Essays on the behavior and regulation of insiders

Antoni Sureda Gomila

TESI DOCTORAL UPF / 2010

DIRECTOR DE LA TESI

Dr. José María Marín Vigueras
Als meus pares
i a na Yolanda.
Acknowledgements

It is a pleasure to thank those who made this thesis possible.

First of all, I would like to express my deep and sincere gratitude to my supervisor, José Marín, for his encouragement, inspiration, guidance, sound advice, and support from the initial to the final stages of the thesis.

I owe the deepest gratitude to my coauthors, Robert H. Edelstein, José Marín, Branko Urosevic, and Nicholas Wonder; without their contributions this thesis would not have been possible.

I am grateful to the Department of Economics and Business of the Universitat Pompeu Fabra for their outstanding courses and for providing a stimulating research environment; and to the Economics Department of Harvard University and the School of Business of the University of Wisconsin-Madison for hosting me as a visiting fellow.

I have received insightful comments and suggestions from Xavier Freixas, Vicente Cuñat, Jacques Olivier, and Jeremy Stein. From Francesco Franzoni I have learnt to deal with financial data. I also thank the participants at the following conferences and seminars: the XIV Foro de Fianzas, especially to David Veredas, UPF’s Finance Lunch Seminars, the 12th Asian Real Estate Society (AsRES) Annual Conference, the 2007 American Real Estate and Urban Economics Association (AREUEA) Annual Conference and Meetings, the Research in Financial Economics seminar at Harvard Business School, a seminar at the University of Alicante; and to the editor and three anonymous referees of the Journal of Real Estate Economics for their valuable comments.

I am also grateful to Marta Araque, Marta Aragay, and Gemma Burballa, for making the paperwork a breeze.

I am indebted to many friends, they have contributed with comments and discussions, with they exemplar determination and success completing their thesis and, above all, making this endeavor much more enjoyable. I big ‘thank you’, Toni, Pablo, Rasa, Ricardo, Judit, Thomas, and Maria Emilia.

I am grateful to the Spanish Ministry of Science and Technology (grant number BES-2003-2083), the Universitat Pompeu Fabra, and the Fundació Banco Herrero for their financial support.

I am eternally grateful for the love and support that I have received from my parents,
they have been and they are the best role models one could possibly have. Finally, I would like to thank Yolanda, who has encouraged me to finish this thesis and for whom I will never find enough words but the ones of love.
Abstract

This thesis consists of three essays on the behavior of corporate insiders and the optimal regulation of insider trading. The first of these three essays examines the welfare effects of insider trading and its attributes as an executive compensation mechanism; in addition, an optimal regulation of insider trading in the light of the model is proposed. The second essay analyzes another facet of insider trading: whether insiders can be a source of liquidity and act as traders of last resort on their companies’ stock; moreover, the effects of transactions by insiders and by the company itself on the distribution of stocks returns are compared empirically. Finally, the topic of the third essays is the dynamics of insiders’ holdings, and how these dynamics are a function of the number of large shareholders in the firm; the conclusions are empirically tested for Real Estate Investment Trusts.

Resum

Aquesta tesi conté tres assajos sobre el comportament dels agents corporatius y la regulació de la compravenda d’accions amb informació privilegiada. El primer examina l’efecte de la negociació amb informació privilegiada y la seva utilitat com a mecanisme de compensació; es proposa una regulació de la negociació amb informació privilegiada. El segon assaig analitza si els agents corporatius són una font de liquiditat y actuen com a comerciants d’últim recurs per a les accions de la seva companyia; també es comparen empíricament els efectes de la negociació per part dels actors corporatius amb els de la negociació per part de les mateixes empreses en la distribució dels rendiments de les accions. L’últim assaig estudia la dinàmica de les carteres d’aquest actors en accions de les seves pròpies empreses, i com aquesta dinàmica es funció del nombre d’actors corporatius a l’empresa; les conclusions es testegen empíricament per a fons d’inversió immobiliària.
Preface

This thesis consists of three self-contained essays examining different aspects of insider trading (IT). Insider trading here is defined as trading by corporate insiders and, in turn, corporate insiders are defined as a company’s officers, directors and any beneficial owners of more than ten percent of a class of the company’s equity securities; this coincides with the SEC definition of corporate insider, whom, in the US, are required to report their trades to the SEC. This wide definition of what constitutes a corporate insider is reflected in these essays: insiders are modeled here as agents that trade on material nonpublic information, as agents that trade on the basis of their understanding and interpretation of public information, as corporate managers taking investment decisions that influence firm’s returns, and as large shareholders that exert corporate monitoring, among others. The common link in the three essays is that they focus on the aspects and consequences of trading by these corporate insiders. The following studies give answers to questions as the following: Should IT in possession of nonpublic information be forbidden or allowed? Can IT be a compensation mechanism? Might IT increase liquidity in security markets? How does insiders’ ownership evolve over time and how do the dynamics depend on the number of insiders?

The first of the three essays is entitled Insider Trading and Welfare when Information Has Social Value. It analyzes the welfare consequences of granting insider trading (IT) rights to a corporate manager as a compensation mechanism in the presence of two agency problems: the first is to induce the manager to gather information with social value, when the collection of this information is costly for her; the second is to induce the manager to implement shareholders’ optimal response to her private information. Our analysis shows that depending on the characteristics of the firm, either completely forbidding IT or allowing it are the optimal IT regimes; a disclose-or-abstain rule is never an optimal regulation. Furthermore, a conflict of interest between society and entrepreneurs might arise, in this case, state intervention forbidding IT would maximize social welfare. This essay has earned a grant from the Fundación Banco Herrero.

The second essay, Insiders As Traders of Last Resort, is a joint project with José Marín. The objective of this piece of research is to explore the role of corporate insiders vs. firms as traders of last resort. We develop a simple model of insider trading in which insiders provide price support, as well as liquidity, in security markets. Consistently with the model predictions, we find that in the US markets insiders’ trading activities have a clear impact on return distributions. Furthermore,
we provide empirical evidence on insiders transactions and firm transactions affecting returns in a different manner. In particular, while insiders’ transactions (both purchases and sales) have a strong impact on skewness in the short run and to a lesser extent in short run volatility, company repurchases only have a clear impact on volatility, both in the short and the long run. We provide explanations for this asymmetry. This paper has been presented in several academic forums.

The title of the third essay is *Ownership Dynamics with Multiple Insiders: The case of REITs* and is the fruit of a joint research project with Robert H. Edelstein, Branko Urosevic, and Nicholas Wonder. We study the ownership dynamics of multiple strategic risk-averse insiders facing a moral hazard problem: on one hand, the company value increases with insider holdings, on the other hand, insiders have an incentive to decrease their stakes over time in order to reduce their risk exposure. We show that, when insiders cannot commit, ex-ante, to an ownership policy, the aggregate insider stake gradually declines towards the competitive allocation. Moreover, both the speed of decline and the long-term equilibrium aggregate insider ownership level are greater for companies with a higher number of insiders, ceteris paribus. We, then, test the model on data from the U.S. Real Estate Investment Trusts (REITs) industry and find that the predictions of the model are supported by the data. This work has been published in the *Journal of Real Estate Economics* and was awarded as the Best Paper for the *American Real Estate Society (ARES)* Foundation in the *12th Asian Real Estate Society (AsRES)* Annual Conference (2007).
Contents

Acknowledgements iii

Abstract v

Preface vii

List of Figures xiii

List of Tables xvi

1 Insider trading and welfare when information has social value 1
  1.1 Introduction ................................................................. 1
  1.2 Relation to previous literature ........................................... 3
  1.3 Model ................................................................. 6
    1.3.1 The firm ................................................................. 7
    1.3.2 The manager ............................................................... 7
    1.3.3 Market participants .................................................... 8
    1.3.4 The manager’s compensation package ................................ 8
    1.3.5 Insider trading regimes ............................................... 9
    1.3.6 The timing ................................................................. 9
  1.4 Equilibrium ............................................................ 9
    1.4.1 Equilibrium when trading by the manager is forbidden ............ 11
    1.4.2 Equilibrium with insider trading ..................................... 15
    1.4.3 Analysis of the equilibrium .......................................... 18
1.5 Welfare Analysis .................................................. 30
1.6 Concluding Remarks ............................................. 33
1.A Proofs of the propositions ..................................... 33
  1.A.1 Proof of proposition 1 ....................................... 34
  1.A.2 Proof of proposition 2 ....................................... 35
  1.A.3 Proof of proposition 3 ....................................... 36
  1.A.4 Proof of proposition 4 ....................................... 37
1.B Numerical analysis of the model .............................. 38

2 Firms vs. Insiders as Traders of Last Resort 41
  2.1 Introduction .................................................... 41
  2.2 The model ..................................................... 46
    2.2.1 Equilibrium and comparative statics ..................... 47
    2.2.2 Effects of trading costs on the price distribution .... 51
  2.3 Empirical analysis ............................................ 53
    2.3.1 Description of the data .................................. 53
    2.3.2 Effects of IT on returns’ distribution .................. 55
  2.4 Conclusions and further research ............................ 62
  2.A Appendix ....................................................... 64
    2.A.1 Proof of lemma 2 ......................................... 64
    2.A.2 Proof of proposition 5 .................................... 65
    2.A.3 Numerical approximation to the equilibrium at $t = 1$ 67

3 Ownership Dynamics with Multiple Insiders 69
  3.1 Introduction .................................................... 69
  3.2 The Model ..................................................... 72
  3.3 Insiders’ Trading Strategies ................................ 75
    3.3.1 The Solution and Comparative Statics ................ 78
  3.4 Empirical Analysis ............................................ 79
    3.4.1 Data Description ......................................... 80
    3.4.2 Aggregate Insider Ownership Evolution ................. 84
    3.4.3 Robustness Checks ....................................... 86
3.5 Conclusions and Future Work ........................................ 93
3.A Proofs of the propositions ........................................... 94
  3.A.1 Preliminaries .................................................... 94
  3.A.2 Proof of Proposition 6 .......................................... 95
  3.A.3 Proof of Proposition 7 .......................................... 96
  3.A.4 Proof of Proposition 8 .......................................... 96
Bibliography ............................................................... 101
## List of Figures

1.1 Summary of the model’s timing ............................................. 7
1.2 Equilibrium in the trading round ........................................ 20
1.3 Sensitivity of investment to manager’s signal ......................... 23
1.4 Precision of the manager’s signal ....................................... 25
1.5 Optimal contract as a function of $c_k$, $c_\varepsilon$, and $\sigma_\pi$ .......... 28
1.6 Optimal $\tau_\varepsilon$ when the contract is optimally chosen, as a function of $c_k$, $c_\varepsilon$, and $\sigma_\pi$ ........................................ 29
1.7 Welfare as a function of $c_k$, $c_\varepsilon$, and $\sigma_\pi$ ................... 32

2.1 Insider and firm trading around large price movements ............... 42
2.2 Timing of events .......................................................... 47
2.3 Equilibrium price ....................................................... 50
2.4 Effects of informed’s trading costs on the price variance and skewness 52

3.1 Timing in the model ...................................................... 75
3.2 Aggregate insider ownership policy varies with the number of insiders 80
3.3 Dynamics of the aggregate insiders’ ownership by the initial number of insiders, $N(0)$ ......................................................... 84
List of Tables

1.1 Components of the equilibrium that are computed analytically or numerically. .................................................. 19

2.1 Summary Statistics .......................................................... 56

2.2 Time series average of cross-sectional correlations between financial constraintness measures and insider trading activity ............. 56

2.3 Fama-MacBeth regressions of short-horizon return variance on insider trading activity ................................................. 58

2.4 Fama-MacBeth IV regressions of short-horizon return variance on insider trading activity ................................................... 59

2.5 Fama-MacBeth regressions of returns skewness on insider trading activity ................................................................. 61

2.6 Fama-MacBeth IV regressions of returns skewness on insider trading activity ................................................................. 63

3.1 Distribution of REITs by \(N(0)\) ............................................. 82

3.2 Descriptive statistics of insider ownership .................................. 83

3.3 Regression of aggregate insiders’ stake on elapsed time since IPO . 85

3.4 Regression of \(A^*\) on the initial number of insiders \(N(0)\) .............. 88

3.5 Regression of speed of adjustment of \(A\) during the first five years of the REIT’s life on the initial number of insiders .................. 89

3.6 Robustness check: Regression of aggregate insiders’ stake including OP units on elapsed time since IPO .............................. 90

3.7 Robustness check: Regression of \(A^*\), including OP units, on the initial number of insiders \(N(0)\) ........................................ 91
3.8 Robustness check: Regression of speed of adjustment of A, including
OP units, during the first five years of the REIT’s life on the initial
number of insiders. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 92
Insider trading and welfare when information has social value

1.1 Introduction

There is a long-lasting debate on whether insider trading (IT), defined as trading in possession of material private information, should be allowed or forbidden and, even now, it is not clear what the optimal IT regime might be. IT regulation, and whether this regulation is enforced, differs across countries. For instance, IT laws are lax in Norway, and Mexico and strict in the US and Ireland; however, there have been enforcement cases in Norway and the US, but never neither in Mexico nor in Ireland\(^1\). There are differences in what is considered illegal IT between American and European regulations: in the US, under Rule 10b-5, anyone in possession of material inside information shall disclose his private information or abstain from trading\(^2\); in Europe, directive 2003/6/EC on insider dealing and market manipulation requires “inside information” to be of a “precise nature”, and forbids both, trading in possession of inside information and the disclosure of this information, “unless such disclosure is made in the normal course of the exercise of his employment, profession or duties”.

This debate on the optimal IT regulation is due, in part, to the fact that the welfare consequences of insider trading are still not well understood. For instance, IT could increase market efficiency and be an effective way to compensate corporate agents for innovations\(^3\); on the other hand, IT puts uninformed investors at an informational disadvantage and, therefore, generates an adverse selection problem. It is not clear,

\(^1\)See Beny (2005) and Ferreira and Fernandes (2009).

\(^2\)The origin of this interpretation can be traced to SEC v. Texas Gulf Sulphur Co, although latter the Supreme Court limited the liability to the cases in which there is a fiduciary duty of the insider to the persons with whom he trades (Chiarella v. US), to tippees (Dirks v. SEC), and to the cases in which there is a fiduciary duty of the insider to the source of the information (the so-called misappropriation theory, endorsed by the Supreme Court in US v. O’Hagan).

\(^3\)Manne (1966)
from a welfare perspective, whether the benefits of IT compensate for its drawbacks. As a contribution to the debate, I analyze the effect of IT on welfare by means of a model that is suitable for welfare analysis, in which insiders can take actions that directly affect firm’s returns\textsuperscript{4}, private information has social value, and all market participants behave rationally and understand the structure of the economy. This framework allows me to analyze which is the optimal IT regime among the considered alternatives, and it can be extended to analyze further executive compensation schemes.

One key element of the model proposed here is the presence of two agency problems between the corporate manager and the firm’s shareholders. Consider the following situation for a corporate manager: first, she must acquire information regarding the enterprise, the market, and the environment in which the firm operates; then, using the available information, she must decide on the policy that the firm will follow. This activity of gathering and processing information is costly for her, which causes an agency problem between the manager and shareholders because the manager might gather less information than what would be socially optimal. The second agency problem arises because, conditional on the information gathered, the optimal response from the point of view of the manager may differ from the optimal response from the point of view of firm’s owners. If shareholders cannot observe directly the information that the manager possesses, they can’t implement their optimal response and have to rely on the manager, who may take investment decisions different from those that would be preferred by the shareholders. In this context, the model can be used to analyze whether IT can improve managerial contracts, and analyze also other relevant trade-offs present in the literature.\textsuperscript{5}

The analysis performed here shows that a disclose-or-abstain rule, as the one derived by common law adjudications from Rule 10b-5, is never an optimal regulation and it destroys investors’ welfare; this is due to two main reasons: the first is that it reduces the incentive for the manager to gather information that is relevant for investment decisions, the second is that early revelation of information reduces investors’ risk sharing opportunities (Hirshleifer effect). Numerical simulations also show that either completely forbidding IT without disclosing manager’s private information or completely allowing it are the optimal IT regimes among the considered alternatives. Which of the two is preferred depends on the characteristics of the firm. For instance, if the cost of capital or the cost of obtaining information are low, it will be optimal to forbid IT, and if these costs are high, it will be optimal to allow the manager to trade. Intuitively, the lower the cost of capital or information, the largest the amount of information that it is optimal to gather, but also the largest is the Hirshleifer effect if this information is partially revealed through IT, therefore, in these cases, it will be preferable to forbid IT; however, in industries with high cost of capital or in which the future is specially difficult to foresee, it will be preferable to allow IT. Similarly, if the volatility of the firm’s profitability is low, it will be optimal to allow IT, and if it is high, it will be optimal to forbid IT; the basic intuition is that the higher

\textsuperscript{4}Given that insider’s actions directly affect the production process, she can be seen as a corporate decision maker and I will refer to her as the manager.

\textsuperscript{5}How the model relates to existing literature is discussed in the section 1.2.
is uncertainty the highest are the hedging needs of the agents, and early revelation of information caused by IT reduces their ability to redistribute risk. Therefore, the main prediction of the paper is that if the cost of capital is high, information is difficult to obtain by the manager, or the volatility of profitability is low, then it is optimal to allow the manager to trade on the basis of her private information; on the other hand, if the cost of capital is low, information is easy to obtain by the manager, or the volatility of profitability is high, it is optimal to forbid the manager from trading and to forbid also the early revelation of inside information\textsuperscript{6}. Last but not least, the model also predicts that the regime that entrepreneurs might choose could not be socially optimal, therefore there is room for insider trading regulation in these cases.

The previous results should be taken cautiously. First, most of the analysis is performed numerically, therefore the results are obtained for a set of parameters around a base case parameterization; nevertheless, this base case parameterization has been chosen to be sensible and in line with previous literature. Second, I don’t consider neither all possible regulations, nor how the regulation could actually be enforced\textsuperscript{7}, nor all possible executive compensation contracts\textsuperscript{8}. Last but not least, the model does not incorporate all the effects and trade-offs that have been discussed in the literature\textsuperscript{9}.

This paper is structured as follows. In the next section I relate this work to the previous literature. The model is described in section 1.3. Section 1.4 defines and describes the equilibrium of the model and, in section 1.5, I perform the welfare analysis. Section 1.6 concludes. Proofs and details regarding the numerical analysis are provided in the appendix.

1.2 Relation to previous literature

As was mentioned in the introduction, the debate on whether IT should be allowed or forbidden has been enduring, tracing back to Manne (1966). Bainbridge (2000) surveys the origins of the debate.

\textsuperscript{6}This model can be used to analyze cross effects like what happens when the cost of capital is high but the volatility of profitability small, however in this paper I do not comment on every possible combination of the parameters

\textsuperscript{7}DeMarzo et al. (1998) analyzed the detection and penalization of insiders engaged in unlawful IT

\textsuperscript{8}Another common element of managerial compensation packages are stock options. Stock options can also induce the manager to gather information and implement riskier strategies, mitigating both agency problem. However, stock options also create the incentive to pursue short-term advantage at the cost of long-term performance. The model proposed here is rich enough to accommodate stock options, but I do not contemplate them as a part of the manager’s compensation package because they have been already studied in previous literature, for instance Bebchuk et al. (2002); Hall and Murphy (2002, 2003), and they are not the focus on this paper.

\textsuperscript{9}For instance, the effects of the interaction of multiple insiders (Edelstein et al. (2010)), whether IT reduces the incentives of other market participants to gather information (Bushman et al. (2005); Fishman and Hagerty (1992); Khanna and Slezak (1994)), or the impact of IT on the cost of capital (Beny (2005); Bhattacharya and Daouk (2002)).
The idea that IT can improve managerial contracts has already been examined in previous literature: for instance, Dye (1984), Bebchuk and Fershtman (1993, 1994), Noe (1997), Hu and Noe (2001), and Bernardo (2001) have analyzed the issue by means of theoretical models; however, these models are handicapped by methodological problems that make them unsuitable for welfare analysis or do not incorporate other relevant aspects of IT. According to Dye (1984), granting IT rights to the manager could be optimal for shareholders, though, in Dye’s model equilibrium is either fully revealing or not consistent with rational expectations; Bebchuk and Fershtman (1993, 1994) show that IT might benefit shareholders either by improving managerial effort or by inducing the manager to choose riskier strategies, but they did not model explicitly the trading process; Hu and Noe (2001) also found that IT can improve executive compensation contracts, but in their model equilibrium price is fully revealing and agents act as price takers; on the other side of the debate, Bernardo (2001) found that shareholders would choose to forbid IT or, in any case, allowing IT would not be socially optimal; this is because if IT is allowed the manager would choose too risky projects and hedging opportunities would be reduced (Hirshleifer effect).

The model proposed here can be seen as an extension of Bernardo (2001), with five major differences. First, in this model the cost of gathering information is not fixed, depends on the desired precision of the signal. Second, the manager can decide on the firm’s investment policy in a more substantial way than to choose among one risky and one riskless project: I include a production function proposed by Dow and Rahi (2003), so that the manager can decide on the level of investment from a continuum of possible values and this investment decision is not observable by outsiders. Third, the manager is aware of her position in the market, and therefore acts as an information monopolist. Four, manager’s minimum holdings are not limited to short selling constraints. Finally, I consider several scenarios as a benchmark, concretely I analyze four information revelation setups:

1. total prohibition of insider trading and no earlier disclosure of private information,

2. total prohibition of insider trading and earlier disclosure of private information,

3. granting insider trading rights to the manager and no earlier disclosure of private information, and

4. granting insider trading rights to the manager and earlier disclosure of private information.

10 This effect could also be harmful to shareholders.

11 It is convenient to use several benchmarks, apart from total IT prohibition, to carefully examine the effects of IT. This is because the revelation of information affects the welfare of investors (as shown in Dow and Rahi (2003), information can both decrease risk sharing opportunities, Hirshleifer effect, or increase them, spanning effect). Some effects of the revelation of information arise in a more general framework than IT: for instance, the requirement of more transparency to corporations would affect the welfare of market participants, and the effect could even be stronger than allowing IT if full revelation of information is required. This makes convenient to use several benchmarks, apart from total IT prohibition, to carefully examine the effects of IT, and the appropriate benchmark should be used when analyzing the optimal IT regulation.
4. granting insider trading rights to the manager and earlier disclosure of private information, *disclose-or-abstain rule.*

Analyzing the cases in which insider trading rights might be limited by short selling restrictions or minimum holdings has the cost of precluding closed form solutions for the equilibrium and ex-ante welfare, and the existence of multiplicity of equilibria, as in Marin and Olivier (2008). Therefore, part of our analysis has to be done numerically. The numerical approximation to the equilibrium is based on the projection method proposed by Bernardo and Judd (2000) and Bernardo (2001).

The relation between IT and executive compensation has also been examined empirically by Roulstone (2003), Zhang et al. (2005), and Brenner (2010). These studies have found evidence that IT and other forms of managerial compensation might be substitutes to some degree, but they are not conclusive regarding the materiality of IT regulation on executive compensation. The consistency of the theoretical results obtained here with these empirical findings will be commented in section 1.4, where the optimal executive compensation packages will be discussed.

Although the main focus of the paper is the welfare consequences of IT in the presence of the two agency problems previously described, it also displays many of the effects of IT on the welfare of economic agents that have been discussed previously in this literature. Among those effects there is the relation between IT and market efficiency. Market efficiency has an impact on welfare because, among others, it facilitates the optimal allocation of resources and alters the hedging opportunities of market participants.

There is still an open debate on whether IT increases or harms market efficiency. It is trivial to build a model in which IT incorporates private information into prices, and there are empirical studies, such as Meulbroek (1992), showing that IT causes price movements and quick price discovery. Furthermore, in addition to incorporating private information to stock prices, insiders can exploit their ability to identify securities that are miss-valued, pushing prices closer to their fundamental value; studies such as Piotroski and Roulstone (2005) and Marin and Sureda-Gomila (2006) have found empirical evidence that insiders trade on the basis of both contrarian beliefs and private information about future cash flow news, and provide price support, as well as liquidity, in security markets. On the other hand, IT also decreases the returns to obtaining information, what might reduce the number of analysts, and possibly make prices less informative; this hypothesis has been examined in Fishman and Hagerty’s (1992), Khanna and Slezak’s (1994) models, and empirically by Bushman et al. (2005).

Larger market efficiency might lead to a better allocation of resources, but also affects the hedging opportunities of market participants, as noted by Hirshleifer (1971), Marin and Rahi (1999), and Dow and Rahi (2003). IT can reduce hedging

---

12In models in which insider’s information cannot affect firm’s returns, as in Medrano and Vives (2004), and information is costly to obtain, an abstain or disclose rule makes insiders to abstain from gathering information. This is not the case in the model proposed here.

13Market efficiency also has implications in the design of managerial contracts: as shown in Holmstrom and Tirole (1993), more informative prices can also improve managerial incentives.
opportunities as in Bernardo (2001), this receives the name of Hirshleifer effect, or increase them, as in Bhattacharya and Nicodano (2001), this receives the name of spanning effect. In the model proposed here, IT increases market efficiency because there are no analysts to be crowded out by the insider; however, in the model proposed here, market efficiency goes against the alternative of a laissez-faire policy towards IT. Indeed, in this model, there is not spanning effect, but Hirshleifer effect, therefore larger market efficiency implies a loss of social welfare caused by a reduction in hedging opportunities. Furthermore, the only agent that makes investment decisions that might affect aggregate consumption is the corporate manager, and she does it when the market is still closed. Therefore, in the context of this model, market efficiency can not lead to a better allocation of resources.

IT also generates an adverse selection problem because uninformed investors are in a disadvantaged position with respect to an insider. This adverse selection problem arises in this model because traders are rational\(^{14}\); they take their information disadvantage into account when they participate in the market. Shareholders can take into account manager’s profits through IT when designing compensation packages, but the adverse selection problem causes a deadweight loss of welfare, in the presence of adverse selection, the rents extracted by the insider are below the wealth loss by uninformed investors.

Finally, there are aspects of IT that are relevant but that I have not incorporated in the model, these aspects have been already examined in previous literature; for instance, the detection and penalization of insiders engaged in unlawful IT\(^{15}\), the effects of the interaction of multiple insiders\(^{16}\), whether IT reduces the incentives of other market participants to gather information\(^{17}\), or the impact of IT on the cost of capital.\(^{18}\)

### 1.3 Model

I model an economy with two assets, one risky asset (the firm) and one riskless asset, which is assumed to be in excess supply and to provide a gross return equal to 1. The firm is set up by a group of entrepreneurs that hire a manager to run the firm; these entrepreneurs also design the manager’s compensation package, \(CP\). The final value of the firm, \(v\), is function of the amount invested, \(k\), and the profitability per unit of investment, \(\pi\). The amount invested is a managerial decision, while the profitability per unit of investment is a random variable. The manager can exert costly effort, \(\tau_e\), to observe a noisy signal of the profitability, \(\theta\), before choosing the level of investment. The firm’s stock is traded in one round before the payoff of the firm is revealed and agents consume their wealth; but after the manager has chosen the level of investment. The participants in the trading round are the

---

\(^{14}\)Noise traders might be hurt by IT, but their willingness to trade is not reduced by it.

\(^{15}\)DeMarzo et al. (1998)

\(^{16}\)Edelstein et al. (2010)

\(^{17}\)Bushman et al. (2005); Fishman and Hagerty (1992); Khanna and Slezak (1994)

\(^{18}\)Beny (2005); Bhattacharya and Daouk (2002).
entrepreneurs, new investors or traders, and potentially the manager. Note that entrepreneurs also participate in the stock market although I do not refer to them as traders. The timing of these events is summarized in 1.1. The components of the model (the firm, the market participants, the manager’s compensation package and the possible insider trading regimes) are described with more detail in the following subsections.

![Figure 1.1. Summary of the model’s timing](image)

### 1.3.1 The firm

Let me consider a firm whose value is given by

\[ v = k \pi - \frac{c_k}{2} k^2, \]  

(1.3.1)

where \( k \) is a measure of investment, \( \pi \) is a random variable measuring profitability, and \( c_k > 0 \) is a parameter for the investment cost. This production function has been proposed by Dow and Rahi (2003). Profitability per unit of investment, \( \pi \), is modeled as a normally distributed random variable, with zero mean and standard deviation \( \sigma_\pi \). The firm’s shares can be traded in a market and I normalize its supply to 1.

### 1.3.2 The manager

The firm is run by a manager, who has two tasks: first, to gather information regarding the firm’s profitability, and second, to choose the level of investment. She, the manager, can exert an effort to estimate \( \pi \). The forecasting technology allows her to collect a noisy signal of the investment profitability, \( \theta = \pi + \sigma_\varepsilon \varepsilon \). The noise component of \( \theta \) is \( \sigma_\varepsilon \varepsilon \), where \( \varepsilon \) is a normally distributed random variable with zero mean, variance equal to 1, and independent of \( \pi \), and the standard deviation of the noise is \( \sigma_\varepsilon \). I define the precision of the manager’ signal as the inverse of the noise variance, \( \tau_\varepsilon = 1/\sigma_\varepsilon^2 \). The precision of the manager’ signal, \( \tau_\varepsilon \), depends on the effort exerted by the manager; this effort is costly and its cost in consumption units\(^{19}\) is

\(^{19}\)See manager’s budget constraint in equation 1.4.2.
given by $g(\tau\varepsilon)$, where $g$ is an increasing function, with $g(0) = 0$ and\textsuperscript{20}. I will assume that $g$ has the form $g(\tau\varepsilon) = c_{\varepsilon} \tau\varepsilon$, with $c_{\varepsilon} > 0$.\textsuperscript{21}

1.3.3 Market participants

After the manager has chosen the level of investment, the firm’s stock is traded. The participants in the market are the entrepreneurs or initial owners of the firm, possible new investors also referred as traders, and potentially the manager if she is allowed to trade. All agents display a CARA utility function and consumption only takes place once the value of the firm is revealed. The manager’s coefficient of absolute risk aversion is denoted by $r_M$ and the entrepreneurs’ by $r_E$. Traders can be either a highly risk averse, with a coefficient of risk aversion $r_H$, to whom I will refer as $H$-type; or slightly risk averse, with coefficient of risk aversion $r_L$, to whom I will refer as $L$-type. In what follows, the subscript $M$ will refer to the manager, $E$ to the entrepreneurs, and $H$ and $L$ to traders from a $H$-type and $L$-type respectively. Agent $i$’s utility function will be denoted by $U_i(\cdot)$.

The fraction of $H$-type is represented by $\lambda$, which is a random variable that follows a uniform distribution on the interval $[0, 1]$ and is independent from all other exogenous random variables in the model. The fraction of $H$-type is not observable, but each trader knows his own type\textsuperscript{22}. The masses of entrepreneurs and traders are normalized to 1, and none of the agents has initial endowments except the entrepreneurs, endowed with the firm’s equity.

All participants behave competitively except the manager, who is aware of her position in the market, and all condition their demand for the stock to their information sets. If there is full revelation of the manager’s signal, all agents information sets will contain $\theta$ and the stock price. Otherwise, the manager’s information set contains $\theta$ and the stock price, entrepreneurs’ only contains the stock price, and traders’ the stock price and their own type. Note that knowing $\theta$ and the stock price is equivalent to knowing $\theta$ and $\lambda$. I will denote agent $i$’s information set by $I_i$.

1.3.4 The manager’s compensation package

Before any other action is done, the firm’s initial owners grant the manager with a compensation package consisting of a fixed salary, a fraction of the firm’s shares, and, if it is convenient for them, the right to trade the firm’s stock when the market is open. Furthermore, even if the manager is allowed to trade, I assume that he cannot end up with stock holdings below a certain threshold. The manager’s compensation package is denoted as a 3-tuple, $\mathcal{CP} = (W_{M,0}, x_{M,0}, IT)$, where $W_{M,0}$ is the fixed

\textsuperscript{20}An example of an information structure satisfying these assumptions is given by Holmstrom and Tirole (1993)

\textsuperscript{21}The cost function is quadratic (and, therefore, convex) on $1/\sigma_{\varepsilon}$, the inverse of the standard deviation of the noise component of $\theta$.

\textsuperscript{22}Uncertainty about the risk aversion of traders is introduced to preclude the prices from fully revealing manager’s private information. This modeling device was proposed by Bernardo (2001).
1.4 Equilibrium

salary, a positive scalar $x_{M,0}$ is the fraction of firm’s shares, and $IT$ is the empty set if the insider is not allowed to trade, $IT = \emptyset$, and a set with the minimum holdings threshold if the insider is allowed to trade, $IT = \{x_M\}$. Note that if there are not minimum holdings, then $x_M = -\infty$; however, under current regulation, insiders are forbidden to short-sell, $x_M = 0$, and it is common that companies impose strictly positive target holdings, $x_M > 0$. Entrepreneurs choose $CP$ in order to maximize their expected utility with the participation constraint that the manager has a reservation value equal to $U_M$ in terms of expected utility.

1.3.5 Insider trading regimes

I will focus on the analysis and comparison of six regimes, that are the combination of the following three trading constraints for the manager:

1. the manager is forbidden to trade,
2. the manager is forbidden to sell, but allowed to purchase firm shares,
3. the manager is allowed to purchase and sell firm shares, but she is not allowed to short-sell;

with the following two information disclosure requirements:

1. the manager’s signal, $\theta$, is publicly disclosed
2. the manager’s signal, $\theta$, is kept private by the manager.

1.3.6 The timing

To sum up, the sequence of events is shown in figure 1.1. I will solve the model by backward induction. I first solve for the equilibrium in the trading round, then I solve the manager’s problem of choosing the level of investment, then the problem of choosing the level of effort, and finally the entrepreneurs’ problem of designing the managers’ compensation package.

In the next section I discuss the notion of equilibrium and the methodology that I will use to compute it.

1.4 Equilibrium

The notion of equilibrium that I will use is the *Rational Expectations Equilibrium (REE)*. In a REE, all agents optimally take their decisions using all the payoff relevant information that they have available, which might include the equilibrium price. In our case, a REE will consist of the manager’s compensation package, the
effort exert by the manager to collect her signal, the investment policy, and the final holdings for each agent and the firm’s price resulting from the trading round. More formally, a REE in our model is defined as:

**Definition 1 (Equilibrium)** An equilibrium is an 8-tuple,

\[(CP, \tau_\varepsilon, k(\cdot), x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot), p(\cdot))\],

where \(CP\) is the manager’s compensation package, \(\tau_\varepsilon\) is the precision of the manager’s signal, \(k(\cdot)\) is the investment policy, \(x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot)\) are the final holdings of the manager, the entrepreneurs, and the traders of \(L\) and \(H\) type, respectively, and \(p(\cdot)\) is the market price of the firm, such that the following conditions are satisfied:

1. **Interim equilibrium condition:** Given \(CP\), \(\tau_\varepsilon\), and \(k(\cdot)\); \((x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot), p(\cdot))\) constitutes an interim equilibrium in the trading round. What constitutes an interim equilibrium will be defined latter, in definitions 2 and 3.

2. **Optimality of investment policy:** Given \(CP\) and \(\tau_\varepsilon\), and knowing that \((x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot), p(\cdot))\) will satisfy 1; \(k(\cdot)\) is a function of \(\theta\), \(k : \mathbb{R} \rightarrow \mathbb{R}\), such that it maximizes the manager’s expected utility conditional on \(\theta\):

   \[E[U_M(W_M)|\theta],\]

   subject to the manager’s budget constraint

   \[W_M + p(x_M - x_{M,0}) + c_\varepsilon \tau_\varepsilon = x_M(k(\theta)\pi - \frac{c_k}{2}k(\theta)^2) + W_{M,0},\]

   where \(W_M\) is the manager’s terminal wealth.

3. **Optimality of manager’s effort:** Given \(CP\) and knowing that \(k(\cdot)\) and \((x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot), p(\cdot))\) will satisfy 1 and 2; \(\tau_\varepsilon\) maximizes the manager’s expected utility:

   \[E[U_M(W_M)],\]

   subject to the manager’s budget constraint 1.4.2.

4. **Optimality of compensation contract:** Given the insider trading regime, and knowing that \(\tau_\varepsilon, k(\cdot),\) and \((x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot), p(\cdot))\) will satisfy 1, 2, and 3; \(x_{M,0}\) and \(W_{M,0}\) are chosen in order to maximize entrepreneurs’ expected utility:

   \[E[U_E(W_E)],\]

   where \(W_E\) is the entrepreneurs’ terminal wealth, subject to the entrepreneurs’ budget constraint:

   \[W_E + W_{M,0} + p(x_E - (1 - x_{M,0})) = x_E v\]
1.4 Equilibrium

the manager’s participation constraint

\[ E[U_M(W_M)] \geq U_M, \tag{1.4.6} \]

and the manager’s budget constraint 1.4.2.

I haven’t defined the notion of *interim equilibrium in the trading round*, I will do it later; the reason is that it will depend on whether the manager is allowed to trade or not. Indeed, if the manager is not allowed to trade, the interim equilibrium will have to satisfy optimality of investors’ demands and the market clearing condition; on the other hand, if the manager can trade, instead of the market clearing condition, the equilibrium will have to satisfy optimality of the manager’s demand subject to the aggregate demand of the rest of investors.

Note also that the insider trading regime is taken as given in 4, as if it were exogenous, this is because I will analyze different insider trading regimes separately and then compare the welfare of entrepreneurs and traders in each regime.

I will solve the model by backward induction, solving first for the equilibrium in the trading round, then for the investment policy and the manager’s effort, and finally for the optimal contract. I will start by examining the regime in which insider trading is forbidden and, after this, the regime in which insider trading is allowed. The analysis of the results is presented in subsection 1.4.3.

### 1.4.1 Equilibrium when trading by the manager is forbidden

In page 9, I mentioned that this paper would focus on the analysis and comparison of the six regimes that result from the combination of three IT regimes (one in which the manager is not allowed to trade; a second in which she is allowed to purchase firm shares, but not to sell them; and a third in which she is allowed to purchase and sell shares, except short-selling them) and two information disclosure requirements (with or without public disclosure of the manager’s private signal, \( \theta \)).

In this subsection I focus on the two regimes in which the manager is forbidden to trade: one regime in which she can keep her signal private and one regime in which, by some means or other, she has to make \( \theta \) public. I will first define the interim equilibrium when the manager is not allowed to trade and, after this, I will provide analytical expression for all the parameters in the overall equilibrium, except for the manager’s compensation package, which is obtained numerically and discussed in subsection 1.4.3.

**Definition 2 (Interim equilibrium)** An interim equilibrium in the trading round when the manager is not allowed to trade is an equilibrium price function \( p(\cdot) \) and a 3-tuple of demand functions for the price-taker investors \((x_E(\cdot), x_H(\cdot), x_L(\cdot))\) such that:
1. The equilibrium price is measurable with respect to the information available to the agents that trade. Therefore \( p : [0, 1] \rightarrow \mathbb{R} \) has to be a Borell-measurable function if \( \theta \) is not publicly known, and \( p : \mathbb{R} \times [0, 1] \rightarrow \mathbb{R} \) has to be a Borell-measurable function if \( \theta \) is made public.

2. Optimality of investors’ demand: entrepreneurs choose their demand functions, \( x_E \), in order to maximize their expected utility

\[
E[U_E(W_E) | I_E],
\]

subject to their budget constraint 1.4.5.

At the same time, traders of \( i \)-type (for \( i = H, L \)) choose their demand functions, \( x_i \), in order to maximize their expected utility

\[
E[U_i(W_i) | I_i],
\]

subject to their budget constraint

\[
W_i + x_i p = x_i v.
\]

3. Finally, the third condition is the market clearing condition:

\[
x_E + \lambda x_H + (1 - \lambda) x_L = 1 - x_{M,0} \tag{1.4.7}
\]

Before solving for the equilibrium in the trading round, I note that the manager chooses the precision of her signal before obtaining any private information; therefore \( \tau_e \) can be inferred by all market participants\(^{23}\), regardless of whether \( \theta \) is publicly revealed or not. If \( \tau_e = 0 \), the optimal investment policy is \( k = 0 \); and as a consequence, the terminal value of the firm and the equilibrium price will also be 0. The equilibrium in the trading round deserves attention only when \( \tau_e > 0 \). I will later see under which conditions \( \tau_e > 0 \). Furthermore, I will also assume that the investment is a linear function of the manager’s signal, \( k = \kappa \theta \). I will show that this is optimal for the manager.

The following proposition characterizes the equilibrium in the trading round:

**Proposition 1 (Characterization of the interim equilibrium)** Let’s assume that the precision of the manager signal, \( \tau_e = \frac{1}{\sigma^2} \), is strictly positive, that not all the firm shares are held by the manager, \( x_{M,0} < 1 \), and that the investment function, \( k \), is a linear function of the manager’s signal, \( k = \kappa \theta \)\(^{24}\). Then:

\(^{23}\)They only have to solve the manager’s problem of choosing effort.

\(^{24}\)I will show in proposition 2 that a sufficient condition for \( k(\cdot) \) being a linear function of \( \theta \) is \( x_{M,0} > 0 \).
1. If there is public revelation of the manager’s signal, $\theta$, the equilibrium investor’s demands will be

$$x_i = \frac{k \beta_{\pi, \theta} \theta - \frac{c_k}{2} k^2 - p}{r_i k^2 \sigma_z^2 \beta_{\pi, \theta}}$$

for $i = I, H, \text{and } L$; and the equilibrium price can be written as

$$p = k \beta_{\pi, \theta} \theta - \frac{c_k}{2} k^2 - r_{IHL} k^2 \sigma_z^2 \beta_{\pi, \theta} (1 - x_{M,0}).$$

2. If there is not public revelation of $\theta$ and if $\mu_v - \frac{\kappa c_k (\sigma_z^2 + \sigma_\varepsilon^2)}{2} > 0$, the equilibrium investor’s demands will be

$$x_i = \frac{2 p \mu_v + \kappa \sigma_z^2 \sigma_\varepsilon^2 - \sqrt{(\kappa \sigma_z^2 \sigma_\varepsilon^2)^2 + 4 p^2 \mu_v^2 + 4 p^2 \sigma_z^2 \sigma_\varepsilon^2}}{2 \kappa r_i \sigma_z^2 \sigma_\varepsilon^2 p},$$

for $i = I, H, \text{and } L$; and the equilibrium price can be written as

$$p = \frac{\kappa \mu_v - \kappa^2 r_{IHL} \sigma_z^2 \sigma_\varepsilon^2 (1 - x_{M,0})}{1 + 2 \kappa \mu_v, r_{IHL} (1 - x_{M,0}) - \kappa^2 r_{IHL}^2 \sigma_z^2 \sigma_\varepsilon^2 (1 - x_{M,0})^2}.$$
compensation package, \( x_M = x_{M,0} \). A closed form expression for the optimal \( k \) is given in the following proposition, which is proved in the appendix.

**Proposition 2 (Investment function)** If the manager is not allowed to trade, and assuming that \( x_{M,0} > 0 \), she will choose a level of investment that will be a linear function of her signal:

\[
k = \kappa \theta,
\]

where

\[
\kappa = \frac{\beta_{\pi, \theta}}{c_k + r_M x_{M,0} \sigma_{\pi}^2 \beta_{\pi, \theta}},
\]

and \( \beta_{\pi, \theta} \) is defined as in 1.4.13. Furthermore, it will happen that \( \mu_v - \frac{\kappa c_k (\sigma_{\pi}^2 + \sigma_{\theta}^2)}{2} > 0 \) where \( \mu_v \) is defined as in 1.4.14.

Before observing \( \theta \), the manager will choose the precision of her signal, \( \tau_{\epsilon} \), in order to maximize her expected utility net of information gathering costs. The following proposition characterizes the equilibrium \( \tau_{\epsilon} \) when the manager is not allowed to trade, satisfying condition 3 in definition 1.

**Proposition 3 (Precision of the manager’s signal)** If the manager is not allowed to trade, the equilibrium precision of the manager’s signal, under condition 3 in definition 1, will be given by

\[
\tau_{\epsilon} = \max \left( 0, \frac{\sqrt{c_{\epsilon} x_{M,0} (r_M^2 x_{M,0} c_{\epsilon} + 2 c_k)} - r_M x_{M,0} \sigma_{\pi}^2}{2 c_{\epsilon} c_k} \right),
\]

where \( \mu_v \) is defined as in 1.4.14.

From equation 1.4.17, it is clear that, if the manager cannot trade and she is not endowed with shares \( (X_{M,0} = 0) \), she will not exert effort and the precision of her signal will be zero. Therefore, in this case, the level of investment will be zero and the project will not be undertaken. Note also that, even if the manager owns the hole the project \( (x_{M,0} = 1) \), it is possible that it is not worth for her to gather any information. Whenever \( x_{M,0} > 0 \), if \( \sigma_{\pi}^2 \) is large enough i.e. there are profit opportunities and the cost of learning, \( c_{\epsilon} \), and the cost of capital \( c_k \) are low enough, it will happen that \( \tau_{\epsilon} > 0 \).

I compute the optimal contract and ex-ante investors’ welfare numerically. As is required in point 4 from definition 1, entrepreneurs will choose \( x_{M,0} \) and \( W_{M,0} \) in order to maximize their expected utility subject to their budget constraint and the manager’s participation constraint. I present the solution to the manager’s compensation problem for all IT regimes in section 1.4.3 and resulting ex-ante investors’ welfare in section 1.5.
1.4.2 Equilibrium with insider trading

The manager is allowed to trade in four of the six regimes that are the focus of this paper. There are two regimes in which she is allowed to purchase firm shares, but not to sell them (one with public disclosure of the manager’s private signal, \( \theta \), and one without), and two corresponding regimes in which she is allowed to purchase and sell shares, but not short-selling them. In this subsection, I give a formal definition of the interim equilibrium in the trading round when the manager is allowed to trade. I also provide an analytic expression for the interim equilibrium in the two regimes in which the manager can trade but has to disclose her private signal, \( \theta \). Given that I solve numerically for the interim equilibrium when the manager is allowed to trade and keeps her signal private, I provide a formal definition of the interim equilibrium suitable for numerical solution. The remaining components of the overall equilibrium are computed numerically, and they are discussed in subsection 1.4.3.

Definition 3 (Interim equilibrium) An interim equilibrium in the trading round when the manager is allowed to trade is an equilibrium price function \( p(\cdot) \) and a 3-tuple of demand functions for the price-taker investors \((x_E(\cdot), x_H(\cdot), x_L(\cdot))\) such that:

1. The equilibrium price is measurable with respect to the information available to the agents that trade. That is: \( p : \mathbb{R} \times [0, 1] \rightarrow \mathbb{R} \) is a Borel-measurable function.

2. Optimality of investors’ demand: entrepreneurs choose their demand functions, \( x_E \), in order to maximize their expected utility

\[
E[U_E(W_E) \mid \mathcal{I}_I],
\]

subject to their budget constraint 1.4.5.

At the same time, traders of i-type (for i = H, L) choose their demand functions, \( x_i \), in order to maximize their expected utility

\[
E[U_i(W_i) \mid \mathcal{I}_i],
\]

subject to their budget constraint

\[
W_i + x_i p = x_i v.
\]

---

25 The two regimes in which IT is completely forbidden are discussed in the previous subsection.
26 As noted before, the notion of interim equilibrium in the trading round depends on whether the manager is allowed to trade or not.
27 The notion of Rational Expectations Equilibrium is generalized to an \( \epsilon \)-Rational Expectations Equilibrium.
28 Table 1.1 summarizes which equilibrium components are obtained from analytical expressions and which are computed numerically.
3. Optimality of manager’s demand: The manager maximizes her expected utility

\[ E[U_M(W_M) | \mathcal{I}_M]; \]

subject to her budget constraint 1.4.2, the market clearing condition

\[ x_M + x_E + \lambda x_H + (1 - \lambda) x_L = 1, \]

and the insider trading restriction \( x_M \geq x^* \).

Note that there is only one non-price-taker trader, the manager; therefore, the manager’s problem can be seen as picking the optimal price on the investors’ aggregated demand function.

Note that if the manager’s signal is not public information, that is, if \( \theta \) is kept secret, and the manager can trade but under some restrictions, \( x_M > -\infty \), it is likely that there will be multiplicity of interim equilibria: multiplicity of equilibria has been found in models with similar restrictions, for instance Basak et al. (2006) or Marin and Olivier (2008). On the other hand, in the cases in which the manager cannot trade or there is full revelation of information, the interim equilibrium is unique.

I have got analytic expressions for the case in which there is public revelation of manager’s private information, this is the case in which all investors can observe the level of investment, \( k \). Proposition 4 characterizes the interim equilibrium when the manager is allowed to trade and the manager’s signal \( \theta \) is publicly known.

**Proposition 4 (Interim equilibrium when \( \theta \) is public)** If the level of investment \( k \) is different from zero, the equilibrium investor’s demands will be

\[ x_i = \frac{k \beta_{\pi, \theta} \theta - \frac{c}{2} k^2 - p}{r_i k^2 \sigma^2 \beta_{\pi, \theta}} \]

for \( i = I, H, \) and \( L \), where \( \beta_{\pi, \theta} \) is defined in equation 1.4.13.

Defining \( r_{IHL} \) as in 1.4.12, the manager’s demand will be

\[ x_M = \max \left\{ \frac{r_{IHL}(1 - x_{M,0})}{2 r_{IHL} + r_M}, x_M \right\} \]

and the equilibrium price can be written as

\[ p = k \beta_{\pi, \theta} \theta - \frac{c}{2} k^2 - r_{IHL} k^2 \sigma^2 \beta_{\pi, \theta}(1 - x_M). \quad (1.4.18) \]

The interim equilibrium in the trading round for the case in which manager’s information is kept private is analyzed numerically, as I do for the solution for the optimal levels of investment, the effort gathering information and the optimal contract. In 1.4.3, I analyze and compare equilibrium and agent’s decisions for the six different legal regimes that I am examining.
1.4 Equilibrium

Given that I can't obtain a closed form solution for the interim equilibrium in the case in which the manager is allowed to trade and there is not revelation of her private information, I have computed a numerical approximation to the RRE based on the projection method used by Bernardo and Judd (2000). As a consequence, in the mentioned case, I will not estimate a RRE, but a \( \epsilon \)-rational expectations equilibrium (\( \epsilon \)-REE). In an \( \epsilon \)-REE, for all states in a set of probability \( 1 - \epsilon \), the decisions of all traders are nearly optimal, with the absolute value of their relative error not larger than \( \epsilon \). More formally:

**Definition 4 (\( \epsilon \)-Rational Expectations Equilibrium)** An \( \epsilon \)-REE in the trading round when the manager is allowed to trade is an equilibrium price function \( p(\cdot) \) and a 3-tuple of demand functions for the price-taker investors \( (x_E(\cdot), x_H(\cdot), x_L(\cdot)) \) such that:

1. The equilibrium price is measurable with respect to the information available to the agents that trade. In other words,
   \[
   p: \mathbb{R} \times [0, 1] \to \mathbb{R} \quad (\theta, \lambda) \to p(\theta, \lambda)
   \]
   is a Borell-measurable function.

2. Optimality of investors’ demand for all states in a set of probability \( 1 - \epsilon \):
   \[
   \left| \mathbb{E}\left[ (v - p)U'_i(x_i(v - p)) \bigg| I_i \right] \right| \leq \epsilon \quad \text{for} \ i = H, L.
   \]

3. Optimality of manager’s demand:
   \[
   x_M = \arg \max E\left[ U_i(x_M(v - p) + px_{M,0} - c_\epsilon \tau_\epsilon + W_{M,0}) \bigg| I_M \right];
   \]
   subject to the market clearing condition
   \[
   x_M + x_E + \lambda x_H + (1 - \lambda) x_L = 1,
   \]
   and the insider trading restriction \( x_M \geq x_M \).

As before, the manager’s problem consist of picking the optimal price on the investors’ aggregated demand function.

Regarding the uniqueness of equilibrium, I have mentioned before that when the manager signal is not public information, that is, when \( \theta \) is kept secret, and the

\[29\]Note that the \( \epsilon \) in the definition of \( \epsilon \)-REE is different and not related to the random variable that models the error in the manager’s signal, \( \epsilon \).
manager can trade but under some restrictions, $x_M > -\infty$, there might be multiplicity of interim equilibria; moreover, an $\epsilon$-REE is not unique by definition. To explore all possible equilibria is behind the objective of this paper, therefore I will focus on the $\epsilon$-REE obtained using the methodology described in the Appendix 1.B.

### 1.4.3 Analysis of the equilibrium

In this section I analyze and compare the equilibrium under the six legal regimes itemized in page 9. Following the sequence in the definition of equilibrium, definition 1, I will start by analyzing the interim equilibrium in the trading round; next, the investment function; after this, the precision of the manager’s signal; and finally, the manager’s contract. Comparing the investors’ welfare in the different regimes will be done in section 1.5.

Most of the analysis of the equilibrium is done numerically; table 1.1 summarizes which components of the equilibrium have been obtained analytically, and which have been calculated numerically. The base case parameterization that I use is given by $r_M = 4$, $r_E = 2$, $r_H = 3$, $r_L = 1$, $c_k = 0.01$, $c_\varepsilon = 0.0001$, $\sigma_\pi = 0.15$, and $U_M = -1$. Bernardo (2001) chose a risk aversion coefficient for the manager of 3, 2 for the $H$-type traders, and 1 for the $L$-type, Bernardo’s choice was based on median CEO shareholdings relative to wealth. The standard deviation of the profitability per unit of investment, $\sigma_\pi$, is assumed to be 15%\(^{30}\). The investment cost, $c_k$, has been chosen such that if the manager could perfectly forecast $\pi$ and could not trade, the level of investment would be $c_k^{-1} \pi = 100\pi$\(^{31}\). The cost of one unit of precision $c_\varepsilon$ equal to $10^{-4}$ implies that having a standard deviation of 5% in the noise component of the manager’s signal, $\varepsilon$, would roughly have a cash equivalent cost of eight hundred thousand dollars for a company with total assets of three hundred million dollars\(^{32}\). Finally the manager’s reservation utility, $U_M$, only impacts the fixed salary and therefore our conclusions will not depend on this parameter, we have taken it to be the equivalent of 0 euros, but I could have taken any other quantity, for instance the median annual CEO compensation. I don’t pretend that these parameter values are those that would face an average company, they are no more that reference points for comparative statics, but I have tried to choose a reasonable collection of parameters.

### Interim equilibrium

Interim equilibrium is characterized in proposition 1 for the regimes in which IT is forbidden, and in proposition 4 for the regimes in which the manager can trade, but must reveal her private information. When the manager is allowed to trade

---

\(^{30}\)It is the historical cross-sectional standard deviation of ROA according to Desai and Jin (2007)

\(^{31}\)In this case, assuming that total assets equal to three hundred million dollars and that $\pi$ is equal to 15%, one monetary unit in our model would be equivalent to twenty million dollars.

\(^{32}\)The cost of gathering information would be $c_\varepsilon \tau_\varepsilon = 10^{-4} 0.05^{-2} = 0.04 \text{ m.u.}$, given that 1 m.u. is equivalent to twenty million dollars the cost would be equivalent to eight hundred thousand dollars.
1.4 Equilibrium

Table 1.1. Components of the equilibrium that are computed analytically or numerically.

<table>
<thead>
<tr>
<th>Legal Regime</th>
<th>Interim equilibrium</th>
<th>Investment policy</th>
<th>Manager’s effort</th>
<th>Compensation contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT forbidden, θ public</td>
<td>Analytical</td>
<td>Analytical</td>
<td>Analytical</td>
<td>Numerical</td>
</tr>
<tr>
<td>Insider sales forbidden, θ public</td>
<td>Analytical</td>
<td>Numerical</td>
<td>Numerical</td>
<td>Numerical</td>
</tr>
<tr>
<td>IT allowed, θ public</td>
<td>Analytical</td>
<td>Analytical</td>
<td>Analytical</td>
<td>Numerical</td>
</tr>
<tr>
<td>Insider sales forbidden, θ private</td>
<td>Analytical</td>
<td>Analytical</td>
<td>Analytical</td>
<td>Numerical</td>
</tr>
<tr>
<td>IT allowed, θ private</td>
<td>Numerical</td>
<td>Numerical</td>
<td>Numerical</td>
<td>Numerical</td>
</tr>
</tbody>
</table>

and can keep her informational advantage the solution for the interim equilibrium is obtained numerically; the numerical approximation to the equilibrium is based on the projection method used by Bernardo and Judd (2000), and it is described in appendix 1.B.

In figure 1.2, I compare the interim equilibrium in the trading round for different insider trading regimes. The objective of this figure is to show that the equilibrium, in the case in which the manager can trade and there is not disclosure of manager’s private information, which I have computed numerically, behaves as it would be expected in the light of REE literature. The structure of the panel is the following. In the first row, I have plotted equilibrium prices, in the second, manager’s holdings, and in the third, entrepreneurs’ holdings. In the first column, I have fixed the manager’s signal, θ, to 0.15 and I represent equilibrium as a function of the fraction of highly risk averse traders, λ. In the second column, I have fixed λ = 0.5 and I represent equilibrium as a function of θ. Equilibrium is computed under our base case parameterization and assuming, in addition, that the precision of the manager signal is τε = 250, the level of investment k = 80θ, and the initial manager’s holdings are xM,0 = 0.1. Note that, latter, this variables will be endogenized.

Equilibrium prices are decreasing in λ because the larger is the aggregate risk aversion of investors, the largest is the risk premium. Note that the slope of the price as a function of λ is the largest when the manager can trade and there is not disclosure of her public information; this is because uninformed investors learn from prices and can not distinguish a lower price due to highly risk averse investors from a lower price due to low firm profitability, therefore a larger λ makes them suspicious that |θ| might be small.

Equilibrium prices are increasing in the manager’s private signal, θ33, for all regimes but when the manager cannot trade and there is not disclosure of θ; in this case, the price cannot depend on θ because manager’s private signal is orthogonal to the information sets of all traders. The slope is maximum when manager’s signal is public. In the case in which the manager can purchase but not sell stocks and there is not disclosure of her private signal, the price is constant for low θ and increasing when θ is large; this is because the manager does not purchase shares for small θ,

33They are increasing in |θ| and the graph is symmetric with respect to the axis θ = 0.
Figure 1.2. Equilibrium in the trading round: our base case parameterization is given by $r_M = 4$, $r_E = 2$, $r_H = 3$, $r_L = 1$, $c_k = 0.01$, $c_\varepsilon = 0.0001$, and $\sigma_\pi = 0.15$. Furthermore, I fix $\tau_\varepsilon = 250$, $k = 80 \theta$, and $x_{M,0} = 0.1$. On the left, $\theta$ is assumed to be equal to 0.15 and, on the right, $\lambda$ is assumed to be equal to 0.5.
and therefore the information that uninformed investors learn from prices does not change for small changes in $\theta$.

Manager’s holdings are increasing in the fraction of highly risk averse investors and her signal. They will be strictly increasing in $\lambda$ whenever she is allowed to trade and her required minimum holdings are less than her desired holdings: this is because, as $\lambda$ increases, she becomes less risk averse compared to other investors. Besides, manager’s holdings will be strictly increasing in $\theta$ whenever she is allowed to trade, her minimum holdings are not binding, and there is not disclosure of her private information. In the cases in which she is allowed to trade and has informational advantage, she will take advantage of this and increase her holdings as $\theta$ increases. When $\theta$ is public information her equilibrium holdings will not depend on this variable.

Finally, entrepreneurs’ demand is also increasing in $\lambda$ because, as the manager, they become less risk averse compared to other investors. The informational disadvantage of entrepreneurs with respect to the manager makes their demand to be decreasing in $\theta$, in particular, it will be strictly decreasing whenever the manager increases her holdings. Other uninformed traders will have equilibrium demands similar to those of entrepreneurs.

**Investment**

The optimal investment policy is solved analytically for the IT regimes in which trading by the manager is forbidden, and numerically for the IT regimes in which trading by the manager is allowed, see table 1.1. This investment policy, when the manager is not allowed to trade, is given by proposition 2. For the cases in which she is allowed to trade, I solve numerically the optimization problem stated in condition 2 in the definition of equilibrium. The methodology used to solve this problem is similar to the one used to solve for the interim equilibrium\(^{34}\), but simpler. The investment policy is approximated using Hermite polynomials, $k(\theta) = \sum_{i=0}^{N} \kappa_i H_i(\theta)$, where $H_i$ is the degree $i$ Hermite polynomial\(^{35}\); then, defining $\frac{dU_M(W_M)}{dk} = 0$ as the uninformed first order condition\(^{36}\), the optimal investment policy is calculated as the solution of the following $(N + 1)$ expectation conditions:

$$E\left[\frac{dU_M(W_M)}{dk} H_i(\theta)\right] = 0 \text{ for } i = 0 \ldots N.$$ 

The expectations and the solution of the previous equations are computed using the same method used to solve for the interim equilibrium; it is described in ap-

\(^{34}\)Described in appendix 1.B.

\(^{35}\)By symmetry, $k()$, has to be an odd function. Given that the parity of the subscript in Hermite polynomials, $H_i$, coincides with the parity of the polynomial, then $\kappa_i = 0$ for all $i$ even. Proposition 2 states that, when the manager is not allowed to trade, the optimal investment policy is linear in $\theta$; numerical analysis has revealed that when the manager can trade the optimal investment policy is also linear or close to linear in $\theta$.

\(^{36}\)The derivative on the left hand side of the first order condition is also computed numerically, using a simple three-point estimation.
The sensitivity of investment to manager’s signal, \(\frac{dk}{d\theta}\), is plotted in Figure 1.3; in the first panel, as a function of the initial manager’s holdings, in the second panel, as a function of the precision of her signal and, in the two panels at the bottom, as a function of two model parameters: the cost of capital and the standard deviation of profitability. The cost of gathering information does not affect \(\frac{dk}{d\theta}\) directly, but through other endogenous variables.

The sensitivity of investment to manager’s signal does not depend on whether the manager must reveal her private information before trading or not: the optimal sensitivity from the manager’s point of view depends on the trading constraints, but not the right to keep her informational advantage.

The larger are the manager’s initial holdings, the largest is the risk that she bears and, therefore, the lowest will be the sensitivity\(^{37}\). Note however, that if the manager is allowed to sell her initial holdings and for \(x_{M,0}\) larger than a certain threshold, the slope is smaller than under other IT regimes: this is because the manager can adjust her risk exposure by trading in the market instead of decreasing the sensitivity of investment. The regime in which only insider purchases are allowed is also the one in which, if initial holdings are hight enough, \(\frac{dk}{d\theta}\) will be the smallest: this is due to the fact that under this IT regime the final manager holdings will always be the largest. The precision of the manager’s signal has the opposite effect on \(\frac{dk}{d\theta}\): a larger precision makes increasing the investment sensibility less risky for the manager. Note that under certainty \(\frac{dk}{d\theta} = c_k^{-1}\); therefore, \(c_k^{-1}\) is an upper bond for \(\frac{dk}{d\theta}\). The sensitivity will decrease, in all IT regimes, as the cost of capital increases. The relation with the variability in the profitability is not monotonic: both the sensitivity and \(\sigma_\pi\) have an impact on the riskiness of the firm; if \(\sigma_\pi\) is small, the desired riskiness will be attained by higher sensitivities; if \(\sigma_\pi\) is big, the desired riskiness will be attained by lower sensitivities.

**Precision of the manager’s signal**

Similarly to the solution for the optimal investment policy, the effort exerted by the manager is solved analytically for the IT regimes in which trading by the manager is forbidden, and numerically for the IT regimes in which trading by the manager is allowed. The precision of the manager’s signal when the manager is not allowed to trade is given by proposition 3. For the cases in which she is allowed to trade, I solve numerically the optimization problem stated in condition 3 in the definition of equilibrium; to do so, the manager’s budget constraint, equation 1.4.2, is substituted into the objective function, equation 1.4.3, therefore, the optimization problem that is actually solved is the following:

\[
\max_{\tau_\varepsilon} \mathbb{E} \left[ U_M \left( x_M (k(\theta) \pi - \frac{c_k k(\theta)}{2}) - p (x_M - x_{M,0}) + W_{M,0} - c_\varepsilon \tau_\varepsilon \right) \right]
\]

\(^{37}\)Notice that, in this graph, the precision of the manager’s signal, \(\tau_\varepsilon\), is fixed to 200. I will show that, if \(\tau_\varepsilon\) is endogenous, it will be increasing in \(x_{M,0}\); given that \(\frac{dk}{d\theta}\) is increasing in \(\tau_\varepsilon\), the effect of \(x_{M,0}\) on \(\frac{dk}{d\theta}\) can be either positive or negative.
Figure 1.3. Sensitivity of investment to manager’s signal: our base case parameterization is given by $r_M = 4$, $r_E = 2$, $r_H = 3$, $r_L = 1$, $c_k = 0.01$, $c_\varepsilon = 0.0001$, and $\sigma_\pi = 0.15$. In all but the first panel $x_{M,0}$ is fixed to 0.5, and in all but the second panel $\tau_\varepsilon$ is assumed to be 200.
knowing that \( k(\cdot) \) and \((x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot), p(\cdot))\) will satisfy 1 and 2 in definition 1. I solve this optimization problem using a golden section search method.

In figure 1.4 I show the precision, \( \tau_\varepsilon \), that the manager will choose as a function of the fraction of shares she receives and the model’s primitives for each of the IT regimes. For all the values of the model primitives analyzed, the effort is largest either when IT is forbidden or either when IT is allowed without a disclose-or-abstain rule. If \( \theta \) is public, the manager not only loses her informational advantage, but also part of her hedging opportunities. It might seem puzzling that under a disclose-or-abstain rule, the effort is larger when the manager is allowed to sell shares: given that she can not profit from private information, she could simply sell a fraction of her shares and exert less effort. The reason is that \( \frac{dk}{d\theta} \) is also larger when the manager can buy and sell stocks than when she can only buy, and therefore it pays to exert more effort in the first case.

The relation between the fraction of company shares in the manager’s compensation package and the effort exerted in gathering information is not monotonic, see the first panel in figure 1.4. This is due to three main effects: the first is that the largest the initial holding the larger the incentive to gather information in order to take a good investment decision; the second is that the largest the initial holding, the largest are the hedging needs, and therefore the larger the incentive to gather less information; the third effect, related to the previous one, is only in place when the manager can profit from trading on her private information: the largest the initial holding the more she will trade for hedging purposes and less she will trade to profit from private information, therefore the lower the incentive to gather information. When IT is forbidden only the first effect is in place, therefore \( \tau_\varepsilon \) will be increasing in \( x_{M,0} \). Under a disclose-or-abstain rule, the two first effects are in place (if the manager is allowed to buy and sell). Finally, if trading in possession of insider information is allowed all three effects will be in place. Therefore, if managers are allowed to buy and sell stocks, the relationship between initial holdings and managerial effort will not be monotonic.

Figure 1.4 also shows the effect of the cost of capital, the cost of gathering information, and the variability in the profitability on the effort exerted by the manager. The effort, \( \tau_\varepsilon \), is decreasing in \( c_k \) because if capital is expensive information is less valuable since it is costly to take advantage of it. This is true except in the case in which trading in possession of private information is allowed and capital is very cheap, the cause of it is that, under these circumstances, the optimal \( \frac{dk}{d\theta} \) would be too high. \( \tau_\varepsilon \) is also decreasing in the cost of gathering information, the amount of information gathered is larger the cheaper is this information. Finally, similarly to what happens with \( x_{M,0} \), the relation with the variability in the profitability is not monotonic if IT is allowed; if IT is forbidden, as can be seen in the figure and in equation 1.4.16, the relation is monotonically increasing.
Figure 1.4. Precision of the manager’s signal, $\tau_\varepsilon$: Our base case parameterization is given by $r_M = 4$, $r_E = 2$, $r_H = 3$, $r_L = 1$, $c_k = 0.01$, $c_\varepsilon = 0.0001$, $\sigma_\pi = 0.15$ and $x_{M0} = 0.3$
Manager’s compensation

The last piece of the model solution is solving the contract problem, that is, condition 4 in the definition of equilibrium, definition 1. This is a constrained optimization problem in which the objective function is the entrepreneurs’ expected utility, equation 1.4.4. Noting that the entrepreneurs’ budget constraint, equation 1.4.5, can be substituted into the objective function, and that the manager’s budget constraint, equation 1.4.2, can be substituted into the manager’s participation constraint, equation 1.4.6, the optimization problem becomes:

$$\max_{x_{M,0}, W_{M,0}} E \left[ U_E \left( x_E v + p \left( 1 - x_{M,0} - x_E \right) - W_{M,0} \right) \right],$$

subject to

$$E \left[ U_M \left( x_M k(\theta) \pi - \frac{c_k}{2} k(\theta)^2 \right) + W_{M,0} - p \left( x_M - x_{M,0} \right) - c_v \tau_e \right] \geq U_M$$

given the insider trading regime, and knowing that $\tau_e$, $k(\cdot)$, and $(x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot), p(\cdot))$ will satisfy conditions 1, 2, and 3 in definition 1.

Observing that the entrepreneurs’ expected utility is strictly decreasing in the manager’s fixed salary, $W_{M,0}$, and the manager’s expected utility is strictly increasing in $W_{M,0}$, the previous problem can be rewritten as

$$\max_{x_{M,0}} E \left[ U_E \left( x_E v + p \left( 1 - x_{M,0} - x_E \right) - W_{M,0} \right) \right], \quad (1.4.19)$$

given the insider trading regime, knowing that $\tau_e$, $k(\cdot)$, and $(x_M(\cdot), x_E(\cdot), x_L(\cdot), x_H(\cdot), p(\cdot))$ will satisfy conditions 1, 2, and 3 in definition 1, and taking into account that $W_{M,0}$ is function of $x_{M,0}$ and that this function is implicitly defined by the following equation

$$E \left[ U_M \left( x_M k(\theta) \pi - \frac{c_k}{2} k(\theta)^2 \right) + W_{M,0} - p \left( x_M - x_{M,0} \right) - c_v \tau_e \right] = U_M$$

The solution of 1.4.19 is computed numerically using a the golden section search method for all insider trading regimes. In figure 1.5 I display graphically the equilibrium contracts varying some of the parameters in our model. On the left hand side, I plot the relation of the model primitives with the amount of shares that is granted to the manager. On the right hand side, I plot the relation of the model primitives with the manager’s fixed salary.

Fixed salary is lower when the manager is allowed to sell shares (with or without disclosure of private information), and they are the lowest when the manager does not have to disclose her private information. However, the right to buy shares, even with private information, does not decrease the fixed salary relative to the case in which IT is forbidden, what reduces is the compensation in the form of equity shares.

---

38 This implies that all the previous optimization problems need to be solved every time that the objective function is evaluated.
The fact that explicit pay is larger in firms in which IT is forbidden relative to firms in which insiders can buy and sell stocks is consistent with Roulstone (2003)\(^{39}\), and Brenner (2010)\(^{40}\) empirical findings, however, the effect would result from the right to sell shares. Furthermore, figure 1.5 also shows that part of the pay increase when IT is forbidden compensates for the loss of gains from exploiting private information and part compensates for the reduction of the managers hedging opportunities.

If IT is forbidden, the amount of shares granted to the manager will be larger when there is revelation of her public information, for most of the parameters values; in the cases in which it is true, the reason is that the revelation of the manager’s signal reduces the hedging opportunities of entrepreneurs, therefore they reduce their holdings increasing those of the manager. Under a disclose-or-abstain rule \(x_{M,0}\) will be larger if the manager is allowed to sell shares than if the manager is only allowed to buy; if the manager is not allowed to sell shares and she want to reduce the risk she is bearing she will do it reducing \(\frac{dk}{d\theta}\), which might hurt entrepreneurs, on the other hand, if she can sell, she will reduce her position instead of reducing \(\frac{dk}{d\theta}\). When trading in possession of private information is allowed, \(x_{M,0}\) will also be larger if the manager is allowed to sell shares; the only exception that can be seen in the figures is when \(\sigma_s\) is small, in this case the hedging needs of the manager will be low and she will trade more to exploit her private information.

The relationship of the manager’s fixed salary with the model primitives is quite simple. The larger the costs, both, the cost of capital and the cost of gathering information, the largest will be the fixed salary paid to the manager in order to meet her reservation utility. On the other hand, the larger is the variability of the profitability, the lower the fixed salary. The relations between the model primitives and the amount of shares granted to the manager will not be necessarily monotonic, this is a consequence of the different effects that influence the optimal amount of shares granted to the manager in the compensation package.

**Precision of the manager’s signal with endogenous contract**

In order to better understand the welfare implications of the different IT regimes, in figure 1.6 I plot managerial effort as a function of market primitives when the contract is optimally chosen \(^{41}\). When the contract is endogenized two types of effect are present: the direct effects of the model primitive, as in figure 1.4, and indirect effects of the primitive through \(x_{M,0}\) the amount of shares granted to the manager. The welfare implications of the different IT regimes are discussed in the next section.

---

\(^{39}\)Roulstone (2003) found that firms that restrict IT outside trading windows following earnings announcements pay a premium in manager's total compensation.

\(^{40}\)Brenner (2010) found an increase, after the prohibition on IT for German firms, in the explicit pay component, but only for firms with the most liquid stock and a few board members; this is the kind of firms in which IT might have been more profitable due to the liquidity of the stock and a small number of competing insiders.

\(^{41}\)This figure differs from figure 1.4 because in 1.4 the contract is taken as a model primitive.
Figure 1.5. Optimal contract as a function of $c_k$, $c_\varepsilon$, and $\sigma_\pi$: Manager’s contracts are pairs $(x_{M,0}, W_{M,0})$ where $x_{M,0}$ is the fraction of shares and $W_{M,0}$ is the fixed salary. Our base case parameterization is given by $r_M = 4$, $r_E = 2$, $r_H = 3$, $r_L = 1$, $c_k = 0.01$, $c_\varepsilon = 0.0001$, and $\sigma_\pi = 0.15$. 
Figure 1.6. Optimal $\tau_\varepsilon$ when the contract is optimally chosen, as a function of $c_k$, $c_\varepsilon$, and $\sigma_\pi$: Our base case parameterization is given by $r_M = 2$, $r_E = 3$, $r_H = 3$, $r_L = 1$, $c_k = 0.01$, $c_\varepsilon = 0.0001$, and $\sigma_\pi = 0.15$. 
### 1.5 Welfare Analysis

In this section, I analyze the welfare consequences of different insider trading regimes. Following what is generally accepted in the related literature, I define the welfare of a participant in the market as her ex-ante expected utility. By construction, the manager’s welfare will be constant and equal to her reservation value, $U_M$, therefore only the welfare of the initial and traders is a quantity of interest. Regarding the traders, note that their welfare is computed as their expected utility before they know their own risk aversion.

Once disregarded the manager’s welfare as uninteresting, we will focus on the analysis of the entrepreneurs’ welfare and the social welfare. Entrepreneurs, being the firm’s initial shareholders and being those that hire the manager and design the contract, might also be the ones that choose the insider trading regime. If the insider trading regime that maximizes entrepreneurs welfare differs from the insider trading regime that maximizes social welfare, then there is room for a regulator to impose the latter. I will assume that the regulator gives the same weight to the welfare of entrepreneurs and traders and, therefore, social welfare will be defined as the sum of these two quantities.\(^\text{42}\)

Before discussing the impact of disclosure of information and insider trading on welfare, let’s see how the model’s primitives affect welfare. As can be seen in figure 1.7, for the collection of parameter values that I consider, welfare is decreasing in the cost of capital, \(c_k\), and the cost of obtaining information \(c_\varepsilon\), but increasing in the standard deviation of the profitability per unit of investment \(\sigma_\pi\).\(^\text{43}\)

Regarding the optimal insider trading regime, it is fairly clear from figure 1.7 that IT regimes in which there is disclosure of the manager’s information (either because disclosure is compulsory or because a disclose-or-abstain rule) are dominated by IT regimes in which the manager can keep her signal private. This effect does not come mainly from differences in the effort exerted by the manager; but from the loss of hedging opportunities that early revelation causes to the investors. This is particularly clear comparing welfare in the two regimes in which IT is forbidden, the two continuous thicker lines in figure 1.7, the loss of welfare due to the early revelation of information is the Hirshleifer effect.

In the range of parameters analyzed, the optimal IT regime is either a regime in which IT is completely forbidden or a regime in which IT is allowed; furthermore, the regime preferred by entrepreneurs does not need to be what maximizes social welfare. Note that in the cases in which allowing IT is optimal, it is optimal to allow the manager to buy and sell shares; however, when forbidding IT is optimal, allowing insider purchases but not insider sales is second best.

If the cost of capital is small, entrepreneurs will forbid the manager to trade, if the cost of capital is high they will allow IT. However, if the cost of capital is high, but

---

\(^\text{42}\)Note that my objective is to see whether, and under which circumstances, the IT policy that entrepreneurs would choose differs from the socially optimal regulation.

\(^\text{43}\)The expected value of the firm is $\kappa \sigma_\pi^2 - \kappa^2 c_\varepsilon \sigma_\pi (\sigma_\varepsilon^2 + \sigma_\pi^2)$, which is increasing in $\sigma_\pi^2$ if $\frac{\kappa c_\varepsilon}{2} < 1$. 
not high enough, entrepreneurs would allow the manager to trade even if it would not be socially optimal. Only for high costs of capital IT will be both, optimal for entrepreneurs and socially optimal. This is because the cheaper the capital the more valuable is information, therefore more of information would be gathered by the manager, \textit{ceteris paribus}; however this also increases the \textit{Hirshleifer effect} if all, or part, of this information is disclosed; this goes in favor of forbidding IT and the early disclosure of information when the cost of capital is low. There is also another effect of the loss of hedging opportunities through the effort exerted by the manager: It can be seen in figure 1.6 that when the cost of capital is low and IT is allowed insiders will gather less information, this is because the manager takes into account the \textit{Hirshleifer effect} when deciding on the level of effort to exert.

The relation between the optimal IT policy and the cost of acquiring information is similar to the relation with the cost of capital. When this cost is large, allowing IT is the optimal policy; but when this cost is low, it might be optimal to forbid the manager to trade. If IT is allowed, managerial effort will be the largest for any value of $c_e$ (for the set of parameters analyzed), and also the fixed salary will be the lowest, see figures 1.6 and 1.5. However, if information is very cheap for the manager to acquire, she will acquire a very precise signal, and if she is allowed to trade, the price will end up revealing a lot about the firm’s prospects, and it will hurt investors trading opportunities: this is the reason why, when $c_e$ is small, forbidding IT might be optimal.

The relation of the volatility of profitability with the optimal IT regime is the opposite of the previous two parameters discussed. If $\sigma_\pi$ is small allowing the manager to trade freely will be optimal, while if $\sigma_\pi$ is large the optimal policy will be to forbid IT. Comparing welfare in the two regimes in which IT is forbidden, it is clear that the larger the volatility of profitability the largest is the welfare due to the reduction in hedging opportunities caused by the early revelation of information; intuitively, the largest the risk the more valuable are the hedging opportunities. Therefore, for large values of $\sigma_\pi$, the \textit{Hirshleifer effect} is large enough so that forbidding IT is optimal.

To sum up, three conclusions can be extracted from figure 1.7. First, a regime which imposes a \textit{disclose-or-abstain} rule is always dominated. Second, if the cost of capital ($c_k$) is high, or the firm operates in an industry in which forecasting by the manager is specially difficult ($c_e$ large), or the volatility of profitability ($\sigma_\pi$) is low, then entrepreneurs will choose to allow the manager to trade on the basis of her private information; on the other hand if the cost of capital is low, information is easy to obtain by the manager, or the volatility of profitability is high, then entrepreneurs will forbid the manager from trading. Third, there are cases in which entrepreneurs might choose to allow the manager to trade on the basis of her private information but this decision is not socially optimal, therefore there is room for IT regulation, forbidding IT, in these cases.
Figure 1.7. Welfare as a function of $c_k$, $c_\epsilon$, and $\sigma_\pi$: our base case parameterization is given by $r_M = 4$, $r_E = 2$, $r_H = 3$, $r_L = 1$, $c_k = 0.01$, $c_\epsilon = 0.0001$, and $\sigma_\pi = 0.15$. 
1.6 Concluding Remarks

In this paper I attempt to contribute into the insider trading (IT) debate by proposing a model in which the welfare consequences of granting insider trading rights to a corporate manager as a compensation mechanism can be stated. The model incorporates many of the effects of IT on the welfare of economic agents that have been discussed in previous literature and, furthermore, it also contemplates two agency problems between the manager and shareholders. These two agency problems are to motivate the manager to exert costly effort to gather information that is socially useful, and to induce her to choose what would be the optimal response to this information from the point of view of shareholders. The model proposed here also delivers a set of implications that could be tested empirically and can be extended to analyze further executive compensation schemes.

Due to the complexity of the model, equilibrium has to be solved numerically. To do so, this paper also contributes to the literature by adapting the projection method to accommodate a participant (the manager) that is aware of her position in the market and acts as an information monopolist.

In the context of the framework proposed here, it is shown that a disclose-or-abstain rule is never an optimal regulation and it destroys investors’ welfare, mainly because of the loss of hedging opportunities that early revelation causes to market participants. Therefore, either completely forbidding IT without disclosing manager’s private information or completely allowing it are the optimal IT regimes. In industries with high cost of capital, in which the future is especially difficult to foresee, or in which profits are highly volatile, it will be optimal for entrepreneurs and investors to allow IT, in the opposite cases, it will be optimal for them to forbid IT. Furthermore, what is socially optimal might differ from what is optimal from the point of view of entrepreneurs; therefore, there might be room for state intervention to forbid insider trading in some cases.

1.A Proofs of the propositions

I will use the following lemma, whose proof can be found in Marín and Rahi (1999).

**Lemma 1** Let be $A$ a symmetric $n \times n$ matrix, $b$ an $n$-vector, $c$ a scalar, and $w$ an $n$-dimensional normal random vector, $w \sim \mathcal{N}(0,\Sigma)$. Then $E\left[\exp(w^T A w + b^T w + c)\right]$ is well-defined if and only if $(I - 2 \Sigma A)$ is positive definite, and in this case

$$E\left[\exp(w^T A w + b^T w + c)\right] = |I - 2 \Sigma A|^{-\frac{1}{2}} \exp\left(\frac{1}{2}b^T (I - 2 \Sigma A)^{-1} \Sigma b + c\right)$$
1.A.1 Proof of proposition 1

First, it is clear that equilibrium prices 1.4.9 and 1.4.11 satisfy the Borell-measurability condition in 1 of definition 2.

In the case in which \( \theta \) is public information, conditional to knowing \( \theta \), the value of the firm is normally distributed; therefore, I can use the properties of the CARA-Normal setup and the fact that \( \theta \in \mathcal{I}_i \) for all types \( i \) of investors, to write condition 2 in definition 2 as

\[
x_i = \arg \max E \left[ x_i (v - p) + p x_{i,0}\theta - \frac{r_i}{2} \text{Var} \left[ x_i (v - p) + p x_{i,0}\theta \right] \right]
= \arg \max x_i E [v|\theta] + p(x_{i,0} - x_i) - \frac{r_i}{2} x_i^2 \text{Var} [v|\theta]
= \arg \max x_i k E [\pi|\theta] - x_i \frac{c_k}{2} k^2 + p(x_{i,0} - x_i) - \frac{r_i}{2} x_i^2 k^2 \text{Var} [\pi|\theta],
\]

solving the first order conditions for the previous problem, I obtain

\[
x_i = \frac{k E [\pi|\theta] - \frac{c_k}{2} k^2 - p}{r_i k^2 \text{Var} [\pi|\theta]}
\]

Noting that \( E [\pi|\theta] = E \pi + \frac{\text{Cov}[\pi,\theta]}{\text{Var}[\theta]} (\theta - E \theta) \), \( \text{Var} [\pi|\theta] = \text{Var} [\pi] - \frac{\text{Cov}[\pi,\theta]^2}{\text{Var}[\theta]} \), \( E \pi = 0 \), \( E \theta = 0 \), \( \text{Var} [\pi] = \sigma^2 \pi \), \( \text{Var} [\theta] = \sigma^2 \pi + \sigma^2 \epsilon \), and \( \text{Cov} [\pi, \theta] = \sigma^2 \pi \); I can write 1.A.1 as 1.4.8.

Finally, using these optimal demands and imposing the market clearing condition 3 in definition 2, it is straightforward that the equilibrium price satisfies 1.4.9.

In the case in which \( \theta \) is not known by the investors and the manager does not trade, the investors’ information sets do not contain any payoff relevant information; therefore I can write the expected utility of an investor of type \( i \) as:

\[
E \left[ - \exp (w^T A_i w + c_i) \right] \text{ for } i \in \{I, H, L\},
\]

where

\[
w = \begin{pmatrix} \theta \\ \pi \end{pmatrix} \sim \mathcal{N}(0, \Sigma),
A_i = \begin{pmatrix} \frac{1}{2} r_i \ x_i \ c_k \ \kappa^2 \\ -\frac{1}{2} r_i \ x_i \ \kappa \end{pmatrix} \text{ for } i \in \{I, H, L\},
\]

\[
\Sigma = \begin{pmatrix} \sigma^2 \pi + \sigma^2 \epsilon \\ \sigma^2 \pi \ \\ \sigma^2 \pi \end{pmatrix},
\]

\[
c_E = -r_E (p(1 - x_{M,0} - x_i) - W_{M,0}), \text{ and}
\]

\[
c_i = r_i p x_i \text{ for } i \in \{H, L\}.
\]

By lemma 1, if

\[
1 + r_i \ x_i \ \kappa \left( \mu_v - \frac{\kappa c_k (\sigma^2 \pi + \sigma^2 \epsilon)}{2} \right) > 0,
\]

(1.A.2)
1.A Proofs of the propositions

and

\[ 1 - r_i^2 x_i^2 \kappa^2 \sigma_i^2 \sigma_i^2 + 2 r_i x_i \kappa \mu_v > 0 \quad (1.A.3) \]

where \( \mu_v = \sigma_v^2 - \frac{\kappa c_k (\sigma_i^2 + \sigma_v^2)}{2} \), then the investors’ problems can be written as

\[
x_E = \arg \max \frac{-\exp \left( -r_E \left(p(1-x_{M,0} - x_E) - W_{M,0}\right) \right)}{\sqrt{1 - r_E^2 x_E^2 \kappa^2 \sigma_v^2 \sigma_v^2 + 2 r_E x_E \kappa \mu_v}},
\]

for the entrepreneurs, and

\[
x_i = \arg \max \frac{-\exp \left( r_i p x_i \right)}{\sqrt{1 - r_i^2 x_i^2 \kappa^2 \sigma_i^2 \sigma_i^2 + 2 r_i x_i \kappa \mu_v}},
\]

for \( H \) and \( L \) type traders.

There are two candidates for optimal demand that satisfy the first order conditions of the previous problems. They are:

\[
2 p \mu_v + \kappa \sigma_v^2 \sigma_i^2 + \sqrt{(\kappa \sigma_i^2 \sigma_v^2)^2 + 4 p^2 \mu_v^2 + 4 p^2 \sigma_i^2 \sigma_v^2} \]

\[
= \frac{2 \kappa r_i \sigma_v^2 \sigma_i^2}{2 \kappa \mu_v \sigma_v^2 \sigma_i^2 p}
\]

and

\[
2 p \mu_v + \kappa \sigma_v^2 \sigma_i^2 - \sqrt{(\kappa \sigma_i^2 \sigma_v^2)^2 + 4 p^2 \mu_v^2 + 4 p^2 \sigma_i^2 \sigma_v^2} \]

\[
= \frac{2 \kappa r_i \sigma_v^2 \sigma_i^2}{2 \kappa \mu_v \sigma_v^2 \sigma_i^2 p},
\]

Note that the first candidate does not satisfy the condition 1.A.3, while the second does. Regarding 1.A.2, note that the sign of the optimal investor demand is the same for all investors, furthermore, given that \( x_{M,0} < 1 \), its sign must be positive by the market clearing condition; therefore, if \( \mu_v - \frac{\kappa c_k (\sigma_i^2 + \sigma_v^2)}{2} > 0 \), condition 1.A.2 will be satisfied. This proves 2 in definition 2.

Using these optimal demands and imposing the market clearing condition 1.4.7, I can see that the equilibrium price is given by

\[
p = \frac{\kappa \mu_v - \kappa^2 r_{IHL} \sigma_i^2 \sigma_i^2 (1 - x_{M,0})}{1 + 2 \kappa \mu_v r_{IHL} (1 - x_{M,0}) - \kappa^2 r_{IHL} \sigma_i^2 \sigma_i^2 (1 - x_{M,0})^2},
\]

where \( r_{IHL} = \frac{r_{E} r_{H} r_{I}}{r_{H} r_{L} + \lambda r_{E} r_{I} + (1 - \lambda) r_{E} r_{H}} \). Condition 3 in definition 2 is satisfied by construction.

1.A.2 Proof of proposition 2

Indeed, using the properties of the CARA-Normal setup, I can write condition 2 in definition 1 as

\[
k = \arg \max E \left[ x_{M,0} (k \pi - \frac{c_k}{2} k^2) | \theta \right] - \frac{r_{M}}{2} \text{Var} \left[ x_{M,0} (k \pi - \frac{c_k}{2} k^2) | \theta \right]
\]

\[
= \arg \max x_{M,0} \left( k E [\pi | \theta] - \frac{c_k}{2} k^2 \right) - \frac{r_{M}}{2} x_{M,0} k^2 \text{Var} [\pi | \theta].
\]
The first order condition of this problem is $x_{M,0} E[\pi|\theta] - k^* x_{M,0} (c_k + r_M x_{M,0} \text{Var}[\pi|\theta]) = 0$, and assuming that $x_{M,0} > 0$, I have

$$k = \frac{E[\pi|\theta]}{c_k + r_M x_{M,0} \text{Var}[\pi|\theta]},$$  \hspace{1cm} (1.A.4)

Noting that $E[\pi|\theta] = E \pi + \frac{\text{Cov}[\pi,\theta]}{\text{Var}[\theta]} (\theta - E \theta)$, $\text{Var}[\pi|\theta] = \text{Var}[\pi] - \frac{\text{Cov}[\pi,\theta]^2}{\text{Var}[\theta]}$, $E \pi = 0$, $E \theta = 0$, $\text{Var}[\pi] = \sigma_\pi^2$, $\text{Var}[\theta] = \sigma_\pi^2 + \sigma_\epsilon^2$, and $\text{Cov}[\pi, \theta] = \sigma_\pi \sigma_\epsilon$, I can rewrite 1.A.4 as in 1.4.15.

Furthermore,

$$\mu_\nu - \frac{\kappa c_k (\sigma_\pi^2 + \sigma_\epsilon^2)}{2} = \sigma_\pi^2 - \kappa c_k (\sigma_\pi^2 + \sigma_\epsilon^2),$$

and substituting $\kappa$ for its value

$$\sigma_\pi^2 - \kappa c_k (\sigma_\pi^2 + \sigma_\epsilon^2) = \sigma_\pi^2 - \frac{\sigma_\pi^2}{1 + \frac{r_M x_{M,0} \sigma_\pi^2 \sigma_\epsilon^2}{c_k (\sigma_\pi^2 + \sigma_\epsilon^2)}},$$

which is strictly larger than zero if $x_{M,0} > 0$.

1.A.3 Proof of proposition 3

Using the expression for the optimal investment function $k(\cdot)$ in 1.4.15, it is easy to check that the manager’s expected utility in 1.4.3 subject to the budget constraint 1.4.2 can be written as

$$E[-\exp(w^T A w + c)],$$

where $w = \begin{pmatrix} \theta \\ \pi \end{pmatrix} \sim \mathcal{N}(0, \Sigma)$, $A = \begin{pmatrix} \frac{1}{2} r_M x_{M,0} c_k \kappa^2 & -\frac{1}{2} r_M x_{M,0} \kappa \\ -\frac{1}{2} r_M x_{M,0} \kappa & 0 \end{pmatrix}$, $\Sigma = \begin{pmatrix} \sigma_\pi^2 + \sigma_\epsilon^2 & \sigma_\epsilon^2 \\ \sigma_\epsilon^2 & \sigma_\pi^2 \end{pmatrix}$, and $c = r_M c_\pi^2$.

Noting that

$$(I - 2 \Sigma A) = \begin{pmatrix} 1 + \kappa r_M x_{M,0} (\sigma_\pi^2 - \kappa c_k (\sigma_\pi^2 + \sigma_\epsilon^2)) & \kappa r_M x_{M,0} (\sigma_\pi^2 + \sigma_\epsilon^2) \\ \kappa r_M x_{M,0} (\sigma_\pi^2 + \sigma_\epsilon^2) & 1 + \kappa r_M x_{M,0} \sigma_\pi^2 \end{pmatrix},$$

is positive definite, by lemma 1, and doing the change of variables $\tau_\epsilon = \frac{1}{\sigma_\pi}$, I can write the manger’s problem of choosing the $\tau_\epsilon$ that maximizes 1.4.3 subject to 1.4.2 as

$$\tau_\epsilon^* = \arg \max \: -e^{r_M c_\pi} \tau_\epsilon \left( \frac{\tau_\epsilon \sigma_\pi^2 + 1}{\tau_\epsilon \sigma_\pi^2 + 1} \right)^{1/2}$$

or equivalently

$$\tau_\epsilon^* = \arg \min \: e^{2 r_M c_\pi} \tau_\epsilon \frac{\tau_\epsilon \sigma_\pi^2 + 1}{\tau_\epsilon \sigma_\pi^2 + 1} \left( \frac{\tau_\epsilon \sigma_\pi^2 + 1}{\tau_\epsilon \sigma_\pi^2 + 1} \right)^{1/2}.$$
Noting that the function to be minimized has a minimum for $\tau_\varepsilon \geq 0$ at
\[
\sqrt{c_\varepsilon x_{M,0}(r_M^2 x_{M,0} c_\varepsilon + 2 c_k) - r_M x_{M,0} \sigma_\pi^2 + 2 c_k c_k \sigma_\pi^2 c_k},
\]
if
\[
\sigma_\pi^2 > \frac{2 c_\varepsilon c_k}{\sqrt{c_\varepsilon x_{M,0}(r_M^2 x_{M,0} c_\varepsilon + 2 c_k) - r_M x_{M,0} c_\varepsilon}},
\]
and it is strictly increasing for $\tau_\varepsilon \geq 0$ otherwise, the result follows.

1.A.4 Proof of proposition 4

First, it is clear that equilibrium price 1.4.18 satisfies the Borell-measurability condition in 1 of definition 3.

From the proof of proposition 1, I know that
\[
x_i = kE[\pi|\theta] - \frac{c_k}{2} k^2 - p r_i k^2 \text{Var}[\pi|\theta] \quad \text{for all } i \in \{I, H, L\};
\]
therefore the investors’ aggregate demand, that I will denote by $x_{IHL}$ is given by
\[
x_{IHL} = kE[\pi|\theta] - \frac{c_k}{2} k^2 - p r_{IHL} k^2 \text{Var}[\pi|\theta],
\]
where $r_{IHL} = r_{HL} r_H r_L + \lambda r_{EL} r_L + (1-\lambda) r_E r_H$.

Given the normality of $v|\theta$, the manager’s problem can be written as
\[
x_M = \arg \max x_M kE[\pi|\theta] - x_M \frac{c_k}{2} k^2 + p(x_{M,0} - x_M) - \frac{r_M}{2} x_M^2 k^2 \text{Var}[\pi|\theta],
\]
subject to $x_{IHL} + x_M = 1$. Which is equivalent to choose $p$ in order to maximize
\[
(1-x_{IHL}) kE[\pi|\theta] - (1-x_{IHL}) \frac{c_k}{2} k^2 + p(x_{M,0} - (1-x_{IHL})) - \frac{r_M}{2} (1-x_{IHL})^2 k^2 \text{Var}[\pi|\theta];
\]
where I have to take into account that $x_{IHL}$ is a function of $p$.

Solving the fist order conditions for the previous problem, I find that the equilibrium price is given by
\[
p = kE[\pi|\theta] - \frac{c_k}{2} k^2 - r_{IHL} k^2 \text{Var}[\pi|\theta] \left(1 - \frac{r_{IHL}(1-x_{M,0})}{2 r_{IHL} + r_M}\right),
\]
and the manager’s demand is
\[
x_M = \frac{r_{IHL}(1-x_{M,0})}{2 r_{IHL} + r_M}.
\]
Note that the manager has a further restriction, \( x_M \geq \bar{x}_M \), therefore the manager’s demand will be

\[
x_M = \max \left\{ \frac{r_{IHL}(1 - x_{M,0})}{2r_{IHL} + r_M}, \bar{x}_M \right\},
\]

and the equilibrium price

\[
p = kE[\pi|\theta] - \frac{c_k}{2}k^2 - r_{IHL}k^2 \text{Var}[\pi|\theta](1 - x_M).
\]

As in the proof of proposition 2, I use that 

\[
E[\pi|\theta] = E[\pi] + \frac{\text{Cov}[\pi,\theta]}{\text{Var}[\theta]}(\theta - E[\theta]), \quad \text{Var}[\pi|\theta] = \text{Var}[\pi] - \frac{\text{Cov}[\pi,\theta]^2}{\text{Var}[\theta]}, \quad E[\pi] = 0, \quad E[\theta] = 0, \quad \text{Var}[\pi] = \sigma^2_\pi, \quad \text{Var}[\theta] = \sigma^2_\pi + \sigma^2_\varepsilon, \quad \text{and Cov}[\pi,\theta] = \sigma^2_\pi
\]
to derive the results in the proposition.

1.B Numerical analysis of the model

As I have mentioned in subsection 1.4.2, the numerical approximation to the equilibrium is based on the projection method used by Bernardo and Judd (2000), with the difference that one of the traders has market power.

To estimate the \( \epsilon \)-REE, the entrepreneurs’ demand, \( x_E(p) \), is approximated by finite-order polynomials, which transforms our problem of computing the equilibrium in an infinite dimensional space into estimating a finite number of parameters. In particular, we define the approximated equilibrium investors’ demand as

\[
\hat{x}_E(p) = \sum_{i=0}^{N} a_i H_i(p)
\]

where \( H_i \) is the degree \( i \) Hermite polynomial and \( N \) is the largest degree of the polynomial approximation. The choice of Hermite polynomials is because they are mutually orthogonal with respect to the normal density function with mean zero, the advantages of such a base of polynomials are discussed in Judd (1992). Our goal is to estimate the parameters \( a_i \) and, to do so, I will impose several conditions derived from the uninformed first order condition and market clearing.

Following Bernardo and Judd (2000) methodology, I numerically impose the conditional expectation first order condition in point 2 of Definition 4 as the \((N + 1)\) expectation conditions

\[
E \left[ (v - p)U'_E(\hat{x}_E(v - p) + p(1 - x_{M,0}) - W_{M,0}) H_i(p) \right] = 0 \text{ for } i = 0 \ldots N, \tag{1.B.1}
\]

where \( p \) is computed to satisfy condition 3 of Definition 4 for each state of the world\(^{44}\) and taking \( \hat{x}_E \) as given\(^{45}\).

\(^{44}\)In fact, I do not compute \( p \) for each state of world but for each Gaussian quadrature point.

\(^{45}\)Note that \( p(\theta, \lambda) \) depends on the parameters \( a_i \), therefore, in each iteration solving 1.B.1, the value of \( p(\theta, \lambda) \) changes.
The expectations are computed using Gaussian quadrature, whose nodes and weights are obtained from the routines `qnwnorm` and `qnwlege`, from COMPECON toolbox, written to accompany Miranda and Fackler (2002). I use 7 Gauss nodes for each normally distributed random variable and 11 Gauss nodes for $\lambda$, which is uniformly distributed, to compute the quadrature; I observe that increasing the number of points does not improve the estimation. Finally, to solve the resulting nonlinear system I use the trust-region dogleg algorithm as implemented in Matlab’s `fsolve`. 
Firms vs. Insiders as Traders of Last Resort

2.1 Introduction

Liquidity shocks can drive asset prices away from fundamental values. Corporate insiders are in a privileged position to assess the severity of the deviations. In the presence of large enough deviations, insiders can take two type of actions: execute trades on their own account (insider trading) or on the company’s account (through buy back programs or seasonal equity offerings). Figure 2.1 provides some preliminary evidence in favor of these two hypothesis. The figures display insiders and corporate trading activity around large returns. The two figures in the left hand side clearly show that insiders purchases and sales pick right after large negative and positive returns, respectively. A similar phenomena, but with some delay, is also observed in the case of seasonal equity offerings and stock repurchases.

In a very recent paper, Hong et al. (2008) study the case of stock repurchases and argue that companies act as traders of last resort and liquidity providers. In this paper we focus on insiders transactions and argue that insiders play a similar but distinct role. In particular, their trades affect both the volatility and the skewness of asset returns and the impact has a shorter life span than those generated by firms’ trades. Furthermore, the impact on volatility is weaker and the impact on skewness stronger for insiders transactions versus firms’ transactions. These results suggest that either corporate managers specialize on a different type of mispricing when trading on their own account versus the company’s account or that the market interprets both type of interventions differently. In the former case we could think of managers with a preference for positions on their own account in the presence of mispricings that revert fast. This speed of adjustment may obey to exogenous reasons (the nature of the shock itself) or may be endogenous as managers may be able to use their position to disclose figures in the income statement that speeds the revelation of the mis-valuation. Regarding the interpretation of each type of traders’ trades by market participants, it is important to notice that while firms...
Figure 2.1. Insider and firm trading around large price movements of individual stocks. This graph show insider sales and seasoned equity offerings around monthly returns smaller than -20%, and insider purchases and firm repurchases around monthly returns larger than 20%. The abscissa axis displays event time in months. The graphs show the time series average of the cross-sectional average by year.
transactions are preannounced (i.e., they are not anonymous), insiders’ trades are only disclosed after they take place\(^1\). This means the, unlike firms, insiders may face a strong adverse selection problem. All these considerations call for the need to develop a full theory that analyzes the tradeoffs involved in trading on the firm’s vs the manager’s account, in the presence of liquidity shocks, when moral hazard and adverse selection considerations are in place. This, however, is beyond the scope of the present paper.

The idea of insiders as traders of last resort may sound suspicious at first sight. First, there are the legal restrictions on insider trading activities. On this front we must realize that only insider trading in possession of material nonpublic information is illegal. Transactions by insiders as traders of last resort are not prohibited in general. An example of such legal insider trades is given by Seyhun (1998a):

\[
\ldots\text{Insiders can clearly trade on the basis of their understanding and interpretation of public information outside the moratorium periods. For instance, assume that the stock price of the firm goes down sharply. The decline of stock price is, after all, public information. Now suppose that insiders do not know anything about their firm that would justify such a price decline. Insiders in this case can comfortably buy stock of their firm (and support the market) without worrying about insider-trading regulations.}
\]

Second, we have witnessed by now more that twenty years of research, both theoretical and empirical, in market microstructure emphasizing somehow the opposite to what we claim here, namely, that insider trading generates volatility and reduces liquidity. Indeed we should expect that the larger the presence of informed traders, the larger the adverse selection in the market and consequently the larger the spreads and the lower the liquidity. This insight has even been documented empirically. For instance, Chung and Charoenwong (1998) show that bid-ask spreads are wider for stocks in which insiders are more active. In our view this is perfectly consistent with our hypothesis. Notice first that insider trading must be publicized. It is indeed the publicity of these trades what resolves uncertainty and information asymmetries in asset markets that results in smaller adverse selection driven spreads and restored liquidity levels.

A third concern is the size of trading by insiders. One may argue that while the size of, say, a company buyback program is big enough to provide actual counterpart to sellers, the typical size of insider purchases is too small for that purpose. While the argument is correct, it ignores what for us is critical: the informational content of the trade \textit{per se}. Insiders trades can be small in terms of share volume, but quite big in terms of information revelation.

A final concern is that insiders may be mainly trading for reasons other that profiting from perceived mispricing, in which case the impact of their trades on returns

---

\(^1\)The SEC requires insiders’ trading to be reported before the end of the second business day following the day of the transaction. Prior to August 29, 2002, reporting requirements were lighter. In particular, reports were required to be filled by the 10th of the month following the transaction.
should be negligible. For instance, portfolio rebalancing (diversification) and keeping a controlling stake are two clear motives for insider trading. In a classic paper, Lakonishok and Lee (2001) reached the conclusion that while insider purchases are driven by information, "insider selling that is motivated by private information is dominated by portfolio rebalancing for diversification purposes". There is, however, more recent evidence linking insider sales to crashes. Marin and Olivier (2008) show robust evidence of a path of insider’s high selling activity in the far past and low selling activity in the near past preceding large price drops. The current state of knowledge, hence, is that the information component in both insider sales and purchases is non negligible.

Although this view of insiders as traders of last resort we propose in this paper is new there is already some encouraging evidence. For instance, Seyhun (1990) shows that insiders bought large amounts of shares after the October 1987 crash. Indeed, our Figure 2.1 provides a stronger picture in this direction: not only insiders purchases pick after big negative returns, but also insiders sales after large positive returns. Marin and Olivier (2008) also provide evidence supportive of our hypothesis. In particular, they find that large drops in the price of a particular stock are more likely after a period of low insider trading volume (i.e. large negative returns happen in the absence of price support by insiders). Insiders also seem to trade in advance of the firm’s trades. Lee et al. (1992) found that insiders buy or decrease their sales prior to fixed price repurchase announcements by their firms; similarly, Jenter (2005) reports that, in years in which a firm issues new equity, its insiders sell between $1.4 and $1.5 million more equity.

The first goal of this paper is to develop a model of insider trading where insiders act as traders of last resort. The model is simple but rich enough to provide testable implication on the impact of insider trading on return distributions. We work out a there period extension of the Grossman-Stiglitz model (Grossman and Stiglitz (1980)) where insiders transactions are disclosed the period after they take place. We also introduce an exogenous trading cost for transactions done by insiders. The first departure is justified on institutional grounds as insiders in actual markets must report their trades before the second business day following the day of the transaction. The second departure makes insiders interventions only worthwhile when the mispricing is large enough. There are several reasons for insiders not to intervene when the mispricing is small. On the one hand, insiders transactions are scrutinized by the SEC, which means that insiders always face a positive probability of being prosecuted. On the other hand, typically insiders’ portfolios are overweighted on their own stock. This means insiders will only find worthwhile to increase their holdings when the mispricing is large enough to compensate their extra poor di-

---

2It is also important emphasizing here that non informational considerations are also relevant in the case of share repurchases. Many companies running out of good investment opportunities have often initiated general payout programs including stock repurchases. In this case the repurchases are not the result of perceived misvaluations. They may be associated to low future return volatility, but the latter is the result of the lack of good risky project rather than the repurchases themselves. Still is seems that misvaluation is the dominating factor. For instance, using survey data Brav et al. and Graham and Harvey conclude that most managers consider misvaluation important or very important when deciding whether to issue new stock or buy it back.
versification. All these considerations point at a cost of trading that, unlike other type of investors, insiders bear. Furthermore, the argument about diversification suggests this cost is larger for insiders purchases than for insiders sales\(^3\). Our model provides several testable implications. We focus on those related to the role of insiders as price supporters. The first prediction of the model is that in the absence of a strong adverse selection problem the short term volatility of the risky asset return is decreasing in both insiders sales and purchases. The second prediction is that the skewness of the risky asset return is increasing on insiders purchases and decreasing on insiders sales. All these predictions are corroborated in the empirical part of the paper.

As previously stated, very recently Hong et al. (2008) have found that firms might act as traders of last resort. Those firms that are less financially constrained can repurchase their own stock when the stock price is significantly lower than its fundamental value. They find evidence of lower short term return variance relative to long term return variance, and larger skewness in the distribution of returns, for those less financially constrained firms, which are those more capable of repurchasing their own shares\(^4\). However, firms don’t have the same ability to become sellers of last resort when their equity becomes overvalued; this is because seasoned equity offerings are more costly and require more time to execute than share repurchases. The latter are much more frequent than the former, Fama and French (2005) estimate that the fraction of firms with seasoned equity offerings in a given year during the period 1983–1992 was 5.7%, and 6.3% for the period 1993–2002; conversely, according to Gruillon and Michaely (2002), 84.2% of the firms that initiated a cash distribution to their shareholders in 2000, also initiated a buyback program. Although it is easier for a firm to repurchase shares than issue new ones, there is evidence that firms also do the latter when they perceive that their shares are overpriced. In the survey of Brav et al. (2005), 86.4% of the surveyed financial executives consider that it is important or very important whether their stock is a good investment relative to its true value when taking a stock repurchase decision; on the other hand, in another survey (Graham and Harvey (2001)) 66.94% of the surveyed considered important or very important the amount by which their stock was overvalued or undervalued when considering issuing common stock.

We have then two clear candidates for traders of last resort: firms and insiders. It is not obvious however if their actions are complementary or substitutes. This motivates our second main goal in this paper which consist on empirically asses the relevance and nature of each type of trading in price supporting and liquidity provision.

The structure of this paper is as follows. In the next section we develop a theoretical framework of insider trading in the presence of liquidity shocks. In Section 2.3 we

\(^3\)In some cases, the cost for insider sales might be larger than the cost of insider purchases: for instance, during lockup periods or when short selling constraints are binding, the cost for insider sales can be infinite.

\(^4\)Hong et al.’s results on skewness are weaker than those on short term variance, more specifically, they do not find significant coefficients when the firm’s financial constrainedness is measured using the Kaplan-Zingales index.
present the empirical study that confirms the model predictions regarding insider trading and compare these finding to those associated with share repurchases. The final Section 2.4 is dedicated to some concluding remarks and the proposal of new lines for future research.

2.2 The model

Let us consider an economy with three dates, \( t = 1, 2, \) and 3, and two assets. The first is a risk free asset that pays a gross rate of return of 1 each period. The second asset pays an uncertain dividend at \( t = 3, d_3, \) where:

\[
d_3 = s + \varepsilon.
\]

This risky asset, to which we refer as the stock, is held by long term investors who want to keep it until it pays its dividend. However, at \( t = 1, \) some of these investors have to trade an exogenous and random amount of shares that, aggregated, equals to \( x. \) With \( P_t \) we denote the price of one share of stock at date \( t, \) for \( t = 1, 2, \) and 3.

Apart from these long term investors, in the economy there are also two other types of agents, informed and uninformed traders. Informed traders have an informational advantage as they observe the dividend related information, \( s, \) before the market opens for trade at \( t = 1. \) We will refer to the informed traders as the informed or the insiders and use the index \( I \) for the variables that refer to them. In the same way, we will refer to the uninformed traders as the uninformed and use the index \( U \) for them. Both type of traders display CARA utility on their terminal wealth with a risk aversion coefficient equal to \( r_i, \) for \( i = I \) and \( U. \) There is a continuum of informed and uninformed traders with masses equal to \( \lambda \) and \( 1 - \lambda \) respectively. We will denote the shares held by the traders at each instant as \( x_{i,t} \) for \( i = I \) and \( U \) and \( t = 1, 2, \) and 3. Note that, in this economy, the market clearing condition can be expressed as

\[
\lambda x_{I,t} + (1 - \lambda) x_{U,t} = -x, \text{ for } t = 1 \text{ and } 2.
\]

Both traders have rational expectations and choose their optimal portfolio conditional on the information that they have at each point in time. We assume that the trade done by insiders at \( t = 1 \) is made public before the market opens for trade at \( t = 2. \) Denoting by \( \mathcal{I}_{i,t} \) the information set of a trader of type \( i \) at \( t \) and given that informed traders are not endowed with securities before the market opens at \( t = 1, \) we have:

\[
\mathcal{I}_{I,1} = \{s, P_1\}, \mathcal{I}_{U,1} = \{P_1\}, \mathcal{I}_{U,2} = \{P_1, P_2, x_{I,1}\}.
\]

Furthermore, we assume that the informed cannot trade at \( t = 2, \) thus \( x_{I,2} = x_{I,1} \) and that they pay a cost per share they trade given by:

\[
k(x_{I,1}) = \begin{cases} 
k_+ x_{I,1} & \text{if } x_{I,1} > 0, \\
-k_- x_{I,1} & \text{if } x_{I,1} < 0. \end{cases}
\]
This trading cost will play an important role in our analysis as it is the parameter that controls for the capacity of insiders to act when a liquidity shock occurs. The larger the \( k' \)'s are, the less active insiders will be in the market place. For instance, a very large \( k_+ \) (\( k_- \)) will severely restrict insiders purchases (sales) and consequently will reduce the role of insiders supporting prices when a negative (positive) shock occurs. In sections 2.2.1 and 2.2.2 we perform several comparative exercises on the \( k' \)'s which constitute the basis for our empirical analysis in section 2.3.

In figure 2.2 we summarize the timing of events in the present model.

\[
\begin{align*}
&\text{t = 1} & \text{t = 2} & \text{t = 3} \\
\text{Liquidity shock x.} & \text{Uninformed learn s.} & \text{Uncertainty is resolved and investors receive the dividend } d_3. \\
\text{Informed learn s.} & \text{Previous trade done by insiders, } x_{i,1}, \text{ and trade.} & \\
\text{Both agents trade, and} & \text{Uninformed learn from prices.} & \\
\text{uninformed learn from prices.} & \text{Informed pay a cost } k_+ \text{ or } k_- \text{ per share.} & \\
\end{align*}
\]

Figure 2.2. Timing of events.

Finally, we assume that all the random variables in the model, \( x, s, \) and \( \varepsilon \) are jointly normally distributed with

\[
\begin{pmatrix}
  x \\
  s \\
  \varepsilon
\end{pmatrix}
\sim N\left( \begin{pmatrix}
  0 \\
  0 \\
  0
\end{pmatrix}, \begin{pmatrix}
  \sigma_x^2 & 0 & 0 \\
  0 & \sigma_s^2 & 0 \\
  0 & 0 & \sigma_\varepsilon^2
\end{pmatrix} \right)
\]

2.2.1 Equilibrium and comparative statics

We solve for the equilibrium prices and holdings by backward induction. All the proofs can be found in the Appendix.

**Equilibrium at t = 2**

Note first that at the final date, \( t = 3 \), all uncertainty is resolved. Agents consume their final wealth and there is no trade. Since the stock pays the certain dividend \( d_3 \) in this date, the price is given by:

\[
P_3 = x + \varepsilon.
\]

At date \( t = 2 \) uninformed traders learn the trades done by insiders in the previous round of trade. Since the informed traders only trade once, we can get a closed form solution for their optimal demand at \( t = 1 \). The following lemma establishes the desired result.
Lemma 2 The optimal demand of an informed trader at \( t = 1 \) is given by

\[
x_{I,1} = \begin{cases} 
  \frac{s-P_1+k_-}{r_I \sigma_z^2} & \text{if } s < P_1 - k_- , \\
  0 & \text{if } P_1 - k_- \leq s \leq P_1 + k_+ , \\
  \frac{s-P_1-k_+}{r_I \sigma_z^2} & \text{if } s > P_1 + k_+.
\end{cases}
\]

Due to the existence of (possibly asymmetric) trading costs there are three possible regions associated to the three possible actions the insider can take: purchases of shares, sales of shares and no trade. These transactions contain information which is relevant for uninformed traders at date \( t = 2 \). In particular, when the insider is active in the market at \( t = 1 \), the uninformed will fully learn the size of the asset payoff related information, \( s \), and the liquidity shock, \( x \), at \( t = 2 \). When the informed does not trade at \( t = 1 \) the uninformed traders update their beliefs but do not reach full knowledge at \( t = 2 \). The following proposition characterizes an equilibrium at \( t = 2 \).

Proposition 5 An equilibrium at \( t = 2 \) is given by the holdings

\[
x_{I,2} = \begin{cases} 
  \frac{s-P_1+k_-}{r_I \sigma_z^2} & \text{if } s < P_1 - k_- , \\
  0 & \text{if } P_1 - k_- \leq s \leq P_1 + k_+ , \\
  \frac{s-P_1-k_+}{r_I \sigma_z^2} & \text{if } s > P_1 + k_+ .
\end{cases}
\]

\[
x_{U,2} = \begin{cases} 
  \frac{-1}{1-\lambda}(x + \lambda \frac{s-P_1+k_-}{r_I \sigma_z^2}) & \text{if } s < P_1 - k_- , \\
  \frac{-1}{1-\lambda}x & \text{if } P_1 - k_- \leq s \leq P_1 + k_+ , \\
  \frac{-1}{1-\lambda}(x + \lambda \frac{s-P_1-k_+}{r_I \sigma_z^2}) & \text{if } s > P_1 + k_+ .
\end{cases}
\]

and the price

\[
P_2 = \begin{cases} 
  s + \frac{r_U \sigma_s^2}{1-\lambda} \left( x + \lambda \frac{s-P_1+k_-}{r_I \sigma_z^2} \right) & \text{if } s < P_1 - k_- , \\
  \frac{r_U (\sigma_s^2+\sigma_z^2)}{1-\lambda} x - s \frac{\phi(\frac{P_1+k_+}{\sigma_s})-\phi(\frac{P_1-k_-}{\sigma_s})}{\Phi(\frac{P_1+k_+}{\sigma_s})-\Phi(\frac{P_1-k_-}{\sigma_s})} & \text{if } P_1 - k_- \leq s \leq P_1 + k_+ , \\
  s + \frac{r_U \sigma_s^2}{1-\lambda} \left( x + \lambda \frac{s-P_1-k_+}{r_I \sigma_z^2} \right) & \text{if } s > P_1 + k_+ .
\end{cases}
\]

where \( \phi \) and \( \Phi \) are the probability density function and the cumulative distribution function of a standard normal random variable.

Since the informed traders cannot trade at \( t = 2 \) their holdings are the same as in period \( t = 1 \). This means that the uninformed do not trade either at \( t = 2 \). The price however is very different depending on whether the informed bought, sold or did not trade at \( t = 1 \). In the regions associated to past insider activity prices reflect the new information the uninformed traders have learned and the risk premium associated to their previous holdings. In the region corresponding to lack
of previous activity by insiders the price is less informative. 

**Equilibrium at** \( t = 1 \)

Note that the equilibrium price 2.2.1 in Proposition 5 is not even a piecewise linear function of the liquidity shock \( x \). This precludes us from finding a closed form solution for the equilibrium at \( t = 1 \). We have no other choice than to solve numerically for the equilibrium. In the Appendix we describe the numerical methodology used.

In order to get a clear picture of the economic forces behind the equilibrium, we examine now the impact of a liquidity shock on the equilibrium price at \( t = 1 \). In figure 2.3 we plot \( P_1 \) as a function of \( x \) on the domain of 3 standard deviations from the mean of \( x \). In each of the four panels, we perform some comparative statics varying one parameter at a time. In the upper left panel we examine the impact of the trading costs the informed traders face, keeping \( k_- = k_+ \), on \( P_1 \). There are two opposing forces on the impact of trading costs on the variance of the price. On the one hand, the larger the trading costs, the less active insiders will be in the market place and hence the smaller their capacity to provide price support. Liquidity shocks have a larger impact on prices what results on a larger slope of the \( P_1 \) function. This effect will imply an increase on the variance of the price as we will corroborate in the next section. On the other hand, the larger the trading cost, the lower the adverse selection in the market (as insiders activity is decreased), and the lower the slope of \( P_1 \). This effect will result on a decrease of the price volatility. From the graph, we see that both effects are present, but the liquidity effect dominates for small trading costs while the adverse selection effect dominates for large trading costs. In 2.2.2 we elaborate on the way in which these two effects affect the variance of the price.

In the upper right plot in figure 2.3 we keep fixed \( k_+ = 0.05 \) and plot \( P_1 \) for different values of \( k_- \). We observe that, whenever \( k_- \neq k_+ \), the impact of liquidity shocks becomes asymmetric and depends on the sign of \( x \). The larger is the cost of selling stocks by the insider, \( k_- \), the larger the impact of a positive liquidity shock on the price compared to a negative liquidity shock of the same magnitude in absolute value. As we will show in the next section, this implies that the larger is \( k_- \) the largest is the skewness of \( P_1 \).

In the two bottom panels of figure 2.3, we perform some comparative statics for \( \sigma^2_\varepsilon \) and \( \sigma^2_s \). The graphs show that the larger is the non predictable part of the dividend, \( \sigma^2_\varepsilon \), the larger the impact of the liquidity shocks in \( P_1 \). Similarly, the larger the volatility of the informed traders’ private signal \( s \) (or, in other words, the larger the asymmetries of information) the larger the adverse selection in the market and, as a consequence, the larger the impact of \( x \) on \( P_1 \).

---

\[\text{In this region there is a price indeterminacy. In particular there are equilibria in which the price may depend on } s. \text{ These equilibria are unreasonable in the sense that they are not measurable with respect to any of the traders’ equilibrium demands. We rule out this type of bubbly equilibrium in this paper. For further details on this type of equilibria that arises when informed traders face trading constraints see Marin and Olivier (2008) and Marín and Olivier (2000).}\]
Figure 2.3. Equilibrium price $P_1$ as a function of the liquidity shock, $x$. The graphs show variations of the base case parametrization that is $\sigma_x^2 = 1$, $\sigma_s^2 = 0.04$, $r_I = 1$, $r_U = 2$, $\lambda = 0.1$, and $k_- = k_+ = 0.1$. The informed’s signal $s$ is fixed to be 0. The first graph, starting at the top left, displays $P_1$ as a function of $x$ for different informed’s trading costs, keeping $k_- = k_+$. In the second graph, we have fixed $k_+ = 0.05$ and we plot $P_1$ for different values of $k_-$. At the first graph of the second row, we vary $\sigma_v^2$, while in the last graph we vary $\sigma_s^2$. 
2.2.2 Effects of trading costs on the price distribution

Our model delivers testable implications regarding the distribution of short term stock returns. Note first, that the variance and skewness of $P_1$ and $P_2$ coincides with the variance and skewness of dollar returns from $t = 0$ (before trading at $t = 1$) to $t = 1$ and $t = 2$, respectively. In figure 2.4 we plot the variance and skewness of $P_1$ and $P_2$ as a function of the trading costs informed traders face. In the plots that display the variance we keep $k_- = k_+$ while in the plots that display the skewness we fix $k_+ = 0.05$ and we graph the effect of moving $k_-$, and variance and skewness computed as:

$$\text{Var} [P_i] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_i^2 \, dx \, ds - \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_i \, dx \, ds \right)^2,$$

and

$$\text{Skew} [P_i] = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( P_i - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_i \, dx \, ds \right)^3 \, dx \, ds}{\text{Var} [P_i]^{\frac{3}{2}},}$$

using the numerical quadrature techniques described in the Appendix.

As argued in the previous section there are two opposing effects in the way in which trading costs affect prices. These two effects are also present on the way in which trading costs affect the variance of the prices. On one hand, the larger the trading costs, the less liquidity will provided by insiders, increasing the impact of liquidity shocks on the price. This effect increases the price volatility. On the other hand, the larger the trading cost, the lower the adverse selection in the market, and the lower the variance of prices. The two top graphs in figure 2.4 show that for small trading costs the first effect dominates, while for large trading costs the second effect dominates. With the parametrization $\sigma^2_x = 1$, $\sigma^2_\varepsilon = 0.01$, $\sigma^2_\eta = 0.01$, $r_I = 2$, $r_U = 1$, and $\lambda = 0.1$, the cutting point happens at a trading cost around 7%.

Figure 2.4 also shows that, for a reasonable set of parameters, an increase in the cost of selling shares by the informed traders, increases the price skewness. Indeed, when the insiders face a large cost for sales and a positive liquidity shock ($x > 0$) happens, they cannot sell stocks and provide liquidity. In the absence of this price support this positive demand shocks generates a increase in the price that results in larger skewness in the price. By symmetry, the skewness of the prices is decreasing in the cost of purchases that informed traders face.

These exercises make clear the way in which trading costs affect the variance and the skewness of the dollar return of the risky asset. Since trading costs are a proxy for the informed traders capacity to buy and sell stock, we can establish testable implication on the relationship between insiders trading activities and return distributions. More specifically the model predicts that the skewness of the return must increase with insiders purchases (decrease with $k_+$) and decrease with insiders sales (increase with $k_-$. The effect on the variance of the returns depends on which of the two opposing effects described above dominates. When the liquidity provision effect dominates (or when the adverse selection effect is weak), both insiders sales and purchases
Figure 2.4. Effects of informed’s trading costs on the price variance and skewness due to liquidity shocks. The graphs show variations of the base case parametrization that is $\sigma_x^2 = 1, \sigma_z^2 = 0.01, \sigma_{\epsilon}^2 = 0.01, r_I = 2, r_U = 1, \lambda = 0.1, \text{ and } k_- = k_+ = 0.05$. The informed’s signal $s$ is fixed to be 0. The two graphs at the top display the variance of $P_1$ and $P_2$ as a function of the informed trading costs (keeping $k_-$ and $k_+$ equal). The two graphs at the bottom display the skewness of $P_1$ and $P_2$ as a function of $k_-$, when $k_+$ is kept constant and equal to 0.05.
reduce the variance of the return. We now turn to our empirical study where these hypothesis are tested. In our empirical exercise we test these hypothesis separately and in combination with the hypothesis put forward in Hong et al. (2008) that relates stock repurchases by firms to the volatility and the skewness of returns.

2.3 Empirical analysis

The main objective of the empirical analysis is to asses whether insider trading affect returns distribution. In particular, we focus on its impact on the short-horizon volatility and the skewness of stock returns. We also compare the results for insider trading with those obtained for the case of share repurchases. We start describing the dataset and defining all variables involved in our study.

2.3.1 Description of the data

Using data from CRSP, COMPUSTAT and Thompson Financial Insider Trading Dataset (TFIT) from January 1986 to December 2003, we construct the variables defined below for each non-financial firm and year pair. The construction of most variables follows Hong et al. (2008), who provide further details on the dataset construction process, and whose notation we follow.\footnote{The only variables that are not defined in Hong et al. (2008) are those related to insider trading activity, IPUR_{i,t} and ISAL_{i,t}, and the measures of return variance using one year of data, DVARY_{i,t} and MVARY_{i,t}.}

Insider trading variables

We consider as insiders all the traders that are defined as such by the Section 16(a) of the Security and Exchange Act of 1934 (SEA), by which large beneficial shareholders and managers of a publicly traded firm are required to file their transactions in the company stock with the Securities and Exchange Commission (SEC). This definition includes the managers of publicly traded companies, in particular the chairman, directors, CEOs, CFOs, officers, presidents, vice presidents, affiliates, members of committees, etc, and large shareholders. Our definition of insiders does not include other people that might posses non-public information about the company, but that are not considered insiders under the Section 16(a) of the Security and Exchange Act of 1934 (SEA). From this definition, it is clear that any information advantage that is given to firms is also an information advantage of insiders.

We define IPUR_{i,t} as the value of all the shares purchased by insiders of firm i during year t divided by the average daily market capitalization of firm i during year t. Similarly, we define ISAL_{i,t} as the value of all the shares sold by insiders of firm i during year t divided by the average market capitalization of firm i during year t.
Measures of return variance and skewness

The measures of return variance and skewness that we construct are those used by Hong et al. (2008). To compute these variables, we use continuously compounded returns.

The measures of return variance are $TVAR_{i,t}$, which is the variance of the two non-overlapping three years returns of firm $i$ using data from year $t$ to $t+5$; $AVAR_{i,t}$, which is the variance of the six non-overlapping one year returns of firm $i$ using data from year $t$ to $t+5$; and $SVAR_{i,t}$, $QVAR_{i,t}$, $MVAR_{i,t}$, $WVAR_{i,t}$, and $DVAR_{i,t}$, which are constructed as $AVAR_{i,t}$, but using semiannual, quarterly, monthly, weekly and daily returns, respectively. All these measures of variance are annualized.

Apart from the previous variables, we also have constructed measures of variance using only data corresponding to year $t$. We call them $DVARY_{i,t}$ and $MVARY_{i,t}$, and they are build as $DVAR_{i,t}$ and $MVAR_{i,t}$, but instead of using data from year $t$ to year $t+5$ we only data corresponding to year $t$.

For each firm and year, we also compute the skewness of quarterly, monthly, weekly, and daily returns of firm $i$ during year $t$, denoted by $QSKEW_{i,t}$, $MSKEW_{i,t}$, $WSKEW_{i,t}$, and $DSKEW_{i,t}$, respectively.$^7$

Other variables

We use the four measures of financial constrainedness employed by Hong et al. (2008). The first is the value of common shares repurchased by firm $i$ during year $t$ adjusted by the firm’s net income, this variable is denoted $REPURCHASES_{i,t}$. It is computed as the purchased common and preferred stock (COMPSTAT annual 115) minus the reduction in the preferred stock liquidation value (the reduction in the annual data item 10), divided by net income (172). The second is the firm’s age $AGE_{i,t}$, computed as the number of years from the first appearance in CRSP. Finally, the last two are the Kaplan-Zingales index, $KZ_{i,t}$, and a reduced version of this index that does not include neither book leverage nor Tobin’s Q, this latter variable, $KZ3_{i,t}$. $KZ$ and $KZ3$ are computed as:

$$KZ_{i,t} = -1.002 \frac{\text{Cash Flow}_{i,t}}{\text{Assets}_{i,t-1}} - 39.368 \frac{\text{Cash Dividends}_{i,t}}{\text{Assets}_{i,t-1}} - 1.315 \frac{\text{Cash Balance}_{i,t}}{\text{Assets}_{i,t-1}} + 3.139 \text{Leverage}_{i,t} + 0.283 Q_{i,t}.$$  

$$KZ3_{i,t} = -1.002 \frac{\text{Cash Flow}_{i,t}}{\text{Assets}_{i,t-1}} - 39.368 \frac{\text{Cash Dividends}_{i,t}}{\text{Assets}_{i,t-1}} - 1.315 \frac{\text{Cash Balance}_{i,t}}{\text{Assets}_{i,t-1}}.$$

Note that $KZ$ and $KZ3$ are increasing with financial constrainedness.

The other variables in our study, all defined for each firm $i$ during year $t$, are

$^7$Starting from a time series of continuously compounded returns $\{r_t\}_{t=1}^T$, we compute its skewness as $\text{Skew} \left[ \{r_t\}_{t=1}^T \right] = \frac{T(T-1)^2}{(T-2)^3} \sum_{t=1}^{T} (r_t - \bar{r})^3 \frac{1}{\sum_{t=1}^{T} (r_t - \bar{r})^2}$, where $\bar{r} = \frac{\sum_{t=1}^{T} r_t}{T}$. 

the logarithm of firm’s average daily market capitalization, \( \text{LOGSIZE}_{i,t} \), its market leverage, \( \text{MLEV}_{i,t} \), the logarithm of the market to book ratio, \( \text{LOGMB}_{i,t} \), the average monthly return, \( \text{RET}_{i,t} \), and the average daily turnover, \( \text{TURNOVER}_{i,t} \). Finally, \( \text{INDUSTRYDUMMIES}_{i,t} \) is a set of dummies for the 48 industries in Fama and French (1997).\(^8\)

**Descriptive statistics**

Table 2.1 contains the time series average of cross-sectional means and standard deviations for the variables that we have previously defined. These summary statistics are similar to those in previous studies, for instance Lakonishok and Lee (2001) or Marin and Olivier (2008) for insider trading activity, and Hong et al. (2008) for the other variables.

Insiders are, on average, net sellers of stock. This suggests that the cost of purchasing shares is larger than the cost of selling them, \( k_+ > k_- \) in our model. However, we must take into account that most of insiders sales could be motivated for diversification purposes and not for misvaluation opportunities.

As Hong et al. (2008), we find that the short term variances are larger than the long term variances, which implies a negative autocorrelation in stock returns. Furthermore, skewness is positive for short horizons and decreases with the time interval, becoming negative for quarterly returns.

The times series average of the cross-sectional correlations between the measures of financial constrainedness and insider trading activity are reported in Table 2.2. This table shows that insider trading activity is larger in younger firms, and that insiders tend to purchase shares of financially constrained firms; hence, when firms have difficulties in providing price support, insiders might provide it. This intimates that insiders’ purchases and firms’ repurchases might be substitutes in providing price support. Note that insider sales are more negatively correlated with KZ than KZ3, this suggests that insiders tend to sell less in firms with high investment opportunities, measured by Tobin’s Q.

### 2.3.2 Effects of IT on returns’ distribution

In this section we examine the effects of insider trading on the short-horizon variance and skewness of stock returns.

---

\(^8\)The measures of insider trading activity, variance, skewness, stock repurchases and the components of the Kaplan-Zingales indexes are winsorized to mitigate the impact of anomalous or extreme observations. Insider trades have been cross-checked with CRSP data to eliminate problematic records as in Lakonishok and Lee (2001). We have excluded from our sample the firms for which we do not have any insider trade in the whole sample period.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>StDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPUR</td>
<td>.00183866</td>
<td>.00645876</td>
</tr>
<tr>
<td>ISAL</td>
<td>.00722405</td>
<td>.01813813</td>
</tr>
<tr>
<td>DVAR</td>
<td>.48864484</td>
<td>.50720914</td>
</tr>
<tr>
<td>WVAR</td>
<td>.33559615</td>
<td>.28896252</td>
</tr>
<tr>
<td>MVAR</td>
<td>.29700674</td>
<td>.25787447</td>
</tr>
<tr>
<td>QVAR</td>
<td>.29543764</td>
<td>.26406154</td>
</tr>
<tr>
<td>SVAR</td>
<td>.28302673</td>
<td>.28190983</td>
</tr>
<tr>
<td>AVAR</td>
<td>.30408996</td>
<td>.34246279</td>
</tr>
<tr>
<td>TVAR</td>
<td>.28311923</td>
<td>.48023924</td>
</tr>
<tr>
<td>DVARY</td>
<td>.55611991</td>
<td>.661616</td>
</tr>
<tr>
<td>MVARY</td>
<td>.35965479</td>
<td>.40246706</td>
</tr>
<tr>
<td>DSKEW</td>
<td>.10654422</td>
<td>1.0255157</td>
</tr>
<tr>
<td>WSKEW</td>
<td>.14004511</td>
<td>.8605122</td>
</tr>
<tr>
<td>MSKEW</td>
<td>.04886174</td>
<td>.81684978</td>
</tr>
<tr>
<td>QSKEW</td>
<td>-.03964613</td>
<td>1.0265205</td>
</tr>
<tr>
<td>REPURCHASE</td>
<td>.16202193</td>
<td>.46403992</td>
</tr>
<tr>
<td>AGE</td>
<td>14.908976</td>
<td>14.406529</td>
</tr>
<tr>
<td>KZ</td>
<td>.8073567</td>
<td>.85847819</td>
</tr>
<tr>
<td>KZ3</td>
<td>-.6765437</td>
<td>1.245824</td>
</tr>
<tr>
<td>LOGSIZE</td>
<td>11.610331</td>
<td>2.0021146</td>
</tr>
<tr>
<td>MLEV</td>
<td>.22520186</td>
<td>.00558925</td>
</tr>
<tr>
<td>LOGMB</td>
<td>.67140609</td>
<td>.04823989</td>
</tr>
<tr>
<td>RET</td>
<td>-.00025596</td>
<td>.86430321</td>
</tr>
<tr>
<td>TURNOVER</td>
<td>.00479945</td>
<td>.22323016</td>
</tr>
</tbody>
</table>

Table 2.2. Time series average of cross-sectional correlations between financial constraintness measures and insider trading activity. Data from 1986-2003

<table>
<thead>
<tr>
<th></th>
<th>REPURCHASE</th>
<th>AGE</th>
<th>KZ</th>
<th>KZ3</th>
<th>IPUR</th>
<th>ISAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>REPURCHASE</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGE</td>
<td>.08478125</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KZ</td>
<td>-.07389364</td>
<td>-1.1764388</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KZ3</td>
<td>-.06999179</td>
<td>-2.1181013</td>
<td>.73401205</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPUR</td>
<td>-.01442049</td>
<td>-.05959318</td>
<td>.02933922</td>
<td>.05206674</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>ISAL</td>
<td>-.01909647</td>
<td>-.14802733</td>
<td>-.03124193</td>
<td>-.01639203</td>
<td>.02590551</td>
<td>1</td>
</tr>
</tbody>
</table>
Effects of IT on short term variance

We first estimate the effect of insider trading and financial constrainedness on the short-horizon return variance, controlling for long-horizon return variance, TVAR, and other variables that have been found relevant in previous studies, following the Hong et al. (2008) setup. In particular, we estimate the model:

\[
\text{STVAR}_{i,t} = \beta_1 \text{CONSTRAINT}_{i,t-1} + \beta_2 \text{IPUR}_{i,t-1} + \beta_3 \text{ISAL}_{i,t-1} + \beta_4 \text{TVAR}_{i,t} + \beta_5 \text{LOGSIZE}_{i,t-1} + \beta_6 \text{MLEV}_{i,t-1} + \beta_7 \text{LOGMB}_{i,t-1} + \beta_8 \text{RET}_{i,t-1} + \beta_9 \text{TURNOVER}_{i,t-1} + \text{INDUSTRYDUMMIES}_{i,t-1} \Delta + \epsilon_{i,t},
\]

for \( i = 1 \ldots N \).

In the estimation of model (2.3.1) we use the Fama-MacBeth type regressions (Fama and MacBeth (1973)) correcting for autocorrelation using Newey-West standard errors (Newey and West (1987)). The measures of short-horizon variance, denoted as STVAR in (2.3.1), are DVAR (daily), WVAR (weekly), MVAR (monthly), QVAR (quarterly), SVAR (semiannual), and AVAR (annual). Recall that all these measures of variance are computed with non-overlapping time periods using data from year \( t \) to \( t + 5 \). The measures of financial constrainedness, denoted by CONSTRAINT in 2.3.1, are stock repurchases (REPURCHASES), firm's age (AGE), and the two Kaplan-Zingales indexes (KZ and KZ3). The estimation results are reported in Table 2.3. Note that each column of the table corresponds to a different measure of financial constrainedness and that the last column does not include any of the previous measures as an explanatory variable.

Insider sales and purchases, when significant, have a negative sign predicting short-horizon variance, which is what we expected according to our model if adverse selection is low. When the measure of short-horizon variance is computed using daily returns, both insider sales and purchases are significant and negative. An increase of two standard deviations in insider purchases and sales leads to a decrease of 0.01587 and 0.01291 in daily variance, respectively, or 3.13% and 2.55% of the cross-sectional variance in DVAR; note that an increase of two standard deviations in firm repurchases leads to a decrease of 4.03% of the cross-sectional variance in DVAR. For longer horizon variances, insider purchases tend to be significant, but not insider sales. Note that insider might sell shares for a variety of reasons, but it is reasonable to think that they only will purchase shares when the market price is below the fundamental price according to insider’s valuation. Insider transactions have an impact on shorter-horizon variances (DVAR or WVAR), but not on longer-horizon variances. Table 2.3 is consistent with Hong et al.’s findings regarding the role of firms being buyers of last resort.

In model (2.3.1), return variances are computed using 6 years of data. It is likely that in this long time period firm’s executive compensation schemes and the board consideration of insider trading activity might change. this would affect insider trading behavior and, consequently, our empirical results. For this reason, we construct measures of daily return variance using only one year of data, DVARY, and a
Table 2.3. Fama-MacBeth regressions of short-horizon return variance on insider trading activity. Variance measures are computed using 5 years of data. Newey-West corrected t-statistics in parentheses.

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>REPURCHASE</th>
<th>AGE</th>
<th>KZ</th>
<th>KZ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CONSTRAINT}_{t-1}$</td>
<td>-0.022**</td>
<td>0.000*</td>
<td>0.035**</td>
<td>0.036**</td>
</tr>
<tr>
<td>$\text{IPUR}_{t-1}$</td>
<td>-0.970</td>
<td>-1.242+</td>
<td>-1.343</td>
<td>-1.363+</td>
</tr>
<tr>
<td>$\text{ISAL}_{t-1}$</td>
<td>-0.421+</td>
<td>-0.341+</td>
<td>-0.322</td>
<td>-0.335</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>DVAR$_{t-1}$</th>
<th>WVAR$_{t-1}$</th>
<th>MVAR$_{t-1}$</th>
<th>QVAR$_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CONSTRAINT}_{t-1}$</td>
<td>-0.019**</td>
<td>0.000</td>
<td>0.028**</td>
<td>0.030**</td>
</tr>
<tr>
<td>$\text{IPUR}_{t-1}$</td>
<td>-0.457</td>
<td>-0.592+</td>
<td>-0.578</td>
<td>-0.593</td>
</tr>
<tr>
<td>$\text{ISAL}_{t-1}$</td>
<td>0.009</td>
<td>0.004</td>
<td>0.052</td>
<td>0.039</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>SVAR$_{t-1}$</th>
<th>AVAR$_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CONSTRAINT}_{t-1}$</td>
<td>-0.017**</td>
<td>-0.000+</td>
</tr>
<tr>
<td>$\text{IPUR}_{t-1}$</td>
<td>-0.325</td>
<td>-0.450+</td>
</tr>
<tr>
<td>$\text{ISAL}_{t-1}$</td>
<td>0.046</td>
<td>0.002</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>GVAR$_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CONSTRAINT}_{t-1}$</td>
<td>-0.018**</td>
</tr>
<tr>
<td>$\text{IPUR}_{t-1}$</td>
<td>-0.340</td>
</tr>
<tr>
<td>$\text{ISAL}_{t-1}$</td>
<td>0.027</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>HVAR$_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{CONSTRAINT}_{t-1}$</td>
<td>-0.015**</td>
</tr>
<tr>
<td>$\text{IPUR}_{t-1}$</td>
<td>-0.147</td>
</tr>
<tr>
<td>$\text{ISAL}_{t-1}$</td>
<td>-0.049</td>
</tr>
</tbody>
</table>

+ significant at 10% level, * significant at 5% level, ** significant at 1% level
2.3 Empirical analysis

variable capturing monthly returns variance, MVARY, as the longer horizon returns measure, using also one year of data. Furthermore, instead of using lagged insider trading activity, we will use the current one, but instrumented by lagged trading and other variables that might affect insider trading activity. We have chosen the instrumental variables approach because of the endogeneity of insider trading activity. The instruments, apart from lagged insider sales and purchases, include lagged measures of return variance, the average monthly stock returns in the previous two years, the logarithm of the firm’s lagged market capitalization, the logarithm of its lagged market to book ratio, the lagged average daily turnover and firm’s age. The specification of this model is

\[
DVARY_{i,t} = \beta_1 \text{CONSTRAINT}_{i,t-1} + \beta_2 \hat{IPUR}_{i,t} + \beta_3 \hat{ISAL}_{i,t} + \beta_4 \text{MVARY}_{i,t} \\
+ \beta_5 \text{LOGSIZE}_{i,t-1} + \beta_6 \text{MLEV}_{i,t-1} + \beta_7 \text{LOGMB}_{i,t-1} + \beta_8 \text{RET}_{i,t-1} \\
+ \beta_9 \text{TURNOVER}_{i,t-1} + \text{INDUSTRYDUMMIES}_{i,t-1} \Delta + \epsilon_{i,t} ,
\]

\[
\hat{IPUR}_{i,t} = \gamma_0 + \gamma_1 \hat{IPUR}_{i,t-1} + \gamma_2 \hat{ISAL}_{i,t-1} + \gamma_3 DVARY_{i,t-1} + \gamma_4 \text{MVARY}_{i,t-1} \\
+ \gamma_5 \text{RET}_{i,t-1} + \gamma_6 \text{RET}_{i,t-2} + \gamma_7 \text{LOGSIZE}_{i,t-1} + \gamma_8 \text{LOGMB}_{i,t-1} \\
+ \gamma_9 \text{AGE}_{i,t-1} + \gamma_{10} \text{TURNOVER}_{i,t-1} + \tilde{\epsilon}_{i,t} ,
\]

\[
\hat{ISAL}_{i,t} = \delta_0 + \delta_1 \hat{IPUR}_{i,t-1} + \delta_2 \hat{ISAL}_{i,t-1} + \delta_3 DVARY_{i,t-1} + \delta_4 \text{MVARY}_{i,t-1} \\
+ \delta_5 \text{RET}_{i,t-1} + \delta_6 \text{RET}_{i,t-2} + \delta_7 \text{LOGSIZE}_{i,t-1} + \delta_8 \text{LOGMB}_{i,t-1} \\
+ \delta_9 \text{AGE}_{i,t-1} + \delta_{10} \text{TURNOVER}_{i,t-1} + \tilde{\tilde{\epsilon}}_{i,t} ,
\]

for \( i = 1 \ldots N \).

which we estimate using Fama-Macbeth approach correcting for autocorrelation using Newey-West standard errors.

Table 2.4. Fama-MacBeth IV regressions of short-horizon return variance on insider trading activity. The measure of short-horizon variance is daily returns variance, DVARY, and we control for monthly returns variance, MVARY; both measures are computed using one year of data. Newey-West corrected t-statistics in parentheses.

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>REPURCHASE</th>
<th>AGE</th>
<th>KZ</th>
<th>KZ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTRAINT_{i,t-1}</td>
<td>0.012**</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( (-2.776) )</td>
<td>( (-0.306) )</td>
<td>( 0.094 )</td>
<td>( -0.038 )</td>
<td></td>
</tr>
<tr>
<td>( \hat{IPUR}_{i,t-1} )</td>
<td>-5.386</td>
<td>-7.017+</td>
<td>-7.121+</td>
<td>-7.112+</td>
</tr>
<tr>
<td>( (-1.313) )</td>
<td>( (-1.903) )</td>
<td>( -1.862 )</td>
<td>( -1.858 )</td>
<td>( -1.959 )</td>
</tr>
<tr>
<td>( \hat{ISAL}_{i,t-1} )</td>
<td>-6.292*</td>
<td>-5.660+</td>
<td>-5.471+</td>
<td>-5.483+</td>
</tr>
<tr>
<td>( (-1.970) )</td>
<td>( (-1.809) )</td>
<td>( -1.688 )</td>
<td>( -1.688 )</td>
<td>( -1.671 )</td>
</tr>
</tbody>
</table>

+ significant at 10% level, * significant at 5% level, ** significant at 1% level
Table 2.4 reports the estimated coefficients for the financial constrainedness measures and insider trading activity of model 2.3.2. In this setup, insider purchases and sales are significant and both reduce the short-horizon variance. Insider purchases fail to be significant when lagged firm repurchases are included in the regressions, and this latter variable is the only measure of financial constrainedness that is significant in this setup.

**Effects of IT on skewness**

The second prediction of our model is that the ability of insiders to purchase shares increases the skewness of short-horizon returns and that the ability of insiders to sell decreases it. These predictions are tested using the following specification, similar to 2.3.1:

\[
\text{SKEW}_{i,t} = \beta_1 \text{CONSTRAINT}_{i,t-1} + \beta_2 \text{IPUR}_{i,t-1} + \beta_3 \text{ISAL}_{i,t-1} \\
+ \beta_5 \text{LOGSIZE}_{i,t-1} + \beta_6 \text{MLEV}_{i,t-1} + \beta_7 \text{LOGMB}_{i,t-1} + \beta_8 \text{RET}_{i,t-1} \\
+ \beta_9 \text{TURNOVER}_{i,t-1} + \text{INDUSTRYDUMMIES}_{i,t-1} \Delta + \epsilon_{i,t},
\]

for \( i = 1 \ldots N \),

(2.3.3)

The dependent variable measuring skewness of stock returns, denoted by SKEW in (2.3.3), is computed using daily data (DSKEW), weekly (WSKEW), monthly (MSKWE), and quarterly (QSKEW). The measures of financial constrainedness, denoted by CONSTRAINT, are the same as in the previous section: stock repurchases (REPURCHASES), firm’s age (AGE), and the two Kaplan-Zingales indexes (KZ and KZ3).

In table 2.5 we report the results of estimating (2.3.3) using the Fama-Macbeth approach correcting for autocorrelation using Newey-West standard errors. The sign of the coefficient for insider purchases is always positive, and always negative for insider sales; this is consistent with the predictions of our model. However, insider purchases tend to be non-significant, but for the case in which the dependent variable is QSKEW. The only measure of financial constrainedness that is significant for all measures of skewness is AGE, but all are significant predicting QSEW. Note that, as in model (2.3.1), lagged insider trading activity might not be a good proxy for the ability of insiders to trade.

Similarly to (2.3.2), we have also instrumented insider trading activity by lagged insider sales and purchases, lagged measures of return variance, the stock returns in the two previous years, the logarithm of the firm’s lagged market capitalization, the logarithm of its lagged market to book ratio, the lagged average daily turnover and firm’s age. The specification of the IV model is similar to (2.3.2):
2.3 Empirical analysis

Table 2.5. Fama-MacBeth regressions of returns skewness on insider trading activity. Newey-West corrected t-statistics in parentheses.

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>REPURCHASE</th>
<th>AGE</th>
<th>KZ</th>
<th>KZ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable is DSKEW&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONSTRAINT&lt;sub&gt;i,t&lt;/sub&gt;−1</td>
<td>0.019</td>
<td>0.005**</td>
<td>0.009</td>
<td>0.002</td>
</tr>
<tr>
<td>(1.512)</td>
<td>(8.766)</td>
<td>(1.375)</td>
<td>(0.262)</td>
<td></td>
</tr>
<tr>
<td>IPUR&lt;sub&gt;i,t&lt;/sub&gt;−1</td>
<td>1.247</td>
<td>1.542</td>
<td>1.731+</td>
<td>1.726+</td>
</tr>
<tr>
<td>(1.031)</td>
<td>(1.641)</td>
<td>(1.719)</td>
<td>(1.712)</td>
<td>(1.616)</td>
</tr>
<tr>
<td>ISAL&lt;sub&gt;i,t&lt;/sub&gt;−1</td>
<td>−1.809**</td>
<td>−1.510**</td>
<td>−1.591**</td>
<td>−1.601**</td>
</tr>
<tr>
<td>(−7.531)</td>
<td>(−7.130)</td>
<td>(−6.789)</td>
<td>(−6.958)</td>
<td>(−8.161)</td>
</tr>
</tbody>
</table>

| Dependent variable is WSKEW<sub>i,t</sub> |  |  |  |  |
| CONSTRAINT<sub>i,t</sub>−1 | 0.014 | 0.003** | 0.004 | −0.003 |
| (1.101) | (7.756) | (0.678) | (−0.376) |
| IPUR<sub>i,t</sub>−1 | 0.512 | 0.646 | 0.830 | 0.839 | 0.603 |
| (0.559) | (0.865) | (0.925) | (0.934) | (0.792) |
| ISAL<sub>i,t</sub>−1 | −1.344** | −1.146** | −1.213** | −1.219** | −1.303** |

| Dependent variable is MSKEW<sub>i,t</sub> |  |  |  |  |
| CONSTRAINT<sub>i,t</sub>−1 | 0.015* | 0.002** | 0.003 | −0.005 |
| (2.514) | (4.661) | (0.498) | (−0.903) |
| IPUR<sub>i,t</sub>−1 | 0.183 | 0.501 | 0.667 | 0.671 | 0.462 |
| (0.226) | (0.828) | (0.837) | (0.840) | (0.754) |
| ISAL<sub>i,t</sub>−1 | −0.912** | −0.871** | −1.004** | −1.007** | −0.981** |
| (−2.862) | (−3.206) | (−3.503) | (−3.531) | (−3.518) |

| Dependent variable is QSKEW<sub>i,t</sub> |  |  |  |  |
| CONSTRAINT<sub>i,t</sub>−1 | 0.016+ | 0.001* | −0.014* | −0.017** |
| (1.688) | (2.464) | (−2.166) | (−2.858) |
| IPUR<sub>i,t</sub>−1 | 1.241* | 0.814 | 1.174+ | 1.183+ | 0.799 |
| (2.291) | (1.612) | (1.925) | (1.952) | (1.558) |
| ISAL<sub>i,t</sub>−1 | −0.280 | −0.189 | −0.303 | −0.296 | −0.244 |
| (−1.023) | (−0.703) | (−1.104) | (−1.090) | (−0.868) |

+ significant at 10% level, * significant at 5% level, ** significant at 1% level
SKEW\(_{i,t} = \beta_1 \text{CONSTRAINT}_{i,t-1} + \beta_2 \hat{\text{IPUR}}_{i,t} + \beta_3 \hat{\text{ISAL}}_{i,t}
\) 
+ \beta_5 \text{LOGSIZE}_{i,t-1} + \beta_6 \text{MLEV}_{i,t-1} + \beta_7 \text{LOGMB}_{i,t-1} + \beta_8 \text{RET}_{i,t-1}
\) 
+ \beta_9 \text{TURNOVER}_{i,t-1} + \text{INDUSTRYDUMMIES}_{i,t-1} \Delta + \epsilon_{i,t},
\) 

IPUR\(_{i,t} = \gamma_0 + \gamma_1 \text{IPUR}_{i,t-1} + \gamma_2 \text{ISAL}_{i,t-1}
\) 
+ \gamma_3 \text{RET}_{i,t-1} + \gamma_4 \text{RET}_{i,t-2} + \gamma_5 \text{LOGSIZE}_{i,t-1} + \gamma_6 \text{LOGMB}_{i,t-1}
\) 
+ \gamma_7 \text{AGE}_{i,t-1} + \gamma_8 \text{TURNOVER}_{i,t-1} + \tilde{\epsilon}_{i,t},
\) 

ISAL\(_{i,t} = \delta_0 + \delta_1 \text{IPUR}_{i,t-1} + \delta_2 \text{ISAL}_{i,t-1}
\) 
+ \delta_3 \text{RET}_{i,t-1} + \delta_4 \text{RET}_{i,t-2} + \delta_5 \text{LOGSIZE}_{i,t-1} + \delta_6 \text{LOGMB}_{i,t-1}
\) 
+ \delta_7 \text{AGE}_{i,t-1} + \delta_8 \text{TURNOVER}_{i,t-1} + \tilde{\tilde{\epsilon}}_{i,t},
\) 
for \(i = 1 \ldots N,\)

Table 2.6 reports the results obtained in the estimation of (2.3.4). Insider purchases increase the skewness and insider sales decrease it, consistently with our model. Insider purchases are significant when the dependent variable is daily or quarterly skewness; insider sales are always significant, but for the case of quarterly skewness. Insider transactions have a larger impact on shorter-horizon skewness (DSKEW) than on longer-horizon skewness. Note that the only measure of financial constrainedness that is significant in all the regressions is firm’s age, share repurchases is significant when the dependent variable is monthly skewness, and the Kaplan-Zingales indexes when the dependent variable is quarterly skewness. In Table 2.6, the evidence in favor of firms being buyers of last resort is weaker than in Tables 2.3 and 2.4, but stronger in favor of insiders being liquidity providers.

### 2.4 Conclusions and further research

When liquidity shocks move asset prices away from fundamental values, corporate insiders are in a privileged position to absorb this demand for liquidity. In order to provide liquidity, insiders can trade on the firm’s account, through buy back programs or seasonal equity offerings, or on their own account. In this paper we provide a theoretical framework and evidence supporting the role of corporate insiders as liquidity providers which complements the evidence provided in Hong et al. (2008) on firms playing a similar role. We identify some differences though in the way these two type of traders provide liquidity. First, while stock repurchases clearly reduce return volatility, insiders transactions may not. This is because, unlike firms, insiders face an adverse selection problem when trading. When the adverse selection effect is stronger than the price support effect, volatility may increase. This explains why our results on volatility are weaker than those linking volatility to stock
Table 2.6. Fama-MacBeth IV regressions of returns skewness on insider trading activity. Newey-West corrected t-statistics in parentheses.

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>REPURCHASE</th>
<th>AGE</th>
<th>KZ</th>
<th>KZ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{CONSTRAINT}_{i,t-1} )</td>
<td>0.019</td>
<td>0.004**</td>
<td>0.010</td>
<td>0.004</td>
</tr>
<tr>
<td>(1.549)</td>
<td>(6.714)</td>
<td>(1.446)</td>
<td>(0.513)</td>
<td></td>
</tr>
<tr>
<td>( \text{IPUR}_{i,t-1} )</td>
<td>6.509</td>
<td>6.556+</td>
<td>7.724*</td>
<td>7.700*</td>
</tr>
<tr>
<td>(1.520)</td>
<td>(1.736)</td>
<td>(2.086)</td>
<td>(2.080)</td>
<td>(2.157)</td>
</tr>
<tr>
<td>(-12.075)</td>
<td>(-5.272)</td>
<td>(-8.699)</td>
<td>(-8.932)</td>
<td>(-7.632)</td>
</tr>
</tbody>
</table>

Dependent variable is DSKEW\(_{i,t}\)

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>REPURCHASE</th>
<th>AGE</th>
<th>KZ</th>
<th>KZ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{CONSTRAINT}_{i,t-1} )</td>
<td>0.015</td>
<td>0.003**</td>
<td>0.005</td>
<td>-0.000</td>
</tr>
<tr>
<td>(1.129)</td>
<td>(8.051)</td>
<td>(0.852)</td>
<td>(-0.044)</td>
<td></td>
</tr>
<tr>
<td>( \text{IPUR}_{i,t-1} )</td>
<td>3.525</td>
<td>3.540</td>
<td>4.162</td>
<td>4.183</td>
</tr>
<tr>
<td>(1.119)</td>
<td>(1.218)</td>
<td>(1.323)</td>
<td>(1.331)</td>
<td>(1.521)</td>
</tr>
<tr>
<td>( \text{ISAL}_{i,t-1} )</td>
<td>-6.849**</td>
<td>-3.858**</td>
<td>-6.192**</td>
<td>-6.159**</td>
</tr>
<tr>
<td>(-9.418)</td>
<td>(-5.122)</td>
<td>(-6.813)</td>
<td>(-6.808)</td>
<td>(-6.691)</td>
</tr>
</tbody>
</table>

Dependent variable is WSKEW\(_{i,t}\)

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>REPURCHASE</th>
<th>AGE</th>
<th>KZ</th>
<th>KZ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{CONSTRAINT}_{i,t-1} )</td>
<td>0.015*</td>
<td>0.002**</td>
<td>0.003</td>
<td>-0.004</td>
</tr>
<tr>
<td>(2.464)</td>
<td>(3.976)</td>
<td>(0.604)</td>
<td>(-0.664)</td>
<td></td>
</tr>
<tr>
<td>( \text{IPUR}_{i,t-1} )</td>
<td>3.313</td>
<td>3.777</td>
<td>3.985</td>
<td>3.983</td>
</tr>
<tr>
<td>(0.952)</td>
<td>(1.290)</td>
<td>(1.255)</td>
<td>(1.326)</td>
<td>(1.441)</td>
</tr>
<tr>
<td>( \text{ISAL}_{i,t-1} )</td>
<td>-5.232**</td>
<td>-2.905*</td>
<td>-5.453**</td>
<td>-5.401**</td>
</tr>
<tr>
<td>(-3.088)</td>
<td>(-2.184)</td>
<td>(-3.533)</td>
<td>(-3.532)</td>
<td>(-3.430)</td>
</tr>
</tbody>
</table>

Dependent variable is MSKEW\(_{i,t}\)

<table>
<thead>
<tr>
<th>CONSTRAINT:</th>
<th>REPURCHASE</th>
<th>AGE</th>
<th>KZ</th>
<th>KZ3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{CONSTRAINT}_{i,t-1} )</td>
<td>0.015</td>
<td>0.001*</td>
<td>-0.014*</td>
<td>-0.017**</td>
</tr>
<tr>
<td>(1.519)</td>
<td>(2.324)</td>
<td>(-2.478)</td>
<td>(-3.222)</td>
<td></td>
</tr>
<tr>
<td>( \text{IPUR}_{i,t} )</td>
<td>4.879*</td>
<td>4.474*</td>
<td>4.720*</td>
<td>4.735*</td>
</tr>
<tr>
<td>(2.410)</td>
<td>(2.110)</td>
<td>(2.071)</td>
<td>(2.095)</td>
<td>(2.242)</td>
</tr>
<tr>
<td>( \text{ISAL}_{i,t} )</td>
<td>-0.842</td>
<td>-0.039</td>
<td>-1.033</td>
<td>-0.980</td>
</tr>
<tr>
<td>(-0.687)</td>
<td>(-0.038)</td>
<td>(-0.836)</td>
<td>(-0.789)</td>
<td>(-0.888)</td>
</tr>
</tbody>
</table>

+ significant at 10% level, * significant at 5% level, ** significant at 1% level
repurchases. On the other hand insider trading plays a more clear role in generating skewness in returns than stock repurchases, specially in the short run.

Our findings are relevant in understanding the liquidity provision process and might enlighten policy makers on the implications of insider trading restrictions, disclosure requirements, and insider transactions publicity on the liquidity of stock markets. Moreover, the impact of insider trading on liquidity, short-horizon variance and skewness is relevant for risk management and asset pricing. Firms that restrict insider trading activity might have less liquid and more volatile stocks. Furthermore, the inability to provide price support on the firm’s account makes financially constrained firms riskier. Finally, the presence of lockup periods, in which insiders cannot sell their holdings, can make stock prices more prone to temporal overpricing.

At the current state of our research agenda, two important questions remain that will be addressed in further work. The first is to improve our understanding of liquidity provision by insiders by focusing on large price corrections due to liquidity shocks. In this event-study type setting we will be in a better position to assess insiders actions and their impact on return distributions. The second is analyzing the effect of earlier disclosure of insider trades, as imposed by Sarbanes-Oxley Act after August of 2002. Furthermore, our results call for the need to develop a full theory that analyzes the tradeoffs involved in insiders’ decision to trade on the firm’s vs their own account, in the presence of liquidity shocks, when both moral hazard and adverse selection considerations are in place. All these extensions, however, are beyond the scope of the present paper.

2.A Appendix

2.A.1 Proof of lemma 2

The informed problem at $t = 1$ is

$$\max_{x_{I,1}} E \left[ -\exp \left( -r_I (x_{I,1} (P_3 - P_1) - k(x_{I,1})) \right) \right] | I_{I,1}$$

it is immediate that the first order conditions derived from this optimization problem imply that the optimal informed demand is

$$x^*_I,1 = \frac{E \left[ P_3 | I_{I,1} \right] - P_1 - \frac{dk}{dx_{I,1}} (x^*_I,1)}{r_I \text{Var} \left[ P_3 | I_{I,1} \right]} = \frac{s - P_1 - \frac{dk}{dx_{I,1}} (x^*_I,1)}{r_I \sigma^2_{s}}$$

whenever $x^*_I,1 \neq 0$ because the cost $k$ is not differentiable at 0.

Note that a sufficient condition to to have $x^*_I,1 > 0$ is $s > P_1 + k_+$; similarly, a sufficient condition to to have $x^*_I,1 > 0$ is $s < P_1 - k_-$. Therefore, the optimal
demand of an informed trader at \( t = 1 \) will be given by

\[
x_{I,1} = \begin{cases} 
  \frac{s - P_1 + k}{r_I \sigma^2} & \text{if } s < P_1 - k_-, \\
  \frac{s - P_1 - k_+}{r_I \sigma^2} & \text{if } s > P_1 + k_+.
\end{cases}
\]

We will show now that whenever \( P_1 - k_- \leq s \leq P_1 + k_+ \) the optimal demand for the informed is \( x_{I,1} = 0 \). Let us assume that the optimal demand is \( x_{I,1} > 0 \), in this case we know that \( x_{I,1} = x_{I,1}^* \), but given that \( s \leq P_1 + k_+ \) we would have \( x_{I,1} = x_{I,1}^* \leq 0 \), which contradicts that \( x_{I,1} > 0 \). In the same way, we can show that it can’t be optimal \( x_{I,1} < 0 \) when \( s \geq P_1 - k_- \). As a consequence, the optimal demand for the informed is \( x_{I,1} = 0 \) whenever \( P_1 - k_- \leq s \leq P_1 + k_+ \).

### 2.A.2 Proof of proposition 5

At \( t = 2 \) the uninformed already knows the informed trade at \( t = 1 \), and being public information we can consider the cases in which the informed trade, \( x_{I,1} \neq 0 \), and the case in which the informed do not trade, \( x_{I,1} = 0 \), separately. Let us solve first the equilibrium in the former case and second in the later.

**Informed trade at \( t = 1 \)**

Given that the uninformed knows that \( x_{I,1} \neq 0 \) and that, in this case, \( x_{I,1} = \frac{s - P_1 + k_+}{r_I \sigma^2} \) by lemma 2. It is clear that knowing the actual value of \( x_{I,1} \neq 0 \) and \( P_1 \) is informationally equivalent to know \( s \). It is immediate to see that the demand of an uninformed will be given by

\[
x_U = \frac{s - P_2}{r_U \sigma^2}
\]

Imposing the market clearing condition, \( \lambda x_{I,2} + (1 - \lambda) x_{U,2} = x \) and the fact that informed agents cannot trade at \( t = 2 \), \( x_{I,2} = x_{I,1} \), it is immediate that

\[
P_2 = s + \frac{r_U \sigma^2}{1 - \lambda} (x + \lambda x_{I,1}).
\]

Finally, by lemma 2,

\[
x_{U,2} = \begin{cases} 
  \frac{-1}{1 - \lambda} (x + \lambda \frac{s - P_1 + k_+}{r_I \sigma^2}) & \text{if } s < P_1 - k_- \\
  \frac{-1}{1 - \lambda} (x + \lambda \frac{s - P_1 - k_+}{r_I \sigma^2}) & \text{if } s > P_1 + k_+.
\end{cases}
\]

and

\[
P_2 = \begin{cases} 
  s + \frac{r_U \sigma^2}{1 - \lambda} (x + \lambda \frac{s - P_1 + k_+}{r_I \sigma^2}) & \text{if } s < P_1 - k_- \\
  s + \frac{r_U \sigma^2}{1 - \lambda} (x + \lambda \frac{s - P_1 - k_+}{r_I \sigma^2}) & \text{if } s > P_1 + k_+.
\end{cases}
\]
In this case, uninformed know $P_1$ and $x_{I,1} = 0$, which implies they know $x$ and that $P_1-k_- \leq s \leq P_1+k_+$. We will define $s' = \frac{s}{\sigma_s}$, $\bar{s} = \frac{P_1-k_-}{\sigma_s}$, and $\bar{s} = \frac{P_1+k_+}{\sigma_s}$. Uninformed maximize the expected utility of their wealth conditional on their information at $t=2$, this expected utility can be written as follows:

\[
E\left[U(W_{U,3}) \mid I_{U,2}\right] = -E\left[\exp\left(-r_U x_{U,1} (P_2 - P_1) + x_{U,2} (\sigma_s s' + \varepsilon - P_2)\right) \mid s' \leq \bar{s}\right] = \\
= \frac{-1}{\Phi(\bar{s}) - \Phi(\bar{s})} \int_\Delta \exp\left(-r_U x_{U,1} (P_2 - P_1) + x_{U,2} (\sigma_s s' + \varepsilon - P_2)\right) \phi(s') ds' = \\
= \frac{-1}{\Phi(\bar{s}) - \Phi(\bar{s})} \int_\Delta \exp\left(-r_U x_{U,2} (\sigma_s s' + \varepsilon) + \frac{r_U^2 x_{U,2}^2}{2} \text{Var}(\sigma_s s' + \varepsilon|s')\right) \phi(s') ds' = \\
= \frac{-1}{\Phi(\bar{s}) - \Phi(\bar{s})} \int_\Delta \exp\left(-r_U x_{U,2} \sigma_s s' - \frac{1}{2} s'^2\right) \phi(s') ds' = \\
\text{where } \phi \text{ and } \Phi \text{ are the probability density function and the cumulative distribution function of a standard normal random variable. Substituting } \phi(s') = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{s'^2}{2}\right) \text{ in the previous expression and taking } \exp\left(-\frac{r_U^2 x_{U,2}^2 \sigma_s^2}{2}\right) \text{ outside the integral} \text{, we can rewrite the expected utility as follows:}
\]

\[
E\left[U(W_{U,3}) \mid I_{U,2}\right] = -\frac{-1}{\Phi(\bar{s}) - \Phi(\bar{s})} \int_\Delta \exp\left(-r_U x_{U,1} (P_2 - P_1) - x_{U,2} (P_2 - \frac{1}{2} r_U x_{U,2}^2 \sigma_s^2)\right) \phi(s') ds' = \\
= -\frac{-1}{\Phi(\bar{s}) - \Phi(\bar{s})} \int_\Delta \exp\left(-r_U x_{U,2} \sigma_s s' - \frac{1}{2} s'^2\right) ds' = \\
= -\frac{-1}{\Phi(\bar{s}) - \Phi(\bar{s})} \int_\Delta \exp\left(-r_U x_{U,2} \sigma_s s' - \frac{1}{2} s'^2\right) ds' = \\
= -\frac{-1}{\Phi(\bar{s}) - \Phi(\bar{s})} \int_\Delta \exp\left(-\frac{1}{2} (s' + r_U x_{U,2} \sigma_s)^2 + \frac{1}{2} r_U^2 x_{U,2}^2 \sigma_s^2\right) ds' = \\
= -\frac{-1}{\Phi(\bar{s}) - \Phi(\bar{s})} \int_\Delta \exp\left(-\frac{1}{2} (s' + r_U x_{U,2} \sigma_s)^2\right) ds'.
\]
With the change of variables \( z = s' + r_U x_U,2 \sigma_s \),
\[
E [ U_U (W_U,3) | I_U,2 ] = -\exp \left( -r_U (x_U,1 (P_2 - P_1) - x_U,2 P_2 - \frac{1}{2} r_U x_U,2^2 (\sigma_e^2 + \sigma_s^2)) \right)
\]
\[
= \Phi(\pi + r_U x_U,2 \sigma_s) - \Phi(s + r_U x_U,2 \sigma_s)
\]
\[
= \exp \left( -r_U (x_U,1 (P_2 - P_1) - x_U,2 P_2 - \frac{1}{2} r_U x_U,2^2 (\sigma_e^2 + \sigma_s^2)) \right)
\]

Note that maximizing \( E [ U_U (W_U,3) | I_U,2 ] \) is equivalent to minimize
\[
\ln \left( \Phi(\pi + r_U x_U,2 \sigma_s) - \Phi(s + r_U x_U,2 \sigma_s) \right) + r_U (x_U,2 P_2 + \frac{1}{2} r_U x_U,2^2 (\sigma_e^2 + \sigma_s^2))
\]
thus, the first order conditions for the uninformed are
\[
\sigma_s \phi(\pi + r_U x_U,2 \sigma_s) - \phi(s + r_U x_U,2 \sigma_s) P_2 + r_U x_U,2 \sigma_s^2 = 0.
\]

The market clearing condition, \((1 - \lambda)x_{U,2} + x = 0\) imposes that \( x_{U,2} = \frac{-x}{1-\lambda} \) and substituting this expression in the first order conditions we obtain the equilibrium price \( P_2 \) when \( P_1 - k_\lambda \leq s \leq P_1 + k_\lambda \):
\[
P_2 = \frac{r_U (\sigma_e^2 + \sigma_s^2)}{1 - \lambda} \left( 1 - \frac{\phi(s - \frac{r_U \sigma_e x}{1-\lambda}) - \phi(s - \frac{r_U \sigma_s}{1-\lambda})}{\phi(\pi + \frac{r_U \sigma_e x}{1-\lambda}) - \phi(s - \frac{r_U \sigma_s}{1-\lambda})} \right)
\]
\[
= \frac{r_U (\sigma_e^2 + \sigma_s^2)}{1 - \lambda} \left( 1 - \frac{\phi(\frac{P_1 + k_\lambda}{\sigma_e} - \frac{r_U \sigma_e x}{1-\lambda}) - \phi(\frac{P_1 - k_\lambda}{\sigma_e} - \frac{r_U \sigma_e x}{1-\lambda})}{\phi(\frac{P_1 + k_\lambda}{\sigma_s} - \frac{r_U \sigma_s x}{1-\lambda}) - \phi(\frac{P_1 - k_\lambda}{\sigma_s} - \frac{r_U \sigma_s x}{1-\lambda})} \right).
\]

### 2.A.3 Numerical approximation to the equilibrium at \( t = 1 \)

The numerical approximation to the equilibrium is based on the projection method used by Bernardo and Judd (2000), as a consequence we are estimating an \( \varepsilon \)-rational expectations equilibrium\(^9\). In an \( \varepsilon \)-rational expectations equilibrium, for all states in a set of probability \( 1 - \varepsilon \), the decisions of all traders are nearly optimal, with the absolute value of their relative error not larger than \( \varepsilon \); and markets almost clear, with the absolute value of the excess demand not larger than \( \varepsilon \).

The equilibrium price, \( P_1(x, s) \), and uninformed demand, \( x_U(P_1) \), are approximated by finite-order polynomials, which transforms our problem of computing the equilibrium in an infinite dimensional space into estimating a finite number of parameters. In particular we define the approximated equilibrium price and uninformed demand

\(^9\)We shall not confuse the \( \varepsilon \) in the definition of \( \varepsilon \)-rational expectations equilibrium with the random variable \( \varepsilon \) in our model.
as
\[ \hat{P}_1(x,s) = \sum_{i=0}^{N} \sum_{j=0}^{N-i} a_{i,j} H_i(x) H_j(s) \]
\[ \hat{x}_U(P_1) = \sum_{i=0}^{N} b_i H_i(x) \]

where \( H_i \) is the degree \( i \) Hermite polynomial and \( N \) is the largest degree of the polynomial approximation. In our case, we have obtained the best results for \( N = 3 \). The choice of Hermite polynomials is because they are mutually orthogonal with respect to the normal density function with mean zero, the advantages of such a base of polynomials are discussed in Judd (1992). Our goal is to estimate the parameters \( a_{i,j} \) and \( b_i \) and, to do so, we will impose several conditions derived from the uninformed first order condition and market clearing.

Following Bernardo and Judd (2000) methodology, we numerically impose the conditional expectation first order condition
\[
E \left[ r_U (s + \varepsilon - P_1) \exp \left( - r_U x_{U,1} (s + \varepsilon - P_1) \right) \left| P_1 \right. \right] = 0
\]

as the \((N + 1)\) expectation conditions
\[
E \left[ r_U (s + \varepsilon - P_1) \exp \left( - r_U x_{U,1} (s + \varepsilon - P_1) \right) H_i(\hat{P}_1(x,s)) \right] = 0, \text{ for } i = 0 \ldots N;
\]

and the market clearing condition is imposed using the conditions
\[
E \left[ (\lambda x_I(\hat{P}_1(x,s),s) + (1-\lambda) \hat{x}_U(\hat{P}_1(x,s)) + x) H_i(x) H_j(s) \right] = 0, \text{ for } i+j = 0 \ldots N.
\]

The expectations are computed using Gaussian quadrature, whose nodes and weights are obtained from the routine `qnwnorm`, that belongs to COMPECON toolbox, written to accompany Miranda and Fackler (2002). We use 9 Gauss nodes to compute the quadrature, we observe that increasing the number of points does not improve the estimation. Finally, to solve the resulting nonlinear system we use the trust-region dogleg algorithm as implemented in `fsolve` function from Matlab’s optimization toolbox.
Ownership Dynamics with Multiple Insiders: The Case of REITs

3.1 Introduction

In this paper we study the dynamics of corporate ownership with multiple risk-averse insiders facing a moral hazard problem. This problem arises naturally within the context of the U.S. real estate investment trust (REIT) industry, among others. The industry has been undergoing substantial structural changes since the 1990s (see, e.g., Edelstein et al. (2005)). As a result of these changes, a typical REIT today has multiple individuals with significant ownership stakes. We study, both theoretically and empirically, dynamics of the aggregate ownership stake of significant REIT shareholders, in particular those shareholders for whom incentive alignment through ownership is important. We will refer to such shareholders as insiders, but the model could also apply to outside block-holders who can exert influence through monitoring.\(^1\) On the theoretical side, our model builds upon the dynamic moral hazard model of DeMarzo and Urosevic (2006) and Edelstein et al. (2005). These models study an optimal ownership policy of a risk-averse large shareholder/insider facing a moral hazard problem, and they determine the corresponding equilibrium share price.\(^2\) In this paper we extend these models to incorporate strategic interactions among multiple insiders. This allows us to explore how the make-up of the aggregate insider stake influences the nature and the speed of its adjustment. On the empirical side, this is the first paper that explores links between the ownership dynamics and the composition of the aggregate insider stake, i.e., the number of insiders in a company. In addition, our paper fills an important void in the REIT

\(^1\)Conditional on shareholdings, however, our large shareholders are assumed to face symmetric agency problems. Therefore the model cannot explain differential impacts of insiders and outside block-holders.

\(^2\)These models build upon one-period models of Admati et al. (1994) and two-period models of Kihlstrom (2001) and Stoughton and Zechner (1998). DeMarzo and Bizer (1993) establish the connection between the durable goods monopolist problem and securities markets and, thus, connection with the Coasian conjecture (see Coase (1972)).
Ownership Dynamics with Multiple Insiders

empirical literature. Namely, while for non-REIT corporations, Mikkelson et al. (1997), Urosevic (2002) and Harjoto and Garen (2005) (for the U.S.) and Franks et al. (2009) (for the U.K.) document a significant and steady decline of the aggregate insider ownership stake after the initial public offering (IPO), to the best of our knowledge, no similar studies have been performed for REITs.

Let us now describe the model. In an economy consisting of one risky firm and a riskless bond, there are $N$ equally risk-averse agents whom we label insiders and a continuum of small outside risk-averse investors, all with CARA preferences. We consider the case in which each insider can commit to an optimal ownership policy and the case in which such commitment is impossible. As in DeMarzo and Urosevic (2006) and Edelstein et al. (2005), the commitment policy is time-inconsistent in the sense that after a planned sale, any insider with marginal valuation below that of the aggregate investor’s will be tempted to trade again. The insider will no longer internalize the value loss on the shares just sold. Put in another way, insiders’ risk aversion creates a wedge between their valuation of company shares and the value placed on the company shares by the outside investors (the market). The optimal time-consistent policy, therefore, is for insiders to gradually adjust their stakes in the company until the perfect risk sharing allocation is achieved. (In contrast to Edelstein et al. (2005), for reasons of tractability, in this paper we do not consider private benefits of control by corporate insiders.) As a result, the competitive allocation coincides with the perfect risk-sharing allocation.

There are two competing forces driving the results of the model. On one hand, insiders face a moral hazard problem so that the expected firm cash flows and, therefore, the company value increases with insider holdings, ceteris paribus. On the other hand, large company ownership implies a significant risk exposure; as a result, insiders have an incentive to decrease their stakes over time. Importantly, the ownership policy of each insider influences the share price and, therefore, every insider’s future ownership decisions. The equilibrium in the economy simultaneously specifies the optimal ownership policies of each insider, as well as the corresponding share price process.

Since outside investors are assumed to be risk-averse, there is an additional strategic reason for a dynamic stake adjustment in this model vis-à-vis the one-agent case. A decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. By selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates among the insiders a “race to diversify.” As a result, in the unique subgame-perfect equilibrium, the speed of adjustment towards the perfect

---

3We use the word “insider” to denote anyone who files SEC insider forms and not necessarily someone who “trades on information.” In our model, insiders do not trade on private information, but instead trade to reduce the amount of idiosyncratic risk that they bear. Other authors have analyzed insider trading on private information in the presence of moral hazard; for example, in Yung (2005), moral hazard arises because a firm’s returns are a function of the insider’s costly effort.

4Benefits of control are likely substantial for REITs, but we see no obvious relation between the effect of those benefits and the number of corporate insiders, which is the focus of our empirical work in this paper.
risk-sharing allocation increases with the number of company insiders $N$. Intuitively, as $N$ increases, the asset price more quickly becomes competitive, though the adjustment towards the long-run equilibrium is gradual.

Our model has three main testable predictions. The first is that the aggregate insider’s stake gradually declines towards the perfect risk-sharing allocation (the long-term equilibrium). The second is that the long-term equilibrium allocation level increases with an increase in the number of a REIT’s insiders. Finally, the third is that the initial speed of adjustment of the aggregate insider stake towards the long-term equilibrium level increases with an increase in the number of a REIT’s insiders. We test these three predictions on a sample of 137 U.S. REITs that went public between January 1986 and December 1999. Using $t$-tests and linear regressions, we find empirical support for all three predictions. We observe a reduction in the aggregate insider ownership by 18% during the first five years and by 42% during the first ten years after the IPO; moreover, the aggregate insider ownership during the tenth year of a REIT’s life and the speed of adjustment during the first five years of a REIT’s life increase with the number of insiders.

Our model adds to a growing literature on firms (not specifically REITs) with several large shareholders. While REITs are particularly well suited for our analysis given their dividend-payout requirements, our qualitative results would likely apply to the non-REIT corporation setting as well. Independently of this work, Pritsker (2005) develops a related model. He also constructs a model of multiple “large traders” in a CARA/normal setting and borrows, as does this paper, modeling techniques from DeMarzo and Urosevic (2006). In many important ways, however, our models are quite different. The key difference is in the economic environment that the two models portray. While both of our papers incorporate diversification as a key motivation to trade, Pritsker (2005) studies market liquidity, shock transmission and market manipulation by large institutional traders with varying degrees of risk aversion in a multi-asset economy with no moral hazard. In contrast, we study the evolution of insider ownership stakes in a single-asset economy in which insiders face a moral hazard problem. Another theoretical work on dynamic trading by multiple large shareholders is Vayanos (2001), which allows for asymmetric information regarding personal allocations but does not include small competitive traders. More distant from our analysis are static analyses of multiple large shareholders including Noe (2002) strategic traders’ incentives to monitor management and thereby become informed, and papers by Gomes and Novaes (2006) and Bennedsen and Wolfenzon (2000) on the optimal large shareholder structure of a firm (from the founder’s perspective) when there may be bargaining problems or coalition building among large insiders who determine investment policy and have opportunities to expropriate value from small investors.

While there is limited previous work on the changing insider ownership of REITs, several previous authors have related ownership to performance. Recent examples include Han (2006), which relates Tobin’s $q$ to insider ownership levels within var-

---

5Another related theoretical work is that of Brunnermeier and Pedersen (2005), who study the issue of predatory trading in a multi-trader context.
Ownership Dynamics with Multiple Insiders

Given that these numbers are twice the level of our average initial aggregate insider stakes (see Table 2 below), we can infer that increases in insider ownership are beneficial for most of the REITs in our sample—the moral hazard model that we include may be more relevant than the control benefits that we exclude from consideration. Capozza and Seguin (2003) and Dolde and Knopf (2007), the latter with a non-linear specification and more recent data, find no relation between REIT insider ownership and returns. Those two papers do report a generally negative relation between general and administrative expenses (perhaps a measure of perk consumption) and ownership. That result also suggests that omitting control benefits may not be crucial. Furthermore, as pointed out by Capozza and Seguin (2003) and others, a relative lack of hostile takeovers in the REIT market is not surprising in light of the legal requirement that five or fewer owners may not hold more than 50% of a REIT’s shares. This suggests that REIT managers are entrenched regardless of their extent of ownership so that control benefits may be independent of shareholdings. A final relation that has been examined recently is that of insider ownership with investment sensitivity to quality of investment opportunities. Hartzell et al. (2006) find investment is more responsive to property-type $q$ when institutional ownership is high and when officer and director ownership is low; our model does not distinguish between institutional block-holders and officer/director owners.

This paper is organized as follows. In the next section we describe the model. Following that we present equilibrium trajectories of the large shareholders’ holdings and derive key empirical predictions, and then we test the model using post-IPO data for U.S. REITs. The final section presents conclusions and suggestions for future research, while proofs are relegated to the Appendix.

### 3.2 The Model

By incorporating several strategic insiders instead of a single strategic agent, this model constitutes an important extension of the work by DeMarzo and Urosevic (2006) and Edelstein et al. (2005). In many other respects, the model structure bears significant similarities. For that reason we omit derivations that closely parallel those in DeMarzo and Urosevic (2006) and Edelstein et al. (2005), instead focusing on those aspects of the model that differ from its one-agent counterparts.

We consider a going-concern publicly traded REIT with a supply of shares that is normalized to one and with a cumulative free cash flow process described by the

---

6Friday et al. (1999) and Capozza and Seguin (2003) estimated piecewise linear relations between $q$ and insider ownership using older data sets. Friday et al. (1999) found that value declined with ownership beyond a 5% pre-specified threshold, while Capozza and Seguin (2003) found value increasing over all ownership ranges. Ghosh and Sirmans (2003) show that CEO shareholdings are negatively related to REIT return on equity, while other large shareholdings and institutional holdings are positively related to ROE.
following diffusion process
\[ dD = \hat{\mu} dt + \hat{\sigma} dz \]
\[ D(0) = 0; \quad (3.2.1) \]

where \( Z \) is standard Brownian motion (the expressions for \( \hat{\mu} \) and \( \hat{\sigma} \) are specified below). Consequently, the cash flows in each period are normally distributed and independent across periods.\(^7\) Shares of the firm trade in the market at the price \( V \), which needs to be determined in equilibrium. In addition to this firm there exists a riskless investment that pays a continuously compounded return of \( r \), with a perfectly elastic supply. We assume for tractability that all cash flows are paid out as dividends. The REIT industry approximates this assumption better than do most, even though the legal requirement that REITs pay out at least 90% of net taxable earnings has limited potency due to the availability of large depreciation deductions.

All agents in the economy maximize CARA expected utility. There are \( N > 1 \) agents that have an ability to monitor the REIT and affect decisions within the firm. We refer to them as insiders. For simplicity we assume that the insiders may have different initial company stakes but are otherwise identical. The coefficient of absolute risk aversion is assumed identical across insiders and is equal to \( \gamma \). In addition to insiders, there exists a continuum of competitive outside investors that can be represented by an aggregate investor; the coefficient of absolute risk aversion of the aggregate investor is denoted by \( \gamma^I \).

All trades occur in a competitive market.\(^8\) Let \( \alpha^l(t) \in [0,1] \) \((l = 1 : N)\) be the fraction of the firm held by the insider \( l \) at time \( t \). We restrict each \( \alpha^l(t) \) to be right-continuous, and we interpret \( \alpha^l(t-) = \lim_{\tau \uparrow t} \alpha^l(\tau) \) as the shares held at the “start” of period \( t \) by the insider \( l \); thus, \( \alpha^l(t) - \alpha^l(t-) \) is the discrete number of shares purchased by the insider \( l \) in period \( t \) or, since the two are interchangeable in this model, the change in insider \( l \)’s ownership stake at time \( t \). Insider \( l \) has an initial endowment \( \alpha^l(0-) = \alpha^l_\text{initial} \). Initial endowments are, generically, different from each other.\(^9\) Let \( A(t) = \sum_{l=1}^{N} \alpha^l(t) \) be the aggregate insider stake at time \( t \) and \( A^\text{-} = \sum_l \alpha^l_\text{-} \) be the initial aggregate insider stake. By market clearing, in equilibrium the aggregate investor’s holdings at time \( t \) are given by the expression \( 1 - A(t), \ t \geq 0 \). Sometimes, it will be convenient to separate in this sum holdings of an insider \( l, \alpha^l \), from the aggregate holdings of all other insiders \( \beta^I(t) = A(t) - \alpha^l(t) \). Furthermore, whenever confusion cannot occur, we shall drop the index \( l \) when describing a generic insider.

---

\(^7\)The unlimited liability implied by normality does not play an important role in the forces driving our results.

\(^8\)That means, in particular, that the insiders in the economy trade with competitive outside investors. In contrast, Vayanos (1999) considers a model with \( N \) strategic traders in the absence of competitive investors.

\(^9\)The initial stakes are exogenous to the model. Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006) endogenize initial stakes in a one-agent moral hazard setting.
Insiders face a simple moral hazard problem. Insider $l$’s costly monitoring effort $e_l$, $l = 1, ..., N$, affects the expected free cash flow of the REIT in a linear fashion, $\hat{\mu}(e) = \sum_{l=1}^{N} e_l$. One way to interpret this is to think of each insider as working on an independent task within the firm. While insiders do not affect the effort of other insiders directly, they do so indirectly by recognizing their own and other insiders’ impact on the process of share price formation (see below). We assume, further, that the variance of the REIT’s cash flows cannot be altered by the actions or holdings of the insiders; we set $\hat{\sigma} = \sigma$. The assumption of fixed volatility may be partially justified in the REIT industry by the requirements that 75% of REIT assets and income sources be real-estate related, though Capozza and Seguin (2003) and Dolde and Knopf (2007) observe relations between ownership levels and systematic risk that they suggest are due to managerial influence on risk. In our model firms with low exogenous volatilities would, as the model solutions to follow will suggest, have lower equilibrium insider holdings, calling into question the direction of a causal relationship. The model would not, however, predict the U-shaped relation observed by Dolde and Knopf (2007).

An insider’s $l$ cost of effort is assumed to be independent of other insiders’ effort choices and quadratic in effort, i.e., $f(e_l) = e^2_l(2\mu)$, where parameter $\mu$ is identical for all insiders. None of the parameters of the model depend on time. Since the effort cannot be contracted on, each insider’s effort choice must be incentive compatible. Because of the CARA/normal setting, each insider’s problem can be expressed in terms of the certainty equivalent. The moral hazard of the insiders is, therefore, reflected in the dependence of the dividend on their aggregate holdings $A$, whereas the motivation for trading is provided by the difference in marginal valuations of the insiders and the aggregate investor.

In the model, insiders live infinitely but trade and effort can influence firm cash flows only for a finite period of time. In particular, while insiders may consume, make effort choices and trade in the riskless security continuously, we assume that they are restricted to trade shares of the firm on a finite set of dates $T = \{t_1 = 0, t_2, ..., t_N = T\}$ common to all insiders. In this case, $\alpha(t) = \alpha(t^-)$ for all $t \in [t_i, t_{i+1})$, where we define $t_{N+1} = \infty$. Technically, a finite number of trading periods is necessary for the uniqueness of the sub-game perfect equilibrium. In practice, companies frequently impose “windows” within which company insiders can trade, say, every quarter. The timing in the model is shown in Figure 3.1.

Insiders’ decisions are sequentially rational so that each agent is playing a multi-period simultaneous-move game with the other insiders in the company. Each insider knows her own, as well as the other insiders’, past trades. They also know that their

---

10Moral hazard plays an important role in ownership decisions of corporate insiders. In particular, Brav and Gompers (2003) find that the moral hazard problem facing corporate insiders is the main reason for the existence of lock up restrictions on share trading immediately following an initial public offering (IPO).

11Private benefits of control are excluded from the model. For a discussion of the effects of private benefits of control in the case of one large shareholder, see Edelstein et al. (2005).

12Urosevic (2002) finds that the average interval between two successive trades by a corporate insider is 109 days, roughly corresponding to quarterly trading windows; also see Seyhun (1998b).
trading decisions today affect the share price and, consequently, the future trading decisions of all insiders in the economy (including their own). Although the outside investors trade competitively as price-takers, they are aware of the strategic interaction among the insiders and the fact that the insiders’ current trading decisions have an impact on the insiders’ current and future trading decisions. Outside investors are rational and make their demands for shares after they observe the insiders’ trading decisions for that time period. In particular, there is no asymmetric information about the dividend process or about the insiders’ trading decisions. In other words, all information about the company and insider trades is revealed instantaneously to the investment community. Finally, we do not explicitly restrict the aggregate investor to trade only on dates T since, in equilibrium, investors would only trade when the insiders trade.

3.3 Insiders’ Trading Strategies

The setup and development of this model is similar to DeMarzo and Urosevic (2006) and Edelstein et al. (2005) with some important differences: (a) they consider one and we consider multiple strategic insiders and their interactions; and (b) in contrast to their models, we restrict the analysis to the case of a stationary linear moral hazard problem where insiders cannot influence volatility and have no benefits of control. Preliminary results of DeMarzo and Urosevic (2006), their Sections 3.1 to 3.3, can be adopted in our case with only minor changes, and we omit most details, including the equilibrium share price. Our focus is on the manner in which the insiders adjust their shareholdings. We will characterize insider trading strategies under three sets of assumptions about the insiders’ behavior. The solution to the final more realistic setting builds on the first two results. In any case, insiders will in general trade away from their exogenously imposed initial allocations. The direction

\[ \text{Figure 3.1. Timing in the model} \]
may typically be toward lower holdings, as insiders’ holdings relative to those of the aggregate investor likely exceed their relative tolerance for risk.

First, suppose that insiders can commit to a series of trades that depend at most on time (in particular, trades cannot be conditioned on previous trades). An insider’s ability to commit influences the trades of the other insiders and the market price at which shares can be sold over time. The following proposition describes a Nash equilibrium in this context.

**Proposition 6** Suppose that at time \( t \), each agent announces a trading policy that depends only on time, \( \alpha^l(t), \tau \geq t \), and cannot be revised in the future. Then, each insider’s strategy is given by the following Nash equilibrium strategy:

\[
\alpha^c(\tau) = \arg \max_{\alpha(\tau)} z(\alpha(\tau), \beta(\tau)) + (\alpha(t^-) - \alpha(\tau)) \nu (\alpha(\tau) + \beta(\tau))
\]

The aggregate insider equilibrium holdings in the commitment case are given by the following expression:

\[
A^c = \frac{(\mu + \gamma^l r \sigma^2)A^- + N \gamma^l r \sigma^2}{\mu + (N + 1) \gamma^l r \sigma^2 + \gamma r \sigma^2}
\]

Since the parameters of the model do not depend on time, the commitment equilibrium allocation does not depend on time either. In addition, from (3.3.1) one can easily check that the first order conditions for the problem read:

\[
\alpha^c = \frac{r \gamma^l \sigma^2}{\mu + r(\gamma + 2 \gamma^l) \sigma^2} (1 - \beta) + \frac{\mu + r \gamma^l \sigma^2}{\mu + r(\gamma + 2 \gamma^l) \sigma^2} \alpha^c
\]

In all of these expressions, the superscript \( l \) has been omitted. One has to bear in mind, however, that (3.3.1) and (3.3.3) stand for \( N \) different equations. In particular, since the initial insider holdings are generically different across the set of insiders, equilibrium insider holdings in the commitment case also differ across insiders. Moreover, an insider’s commitment equilibrium stake increases with her initial holdings. That means that whenever insiders can commit to an optimal allocation, those who initially hold larger stakes would tend to optimally choose a higher level of ownership to commit to. In addition, an insider’s commitment holdings decreases with an increase in other insiders’ holdings. That means that by holding a smaller company stake each insider aims to raise the market risk premium, thus lowering the stock price and, consequently, the motivation of the other insiders to trade down in the future. This effect will play an important role in the time-consistent model.

In the commitment equilibrium, each insider anticipates the impact that her trades, as well as those of other insiders, will have on the stock price. On the other hand, if insiders ignore such impact, the following proposition holds:
Proposition 7 Define $\alpha^p = \gamma I / (\gamma + N \gamma I)$. Then, if $\mu < r \gamma \sigma^2$, a Walrasian equilibrium exists in which each insider is a price-taker, the equilibrium trading strategy for each insider is given by $\alpha^p$ and the aggregate price-taking equilibrium allocation is

$$A^p = N \alpha^p = \frac{\gamma I}{N + \gamma I} \quad (3.3.4)$$

From Proposition 7 it follows that the price-taking equilibrium exists when the (beneficial) incentive effect associated with shareholding is smaller than the insiders’ aversion towards risk. The aggregate perfect risk sharing allocation $A^p$ can be seen as the competitive allocation of one insider with $N$ times higher risk tolerance. Notice that (3.3.4) increases in the number of insiders $N$. That leads to an important testable prediction of our model. Note, also, that when $N = 1$, Expressions (3.3.2) and (3.3.4) coincide, respectively, with the commitment and the competitive allocations in DeMarzo and Urosevic (2006).

Proposition 6 presented the optimal trading strategies assuming that all insiders could commit \textit{ex ante} to future trades. In that case, the current share price depends on all future trades that the insiders will make. If we no longer allow the insiders to commit to future trades, each insider’s trading strategy must be time-consistent. The previous results suggest that for a generic insider the commitment policy $\alpha^c$ is not time-consistent. To see the intuition for this, note that Equation (3.3.3) implies that $\alpha^c$ is increasing with the initial insider shareholdings and always exceeds the competitive allocation, provided the initial allocation did so. Therefore, an insider who has completed an initial sale of shares to reach level $\alpha^c$ would recalculate her optimal shareholdings using this $\alpha^c$ in place of the initial allocation and have an incentive to sell again. This second sale and the resulting change in effort impose a negative externality on the initial buyers of shares that the insiders do not consider when making a second sale. The same is true for each individual insider.

To solve for the equilibrium without commitment, note that the value of the shares at any time $t$ must depend on the investors’ expectations of the insiders’ future trading decisions. Investors must anticipate the insiders’ \textit{ex post} incentives to trade. At each time, each insider recognizes that her trading decision today impacts not only her future trading decisions but also those of all of the other insiders.

The problem is solved by backward induction. For that reason, first consider the insiders’ decision-making process at time $T$ (the last trading date). Recall that the insiders have the opportunity to trade only on the discrete dates $T = \{t_1 = 0, t_2, \ldots, t_N = T\}$. Implicitly, the insiders commit not to trade during the intervals $(t_i, t_{i+1})$. For simplicity, we assume that time intervals $D$ between trades are constant (except for the last trading interval that is infinite) and introduce the capitalization factor $\delta_t$ where $\delta_t = \delta = (1 - e^{-r\Delta}) / r$, except in the case when $t_{N+1} = \infty$, when we set $\delta_T = 1 / r$. At time $T$ insiders, by assumption, commit not to trade again. Denote a generic insider’s holdings at time $t$ as $\alpha_t$ and the aggregate holdings of all of the other insiders by $\beta_t$. Let $J_t$ be the optimal certainty equivalent of an insider at time $t$, \textit{i.e.}, the value function. Finally, let the share price at time $t$ be denoted as $V_t$. Then, given a vector of the initial insiders’ holdings, one can specify the
recursive solution. Furthermore, under the assumptions of the model this recursive equilibrium is a unique sub-game perfect equilibrium. It is characterized by the following proposition:

**Proposition 8** For each \( t \leq T \), the value function of the dynamic programming problem above is a quadratic form in variables \((\alpha_t, \beta_t)\):

\[
J_{t+1} = J_{t+1}^{\alpha,\alpha} \alpha_t \alpha_t + J_{t+1}^{\alpha,\beta} \alpha_t \beta_t + J_{t+1}^{\beta,\alpha} \beta_t \alpha_t + J_{t+1}^{\beta,\beta} \beta_t \beta_t + J_{t+1}^{0,0}; \tag{3.3.5}
\]

while the share price is an affine function in \( A_t = \alpha_t + \beta_t \)

\[
V_t = v_{0,t} + v_t A_t. \tag{3.3.6}
\]

The optimal holdings of each insider at time \( t \) are determined as an affine transformation of her own and other insiders’ holdings at time \( t - 1 \):

\[
\alpha_t = l_t^{\alpha,\alpha} \alpha_{t-1} + l_t^{\alpha,\beta} \beta_{t-1} + l_t^{\alpha,0} \\
\beta_t = l_t^{\beta,\alpha} \alpha_{t-1} + l_t^{\beta,\beta} \beta_{t-1} + l_t^{\beta,0} \tag{3.3.7}
\]

A complete set of recursive relations and the appropriate boundary conditions that determine the coefficients in (3.3.5)-(3.3.7) are given in the Appendix (Equations (A10) to (A13)). Under the assumptions of the model (in particular a \( \mu \geq 0 \) and a constant volatility), this equilibrium is the unique sub-game perfect equilibrium in the economy.

A straightforward but tedious calculation shows that, when \( N \to 1 \), the solution given by Proposition 3 coincides with the solution in DeMarzo and Urosevic (2006). Therefore, the equilibrium of Proposition 3 generalizes to the multi-agent setting the time-consistent equilibrium in DeMarzo and Urosevic (2006).

### 3.3.1 The Solution and Comparative Statics

Proposition 8 specifies the procedure for determining the optimal aggregate insider ownership policy when commitment is not possible. The solution is obtained in terms of a system of coupled recursive relations. Since the aggregate investor is assumed to be risk-averse, the time-consistent solution can, in general, be analyzed only numerically.\(^{14}\) Now we will convey the results of such an analysis. In particular, we describe three important properties of the model:

**Property 1:** The aggregate stake gradually declines towards the perfect risk sharing allocation (the long-term equilibrium).

\(^{14}\)In a limit when the outside investors are risk-neutral, the problem effectively decouples into \( N \) single-agent problems, each of them equivalent to a problem discussed in DeMarzo and Urosevic (2006) and Edelstein et al. (2005), which can be solved exactly.
Property 2: The long-term equilibrium allocation level increases with an increase in number of REIT insiders, *ceteris paribus*.

Property 3: The initial speed of adjustment of the aggregate insider stake towards the long-term equilibrium level increases with an increase in number of REIT insiders, *ceteris paribus*.

Property 1 follows from the fact that insiders cannot credibly commit to a level of ownership above the competitive allocation. The second property (see equation 3.3.4) follows intuitively from the fact that with an increase in a number of insiders, \(N\), each insider’s risk exposure diminishes. This allows insiders to absorb more risk in aggregate. In order to explain Property 3, note that a decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. Therefore, by selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates among insiders a “race to diversify”. As a result, in the unique sub-game perfect equilibrium, the speed of adjustment toward the perfect risk sharing allocation increases with the number of insiders in the company. Intuitively, as there are more strategic agents in the economy, prices more quickly become competitive, although the adjustment towards the long-run equilibrium is gradual.

The model confirms our intuition. In Figure 3.2, we present the dynamics of the aggregate insider stakes when the number of insiders varies from \(N = 1\) to \(N = 5\).

While an increase in the number of insiders, *ceteris paribus*, raises the speed of adjustment towards the competitive allocation (3.3.4), it raises that level as well. This leads to an interesting empirical prediction. To wit, if outside investors are risk-averse, one would expect that companies with a relatively large number of (identical) insiders, *ceteris paribus*, should have a relatively short period of steep insider ownership adjustment towards a relatively high aggregate insider ownership level thereafter. In contrast, when the number of insiders in a company is relatively small, *ceteris paribus*, one would expect to observe slower adjustment towards a relatively low level of the aggregate insider ownership stake. Thus, if outside investors are risk-averse, the dynamics of the aggregate insider stake depends on the number of corporate insiders.

### 3.4 Empirical Analysis

In this section we test the three key model predictions:

Prediction 1: The aggregate insider stake gradually declines towards a long-term equilibrium level.

Prediction 2: The long-term equilibrium level of the aggregate insiders’ stake increases with the number of insiders.
Figure 3.2. Aggregate insider ownership policy varies with the number of insiders (risk-averse investors). Here, $\gamma^I \sigma^2 r = 5$, $\gamma = 15 \gamma^I$, $\mu = 100$, $r = 4\%$, and the number of insiders varies from $N = 1$ to $N = 5$. Notice that the speed of adjustment increases as the number of insiders in the company increases. At the same time, the long-term aggregate equilibrium allocation also increases. As a result, as the number of insiders increases, the aggregate insider stake adjusts relatively quickly to a relatively high long-term equilibrium level. Trading is quarterly.

**Prediction 3:** The initial speed of adjustment of the aggregate insiders’ stake also increases with the number of insiders.

In order to test empirically the predictions of the theoretical model, we study the evolution of the aggregate insiders’ ownership stake for 137 publicly traded U.S. REITs during the first ten years after their IPO. We find support for Predictions 1-3: there is a significant reduction in the aggregate insiders’ stake post-IPO. Moreover, the long-term aggregate ownership level of REIT insiders, as well as the initial speed of adjustment, is higher for REITs with a larger number of insiders than in REITs with a smaller number of insiders.

### 3.4.1 Data Description

Our initial universe of companies consists of publicly traded U.S. REITs companies that appear in the Center for Research in Security Prices (CRSP) data file. All the market data fields come from the CRSP database.

We obtain insider ownership data using the Thomson Financial Insiders Filings (TFIF) database on insider transactions between January 1986 and December 2004. According to Section 16(a) of the Security and Exchange Act of 1934 (SEA), large
beneficial shareholders and managers of a publicly traded firm are required\textsuperscript{15} to file their transactions in the company stock with the Securities and Exchange Commission (SEC); these reports are collected in the TFIF dataset. Insiders are supposed to report the number of shares owned at the time of IPO filing; using this information jointly with information on the number of shares acquired or disposed of at each transaction, we obtain estimates of insider positions in common shares of REITs.\textsuperscript{16} After merging the estimated insider holdings at the end of each calendar month with the monthly data on shares outstanding from the CRSP file (correcting for stock splits), we compute the ownership stake of each insider at the end of each month. Finally, other variables such as operating partnership (OP) units and REIT type are obtained from the SNL dataset.

For the purposes of our analysis of the first and second predictions we focus on companies with at least ten years of post-IPO\textsuperscript{17} data (a total of 59 REITs); for our analysis of the third prediction we focus on companies with five years of data post-IPO (a total of 130 REITs). Since we are interested in the behavior of insiders with significant company stakes, we eliminate from further consideration in the five-years sample companies in which no insider owned more than 1\% of company shares outstanding at any time during the first five years post-IPO, \textit{i.e.} companies with no insiders with a significant stake post-IPO; in addition, we eliminate from further consideration in the ten-years sample companies in which no insider owned more than 1\% of company shares outstanding at any time during the first ten years post-IPO.\textsuperscript{18}

Regarding the distribution of IPOs in our sample, whether measured by number of IPOs or by market capitalization, there is a concentration in the 1993-1994 and 1997-1998 periods. The largest IPO in our sample is that of Equity Office Properties Trust in 1997; the value of this REIT at the end of the first trading month was 4,016.7 million dollars (expressed in real terms as January 1995 dollars). Although there were four IPOs in 1999, their size was quite small, only 110.4 million dollars (again expressed in January 1995 dollars).

In Table 3.2 we show the distribution of companies in our sample by the initial number of significant insiders ($N(0)$). Most REITs in the sample have more than one significant insider; this is a motivation to construct a model such as ours, which considers the strategic interaction between and among these multiple insiders. The number of large insiders usually is not large; no more than 10\% of REITs have more

\textsuperscript{15}The threshold for reporting is 5\% ownership by non-managers, but some external shareholders choose to report their trades even when they hold less than 5\% of a firm, as explained by Lakonishok and Lee (2001).

\textsuperscript{16}The procedure that we follow to determine the total holdings for a particular insider for each document is the following: To determine direct holdings we use those that are reported with the largest sequence number. To determine indirect holdings (holdings of a trust, spouse, children) we aggregate all of the reported indirect holdings in the same document. Note that the same document must contain both direct and indirect holdings, if the latter exists.

\textsuperscript{17}We consider the IPO date to be the date of a company’s first appearance in the CRSP database.

\textsuperscript{18}There are seven REITs in which no insider owned more than 1\% of company shares outstanding at any time during the first five years post-IPO, but they did at some time during the following five years. For this reason, we do not include all 137 REITs in our five-year sample.
than six significant insiders. Finally, there are three REITs in our sample that did not have a single significant insider at the IPO date.

Table 3.1. Distribution of REITs by \( N(0) \): The table shows the distribution of REITs by the number of insiders at the IPO date. We consider only those insiders that, at some time during the first five or ten years post-IPO, owned more than 1% of shares outstanding. Our universe of companies consists of 137 REITs that went public between January 1986 and December 1999, that appear in CRSP data files, for which we have been able to construct time series of insiders’ ownership and for which at least one insider has a stake higher than 1% at some time in the first five or ten years after the IPO date.

<table>
<thead>
<tr>
<th>( N(0) )</th>
<th>Subsample for which we have five years of data</th>
<th>Subsample for which we have ten years of data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of REITs</td>
<td>Percentage</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>2.31</td>
</tr>
<tr>
<td>1</td>
<td>37</td>
<td>28.46</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>22.31</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>16.92</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>7.69</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>7.69</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>8.46</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>3.08</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>1.54</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.77</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>0.77</td>
</tr>
</tbody>
</table>

The summary statistics on insider ownership are reported in Table 3.1. We report, for each of the first five or ten years after an IPO (depending on the sample), the mean and the standard deviation (the latter quantities are given in italics) of the number of insiders with significant stakes \( (N(t)) \), the aggregate insider stake \( (A(t)) \), the average insider stake \( (A(t)/N(t)) \), the largest stake owned by an insider \( (\alpha_{\text{max}}(t)) \) and the stake of an insider with the largest holdings at the IPO date \( (\alpha_{\text{inimax}}(t)) \). \( t \) is the number of months that elapsed after the IPO. The average number of insiders with significant company stakes remains roughly constant (around three insiders per REIT), with a small increase over time. Just as the model predicts, the mean values of \( A \), \( \alpha_{\text{max}} \), \( A/N \) and \( \alpha_{\text{inimax}} \) decrease over time; on average \( A \) decreases by 18% during the first five years and by 42% during the first ten years after the IPO. The standard deviations of these quantities are large, which indicates a large cross-sectional variation across REITs. Furthermore, we know from this table that not only the initially largest shareholder sells his or her shares; around three quarters of the reduction of the aggregate insiders’ stake in ten years is due to a reduction in the initially largest shareholders holdings, with the remaining quarter due to selling by other large shareholders.

In Figure 3.3 we plot, for companies with initial number of significant insiders, \( N(0) \), equal to one, two, five or six, the aggregate insider stake as a function of the number
Table 3.2. Descriptive statistics of insider ownership: The table offers mean and standard deviations (the latter quantity is in italics) of the number of insiders ($N$), the aggregate insider stake ($A$), the average insider stake ($A/N$), the largest stake owned by an insider ($\alpha_{\text{max}}$) and the stake of an insider with the largest holdings at the IPO date ($\alpha_{\text{inimax}}$). The first column ($t$) is time elapsed after the IPO (in months). The statistics are reported at the IPO date and every year up to year five after the IPO. We consider only those insiders that, at some time during the first five or ten years post-IPO, owned more than 1% of shares outstanding. Our universe of companies consists of 137 REITs that went public between January 1986 and December 1999, that appear in CRSP data files, for which we have been able to construct time series of insiders’ ownership and for which at least one insider has a stake higher than 1% at some time in the first five or ten years after the IPO date.

<table>
<thead>
<tr>
<th>$t$</th>
<th>Subsample for which we have five years of data</th>
<th>Subsample for which we have ten years of data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N(t)$</td>
<td>$A(t)$</td>
</tr>
<tr>
<td>0</td>
<td>2.938</td>
<td>0.173</td>
</tr>
<tr>
<td>12</td>
<td>3.115</td>
<td>0.163</td>
</tr>
<tr>
<td>24</td>
<td>3.2</td>
<td>0.155</td>
</tr>
<tr>
<td>36</td>
<td>3.215</td>
<td>0.151</td>
</tr>
<tr>
<td>48</td>
<td>3.215</td>
<td>0.148</td>
</tr>
<tr>
<td>60</td>
<td>3.223</td>
<td>0.142</td>
</tr>
<tr>
<td>72</td>
<td>3.644</td>
<td>0.121</td>
</tr>
<tr>
<td>84</td>
<td>3.644</td>
<td>0.124</td>
</tr>
<tr>
<td>96</td>
<td>3.695</td>
<td>0.124</td>
</tr>
<tr>
<td>108</td>
<td>3.695</td>
<td>0.122</td>
</tr>
<tr>
<td>120</td>
<td>3.678</td>
<td>0.111</td>
</tr>
</tbody>
</table>

of months elapsed after the IPO. We restrict ourselves to these values for $N(0)$ for the clarity of the graph. We observe that the aggregate company stake declines over time and that the speed of adjustment towards the long-run equilibrium is, on average, faster for companies with a higher initial number of significant insiders than for companies with a lower initial number of significant insiders. Finally, in REITs with a higher initial number of significant insiders, the long-run aggregate insider stake tends to be higher, on average, than for companies with a smaller number of significant insiders.
Ownership Dynamics with Multiple Insiders

Figure 3.3. Dynamics of the aggregate insiders’ ownership by the initial number of insiders, $N(0)$: We consider insiders that own at least 1% of shares outstanding at some time during the first ten post-IPO years. Our universe consists of 59 REITs that went public between January 1986 and December 1994, that appear in CRSP data files and for which we have been able to construct time series of insiders’ ownership. In this graph, we use data only for the 35 REITs that had one, two, five or six significant insiders at the IPO date.

3.4.2 Aggregate Insider Ownership Evolution

The first prediction is that the aggregate insider stake gradually declines over time. To test for the decline, we regress the aggregate insider stake, $A(t)$, on elapsed time from IPO, $t$ and $\ln(t)$. We use the logarithm of elapsed time because the insiders’ stake does not decrease linearly over time; the decrease is initially speedier. We express $A(t)$ in percentage in order to facilitate economic interpretation. Table 3.3 reports the results, confirming the decrease in aggregate insiders’ holdings over time since the IPO. The coefficients for $t$ and $\ln(t)$ are both significant at a 1% confidence level, and, on average, insiders reduce their aggregate holdings by around 0.05% monthly.

The second prediction of the model can be formulated as follows: the long-run aggregate insider stake is larger for companies with a larger number of insiders and smaller for companies with a smaller number of insiders. We estimate the long-term equilibrium, $A^*$, as the average aggregate insiders’ ownership during the 10th year after the IPO, that is $A^* = \frac{1}{12} \sum_{t=109...120} A(t)$. In Table 3.4 we report the results of regressing the long-run aggregate insider stake, $A^*$, on the initial number of insiders. The coefficient for $N(0)$ is positive and significant. The finding is consistent with our model, that average long-run aggregate insider stake is increasing with the initial number of insiders. According to this regression, the marginal contribution of an additional insider to the aggregate equilibrium holdings is about 1.3%.
3.4 Empirical Analysis

Table 3.3. Regression of aggregate insiders’ stake on elapsed time since IPO: We consider only insiders that, at some time during the first five or ten years after the IPO, own more than 1% of the shares outstanding. Our universe of companies consists of 137 REITs that went public between January 1986 and December 1999, that appear in CRSP data files, for which we have been able to construct time series of insiders’ ownership and for which at least one insider has a stake higher than 1% at some time in the first five or ten years after the IPO. In this table $A(t)$ is expressed in percentage, instead of as a fraction, in order to facilitate the economic interpretation of the coefficients.

<table>
<thead>
<tr>
<th>Subsample for which we have five years of data</th>
<th>Subsample for which we have ten years of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$A(t)$</td>
<td>$A(t)$</td>
</tr>
<tr>
<td>ln($t$)</td>
<td>$-0.852^{**}$</td>
</tr>
<tr>
<td></td>
<td>($-0.048^{**}$</td>
</tr>
<tr>
<td></td>
<td>($0.056^{**}$</td>
</tr>
<tr>
<td>$t$</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
</tr>
</tbody>
</table>

Robust $t$-statistics are in parentheses.
+ significant at 10%; * significant at 5%; ** significant at 1%.

Because long-term equilibrium holdings may be a function of the initial aggregate insider’s holdings), vis-à-vis the REIT’s overall capitalization and expected returns, we control for the REIT size, the initial aggregate insider’s holdings, $A(0)$, and the returns reported by the FTSE NAREIT Equity REITs Index during years eight and nine after the IPO. Our statistical results, as shown in Table 3.4, are unchanged.

The third prediction of our model is that the speed of adjustment of the aggregate insider stake towards the long-term equilibrium increases as a function of the original number of insiders, $N(0)$. We define the speed of adjustment during the first five years after the IPO by the expression: $\text{SpdAdj} = -(\ln(A(60)) - \ln(A(0)))$.

In Table 5 we report the results of regressing the initial speed of adjustment for the original number of insiders, $N(0)$. As our model suggests, the coefficient for $N(0)$ is positive and significant, controlling for the REIT’s size. According to this regression, the marginal contribution of one more original insider to the speed of adjustment during the first five years of the REIT’s life is about 17%.

In summary, the empirical evidence is consistent with the main empirical predictions.
of our theoretical model.

### 3.4.3 Robustness Checks

We perform several robustness checks. First, we include operating partnership (OP) units in the analysis. This is done by including OP units in the aggregate insider ownership and, separately, by excluding UPREITs from the sample. Second, we control for REIT cohort effects and focus on the most recent sub-sample. Finally, we control for additional variables including REIT initial market capitalization or REIT type.

#### Operating Partnerships

The umbrella partnership, or UPREIT, structure has been popular among more recently created REITs. This form enables individuals to transfer their property to an umbrella partnership, also known as an operating partnership, in exchange for operating partnership (OP) units. In general, OP unit holders defer capital gains taxation until the holder of the OP units chooses to convert them to common shares. OP units generally receive the same distributions (dividends) as do ordinary common stock shares. An insider should consider her OP units as a part of her risky stake in a REIT when determining a divestment plan; however, OP unit holders usually have significant tax disincentives, causing them to “hold” their OP units.

Each of the OP units for our REITs is exchangeable for a single common share, and we therefore treat the OP units as equivalent to shares. Further, data limitations force us to assume that all holders of minority OP interests (i.e., interests in the operating partnership other than that held by the REIT itself) are insiders whom we have already identified by their direct holdings of common shares. We hence redefine aggregate insider ownership as the sum of common shares in insiders’ hands plus the share equivalents of OP units, as a fraction of the total outstanding common shares plus OP units:

\[
\text{aggregate insider’s ownership} = \frac{\text{aggregate insiders’ holdings} + \text{OP units outstanding}}{\text{common shares outstanding} + \text{OP units outstanding}}.
\]

Furthermore, given that the timing of taxable events frequently drives the conversion of OP units to common shares and cash, the inclusion of UPREITs in the analysis may be a source of noise. The results of the analysis including OP units and excluding UPREITs are similar to those in the previous section.

The statistical results of the regressions including OP units in insider ownership are given in Tables 3.6, 3.7 and 3.8. The statistical findings contained in Tables 3.6, 3.7 and 3.8 are similar to those found in Tables 3.3, 3.4 and 3.5, respectively. The speed of adjustment regressions in Table 3.8 (compared to Table 3.5) are not statistically significant, perhaps explained by the tax motivation timing of the sales for OP units. On balance, the inclusion of OP units provides additional supportive evidence for
our model’s three hypotheses.

The results of the regressions excluding UPREITs (tables omitted for brevity and available upon request) are not conclusive because of the predominance of UPREITs, which leads to a small sample size, and the influence of two outliers, Transcontinental Realty Investors and Fog Cutter Capital Group. For instance, insider ownership in Transcontinental Realty Investors rose from an initial level of less than 5% to around 80% after five years.

Robustness Over Time

In a second set of robustness checks (tables also omitted for brevity and available upon request), we examine the stability of the model’s implications over time. We have repeated the previous analysis excluding all REITs that became public before 1994, and the signs of all the coefficients are consistent with our hypothesis, although we lose the significance of some estimates, an outcome that may be, among other structural REIT market changes, attributable to the smaller sample.

Furthermore, we have examined whether the REIT’s cohort affects the results, given the temporal clustering in REITs IPOs. Hence, we have analyzed separately the sets of REITs that became public in 1994 and 1997, two of the years with the largest IPO activity. The results of the analysis by cohort are the following. First, the decline in ownership during the first five years is statistically significant at 1% level in both sub-samples. Second, the dependence of long term equilibrium on the number of insiders can be tested only for the 1994 sample and, though the coefficients are not significant (we have only 25 REITs in this subset with the necessary data), all of them have the correct sign and are economically significant. One additional initial insider would increase the long-term equilibrium aggregate insider ownership by more than 1%. Third, the hypothesis that the initial speed of adjustment is increasing in the number of insiders holds for the 1994 sample, but oddly not for the 1997 subsample; in this case we also attribute this to the fact that we have data for only 12 REITs that became public in 1997.

Other Robustness Analyses

In a last round of robustness analyses, we have included the market capitalization and REIT type as control variables. We have repeated the previous regressions, including the initial REIT’s size without changing the statistical results. Finally, controlling either for the REIT’s property focus or the more finely divided property subtype from the SNL dataset does not affect the statistical significance of the decline in insider ownership.
Table 3.4. Regression of $A^*$ on the initial number of insiders $N(0)$: Regression of the long-run aggregate insider stake, $A^*$, on the initial number of insiders, $N(0)$, the initial aggregate insiders’ holdings, $A(0)$, natural logarithm of the REIT’s size, $Size$, and the returns on the FTSE NAREIT Equity REITs Index during years eight and nine after the IPO, $Ret$. The regression is estimated using robust standard errors. We estimate the long-term equilibrium, $A^*$ as the average aggregate insiders’ stake between 109 and 120 months after the IPO. We consider only insiders that, at some time during the ten years after the IPO own more than 1% of the shares outstanding. Our universe contains 59 REITs that went public between January 1986 and December 1994, that appear in CRSP data files and for which we have been able to construct time series of insiders’ ownership.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(0)$</td>
<td>0.026**</td>
<td>0.017*</td>
<td>0.013+</td>
<td>0.017*</td>
<td>0.013+</td>
</tr>
<tr>
<td></td>
<td>(3.56)</td>
<td>(2.40)</td>
<td>(1.85)</td>
<td>(2.28)</td>
<td>(1.79)</td>
</tr>
<tr>
<td>$A(0)$</td>
<td>0.210**</td>
<td>0.219**</td>
<td>0.210**</td>
<td>0.219**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.98)</td>
<td>(2.98)</td>
<td>(2.96)</td>
<td>(2.95)</td>
<td></td>
</tr>
<tr>
<td>$Size$</td>
<td>-0.025*</td>
<td>-0.025*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.40)</td>
<td>(2.47)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Ret$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.12)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.28)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.028</td>
<td>0.019</td>
<td>0.520*</td>
<td>0.015</td>
<td>0.515*</td>
</tr>
<tr>
<td></td>
<td>(1.07)</td>
<td>(0.82)</td>
<td>(2.39)</td>
<td>(0.39)</td>
<td>(2.22)</td>
</tr>
<tr>
<td>Observations</td>
<td>59</td>
<td>59</td>
<td>59</td>
<td>59</td>
<td>59</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.18</td>
<td>0.26</td>
<td>0.35</td>
<td>0.26</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Robust $t$-statistics are in parentheses.
+ significant at 10%; * significant at 5%; ** significant at 1%.
Table 3.5. **Regression of speed of adjustment of A during the first five years of the REIT’s life on the initial number of insiders:** We define the initial speed of adjustment of \( A \) as \( \text{SpdAdj} = -(\ln(A(60)) - \ln(A(0))) \). \( Size \) is the average, from monthly observations during the first five years of the REIT’s life, of the natural logarithm of the REIT’s size. The regression has been estimated with robust standard errors. We consider only insiders that, at some time during the first five years of the REIT’s life, own more than 1% of the shares outstanding. Our universe contains 130 REITs that became public between January 1986 and December 1999, that appear in CRSP data files and for which we have been able to construct time series of insiders’ ownership. Three of these REITs have not been included in the regression because the aggregate ownership of significant insiders was zero at the IPO date.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N(0) )</td>
<td>0.174**</td>
<td>0.168*</td>
</tr>
<tr>
<td></td>
<td>(2.68)</td>
<td>(2.50)</td>
</tr>
<tr>
<td>( Size )</td>
<td></td>
<td>0.196</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.63)</td>
</tr>
<tr>
<td>( \text{Constant} )</td>
<td>-0.495</td>
<td>-4.165+</td>
</tr>
<tr>
<td></td>
<td>(1.55)</td>
<td>(1.94)</td>
</tr>
<tr>
<td>Observations</td>
<td>127</td>
<td>127</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.04</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Robust \( t \)-statistics are in parentheses.

+ significant at 10%; * significant at 5%; ** significant at 1%.
Table 3.6. Robustness check: Regression of aggregate insiders’ stake including OP units on elapsed time since IPO. We consider only insiders that, at some time during the first five or ten years after the IPO, own more than 1% of the shares outstanding. Our universe of companies consists of 67 REITs that went public between January 1990 (we have SNL data starting at 1990) and December 1999, that appear in CRSP and SNL data files, for which we have been able to construct time series of insiders’ ownership and for which at least one insider has a stake higher than 1% at some time in the first five or ten years after the IPO. OP units data has been obtained from the SNL database.

<table>
<thead>
<tr>
<th>Subsample for which we have five years of data</th>
<th>Subsample for which we have ten years of data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>ln(t)</td>
<td>$A(t)$</td>
</tr>
<tr>
<td>-0.013**</td>
<td>-0.041**</td>
</tr>
<tr>
<td>(3.91)</td>
<td>(13.71)</td>
</tr>
<tr>
<td>$t$</td>
<td>-0.001**</td>
</tr>
<tr>
<td>(4.68)</td>
<td>(15.78)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.296**</td>
</tr>
<tr>
<td>(25.89)</td>
<td>(46.52)</td>
</tr>
<tr>
<td>Observations</td>
<td>5349</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Robust t-statistics are in parentheses.
+ significant at 10%; * significant at 5%; ** significant at 1%.
Table 3.7. Robustness check: Regression of $A^*$, including OP units, on the initial number of insiders $N(0)$. Regression of the long-run aggregate insider stake, $A^*$, on the initial number of insiders, $N(0)$, the initial aggregate insiders’ holdings, $A(0)$, natural logarithm of the REIT’s size, $Size$, and the returns on the FTSE NAREIT Equity REITs Index during years eight and nine after the IPO, $Ret$. The regression is estimated using robust standard errors. We estimate the long-term equilibrium, $A^*$, as the average aggregate insiders’ stake between 109 and 120 months after the IPO. We consider only insiders that, at some time during the ten years after the IPO, own more than 1% of the shares outstanding. Our universe contains 42 REITs that went public between January 1990 and December 1994, that appear in CRSP and SNL data files and for which we have been able to construct time series of insiders’ ownership including OP units. OP units data has been obtained from the SNL database.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N(0)$</td>
<td>0.017+</td>
<td>0.013+</td>
<td>0.012+</td>
<td>0.009</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(2.00)</td>
<td>(1.89)</td>
<td>(1.70)</td>
<td>(1.33)</td>
<td>(1.05)</td>
</tr>
<tr>
<td>$A(0)$</td>
<td>0.329**</td>
<td>0.348**</td>
<td>0.354**</td>
<td>0.385**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.44)</td>
<td>(5.15)</td>
<td>(4.68)</td>
<td>(5.62)</td>
<td></td>
</tr>
<tr>
<td>$Size$</td>
<td>-0.015</td>
<td>-0.020*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.39)</td>
<td>(2.07)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Ret$</td>
<td>0.386+</td>
<td>0.472*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.91)</td>
<td>(2.55)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.114**</td>
<td>0.028</td>
<td>0.325</td>
<td>-0.061</td>
<td>0.335+</td>
</tr>
<tr>
<td></td>
<td>(3.61)</td>
<td>(1.03)</td>
<td>(1.45)</td>
<td>(1.38)</td>
<td>(1.71)</td>
</tr>
<tr>
<td>Observations</td>
<td>42</td>
<td>34</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.09</td>
<td>0.51</td>
<td>0.53</td>
<td>0.56</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Robust $t$-statistics are in parentheses.
+ significant at 10%; * significant at 5%; ** significant at 1%.
Table 3.8. Robustness check: Regression of speed of adjustment of A, including OP units, during the first five years of the REIT’s life on the initial number of insiders. We define the initial speed of adjustment of A as SpdAdj = −(ln(A(60)) − ln(A(0))). Size is the average, from monthly observations during the first five years of the REIT’s life, of the natural logarithm of the REIT’s size. The regression has been estimated with robust standard errors. We consider only insiders that, at some time during the first five years of the REIT’s life, own more than 1% of the shares outstanding. Our universe contains 67 REITs that became public between January 1990 and December 1999, that appear in CRSP and SNL data files and for which we have been able to construct time series of insiders’ ownership. OP units data has been obtained from the SNL database.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SpdAdj</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N(0)</td>
<td>0.065</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>(0.95)</td>
<td>(0.97)</td>
</tr>
<tr>
<td>Size</td>
<td></td>
<td>0.072</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.78)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.012</td>
<td>-1.416</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.72)</td>
</tr>
<tr>
<td>Observations</td>
<td>67</td>
<td>67</td>
</tr>
<tr>
<td>R2</td>
<td>0.01</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Robust t-statistics are in parentheses.
+ significant at 10%; * significant at 5%; ** significant at 1%.
3.5 Conclusions and Future Work

This paper develops a model of optimal ownership dynamics of multiple equally risk-averse insiders facing a moral hazard problem and tests the predictions of the model with data from the U.S. REIT industry. On the theoretical side, the paper extends the related one-agent models (DeMarzo and Urosevic (2006) and Edelstein et al. (2005)) to the situation with multiple strategic insiders. A solution for the equilibrium share price and the dynamics of the aggregate insider ownership stake is derived in two cases: when insiders can credibly pre-commit not to deviate from their optimal ownership policies; and in the more realistic case when such a commitment is not credible (i.e., the time-consistent case). In the latter case there is an additional strategic reason for a dynamic aggregate stake adjustment. A decrease in the aggregate insider stake raises the market risk premium and thus lowers the company valuation. Therefore, by selling more today, each insider hopes to decrease the incentive for others to sell in the future (since they will receive a lower share price). This creates a “race to diversify” and in equilibrium, the speed of adjustment toward the perfect risk sharing allocation increases with the number of insiders in the company. As a result we would expect: (a) the aggregate insider stake declines over time, (b) the long-run aggregate insider stake increases with the number of insiders and (c) the speed of adjustment towards the long-run equilibrium level also increases with the number of insiders.

Theoretical predictions of the model are, then, tested on data from the U.S. REIT industry. To the best of our knowledge, no empirical work prior to this one has studied the evolution of insider ownership of REITs. Related empirical studies on REITs, such as Cannon and Vogt (1995), Friday et al. (1999) and Capozza and Seguin (2003), focus on static cross-sectional analysis of impact of ownership concentration on REIT prices, not on predicting REIT insider ownership dynamics. In addition, no paper previously studied the dependence of the long-run level of insider ownership and/or the speed of adjustment towards that level on the number of insiders in a company (for either REITs or non-REITs).

There are several possible interesting extensions both of the theoretical model and the empirical analysis that would be worthwhile to pursue in the future. On the theoretical side, the model rests on a number of other simplifying assumptions that would be interesting to relax. For tractability, for example, we assumed that the only strategic interaction between insiders occurs through their market risk premium impact. In our model an insider is merely a large stakeholder, but “insiders” may differ in many other ways. Including other types of “insider” interactions would make the model less tractable but, nevertheless, interesting to pursue. Also, De-Marzo and Urosevic (2006) and, especially, Edelstein et al. (2005) demonstrated that the private benefits of control may have a significant impact on an insider’s dynamic trading policy. Incorporating private benefits of control would enable one

---

19 For example, one could include in the expression for the expected dividends an interaction term proportional to $e_i e_j$. In that case, even when investors are risk-neutral, one would expect the model to exhibit a non-trivial strategic interaction between the agents.
to gain insight into the dynamics of strategic corporate control issues.\footnote{For example, it would be interesting to consider a dynamic strategic game between one owner who can extract private benefits of control (say, a manager) and another insider who cannot extract benefits of control but still is subject to a moral hazard problem (say, a worker). Such a model could be of particular use when attempting to explain the evolution of ownership in transition economies. The work in these areas is currently in progress.} With or without benefits of control, executive stock options and other forms of incentive compensation may interact with insider holdings of stock. This could be treated formally, or empirical tests could account for effective shareholding through options. Finally, the model is mute on the issue of the initial insider stake creation. These stakes appear naturally in an IPO mechanism (see Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006)). It would be interesting to apply this model in the context of the IPO literature. On the empirical side, it is natural to extend the analysis from REITs to non-REIT corporations. In particular, it may be interesting to pursue empirical work in the context of the empirical IPO literature.

### 3.A Proofs of the propositions

#### 3.A.1 Preliminaries

At each instant, insiders simultaneously choose their efforts in order to maximize the instantaneous certainty equivalent of their payoff:\footnote{The derivation of this expression follows along the same lines as in the one-agent case and is omitted for brevity (see the third section of DU (2006) for more details).}

\[
\begin{align*}
  z_l'(\alpha_l(t), \beta_l(t)) &= \max_{\alpha_l(t)} \alpha_l(t) \hat{\mu}(e_l) - f(e_l) - \frac{1}{2} \alpha_l^2(t) \gamma r \sigma^2, \ l = 1, \ldots, N. \quad (3.A.1)
\end{align*}
\]

The instantaneous certainty equivalent is equal to the total expected dividends received by the insider, net of her cost of effort and adjusted for the insider’s risk aversion. Here, \( \alpha_l^2(t) \sigma^2 \) captures the variance of the dividends received by an insider, \( \gamma \) is the insider’s risk aversion, and the scaling by the interest rate \( r \) appears since the insider can smooth shocks over time. Given \( \hat{\mu}(e) \) and \( f(e_l) \), the maximization problem in (3.A.1) is solved by \( e_l'(t) = \alpha_l'(t) \mu_l, \ l = 1, \ldots, N \). Thus, the certainty-equivalent payoff flow to each insider can be rewritten entirely in terms of her and other insiders’ holdings:

\[
\begin{align*}
  z_l'(\alpha_l(t), \beta_l(t)) &= (\mu_l - \gamma r \sigma^2) \alpha_l^2(t)/2 + \mu_l \alpha_l'(t) \beta_l(t), \ l = 1, \ldots, N. \\
\end{align*}
\]

Whenever confusion does not arise we shall drop in the above expression the index \( l \) and write it, instead, for a generic insider, as

\[
\begin{align*}
  z'(\alpha(t), \beta(t)) &= (\mu_l - \gamma r \sigma^2) \alpha^2(t)/2 + \mu_l \alpha(t) \beta(t) \quad (3.A.2)
\end{align*}
\]

The instantaneous certainty equivalent of an insider is a function of that insider’s
holdings, given the aggregate holdings of all other insiders. The quantity \( \mu \geq 0 \) measures the expected free cash flow sensitivity with respect to the change in corporate ownership and thus parameterizes the importance of the moral hazard problem in this model.

Equation (3.A.2) determines the total risk-adjusted payoff to an insider from holding a fraction \( a \) of the firm. It is useful to restate this in terms of the marginal value of ownership to the insider:

\[
\frac{\partial z}{\partial \alpha}(\alpha(t), \beta(t)) = \mu A(t) - \alpha(t) \gamma r \sigma^2.
\]  

(3.A.3)

That is, the marginal value of a share to the insider is simply the expected dividend per share, \( \mu A(t) \), adjusted by the insider’s “risk premium” given holdings \( \alpha(t) \). In fact, to find the total risk-adjusted payoff it is sufficient to know \( \frac{\partial z}{\partial \alpha} \), since \( z(\alpha, \beta) = \int_0^\alpha d\alpha \frac{\partial z}{\partial \alpha}(\hat{\alpha}, \beta) \).

An analogous expression can be derived for the aggregate investor.\(^{22}\) Namely, the marginal value of a REIT share from the aggregate investor’s perspective is given by the expression

\[
\nu(A(t)) = \mu A(t) - (1 - A(t)) \gamma^I r \sigma^2
\]  

(3.A.4)

That is, the marginal value of a share to the aggregate investor is the expected dividend per share, \( \mu A(t) \) but, in contrast to (3.A.3), adjusted by the aggregate investor’s risk premium given holdings \( (1 - A(t)) \). Equations (3.A.3) and (3.A.4) summarize the primitives of the model.

3.A.2 Proof of Proposition 6

In this equilibrium, each agent maximizes her certainty equivalent taking into account other agents’ equilibrium allocations. Denoting for simplicity \( \alpha^I = \alpha \). Expression (3.3.1) follows from the expression for an insider’s certainty equivalent (analogous to DeMarzo and Urosevic (2006) Lemma 1) upon substituting an expression similar to DeMarzo and Urosevic (2006) Equation (20) for the share price. Indeed:

\[
k(\alpha(t)) = \int_{[t, \infty]} e^{-r(t-\tau)}[z(\alpha(t), \beta(t))d\tau - d\alpha(t)V(\alpha(t))]
\]

\[
= \int_{[t, \infty]} e^{-r(t-\tau)}z(\alpha(t), \beta(t))d\tau - \int_{[t, \infty]} e^{-r(s-t)} \int_s^\infty e^{-r(\tau-s)} \nu(\alpha(\tau) + \beta(\tau))d\tau d\alpha(s)
\]

\[
= \int_{[t, \infty]} e^{-r(t-\tau)}z(\alpha(t), \beta(t))d\tau - \int_t^\infty e^{-r(\tau-t)} \left( \int_t^{[t, \tau]} d\alpha(s) \right) \nu(\alpha(\tau) + \beta(\tau))d\tau
\]

\[
= \int_t^\infty e^{-r(\tau-t)}z(\alpha(t), \beta(\tau))d\tau + \int_t^\infty e^{-r(\tau-t)}(\alpha(t) - \alpha(\tau))\nu(\alpha(\tau) + \beta(\tau))d\tau,
\]

\(^{22}\)The derivation of this expression practically coincides with the one-agent case of DeMarzo and Urosevic (2006) and is, thus, omitted for brevity.
Given her own initial allocation $\alpha^-$ and the other agents’ aggregate equilibrium allocation choice $\beta$, an agent’s best response is given by the first order condition:

$$
\alpha^c = \frac{r\gamma^I \sigma^2}{\mu + r(\gamma + 2\gamma^I)\sigma^2}(1 - \beta) + \frac{\mu + r\gamma^I \sigma^2}{\mu + r(\gamma + 2\gamma^I)\sigma^2}\alpha^-
$$

(3.A.5)

Summing up $N$ identical equations 3.A.5, taking into account that $\sum_i \alpha^i = A$ and $\sum_i \beta^i = (N - 1)A$, and solving for the aggregate allocation $A^c$ one obtains (3.3.2).

3.A.3 Proof of Proposition 7

From Lemma 1 in DeMarzo and Urosevic (2006), and a calculation similar to that in the proof of Proposition 1, it follows that if the price-taking equilibrium exists, then the equilibrium allocation is given, for each insider, by the following expression (here, each insider is taking $\beta$ and $A^p$ as given):

$$
\arg \max_{\alpha} z(\alpha, \beta) - \alpha \nu(A^p(\tau)).
$$

This implies that, in such equilibrium, the optimality conditions $\gamma \alpha^p - \gamma^I (1 - A^p) = 0$ need to be satisfied for each insider. In addition, an equilibrium condition is needed that states that the sum of each insider’s holdings $\alpha^p$ is equal to the aggregate holdings $A^p$. Summing up $N$ identical equations and solving for $A^p$ renders (3.3.4) as well as $\alpha^p = A^p/N$. The second-order condition reads: $\frac{\partial^2 z}{\partial \alpha^2}(\alpha, \beta) = (\mu - r\gamma\sigma^2) < 0$ which completes the proof.

3.A.4 Proof of Proposition 8

By backward induction from DeMarzo and Urosevic (2006) Equation (20), it follows that the share price at time $T$ can be written as:

$$
V(\bar{\alpha}(T)) = V_T(\alpha_T, \beta_T) = v_{0T} + v_T A_T = v_{0T} + v_T(\alpha_T + \beta_T),
$$

$$
v_{0T} = -\gamma^I \sigma^2, \ v_T = \mu/r + \gamma^I \sigma^2.
$$

(3.A.6)

Note that the share price at the last trading date is an affine function of the aggregate insider holdings. Next, each insider chooses her holdings at time $T$ in such a way as to maximize the certainty equivalent at time $T$, $J_T$:

$$
J_T = \max_{\alpha_T} z(\alpha_T, \beta_T)\delta_T + V_T(\alpha_T, \beta_T)(\alpha_{T-1} - \alpha_T)
$$

(3.A.7)

From 3.A.7, the following first order conditions are obtained:

$$
\frac{1}{r} \frac{\partial z}{\partial \alpha}(\alpha_T, \beta_T) + \frac{\partial V}{\partial \alpha}(\alpha_T, \beta_T)(\alpha_{T-1} - \alpha_T) - V(\alpha_T, \beta_T) = 0
$$
Using the fact that $V_T(\alpha_T, \beta_T)$ is an affine function of $\alpha_T$ and $\beta_T$ and that $z$ is a quadratic function in $\alpha_T$ and linear in $\beta_T$ (see 3.A.2), the first order conditions become:

$$n_1 T \alpha_T + n_2 T \beta_T + n_3 T \alpha_{T-1} + n_4 T = 0,$$

where

$$n_1 T = \mu / r - \gamma \sigma^2 - 2 v_T, \quad n_2 T = \mu / r - v_T, \quad n_3 T = v_T, \quad n_4 T = -v_0 T$$

(3.A.8)

Summing up the equations 3.A.8 and solving for the aggregate allocation, one can show that the aggregate insider stake at time $T$ is an affine function of the aggregate insider holdings in the previous period:

$$A_T = -\frac{n_3 A_{T-1}}{n_5 T} - \frac{N n_4 T}{n_5 T}$$

(3.A.9)

Substituting 3.A.9 back into 3.A.8 leads to the following specifications of $\alpha_T$ and $\beta_T$ in terms of the insiders’ holdings at time $T - 1$:

$$\alpha_T = l_{T}^{\alpha, \alpha} \alpha_{T-1} + l_{T}^{\alpha, \beta} \beta_{T-1} + l_{T}^{\alpha, 0},$$

$$\beta_T = l_{T}^{\beta, \alpha} \alpha_{T-1} + l_{T}^{\beta, \beta} \beta_{T-1} + l_{T}^{\beta, 0}.$$  

(3.A.10)

Here, coefficients $l$ are defined as:

$$l_{T}^{\alpha, \alpha} = \frac{n_3 T (n_2 T (2 - N) - n_1 T)}{(n_1 T - n_2 T) n_5 T},$$

$$l_{T}^{\alpha, \beta} = \frac{n_2 T n_3 T}{(n_1 T - n_2 T) n_5 T},$$

$$l_{T}^{\alpha, 0} = \frac{n_4 T}{n_5 T},$$

$$l_{T}^{\beta, \alpha} = \frac{n_3 T}{n_5 T} - l_{T}^{\alpha, \alpha},$$

$$l_{T}^{\beta, \beta} = \frac{n_3 T}{n_5 T} - l_{T}^{\alpha, \beta},$$

$$l_{T}^{\beta, 0} = \frac{n_4 T}{n_5 T} N - l_{T}^{\alpha, 0}.$$  

Therefore, the optimal holdings for each insider at time $T$ depend on her own past holdings as well as on those of the other insiders. Importantly, this dependence is affine. In addition, each insider’s optimal holdings are positively correlated with her own holdings at the preceding time period, i.e., $l_{T}^{\alpha, \alpha} > 0$, and negatively correlated with all of the other insiders’ holdings at the preceding time period, i.e., $l_{T}^{\alpha, \beta} < 0$.

In order to proceed to $t = T - 1$, note that the share price at time $t = T - 1$ is, again, an affine function of the aggregate holdings. Indeed, from DeMarzo and Urosevic (2006) Equation (20) and 3.A.10 it follows that

$$V_{T-1}(\alpha_{T-1}, \beta_{T-1}) = \delta \nu (A_{T-1}) + e^{-r \Delta T} V_T (\alpha_T(\alpha_{T-1}, \beta_{T-1}), \beta_T(\alpha_{T-1}, \beta_{T-1})).$$
Using the definition of $\nu$ and utilizing 3.A.6 and 3.A.9, one can again express $V_{T-1}$ as a function of the aggregate holdings:

$$V_{T-1}(\alpha_{T-1}, \beta_{T-1}) = v_{0T-1} + v_{T-1}A_{T-1} = v_{0T-1} + v_{T-1}(\alpha_{T-1} + \beta_{T-1})$$

$$v_{0T-1} = -\delta\gamma^2\sigma^2 r + e^{-r\Delta}(v_{0T} - N v_T n_4T / n_5T),$$

$$v_{T-1} = \delta(\mu + \gamma^2\sigma^2 r) - e^{-r\Delta}(n_3T v_T / n_5T) \quad (3.11)$$

The value function $J_T$, obtained upon the substitution of the expressions for optimal holdings 3.A.10 into the objective function 3.A.7, is a quadratic form in $(\alpha_{T-1}, \beta_{T-1})$:

$$J_T = J^a_T \alpha_{T-1} \alpha_{T-1} + J^b_T \beta_{T-1} \beta_{T-1} + J^c_T \alpha_{T-1} \beta_{T-1} + J^d_T \alpha_{T-1} + J^e_T \beta_{T-1} + J^f_T. \quad (3.12)$$

Here, with some abuse of notation, we introduced on the right-hand side coefficients $J$s while on the left hand side $J$ is the value function. In order to establish 3.A.12, it is sufficient to note that affine transformations map a quadratic form into another quadratic form. Then, 3.A.12 follows immediately from 3.A.6, 3.A.7 and 3.A.10.

So far, we have determined the optimal insiders’ holdings at time $T$ given their holdings as well as the holdings of all of the other insiders at time $T-1$. Clearly, at time $t = T - 1$, insiders are facing a very similar problem, as portrayed in 3.A.13:

$$J_{T-1} = \max_{\alpha_{T-2}} z(\alpha_{T-1}, \beta_{T-1}) \delta + V_{T-1}(\alpha_{T-1}, \beta_{T-1})(\alpha_{T-2} - \alpha_{T-1}) + e^{-r\Delta} J_T. \quad (3.13)$$

Note that the last term in 3.A.13 is a quadratic form in $(\alpha_{T-1}, \beta_{T-1})$ (see 3.A.12). Proceeding by backward induction one can obtain, eventually, the insiders’ optimal holdings at time $t = 1$ as a function of their initial holdings. Indeed, suppose that the relationships 3.3.5-3.3.7 and 3.A.14-3.A.17 are valid for $t+1$. One can easily see that, then, that they are valid for $t$ as well. Indeed, each insider’s maximization problem reads:

$$J_t = \max_{\alpha_t} \delta z(\alpha_t, \beta_t) + V_t(\alpha_t, \beta_t)(\alpha_{t-1} - \alpha_t) + e^{-r\Delta} J_{t+1}$$

where, by assumption, 3.3.5 and 3.3.6 hold. Consequently, the optimal insiders’ holdings are easily seen to yield 3.3.7 as well as 3.A.14-3.A.17. Utilizing these expressions it is immediate to see that the equilibrium share price at time $t - 1$ is a linear function of the aggregate insider holdings at time $t$ and 3.A.16. Using (3.3.7) and the linearity of the share price, value function $J_t$ can be re-written as a quadratic form in variables $(\alpha_{t-1}, \beta_{t-1})$. Reading off the appropriate coefficients in $J_t$ establishes 3.A.17. In order to establish the second order conditions, note that they are equivalent to $n_{it} < 0$, where $n_{it}$ is given by the first expression in 3.A.15. Using 3.A.14-3.A.17, as well as the fact that $\mu \geq 0$ and that volatility and the insiders’ risk aversion are positive constants, one shows, working backwards period by period, that $n_{it} < 0$ and, thus, that the equilibrium is a unique sub-game perfect equilibrium under the assumptions of the model. Note that Assumptions A-C in DeMarzo and Urosevic (2006), under which they establish the existence and uniqueness of a sub-game perfect equilibrium for $N = 1$, coincide with $\mu \geq 0$ and
the constant volatility assumptions under which unique sub-game perfection exists for \( N > 1 \) in this model.

The coefficients in (3.3.7) are determined as follows:

\[
\begin{align*}
I_t^{\alpha,\alpha} &= \frac{n_{3t}(n_{2t}(2 - N) - n_{1t})}{(n_{1t} - n_{2t})n_{5t}}, \\
I_t^{\alpha,\beta} &= \frac{n_{2tn_{3t}}}{(n_{1t} - n_{2t})n_{5t}}, \\
I_t^{\alpha,0} &= -\frac{n_{4t}}{n_{5t}}, \\
I_t^{\beta,\alpha} &= -\frac{n_{3t}}{n_{5t}} - I_t^{\alpha,\alpha}, \\
I_t^{\beta,\beta} &= -\frac{n_{3t}}{n_{5t}} - I_t^{\alpha,\beta}, \\
I_t^{\beta,0} &= -\frac{n_{4t}}{n_{5t}} N - I_t^{\alpha,0}.
\end{align*}
\] (3.A.14)

The coefficients \( n_{it} \) are defined by the following relations:

\[
\begin{align*}
n_{1t} &= \delta(\mu_1 - \gamma \sigma^2 r) - 2v_t + 2e^{-r\Delta}f_{t+1}^{\alpha,\alpha}, \\
n_{2t} &= \delta \mu_t - v_t + e^{-r\Delta}f_{t+1}^{\alpha,\beta}, \\
n_{3t} &= v_t, \\
n_{4t} &= -v_0 + e^{-r\Delta}f_{t+1}^{\alpha,0}, \\
n_{5t} &= n_{1t} + (N - 1)n_{2t}.
\end{align*}
\] (3.A.15)

so that \( A_t = -\frac{n_{3t}}{n_{5t}} A_{t-1} - \frac{n_{4t}}{n_{5t}} \).

The following are the recursive relations that define the coefficients in (3.3.6):

\[
\begin{align*}
v_{0t-1} &= -\delta \gamma \sigma^2 r + e^{-r\Delta}[v_0 - Nv_t n_{4t}/n_{5t}], \\
v_{t-1} &= \delta(\mu + \gamma \sigma^2 r) - e^{-r\Delta}(n_{3t}v_t/n_{5t})
\end{align*}
\] (3.A.16)
The set of six recursive relations that defines the coefficients in (3.3.5) is listed below:

\[ J_t^{\alpha_0} = \delta(\mu_1 - \gamma t)(l_t^{\alpha_0})^2/2 + \delta \mu_1 J_t^{\alpha_0} l_t^{\alpha_0} + v_t(1 - l_t^{\alpha_0})(l_t^{\alpha_0} + l_t^{\beta_0}) \]
\[ J_t^{\alpha_0} = \delta(\mu_1 - \gamma t)(l_t^{\alpha_0})^2 + \delta \mu_1 J_t^{\alpha_0} l_t^{\alpha_0} + v_t(1 - l_t^{\alpha_0})(l_t^{\alpha_0} + l_t^{\beta_0}) \]
\[ J_t^{\alpha_0} = \delta(\mu_1 - \gamma t)(l_t^{\alpha_0})^2/2 + \delta \mu_1 J_t^{\alpha_0} l_t^{\alpha_0} + v_t(1 - l_t^{\alpha_0})(l_t^{\alpha_0} + l_t^{\beta_0}) \]

The boundary conditions read as follows: coefficients \( J_{t+1} \) vanish at time \( t = T \); the boundary values for the stock price coefficients are given by (3.3.6); and the initial insiders’ allocations are given by an (exogenous) vector \( \alpha^- \).
Bibliography


