. TWO COUNTRIES

From now on, denote $\psi \equiv \psi_H$. Interest rates are denoted $r_i$, $i \in \{H, L\}$, for loans within country $i$, and $r_f$ for loans to a foreign borrower. Last, $\Delta I_L^L$ and $\Delta I_B^H$ denote the liquidation of a country $i$ lender respective borrower.

3.4.1 The IBM in the illiquid country

In the market for liquidity in country $H$, there is measure of $1 - q_H$ lenders. The measure of borrowers depends on $\psi$: if a fraction $\psi$ of $\{\bar{s}, \bar{s}\}$-banks borrows abroad, then the only banks asking for a loan in $H$ are all $\{\bar{s}, \bar{s}\}$-banks and $(1 - \psi) \{\bar{s}, \bar{s}\}$-banks. The measure of banks from country $H$ borrowing in $H$ is then

$$f_H(\psi) = q_H (1 - \psi)(pr\{\bar{s}, \bar{s}\} + pr\{\bar{s}, \bar{s}\}). \quad (3.4)$$

Note that because of Lemma 3, the probability of solvency of banks borrowing in country $H$ does not change with $\psi$.

We now define the aggregate demand for liquidity $\Omega_H$ in country $H$ as a function of $\psi$:

$$\Omega_H(\psi) \equiv (1 - q_H)\pi_L + f_H(\psi)\pi_H - (1 - I) [1 - q_H + f_H(\psi)]$$

Clearly, a higher fraction of borrowers asking for a loan abroad, $\psi$, leads to a lower excess demand for liquidity $^W$ in country $H$. Similar to the one-country case, liquidation by both borrowers and lenders, and hence the total liquidity supply by borrowers and lenders $\Lambda_H(1 + r_H)$, is a non-decreasing function of the interest rate $1 + r_H$

$$\Lambda_H(1 + r_H) \equiv (1 - q_H)l(\Delta I_L^H)(1 + r_H) + f_H(\psi)l(\Delta I_B^H)(1 + r_H).$$

The equilibrium interest rate has to be such that the liquidity supply by market participants is just high enough to cover the liquidity shortage. It solves

$$\Lambda_H(1 + r_H) = \Omega_H(\psi). \quad (3.5)$$
Inter-bank Liquidity Channels

Figure 3.4: For high $q_H$, liquidity is scarce for all $\psi$. For lower $q_H$, there is excess liquidity for high $\psi$ and the interest rate falls abruptly.

and thus is non-decreasing in the liquidity demand $\Omega_H(\psi)$. For $\Omega_H(\psi) < 0$, there is excess liquidity, and competition among lenders drives the interest rate down to $1 + r_H = 1/p_D$. For $\Omega_H(\psi) > 0$, a higher interest rate is necessary to induce liquidation by banks. Since $\Omega_H(\psi)$ is decreasing in $\psi$, it follows that the interest rate $1 + r_H$ is non-increasing in $\psi$. It is derived in appendix B and illustrated in figure 3.4.

### 3.4.2 The IBM in the liquid country

Now let us turn to the market for loans in country $L$. The total measure of borrowers in that market is $q_L \theta + f_L(\psi)$ where

$$f_L(\psi) \equiv q_H [\psi \, pr\{s, \overline{s}\} + pr\{\overline{s}, \overline{s}\}]$$

(3.6)

denotes the measure of banks from country $H$ wanting to borrow in country $L$. Note that all borrowers from the illiquid country with $\{s, s\}$ demand a loan abroad since they are not able to obtain any liquidity in their own country. From (3.6), it is immediate that the proportion of $\{\overline{s}, \overline{s}\}$-borrowers changes with $\psi$. For $\psi = 0$, all foreign banks demanding a loan in $L$ are those with the signal pair $\{\overline{s}, \overline{s}\}$. The higher $\psi$, i.e. the larger the fraction of $\{\overline{s}, \overline{s}\}$-banks borrowing in country $L$, the higher is the probability of solvency of a foreign borrower. We obtain
3.4. TWO COUNTRIES

Lemma 5 The updated probability of solvency of a foreign bank is non-decreasing in $\psi$.

In particular, for $\psi = 0$, the probability of solvency for borrowers from abroad is equal to $p_D = p(solv|g)$. By assumption 1 this means that it is not profitable to give loans to these banks, since the interest rate would have to be too high. We can define $\bar{\psi}$ as the minimal $\psi$ for which the interest rate demanded can be paid. Then, lenders from country $L$ are willing to lend to borrowers from abroad only if $\psi \geq \bar{\psi}$. For $\psi < \bar{\psi}$, however, adverse selection in the market for foreign loans impedes $\{\bar{s}, \bar{s}\}$-banks to obtain a loan from abroad.

Now, let us consider only $\psi$ for which $\bar{\psi} \leq \psi \leq 1$. The aggregate liquidity shortage in country $L$ is

$$\Omega_L(\psi) = (1 - q_L)\pi_L + (q_L \theta + f_L(\psi))\pi_H - (1 - I)[1 - q_L + q_L \theta + f_L(\psi)]$$

which is an increasing function in $\psi$. The equilibrium interest rates have to satisfy

$$\Lambda_L(1 + r_L, 1 + r_f) = \Omega_L(\psi) \quad (3.7)$$

and

$$p_f(\psi)(1 + r_f) = \bar{p}_D(1 + r_L) \quad (3.8)$$

where

$$\Lambda_L(1 + r_L, 1 + r_f) \equiv (1 - q_L)l(\Delta I_L^L)(1 + r_L, 1 + r_f) + q_L \theta l(\Delta I_B^L)(1 + r_L) + f_L(\psi)l(\Delta I_B^H)(1 + r_f)$$

denotes the liquidity supply by lenders, domestic borrowers, and foreign borrowers in country $L$. The first equation ensures that liquidity supply equals the demand for liquidity. The second equation states that in equilibrium, the lenders are indifferent between lending to foreign and domestic borrowers.
Consider the rate charged to foreign borrowers, $1 + r_f$. For $\Omega_L(\psi) < 0$, i.e. an excess supply of liquidity, the interest rates are such that the lender just breaks even when lending: $p_f(\psi)(1 + r_f) = 1$. But since $p_f'(\psi) \geq 0$, in this region the interest rate $r_f$ is downward sloping in the aggregate demand for liquidity.

As soon as $\psi$ is so large that liquidity is scarce in country $L$, i.e. for $\Omega_L(\psi) \geq 0$, the equilibrium interest rate increases to induce the market participants to liquidate the needed amounts. In appendix B, the interest rate is derived formally. Depending on parameters, it is either increasing, or non-monotonous for $\Omega_L(\psi) \geq 0$.

### 3.4.3 The Equilibrium in the IIBM

We now consider the simultaneous equilibrium on both markets for liquidity. We have characterized the equilibrium interest rates on both markets depending on $\psi$, the fraction of $\{\bar{s}, \bar{s}\}$-borrowers from country $H$ borrowing
abroad. Depending on where they are offered a loan at the cheaper interest rate, these borrowers choose in which country to borrow. The relationship between the interest rates $r_H$ and $r_f$ for different values of $\psi$ is therefore crucial in establishing possible equilibria. We now characterize the $\psi$ for which a simultaneous equilibrium is reached.

Define an equilibrium in the inter-bank market as a quadruple $\{\psi, r_H, r_f, r_L\}$, specifying the fraction $\psi$ of {s, s}-banks from country $H$ borrowing abroad as well as the interest rates demanded.

**Lemma 6** The equilibrium fraction of {s, s}-borrowers demanding a loan abroad $\psi^*$ can be one of the following:

1. $\psi^* = 0$
2. $\psi^* = 1$ only if $r_H(1) \geq r_f(1)$.
3. $0 < \psi^* < 1$ only if $r_H(\psi^*) = r_f(\psi^*)$.

The first type of equilibrium corresponds to the case where markets for liquidity are separated between countries: if no bank borrows abroad ($\psi = 0$), then any bank from country $H$ asking for a loan in country $L$ would be turned down. Borrowing in the home country is trivially the best strategy, and $\psi = 0$ is an equilibrium. Similarly, in the second type of equilibrium, all borrowers prefer to borrow in country $L$. If foreign interest rate $r_f$ is lower than the domestic one, then this is indeed the best strategy, such that $\psi = 1$ is an equilibrium. Finally, the third type of equilibrium can prevail if the two interest rate functions cross for some $0 < \psi < 1$. Since the rates are equal at this point, borrowers are indifferent between borrowing in either country (such that no-one has an incentive to deviate.)

Figure 3.6 illustrates the interest rates $1 + r_H$ and $1 + r_f$ as functions of $\psi$ for different parameter constellations.
In all four cases illustrated, an equilibrium with separated inter-bank markets is possible (point $A$). In case $(i)$, the rate charged abroad is strictly higher than the one in country $H$ for all values of $\psi$. Here, borrowers always prefer to borrow in country $H$, and point $A$ is the only equilibrium.

In the remaining cases $(ii)$, $(iii)$, and $(iv)$, the curves $1+r_{H}(\psi)$ and $1+r_{f}(\psi)$ cross for some $\psi > 0$. Here, there exist equilibria of the third type with an active IIBM: at the crossing points (points $B$ and $C$), borrowers face the same interest rates in both countries, and they are indifferent as to where to demand a loan. Finally, in case $(iv)$, there is an equilibrium of type two at point $D$, where all banks prefer to borrow abroad.

Under which circumstances can an equilibrium with an IIBM be obtained? We discussed before that foreign lenders have to update beliefs about the foreign borrowers’ solvency, conditional on the demand for foreign loans. As seen in the previous subsection, the inferior information leads to a premium on the interest rate charged to foreign borrowers. This premium affects the shape of $r_{f}(\psi)$: the higher the premium, the more likely...
3.4. TWO COUNTRIES

Figure 3.7: The regions for which an integrated inter-bank market can exist

the economy is in the case of figure (i) where the only equilibrium is one of separated inter-bank markets.

On the other hand, an equilibrium with an IIBM is more likely for higher $1 + r_H(\psi)$, and thus, when the demand for liquidity in country $H$, $\Omega_H$, is high. The demand for liquidity in country $H$ can be measured by $q_H$. Taken all other parameters as given, we can define $\tilde{q}_H(\varepsilon)$ as the minimal $q_H$ for which an IIBM is attainable for each value of $\varepsilon$. $\tilde{q}_H(\varepsilon)$ has been evaluated numerically\(^5\) and is displayed in figure 3.7.

From figure 3.7, it is obvious that an equilibrium with cross-border lending is possible only if the difference in information $\varepsilon$ is small relative to the aggregate liquidity demand in country $H$. If this is indeed the case, there will be a fraction $\psi$ of $\{\bar{s}, \bar{s}\}$-banks borrowing abroad for which interest rates on both countries are equal. If, however, $\varepsilon$ was very high, then the function $r_f(\psi)$ can lie strictly above $r_H(\psi)$, and the only possible equilibrium would be to have two separate inter-bank markets, i.e. $\psi = 0$.

From figure 3.6, we see that if an equilibrium with an IIBM exist, there is a multiplicity of equilibria. These equilibria differ in the extent of cross-border transfers as well as in the welfare generated for the economy.

\(^5\)I used the following parameter values: $p = 0.7$, $R = 2$, $q_L = 0.4$, $\pi_L = 0.2$, $\pi_H = 0.8$, $\alpha_D = 1$, $I = 0.4$. 

In all four cases illustrated, an equilibrium with separated inter-bank markets is possible (point A). In case (i), the rate charged abroad is strictly higher than the one in country $H$ for all values of $\psi$. Here, borrowers always prefer to borrow in country $H$, and point A is the only equilibrium.

In the remaining cases (ii), (iii), and (iv), the curves $1 + r_H(\psi)$ and $1 + r_f(\psi)$ cross for some $\psi > 0$. Here, there exist equilibria of the third type with an active IIBM: at the crossing points (points B and C), borrowers face the same interest rates in both countries, and they are indifferent as to where to demand a loan. Finally, in case (iv), there is an equilibrium of type two at point D, where all banks prefer to borrow abroad.

Under which circumstances can an equilibrium with an IIBM be obtained? We discussed before that foreign lenders have to update beliefs about the foreign borrowers’ solvency, conditional on the demand for foreign loans. As seen in the previous subsection, the inferior information leads to a premium on the interest rate charged to foreign borrowers. This premium affects the shape of $r_f(\psi)$: the higher the premium, the more likely
3.4. TWO COUNTRIES

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all other parameters as given, we can define $\hat{q}_H(\varepsilon)$ as the minimal $q_H$
for which an IIBM is attainable for each value of $\varepsilon$. $\hat{q}_H(\varepsilon)$ has been evaluated
numerically\(^5\) and is displayed in figure 3.7.

From figure 3.7, it is obvious that an equilibrium with cross-border lending
is possible only if the difference in information $\varepsilon$ is small relative to the
aggregate liquidity demand in country $H$. If this is indeed the case, there
will be a fraction $\psi$ of $\{\bar{s}, \bar{s}\}$-banks borrowing abroad for which interest rates
on both countries are equal. If, however, $\varepsilon$ was very high, then the function
$r_f(\psi)$ can lie strictly above $r_H(\psi)$, and the only possible equilibrium would
be to have two separate inter-bank markets, i.e. $\psi = 0$.

From figure 3.6, we see that if an equilibrium with an IIBM exist, there
is a multiplicity of equilibria. These equilibria differ in the extent of cross-
border transfers as well as in the welfare generated for the economy.

\(^5\)I used the following parameter values: $p = 0.7$, $R = 2$, $q_L = 0.4$, $\pi_L = 0.2$, $\pi_H = 0.8$, $\alpha_D = 1$, $I = 0.4$. 
Proposition 1 Consider three equilibria $A$, $B$, and $C$ where $\psi_A = 0$ and $0 < \psi_B < \psi_C < 1$. Then

1. aggregate welfare is higher in equilibrium $C$ than in equilibrium $B$,
2. aggregate welfare is higher in equilibrium $C$ than in equilibrium $A$ if $q_H$ is small or $\varepsilon$ is small.

Since the interest rate in country $H$, $1+r_H$, is decreasing in $\psi$, the interest rates at the three equilibria satisfy $r_H(\psi_C) < r_H(\psi_B) < r_H(\psi_A)$. Part 1 of the proposition follows from the fact that lower interest rates imply a lower level of liquidation. Because the liquidation technology is concave, it is better to have lower interest rates with an intermediate level of liquidation in both countries, than high interest rates with a lot of liquidation in country $H$, but little in country $L$.

The comparison between equilibria $A$ and $C$ is not as straightforward. Here, we have to take into account that there are some banks who obtain a loan in equilibrium $C$ but not in equilibrium $A$. These are banks with the signal pair $\{s, \bar{s}\}$. According to our assumptions it is inefficient not to liquidate these banks. Since their level of liquidation is higher in equilibrium $A$ than in $C$, equilibrium $A$ is more efficient in this sense. However, since $r_H(\psi_C) < r_H(\psi_A)$, the argument from part 1 of the proposition also applies here. The relative size of the two effects is crucial for which equilibrium leads to a higher aggregate welfare.\(^6\)

The analysis shows that a high level of cross-border information is essential for an integrated inter-bank market to exist. However, even when the difference in information across borders is sufficiently low, there is no guarantee that private market forces reach the most efficient equilibrium.

Furthermore, in all possible equilibria, an inefficiency remains that is due to the informational asymmetry between countries. Because of the concavity

\(^6\)A similar argument applies to a welfare comparison with an equilibrium at $\psi = 1$ (point $D$ in figure 3.6). The proof is left out here because of its similarity to the proof of Proposition 1.
3.5. EXTENSIONS

of the liquidation technology, the most efficient outcome would involve all banks with the domestic good signal to liquidate the same amount. However, since borrowers from country $H$ pay a premium that reflect the asymmetry in information, they liquidate more than banks in country $L$ in either one of the equilibria.

3.5 Extensions

3.5.1 Correspondent Banking

Consider the case (iv) of figure 3.6. Here, there is an equilibrium where all banks of country $H$ who are able to borrow abroad do so, i.e. $\psi^* = 1$ (point $D$). The interest rate they have to pay on the inter-bank market in country $L$, $r_f$, is lower than the one that banks borrowing in country $H$ have to pay.

This opens possibilities for arbitrage: those able to borrow abroad could borrow in $L$, and then use the liquidity obtained to lend it to those banks in their home country who are not able to borrow abroad. Then, banks who are restricted to borrow on the domestic market are able to obtain liquidity from abroad via other domestic banks. To explore this line of thought, we relax assumption 2 and allow banks to borrow and lend at the same time.

Consider a bank from the illiquid country with the signal pair $\{s, \overline{s}\}$, where $s$ can be either one of the signals. Suppose that independently of its own loan demand or supply, this bank borrows some amount $z$ in the country with excess liquidity, in order to re-lend it to banks in its domestic country. We call this bank a correspondent bank (or CB). Like all banks borrowing abroad, she pays an interest rate $r_f$ on the foreign market for liquidity. After obtaining the loan, she lends $z$ to those borrowers from country $H$ who could not borrow abroad but about whom the CB has obtained the good signal. These are banks with the signal pair $\{\overline{s}, \overline{s}\}$. Denote the rate at which the CB gives the loan to these banks by $r_{CB}$, and the measure of banks that engage in correspondent banking by $\varphi$. 
First note that with \( z > 0 \), the demand or supply on both markets for liquidity changes: the interest rates \( r_f \) and \( r_L \) charged by lenders in country \( L \) increase as a result of a higher demand for liquidity. On the other hand, the interest rate in country \( H \) will adjust downwards, because of the higher inflow of foreign funds into the country. Denote the resulting interest rates by \( r_H(z) \), \( r_f(z) \), and \( r_{CB}(z) \). Here, \( Z \equiv \varphi z \) is the total amount of liquidity channeled by correspondent banking.\(^7\)

For correspondent banking to be possible, two conditions need to be fulfilled: Firstly, the interest rate charged by the \( CB \) to borrowers cannot be higher than the one that these banks face on the domestic market for liquidity, otherwise, they would not be willing to borrow from the \( CB \). Thus, we need

\[
r_{CB}(z) \leq r_H(z).
\]

Secondly, correspondent banking needs to be profitable. For this, the cost of borrowing in country \( L \), \( 1 + r_f(z) \), cannot exceed the average return from lending to borrowers in country \( H \), \( \bar{p}_D(1 + r_{CB}(z)) \). Taking both conditions together and using the fact that \( r_f'(z) \geq 0 \) and \( r_H'(z) \leq 0 \), we find that correspondent banking is possible only if at \( \psi = 1 \),

\[
1 + r_f(Z = 0) \leq \bar{p}_D(1 + r_H(Z = 0)).
\]

In other words, the interest rate differential between \( r_f \) and \( r_H \) at \( \psi = 1 \) needs to be sufficiently high.

The optimal amount transferred by correspondent banking, \( Z^* \), is such that there are no more gains from correspondent banking. This occurs when \( Z \) is so high that \( CBs \) are indifferent between engaging in correspondent banking or not, and borrowers in country \( H \) are indifferent between borrowing from both sources, i.e. for

\[
1 + r_f(Z^*) = \bar{p}_D(1 + r_{CB}(Z^*)) = \bar{p}_D(1 + r_H(Z^*)).
\]

\(^7\)We leave out any indexation for \( \psi \), since we are regarding the case of \( \psi = 1 \).
From this discussion it is clear that correspondent banking has a positive effect on welfare, since it helps channeling liquidity to where it is most needed: it is efficient that banks with the signals \( \{\bar{s}, \underline{s}\} \) liquidate as little as possible. This fact together with a concave liquidation technology implies that having correspondent banking in an equilibrium at \( \psi = 1 \) is welfare-improving.\(^8\) However, the inefficiency due to the asymmetry of information across countries is not completely removed since there continues to remain an interest rate differential.

### 3.5.2 International Banks

If there are banks who are equally much active in the financial markets of both countries, they could also engage in correspondent banking. An international bank (IB) would have the advantage of being able to operate as a domestic bank in both markets. Firstly, it would receive the domestic signal about banks in both countries. Secondly, banks in both countries would receive the domestic signal about the IB. Consequently, the IB is able to obtain a loan at domestic rates in both countries.

Similar to section 3.5.1, the IB can borrow in one market and lend to the other, using its informational advantage over other market participants. In the setup of the last subsection, a bank was able to engage in correspondent banking only in an equilibrium at \( \psi = 1 \). An IB, on the other hand, might be able to do so in any equilibrium.

Take for instance an equilibrium at which \( r_H(\psi) = r_f(\psi) \) at \( 0 < \psi < 1 \). The IB can obtain liquidity in the country with excess liquidity at interest rate \( r_L \). It can re-lend the money obtained to borrowers in country \( H \) with the signals \( \{\bar{s}, \underline{s}\} \) or \( \{\bar{s}, \bar{s}\} \). Similar to the discussion in section 3.5.1, these banks are willing to borrow from the IB at rate \( r_{IB} \) only if \( r_{IB} \leq r_H \), while the international bank would lend to them only if \( 1 + r_L \leq \bar{p}_D(1 + r_{IB}) \). In sum, a necessary condition for international banks to transfer liquidity to

\(^8\)The proof is analogous to the one of proposition 1.
these banks is

\[ 1 + r_L(\psi) \leq \bar{p}_D(1 + r_H(\psi)). \]

The difference to correspondent banking by a local bank is that a CB borrows at the interest rate \( r_J \), while the IB borrows at the lower rate \( r_L \). Thus, for the IB, arbitrage is possible for a wider set of parameter constellations, and for different types of equilibria.

### 3.6 Conclusion

In this paper, we developed a model of inter-bank markets in an international context. We showed that having a single currency does not guarantee the emergence of integrated markets. Furthermore, we found that if a single market is possible, there is a multiplicity of equilibria, where some of these equilibria are characterized by an inefficient level of cross-country lending.

The analysis showed that the emergence of a unified inter-bank market is only possible if the differences in information across countries is not too large. Considering the case of the European Union, we conclude that creating transparency in the financial markets should be an important task for authorities.

However, as long as there are differences in information, the efficient outcome is not guaranteed because of multiple equilibria. In particular, even for increasingly better information, the economy might stay at the inefficient equilibrium with no integrated markets. Therefore, the markets do not guarantee that the most efficient allocation is reached.

The analysis also has important implications for the conduct of monetary policy, since the aggregate liquidity shortage in the area of the single currency depends crucially on the way the interbank market is developed.

Finally, we showed that having banks that operate on an international basis can play an important role in channeling liquidity across countries.
Bibliography


Appendix A

Proof of Lemmata and Propositions

A.1 Lemma 1

The lender's problem at time 1 is\(^1\)

\[
\max_{\Delta I} E\Pi_L = p\{R(I - \Delta I_L) + \bar{p}_D(1 + r)L_S + \rho - (1 - \pi_L)\}
\]

s.t. \(\rho = 1 - I + l(\Delta I_L) - \pi_L - L_s.\)

Maximization yields

\[
\begin{align*}
\frac{\partial E\Pi_L}{\partial \Delta I_L} &= -pR + \lambda'(\Delta I_L) \\
\frac{\partial E\Pi_L}{\partial L_s} &= \bar{p}_D(1 + r) - \lambda \\
\frac{\partial E\Pi_L}{\partial \rho} &= p - \lambda
\end{align*}
\]

where \(\lambda\) is the Lagrange multiplier associated with the constraint.

Clearly, lending is more profitable than storing iff \(\bar{p}_D(1 + r) > 1\). Furthermore, since \(l'(\Delta I) \leq 1\), liquidating and lending is profitable only if

---

\(^1\)We assume that both lenders and borrowers obtain a positive profit at time 2 if and only if their risky technology was successful.
\( \bar{p}_D(1 + r) \geq R \). The optimal amount of liquidation is given by

\[
\left\{ \begin{array}{ll}
\frac{R}{\bar{p}_D(L)} = \bar{p}_D(1 + r) & \text{for } \bar{p}_D(1 + r) \geq R \\
\Delta L = 0 & \text{for } \bar{p}_D(1 + r) < R
\end{array} \right.
\tag{A.1}
\]

On the other hand, if \( \bar{p}_D(1 + r) < 1 \), then storing is more profitable than lending. Since \( l'(\Delta L) \leq 1 \), the marginal cost of liquidation \( \frac{R}{\bar{p}_D(L)} \) is then always strictly higher than the return from storing, such that the optimal amount of liquidation is zero.

A borrower’s problem is to choose liquidation \( \Delta I_B^H \) and loan demand \( L_D \) as to maximize

\[
E \Pi_B = \bar{p}_D \{ R(I - \Delta I_B) - (1 + r)L_D - (1 - \pi_H) \}
\]

s.t. \( L_D = \pi_H - (1 - I) - l(\Delta I_B) \).

Similarly to above, we find that he chooses to liquidate \( \Delta I_B \) given by

\[
\left\{ \begin{array}{ll}
\frac{R}{\bar{p}_D(L)} = (1 + r) & \text{for } (1 + r) \geq R \\
\Delta I_B = 0 & \text{for } (1 + r) < R
\end{array} \right.
\tag{A.2}
\]

Equations (A.1) and (A.2) imply that the optimal amount of liquidation is a function of the interest rate. Using the logarithmic liquidation function, we find

\[
l(\Delta I_L)(1 + r) = \max \left\{ 0, \ln \left( \frac{\bar{p}_D(1 + r)}{R} \right) \right\}
\tag{A.3}
\]

\[
l(\Delta I_B)(1 + r) = \max \left\{ 0, \ln \left( \frac{1 + r}{R} \right) \right\}
\]

The total supply of liquidity by lenders and borrowers at time 1 is

\[
\Lambda(1 + r) = (1 - q)l(\Delta I_L)(1 + r) + q\theta l(\Delta I_B)(1 + r).
\]

Replacing \( l(\Delta I_L) \) and \( l(\Delta I_B) \) from (A.3), we can write \( \Lambda \) as

\[
\Lambda(1 + r) = \left\{ \begin{array}{ll}
0 & \text{for } 1 + r \leq R' \\
q\theta \ln \left( \frac{1 + r}{R} \right) & \text{for } R < 1 + r \leq \frac{R}{\bar{p}_D} \\
(1 - q) \ln \left( \frac{\bar{p}_D(1 + r)}{R} \right) + q\theta \ln \left( \frac{1 + r}{R} \right) & \text{for } \frac{R}{\bar{p}_D} < 1 + r
\end{array} \right.
\]

It is easy to see that \( \Lambda'(1 + r) \geq 0. \)
A.2 Lemma 2

Consider a bank that cannot obtain any loans on the interbank market (constrained bank). The minimal amount that the bank needs to liquidate in order to stay in operation is $\triangle I_C$ so that $l(\triangle I_C) = \pi_H - (1 - I)$. Suppose now that it liquidates $\triangle I_C + \delta$ with $0 \leq \delta \leq I - \Delta I_C$, storing the additional proceeds from liquidation $\bar{l}(\delta) \equiv l(\triangle I_C + \delta) - l(\triangle I_C)$. Denoting $p_D \equiv p(solv|s_D = \bar{s})$, its profits are

$$\Pi_C = \begin{cases} R(I - \triangle I_C - \delta) + \bar{l}(\delta) - (1 - \pi_H) & \text{with probability } p_D \\ \max\{0, l(\delta) - (1 - \pi_H)\} & \text{otherwise} \end{cases}$$

Since $R\delta > \delta > \bar{l}(\delta)$, it is easy to see that the expected profits in case of success is decreasing in $\delta$. Furthermore, in case of failure, the profit is never positive because $\bar{l}(\delta) - (1 - \pi_H) < \delta - (1 - \pi_H) < I - \Delta I_C - (1 - \pi_H) = 0$. Thus, the bank would always liquidate exactly $\triangle I_C$.

A.3 Lemma 3

Suppose that the domestic bank can infer the foreign signal. It can then update its beliefs using both signals. For example, for $\{\bar{s}, \bar{s}\}$ we have

$$p(solv|\{\bar{s}, \bar{s}\}) = \frac{\text{prob}(solv \text{ and } \{\bar{s}, \bar{s}\})}{\text{prob}(\{\bar{s}, \bar{s}\})} = \frac{p \alpha_D(1 - \beta)}{p \alpha_D(1 - \beta) + (1 - p)(1 - \alpha_D)(1 - \beta)} = p(solv|s_D = \bar{s})$$

Thus, the domestic bank does not learn anything about the other bank’s type by observing the foreign signal. The reason is that the signals are dependent. The same argument applies for the other possible signal combinations. We obtain

$$p(solv|\{\bar{s}, \bar{s}\}) = p(solv|\{\bar{s}, \bar{s}\}) = p(solv|s_D = \bar{s}) \quad (A.4)$$

$$p(solv|\{\bar{s}, \bar{s}\}) = p(solv|\{\bar{s}, \bar{s}\}) = p(solv|s_D = \bar{s})$$
This implies at the same time that if the foreign bank can infer the domestic signal, it would use only the domestic signal, but not its own.

A.4 Lemma 4

We proceed in several steps:

Step 1: Suppose a bank has received \( s \) both about a domestic and a foreign borrower. Then, the probability of solvency of a foreign borrower (\( p_F \)) is lower than the one for a domestic borrower (\( p_D \)).

Consider first a domestic borrower. The lender gives loans only to banks with \( s \). Even if he could infer the foreign signal about these banks, from Lemma 3 it follows that he never uses those signals such that \( p_D = p(solv|\{s, s_i\}) = \bar{p}_D \).

Now consider the foreign borrower. A foreign borrower who has obtained the bad signal in his home country will always try to obtain a loan abroad. Thus, the population of foreign borrowers always consists of a positive fraction of banks with \( s_D = s \). Denote the fraction of foreign borrowers with \( s_D = s \) by \( \rho_1 \) and those with \( s_D = \bar{s} \) by \( \rho_2 \). The probability of solvency of all foreign borrowers is then \( p_F = \left( \bar{p}_D \rho_1 + p_D \rho_2 \right) / (\rho_1 + \rho_2) \). Since \( \rho_2 < 0 \), \( p_F \) is strictly lower than \( \bar{p}_D \) and thus \( p_D > p_F \).

Step 2: Show that in an equilibrium with an IIBM, lenders charge a higher interest rate to foreign borrowers than to domestic borrowers.

Denote \( r^*_i \) the interest rate from a country \( j \) lender to a country \( i \) borrower. We have to show that \( r^*_i > r^*_j \) for \( i \neq j \). Suppose \( r^*_i > r^*_j \) does not hold. Then, step 1 implies that the return from lending to a domestic borrower, \( \bar{p}_D (1 + r^*_j) \), is strictly higher than the one from lending to a foreign borrower, \( p_F (1 + r^*_j) \). But then, lending to the foreign borrower would not be an equilibrium strategy, and there would be no IIBM.

Step 3: Show that \( \psi_L = 0 \) is an equilibrium strategy.
A.5. Lemma 5

Suppose that $\psi_H = 0$, i.e. no borrower from country $H$ borrows in country $L$. Then, for $\psi_L = 0$, there is excess liquidity in country $L$, but a shortage in $H$. Borrowers from country $L$ therefore pay the lowest possible interest rate when they borrow at home, but not when borrowing in $H$, and thus $r^L_H > r^L_L$. Moreover, as $\psi_L$ increases, the demand for liquidity in country $H$ becomes larger, and thus the interest rate $r^L_H$ increases further, while $r^L_L$ remains constant. Then, it is always cheaper for $L$-borrowers to borrow in $L$, and hence $\psi_L = 0$.

Now suppose that $\psi_H > 0$. Because borrowers use the cheapest source of liquidity available to them, this can only be true if $r^L_H \leq r^H_H$. Furthermore, by step 2 we have $r^H_H < r^L_H$ and $r^L_L < r^H_H$. Taken together, these conditions imply $r^L_L < r^L_H$. But then, borrowing in country $L$ is always cheaper than borrowing in country $H$. Thus, $\psi_L = 0$ is an equilibrium strategy.

A.5 Lemma 5

The measure of banks with either one of the signal pairs $\{s_D, s_F\}$, is

\[
\begin{align*}
pr\{\bar{s}, \bar{s}\} &= \theta(1 - \beta) & pr\{\bar{s}, s\} &= \theta\beta \\
pr\{s, \bar{s}\} &= (1 - \theta)\beta & pr\{s, s\} &= (1 - \theta)(1 - \beta)
\end{align*}
\]

For a given $\psi$, the updated probability of solvency of a foreign borrower is then

\[
p_f(\psi) = \frac{\psi p(\text{solvency and } \{\bar{s}, \bar{s}\}) + p(\text{solvency and } \{s, \bar{s}\})}{\psi pr\{\bar{s}, \bar{s}\} + pr\{s, \bar{s}\}} \\
= \frac{\psi \alpha(1 - \beta) + p(1 - \alpha)\beta}{\psi(1 - \beta)\theta + (1 - \theta)\beta}.
\]

It is easy to see that $p'_f(\psi) > 0$.■
A.6 Proposition 1

A.6.1 part (1)

We focus here on the case that both at B and at C there is excess liquidity in country L but not in country H. Thus, we take $0 < \psi_B < \psi_C < 1$ and assume $\Omega_L(\psi) \leq 0$, and $\Omega_H(\psi) > 0$ for $\psi \in \{\psi_B, \psi_C\}$.\(^2\)

\hspace{1cm} step 1: derive aggregate welfare $W_B$ and $W_C$

Aggregate welfare in the economy is the sum of consumer welfare and expected profits, less payments made by the Deposit Insurance Company (DIC).

Lenders from country $i$, $i \in \{L, H\}$ have expected profits

$$E\Pi^i_L = p \{ R(I - \Delta I^i_L) + \bar{p}_D(1 + r_i)I^i_S + \rho_i - (1 - \pi_L) \}$$

where $\rho^i = 1 - I + l(\Delta I^i_L) - \pi_L - L_i$.\(^3\) If the risky technology fails, the bank has reserves $\bar{p}_D(1 + r_i)I^i_S$ at time 2. The DIC thus faces expected payments to this bank's consumers of $EP^i_L = (1 - p)[(1 - \pi_L) - \bar{p}_D(1 + r_i)I^i_S]$. Summing both terms, we obtain the expected welfare created by lenders from country $i$

$$EW^i_L = E\Pi^i_L - EP^i_L = pR(I - \Delta I^i_L) + \bar{p}_D(1 + r_i)I^i_S + \rho_i - (1 - \pi_L).$$

Borrowers from country $i$ with the $s_D = s$ have expected profits $E\Pi^i_B = E(p|s) \{ R(I - \Delta I^i_B) - (1 + r_i)I^i_B - (1 - \pi_H) \}$ where $E(p|s)$ denotes the probability of solvency given $s_D = s$.\(^4\) Borrowers always obtain zero profits when the risky technology fails. Therefore, the DIC has to pay the full amount $1 - \pi_H$ to consumers in this event, $EP_B = (1 - E(p|s))(1 - \pi_H)$, and thus

$$EW^i_B(s) = E\Pi^i_B(s) - EP_B(s)$$

\(^2\)The case of liquidity shortage at point C can be derived analogously.

\(^3\)Note that it is irrelevant whether L-lenders give a loan to domestic banks with an expected repayment of $\bar{p}_D(1 + r^L_f)$ or to foreign borrowers at $p_f(\psi)(1 + r_f)$ since both terms are identical in equilibrium.

\(^4\)Again it is irrelevant whether the banks borrows abroad or in the home country, since in equilibrium, $r_H = r_f$. 
A.6. **PROPOSITION 1**

\[ W = \sum_{i=H,L} \{(1 - q_i)E(W_i^L) + q_i\theta E(W_i^B(\bar{s}))\} + q_H(1 - \theta)\beta E(W_B^H(\bar{s})) + C \]

where \( C \) is a constant that includes consumer welfare as well as welfare created by constrained banks, neither of which changes with \( \psi \).

The conditions for market clearing in the markets for liquidity in both countries are given by equations (3.5) and (3.7). It can be shown easily that market clearing implies that the expected loan payments/repayments in this equation cancel out. Therefore, the aggregate amount of transfers made between banks does not affect welfare. Total welfare is then

\[ W = \sum_{i=H,L} (1 - q_i) (pR(I - \Delta I_i^L) + \rho_i - (1 - \pi_i)) \]

\[ + \sum_{i=H,L} q_i\theta \left( \bar{p}_D R(I - \Delta I_B^L) - (1 - \pi_i) \right) \]

\[ + q_H(1 - \theta)\beta \left( \bar{p}_D R(I - \Delta I_B^H) - (1 - \pi_H) \right) + C \]

Because there is excess liquidity in country \( L \), but not in country \( H \), we have \( \Delta I_L = \Delta I_B^L = 0 \), \( \rho_L = 1 - I - \pi_L - L^L \) and \( \rho_H = 0 \).

Now consider \( \psi_B < \psi_C \), with welfare \( W_B \) and \( W_C \), respectively. Denote liquidation in country \( H \) by agent \( k \) at \( \psi \), as \( \Delta I_k^L \). From equation (A.5) we find

\[ W_C - W_B = (1 - q_H)pR \left[ \Delta I_L^P - \Delta I_L^C \right] + (1 - q_L) \left[ \rho_L^C - \rho_L^P \right] \]

\[ + q_H \left( \delta \bar{p}_D + (1 - \theta)\beta \bar{p}_D \right) R \left[ \Delta I_B^P - \Delta I_B^C \right] . \]

We need to show that this difference is positive.

step 2: find an expression for \( (1 - q_L) \left( \rho_L^C - \rho_L^P \right) \)

The aggregate liquidity shortage in the two countries is

\[ \bar{\Omega} = \Omega_L(\psi) + \Omega_H(\psi) \]

\[ = \Omega_L(0) + \Omega_H(0) + q_H (\theta + (1 - \theta)\beta) [\pi_H - (1 - I)] \]
which is independent of $\psi$.

Since $\Delta I_L^L = \Delta I_B^B = 0$ but $\Delta I_B^H > 0$, the excess liquidity in country $L$ is the liquidation value minus the liquidity shortage, or $(1 - q_L)\rho_L = f_L(\psi)(\Delta I_B^H) - \Omega_L(\psi)$. In country $H$, total liquidation equals the liquidity shortage as given by equation (3.5). Replacing these expressions for $\Omega_L(\psi)$ and $\Omega_H(\psi)$ into (A.7), we obtain

$$
\bar{\Omega} = (1 - q_H)(\Delta I_L^H) + q_H(\theta + (1 - \theta)\beta)(\Delta I_B^H) - (1 - q_L)\rho_L.
$$

This has to be true for $\psi \in \{\psi^B, \psi^C\}$. Subtracting the expressions for $\psi^B$ and $\psi^C$ yields

$$
(1 - q_L)(\rho_L^B - \rho_L^C) = (1 - q_H)(l(\Delta I_B^B) - l(\Delta I_B^C)) + q_H(\theta + (1 - \theta)\beta)(l(\Delta I_B^B) - l(\Delta I_B^C)) \quad \text{(A.8)}
$$

**step 3: show that** $\theta \bar{p}_D + (1 - \theta)\beta \bar{p}_D \geq \theta p + (1 - \theta)\beta p$

The ex-ante probability of solvency has to fulfill $p = \bar{p}_D \cdot \text{prob}(\bar{s}) + \bar{p}_D \cdot \text{prob}(\bar{a}) = \bar{p}_D \theta + \bar{p}_D(1 - \theta)$. Then, $\theta \bar{p}_D + (1 - \theta)\beta \bar{p}_D = p - (1 - \theta)(1 - \beta)\bar{p}_D \geq p - (1 - \theta)(1 - \beta)p = \theta p + (1 - \theta)\beta p$. Replace this expression as well as (A.8) into equation (A.6) to obtain

$$
W_C - W_B \geq (1 - q_H)[pR(\Delta I_L^B - \Delta I_L^C) - l(\Delta I_B^B) - l(\Delta I_B^C)] + q_H(\theta + (1 - \theta)\beta)[pR(\Delta I_B^B - \Delta I_B^C) - l(\Delta I_B^B) - l(\Delta I_B^C)]
$$

The last step is to show that both terms are positive.

**step 4: show that** $pR(\Delta I_k^B - \Delta I_k^C) - [l(\Delta I_k^B) - l(\Delta I_k^C)]$ for $k \in \{L, B\}$

Follows from $pR > 1$, since

$$
l(\Delta I^B) - l(\Delta I^C) = \int_{\Delta I^C}^{\Delta I^B} l'(\Delta I)d\Delta I < \int_{\Delta I^C}^{\Delta I^B} pRd\Delta I = pR(\Delta I^B - \Delta I^C).
$$

Hence, $W_C > W_B. \blacksquare$
A.6.2 part (2)

For equilibrium \( A \), we can derive aggregate welfare in the same way as done for part (1).

\[
W_A = \sum_{i=H,L} (1 - q_i) \left( \rho R(I - \Delta I^i_L) + \rho_i - (1 - \pi_L) \right) + \sum_{i=H,L} q_i \theta (\bar{p}_D R(I - \Delta I^i_B) - (1 - \pi_H)) + q_H(1 - \theta)\beta \left( \bar{p}_D R(I - \Delta I^H_C) - (1 - \pi_H) \right) + C
\]

Here, \( \Delta I^i_C \) refers to the liquidation if the bank is constrained, i.e. not able to borrow.

The only difference between (A.5) and (A.9) is the term related to \( \{s, s\} \)-borrowers from country \( H \): while in equilibrium \( C \), they were able to obtain a loan in the IIBM, now they are forced to obtain cover their entire liquidity needs by liquidation. Thus, they are constrained banks with a liquidation of \( \Delta I^H_C < \Delta I^H_B \). Furthermore, they do not enter the market for liquidity in either country, and hence \( \Omega_C = \Omega_A + q_H(1 - \theta)\beta(\Delta I^A_C) \).

Following the same steps as in part 1, we arrive at

\[
W_C - W_A \geq (1 - q_H) \left[ pR (\Delta I^A_L - \Delta I^C_L) - (l(\Delta I^A_L) - l(\Delta I^C_L)) \right] + q_H \theta \left[ \bar{p}R (\Delta I^A_B - \Delta I^C_B) - (l(\Delta I^A_B) - l(\Delta I^C_B)) \right] + q_H(1 - \theta)\beta \left[ pR (\Delta I^A_C - \Delta I^C_B) - (l(\Delta I^A_C) - l(\Delta I^C_B)) \right]
\]

We have already shown that the first two terms are positive. However, since

\[
l(\Delta I^A_C) - l(\Delta I^C_B) = \int_{I^B_C}^{I^A_C} l'(\Delta I)d\Delta I > \int_{I^B_C}^{I^A_C} \bar{p} Rd\Delta I = pR (\Delta I^A_C - \Delta I^C_B),
\]

the last one is negative. In the proof to part 1, we were able to combine the last two terms and show that the sum was positive, because liquidation was the same. Here, however, the higher level of liquidation \( \Delta I^A_C < \Delta I^B_C \) implies a positive effect on welfare in equilibrium \( A \). The size of the third term
relative to the first two is thus important in establishing which equilibrium leads to a higher aggregate welfare. For $q_H$ small or $\beta$ small, the third term is small such that welfare would be higher in equilibrium $C$.■
Appendix B

The interest rate

One country In the text it was already discussed that for $\Omega < 0$, $1 + r = 1/\bar{p}_D$. Now suppose that $\Omega \geq 0$. We see that for any liquidation to occur, interest rates have to be at least as high as $R$. Thus, at $\Omega = 0$, the interest rates must jump from $1/\bar{p}_D$ up to $R$, because positive amounts of liquidation are needed to cover a liquidity shortage in the economy. Suppose $R < 1 + r < \bar{p}_D R$. At this interest rate, only borrowers wish to liquidated positive amounts $l(\Delta I_B) = \ln \left( \frac{1+r}{R} \right)$, but not lenders: $l(\Delta I_L) = 0$. Replacing these into equation (3.3), we can solve for the interest rate that clears the market $1 + r = \exp \left\{ \frac{\Omega}{\theta} \right\} R$. Similarly, for $1 + r > \bar{p}_D R$, also lenders choose positive liquidation. It is easy to see that the market clearing interest rate then is $1 + r = \exp \left\{ \frac{\Omega - (1-q) \ln(p)}{(1-q) + \theta} \right\} R$.

Denote $\hat{\Omega}$ the liquidity shortage for which lenders would start to liquidate, i.e. for which $1+r = \bar{p}_D R$. $\hat{\Omega}$ is given by $\hat{\Omega} = q_H \theta \ln \left( \frac{1+r}{R} \right) = q_H \theta \ln \left( \frac{\bar{p}_D R}{R} \right) = q_H \theta \ln (\bar{p}_D)$.

We can summarize

$$1 + r = \begin{cases} \frac{1}{\bar{p}_D} & \text{for } 0 \leq \Omega \\ \exp \left\{ \frac{\Omega}{\theta} \right\} R & \text{for } 0 < \Omega \leq \hat{\Omega} \\ \exp \left\{ \frac{\Omega - (1-q) \ln(p)}{(1-q) + \theta} \right\} R & \text{for } \hat{\Omega} < \Omega. \end{cases}$$

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Two countries. To establish the interest rate in the illiquid country in the two-country case, essentially the same argument is used, only with $\Omega = \Omega_H(\psi)$ and a fraction of borrowers $f_H(\psi)$ instead of $q\theta$. We obtain:

\[
1 + r_H = \begin{cases} 
\frac{1}{\bar{p}_D} & \text{for } \Omega_H(\psi) < 0 \\
\exp \left\{ \frac{\Omega_H(\psi)}{f_H(\psi)} \right\} R & \text{for } 0 \leq \Omega_H(\psi) < \Omega(\psi) \\
\exp \left\{ \frac{\Omega_H(\psi) - (1-q_H) \ln(p)}{1-q_H + f_H(\psi)} \right\} R & \text{for } \Omega(\psi) \leq \Omega_H(\psi)
\end{cases} \tag{B.1}
\]

where $\Omega(\psi) \equiv f_H(\psi) \ln(1/\bar{p}_D)$ denotes the point from which on lenders decide to liquidate.

Similarly, the interest rate for foreign loans in country $L$ can be derived. However, there are two differences to the previous cases. First, for a liquidity shortage $\Omega_L(\psi) < 0$, the foreign interest has to satisfy $1 + r_f = \frac{1}{p_f(\psi)}$. From Lemma 5, the updated probability of solvency is increasing in $\psi$ and also in $\Omega_L(\psi)$. Therefore, $1 + r_f$ is decreasing for $\Omega_L(\psi) < 0$.

Secondly, there are two types of borrowers in country $L$, both facing different interest rates. Borrowing is more expensive for borrowers from country $H$, and this fact induces them to liquidate more than borrowers from country $L$. In the same way as above, we find $\Omega_1(\psi) \equiv f_L(\psi) \ln(1/p_D)$ and $\Omega_2(\psi) = q_L \theta \ln(1/p_D) + f_L(\psi) \ln(1/p_f(\psi))$ as the points from which on $L$-borrowers and $L$-lenders start to liquidate, respectively. The rate charged by lenders in country $L$ to borrowers from country $H$ is

\[
1 + r_f = \begin{cases} 
\frac{1}{p_f(\psi)} & \text{for } \Omega_L(\psi) < 0 \\
\exp \left\{ \frac{\Omega_L(\psi)}{f_L(\psi)} \right\} R & \text{for } 0 \leq \Omega_L(\psi) < \Omega_1(\psi) \\
\exp \left\{ \frac{\Omega_L(\psi) - q_L \theta \ln(p_f(\psi))}{q_L \theta + f_L(\psi)} \right\} R & \text{for } \Omega_1(\psi) \leq \Omega_L(\psi) < \Omega_2(\psi) \\
\exp \left\{ \frac{\Omega_L(\psi) - (1-q_L) \ln(p_f(\psi)) - q_L \theta \ln(p_f(\psi))}{1-q_L + q_L \theta + f_L(\psi)} \right\} R & \text{for } \Omega_2(\psi) \leq \Omega_L(\psi).
\end{cases} \tag{B.2}
\]

The proof is done by taking partial derivatives of $1 + r_f$ with respect to $\psi$. 
Note also that $H$-borrowers can also liquidate when there is excess liquidity in country $L$. That is because for $\psi$ small, the foreign interest rate can be very high such that liquidation is cheaper than demanding a loan. Finally, for $\varepsilon = 0$, the interest rate is monotonously non-decreasing, because $\varepsilon = 0$ implies that $p_f(\psi) = \bar{p}_D$. 
Chapter 4

Optimal Shareholder Structure when Private Benefits from Control are Divisible

4.1 Introduction

A standard characterization of corporate ownership presumes that ownership is dispersed among many small shareholders, and that corporate control is performed via the market for takeovers. However, this description is not accurate for many economies. Germany, France, and Japan are examples of economies that are sometimes dubbed 'bank-based'. In these economies, shares are typically not widely held as in 'market-based' economies like the US or the UK. The ownership structures are instead very concentrated, and large shareholdings are common. For example, Iber [13] reports that in Germany, in more than 85% of the 300 largest publicly traded companies, one single shareholder holds more than 25% of the shares. In the US, on the other hand, in only 20% of large listed companies do the largest blocks exceed 10% of total equity\(^1\). Furthermore, in Germany only a small fraction of large companies are listed on a stock exchange. Instead, most firms are pri-

\(^1\)see Holderness and Sheehan [12].
Another distinctive feature is the role played by banks as equity holders. While in the US, bank equity ownership in nonfinancial institutions is prohibited, in Germany, banks are commonly among the shareholders with the largest blocks\(^2\). To my knowledge, no theoretical analysis exists on the determinants of the bank’s blocksize and the evolution of shareholder structures in bank-based economies. This paper contributes to filling this gap.

In the literature on ownership structures in US-type economies, the emphasis lies first of all on problems that arise from the separation of ownership and control. The standard argument is that dispersed shareholders do not have the appropriate incentives to monitor the company’s management as the possible gain from improved performance will be small for each of them. Shleifer and Vishny [23] show how the presence of large block-holders can increase the value of the firm by limiting the agency problems. In recent studies, the role of large block-holders in monitoring and their incentives to acquire blocks has received new attention. Kahn and Winton [14] determine an investor’s optimal blocksize as a function of firm’s characteristics, and Maug [17] studies the interdependence of the market’s liquidity and the optimal blocksize.

These models are inappropriate to study the German case for several reasons. First, the issue of separation of ownership of control is not so relevant in bank-based economies because of the presence of large shareholders. Second, the models do not take into account the role of banks in corporate governance. Third, in most existing models the market for shares is assumed to be liquid enough such that investors are able to acquire the desired stake-size from liquidity traders. However, in Germany, this might prove difficult when shares are not widely held, but instead in the hands of few shareholders with significant blocksizes. The acquisition of large blocks is therefore likely to be determined by other factors. Indeed, a study by the German

\(^2\)see Allen and Gale [2].

\(^3\)Kester [15] reports that of the top 100 corporations, banks controlled the votes of nearly 40% of equity (including proxy votes).
antitrust authority (Monopolkommission [18]) reports that blocks are often sold within special transactions when family owners sell out. It seems thus that block sizes and combinations are usually chosen by the sellers rather than investors.

In this paper, we develop a theory about the determinants of ownership structure. We base the model on two crucial assumptions. The first is that banks are allowed to hold equity, and moreover, have special abilities in improving firm performance. Secondly, we assume that owners of large blocks derive utility from being in control. With these ingredients, we determine the optimal selling strategy of the initial owner of a company.

The role of banks in corporate control is a widely discussed topic. In the literature on the comparison of financial systems, it is often argued that the influence of banks on corporate decision making might somehow be a substitute for the takeover mechanism that is common in market-based systems. Not all authors agree on this notion. Edwards and Fischer [8], for example, argue that the role of banks has been overstated in the literature, and that other factors are more likely to determine the control process. However, several empirical studies support the view that banks have a role in corporate governance. An analysis by Cable [6] finds that when banks are represented on supervisory boards, they monitor better than other board members. Also, Gorton and Schmidt [10] provide evidence that bank shareholders have a positive impact on firm performance.

Based on this evidence, we assume that banks have superior monitoring abilities, but we do not attempt to explain why they do. One argument is that universal banks usually have multiple relationships with a company, because the firm has accounts with the bank or the bank also is a lender. As a result, they can both have better information about the true state of

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5Monitoring here stands for a variety of activities that can increase firm value, such as replacing management, reorganizing internal organizations, or changing the financial structure.
the firm and about its activities, but they might also have more power to constrain management. Furthermore, as argued by Porter [21], banks might be less myopic in their investment decisions but instead seek to maximize long term firm value.

The second basic assumption underlying my model concerns benefits from having voting power. Voting rights are valuable to investors and should be reflected in the share price. Indeed, empirical studies show that blocks of shares usually trade at a premium, reflecting value from being in control. For shares listed at the AMEX stock exchange, Barclay and Holderness [3] found an average premium on blocks of around 20%. Premia are found to be non-decreasing in the percentage of the firm’s assets acquired with the block. However, they do not increase proportionally with the size of the block. Zingales [25], in a study of the Milan stock exchange, finds even greater evidence: private benefits of control are estimated to be up to 60% of the value of nonvoting equity. For Germany, Boehmer [5] finds that block sizes very often coincide with control thresholds (i.e. 25%, 50% etc.).

Benefits of control can arise from multiple sources: for individual investors, they can represent the consumption of perquisites from control or the ability to divert funds for own purposes. For corporate investors, on the other hand, synergies in production or transfer of goods at below market prices can play a role.

In most previous studies which take into account benefits from control, benefits are consumed either by the founder of the company, or by the majority shareholder (see, Pagano and Röell [20], or Bebchuk [4]). Conflicts of interest then arise between the controlling party and the minority shareholders, who care about cash flow only. However, the empirical evidence shows that already small blocks carry a premium, suggesting that benefits are consumed not only by the largest shareholder, but instead divided between block-owners according to their voting power. In this model, we

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6 In the sample of Barclay and Holderness [3], five-percent-blocks already trade at a premium.
assume that the share of total benefits that an individual can secure for himself depends on his strategic importance within the firm. Following a study by Zwiebel [27], we use the Shapley value to represent the division of benefits among shareholders.

We assume that investors buy blocks of shares only for two reasons: either, they are agents with monitoring powers (banks), who invest in order to create a higher level of cash flow. Furthermore, there are investors who buy blocks in order to obtain benefits of control. For simplicity, we assume that the former ones do not care about benefits, and the latter ones are unable to monitor. When the initial firm owner decides on the ownership structure of the firm, he then has to take into account that selling shares will change the division of benefits among shareholders, and that a bank investor will exert monitoring effort according to its incentives.

Given that banks are the only agents that can improve firm performance, it would seem obvious that a firm owner would sell as much as possible to them. However, in practice we observe that companies are owned by several large block-holders, among those often a bank, but also other firms. Why would a firm owner decide to sell to several agents? One reason could be that agents prefer to hold smaller stakes in companies because they are risk-averse. However, this argument is not consistent with the fact that blocks trade at a premium which is increasing in the size. If risk-aversion was very important, the control premium should be decreasing with the size of the block (contrary to the evidence found), and owners would prefer to sell the firm publicly to small shareholders. Furthermore, banks are unlikely to hold a smaller fraction of shares than optimal because of cash-constraints.\footnote{According to a study by the Deutsche Bundesbank [7], banks often acquired holdings via special transactions, when the acquired firm had liquidity difficulties. Portfolio considerations were almost never the deciding factor.}

In this model, we offer an alternative argument for the dispersion of shares among several block-owners. It is argued that a firm owner can choose the degree of dispersion in order to signal the quality of his firm to outside
investors. The basic argument is that a higher dispersion of shares among non-monitoring investors lowers their joint strategic position. Dispersion thus amounts to giving up revenue of selling benefits of control. The higher the value of benefits relative to the underlying cash flow value, the more costly will this strategy be. Thus, owners of firms with a high cash flow can signal it by choosing dispersion. If the owner convinces the investors that his firm generates a high future cash-flow, he can obtain both a higher price on the shares sold and more monitoring by the bank.

The model explains why small blocks carry a premium reflecting control benefits, but blocks sold to banks trade at a discount. Also, the model reflects evidence that banks often are the second largest shareholder after the family owners. The model predicts that firms with a higher performance will choose a higher degree of dispersion, and that this will be the case especially when differences in firms are large.

The literature on initial public offerings is somewhat related to our model, because both types of models investigate the initial owner's decision on the dispersion of shares. The major difference is that we are not so much interested in public offerings, but just as much in private purchases, which are much more common in Germany than in the US. Moreover, as reported by Goergen [9], ownership does not become dispersed even when firms go public, such that the effects involved are likely to be different. In a related study by Zingales [26], costs and benefits of going public are analyzed. Going public maximizes proceeds from cash-flow rights, while selling shares privately generates higher revenues from selling control rights. Pagano and Röell [20] identify a trade-off of facing too much monitoring when privately selling to a large investors, or having to bear large costs of going public. Finally, Allen and Faulhaber [1] as well as Grindblatt and Huang [11] study IPO sig-

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8 Barclay and Holderness [3] report that when the firm was in severe financial difficulties, blocks often sold at a discount, reflecting the cost of needed monitoring or the increased threat of litigation.
9 see Boehmer [5]
nailing models where issuers signal their type both by retaining shares and underpricing.

In section 2, the model is described. Section 3 sets up the basic maximization problem of the firm's owner. Then the benchmark case of symmetric information about the firm's type is analyzed. Section 4 studies the selling strategy of the firm's owner under asymmetric information. We will consider two signalling strategies: the first is to signal by choosing a higher dispersion of shares among investors, and the second is to retain shares. In section 5, the outcomes in the possible equilibria are compared. Finally, section 6 discusses extensions and concludes.

4.2. The model

Assume that a firm initially belongs to one single owner. The company is in a crisis, and without any intervention, would yield an expected cash flow of zero. The owner has the possibility to sell parts of his firm to different investors by issuing shares. His strategy \( S \) consists of choosing the combination of shareholdings. There are two types of potential shareholders: banks, who are assumed to have special monitoring powers, and non-bank investors. We restrict the analysis here such that the owner can sell to maximally one of each type of investors, i.e. he sells shares to maximally two investors.\(^\text{10}\)

Assume that all agents are risk-neutral and derive utility from two sources: First, they profit from the cash flow rights of shares, proportionally to the fraction of the company they own. Second, block-holders can derive utility from receiving private benefits of control. Banks differ from other shareholders in two respects: not only do they have superior abilities to monitor the firm, but they furthermore care less about private benefits from control than other agents do. For simplicity we assume that they do not care at all about

\(^{10}\text{Indeed, Gorton and Schmidt report that in only 7\% of all large limited public companies considered in their sample did more than one bank hold blocks.}\)
the benefits and that they are the only ones who can effectively monitor and improve firm performance.

Firms can be either of 'good' type with a high potential firm value but low private benefits, or of 'bad' type with low expected cash flow but high benefits. The type is private information to the owner. Denote the future cash flow of a type $i$ firm $\beta_i V$, with $V \in \{0, V\}$ and the benefit $\alpha_i Q$, where $i \in \{G, B\}$, $\alpha_G < \alpha_B$, and $\beta_G > \beta_B$. For computational simplicity, we choose $\alpha_G = \alpha$, $\alpha_B = 1$ as well as $\beta_G = 1$ and $\beta_B = \beta$. The positive cash flow $\beta_i V$ of a firm can be realized only if the firm is monitored. Without loss of generality we assume that if the firm is not monitored, its cash flow will remain zero.

A bank can exert monitoring effort $e \in [0, 1]$. Effort influences the probability that the firm generates a positive cash flow in the next period.$^{11}$ For simplicity, we take

$$\tilde{V} = \begin{cases} V & \text{with probability } e \\ 0 & \text{with probability } 1 - e \end{cases}$$

ExERTing effort generates some costs $c(e)$, where $c$ is increasing and convex in effort, and $c(0) = 0$. Furthermore, we assume that $c''(e) \leq 1$.

The private benefit of control $\alpha_i Q$ is divisible and allocated among shareholders according to their strategic position in the company. The allocation is represented by the vector of Shapley Values $\Phi(S)$.\(^{12}\) Shareholder $k$ then receives benefits $\Phi_k(S)\alpha_i Z$. To distinguish small shareholders from blockholders, we assume that it is costly to be able to enjoy these benefits from control (e.g. there is a cost to attend shareholder meetings etc.), such that a minimal amount of shares $\delta$ need to be held in order to be able to enjoy benefits of control. We assume that all decisions are made by majority rule.

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$^{11}$For simplification we assume that the impact of effort on cash flow is not influenced by the bank's strategic position in the firm.

$^{12}$The Shapley value for player $i$ in an $n$-player game with the characteristic function $v$ is given by $\Phi_i = \frac{1}{n!} \sum_{T \subseteq N \setminus i} t!(n - t - 1)! (v(T \cup \{i\}) - v(T))$ where $t$ is the number of agents in coalition $T$ and $N$ the set of all $n$ players (see Myerson [19]).
4.3 SOLVING THE MODEL

A selling strategy of the owner of a type-i firm is described by $S_i = \left( \frac{n_b}{n}, \frac{n_k}{n} \right)$ where $n$ is the total number of shares, $n_b$ is the number of shares sold to the bank, and $n_k$ is the number of shares sold to the non-bank investor. Without loss of generality, we take the total number of shares, $n$, to be odd.

We assume that the buyers are competing to buy shares and that the agents’ alternative investment opportunities yield zero return.

The timing is the following: first, the owner chooses his strategy $S_i$, and offers the blocks he wants to sell at prices $P_b$ to the bank, and $P_k$ to the non-monitoring investor. Then, the bank chooses how much effort to exert. Finally, cash flows are realized, and the shareholders receive payoffs in the form of benefits of control and cash flow.

4.3 Solving the model

We solve the model backwards, and start by considering the effort choice of the bank, given the strategy chosen by the owner in the first stage of the game, $S_i$, and his beliefs about the firm’s type $j$. For the bank, exerting effort $e$ will lead to an expected cash flow income of $e\beta_j V$. Its maximization problem is

$$\max_e \frac{n_b}{n} e \beta_j V - c(e).$$

In an interior solution, we then have

$$c'(e^*) = \frac{n_b}{n} \beta_j V.$$

The effort exerted by the bank thus depends on the product of the fraction of the firm owned, $\frac{n_b}{n}$, as well on his belief about the type. For what follows, we will denote $e^* = e\left(\frac{n_b}{n}, \beta_j\right)$, as $V$ is kept constant throughout the analysis. It is easy to see that $e^*$ is increasing in both its arguments, as

$$\frac{\partial e^*}{\partial n_b/n} = -\frac{\beta_j}{c''(e^*)} > 0 \quad \text{and} \quad \frac{\partial e^*}{\partial \beta_j} = -\frac{-n_b/n}{c''(e^*)} > 0.$$

Hence, the larger the block held by the bank, and the higher the bank believes $\beta$ to be, the higher his own expected cash flow as a reward on his effort.
Now consider the owner's choice of $S_t$ in the first period. His total profit is the sum of the cash flow rights, the benefits he receives from his retained shares, and the revenues he obtains from selling to the investors, or

$$\max_{S_t} \pi_{st}(S_t, j) = \frac{n_s}{n} c\left(\frac{1}{n}, \beta \right) \beta_j V + \Phi_s(S_t) \alpha_t Q + P_b(S_t, j) + P_k(S_t, j).$$

Having assumed that the seller has all the bargaining power, he will choose prices such that investors' expected profits

$$\pi_b(S_t, j) = \frac{n_b}{n} c\left(\frac{n_b}{n}, \beta \right) \beta_j V - c\left(\frac{n_b}{n}, \beta \right) - P_b(S_t, j)$$

$$\pi_k(S_t, j) = \frac{n_k}{n} c\left(\frac{n_b}{n}, \beta \right) \beta_j V + \Phi_k(S_t) \alpha_j Q - P_k(S_t, j)$$

are zero. Replacing these two constraints, the general problem for a seller of type $j$ reduces to

$$\max_{S_t} \pi_{st}(S_t, j) = c\left(\frac{n_b}{n}, \beta \right) \left[\frac{n_s}{n} \beta_t + \frac{n_b + n_k}{n} \beta_j \right] V - c\left(\frac{n_b}{n}, \beta \right)\left[\Phi_s(S_t) \alpha_t + \Phi_k(S_t) \alpha_j\right] Q.$$

For what follows, it will be convenient to split profits according to its source: denote by $C_t(S_t, j)$ the revenues that the seller obtains from expected enhanced cash flow, which correspond to the first two terms in the profit function. Furthermore, we denote the last term in the function $B_t(S_t, j)$, which is the utility he obtains from enjoying or selling private benefit. Then,

$$\pi_{st}(S_t, j) = C_t(S_t, j) + B_t(S_t, j)$$

4.3.1 Benchmark: Symmetric Information

Here, the buyer's expectations about the type always equal the true type, i.e. $i = j$. The seller's objective function simplifies to

$$\max_{S_t} \pi_{st}(S_t, i) = c\left(\frac{n_b}{n}, \beta \right) \beta_i V - c\left(\frac{n_b}{n}, \beta \right) + \left[\Phi_s(S_t) + \Phi_k(S_t)\right] \alpha_t Q.$$

As a first step we establish the following result:
4.3. SOLVING THE MODEL

**Lemma 7** The seller cannot increase his profits by selling to the non-monitoring investor.

Obviously, if the owner did not sell any shares to the bank, the future expected cash flow would be zero. In that case, selling shares to the non-bank investor would amount only to a reallocation of benefits between him and the investor. As the investor pays for receiving benefits, and total benefits $\alpha_i Q$ are constant, the owner is indifferent between selling to the non-bank investor or not.

On the other hand, if the seller does sell shares to the bank, then altering the shares sold to the non-bank investor can reduce profits. The reason is that now a change in the allocation of benefits matters, because any benefit given to the bank is lost to the seller (as the bank does not care about it). In order to give as little strategic power to the bank as possible, the owner will always be better off keeping more shares to himself than giving it to a non-monitoring investor. This strategy increases his own strategic position and reduces that of the bank.\(^\text{13}\) A formal proof of the argument can be found in the appendix.

To find the optimal vector of shareholdings, we can therefore focus on the case $n_k = 0$. When deciding on how many shares to sell to the bank, the seller faces the following trade-off: on the one hand, the larger the block he sells to the bank, the higher will be its monitoring effort (since $\partial e / \partial n_b > 0$), and thus the higher is the revenue from expected cash flow $C_i(S_{i,t})$. On the other hand, his voting power in the firm will be reduced, such that he will face a lower revenue from private benefits $B(S_{i,t})$.

A feature of the Shapley Values is that it is not continuous. As a consequence, $B(S_{i,t})$ exhibits jumps, such that the profit function is not differentiable, and standard solution techniques cannot be used. Investigating $B(S_{i,t})$ more closely, one can see that it in fact can take only two values: for

\(^\text{13}\)A crucial fact for this argument is that the bank's monitoring power is independent of its strategic position, but only depends on the number of shares owned.
Optimal Shareholder Structure

\( n_b/n > 1/2 \), the bank has the majority in the firm. In this case, \( B(S_i, i) = 0 \) because the owner does not get any benefits. If \( n_b/n < 1/2 \), then the seller has full control and extracts the maximal amount of benefits for himself, hence \( B(S_i, i) = \alpha_i Q \). To maximize utility, we therefore have to find the optimal \( n_b \) within each interval, and then compare utility for both.

Within an interval, \( B(S_i, i) \) is constant such that only \( C_i(S_i, i) \) depends on \( n_b \). \( C_i(S_i, i) \) however is strictly increasing in \( n_b/n \). Therefore, within both intervals, maximal utility can be achieved by selling the largest possible number of shares to the bank. These are

\[
\frac{n_b}{n} = \begin{cases} 
1 & \text{for } \frac{n_b}{n} > 1/2 \\
\frac{n-1}{2n} & \text{for } \frac{n_b}{n} < 1/2.
\end{cases}
\]

The only relevant strategies are therefore \( S_1 = (1,0) \) and \( S_2 = (\frac{n-1}{2n},0) \). The profits are computed as

\[
\begin{align*}
\pi_x(S_1, i) &= e(1, \beta_i) \beta_i V - c(e(1, \beta_i)) \\
\pi_x(S_2, i) &= e \left( \frac{n-1}{2n}, \beta_i \right) \beta_i V - c \left( e \left( \frac{n-1}{2n}, \beta_i \right) \right) + \alpha_i Q.
\end{align*}
\]

Thus, the owner will choose to sell the entire firm (strategy 1) if and only if

\[
\alpha_i Q \leq \left( e(1, \beta_i) - e \left( \frac{n-1}{2n}, \beta_i \right) \right) \beta_i V - \left( c(e(1, \beta_i)) - c \left( e \left( \frac{n-1}{2n}, \beta_i \right) \right) \right).
\]

Note that the owner of a bad firm is more inclined to sell only parts of the firm, because he is more reluctant to give up private benefits. In what follows, we are interested in a case where private benefits are large enough to influence the decision process. For this reason, we will focus on the case where both types prefer to sell \( \frac{n-1}{2n} \) under symmetric information for all possible \( \alpha \) and \( \beta \). A sufficient condition for this to happen is that private benefits are relatively large even for the good type, or

\[
Q > \frac{1}{\alpha} \left( e(1, 1) - e \left( \frac{n-1}{2n}, 1 \right) \right) V - \frac{1}{\alpha} \left( c(e(1, 1)) - c(e \left( \frac{n-1}{2n}, 1 \right)) \right) \equiv \overline{Q}.
\]

(4.3)
4.4 Asymmetric Information

In this section we consider the problem of choosing the optimal shareholder structure when investors cannot observe the firm's type. Here, we have a classical adverse selection problem: investors will demand a price that reflects the population average of firm types. This price will be lower than the one that the good type would have been able to obtain, had his type been observable. Moreover, the bank-investor will exert less monitoring effort. We will consider two possible strategies for the owner to signal his type to investors. Finally, we compare the owner's expected utility under each of these strategies.

4.4.1 Separation by Dispersion

In the benchmark case with symmetric information, we had shown that selling to non-monitoring investors could never increase the seller's profit. In this section, we will show that doing so can serve as a signal about the firm's type.

The idea is the following: increasing the dispersion of shares while holding the fraction of votes controlled by the bank constant will result in a better strategic position for the bank. The total benefit allocated between non-bank shareholders will decrease, and this implies a loss in revenues from private benefits for the seller. These losses are larger for the bad type, and therefore he might decide not to imitate the good type's strategy. However, due to the complex structure of our problem, there are several effects of this strategy, not all of which go in the desired direction. That is why the good type owner will not always be able to signal his type.

To analyze Signalling by Dispersion (SD), we consider strategies of the form $S_G = (x, y)$ and $S_B = (x, 0)$ where we define $x = \frac{n-1}{2n}$ as the fraction of the firm sold to the bank, and $y$ the fraction sold to the non-bank investor. Thus, while the bad type chooses the strategy that would be optimal if his type was observable, the good type chooses to sell to one additional investor.
We will analyze the case where \( y = \frac{\delta}{n} \), i.e. where the amount of shares sold to the non-bank investor is equal to the minimal amount that he needs in order to be able to enjoy private benefits. While this sounds restrictive, we will in fact later show that the only effect of choosing \( y > \frac{\delta}{n} \) will be that a signalling equilibrium will be harder to obtain.\(^{14}\) Furthermore, denote 
\[ c(e(x, \beta_j)) = c(x, \beta_j). \]

The strategies \( S_G \) and \( S_B \) can form a separating equilibrium only if (i) neither type of seller has an incentive to deviate from his equilibrium strategy; (ii) prices are such that investors buy the blocks of shares offered, and (iii) investors' beliefs about the type coincide with the true types.\(^{15}\)

Let us start by analyzing the incentive compatibility constraints for both types. These demand that imitation of the other type's strategy is not profitable for either type, given the equilibrium beliefs that choosing the good strategy implies that the firm is of the good type (and vice versa). The constraints are, for the good and the bad type, respectively,
\[
\begin{align*}
\pi_G(S_G, G) & \geq \pi_G(S_B, B) \\
\pi_B(S_B, B) & \geq \pi_B(S_G, G).
\end{align*}
\]

One interpretation of these constraints is that when considering the switch from strategy \( S_G \) to \( S_B \), the benefits should outweigh the costs for the good type, but not for the bad type. Denoting \( \Delta \pi_i = \pi_i(S_G, G) - \pi_i(S_B, B) \) the change in profits when changing from \( S_G \) to \( S_B \), we can indeed rewrite the incentive compatibility constraints as
\[
\Delta \pi_G \geq 0 \geq \Delta \pi_B.
\]

We see that a necessary condition for a signalling equilibrium is that the switch to \( S_G \) is less profitable for \( B \) than for \( G \). This condition is similar to the single-crossing property. In our model, a switch in strategies has several

\(^{14}\)In this analysis we abstract from \( n_k < \delta \), i.e. the presence of small shareholders.

\(^{15}\)In the equilibrium considered here, we assume that investors believe \( j = G \) if and only if they observe strategy \( S_G \). Choosing any other strategy leads to beliefs \( j = B \).
4.4. ASYMMETRIC INFORMATION

effects, concerning both cash-flow related profits as well as benefit related ones. These are (i) a decrease in revenues from benefits $\Delta B_i$, (ii) a higher cash flow because of more monitoring $\Delta M_i$, and (iii) a higher price for shares sold to investors, $\Delta P_i$, such that $\Delta \pi_i = \Delta B_i + \Delta M_i + \Delta P_i$. We will analyze each effect separately.

**Effect 1 (benefit-effect):** To determine the effects of dispersion on the benefit-term, we need to compute the Shapley Value associated with the good type's strategy:

**Lemma 8** Consider the strategy $S = (\frac{n-1}{2n}, \frac{n_k}{n})$. Then, for any $\frac{n_k}{n} \in [1, \frac{n-1}{2n}]$, the vector of Shapley values generated by this strategy is $\Phi(S) = (1/3, 1/3, 1/3)$.

The lemma states that when the bank lacks only one share to obtain the majority of shares, then the remaining $\frac{n_k}{2n}$ shares can be split up in any way between the other two shareholders such that all shareholders obtain a Shapley value of 1/3. The only condition is that the other shareholders have a least one share each. This result highlights the fact that the Shapley Value reflects the strategic position of each voter: in the above situation, one single vote is the crucial one in obtaining majority. It is now straightforward to compute the benefit-related term in the seller's utility function:

**Lemma 9** When a type-i seller chooses strategy $S_G$, and investors believe that the firm is of the good type, then

$$B_i(S_G, G) = [\Phi_s \alpha_i + \Phi_k \alpha] Q = \frac{(\alpha_i + \alpha)}{3} Q.$$

Clearly, $B_i(S_G, G)$ is always higher for the bad type, as $\alpha_G < \alpha_B = 1$. Comparing the changes in benefits from changing to strategy $S_G$, we find that

$$\Delta B_G = -\frac{\alpha}{3} Q \quad \text{and} \quad \Delta B_B = \frac{(\alpha - 2)}{3} Q$$

$$BE \equiv \Delta B_G - \Delta B_B = (1 - \alpha)\frac{2}{3} Q > 0. \quad (4.6)$$
Thus changes are negative for both types, but the loss is higher for the bad type. This latter result is what we call the "benefit-effect". Therefore, we have proved the following lemma:

**Lemma 10** When changing from $S_B$ to $S_G$, the loss in benefits is higher for the bad type.

This is precisely what we wanted in order for separation to be possible. Now we turn to the two sources of changes in cash-flow related income.

**Effect 2 (monitoring-effect)** The second effect from changing to strategy $S_G$ arises because in equilibrium it changes the bank’s beliefs about the firm’s type. If the bank believes $j = G$, then its monitoring effort will be higher, as $\frac{\partial e^*/\partial \beta}{\partial \beta} > 0$. However, the resulting increase in cash flow is higher for the good type. This is so because the good type will receive an increase in cash flow of $\Delta M_G = (e(x, 1) - e(x, \beta))V - (c(x, 1) - c(x, \beta))$, while the bad type only gets an increase of $\Delta M_B = (e(x, 1) - e(x, \beta))\beta V - (c(x, 1) - c(x, \beta))$, because his firm has a lower potential in cash flow.

Denote by $ME$ the difference of this effect between both types:

$$ME = \Delta M_G - \Delta M_B = (e(x, 1) - e(x, \beta))(1 - \beta)V > 0. \quad (4.7)$$

This effect also goes in the desired direction: the good type profits more from a switch to $S_G$ than the bad type.

**Effect 3 (price-effect):** The last effect concerns the price obtained for shares sold, again resulting from a change in beliefs. Both types profit from the fact that the investors are willing to pay a higher price on its shares if they think that the firm is of good type. However, the magnitudes are different for both types.

The good type gains from a switch to strategy $S_G$, because now he obtains a price from the bank that reflects the true underlying cash flow of the share bundle. When the bank believed the firm was of type $B$, it was only willing
to pay for his expected cash flow $xe(x, \beta)\beta V$ while the true cash flow was $xe(x, \beta)V$. A switch to strategy $S_G$ therefore leads to an increase in profits by $\Delta P_G = e(x, \beta)x(1 - \beta)V$.

The bad type, on the other hand, receives the correct price when choosing strategy $S_B$. By changing to $S_G$, he makes an additional profit, because from both investors he receives a price that is higher than the underlying cash-flow value. He then has an additional gain of $\Delta P_B = e(x, 1)(x + y)(1 - \beta)V$.

Denote the difference in revenue gains between both types $PE$ (price effect):

$$PE \equiv \Delta P_B - \Delta P_B = [e(x, \beta)x - e(x, 1)(x + y)](1 - \beta)V. \quad (4.8)$$

It is easy to see that $PE < 0$. Hence, this third effect is larger for the bad type. In other words, the increase in profits due to a higher share price is higher for the bad type. This effect is the one that makes signalling not always possible: the price effect needs to be smaller than the first two.

A sufficient condition for equation (4.5) to hold is that the total cash-flow effect $(ME + PE)$ is positive. We can see that the restriction is essentially on either $\beta$ or $y$, as

$$ME + PE = [e(x, 1)(1 - x - y) - e(x, \beta)(1 - x)].$$

This term is decreasing in $\beta$, because $\partial e/\partial \beta > 0$. Thus, for $\beta$ small, the effect is positive (as desired). Alternatively, we can require that the fraction of shares sold to the non-monitoring investor $y$ is small. For all $y < \gamma(\beta)$ we obtain $ME + PE \geq 0$ and hence $\Delta \pi_G - \Delta \pi_B = ME + PE + PE \geq 0$. Under this condition, the good type therefore faces lower costs than the bad type from switching from $S_G$ to $S_B$.

**Lemma 11** Denote

$$\gamma(\beta) = \frac{e(x) - e(x, \beta)}{e(x)}(1 - x). \quad (4.9)$$

Then, $y \leq \gamma(\beta)$ is a sufficient condition for the single-crossing property to be fulfilled, i.e. for $\Delta \pi_G - \Delta \pi_B \geq 0$. 
Note that $\gamma(\beta)$ is a decreasing function. In particular, $\gamma(0) = 1 - x$ (i.e. all shares not held by the bank could be sold to the non-bank investor, and signalling would still be possible) while $\gamma(1) = 0$ (in this case, signalling is impossible). Again, the smaller $\beta$, the larger the parameter region for which we can obtain a $SD$-equilibrium. We see that the specific shape of the cost- and thus the effort-function is essential for whether the single crossing property is satisfied. From now on, we focus on $\beta$ low such that the condition holds.

Using the notation developed, we can now state the incentive-compatibility constraints as constraints on $Q$:

\[ Q \leq \frac{3}{\alpha} [\Delta M_G + \Delta P_G] \equiv Q_G \]

for the good type, and

\[ Q \leq \frac{3}{2 - \alpha} [\Delta M_B + \Delta P_B] \equiv Q_B \]

for the bad type. Both constraints depend on $\alpha$ and $\beta$. It is easy to check that the upper bound on $Q$, $Q_G$, is decreasing in $\beta$, while the lower bound $Q_B$ is first increasing, then decreasing. This again confirms that the region for which signalling is possible is largest for $\beta$ small. Furthermore, we find $\partial Q_G / \partial \alpha \geq 0$ while $\partial Q_B / \partial \alpha \leq 0$, such that for higher $\alpha$, the constraints move closer together and the area for which signalling is possible becomes smaller. The constraint for the good type becomes more restrictive for higher $\alpha$, because then his total private benefits are relatively high, and he is less willing to give up the benefit in order to signal. The bad type, on the other hand, profits more when $\alpha$ is high, because pretending to be of the good type then yields a higher benefit-related premium from the non-bank investor. Thus, a low $\alpha$ makes a signalling equilibrium generally easier to obtain.

**Proposition 2 (Signalling by Dispersion ($SD$))** As long as the condition of Lemma 5 is satisfied, the owner of the good firm can signal his type by
choosing a higher dispersion of shares for all $Q$ such that $Q_G(\beta) \geq Q \geq Q_G(\beta)$.

This result is a novelty in the literature. Since the seminal work of Leland and Pyle [16], several studies\textsuperscript{16} have explored the possibility that an owner can signal his superior type by keeping more shares to himself. Here, on the other hand, we have shown that it can be optimal to sell more shares and retain fewer.\textsuperscript{17}

Contrary to a usual analysis of signalling, here we have basically only one choice variable that can take two values: sell to a non-monitoring investor or not. This is because we have restricted the number of these investors to one, in order to reduce computational complexity. If we allowed for several non-monitoring shareholders, we could determine the profit-maximizing number of these shareholders for which the incentive compatibility constraints are satisfied. We can show that this in fact augments the parameter regions for which signalling is possible. However, as the basic argument is clear with one non-bank shareholder, we abstracted here from doing so.

### 4.4.2 Signalling by retention

In this subsection, we focus on possible outcomes when the owner's strategy is restricted to selling shares only to the bank. The maximization problem of a type-$i$ seller becomes

$$\max_{S_i} \pi_{si}(S_i, j) = e\left(\frac{n_b}{n}, \beta_j\right) \left[\left(1 - \frac{n_b}{n}\right)\beta_i + \frac{n_b}{n}\beta_j\right] V - c\left(e\left(\frac{n_b}{n}, \beta_j\right)\right) + \Phi_s(S_i)\alpha_i Q$$

Leland and Pyle [16] were the first to develop a signalling model in similar circumstances as here. In their model, a risk-averse entrepreneur wants to sell his project to a risk-neutral investor. To signal that the expected return on his shares is high, he retains a fraction of shares to himself. This strategy is costly, as the entrepreneur continues to bear some risk. However, the good

\textsuperscript{16}See Allen and Faulhaber[1], Grindblatt and Huang [11], and Welch [24].

\textsuperscript{17}A discussion of a Leland-and-Pyle type of signalling is undertaken in the next section.
entrepreneur incurs a lower cost than a bad one because the expected payoff on the retained shares is higher.

In our model, there is also scope for this type of signalling to work: the good type owner could choose to retain more shares than the bad type. The strategy is costly to both types, because it will induce a lower monitoring effort from the bank, but the cost might be higher for the bad type because he receives a lower return on the shares he does not sell. However, matters are not so simple in our setup, because retaining shares has two additional effects: (1) the reduction in monitoring effort $\Delta e$ could also be more costly for the good type because foregone profit is higher: $\Delta eV > \Delta e\beta V$, and (2) the seller might receive a higher share of private benefits. This is more profitable for the bad type.

These additional effects work against Signalling by Retention of shares ($SR$), because both imply a lower cost (or a higher benefit) for the bad type. Therefore, $SR$-equilibria do not always exist.

We are focusing on the case where the bad type chooses to sell $\frac{n_k}{n} = \frac{n-1}{2n}$ under symmetric information. For $SR$-signalling, we then analyze an equilibrium in the selling strategies $S_G = (x\gamma, 0)$ and $S_B = (x, 0)$, with $\gamma \in [0, 1)$, and where $x = \frac{n-1}{2n}$. The incentive compatibility constraints for a separating equilibrium for both types are

$$C_G(S_G; G) \geq C_G(S_B; B) \quad C_B(S_B; B) \geq C_B(S_G; G).$$

It suffices to compare the cash-flow related terms because revenues from private benefits are the same for both strategies. Because $\partial C_i(S_j, j)/\partial e > 0$, the first constraint gives a lower bound $\gamma$ on $\gamma$ while the second one gives an upper bound $\overline{\gamma}$. However, it depends on the specific shape of the cost function whether $\gamma \leq \overline{\gamma}$, i.e. whether we can find a $\gamma$ that satisfies both constraints simultaneously.\(^{18}\) As long as indeed $\gamma \leq \overline{\gamma}$, a continuum of signalling

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\(^{18}\)This is equivalent to demanding that a condition like the single-crossing property is satisfied.
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equilibria exists for all $\gamma \in [\gamma_l, \gamma_r]$, and for different out-of-equilibrium beliefs.

Following Leland and Pyle, we assume that the owner chooses among the admissible $\gamma$ the one that maximizes his expected utility, which is the one that satisfies the ICC for the bad type with equality, $\gamma_r$.

We get the following results:

**Proposition 3** The good owner can signal his firm’s type by retaining shares only if $xe(x, \beta) - x\gamma e(x\gamma, 1) \geq e(x, \beta) - e(x\gamma, 1)$. 

**Corollary 1** The greater the difference in potential cash flows between the two types, the more shares the good type has to retain in a signalling equilibrium.

As in the previous section, the condition in the proposition is essentially on the weights of the different effects involved. The left-hand side of the inequality reflects the changes in prices on the shares sold to the bank. It is very similar to the price-effect of the previous section, only now it is the desired effect. In the proof to the corollary, we show that we need $\gamma \leq \beta$ for the bad type’s incentive compatibility constraint to be satisfied. Therefore, the increase in profits because of an increase in prices is always larger for the good type.

The right-hand side of the same inequality corresponds to the net monitoring effect. Note that in equilibrium, now we have two effects concerning monitoring: when switching to $SG$, on the one hand there will be less monitoring because the bank holds fewer shares. On the other hand, it has larger monitoring incentives because it believes the firm is of good type. As $\beta \geq \gamma$, the net monitoring effect is negative, and the bad type suffers less from it. The condition in Proposition 7 states that this net monitoring effect should not be too large compared to the price effect, such that the overall cost of this strategy is lower for the good type.

The corollary says that the optimal $\gamma$ is increasing in $\beta$: the larger the difference in the cash flow of the two types ($\beta$ small), the more the good type
needs to retain. The argument behind this relationship is again the relative importance of the price- and the monitoring effect. To see this, consider a very small $\beta$. Then, the bad type will lose very little from the decrease in monitoring compared to the good type, and the monitoring effect is very large in absolute terms. In order to compensate for this, the good type has to choose $\gamma$ such that the price effect is large enough to outweigh the monitoring effect. The price effect however is largest for small $\gamma$ (as we see from the left-hand side of the condition in Proposition 7). This means that for small $\beta$, the signalling strategy can be very costly for the good type, as he has to give up a lot of monitoring.

Before we turn to the issue of which equilibrium to choose, we establish one further result, which is proved in the appendix:

**Proposition 4** The owner would never choose to signal by choosing a combination of both signalling strategies.

### 4.4.3 Utility Comparison

In this section we discuss which of the above strategies an owner would prefer. The difficulty with comparison of the two lies in the question of existence. We saw that neither type of signalling equilibrium existed for all the entire parameter space, or for all types of cost functions. We obtained bounds on admissible parameters in form of the incentive-compatibility constraints. However, these are very different restrictions for both types of equilibria. For example, the constraints for a $SD$-equilibrium depended on $\alpha$ and $Q$, while these variables did not affect at all the possibility to signal in a $SR$-equilibrium.

Let us now suppose that both types of equilibria exist. The good type's equilibrium utility for a signalling-by-dispersion ($SD$) as well as a signalling-by retention equilibrium ($SR$) is:

$$\pi^G_{SD} = e(x, 1)V - c(x, 1) + 2/3\alpha Q$$

$$\pi^G_{SR} = e(x\gamma, 1)V - c(x\gamma, 1) + \alpha Q.$$
4.5 Conclusion

One can see immediately that the different types of signalling bring about different types of costs: in a \( SD \)-equilibrium, the good type gives up benefits, but in a \( SR \)-equilibrium, he gives up cash-flow returns. Thus, if the size of benefits is relatively small compared to cash flow (i.e. \( \alpha Q/V \) small), then signalling by dispersion is the less costly way of signalling. For \( \alpha Q/V \) large, on the other hand, signalling by retention is more likely to give a higher utility.

Remark 7 The higher the value of potential cash flow \( V \) relative to total private benefits \( \alpha Q \), the more attractive is Signalling by Dispersion to the good owner.

Furthermore, we are interested to see how changes in \( \alpha \) and \( \beta \) affect the comparison. First, we find that utility increases with \( \alpha \) for both equilibria types as the benefit consumed increases in \( \alpha \). Concerning \( \beta \), the size of the bad type’s potential cash flow relative to that of the good type, we find

\[
\frac{\partial \pi^D}{\partial \beta} = 0
\]

\[
\frac{\partial \pi^R}{\partial \beta} = \frac{\partial e}{\partial \gamma} (V - c'(e_r x)) = \frac{\partial e}{\partial \gamma} V (1 - \gamma x) \geq 0.
\]

We see that if Signalling by Dispersion is possible, then the utility obtained is independent of \( \beta \). Utility in \( SR \)-equilibrium does depend on \( \beta \), however, as the optimal \( \gamma \) is a function of it. Since \( \frac{\partial \gamma}{\partial \beta} > 0 \), the owner’s utility increases in \( \beta \). Therefore, for very high \( \beta \), \( SR \)-signalling will be attractive, because the bank will get close to \( x \) shares. However, for \( \beta \) low, a \( SR \)-equilibrium is likely to be very costly.

Remark 8 If the difference between the two type’s cash flow is very large, the good type would prefer Signalling by Dispersion over Signalling by Retention.

4.5 Conclusion

We have analyzed the selling decision of the single owner of a company. While selling stakes to a bank with monitoring powers increases the com-
pany's expected cash flow, it reduces the seller's utility from enjoying private benefits of control. The owner's decision on the number of shares to sell thus depends on the size of benefits relative to nominal firm value. Furthermore, we explained the presence of minority shareholders with limited monitoring abilities. We have argued that if the firm's type is private information to the initial owner, selling minority stakes can be a way of signalling that profitability is high relative to total benefits. Moreover, we have shown that this strategy is most likely to be chosen when differences in cash flow between firms are high.

We have focused on economies like Germany, where banks often hold large blocks of shares and are involved in the supervisory process of a firm. We explained why in this environment, we typically observe several large shareholders, among them very often one bank. However, the analysis is also applicable to other economies, in the context of initial public offerings.

The pricing of blocks derived in the model is consistent with the evidence that even minority blocks usually trade at a premium. However, if firms are in distress before the purchase, large blocks trade at a considerable discount, presumably to compensate the investors for the required restructuring needed to improve performance. Our results support these findings.

Several assumptions were made in order to simplify the analysis. I assumed that the owner could sell to maximally two investors. It is straightforward to introduce several shareholders. Allowing for more than one non-bank investor in fact supports the theory of signalling by dispersion, because the region for which separating equilibria are attainable is enlarged. The presence of several banks would be consistent with evidence showing that blocks of 25% or 10% are very common, instead of the 50% threshold found in the analysis.
Bibliography


