Essays on Non-Price Competition and Macroeconomics

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Abstract

My dissertation is a collection of three essays that study various aspects of non-price competition among firms using fully microfounded general equilibrium models. The first two chapters, both coauthored with Benedetto Molinari, introduce advertising expenditures by firms into a dynamic and stochastic general equilibrium model (DSGE), in order to address the question of whether and how aggregate advertising expenditures provide important effects upon the aggregate economy. In particular, the first chapter provides a short-run analysis, by focusing on the implications of aggregate adverting expenditure upon the business cycle. The second chapter, in turn, focuses on long-run effects of advertising, by analyzing the implications upon the steady-state equilibrium of aggregate advertising expenditures by firms. The last chapter, by using a modified version of the canonical New Keynesian model, investigates the effect upon inflation dynamics of non-price competition among firms.

Resumen

Esta tesis contiene tres ensayos que estudian varios aspectos de la competencia no en precio entre las impresas, utilizando modelos de equilibrio general micro-fundados. En los primeros dos capítulos, ambos coautorados con Benedetto Molinari, se introducen gastos en publicidad de las empresas en un modelo dinámico y estocástico de equilibrio general, a través del cual, se estudian las implicaciones de la publicidad en la economía agregada. El primer capítulo se focaliza en los efectos de corto plazo de la publicidad, analizando las implicaciones con respecto al ciclo económico. El segundo capítulo, estudia los efectos de largo plazo de la publicidad, con el objetivo de analizar las implicaciones sobra el estado estacionario del economía. En el último capítulo se utiliza una versión modificada del modelo Neo-Keynesiano que estudia los efectos de la competencia no en precio en relación la dinámica de la inflación.

Preface

In 2005 firms spent 230 billion dollars to advertise their products in the U.S. media, around 1000 dollars per U.S. citizen. The U.S. advertising industry accounts for 2.2% of GDP, absorbs around 20% of firms' budgets for new investments, and uses 13% of their corporate profits. Similar magnitudes characterize the advertising sector of other industrialized countries, such as the United Kingdom, Germany and Japan. Despite the sizeable amount of resources absorbed, advertising has traditionally been analyzed in microeconomic contexts, receiving scarce attention in the macroeconomics literature. Advertising is typically viewed as a selling cost that potentially redistributes consumers' demand across firms without affecting the total market size, and that therefore does not play any significant role in macroeconomic theory. In this dissertation, we challenge such opinions by arguing that advertising, in particular, and non-price competition, in general, might instead provide important effects upon the aggregate economy.

The rationale for firms' advertising decisions has been identified in the literature as the positive effect of advertisements on sales. Firms realize that the demand they face is not exogenously a product of consumers' preferences, but instead that it can be tilted toward their own products through advertisements. Building on this fact, we ask whether such relationships would hold in the aggregate. Since the reason for advertising is to increase consumers' demand, as targeted advertising increases the sales of single goods, will aggregate advertising enhance aggregate consumption? If so, will it also increase aggregate demand and production? Moreover, does advertising affect other aspects of the aggregate economy?

The literature on advertising has often speculated about the way advertising would affect macro variables. The basic argument supporting this idea relies on the indirect effect that advertising may have on the aggregate demand. Although advertising itself is a relatively small sector of the aggregate production, yet by its own nature it may have a relevant effect the aggregate consumption. Since consumption is a major component of the aggregate demand, through this channel advertising may possibly create important distortions in the economy. In this dissertation, we push further this argument claiming that such distortions can be properly assessed only in a dynamic general equilibrium context. Suppose, for instance, that advertising stimulates aggregate consumption at the expense of saving. Then, it would contemporaneously increase consumption and crowd out investment, therefore having an unclear net effect on the aggregate demand. A partial equilibrium analysis would clearly miss to account for this trade-off effect. Moreover, by possibly reducing investment, advertising may restrict future production capacity, thus creating a distortion between future demand and supply of goods. A static model would miss this connection. Also, advertising may imply a reallocation of resources across sectors, thereby indirectly creating pressures on prices in the productive factor markets, thus distorting the aggregate supply.

In order to cope with all the effects mentioned above, we introduce advertising in a neoclassical growth model with monopolistically competitive firms. This model allows us to precisely identify the conditions under which the presence of advertising significantly affects the aggregate economy, both in the short-run (Chapter 1) and in the long-run (Chapter 2). The main contributions of our analysis to the debate on the macroeconomic effects of advertising can be summarized as follows. First, by means of a bayesian estimator, we provide evidence supporting the hypothesis that advertising is one of the determinant of aggregate consumption. Second, because of its effect on consumption, we show that advertising operates as an endogenous amplification mechanism for any stochastic shock hitting the economy. Third, we show that the presence of advertising expenditures by firms modifies the property of the stationary equilibrium of the economy, increasing the equilibrium level of hours worked, output and its components. Finally, we provide a general equilibrium framework that rationalizes the potential linkage between advertising and labor supply.

In fact, from the standpoint of households, advertising operates in our framework as an endogenous tastes shock that, by increasing the marginal evaluation of consumption, makes the households more inclined to substitute from leisure into consumption. All else being equal, this implies that an increase in aggregate advertising shifts the labor supply to the right, thereby making the consumer willing to work more in order to consume more.

Beyond the macroeconomic effects of advertising, the potential linkage between advertising and labor supply appears of particular interest in the light of the literature on differentials in hours worked across countries, e.g. Alesina, Glaeser and Sacerdote (2005) or Prescott (2004). Our analysis contributes to this literature showing that advertising is one determinant of such differentials. In addition, such prediction of the model is empirically supported by data from several OECD countries. In this perspective, we document a novel stylized fact: in the last decade per-capita advertising expenditures are positively correlated with hours worked across OECD countries.

Another interesting feature of our model is that advertising affects the demand price elasticity of each variety. This feature has a natural interpretation in terms of the degree of substitutability among goods. In our framework, in fact, an increase in advertising expenditures by a firm directly affects the consumers' tastes, making that product more valuable in terms of utility. As such, the consumers' cost of switching from that good to another, for example, as the former becomes more expensive, increases. Equivalently, the degree of substitutability between that good and the rival products decreases. This is a typical feature of non-price competition tools, such as advertising, customer services and investment in quality. As emphasized by the industrial organization literature, through these activities, firms may successfully build customers' loyalty for their products, thereby gaining monopolistic and pricing power. This feature is particularly interesting in light of the New Keynesian theory. This literature has in fact emphasized firms' pricing behavior as a key determinant for both inflation dynamics and the persistence of the real effects of monetary policy shocks. From this perspective, therefore, by interacting with the firms' pricing behavior, non-price competition among firms may also affect inflation dynamics. The current New Keynesian literature overlooks this interesting linkage precisely because it assumes that firms compete for the market with no tools other than their relative prices.

This issue is addressed in the last chapter of this dissertation, which provides an analysis of the implications for inflation dynamics of introducing non-price competition into a New-Keynesian model featuring both nominal rigidities, in the form of staggered prices, and real rigidities, in the form of strategic complementarities in price setting. The main result is that non-price competition dampens the effects of real rigidities on inflation. This result stems from the strategic complementarity between price and non-pricing policies, which mitigates the effect of price movements on a firm's market share and therefore it reduces the opportunity cost of changing price. The implication is that the addition of non-price competition makes inflation more sensitive to movements in real marginal costs relative to the case with only price competition. In other words, under non-price competition the Phillips curve slope is steeper than it would have been otherwise.

From the perspective of New Keynesian theory, our results are relevant because they show that allowance for non-price competition among firms generates a mechanism that dampens the overall impact of real rigidities on inflation dynamics. This issue is particularly important, as real rigidities have became popular among New Keynesian theorists precisely because they provide a mechanism to amplify the effect of nominal disturbances and, all else being equal, to reduce the size of the Phillip curve's slope. In light of these features, real rigidities in price-setting, also refereed to as strategic complementarities, are now recognized as important theoretical ingredients of modern-day New Keynesian models. For instance, Eichenbaum and Fisher (2007) have shown that extending the canonical Calvo model by assuming firm-specific capital and demand functions to have non-constant elasticity of demand (quasi-kinked demand) allows one to recover estimates of the Phillip's curve's slope with a realistic degree of nominal rigidities. Smetz and Wouters (2007) have used quasi-kinked demands function in an estimated monetary DSGE model. Sbordone (2008) extends the Kimball model to study the effect of globalization on inflation dynamics. Our analysis casts some doubt regarding the robustness of such conclusions, showing that abstracting from non-price competition, as canonical model do, may potentially overstate the overall impact of strategic complementarities on inflation dynamics. This therefore suggests that enriching the New Keynesian framework to include non-price competition among firms may be a promising feature in order to improve our understanding on the key determinants of inflation dynamics. This should be particularly true in economy, as the US one, in which non-price competition appears to be an important dimension of the inter-firm rivalry.

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Chapter 1

Advertising and Business Cycle Fluctuations

(Joint with Benedetto Molinari)

1.1 Introduction

In 2005 firms spent 230 billion dollars to advertise their products in the U.S. media, around 1000 dollars per U.S. citizen. The U.S. advertising industry accounts for 2.2% of GDP, absorbs around 20% of firms' budgets for new investments, and uses 13% of their corporate profits.[1] Despite the sizeable amount of resources absorbed, advertising has traditionally been analysed in microeconomic contexts, receiving scarce attention in the macroeconomics literature. Advertising is typically viewed as a selling cost[2] that potentially redistributes consumers' demand across firms without affecting the total market size, and that therefore does not play any significant role in macroeconomic theory.[3]

This paper challenges such opinions by arguing that advertising can have a significant impact on the aggregate dynamics after accounting for its effect on the demand for goods. The rationale for firms' advertising decisions has been identified in the literature as the positive effect of advertisements on sales. Firms realise that the demand they face is not exogenously a product of consumers' preferences, but instead that it can be tilted toward their own products through advertisements. The effectiveness of advertising in enhancing demand is not only revealed by firms' willingness to spend money on it, but is also supported by a large number of empirical studies.[4] Overall, a positive relationship between firms' advertising and sales is widely accepted based on robust empirical evidence.

Building on this fact, we ask whether such relationships would hold in the aggregate. Since the reason for advertising is to increase consumers' demand, as targeted advertising increases the sales of single goods, will aggregate advertising enhance aggregate consumption? If so, will it also increase aggregate demand and production? Moreover, does advertising affect other aspects of the aggregate economy? In analysing aggregate advertising, we first focus on the relationship between advertising and consumption because of the pivotal role that consumption plays in assessing the impact of advertising on the aggregate dynamics. As we will show, aggregate consumption is the main avenue by which a variation in aggregate advertising can have an economy-wide effect. If this causative channel is shut down, the macroeconomic effect of aggregate advertising becomes negligible.

The question of whether aggregate advertising is a determinant of aggregate consumption has already been posed in the literature, and the widespread opinion is that it is not. Building on Solow (1968) and Simon (1970), macro-economists argued that it would be incorrect to assume

[1] Statistics refer to the year 2005. Investments are fixed non-residential investments (source: Bureau of Economic Analysis of the U.S.). Profits are taken from The Economist (Economic and Financial Indicators).

[2] A selling cost is defined as a cost that firms bear in order to enhance demand, but that neither enters as a factor in the production function like investment in equipment and machinery does, nor affects production technology like R&D does.

[3] From this perspective, advertising is intended as a combative and dissipative cost. However, it is interesting to note that the Industrial Organisation literature widely accepts the idea that advertising is market-enhancing at the industry level. For instance, see Friedman (1983) or Martin (1993, Ch. 6).

[4] A survey of these studies can be found in Bagwell (2005) and Schmalensee (1972).

aggregate advertising and aggregate consumption to have a causal relationship identical to that between targeted advertising and sales, since advertising raises a firm's level of demand by stealing customers from competitors, not by increasing the overall size of markets. Because of this "competition" effect, advertising affects the composition but not the size of aggregate consumption. This view is usually referred to as *spread-it-around* advertising. In the literature, however, there is also an opposite view that supports the enhancing effect of advertising on aggregate consumption, the *market-enhancing* hypothesis (Galbraith, 1958). Several papers have attempted to empirically test the relationship between advertising and the amount of consumption, among them Ashley, Granger, and Schmalensee (1980), Jacobson and Nicosia (1981), or more recently, Jung and Seldom (1995). Despite the large amount of empirical evidence considered, none of these studies were conclusive. Additionally, the literature lacks a theoretical model that could be used to analyse aggregate advertising such as the one developed in this paper,[5] which reveals evidence that a positive relationship between advertising and consumption alone is not enough to predict the overall effect of advertising on aggregate demand. Moreover, once we assume such a relationship to hold, we find that advertising has several other significant effects on equilibrium.

This paper analyses aggregate advertising using a general equilibrium model that incorporates the two hypotheses mentioned above, and uses the model with a twofold objective. First, we aim to analyse the effect of advertising on the aggregate dynamics under each of the two hypotheses. While the impact of spread-it-around advertising has been shown to be negligible in the aggregate, market-enhancing advertising can have a significant impact by generating a *work and spend* cycle, where a consumer who wants to consume more because of the advertising incentive but faces an intertemporal budget constraint ends up working more hours. Second, we aim to estimate the model parameters in order to test which hypothesis fits better with U.S. postwar macroeconomic data. The results show that aggregate advertising does affect aggregate consumption, as originally suggested by Galbraith.

This paper considers advertising as a way of manipulating consumer's preferences.[6] As in Dixit and Norman (1978) and Benhabib and Bisin (2002), advertising is modeled as increasing the marginal utility of the advertised good through a modification of parameters in the utility function. Note, however, that this assumption by itself is not a sufficient condition to conclude that aggregate advertising enhances aggregate demand. If the consumer used savings to pay for the extra consumption generated by advertisements, then advertising would at the same time increase consumption and crowd out investments, and the net effect on the demand would be unclear. Also, if advertising shifted purchases towards more expensive goods, then an increase in advertising could imply a reduction in real consumption, and therefore in the aggregate demand. Moreover, advertising is not just a matter of demand; it can affect economic activity in various ways– for instance, increasing substitutability among goods and therefore affecting the market power of firms (price effects), or in a dynamic framework, reducing consumer's savings and hence reducing future demand.

In order to cope with all the effects mentioned above, we embed the candidate utility function with advertising into a dynamic stochastic growth model with monopolistic competition, which

[5] There are some exceptions in this regard. Benhabib and Bisin (2002, manuscript) analyse under which conditions advertising can affect the aggregate labour supply in a neoclassical general equilibrium model, and Grossmann (2007) studies the link between advertising and in-house R&D expenditures in a quality-ladder model of endogenous growth.

[6] There is controversy over how to integrate advertising into consumer's choice theory. In general, there are three different views in the literature about what advertising does: the Persuasive, the Informative, and the Complementary views. See Bagwell (2005) for an excellent survey. Taste-shifter advertising as it is modelled here fits with the Persuasive view of advertising as originally proposed by Marshall (1890,1919), Chamberlain (1933), Robinson (1933), and Kaldor (1950) and as used later on by Dixit and Norman (1978) and Benhabib and Bisin (2002).

is then simulated to analyse the general equilibrium effects of advertising. In general, advertising absorbs resources, can increase firms' monopolistic power, and can eventually shift upward the responses of consumption, labour, and output to exogenous shocks. In particular, we find that market-enhancing advertising operates through three channels. The first one is the *work and spend cycle*: in the presence of advertising, people work more in order to be able to afford greater consumption, where the perceived need for higher consumption is due to the advertising signals they are exposed to. The second mechanism operates through prices. Advertising increases firms' markup, therefore reducing consumer's wages, and with all else being equal, the quantity of labour supplied. The third operates through the resource constraint. By absorbing resources, advertising puts a wedge between gross production and net GDP, which is defined as consumption plus investment.

We show that for a reasonable set of calibrations, the first mechanism prevails over the other two. At equilibrium, both labour and output increase, where part of the extra production is used to produce advertising and the rest is sold as consumption. As a consequence, after an exogenous shock, the responses of consumption and labour from the model are larger than those from a benchmark model economy where advertising is banned. Thus, advertising could tend to accentuate the amplitude of business cycle fluctuations, as argued by Kaldor. We quantify the impact of advertising on fluctuations by comparing the welfare costs of fluctuations when firms can advertise their products to those when advertising is banned. The welfare analysis points out that a 2% of GDP level of spending on advertising increases the cost of fluctuations to the consumer by 124%.

The method we use to include advertising in the utility function is akin to that used in the macroeconomic literature on consumption habits.[7] Like external deep habits, advertising creates dissatisfaction in the consumer about his actual level of consumption, pushing him to buy more. This modelling strategy has the side result that the optimal demand for goods for consumption turns out to be a function of past sales, as in the model with deep habits or in customers' market models. The result appears particularly appealing because it rationalises the persistence observed in actual data of consumption within a context of profit-maximising firms with rational representative consumer. From this perspective, advertising improves on models with habits because consumption persistence arises endogenously at equilibrium based on the interaction between firms' optimal advertising policies and consumer's optimal demand for goods, while in the case of habits it is exogenously assumed in the utility function.

The paper is structured as follows. Section 1.2 characterises the cyclical behaviour of quarterly aggregate advertising expenditures in the U.S. postwar economy. Section 1.3 introduces the DSGE model for advertising and shows that firms optimally use advertising as a complementary tool for price-setting. This section also presents the side contribution of a dynamic version of the Dorfam-Steiner (1954) theorem of optimal advertising spending with monopolistic competitive markets. Simulation results are reported in Section 1.4. Section 1.5 estimates a log-linearised version of the model to test for the effect of advertising on aggregate consumption. Finally, a counterfactual exercise and the variance decomposition of the estimated model are used to show that advertising improves the internal propagation mechanism of the standard real business cycle model. Section 1.6 presents our conclusions.

1.2 Stylized Facts

In what follows we define *aggregate advertising* as the total spending of domestic and foreign firms that advertise their products in U.S. media. Quarterly data for aggregate advertising are not

[7] See Abel (1990) and Ravn, Schmitt-Grohe and Uribe (2006).

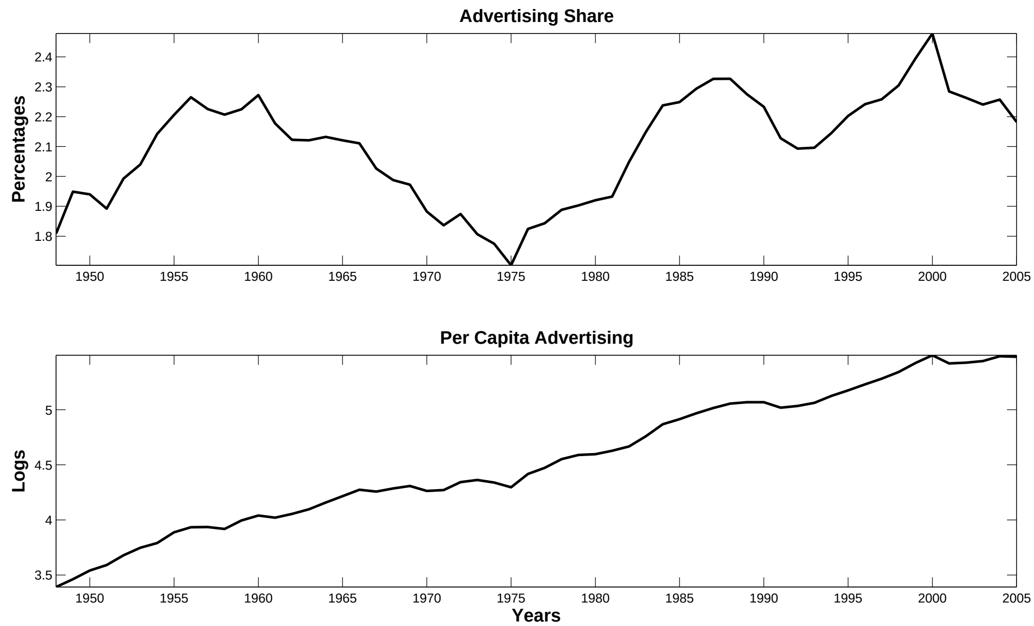

Figure 1.1: **Advertising in Postwar U.S. economy.** Panel 1. Advertising as share of GDP. Panel 2. Per-capita real advertising. Coen's annual data, sample from 1948 to 2005.

included among standard business cycle indicators. Appendix A lists the sources used to collect the data. The resulting database is novel in the literature, and is to our knowledge the only up-to-date free-of-charge quarterly series for U.S. aggregate advertising.[8] Our data report firms' expenditures for advertisements in 7 media types, namely cable and network television, radio, newspapers, magazines and Sunday magazines, billboards, direct mail, and outdoor advertising. The sample starts in the first quarter of 1976 and ends in the second quarter of 2006 (122 quarters).

In order to check whether the series provided is actually representative of all U.S. aggregate advertising expenditures, we compute the cumulative yearly expenditures from our data set and compare them with annual data for total advertising expenditures constructed by Robert Coen of Universal McCann; advertising experts consider this to be the most reliable source of data on aggregate advertising. In the considered sample, our series accounts on average for 30% of Coen's aggregate advertising, with a minimum of 25%, and an in-sample standard deviation of 2.95%.

Coen's annual data are also useful in assessing the magnitude of *aggregate advertising*. Figure (1.1) plots the ratio of advertising over GDP (panel 1), which measures the relative importance of advertising as a component of GDP, and per-capita real advertising expenditures (panel 2), which are commonly used in the literature as a measure of the number of advertising messages that reach the consumer – i.e., a proxy for the intensity of advertising in the economy. The first statistics fluctuate around 2.1% throughout the sample, with a maximum peak in year 2000, while the second show a steady and strong upward trend, implying that the number of advertising messages per individual has constantly grown during the second half of the last century.

The novel series of quarterly data is used in figure (1.2) to represent the cyclical component of

[8] The U.S. Federal administration used to collect quarterly data for aggregate advertising, but it stopped after 1968 when advertising was dismissed from the list of relevant variables used by the Fed to analyse the cycle.

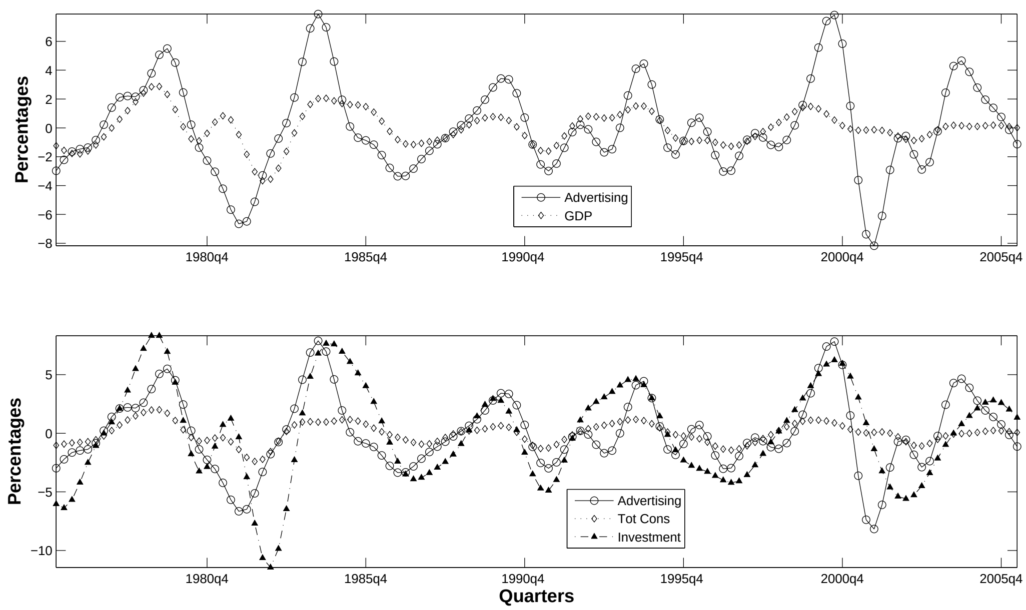

Figure 1.2: **Advertising and the main Business Cycle Indicators.** Quarterly figures. Data sample from 1976q1 to 2006q2.

real advertising expenditures along with that of real GDP, real total consumption, and real fixed private investment.[9] Basically, the figure shows that (i) advertising is pro-cyclical; and that (ii) it is more volatile than GDP and consumption and less so than investment. Table 1.1 reports some related statistics, which confirm these findings: advertising displays a high and positive correlation with GDP (0.59), and it is 2.62 times more volatile than GDP. In addition, it appears to be very persistent over the cycle, with a point estimate of first-order autocorrelation of 0.89. Besides, the positive correlation (0.26) between the advertising-GDP ratio and GDP itself suggests that advertising cannot be simply assumed as a constant proportion of output.

Regarding the other aggregates, advertising displays the strongest correlation with total consumption expenditures (0.68), and it has a very high standard deviation, with a point estimate 3.64 times higher than the one for consumption. Specifically, advertising is 4 times more volatile than non-durable consumption, slightly more volatile than expenditures in durable goods (the relative standard deviation is equal to 1.12), and 23% less volatile than investment.

Since we have only a partial series of aggregate advertising expenditures, we check the robustness of the previous findings by computing the same statistics with Coen's annual data. Results are provided in the second panel of Table 1.1. Annual data confirm the quarterly evidence: aggregate advertising is pro-cyclical $-$ $corr\,(Adv_t, GDP_t) = 0.72$ $-$, and more volatile than GDP $-$ $\sigma(Adv_t)/\sigma(GDP_t) = 1.62$.

Finally, we analyse the dynamic cross-correlations between advertising, GDP, consumption,

[9]All the quarterly figures used in this section are in logs and per capita units. In figure (1.2), the cyclical components have been extracted using a Band Pass (BP) filter with 6-32 as bands. For advertising, we previously eliminated the seasonal component from raw data using the X11 filter. Also, to control for spurious facts, we calculated all the statistics in this section with both BP and Hodrick-Prescott filters. The main empirical evidence presented hereafter does not change when one or the other filter is used.

Table 1.1: **Second order moments**

X_t	$\frac{\sigma(X_t)}{\sigma(Gdp_t)}$	$corr(X_t, Adv_t)$	$corr(X_t, GDP_t)$	$corr(X_t, X_{t-1})$
Quarterly Data				
Advertising	2.62	1	0.59	0.90
GPD	1	0.59	1	0.93
Consumption	0.72	0.68	0.91	0.94
Non-Dur.	0.60	0.67	0.79	0.93
Durables	2.33	0.60	0.90	0.92
Investment	3.41	0.64	0.93	0.94
$\frac{\mathbf{Adv}}{\mathbf{GDP}}$	2.18	0.93	0.26	0.88
Annual Data				
GDP	1	0.72	1	0.08
Advertising	1.62	1	0.72	0.12
$\frac{\mathbf{Adv}}{\mathbf{GDP}}$	1.14	0.79	0.15	0.01

Note: $\sigma(.)$ is in-sample standard deviation. Annual data have been detrended using the BP(2,8)

Table 1.2: **Dynamic cross correlations.**

$corr\left(\mathbf{X}_t, \mathbf{Gdp_{t+k}}\right)$									
k	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
Advertising	0.01	0.20	0.38	0.52	0.59	0.60	0.55	0.47	0.38
Consumption	0.16	0.39	0.62	0.81	0.91	0.90	0.78	0.58	0.35
Investment	0.27	0.54	0.76	0.91	0.93	0.84	0.66	0.42	0.18
$corr\left(\mathbf{X}_t, \mathbf{Adv_{t+k}}\right)$									
Consumption	0.35	0.46	0.57	0.65	0.68	0.63	0.51	0.34	0.13
Non-Dur.	0.34	0.47	0.59	0.67	0.67	0.60	0.46	0.28	0.08
Durables	0.16	0.26	0.38	0.50	0.60	0.64	0.58	0.44	0.25
Investment	0.51	0.63	0.70	0.71	0.64	0.51	0.32	0.12	-0.09

and investment. Dynamic correlations are useful in providing empirical evidence in order to support or dismiss the idea that advertising can be a leading indicator of the cycle. As we see from Table 1.2, advertising only slightly leads GDP: the cross-correlation coefficient is almost the same at k=0 (0.59) and k=1 (0.60). Also, advertising appears to move contemporaneously with consumption (i.e., the strongest correlation occurs at k=0), while it strongly leads investment (higher correlations occur at k=-2 and k=-1). Overall, the dynamic cross-correlations seem to dismiss the idea that advertising can be used as a leading indicator of the cycle. The fact that advertising slightly leads GDP could be due to the fact that it moves with consumption, which itself has been shown to slightly lead GDP in actual data.[10]

Overall, the main findings of this section can be summarised as follows:

- The amount of resources invested in advertising in the U.S. accounts for roughly 2% of GDP.
- Advertising is strongly procyclical and positively correlated with both consumption and investment.
- Advertising is highly volatile, more volatile than GDP and consumption, but less volatile than investment. Also, it is persistent over the cycle.

1.3 A DSGE model with Advertising

This section describes the model economy and displays the problems of households and firms. The market consists of a continuum of differentiated goods produced by monopolistically competitive producers that possess the technology to advertise their products. Advertising is assumed to generate an *urge to consume* the advertised good. We obtain this effect by introducing advertising as an argument of the utility function that is complementary to consumption (we support this modelling strategy in section 1.3.5). We then embed the modified utility function with advertising into an otherwise standard dynamic stochastic growth model with no nominal or real friction, and we study the dynamics of this model in reaction to: (i) a shock to production technology; (ii) a shock to preferences; (iii) a shock to exogenous government spending; (iv) an idiosyncratic shock to the production of advertising.

1.3.1 The household and the role of advertising

We assume that a representative consumer exists with preferences defined for consumption and hours worked, which are described by the utility function:

$$U(\tilde{C}_t, H_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\tilde{C}_t^{(1-\sigma)} - 1}{1-\sigma} - \xi_t \frac{H_t^{1+\phi}}{1+\phi} \right] \tag{1.1}$$

where $\tilde{C}_t$ is the consumption aggregate, H_t is the time devoted to work, and ξ_t is a preference shock. The composite consumption aggregate $\tilde{C}_t$ is defined as follows:

$$\tilde{C}_t = \left(\int_0^1 \left(c_{i,t} + B\left(g_{i,t}\right)\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.2}$$

[10]This evidence is not clear in our data, where the correlation between consumption and output is almost the same at k=0 and k=1, but it has been analysed and supported in several papers, e.g. Wen and Benhabib (2004).

where $\varepsilon > 1$ is the pseudo-elasticity of substitution across varieties; $g_{i,t}$ is the goodwill associated with good i, where goodwill is meant to represent the stock of the firm's advertising accumulated over time; and $B(\cdot)$ is a decreasing and convex function controlling for the impact of goodwill on consumer's preferences, satisfying $B(0) = a \geq 0$.[11] We introduce the concept of goodwill because several empirical studies have shown that advertising campaigns affect product sales for several periods, evidence that seems robust across different goods, countries, and time periods.[12]

Building on Arrow and Nerlove (1962), we model the dynamic effect of advertising by assuming that current and past advertising combine to create a reputation for a good, the producer's *goodwill*, which is defined as the intangible stock of advertising that affects the consumer's utility at time t, as shown in (1.2). The stock of goodwill evolves according to the law of motion:

$$g_{i,t} = z_{i,t} + (1 - \delta_g)\, g_{i,t-1} \tag{1.3}$$

where $z_{i,t}$ is a firm's investment in new advertising at time t and $\delta_g \in (0, 1)$ is the depreciation rate of the goodwill. The law of motion (1.3) implies that current sales could be affected not only by current advertising expenditures, but also by past advertising, with a decreasing intensity over time.

In this setup, the positive link between the producer's goodwill and sales operates through the marginal utility of consumption. Notice that from (1.2) follows:

$$\frac{\partial^2 \tilde{C}_t}{\partial c_{i,t} \partial g_{i,t}} \propto -\frac{1}{\varepsilon} \left(c_{i,t} + g_{i,t}\right)^{\frac{-(1+\varepsilon)}{\varepsilon}} B'(g_{i,t}) \geq 0 \tag{1.4}$$

where the last inequality comes from the assumption that $B(\cdot)$ is decreasing in $g_{i,t}$. This setup reflects what is known in the literature as the persuasive role of advertising: advertisements create some added value for the good that would otherwise not exist. Consequently, the promoted good is worth more to consumers, as if it were a new or different good. The intuition behind this effect is that advertising creates dissatisfaction in the consumer about his current level of consumption. The consumption aggregate (1.2) is modelled in the spirit of the "catching up with the Joneses" hypothesis of Abel (1990), or better, is based on the single-good habits version proposed by Ravn Schmitt-Grohe and Uribe (2006).[13]

The rest of the model is standard. We assume that the representative consumer holds one asset, the capital stock K_t, which he rents to firms, and that he supplies labour services per unit of time. Labour and capital markets are perfectly competitive, with a wage W_t paid per unit of labour services and a rental rate R_t paid per unit of capital. In addition, the consumer receives net profits Π_t from firms and pays lump sum taxes T_t to finance the exogenous government spending. Under these assumptions, the representative agent's nominal budget constraint is defined as:

$$\int_0^1 p_{i,t}\left(c_{i,t} + i_{i,t}\right) di \leq W_t H_t + R_t K_{t-1} + \Pi_t - T_t \tag{1.5}$$

[11] The consumption aggregate (1.2) is a Stone-Geary-type non-homothetic utility function. Depending on whether the term $B(g_{i,t})$ is assumed to be positive or negative, the utility displays a saturation point or a subsistence level with respect to each variety consumed.

[12] In particular, see Clarke (1976) for an empirical study of the dynamic effects of advertising in the U.S. and Bagwell (2005) for a survey.

[13] As in the case of external habits, goodwill works in the utility as a negative externality for the consumer. With respect to other theories of advertising, one advantage of this modelling strategy is that it allows advertising to affect consumer behaviour maintaining a certain analytical tractability when solving for the general equilibrium.

The utility maximisation problem for the representative consumer can be stated as a matter of choosing the processes $\tilde{C}_t$, H_t in order to maximise the utility function (1.1) subject to the standard law of the motion of capital, i.e., $K_{t+1} = (1 - \delta_k) K_t + I_t$, and the budget constraint (1.5).[14] Note that in our setup the consumer does not choose the desired goodwill, but instead passively receives the whole amount of advertising determined by the firms.[15] The first-order conditions for an interior maximum are:

$$\frac{\tilde{C}_t^{-\sigma}}{P_t} = \lambda_t \tag{1.6}$$

$$\lambda_t = \beta E \left\{ \lambda_{t+1} \left[R_t + (1 - \delta_k) \right] \right\} \tag{1.7}$$

$$\xi_t H_t^{\phi} = W_t \lambda_t \tag{1.8}$$

where λ_t is the Lagrange multiplier associated with the budget constraint, and P_t is the aggregate price index. Equation (1.7) is the familiar Euler equation that gives the intertemporal optimality condition, while equation (1.8) describes the labour supply schedule.

The optimality conditions (1.6), (1.7), and (1.8) mimic those of the standard neoclassical growth model, but with the remarkable difference that the definition of the shadow price λ_t depends not only on aggregate consumption but also on aggregate goodwill. Consequently, consumer's decisions about labour and investment are affected by the level of aggregate advertising.[16]

This mechanism plays a pivotal role in determining the general equilibrium results that we will explore in the next section. A partial equilibrium analysis is useful for understanding how advertising affects demand. Suppose, for instance, that advertising expenditures increase exogenously for a sufficiently large fraction of firms. Given our assumptions, $\int B\left(g_{i,t}\right) di$ decreases, and as a consequence, the consumer's shadow price λ_t increases. Consider now the labour supply schedule (1.8). An increase in λ_t implies that the agent values consumption more than leisure, since for any given wage the marginal rate of substitution increases. Hence, the labour supply schedule shifts to the right, or the agent is willing to work more in order to consume more.

An increase in λ_t also affects the consumer's saving decisions by changing the intertemporal elasticity of substitution in the Euler equation (1.7). However, since (1.7) is a function of the ratio of current λ_t over future λ_{t+1} marginal utility, the sign of the effect of higher advertising depends on the relative response of current over future goodwill. In this simple example, the eventual effect is easily predictable. The goodwill is an $AR(1)$ process, and we assumed a one-time increase in advertising: current consumption will increase. In general, an increase in advertising due to an exogenous shock, while unambiguously shifting the labour supply to the right, has an effect on the saving function that is determined by the dynamic response of expected future goodwill to a shock, which itself depends on several different general equilibrium effects that combine together. In particular, however, whenever the growth rate of the goodwill is positive, the consumer finds it

[14] To solve the maximisation problem, it is useful to write the budget constraint in the Lagrangian as a function of $\tilde{C}_t$, I_t. Note that at the optimum, $\int_0^1 p_{i,t} i_{i,t} di = P_t I_t$ and $\int_0^1 p_{i,t} c_{i,t} di = P_t \tilde{C}_t - \int_0^1 p_{i,t} g_{i,t} di$.

[15] This feature distinguishes our model from Becker's (1993) *complementary* theory of advertising. Following the Persuasive view of advertising, we assume that the agent passively receives the advertising signals without being aware of the effect they have on his preferences. On the contrary, in Becker (1993), the agent actively demands the informative content of advertising, since it raises the utility he gets from consumption.

[16] In particular, insofar as $\widetilde{C}_t$ has a negative first derivative with respect to the aggregate goodwill, then advertising will increase both the marginal utility of aggregate consumption and the opportunity cost of leisure.

more convenient to postpone his consumption, since he foresees that his marginal utility will be higher in the future. Conversely, when the growth rate of the goodwill is negative, the consumer experiences an *urge to consume* and increases his demand for current consumption.

Overall, this analysis suggests that from the standpoint of a consumer, aggregate advertising can be interpreted as an exogenous state variable that modifies its own supply of labour and savings modifying, respectively, the elasticity of the wage and the intertemporal elasticity of substitution.

1.3.2 Firms

There is a continuum of firms indexed by $i \in [0,1]$, each producing a differentiated product that is sold as an item for consumption, an investment, or a government good.

The optimal demand function for consumption goods is the solution to the consumer's problem of minimising consumption expenditures subject to the aggregate constraint (1.2), i.e.,

$$c_{i,t} = \max\left\{\left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon}\tilde{C}_t - B(g_{i,t})\ ;\ 0\right\} \tag{1.9}$$

where

$$P_t = \left[\int_0^1 p_{i,t}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{1.10}$$

is the nominal price index. Equation (1.9) has two key implications for this paper, which we shall explore in turn.

Firstly, the demand for consumption goods increases with the level of advertising. A positive investment in $z_{i,t}$ raises the stock of goodwill $g_{i,t}$, which in turns decreases $B(g_{i,t})$, thus shifting the demand function (1.9) to the right. The prediction of a positive relationship between sales and advertising is in line with a large number of empirical studies about advertising at the firm level,[17] and in our model derives from the assumption that advertising enters into the utility function. However, it is worth noting that this assumption is not arbitrary once we restrict our attention to models with Walrasian demand functions and perfect information. In this case, the only way that advertising can enhance demand is through a modification of the preference relation.[18]

Secondly, the price elasticity of the demand diminishes with the level of advertising. Specifically, the demand function (1.9) is composed of two terms: the first one, $(P_{i,t}/P_t)^{-\varepsilon}\widetilde{C}_t$ with elasticity ε, and the second one $B(g_{i,t})$, which is inelastic. Overall price elasticity is then a combination

[17] Actually, a positive relationship between advertising and sales is one of the few non-controversial pieces of evidence regarding advertising. See Bagwell (2005), section 3.2, for more references.

[18] The argument proceeds by contradiction. First, recall that advertising according to our assumptions is not a productive factor, nor does it affect the production technology. As a result, it does not alter the quality of goods, thus implying that pre- and post-advertising, the consumer chooses among the same bundle of goods. Moreover, the condition that advertising does not affect the marginal cost, again because it does not enter into the production function, together with the assumption of perfect information, rules out any direct effect of advertising on prices. In this case, any bundle the agent chooses post-advertising must also have been affordable pre-advertising. Now, suppose that advertising shifts the demand, meaning that the consumer chooses two different bundles of goods pre- and post-advertising, but that the preferences relation remains unchanged pre and post advertising. Since the bundle chosen post-advertising was affordable pre-advertising, it must yield lower utility than the one chosen pre-advertising, since the preference relation is unchanged. As a result, post-advertising the agent is choosing a bundle that is not preferred to the pre-advertising one, violating the Weak Axiom of Revealed Preferences. Hence, if the agent chooses two different bundles pre and post advertising, then the preferences relation must change pre- and post-advertising, which justifies the assumption of advertising as an argument of the utility function. Note that, in general, for this argument to hold true, the model utility function must be derived from a *strictly* convex preference relation.

between the elasticity of these two terms, and we can show that its value will depend on the relative importance of the goodwill over the total demand, i.e.,

$$\eta\left(c_{i,t}, g_{i,t}\right) = \left|\frac{\partial c_{i,t}}{\partial p_{i,t}} \frac{p_{i,t}}{c_{i,t}}\right| = \varepsilon\left(1 + \frac{B(g_{i,t})}{c_{i,t}}\right) \tag{1.11}$$

In particular, notice that the elasticity of demand (1.11) is always smaller than the elasticity of the demand without advertising, i.e., with $g_{i,t} = 0$, since $B(g_{i,t})$ is decreasing over $g_{i,t}$. This feature of the model replicates a well-known effect of advertising in the literature: firms advertise their products to develop consumers' loyalty. The intuition is that advertising, although it does not modify the quality of the advertised good, increases the differentiation among goods *perceived* by consumers. Thus, firms can use advertisements to manipulate the elasticity of the demand, thus increasing market power, and eventually profits.

The goods produced by firms are sold as consumption, investment, and government purchases. Unlike consumption, investments and government purchases are assumed not to be affected by advertising.[19] The assumption about the demand for investment goods fits naturally into our setup because by assumption we modelled a positive effect of advertising on consumer's willingness to consume, whereas investment represents the alternative option for the consumer who does not want to consume. The second assumption about government spending is conservative with respect to the results we will find, since it can be shown that modelling a positive effect of advertising on government expenditures would strengthen the effect of advertising on the aggregate dynamics.

Altogether, the demand for consumption $c_{i,t}$, investment $i_{i,t}$, and government expenditures $f_{i,t}$ forms the total demand of firm i at time t, i.e.:

$$y_{i,t} \equiv c_{i,t} + i_{i,t} + f_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \left(\tilde{C}_t + I_t + F_t\right) - B(g_{i,t}) \tag{1.12}$$

Accordingly, firm i chooses a price for sales and a level of advertising in order to maximise the discounted flow of future profits subject to the constraint given by (1.12). The optimal policy rules, which are derived formally in Appendix B, are:

$$p_{i,t} = \frac{\varepsilon\left(1 + \frac{B(g_{i,t})}{y_{i,t}}\right)}{\varepsilon\left(1 + \frac{B(g_{i,t})}{y_{i,t}}\right) - 1}\, \varphi_t \equiv \mu_{i,t}\, \varphi_t \tag{1.13}$$

$$-\left(p_{i,t} - \varphi_t\right) B'(g_{i,t}) + E_t\left[(1 - \delta_g)\left(\nu_{i,t+1} r_{t,t+1}\right)\right] = \nu_{i,t} \tag{1.14}$$

where φ_t is the marginal cost of production and $\nu_{i,t}$ is the marginal cost of producing new advertising $z_{i,t}$.

Equation (1.13) describes the familiar pricing policy in monopolistic competition models: the firm exploits its monopolistic power by charging a positive markup $\mu_{i,t}$ over the marginal cost. Unlike the standard case, $\mu_{i,t}$ is not constant but increases with the level of goodwill due to the negative relation between price elasticity (1.11) and goodwill.

Equation (1.14) describes the optimal advertising policy. It states that the firm invests in advertising until the marginal benefit from an extra dollar of advertising equals the marginal costs of producing it. Given the dynamic nature of goodwill, the marginal benefit on the LHS of (1.14) has two components: the increase in current revenues associated with a marginal increase in

[19] These demands are derived in Appendix B.

advertising, and the discounted opportunity cost of not producing tomorrow the surviving goodwill produced today.

According to (1.14), advertising is sensitive to variations in the conditions of both supply and demand. On the one hand, reductions in marginal costs lead to higher investments in advertising. On the other hand, the marginal benefit of advertising depends on markup that is positively affected by aggregate demand (see equation 1.13). Hence, any exogenous increase in the demand simultaneously raises markup and advertising.

Besides, note that (1.13) and (1.14) together imply that advertising and price-setting are complementary policies, in accordance with the theory of optimal advertising as the outcome of firms playing a supermodular game, as shown in Tremblay (2005).[20]

Interestingly, we can establish an equivalence result between the optimal advertising policy (1.14) and the seminal Dorfman-Steiner (1954) theorem about firms' optimal spending on advertising, which states that the optimal budget for advertising expenditures is equal to the ratio between the elasticity of the demand with respect to advertising and the elasticity of demand with respect to price. The equivalence result is contained in the following proposition.

Proposition 1. *Let the demand function for firm i be defined as in (1.12), and denote $\eta_{g,t}(i)$ and $\eta^*_{p,t}(i)$ as the elasticity of the demand with respect to goodwill and the elasticity of the demand with respect to price, respectively. Then, the optimal level of goodwill for firm i will be a proportion of the ratio of $\eta_{g,t}(i)$ over $\eta^*_{p,t}(i)$.*

Proof. First notice that from (1.12) follows:

$$-B'(g_{i,t}) = \eta_{g,t}(i)\frac{y_{i,t}}{g_{i,t}}$$

Using this result into (1.14) to substitute out $B'(g_{i,t})$ and rearranging, it yields:

$$\frac{g_{i,t}}{y_{i,t}} = \eta_{g,t}(i)\left\{\frac{p_{i,t}-\varphi_t}{\nu_{i,t}-E_t\left[(1-\delta_g)\left(\nu_{i,t+1}r_{t,t+1}\right)\right]}\right\}$$

or, substituting out $p_{i,t}$ using the optimal pricing rule (1.13),

$$\frac{g_{i,t}}{y_{i,t}} = \frac{\eta_{g,t}(i)}{\eta^*_{p,t}(i)}\Omega_{t,i} \tag{1.15}$$

where $\Omega_{i,t} = \frac{\varphi_t}{\nu_{i,t}-E_t[(1-\delta_g)(\nu_{i,t+1}r_{t,t+1})]}$ and $\eta^*_{p,t}(i) = (\eta(y_{i,t},p_{i,t})-1)$. Thus, the optimal goodwill intensity is proportional to the ratio $\eta_{g,t}(i)/\eta^*_{p,t}(i)$. □

It is straightforward to see that equation (1.15) is a general result that nests the Dorfman-Steiner theorem as a particular case when $\delta_g = 1$ and $\nu_{i,t} = \varphi_{i,t}$, i.e., the goodwill fully depreciates in each period, and firms use the same technology to produce goods and advertising.

[20] Also, note that in the extreme case where advertising has no effect on demand, (i.e., $B'(\cdot) = 0$), equation (1.14) implies that optimal advertising is equal to zero. Therefore, in this framework the only incentive for firms to advertise is the potential that they will have to manipulate demand. In particular, no strategic reason is considered, like for instance entry deterrence.

1.3.3 Advertising and consumption persistence

Another result embedded in equation (1.15) is that advertising generates persistence in the dynamics of consumption. To prove this claim, first note that at the symmetric equilibrium:[21] $\eta^*_{p,t}(i) = \eta^*_{p,t}(j)$, $\eta_{g,t}(i) = \eta_{g,t}(j)$, and $\nu_{i,t} = \nu_{j,t}\ \forall\, i, j$. Then, rewrite equation (1.15) as:

$$g_{i,t} = \Phi_t y_{i,t} \tag{1.16}$$

Using (1.16) lagged one period to work out $g_{i,t-1}$ from the law of motion of goodwill (1.3), and plugging the result into the demand function (1.9) to work out $g_{i,t}$, we obtain:

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \left(\widetilde{C}_t + I_t + F_t\right) + \psi\left(\Phi_t, y_{i,t-1}, z_{i,t}\right) \tag{1.17}$$

where $\psi(\cdot)$ is a non-linear function with a non-negative partial derivative with respect to the last two arguments.[22]

Equation (1.17) reveals that the demand faced by each producer depends on past sales, as in customers market models or models that include habits of consumption.[23] This result is determined by two properties of our setup: (i) the assumption that the consumer's preferences are affected by advertising; and (ii) the dynamic nature of the goodwill. As noted in the introduction, advertising naturally rationalises the presence of consumption persistence within the theory of rational consumer and profit-maximising firms. Compared with other models that have explained consumption persistence, the advantage of using advertising is that persistence is endogenously determined at equilibrium based on the optimising behaviour of firms, once we explicitly incorporate into the model the observable and well-known phenomenon of advertising, instead of being derived from an *ad hoc* assumption like "the costumers" in the market or "the habits" in the utility.

1.3.4 The Symmetric Equilibrium

In this model the market for production factors is perfectly competitive, and all firms share the same production technology. Thus, all firms face the same marginal cost.[24] Moreover, all goods have the same pre-advertising elasticity of substitution, i.e., ε. These two conditions imply together that a symmetric equilibrium exists where all firms set the same price, produce the same quantities, and invest the same amount of resources in advertising.[25] In addition, the equilibrium (common) price of goods is normalised to unity in each period, i.e., $p_t = 1 \quad \forall\, t$. Thus, all other prices in the model (e.g., wage, rental rate) are expressed in terms of contemporaneous consumption.

Let X_t be the vector of all endogenous variables,[26] then the symmetric equilibrium is a process $\{X_t\}_{t=0}^{\infty}$ that satisfies: (1.6)-(1.8), (1.13)-(1.14), plus the production function of consumption goods and advertising, the optimal factors demand for productions,[27] the laws of motion of capital and goodwill, the market clearing condition on the goods market, $Y_t = C_t + I_t + F_t$, and the market clearing condition on the labour market, $H_t = H_{p,t} + H_{a,t}$.

[21] See section 1.3.4.

[22] This follows immediately from the fact that $B(\cdot)$ is assumed to be strictly decreasing together with the fact that non-negative optimal advertising requires $\Phi_t > 0$.

[23] For more about this issue, see Ravn, Schmitt-Grohé and Uribe (2006), and the "habit persistence" entry of the Palgrave Economic Dictionary, written by Schmitt-Grohé and Uribe (2006).

[24] The reader can check this by inspecting the RHS of equation (1.7.3).

[25] This equilibrium requires the extra assumption that the initial stock of goodwill is the same across firms.

[26] Specifically, $X_t = (\lambda_t,\ G_t,\ \mu_t,\ Z_t,\ H_t,\ H_{a,t},\ H_{p,t},\ C_t,\ K_t,\ I_t,\ Y_t,\ R_t,\ W_t,\ Q_{t,t+1})$.

[27] See Appendix B for details.

1.3.5 Advertising in Utility Function: Functional Forms Assumptions

In order to fully specify the utility function, we need to parameterise the function $B(\cdot)$ in a way that satisfies all assumptions made so far (see equation (1.2) and the following discussion). In addition, we are interested in some specification of $B(\cdot)$ that nests market-enhancing and spread-it-around advertising.

In the following, we assume that the function $B(g_{i,t})$ is defined as:

$$B(g_{i,t}) \equiv S(g_{i,t}) + \gamma \int_0^1 (1 - S(g_{i,t}))\, di \quad with\ \gamma \in [0,1] \tag{1.18}$$

where

$$S(g_{i,t}) \equiv \frac{1}{1 + \theta g_{i,t}} \tag{1.19}$$

It is easy to verify that the function (1.19) is strictly decreasing and convex in goodwill. More importantly, the goodwill in (1.18) enters in quasi-difference from its market average, meaning that the effectiveness of firm's advertising on its own demand will depend on the level of advertising of competitors.[28]

Now, we consider in detail the role of γ. At symmetric equilibrium, (1.18) and (1.19) imply:

$$B(G_t) \equiv \frac{1 + \gamma\theta G_t}{1 + \theta G_t}$$

If $\gamma = 1$, then $B(g_{i,t}) = 1$. Thus, the aggregate goodwill disappears from the marginal utility of consumption (see equations (1.6) and (1.2)), and does not directly affect the representative consumer's decisions about labour supply (1.7) and savings (1.8), and therefore the spread-it-around hypothesis holds. In this case, the effect of advertising on the whole economy is easily predictable. It absorbs resources without enhancing demand, and it has no direct effect on prices. Thus, it is a deadweight loss both for firms and for the consumer. Note that firms are still employing resources to advertise their products because in the non-cooperative solution, they do not internalise the effect of their decisions on the mean level of advertising. As a result, they keep wasting money in an unproductive way while the effect of their advertisements on their own demand is offset by other firms' advertising.

If $\gamma = 0$ the goodwill enters in level in the utility function. Accordingly, each firm's advertising affects the marginal utility of its product no matter what the other firms do. In this case advertising directly affects consumption, labor, investment, and firms' markup, and its overall effect in the general equilibrium will be the object of the analysis in next section 1.4. Finally, any value of $\gamma \in (0, 1)$ implies a convex combination between the two extreme cases (complete spread-it-around vs. market enhancing).

A consideration apart deserves the choice of $S(\cdot)$. Equation (1.19) implies that the marginal utility of consumption is bounded (hence we will refer to it as "bounded marginal utility").[29] Due to this bound, in the demand function (1.9) there exists a maximum price above which the demand is zero: when the price is too high the marginal benefit of consuming that good is smaller than its cost, and the consumer drops it from his basket of purchases. In this fashion, firms have incentive to advertise their products for reducing the bound. In the absence of advertising, the bound is constantly equal to 1, while with advertising the bound depends on the level of goodwill, whose

[28]This formulation implies that advertising is combative.

[29]As a matter of fact, preference featuring bounded marginal utility have been already used in the literature. See for instance Melitz and Ottaviano (2008)

effect is larger with larger θ. Hence, this parameter is interpreted as a measure of the effectiveness of advertising in manipulating consumer's tastes.

1.4 Impulse-Response Analysis

This section considers a log-linear approximation of the model's policy functions in the neighbourhood of the non-stochastic steady state. Rational expectations are solved to obtain the dynamic responses of endogenous variables as functions of state variables. We characterise the response of the model's variables to several exogenous shocks, namely: a technology shock (figure 1.3), a preferences shock (figure 1.4), a shock on exogenous government spending (figure 1.5), and an idiosyncratic shock to the production function of advertising (figure 1.6).

To compute the impulse-response functions (hereafter IRFs), we need to assign values to the parameters $\{ \beta, \sigma, \phi, \Xi, \varepsilon, \theta, \alpha, \rho_z, \rho_a, \rho_f, \rho_h, \sigma_h, \sigma_a, \sigma_f, \delta_g, \delta_k, \gamma\}$. The parameters that are standard in real business cycle (hereafter RBC) models are calibrated using the values commonly used in the literature, while the others are chosen such that steady states of model variables match selected long-run moments of U.S. postwar data. In particular, the discount parameter β is set to $(1.04)^{-.25}$, implying a yearly nominal interest rate of about 4%. The depreciation rate of capital δ_k is equal to 2.5% per quarter, and the gross elasticity of substitution across varieties is equal to 6. Following Prescott (1986), the preference parameter Ξ is chosen to ensure that in the steady state, the consumer devotes 1/4 of his time to labour activities. Following Ravn, Schmitt-Groh and Uribe (2006), we set the intertemporal elasticity of substitution to 0.5, the labour elasticity of output α to 0.75, the Frisch elasticity of labour supply to 1.3, and the government expenditures-GDP ratio s_f to 0.12. These restrictions imply that the preference parameters σ and ϕ are 2 and 0.77, respectively, and the steady state labour share is 0.71.[30]

The values of advertising related parameters have been assigned using the following strategy. The goodwill depreciation rate has been fixed to 0.3, implying that the half life of goodwill stock is about two quarters. This value is consistent with the empirical evidence provided in Clarke (1976): the effect of advertising on the firm's demand basically vanishes after one year. As a benchmark case, we set the parameter γ to zero, while the intensity of advertising in the utility function θ is chosen such that conditional to all other parameters, the steady-state value of the advertising over GDP ratio is equal to 2.27%, consistent with the U.S. average over the period 1948-2005.[31]

The autoregressive parameters for all the endogenous process have been set to 0.95. This number is intermediate among the values normally used in the RBC literature. For the simulations, following Rebelo and King (1998) and Collard (2006), we set the standard deviations of technology shock σ_a and government expenditures shock σ_f to 0.0079 and 0.0089, respectively. Finally, the standard deviation of the preference shock σ_h is chosen such that the volatility of hours worked in the model matches its empirical counterpart of 0.91%.[32] The time period in the model is one

[30]In our framework, the steady state labour share denoted by s_h takes the following form:

$$\begin{aligned} s_h &= \frac{W\,(H_p + H_a)}{Y} \\ &= \alpha\mu^{-1}\left[1 + \frac{H_a}{H_p}\right] \end{aligned}$$

so that the usual relationship between the intensity of labour in the production function and the labour share no longer necessarily holds. Note that in the last equation, μ denotes the average long run markup.

[31]This number refers to the ratio of advertising expenditures to net GDP, where exports are subtracted from GDP because exported goods are not sold based on domestic advertising.

[32]This number refers to the standard deviation of the bandpass filtered hours worked in our sample.

Table 1.3: **Calibration**

Parameter	Value	Description
β	.9902	Subjective discount factor
ε	6	Elasticity of substitution across varieties
δ_k	0.025	Capital depreciation rate
Ξ	2.49	Steady State of the preference shock
δ_g	0.3	Goodwill depreciation rate
ϕ	0.77	Preference parameter
θ	2.54	Intensity of advertising in the utility function
α	0.75	Labor elasticity of output
σ	2	Preference parameter
s_f	0.12	Government expenditures-Gdp ratio
$\rho_a, \rho_h, \rho_g, \rho_z$	0.95	Persistence of exogenous shocks
σ_a	$7.9e-3$	Standard error of the technology shock
σ_f	$9.8e-3$	Standard error of the government spending shock
σ_h	$6.2e-3$	Standard error of the preference shock

quarter. Table 1.3 summarises the set of calibrated parameters.

We plot the IRFs for different values of the spread-it-around parameter γ, and we use the associated model economy where advertising is banned as a benchmark to evaluate the impact of advertising. The IRFs appear in Figures (1.3) $-$ (1.6), and we will emphasise a number of these results.

First, advertising responds positively to any shock considered. This result follows directly from equation (1.16), which establishes a positive relationship between aggregate goodwill and aggregate demand. Whenever a shock increases demand, the marginal benefit of goodwill also increases, pushing firms to invest more in advertising. In particular, of the shocks considered, advertising reacts mostly to the technology shock, as is apparent by comparing figures (1.3) and (1.4). The response of advertising to a 1% technology shock is twice as large as the response to a 1% preference shock. This is due to the double effect of an unexpected increase in productivity; the firm revises its advertising spending on the one end because the demand increases, and on the other end because the marginal cost of advertising diminishes. Note that this second effect is further amplified by the dynamic nature of advertising, which modifies the optimal plan of producing advertising to stock future goodwill.[33]

Second, advertising in the model is pro-cyclical, as it becomes apparent comparing the IRFs of advertising and output in figures (1.3) $-$ (1.6). After each shock, the pairs of IRFs display the same sign both at impact and afterward during the transition back to the steady state, thus replicating the positive correlation between advertising and GDP observed in real data. Note that

[33]Clearly, in the event of a transitory positive technology shock, producing advertising today becomes cheaper than doing it tomorrow, thus pushing firms to produce today the advertising that they will need to maintain future goodwill at the optimal level.

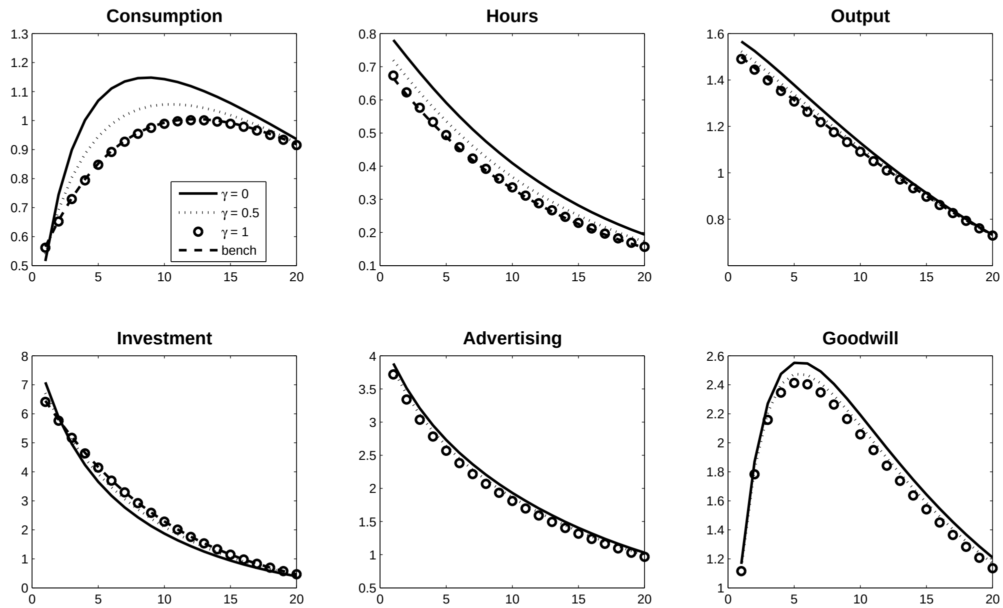

Figure 1.3: **Impulse Response Functions to technology shock.** Each plot displays percent deviation from steady state of the corresponding variable in response to a 1% increase in the rate of productivity.

this feature of the model is independent of the value assigned to γ.[34]

Third, spread-it-around and market-enhancing advertising play two very different roles in the aggregate dynamics. When $\gamma = 1$ (spread-it-around), the IRFs of the main economic aggregates essentially coincide with the benchmark ones (compare dashed versus circle lines): i.e., the effect of advertising on the aggregate becomes negligible. As intuition suggests, in this case advertising does not influence consumer's decisions, and its effect on the aggregate dynamics is determined only by the excess of labour demand that results from producing advertising. The numerical analysis shows that such an effect is negligible since the absorbtion of resources due to advertising is too small to affect the other aggregates in a relevant way. Hence, the conjecture of Simon and Solow is confirmed: under the spread-it-around hypothesis, advertising is irrelevant in the aggregate.

Oppositely, when $\gamma \neq 1$ (market-enhancing), advertising operates on the aggregate dynamics as a mechanism that amplifies and propagates the effects of exogenous shocks (compare the continuous versus dashed lines), and the effect turns out to be stronger when γ is lower (continuous versus dotted lines). The effect of advertising on the labour supply is most important.[35] Despite the fact that wages are lower at equilibrium than in the benchmark model, the upward pressure that advertising puts on the supply of labour is so strong that worked hours increase.[36] This mechanism is called the *work and spend cycle* (Schor, 1992), and it has been empirically supported by Brack and Cowling (1983) for the U.S., and by Fraser and Paton (2003) for the UK.

As a result of the higher level of hours worked, production increases, and the response of output to any shock considered is stronger than the benchmark. The effect of advertising appears

[34]It has often been argued in the literature that the correlation between GDP and advertising is not a relevant statistic to the disentangling of market-enhancing advertising from spread-it-around advertising. This also remains true in our model.

[35]See Molinari and Turino 2007 for a detailed analysis.

[36]To save space, the impulse-response functions of wages are not reported, but they are available upon request.

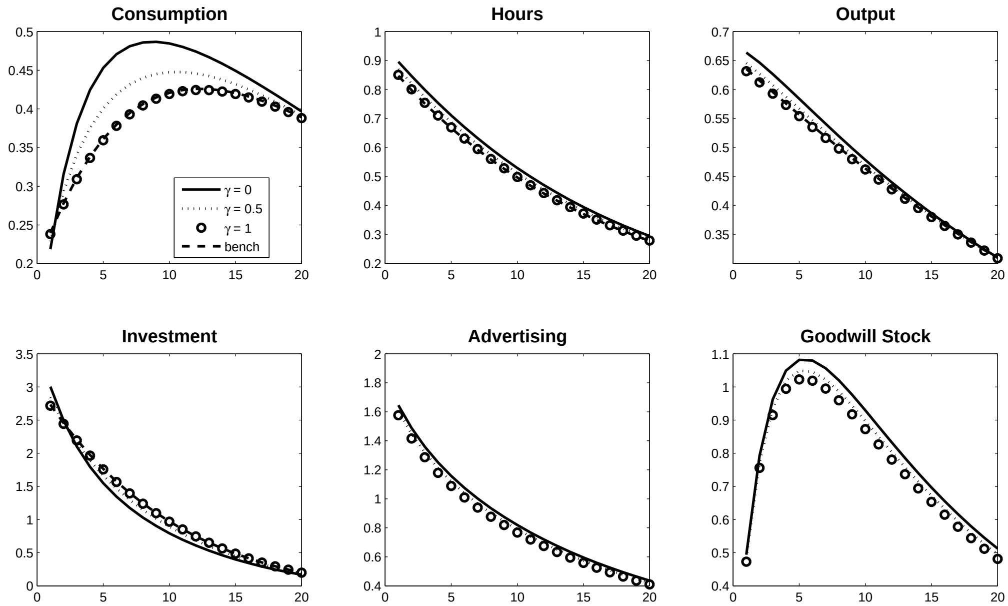

Figure 1.4: **Impulse Response Functions to preferences shock.** Each plot displays percent deviation from steady state of corresponding variable in response to a 1% decrease in the preference shock.

quantitatively relevant when $\gamma = 0$. Figure (1.6) shows that an unexpected 1% increase in the productivity of advertising affects fluctuations in a non-negligible way; consumption increases by 0.25%, output by 0.12%, investment by 1.07%, and labour by 0.18%.[37]

Overall, the analysis of the aggregate dynamics reveals that demand-shifting advertising works as a built-in mechanism of the transmission of exogenous shocks. The case of a positive technology shock is particularly interesting in light of the RBC literature. Intuitively, an unexpected increase in productivity leads to a higher level of desired goodwill.[38] In turn, a higher level of goodwill implies an upward shift in the demand of consumption goods, according to (1.17). Hence, after a technology shock, the level of the aggregate demand increases not only because of the traditional transmission channels (higher wage and higher present value of wealth), but also because of the higher spending on advertising, which makes the consumer willing to consume more for any given price. In the RBC literature, it has often been argued that technology shocks cannot generate business cycles such as those observed in actual data. We suggest that the extra propagation channel provided by advertising could help to reconcile the theory with the data. Section 1.5 will test this assertion.

Lastly, we want to emphasise the behaviour of consumption, whose dynamics in the model with respect to the benchmark display a lower response to the impact of the shock, but a larger, hump-shaped impulse response function during the transition. Given that by assumption advertising raises the marginal utility of consumption in the model, the relative reduction of consumption at impact may not be intuitive and deserves further explanation. The basic intuition is that with advertising the consumer experiences temporal variations in the intertemporal elasticity of substitution, which determines the relative reduction of consumption with respect to the benchmark

[37]Recall that advertising in the model is not a component of the output, which is defined as consumption plus investment and government spending.

[38]This result follows immediately from the optimal advertising policy (1.14).

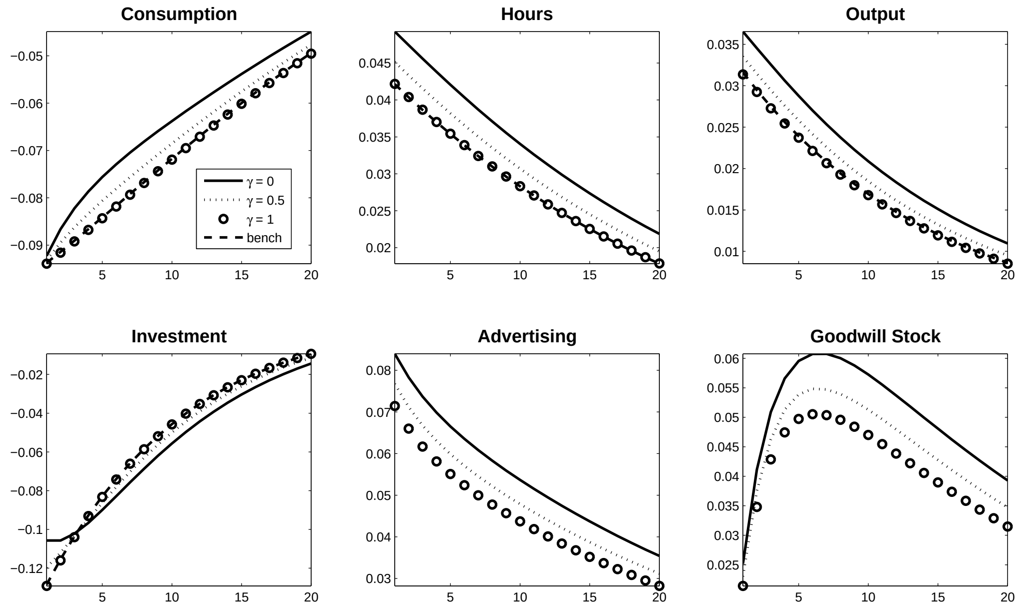

Figure 1.5: **Impulse Response Functions to government spending shock.** Each plot displays percent deviation from steady state of corresponding variable in response to a 1% shock on government spending.

economy. To see this point, it is useful to rewrite the log-linearised Euler Equation in terms of expected consumption growth, that is:

$$E_t\{\Delta\widehat{c}_{t+1}\} = (1-\gamma)\eta_{c,g}E_t\{\Delta\widehat{g}_{t+1}\} + \frac{\eta_{c,p}}{\varepsilon}\frac{1}{\sigma}E_t\{\widehat{r}_{t+1}\} \tag{1.20}$$

where $\eta_{c,p}$ and $\eta_{c,g}$ are, respectively, the steady state demand elasticity with respect to price and goodwill. According to (1.20), aggregate advertising modifies the savings decision along two different dimensions. On the one hand, since the elasticity $\eta_{c,g}$ is positive (the first term on the RHS of (1.20), expected variations in the stock of goodwill modify the expected consumption growth in the same direction. Intuitively, the consumer correctly anticipates the effect of future goodwill on the utility of future consumption and modifies the degree of consumption smoothness over time accordingly. Clearly, the effect is stronger for lower γ, since for values of γ that approach 1, the evaluation of future utility becomes independent from the level of goodwill. On the other hand, the elasticity of expected consumption growth to the interest rate is lower than that used in the benchmark model (second term in the RHS of 1.20), since the long run price elasticity $\eta_{c,p}$ is lower than the benchmark one ε.[39] Hence, if interest rate and goodwill growth respond to a shock in the same direction, the two effects tend to offset each other, and the overall impact will depend on which one prevails.

In our calibrations, the first effect dominates the second, making the response of consumption to a shock larger than in the benchmark model. This feature of the model becomes apparent upon inspecting the behaviour of investment. In all the cases considered, the IRF of investment

[39] Overall, since the impulse response function of the goodwill stock is hump-shaped, during the transition to the steady state, the consumer experiences a time-varying intertemporal elasticity of substitution, as previously claimed. Accordingly, the sensitivity of the savings function to the interest rate is not constant as in the benchmark model.

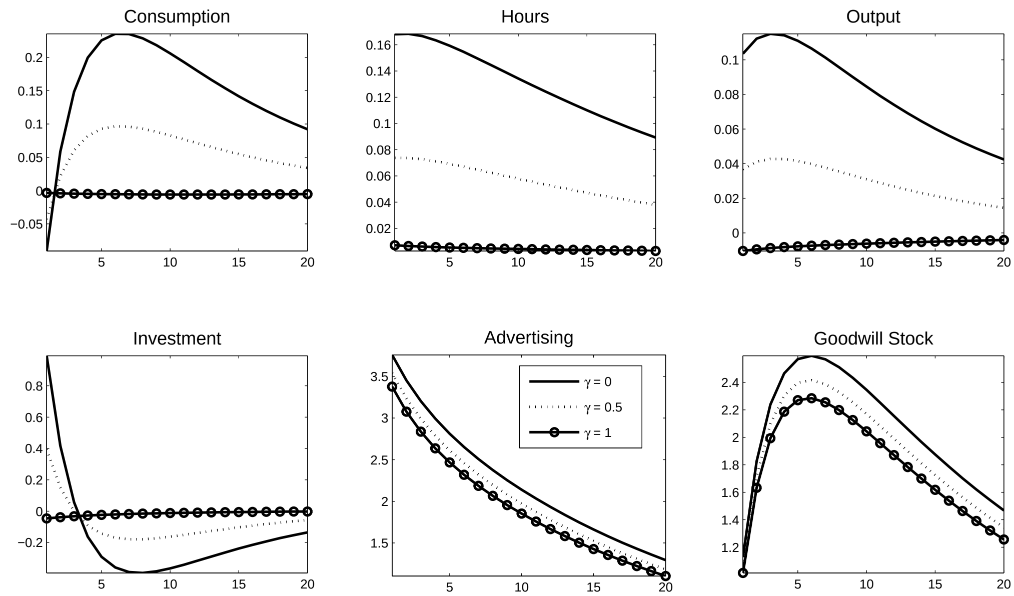

Figure 1.6: **Impulse Response Functions to a shock on advertising production function.** Each plot displays percent deviation from steady state of corresponding variable in response to a 1% shock on the production function of advertising.

is stronger than the benchmark one up to the quarter in which the goodwill peaks. This effect is particularly interesting because it shows that contrary to the conventional wisdom, a positive link between consumption and advertising does not necessarily need to be associated with a crowding-out effect on investment. In fact, once we account for the dynamic effect of advertising and let the supply of labour be endogenously determined by the consumer, an equilibrium in which consumption, hours and investment all increase is indeed possible.

1.5 Model Estimation

This section estimates the DSGE model with advertising using a quarterly time series of U.S. macroeconomic data and a Bayesian estimation method. The data sample goes from the first quarter of 1976 to the second quarter of 2006, the interval over which we have data on aggregate advertising.

Bayesian estimation is preferred over other techniques for several reasons. First, it naturally accommodates the unobservable goodwill in the estimation algorithm, which is a crucial variable for estimating advertising related parameters. Second, it is preferred to Maximum Likelihood estimation because we want to pin down model parameters exploiting combined information from business cycle frequency data and long-run moments of the data, as we showed that advertising also has a relevant effect in the long run.[40] Bayesian priors are a convenient tool for including extra information in the estimation. Third, the effect of advertising spreads in the economy through various transmission channels that can only be assessed completely by computing the general equilibrium solution of the model, as we showed in section 1.4. Therefore, any estimation that

[40] This point is made in a companion paper. See Molinari and Turino (2007).

exploits only partial equilibrium relationships, such as a GMM estimation of the Euler Equation, would neglect to consider some potentially important information.

To estimate the model, we proceed as follows. First, we derive the VARMA representation of the log-linearised model used in section 1.4:[41]

$$\widehat{x}_t = \Psi_x(\omega)\,\widehat{x}_{t-1} + \Psi_\epsilon(\omega)\,\epsilon_t \tag{1.21}$$

where $\widehat{x}_t$ is the vector of partially latent endogenous variables in log-deviations from their steady state values, and $\Psi_x(\omega)$ and $\Psi_\epsilon(\omega)$ are matrices containing the structural parameters of the model, which are listed in vector ω.[42] Then, we add four measurement equations that link model variables to four key observable macroeconomic variables. In particular, we use log differences of real consumption, real output net of exports, total hours worked, and aggregate real advertising expenditures.[43] The corresponding vector of measurement equations is:

$$x_t^* = \begin{bmatrix} dlCONS_t \\ dlGDP_t \\ dlHOURS_t \\ dlADV_t \end{bmatrix} = \begin{bmatrix} \overline{\tau} \\ \overline{\tau} \\ 0 \\ \overline{\tau} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{h}_t - \hat{h}_{t-1} \\ \hat{z}_t - \hat{z}_{t-1} \end{bmatrix} \tag{1.22}$$

where $\overline{\tau} = log(\tau)$ is the common quarterly growth rate trend, and τ is the theoretical deterministic trend of technology. Finally, using the state-space representation (1.21)-(1.22), model parameters are estimated in order to maximise the likelihood of observed data, or in other words, we choose the vector ω that maximises the log posterior kernel of ω conditional to x_t^*.[44]

The shocks used to estimate the model are different from those used for simulations in section 1.4, since we eliminated the shock from the production function of advertising and introduced a shock in the elasticity of demand, which is usually interpreted in the literature as a cost push shock. The new shock is introduced so that the number of structural shocks equals the number of observable variables to which the model is fitted, while the shock on advertising is dismissed because it appeared to work in the estimation algorithm as a measurement error that absorbs all the variations in actual advertising series that could not be explained by the model. We wanted instead to use the information contained in the series of advertising data to influence all of the estimates, in order to use joint information from all the observables to identify all the parameters.

In the estimation, we keep some parameters fixed, namely the discount rate β, the gross elasticity of demand ϵ, the depreciation rate of capital δ_k, the depreciation rate of goodwill δ_g, and the steady state value of hours worked H. The first three parameters are typically difficult to identify in RBC models, while δ_g turns out to be not identifiable separately from θ. H is fixed because we are using data on worked hours in first difference, which leaves the mean level of hours worked undetermined.[45] Fixed parameters are calibrated according to the values in table 1.3. The other parameters are estimated by combining the information contained in actual data x_t^* with that contained in the priors. Prior distributions are chosen according to what is used in the literature, while prior means are chosen according to table 1.3. Details about the priors are given in tables 1.4 and 1.5.

[41] See the appendix C for a detailed explanation of the model used in the estimation.

[42] Specifically, $\omega = \{\beta, \sigma, \phi, \Xi, \varepsilon, \theta, \alpha, \alpha_z, \rho_h, \rho_a, \rho_f, \rho_p, \sigma_h, \sigma_a, \sigma_f, \sigma_p, \delta_g, \delta_k, \gamma\}$.

[43] Details of the data set are available in Appendix A.

[44] The log posterior kernel $ln\mathcal{K}(\omega \mid x_t^*)$ is a linear combination of our prior knowledge about the distribution of ω, $lnp(\omega)$, and the log likelihood of ω conditional to the observed data, $ln\mathcal{L}(\omega \mid x_t^*)$.

[45] The estimation works as follows: at each iteration where the algorithm tries a new vector ω, the steady state value of the preference shock Ξ adjusts for H to remain fixed.

Table 1.4: **Prior Distributions of Structural Parameters**

Parameter	Density	Domain	Mean	90% interval
σ	Gamma	$\mathbb{R}_+$	2.00	[1.39, 2.70]
ϕ	Gamma	$\mathbb{R}_+$	0.77	[0.19, 1.46]
θ	Gamma	$\mathbb{R}_+$	2.50	[1.88, 3.19]
γ	Uniform	$[0,1]$	0.5	[0.10, 0.90]
α	Beta	$[0,1)$	0.75	[0.68, 0.81]
α_z	Beta	$[0,1)$	0.75	[0.68, 0.81]
$\overline{\tau}$	Normal	$\mathbb{R}$	.005	[.0017, .0083]

Table 1.5: **Prior Distributions of Shock Processes**

Parameter	Density	Domain	Mean	90% interval
σ_y	InvGamma	$\mathbb{R}_+$	.008	[.001, .022]
σ_h	InvGamma	$\mathbb{R}_+$	.034	[.007, .096]
σ_f	InvGamma	$\mathbb{R}_+$	.099	[.020, .279]
σ_{mk}	InvGamma	$\mathbb{R}_+$	.039	[.008, .109]
ρ_y	Beta	$[0,1)$	0.6	[0.25, 0.90]
ρ_h	Beta	$[0,1)$	0.6	[0.25, 0.90]
ρ_f	Beta	$[0,1)$	0.6	[0.25, 0.90]
ρ_{mk}	Beta	$[0,1)$	0.6	[0.25, 0.90]

In general, our priors for structural parameters are quite flat. The prior on θ is a gamma distribution − i.e., θ is bounded away from zero − with a mean of 2.50 and a variance of 0.4. Given $\delta_g = 0.3$, this value of θ implies a ratio of advertising over GDP equal to 0.02, in line with the empirical evidence presented in Section 2. The prior for γ is a uniform (0,1) distribution, which reflects our neutral stand between spread-it-around and market-enhancing advertising. For the processes of the shocks, we use standard priors, following Smets and Wouters (2007), Chang Doh and Schorfheide (2006), and An and Schorfheide (2007).

The algorithm for the Bayesian estimation works as follows. First, the posterior kernel is maximised in order to find the mode of the posterior distribution. Second, starting from a random perturbation around the mode, a random-walk Metropolis-Hastings algorithm is used to sample from the posterior distribution.[46] We run this algorithm 4 times from different perturbation points, eventually building 4 chains of 70,000 draws each. This strategy seems to ensure a relatively fast convergence of the Markov chains generated from the algorithm, at least compared to what is usually reported in related literature. Convergence diagnostics indicate that around 40,000 drawings are sufficient to attain convergence for all the parameters.[47] Finally, we report selected

[46]The variance of the jumping distribution is the inverse of the Hessian from the maximisation of the mode, multiplied by 0.35. The acceptance rate is around 35%. The initial perturbation is a single draw from a normal distribution with zero mean and a variance equal to 5 times the variance of the jumping distribution. This larger variance helps to ensure that together, the chains cover the entire parameter space.

[47]See Appendix C.2 for details about the convergence diagnostic. To double-check our results, we also estimate

Table 1.6: **Posterior Distributions of Structural Parameters**

	Prior		*Posterior*	
Parameter	Mean	90% interval	Mean	90% interval
σ	2.00	[1.39, 2.70]	2.48	[1.87 3.08]
ϕ	0.77	[0.19, 1.46]	2.52	[1.76 3.37]
θ	2.50	[1.88, 3.19]	2.62	[1.96 3.27]
γ	0.50	[0.10, 0.90]	0.39	[0.00 0.76]
α	0.75	[0.68, 0.81]	0.67	[0.61 0.73]
α_z	0.75	[0.68, 0.81]	0.73	[0.69 0.77]
$\overline{\tau}$	.005	[.0017, .0083]	.0035	[.0031 .0041]

Table 1.7: **Posterior Distributions of Shock Processes**

	Prior		*Posterior*	
Parameter	Mean	90% interval	Mean	90% interval
σ_y	.008	[.001, .022]	.006	[.0056 .0070]
σ_h	.034	[.007, .096]	.019	[.0146 .0248]
σ_f	.099	[.020, .279]	.047	[.0294 .0655]
σ_{mk}	.004	[.008, .109]	.020	[.0157 .0246]
ρ_y	0.6	[0.25, 0.90]	0.93	[0.886 0.976]
ρ_h	0.6	[0.25, 0.90]	0.94	[0.917 0.974]
ρ_f	0.6	[0.25, 0.90]	0.95	[0.926 0.984]
ρ_{mk}	0.6	[0.25, 0.90]	0.83	[0.763 0.903]

statistics for the posterior distributions by computing the average of correspondent moments from all the chains, wherein we discard the initial 40% of observations from each chain.

1.5.1 Results

Tables 1.6 and 1.7 report the mean and 90% interval from posterior distributions, and Appendix C.3 provides a set of figures of prior and posterior distributions represented together for each parameter.

Our first concern is for the estimates of γ. As we showed in section 1.4, γ is a crucial parameter for assessing whether and how advertising matters in the aggregate. Any estimate of γ significantly close to 1 would imply that the model fits the data better with spread-it-around advertising, while any estimate significantly lower than 1 would imply that market-enhancing advertising is preferred. The estimate of γ seems quite informative, and the posterior has less variance than the

the model running the algorithm to build 2 chains of 500,000 draws each. The estimates remained close to the values found with the shorter chains, but the estimation took much longer, since the number of draws increased from 280,000 to 1 million.

Table 1.8: **Posterior Distributions for $\{\sigma, \phi\}$ in different models.**

Model	σ	ϕ	*LogMarginal DataDensity*
Dixit-Stiglitz	1.55 [1.12, 1.98]	2.48 [1.54, 3.33]	1,312
Bounded Marginal Utility	1.88 [1.24, 2.47]	2.65 [1.85, 3.14]	1,314
B.M.U. with Advertising	2.48 [1.86, 3.07]	2.52 [1.76, 3.37]	1,530
B.M.U. with Advertising restricted $\overline{\gamma} = 1$	2.43 [1.83, 3.01]	2.63 [1.76, 3.43]	1,528

Note: 90% confidence interval in parentheses.

corresponding prior. The posterior mean is 0.39, and the upper bound of 1 is rejected with certainty based on the data.[48] This value suggests that aggregate advertising is a significant explanatory variable of aggregate consumption. Hence, market-enhancing advertising is preferred to spread-it-around advertising, thereby yielding all the consequences of the importance of aggregate advertising highlighted in section 1.4.

Our interpretation of this result hinges on the effect of advertising on the marginal utility of consumption. Typically, RBC models tend to predict an excess of consumption smoothing with respect to what is observed in the data.[49] Here, advertising has an effect on the marginal utility opposite to the effect of consumption, while actual advertising data co-move with those of consumption. Thus, the effect of cyclical variations in advertising offsets that of consumption leaving the overall argument of the marginal utility more stable over the cycle, and in particular, less volatile than the single series of consumption. This feature reconciles the evidence of stable marginal utility and volatile consumption data.

Our second concern is about θ, which is the parameter that controls for the effect of advertising on preferences. Again, the estimate seems informative since the posterior has less variance than the corresponding prior. Its posterior mean suggests that a value of 2.5 is reasonable for this parameter, supporting the previous calibration. This estimate implies a ratio of advertising to GDP of 1.6% in the estimated model, which is a notable result because it is obtained using first difference data and remains stable when we increase the variance of the prior of θ.

Among the other parameters estimates, the mean values from posterior distributions of σ and ϕ raise our attention because they are relatively high compared with other estimates appeared in the literature, e.g., Smets and Wouters (2007), or with the calibrations typically used in the RBC literature. In order to better understand whether this result depends on our model or on the data set employed, we estimate two more RBC models without advertising, one based on a utility function with the standard Dixit-Stiglitz consumption aggregate, and one with a Bounded Marginal utility. Mean estimates of σ and ϕ are reported in the table (1.8).

[48] We estimated several specifications of the model; in all cases, $\widehat{\gamma}$ was significantly different from 1, ranging over $\widehat{\gamma} \in (0.00, 0.39)$.

[49] See Attanasio (1989).

Table 1.9: **Counterfactual Analysis**

	All	Tech. Shock	Gov. Shock	Pref. Shock	Mark Up Shock
Consumption	10.6	20.5	-1.91	19.9	798
GDP	0.40	0.86	5.38	0.38	24.1
Investment	-1.48	-4.18	-0.34	-4.65	251
Hours	1.16	9.72	6.22	1.13	-50.5
Mark-Up	8.91	9.71	11.8	10.2	8.50
Welfare Loses[a]	1.31	-0.37	0.10	0.40	1.18
	(125%)	(17.1%)	(359%)	(11.2%)	(98.8%)

The figures indicate the percent increase in the volatility of corresponding variable when going from the counterfactual economy to the model economy. They are computed as $(var(x|adv > 0)/var(x|adv = 0) - 1) * 100$, and $var(x|\cdot)$ are simulated standard deviations obtained as averages out of 3000 simulated samples of 151 periods length. Simulated data are detrended with BP filter BP(6,32). *All* refers to the full model with parameters at estimated value. Other columns refer to the same model where all the shocks are shut down but the one indicated in the heading of the column.

[a]Welfare losses in units of steady state consumption. In parenthesis, the increase in welfare losses when switching from the counterfactual economy without advertising to the model economy.

While the evidence on ϕ is mixed, the estimates of σ clearly indicates that σ is always higher with advertising. This result can be explained again by observing the behavior of the marginal utility of consumption. Since in the model with advertising there is no one-to-one correspondence between the volatility of marginal utility and consumption, then the estimation algorithm does not pin down the value of σ to match the volatility of the marginal utility with that of consumption, as usually happens in standard RBC models.

As final remark, we point out that the estimates of shocks processes appear in line with the ones found in the empirical literature for similar DSGE models, e.g. Smets and Wouters (2007), suggesting that our model is able to treat uncertainty in the data with the same degree of accuracy of other estimations reported in the literature.

1.5.2 Applications

In section 1.4, we argued that advertising works in the model economy as a built-in mechanism of propagations of exogenous shocks and, in particular, as a mechanism of transmission of technology shocks to the demand. In the following, we make use of the estimated model to test our assertion. We examine how standard deviations of model variables would change if advertising were suddenly banned from the economy. The counterfactual exercise computes the percent difference of standard deviations from the model economy to the counterfactual economy without advertising. Table 1.9 reports the figures. Last line of table 1.9 gives an overall assessment about the impact of advertising on the aggregate dynamics, by comparing the welfare costs of fluctuations from the model economy with those from the counterfactual economy. As usual in the literature on welfare,[50] we define the costs of fluctuations as the units of steady state consumption that the consumer would be willing to pay to eliminate all the variability from his consumption stream. Such costs are computed as the difference between the 2^{nd} order Taylor approximated utility of

[50]See Erceg, Henderson and Levin (2000).

the agent endowed with the equilibrium bundle $\{C_t, H_t\}$ and the 2^{nd} order Taylor approximated utility of an agent that consumes at the steady state level and works the steady state amount of hours in every period.

In general, advertising is confirmed to amplify the volatility of endogenous variables, where the result is stronger for consumption and markup, +10.6% and +8.91%, respectively, relatively mild for worked hours, +1.16%, and does not apply to investment, −1.48%. The difference between the mild effect on labour encountered here and the relatively large effect observed in the IRF of labour in section 1.4 is explained by the difference between the estimated mean value of ϕ and the calibration used in section 1.4. The estimated Frish labour elasticity is much lower than in the calibration, implying a series of worked hours that is fairly stable over the cycle and reacts little to any shocks.

The welfare analysis supports the results of the counterfactual exercise. The increase in the costs of fluctuations clearly indicates that when firms advertise their products, the consumer is willing to pay an higher percentage of his consumption to get rid of fluctuations. The extra cost due to advertising ranges between 11.2% and 359%, and worths 124% in the baseline case where all the shocks are active. Overall, our results confirm that even a small advertising industry can have a relevant impact on the aggregate dynamics and, in particular, on fluctuations of the endogenous variables.

To isolate and shed light on the effect of technology shocks alone, we also provide a counterfactual analysis where the model is simulated by shutting down all but one shock at a time. As expected, technology shocks generate more volatile variables in the model economy than in the benchmark. In particular, the effect on the variance of consumption and hours worked is stronger than in the case where all shocks are active. Only the effect on the variance of output remains mild, which is clearly due to the lower volatility of investment, which compensates the higher volatility of consumption. Beyond the impact on technology, advertising also stimulates consumption by partially offsetting the usual crowding out effect of government spending shocks on consumption. A positive government spending shock that increases demand leads firms to raise investments in advertising. This in turn creates urge to consume and pushes the supply of labour upward, which together reduce the crowding out effect on consumption.

The greater importance of technology shocks in generating the cyclical fluctuations in model variables can be further investigated using the variance decomposition of estimated model variables. Figure (1.7) depicts the variance decomposition of the estimated model compared to that of the estimated benchmark, i.e., models 3 and 2 of table 1.8, and confirm previous results. Technology shocks account for a bigger proportion of the volatility of consumption, output, and labor. In particular, with respect to the benchmark, technology shocks account for roughly 10% more of consumption volatility and 17% more of output volatility, whereas exogenous preference shocks account for 15.3% less of consumption volatility and around 15% less of output volatility.

1.6 Conclusions

This paper provides a model to understand the observed behavior of firms' expenditures on advertising over the business cycle and to quantify the effect of advertising on the aggregate dynamics. To this end, we first characterised the pattern of actual U.S. data of aggregate advertising expenditures over the business cycle. Second, we built a model that can rationalise this pattern within the neoclassical growth model theory. Third, we showed that under some conditions advertising can have a relevant impact on aggregate dynamics, and we isolated this impact with a simulation exercise which makes use of a calibrated version of the model. Fourth, we showed that a log-linearised version of the model fits well with actual data of aggregate advertising. Finally,

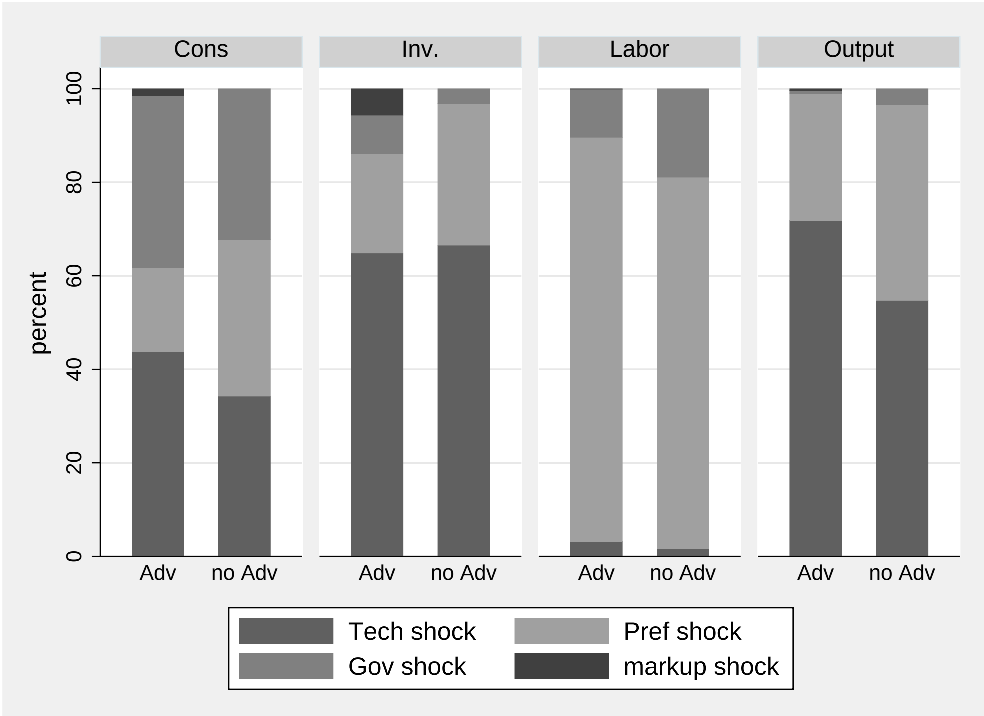

Figure 1.7: **Total variance decomposition for selected variables.**

we estimated the model to test whether the conditions mentioned above do actually hold in the U.S. economy. In our framework, this was equivalent to testing the spread-it-around (Solow, 1968) versus market-enhancing (Galbraith, 1958) hypotheses of aggregate advertising. We find that the data fit the second hypothesis better. From an econometric point of view, this result hinges on the significance of aggregate advertising as an explanatory variable of the volatility of aggregate consumption.

The general equilibrium results from the model show that the effectiveness of advertising in enhancing aggregate consumption yields an important corollary: advertising affects agents choices about desired savings and labor supplies. Through this channel, advertising is shown to have a non-negligible impact on the aggregate dynamics of the whole economy. Despite the modest size of the advertising industry in comparison with total production, i.e., roughly 2% of GDP in the U.S., its short run impact on business cycle fluctuations turns out to be quantitatively important, exacerbating the welfare costs of fluctuations by 124%. Overall, the model goes in the direction suggested by Kaldor (1950): advertising works to amplify and propagate fluctuations of economic activity.

One final remark is worth underlining. The model proposed in this paper uses the worst-case scenario to obtain effects of advertising on the aggregate. In a model with nominal rigidities, the consumer would further increase the supply of labor in response to new advertising because of his low wage variability, which would strengthen the work and spend cycle mentioned above. Also, in a model with fully flexible prices like the one used here, any increase of the markup due to advertising makes investment goods more expensive, thus reducing the real return on capital and therefore household savings. As a result, the general equilibrium effect of advertising on investment is negative, and it partially offsets the positive effect of the work and spend cycle, thus reducing the overall effect of advertising on aggregate demand.[51]

Lastly, we show that advertising in this model behaves as an endogenous taste shock whose intensity is controlled by firms and varies whenever a shock to productivity occurs. This feature leads to the observation that technology shocks shift the aggregate demand through advertising. In fact, we find evidence that disregarding the advertising channel in an RBC model may lead to an underestimation of the effect of technology shocks in generating business cycle fluctuations.

[51] In an early draft of this paper, we showed that in a two-sector model where consumption and investment consist of different goods, advertising increases not only consumption but also investment, since it lowers the relative price of investment goods. Thus, the overall effect on the aggregate demand was stronger than the effects presented in this paper.

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1.7 Appendix

Sources of Data

Data on Advertising

Advertising expenditures in TV, Cable, Radio, Magazines, and Outdoor:
Ad$Summary, quarterly issues from 1976:1 to 2006:2, issued by Media Market New York City

Advertising expenditures in newspaper:
Newspaper Association of America. Data available on the official website of the Association: http://www.naa.org/

Annual advertising expenditures and its components:
Universal McCann, Robert Coen's Annual Report, Estimated Annual U.S. Advertising Expenditures from 1958 to 2006.

Macroeconomic Data

Source: Database "FRED II" of the Federal Reserve Bank of St. Luise available at the website: http://research.stlouisfed.org/fred2

Real Gross Domestic Product (GDPC96)

Real Exports of Goods & Services (EXPGSC96)

Real Personal Consumption Expenditures (PCECC96)

Real Personal Consumption Expenditures: Durable Goods (PCDGCC96)

Real Personal Consumption Expenditures: Nondurable Goods: (PCNDGC96)

Real Private Fixed Investment (FPIC96)

GDP Implicit Price Deflator (GDPDEF)

Civilian Employment-Population Ratio (EMRATIO)

Civilian Non-Institutional Population (CNP160V)

Source: Bureau of Labor Statistics
Available at the website: http://www.bls.gov/data/home.htm

Total Private Average Weekly Hours of Production Workers (CES050007)

Total Non-farm Employment (CES050001)

Note: The series of worked hours used in the estimation is

$$H = \frac{CES050007 * EMRATIO}{168}$$

where 168 normalizes weekly hours to agents' total endowment of hours in a week. Alternatively we use the series:

$$H = \frac{CES050001}{CNP160V * 168}$$

Technical Appendix

Firm's costs minimization problem

To produce its good each firms employs two inputs, labor and capital, combined according to the following production function:

$$y_{i,t} = A_t k_{i,t}^{1-\alpha} \left(\Gamma_t H_{p,t}(i)\right)^{\alpha} \tag{1.7.1}$$

where $y_{i,t}$, $k_{i,t}$, $H_{p,t}(i)$, denote respectively firm's output, capital stock, and production-related labor. A_t measures the stochastic technological progress of the Total Factor Productivity, and Γ_t is the labor augmenting technological progress, which follows a deterministic trend, i.e. $\Gamma_t = \tau\Gamma_{t-1}$.

Firm's demand of production-related inputs is the solution to the dual problem of total cost minimization, given by $W_t hp_{i,t} + R_t k_{i,t}$, and subject to the production function constraint (1.7.1).

As result, firm's total cost function, and marginal cost are given respectively by:

$$CT(y_{i,t}) = \frac{D}{A_t} W_t^{\alpha} R_t^{1-\alpha} (y_{i,t}) \tag{1.7.2}$$

and

$$\varphi_{i,t} = \frac{D}{A_t} W_t^{\alpha} R_t^{1-\alpha} \tag{1.7.3}$$

where $D = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \frac{1}{(1-\alpha)}$ is a positive constant.

Also, each firm promotes its sales by incurring in advertising expenditures. As in Grossmann (2007), we assume that advertising is produced by the marketing department of the firm using the following technology:

$$z_{i,t} = A_t \Gamma_t \left(H_{a,t}(i)\right)^{\alpha} U_t^z \tag{1.7.4}$$

where $z_{i,t}$, $H_{a,t}(i)$ denote respectively the new investment in advertising and the marketing-related labor. U_t^z is a purely transitory idiosyncratic shock on advertising productivity.

Profits maximization problem

Each producer faces three demands for its product. One for consumption, i.e. (1.9), one for investment, and one for government purchases.

The demand function for investment goods derives from the solution to consumer's dual problem of expenditures minimization, subject to the technology used to combine the goods into a unit of capital,

$$I_t \geq \left(\int i_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.7.5}$$

and it will be

$$i_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} I_t \tag{1.7.6}$$

The demand function for government purchases derives from the solution of the consumer's dual problem of expenditures minimization, subject to the constraint:

$$F_t \geq \left(\int f_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.7.7}$$

where for simplicity we set the bound in utility to zero. It will be:

$$f_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} F_t \tag{1.7.8}$$

Thus, the total demand for good i can be written as:

$$y_{i,t} \equiv c_{i,t} + i_{i,t} + f_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon} \left(\tilde{C}_t + I_t + F_t\right) - B(g_{i,t}) \tag{1.7.9}$$

where (1.7.9) uses (1.9) (1.7.8) and (1.7.6).

Firm's problem of profits maximization can be stated as choosing a sequences of prices $p_{i,t}$ and advertising-related labor $H_{a,t}(i)$ in order to maximize:

$$\max_{\{H_{a,t}(i),p_{i,t}\}} E\sum_{t=0}^{\infty} Q_{0,t}\left(p_{i,t}y_{i,t} - CT(y_{i,t}) - W_t H_{a,t}(i)\right) \tag{1.7.10}$$

subject to

$$g_{i,t} = z_{i,t} + (1-\delta_g)\, g_{i,t-1}$$

$$z_{i,t} = A_t \Gamma_t \left(H_{a,t}(i)\right)^{\alpha} U_t^z$$

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\varepsilon}\left(\tilde{C}_t + I_t + F_t\right) - B(g_{i,t})$$

where $Q_{0,t}$ is the discount factor. $CT(y_{i,t})$ is defined as in equation (1.7.2).

The first order conditions for an interior maximum of (1.7.10) are:

$$P_{i,t} = \frac{\varepsilon\left(1+\frac{B(g_{i,t})}{y_{i,t}}\right)}{\varepsilon\left(1+\frac{B(g_{i,t})}{y_{i,t}}\right)-1}\varphi_t \equiv \mu_{i,t}\varphi_t \tag{1.7.11}$$

$$\nu_t = \frac{W_t}{\alpha \Gamma_t U_t^z A_t} H_{a,t}(i)^{1-\alpha} \tag{1.7.12}$$

$$-\left(p_{i,t}-\varphi_t\right) B'(g_{i,t}) + E\left[(1-\delta_g)\left(\nu_{t+1}Q_{t,t+1}\right)\right] = \nu_t \tag{1.7.13}$$

Estimation

The estimated model

To ensure that the economy evolves along a balanced growth path we have to modify consumer's preference. First, we assume that representative household derives utility from the object $\tilde{C}_t$ relative to the level of technology, Γ_t. That is, equation (1.1) is replaced with the following utility function:

$$U(\tilde{C}_t, H_t) = E_0\sum_{t=0}^{\infty}\beta^t\left[\frac{\left(\tilde{C}_t/\Gamma_t\right)^{(1-\sigma)}-1}{1-\sigma} - \xi_t\frac{H_t^{1+\phi}}{1+\phi}\right] \tag{1.7.14}$$

As in An and Schorfheide (2007), the term Γ_t in the previous equation can be interpreted as an exogenous habit stock component. Second, we modify equation (1.18) as follows:

$$B(g_{i,t}) \equiv \Gamma_t\left(S(g_{i,t}) + \gamma\int_0^1 \left(1 - S\left(g_{i,t}\right)\right) di\right) \tag{1.7.15}$$

where the function $S\left(g_{i,t}\right)$ is now defined as

$$S(g_{i,t}) \equiv \frac{1}{1+\theta g_{i,t}/\Gamma_t} \tag{1.7.16}$$

One can easily show that these assumptions, together with the assumed production functions, guarantee that existence of a balanced growth equilibrium in which all the endogenous variable growth a the same rate, τ, with the exception of mark-up, interest rate and labor which are instead constant.

It is therefore convenient to express the model in terms of detrended variables, for which there exists a deterministic steady state.[52] Let $\widehat{X_t} = X_t/\tau$ denote the ratio of a variable X_t with respect to its deterministic trend, τ. The model can be expressed as:

$$\widehat{\tilde{C}}_t = \widehat{C}_t + \frac{1+\gamma\theta\widehat{G}_t}{1+\theta\widehat{G}_t} \tag{1.7.17}$$

$$\widehat{\tilde{C}}_t^{-\sigma} = \frac{\beta}{\tau}E_t\left[\widehat{\tilde{C}}_{t+1}^{-\sigma}\left(R_{t+1}+1-\delta_k\right)\right] \tag{1.7.18}$$

$$\xi_t H_t^{\phi} = \widehat{W}_t\widehat{\tilde{C}}_t^{-\sigma} \tag{1.7.19}$$

$$\widehat{W}_t = \alpha\mu_t^{-1}\left(\frac{\widehat{Y}_t}{H_{p,t}}\right) \tag{1.7.20}$$

$$R_t = (1-\alpha)\,\mu_t^{-1}\left(\frac{\widehat{Y}_t}{\widehat{K}_{t-1}}\right)\tau \tag{1.7.21}$$

$$\mu_t = \frac{\varepsilon_t\left(1+\frac{1+\gamma\theta\widehat{G}_t}{\left(1+\theta\widehat{G}_t\right)\widehat{Y}_t}\right)}{\varepsilon_t\left(1+\frac{1+\gamma\theta\widehat{G}_t}{\left(1+\theta\widehat{G}_t\right)\widehat{Y}_t}\right)-1} \tag{1.7.22}$$

$$\left(1-\mu_t^{-1}\right)\frac{\theta}{\left(1+\theta\widehat{G}_t\right)^2} + E_t\left[\left(1-\delta_g\right)Q_{t,t+1}\nu_{t+1}\right] = \nu_t \tag{1.7.23}$$

$$Q_{t,t+1} = \frac{\beta}{\tau}\left(\frac{\widehat{\tilde{C}}_{t+1}}{\widehat{\tilde{C}}_t}\right)^{-\sigma} \tag{1.7.24}$$

$$H_t = H_{a,t} + H_{p,t} \tag{1.7.25}$$

$$\nu_t = \frac{\widehat{W}_t H_{a,t}}{\alpha_z\widehat{Z}_t} \tag{1.7.26}$$

$$\widehat{Z}_t = A_t H_{a,t}^{\alpha_z} \tag{1.7.27}$$

$$\widehat{G}_t = \frac{(1-\delta_g)}{\tau}\widehat{G}_{t-1} + \widehat{Z}_t \tag{1.7.28}$$

$$\widehat{K}_t = \frac{(1-\delta_k)}{\tau}\widehat{K}_{t-1} + \widehat{I}_t \tag{1.7.29}$$

$$\widehat{Y}_t = \widehat{C}_t + \widehat{I}_t + \widehat{F}_t \tag{1.7.30}$$

$$\widehat{Y}_t = A_t\widehat{K}_{t-1}^{1-\alpha}H_{p,t}^{\alpha}\tau^{\alpha-1} \tag{1.7.31}$$

$$log(A_t) = \rho_a log(A_{t-1}) + \epsilon_t^a \tag{1.7.32}$$

$$log(\xi_t) = (1-\rho_h)\,log(\xi) + \rho_h log(\xi_{t-1}) + \epsilon_t^h \tag{1.7.33}$$

[52] An equilibrium in which all the stochastic innovation are zero all the time.

$$log(F_t) = (1-\rho_f)\, log(F) + \rho_f log(F_{t-1}) + \epsilon_t^f \tag{1.7.34}$$

$$log(\varepsilon_t) = (1-\rho_p)\, log(\varepsilon) + \rho_p log(\varepsilon_{t-1}) + \epsilon_t^p \tag{1.7.35}$$

where the exogenous shocks processes are assumed to satisfy: $\rho_a, \rho_h, \rho_f, \rho_{mk} \in [0,1)$ and

$$\begin{pmatrix} \epsilon_t^a \\ \epsilon_t^h \\ \epsilon_t^f \\ \epsilon_t^p \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} ; \begin{pmatrix} \sigma_a^2 & 0 & 0 & 0 \\ 0 & \sigma_h^2 & 0 & 0 \\ 0 & 0 & \sigma_f^2 & 0 \\ 0 & 0 & 0 & \sigma_p^2 \end{pmatrix} \right]$$

The system of equations (1.7.17)-(1.7.35) fully describes the model economy. Let $\hat{x}_t = log(\widehat{X}_t) - log(X)$ denote the percentage deviation of the variable $\widehat{X}_t$ with respect to its deterministic steady-state. The VARMA representation of the model is determined by solving the linear system of first order stochastic difference equations obtained by log-linearized equations (1.7.17)-(1.7.35) around the deterministic steady-state:

$$\widehat{x}_t = \Psi_x(\omega)\, \widehat{x}_{t-1} + \Psi_\epsilon(\omega)\, \epsilon_t \tag{1.7.36}$$

where $\widehat{x}_t$ is a vector containing all the endogenous variables in percentage deviation with respect to its steady state, ϵ_t is a vector containing all the exogenous innovations, and Ψ_x and Ψ_ϵ are matrixes whose entries are functions of the model structural parameters.

To estimate the model, the VARMA representation is augmented by adding four measurement equations that link model variables to four key macroeconomic observable variables. In particular, we use log difference data of: real consumption, real output net of exports, total hours worked, and aggregate real advertising expenditures. The corresponding vector of measurement equations is:

$$x_t^* = \begin{bmatrix} dlCONS_t \\ dlGDP_t \\ dlHOURS_t \\ dlADV_t \end{bmatrix} = \begin{bmatrix} \overline{\tau} \\ \overline{\tau} \\ 0 \\ \overline{\tau} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{h}_t - \hat{h}_{t-1} \\ \hat{z}_t - \hat{z}_{t-1} \end{bmatrix} \tag{1.7.37}$$

where $\overline{\tau} = log(\tau)$ is the common quarterly growth rate trend. Equations (1.7.36)-(1.7.37) form the state-space representation of the model economy through which the structural parameters are estimated in order to maximize the likelihood of observed data conditional to our model.

Convergence diagnostics for selected parameters

Figure 1.8 plots the tests of convergence for our Markov chains. Following section 3 in Gelman and Brooks (1998), we employ three criteria of converge for each parameter (i.e. univariate diagnostic):

Interval First column of figure 1.8. For each chain, take the 80% central interval of the draws from the simulation, compute the length of the interval over the parameter value, and form the mean of the interval lengths. This is the red line, i.e. the mean length of the within-chain intervals, calculated and plotted for increasing number of draws n. After, from the whole set of draws gained from all the chains (280,000 draws), calculate the 80% central interval length. This is the blue line, i.e. the length of total-sequence interval, plotted for increasing number of draws n.

m2 Second column of figure 1.8. Repeat the same procedure, calculating the central second moment of the chain instead of the interval. Hence, the red line is the mean of central second moments, i.e. the mean of the variance of chains, while the blue line is the total-sequence (all chains) variance.

m3 Third column of figure 1.8. Repeat the same procedure as in *m2*, calculating the central third moment instead of the second moment.

Gelman and Brooks (1998) showed that in the three criteria the ratio between the two statistics goes to 1 as convergence is approached. Thus, we expect the blue and the red lines to stabilize and shrink to the same value for an increasing number of n on the x-axis.

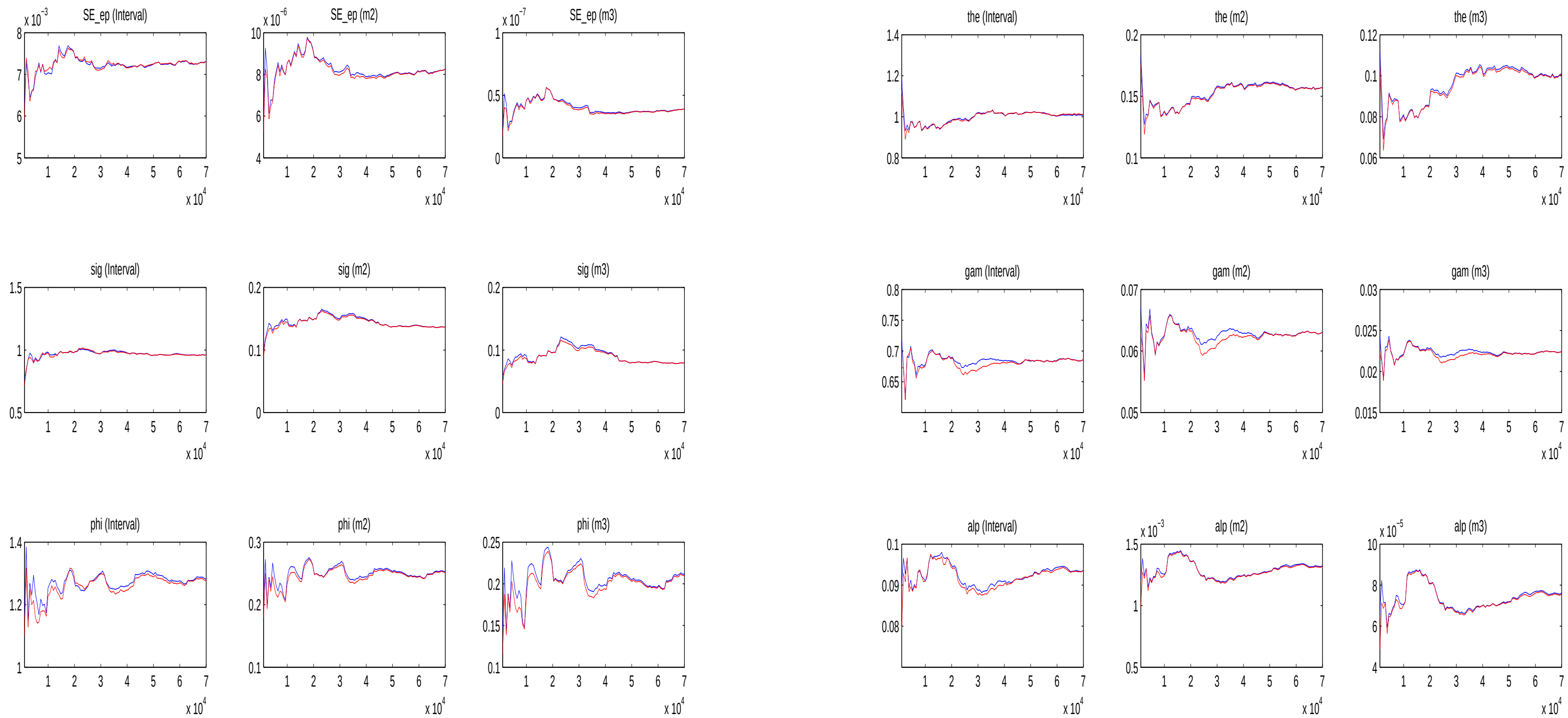

Figure 1.8: Gelman-Brooke Test

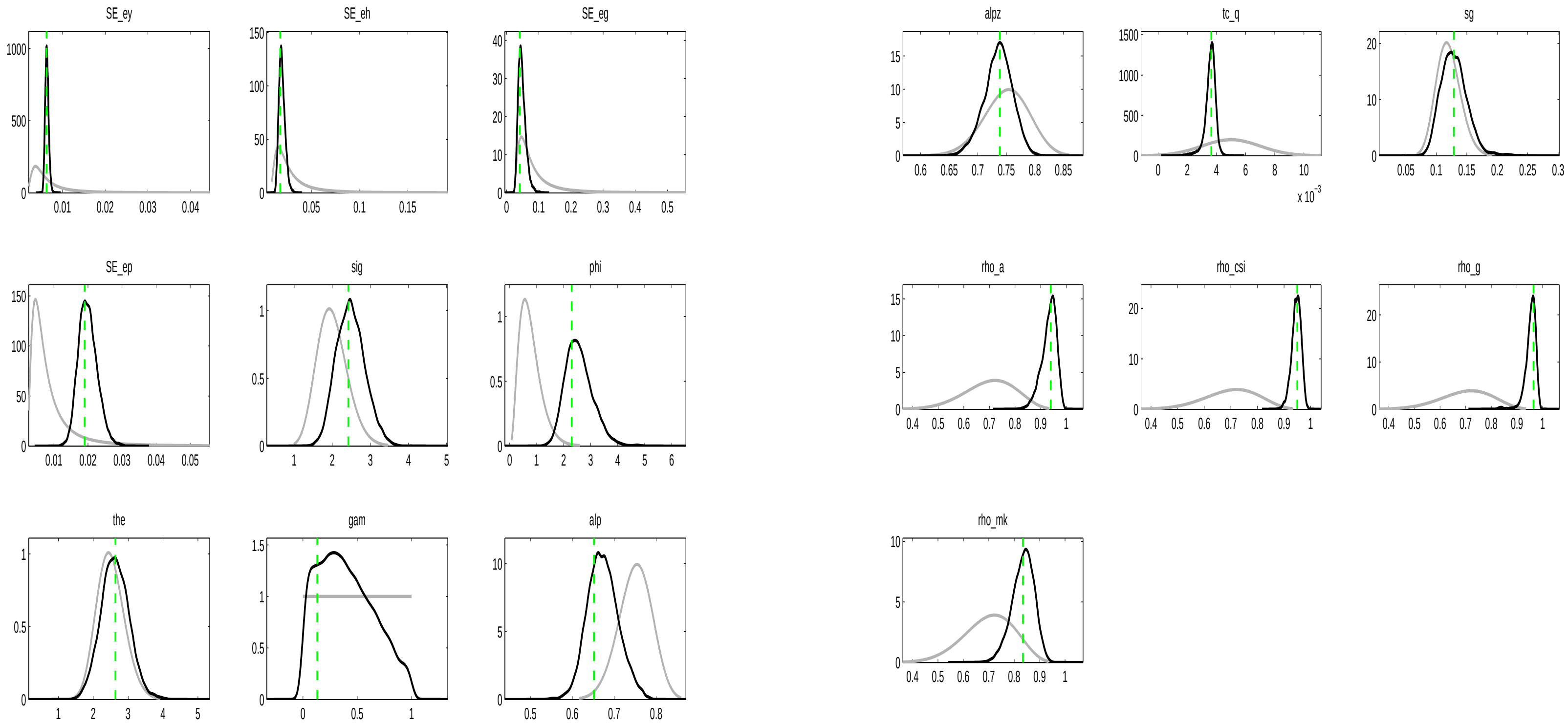

Figure 1.9: **Priors and Posteriors distributions.** Priors are plotted in gray, Posteriors in black, and the mode values computed to initiate the Chains are in dashed green.

Chapter 2

Advertising, Labor Supply and the Aggregate Economy. A Long Run Analysis.

(Joint with Benedetto Molinari)

2.1 Introduction

The goal of this paper is to study the influence of persuasive advertising in a neoclassical growth model with endogenous labor supply and monopolistically competitive firms. Building on Dixit and Norman (1978), we introduce advertising into the representative agent's framework by assuming that consumers' tastes are endogenously determined, depending upon firms' spending on advertising. This assumption generates a positive linkage between the demand of consumption goods and producers' advertising, as result of the agent's optimization behavior. Such a linkage provides, in turn, a rationale to firms' spending on advertising, which is endogenously determined in the general equilibrium setup by profit maximizing firms. This is the major novelty of our paper.

The main reason to study the influence of advertising in a general equilibrium framework is to investigate its potential effects upon the aggregate economy. The literature on advertising has often speculated about the way advertising would affect macro variables.[1] The basic argument supporting this idea relies on the indirect effect that advertising may have on the aggregate demand. Although advertising itself is a relatively small sector of the aggregate production, yet by its own nature it may have a relevant effect the aggregate consumption.[2] Since consumption is a major component of the aggregate demand, through this channel advertising may possibly create important distortions in the economy. In this paper we push further this argument claiming that such distortions can be properly assessed only in a dynamic general equilibrium context. Suppose, for instance, that advertising stimulates aggregate consumption at the expense of saving. Then, it would contemporaneously increase consumption and crowd out investment, therefore having an unclear net effect on the aggregate demand. A partial equilibrium analysis would clearly miss to account for this trade-off effect. Moreover, by possibly reducing investment, advertising may restrict future production capacity, thus creating a distortion between future demand and supply of goods. A static model would miss this connection. Also, advertising may imply a reallocation of resources across sectors, thereby indirectly creating pressures on prices in the productive factors markets, thus distorting the aggregate supply.

The model provided in this paper allows us to identify the conditions under which the presence of advertising significantly affects the aggregate economy in the long run, modifying in particular the properties of its stationary equilibrium. To preview our results, we will show that the effect of advertising on the main economic aggregates depends crucially upon the endogenous response of labor. In fact, from the standpoint of households, advertising operates in our framework as an endogenous tastes shock that, by increasing the marginal evaluation of consumption, makes the households more inclined to substitute from leisure into consumption. All else being equal, this

[1] See Bagwell (2005) for a exhaustive survey on the related literature.

[2] See Molinari and Turino (2009) for more recent analysis that supports the importance of advertising in the aggregate economy.

implies that an increase in aggregate advertising shifts the labor supply to the right, thereby making the consumer willing to work more in order to consume more. In the general equilibrium, the larger supply of labor also increases the equilibrium level of GDP and its components. If, instead, we assume an exogenous labor supply, the results are the opposite: by reducing the equilibrium level of GDP and increasing the markup, advertising essentially exacerbates the distortions caused by the monopolistically competitive market.

The effect of advertising on the representative consumer's labor supply operates as a powerful mechanism that magnifies the long run impact of advertising on the main economic aggregates. By calibrating the model to the US economy, our framework predicts that the equilibrium level of hours worked in the US would have been about 9% lower if US firms had not advertised their products. Similarly, GDP and its components are significantly lower in a model economy without advertising. The other major contribution of this paper is to show that such prediction of the model is empirically supported by data from several OECD countries. In this perspective, we document a novel stylized fact: in the last decade per-capita advertising expenditures are positively correlated with per-capita output, consumption and hours worked across OECD countries.

The relationship established in this paper between aggregate advertising and the labor supply appears of particular interest in the light of the literature on differentials in hours worked across countries, e.g. Alesina, Glaeser and Sacerdote (2005) or Prescott (2004). Our paper contributes to this literature showing that advertising is one determinant of such differentials. In particular, we provide two numerical exercises that contrast model's predictions with actual data, showing that the model with advertising improves upon the neoclassical one in explaining both within-country and cross-country variability of labor supply. On the one hand, by allowing only for cross-country heterogeneity in the advertising sector, our model is able to explain between 25% to 33% of the observed differences in hours worked between some selected European countries and the US. On the other hand, by performing a business cycle accounting exercise in the spirit of Chari, Kehoe and McGrattan (2007), we show that the model with advertising predicts an increasing pattern for US worked hours during the boom in the 1990s that resembles the one observed in actual data. We also show that this prediction of the model sensibly improves upon the one of the canonical RBC model, which fails to predict any upward trend in hours worked.

Our paper is not the first one to advocate a potential relationship between advertising and labor supply. Among the others, Brack and Cowling (1983) provided empirical evidence in favor of such linkage for the US economy and, more recently, Fraser and Paton (2003) empirically supported the same relationship for the UK economy. Where our paper improves upon this literature is by providing a theoretical framework to rationalize such relationship and, using this framework, to show that advertising can have an important effect not only on the labor supply, but also on several macro aggregates.

An interesting implication of our model is that the presence of advertising results in a higher level of both hours worked and output, even though the steady state equilibrium is, at the same time, characterized by a larger aggregate markup. Unlike standard results, this feature provides a theoretical counterexample showing that an increase in market power is not necessarily associated with lower level of hours worked and output. Among other things, this feature suggests that advertising, although persuasive, can be nevertheless welfare-improving in our framework. In fact, by reducing aggregate leisure in the economy, it might mitigate the distortion associated with firms' market power, thereby bringing the economy closer to the competitive equilibrium. In order to address this issue, we provide a welfare analysis that takes into account the endogenous nature of consumer's tastes, as in Benhanib and Bisin (2002), showing that the consumer is worse off with advertising. However, unlike standard results on the welfare consequences of firms' market power, welfare losses in our model are driven by the *overworking* effect due to advertising, and not by the larger markup of firms.

The paper is organized as follows. Section 2.2 documents the empirical evidence. Section 2.3 describes the model economy. Section 2.4 quantifies the long-run effects resulting from the presence in the economy of advertising activities by firms. Section 2.5 contrasts the model's predictions and actual data on hours worked both in a within-country a cross-country perspective. Section 2.6 provides the results for the welfare analysis. Section 2.7 concludes. All proofs are relegated to the Appendix.

2.2 Empirical Evidence

This section provides empirical stylized facts for advertising industries in several OECD countries. In what follows, we will define *aggregate advertising* in a specific country as the total spending of domestic and foreign firms that advertise their products in that country's media. Our dataset contains annual figures for macro aggregates, hours worked, population and advertising expenditures. Data are collected using several different sources. Details are provided in the appendix.

We will begin our analysis by documenting the relative importance of advertising sector in the aggregate economy. To this end, figure 2.1 provides graphs of the ratio of advertising expenditures to GDP (in short, advertising share) in the United States, Japan, Germany, and UK. Two main features are worth emphasizing. First, by absorbing not less than 1% of GDP, the advertising industry is a sizeable sector in all the countries considered. For example, in 2005, firms spent 230 billion dollars to advertise their products in the US media, around 1000 dollars per US citizen. There are however remarkable differences across country. Germany and the UK displays a very similar pattern, with an advertising share that on average accounts slightly less than 1.4% of GDP. The US is the economy in which the greatest amount of resources is absorbed by advertising: on average, its advertising share accounts for more than 2% of GDP, while Japan, accounting for almost 1.15% of GDP, is the country with the smallest advertising share. Second, there is not a clear trend in all the figures we have considered, rather it seems that the advertising share has fluctuated around a constant mean, which, in turn, implies that the average growth rate of advertising and GDP should be approximately the same. The observed fluctuations are probably due to cyclical episodes. In fact, as shown in Molinari and Turino (2009), over the course of the business cycle, advertising is a procyclical and highly volatile variable.

As a marketing activity, advertising is typically intended as a form of investment for firms, an intangible asset that, by affecting the demand for more than one period, provides revenues that extend into the future.[3] From this point of view, comparing advertising expenditures with aggregate investment in tangible capital is useful to assess the relative importance of this asset for the firms' investment policy. To this end, in table 2.1 we have reported the ratio of advertising expenditures to non-residential fixed investment, which, in turn, represents a raw measure of all the productive investment in the economy. As we can see, advertising, ranging from a minimum of 4.2% for Italy to a maximum of 16.4% for the US, accounts for a relevant part of total productive investment in all the considered economies, displaying, as in the previous analysis, a substantial degree of variation across countries. This further highlights the importance of the advertising sector in the aggregate economy. The data for the US are particularly striking, indicating that, in this country, advertising, as non-price competition tool, is a particularly important component of the firms' investment budget. In fact, our data show that, in the US, firms spent in advertising activities an amount of resources equivalent to more than 16% of all the investments in tangible capital.

We next explore the potential linkage between advertising and economic activity. To this end,

[3] See Arrow and Nerlove (1962).

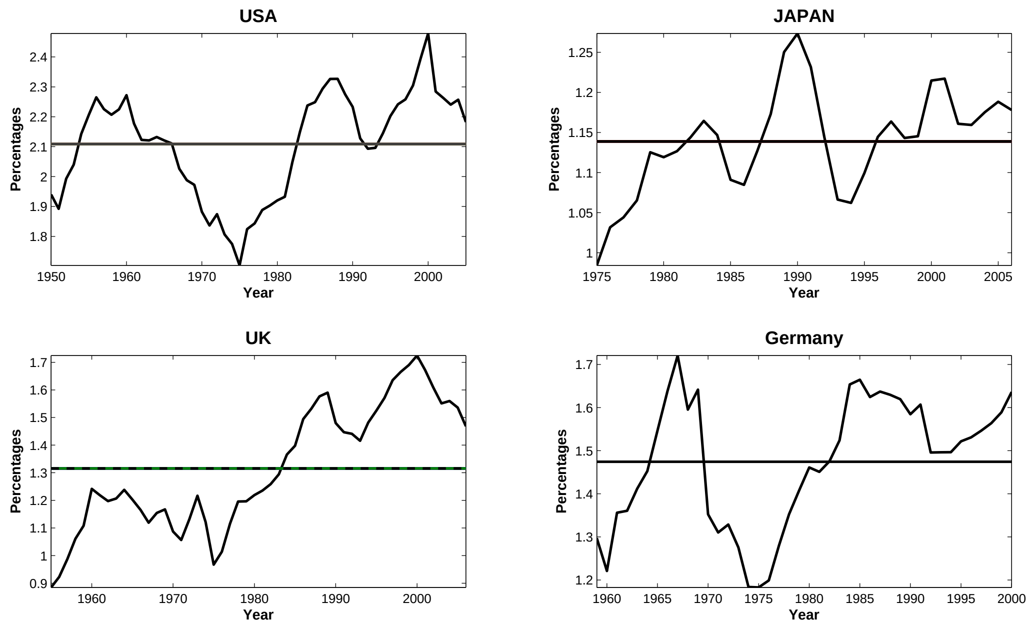

Figure 2.1: **Advertising share in the US, Japan, the UK and Germany**. Advertising share is calculated as the ratio of total advertising expenditures (all the media) to GDP. The horizontal line indicates the sample average mean.

in table 2.1 we have also reported the per-capita real GDP. To render the figures comparable, all the variables are in dollars with constant purchasing power parity and constant prices. As evident from the table, independently of whether we use as indicator the share of GDP or per-capita expenditures, our data show a clear positive cross-country correlation between per-capita real GDP and advertising. In each block of the table, countries with the highest level of advertising are also characterized by the highest level of per capita GDP. This interesting feature holds true by extending our sample to 18 OECD countries. As shown in figure 2.2 (panel A), there exists in fact a strong positive cross-country correlation between per capita GDP and per-capita advertising. The estimated elasticity is in fact positive (0.45) and statistically significant at the 5% level, with an R^2 coefficient of 0.42 (see table 2.2, column A).

While the previous analysis provides an interesting stylized fact, the natural locus to study the macroeconomic effects of advertising is the potential connection with aggregate consumption, rather than the connection with GDP. Advertising is in fact a selling cost for firms: that is a marketing activity devoted to increase their customer bases by boosting sales. As pointed out by Galbraith (1967), these activities, by promoting new and larger desires for material consumption, may also have important market enhancing effects, thereby affecting not only the distribution of market shares across firms (as suggested by the conventional wisdom[4]) but also the market size of consumption goods and eventually aggregate consumption.[5] This feature has been recognized by the specialized literature, and, in fact, most of the empirical studies on the macroeconomic effects of advertising focus on its potential link with aggregate consumption. In general, there

[4]See Simon (1970).

[5] As a result, because of advertising, in the Galbraithian vision, the economy may become more consumption based.

Table 2.1: **Summary statistics for selected countries**.

Country	Time	$\frac{Adv}{GDP}\%$	$\frac{Adv}{Pop}$	$\frac{Adv}{Inv}\%$	$\frac{GDP}{Pop}$	$\frac{Cons}{GDP}\%$	$\frac{Cons}{Pop}$
USA	1984-2005	2.27	1.09	16.4	48.1	67.7	33.0
GBR		1.54	0.56	10.7	36.1	61.0	22.3
DEU		1.49	0.53	9.82	35.6	57.0	20.5
JPN		1.16	0.40	5.20	34.9	54.1	19.2
CAN	1996-2005	0.90	0.38	6.27	42.1	55.3	23.3
FRA		0.69	0.28	4.74	39.9	54.9	22.6
ITA		0.67	0.25	4.17	37.5	58.6	22.2

Note: *Inv* refers to total private fixed investment net of housing while *Pop* is the total person aged 15-64. All the variables are expressed in dollars with constant ppp and constant prices,

exists evidence of a bi-directional causation (in the Granger sense) between advertising and consumption.[6] Those studies focus on individual countries, while our data allow for a cross-countries analysis. To explore this issue, panel B of figure 2.2 plots per-capita consumption expenditures versus per-capita advertising for 18 OECD countries. As we can see, there exists a positive and significant relationship between these two variables, thereby providing cross-country evidence on the connection between advertising and aggregate consumption. The estimated elasticity is in fact positive (0.55) and statistically significant at the 5% level, with an R^2 of 0.61. Furthermore, this relationship holds true even when we use the share of GDP as a measure of consumption (see table 2.1).

In comparing the United States with European countries, it has been often noted that Americans work more than Europeans. The literature has provided several different explanations for this interesting stylized fact. Prescott (2004) suggested that the observed differences in hours worked between Europe and USA can be explained by differences in marginal tax rates on labor income. Alesina et al. (2005), indicated that the major differences between Europe and USA are largely due to the European labor market regulations advocated by politically powerful unions. Blanchard (2004) argues that Europeans enjoy leisure more than Americans do. Cowling and Poolsomnute (2007) take a different stand on the issue, arguing that *"the intensity of creation of wants through advertising and marketing might be an influence on decisions made by Americans about how much time they should devote to paid work, and how much time to leisure".* The argument of the authors is based on the vision that advertising *"creates a continuing dissatisfaction with current levels of consumption, that may encourage people to offer a larger fraction of their time for the generation of income in order to satisfy their increased demands for material consumption"*. As a consequence, the pressure to consume provided by advertising will also affect labor supply decisions. Evidence for such a phenomenon is documented in Brack and Cowling (1983) for the US labor supply and, more recently, in Fraser and Paton (2003) for the UK economy.

In order to explore this issue in a cross-country dimension, in panel B and C of figure 2.2, we have reported per capita hours worked (in logs) versus per-capita advertising and advertising share, respectively. In both cases, the figure shows a positive cross-country correlation, with an estimated elasticity that is, in fact, positive and statistically significant at 5% level (see table 2). However,

[6]See for instance in Ashley et al. (1980) or, more recently, in Jung and Seldom (1994). For a structural approach based on a fully fledged DSGE model with advertising, see Molinari and Turino (2009).

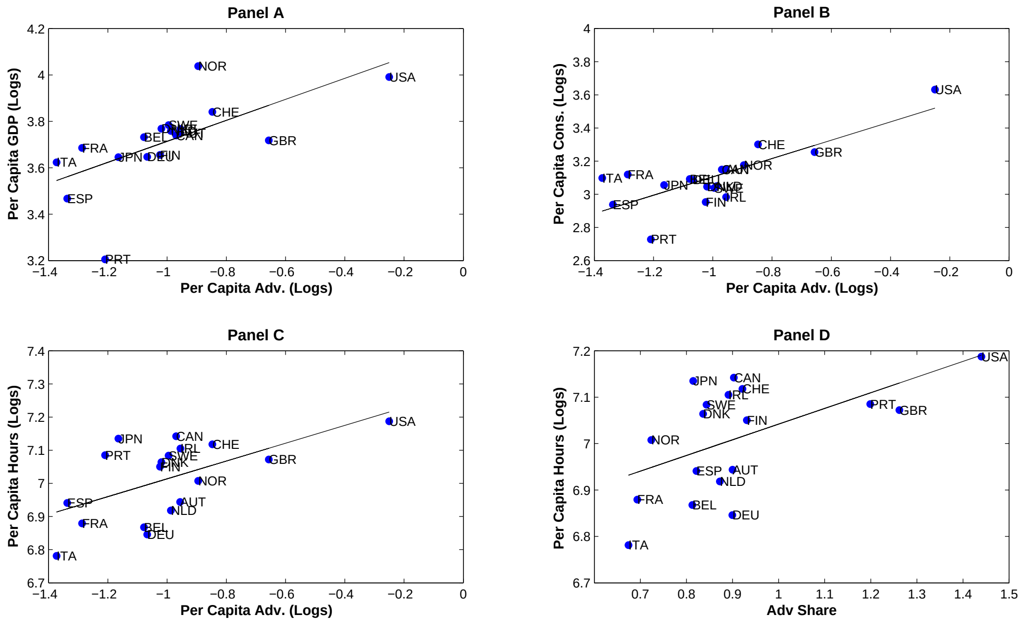

Figure 2.2: **Scatter plots.**. Panels A, B, and C graph respectively the logs of per capita GDP, Consumption, and Hours against per capita advertising. Panel D graphs the logs of per capita hours versus advertising share (in percentage). Period 1996-2005. Source for aggregate advertising expenditures: WARC. See the data appendix for details.

we note that the correlation between advertising share and hours worked (see panel D) appears to be less clear than in the other cases: the R^2 coefficient is significantly lower than all the other estimates, and, in particular, the relationship appears to actually be driven by the observations for the US, UK, and Portugal. For this reason, we repeat the experiment by excluding those countries from the sample. As shown in column E of table 2, this does not dramatically change our results: although in this case the estimate is much less precise than before, the estimated elasticity is still positive and statically significant at a 10% level.

To summarize, our analysis has shown that advertising is a sizeable sector the most industrialized countries, displaying a substantial degree of cross-country variations. The US is the economy in which advertising absorbs the greatest amount of resources, showing a remarkable difference compared with the other countries. As a new stylized fact, we provide evidence in favor of a positive cross-country connection between advertising and the main macro aggregates, such as GDP, consumption and hours worked.

2.3 The Model

The basic structure of the model is the standard neoclassical growth model augmented with a monopolistically competitive structure of product markets a la Dixit-Stiglitz. Following Dixit and Norman (1978), we introduce advertising into the representative agent's framework by assuming that consumers' tastes are endogenously determined, depending upon the aggregate expenditures in advertising activities by firms. This assumption allows one to obtain the positive linkage between demand for consumption goods and producers' advertisements as a result of the individual optimization behavior, thereby introducing advertising activities into a dynamic general equilibrium

Table 2.2: **Simple Regressions**

Regressors	(A) $\frac{GDP}{Pop}$	(B) $\frac{Cons}{Pop}$	(C) $\frac{Hours}{Pop}$	(D) $\frac{Hours}{Pop}$	(E) $\frac{Hours}{Pop}$
Per-Capita Adv	0.452	0.553	0.269	-	-
	(0.004)	(0.000)	(0.011)	-	-
Advertising Share	-	-	-	0.338	0.717
	-	-	-	(0.015)	(0.058)
Constant	4.166	3.659	7.282	6.704	6.392
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	0.420	0.612	0.343	0.318	0.250

Columns A-D report the estimated relationship graphed in figure 2.2. Column E report estimates for the relationship by hours worked and advertising share by excluding US UK, and Portugal (PRT) from the sample. All the variables, with the exception of advertising share, are in logs. P-values are reported in parenthesis. 18 OECD countries. Average mean over 1996-2006. Source of advertising expenditures data: WARC.

setup as an endogenous firms' decision policy.

2.3.1 Households

The economy consists in a continuum of differentiated goods indexed by $i \in [0,1]$, each produced by a monopolistically competitive firm and over which consumer preferences are defined. More precisely, we assume that the representative consumer has preferences for consumption and hours worked described by the following utility function:

$$U(\widetilde{C}_t, H_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\widetilde{C}_t / A_t\right)^{(1-\sigma)} - 1}{1-\sigma} - \xi \frac{H_t^{1+\phi}}{1+\phi} \right] \tag{2.3.1}$$

where $\widetilde{C}_t$ is the consumption aggregate, H_t is the time devoted to work, and ξ is a parameter affecting the disutility of labor. To ensure that the economy evolves along a balanced growth path, we assume that representative household derives utility from the object $\widetilde{C}_t$ relative to the level of technology, A_t that evolves at the constant rate $\gamma_a > 1$. As in An and Schorfheide (2007), we interpret this term as an exogenous habit stock component. The composite consumption aggregate $\widetilde{C}_t$ is defined as follows:

$$\widetilde{C}_t = \left(\int_0^1 \left(c_{i,t} + B\left(g_{i,t}, A_t\right)\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3.2}$$

where $\varepsilon > 1$ is the pseudo-elasticity of substitution across varieties; $g_{i,t}$ is the goodwill associated with good i, where goodwill is meant to represent the stock of the firm's advertising accumulated over time; and $B(\cdot)$ is a decreasing and convex function of the goodwill stock controlling for the effect of advertising on consumer's preferences and satisfying $B(0, A_t) \geq 0 \ \forall \ A_t \in \mathbb{R}_+$.[7] As we will see shortly, the dependence of $B(\cdot)$ on the rate of technology, A_t is a necessary condition to guarantee the existence of a balanced growth path equilibrium.

Building on Arrow and Nerlove (1962), we model the dynamic effect of advertising by assuming that current and past advertising for a good combine to create the producer's *goodwill*, which, in turn, is defined as the intangible stock of advertising that affects the consumer's utility at time t, as shown in (2.3.2). The stock of goodwill evolves according to the law of motion:

$$g_{i,t} = \omega z_{i,t} + (1 - \delta_g)\, g_{i,t-1} \tag{2.3.3}$$

where $z_{i,t}$ is a firm's investment in new advertising at time t, $\delta_g \in (0, 1)$ is the depreciation rate of the goodwill and $\omega > 0$ is a parameter determining the advertising efficiency that might reflect institutional aspects, such as specific regulations (i.e., advertising bans) or taxation.

Equation (2.3.1) is the key relationship to introduce advertising in the model. It represents a non-homothetic version of the consumption aggregator originally proposed by Dixit and Norman (1978) to study the welfare implication of advertising.[8] In this formulation, advertising is intended to be purely persuasive in the sense that it affects the consumers' choice by modifying his/her tastes without conveying any information about the characteristics of the good.[9] For each variety i, this effect is controlled by function $B(\cdot)$, whose properties are restricted in order to guarantee a positive linkage between firm's advertisements and sales of its own product. This feature is apparent by explicitly deriving the demand curve for each individual variety. The latter is the solution to the dual problem of minimizing consumption expenditures subject to the aggregation constraint (2.3.2), that is:

$$c_{i,t} = max\left\{ \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} \widetilde{C}_t - B(g_{i,t}, A_t); 0 \right\} \tag{2.3.4}$$

where

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$

is the nominal price index.[10] Given (2.3.4), it is easy to verify that the monotonically decreasing

[7]The consumption aggregate (2.3.2) is a Stone-Geary-type non-homothetic utility function. Depending on whether the term $B(g_{i,t})$ is assumed to be positive or negative, the utility displays a saturation point or a subsistence level with respect to each variety consumed.

[8]Recently, non-homothetic preferences have received increasing attention in macroeconomics. Ravn et al. (2006) have modeled habits formation at the level of individual varieties by using a non-homothetic consumption aggregator, which is essentially the same as the one we have assumed. There are indeed several similarities between our framework and the recent "deep habits" literature. See Molinari and Turino (2009) for a discussion on this point.

[9] Although the way advertising affects the consumers' decision is a rather controversial issue, we choose to focus on persuasive advertising for several reasons. First, as emphasized in Kaldor (1950), advertising, since it is pursued by an interested party, largely tries to persuade rather than to inform consumers and therefore is persuasive in nature. Second, recent studies of behavioral economists provided evidence on how consumers' tastes are distorted by advertising. Among others, Gabaix and Laibson (2004) have shown that, in context of informative advertising, information revelation may break down in the presence of consumers that fail to foresee "shrouded attributes," such as hidden fees or maintenance costs. Finally, focusing on non-informative advertising allows us to eliminate complications, such as modeling informational asymmetries, that are impossible to analyze with a representative household framework. This makes the analysis relatively simple.

[10]Notice that in the optimum, equation (2.3.4) implies:

$$\int_0^1 P_{i,t} c_{i,t} di = P_t \tilde{c}_t - \int_0^1 P_{i,t} B(g_{i,t}, A_t) di$$

behavior of the function $B(\cdot)$ implies that the demand of each variety is increasing in the firm's advertising effort. This feature hinges on the property that consumer's preferences are endogenously determined in our model. In fact, an increase in the advertising spending by a firm raises the goodwill stock of its own product and, at the same time, affects the consumers' taste by increasing the marginal utility of that particular variety. As a result, the consumers' willingness to pay for that good rises, thereby causing an upward shift in the demand schedule. Furthermore, the convexity of the function $B(\cdot)$ guarantees a sort of saturation effect by inducing decreasing returns of advertising.

It is interesting to note that our framework implicitly embeds the combative nature of advertising. By differentiating equation (2.3.4) with respect to the goodwill stock and by assuming that a fraction $\lambda > 0$ of firms change their advertising levels, we get:

$$\frac{\partial c_{i,t}}{\partial g} = \int_0^\lambda \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} \frac{\partial B(g_{i,t}, A_t)}{\partial g} di \; < \; 0 \, \forall \, i \, \in [\lambda, 1]$$

Therefore, the effect on the demand that a firm faces when other firms increase the advertising levels is negative. Consequently, for a given level of consumption expenditures, any asymmetrical distribution in the goodwill stocks merely redistributes demand among firms, thereby causing an asymmetrical distribution in market shares.

The rest of the model is standard. We assume that the capital stock, K_t, held by the representative consumer evolves over time according to the following law of motion

$$K_{t+1} = I_t + (1 - \delta_k) K_t \tag{2.3.5}$$

where the investment in period t, I_t, is assumed to be a composite good produced by aggregating differentiated goods via the following technology:

$$I_t = \left(\int_0^1 (i_{i,t})^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3.6}$$

As before, for any level of I_t, purchases of each variety $i \in [0, 1]$ in period t must solve the dual problem of minimizing total investment expenditures subject to the aggregation constraint (2.3.6), that is:

$$i_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} I_t \tag{2.3.7}$$

Notice that advertising does not directly affect the investment expenditures decision. This assumption implies that any link between advertising and investment is indirectly driven by general equilibrium effects. This allows us to embed in the model a possible crowd out effect of advertising on investment, so that the actual impact of advertising on the aggregate demand is *a priori* ambiguous.

The consumer supplies labor services per unit of time and rents whatever capital it owns to firms. The labor and capital markets are perfectly competitive, so that each consumer takes as given the wage rate W_t paid per unit of labor services and the rental rate R_t paid for unit of capital. In addition, the consumer receives pure profit from the ownership of firms, Π_t. The flow budget constraint faced by the representative consumer is then given by the following equation:

$$\int_0^1 p_{i,t}\,(c_{i,t} + i_{i,t})\,di \leq W_t H_t + R_t K_t + \Pi_t \tag{2.3.8}$$

The intertemporal maximization problem for the representative consumer can be stated as consisting in choosing processes $\widetilde{C}_t$, H_t so as to maximize the utility function (2.3.1) subject to (2.3.5) and (2.3.8). The first-order necessary conditions for an interior maximum of U are

$$A_t^{(\sigma-1)}\widetilde{C}_t^{-\sigma} = \lambda_t \tag{2.3.9}$$

$$\lambda_t = \beta\left\{\lambda_{t+1}\left[R_t + (1-\delta_k)\right]\right\} \tag{2.3.10}$$

$$\xi H_t^{\phi} = W_t \lambda_t \tag{2.3.11}$$

where λ_t is the Lagrange multiplier for the budget constraint (2.3.8). Equation (2.3.10) is the familiar Euler equation that gives the intertemporal optimality condition. Equation (2.3.11), under the assumption of a perfect competitive labor market, describes instead the supply of hours.

The optimality conditions (2.3.9), (2.3.10), and (2.3.11) mimic those of the standard neoclassical growth model, but with the remarkable difference that the definition of the shadow price λ_t depends not only on aggregate consumption but also on aggregate goodwill. Consequently, consumers' decisions about labor and investment are affected by the level of aggregate advertising.[11]

This mechanism plays a pivotal role in determining the general equilibrium results that we will explore in the next section. A partial equilibrium analysis is useful for understanding how advertising affects demand. Suppose, for instance, that advertising expenditures increase exogenously for a sufficiently large fraction of firms. Given our assumptions, $\int B\,(g_{i,t}, A_t)\,di$ decreases, and, as a consequence, the consumer's shadow price λ_t increases. Consider now the labor supply schedule (2.3.11). An increase in λ_t implies that the agent values consumption more than leisure, since the marginal rate of substitution increases for any given wage. Hence, the labor supply schedule shifts to the right, i.e., the agent is willing to work more in order to consume more.

An increase in λ_t also affects the consumer's saving decisions by changing the intertemporal elasticity of substitution in the Euler equation (2.3.10). However, since (2.3.10) is a function of the ratio of current λ_t over future λ_{t+1} marginal utility, the sign of the effect of higher advertising depends on the relative response of current over future goodwill. In this simple example, the eventual effect is easily predictable. The goodwill is an $AR(1)$ process, and we assumed a one-time increase in advertising: current consumption will increase. In general, an increase in advertising due to an exogenous shock, while unambiguously shifting the labor supply to the right, has an effect on the saving function that is determined by the dynamic response of expected future goodwill to a shock, which itself depends on several different general equilibrium effects that combine together. In particular, however, whenever the expected growth rate of the goodwill is positive, the consumer finds it more convenient to postpone his consumption, since he foresees that his marginal utility will be higher in the future. Conversely, when the growth rate of the goodwill is negative, the consumer experiences an *urge to consume* and increases his demand for current consumption.

[11] In particular, insofar as $\widetilde{C}_t$ has a negative first derivative with respect to the aggregate goodwill, then advertising will increase both the marginal utility of aggregate consumption and the opportunity cost of leisure.

2.3.2 Firms

In this model, firms make decisions on pricing policy, production plans, and budgets for advertising activities. We assume that each firm uses two types of input: labor and capital. To produce goods, firms have access to a common technology of the following form:

$$y_{i,t} = k_{i,t}^{1-\alpha} \left(h_{p,t}(i)A_t\right)^{\alpha} - A_t F \tag{2.3.12}$$

where $y_{i,t}$, $k_{i,t}$, and $h_{p,t}(i)$ respectively denote firm i's output, capital stock, and the amount of production-related labor. A_t measures the (labor augmented) technological progress evolving at a positive constant rate γ_a; $\alpha \in (0,1)$ and F is a fixed cost.

Each firm may promote its products by incurring advertising expenditures. As in Grossmann (2008), we assume that firms produce advertising in house by using a common technology that requires only labor, that is:

$$z_{i,t} = A_t \left(h_{a,t}(i)\right)^{\alpha} \tag{2.3.13}$$

where $z_{i,t}$ and $h_{a,t}(i)$ respectively denote firm i's advertising effort and the amount of marketing-related labor. By getting rid of complications related to the specification of an advertising sector, this assumption greatly simplifies the analysis without affecting our main conclusions.

By virtue of equations (2.3.4) and (2.3.7), the demand schedule faced by each firm can be written as follows:

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} \left(\widetilde{C}_t + I_t\right) - B(g_{i,t}, A_t) \tag{2.3.14}$$

The non-homotheticity of the consumption aggregate (2.3.2) implies that the demand schedule features a non-constant elasticity of demand. Using (2.3.14), it is in fact easy to verify the following:

$$\xi\left(y_{i,t}, g_{i,t}\right) = \left|\frac{\partial y_{i,t}}{\partial P_{i,t}} \frac{P_{i,t}}{y_{i,t}}\right| = \varepsilon \left(1 + \frac{B(g_{i,t}, A_t)}{y_{i,t}}\right) \tag{2.3.15}$$

which shows that the price elasticity of demand depends, over time, upon the ratio $B(g_{i,t}, A_t)/y_{i,t}$. This feature hinges on the property that the total demand faced by one firm is composed of two terms, one of them perfectly inelastic with respect to the price, while the other one has a constant elasticity. As a consequence, the resulting price elasticity is a combination between the elasticities of the two components, so that its value depends, over time, on the relative importance of the inelastic term over total demand.

Most importantly, the elasticity of demand of each variety decreases with the producer's investment in advertising. This feature has a natural interpretation in terms of the degree of substitutability among goods. Intuitively, we can think that advertising activities by a firm, by attaching peculiar attributes to the product, increase the consumers' perceived differentiation with respect to rival products. Consider, for instance, a producer that increases its expenditures for advertising. In our framework, this directly affects the consumers' tastes, making that product more valuable in terms of utility. As such, the consumers' cost of switching from that good to another, for example, as the former becomes more expensive, increases. Equivalently, the degree of substitutability between that good and the rival products decreases. Because of this perception of product differentiation, consumers are then willing to pay a higher price for that good, and, for a given price, the producer's market share increases. Alternatively, if firms advertise their products, the demand will decrease less in correspondence of an increase in their relative prices or, equivalently, the price elasticity decreases.

The demand for production-related inputs is the solution of the dual problem of minimizing total cost, given by $W_t n_{p,t}(i) + R_t k_{i,t}$, subject to the production constraint (2.3.12). The resulting optimal ratio of factors is of the form:

$$\frac{k_{i,t}}{h_{p,t}(i)} = \left(\frac{1-\alpha}{\alpha}\right)\frac{W_t}{R_t} \tag{2.3.16}$$

The corresponding total cost function and the associated marginal production cost are respectively given by the following equations:

$$CT(y_{i,t}) = \frac{D}{A_t^{\alpha}} W_t^{\alpha} R_t^{1-\alpha}\left(y_{i,t} + A_t F\right) \tag{2.3.17}$$

$$\varphi_{i,t} = \frac{D}{A_t^{\alpha}} W_t^{\alpha} R_t^{1-\alpha} \tag{2.3.18}$$

where $D = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}\frac{1}{\alpha}$ is a positive constant.

The firm i's intertemporal problem can be stated as choosing sequences of prices $P_{i,t}$ and the amount of advertising-related labor $h_{a,t}(i)$ in order to maximize the discounted value of all future profit flows. Formally, firm i solves the following problem:

$$\max_{h_{a,t}(i), P_{i,t}} \sum_{t=0}^{\infty} r_{0,t}\left(\frac{\pi_{i,t}}{P_t}\right)$$

subject to

$$\pi_{i,t} = P_{i,t} y_{i,t} - CT(y_{i,t}) - W_t h_{a,t}(i)$$

$$g_{i,t} = \omega z_{i,t} + (1-\delta_g)\, g_{i,t-1}$$

$$z_{i,t} = A_t\left(h_{a,t}(i)\right)^{\alpha}$$

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon}\left(\widetilde{C}_t + I_t\right) - B(g_{i,t}, A_t)$$

where $r_{0,t}$ is the firm's discount factor,[12] and $CT(y_{i,t})$, defined as in (2.3.17). The first-order conditions for an interior maximum are the following:

$$P_{i,t} = \frac{\varepsilon\left(1 + \frac{B(g_{i,t},A_t)}{y_{i,t}}\right)}{\varepsilon\left(1 + \frac{B(g_{i,t},A_t)}{y_{i,t}}\right) - 1}\varphi_t \equiv \mu_{i,t}\varphi_t \tag{2.3.19}$$

$$\phi_t = \frac{W_t}{\alpha\omega A_t} h_{a,t}^{1-\alpha} \tag{2.3.20}$$

[12] Under the assumptions of a perfect financial market and households holding the ownership of the firms, the stochastic discount factor is defined as:

$$r_{0,t} = \beta^k\left(\frac{\lambda_t}{\lambda_0}\right)$$

where $\beta \in (0,1)$ is the consumer's subjective discount factor, and λ_t is the consumer's shadow price, defined as in equation (2.3.9).

$$\phi_t = -\frac{\partial B(g_{i,t}, A_t)}{\partial g_{t,i}} (P_{i,t} - \varphi_t) + (1 - \delta_g) (\phi_{t+1} r_{t,t+1}) \tag{2.3.21}$$

Equation (2.3.19) describes the familiar firm's pricing policy. The firm exploits its monopolistic power by charging a positive markup ($\mu_{i,t}$) over the marginal cost. Equation (2.3.20) defines the shadow price ϕ_t as the marginal cost of producing advertising. Equation (2.3.21) is the optimal advertising policy, stating that a firm chooses the optimal level of goodwill by equating its marginal benefit with its marginal costs. Substituting equation (2.3.20) into (2.3.21), solving the resulting equation forward and using the advertising production function (2.3.13) yields:

$$z_{i,t}^{\frac{1-\alpha}{\alpha}} = \alpha\omega \frac{A_t^{\frac{1}{\alpha}}}{W_t} \sum_{j=0}^{\infty} (1 - \delta_g)^j \; r_{t,t+j} \left[- \frac{\partial B(g_{t+j,i}, A_{t+j})}{\partial g_{t+j,i}} (p_{i,t+j} - \varphi_{t+j}) \right] \tag{2.3.22}$$

This expression shows that advertising decisions by firms are sensitive to both cost and demand conditions. On one hand, it clarifies that exogenous reductions of the wage rate or an exogenous increase in the technology process pushes down the advertising marginal cost, thereby raising the firm's incentive to advertise its own product. On the other hand, the marginal benefit of advertising positively depends on the net revenues, which in turn, from equation (2.3.19), are also affected by demand. Therefore, any exogenous variation of the demand condition affects, through the price and in the same direction, the advertising spending of a firm. Interestingly, the marginal benefit of advertising increases in the stochastic discount factor $r_{t,t+j}$. This increase implies that any exogenous changes in the discount factor, such as, for instance, variation in the interest rate due to monetary authority as well as variations due to exogenous shocks driving economic fluctuations, affect the advertising expenditures decisions.

Finally, it is interesting to note that, in our framework, the firms' pricing and advertising policies are in fact directly related. According to equation (2.3.22), a firm will find it convenient to increase its advertising budget in response to an increase in the unit net revenue from sales caused by a higher relative price. Using the terminology of Iwasaki *et al.* (2008), this means that in our framework, firms play a super-modular game, since their pricing and advertising policies are complementary strategies.[13]

2.3.3 The Symmetric Equilibrium

The equilibrium for the model economy is derived by imposing a clearing condition in all the markets. Let Y_t denotes total output obtained by integrating (2.3.12) over the firms' index. The clearing of the goods market requires:

$$Y_t = C_t + I_t \tag{2.3.23}$$

while equilibrium in the market for labor factor requires:

$$H_t = \int_0^1 (H_{a,t}(i) + H_{p,t}(i)) \, di \tag{2.3.24}$$

Adding these conditions to all the other optimality equations for the economy's agent also imply clearing in the capital market. Given the symmetry embedded in our model, in equilibrium all

[13] Super-modular games are a general class of noncooperative games where n players simultaneously choose a set of strategies. See Milgrom and Roberts (1990) for further details. Iwasaki *el al.* (2008) discuss the general property of advertising that unequivocally leads to a supermodular game in the context of an oligopolistic market in which firms simultaneously choose their advertising budgets and pricing policy.

firms will set the same price, produce the same quantities, and invest the same amount of resources in advertising. Hereafter, we will therefore restrict our analysis by focusing on the symmetric equilibrium of the model. In addition, we will normalize the price of consumption goods, $p_{i,t}$, to 1, so that all the remaining prices are defined in terms of contemporaneous consumption.

The next proposition summarizes sufficient conditions to guarantee that a balanced growth path exists: that is, an equilibrium in which all the variables grow at a constant rate, with the exception of the interest rate, labor and the aggregate markup, which are instead constant.

Proposition 1. *Consider an economy in which monopolistically competitive firms may promote their products by incurring advertising expenditures. Suppose furthermore that consumer preferences are defined as in equation (2.3.1) and that the technology for producing goods and advertising are respectively given as in equations (2.3.12) and (2.3.13). Then, a sufficient condition for a balanced growth path equilibrium to exist is that whenever:*

$$\frac{G_t}{A_t} = \frac{G_{t+1}}{A_{t+1}}$$

implies both:

$$\frac{B(G_{t+1}, A_{t+1})}{B(G_t, A_t)} = \gamma_a$$

and

$$\frac{\partial B(G_{t+1}, A_{t+1})}{\partial G_{t+1}} = \frac{\partial B(G_t, A_t)}{\partial G_t}$$

Thus, assuming that the function $B(\cdot)$ satisfies the requirements of proposition 1, it is convenient to express the model economy in terms of detrended variables, for which there exists a deterministic steady state. Therefore, denoting with $\widehat{S}_t = S_t/A_t$ the original variable S_t detrended by the rate of technology, A_t, and letting $X_t = (\widehat{G}_t, \mu_t, \widehat{Z}_t, H_t, H_{a,t}, H_{p,t}, \widehat{C}_t, \widehat{K}_t, \widehat{I}_t, \widehat{Y}_t, R_t, \widehat{W}_t, \widehat{\widehat{C}}_t)$ be the vector of the all the endogenous variables, a symmetric equilibrium for the model economy can be now defined as a pair of initial conditions $(K_0, G_0) \in \mathbb{R}^2_+$ and a process $\{X_t\}_{t=0}^{\infty}$ that satisfies the following system of equations:

$$\widehat{W}_t = \alpha \mu_t^{-1} \left(\frac{\widehat{K}_t}{H_{p,t}} \right)^{1-\alpha} \tag{2.3.25}$$

$$R_t = (1-\alpha)\, \mu_t^{-1} \left(\frac{H_{p,t}}{\widehat{K}_t} \right)^{\alpha} \tag{2.3.26}$$

$$\widehat{\widehat{C}}_t^{-\sigma} = \frac{\beta}{\gamma_a} \left\{ \widehat{\widehat{C}}_{t+1}^{-\sigma} \left[R_{t+1} + (1-\delta_k) \right] \right\} \tag{2.3.27}$$

$$\widehat{\widehat{C}}_t = \widehat{C}_t + \widehat{B}(\widehat{G}_t, A_t) \tag{2.3.28}$$

$$H_t = H_{a,t} + H_{p,t} \tag{2.3.29}$$

$$\widehat{Y}_t = H_{p,t}^{\alpha} \widehat{K}_t^{1-\alpha} - F \tag{2.3.30}$$

$$\widehat{Z}_t = H_{a,t}^{\alpha} \tag{2.3.31}$$

$$\widehat{G}_t = \frac{(1-\delta_g)}{\gamma_a} \widehat{G}_{t-1} + \omega \widehat{Z}_t \tag{2.3.32}$$

$$\gamma_a \widehat{K}_{t+1} = (1-\delta_k)\,\widehat{K}_t + \widehat{I}_t \tag{2.3.33}$$

$$\widehat{Y}_t = \widehat{C}_t + \widehat{I}_t \tag{2.3.34}$$

$$\xi H_t^{\phi} = \widehat{W}_t \widehat{\widehat{C}}_t^{-\sigma} \tag{2.3.35}$$

$$\widehat{W}_t H_{a,t}^{1-\alpha} = -\alpha\omega\,\frac{\partial B(\widehat{G}_t, A_t)}{\partial G_t}\,(1-\mu_t^{-1}) + \frac{\beta(1-\delta_g)}{\gamma_a}\left(\frac{\widehat{\widehat{C}}_{t+1}}{\widehat{\widehat{C}}_t}\right)^{-\sigma}\widehat{W}_{t+1} H_{a,t+1}^{1-\alpha} \tag{2.3.36}$$

$$\mu_t = \frac{\varepsilon\left(1+\frac{\widehat{B}(\widehat{G}_t,A_t)}{\widehat{Y}_t}\right)}{\varepsilon\left(1+\frac{\widehat{B}(\widehat{G}_t,A_t)}{\widehat{Y}_t}\right)-1} \tag{2.3.37}$$

2.3.4 The Steady State

The steady state equilibrium is derived from the above system of equations by assuming that the vector X_t is constant over time. As explained in the appendix, in such an equilibrium, all the endogenous variables can be expressed as functions of the vector $J_t = (H_t, \mu_t, H_{a,t})$. Moreover, letting $V_t = (H_{p,t}, R_t, \widehat{K}_t, \widehat{W}_t, \widehat{Y}_t, \widehat{I}_t, \widehat{\widehat{C}}_t, \widehat{G}_t, \widehat{Z}_t, \widehat{\widehat{C}_t})$, we can conveniently summarize the equilibrium relationships (2.3.25)-(2.3.34) by introducing a map $V : \mathbb{R}_+^3 \to \mathbb{R}_+^{10}$ such that $V_t = V(H_t, \mu_t, H_{a,t})$. This allows us to characterize a steady state equilibrium as follows:

Proposition 2. *A stationary perfect foresight equilibrium is as a sequence $\{V_t, J_t\}_{t=0}^{\infty}$ such that $V_t = V(H, \mu, H_a)\ \forall\ t$ and where the vector $J_t = (H, \mu, H_a) \in \mathbb{R}_+^3$ satisfies*

$$-\omega\frac{\partial B(\widehat{G}, A)}{\partial G} H_a^{\alpha-1} = \left(\frac{\upsilon_1}{\mu-1}\right)\mu^{\frac{1-\alpha}{\alpha}}$$

$$H = (1-\alpha)\,H_a + \varepsilon\widehat{B}(\widehat{G}, A)\left[\frac{\mu^{\frac{1}{\alpha}}(\mu-1)}{\mu(1-\varepsilon)+\varepsilon}\right]\left(\frac{R}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}}$$

$$\xi H^{\phi} = \alpha\left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}\mu^{-\frac{1}{\alpha}}\left(\widehat{C} + \widehat{B}(\widehat{G}, A)\right)^{-\sigma}$$

where $\upsilon_1 = \left(\frac{\gamma_a - \beta(1-\delta_g)}{\gamma_a}\right)\left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}$ and where R, $\widehat{C}$ and $\widehat{G}$ and defined as in equations (2.8.4), (2.8.9) and (2.8.11) in the appendix.

We can therefore identify tree channels through which advertising affects the steady state equilibrium. First, given equation (2.3.37), the long run markup depends upon aggregate advertising. This is a price channel through which advertising activities by firms, by modifying the optimal choices of all the economy's agents, end up affecting all the other endogenous variables. Second, by virtue of equation (2.3.29), the production of advertising activities absorbs labor, thereby reducing the total amount of resources available for producing goods. Unless the steady state results in a greater amount of hours worked, this channel implies that the equilibrium level of GDP is lower than it would have been without advertising. Finally, the aggregate goodwill stock affects the marginal evaluation of consumption. As we have discussed in section 2.3.1, this mechanism modifies the representative consumer's decisions about both labor and saving, thereby modifying the aggregate supply of productive factors. We will see next that the pressure provided by this mechanism upon the supply of hours turns out to be crucial in determining the macroeconomic effects of advertising expenditures.

The next proposition summarizes long-run effects that unequivocally result from the presence in the economy of advertising expenditures by firms.

Proposition 3. *In the steady equilibrium, the ratio of consumption to GDP and the labor income share increase with advertising.*

Accordingly, advertising activities by firms affect the steady state equilibrium by generating a redistribution of resources from capital to labor. As stated in the proposition, the share of GDP that is remuneration for labor services is in fact higher than it would have been without advertising. This feature hinges on the property that, at the steady state, the interest rate is independent on advertising (see equation (2.3.27)), so that all the adjustments between productive factors come through changes in the wage rate. This asymmetrically affects the factor markets, resulting in a general equilibrium in which the labor share increases. In addition, by virtue of the resource constraint (2.3.34), this also implies that the ratio of investment to GDP unequivocally decreases with advertising, thereby making, as in the Galbraithian vision (1967), the economy more consumption-based.

2.4 Quantitative Properties

The mechanisms that we have discussed in the previous section provide competitive pressures on the endogenous variable, thus making the actual impact of advertising on the aggregate economy not easily predictable. Consider, for instance, the supply of hours. Although we know the *ex-ante* advertising shifts the labor supply schedule to the right, the general equilibrium effect is instead ambiguous. Given equation (2.3.35), movements in consumption may in fact offset the impact of advertising on the marginal utility, so that the labor supply may shift upwards, downwards or remain unchanged. This depends on the relative strength of different mechanisms at play and, eventually, on the model parametrization. Furthermore, the system of equations representing the steady state equilibrium is non-linear, and, therefore, an explicit solution for the vector of endogenous variables cannot be found. Therefore, in order to evaluate the aggregate effects resulting from the presence of advertising expenditures in the economy, in what follows, we will perform several numerical experiments.

2.4.1 Calibration

In order to explore the quantitative properties of our model economy, we need to assign specific numerical values to all the structural parameters. To this end, we need first to parameterize the function $B(\cdot)$. Hereafter, we will therefore restrict the analysis by considering functions of the following form:

$$B(g_{t,i}, A_t) = \frac{A_t}{1 + \theta \frac{g_{t,i}}{A_t}} \tag{2.4.1}$$

where $\theta > 0$ is a parameter that controls for the effect of advertising on the consumer's preferences. It is easy to verify that this function satisfies all the assumptions we have made so far: it is increasing and convex in the goodwill stock, $g_{t,i}$, and, in addition, satisfies the requirements of the proposition 1. Among other things, note that this formulation implies that the marginal utility of consumption is bounded. Because of this bound, the demand schedule (2.3.4) features a maximum price above which the demand is zero: when the price is too high the marginal benefit of consuming that good is smaller than its cost, and the consumer drops it from his basket of purchases. In this

Table 2.3: **Calibration**

Parameter	Value	Description
β	.9952	Subjective discount factor
ε	6	Elasticity of substitution across varieties
δ_k	0.03	Capital depreciation rate
ξ	2.351	Preference Parameter
δ_g	0.3	Goodwill depreciation rate
ϕ	0.77	Inverse of Frisch Elasticity of Labor Supply
θ	2.62	Intensity of advertising in the utility function
α	0.75	Labor elasticity of output
σ	2	Inverse of Intertemporal Elasticity of Substitution
ω	0.949	Advertising Efficiency
γ_a	1.005	Growth Rate of Technology

fashion, firms have incentive to advertise their products to reduce the bound. In the absence of advertising, the bound is constantly equal to 1, while with advertising, the bound depends on the level of goodwill, whose effect is larger with larger θ. Hence, this parameter is interpreted as a measure of the effectiveness of advertising in affecting consumer's tastes.

Now, we explain in detail how to calibrate the model economy. Given (2.4.1), the model structural parameters is given by the vector $\Xi = \{\beta, \sigma, \phi, \xi, \varepsilon, \theta, \alpha, \delta_g, \delta_k, \gamma_a, \omega, \theta\}$. The parameters that are standard in real business cycle models are calibrated using the values commonly used in the literature, while the others are chosen such that steady states of model variables match selected long-run moments of US postwar data. In particular, the discount parameter β is set to $(1.04)^{-.25}$, implying a yearly nominal interest rate of about 4%. The growth rate of technology, γ_a is set to 1.005, so that the annual growths rate of GDP is 2%. The depreciation rate of capital δ_k is equal to 3% per quarter, and the gross elasticity of substitution across varieties is equal to 6. Following Prescott (1986), the preference parameter ξ is chosen to ensure that, in the steady state, the consumer devotes 1/4 of his time to labor activities. Following Ravn, Schmitt-Grohe and Uribe (2006), we set the intertemporal elasticity of substitution to 0.5, the labor elasticity of output α to 0.75, and the Frisch elasticity of labor supply to 1.3. These restrictions imply that the preference parameters σ and ϕ are 2 and 0.77, respectively.

The values of advertising related parameters have been assigned using the following strategy. The goodwill depreciation rate has been fixed to 0.3, implying that the half life of goodwill stock is about two quarters. This value is consistent with the empirical evidence provided in Clarke (1976): the effect of advertising on the firm's demand basically vanishes after one year. The intensity of advertising in the utility function θ is set to 2.62, a value that is consistent with the empirical evidence reported in Molinari and Turino (2009), while a numerical value to the parameter ω is assigned such that, conditional to all other parameters, the steady-state value of the advertising over GDP ratio is equal to 2.27%, consistent with the US average over the period 1948-2005.[14]

The time period in the model is one quarter. Table 2.3 summarizes the set of calibrated

[14] This number refers to the ratio of advertising expenditures to net GDP, where exports are subtracted from GDP, because exported goods are not sold based on domestic advertising.

parameters.

2.4.2 Steady States Effects

We now quantitatively characterize the steady state equilibrium of our model economy. In order to disentangle the effects resulting from the presence of advertising expenditures by firms, we will compare our model with a benchmark framework in which firms cannot advertise their products. To this end, we will first perform a static comparative exercise by analyzing the effect upon the steady state of alternative values for the advertising efficiency parameter, ω. All else being equal, by virtue of equation (2.3.36), this parameter controls for the optimal level of goodwill by firms, thereby determining the aggregate amount of resources devoted to advertising expenditures. Furthermore, setting this parameter to zero, for example because advertising is completely banned, implies that firms has no incentive to promote their products, so the optimal level of advertising is zero and therefore the steady state equilibrium coincides with the benchmark one. Figure 2.3 displays the results by plotting the graphs for the main endogenous variables expressed as percentage deviations from their benchmark values without advertising. The first panel of table 2.4 summarizes the quantitative effects on several endogenous variables by calibrating all the parameters to the US economy. As robustness check, the table also provides the results obtained by setting the elasticity ε to alternative values.

Several remarks are in order. First, we note that the advertising share monotonically increases with ω. For larger values of this parameter, the rate of transformation of advertising to goodwill increases, so that one unit of advertising becomes more effective in enhancing demand. This raises the firms' incentive to promote their products through marketing activities, thereby increasing the share of their sales devoted to advertising expenditures. In the aggregate, this implies that the amount of resources absorbed by the advertising also increases with ω.

Second, the presence of advertising results in a steady state equilibrium characterized by a higher level of hours worked, output and its components. Moreover, all these variables monotonically increase with ω. Our model therefore predicts that if we compare two economies that are identical in all structural parameters but ω so that one of them is characterized by a larger advertising share, then in the economy with the larger advertising share, we should also observe a higher level of hours worked, output and consumption. This is clearly consistent with the positive cross-country correlations we documented in section 2.2. Furthermore, the size of the effects provided by the presence of advertising in the steady state equilibrium turns out to be quantitatively important. As summarized in table 2.4, our model in fact predicts that, without advertising, the equilibrium level of hours worked in the US economy would have been between 7.31% and 10.9% lower, depending on the value we chose for the gross elasticity of substitution among varieties, ε. Similar magnitudes characterize variations in output, consumption and investment. However, the ratio of advertising-related labor to total hours worked, which we called resource absorption, also increases with ω, thereby reducing the amount of labor available for producing consumption goods. As a result, the increase in the equilibrium level of GDP, ranging from a minimum of 6.31% to a maximum of 9.90%, is sizeable, but lower than the increase in total hours worked. In addition, the redistribution effect we have discussed in section 2.3.4 implies a substantial difference in the variation of consumption and investment in spite of the fact that the increase in consumption share is instead quantitatively more contained.

The mechanism providing upward pressure upon the supply of hours through movements in the aggregate goodwill stock is crucial in generating the effects we have discussed so far. This is apparent by noticing that the real wage monotonically decreases with ω, thereby indicating that the increase in total hours worked is necessarily driven by the excess of supply that occurs in

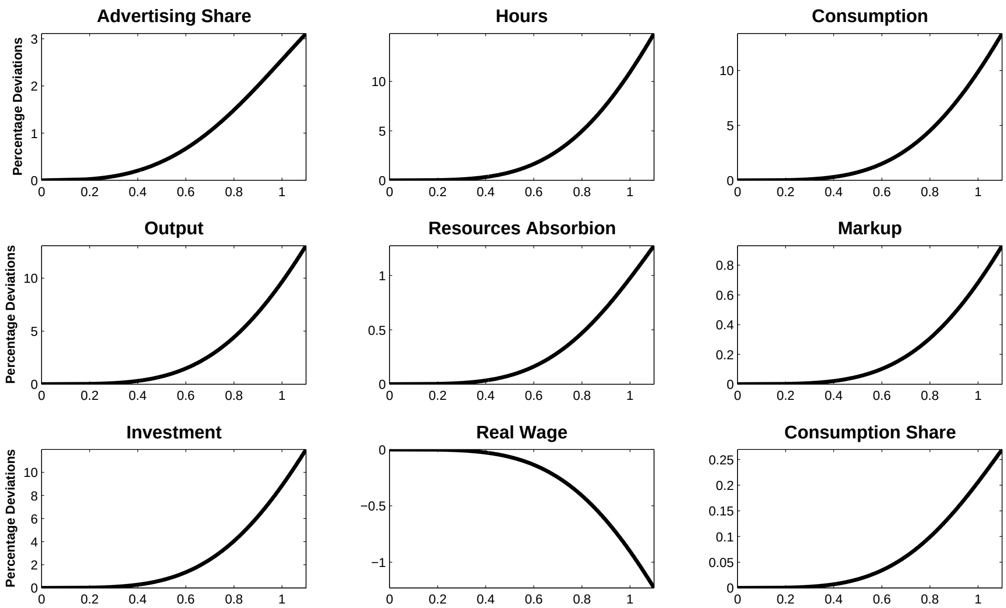

Figure 2.3: **Steady state allocations as a function of advertising productivity ω. Endogenous labor supply**. All the variables are expressed as percentage deviation with respect to the benchmark model without advertising (i.e. $\omega = 0$). Resource absorption refers to the ratio of advertising-related hours to total hours worked.

the labor market. As a result, the greater availability of labor causes a size effect that ends up increasing GDP, and its components.

To further emphasize this feature, it is useful comparing our model with two alternative formulations in which we first assume an exogenous labor supply and then we shut down the effect of aggregate goodwill on the marginal utility of consumption. In either of these cases, advertising does not affect directly the supply of hours, and therefore all the general equilibrium effects are driven by price movements and resource absorption due to advertising. Regarding the second formulation, we will follow Molinari and Turino (2009) by modifying function $B(\cdot)$ such that:

$$\frac{B(g_{i,t}, A_t)}{A_t} = S(g_{i,t}, A_t) + \int_0^1 \left(1 - S\left(g_{i,t}, A_t\right)\right) di \tag{2.4.2}$$

where

$$S(g_{t,i}, A_t) = \frac{1}{1 + \theta \frac{g_{t,i}}{A_t}}$$

Under this specification, for any good i the effectiveness of firm's advertising on its own demand depends not only upon the producer's goodwill stock, $g_{t,i}$, but also upon the level of advertising of competitors. In the symmetric equilibrium, in particular, function B(.) is equal to 1 for any value of $\widehat{G}_t$, and therefore the marginal utility of consumption becomes independent on the aggregate stock of goodwill, so that all of the effects of advertising upon the labor supply, as well on aggregate consumption, are indirect. Among other things, this formulation implies that advertising is a zero-sum game for firms since it just redistributes demand across firms without affecting the market

Table 2.4: **Results and Models Comparisons**

ε	ΔY	ΔC	ΔI	ΔH	$\Delta\mu$	$\Delta(C/Y)$
			Baseline Model			
5	6.31	6.50	5.66	7.31	0.55	0.17
6	8.09	8.27	7.42	9.09	0.54	0.17
7	9.90	10.1	9.23	10.9	0.52	0.17
			Exogenous Labor Supply			
5	-0.73	-0.50	-1.53	0	0.34	0.23
6	-0.54	-0.37	-1.14	0	0.26	0.17
7	-0.56	-0.39	-1.17	0	0.27	0.17
			Purely Combative Advertising			
5	-0.17	-0.01	-0.74	0.01	-0.01	0.16
6	-0.17	-0.01	-0.75	0.01	-0.01	0.16
7	-0.18	-0.01	-0.76	0.01	-0.01	0.17

Note: Δx refers to the percentage deviation of the original variable x with respect to its benchmark value without advertising ($\gamma = 0$). Panel 1 displays the results for the baseline model. Panel 2 displays the results for the model with an exogenous labor supply. Panel 3 provides the results for the model in which function $B(\cdot)$ is specified as in equation (2.4.2).

size. We will refer to this case as purely combative advertising. Results are summarized in table 2.4.

Two main features are worth emphasizing. First, compared with the baseline specification of our model, we note that shutting down the effect of advertising on the representative consumer's labor supply, either because labor decisions are exogenous or because advertising does not affect the marginal utility of consumption, implies dramatically different results. In either of the cases, our model now predicts that the presence of advertising results in a lower level of output, consumption and investment. However, while with an exogenous labor supply advertising essentially exacerbates the distortion associated with the monopolistic competitive structure of the goods market,[15] with purely combative advertising, we note that total amount of hours worked instead increases. In this case, the effect is driven by an excess of demand that occurs in the labor market because of the production of advertising activities. Second, we note the size of the effects provided by advertising upon the main aggregates is, in both cases, substantially smaller than the ones we obtained in our baseline framework. For example, notice that the GDP variation (in absolute value) is about 14 times lower when the labor supply is assumed to be exogenous. Therefore, the linkage upon the representative consumer's labor supply and the aggregate goodwill stock is also crucial to generate quantitatively significant effects. In fact, this channel operates as a powerful amplification mechanism, which indirectly magnifies the impact of advertising to GDP. For example, notice that in the baseline calibration, our model predicts that a 2% amount of aggregate resources absorbed by the advertising sector implies an increase in GDP of 8%, which is 4 times larger.

[15] Notice that in fact the aggregate markup also increases.

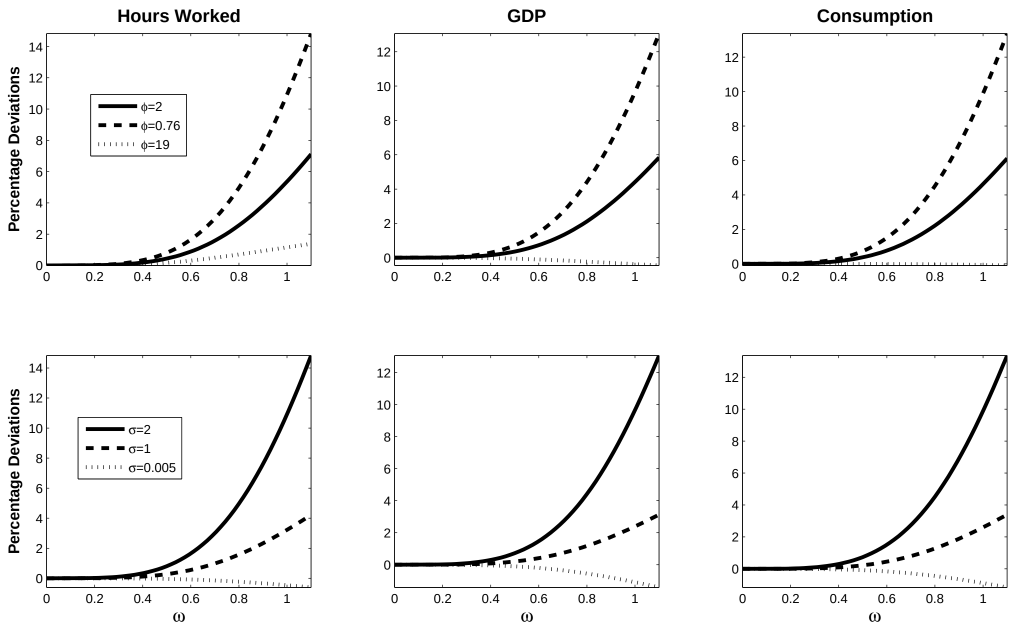

Figure 2.4: **Steady State Hours Worked, Output, and Consumption as function of advertising productivity for various value of ϕ, and σ.** All the variables are expressed as percentages deviation from the benchmark value in the model without advertising expenditures ($\omega = 0$)

Another interesting implication of our model is that the presence of advertising results in a higher level of both hours worked and output even though the steady state equilibrium is, at the same time, characterized by a larger aggregate markup (see figure 2.3). Unlike standard results,[16] this feature provides a theoretical counterexample showing that an increase in the market power is not necessarily associated with a lower level of hours worked and output. To get an intuition for this, it is worth comparing our model with the canonical framework. To this end, notice that in the standard Dixit-Stiglitz model with monopolistic competition in the goods market, the market power results in a wedge that affects the consumers' intra-temporal condition in a way that makes the consumers less willing to substitute from leisure into consumption. This effect results in a suboptimal level of hours and, consequently, output. Instead, in our framework, advertising by modifying the consumers' tastes raises the willingness to pay for the goods and, at the same time, increases the marginal utility of consumption. As a result, firms' gain market power that they exploit by charging a higher markup over the marginal cost, while the consumers feel disaffection with the current level of consumption that leads to a higher supply of hours. The negative effect related to the increase of firms' market power is offset by the stronger substitutional effect induced by advertising, so that the equilibrium level of hours worked increases. Among other things, this feature indicates that, in our framework, advertising, although persuasive, can nevertheless be welfare-enhancing. We will return to this point next.

As a final issue, we evaluate the extent to which the predictions of our framework are sensitive to alternative calibrations for the model's structural parameters. In the analysis, we restrict our attention to the parameters affecting the labor supply schedule, that is, the inverse of intertemporal

[16] See, for instance, Blanchard and Kyotaky (1987).

elasticity, σ, and the inverse of the Frisch elasticity of labor supply, ϕ. Figure 2.4 illustrates the results, displaying, for alternative values of ϕ and σ, the graphs of hours, output and consumption as functions of ω. All the variables are expressed as percentage deviations from their benchmark values.

As shown in the first panel of figure 2.4, we note that the size of the steady state effects declines with larger ϕ. Intuitively, increasing this parameter implies a lower value for the Frisch elasticity and therefore the supply of hours moves less for equal variations in both wage and marginal utility of consumption.[17] As a result, the downward pressures provided by the increase in the aggregate markup and the resources absorption due to advertising activities offset the effect generated by the increase in the supply of hours, thereby reducing, for any given ω, the size of the steady state effect on output and consumption. In fact, for a large enough value of ϕ, the equilibrium level of these two aggregates decreases even though the equilibrium level of hours worked increases.

Regarding the effect of changing the intertemporal elasticity of substitution, we note that, compared with the baseline calibration, the effect of advertising on the main aggregates declines with lower σ. In this case, all else being equal, the substitution effect between consumption and leisure becomes smaller, and, therefore, the consumer's willingness to work declines. As such, the labor supply reacts less to equal movements in the aggregate goodwill stock, so that total hours worked, output and consumption all increase less. Moreover, when σ is set to a sufficiently low value, we note that the main results are reverted. As shown in the picture (see the dotted line), now hours worked and the main aggregates monotonically decrease with ω. In such a circumstance, while the aggregate markup is still affected, the labor supply becomes almost independent on the aggregate goodwill stock, so that advertising ends up exacerbating the distortions associated with the monopolistically competitive structure of the goods market. As a result, hours worked, output and consumption all decreases with larger ω.

To summarize, in this section we have shown that, with advertising expenditures, the equilibrium level of hours worked, outputs and its component are, at the steady state, larger than they would have been otherwise. Moreover, as long as the representative consumer's labor supply is sufficiently elastic to movements in the marginal utility of consumption, the sizes of these effects are quantitatively important. However, the assumptions of endogenous labor supply and advertising with market-enhancing effects are both crucial for these results. On the contrary, the presence of advertising ends up exacerbating the distortions associated with the monopolistically competitive structure of the goods market, providing quantitatively small effects.

2.5 Advertising and Labor Supply

Among the theoretical results we have shown so far, the connection between the aggregate goodwill stock and individual decisions about working activities is a particularly interesting feature. It in fact suggests that aggregate advertising may potentially be an important determinant of the economy's labor supply. In section 2.2, in particular, we have shown that the amount of resources aborted by the advertising sector features a substantial within-country variation and, at the same time, shows remarkable differences across countries. According to our theoretical results, this suggests that advertising, providing pressures that vary over time and across countries, may potentially play a significant role in explaining the observed within-country variation of the labor supply as well as the cross-country difference in per-capita hours worked. This feature is particularly important in the light of the fact that understanding the determinants of labor supply

[17] In fact, in the limiting case in which the labor supply is totally inelastic ($\phi = \infty$), we would find exactly the same results as in the case of exogenous labor supply, that is, constant hours worked and a lower level of both consumption and output.

decisions is an issue that has spurred a wide amount of research in all branches of economic theory. Our framework identifies another mechanism affecting the supply of labor that has so far been overlooked by economic theory. It is therefore interesting to study this mechanism more deeply. To this end, in the following sections, we will focus on studying this connection, by contrasting the ability of our model in explaining both the within-country and cross-country variation in labor supply.

2.5.1 The US boom in the 1990s

In the 1990s, the US economy experienced a decade of sustained economic growth. As evident from figure 2.5, during this period, per-capita hours worked show an increasing pattern, returning to the level of early 1990s only with the recession of the year 2001. As emphasized in McGrattan and Prescott (2007), the basic neoclassical growth model, while accounting well for the post-war US economy prior to the 1990s, fails in reproducing this economic boom. The reason for this failure is that, by incorporating variation in total factor productivity (TFP) and marginal taxes for labor income, the model predicts an after-tax real wage below its secular trend, which, in turn, implies a decline in the predicted hours worked, when in fact it was growing. Other factors thus appear to be crucial in determining the behavior of the US during that period of time. In fact, McGrattan and Prescott identified the huge increase in intangible investments, such as advertising or expenditures in research and development, as the crucial mechanism to explain the boom that occurred in that period of time.

Among other things, the 1990s US boom appears to be an interesting case study for our purposes. During the same period, the US advertising expenditures had experienced a sustained growth. This growth is apparent in figure 2.1, which shows that advertising, relative to GDP, has constantly grown over the 1990s, reaching its maximum peak over 50 years in 1999. According to our theoretical framework, this should have provided important pressure on the labor supply, and, therefore, advertising might be partly responsible for the observed *puzzling* dynamics in hours worked.

In order to address this issue, we will perform a business cycle accounting (BCA) exercise along the line of Chari, Kehoe, and McGrattan (2007). Namely, we will make use of data on investment, GDP, hours worked, advertising expenditures, and taxes in order to recover from the equilibrium conditions of our model three exogenous wedges that allow for a perfect match between data and the model's predications.[18] More specifically, we will use the intratemporal condition (2.3.9) to recover the labor wedge, the Euler equation (2.3.10) to recover the investment wedge, and the optimal advertising policy (2.3.21) to recover the advertising wedge. Furthermore, in order to recover a sequence for TFP, we will modify both production functions (2.3.12) and (2.3.13) by adding a multiplicative term that captures purely transitory variations in productivity. Then, we will compare data for hours worked with the model's predictions by shutting down the labor wedge and instead allowing taxes, TFP, and other wedges to vary. Since within the basic neoclassical model the labor wedge has been proved to be the main determinant of the labor dynamics,[19] by doing so we will be able to disentangle the effect of advertising on the labor supply during the 1990s. Finally, notice that in performing the BCA exercise, we will slightly modify our model by introducing taxes on labor income, τ_t, so that the representative consumer's intratemporal condition becomes:

$$\xi H_t^{\phi} = (1-\tau_t)\widehat{W}_t \left(\widehat{C}_t + \widehat{B}(\widehat{G}_t, A_t)\right)^{-\sigma} \tag{2.5.1}$$

[18]Details on the procedure are provided in appendix.

[19]See for instance Shimer (2009)

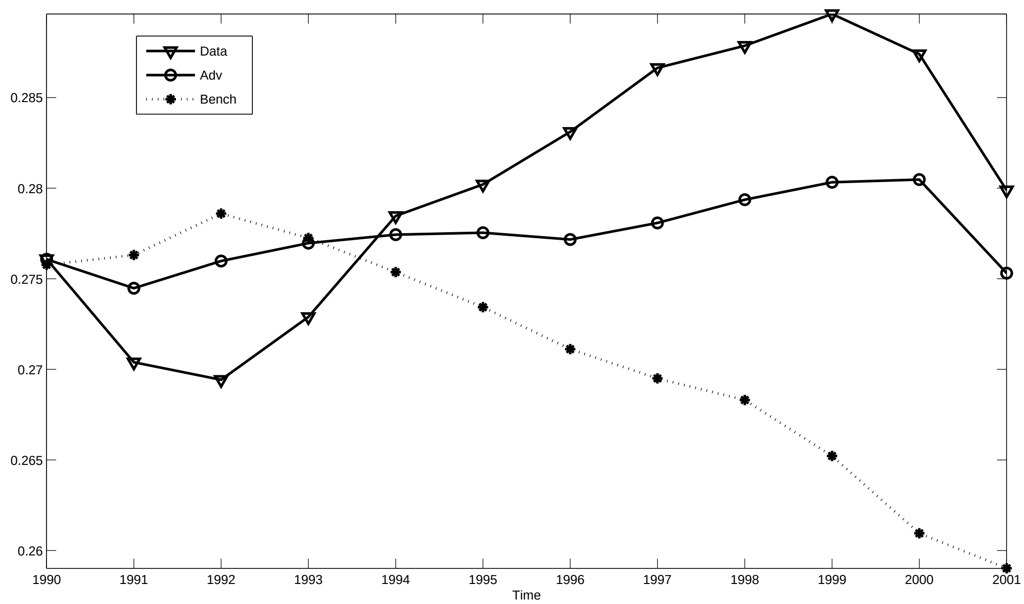

Figure 2.5: **Hours worked during the US boom in the 1990s**. Model's predictions vs actual data. All the data are taken from McGrattan and Prescott (2007). Bench refers to the model without advertising ($\omega = 0$).

Figure 2.5 compares the US per capita hours worked and the model's prediction for the same variable. As a benchmark case, we have also reported the predicted per capita hours worked in the model without advertising ($\omega = 0$). As apparent from the picture, we note that the canonical neoclassical growth model predicts a constant decline in hours worked, when they were in fact increasing. As in McGrattan and Prescott, by setting a constant labor wedge, the model fails to predict the boom of the 1990s. Adding advertising to the model, however, has the effect of remarkably improving the model's predictions. Now, the model predicts a constant increase in per-capita hours worked. To get an intuition for this result, it is worth bearing in mind that our framework naturally provides a theory for the household's disutility of work to be time varying. This is apparent from equation (2.5.1), which shows that for a given level of consumption the elasticity of the labor supply with respect to the after-tax real wage depends, over time, upon the aggregate level of goodwill. In particular, for a given wage rate, the larger the aggregate goodwill stock, the stronger the consumers' willingness to work. Given the sustained growth we observed in the advertising share, this effect appears to be particularly strong during the 1990s. In fact, while we do not find any significant difference in the recovered TFP, the labor wedge in the model with advertising is remarkably lower than the one we obtain in the benchmark.[20] Hence, the effect provided upon the marginal utility of consumption of movements in the aggregate stock overcompensate the effect caused by the dynamics of the after-taxes real wage, thereby implying, as in the data, an increasing pattern in per-capita hours worked. The fits of our model is far from being perfect, thereby implying that other factors may have also played a role in determining the dynamics of labor. Nevertheless, the experiment shows that our model substantially improves upon the neoclassical one, thereby suggesting that, at least for the US economy, the connection

[20]See figure 2.7 in the appendix.

Table 2.5: **Cross-Country comparison, baseline model predictions.**

	Labor Supply (US=1)			Prediction Factors		
Country	Actual	Predicted	Diff	Adv/Gdp	Tax Rate τ	TFP (US=1)
(I) Heterogeneity in Advertising						
Germany	0.71	0.93	0.25	1.28	0	1
France	0.73	0.91	0.32	0.97	0	1
Italy	0.66	0.91	0.26	0.89	0	1
United Kingdom	0.89	0.96	0.33	2.11	0	1
USA	1	1	-	2.76	0	1
(II) Heterogeneity in Advertising, TFP and Taxes						
Germany	0.71	0.71	0.99	1.28	0.59	0.89
France	0.73	0.70	1.11	0.97	0.59	0.92
Italy	0.66	0.65	1.03	0.89	0.64	0.98
United Kingdom	0.89	0.91	0.78	2.11	0.44	1.06
USA	1	1	-	2.76	0.44	1
(III) Heterogeneity in Taxes and TFP (Benchmark Model)						
Germany	0.71	0.80	0.69	0	0.59	0.89
France	0.73	0.80	0.75	0	0.59	0.92
Italy	0.66	0.74	0.76	0	0.64	0.98
United Kingdom	0.89	0.96	0.36	0	0.44	1.06
USA	1	1	-	0	0.44	1

Note: Labor Supply and Total Factor Productivity are both relative to the corresponding US values. $Diff$ = $(1 - H^{actual})$ / $(1 - H^{predicted})$ indicates the fraction of actual difference in hours worked between a country x and the US economy that is explained by the model. Panel (I) provides the results for the baseline model by assuming cross-country heterogeneity in the advertising sector. Panel (II) same as panel (I) but letting both taxes on labor income and TFP to vary. Panel (III) displays the results for the benchmark model ($\gamma = 0$) by assuming cross-country heterogeneity in both marginal tax on labor income and TFP. Marginal taxes on labor income are taken from Prescott (2004). Details are provided in appendix.

between advertising and labor supply might operate as an important mechanism in determining the aggregate level of hours worked.

2.5.2 Cross-country comparison

We will next assess the ability of our model to explain the observed cross-country differences in hours worked. In order to address this issue, for a given set of countries, we will compare actual data for average per-capita hours worked with the model's predictions, where the latter will be obtained by computing the steady state equilibrium. The analysis starts by assuming that the only source of cross-country heterogeneity is given by the parameter controlling for the rate of transformation of advertising in goodwill, ω. Then, we will repeat the experiment by introducing other sources of cross-country heterogeneity that have been identified in the literature as crucial variables in the study of the labor supply, namely TFP and marginal tax on the labor income. All the quantitative results are obtained by setting the structural parameters to their baseline values, with the exception of the advertising efficiency, ω and the preference parameter ξ. The former is chosen by targeting the country-specific advertising share, while the latter is calibrated in order to match the average hours worked in the US economy during the time period 1996-2006. By doing so, any predicted difference in per-capita hours worked can be imputed to the chosen source of cross-country heterogeneity. The choice of the time period is instead constrained to the availability of data for advertising. Table 2.5 provides the results for the US, Germany, France, Italy, and the UK.

Several remarks are in order. First, analyzing actual data reveals that, over the considered period of time, both per-capita hours worked and advertising share display a substantial degree of cross-country variation. In either case, the US is the leading country. This is therefore an interesting case study to assess the potential role of advertising to explain the huge differences in hours worked between the US and Europe. Second, comparing the predictions of our model with the data shows that differences in the size of advertising sector explain an important part of the cross-country variability in hours worked: allowance for this source of heterogeneity explains alone between 1/4 and 1/3 of actual data (see the first panel of the table). Finally, our experiment confirms to a large extent the results provided in Prescott (2004): over the decade 1996-2006, as in the early 1990s, cross-country heterogeneity in both taxes on labor income and TFP still appears to be crucial to explain cross-country differences in hours worked.[21] As is apparent in panel 2 of the table, allowance for those sources of heterogeneity notably improves the predictions of our framework. In this case, the results are remarkable: the model's fit is in fact almost perfect. However, it is worth noting that the heterogeneity in the size of the advertising sector still plays an important role in explaining variability of labor across countries, even when we allow for taxes and TFP to vary. This is apparent by noting that the model's fit worsens once we drop advertising decisions from the model (see panel 3 of the table). This therefore suggests that during the decade 1996-2006, although taxes and TFP are surely important, other factors might have also contributed to the differences in hours worked between the US and Europe. Our framework identifies advertising as one of them.

To conclude, it is worth noticing that the mechanism behind the results we have shown so far is consistent with the idea, suggested by Blanchard (2004), that cross-country differences in hours worked might simply reflect cultural differences in the evaluation of leisure. According to his vision, if in fact Europeans enjoy leisure more than Americans do, then any equal increase of wage would asymmetrically affect their labor supplies, since the income effect would be stronger in Europe than in the US. In our model, the mechanism is exactly the same, but with the remarkable difference that the effect of advertising upon the marginal evaluation of leisure is endogenous rather than exogenously determined by cultural considerations. In fact, our model predicts that even if Americans and European had the same ex ante preferences toward consumption and leisure, the former would have still worked more because of the stronger desires for material consumption endogenously provided by the greater intensity in advertising activities by firms.

2.6 Welfare Analysis

In section 2.4.2, we have shown that the presence of advertising results in a steady state equilibrium characterized by a larger level of both consumption and hours, in spite of the fact

[21] TFP is recovered from the data using the model equilibrium conditions. Marginal taxes on labor income are taken from Prescott (2004). Note that in all experiments we have adjusted data in order to be consistent with the model. See the appendix for further details.

that the economy experiences a larger aggregate markup at the same time. This feature suggests that advertising, although persuasive, might nevertheless be welfare-improving since, by inducing a lower level of aggregate leisure in the economy, it may potentially mitigate the distortion induced by the monopolistically competitive structure of the goods markets.

However, in our framework, evaluating the welfare consequences of advertising is complicated by the fact that preferences are endogenously determined. In fact, as emphasized in Dixit and Norman (1978), with persuasive advertising affecting the consumers' taste, there are at least two natural yardsticks for welfare comparisons: the pre-advertising and post-advertising tastes[22] and it is not clear *a priori* which of them is the most appropriated. For this reason, we will follow Benhabib and Bisin (2002) by introducing here a welfare criterion that takes into account both the pre- and post-advertising preferences. More precisely, let $(C(\omega), H(\omega))$ and $U(C(\omega), H(\omega), \omega)$ respectively denote the representative consumer's allocations pair and his equilibrium utility function, [23] we will make use of the following criterion:

Definition 1. *We say that the consumer's welfare increases due to advertising if and only if it increases with respect to post-advertising preferences so that*

$$U(C(\omega), H(\omega), \omega) \geq U\left(C\left(0\right), H(0), \omega\right)$$

and it also increases with respect to pre-advertising preferences

$$U(C(\omega), H(\omega), 0) \geq U(C(0), H(0), 0)$$

with at least one inequality holding strictly.

Accordingly, for any given $\omega > 0$, we will say that the consumer is better off with advertising if, independently of whether we use pre- or post-advertising preferences as a welfare yardstick, he always prefers the post-advertising allocation $(C(\omega), H(\omega))$ to the pre-advertising one $(C(0), H(0))$.

The top panel of figure 2.6 provides the graphs for the representative consumer's utility function for alternative values of parameter ω. All the remaining parameters are calibrated to their baseline values, so that as shown in section 2.4.2, the equilibrium levels of both hours worked and consumption monotonically increase with ω. In order to facilitate comparisons, the bottom panel of the picture provides graphs for welfare gain (in terms of steady-state consumption) associated with a policy that totally bans advertising.[24]

Several remarks are in order. First, while the welfare gain is strictly increasing with ex-ante preferences, we note that, with ex-post preferences, it instead displays an hump-shaped pattern, becoming negative for values of ω that are larger than the cutoff point ω^* (see the graph at the right-bottom corner). According to our criterion, this shows that for a wide range of values for ω,[25] the consumer is unequivocally worse off with advertising. Thus, in the general equilibrium, the presence of advertising activities by firms ends up exacerbating the welfare losses caused by the monopolistic competitive structure of the goods market. However, unlike canonical results, given that the equilibrium level of consumption increases with ω, in our framework, welfare losses are driven by an "overworking" effect induced by advertising.

Second, if we assume the ex-post preference as a standard for the judgment, our experiment shows that persuasive advertising does not necessarily imply a welfare loss for the society. Quite the contrary, for a large enough value of ω, consumers are instead better off with advertising.

[22]That is, the respective utility functions from which the pre and the post-advertising demand can be derived.

[23]So that the pre-advertising allocations and taste are the equilibrium values corresponding to the case in which we set $\omega = 0$

[24]More precisely, for any given ω, the welfare gain is defined as the value of λ that solves the equation: $U(\lambda C(\omega), H(\omega), \omega) = U(C(0), H(0), \omega)$

[25]More precisely, for any $\omega \in [0, \omega^*]$.

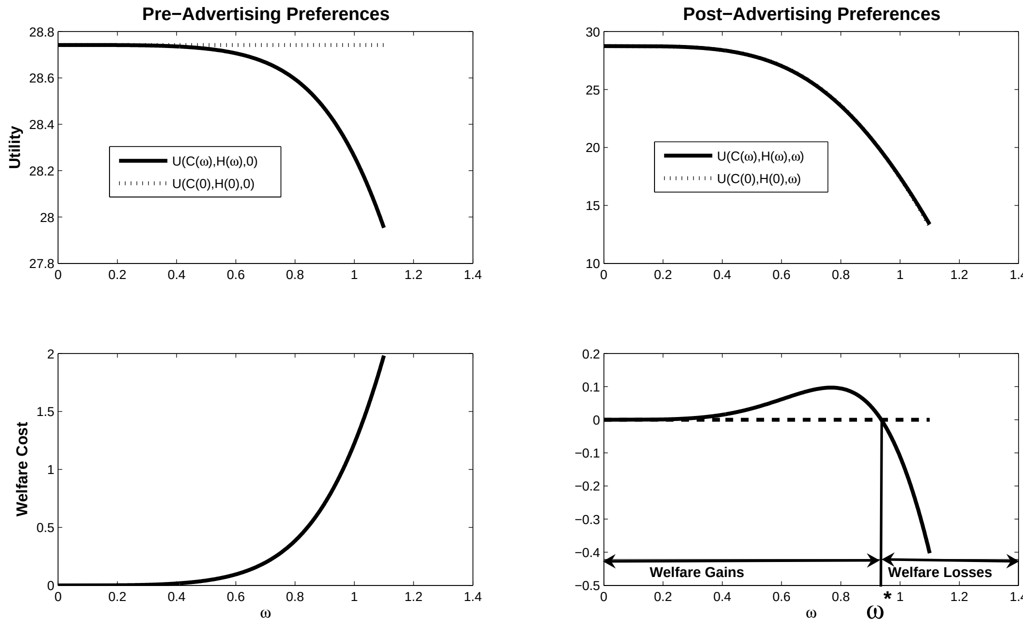

Figure 2.6: **Welfare Analysis**. Steady state utility functions. Left panel: pre-advertising preferences. Right panel: post-advertising preferences. The bottom panel illustrates the welfare gain, in terms of percentage of steady-state consumption, of a policy that completely bans advertisements.

Finally, our model predicts that consumers' satisfaction and per capita income are negatively correlated. As we have seen in section 2.4, conditional to the baseline calibration, total output is in fact an increasing function of advertising efficiency while, as shown in figure 2.6, the utility associated with the equilibrium allocations is instead decreasing, and this independently of whether we focus on pre or post-advertising preferences. This result is consistent with the fact that, across countries, no clear relationship between average income and average happiness can be found (see Graham (2005)). In fact, the literature on the economics of happiness suggests that other factors, such as aspirations or relative income considerations, are at play. For instance, Laynards (2005) highlights the extent to which people's happiness is affected by status. Also, the increasing flow of information about the living standards of others can increase frustration with relative income differences. From this perspective, individuals may engage in consumption not only for its intrinsic value, but also for its value in signaling their relative position in the income distribution. Heffetz (2009) provides evidence for such a phenomenon, showing that households' consumption decisions are also affected by the so-called *socio-cultural visibility* of consumer expenditures, which he defines as *the speed with which members of society notice a household's expenditures on different commodities*. Our framework embeds many of these features, providing a mechanism that endogenizes welfare losses that are driven by increasing aspirations for material consumption. It is interesting to note that, although we do not assume preferences for relative consumption (or status), the core mechanism of our model is consistent with the potential connection between advertising and socio-cultural visibility of consumption goods. As shown in Krahmer (2006), if in fact consumers use brands for image reasons (visibility), then advertising, by informing individuals of the brand name, renders a good potential signaling device, thereby inducing households to engage in conspicuous consumption. As in our framework, by inducing disaffection with the current level of consumption, this mechanism creates a connection between advertising and aggregate consumption expenditures. Moreover, in our framework, as in models with preferences for relative consumption (see Fisher and Hof (2000)), if consumers' decisions about working activities are endogenous, than this mechanism, by producing an overworking effect, might reduce the individual well-being even though he consumes more.

2.7 Conclusion

In this paper, we have studied the influence of persuasive advertising in a neoclassical growth model, showing that advertising may significantly affect the aggregate economy. In particular, we have shown that, with advertising expenditures, the equilibrium level of hours worked, outputs, and its components are, at the steady state, larger than they would have been otherwise. Moreover, as long as the representative consumer's labor supply is sufficiently elastic to movements in the marginal utility of consumption, the size of these effects are quantitatively important. However, the assumptions of endogenous labor supply and advertising with market-enhancing effects are both crucial for these results. On the contrary, the presence of advertising ends up exacerbating the distortions associated with the monopolistically competitive structure of the goods market, providing quantitatively small effects. Therefore, in order to properly identify the macroeconomic effects of advertising, the potential link between advertising and the labor supply needs to be empirically documented. Despite the evidence reported in Brack and Cowling (1983) and Fraser and Paton (2003), we are still missing an appropriated empirical analysis involving a cross-country comparison. Exploring this issue is the next priority of our research agenda.

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2.8 Appendix

Data

Advertising expenditures data

- **Germany**: Investment in advertising including expenditures on salaries, media and the production of means of publicity for the period 1950-2000. Sources Rehme G., and Weiser S. (2007) table 7.
- **United Kingdom**: Annual advertising expenditures all the media. The data for 1950 to 1991 are provided to us by courtesy of Stuart Fraser. The data from 1991 to 2005 are taken from IPA (www.ipa.co.uk).
- **USA**: The data for 1948 to 1999 are obtained from an updated version of Robert J. Coens (McCann-Erikson, Inc.) original data published in Historical Statistics of the United States, Colonial Times to 1970. The data for 2000 to 2005 are obtained from the Newspaper Association of America (http://www.naa.org). The aggregate data include spending for advertising in newspapers, magazines, radio, broadcast television, cable television, direct mail, billboards and displays, Internet, and other forms.
- **Japan**: Data from 1975 to 2005. Source DENTSU (www.dentsu.com)
- **Others OECD countries** data from 1996 to 2006. Source World Advertising Research Center (WARC). Advertising Media and Forecasts.

Macro aggregates.

- Output, Consumption and Investment are from the OECD dataset. Investment are net of housing.
- Per capita hours worked are taken from Groningen Growth and Development Centre and the Conference Board, Total Economy Database, January 2007, http://www.ggdc.net
- Data for marginal taxes on labor income are taken taken from Prescott (2004)
- Macro aggregates and taxes on labor income for the business cycle accounting exercise are taken from McGrattan and Prescott (2007).
- Country specific capital for the cross-country analysis are taken from Kiel Institute for the world economy (http://www.ifw-kiel.de/forschung/datenbanken/netcap).

Proof of proposition 1

A balanced growth path is defined as an equilibrium such that all the endogenous variables grow at a constant rate with the exception of labor variables, markup, and interest rate which, instead, stay constant. We want to show that any function $B(\cdot)$ satisfying the requirements stated in proposition 1 implies the existence of such an equilibrium. Before proceeding, it is worth noting that the assumption of labor-augmenting Cobb-Douglas technology for the production of goods, the capital accumulation equation (2.3.33) and the good market clearing condition (2.3.34) imply that the steady-state rates of growth of output, capital, investment and consumption are all equal to the growth rate of labor augmenting technical progress, γ_a. By virtue of the labor market clearing condition (2.3.28), constancy of total hours worked implies that both advertising and production related labor stay constant. Given the production function of advertising activities (2.3.31) and the accumulation equation (2.3.3), this also implies that the goodwill stock and advertising have the same rate of growth γ_a. Thus, denoting one plus the growth rate of a variable X as γ_X, along the balanced growth path it must be true that:

$$\gamma_Y = \gamma_K = \gamma_I = \gamma_C = \gamma_Z = \gamma_G = \gamma_a \tag{2.8.1}$$

and

$$\gamma_H = \gamma_{H_a} = \gamma_{H_p} = 1 \tag{2.8.2}$$

Using the terminology of King, Plosser and Rebelo (1988), equations (2.8.1)-(2.8.2) describe the technologically feasible steady state. Notice, moreover, that the same conditions also implies: (i) the marginal product of capital is constant over time; (ii) the marginal product of labor grows at the rate γ_a.

To prove the statement, we need to show can that any function $B(\cdot)$ that satisfies the restrictions stated in the proposition implies that conditions (2.8.1)-(2.8.2) are compatible with all the optimality conditions of agents in the economy. To this end, notice first that rewriting the intratemporal condition (2.3.35) as follows:

$$\xi H_t^{\phi} = \frac{W_t}{A_t}\left(\frac{\widetilde{C}_t}{A_t}\right)^{-\sigma}$$

indicates that for hours worked to be constant it is required that $\widetilde{C}_t$ and A_t grow at the same rate γ_a. Thus, by rewriting its rate of growth as follows:

$$\frac{\widetilde{C}_t}{\widetilde{C}_{t-1}} = \left(\frac{C_t}{C_{t-1}} - \frac{B(G_t, A_t)}{B(G_{t-1}, A_{t-1})}\right)\frac{C_{t-1}}{\widetilde{C}_{t-1}} + \frac{B(G_t, A_t)}{B(G_{t-1}, A_{t-1})}$$

we note that $\widetilde{C}_t$ grows at the rate γ_a if and only if also $B(G_t, A_t)$ does. Given that the goodwill stock grows at the steady state rate γ_a, for this requirement to be satisfied it is sufficient that:

$$\frac{B(G_{t+1}, A_{t+1})}{B(G_t, A_t)} = \gamma_a$$

whenever:

$$\frac{G_t}{A_t} = \frac{G_{t+1}}{A_{t+1}}$$

In addition, by virtue of equations (2.3.25), (2.3.26), (2.3.27) and (2.3.37), this condition also implies that: (i) both the aggregate mark-up and the interest rate stay constant; (ii) the representative consumer's Euler equation is satisfied; (iii) the real wage grows at the rate γ_a. To conclude the proof it remains to show that equation (2.3.21) is satisfied along the balanced growth path equilibrium. To this end, notice that by virtue of equation (2.3.20), the advertising marginal cost is constant, thereby implying that equation (2.3.21) is satisfied if and only if the derivative $\partial B(g_{i,t}, A_t) \,/\, \partial g_{t,i}$ is also constant. As before, a sufficient condition for this requirement to be satisfied is that whenever:

$$\frac{G_t}{A_t} = \frac{G_{t+1}}{A_{t+1}}$$

implies:

$$\frac{\partial B(G_{t+1}, A_{t+1})}{\partial G_{t+1}} = \frac{\partial B(G_t, A_t)}{\partial G_t}$$

This concludes the proof.

Proof of proposition 2

We begin the proof by showing how to construct a map $V : \mathbb{R}^3_+ \to \mathbb{R}^{10}_+$ that links a subset of endogenous variables to the vector $J_t = (H, \mu, H_a)$. To this end, let us assume that the economy is at the steady-state. From the clearing condition in the labor market we can derive en expression for production-related labor:

$$H_p = H - H_a \equiv H_p(H, \mu, H_a) \tag{2.8.3}$$

and for the Euler equation (2.3.27) an expression for the long run interest rate:

$$R = \frac{1-\beta(1-\delta_k)}{\beta} \equiv R(H, \mu, H_a) \tag{2.8.4}$$

By making use of equation (2.3.26), the ratio of production-relate labor to capital can be expressed as follows:

$$\frac{H_p}{\widehat{K}} = \left(\frac{R}{1-\alpha}\right)^{\frac{1}{\alpha}} \mu^{\frac{1}{\alpha}}$$

which, in turn, allows us to explicitly derive an expression for both capital stock and wage rate. That is:

$$\widehat{K} = \left(\frac{R}{1-\alpha}\right)^{-\frac{1}{\alpha}} \frac{\mu^{-\frac{1}{\alpha}}}{H - H_a} \equiv \widehat{K}(H, \mu, H_a) \tag{2.8.5}$$

$$\widehat{W} = \alpha\mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \equiv \widehat{W}(H, \mu, H_a) \tag{2.8.6}$$

No entry condition implies that the fixed cost is of the form:

$$F = \left(1 - \frac{1}{\mu}\right) H_p^{\alpha} K^{1-\alpha} - W H_a$$

Substituting this equation into the definition of output (2.3.30), and using equations (2.3.25) and (2.3.29) into the resulting expression yields:

$$\widehat{Y} = \mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \left[H - (1-\alpha) H_a\right] \equiv \widehat{Y}(H, \mu, H_a) \tag{2.8.7}$$

An expression for the equilibrium level of investment can be derived using the low of motion for the capital stock. Accordingly:

$$\widehat{I} = \bar{\delta}_k \, \mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} (H - H_a) \equiv \widehat{I}(H, \mu, H_a) \tag{2.8.8}$$

where $\bar{\delta}_k = \gamma_a - (1 - \delta_k)$. Thus, using the goods market clearing condition, the equilibrium level of consumption can be rewritten as follows:

$$\widehat{C} = \mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \left[(1-\bar{\delta}_k)H - \left(1 - \alpha - \bar{\delta}_k\right) H_a\right] \equiv \widehat{C}(H, \mu, H_a) \tag{2.8.9}$$

Finally, by virtue of equations (2.3.28),(2.3.31) and (2.3.32) we get:

$$\widehat{Z} = H_a^{\alpha} \equiv \widehat{Z}(H, \mu, H_a) \tag{2.8.10}$$

$$\widehat{G} = \omega \frac{H_a^{\alpha}}{\delta_g} \equiv \widehat{G}(H, \mu, H_a) \tag{2.8.11}$$

$$\widehat{\widetilde{C}} = \widehat{C} + \widehat{B}(\widehat{G}, A) \equiv \widehat{\widetilde{C}}(H, \mu, H_a) \tag{2.8.12}$$

Therefore, letting $V_t = (\, H_{p,t},\ R_t, \widehat{K}_t,\ \widehat{W}_t,\ \widehat{Y}_t,\ \widehat{I}_t,\ \widehat{\widetilde{C}}_t,\ \widehat{G}_t,\ \widehat{Z}_t,\ \widehat{\widetilde{C}_t})$, we can conveniently summarize the equilibrium relationships (2.8.3)-(2.8.12) by introducing a map $V : \mathbb{R}_+^3 \to \mathbb{R}_+^{10}$ such that $V_t = V\,(H_t, \mu_t, H_{t,a})$.

To conclude the proof, we next derive a set of equilibrium conditions that allows us to identify among all the vectors $J_t = (H, \mu, H_a)$ those representing a steady-state equilibrium. To this end, notice first that the intratemporal condition (**??**) and the optimal advertising policy (2.3.36) can be respectively rewritten as follows:

$$\xi H^{\phi} = \alpha \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \mu^{-\frac{1}{\alpha}} \left(\widehat{C} + \widehat{B}(\widehat{G}, A)\right)^{-\sigma} \tag{2.8.13}$$

$$-\,\omega \frac{\partial B(\widehat{G}, A)}{\partial G} H_a^{\alpha-1} = \left(\frac{\upsilon_1}{\mu - 1}\right) \mu^{\frac{1-\alpha}{\alpha}} \tag{2.8.14}$$

where $\upsilon_1 = \left(\frac{\gamma_a - \beta(1-\delta_g)}{\gamma_a}\right) \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}$ and where R, $\widehat{C}$ and $\widehat{G}$ and defined as in equations (2.8.4), (2.8.9) and (2.8.11). The optimal price policy instead implies that the price elasticity takes the following form:

$$\varepsilon \left(1 + \frac{\widehat{B}(\widehat{G}, A)}{\widehat{Y}}\right) = \frac{\mu}{\mu - 1}$$

which, in turn, allows us to express aggregate output as follows:

$$\widehat{Y} = \varepsilon \widehat{B}(\widehat{G}, A) \left[\frac{\mu - 1}{\mu(1-\varepsilon) + \varepsilon} \right]$$

Thus, substituting equation (2.8.7) into the previous one, and solving for H, yields:

$$H = (1-\alpha)\, H_a + \varepsilon \widehat{B}(\widehat{G}, A) \left[\frac{\mu^{\frac{1}{\alpha}}\, (\mu - 1)}{\mu\, (1-\varepsilon) + \varepsilon} \right] \left(\frac{R}{1-\alpha} \right)^{\frac{1-\alpha}{\alpha}} \tag{2.8.15}$$

Hence, at the steady-state the vector $J_t = (H, \mu, H_a)$ is the solution of the system of equations (2.8.13)-(2.8.15). Therefore, a stationary perfect foresight equilibrium for the model economy can be now defined as a sequence $\{V_t, J_t\}_{t=0}^{\infty}$ such that $V_t = V(H, \mu, H_a)\ \forall\ t$ and where the vector $J_t = (H, \mu, H_a) \in \mathbb{R}^3_+$ satisfies equations (2.8.13)-(2.8.15).

Proof of proposition 3

Let $s_h(H, H_a)$ denotes the labor income share as a function of total hours worked, H, and advertising-related labor, H_a. By virtue of equations (2.8.6) and (2.8.7) it follows:

$$s_h(H, H_a) \equiv \frac{W\ H}{Y} = \alpha \left[\frac{H}{H - (1-\alpha) H_a} \right] > \alpha \equiv\ s_h(H, 0)\ \forall\ H_a\ \in\ (0, H)$$

which shows that with advertising the labor income share is unequivocally larger than otherwise (i.e. $H_a = 0$). To prove that the consumption share increases with advertising, it is enough to note that equations (2.8.8) and (2.8.9) jointly imply:

$$\frac{I}{Y}(H, H_a) = \frac{\bar{\delta}_k (1-\alpha)}{R} \left[\frac{H - H_a}{H - (1-\alpha) H_a} \right] < \frac{\bar{\delta}_k (1-\alpha)}{R} \equiv\ \frac{I}{Y}(H, 0)\ \forall\ H_a\ \in\ (0, H)$$

Details on the BCA exercise

Simulation of the model is performed by following several steps and using yearly data over the period of time 1990-2003. Figures for macro-aggregates are taken from McGrattan and Prescott (2007). .

Step 1. Modifying the model

To perform the BCA exercise, we begin by slightly modifying the model presented in section 2.3. More precisely, we introduce proportional taxes on labor income, τ_t, so that the representative consumer's intratemporal condition becomes:

$$\xi H_t^{\phi} = (1-\tau_t) \widehat{W}_t \left(\widehat{C}_t + \widehat{B}(\widehat{G}_t, A_t) \right)^{-\sigma} \tag{2.8.16}$$

and we modify both the production functions for goods and advertising activities in order to introduce a factor, Υ_t, capturing purely transitory variations in total factor productivity (TFP). That is:

$$\widehat{Y}_t = \Upsilon_t H_{p,t}^{\alpha} \widehat{K}_t^{1-\alpha} - F \tag{2.8.17}$$

$$\widehat{Z}_t = \Upsilon_t H_{a,t}^{\alpha} \tag{2.8.18}$$

Step 2. Calibration

As second step, we need to assign numerical values to all the model structural parameters. Some of them are set to their baseline values as reported in table (2.3), while the other are calibrated in order to: (i) take into account the yearly frequency of the data; (ii) match specific moments for the 1990 US (detrended) data. More specifically, we set the growth rate of technology, γ_a to 1.02, implying a yearly growth rate of

GDP of 2%; the depreciation rate δ_g is fixed to 0.7, implying a half-life for the goodwill stock of about half year; the elasticity ε is set to 9, implying, as in the baseline case, an average mark-up equal to 5%. Parameters σ, θ, and ϕ are set to their baseline values. The remaining parameters $(\beta;\omega;\alpha;\delta_k;\xi;\Upsilon)$ are calibrated by using the US level of detrended variable in 1990 for GDP (normalized to 1), capital stock, hours worked, consumption, advertising expenditures, and by setting the yearly real interest rate to 1.041.

Step 3. Constructing the series of exogenous and endogenous variables

In order to simulate the model, we need first to construct sequences of all the endogenous and exogenous variables, including three exogenous wedges. To this end, we make use of data for output $\widehat{Y}_t$, investment $\widehat{I}_t$, advertising expenditures $\widehat{Z}_t$, hours worked H_t and taxes on labor income, τ_t. Exogenous wedges are introduces into the model in order to force the agents' optimality conditions so that, by computing the perfect foresight equilibrium, the model's predictions coincide with the observable data.

More specifically, we use the accumulation equation (2.3.33) with observations for the initial capital stock (the 1990 value) and investment to construct sequences of capital stocks. A series of aggregate consumption is obtained by using the resources constraint (2.3.34) with observations for detrended output and investment. Sequences of TFP is recovered by first noting that, by virtue of equations (2.3.29) and (2.8.18), the production function can be rewritten as follows:

$$\begin{aligned}\widehat{Y}_t &= \Upsilon_t H_{p,t}^{\alpha}\widehat{K}_t^{1-\alpha}-F\\ &= \Upsilon_t\left(H_t-H_{a,p}\right)^{\alpha}\widehat{K}_t^{1-\alpha}-F\\ &= \Upsilon_t\left(H_t-\left(\widehat{Z}_t/\Upsilon_t\right)^{\frac{1}{\alpha}}\right)^{\alpha}\widehat{K}_t^{1-\alpha}-F\end{aligned}$$

so that Υ_t can be expressed as:

$$\Upsilon_t=\left[\frac{(\widehat{Y}_t+F)^{\frac{1}{\alpha}}+\widehat{K}_t^{\frac{1-\alpha}{\alpha}}\widehat{Z}_t^{\frac{1}{\alpha}}}{\widehat{K}_t^{\frac{1-\alpha}{\alpha}}H_t}\right]^{\alpha} \tag{2.8.19}$$

Sequences of $H_{a,t}$ and $H_{p,t}$ are derived by using equations (2.3.29) and (2.8.18) with observations for $\widehat{Z}_t$, H_t and Υ_t. To perfectly match data for detrended advertising expenditures we modify the accumulation equation (2.3.32) by introducing an advertising wedge, $X_{z,t}$, as follows:

$$\widehat{G}_t=\frac{(1-\delta_g)}{\gamma_a}\widehat{G}_{t-1}+\omega X_{z,t}\widehat{Z}_t$$

This modifies the optimal advertising policy, which now is given by:

$$\frac{\widehat{W}_t H_{a,t}^{1-\alpha}}{X_{z,t}}+\alpha\omega_t\,\frac{\partial B(\widehat{G}_t,A_t)}{\partial G_t}\,(1-\mu_t^{-1})+\frac{\beta(1-\delta_g)}{\omega\gamma_a}\,\left(\frac{\widehat{\widetilde{C}}_{t+1}}{\widehat{\widetilde{C}}_t}\right)^{-\sigma}\frac{\widehat{W}_{t+1}H_{a,t+1}^{1-\alpha}}{X_{z,t+1}} \tag{2.8.20}$$

A sequence of labor wedges, $X_{h,t}$, is recovered by combining the representative consumer's intratemporal condition (2.3.35) with the optimal demand of production related labor (2.3.25) as follows:

$$X_{h,t}=\alpha\frac{(1-\tau_t)}{\xi\mu_t}\left(\frac{Y_t}{H_{p,t}}\right)\left[\frac{\tilde{C}_t^{-\sigma}}{H_t^{\phi}}\right] \tag{2.8.21}$$

while the investment wedge, $X_{k,t}$, is obtained from the representative consumer's intertemporal condition (2.3.27) as follows:

$$X_{k,t+1}=X_{k,t}\left\{\frac{\beta\left\{\tilde{C}_{t+1}^{-\sigma}\left[R_{t+1}+(1-\delta_k)\right]\right\}}{\tilde{C}_t^{-\sigma}}\right\} \tag{2.8.22}$$

where the sequence $X_{k,t+1}$ is constructed by assuming $X_{k,1}=1$ and R_{t+1} is defined as the following:

$$R_{t+1}=\frac{(1-\alpha)}{\mu_{t+1}}\frac{Y_{t+1}}{K_{t+1}} \tag{2.8.23}$$

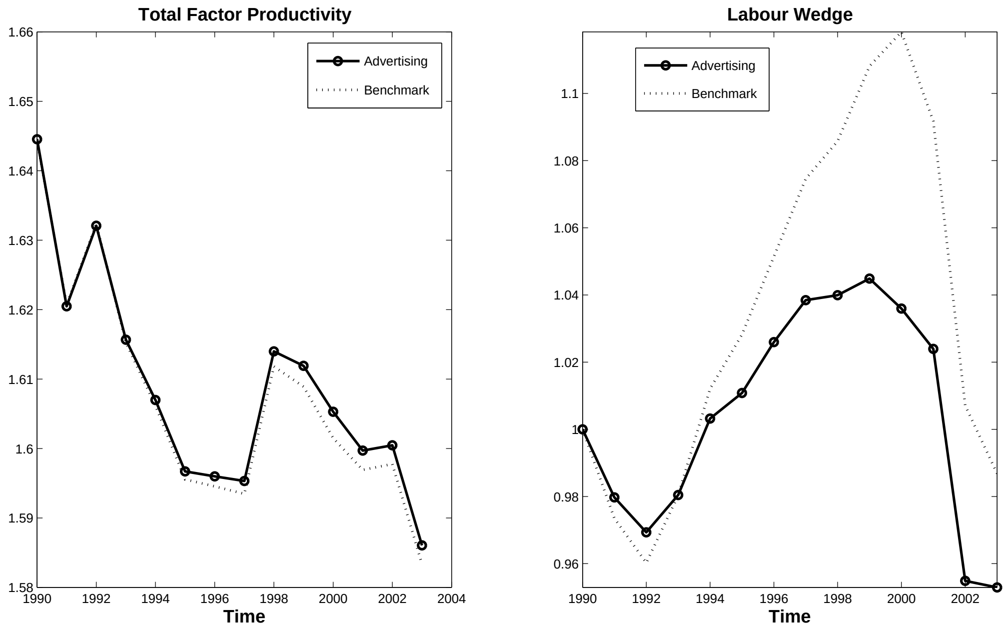

Figure 2.7: **Total Factor Productivity and the Labor Wedge**. *Note*: Benchmark refers to the model without advertising expenditures ($\gamma = 0$).

Finally, using equations (2.8.20)-(2.8.22) and equations (2.3.25), (2.3.26), (2.3.28) and (2.3.37) with observations for GDP, Investment, Consumption, labor related variables, TFP and taxes on labor income we simultaneously recover sequences of $\widehat{W}_t$, R_t, μ_t, $\widehat{\widehat{C}}_t$, G_t, $X_{z,t}$, $X_{h,t}$ and $X_{k,t}$.

By construction, if we compute a perfect-foresight equilibrium path for this model, assuming households take as given time paths for TFP, tax on labor income and wedges, we get a perfect match between the model predictions and the data. Figure provides graph for the estimated series for the labor wedge and TFP in our framework, along with the corresponding sequences obtained in the benchmark case without advertising.

Step 4. Simulation

To analyze the ability of our model to explain the dynamics of hours worked during the 1990s, we compute the perfect-foresight equilibrium path by setting the labor wedge constantly equal to 1. By doing so, we are able to disentangle the effect upon the labor supply of advertising expenditures by comparing the model's predictions with data. Figure 2.5 provides the results.

Details on Cross-Country comparison.

To perform the cross-country analysis, we compute the steady state equilibrium of our model economy by allowing for cross-country heterogeneity in the advertising sector, TFP and taxes on labor income. As in the case of the BCA exercise, we need first to construct a sequence for country-specific TFP. To this end, we make use of equation (2.8.19) with yearly data on detrended GDP and capital stock, where the latter is constructed by using the accumulation equation (2.3.33) with detrended data of investment. Data for the country-specific initial capital stock is taken from Kiel Institute for the world economy (http://www.ifw-kiel.de/forschung/datenbanken/netcap). In computing the country-specific steady-state, in order to introduce cross-country heterogeneity in TFP, we set the TFP parameter, Υ, to the country average over the period of time 1996-2006.

Chapter 3

Non-Price Competition, Real Rigidities and Inflation Dynamics

3.1 Introduction

In the last decade, the analytical progress achieved in the New Keynesian literature has been remarkable. Many of the early assumptions have been relaxed, leading to medium-scale macroeconomic models that are now able to capture several features of real-world data. As such, the New Keynesian framework has became a workhorse for the analysis of monetary policy in both academic and central-banking circles.

Nevertheless, modern-day New Keynesian models still assume, as did their early counterparts, that firms compete in the market with no tools other than their relative prices. This literature particularly lacks a formal study that explicitly evaluates the consequences of extending competition between firms to the non-price dimension. In fact, in general equilibrium models with nominal rigidities, the firms' behavior is typically modeled by abstracting from any decisions concerning non-price factors, such as investment in quality, advertising and customer services. This paper tries to fill this gap by extending the canonical New Keynesian framework to an environment where firms face, at the same time, both price and non-price competition.

In this paper, non-price competition refers to any activity by firms that shifts the demand for their goods (and/or those of their competitors), for any given set of prices. Several aspects make this dimension of the firms' competition a theoretically interesting feature in models with nominal rigidities. First, assuming, as in the New Keynesian literature, that firms compete for the market only through their relative prices is a simplification that is not always realistic. This is particularly true in oligopolistic or monopolistically competitive industries in which non-price tools often drive the major source of inter-firm competition. Furthermore, real-world data suggest that firms in fact devote large amounts of resources to these activities. In the US economy, for example, aggregate expenditures on advertising and research and development (R&D) in the second half of the last century have accounted, on average, for almost 5% of GDP.[1] Similar magnitudes characterize advertising and R&D markets in other industrialized countries.[2]

Second, although we have identified non-price tools as demand shifters independent of prices, non-price competition among firms may still have indirect effects on their pricing behavior. As emphasized in the industrial organization literature, a typical feature of non-price tools is that they may modify the degree of substitutability among goods.[3] For instance, by spending on advertising, a firm may successfully persuade customers about peculiar characteristics of its product, or by investing in quality (product innovation), it may increase the degree of differentiation between its product and those of its rivals. Through these activities, therefore, firms may successfully build

[1] This statistic refers to data taken from 1950 to 2003. Data for R&D and advertising expenditures are respectively taken from the National Science Foundation (www.nsf.org) and the Newspaper Association of America (http://www.naa.org). Figures for GDP are taken from Federal Reserve Bank of St. Louis (http://research.stlouisfed.org/fred2/).

[2] See Molinari and Turino (2007) for an international comparison.

[3] In fact, in this literature, non-price competition tools, such as advertising and R&D for product innovation, are often modeled as affecting the degree of substitutability among goods. See, for instance, Lambertini and Mantovani (2008) and the references they provide.

customers' loyalty for their products, thereby gaining monopolistic and pricing power. This feature is particularly interesting in light of the New Keynesian theory. This literature has in fact emphasized firms' pricing behavior as a key determinant for both inflation dynamics and the persistence of the real effects of monetary policy shocks. From this perspective, therefore, by interacting with the firms' pricing behavior, non-price competition may also affect inflation dynamics. The current New Keynesian literature overlooks this interesting linkage precisely because it assumes that firms compete for the market with no tools other than their relative prices. Therefore, the analysis provided in this paper will focus in particular on understanding whether and how non-price competition among firms can affect inflation dynamics.

To address this issue, we develop a variant of the Calvo model (1983) by extending the canonical framework to an environment where firms also engage in non-price competition. Building on Spence (1977), this feature is introduced in the model by assuming that consumers' tastes are endogenously determined, depending on the distribution of non-price activities across all the firms. To emphasize that our results depend only on the interaction between non-price policy and pricing behavior, we further assume that firms engage in a purely combative non-price competition context, in the sense that non-price activities by a firm expand the demand for its products by drawing existing customers away from rival products.

The analytical framework of this paper departs from the assumption of constant elasticity of substitution among goods, which is typically made in a Calvo-style model, and instead assumes a more generic specification, such as that used in Kimball (1995), that allows for demand functions featuring a non-constant price elasticity (quasi-kinked demand). Several reasons justify this modeling strategy. First, adopting Kimball preferences allows us to obtain in a relatively simple way the linkage between non-price competition activities by firms and product differentiation among goods of different producers. As we will see next, with non-constant elasticity of demand, changes in non-price activities that affect the distribution of market shares across firms in turn affect the elasticity of demand faced by each individual producers, and hence the degree of substitutability of their products. Second, an environment where firms face demand functions featuring a non-constant price elasticity appears to be the most appropriate theoretical ground for evaluating non-price competition in a model with nominal rigidities. In this context, in fact, a producer, when increasing its own relative price, faces a higher opportunity cost, as the loss of customers resulting from the downward sloping demand curve is amplified by the price elasticity being an increasing function of the relative price. In a such a circumstance, therefore, it should be potentially more convenient for a producer trying to boost profits to affect demand through non-price tools, which do not directly involve price movements.

Within this analytical framework, the joint role of non-price competition and firms' pricing policies in the determination of the aggregate price level is analyzed by focusing on the reduced-form inflation dynamics represented by the New Keynesian Phillips curve. To preview our results, we find that non-price competition does affect inflation dynamics, by increasing the inflation-marginal cost coefficient. This result hinges on the property that, under very general assumptions, non-price competition generates a mechanism that dampens the overall degree of real rigidity in price-setting. In our framework, in fact, pricing and non-pricing policies are strategic complements, so that, through non-price tools, a firm mitigates the effect upon its market share made by price movements. This reduces the opportunity cost that price-setters face in changing their relative price, mitigates the degree of real rigidity and eventually increases the size of price changes. As a result, if firms engage in non-price competition, inflation becomes more sensitive to movements in marginal cost.

From the perspective of New Keynesian theory, our results are relevant because they show that allowance for non-price competition among firms generates a mechanism that dampens the overall impact of real rigidities on inflation dynamics. This issue is particularly important, as real rigidities have became popular among New Keynesian theorists precisely because they provide a mechanism to amplify the effect of nominal disturbances and, all else being equal,to reduce the size of the Phillip curve's slope. In light of these features, real rigidities in price-setting, also refereed to as strategic complementarities, are now recognized as important theoretical ingredients of modern-day New Keynesian models. For instance, Eichenbaum and Fisher (2007) have shown that extending the canonical Calvo model by assuming firm-specific capital and demand functions to have non-constant elasticity of demand (quasi-kinked demand) allows one to recover estimates of the Phillip's curve's slope with a realistic degree of nominal rigidities. Smetz and Wouters (2007) have used quasi-kinked demands function in an estimated monetary DSGE model. Sbordone (2008) extends the Kimball model to study the effect of globalization on inflation dynamics. Our analysis casts some doubt regarding the robustness of such conclusions, showing that abstracting from non-price competition, as canonical model do, may potentially overstate the overall impact of strategic complementarities on inflation dynamics. This therefore suggests that enriching the New Keynesian framework to include non-price competition among firms may be a promising feature in order to improve our understanding on the key determinants of inflation dynamics. This should be particularly true in economy, as the US one, in which non-price competition appears to be an important dimension of the inter-firm rivalry.

The paper is organized as follows. Section 3.2 describes the model economy and provides the main results; Section 3.3 concludes.

3.2 A simple economy with non-price competition

In this section we lay out a baseline framework that captures the key features of non-price competition in a context of nominal and real rigidities. The specific framework we develop is a variant of the canonical model discussed in Kimball (1995). The main difference is that in our model, firms may also engage in non-price competition by spending on activities that expand their customer bases. This modification results in non-trivial consequences for inflation dynamics, affecting in particular the inflation-marginal cost relationship. As a general result, we show that any activity by firms that boosts demand for their products, without directly affecting their prices, dampens at the same time the overall degree of real rigidities in price-settings.

3.2.1 Households

The economy consists in a continuum of differentiated goods indexed by $i \in [0, 1]$, each produced by a monopolistically competitive firm and over which consumer preferences are defined. More specifically, the household derives utility from an object C_t, which is implicitly defined by a relation of the form

$$\int_0^1 \psi\left(\frac{b_t(i)c_t(i)}{C_t}\right) di = 1 \tag{3.2.1}$$

where $c_t(i)$ denotes the quantity consumed of variety i; $\psi(.)$ is an increasing and strictly concave function, with $\psi(1) = 1$, while $b_t(i) \geq 0$ satisfies the following condition:

Assumption 1.

$$b_t(i) = \nu\left(z_t(i)\right) \tag{3.2.2}$$

where ν is a positive, strictly increasing and strictly concave function, with $\nu(1) = 1$; $z_t(i) = Z_t(i)/Z_t$ denotes firm i's non-price activities relative to the market average.

Equation (3.2.1) extends Kimball's (1995) preferences to an environment where firms engage in non-price competition. To embed this feature into the model, we follow Spence (1977) by assuming that consumers' preferences are endogenously determined, depending on the distribution of non-price activities across firms. For each variety i, this linkage is controlled by the term $b_t(i)$, whose properties are restricted in order to guarantee that the desired requirements for non-price competition are fulfilled in this general setup. This feature will be apparent by deriving the demand curve for each individual variety. The latter is the solution to the dual problem of minimizing consumption expenditures subject to the aggregation constraint (3.2.1), that is:

$$c_t(i) = \frac{C_t}{b_t(i)} g\left(\frac{P_t(i)C_t}{\lambda_t b_t(i)}\right) \tag{3.2.3}$$

where $g(\cdot) = \psi'^{-1}(\cdot)$ denotes the inverse function of ψ', $P_t(i)$ is the price of good i, and λ_t is the Lagrange multiplier for the constraint (3.2.1) which is implicitly defined as follows:

$$\int_0^1 \psi\left(g\left(\frac{P_t(i)C_t}{\lambda_t b_t(i)}\right)\right) di = 1 \tag{3.2.4}$$

For future reference, notice that here, in contrast with the standard Dixit-Stiglitz framework, the Lagrange multiplier, λ_t, does not need to coincide with the price index P_t. The latter is usually defined as the cost of a unit of the composite good, that is:

$$P_t = \frac{1}{C_t}\int_0^1 P_t(i)c_t(i)di \equiv \int_0^1 \frac{P_t(i)}{b_t(i)}\psi'^{-1}\left(\frac{P_t(i)C_t}{\lambda_t b_t(i)}\right) di \tag{3.2.5}$$

Given (3.2.3), the monotonically increasing behavior of $b_t(i)$ implies that the demand of each variety is also increasing in the producer's relative non-price activities. To see this, notice that differentiating the demand schedule (3.2.3) with respect to $z_t(i)$ yields

$$\frac{\partial c_t(i)}{\partial z_t(i)} = \frac{c_t(i)}{b_t(i)}\left(\varepsilon_{p,t}(i) - 1\right) v'(z_t(i)) \tag{3.2.6}$$

where $\varepsilon_{p,t}(i)$ is the demand price elasticity for good i which will be formally introduced shortly. Since for the firms' optimization problem to be well-defined it is required that $\varepsilon_{p,t}(i) > 1$, we find that $\partial c_t(i)/\partial z_t(i) > 0 \; \forall \; i \in [0,1]$.

This property hinges on the assumption that consumers' tastes are endogenously determined. In fact, in our framework the consumers' marginal utility of each variety is also increasing in the producer's relative expenditures on non-price activities. To see this, notice that applying the implicit function theorem to equation (3.2.1) in order to derive an expression for the marginal utility of each variety, and differentiating the resulting equation with respect to $z_t(i)$ yields:

$$\frac{\partial^2 C_t}{\partial c_t(i)\partial z_t(i)} = \frac{\psi'(x_t(i))}{D_\psi}\left[1 - \frac{1}{\varepsilon_{p,t}(i)}\right] v'(z_t(i)) > 0 \; \forall \; i \; \in [0,1]$$

where D_ψ is a positive constant[4] and $x_t(i) = \left(c_t(i)b_t(i)\right)/C_t$. Hence, any asymmetrical distribution

[4]To see this, notice first that $D_\psi = \int_0^1 \psi'\left(x_t(i)\right) x_t(i)di$. By assumption $\psi_t(x_t(i))$ is a strictly increasing and strictly convex of function of $x_t(i)$, so the integrand $\psi'\left(x_t(i)\right) x_t(i)$ is then a positive and continuous function. As such, the above integral is a positive real number.

of non-price activities across firms affects consumers' demand precisely because it modifies the consumers' marginal evaluation of each variety. For equal prices, in particular, a household will devote a higher fraction of its income in products whose producers spent the most to gain an advantage in the non-price competition in their market. Alternatively, the demand of each variety will increase in response to an increment in the producer's non-price expenditures.

This feature has a natural interpretation in terms of the degree of substitutability among goods. Intuitively, we can think that non-price activities by a firm, such as marketing promotions or investment in quality, by attaching peculiar attributes to the product, increase the consumers' perceived differentiation with respect to rival products. Consider, for instance, a producer that increases its relative expenditures on non-price competition. In our framework, this directly affects the consumer's tastes, making that product more valuable in terms of utility. As such, the consumers' cost of switching from that good to another, for example, as the former becomes more expensive, increases. Equivalently, the degree of substitutability between that good and the rival products decreases. Because of this perception of product differentiation, consumers are then willing to pay a higher price for that good and, for a given price, the producer's market share increases.

As a second property, we note that the demand for each product is decreasing in the average expenditure on non-price competition. This is an immediate implication of the assumption that for any variety i, the factor $b_t(i)$ depends upon the producer's relative expenditures $z_t(i)$. In fact, direct differentiation of (3.2.3) yields:

$$\frac{\partial c_t(i)}{\partial Z_t} = -\frac{c_t(i)}{Z_t}\left(\varepsilon_{p,t}(i) - 1\right)\varepsilon_{b,z}(i) < 0$$

where $\varepsilon_{b,z}(i) > 0$ denotes the percentage change in $b_t(i)$ resulting from a percentage variation of $z_t(i)$. Hence, our formulation captures the fact that firms engage in a combative non-price competition context in the sense that an increase in expenditure for non-price tools of a sufficiently large fraction of firms creates a negative externality on the demand faced by other firms. This means that in our framework non-price competition is a zero-sum game, since non-price activities by a firm increase demand by drawing existing consumers away from rival products. Therefore, any asymmetrical distribution in non-price activities merely redistributes demand among firms, thereby causing an asymmetrical distribution in market shares.

One important feature of the Kimball aggregator (3.2.1) is that it generalizes the standard Dixit-Stiglitz preferences,[5] allowing for a non-constant elasticity of demand with respect to relative price. Denoting the latter with $\varepsilon_{p,t}(i)$, one can easily prove the following:

$$\varepsilon_{p,t}(i) = -\frac{\psi'(x_t(i))}{x_t(i)\psi''(x_t(i))} \tag{3.2.7}$$

which shows that the CES aggregator is in fact a special case of (3.2.1) obtained by specifying the function $\psi(x_t(i))$ so that the ratio $\psi'(x_t(i))/\left[x_t(i)\psi''(x_t(i))\right]$ is constant $\forall\ x_t(i)$. As Kimball (1995) has shown, it is always possible to find a specific functional form for $\psi(x_t(i))$ that matches any desired dependence of the elasticity of demand on the firm's relative output. Here, in order to introduce strategic complementarity in price-setting, we are interested in the functional form for $\psi(x_t(i))$ that generates '*quasi-kinked*' demand functions, characterized by the property that for the firm at its normal market share, it is easier to lose customers by increasing its relative price than to gain customers by lowering its relative price. This requires that the price elasticity is decreasing in the firm's relative sales (market share). However, the assumptions we have made

[5]The Dixit-Stiglitz preferences are obtained, as a special case, by setting $\psi(x_t(i)) = x_t(i)^{\varepsilon/(\varepsilon-1)}$, with $\varepsilon > 1$.

so far do not have any implications for the sign of $\varepsilon'_{p,t}(i)$. Therefore, hereafter we confine our attention to functional forms satisfying the following condition:

Assumption 2.

$$\frac{\partial \varepsilon_{p,t}(i)}{\partial m_{y,t}(i)} = \varepsilon'_{p,t}(i) b_t(i) < 0 \tag{3.2.8}$$

where $m_{y,t}(i) = c_t(i)/C_t$ is the firm i's market share, and $\varepsilon'_{p,t}(i) = \partial \varepsilon_{p,t}(i)/\partial x_t(i)$.

The next proposition summarizes an important implication resulting from non-price competition in an environment where producers face quasi-kinked demand functions.

Proposition 1. *Let the households' consumption aggregate be of the form (3.2.1). Then, for any functions $\psi(x_t(i))$ and $b_t(i)$ satisfying assumptions 1-2, the demand price elasticity of each variety i, $\varepsilon_{p,t}(i)$, is decreasing in the producers' relative non-price activities.*

Proof. In order to prove this statement, note first that,

$$\begin{aligned}
\frac{\partial \varepsilon_{p,t}(i)}{\partial z_t(i)} &= \frac{\partial \varepsilon_{p,t}(i)}{\partial x_t(i)} \frac{\partial x_t(i)}{\partial b_t(i)} \frac{\partial b_t(i)}{\partial z_t(i)} \\
&= \varepsilon'_{p,t}(i)\, m_{y,t}(i) \left(1 + \frac{\partial c_t(i)}{\partial b_t(i)} \frac{b_t(i)}{c_t(i)}\right) v'(z_t(i)) \\
&= \varepsilon'_{p,t}(i)\, m_{y,t}(i) \varepsilon_{p,t}(i)\, v'(z_t(i))
\end{aligned} \tag{3.2.9}$$

given that the assumption 1 requires $v'(z_t(i)) > 0$ and that $m_{y,t}(i)$ and $\varepsilon_{p,t}(i)$ are both strictly positive, this implies:

$$sign\left(\frac{\partial \varepsilon_{p,t}(i)}{\partial z_t(i)}\right) = sign(\varepsilon'_{p,t}(i))$$

Therefore, for any function $\psi(x_t(i))$ that satisfies assumption 2, it must be true that

$$\frac{\partial \varepsilon_{p,t}(i)}{\partial z_t(i)} \; < \; 0 \; \forall \; z_t(i) > 0$$

since $sign(\varepsilon'_{p,t}(i)) < 0 \; \forall \; x_t(i) > 0$. □

Accordingly, for each variety i, the producer's relative expenditures for non-price competition not only shift the demand schedule but also reduce price elasticity. While the first effect is a direct consequence of assumption 1, the second is instead entirely due to assumption 2. As is apparent from equation (3.2.9), with CES preferences, an increase in the firm's relative non-price activities would just generate a parallel upward shift in the demand schedule without affecting the slope. Furthermore, the effect of non-price activities on the demand price elasticity uniquely depends upon the linkage between the latter and the firm's relative price. In fact, assuming strategic substitutability in price-setting[6] would instead have the effect of making the demand price elasticity an increasing function of firm's non-price activities. Therefore, it turns out that in our framework assumption 2 is a necessary condition to obtain a positive linkage between the intensity of non-price competition and firms' market power.

[6]That is, assuming that $\partial \varepsilon_p(i)/\partial m_{y,t}(i) > 0$.

3.2.2 Firms

In this model, firms make decisions on pricing policy, production plans and budgets for non-price competition. In order to explicitly analyze firms' behavior, we therefore require further assumptions concerning the technology available for producing goods, the type of market and technology characterizing non-price activities and the source of nominal rigidities. To this end, hereafter we confine our attention to a model economy satisfying the following two conditions:

Assumption 3. *Prices are set in staggered contracts with random duration as in Calvo (1983): in any period, each firm faces a constant probability* $(1-\theta)$ *of being able to re-optimize and charge a new price.*

Assumption 4. *There exists an economy-wide labor market, with nominal wage* W_t*, from which firms hire labor for the production of both consumption goods and non-price activities. All the firms have access to a common technology of the form* $A_t f_s(h_{s,t}(i))$ *with* $s = \{y, z\}$*, where* $f_s(.)$ *is a strictly increasing and concave function;* $h_{s,t}$ *denotes hours for producing goods* ($s = y$) *or non-price activities* ($s = z$)*, and* A_t *is a (stochastic) factor that describes the evolution of technology.*

Assumption 3 introduces nominal rigidities by assuming staggered prices a la Calvo; every period, only a fraction $(1-\theta)$ of the firms can set a new price, independently of the past history of price changes. Assumption 4 instead describes the technological structure of the model economy. It states that firms produce goods as well as non-price activities by using a common technology that requires only labor. While we could proceed by making rather general assumptions about the market for these non-price activities, we choose to simplify the analysis by assuming, as in Grossmann (2007), that firms produce these tools in-house. However, this choice is inconsequential to our main conclusions.[7]

The firms' optimization problem can be solved in an equivalent two-step procedure. In the first step, a firm chooses the amount of labor for the production of goods and non-price activities by minimizing the total cost function subject to technology constraints. Given assumption 4, the first-order conditions for an interior minimum are the following:

$$\eta_t(i) = \frac{w_t}{A_t f_y'(h_{y,t}(i))} \tag{3.2.10}$$

$$\varphi_t(i) = \frac{w_t}{A_t f_z'(h_{z,t}(i))} \tag{3.2.11}$$

where w_t denotes the real wage, while $\eta_t(i)$ and $\varphi_t(i)$ are the marginal costs for producing consumption goods and non-price activities, respectively. While we have assumed an economy-wide labor market, equation (3.2.10) indicates that decisions concerning demand of labor input are instead firm-specific. This feature implies that production marginal cost depends not only on economy-wide factors but also on the firm's own output. In a model based on the Calvo pricing mechanism,[8] this generates strategic complementarity in price-setting that reduces the size of price changes and, all else being equal, lowers the sensitivity of inflation to marginal cost. To see why, take the example of a contractionary monetary shock. In this case, adjusters would find it convenient to reduce their prices in order to boost their profits. However, if marginal cost is firm-specific, adjusters trying to boost its sales by undercutting others would also be increasing

[7] One can easily show that this restriction is equivalent to assuming a perfectly competitive market for non-price activities, where producing firms use the same technology as an assumption 4.

[8] See Sbordone (2002) for further details.

their own production marginal cost. This reduces profits from undercutting, thereby making price-setters less inclined to undercut the fixed prices of their competitors. As a consequence, the size of price changes declines, thereby reducing the sensitivity of inflation to movements in real marginal costs.

In the second step, a firm seeks to maximize profits by choosing price and non-price expenditures, $Z_t(i)$. Given the cost-minimizing conditions, the instantaneous real profit at date t for firm i can then be written as

$$\pi(p_t(i), Z_t(i), \xi_t) = p_t(i)y_t(i) - TC\left(Z_t(i), y_t(i), \xi_t\right) \tag{3.2.12}$$

where $p_t(i) = P_t(i)/P_t$ is the firm relative price at date t, $TC(.)$ is the cost function, ξ_t is a vector containing all the exogenous variables affecting profits, and $y_t(i)$ the demand curve for the good i which, according to the households' expenditures minimization problem, is given by:

$$y_t(i) = \frac{Y_t}{b_t(i)} g\left(\frac{P_t(i)Y_t}{\lambda_t b_t(i)}\right) \tag{3.2.13}$$

where Y_t denotes aggregate demand.

It should be noted here that the assumptions we have made so far are not enough to guarantee that the profit function attains its maximum at some interior point. Therefore, in order to make the analysis of the first order conditions meaningful, hereafter we restrict the space of admissible functions by considering only functional forms for $\psi(x_t(i))$ and $b_t(i)$ that satisfy the following condition:

Assumption 5. *The profit $\pi(p_t(i), Z_t(i), \xi_t)$ is a single-picked function of its first two arguments, with a maximum at some positive vector $(p_t(i), Z_t(i))$ for any values of its other arguments.*

Given (3.2.12), the second step of the firms' optimization problem therefore consists of the following program:

$$\max \sum_{k=t}^{\infty} E_t \left\{ Q^r_{t,t+k} \left[P_{t+k}(i)y_{t+k}(i) - TC(Z_{t+k}(i), y_{t+k}(i), \xi_{t+k}) \right] \right\}$$

s.t.

$$\begin{aligned}
y_{t+k}(i) &= \frac{Y_{t+k}}{b_{t+k}(i)} g\left(\frac{p_{t+k}(i)Y_{t+k}}{\lambda_{t+k} b_{t+k}(i)}\right) \\
b_{t+k}(i) &= \nu\left(\frac{Z_{t+k}(i)}{Z_{t+k}}\right) \\
p_{t+k}(i) &= \begin{cases} p_{t+k-1}(i)\Pi^{-1}_{t+k} & \text{with probability } \theta \\ p^*_{t+k}(i) & \text{with probability } (1-\theta) \end{cases}
\end{aligned}$$

where $Q^r_{t,t+k} = Q_{t,t+k}\Pi_{t+k}$ is the real stochastic discount factor[9], $\Pi_{t+k} = P_{t+k}/P_{t+k-1}$ is the inflation rate at date $t+k$, and $p^*_{t+k}(i)$ denotes the optimal relative price chosen by a resetting firm at date $t+k$. The first-order condition with respect to non-price activities is given by:

$$\frac{\partial y_t(i)}{\partial Z_t(i)} p_t(i) = \varphi_t(i) + \frac{\partial y_t(i)}{\partial Z_t(i)} \eta_t(i) \tag{3.2.14}$$

[9]Under the maintaining assumptions of perfect financial market and households that holds the ownership of the firms, the stochastic discount factor is defined as:

$$Q^r_{t,t+k} = \beta^k \frac{\Lambda_{t+k}}{\Lambda_t};$$

where β is the subjective discount factor and Λ_{t+k} is the marginal evaluation of consumption at date $t+k$.

Accordingly, in order to maximize profit, a firm chooses the non-price competition budget such that the increase in revenues resulting from an additional unit of non-price expenditures is equal to the corresponding increase in total cost. This equation implies an optimal rule for non-price activities in terms of its ratio to total sales. To see this, notice that differentiating (3.2.13) with respect to $Z_t(i)$, plugging the resulting equation into (3.2.14) to substitute out the derivative of demand with respect to non-price activities, and rearranging the terms yields:

$$\frac{Z_t(i)}{y_t(i)} = \varepsilon_{z,t}(i)\left(\frac{p_t(i) - \eta_t(i)}{\varphi_t(i)}\right) \tag{3.2.15}$$

where $\varepsilon_{z,t}(i)$ denotes the elasticity of the demand function with respect to spending for non-price tools, which in turn is given by:

$$\varepsilon_{z,t}(i) = (\varepsilon_{p,t}(i) - 1)\,\varepsilon_{b,z}(i) \tag{3.2.16}$$

For any firm i, the intensity of non-price competition is therefore proportional to the ratio of average net revenues from sales to non-price marginal cost, where the factor of proportionality is given by the elasticity of demand with respect to non-price tools, $\varepsilon_{z,t}(i)$. Interesting, this equation reveals that firms' pricing and non-pricing policies are in fact directly related. According to (3.2.15), a firm will find it convenient to increase its non-price budget in response to an increase in the unit net revenue from sales caused by a higher relative price. Using the terminology of Iwasaki *et al.* (2008), this means that in our framework, firms play a super-modular game, since their pricing and non-pricing policies are complementary strategies.[10]

The first-order condition for price-setting is given by

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{Q^r_{t,t+k} y_{t+k}(i)}{p_{t+k}(i)} (\varepsilon_{p,t+k}(i) - 1) \left[p_{t+k}(i) - \mu_{t+k}(i)\eta_{t+k}(i)\right] \right\} = 0 \tag{3.2.17}$$

where $\mu_{t+k}(i)$ denotes the firm's desired markup,[11] which is given as follows:

$$\mu_{t+k}(i) = \frac{\varepsilon_{p,t+k}(i)}{\varepsilon_{p,t+k}(i) - 1} \tag{3.2.18}$$

The presence of a demand featuring a non-constant price elasticity modifies the optimal choice of a price setter along a crucial dimension; the desired markup becomes an increasing function of the firm's relative output. As it is well-known (see Woodford (2003)), for a re-optimizing firm, this mechanism increases the cost of deviating from the prices charged by others, making firms more reluctant to change their prices when this action is allowed. As we will see next, in the aggregate this mechanism reduces the sensitivity of inflation to marginal costs, leading to a flattened Phillips curve. However, this result is obtained under the crucial assumption that firms compete for the market with no instruments but their relative output prices. As we have seen before, extending the Kimball framework to an environment with non-price competition implies that the demand price elasticity is by itself a decreasing function of non-price activities, and the firm's desired mark-up is thus increasing in these tools. This clearly affects the firms' pricing behavior, since it modifies the opportunity cost of changing relative prices. However, the extent to which this mechanism affects the inflation dynamics depends, in turn, on the kind of relation existing between pricing and non-pricing policies. This is what is analyzed in the next section.

[10]Super-modular games are a general class of noncooperative games where n players simultaneously choose a set of strategies. See Milgrom and Roberts (1990) for further details. Iwasaki *el al.* (2008) discuss the general property of advertising that unequivocally leads to a supermodular game in the context of an oligopolistic market in which firms simultaneously choose their advertising budgets and pricing policy.

[11]That is the optimal mark-up a firm would have chosen in the context of flexible prices.

3.2.3 The New Keynesian Phillips Curve

To derive an expression for the New Keynesian Phillips curve in the context of non-price competition, we restrict the attention to a linear approximation of the equilibrium dynamics around a zero-inflation steady-state. Hereafter, a hat on a variable denotes the log deviation of the original variable with respect to its steady state, while a variable evaluated at the steady-state is indicated by suppressing indexes i and t from the original variable.

Log-linearizing the first-order condition for the optimal price chosen by a resetting firm yields:[12]

$$\Gamma \sum_{k=0}^{\infty} (\theta\beta)^k E_t \tilde{p}_{t+k}(i) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \hat{\eta}_{t+k} + (\varepsilon_p \varepsilon_\mu + (\varepsilon_p - 1) s_y) \sum_{k=0}^{\infty} (\theta\beta)^k \varepsilon_{b,z} E_t \tilde{z}_{t+k}(i) \qquad (3.2.19)$$

where $\varepsilon_\mu > 0$ is the elasticity of the desire markup function with respect to the firm market share,[13] $\beta \in (0,1)$ denotes the long-run stochastic discount factor, $s_y > 0$ is the steady-state elasticity of the production marginal cost with respect to firms' relative output, $\Gamma = [1 + \varepsilon_p (\varepsilon_\mu + s_y)]$, $\tilde{p}_{t+k}(i) = \hat{p}_t^*(i) - \hat{P}_{t+k}$ and $\tilde{z}_{t+k}(i) = \hat{Z}_{t+k}(i) - \hat{Z}_{t+k}$.

The standard case without non-price competition can be recovered from the previous equation by setting $\varepsilon_{b,z}$ equal to zero. In such a case, with standard manipulation, one can obtain the following familiar form of inflation dynamics as a function of the expected future inflation and aggregate marginal cost:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \zeta \hat{\eta}_t \qquad (3.2.20)$$

where ζ is defined as:

$$\zeta = \frac{(1-\beta\theta)(1-\theta)}{\theta} \left(\frac{1}{1 + \varepsilon_p (\varepsilon_\mu + s_y)} \right) \qquad (3.2.21)$$

Equation (3.2.21) formalizes the effect of real rigidities in price-setting on inflation dynamics. The slope ζ becomes the product of two terms, where the first corresponds to the slope we would obtain in the standard Dixit-Stiglitz case (i.e., $\varepsilon_\mu = 0$) with a constant return to scale ($s_y = 0$), while the second is known as the strategic complementarity term. Hence, in our setup there are two channels through which real rigidities affect the inflation-marginal cost relation: (i) the sensitivity of the firm's desired markup to relative prices, ε_μ; (ii) the sensitivity of marginal cost to the firm's relative output, s_y. In both cases, the effect is stronger with increases in demand price elasticity, ε_p.

Returning to our point, to precisely evaluate how the presence of non-price competition affects inflation dynamics, we first need to model the evolution of the firm i's relative expenditures for non-price activities. In fact, equation (3.2.19) involves the latter at a sequence of future dates, which in turn depends upon the optimal policy (3.2.14). Therefore, using the theory we have developed in the previous section, we find a result of the following form.

Proposition 2. *Let the households' consumption aggregate be of the form (3.2.1), and suppose that the model economy has the structure described by assumptions 3 and 4. Then, for any functions $\psi(x)$ and $b_t(i)$ satisfying assumptions 1, 2 and 5, the New Keynesian Phillips curve has the*

[12] See the Appendix for further details on the derivation.

[13] More precisely, $\varepsilon_\mu(i)$ denotes the elasticity of the markup function with respect to $x_t(i)$. However, denoting with $\varepsilon_{\mu,m_{y,t}}(i)$ the elasticity with respect to firm i's market share, the following equality holds:

$$\varepsilon_{\mu,m_{y,t}}(i) = \varepsilon_\mu(i) b_t(i)$$

so that at the steady-state the two expressions coincide.

representation:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \zeta_{np}\,\hat{\eta}_t \tag{3.2.22}$$

with:

$$\zeta_{np} = \frac{(1-\beta\theta)(1-\theta)}{\theta}\left\{\frac{1}{1+\varepsilon_p\left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right]}\right\} \tag{3.2.23}$$

where the coefficients ϕ_0 and ϕ_1 are strictly positive functions of the model structural parameters, such that $\phi_0 > \phi_1$.

In addition, the slope ζ_{np} possess the following properties:

(i) $\zeta_{np} > \zeta$, where ζ is defined as in (3.2.21).

(ii) $\frac{\partial \zeta_{np}}{\partial \varepsilon_\mu} < 0$. Moreover:

$$\zeta_{np} > \frac{(1-\theta\beta)(1-\theta)}{\theta}\left\{\frac{\varepsilon_{b,z}^2\varepsilon_p}{-v''(1)+\varepsilon_{b,z}\left[s_y\varepsilon_{b,z}+s_z+2\varepsilon_{b,z}\right]}\right\} > 0\ \forall\ \varepsilon_\mu \in (\varepsilon_\mu^*, \infty)$$

where $\varepsilon_\mu^* = \inf_{\varepsilon_\mu} \mathcal{B}$, with $\mathcal{B} \subseteq \mathbb{R}_+$ is a set of real numbers defined as in the appendix.[14]

Proof. See the appendix. □

Several remarks are in order. First of all, the proposition confirms that for a firm, pricing and non-pricing policies are strategic complements.[15] As shown in the Appendix, the firm i's relative expenditures on non-price activities can be expressed as a function of its own relative price as:

$$\tilde{z}_t(i) = \phi_z \tilde{p}_t(i) \tag{3.2.24}$$

where ϕ_z is a strictly positive function of the model's structural parameters.[16] For any firm i, therefore, relative spending in non-price activity accommodates movements in relative prices. The elasticity ϕ_z measures in particular the extent to which a firm manages its non-price policy in order to offset real rigidities in price-settings.[17] In fact, it is possible to show that without assuming firm-specific marginal costs and quasi-kinked demand functions, the coefficient ϕ_z is equal to zero, and, up to a order linear approximation, the firms' expenditures for non-price competition are independent of their relative prices.[18]

One way to think about this is that strategic complementarities strengthen firms' incentives to engage in non-price competition. To see this, suppose that a price setter deviates from the flexible price equilibrium by charging a higher price. This immediately increases the demand price elasticity, thus amplifying the loss of customers resulting from the flatter demand schedule. At the

[14] Here, the coefficient s_z denotes the long-run elasticity of non-price average marginal cost, while v"(1) denotes the second derivative of $b_t(i)$ evaluated at the steady-state.

[15] Given that $\varepsilon_{b,z} > 0$, this follows by combining properties (i) of proposition with equation (3.2.19).

[16] See the appendix for further details

[17] It is interesting to note that the elasticity ϕ_z can be decomposed as follows:

$$\phi_z = \phi_\varepsilon + \phi_{sy}$$

The coefficient ϕ_ε measures the extent to which the optimal investment in non-price tools moves in order to compensate for the impact of the relative prices on the demand price elasticity. The coefficient ϕ_{sy} instead relates movements in non-price activities to variation in the marginal cost for producing consumption goods.

[18] In such a case, fluctuations in expenditures for non-price tools are only due to aggregate shocks affecting the opportunity cost and benefits of investing in this tools.

same time, because of the lower market share, marginal cost decreases, and the production of one unit of consumption goods becomes less expensive. In such a circumstance, the firm's incentive to increase its relative expenditures on non-price tools strengthens. On the one hand, because of the negative effect on the demand price elasticity, by increasing the relative expenditures on non-price tools, a firm may partially offset the loss of customers caused by the higher relative price. On the other hand, the lower marginal cost for producing goods increases the marginal revenue of non-price activities, therefore making the latter more profitable.

As a second result, we note that in our framework, it is again possible to derive an expression that relates, as in the canonical model, the inflation rate to the average marginal cost and the expected future inflation.[19] However, all else being equal, if firms engage in non-price competition, we find that the sensitivity of the inflation rate to marginal cost increases.[20] Hence, by amplifying the overall response of inflation to output, non-price competition among firms does affect inflation dynamics.

To get an intuition for this result, notice that the interaction between firms' pricing behavior and non-price policy generates a mechanism that dampens the degree of strategic complementarities in price-setting. In fact, by comparing the slope coefficient ζ_{np} with the one we obtain in the Kimball model, we note that while the nominal rigidities term is exactly the same, the real rigidities term is instead larger in our framework. This is an immediate implication of non-price policy being strategically complementary to the firm's pricing behavior. In fact, by accommodating the pricing policy with non-price tools, a firm essentially mitigates the effect upon its market share of movements in its relative price. This follows by noticing that, for each variety i, the producer's market share is given by:

$$\hat{y}_t(i) - \hat{Y}_t = (-\varepsilon_p + \varepsilon_z \phi_z)\tilde{p}_t(i) \tag{3.2.25}$$

Accordingly, with non-price competition, the firm's relative output becomes less sensitive to movements in its relative price.[21] As a consequence, the firms' desired markup and its marginal cost for producing goods react less to movements in relative output price, making the profit function less steep around the point at which everybody charges the same price. Hence, the firms' opportunity cost to move their relative output prices, when this is allowed, declines, thus amplifying the size of price changes. As a result, if firms engage in non-price competition, inflation becomes more sensitive to movements in the real marginal cost.

Third, although the economic environment is characterized by real rigidities, we note that in our framework, the price-setting decision of different producers might become strategic substitutes. This feature is particularly striking, since in the Kimball model this is only possible by assuming that the price elasticity of demand decreases in relative prices. To see this, take for simplicity the case in which the non-constant elasticity of demand is the only source of real rigidity, and notice that the firms' desired markup reads as:

$$\hat{\mu}_t(i) = \varepsilon_\mu \varepsilon_p [\varepsilon_{b,z}\phi_z - 1]\tilde{p}_t(i) \tag{3.2.26}$$

Strategic complementarity in price-setting implies that the elasticity of the firm's desired markup with respect to its relative output price is negative. Equation (3.2.26) shows that this requirement is not obviously satisfied if firms engage in non-price competition. In fact, the factor $\varepsilon_{b,z}\phi_z$ does not necessarily need to be lower than one. As such, producers may face strategic complementarity in price-setting, despite the fact that we have assumed that the elasticity of demand is

[19]The New Keynesian Phillips Curve with non-price competition and the standard one are therefore observationally equivalent.

[20]See property (i) of the proposition.

[21]This is true since the factor $\varepsilon_z \phi_z$ is positive.

increasing in the firms' relative prices. In such a circumstance, the dampening effect of non-price competition would completely offset real rigidity in price-setting, thereby leading to an inflation-marginal cost coefficient even larger than the Dixit-Stiglitz one.

Fourth, the slope ζ_{np} is a bounded and decreasing function of the elasticity ε_μ.[22] Unlike the Kimball model, however, in our framework, the lower bound is strictly positive. This property has two important and related implications that contrast with the canonical framework : (i) the overall impact of quasi-kinked demand functions on the persistence of the real effects of monetary policy shocks is bounded; and (ii) it is not obvious whether, for plausible calibrations of all the remaining parameters, it is still possible to pin down a realistic value for the slope coefficient ζ_{np} by adjusting accordingly the elasticity ε_μ.

Oppositely,[23] assuming Kimball preferences in Calvo-style models without non-price competition, by allowing for a non-constant elasticity of demand, generates a kind of strategic complementary that instead allows one to get any desired level of persistence in the real effect of monetary policy shock and, for any given estimate of the Phillips curve's slope, to pin down a more plausible degree of nominal rigidity. This feature is apparent from equation (3.2.21), which shows that the slope coefficient ζ is strictly decreasing in the elasticity of the markup function, ε_μ, converging to zero as the latter goes to infinity. As such, for any given value of the Calvo probability θ, it is always possible to pin down ε_μ in order to set the slope coefficient ζ to any desired level.

This property does not hold true in our framework, since with non-price competition, an increase in the elasticity ε_μ generates two different effects: (i) as in standard models, each price-setter faces a higher opportunity cost of changing price, since demand becomes more steep around each firm's normal market share; and (ii) for each firm, an additional unit of relative spending toward non-price competition becomes more effective in lowering the demand price elasticity. In fact, in the equilibrium in which everybody charges the same price, the following relation holds:

$$\left.\frac{\partial \varepsilon_p(i)}{\partial \tilde{z}_t(i)}\right|_{x_t(i)=1} = -\varepsilon_\mu \varepsilon_p^2 \varepsilon_z \tag{3.2.27}$$

which shows that the effect of non-price activities on the elasticity of demand is stronger as ε_μ increases.[24] The second effect, missing in canonical models, offsets the first one, thereby attenuating the overall impact upon the inflation-marginal cost coefficient of higher values of ε_μ.

As a final remark, we note that the coefficient ϕ_0 is greater than ϕ_1 for any admissible vector of the model's structural parameters. This means that the dampening effect of non-price competition affects more real rigidity coming from quasi-kinked demand functions than that caused by decreasing returns to scale. In fact, as is apparent from equation (3.2.23), the two coefficients ϕ_0 and ϕ_1 respectively capture the partial effect of non-price competition on the inflation-marginal cost relationship through the real rigidity components ε_μ and s_y.[25]

[22] This follows from property (ii) of the proposition.

[23] See for instance Woodford (2003).

[24] Equation (3.2.27) is derived as follows. First, evaluating equation (3.2.9) at the steady state implies:

$$\left.\frac{\partial \varepsilon_p(i)}{\partial \tilde{z}_t(i)}\right|_{x_t(i)=1} = \left.\frac{\partial \varepsilon_p(i)}{\partial x_t(i)}\right|_{x_t(i)=1} \varepsilon_p \varepsilon_{b,z}$$

Second, making use of the definition of ε_μ, we find:

$$\left.\frac{\partial \varepsilon_p(i)}{\partial x_t(i)}\right|_{x_t(i)=1} = -\varepsilon_\mu(\varepsilon_p - 1)\varepsilon_p$$

Finally, combining together these two expressions and using (3.2.16) in the resulting equation yields (3.2.27).

[25] To see this compare equation (3.2.21) with equation (3.2.23).

3.3 Conclusion

This paper has analyzed the implications of non-price competition among firms in a Calvo-style model characterized by strategic complementarity in price-setting. Under very general assumptions, it has been shown that any activity by firms that expands their customer bases without directly affecting their prices dampens at the same time the degree of real rigidity in price-setting. This provides an answer to the main question of this paper: by increasing the inflation-marginal cost coefficient, non-price competition among firms does affect inflation dynamics.

Among other things, the theoretical results provided in this paper highlight the fact that non-price competition generates a mechanism that dampens the overall impact of real rigidities on the inflation-marginal cost coefficient. This effect could be strong enough to completely offset real rigidity in price-setting. In such an extreme case, although the model economy is characterized by quasi-kinked demand functions and local factors market, one would find an inflation marginal cost coefficient even larger than the Dixit-Stiglitz one. This suggests that abstracting from non-price competition, as canonical models do, may potentially overstate the overall impact of real rigidities on inflation dynamics. This issue is particularly important, as real rigidities have became popular among New Keynesian theorists precisely because they provide a mechanism to amplify the effect of nominal disturbances and, all else being equal, to reduce the size of the Phillips curve's slope. Our analysis casts some doubt regarding the robustness of such conclusions, at least with respect to alternative specifications of the firms' competition. However, the overall impact of non-price competition on inflation dynamics depends upon different mechanisms whose relative strength in turn depends on the model's parametrization. Addressing this issue therefore requires the specification and calibration of a full-fledged model through which it could be possible to evaluate the implication for different sources of real rigidity. This issue goes far beyond the scope of this paper, and we have therefore left it for future research.

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3.4 Appendix

Proof of proposition 2

I) The New Keynesian Phillips curve

We will first show that in our framework it is possible to derive a New Keynesian Phillips curve of the form

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \zeta_{np}\hat{\eta}_t$$

where

$$\zeta_{np} = \frac{(1-\beta\theta)(1-\theta)}{\theta}\left\{\frac{1}{1+\varepsilon_p\left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right]}\right\}$$

Step 1. Deriving equation (3.2.19)

Let us start by noticing that in the non-stochastic steady state with zero inflation the following conditions hold:

$$\eta_s = \frac{1}{\mu_s} \equiv \frac{(\varepsilon_p - 1)}{\varepsilon_p}$$

$$Q^r_{s,s+k} = \beta^k$$

where η_s and μ_s are respectively the long-run average production marginal cost and average mark-up, while $Q^r_{s,s+k}$ is the stochastic discount factor evaluated at the steady state. Plugging these relationships into the log-linearized first order condition for price-setters yields:

$$\sum_{k=0}^{\infty}(\theta\beta)^k E_t\tilde{p}_{t+k}(i) = \sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{\eta}_{t+k}(i) + \sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{\mu}_{t+k}(i) \tag{3.4.1}$$

Our next goal is to find an expression for both the firm's desired mark-up, $\hat{\mu}_t(i)$, and the firm's production marginal cost, $\hat{\eta}_t(i)$. Using equation (3.2.18), one can easily prove the following:

$$\hat{\mu}_t(i) = -\frac{\hat{\varepsilon}_{p,t}(i)}{(\varepsilon_p - 1)}$$

where $\hat{\varepsilon}_{p,t}(i)$ is the log-linearized elasticity of demand. The latter is given by:

$$\begin{aligned}
\hat{\varepsilon}_{p,t}(i) &= \left.\frac{\partial\varepsilon_{p,t}(i)}{\partial x_t(i)}\right|_{x_t(i)=1} \frac{\left[\left(\hat{y}_t(i) - \hat{Y}_t\right) + \varepsilon_{b,z}\tilde{z}_t(i)\right]}{\varepsilon_p} \\
&= -\varepsilon_\mu(\varepsilon_p - 1)\left[\left(\hat{y}_t(i) - \hat{Y}_t\right) + \varepsilon_{b,z}\tilde{z}_t(i)\right]
\end{aligned}$$

where the last equality follows by using the definition of ε_μ to substitute out the derivative $\partial\varepsilon_{p,t}(i)/\partial x_t(i)$. An expression for the firm i's log-linearized market share, $\hat{y}_t(i) - \hat{Y}_t$, can be derived from (3.2.3) as:

$$\hat{y}_t(i) - \hat{Y}_t = -\varepsilon_p\tilde{p}_t(i) + \varepsilon_{b,z}(\varepsilon_p - 1)\tilde{z}_t(i) \tag{3.4.2}$$

Therefore:

$$\hat{\varepsilon}_{p,t}(i) = -\varepsilon_\mu(\varepsilon_p - 1)\varepsilon_p\left(\varepsilon_{b,z}\tilde{z}_t(i) - \tilde{p}_t(i)\right) \tag{3.4.3}$$

$$\hat{\mu}_t(i) = \varepsilon_\mu\varepsilon_p\left(\varepsilon_{b,z}\tilde{z}_t(i) - \tilde{p}_t(i)\right) \tag{3.4.4}$$

Next, in order to find an expression for the firm specific production marginal cost, it is convenient to rewrite equation (3.2.10) as follows:

$$\eta_t(i) = \frac{w_t}{A_t}\Phi_y\left(\frac{y_t(i)}{A_t}\right) \tag{3.4.5}$$

where

$$\Phi_y\left(\frac{y_t(i)}{A_t}\right) = \frac{1}{f'_y\left(f_y^{-1}\left(\frac{y_t(i)}{A_t}\right)\right)}$$

This formulation allows to express the elasticity of production marginal cost with respect to the firm's output, $s_{y,t}(i)$ as follows:

$$s_{y,t}(i) = \left[\frac{\Phi'_y\left(\frac{y_t(i)}{A_t}\right)}{A_t\Phi_y\left(\frac{y_t(i)}{A_t}\right)}\right] y_t(i) \tag{3.4.6}$$

while the average marginal production cost can be defined as:

$$\eta_t = \frac{w_t}{A_t}\int_0^1 \Phi_y\left(\frac{y_t(i)}{A_t}\right) di \tag{3.4.7}$$

Log-linearizing equations (3.4.5) and (3.4.7) and solving for $\hat{\eta}_t(i)$ yields:

$$\hat{\eta}_t(i) = \hat{\eta}_t + s_y\left(\hat{y}_t(i) - \hat{Y}_t\right)$$

Therefore, using (3.4.2) we can express the firm's specific production marginal cost as:

$$\hat{\eta}_t(i) = \hat{\eta}_t + s_y\left[(\varepsilon_p - 1)\varepsilon_{b,z}\tilde{z}_t(i) - \varepsilon_p\tilde{p}_t(i)\right] \tag{3.4.8}$$

Finally, substituting equations (3.4.4) and (3.4.8) into (3.4.1) and rearranging the resulting equation yields:

$$\Gamma\sum_{k=0}^{\infty}(\theta\beta)^k E_t\tilde{p}_{t+k}(i) = \sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{\eta}_{t+k} + (\varepsilon_p\varepsilon_\mu + (\varepsilon_p - 1)s_y)\sum_{k=0}^{\infty}(\theta\beta)^k \varepsilon_{b,z}E_t\tilde{z}_{t+k}(i) \tag{3.4.9}$$

where $\Gamma = [1 + \varepsilon_p(\varepsilon_\mu + s_y)]$.

Step 2. Finding an expression for $\tilde{\mathbf{z}}_{\mathbf{t}}(\mathbf{i})$

To model the evolution of the relative spending for non-price competition it is convenient to substitute (3.2.6) into (3.2.14) in order to rewrite the optimal policy for non-price tools as follows:

$$\frac{y_t(i)}{v(z_t(i))}(\varepsilon_{p,t}(i) - 1)v'(z_t(i))(p_t(i) - \eta_t(i)) = \varphi_t(i)$$

By log-linearizing this expression, after a bit of algebra, one obtains:

$$\varepsilon_p^2\varepsilon_\mu\tilde{p}_t(i) - \left[\frac{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - v''(1) + \varepsilon_{b,z}^2(2 - \varepsilon_p)}{\varepsilon_{b,z}}\right]\tilde{z}_t(i) - (\varepsilon_p - 1)\hat{\eta}_t(i) = \hat{\varphi}_t(i)$$

From this expression take the average across all the firm and subtract the resulting equation to find:

$$\varepsilon_p^2\varepsilon_\mu\tilde{p}_t(i) - \left[\frac{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - v''(1) + \varepsilon_{b,z}^2(2 - \varepsilon_p)}{\varepsilon_{b,z}}\right]\tilde{z}_t(i) - (\varepsilon_p - 1)(\hat{\eta}_t(i) - \hat{\eta}_t) = (\hat{\varphi}_t(i) - \hat{\varphi}_t)$$

which, by using equation (3.4.8) to work out $(\hat{\eta}_t(i) - \hat{\eta}_t)$, can be equivalently rewritten as follows:

$$\left[\varepsilon_p^2\varepsilon_\mu + \varepsilon_p(\varepsilon_p - 1)s_y\right]\tilde{p}_t(i) - \left[\frac{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - v''(1) + \varepsilon_{b,z}^2(2 - \varepsilon_p) + \varepsilon_{b,z}^2(\varepsilon_p - 1)^2 s_y}{\varepsilon_{b,z}}\right]\tilde{z}_t(i) = (\hat{\varphi}_t(i) - \hat{\varphi}_t) \tag{3.4.10}$$

As we have done for the marginal production cost, we can use (3.2.11) to express $\varphi_t(i)$ as:

$$\varphi_t(i) = \frac{w_t}{A_t}\Phi_z\left(\frac{Z_t(i)}{A_t}\right) \tag{3.4.11}$$

so that the elasticity $s_{t,z}(i)$ and the average marginal cost for producing non-price competition tools, φ_t are respectively given by:

$$s_{z,t}(i) = \left[\frac{\Phi_z'\left(\frac{Z_t(i)}{A_t}\right)}{A_t\Phi_z\left(\frac{Z_t(i)}{A_t}\right)}\right] Z_t(i) \tag{3.4.12}$$

$$\varphi_t = \frac{w_t}{A_t}\int_0^1 \Phi_z\left(\frac{Z_t(i)}{A_t}\right) di \tag{3.4.13}$$

Therefore, log-linearizing equations (3.4.11) and (3.4.13) to obtain an expression for $(\hat{\varphi}_t(i) - \hat{\varphi}_t)$, substituting the resulting equation into (3.4.10) and solving for $\tilde{z}_t(i)$, yields:

$$\tilde{z}_t(i) = \phi_z\tilde{p}_t(i) \tag{3.4.14}$$

where

$$\phi_z = \frac{\varepsilon_{b,z}\left[\varepsilon_p^2\varepsilon_\mu + \varepsilon_p(\varepsilon_p - 1)s_y\right]}{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - \upsilon''(1) + \varepsilon_{b,z}^2(2-\varepsilon_p) + \varepsilon_{b,z}s_z + \varepsilon_{b,z}^2(\varepsilon_p - 1)^2 s_y}$$

Notice that, using (3.4.14), we can rewrite the firm i's market share and desired markup as follows:

$$\hat{y}_t(i) - \hat{Y}_t = (-\varepsilon_p + \varepsilon_z\phi_z)\,\tilde{p}_t(i)$$

$$\hat{\mu}_t(i) = \varepsilon_\mu\varepsilon_p\,(\varepsilon_{b,z}\phi_z - 1)\,\tilde{p}_t(i)$$

Step 3. Deriving the New Keynesian Phillips Curve

Equation (3.4.14) implies that

$$E_t\tilde{z}_{t+k}(i) = \phi_z E_t\tilde{p}_{t+k}(i)\ \forall\ k\ >\ 0$$

Substituting this equality into (3.4.9) yields:

$$\begin{aligned}
\Gamma\sum_{k=0}^{\infty}(\theta\beta)^k E_t\tilde{p}_{t+k}(i) &= \sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{\eta}_{t+k} + (\varepsilon_p\varepsilon_\mu + (\varepsilon_p - 1)s_y)\sum_{k=0}^{\infty}(\theta\beta)^k \phi_z\varepsilon_{b,z}E_t\tilde{p}_{t+k}(i) \\
&\equiv \sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{\eta}_{t+k} + \varepsilon_p\left(\varepsilon_\mu\phi_0 + s_y\phi_1\right)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\tilde{p}_{t+k}(i)
\end{aligned}$$

where the two coefficients ϕ_0 and ϕ_1 are respectively given by:

$$\phi_0 = \frac{\varepsilon_{b,z}^2\varepsilon_\mu\varepsilon_p^2 + \varepsilon_{b,z}^2\varepsilon_p s_y(\varepsilon_p - 1)}{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - \nu''(1) + \varepsilon_{b,z}^2(2-\varepsilon_p) + \varepsilon_{b,z}s_z + \varepsilon_{b,z}^2(\varepsilon_p - 1)^2 s_y} \tag{3.4.15}$$

$$\phi_1 = \frac{\varepsilon_{b,z}^2(\varepsilon_p - 1)\varepsilon_\mu\varepsilon_p + \varepsilon_{b,z}^2 s_y(\varepsilon_p - 1)^2}{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - \nu''(1) + \varepsilon_{b,z}^2(2-\varepsilon_p) + \varepsilon_{b,z}s_z + \varepsilon_{b,z}^2(\varepsilon_p - 1)^2 s_y} \tag{3.4.16}$$

Therefore, we get:

$$\{1 + \varepsilon_p\left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right]\}\sum_{k=0}^{\infty}(\theta\beta)^k E_t\tilde{p}_{t+k}(i) = \sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{\eta}_{t+k}$$

As in the canonical models, this equation relates the price chosen at time t to only the future expected path of average production marginal cost.[26] Therefore, by using standard manipulations,[27] we finally get the following inflation equation:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \zeta_{np} \hat{\eta}_t$$

where

$$\zeta_{np} = \frac{(1-\beta\theta)(1-\theta)}{\theta} \left\{ \frac{1}{1+\varepsilon_p \left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right]} \right\}$$

II) Proving property (i)

Our next goal is to show that if the model economy satisfies assumptions 1-5 then

$$\zeta_{np} > \zeta$$

where ζ is defined as in equation (3.2.21).
Proof: By comparing equation (3.2.21) with (3.2.23), one can easily show that this statement is equivalent of proving that for any model economy satisfying assumptions 1-5, it must be true that:

$$0 < 1 + \varepsilon_p \left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right] < 1 + \varepsilon_p \left[\varepsilon_\mu + s_y\right]$$

In virtue of (3.4.15) and (3.4.16), this condition depends only upon a subset of all the model structural parameters, that is the vector $\vartheta = [\varepsilon_p, \varepsilon_{b,z}, s_z, \varepsilon_\mu, s_y, v''(1)]$. Therefore, denoting with $\mathfrak{F}$ the set of all the vectors ϑ that satisfy assumption 1-5, the statement can be proved by showing that:

$$0 < 1 + \varepsilon_p \left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right] < 1 + \varepsilon_p \left[\varepsilon_\mu + s_y\right] \ \forall \, \vartheta \in \mathfrak{F}$$

Our first issue is to characterize the set $\mathfrak{F}$. To this end, notice that each entry of ϑ is a long run elasticity that is obtained by evaluating the corresponding function at the non-stochastic steady-state with zero inflation. The latter is a symmetric equilibrium in which every producer charges the same price ($p_t(i) = 1$) and incurs in the same expenditures for non-price competition ($z_t(i) = 1$); in other words such that:

$$x_t(i) = 1 \, \forall \, i \in [0,1]$$

Therefore, we can explicitly characterize the set $\mathfrak{F}$ by deriving all the restrictions that assumptions 1-5 impose on the vector of parameters ϑ by focusing only on this symmetric equilibrium with zero inflation. To this end, notice first that since $\varepsilon_p > 1$ and assumptions 1-4 imply both $[\varepsilon_{b,z}, s_z, \varepsilon_\mu, s_y] \in \mathbb{R}^4_+$ and $v''(1) < 0$, we must have:

$$\mathfrak{F} \subseteq (1,\infty) \times \mathbb{R}^4_+ \times (-\infty, 0)$$

Assumption 5, in turn, imposes the existence of a unique global maximum for the profit function (3.2.12). Since the latter is twice continuously differentiable, this condition can be equivalently stated by requiring that the Hessian matrix evaluated at a maximization point is negative definite. More precisely, denoting with $\pi(x_t(i))$ the firm i's profit function and with $H_\pi(x_t(i))$ the associated Hessian matrix, that is:

$$H_\pi(x_t(i)) = \begin{bmatrix} \frac{\partial^2 \pi(x_t(i))}{\partial^2 Z_t(i)} & \frac{\partial^2 \pi(x_t(i))}{\partial Z_t(i) \partial p_t(i)} \\ \\ \frac{\partial^2 \pi(x_t(i))}{\partial p_t(i) \partial Z_t(i)} & \frac{\partial^2 \pi(x_t(i))}{\partial^2 p_t(i)} \end{bmatrix}$$

we must have:

- both $\left.\frac{\partial^2 \pi(x_t(i))}{\partial^2 p_t(i)}\right|_{x_t(i)=1} < 0$ and $\left.\frac{\partial^2 \pi(x_t(i))}{\partial^2 Z_t(i)}\right|_{x_t(i)=1} < 0$
- $det(H_\pi(1)) > 0$

[26] The only difference with the standard Kimball model is in fact represented by the term $\{1 + \varepsilon_p \left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right]\}$, since in our framework it also involves the two coefficients ϕ_0 and ϕ_1.
[27] See Galí (2008) for a formal derivation.

Let us start by evaluating $\partial^2\pi(x_t(i)) \,/\, \partial^2 p_t(i)$. In doing this, we make use of the followings relationships:

$$\frac{\partial y_t(i)}{\partial p_t(i)} = -\frac{\varepsilon_{p,t}(i)y_t(i)}{p_t(i)}$$

$$\frac{\partial \varepsilon_{p,t}(i)}{\partial p_t(i)} = \varepsilon'_{p,t}(i)\frac{b_t(i)}{Y_t}\frac{\partial y_t(i)}{\partial p_t(i)}$$

$$\frac{s_{y,t}(i)}{y_t(i)} = \left[\frac{\Phi'_y\left(\frac{y_t(i)}{A_t}\right)}{A_t\Phi_y\left(\frac{y_t(i)}{A_t}\right)}\right]$$

Therefore, by double differentiating the profit function (3.2.12) with respect to the relative price $p_t(i)$ we find:

$$\frac{\partial^2\pi(x_t(i))}{\partial^2 p_t(i)} = \frac{\partial y_t(i)}{\partial p_t(i)}\left[1-\varepsilon_{p,t}(i)+\frac{s_{y,t}(i)\varepsilon_{p,t}(i)}{p_t(i)}\eta_t(i)+\frac{\eta_t(i)}{p_t(i)}\left(1+\varepsilon_{p,t}(i)\right)-\varepsilon'_{p,t}(i)x_t(i)\left(1-\frac{\eta_t(i)}{p_t(i)}\right)\right]$$

Thus, denoting with Y_s the aggregate level of output, in the steady-state equilibrium it must be true that

$$\begin{aligned}\left.\frac{\partial^2\pi(x_t(i))}{\partial^2 p_t(i)}\right|_{x_t(i)=1} &= -\varepsilon_p Y_s\left[s_y(\varepsilon_p-1)+\frac{\varepsilon_p-1}{\varepsilon_p}-\frac{\varepsilon'_p}{\varepsilon_p}\right]\\ &= -\varepsilon_p Y_s\left[s_y(\varepsilon_p-1)+\frac{\varepsilon_p-1}{\varepsilon_p}+(\varepsilon_p-1)\varepsilon_\mu\right]<0\end{aligned}$$

Given that $\varepsilon_p > 1$ and that Y_s is non-negative, this condition requires:

$$\left[s_y(\varepsilon_p-1)+\frac{\varepsilon_p-1}{\varepsilon_p}+(\varepsilon_p-1)\varepsilon_\mu\right]>0$$

However, since assumptions 2 and 4 imply that ε_μ and s_y are both strictly positive, this condition is always satisfied without imposing further restrictions to the parametric space.

Let us now evaluate the term $\partial^2\pi(x_t(i))/\partial^2 Z_t(i)$. By double differentiating the profit function with respect to $Z_t(i)$ we obtain:

$$\begin{aligned}\frac{\partial^2\pi(x_t(i))}{\partial^2 Z_t(i)} &= (p_t(i)-\eta_t(i))\frac{\partial^2 y_t(i)}{\partial^2 Z_t(i)}-\frac{s_{y,t}(i)}{y_t(i)}\left(\frac{\partial y_t(i)}{\partial Z_t(i)}\right)^2\eta_t(i)-\frac{s_{z,t}(i)}{Z_t(i)}\varphi_t(i)\\ &= (p_t(i)-\eta_t(i))\left[\frac{\partial^2 y_t(i)}{\partial^2 Z_t(i)}-\frac{\partial y_t(i)}{\partial Z_t(i)}\frac{s_{z,t}(i)}{Z_t(i)}-\frac{s_{y,t}(i)}{y_t(i)}\left(\frac{\partial y_t(i)}{\partial Z_t(i)}\right)^2\left(\frac{\eta_t(i)}{p_t(i)-\eta_t(i)}\right)\right]\end{aligned}$$

where the last equality follows by using the optimal policy (3.2.14) in order to eliminate the term $\varphi_t(i)$. Using (3.2.13) we obtain:

$$\frac{\partial y_t(i)}{\partial Z_t(i)} = \frac{y_t(i)}{b_t(i)}\left(\varepsilon_{p,t}(i)-1\right)\frac{v'(z_t(i))}{Z_t}$$

and therefore, by differentiating this expression with respect to $Z_t(i)$, we find:

$$\frac{\partial^2 y_t(i)}{\partial^2 Z_t(i)} = \frac{y_t(i)}{b_t(i)}\frac{(\varepsilon_p(i)-1)}{Z_t^2}\left[\left(\varepsilon_{p,t}(i)-2\right)\frac{(v'(z_t(i)))^2}{b_t(i)}+\varepsilon_{p,t}(i)\frac{\varepsilon'_{p,t}(i)}{(\varepsilon_{p,t}(i)-1)}m_{y,t}(i)(v'(z_t(i)))^2+v''(z_t(i))\right]$$

Notice that in the derivation of this expression we have made use of equation (3.2.9) in order to evaluate $\partial\varepsilon_{p,t}(i)/\partial Z_t(i)$. Hence, denoting with Z_s the long-run aggregate level of expenditures for non-price competition, it must be true that

$$\begin{aligned}\left.\frac{\partial^2 y_t(i)}{\partial^2 Z_t(i)}\right|_{x_t(i)=1} &= \frac{Y_s}{Z_s^2}\left(\frac{\varepsilon_p-1}{\varepsilon_p}\right)\left[\left(\varepsilon_p-2\right)\varepsilon_{b,z}^2+\varepsilon_p\frac{\varepsilon'_p}{(\varepsilon_p-1)}\varepsilon_{b,z}^2+v''(1)-s_z\varepsilon_{b,z}-s_y(\varepsilon_p-1)^2\varepsilon_{b,z}^2\right]\\ &= -\frac{Y_s}{Z_s^2}\left(\frac{\varepsilon_p-1}{\varepsilon_p}\right)\left[\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2-v''(1)+s_z\varepsilon_{b,z}+s_y(\varepsilon_p-1)^2\varepsilon_{b,z}^2+\left(2-\varepsilon_p\right)\varepsilon_{b,z}^2\right]<0\end{aligned}$$

Given that $\varepsilon_p > 1$ and Y_s and Z_s are both non-negative, this condition holds if and only if:

$$\left[\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - v''(1) + s_z\varepsilon_{b,z} + s_y(\varepsilon_p - 1)^2\varepsilon_{b,z}^2 + (2-\varepsilon_p)\,\varepsilon_{b,z}^2\right] > 0 \tag{3.4.17}$$

However, the term $(2-\varepsilon_p)\,\varepsilon_{b,z}^2$ is negative for $\varepsilon_p > 2$ and therefore (3.4.17) is not satisfied for every $\vartheta \in \mathfrak{D} = (1,\infty)\times\mathbb{R}_+^4\times(-\infty,0)$.Thus, the space of admissible vector ϑ is restricted to a set $\mathfrak{F}_0$ such that in the symmetric equilibrium with zero inflation condition (3.4.17) holds, that is:

$$\mathfrak{F}_0 = \left\{\vartheta\in\mathfrak{D} \mid \varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - v''(1) + s_z\varepsilon_{b,z} + s_y(\varepsilon_p-1)^2\varepsilon_{b,z}^2 + (2-\varepsilon_p)\,\varepsilon_{b,z}^2 > 0\right\}$$

As a final issue, we have to characterize the restrictions imposed by the condition $det(H_\pi(1)) > 0$. In order to do this, we have first to determine the partial mixed derivative $\partial^2\pi(x_t(i))/\partial p_t(i)\partial Z_t(i)$. To this end, notice first that direct differentiation of (3.2.12) implies:

$$\frac{\partial\pi(x_t(i))}{\partial p_t(i)} = y_t(i)\,(1-\varepsilon_{p,t}(i)) + \frac{w_t}{A_t}\Phi_y\left(\frac{y_t(i)}{A_t}\right)\frac{\varepsilon_{p,t}(i)y_t(i)}{p_t(i)}$$

So differentiating this equation with respect to $Z_t(i)$ yields:

$$\frac{\partial^2\pi(x_t(i))}{\partial p_t(i)\partial Z_t(i)} = \frac{\partial y_t(i)}{\partial Z_t(i)}\left\{1-\varepsilon_{p,t}(i)\left(1-\frac{\eta_t(i)}{p_t(i)}\right)\left[1+\frac{\varepsilon'_{p,t}(i)x_t(i)}{\varepsilon_{p,z}(i)-1}\right] + \frac{s_{y,t}(i)\varepsilon_{p,t}(i)}{p_t(i)}\eta_t(i)\right\}$$

Thus

$$\left.\frac{\partial^2\pi(x_t(i))}{\partial p_t(i)\partial Z_t(i)}\right|_{x_t(i)=1} = \frac{\varepsilon_{b,z}(\varepsilon_p-1)Y_s}{Z_s}\left[s_y(\varepsilon_p-1)+\varepsilon_p\varepsilon_\mu\right]$$

Therefore, in the steady-state with zero inflation the $det(H_\pi(1)) > 0$ if and only if

$$\left[\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 + s_y(\varepsilon_p-1)^2\varepsilon_{b,z}^2 + \varpi\right]\left[s_y(\varepsilon_p-1) + \frac{\varepsilon_p-1}{\varepsilon_p} + (\varepsilon_p-1)\varepsilon_\mu\right] > \varepsilon_{b,z}^2(\varepsilon_p-1)\left[s_y(\varepsilon_p-1)+\varepsilon_p\varepsilon_\mu\right]^2$$

where ϖ is an auxiliary variable given by:

$$\varpi = -v''(1) + s_z\varepsilon_{b,z} + (2-\varepsilon_p)\,\varepsilon_{b,z}^2$$

With simple algebra, it is possible to show that the previous condition reduces to:

$$1+\varepsilon_p\frac{(\varepsilon_\mu+s_y)\varpi + \varepsilon_\mu\varepsilon_{b,z}^2 s_y}{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 + \varpi + \varepsilon_{b,z}^2(\varepsilon_p-1)^2 s_y} > 0 \tag{3.4.18}$$

Therefore, denoting with $\mathfrak{F}_1$ the set of all the vectors satisfying condition (3.4.18), it must be true that:

$$\mathfrak{F} = \mathfrak{F}_0 \cap \mathfrak{F}_1 \tag{3.4.19}$$

The set $\mathfrak{F}$ fully characterizes the space of all the vectors ϑ satisfying assumptions 1-5. Moreover, for any vector $\vartheta\in\mathfrak{F}$ it must be also true that:

1. ϕ_0 and ϕ_1 are both strictly positive
2. $1+\varepsilon_p\left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right] > 0$
3. $\phi_0 > \phi_1$

Given equations (3.4.15) and ((3.4.16), the first condition follows immediately from (3.4.17), while the second condition follows from (3.4.18) by noticing that:

$$1+\varepsilon_p\left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right] = 1+\varepsilon_p\frac{(\varepsilon_\mu+s_y)\varpi + \varepsilon_\mu\varepsilon_{b,z}^2 s_y}{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 + \varpi + \varepsilon_{b,z}^2(\varepsilon_p-1)^2 s_y} \tag{3.4.20}$$

The third condition follows by noticing that:

$$\phi_0 - \phi_1 = \frac{\varepsilon_{b,z}^2\varepsilon_\mu\varepsilon_p + \varepsilon_p^2(\varepsilon_p-1)s_y}{\varepsilon_p^2\varepsilon_\mu\varepsilon_{b,z}^2 - \nu''(1) + \varepsilon_{b,z}^2(2-\varepsilon_p) + \varepsilon_{b,z}s_z + \varepsilon_{b,z}^2(\varepsilon_p-1)^2 s_y} \;>\; 0$$

Thus, combining together the first two conditions we find:

$$0 < 1+\varepsilon_p\left[(1-\phi_0)\varepsilon_\mu + (1-\phi_1)s_y\right] < 1+\varepsilon_p\left[\varepsilon_\mu + s_y\right]\forall\,\vartheta\;\in\;\mathfrak{F}$$

This proves the statement.

III) Proof of property (ii)

As final issue, we want to prove that under assumption 1-5 the slope ζ is a decreasing and bounded function of ε_μ, converging to a strictly positive number as ε_μ goes to infinity.

Proof: Choose an arbitrary vector $\tau = [\varepsilon_p, \varepsilon_{b,z}, s_z, s_y, v''(1)] \in (1, \infty) \times \mathbb{R}^3_+ \times (-\infty, 0)$ and let $\zeta_{np}(\varepsilon_\mu)$ denotes the slope coefficient (3.2.23) as a function of only the elasticity ε_μ. To ensure that assumption 1-5 are satisfied, define the set $\mathfrak{B}$ as:

$$\mathfrak{B} = \left\{\varepsilon_\mu \in \mathbb{R}_+ : \zeta_{np} > \zeta\, ; \tau \in (1, \infty) \times \mathbb{R}^3_+ \times (-\infty, 0)\right\}$$

so that $\varepsilon_\mu \in (\varepsilon^*_\mu, \infty)$, with $\varepsilon^*_\mu = \inf \mathfrak{B}$. The monotonically decreasing behavior of $\zeta_{np}(\varepsilon_\mu)$ follows from (3.2.23) by noticing that direct differentiation of (3.4.20) yields:

$$\frac{\partial 1 + \varepsilon_p\left[(1-\psi_0)\varepsilon_\mu + (1-\psi_1)s_y\right]}{\partial \varepsilon_\mu} = \varepsilon_p \left[\frac{\varpi - \varepsilon^2_{b,z} s_y(\varepsilon_p - 1)}{\varepsilon^2_p \varepsilon_\mu \varepsilon^2_{b,z} + \varpi + \varepsilon^2_{b,z}(\varepsilon_p - 1)^2 s_y}\right]^2 > 0 \; \forall \varepsilon_\mu \in (\varepsilon^*_\mu, \infty)$$

This condition and (3.4.18) jointly imply:

$$0 < \zeta_{np}(\varepsilon_\mu) < \zeta_{np}(\varepsilon^*_\mu) \; \forall\, \varepsilon_\mu \in (\varepsilon^*_\mu, \infty)$$

which in turn proves that $\zeta_{np}(\varepsilon_\mu)$ is a bounded function. To show that the lower bound is strictly positive, it is enough to prove that $\zeta_{np}(\varepsilon_\mu)$ converges to a positive real number as ε_μ goes to infinity. To this end, notice that:

$$\begin{aligned}
\lim_{\varepsilon_\mu \to \infty} 1 + \varepsilon_p\left[(1-\psi_0)\varepsilon_\mu + (1-\psi_1)s_y\right] &= 1 + \lim_{\varepsilon_\mu \to \infty} \varepsilon_p \frac{(\varepsilon_\mu + s_y)\varpi + \varepsilon_\mu \varepsilon^2_{b,z} s_y}{\varepsilon^2_p \varepsilon_\mu \varepsilon^2_{b,z} + \varpi + \varepsilon^2_{b,z}(\varepsilon_p - 1)^2 s_y} \\
&= 1 + \frac{\varpi + \varepsilon^2_{b,z} s_y}{\varepsilon_p \varepsilon^2_{b,z}} \\
&= \frac{-v''(1) + \varepsilon_{b,z}\left[s_y \varepsilon_{b,z} + s_z + 2\varepsilon_{b,z}\right]}{\varepsilon_p \varepsilon^2_{b,z}} \in \Re_+
\end{aligned}$$

which, given (3.2.23), in turn implies:

$$\lim_{\varepsilon_\mu \to \infty} \zeta_{np}(\varepsilon_\mu) = \frac{(1-\theta\beta)(1-\theta)}{\theta} \left\{\frac{\varepsilon^2_{b,z}\varepsilon_p}{-v''(1) + \varepsilon_{b,z}\left[s_y \varepsilon_{b,z} + s_z + 2\varepsilon_{b,z}\right]}\right\} > 0$$

Therefore, we can conclude that:

$$\zeta_{np} > \frac{(1-\theta\beta)(1-\theta)}{\theta} \left\{\frac{\varepsilon^2_{b,z}\varepsilon_p}{-v''(1) + \varepsilon_{b,z}\left[s_y \varepsilon_{b,z} + s_z + 2\varepsilon_{b,z}\right]}\right\} > 0 \; \forall\, \varepsilon_\mu \in (\varepsilon^*_\mu, \infty)$$