Optimal Policy, Heterogeneity and Limited Commitment

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To my parents

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Abstract

This thesis contributes to the literature on optimal fiscal and monetary policy. First, I analyze how the tax-smoothing result obtained in models of optimal fiscal policy is altered in a context of international risk sharing with limited commitment. I find that the presence of limited commitment alters substantially the dynamics of the fiscal variables with respect to the full commitment case. In particular, the volatility of the tax rate increases. Second, I study the optimal monetary and fiscal policy mix in a model in which agents are subject to idiosyncratic uninsurable shocks to their labor productivity. I find that, for a utilitarian government, the monetary policy-maker sets nominal interest rates to zero. Although the aggregate welfare costs of inflation are small, individual costs and benefits are large. Net winners from inflation are poor, less productive agents, while middle-class and rich households are always net losers.

Resumen

Esta tesis contribuye a la literatura de política fiscal y monetaria óptima. Primero analizo cómo el resultado de tasas impositivas suaves, que habitualmente se obtiene en modelos de política fiscal óptima, se ve alterado en un contexto de división internacional de riesgo con compromiso parcial. Encuentro que la presencia de compromiso parcial altera significativamente la dinámica de las variables fiscales, con respecto al caso de compromiso total. La volatilidad de la tasa impositiva aumenta. En segundo lugar, estudio la política fiscal y monetaria en un modelo con shocks idiosincráticos no asegurables a la productividad laboral. Encuentro que, cuando el gobierno es utilitarista, la autoridad monetaria fija la tasa de interés nominal a cero. Los efectos agregados de bienestar son pequeños, mientras que los efectos individuales son grandes. Los beneficiarios de la inflación son agentes pobres con baja productividad, mientras que los agentes de clase media y alta siempre son perdedores.

Foreword

Macroeconomists have long been interested in understanding the impact and consequences of fiscal and monetary policies and, on a related note, in the optimal determination of such policies. Some early works, such as Friedman (1969) and Phelps (1973) have opened a fascinating debate about how the monetary authority should conduct monetary policy. Phelp's idea that the inflation tax should be actively used to reduce the distortions associated to other taxes seems intuitive enough to be embraced. However, Friedman argues that, in the case of inflation, the nature of the tax base (money holdings) calls for a unique result: minimizing distortions implies setting the inflation tax to zero. Although less intuitive at first sight, Friedman's idea has been proven to be quite robust to different specifications and has become a reasonably well accepted result in the profession.

Lucas and Stokey (1983) study in a seminal paper the optimal fiscal and monetary policy plan in a dynamic stochastic general equilibrium setup. Their general prescriptions are remarkably powerful: taxes should be smooth because, in this way, consumption is also smooth. Moreover, the government uses bond holdings as an insurance mechanism that allows it to smooth distortions not only over time, but across states of nature as well.

In the last two decades, the macroeconomic literature has produced a large number of contributions that have extended these arguments in various directions. Issues such as time-consistency, market incompleteness, instrument availability and capital taxation, among others, have been analyzed in detail. A few examples of such studies are Aiyagari et al. (2002), Chari et al. (1991), Chari et al. (1994), Chari et al. (1996) and Correia and Teles (1999).

Moreover, in the recent past there has been increasing interest in relaxing some of the key assumptions implicit in the classical contributions, so as to make the working models closer to reality. In a large extent, this has been possible thanks to the advances in computational power and the diffusion of computational techniques for economics, which have allowed economists to solve problems that were unfeasible in the past.

In a nutshell, the main effort has been put in understanding the consequences of household heterogeneity, private information and non-rational expectations. In many cases, the departure from these assumptions has proven to change the optimal policy plans in a qualitative as well as in a quantitative dimension, at least in the short run. This fact clearly poses the need for future research along these directions.

This thesis contributes to the literature of optimal policy in environments that differ from the classical setup. In the first chapter, I analyze how the tax-smoothing result obtained in models of optimal fiscal policy is altered in a context of international risk sharing with limited commitment. The study is motivated by the empirical observation that, in developing countries, the process of tax revenues over GDP seems seems to be considerably more volatile than in developed economies, even when controlling for government spending over GDP.

I consider the problem of a benevolent government in a small open economy that has to choose optimally distortionary taxes on labor income and transfers from the rest of the world. The contract between the government and the rest of the world is designed so that at any point in time, neither agent has incentives to exit the contract and there is no net transfer of wealth between them.

My analytical results suggest that the presence of limited commitment alters substantially the dynamics of the fiscal variables with respect to the full commitment case. In particular, the volatility of the tax rate is increased with respect to the case of full commitment since taxes respond strongly to the incentives to default of both agents. Moreover, optimal taxes are procyclical. When the government expenditure shock is low, the domestic economy wants to exit the contract. To prevent default in equilibrium, the utility of households has to increase and the tax rate decreases. Conversely, when the government expenditure shock is high, the rest of the world has incentives to default. In this case the tax rate in the domestic economy increases to pay back the external debt and induce the rest of the world not to leave the contract.

I calibrate the model to match the argentinean government expenditure process and find that the presence of limited commitment in international risk-sharing agreements increases the volatility of tax revenues over GDP significantly from a quantitative point of view. Finally, I show that the setup proposed can be reinterpreted as one in which the government of the domestic economy issues debt in domestic and international capital markets subject to debt limits.

In the second chapter I study the optimal monetary and fiscal policy mix in a model in which agents are subject to idiosyncratic uninsurable shocks to their labor productivity. I identify two main effects of anticipated inflation absent in representative agent frameworks. First, inflation stimulates savings for precautionary reasons. Hence, a higher level of anticipated inflation implies a higher capital stock in steady state, which translates into higher wages and lower taxes on labor income. This benefits poor, less productive agents. Second, inflation acts as a regressive consumption tax, which favors rich and productive agents.

I calibrate the model economy to the U.S. economy and compute the optimal policy mix. Some key targets are the correlation between money and asset holdings, the fraction of consumption expenditures made with cash and the Gini coefficient of the asset distribution. I define the benchmark economy to be one that displays an annual rate of inflation of 2%. I find that, for a utilitarian government, the optimal monetary policy sets nominal interest rates to zero, a result known in the literature as the *Friedman rule*. This result, which usually holds in representative-agent economies, survives the introduction of heterogeneity and uninsurable idiosyncratic risk in the model.

Although the aggregate welfare costs of inflation are small, individual costs and benefits are large[1]. Net winners from inflation are poor, less productive agents, while middle-class and rich households are always net losers. This is due to the fact that the effect of inflation over capital accumulation and, thus, over wages, dominates quantitatively.

[1]They can amount to minus/plus 4% of permanent consumption.

Contents

1 Optimal Fiscal Policy in a Small Open Economy with Limited Commitment

1.1 Introduction

A fundamental question in macroeconomics is how a policymaker has to set distortionary taxes in order to finance an exogenous public expenditure shock. The answer to this question depends on both the degree of openness of the economy and on the commitment technology of the government to fulfill its obligations towards foreign investors.

Consider a setup in which a small open economy, which we call the *home country*, can trade assets with the rest of the world. The government of the home country has to collect revenues optimally in order to finance an exogenous stream of public expenditure, while the rest of the world is subject to no shock. In this case, a benevolent government of the home country would set taxes roughly constant over the business cycle. When a bad shock hits the economy, the government can borrow from abroad and pay back the debt later on, when the economy faces instead a good shock. In this way, the possibility to do risk-sharing with the rest of the world implies that the deadweight losses associated to distortionary taxation are minimized. In the extreme case in which the rest of the world is risk neutral, the optimal tax rate is perfectly flat and all fluctuations in public expenditure can be absorbed by international capital flows. It follows that, at least from a theoretical point of view, tax volatility in small open economies should be lower than the tax volatility in large or closed economies, thanks to the insurance role played by international borrowing and lending.

Nevertheless, this conclusion does not seem to be validated in the data. Table 1.1 shows some statistics for government expenditure and average tax rate series in Argentina and in the USA.[1]. Although the variability of the government expenditure series is roughly the same in the two countries, tax rates in Argentina are much more volatile than in the USA: the standard deviation of the series for Argentina is almost 60% higher than the one for the US economy. As can be seen from Table 1.2, this empirical evidence applies to other countries as well, for the same sample period.

In this chapter we introduce sovereign risk into a standard optimal fiscal policy open economy model as the one described before by relaxing the assumption of full commitment from the home country and the rest of the world towards their contracted

[1]The series for USA are from the Bureau of Economic Analysis of the US Department of Commerce. In the case of Argentina, the data we use is from the IMF, INDEC and Ministerio de Economia. We use quarterly data of current government expenditure net of interest payments plus gross government investment as a measure of government expenditure, and total tax revenues plus contributions to social security as a measure of total tax revenues. The average tax rate is calculated as the ratio between tax revenues over GDP. Due to reliability/availability of data for Argentina, we use data for the period $1993-2005$.

Table 1.1: Fiscal variables for the USA and Argentina

	USA		ARG	
	Govt. expend.	Tax rate	Govt. expend.	Tax rate
Mean	0.1755	0.1850	0.1704	0.1828
St. deviation	0.0092	0.0130	0.0098	0.0214
Coef. of variation	0.0525	0.0704	0.0573	0.117

Table 1.2:

Country	Tax rate coefficient of variation
Bulgaria	0.104
Guatemala	0.136
Nicaragua	0.139
Venezuela	0.13

obligations. We show that this framework provides a theoretical justification for the tax rate volatility observed in small open economies that have commitment problems to repay their external obligations.

In the model the home country is populated by risk adverse households. The fiscal authority has to finance an exogenous public expenditure shock either through distortionary labor income taxes or by issuance of internal and/or international debt. The rest of the of world is inhabited by risk-neutral agents that receive a constant endowment and have to decide how much to consume and how much to borrow/lend in the international capital market. We assume that neither the government in the home country nor the rest of the world can commit to pay back the debt contracted among themselves.

A contract, signed by the two countries, regulates international capital flows. The terms of the contract depend on the commitment technology available to the two parts to honor their external obligations. When both countries can fully commit to stay in the contract in all states of nature, the only condition to be met is that *ex-ante* there is no exchange of net wealth among them. Instead, when the countries may at some point decide to leave the contract, further conditions need to be imposed. In particular, since default takes place if the benefit a country obtains from staying in the contract is smaller than its outside option, the contract must specify an adjustment in the allocation necessary to rule out default in equilibrium.

We show that the presence of sovereign default risk, i.e., the possibility that a country may exit the contract with the other country, limits the amount of risk-sharing among countries. Consequently, the classical tax-smoothing result is broken since now the optimal tax rate depends on the incentives to default of both countries. In particular, when the home country wants to exit the contract it has to be compensated so that the benefits of staying in it equal the value of its outside option. Therefore, consumption and leisure have to increase, and the tax rate decreases. On the other hand, when the rest of the world has incentives to default, the tax rate in the home country increases to pay back the external debt and induce the rest of the world not to leave the contract.

In our model, the home country has incentives to exit the contract when the realization of the public expenditure shock is low. There are two reasons behind this feature. First, the benefits from staying in the contract decrease, since in this case the home country has to repay its foreign debt. The second reason relies on our definition of the outside option for the home country. We assume that, if the home country defaults, its government is forced to run a balanced-budget thereafter. The outside option is the expected life-time utility of the households under this fiscal policy plan. When the shock is low, the tax rate is low as well, so the outside option increases. Therefore, an important corollary of the analysis is that the optimal fiscal policy is pro-cyclical: tax rates decrease when the country has incentives to leave the contract, and this happens when public consumption is low. This conclusion is in line with recent evidence for developing countries (see e.g. Ilzetzki and Vegh (2008) and Cuadra and Sapriza (2007)).

Some possible alternative explanations for the high volatility of tax rates observed in developing economies rely on the quality of their institutions and the sources of tax collection. It is argued that developing countries are more prone to switches in political and economic regimes that, almost by definition, translate into unstable tax systems. Moreover, in booms these countries' often tax heavily those economic sectors that are responsible for the higher economic activity[2]. As a consequence, when economic conditions deteriorate, necessarily tax revenues go down dramatically. We are aware that these considerations are relevant sources of tax variability and that our study does not incorporate them in the analysis. However, we do not intend to provide an exhaustive description of such sources. In this sense, by focusing on sovereign risk and incomplete international capital markets as causes for the high tax rate volatility of developing economies we are carrying out a partial analysis of the phenomenon.

In the recent years there have been some attempts to add default to dynamic macroeconomic models. A number of papers (Arellano (2008), Aguiar and Gopinath (2006), Hamann (2004)) have introduced sovereign default in otherwise standard business cycle models in order to quantitatively match some empirical regularities of small open developing economies. More specifically, they adapt the framework of Eaton and Gersovitz (1981) to a dynamic stochastic general equilibrium model. These models are usually

[2]As an example, in the recent years Argentina has been experiencing rapid export-led growth, mainly due to exports of commodities such as soya. In this period, the government's main source of tax revenues has come from taxation of these exports.

able to explain with relative success the evolution of the interest rate, current account, output, consumption and the real exchange rate. Nevertheless, since they all consider endowment economies, they fail to capture the effects of default risk over the taxation scheme. Our contribution is to extend the analysis to be able to characterize the shape of fiscal policy and the links between the risk of default and taxes in a limited commitment framework.

Many papers have introduced the idea of limited commitment to study many important issues. Among others, Kehoe and Perri (2002) introduce credit arrangement between countries to reconcile international business cycle models with complete markets and the data, Krueger and Perri (2006) look at consumption inequality, Chien and Lee (2008) look at capital taxation in the long-run, Marcet and Marimon (1992) study the evolution of consumption, investment and output, and Kocherlakota (1996) analyzes the properties of efficient allocations in a model with symmetric information and two-sided lack of commitment. To our knowledge, none of them has focused on the impact of the possibility of default on the volatility of optimal taxation.

The closest papers to ours are probably those by Cuadra and Sapriza (2007), Pouzo (2008) and Scholl (2009). The first paper focuses on matching some stylized facts in developing countries, namely the positive correlation between risk premia and the level of external debt, higher risk premia during recessions and the procyclicality of fiscal policy in developing economies. The second paper studies the optimal taxation problem in a closed economy under incomplete markets allowing for default on internal debt. Finally, the third paper analyzes the problem of a donor that has to decide how much aid to give to a government that has an incentive to use these external resources to increase its own personal consumption without decreasing the distortive tax income it levies on private agents.

We differentiate from these papers along various dimensions. In the first place, we consider the full commitment solution instead of the time-consistent one. We do this to isolate the effect of endogenously incomplete markets on the optimal fiscal plan, while giving the government all the usual tools to distribute the burden of taxation across periods and states of the world. In particular, in our framework there is a complete set of state-contingent bonds the government can issue internally. This has important implications for consumption smoothing as it allows the government to distribute the burden of taxation across states. Finally, in contrast with the assumption in Scholl (2009), we focus on the scenario in which the government of the small open economy is benevolent, i.e., its objective is to maximize the expected life-time utility of its citizens.

The rest of the chapter proceeds as follows. Section 1.2 describes the model. Section 1.3 shows how the optimal fiscal plan is affected by the possibility of default in the case study of a perfectly anticipated one-time fiscal shock. In section 1.4 we solve the model for the general case of correlated government expenditure shock. Section 1.5 is devoted to show that our economy can be reinterpret as one in which the government can issue debt subject to debt limits, both on internal and external debt. Section 1.6 offers some empirical evidence on the relationship between tax volatility and default risk. Section 1.7 concludes.

1.2 The Model

We assume that the economy is constituted by two countries: the home country (HC) and the rest of the world (RW). The HC is populated by risk-averse agents, which enjoy consumption and leisure, and by a benevolent government that has to finance an exogenous public expenditure shock either by levying distortionary taxes, by issuing state-contingent internal bonds, or by receiving transfers from the RW. The RW is populated by risk-neutral agents that receive a fixed endowment each period. These resources can be either consumed or lent to the HC. Being an endowment economy without shocks, there is no government activity in this country.

a. The contract

The government of the HC can do risk-sharing with the RW by contracting transfers[3]. Let T_t be the amount of transfers received by the HC at time t. There are three conditions that have to be met by $\{T_t\}_{t=0}^{\infty}$.

First, the expected present discounted value of transfers exchanged with the RW must equal zero:

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{1.1}$$

where β is the discount factor of households in the RW and the HC. This condition rules out the possibility that the government of the HC uses resources from the RW for reasons other than the risk-sharing one. In other words, we do not allow for net redistribution of wealth between countries at time 0. We call this condition the *fairness condition*, since it implies that *ex-ante* the contract is fair from an actuarial point of view[4].

[3] In section 1.5 we show that these transfers can be reinterpreted as bonds traded in the international capital market.

[4] This condition implies that the contract is actuarially fair only if the RW has full commitment. This is due to the fact that, if the RW has limited commitment, the risk-free interest rate will not always be $1/\beta$ (see Section 1.5 for further details). This condition is useful because it allows us to pin down the allocations. However, one can impose other similar conditions that will yield different allocations.

If we assumed that the two parts in the contract have full commitment to pay back the debt contracted with each other, equation (1.1) would be the only condition regulating international flows. The allocations compatible with this situation will be our benchmark for comparison purposes. However, when the government in the HC does not have a commitment technology, it may decide to leave the contract if it finds it profitable to do so. Denote by V_t^a the value of the government's outside option, i.e., the expected life-time utility of households in the HC if the government leaves the contract, and by V_t the continuation value associated to staying in the contract in any given period t. Then, in order to rule out default in equilibrium, the following condition has to be satisfied

$$V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V_t^a \ \forall t \tag{1.2}$$

This condition constitutes a *participation constraint for the HC*. We assume that, if the government chooses to leave the contract at any given period, it remains in autarky from then on. Moreover, when the government defaults on its external obligations, it also default on its outstanding domestic debt. Consequently, the government is forced to run a balanced budget thereafter[5]. Alternative assumptions to identify the costs of default could be made, for example that the government cannot use external funds, but it still has access to the domestic bonds market to smooth the distortions caused by the expenditure shock. We have chosen the current specification for two reasons. First, this allows us to keep the problem tractable, both from an analytical and a numerical point of view. Second, this specification is consistent with the interpretation that the government is subject to debt limits, as shown in section 1.5.

Similar to the case of the HC, the RW also lacks a commitment technology and can potentially exit the contract at any point in time. Therefore, we need to impose a *participation constraint for the RM*:

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \ \forall t \tag{1.3}$$

This condition is analogous to (1.2) and states that, at each point in time and for any contingency, the expected discounted value of future transfers the HC is going to receive cannot exceed an exogenous threshold value $\underline{B}$. This restriction, together with the fairness condition, poses an upper limit on how much indebted the RW can get.

As long as conditions (1.1), (1.2) and (1.3) are satisfied, the government of the HC can choose any given sequence $\{T_t\}_{t=0}^{\infty}$ to partially absorb its expenditure shocks.

[5]It follows that the only state variable influencing the outside option is the government expenditure shock. Therefore $V_t^a = V_t^a(g_t)$.

b. Households in the HC

Households in the HC derive utility from consumption and leisure, and each period are endowed with one unit of disposable time. The production function is linear in labor and one unit of labor produces one unit of the consumption good. Therefore, wages $w_t = 1\ \forall t$.

The representative agent in the HC maximizes her expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

subject to the period-by-period budget constraint

$$b_{t-1}(g_t) + (1-\tau_t)(1-l_t) = c_t + \sum_{g^{t+1}|g^t} b_t(g_{t+1}) p_t^b(g_{t+1}) \tag{1.4}$$

where c_t is private consumption, l_t is leisure, $b_t(g_{t+1})$ denotes the amount of bonds issued at time t contingent on the government shock in period $t+1$, τ_t is the flat tax rate on labor earnings and $p_t^b(g_{t+1})$ is the price of a bond contingent on the government shock realization in the next period.

The optimality condition with respect to the state-contingent bond is:

$$p_t^b(g_{t+1}) = \beta \frac{u_{c,t+1}(g_{t+1})}{u_{c,t}} \pi(g^{t+1}|g^t) \tag{1.5}$$

.

where $\pi(g^{t+1}|g^t)$ is the conditional probability of the government expenditure shock. Combining the optimality conditions with respect to consumption and leisure we obtain the intratemporal condition

$$1 - \tau_t = \frac{u_{l,t}}{u_{c,t}} \tag{1.6}$$

c. Government of the HC

The government finances its exogenous stream of public consumption $\{g_t\}_{t=0}^{\infty}$ by levying a distortionary tax on labor income, by trading one-period state-contingent bonds with domestic consumers and by contracting transfers with the RW. The government's budget constraint is

$$g_t = \tau_t(1-l_t) + \sum_{g^{t+1}|g^t} b_t(g_t+1) p_t^b(g_t+1) - b_{t-1}(g_t) + T_t \tag{1.7}$$

d. Equilibrium

We proceed to define a *competitive equilibrium with transfers* in this economy.

Definition 1. *A competitive equilibrium with transfers is given by allocations* $\{c, l\}$, *a price system* $\{p^b\}$, *government policies* $\{g, \tau^l, b\}$ *and transfers* T *such that*[6]*:*

1. *Given prices and government policies, allocations satisfy the household's optimality conditions* (1.4), (1.5) *and* (1.6).
2. *Given the allocations and prices, government policies satisfy the sequence of government budget constraints* (1.7).
3. *Given the allocations, prices and government policies, transfers satisfy conditions* (1.1), (1.2) *and* (1.3).
4. *Allocations satisfy the sequence of feasibility constraints:*

$$c_t + g_t = 1 - l_t + T_t \tag{1.8}$$

e. Optimal policy

The government of the HC behaves as a benevolent Ramsey Planner and chooses tax rates, bonds and transfers $\{c_t, b_t, T_t\}_{t=0}^{\infty}$ in order to maximize the representative household's life-time expected utility, subject to the constraints imposed by the definition of competitive equilibrium.

Before studying the consequences of introducing default in terms of the optimal fiscal plan, it is instructive to analyze the benchmark scenario in which both the government in the HC and the RW have a full commitment technology.

Full commitment

If both the HC and the RW can commit to honor their external obligations in all states of nature, then conditions (1.2) and (1.3) need not be specified in the contract. Then, the problem of the Ramsey planner is

$$\max_{\{c_t, l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

[6] We follow the notation of **?** and use symbols without subscripts to denote the one-sided infinite sequence for the corresponding variable, e.g., $c \equiv \{c_t\}_{t=0}^{\infty}$.

s.t.

$$b_{-1}u_{c,0} = E_0 \sum_{t=0}^{\infty} \beta^t (u_{c,t}c_t - u_{l,t}(1 - l_t)) \tag{1.9}$$

$$c_t + g_t = 1 - l_t + T_t \tag{1.10}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t T_t = 0 \tag{1.11}$$

The optimality conditions for $t \geq 1$ are:

$$u_{c,t} + \Delta(u_{cc,t}c_t + u_{c\cdot t} + u_{cl,t}(1 - l_t)) = \lambda \tag{1.12}$$

$$u_{l,t} + \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = \lambda \tag{1.13}$$

where λ is the multiplier associated with constraint (1.11), and Δ is the multiplier associated with the implementability condition (1.9). The next proposition characterizes the equilibrium.

Proposition 1. *Under full commitment, consumption, labor and taxes are constant* $\forall t \geq 1$*. Moreover, if* $b_{-1} = 0$*,* $b_t(g_{t+1}) = 0$ $\forall t$*,* $\forall g_{t+1}$ *and the government perfectly absorbs the public expenditure shocks through transfers* T_t*.*

Proof. Using optimality conditions (1.12) and (1.13) we have two equations to determine two unknowns, c_t and l_t, given the lagrange multipliers λ and Δ. Since these two equations are independent of the current shock g_t, the allocations are constant $\forall t \geq 1$. From the intratemporal optimality condition of households (1.6) it can be seen that the tax rate τ_t^l is also constant $\forall t \geq 1$[7]. Finally, the intertemporal budget constraint of households at time 0 (equation (1.9)) can be written as

$$\frac{1}{1-\beta}(u_c c - u_l(1 - l)) = 0$$

Notice that, for any given time $t+1$, domestic bond holdings $b_t(g_{t+1})$ are obtained from the intertemporal budget constraint of households in that period, i.e.,

$$b_t(g_{t+1})u_{c,t+1} = E_{t+1} \sum_{j=0}^{\infty} \beta^j (u_{c,t+1+j}c_{t+1+j} - u_{l,t+1+j}(1 - l_{t+1+j}))$$

[7]Notice that at $t = 0$ the optimality conditions of the Ramsey planner differ from those at $t \geq 1$. This is the standard source of time-inconsistency in this type of problems.

However, since the allocations are constant over time, it is the case that

$$b_t(g_{t+1})u_c = E_{t+1}\sum_{j=0}^{\infty}\beta^j(u_c c - u_l(1-l)) = \frac{1}{1-\beta}(u_c c - u_l(1-l)) = 0$$

Therefore, $b_t(g_{t+1}) = 0\ \forall g_{t+1}$ and, from the feasibility constraint (1.10) it follows that all fluctuations in g_t must be absorbed by T_t.

□

Proposition 1 illustrates the effect of full risk-sharing on the optimal fiscal policy plan: being consumption and leisure constant along the business cycle, the optimal tax rate is constant as well. The government in the HC uses transfers from the RW to absorb completely the exogenous shock. When g_t is higher than average, the government uses transfers to finance its expenditure; conversely, when g_t is below average, the government uses the proceeds from taxation to pay back transfers received in the past[8]. The RW, which is a risk neutral agent, provides full insurance to the domestic economy.

Limited Commitment

We consider the case in which neither the government in the HC nor the RW can commit to repay external debt. The problem of the Ramsey planner is identical to the one in the previous section, but now conditions (1.2) and (1.3) have to be explicitly taken into account:

$$\max_{\{c_t,l_t,T_t\}_{t=0}^{\infty}} E_0\sum_{t=0}^{\infty}\beta^t u(c_t,l_t)$$

subject to

$$c_t + g_t = (1-l_t) + T_t \tag{1.14}$$

$$E_0\sum_{t=0}^{\infty}\beta^t(u_{c,t}c_t - u_{l,t}(1-l_t)) = u_{c^1,0}(b_{-1}) \tag{1.15}$$

$$E_0\sum_{t=0}^{\infty}\beta^t T_t = 0 \tag{1.16}$$

[8]In Appendix A.1 we study the case in which the utility function is logarithmic in its two arguments. In such a case, it is easy to see that transfers behave exactly as described here.

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \quad (1.17)$$

$$E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} \leq \underline{B} \ \forall t \quad (1.18)$$

Since the participation constraint at time t (1.17) includes future endogenous variables that influence the current allocation, standard dynamic programming results do not apply directly. To overcome this problem we apply the approach described in Marcet and Marimon (2009) and write the Lagrangean as:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [(1+\gamma_t^1) u(c_t, l_t) - \psi_t(c_t + g_t - (1-l_t) - T_t) \\ - \mu_t^1(V_t^a) + \mu_t^2(\underline{B}) - \Delta(u_{c,t}c_t - u_{l,t}(1-l_t)) - T_t(\lambda + \gamma_t^2)] + \Delta(u_{c,0}(b_{-1}))$$

where

$$\gamma_t^1 = \gamma_{t-1}^1 + \mu_t^1$$

$$\gamma_t^2 = \gamma_{t-1}^2 + \mu_t^2$$

for $\gamma_{-1}^1 = 0$ and $\gamma_{-1}^2 = 0$. Δ is the Lagrange multiplier associated to equation (1.15), ψ_t is the Lagrange multiplier associated to equation (1.14), λ is the Lagrange multiplier associated to equation (1.16), μ_t^1 is the Lagrange multiplier associated to equation (1.17) and μ_t^2 is the Lagrange multiplier associated to equation (1.18). γ_t^1 and γ_t^2 are the sum of past Lagrange multipliers μ^1 and μ^2 respectively, and summarize all the past periods in which either constraint has been binding. Intuitively, γ^1 and γ^2 can be thought of as the collection of past compensations promised to each country so that it would not have incentives to leave the contract.

In can be shown that, for $t \geq 1$[9], the solution to the problem stated above is given by time-invariant policy functions that depend on the *augmented* state space $\mathcal{G} \times \Gamma^1 \times \Gamma^2$, where $G = \{g_1, g_2, \ldots, g_n\}$ is the set of all possible realizations of the public expenditure shock g_t and Γ^1 and Γ^2 are the sets of all possible realizations of the costate variables γ^1 and γ^2, respectively. Therefore,

[9]Once again, for $t = 0$ the FOCs of the problem are different. Applying Marcet and Marimon (2009), the problem only becomes recursive from $t \geq 1$ onwards.

$$\begin{bmatrix} c_t \\ l_t \\ T_t \\ \mu_t^1 \\ \mu_t^2 \end{bmatrix} = H(g_t, \gamma_{t-1}^1, \gamma_{t-1}^2) \qquad \forall t \geq 1$$

More specifically, the government's optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1-l_t)) = 0 \tag{1.19}$$

$$u_{l,t}(1+\gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1-l_t)) = 0 \tag{1.20}$$

$$\psi_t = \lambda + \gamma_t^2 \tag{1.21}$$

Other optimality conditions are:

$$c_t + g_t = (1 - l_t) + T_t \tag{1.22}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{1.23}$$

$$\mu_t^1 (E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) - V^a(g_t)) = 0 \tag{1.24}$$

$$\mu_t^2 (E_t \sum_{j=0}^{\infty} \beta^j T_{t+j} - \underline{B}) = 0 \tag{1.25}$$

$$\gamma_t^1 = \mu_t^1 + \gamma_{t-1}^1 \tag{1.26}$$

$$\gamma_t^2 = \mu_t^2 + \gamma_{t-1}^2 \tag{1.27}$$

$$\mu_t^1 \geq 0 \tag{1.28}$$

$$\mu_t^2 \geq 0 \tag{1.29}$$

Two observations are worth mentioning. First, from (1.19), (1.20) and (1.21) it is immediate to see that now the presence of γ_{t-1}^1 and γ_{t-1}^2 makes the allocations state-dependent. Moreover, being γ_{t-1}^1 and γ_{t-1}^2 functions of all the past shocks hitting the economy, the allocations are actually history-dependent. Second, the presence of these Lagrange multipliers makes the cost of distortionary taxation state-dependent. While in the full-commitment case this cost is constant over time and across states, in the limited commitment case it changes depending on the incentives to default that the HC and the RW have[10]. We will discuss this is further detail in section 1.5.

The next proposition characterizes the equilibrium for a logarithmic utility function.

Proposition 2. *Consider a utility function logarithmic in consumption and leisure and separable in the two arguments:*

$$u(c_t, l_t) = \alpha * log(c_t) + \delta * log(l_t) \tag{1.30}$$

with $\alpha > 0$ and $\delta > 0$. Define $t < t'$:

1. *If the participation constraint (1.17) binds such that $\gamma_t^1 < \gamma_{t'}^1$, then $c_t < c_{t'}$, $l_t < l_{t'}$ and $\tau_t > \tau_{t'}$.*

2. *If the participation constraint (1.18) binds such that $\gamma_t^2 < \gamma_{t'}^2$, then $c_t > c_{t'}$, $l_t > l_{t'}$ and $\tau_t < \tau_{t'}$.*

Proof. See Appendix A.2. □

Proposition 2 states the way the allocations and tax rates adjust in order to make the contract incentive-compatible for the HC and the RW. The optimal tax rate decreases whenever constraint (1.17) is binding and increases when constraint (1.18) binds instead[11]. Since the government in the HC has incentives to leave the contract when the government expenditure shock is low, because the the value of the ouside option in that case is high, the model implies a procyclical fiscal policy: the tax rate decreases following a low realization of the public expenditure process and increases when a the realization instead is high. This conclusion is in line with some recent empirical evidence for developing countries (see Ilzetzki and Vegh (2008) and Cuadra and Sapriza (2007)).

1.3 An example of labor tax-smoothing

In order to understand better the impact of limited commitment on the ability of the government to smooth taxes, in this section we analyze the case study of a perfectly anticipated government expenditure shock. Suppose that government expenditure is

[10]It can be shown that, in the full commitment case, this cost is given by Δ, while in the limited commitment one is determined by $\frac{\Delta}{1+\gamma_t^1}$.

[11]Notice that it cannot be the case that the two participation constraints bind at the same time.

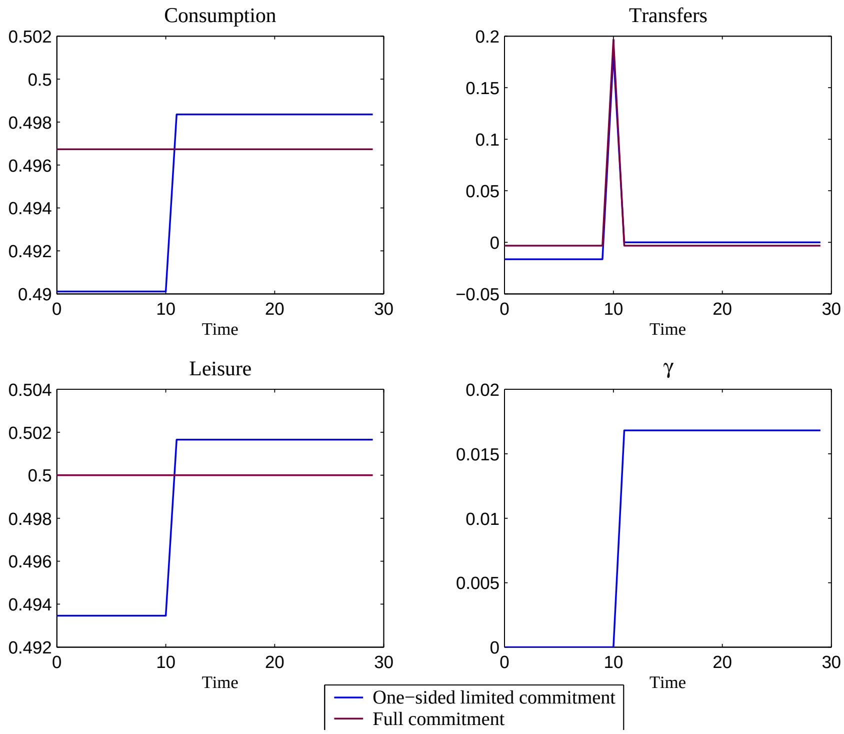

Figure 1.1: Example: $g_t = 0$ for $t \neq T$, and $g_T > 0$

known to be constant and equal to 0 in all periods except in T, when $g_T > 0$. In order to simplify the analysis, throughout this section we assume that only the HC can default, while the RW has a commitment technology. Moreover, we assume that $b_{-1} = 0$ and that households have a logarithmic utility function as (1.30).

Since equilibrium allocations depend on γ_t^1, understanding the dynamics of the incentives to default is crucial. The next proposition states that, given the assumptions previously made, the participation constraint (1.17) binds only at $t = T + 1$[12].:

Proposition 3. *Suppose that government expenditure is known to be constant and equal to* 0 *in all periods except in* T*, when* $g_T > 0$*. Assume further that* $b_{-1} = 0$*. Then, the participation constraint* (1.17) *binds at exactly* $T + 1$*.*

Proof. See Appendix A.3. □

From the results of Proposition 2 we can characterize the allocations for $t < T+1 \leq t'$. Given that $\gamma_t^1 < \gamma_{t'}^1$, it follows that $c_t < c_{t'}$, $l_t < l_{t'}$ and $\tau_t > \tau_{t'}$[13]. The limited commitment by the government exerts a permanent effect on the tax rate and alters

[12] The reader may wonder why the participation constraint binds just after the shock. The reason is that agents know the bad shock will happen in T, so this decreases the outside option value in every period before the shock effectively takes place. Once the shock is over, the autarky value goes up.

[13] In Appendix A.5 we show that $\Delta < 0$ in this case.

Table 1.3: Parameter values

Preferences	$\alpha = \delta = 1$	
Intertemporal discount factor	$\beta = 0.98$	
Government expenditure	T = 10	$g_T = 0.2$

its entire dynamics, since the tax rate level after the shock is permanently lower than before the shock.

The intuition for this result is as follows. Since at $T + 1$ the continuation value of staying in the contract has to increase in order to prevent default, utility of households in the HC has to increase. By the intratemporal optimality condition, a positive tax rate implies that the marginal utility of consumption is higher than the marginal utility of leisure. Therefore, increasing consumption is relatively more efficient than increasing labor and, as a consequence, the tax rate decreases.

a. The example in numbers

In this section we solve numerically the example depicted above. Table 1.3 contains the parameters values used in the simulation.

Figures 1.1 and 1.2 show the evolution of the allocations c_t, l_t, the tax rate τ_t, international capital flows T_t, domestic bonds b_t and the costate variable γ_t^1. We compare the allocations with limited commitment to the ones under full commitment by the government towards the RW. There are two forces determining the dynamics of the economy. On one side the government has to finance the higher and expected expenditure outflow at T in the most efficient way; on the other, the participation constraint has to be satisfied. For $t \leq T$ the higher and expected shock at T keeps the continuation value of autarky low, and for this reason leaving the contract is not optimal. Therefore, before T the government accumulates assets towards the RW, and uses them to finance part of the high expenditure outflow in T. The remaining part is covered both through tax revenues and through transfers from the RW. After the high shock has taken place, the outside option value increases. In order to prevent default, the government lowers the tax rate to allow domestic households to enjoy a higher level of consumption and leisure. Moreover, from $T + 1$ onwards, the transfer with the RW adjusts to guarantee that the expected present discounted value of net international flows is zero.

Notice the difference with the full commitment scenario, where the allocations are constant and transfers absorb completely the shock. The high inflow in period T is repaid forever by the government through small outflows after the shock. Taxes remain constant *even in period* T and do not react to the shock at all. The limited commitment

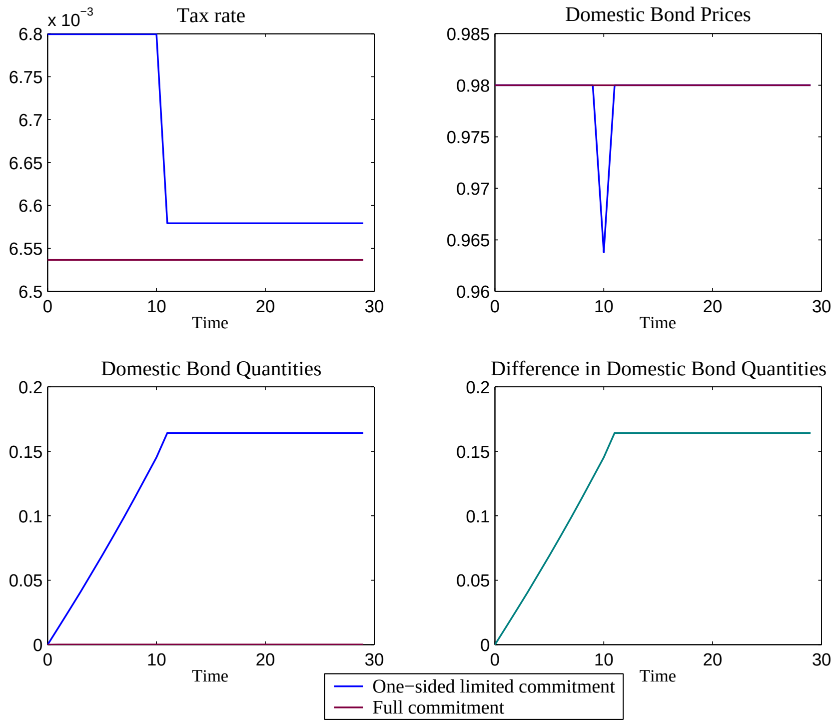

Figure 1.2: Example: $g_t = 0$ for $t \neq T$, and $g_T > 0$

technology constraints the amount of insurance offered by international capital markets, and perfect risk-sharing among countries is no longer possible. Consequently, the negative expenditure shock has to be absorbed through external debt and higher tax revenues in the initial periods.

1.4 Numerical results

In section 1.2 we have characterized the equilibrium allocations arising from the Ramsey planner's maximization problem under two different scenarios. First, we studied the case in which the two parts in the contract have full commitment. In this case, we have seen that the risk-neutral households of the RW fully insure the HC and, consequently, consumption, labor and tax rates are perfectly constant in the RW. When both parts have limited commitment, however, it is no longer possible to do perfect risk-sharing and the allocations are no longer constant.

In this section we proceed to solve the model numerically assuming that the government spending follows an $AR(1)$ process. We calibrate the parameters of this process to the argentinean economy. We use quarterly series of current government expenditure net of interest payments plus gross government investment as our measure of government expenditure for the period 1993-I to 2005-IV.

Table 1.4: Parameter values

Preferences	$\alpha = \delta = 1$
Intertemporal discount factor	$\beta = 0.98$
Government expenditure process	$g_t = g * + \rho^g g_{t-1} + \epsilon_t$
$g*$	$0.1820 * 0.33$
ρ^g	0.9107
σ_g^2	$0,1320 * 0.0607$
$\underline{B}$	0.031
$b_{-1} = b_{-1}^G$	0

Given that we need to calibrate the process for government expenditure, we estimate an AR(1) process in levels for the argentinean data. We find that, for the broader measure of real government expenditure, $\hat{\rho} = 0,9107$ for a specification as

$$g_t = \alpha + \rho g_{t-1} + \epsilon_t$$

We also need to obtain a value for the variance of the shock associated to g_t. Since the variance of g_t and, similarly, of ϵ_t are influenced by the units in which government expenditure is measured, in order to make meaningful international comparison across countries we need to find a statistic that is not influenced by neither the currency in which expenditure is denominated nor the size of the government itself. We therefore use the coefficient of variation (CV), defined as

$$CV = \frac{\text{Std. Dev}}{\text{Mean}}$$

In the data for Argentina, $CV = 0,1320$. We estimate the mean of g_t as the value of g_t in steady state, given the mean of $\frac{g_t}{GDP_t}$ in the data. This value is $\overline{\frac{g}{GDP}} = 0,182$. Since our problem does not have a well defined steady state, we consider, as others in the literature, that $1 - l_t = \frac{1}{3}$ in steady state. Then $\overline{\frac{g}{GDP}} = \overline{\frac{g}{1-l}} = 0,182$. Therefore $\overline{g} = 0,33*0,182 = 0,0607$. Finally, the variance of $g_t = (0,1320*0.0607)^2 = 0,0000641$. We obtain the variance of ϵ_t in the following way:

$$\sigma_\epsilon^2 = \sigma_g^2(1 - \rho^2)$$

Figures 1.3, 1.4 and 1.5 show the allocations, co-state variables and fiscal variables respectively for the calibrated government expenditure shock, for the case in which both the government and the international institution have limited commitment (blue line). For comparison purposes, we show the same variables under full commitment

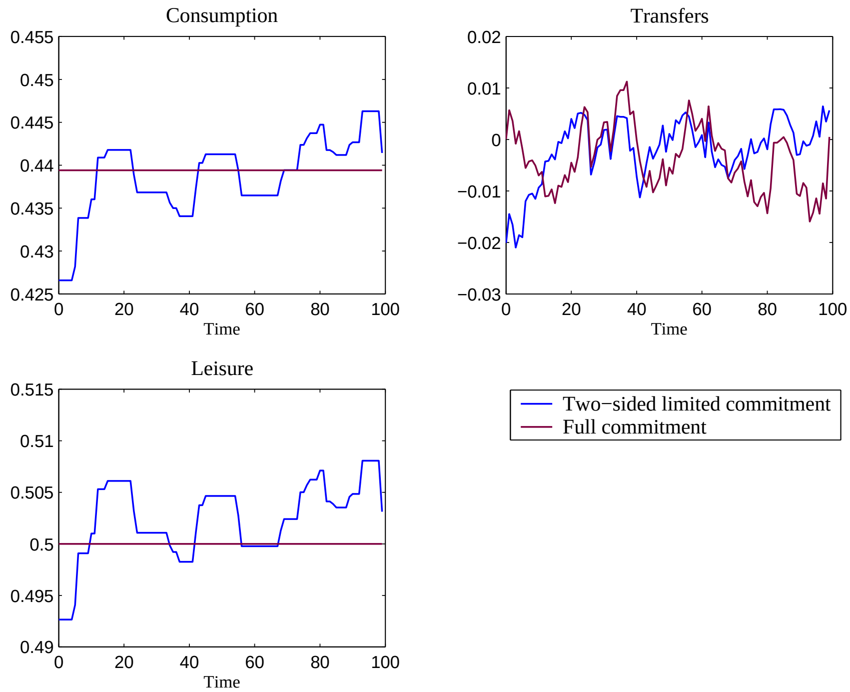

Figure 1.3: Two-sided limited commitment - Allocations

(red line). Compared to the case in which international flows allow the government to smooth completely the distortion in the consumption-leisure choice, under partial commitment the current realization of the shock influences the equilibrium. Tables 1.5 and 1.6 summarize some statistics for the allocations and the fiscal variables for the cases of full and limited commitment.

Table 1.5: Statistics of allocations for the first 50 periods

	Partial Comm.			Full Comm.		
	Mean	St.Dev.	Autocorr.	Mean	St.Dev.	Autocorr.
consumption	.43	.004	.92	.44	0	1
leisure	.5	.003	0.6	.92	0	1
labor tax rate	.13	.001	.92	.12	0	1
international flows	-.005	.006	.86	-.01	.003	.7

Three observations are worth making. First, while the average values of the allocations are roughly the same in the two frameworks, their variance is much higher under partial commitment. Second, the allocations under full commitment are uncorrelated with the government expenditure shock; however, under partial commitment this correlation is negative. The reason is that an increase in public consumption induces the standard crowding out effect to operate and, additionally, the participation constraint of the

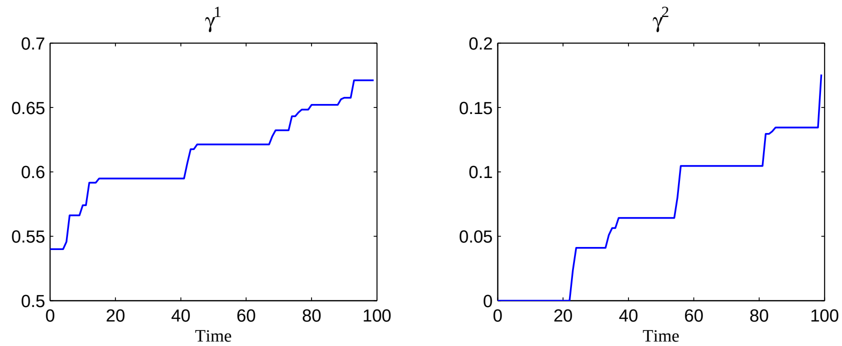

Figure 1.4: Two-sided limited commitment - Co-state variables

Table 1.6: Correlation with past promises

	Partial Comm.		Full Comm.	
	$Corr(x, \gamma^1)$	$Corr(x, g)$	$Corr(x, \gamma^1)$	$Corr(x, g)$
consumption	.5	-48	-	0
labor tax rate	-.97	.36	-	0

RW to be binding. This second channel reinforces the decrease in private consumption due to the first effect. Third, there is a positive correlation between consumption in the HC and the Lagrange multiplier associated to the participation constraint of the HC. The same is true for the correlation between international flows and the incentives to default. In periods in which it is optimal for the government of the HC to leave the contract, it receives a positive amount of transfers from abroad so as to adjust the continuation value of staying in the contract upwards to equate it with the continuation value associated to the outside option. Thus, default does not take place in equilibrium.

1.5 Borrowing constraints

In this section we show that it is possible to reinterpret the problem depicted in the previous sections as one in which the the HC and the RW trade one-period state-contingent bonds in the international financial market, but their trading is limited by borrowing constraints. To do so, we follow the strategy proposed by Alvarez and Jermann (2000) and Abraham and Cárceles-Poveda (2009)[14]. We show that, if we impose limits only on international borrowing, the allocations in the two setups do not coincide. An ad-

[14]In the Appendix we show that the government's problem coincides with the one of an international institution in charge of distributing resources among the HC and the RW, taking into account the aggregate resource constraint, the implementability condition of the HC, and the fact that countries have limited commitment. Therefore, the problem laid out in section 1.2 can be thought of one in which a central planner determines the constrained efficient allocations.

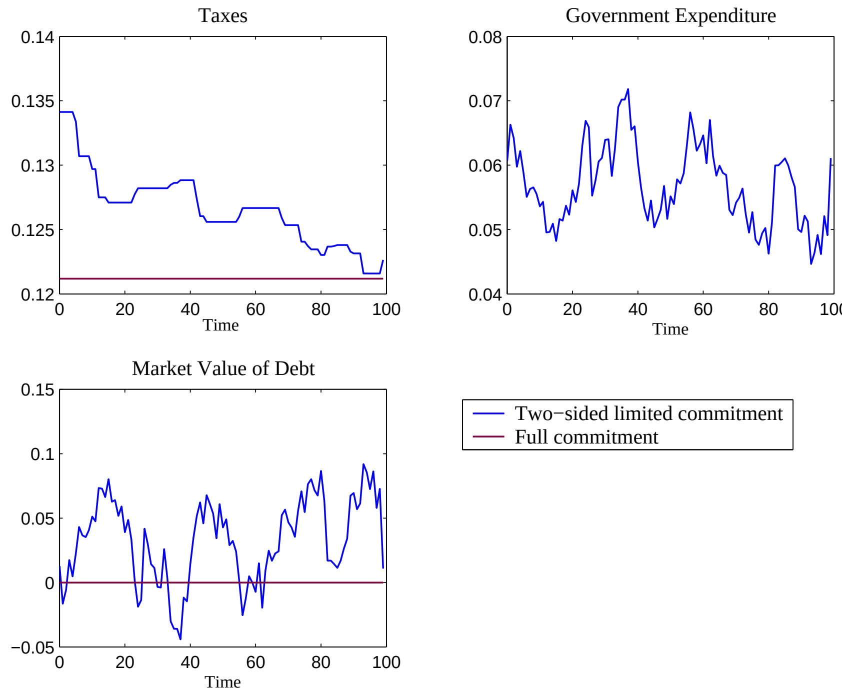

Figure 1.5: Two-sided limited commitment - Fiscal variables

ditional constraint on the value of domestic debt that the government of the HC can issue is required.

In what follows, we will denote with a superscript 1 variables corresponding to the HC, and with superscript 2 variables corresponding to the RW. $Z_t^1(g_{t+1})$ is the international bond bought at t by the government of the HC contingent on next period's realization of the government expenditure shock. Symmetrically, call $Z_t^2(g_{t+1})$ the international bond bought at t by households of the RW contingent on next period's realization of the government expenditure shock. Denote the price of these bonds by $q_t(g_{t+1})$, and assume that there are lower bounds, denoted by $A_t^1(g_{t+1})$ and $A_t^2(g_{t+1})$, on the amount of bonds that the government of the HC and the households in the RW can hold, respectively.

In the current setup, the problem of the households in the HC is exactly identical to the one described in section b., so we do not reproduce it here. The problem of the government in the HC is slightly different from the one in previous sections. In order to finance its public expenditure, in addition to distortionary taxes on labor income and domestic bonds, now the government has available one-period state-contingent bonds traded with the RW. The budget constraint of the government is:

$$g_t + \sum_{g^{t+1}|g^t} Z_t^1(g_{t+1}) q_t(g_{t+1}) - Z_{t-1}^1(g_t) = \tau_t(1-l_t) + \sum_{g^{t+1}|g^t} b_t(g_{t+1}) p_t^b(g_{t+1}) - b_{t-1}(g_t) \quad (1.31)$$

The government faces a constraint on the amount of debt that can issue in the international financial market:

$$Z_t^1(g_{t+1}) \geq A_t^1(g_{t+1}) \tag{1.32}$$

The problem of households in the RW that trade bonds with the government in the HC now is

$$\max_{\{c_t^2, T_t^2\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t c_t^2$$

$$y + Z_{t-1}^2(g_t) = c_t^2 + \sum_{g_{t+1}} q_t(g_{t+1}) Z_t^2(g_{t+1}) \tag{1.33}$$

$$Z_t^2(g_{t+1}) \geq A_t^2(g_{t+1}) \tag{1.34}$$

Notice that the RW is also constraint in the amount of debt it can trade with the RW. The optimality conditions of this problem are equation (1.33) and

$$q_t(g_{t+1}) = \beta(1 + \gamma_t^2)\pi(g^{t+1}|g^t) \tag{1.35}$$

$$\gamma_t^2(Z_t^2(g_{t+1}) - A_t^2(g_{t+1})) = 0$$

$$\gamma_t^2 \geq 0$$

where γ_t^2 is the Lagrange multiplier associated to the borrowing constraint (1.34).

Definition 2. *A competitive equilibrium with borrowing constraints is given by allocations* $\{c^1, c^2, l\}$*, a price system* $\{p^b, q\}$*, government policies* $\{g, \tau, b\}$ *and international bonds* $\{Z^1, Z^2\}$ *such that:*

1. *Given prices and government policies, allocations* c *and* l *satisfy the HC household's optimality condition* (1.4), (1.5) *and* (1.6).
2. *Given the allocations and prices, government policies and bonds* Z^1 *satisfy the sequence of government budget constraints* (1.31) *and borrowing constraints* (1.32).
3. *Prices* q *and bonds* Z^2 *satisfy the RW optimality conditions* (1.35) *and* (1.34).

4. Allocations satisfy the sequence of feasibility constraints:

$$c_t^1 + g_t + \sum_{g^{t+1}|g^t} Z_t^1(g_{t+1}) q_t(g_{t+1}) = 1 - l_t + Z_{t-1}^1(g_t) \tag{1.36}$$

$$c_t^2 + \sum_{g^{t+1}|g^t} Z_t^2(g_{t+1}) q_t(g_{t+1}) = y + Z_{t-1}^2(g_t) \tag{1.37}$$

5. International financial markets clear:

$$Z_t^1(g_{t+1}) + Z_t^2(g_{t+1}) = 0$$

We need to specify borrowing constraints that prevent default by prohibiting agents from accumulating more contingent debt than they are willing to pay back, but at the same time allow as much risk-sharing as possible. Define first

$$V_t^1(Z_{t-1}^1(g_t), g_t) = u(c_t^1, l_t) + \beta E_{t|g_t} V_{t+1}^1(Z_t^1(g_{t+1}), g_{t+1})$$

$$V_t^2(Z_{t-1}^2(g_t), g_t) = c_t^2 + \beta E_{t|g_t} V_{t+1}^2(Z_t^2(g_{t+1}), g_{t+1})$$

Then we define the notion of borrowing constraints that are not too tight:

DEFINITION 3. *An equilibrium has borrowing constraints that are not too tight if*

$$V_{t+1}^1(A_t^1(g_{t+1}), g_{t+1}) = V_{t+1}^a \ \forall t \geq 0, g_{t+1}$$

and

$$V_{t+1}^2(A_t^2(g_{t+1}), g_{t+1}) = \underline{B} \ \forall t \geq 0, g_{t+1}$$

where V_{t+1}^a *and* $\underline{B}$ *are defined as in section a..*

We continue to assume that the government of the HC behaves as a benevolent Ramsey Planner and chooses tax rates, domestic and international $\{\tau_t, b_t, Z_t^1\}_{t=0}^{\infty}$ in order to maximize the representative household's life-time expected utility, subject to the constraints imposed by the definition of competitive equilibrium with borrowing constraints. We can write the problem of the government as

The problem of the government in the HC is

$$\max_{\{c_t^1, l_t, Z_t^1\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^1, l_t) \tag{1.38}$$

s.t.

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{c^1,t} c_t^1 - u_{l,t}(1-l_t)) = b_{-1} u_{c^1,0} \qquad (1.39)$$

$$1 - l_t + Z_{t-1}^1(g_t) = c_t^1 + g_t + \sum_{g_{t+1}} q_t(g_{t+1}) Z_t^1(g_{t+1}) \qquad (1.40)$$

$$Z_t^1(g_{t+1}) \geq A_t^1(g_{t+1}) \qquad (1.41)$$

Proposition 4. *When the only borrowing constraints imposed on the competitive equilibrium are* (1.32) *and* (1.34), *the allocations satisfying* (1.19)-(1.21) *do not solve the government's problem* (1.38)-(1.41).

Proof. The proof is immediate. Taking the first order conditions

$$u_{c_t^1} - \Delta(u_{c_t^1} c_t^1 + u_{cc_t^1} c_t^1 + u_{c^1 l,t}(1-l_t)) = \lambda_{1,t} \qquad (1.42)$$

$$u_{l_t} - \Delta(u_{c^1 l,t} c_t^1 + u_{l,t} l_t + u_{ll,t}(1-l_t)) = \lambda_{1,t} \qquad (1.43)$$

Clearly, the allocations satisfying equations (1.42)-(1.43) cannot coincide with the solution of the system of equations (1.19)-(1.20), because in the latter case the weight attached to the term $u_{c_t^1} c_t^1 + u_{cc_t^1} c_t^1 + u_{c^1 l,t}(1-l_t)$ is constant and equal to Δ, while in the former it is given by $\frac{\Delta}{1+\gamma_t^1}$ and varies over time. □

This proposition states that the economy with transfers cannot be reinterpreted as an economy in which there are international bond markets and limits to international debt issuance only.

From the proof of the proposition, it is again evident what has already been pointed out in section e.. When there is full commitment, the cost of distortionary taxation is given by the Lagrange multiplier associated to the implementability constraint, Δ. This cost is constant due to the presence of complete bond markets. However, when we relax the assumption of full commitment and consider instead the case in which the government of the HC has limited commitment, this cost becomes state-dependent and is given by $\frac{\Delta}{1+\gamma_t^1}$. The reason for this is that now the government faces endogenously incomplete international bond markets. Since allocations and tax rates vary permanently every time the participation constraint of the HC binds, so does the burden of taxation.

The previous discussion leads us to impose borrowing constraints on the value of domestic debt in addition to the constraints on international debt[15]. The next proposition states that, in this case, it is possible to establish a mapping between the economy with transfers and the one with borrowing constraints on domestic as well as international debt:

[15] Sleet (2004) also defines a borrowing constraint in terms of the value of debt.

Proposition 5. *Suppose that, in addition to constraints* (1.32) *and* (1.34)*, we impose a lower bound on the value of state contingent domestic debt (expressed in terms of marginal utility)*

$$b^1_{t-1}(g_t)u_{c^1,t} = E_t \sum_{j=0}^{\infty} \beta^j (u_{c^1,t+j}c^1_{t+j} - u_{l,t+j}(1 - l_{t+j})) \leq B_{t-1}(g_t) \tag{1.44}$$

Then the allocations solving the system (1.19)-(1.21) *also solve the government's problem* (1.38)-(1.41).

Proof. See Appendix A.6 □

This result provides a rationale for our specification of the outside option of the government in the HC. In section 1.2 we assumed that if the government defaulted, it would lose access to the international and domestic bond markets and would remain in financial autarky thereafter. It seems natural then to impose constraints on the amount of debt that it can issue in both markets.

1.6 Stylized Facts

In this section we want to check if in the data tax rate volatility is affected by the availability of external sources to finance domestic shocks. We use the Emerging Markets Bond Index (EMBI) to measure the degree of insurance against internal shocks governments in emerging markets can get offshore. EMBI tracks total returns for traded external debt instruments, and gives a measure of the riskiness of the sovereign bonds issued by a country. We compute the annual average of EMBI for 6 emerging economies (Argentina, Mexico, Nigeria, Venezuela, Panama, Peru) from 1995 until 2001 and we look at the relationship with the average tax rate volatility referring to the same period.[16][17] The idea is that the higher the (mean) EMBI specific to a country, the higher the perceived investment risk that investors from abroad associate to that country, and the lower the amount of international flows the country can use to hedge against government revenues shocks.

Figure 1.6 plots this relationship. The horizontal axis measures tax rate standard deviation, and the vertical axis measures the EMBI. The graph shows that the higher the EMBI the more the government has to vary taxes to satisfy its budget constraint: in a sense the government can rely less on international debt to minimize the taxation distortion.

[16] We restrict our analysis to this period and to these countries for a question of availability of data.

[17] We consider a broad measure of tax rate as we define it as the ratio between total government revenues over GDP.

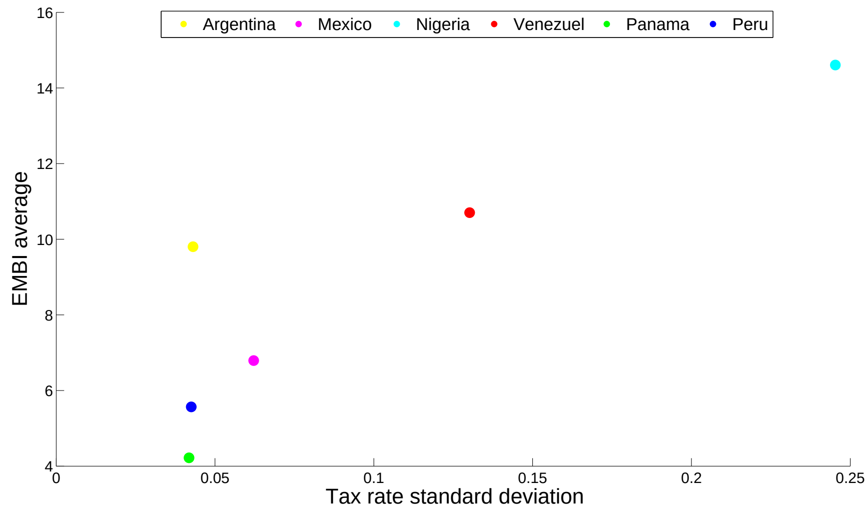

Figure 1.6:

Although it supports the main result of this chapter, the evidence we suggest is very preliminary. Data on fiscal variables for emerging economies are difficult to obtain and, when available, the series are either very short or they are not always reliable. For this reason we can not perform any time series analysis. Apart from this problem, what really matters for tax rate variability in the model is the availability of international lending/borrowing and it is not obvious how to measure this variable. As we use EMBI, which refers to emerging markets, we cannot run any cross-sectional estimation as there are too few observations. Nevertheless using some other indicator for limited commitment across countries would allow us to address the empirical estimation of the model in a more formal way. We leave this issue as future research.

1.7 Conclusions

A key issue in macroeconomics is the study of the optimal determination of the tax rate schedule when the government has to finance (stochastic) public expenditure and only has available distortionary tools[18]. Under this restriction, a benevolent planner seeks to minimize the intertemporal and intratemporal distortions caused by taxes. Since consumption should be smooth, a general result is that taxes should also be smooth across time and states.

When considering a small open economy that can borrow from international risk-neutral lenders and both parts can fully commit to repay the debt, this result is amplified because there is perfect risk-sharing. Consumption and leisure are perfectly flat, thus

[18]When the government can levy non-distortionary taxes, such as lump-sum taxes, the Ricardian Equivalence holds and the first best can be achieved.

the tax rate is also flat. The domestic public expenditure shock is absorbed completely by external debt and there is no role for internal debt. In light of this result, one would expect small open economies to have less volatile tax rate schedules than large economies. However, the available data seems to contradict this intuition.

When we relax the assumption of full commitment from both the small open economy and from international lenders towards their international obligations, perfect risk-sharing is no longer possible. The presence of limited commitment hinders the ability of the government to fully insure against the public expenditure shock through use of international capital markets. Consequently, the government has to resort to taxes and internal debt in order to absorb part of the shock, and it is no longer possible to have constant allocations and tax rates.

Our simulation results show that the volatility of the tax rate increases substantially when there is limited commitment. Moreover, fiscal policy is procyclical: when the government expenditure shock is low, the country has incentives to leave the contract with the international institution. Therefore, taxes should decrease in order to allow consumption and leisure to be higher and, in this way, increase household utility. On the contrary, when the government expenditure is high, taxes need to be high as well to repay the external debt.

These two features of our model are in line with what we observe in the data for developing countries, which are the ones more likely to lack a commitment technology towards international obligations. In section 1.6 we confirm this by looking at some stylized facts regarding fiscal variables, public debt and sovereign risk. We find that there seems to be a positive relation between the volatility of the average tax rate and that of a measure of the country risk premium.

The results presented in this chapter suggest that the volatility and cyclicality of tax rates observed in developing countries is not necessarily an outcome of reckless policy-making, as one could think a priori. We have shown that, in order to establish the optimal fiscal policy plan in small developing countries, it is important to take into account the degree of commitment that the economy has towards its external obligations, as this element is crucial in determining the extent of risk-sharing that can be achieved.

A question that remains unanswered is where the allocations converge in the long run. It could be the case that the participation constraints continue to bind in some states of nature, even in the long run. In that case γ^1 and γ^2 diverge to infinity. The other possibility is that the economy arrives to a point in which neither the HC nor the RW has further incentives to leave the contract, and from that moment onwards c, l, τ, γ^1 and γ^2 remain constant. In that case, there is partial risk-sharing only in the short-run. This is an issue that we plan to address in the future.

Finally, the theoretical results outlined in the chapter suggest a new mechanism to check in the data for developing countries. Our claim is that governments that are suspected to have commitment problems in repaying their debt necessarily have to set more volatile taxation schemes than more reliable ones. We have presented some very preliminary evidence that this might be supported by the data, but evidently a deeper analysis is called for.

2 Seigniorage and distortionary taxation in a model with heterogeneous agents and idiosyncratic uncertainty

2.1 Introduction

The seminal papers by Friedman (1969) and Phelps (1973) opened a wide debate in the last decades over the issue of the optimal monetary and fiscal policy combination in representative agent frameworks. Friedman argued that optimality required setting the nominal interest rate to zero, so that the return on money holdings was equated to the return on any other interest-bearing nominal asset. This is known in the literature as the *Friedman rule*. Phelps, on the other hand, indicated that in economies in which lump-sum taxes are not available, the policy maker should tax all goods, including money. Moreover, since the money demand function is typically more inelastic than the demand for consumption goods, Phelps concluded that money should be taxed heavily. This apparent contradiction in the optimal policy prescription motivated some classical papers such as Chari et al. (1996) and Correia and Teles (1996) among many others. With some exceptions, the general consensus appears to be that Central Banks should follow the Friedman rule.

However, by construction all these early contributions overlook issues of heterogeneity and redistribution. By working with the representative agent assumption, their analysis of optimal policy focuses only on efficiency in distorting relative prices. Therefore, a crucial aspect of inflation is neglected, which is the fact that it does not affect all individuals in the same way.

In this chapter we tackle this issue by building a heterogeneous agent model with uninsurable idiosyncratic shocks to labor productivity. We consider the problem of a benevolent government that has to finance an exogenous and constant stream of public expenditure. The available instruments are the inflation tax and a tax on labor income and, given that labor supply is endogenous, both instruments are distortionary. We look for the determination of the optimal monetary and fiscal policy mix in such an environment, assuming the government assigns equal weight to all agents in the economy[1].

Since we are interested in identifying and studying the effects of inflation over individuals with different income and wealth profiles, we need a model in which agents differ in these two dimensions. In order to assess the effect exerted by inflation on the incentives of households to consume, work and save, and consequently on aggregate long-run capital and output, we depart from the complete-markets assumption by assuming that

[1]Examples of other papers that use the same social welfare function are Domeij and Heathcote (2004), Floden (2001), Floden and Linde (2001) and Heathcote et al. (2008a).

agents cannot insure against their idiosyncratic shocks and are subject to a borrowing constraint. Finally, previous literature has pointed to the fact that inflation can be regarded as a regressive tax on consumption[2]. We introduce this in the analysis by means of a transaction technology alternative to money in which richer, more productive agents have comparative advantages relative to poorer, less productive ones. In addition, we exacerbate these advantages by assuming easier access to the transaction technology for more productive agents.

In the setup we propose there are two effects from inflation that are not present in representative-agent frameworks. On the one hand, more productive agents, by having easier access to alternative transaction technologies, can shelter better from inflation. This shifts the burden of taxation from richer, more productive agents to poorer households, thus benefiting the former group. If the planner cares sufficiently for poor agents, this effect goes in favor of the Friedman rule. On the other hand, in economies in which households cannot insure against their idiosyncratic shocks, inflation amplifies the motive for precautionary savings. When the bad shock hits and the individual is very poor (i.e., is close to hitting the borrowing constraint) the inflation tax reduces consumption and leisure and, consequently, utility, thus creating an incentive to save. The higher savings level translates into higher capital in steady state, higher wages and lower labor tax rates. By this means, a higher level of inflation increases welfare of poor, low productivity households that rely almost entirely on labor income. When the government cares about poor households, this effect goes against the Friedman rule. Considering these two effects jointly, we see that they operate in opposite directions. Deviating from the Friedman rule assures poor agents a higher labor income, while middle-class and rich agents have to endure lower levels of capital income. Nevertheless, they are benefited by a reduction in their tax burden associated to the increase in the inflation tax which, as explained before, is a regressive tax.

There is a distortion associated to inflation that is already present in environments with a representative agent, which is related to the uniform taxation argument from the public finance literature. When consumption goods can be bought either with cash or with an alternative transaction technology, deviating from the Friedman rule implies taxing the goods bought with cash more than the rest of the goods. If all goods enter in the utility function of the household in identical manner, this is not efficient[3].

From the previous discussion it can be concluded that, in economies with uninsurable idiosyncratic uncertainty and heterogeneous agents, the determination of the optimal monetary and fiscal policy mix remains a quantitative question. In order to provide a suitable answer, we calibrate the model economy to match some selected statistics of the U.S. economy. Some key targets are the correlation between money and asset holdings,

[2]See Erosa and Ventura (2002).

[3]To be more precise, for this argument to hold the utility function has to be separable in consumption and leisure and the subutility over consumption goods has to be homothetic.

the fraction of consumption expenditures made with cash and the Gini coefficient of the asset distribution. We define the benchmark economy to be one that displays an annual rate of inflation of 2%[4].

Given our parameterization we find that for a utilitarian planner[5] the Friedman rule is optimal despite the introduction of heterogeneity and uninsurable idiosyncratic risk. The aforementioned beneficial effects of inflation on welfare are not sufficiently large, from a quantitative point of view, to offset its detrimental effects. Thus it is optimal to set the nominal interest rate equal to zero. Next, we perform a welfare analysis comparing the benchmark economy to an economy in which the Friedman rule is implemented. we find that the aggregate welfare gains of switching from the benchmark policy regime to the optimal one are rather small. The percent of life-time consumption that agents in the benchmark economy are prepared to give up to get the policy change is 0.51%. Following Floden (2001) we decompose these welfare gains into gains from increased levels of consumption, reduced uncertainty and reduced inequality. We find that most of the gains are due to increased levels of consumption and, to a lesser extent, to reduced uncertainty.

Despite these seemingly small aggregate welfare gains, the individual welfare gains and loses can be very large. A surprising finding is that poor, less productive agents are net losers from the policy change[6]. When inflation decreases, so does aggregate capital and, with it, wages. This effect can be very harmful for these agents: we find that, for the poorest individual, consumption should decrease permanently by about 4% in the benchmark economy for him to be indifferent between living in any of the two worlds proposed. On the contrary, middle-class and rich individuals are net winners from the change in regime. Rich, low- productivity households should obtain a permanent increase in consumption of around 4% to be indifferent between the two regimes. These large individual effects cancel out almost completely in the aggregate, thus yielding the small overall effects described before.

Although this chapter is not the first one to look at inflation in heterogeneous-agent environments, it is, to our knowledge, the first one that identifies in a unified framework different effects of inflation that had been described separately and derives the optimal policy within such framework.

The contributions of Erosa and Ventura (2002) and Algan and Ragot (2006) study the redistributive aspects of inflation. Both papers point out different mechanisms through which inflation affects the agents' welfare depending on their level of wealth and labor productivity, namely, that inflation acts as a regressive tax on consumption and that it stimulates savings. They reach contradictory conclusions: while the former states that

[4]This is a reasonable annual inflation target for the Fed.

[5]A utilitarian planner is one that assigns equal weights to all households in the computation of the social welfare function.

[6]See Algan and Ragot (2006) for a similar result.

inflation is relatively more harmful to poorer households, the latter claims that the higher level of capital in steady state translates into higher wages and higher welfare for poor, labor-income dependent individuals. Neither of these papers addresses the issue of inflation from a normative point of view.

There are a number of papers that deal with the determination of the optimal inflation rate when taking into account issues of heterogeneity. Akyol (2004) studies an endowment economy in which only high productivity households hold money in equilibrium. Seigniorage revenues can be used to finance redistributive (anonymous) transfers and interest payments on government debt. Therefore, the main role of inflation is to redistribute resources from agents with high endowment shocks to those holding bonds, which improves risk-sharing. The author finds that the optimal inflation rate is of about 10%. Although in this chapter not all high-productivity agents are also rich, the correlation between wealth and productivity is very high. Then, the idea that richer agents are the ones holding money in equilibrium is at odds with some stylized facts on transactions and cash holdings reported in Erosa and Ventura (2002), which indicate the opposite. In this chapter, we construct the model such that its predictions in terms of cash holdings and proportion of purchases made with cash are in line with what the data suggests.

Albanesi (2005) and Bhattacharya et al. (2007) propose frameworks in which agents are ex-ante heterogeneous and there is no idiosyncratic uncertainty. In particular, they assume there are two types of households in the economy. While in Albanesi (2005), agents of different types have different labor productivities, Bhattacharya et al. (2007) assume that agents differ in the marginal utility they derive from real money balances. Albanesi (2005) finds that it may be optimal to depart from the Friedman rule, depending on the weight that the government assigns to each type of agent. On the other hand, Bhattacharya et al. (2007) conclude that, because inflation redistributes resources from one type of agent to the other, both types may benefit if the central bank deviates from the Friedman rule. Both papers abstract from capital, but given the lack of idiosyncratic uninsurable risk, monetary policy would not affect the long-run capital stock. This is a crucial element that we include in our analysis.

Some papers in the search literature study the implications of anticipated inflation, as for example Lagos and Rocheteau (2005) and Bhattacharya et al. (2005). These studies view monetary policy as a mechanism to induce agents to exert the correct amount of search effort. Search effort is related to the quantity of output produced and, consequently, welfare. The scope of this literature is substantially different from ours. Although we do regard money as a means of payment, our interest in inflation is related to the fact that it distorts the consumption, leisure and savings decisions. Thus, we do not address in detail how money facilitates transactions and to what extent monetary policy can enhance this role but, rather, focus on the effects of money growth on allocations.

Finally, da Costa and Werning (2008) propose a framework in which agents have private information on their labor productivity. The analysis abstracts from idiosyncratic risk by assuming that differences in productivity are permanent. Moreover, agents do not hold capital. The authors assume that money and work effort are complements, so that the demand for money, conditional on the expenditure of goods, weakly increases with the amount of work effort. They find that the Friedman rule is optimal if labor income is positively taxed. The reason for this result is that deviating from the Friedman rule does not aid the planner in designing a mechanism to ensure that individuals do not underreport their productivity, i.e., it does not help relaxing the incentive-compatibility constraints in the planner's problem. Although this study and ours reach similar conclusions in terms of policy prescription, the reasons behind this result are very different in the two setups. We assume the government cannot observe an agent's productivity by restricting the set of fiscal instruments to an anonymous tax on labor, so in our framework agents do not have incentives to underreport by construction. Instead, we focus on the effects of inflation on households that differ in wealth as well as labor productivity.

The remainder of the chapter is organized as follows. Section 2 describes the model. In section 3 we discuss the calibration strategy. Section 4 contains a description of the different effects operating in the model and shows the results in terms of optimal policy. In section 5 we perform the welfare analysis. Finally, section 6 concludes and discusses lines for future research.

2.2 The model

The model is close to Erosa and Ventura (2002) but with two important differences. First, in our model the labor supply is endogenous. Second, we introduce the idea that more productive agents have easier access to transaction technologies alternative to the use of cash, with respect to less productive agents. These two modifications to the basic setup alter completely the analysis of the effects of inflation over different population sectors. We defer the discussion of these effects until the next section.

Households face idiosyncratic labor productivity shocks, that we denote by ε_t. There is no aggregate uncertainty. Consequently, the economy is at its steady state and all aggregate real variables remain constant. For simplicity, we will omit the time subscript from aggregate real variables.

a. Households

The economy is populated by a continuum of mass 1 of ex-ante identical and infinitely lived households. Households are endowed with one unit of time each period and derive utility from consumption of final goods and leisure.

Markets are incomplete, in the sense that it is not possible to trade bonds which payoffs are contingent on the realization of the idiosyncratic shock. Moreover, we assume that agents cannot borrow. Consequently, agents can only save by holding one-period riskless assets and money. We denote by W_t the total nominal wealth an individual has in period t, where W_t is the sum of total money holdings M_t and assets A_t.

Agents consume a continuum of final goods indexed by $j \in [0,1]$. We assume that the consumption aggregator, denoted by c, takes the form $c = \inf_j \{c(j)\}$[7]. The choice of the aggregator implies that agents consume an equal amount of each good j, therefore

$$c = c(j) = c(m)\ \forall j, m \in [0,1]$$

Agents choose optimally whether to buy final goods with cash or with a costly transaction technology, which we will call credit. In order to buy an amount c of good j with credit, the consumer must purchase $\gamma(j)$ units of financial services.

Following Lucas and Stokey (1983), we assume that the financial market closes first and the goods market follows. More specifically, at the beginning of period t, after observing the current shock ε_t, agents adjust their portfolio compositions by trading money and assets in a centralized securities market and pay the credit obligations that they contracted in the previous period. In this sense, the transaction technology we consider does not allow households to transfer liabilities from one period to the other. Instead, it represents a way in which consumers can transform their interest-bearing assets into a means of payment that is not subject to the inflation tax.

The gross nominal return of a one-period bond A_t is the nominal interest rate R. Notice that the gross nominal return of money is 1, so money is (weakly) dominated in rate of return by assets. Nevertheless, households value money because it provides liquidity services to buy consumption goods.

After trade in the securities market has taken place, the goods market opens. At this stage, households buy consumption goods with money or credit, decide how much to work and save a fraction of their total income in the form of nominal wealth W_{t+1} that they will carry to the next period to transform it into assets and money in the securities market. Household i's budget constraint at the time the goods market opens is

$$p_t c_t^i + q_t \int_0^{z_t^i} \gamma^i(j) dj + W_{t+1}^i = R A_t^i + M_t^i + (1-\tau^l)\omega p_t (1-l_t^i)\varepsilon_t^i \tag{2.1}$$

where p_t is the unitary price of the final good, q_t is the price of a unit of financial services, $\int_0^{z_t^i} \gamma^i(j) dj$ is the total amount of credit bought, ω is the real wage for one

[7]We choose this aggregator for simplicity reasons only. Working with a more general aggregator such as a Dixit-Stigliz aggregator does not alter our results qualitatively.

efficiency unit of labor, l_t^i is time devoted to leisure and τ^l is an anonymous linear tax on labor income.

As mentioned before, households need either cash or credit in order to buy consumption goods. Agents choose which goods they will buy with credit and which with cash, and $z_t^i \in [0,1]$ stands for the fraction of goods bought with credit by household i[8]. Since all goods that are not bought with credit need to be paid with money, household i faces the following cash-in-advance constraint in the goods market:

$$p_t c_t^i (1 - z_t^i) \leq M_t^i \tag{2.2}$$

Notice that, if $R > 1$, money is strictly dominated in rate of return by assets and, consequently, constraint (2.2) will always be binding because agents can adjust their money holdings *after* the idiosyncratic shock is observed.

The problem that household i solves can be stated as

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i)$$

subject to

$$p_t c_t^i + q_t \int_0^{z_t^i} \gamma^i(j)dj + W_{t+1}^i = RA_t^i + M_t^i + (1-\tau^l)\omega p_t (1 - l_t^i)\varepsilon_t^i$$

$$p_t c_t^i (1 - z_t^i) \leq M_t^i$$

$$A_t^i \geq 0$$

In the appendix we show the optimality conditions associated to this problem.

b. Firms

There are two types of competitive firms in this economy: firms that produce consumption goods and financial firms that produce transaction services. We assume that all markets are perfectly competitive and, in consequence, firms make zero profits in equilibrium.

[8] To be more precise, agents will buy goods indexed from 0 to z_t^i with credit and the rest of the goods (from z_t^i to index 1) with cash.

Consumption goods sector

Let K denote the aggregate capital stock and L^g aggregate labor in efficiency units employed in the goods sector. Then the production technology in the consumption goods sector can be written as

$$y_t = F(K, L^g)$$

where $F(\cdot)$ is a neoclassical production function[9]. Optimality conditions of the firm are

$$r = F_K - \delta \tag{2.3}$$

$$\omega = F_L^g \tag{2.4}$$

where r is the before-tax real return on capital and δ is the depreciation rate.

Financial services sector

We assume that a household with labor productivity ε_t^i can acquire a fraction of goods $\tilde{z}_t^i$ with credit at zero marginal cost, and that $\tilde{z}_t^i$ depends on the potential labor income of an agent. More specifically, $\tilde{z}_t^i = f(\varepsilon_t^i)$ with $\frac{\partial \tilde{z}_t^i}{\varepsilon_t^i} > 0$. In words, this assumption means that the fraction of goods that can be purchased with credit at zero marginal cost increases with the productivity of an agent. Then, more productive agents have an advantage in the use of credit with respect to less productive ones. Figure 2.1 shows the marginal cost of credit for goods $i \in [0, 1]$ for agents n and m with $\varepsilon_t^n > \varepsilon_t^m$.

The assumption that $\tilde{z}_t^i$ depends positively on the labor productivity of agent i is a shortcut to reflect the better access to commercial credit markets that high-income, rich households enjoy when compared to poor, low-income ones. Think of a similar scenario to the one proposed here, but now at the beginning of each period, after observing her individual shock, household i decides whether to repay her credit obligations contracted in the last period. If the household chooses not to repay she is excluded from the credit market for the next period, otherwise it can apply for a new credit line. The household will decide to pay the credit obligations contracted in period $t-1$ only if the value of the credit in t is higher or equal than what she owes from $t-1$. Obviously, the amount of credit in t is determined by the wealth and the labor productivity of the agent in t. In $t-1$, when the agent applies for the loan, the financial services firm will charge an interest rate that reflects the risk that the agent defaults in the next period. This risk

[9] We assume that the production function is identical for any type of consumption good $i \in [0, 1]$ so the relative price of any two types of goods i and j is 1 $\forall i, j$.

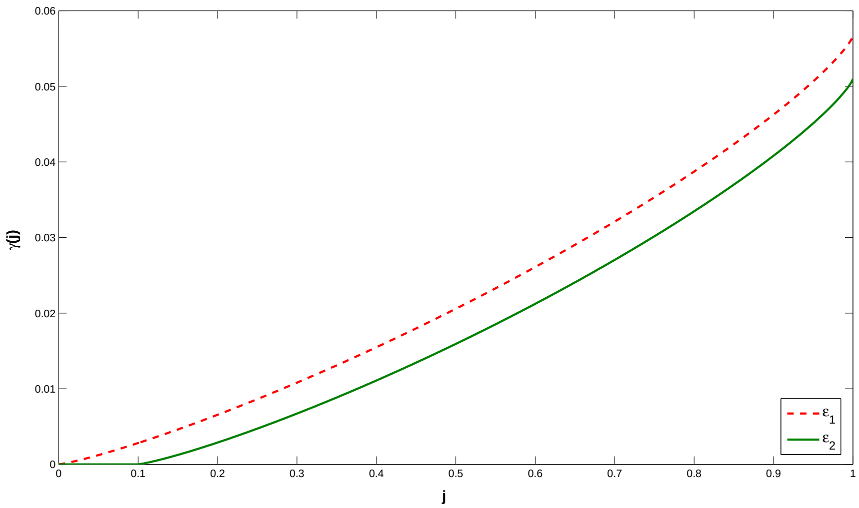

Figure 2.1: Marginal cost of credit $\gamma(j)$ for $\varepsilon_1 < \varepsilon_2$

will be decreasing in wealth and productivity since richer, more productive agents are more likely to use credit more intensely in the next period.

With this argument in mind, it is natural to think that the cost of credit depends positively on the earnings capacity of an individual. A natural way to capture this feature in the model is through $\tilde{z}_t^i$. Of course, given our previous discussion, one would naturally think that $\tilde{z}_t^i$ should depend on W_t^i as well. Nevertheless, adding this dependence complicates the numerical solution of the model and, since ε_t^i and W_t^i are highly correlated anyway, it presumably does not change the results substantially.

For goods $j \in [\tilde{z}_t^i, 1]$ the nominal marginal cost of the use of credit is $q_t \gamma^i(j)$. The total cost of credit for agent i with productivity ε_t^i that buys a fraction $z_t^i > \tilde{z}_t^i$ of goods with credit is

$$q_t \int_0^{z_t^i} \gamma^i(j) dj = q_t \int_{\tilde{z}_t^i}^{z_t^i} \gamma^i(j) dj$$

Following Erosa and Ventura (2002), the function $\gamma(i)$ is strictly increasing in i for $i > \tilde{z}_t$ and satisfies $lim_{i \to 1} \gamma(i) = \infty$[10]. This assumption guarantees that some goods will be purchased with cash so that there is a well-defined demand for money.

The technology to produce transaction services requires one unit of labor (in efficiency units) per unit of service produced. We denote by L^c the amount of labor in efficiency units employed in the production of transaction services:

[10]The meaning and role of $\tilde{z}_t$ will be analyzed in detail later.

$$\int \int_{\tilde{z}_t^i}^{z_t^i} \gamma^i(j) dj d\lambda = L^c$$

where λ is the distribution of agents in the economy. Firms in the sector charge the price q_t per unit of credit sold. Competition ensures that firms make zero profits and set their prices such that $\omega p_t = q_t$[11].

c. Government

The government that has to finance an exogenous stream of public spending through distortionary taxes on aggregate labor income and asset returns and through seigniorage revenues. The nominal budget constraint of the government is

$$p_t g + M_t = \tau^l \omega p_t L + \tau^k r p_t K + M_{t+1} \tag{2.5}$$

where L is total aggregate labor in efficiency units $L = L^g + L^c$ and τ^k is the tax rate on asset returns.

d. Equilibrium

In our economy, each agent is characterized by the pair (w_t, ε_t) where w_t is wealth in real terms. Let $W \equiv [0, \bar{w}]$ be the compact set of all possible wealth holdings where $\bar{w}$ is an upper bound on wealth and the lower bound is determined by the no borrowing condition[12]. Let $E \equiv \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$ be the set of all possible realizations of the labor productivity shock ε_t. The shock follows a Markov process with transition probabilities $\pi(\varepsilon', \varepsilon) = \Pr(\varepsilon_{t+1} = \varepsilon' | \varepsilon_t = \varepsilon)$. Define the state space S as the cartesian product $S = W \times E$ with Borel σ-algebra $\mathcal{B}$ and typical subset $\mathcal{S} = (\mathcal{W} \times \mathcal{E})$. The space $(S, \mathcal{B})$ is a measurable space, and for any set $\mathcal{S} \in \mathcal{B}$ $\lambda(\mathcal{S})$ is the measure of agents in the set $\mathcal{S}$. Denote Λ as the set of all probability measures over $(S, \mathcal{B})$. Then,

$$\lambda_{t+1}(\mathcal{W} \times \mathcal{E}) = \int_S Q((w, \varepsilon), \mathcal{W} \times \mathcal{E}) d\lambda_t(w, \varepsilon)$$

[11]The zero profit condition can be written as

$$q_t \int \int_0^{z_t^i} \gamma(j) dj d\lambda = p_t \omega L^c$$

[12]Notice that the no borrowing condition means that

$$a_t \geq 0 \tag{2.6}$$

Nevertheless, since $w_t = a_t + m_t$ and a_t and m_t are decision variables of the agent, only by imposing $w_t \geq 0$ we make sure that condition (2.6) is satisfied always.

where $Q((w,\varepsilon),\mathcal{W}\times\mathcal{E})$ is the probability that an individual with current state (w,ε) is in the set $\mathcal{W}\times\mathcal{E}$ next period:

$$Q((w,\varepsilon),\mathcal{W}\times\mathcal{E}) = \sum_{\varepsilon'\in\mathcal{E}} I\{w'(w,\varepsilon)\in\mathcal{W}\}\pi(\varepsilon',\varepsilon)$$

Here $I(\cdot)$ is the indicator function and $w'(w,\varepsilon)$ is the optimal savings policy of an individual in state (w,ε).

Definition 4. *A stationary equilibrium is given by functions* $\{g^c, g^l, g^z, g^{w'}, g^a, L^g, L^c\}$, *a price system* $\{p_t,\omega,q_t,r\}_{t=0}^{\infty}$ *and government policies* $\{\tau^l,\tau^k,R,M_t\}_{t=0}^{\infty}$ *such that*

1. *Given prices and government policies, the allocations solve the household's problem.*
2. $r = F_K(K,L^g)-\delta$, $\omega = F_{L^g}(K,L^g)$
3. *Given the allocations and price system, the government budget constraint is satisfied.*
4. *Markets clear:*

$$\int g^c d\lambda + g + \delta K = F(L^g,K)$$
$$\int\int_{\tilde{z}_t}^{z_t}\gamma(j)djd\lambda = L^c$$
$$L^g + L^c = \int \varepsilon(1-g^l)d\lambda$$
$$K = \int g^a d\lambda$$

5. *The measure of households is stationary:*

$$\lambda^*(\mathcal{W}\times\mathcal{E}) = \int_S Q((w,\varepsilon),\mathcal{W}\times\mathcal{E})d\lambda^*(w,\varepsilon)$$

e. Optimal Policy

The government is benevolent and seeks to set taxes and seigniorage so as to maximize a social welfare function. We define the objective function in the maximization problem of the government to be the *utilitarian social welfare function*, U:

$$U = \int E_t V(\{c_s,l_s\}_{s=t}^{\infty})d\lambda \tag{2.7}$$

where $V(\{c_s, l_s\}_{s=t}^{\infty}) = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s, l_s)$ is life-time utility at time t. As we can see in expression (2.7), all households receive an equal weight for the computation of social welfare. A standard interpretation for this criterion is that the planner maximizes welfare under the veil of ignorance; that is, ex-ante welfare for a hypothetical household before knowing in which point of the distribution she is.

The problem of the government can be stated as

$$\max_{\tau^l, R} \int E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i) d\lambda$$

subject to

Competitive equilibrium (equations (B.7) - (B.11))

$$p_t g + M_t = \tau^l \omega p_t L + \tau^k r p_t K + M_{t+1}$$

2.3 Effects of inflation

It is well known that in representative-agent frameworks the Friedman rule is the optimal policy recommendation for a wide variety of models (see Chari et al. (1996)). This result extends to our model economy if we shut down heterogeneity among households. In the case in which $\varepsilon_t^i = \varepsilon \ \forall i$ a benevolent planner sets $R = 1$. The intuition behind this result lies in the uniform commodity taxation argument from the public finance literature. Notice that in our framework goods bought with cash and with credit enter the utility function in identical manner[13]. Therefore, the tax on labor income implicitly taxes all goods, whether bought with cash or with an alternative transaction technology, at an identical rate. Setting $R > 1$ entails taxing more those goods bought with cash, which is not efficient. Given the representative-agent assumption, in the model we are describing there are no issues of redistribution or self-insurance. Moreover, the capital-labor ratio is pinned down by the intertemporal discount factor β, so inflation does not affect the return on capital or the wage rate. Thus, efficiency in the taxation of different goods is the only aspect that the planner should take into account when designing the optimal policy plan.

When we introduce idiosyncratic uninsurable risk the analysis changes substantially. Now inflation has effects over the level of capital in steady state. In addition, due to the presence of an alternative transaction technology like the one have introduced,

[13]The argument that follows holds always that utility is separable in consumption and leisure, and the utility over consumption goods is homothetic, see Chari et al. (1996) for a formal proof in a similar model.

inflation acts as a regressive tax on consumption. We proceed to describe these effects in detail. We argue that, once these effects are taken into consideration, the determination of the optimal policy mix remains an open question that needs a quantitative answer.

a. Inflation as a regressive tax on consumption

As described by Erosa and Ventura (2002), inflation can act as a regressive consumption tax when, in a heterogeneous agent setup such as ours, we allow agents to substitute cash by an alternative transaction technology that displays economies of scale. For the moment we abstract from the presence of idiosyncratic risk, since all the analysis holds by allowing for heterogeneity in labor productivities only.

Without loss of generality, assume that an agent's productivity ε^i is constant $\forall t$, $\varepsilon^i \in E = [\varepsilon^1, \varepsilon^2]$ with $\varepsilon^1 < \varepsilon^2$ and there is an equal mass of each type of agent in the population [14]. Furthermore, assume for simplicity that initial wealth holdings w_0^1 and w_0^2 are such that the economy is in steady state from $t = 0$ onwards. Then, optimality for agent i requires that

$$R = 1 + \frac{\omega\gamma(z^i)}{c^i} \tag{2.8}$$

The second term on the right hand side of the previous expression is the unitary cost of credit for the threshold good z^i. It is clear from expression (2.8) and from the functional specification of the transaction technology (2.11) that this unitary cost decreases when the volume transacted increases. Thus, the transaction technology displays economies of scale.

Assume for now that $\tilde{z}^i = 0$ for $i = 1, 2$. From (2.8) it is immediate to see that $z^1 = z^2 = 0$ when $R = 1$, i.e, when the planner follows the Friedman rule both types of agents buy all consumption goods using cash, since holding cash does not bring about any opportunity cost. On the contrary, if $R > 1$, $z^2 > z^1 > 0$, given that $c^2 > c^1$ because agent 2 enjoys a higher labor income and, therefore, a higher level of consumption[15]. Then it follows that the more productive agents use the credit technology more intensely. This feature of the model is consistent with the evidence on transaction patterns and portfolio holdings that Erosa and Ventura (2002) report in their paper, which can be summarized in three main facts: high income individuals buy a smaller fraction of their consumption with cash, the fraction of wealth in the form of liquid assets held by a household decreases with her wealth and income and, finally, a non-negligible fraction of households does not own a credit card.

[14] The results of this section are robust to changes in the number of productivity states and in the composition of the population.

[15] Strictly speaking, this is only true for particular levels of initial wealth w_0^1 and w_0^2. Here we are implicitly making the assumption that the more productive agent is at least as rich as the less productive one.

Due to the presence of economies of scale in the transaction technology, buying goods with credit is relatively more expensive for less productive agents. Because these agents buy a larger fraction of goods with cash, they need to hold a relatively larger fraction of their income in liquid assets. It is in this sense that inflation acts as a regressive tax on consumption, since setting $R > 1$ corresponds to taxing low-income individuals more.

If $\tilde{z}^2 > \tilde{z}^1$ this asymmetric effect of the inflation tax is exacerbated. As we discussed in section b., the introduction of $\tilde{z}^i$ is a shortcut to model differences in the access to commercial credit markets that high-income, rich households enjoy when compared to poor, low-income ones. The regressive nature of the inflation tax implies that *for high productivity households it is optimal to set a gross nominal interest rate higher than 1*, since in this way the burden of taxation is shifted to poor, unproductive individuals. To see the intuition behind this statement, think of the limit case in which $\tilde{z}^2 = 1$. In this case the inflation tax does not affect agents of type 2 in any way, so they would want the government to set it as high as necessary to finance completely its expenditure from seigniorage revenue.

In the appendix we show (numerically) that, in the current setup, a benevolent government (a Ramsey planner) that assigns a sufficiently high Pareto weight on type 2 agents would find it optimal to deviate from the Friedman rule and set $R > 1$.

b. Inflation as a motive for precautionary savings

The effect described in the previous section is at work due to the assumption of heterogeneity and the transaction technology we have specified, but it does not depend on the presence of uninsurable idiosyncratic risk. In this section we argue that inflation accentuates such risk and that, consequently, households save more when inflation is high.

Consider first the case in which there is no uncertainty (aggregate or idiosyncratic). Then the capital/labor ratio is determined in steady state by the discount factor β and is completely independent of the inflation rate. In this sense *inflation is neutral* and it does not affect the wage rate or the real interest rate in steady state. This is also true if we allow for idiosyncratic uncertainty but assume that households can trade a complete set of Arrow securities contingent on the realization of the labor productivity shock. It is easy to show that in this case agents can do full risk sharing and, if there is no aggregate uncertainty and utility is separable in consumption and leisure, enjoy a constant level of consumption independently of their current labor productivity. As in the case with no uncertainty, β determines the aggregate capital-labor ratio, ω and r[16].

[16] We have abstracted from the possibility that there is aggregate uncertainty. In this case, if there are incomplete markets with respect to the aggregate shock, inflation can have an active role as a mechanism to complete the markets. See Chari et al. (1991) for a discussion.

In models with incomplete markets and borrowing constraints, agents save not only to smooth consumption by transferring resources from one period to the other, but also to insure themselves against bad realizations of the shock that may push them close to the borrowing constraint and force them to consume very little. The absence of complete markets and the presence of borrowing constraints lead agents to save for precautionary reasons[17]. Moreover, the more uncertain future income (and consumption) becomes, the stronger the motive for precautionary savings. The increase in savings translates into an increase in the capital stock in steady state, with the consequent decrease in the real interest rate and increase in the wage rate.

In the economy we have described in previous sections, a higher level of steady state inflation implies that future consumption is more uncertain and, consequently, reinforces the incentives to save. To see this, consider the no-borrowing constraint of household i, which says that $A_t^i \geq 0$. As shown in the appendix, this constraint can be re-written as:

$$c_t^i(1 - z_t^i)(1 + \pi) \leq w_t^i \tag{2.9}$$

where π is the inflation rate. Consider a steady state with π^A, in which household i at time t hits the constraint:

$$c_t^{i,A}(1 - z_t^{i,A})(1 + \pi^A) = w_t^{i,A}$$

If inflation were higher, say $\pi^B > \pi^A$, to sustain the same level of consumption $c_t^{i,A}$ and the same fraction of goods bought with credit $z^{i,A}$, household i would need to have a higher level of wealth in order to satisfy constraint (2.9). Similarly, for a household i that has wealth holdings $w_t^{i,A}$ in an economy where the inflation rate is $\pi^B > \pi^A$, either $c_t^{i,B} < c_t^{i,A}$, $z_t^{i,B} > z_t^{i,A}$, or a combination of both. Raising z_t^i entails working more to be able to pay for the higher credit expenses; since the intratemporal optimality condition (B.10) has to be satisfied, consumption needs to decrease, so it has to be the case that $c_t^{i,B} < c_t^{i,A}$. This means that the higher level of inflation π^B renders consumption more uncertain.

The previous discussion points out to the fact that inflation raises the incentives to save and, as a consequence, the level of steady- state capital. Thus, an economy with higher inflation displays a lower real interest rate and higher wages. Also, because the budget constraint of the government (2.5) has to be satisfied, the higher seigniorage revenue calls for a decrease in τ^l. Poor agents, who rely almost entirely on their labor income and whose marginal utilities of consumption and leisure are very large, find this beneficial because a small increase in disposable income translates into a sizeable

[17]When the marginal utility is convex, i.e., utility displays a positive third derivative, independently of the presence of borrowing constraints, agents save because of prudence.

increase in utility. On the other hand, middle-class and rich households are harmed by the reduction in their capital income derived from the lower real interest rate.

2.4 Calibration and functional specification

The model described here cannot be solved analytically. Consequently, we need to resort to numerical techniques to obtain a solution. In what follows, we describe the calibration strategy and functional specification we work with.

The length of the model period is one year. We define the benchmark steady state as one in which the government sets its policy in accordance with what is observed for the U.S economy. In particular, we set inflation to be 2% annually in the benchmark steady state. Next, we select the model parameters so that in the benchmark steady state equilibrium the model economy matches some selected features of the U.S data. While some of the parameters can be set externally, others are estimated within the model and require solving for equilibrium allocations[18]. We summarize the values of externally set and internally calibrated parameters in Tables a. and b., as well as the targets they are related to and the values for the targets obtained from the model. Notice that, in the case of internally determined parameters, all parameters affect all calibration targets. Nevertheless, since usually each parameter affects more directly only one target, we report in the table the target that it is more related to[19].

a. Parameters set exogenously

Preferences

Following Domeij and Flodén (2006), the utility function we use is

$$u(c_t, l_l) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \theta_0 \frac{(1-l_t)^{1+\frac{1}{\theta_1}}}{1+\frac{1}{\theta_1}} \tag{2.10}$$

This specification is convenient because the Frisch elasticity of labor supply is equal to θ_1. We set $\theta_1 = 0.59$, which is in line with Domeij and Flodén (2006) estimates[20]. It is common to find in the literature that $\sigma \in [1, 2]$, so we set $\sigma = 1.5$[21].

[18] The distinction between externally set and internally calibrated parameters corresponds to Heathcote et al. (2008b).

[19] As Pijoan-Mas (2006) explains, this calibration strategy can be seen as an exactly identified simulated method of moments estimation.

[20] These authors claim that previous estimates of the labor-supply elasticity are inconsistent with incomplete market models because borrowing constraints are not considered in the analysis, in particular, they are downward-biased.

[21] Examples of papers that use these values are Erosa and Ventura (2002), Campanale (2007), Castañeda et al. (2003) and Domeij and Heathcote (2004), among many others.

Technology

Consumption goods sector: The technology for the production of consumption goods is a standard Cobb-Douglas function

$$y = K^{\alpha} L^{g1-\alpha}$$

α is set such that the labor income share of GDP, $wL/Y = 1 - \alpha = 0.64$.

Transaction services sector: We adopt the following credit technology, which is a modified version of what Dotsey and Ireland (1996), Erosa and Ventura (2002) and Albanesi (2005) use:

$$\gamma^i(j) = \gamma_0 \left(\frac{j - \tilde{z}_t^i}{1 - j} \right)^{\gamma_1} \tag{2.11}$$

γ_0 and γ_1 are internally calibrated, as explained in section b..

As was explained before, $\tilde{z}_t^i$ depends on the labor productivity of an agent, ε_t^i. A proposed function for $\tilde{z}_t^i$ is

$$\tilde{z}_t^i = \mu_0 + \mu_1 \left((1 - \bar{l}) \omega \varepsilon_t^i \right)$$

where $(1-\bar{l})$ are average hours worked, which we set to be 1/3 of disposable time. Notice that we are making $\tilde{z}_t^i$ depend on *potential* gross labor income rather than on actual labor income. This simplifies greatly the analysis. If $\tilde{z}_t^i$ depended on current labor income and/or wealth w_t, then the agent would take into account that by changing her labor/leisure and consumption/savings decisions, she would be affecting the cost of credit she faces. This interaction complicates the task of obtaining the allocations c_t, l_t, z_t and w_{t+1} from the optimality conditions of the household[22]. We leave this exercise for future research.

μ_0 and μ_1 are determined so that agents with the lowest productivity level possible have $\tilde{z}_t^i = 0$ while agents with the highest productivity level have $\tilde{z}_t^i = 0.2$ in the benchmark parameterization.

Government

We adopt a very standard parameterization for fiscal variables. Usually it is accepted that the ratio of government expenditure over GDP lies in the interval $[0.19, 0.22]$[23].

[22] In the Appendix we describe in detail the numerical algorithm used to solve the model.

[23] Some studies that use values in this range are Erosa and Ventura (2002), Campanale (2007) and Floden and Linde (2001).

Table 2.1: Parameters set exogenously

Parameter	Value
σ	1.5
θ_1	0.59
μ_0	-0.0117
μ_1	0.0835
G/Y	0.2003
τ^k	0.397
π	0.02

We set $G/Y = 0.2$. To make our study comparable to other papers in the optimal fiscal policy literature, we introduce a tax on asset returns τ^k. Nevertheless, we assume it is fixed at a value of 0.397, which is in line with Domeij and Heathcote (2004) and Floden and Linde (2001). The reason to introduce this tax is that, otherwise, almost all public revenues would have to come from labor income taxation, causing the distortion on the labor/leisure decision to be very large. Finally, we set inflation to be 0.02 in the benchmark parameterization.

b. Internally calibrated parameters

Preferences and technology

We need to pin down θ_0 from the utility function (2.10), the discount factor β, the depreciation rate δ and the two parameters γ_0 and γ_1 from the credit technology (2.11). The targets we choose are the following: the average fraction of disposable time devoted to work should be $1/3$[24], the capital to output ratio should be $K/Y = 3$[25], the investment to output ratio should be $I/Y = 0.25$, the correlation between money and asset holdings should be $corr(m, a) = 0.16$[26] and the fraction of consumption expenditures made with cash should be $\int(1 - z_t^i)c_t^i d\lambda = 0.8$[27].

[24]See Pijoan-Mas (2006) and Castañeda et al. (2003) for a similar choice.

[25]Pijoan-Mas (2006), Campanale (2007), Conesa and Krueger (1999) and Castañeda et al. (2003)among others.

[26]The correlation between money and asset holdings is computed using the 2004 Survey of Consumer Finances. Since cash holdings are not reported in the survey, we use the amount of money held in checking accounts as a proxy for money holdings. All the remaining sources of net worth are considered to be assets a_t.

[27]Erosa and Ventura (2002) report that 80% of transactions are made with cash (M1). Since we do not have a measure of the number of transactions in our model, we use as a proxy the value of transactions. Similarly, Algan and Ragot (2006) report that $M1/C \simeq 0.78$ for the 1960-2000 period. We use a value of 0.8 which is in the middle of these two.

Table 2.2: Parameters set endogenously

Parameter	Value	Target (data)	Model
θ_0	25	$\int(1-l)d\lambda = 0.33$	0.34
β	0.952	K/Y=3	2.99
γ_0	0.035	corr(M,K)=0.16	0.175
γ_1	0.2	$\frac{\int(1-z_t^i)c_t^i d\lambda}{\int c_t^i d\lambda} = 0.8$	0.79
ε^1	0.3547	Gini $K = 0.78$	0.77
ε^2	0.9428	Fraction of wealth of Q1+Q2 = 0.0335	0.08
α	0.36	$\omega L/Y = 0.64$	0.64
δ	0.083	I/Y=0.25	0.25
p	0.921	ρ=0.92	0.92
q	0.9886	$\sigma_\epsilon = 0.21$	0.21

Labor productivity process

Our main aim is to define what the optimal mix of fiscal and monetary policy should be in the presence of idiosyncratic uninsurable risk. As we will see in the following sections, individuals with different levels of wealth and labor income are affected differently by different policies. Moreover, the optimal policy prescription depends crucially on the presence and the extent in which agents are exposed to the idiosyncratic shock. Consequently, the definition of the process for the labor productivity shock ε_t is critical to the analysis.

We follow the approach of Domeij and Heathcote (2004) and set two goals that our specification of the shock should accomplish. The first goal is that the persistence and variance of earnings shocks in the model are consistent with empirical estimates from panel data. The second is that in equilibrium the model yields a distribution of households across wealth that resembles in some aspects the distribution observed in the US.

We assume that the labor productivity process can display only three values, i.e., $E = \{\varepsilon^1, \varepsilon^2, \varepsilon^3\}$ where $\varepsilon^1 < \varepsilon^2 < \varepsilon^3$. The transition probability matrix corresponding to the shock adds 6 free parameters[28] which, added to the three productivity levels, sum up to 9 parameters that need to be pinned down.

In order to restrict the number of free parameters available for calibration, we assume the following: households cannot jump directly from the lowest productivity state to the

[28]Since the shock is a 3-state Markov chain, the transition probability matrix is a 3-by-3 matrix so it has 9 values to be determined. Nevertheless, the rows of the matrix have to add up to one, therefore the number of free parameters is 6.

highest one and viceversa, and they face equal probability of going from the medium productivity state to the low one as to the high one. These restrictions yield the following transition probability matrix:

$$\Pi = \begin{pmatrix} p & 1-p & 0 \\ \frac{1-q}{2} & q & \frac{1-q}{2} \\ 0 & 1-p & p \end{pmatrix} \tag{2.12}$$

Finally, we impose average productivity in the economy to be equal to 1. This leaves us with 4 free parameters to pin down.

As mentioned before, our first goal is to have a process that replicates the persistence and variance of earnings shocks present in panel data. As described in Floden (2001) and Pijoan-Mas (2006), wages (in logs) can be decomposed into two components. The first component, which we call η, is constant for a given individual and represents ability, education and all other elements that influence wages and can be depicted as fixed idiosyncratic characteristics of an agent. The second component, ν_t, is a stochastic individual component meant to capture idiosyncratic uncertainty in the earnings process. It basically reflects changes in the employment status of each agent, job changes to positions that match better or worse the individual's ability, health shocks that affect productivity, etc. This last component corresponds to the notion of (log of) ε in our model.

Floden and Linde (2001) have estimated the following process for $\nu_{i,t}$:

$$\nu_t^i = \rho^\nu \nu_{t-1}^i + \zeta_t^i \text{ with } \zeta_t^i \sim N(0, \sigma_\zeta^2) \tag{2.13}$$

and found $\rho^\nu = 0.92$ and $\sigma_\zeta = 0.21$. We use these as the targets for the persistence and variance of our productivity shock[29].

Our second goal is to have realistic heterogeneity in terms of wealth in equilibrium. As a consequence, we set as targets the Gini coefficient of the asset (total wealth minus money holdings) distribution, which is approximately 0.78 according to 2004 SCF data, and the fraction of total wealth in the hands of the two poorest quintiles of population, which is about 3.35% using the same data. The last target is important because, a priori, inflation is likely to affect more poorer agents that consume a relatively much larger fraction of their income than richer ones and thus need to have most (if not all) of their wealth in the form of money.

[29]Different studies suggest that ρ^ν should belong to the $[0.88, 0.96]$ interval, while σ_ζ should be between 0.12 and 0.25. See Domeij and Heathcote (2004) for references.

Table 2.3: Comparison of steady states

	Benchmark	Optimal
R	1.0426	1
r	0.0367	0.0376
K/Y	2.99	2.976
ω	1.1873	1.1823
τ^l	0.2308	0.259
Welfare	-39.9484	-39.7701

2.5 Results

a. Optimal policy

In the previous sections we have discussed the role that the inflation tax has as a regressive tax on consumption and as an incentive for capital accumulation. The two effects affect asymmetrically different sectors of the population: while the former benefits richer, more productive agents, the latter increases welfare of the poor, unproductive ones.

Having exposed all the mechanisms by which inflation affects the agents in our economy, it should be clear by now that the determination of the optimal policy mix is a question that does not have an immediate answer. There are a variety of effects operating simultaneously, affecting different agents in contradictory ways. The only way to provide an answer is to find the optimal policy numerically, once we have a reasonably calibrated model economy.

The main result of our analysis is the following:

Result 1. *For a utilitarian social welfare function, that is, one that assigns equal weight to all individuals, the Friedman rule is optimal and the government sets* $R = 1$.

The previous result suggests that, despite the fact that some agents win with relatively high levels of inflation (compared to the one that arises when $R = 1$), the efficiency motive associated to uniform taxation dominates and the optimal policy prescription is the Friedman rule. Therefore, the optimality of the Friedman rule is not only robust to the introduction of distortionary taxation, as explained by Chari et al. (1996), but also to considerations of heterogeneity and uninsurable idiosyncratic risk, as we have shown here.

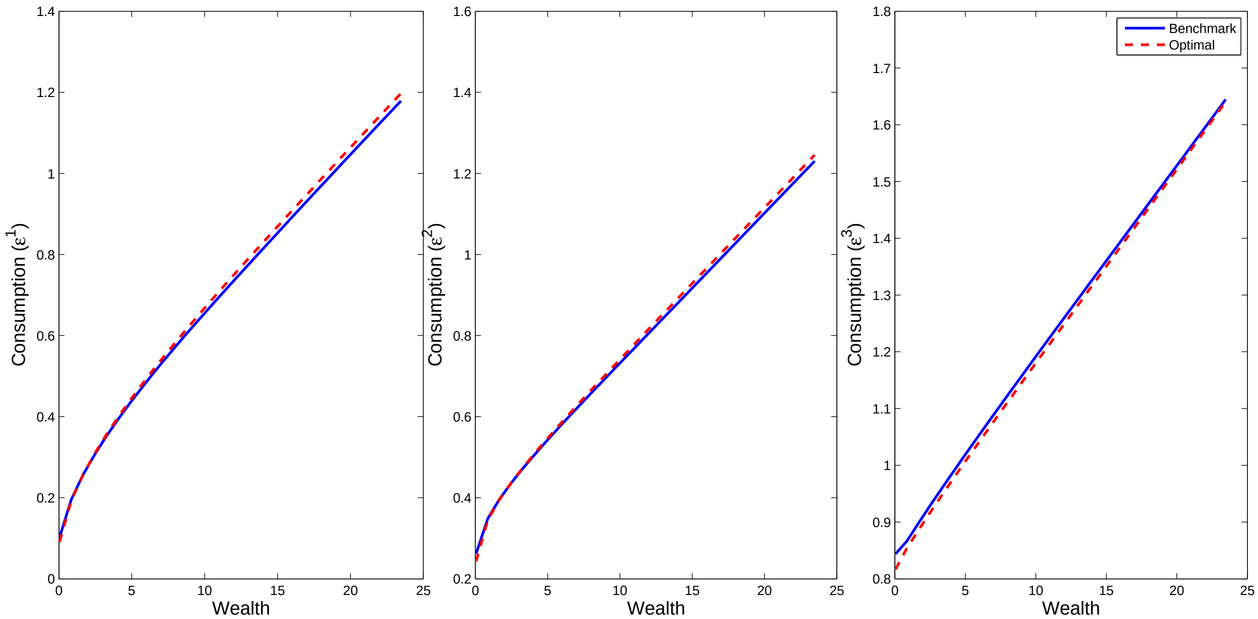

Figure 2.2: Consumption

b. Steady state comparison

Table 2.3 shows some statistics for the benchmark and the optimal policy steady states, respectively. From the table we see that in the optimal policy steady state the capital to labor ratio is smaller than in the benchmark economy, thus the lower real interest rate and higher wage rate. This is a direct consequence of the fact that inflation reinforces the motive for precautionary savings, as discussed in the previous section. The lower wage rate, lower savings and lower seigniorage revenue force the government to increase the tax rate on labor income in order to balance its budget. Therefore τ^l is higher in the optimal policy economy.

Figures 2.2 and 2.3 show consumption and leisure policy functions in the benchmark economy and in the optimal policy one, for the three levels of labor productivity. It can be observed that, for ε^1 and ε^2 and very low levels of wealth, consumption and leisure are higher in the benchmark economy, the reason being the higher labor income that poor households enjoy. When wealth increases the return on capital holdings starts being a relevant source of income for the household. Since the real interest rate is lower in the benchmark economy, consumption and leisure decrease. For high productivity households the picture looks different. These households are always enjoying a high level of consumption, and even for very little levels of wealth their labor income is sufficiently high to finance high consumption and savings. The decrease in uncertainty as a consequence of lower inflation levels present in the optimal policy economy diminishes

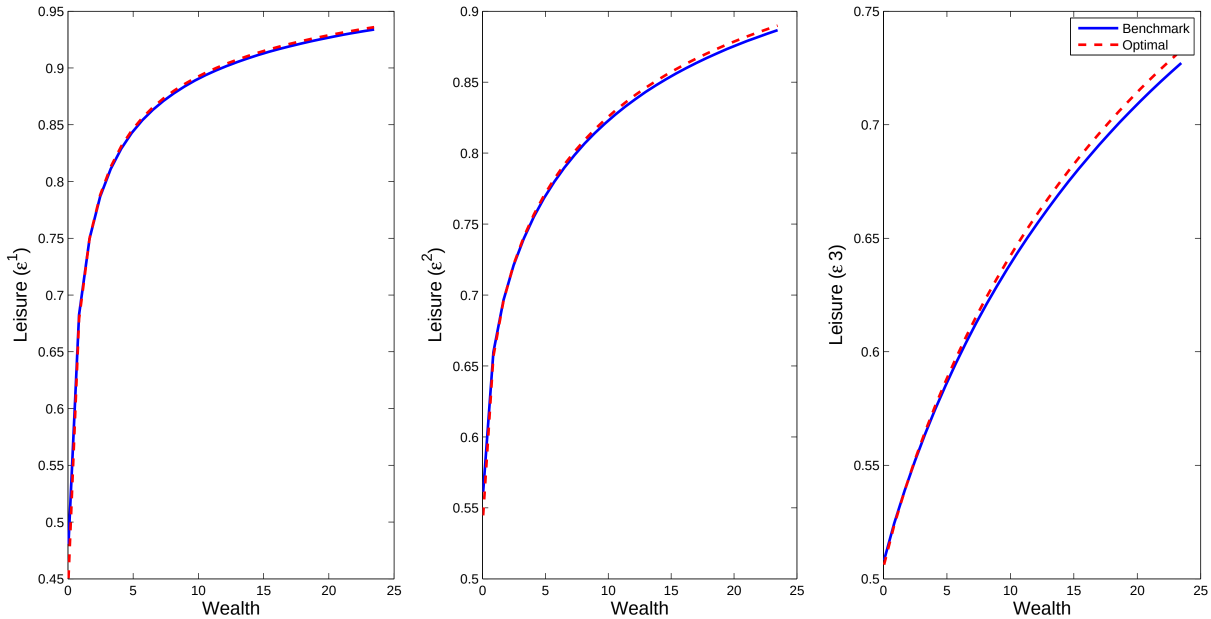

Figure 2.3: Leisure

the incentives to save for precautionary reasons. Therefore, very productive agents can afford to work less and enjoy higher levels of leisure, even if this means giving up some of their consumption.

Figure 2.4 plots the difference in savings between the benchmark economy and the optimal policy one. It is immediate to see that, for agents with ε^1 and ε^2, this difference is always negative, thus savings are higher in the optimal policy steady state. For agents with ε^3, however, the contrary statement is true. Again, this result hinges on the fact that high productivity agents need to save less for self-insurance reasons when $R = 1$. Agents with low and medium productivity and very little wealth need to save more because hitting the constraint is more harmful in this case. Notice that this is the reason why the curve of the difference in savings first rises and then goes down. As wealth increases, their total income goes up and they are able to better self-insure by saving more.

Finally, figure 2.5 plots the use of the transaction technology in the benchmark economy. Notice that, unless the borrowing constraint is binding, when $R = 1$ an agent will not use the transaction technology because the opportunity cost of holding money is zero[30], so the z^i policy function is trivial in the optimal policy economy. If the borrowing constraint is binding, from inspection of equation (2.9) it is clear that an agent will use credit, even if $R = 1$, because doing so allows her to relax such constraint.

[30]To be precise, agent i will be indifferent between using money or buying up to good $\tilde{z}_t^i$ with credit, since both entail zero costs.

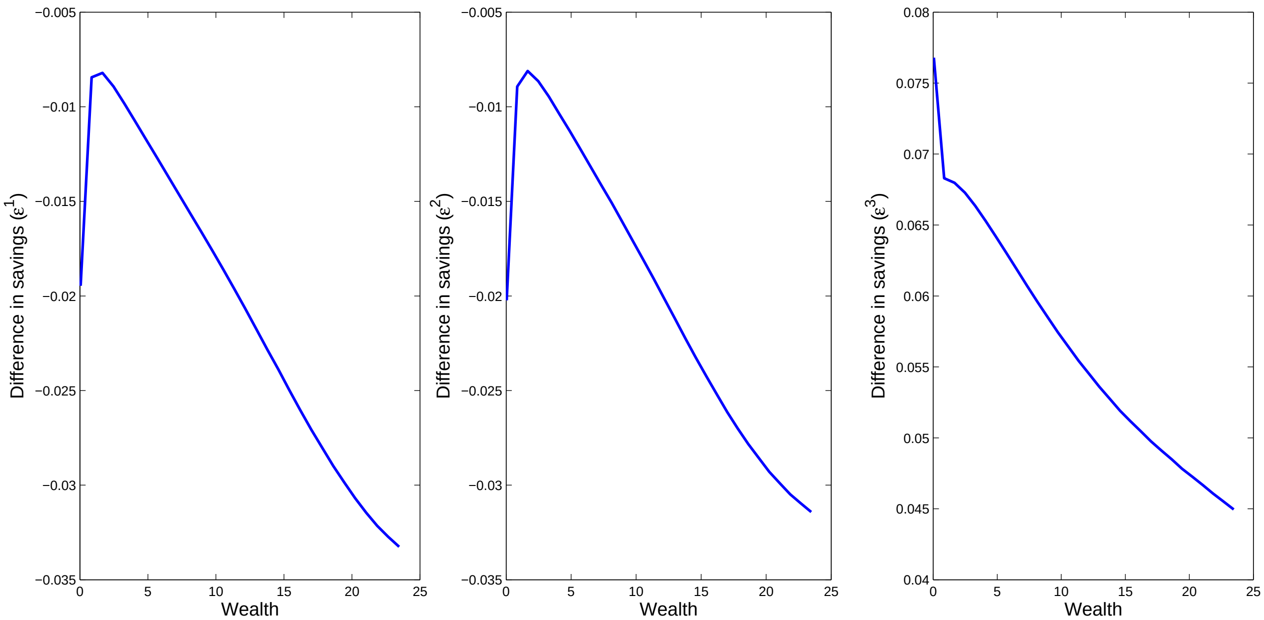

Figure 2.4: Savings

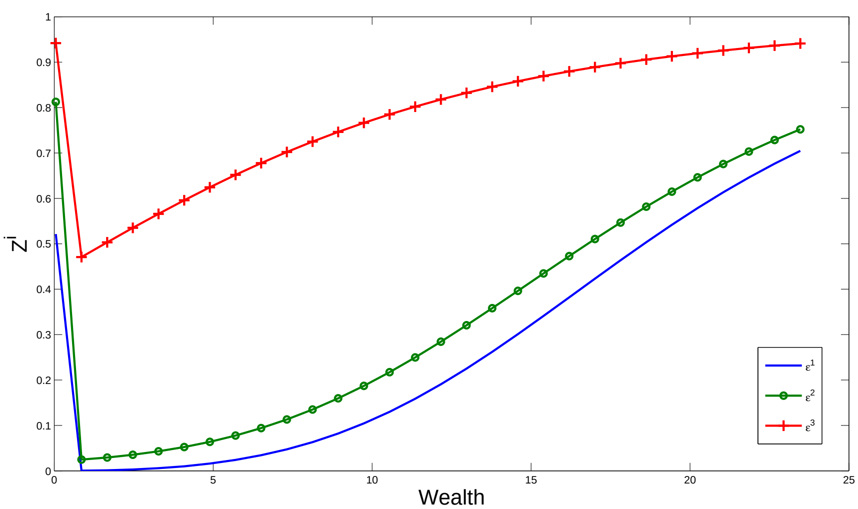

Figure 2.5: Transaction technology use

Figure 2.5 shows that the use of the transaction technology becomes more intensive with higher wealth holdings. The reason for this lies in the increasing returns to scale nature of the transaction technology we have adopted. Since higher wealth implies higher consumption for all levels of labor productivity, the unitary cost of credit goes down as w^i goes up, so z^i goes up as well. Increasing returns to scale are also responsible for the three lines, corresponding to the three labor productivity levels, becoming closer together as wealth increases, since for high w^i the differences in consumption become smaller.

2.6 Welfare analysis

We proceed to compare aggregate welfare in the benchmark economy (with a level of inflation of 2% annually) and in the economy in which the optimal policy is implemented. In this section we are comparing welfare in two different steady states, and we are not saying anything as to what happens during the transition from one to the other if there is a reform on policy. Obviously, studying the transition is a very interesting exercise, specially if we want to determine the "optimal transition", i.e., the transition such that no agent loses from the policy reform. As shown by Greulich and Marcet (2008) the optimal transition can imply a policy during the transition very different from the long-run policy prescription. We leave the analysis of the transition for future research.

We need to start with some definitions. The overall utilitarian welfare gain of the policy reform, ϖ_U is such that

$$\int E_t V(\{(1+\varpi_U)c_s^B, l_s^B\}_{s=t}^{\infty})d\lambda^B = \int E_t V(\{c_s^O, l_s^O\}_{s=t}^{\infty})d\lambda^O$$

where the superscript B stands for the benchmark economy and O for the economy with the optimal policy. ϖ_U can be thought of as the percent permanent change in consumption that agents in economy B should receive to be indifferent between living in economy B or in economy O.

Notice that the utilitarian social welfare (eq. (2.7)) can increase for three reasons. The first is when consumption or leisure increase for all agents. This is called the *level effect*. The second, called *inequality effect*, is when inequality is reduced, since $u(\cdot)$ (and therefore $V(\cdot)$) is concave. Finally, since agents are risk-averse, if uncertainty is reduced U increases. This is the *uncertainty effect*. Following Floden (2001), we can approximately decompose the utilitarian welfare gain into the welfare gains associated to the three effects mentioned before. In order to do this, define the certainty-equivalent consumption bundle $\bar{c}$ as:

Table 2.4: Welfare gains

$\varpi_U = 0.0051$	$\varpi_{lev} = 0.0044$	$\varpi_{unc} = 0.0015$	$\varpi_{ine} = -0.00005$

$$V(\{\bar{c}, l_s\}_{s=t}^{\infty}) = E_t V(\{c_s, l_s\}_{s=t}^{\infty})$$

Call $C = \int c d\lambda$, $Leis = \int l d\lambda$ and $\bar{C} = \int \bar{c} d\lambda$ average consumption, leisure and certainty-equivalent consumption, respectively. Then the cost of uncertainty p_{unc} can be defined as

$$V(\{(1 - p_{unc})C, Leis\}_{s=t}^{\infty}) = V(\{\bar{C}, Leis\}_{s=t}^{\infty})$$

This is the fraction of average consumption that an individual with average consumption and leisure would be willing to give up to avoid all the risk from labor productivity fluctuations. When uncertainty increases, $\bar{C}$ decreases and, since C and $Leis$ remain unchanged, p_{unc} necessarily goes up.

Define the cost of inequality p_{ine} as

$$V(\{(1 - p_{ine})\bar{C}, Leis\}_{s=t}^{\infty}) = \int V(\{\bar{c}, l_s\}_{s=t}^{\infty}) d\lambda$$

If we redistribute consumption from a rich household to a poor one, $\bar{C}$ and $Leis$ remain unchanged. However, the right-hand side of the previous expression increases, so p_{ine} has to go down. Finally, define leisure-compensated consumption in economy O, $\hat{C}^O$ as

$$V(\{\hat{C}^O, Leis^B\}_{s=t}^{\infty}) = V(\{C^O, Leis^O\}_{s=t}^{\infty})$$

which is the average consumption level that would make life-time utility in economy O equal to the one in an economy with the average leisure of economy B.

We are now ready to define the welfare gains associated to each one of the effects described before:

- The welfare gain of increased levels, ϖ_{lev} is

$$\varpi_{lev} = \frac{\hat{C}^O}{C^B} - 1$$

- The welfare gain of reduced uncertainty is

$$\varpi_{unc} = \frac{1 - p_{unc}^O}{1 - p_{unc}^B} - 1$$

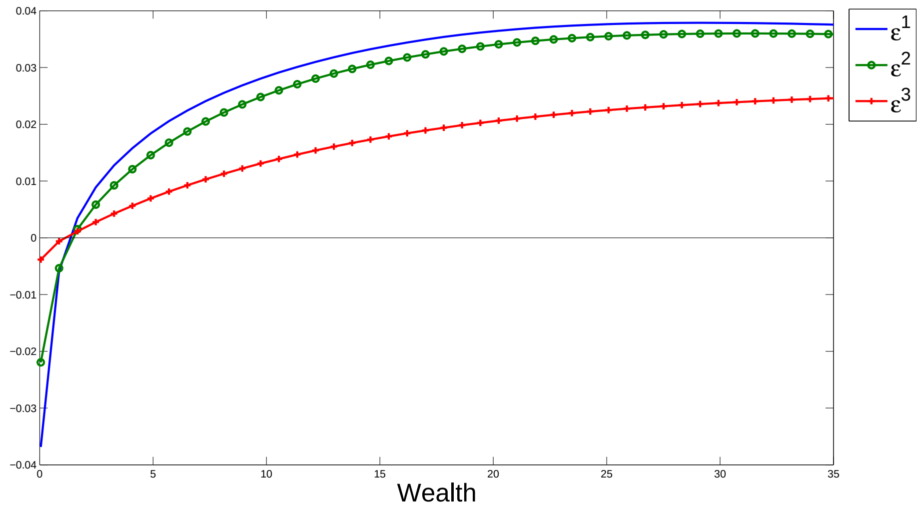

Figure 2.6: Individual welfare gains

- The welfare gain of reduced inequality is

$$\varpi_{ine} = \frac{1 - p^{O}_{ine}}{1 - p^{B}_{ine}} - 1$$

Table 2.4 shows the welfare gains in our setup. As we can see, the aggregate utilitarian welfare gains are very small, only 0.51% of life-time consumption. The majority of these gains are due to the change in consumption levels (0.44% of consumption), and the remaining is because of the decrease in uncertainty that is associated with the optimal policy (0.15% of consumption). The welfare gains associated to the decrease in inequality are, actually, welfare costs, and are negligible.

We proceed now to compute the individual welfare gains from the change in policy. We perform the following exercise: we calculate the percentage permanent increase in consumption ϖ^i that we should give to a household i with (w^i, ε^i) to be indifferent between living in the benchmark economy B or living in the optimal policy economy O with the same level of labor productivity and wealth. Figure 2.6 shows ϖ^i as a function of w^i and ε^i. We can see from the graph that, although the aggregate welfare gain is small, individual welfare gains and loses can be very large, depending on an agent's productivity and wealth holdings. It is by aggregation that the individual effects cancel out, thus yielding a mild aggregate effect.

From inspection of figure 2.6 we can determine who are the net winners and net losers from the reform. Because of the smaller level of steady state capital in the optimal policy economy, the wage rate is lower and labor taxes are higher. Then, very poor agents always lose with the change in policy, irrespective of their labor productivity, the reason being that poor agents rely almost entirely on their labor income to pay for consumption goods, so changes in labor income matter substantially for them. The welfare loss amounts to about a 4% of permanent consumption for low productivity agents, while it is less than 1% for high productivity households. The difference in these effects relies on the fact that utility is concave and agents with high ε are income-rich, so they can afford higher levels of consumption and leisure. On the contrary, net winners from the policy change are middle-class and rich households, again irrespective of their labor productivity, who see their returns on capital increased because of the higher real interest rate. Again, because of the concavity of the utility function, rich and low-productivity agents are the ones that benefit the more. Their welfare gains can reach a maximum of around 4% of permanent consumption, while for high-productivity households the maximum is about 2.5%.

2.7 Conclusions

The determination of the optimal monetary policy prescription in the long run is a crucial issue for policy makers as well as for academics. Arguably, central banks set their inflation targets according to some criteria related to the maximization of social welfare. The natural question that arises is what the long-run optimal inflation target should be.

The standard literature in optimal monetary and fiscal policy, by focusing on representative agent environments, looks into this problem in a partial way and only considers issues of efficiency in distributing the distortions associated to taxation. In this chapter we have relaxed the representative-agent assumption by allowing for heterogeneity and uninsurable idiosyncratic risk. This allows us to include in the analysis issues of redistribution of the tax burden and long-run effects of different tax schemes over capital and output that cannot be addressed in the traditional framework.

We make the standard modeling assumption that agents demand cash because it provides liquidity services. Moreover, we allow them to use an alternative costly transaction technology by which they economize on their money holdings. This transaction technology reconciles the model with some stylized facts reported in the literature regarding transaction patterns for different sectors of population. We are able to identify the effects of inflation as a regressive tax on consumption and as a motive to increase savings for precautionary reasons.

We calibrate the model to the U.S. economy and find that the optimal policy prescription that arises from the exercise is the Friedman rule. This result provides robustness to what is a classical result in representative-agent models. A surprising implication of the analysis is that, despite the fact that inflation taxes relatively more consumption of poor agents, these agents actually win with inflation, while middle-class and rich agents lose. Therefore, this analysis challenges the conventional wisdom that inflation hurts the poor and benefits the rich.

The analysis presented here opens many avenues for future research. Probably one of the most natural extensions is the study of the transition between the steady state with the benchmark policy and the one in which the optimal policy is implemented. Studying the transition allows to perform a more accurate analysis of the welfare gains from the change in policy for different individuals. Moreover, studying the *optimal* transition, i.e., the transition taking into account that all agents should benefit from the reform, can lead to policy plans very different than what is optimal in the long run, as is shown in Greulich and Marcet (2008).

On a related note, Doepke and Schneider (2006) have driven attention to the fact that unexpected inflation can have large redistributive effects for individuals with different portfolio holdings. As a further step, we would like to introduce aggregate fluctuations to a framework similar in spirit to the one we consider here, but taking into account this heterogeneity of portfolio holdings among different individuals. This type of environment is suitable for studying optimal monetary and fiscal policy as stabilizing mechanisms for macroeconomic fluctuations, an issue that has not been addressed in this chapter.

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A Appendix to Chapter 1

A.1 Optimal policy under full commitment: logarithmic utility

In this section we show a particular case of Proposition 1 when the utility function of households is logarithmic both in consumption and leisure, which corresponds to the utility function used for the numerical exercises in the paper.

Consider a utility function of the form:

$$u(c_t, l_t) = \alpha * log(c_t) + \delta * log(l_t)$$

with $\alpha > 0$ and $\delta > 0$. Assume that initial wealth $b_{-1} = 0$. Then the allocations and government policies can be easily computed from the optimality conditions (1.9) to (1.13). From the intertemporal budget constraint of households (1.9) it can be derived that:

$$l = \frac{\delta}{\alpha + \delta} \tag{A.1}$$

Plugging in this expression in (1.12), $c = \frac{\alpha}{\lambda}$. Combining this expression for consumption, together with (A.1), (1.10) and (1.11) we arrive to the following expression:

$$\frac{1}{1-\beta}\left(\frac{\alpha}{\lambda} - 1 + \frac{\delta}{\alpha + \delta}\right) + E_0 \sum_{t=0}^{\infty} \beta^t g_t = 0$$

Define the last term of the previous expression as

$$E_0 \sum_{t=0}^{\infty} \beta^t g_t \equiv \frac{1}{1-\beta}\tilde{g}$$

where $\tilde{g}$ is known at $t = 0$. Then

$$\lambda = \frac{\alpha + \delta}{1 - \frac{\alpha+\delta}{\alpha}\tilde{g}}$$

Substituting in the expression for c, we obtain

$$c = \frac{\alpha - (\alpha + \delta)\tilde{g}}{\alpha + \delta} \tag{A.2}$$

From the feasibility constraint (1.10), transfers are given by the difference between the actual realization of public expenditure g_t and its expected discounted value $\tilde{g}$:

$$T_t = g_t - \tilde{g} \tag{A.3}$$

Finally, from the intratemporal optimality condition of households (1.6) we can obtain an expression for the tax rate:

$$\tau = \frac{\delta(\alpha + \delta)}{\alpha}\tilde{g} \tag{A.4}$$

A.2 Proof of Proposition 2

In order to prove Proposition 2, we first need to establish some intermediate results. We begin with a discussion about the sign of Δ, the Lagrange multiplier associated to the intertemporal budget constraint in the Ramsey planner's problem.

a. The Ramsey problem with limited commitment

For ease of exposition, we will assume that only the HC has limited commitment. Since the problem of the Ramsey planner is identical to the one in section e., but without imposing constraint 1.18, we do not reproduce it here.

The optimality conditions for $t \geq 1$ are:

$$u_{c,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = 0 \tag{A.5}$$

$$u_{l,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = 0 \tag{A.6}$$

$$\psi_t - \lambda = 0 \tag{A.7}$$

$$c_t + g_t = (1 - l_t) + T_t \tag{A.8}$$

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) \geq V^a(g_t) \forall t \tag{A.9}$$

$$\mu_t^1(E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j}) - V^a(g_t)) = 0 \tag{A.10}$$

$$\gamma_t^1 = \mu_t^1 + \gamma_{t-1}^1 \tag{A.11}$$

$$\mu_t^1 \geq 0 \tag{A.12}$$

Multiplying equations (A.5) and (A.6) by c_t and $-(1-l_t)$ respectively, and summing:

$$\begin{aligned}&(1+\gamma_t^1-\Delta)(u_{c,t}c_t - u_{l,t}(1-l_t)) - \psi_t(c_t-(1-l_t))\\&-\Delta\underbrace{\left(u_{cc,t}c_t^2 - 2u_{cl,t}(1-l_t)c_t + u_{ll,t}(1-l_t)^2\right)}_{A_t} = 0\end{aligned} \tag{A.13}$$

Notice that given that the utility function is strictly concave, expression A is strictly negative. By a similar procedure we can write down an equivalent expression at $t=0$:

$$\begin{aligned}&(1+\gamma_0^1-\Delta)(u_{c,0}(c_0-b_{-1}) - u_{l,0}(1-l_0)) - \psi_0(c_0-(1-l_0)-b_{-1})\\&-\Delta\underbrace{\left(u_{cc,0}(c_0-b_{-1})^2 - 2u_{cl,0}(1-l_0)(c_0-b_{-1}) + u_{ll,0}(1-l_0)^2\right)}_{A_0} = 0\end{aligned} \tag{A.14}$$

Multiplying (A.13) by $\beta^t\pi(s^t)$[1], summing over t and s^t and adding expression (A.14):

$$\begin{aligned}&E_0\sum_{t=0}^{\infty}\beta^t(1+\gamma_t^1-\Delta)(u_{c,t}c_t - u_{l,t}(1-l_t)) - (1+\gamma_0-\Delta)u_{c,0}b_{-1}\\&-\Delta Q - E_0\sum_{t=0}^{\infty}\beta^t\psi_t(c_t-(1-l_t)) + \psi_0 b_{-1} = 0\end{aligned}$$

where Q is the expected value of the sum of negative quadratic terms A_t. Using the implementability constraint (1.15) and the resource constraint (1.14) we obtain equation (A.15)

$$\begin{aligned}&E_0\sum_{t=0}^{\infty}\beta^t(\gamma_t^1-\gamma_0^1)(u_{c,t}((1-l_t)+T_t-g_t) - u_{l,t}(1-l_t))\\&-\Delta Q + E_0\sum_{t=0}^{\infty}\beta^t\psi_t(g_t-T_t) + \psi_0 b_{-1} = 0\end{aligned} \tag{A.15}$$

For later purposes, using the intratemporal optimality condition of households (1.6) we can reexpress this equation as[2].

[1] $\pi(s^t)$ is the probability of history s^t taking place given that the event s_0 has been observed.

[2] Notice that if the participation constraint was never binding, then $\gamma_t^1 = \gamma_0^1 = 0$ and we would recover an identical condition to the one obtained in the Lucas and Stokey (1983) model.

$$E_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t^1 - \gamma_0^1) u_{c,t} (\tau_t (1 - l_t) - g_t + T_t) - \Delta Q + E_0 \sum_{t=0}^{\infty} \beta^t \lambda g_t + \lambda b_{-1} = 0 \quad \text{(A.16)}$$

Notice that, in the case of full commitment, expression (A.15) simplifies to

$$-\Delta Q + \lambda \left(E_0 \sum_{t=0}^{\infty} \beta^t g_t + b_{-1} \right) = 0 \quad \text{(A.17)}$$

Since $\lambda = \psi_t > 0 \ \forall t$, it is straightforward to see that when the present value of all government expenditures exceeds the value of any initial government wealth, the Lagrange multiplier $\Delta < 0$.

In the presence of limited commitment, however, there is an extra term involving the costate variable γ_t^1 which prevents us from applying the same reasoning. Nevertheless, we will show that this is the case for the specific example of section 1.3, and we will assume this result extends to the general setup. In the numerical exercise we perform in section 1.4 we confirm that this assumption holds.

We show now under which conditions $\Delta = 0$. Setting $\Delta = 0$, from equations (A.5) and (A.6) we know that

$$u_{c,t}(1 + \gamma_t) = u_{l,t}(1 + \gamma_t) \quad \text{(A.18)}$$

$$u_{c,t} = u_{l,t} \quad \text{(A.19)}$$

This last expression and equation (1.6) in the text imply that $\tau_t = 0 \ \forall t$. Inserting these results into equation (A.16):

$$E_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t - \gamma_0)(u_{c,t}(T_t - g_t) + E_0 \sum_{t=0}^{\infty} \beta^t \lambda (g_t - T_t) + \lambda b_{-1} = 0$$

Using (A.18)

$$\Rightarrow E_0 \sum_{t=0}^{\infty} \beta^t (-\gamma_t + \gamma_0 + 1 + \gamma_t) u_{c,t} (g_t - T_t) + u_{c,0}(1 + \gamma_0) b_{-1} = 0$$

$$\Rightarrow E_0 \sum_{t=0}^{\infty} \beta^t \frac{u_{c,t}}{u_{c,0}} (g_t - T_t) = -b_{-1} \quad \text{(A.20)}$$

We can rewrite (A.20) as

$$\sum_{t=0}^{\infty}\sum_{s^t} p_t^0(g_t - T_t) = -b_{-1} = b_{-1}^g \tag{A.21}$$

where p_t^0 is the price of a hypothetical bond issued in period 0 with maturity in period t contingent on the realization of s_t. Equation (A.21) states that when the government's initial claims b_{-1}^g against the private sector equal the present-value of all future government expenditures net of transfers, the Lagrange multiplier Δ is zero. Since the government does not need to resort to any distortionary taxation, the household's present-value budget does not exert any additional constraining effect on welfare maximization beyond what is already present through the economy's technology.

Finally, we will follow Ljungqvist and Sargent (2000) and assume that if the government's initial claims against the private sector were to exceed the present value of future government expenditures, the government would return its excess financial wealth as lump-sum transfers and Δ would remain to be zero.

b. Proof of Proposition 2

We begin by proving the first part of the Proposition. Given a logarithmic utility function as (1.30), optimality conditions (1.19) to (1.21) become

$$\frac{\alpha}{c_t}(1+\gamma_t^1) - (\lambda+\gamma_t^2) - \Delta\left(-\frac{\alpha}{c_t^2}c_t + \frac{\alpha}{c_t}\right) = 0$$

$$\Longrightarrow c_t = \frac{\alpha(1+\gamma_t^1)}{\lambda+\gamma_t^2} \tag{A.22}$$

$$\frac{\delta}{l_t}(1+\gamma_t^1) - (\lambda+\gamma_t^2) - \Delta\left(\frac{\delta}{l_t} + \frac{\delta}{l_t^2}(1-l_t)\right) = 0$$

$$\Longrightarrow l_t = \frac{\delta(1+\gamma_t^1) \pm \sqrt{\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}}{2(\lambda+\gamma_t^2)} \tag{A.23}$$

Notice from equation (A.23) that if $\Delta < 0$ then we need to take the square root with positive sign in order to have $l_t > 0$. To show that consumption and leisure increase with γ_t^1, we take the derivatives of c_t and l_t with respect to γ_t^1

$$\frac{\partial c_t}{\partial \gamma_t^1} = \frac{\alpha}{\lambda+\gamma_t^2} > 0$$

$$\frac{\partial l_t}{\partial \gamma_t^1} = \frac{\delta + \left(\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)\right)^{-\frac{1}{2}}\delta^2(1+\gamma_t^1)}{2(\lambda+\gamma_t^2)} > 0$$

We can write the intratemporal optimality condition of households (1.6) as

$$\tau_t = \frac{u_{c,t} - u_{l,t}}{u_{c,t}} = 1 - \frac{\delta c_t}{\alpha l_t} \tag{A.24}$$

Given $t < t'$, assume $\gamma_t^1 < \gamma_{t'}^1$ while $\gamma_t^2 = \gamma_{t'}^2$. Now we compare the tax rates at t and t', and show that τ_t decreases with γ_t^1 by contradiction. Then, using (A.24)

$$\tau_{t'} - \tau_t = \frac{\delta}{\alpha}\left(\frac{c_t}{l_t} - \frac{c_{t'}}{l_{t'}}\right) > 0$$

It follows that it must be the case that $c_t l_{t'} - c_{t'} l_t > 0$. After some algebra this condition translates into

$$\left(\frac{1+\gamma_t^1}{1+\gamma_{t'}^1}\right)^2 > \frac{\delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}{\delta^2(1+\gamma_{t'}^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)}$$

$$(1+\gamma_t^1)^2 > (1+\gamma_{t'}^1)^2$$

which is clearly a contradiction. Thus, τ_t increases with γ_t^1.

Now we proceed to prove the second part of the Proposition. We can immediately check that c_t decreases with γ_t^2 by taking partial derivates:

$$\frac{\partial c_t}{\partial \gamma_t^2} = -\frac{\alpha(1+\gamma_t^1)}{(\lambda+\gamma_t^2)^2} < 0$$

Suppose l_t is an increasing function of γ_t^2. Then the partial derivative of l_t w.r.t γ_t^2 must be positive

$$\frac{\partial l_t}{\partial \gamma_t^2} = \frac{-2\Delta\delta(\lambda+\gamma_t^2)A^{-\frac{1}{2}} - \delta(1+\gamma_t^1) - A^{\frac{1}{2}}}{4(\lambda+\gamma_t^2)^2} > 0$$

where $A = \delta^2(1+\gamma_t^1)^2 - 4\Delta\delta(\lambda+\gamma_t^2)$. This last expression implies that

$$-2\Delta\delta(\lambda+\gamma_t^2) > \delta(1+\gamma_t^1)A^{\frac{1}{2}} + A$$

Then,

$$2\Delta\delta(\lambda+\gamma_t^2) - \delta^2(1+\gamma_t^1)^2 > A^{\frac{1}{2}}\delta(1+\gamma_t^1)$$

Since the left hand side of the previous expression is negative, while the right hand side is positive, this statement is clearly a contradiction. Then it must be the case that l_t is a decreasing function of γ_t^2.

Finally, suppose that $t' > t$, $\gamma^2_{t'} > \gamma^2_t$ but $\gamma^1_{t'} = \gamma^1_t$. Assume that τ_t is a decreasing function of γ^2_t. Then, using (A.24), it must be the case that

$$u_{c,t'}u_{l,t} < u_{c,t}u_{l,t'}$$

This implies that

$$\begin{aligned}&\frac{\delta(1+\gamma^1_t)+\sqrt{\delta^2(1+\gamma^1_t)^2-4\delta\Delta(\lambda+\gamma^2_{t'})}}{2(\lambda+\gamma^2_{t'})}\frac{\alpha(1+\gamma^1_t)}{\lambda+\gamma^2_t} < \\ &\frac{\delta(1+\gamma^1_t)+\sqrt{\delta^2(1+\gamma^1_t)^2-4\delta\Delta(\lambda+\gamma^2_t)}}{2(\lambda+\gamma^2_t)}\frac{\alpha(1+\gamma^1_t)}{\lambda+\gamma^2_{t'}}\end{aligned} \tag{A.25}$$

Simplifying and remembering that $\Delta < 0$, the previous inequality is a contradiction. Therefore, τ_t increases with γ^2_t. This completes the proof.

A.3 Proof of Proposition 3

Notice first that at $t = 0$ and for $\gamma^1_0 = 0$, the continuation value of staying in the contract has to be (weakly) greater than the value of the outside option (financial autarky):

$$\sum_{t=0}^{\infty}\beta^t u(c_t, l_t) \geq \sum_{t=0}^{\infty}\beta^t u(c_{t,A}, l_{t,A}) \tag{A.26}$$

The reason for this statement is that, for the government, subscribing the contract with the rest of the world represents the possibility to do risk-sharing and, consequently, to smooth consumption of domestic households. Since utility is concave, a smoother consumption path translates into a higher life-time utility value. Obviously, this result hinges on the fact that the initial debt of the government is zero and that equation (1.1) must hold[3].

Now we show that equation (1.17) holds with strict inequality for $1 \leq t \leq T$. It is important to bear in mind that the allocations could change in time only due to a different γ^1_t. Since $\gamma^1_{t-1} \leq \gamma^1_t$ $\forall t$, then $u(c_{t-1}) \leq u(c_t)$. Assume that $\mu^1_1 > 0$. This implies that, if μ^1_1 was equal to zero, the PC would be violated, that is,

[3]If, for example, the initial level of government debt b_{-1} was very high, then the government could find it optimal to default on this debt and run a balanced budget thereafter. On the other hand, if condition (1.1) was not imposed, then the contract could mean a redistribution of resources from the HC to the RW that could potentially lead the HC to have incentives not to accept the contract.

$$u(c_0,l_0)+\sum_{t=2}^{T-1}\beta^{t-1}u(c_t,l_t)+\beta^{T-1}u(c_T,l_T)+\sum_{t'=T+1}^{\infty}\beta^{t'-1}u(c_{t'},l_{t'})$$
$$<\sum_{t=0}^{T-2}\beta^t u(c_A,l_A)+\beta^{T-1}u(c_{A'},l_{A'})+\sum_{t=T}^{\infty}\beta^t u(c_A,l_A) \tag{A.27}$$

Equation (A.26) can be rewritten as

$$\sum_{t=0}^{T}\beta^t u(c_t,l_t)+\sum_{t'=T+1}^{\infty}\beta^{t'}u(c_{t'},l_{t'})$$
$$>\sum_{t=0}^{T-1}\beta^t u(c_A,l_A)+\beta^T u(c_{A'},l_{A'})+\sum_{t=T+1}^{\infty}\beta^t u(c_A,l_A) \tag{A.28}$$

Subtracting (A.28) from (A.27):

$$\beta[u(c_2,l_2)-u(c_1,l_1)]+\beta^2[u(c_3,l_3)-u(c_2,l_2)]+\ldots+\beta^{T-1}[u(c_T,l_T)-u(c_{T-1},l_{T-1})]+$$
$$\beta^T[u(c_{T+1},l_{T+1})-u(c_T,l_T)]+\beta^{T+1}[u(c_{T+2},l_{T+2})-u(c_{T+1},l_{T+1})]+\ldots$$
$$<\beta^{T-1}[u(c_{A'},l_{A'})-u(c_A,l_A)]+\beta^T[u(c_A,l_A)-u(c_{A'},l_{A'})] \tag{A.29}$$

Reordering terms we arrive at:

$$\beta\underbrace{[u(c_2,l_2)-u(c_1,l_1)]}_{\geq 0}+\beta^2\underbrace{[u(c_3,l_3)-u(c_2,l_2)]}_{\geq 0}+\ldots+\beta^{T-1}\underbrace{[u(c_T,l_T)-u(c_{T-1},l_{T-1})]}_{\geq 0}+$$
$$+\beta^T\underbrace{[u(c_{T+1},l_{T+1})-u(c_{T-1},l_{T-1})]}_{\geq 0}+\beta^{T+1}\underbrace{[u(c_{T+2},l_{T+2})-u(c_{T+1},l_{T+1})]}_{\geq 0}+\ldots$$
$$<\underbrace{[u(c_{A'},l_{A'})-u(c_A,l_A)]}_{<0}\underbrace{(\beta^{T-1}-\beta^T)}_{>0} \tag{A.30}$$

Expression (A.30) is clearly a contradiction, since the left hand side of the inequality is greater or equal to 0, but the right hand side is strictly smaller than 0. We conclude then that it cannot be that $\mu_1^1>0$. Therefore, equation(1.17) is not binding in period $t=1$. The same reasoning can be extended to periods $t=2,3,\ldots,T$. Therefore, $\gamma_t^1=\gamma_0^1=0$ for $t=1,2,...,T$ and the allocations $\{c_t\}_{t=0}^T,\{l_t\}_{t=0}^T$ are constant.

Notice that, from $T+1$ onwards, $g_t=0$ so the allocations do not change. Therefore, $\gamma_t^1=\gamma_{T+1}^1$ for $t=T+2,T+3,\ldots,\infty$.

Finally, we show that $\mu^1_{T+1} > 0$[4]. We prove this by contradiction. Assume that $\mu_{T+1} = 0$. From the previous discussion, this implies that $\gamma^1_t = 0\ \forall t$. Then the allocations are identical to the case of limited commitment, and from the results of section A.1, we know that $T_t < 0$ for $t \neq T$ and $T_T > 0$. Thus, from the feasibility constraint (1.14) we can see that $c_{T+1} < c_A$ and $l_{T+1} < l_A$. But this implies that utility $u(c_{T+1}, l_{T+1}) < u(c_A, l_A)$, so

$$\frac{1}{1-\beta}u(c_{T+1}, l_{T+1}) < \frac{1}{1-\beta}u(c_A, l_A)$$
$$\sum_{j=0}^{\infty}\beta^j u(c_{T+1+j}, l_{T+1+j}) < \sum_{j=0}^{\infty}\beta^j u(c_A, l_A)$$

which clearly contradicts with the fact that $\mu^1_{T+1} = 0$. Therefore, it must be the case that $\mu^1_{T+1} > 0$. This completes the proof.

A.4 Proof that $\Delta < 0$ in Section 1.3

Since in the example of Section 1.3 we have a full analytical characterization of the equilibrium, it is possible to determine the sign of Δ.

Given our assumption about the government expenditure shock and the result of Proposition 3, equation (A.16) can be written as

$$\sum_{t=T+1}^{\infty}\beta^t(\gamma_{T+1} - \gamma_0)u_{\bar{c}}(\bar{\tau}(1-\bar{l}) + \bar{T}) - \Delta Q + \beta^T\lambda g_T = 0 \qquad \text{(A.31)}$$

where $\bar{c}, \bar{l}, \bar{\tau}$ and $\bar{T}$ are the constant allocations and fiscal variables from $t = T+1$ onwards. In order to determine the sign of the first term of the previous expression, we recall the period by period budget constraint of the government for $t \geq T+1$:

$$(\beta - 1)\bar{b}^G = \bar{\tau}(1-\bar{l}) + \bar{T}$$

The sign of the first term of equation (A.16) depends on whether government bonds are positive or negative after the big shock has taken place. From equation (1.15)

[4] Notice that, given that our shock in this example is not a Markov process, neither γ_t nor the allocations c_t and l_t are time-invariant functions of the state variables g_t, γ_{t-1} but, on the contrary, the depend on t.

$$
\begin{aligned}
&\sum_{j=0}^{T} \beta^t (u_{\tilde{c}}\tilde{c} - u_{\tilde{l}}(1-\tilde{l})) + \sum_{j=T+1}^{\infty} \beta^t (u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l})) = 0 \\
\Rightarrow &\frac{1-\beta^{T+1}}{1-\beta}\left(\alpha - \frac{\delta}{\tilde{l}}(1-\tilde{l})\right) + \frac{\beta^{T+1}}{1-\beta}\left(\alpha - \frac{\delta}{\bar{l}}(1-\bar{l})\right) = 0
\end{aligned} \tag{A.32}
$$

where $\tilde{c}$ and $\tilde{l}$ are the constant allocations from $t = 0$ to $t = T$. We know that the participation constraint binds in period $T+1$ and consequently $\bar{l} > \tilde{l}$. But this implies that

$$
\begin{aligned}
\alpha - \frac{\delta}{\tilde{l}}(1-\tilde{l}) < 0 \\
\alpha - \frac{\delta}{\bar{l}}(1-\bar{l}) > 0
\end{aligned} \tag{A.33}
$$

because the two terms of (A.32) have to add up to zero. Now we recover b_t for $t \geq T+1$ from the intertemporal budget constraint (1.15) of households at time $T+1$:

$$
\begin{aligned}
u_{\bar{c}}\bar{b} =& \sum_{j=0}^{\infty} \beta^j (u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l})) = \frac{1}{1-\beta}(u_{\bar{c}}\bar{c} - u_{\bar{l}}(1-\bar{l})) \\
=& \frac{1}{1-\beta}\left(\alpha - \frac{\delta}{\bar{l}}(1-\bar{l})\right) > 0
\end{aligned}
$$

If $\bar{b} > 0$, $\bar{b}^G < 0$ so the first term in equation (A.31) is positive. But then from this equation it is immediate to see that $\Delta < 0$.

A.5 The International Institution Problem and the Government Problem: Equivalence of Results

Suppose that there exists an international financial institution that distributes resources among the HC and the RW, taking into account the aggregate feasibility constraint, the implementability condition (1.15), and participation constraints (1.17) and (1.18). The Lagrangian associated to the international institution is

$$
\begin{aligned}
&\max_{\{c_t^1, c_t^2, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (\alpha u(c_t^1, l_t^1) + (1-\alpha) u(c_t^2) + \\
&+\tilde{\mu}_{1,t}(E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}^1, l_{t+j}^1) - V_t^{1,a}) + \tilde{\mu}_{2,t}(\underline{B} - E_t \sum_{j=0}^{\infty} \beta^j T_{t+j}) + \\
&-\tilde{\Delta}(u_{c^1,t} c_t^1 - u_{l_{1_t}}(1-l_t^1)) + \tilde{\psi}_t(c_t^1 + c_t^2 + g - (1 - l_t^1 + y)))
\end{aligned} \tag{A.34}
$$

where α is the Pareto weight that the international institution assigns to the HC. Since by assumption households in the RW are risk-neutral, $u(c_t^2) = c_t^2$. The feasibility constraint in the RW implies that $c_t^2 = y - T_t$. Substituting this into A.34 and applying **?** we can recast problem A.34 as

$$\max_{\{c_t^1, c_t^2, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t((\alpha + \tilde{\gamma}_{1,t})u(c_t^1, l_t^1) + (1-\alpha)(y - T_t) +$$
$$-\tilde{\mu}_{1,t}V_t^{1,a} + \tilde{\mu}_{2,t}\underline{B} - \tilde{\gamma}_{2,t}T_t) - \tilde{\Delta}(u_{c^1,t}c_t^1 - u_{l_{1_t}}(1-l_t^1)) + \tilde{\psi}_t(c_t^1 + g - (1 - l_t^1 + T_t)) \tag{A.35}$$

Dividing each term by α does not change the solution, since α is a constant. Let $\frac{\tilde{\gamma}_t^i}{\alpha} \equiv \gamma_t^i$, for $i = 1, 2$, $\frac{\tilde{\Delta}}{\alpha} \equiv \Delta$ and $\frac{\tilde{\psi}_t}{\alpha} \equiv \psi_t$.

The first-order conditions are

$$u_{c,t}(1 + \tilde{\gamma}_t^1) - \psi_t - \Delta(u_{cc,t}c_t + u_{c,t} - u_{cl,t}(1 - l_t)) = 0 \tag{A.36}$$

$$u_{l,t}(1 + \gamma_t^1) - \psi_t - \Delta(u_{cl,t}c_t + u_{l,t} - u_{ll,t}(1 - l_t)) = 0 \tag{A.37}$$

$$\psi_t = \frac{1-\alpha}{\alpha} + \gamma_t^2 \tag{A.38}$$

where $c_t \equiv c_t^1$. Posing $\lambda \equiv \frac{1-\alpha}{\alpha}$ makes the system of equations (A.36)-(A.38) coincide with (1.19)-(1.21).

A.6 Proof of Proposition 5

The Lagrangean for the government in the HC can be written as

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t^1, l_t^1) + (\Delta + \Lambda_{1,t})(u_{c^1,t}c_t^1 - u_{l^1,t}(1 - l_t^1)) - \lambda_{1,t}B_{t-1}(g_t) + \lambda_{2,t}(-A_t^1(g_{t+1})$$
$$+ T_t^1(g_{t+1})) + \lambda_{3,t}(1 - l_t^1 - \sum_{g_{t+1}} q_t T_t^1(g_{t+1}) + T_{t-1}^1 - c_t^1 - g_t) - \Delta b_{-1}u_{c^1,0}\}$$

where $\Lambda_{1,t} = \Lambda_{1,t-1} + \lambda_{1,t}$.

The optimality conditions are

$$u_{c^1,t} + (\Delta + \Lambda_{1,t})(u_{cc^1,t}c_t^1 + u_{c^1,t}) = \lambda_{3,t} \tag{A.39}$$

$$u_{l^1,t} + (\Delta + \Lambda_{1,t})(u_{l^1,t} - u_{ll^1,t}(1 - l_t^1)) = \lambda_{3,t} \quad \text{(A.40)}$$

$$q_t(g_{t+1}) = \frac{\beta\lambda_{3,t+1}(g_{t+1})\pi((g_{t+1})) + \lambda_{2,t}}{\lambda_{3,t}} \quad \text{(A.41)}$$

Denote by $\overline{T}_t^2$ the limit on transfers received by the RW, in the original problem of section 1.2. The problem of the household in the RW is

$$\max_{\{c_t, T_t^2\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{(A.42)}$$

s.t.

$$T^2(g_{t+1}) > \overline{T}_t^2 \quad \text{(A.43)}$$

$$y + T_{t-1}^2(g_t) = c_t^2 + \sum_{g_{t+1}} q_t T_t^2(g_{t+1}) \quad \text{(A.44)}$$

The Lagrangean for this problem can be written as

$$\begin{aligned}\mathcal{L} =& E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t^2) + \lambda_t \left(y - \sum_{g_{t+1}} q_t T_t^2(g_{t+1}) + T_{t-1}^2 - c_t^2 \right) \\ &+ \gamma_t^2 (T^2(g_{t+1}) - A_t^2(g_{t+1}))\}\end{aligned}$$

The first order conditions are

$$u_{c^2,t} = \lambda_t \quad \text{(A.45)}$$

$$q_t = \frac{\beta u_{c^2,t+1}\pi((g_{t+1})) + \gamma_t^2}{u_{c^2,t}} \quad \text{(A.46)}$$

It is easy to show that equations (A.39) and (A.40) coincide with equations (1.19) and (1.20) with

$$\frac{\Delta}{1+\gamma_t^1} = (\Delta + \Lambda_{1,t}). \quad \text{(A.47)}$$

Proposition 5 says that a binding participation constraint in the HC can be seen as a binding constraint on the value of domestic debt the government can issue.

The price of the bond exchanged between the two countries is equal to

$$q_t(g_{t+1}) = max\{\frac{\beta u_{c^2,t+1}\pi((g_{t+1})) + \gamma_t^2}{u_{c^2,t}}, \frac{\beta\lambda_{3,t+1}(g_{t+1})\pi((g_{t+1})) + \lambda_{2,t}}{\lambda_{3,t}}\} \quad \text{(A.48)}$$

B Appendix to Chapter 2

B.1 Optimality conditions of the household

The problem of household i is

$$\max\; E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i) \tag{B.1}$$

subject to

$$p_t c_t^i + q_t \int_0^{z_t^i} \gamma^i(j)dj + W_{t+1}^i = RA_t^i + M_t^i + (1-\tau^l)\omega p_t(1-l_t^i)\varepsilon_t^i \tag{B.2}$$

$$p_t c_t^i(1-z_t^i) \leq M_t^i \tag{B.3}$$

$$A_t^i \geq 0 \tag{B.4}$$

We define

$$a_t = \frac{A_t}{p_{t-1}}$$
$$w_t = \frac{W_t}{p_{t-1}}$$
$$m_t = \frac{M_t}{p_{t-1}}$$

We can rewrite equations (B.2) and (B.3) in real terms by diving both sides of the equations by p_t:

$$c_t^i + q + \omega \int_0^{z_t^i} \gamma^i(j)dj + w_{t+1}^i = (1+\tilde{r})a_t^i + \frac{m_t^i}{1+\Pi} + (1-\tau^l)\omega(1-l_t^i)\varepsilon_t^i \tag{B.5}$$

$$c_t^i(1-z_t^i) \leq \frac{m_t^i}{1+\Pi} = \frac{m_t^i R}{1+\tilde{r}} \tag{B.6}$$

where we have used the fact that $\frac{p_t}{p_{t-1}} = 1+\Pi$, $q_t = \omega p_t$ and $R = (1+\tilde{r})(1+\Pi)$. $\tilde{r} = r(1-\tau^k)$ is the after-tax real return on capital.

Plugging equation (B.6) into (B.5), using $w_t^i = a_t^i + m_t^i$ and rearranging, we obtain

$$c_t^i(1-(1-z_t^i)(1-R))+\omega\int_0^{z_t^i}\gamma^i(j)dj+w_{t+1}^i=(1+\tilde{r})w_t^i+(1-\tau^l)\omega(1-l_t^i)\varepsilon_t^i \quad \text{(B.7)}$$

Similarly, equation (B.4) can be rewritten as

$$c_t^i(1-z_t^i)\frac{R}{1+\tilde{r}}-w_t^i\leq 0 \quad \text{(B.8)}$$

The problem of the household becomes maximizing (B.1) subject to constraints (B.7) and (B.8).

The optimality conditions of the household are:

$$R=1+\frac{\omega\gamma(z_t^i)}{c_t^i} \quad \text{(B.9)}$$

$$\frac{u_{l,t}}{\varepsilon_t^i u_{c,t}}\Gamma_t=(1-\tau^l)\omega \quad \text{(B.10)}$$

where

$$\Gamma_t=\left(1+(1-z_t^i)\frac{\omega\gamma(z_t^i)}{c_t^i}\right)$$

along with the Euler equation. Call μ_t the multiplier associated to constraint (B.8). If the borrowing constraint in period $t+1$ is not binding, i.e., if $\mu_{t+1}=0$, then the corresponding Euler equation is

$$\frac{u_{c,t}}{\Gamma_t}=\beta(1+\tilde{r})E_t\frac{u_{c,t+1}}{\Gamma_{t+1}} \quad \text{(B.11)}$$

If, on the contrary, $\mu_{t+1}>0$ the Euler equation becomes

$$\frac{u_{c,t}}{\Gamma_t}=\beta(1+\tilde{r})E_t\frac{u_{c,t+1}}{\Gamma_{t+1}}\left(1+\frac{1}{R}\left((1-R)+\frac{\omega\gamma(z_{t+1}^i)}{c_{t+1}^i}\right)\right) \quad \text{(B.12)}$$

Given that equation (B.8) can be binding in t and/or in $t+1$, 4 possible cases need to be considered when solving for the allocations of agent i:

- $\mu_t=0$ and $\mu_{t+1}=0$. The relevant equations for obtaining the allocations are (B.9), (B.10), (B.11) and the budget constraint (B.7).
- $\mu_t>0$ and $\mu_{t+1}=0$. The relevant equations for obtaining the allocations are (B.8), (B.10), (B.11) and the budget constraint (B.7).

- $\mu_t = 0$ and $\mu_{t+1} > 0$. The relevant equations for obtaining the allocations are (B.9), (B.10), (B.12) and the budget constraint (B.7).

- $\mu_t > 0$ and $\mu_{t+1} > 0$. The relevant equations for obtaining the allocations are (B.8), (B.10), (B.12) and the budget constraint (B.7).

B.2 Optimal fiscal and monetary policy with constant heterogeneity and no idiosyncratic risk

As in section a., assume that an agent's productivity ε^i is constant $\forall t$, $\varepsilon^i \in E = [\varepsilon^1, \varepsilon^2]$ with $\varepsilon^1 < \varepsilon^2$ and there is an equal mass of each type of agent in the population [1]. Furthermore, assume for simplicity that the initial wealth holdings w_0^1 and w_0^2 are such that the economy is in steady state from $t = 0$ onwards.

Consider the case of a benevolent government (a Ramsey planner) that has to decide on the level of R and τ^l in our economy, in order to maximize a social welfare function given by the weighted sum of the utilities of both types of agents. The problem of the government can be written as:

$$\max_{\{c^i, l^i, z^i\}} \sum_{t=0}^{\infty} \beta^t u(c^i, l^i)$$

s.t.

$$\frac{1}{1-\beta}\left[u_{c^i}c^i + \frac{u_{c^i}c^i}{c^i + (1-z^i)\gamma(z^i)F_{L^g}} F_{L^g} \int_{\tilde{z}^i}^{z^i} \gamma(j)dj - u_{l^i}(1-l^i)\right] \tag{B.13}$$

$$= \frac{u_{c^i}c^i(1+\tilde{r})}{c^i + (1-z^i)\gamma(z^i)F_{L^g}} W_{-1}^i \;\; i = 1,2 \tag{B.14}$$

$$c^1 + c^2 + g + \delta K = F(K, L^g) \tag{B.15}$$

$$\int_{\tilde{z}^1}^{z^1} \gamma(j)dj + \int_{\tilde{z}^2}^{z^2} \gamma(j)dj = (1-l^1)\bar{\varepsilon^1} + (1-l^2)\bar{\varepsilon^2} - L^g \tag{B.16}$$

$$R = 1 + \frac{F_{L^g}\gamma(z^1)}{c^1} = 1 + \frac{F_{L^g}\gamma(z^2)}{c^2} \tag{B.17}$$

[1] The results of this section are robust to changes in the number of productivity states and in the composition of the population.

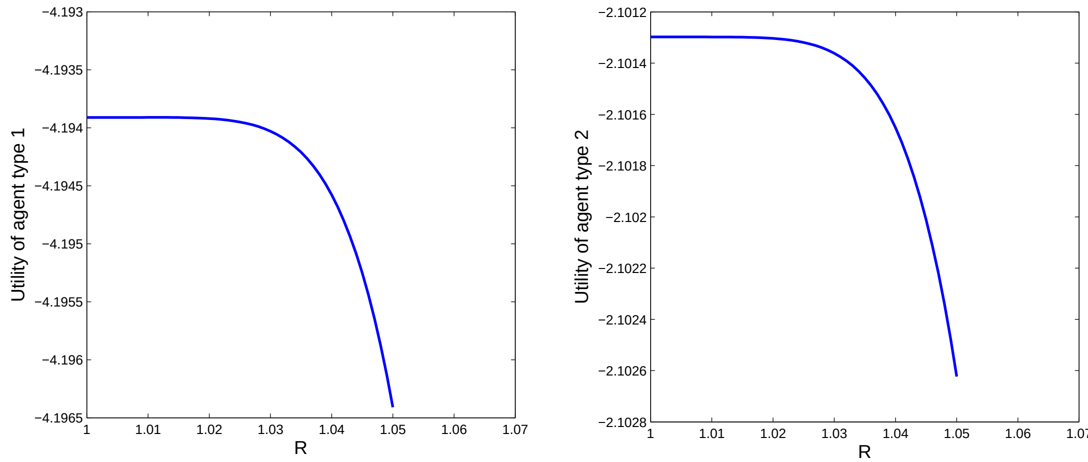

Figure B.1:

$$\frac{u_{l^1}}{\varepsilon^1 u_{c^1}}\left(1+\frac{(1-z^1)F_{L^g}\gamma(z^1)}{c^1}\right)=\frac{u_{l^2}}{\varepsilon^2 u_{c^2}}\left(1+\frac{(1-z^2)F_{L^g}\gamma(z^2)}{c^2}\right) \tag{B.18}$$

where $K = K^1 + K^2$, $L^g = L^{g,1} + L^{g,2}$ and ψ^i is the Pareto weight that the Ramsey planner assigns an agent of type i.

Equations (B.14), (B.15) and (B.16) correspond to the implementability constraints and resource constraints respectively. Notice that, when there is more than one type of agent in the economy, we need to consider one implementability constraint *for each type of household*[2].

We assume that the tax system is *anonymous*, in the sense that the tax rates on labor τ^l and τ^k are not agent-specific. τ^k is exogenously imposed at a certain level for all individuals. Equation (B.18) imposes the condition that tau_l is equal for both types of households[3]. Finally, given that the gross nominal interest rate R has to be the the same across individuals, equation (B.17) needs to be imposed.

In this case, the solution to the Ramsey problem varies with the determination of $\tilde{z}^i$ and with the Pareto weight ψ^i that corresponds to each type of agent. Consider first the case in which $\tilde{z}^i = \tilde{z}$ for $i = 1, 2$. Our numerical exercise yields the following result:

RESULT 2. *In the heterogeneous-agent case with no idiosyncratic uncertainty and $\tilde{z}^i = \tilde{z}$ for $i = 1, 2$, the Friedman rule is optimal for all possible Pareto weights $\psi^1, \psi^2 \in [0, 1]$.*

[2]By virtue of Walras' Law, the budget constraint of the government is automatically satisfied.

[3]To see why this equation implies that τ_l should be equal across individuals, notice that the optimality conditions of household i require that

$$\frac{u_{l^i}}{u_{c^i}\varepsilon^i}\left(1+\frac{(1-z^i)\gamma(z^i)}{c^i}\right)=\omega(1-\tau^l)$$

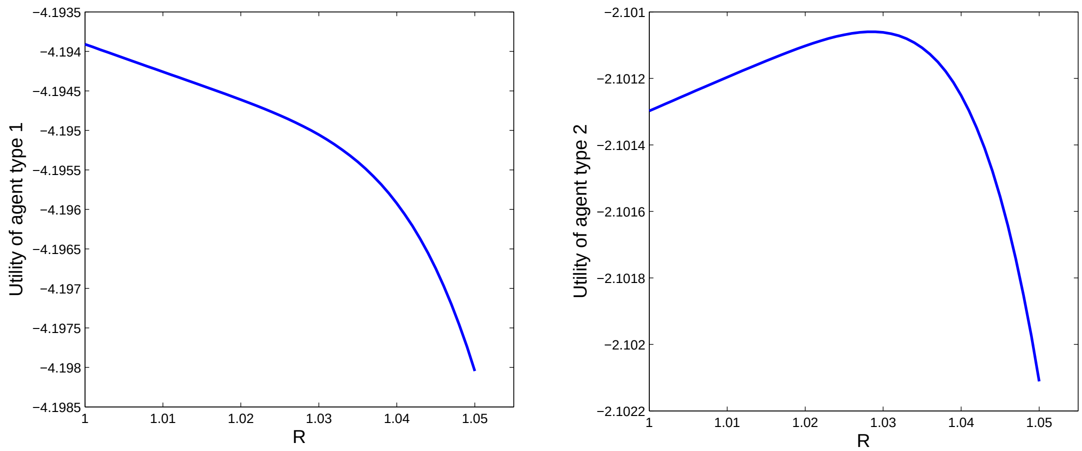

Figure B.2:

The intuition for the result lies in the uniform taxation argument explained in the main text of the paper. When we allow $\tilde{z}^i = f(\varepsilon^i)$ the optimal policy prescription varies depending on the Pareto weights we consider, as summarized below:

Result 3. *Assume $\tilde{z}^i = f(\varepsilon^i)$ and $\frac{\partial \tilde{z}^i}{\varepsilon^i} > 0$, so that $\tilde{z}^1 < \tilde{z}^2$. Then, for a high enough Pareto weight on the more productive agents, ψ^2, the planner deviates from the Friedman rule and sets $R > 1$.*

Figures B.1 and B.2 illustrate what is stated in results 2 and 3. These figures show the utilities of the two types of agents for $R \in [1, 1.05]$ for the cases in which $\tilde{z}^i = 0$ for $i = 1, 2$ and $\tilde{z}^2 = 0.2 > \tilde{z}^1 = 0$, respectively. As we can see, in the first case the utility of both agents is decreasing in R, while in the second case the utility of agents with ε^2 is humped-shaped, increasing first with R, reaching a maximum and then decreasing for higher values of the nominal interest rate. The behavior of this utility causes the utility of the planner to be humped-shaped as well, provided that the weight on agents with ε^2 is high enough.

To grasp the intuition behind result 3, rewrite equation (B.10) as

$$\frac{u_{l^i}}{\varepsilon^i u_{c^i}} = \frac{\omega(1-\tau^l)}{1+(1-z^i)(R-1)} \tag{B.19}$$

Now assume that $\tilde{z}^2$ is large, that $\psi^2 \to 1$ and $\psi^1 \to 0$. In this case the denominator of the right hand side of expression (B.19) will be close to 1, even if R is very large. Since the planner cares only about agent 2, it wants to distort as little as possible the intratemporal decision of agent 2, therefore it wants the wedge between the marginal utility of leisure and that of consumption to be as close to one as possible. From expression (B.19) it is clear that the way to achieve this is to set τ^l as low as possible while relying heavily on seigniorage revenues to finance its budget. Since agents with ε^2 can shelter from inflation by recurring to credit, this is clearly optimal.

B.3 Computational algorithm

As explained in the main text, the model depicted here cannot be solved analytically. Consequently, we need to use numerical tools in order to obtain a solution.

The problem of the household includes a borrowing constraint, which is occasionally binding constraint. This constraint translates into strong non-linearities in the policy functions of the household. Since we are interested in computing welfare gains and loses at the individual level, we should compute these policy functions accurately. Thus, we choose not to use perturbation techniques but, instead, implement a collocation algorithm to obtain such functions.

The solution algorithm is as follows:

1. Define a grid for w_t^i.

2. Set an initial guess for the real interest rate r^g and an initial guess for the vector of parameters Ω^g that we want to calibrate.

3. For a given r^g and Ω^g, approximate the individual policy function:

$$w_{t+1}^i = f(w_t^i, \varepsilon_t^i) \simeq \tilde{f}(w_t^i, \varepsilon_t^i, \Lambda)$$

   We approximate the policy function with linear splines and discretize the state space using 80 grid points. We solve for Λ by collocation, which implies that we need to find Λ such that the Euler equation (B.11)-(B.12) is satisfied at every (w_t^i, ε_t^i) in our grid. Therefore, we obtain a system of non-linear equations that we solve through a successive approximations strategy.

4. Simulate the economy for 5000 ex-ante identical individuals, for 700 periods in order to obtain an invariant distribution.

5. Obtain aggregate capital by integrating asset holdings across individuals:

$$K = \int a d\lambda$$

   Compute r^f.

6. If $r^f = r^g$ stop, otherwise update r^g and iterate on 3-5 until convergence.

7. Compute the desired moments from the model, if they coincide with the targets from the data, stop, otherwise update Ω^g and iterate on 3-6 until convergence.

8. Repeat 2-7 for a fine grid of R, compute welfare and choose R such that social welfare is maximized.