

# **DOCTORAL THESIS**

Title

**“THREE-DIMENSIONAL SPATIAL DISTRIBUTION OF SCATTERERS  
IN THE CRUST BY INVERSION ANALYSIS OF S-WAVE CODA  
ENVELOPES. A CASE STUDY OF GAURIBIDANUR SEISMIC ARRAY  
SITE (SOUTHERN INDIA) AND GALERAS VOLCANO (SOUTH-  
WESTERN COLOMBIA)”**

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Aquesta Tesi Doctoral ha estat defensada el dia  
al Centre Observatori de l'Ebre  
de la Universitat Ramon Llull  
davant el Tribunal format pels Doctors sotasignants, havent obtingut la qualificació:



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*“I think however, that there isn’t any solution to this problem of education other than to realize that the best teaching can be done only when there is a direct individual relationship between a student and a good teacher, a situation in which the student discusses the ideas, thinks about the things, and talks about the things. It’s impossible to learn very much by simply sitting in a lecture, or even by simply doing problems that are assigned. But in our modern times we have so many students to teach that we have to try to find some substitute for the ideal. Perhaps my lectures can make some contribution. Perhaps in some small place where there are individual teachers and students, they may get some inspiration or some ideas from the lectures. Perhaps they will have fun thinking them through or going on to develop some ideas further.”*

The Lectures on Physics, Richard Feynmann, June, 1963



# 1 INTRODUCTION

Seismology is the primary tool for the study of the earth's interior. Because few kilometres in depth can be drilled, all the information on deeper depths comes from indirect methods. Seismograms provide the data used for mapping the earth's interior and for studying the distribution of physical properties. The analysis of seismograms is also useful for assessing the societal hazards posed by earthquakes.

Because of the complexity of the processes involved, the approach taken, in general, is to describe them with simplified models that seek to represent key elements of the process under consideration. A hierarchy of different approximations, as appropriate, are used as starting models for more detailed investigations. The most accurate earth model used in Seismology is a laterally heterogeneous sphere. This model is often approximated as being spherically symmetric, with properties varying only with radius. This spherically symmetric model can be further approximated for many purposes as a stratified half-space, in which properties vary only with depth, or as a layered half-space composed of discrete uniform layers (Stein and Wysession, 2003, [1]).

A type of model to represent the earth medium is often chosen, and then seismological and other data is used to estimate the parameters of this model. Thus, a characteristic activity of Seismology is to solve inverse problems. Inverse problems are complicated to solve, because seismograms reflect the combined effect of the source and medium, neither of which is known exactly. Moreover, the inverse problems often have no unique solutions and the model parameters that describe the observations well do not have to reflect the physical reality necessarily. As a consequence, it is necessary to consider issues of precision, accuracy, and uncertainty.

A homogeneous, isotropic, elastic, layered half-space is often used in crust and upper mantle studies, where the distance between source and receiver is less than few hundred kilometres. For larger source-receiver distances, spherical geometry is required. More complex and accurate models consider the anisotropic and anelastic behaviour of the earth, and lateral variations, or heterogeneities.

In addition to reflection and transmission at discrete interfaces, the reasons why seismic waves attenuate or decrease in amplitude as they propagate are: anelasticity (or deviation from elasticity), geometrical spreading, multipathing, and scattering. Anelasticity, also called intrinsic absorption, implies the conversion of seismic energy into heat and it differs from the other processes in that energy is lost, not just moved onto a different path. The geometrical spreading effect is due to the redistribution of energy that occurs as the wave front expands or contracts during seismic waves' propagation. Multipathing implies a focusing and defocusing of seismic waves by lateral variations in velocity.

Scattering is due to the interaction of seismic waves with the heterogeneities of the medium and it occurs depending on the ratio of the heterogeneity size to the wavelength and the distance the wave travels through the heterogeneous region (Aki and Richards, 1980, [2]). When the heterogeneity is large compared to the wavelength, the wave is regarded as following a distinct ray path that is distorted by multipathing. When the heterogeneities are closer in size to the wavelength, scattering occurs. When the heterogeneities are much smaller than the wavelength, they simply change the medium's overall properties.

Scattering is especially important in the continental crust, which has many small layers and reflectors resulting from continental evolution. These structures do not affect waves with wavelengths longer than tens of kilometres, but they act as point scatterers for shorter wavelength waves. Scattering is the cause of the presence, in high frequency (>1Hz) seismograms, of continuous wave trains following the direct S-wave which are known as coda waves. Array observations have shown that they are incoherent waves scattered by randomly distributed heterogeneities having random sizes and contrast of physical properties (Sato and Fehler, 1998) [10].

A number of models have been developed to explain the relationship between coda-waves' envelopes and the spectral structure of the random heterogeneity in the earth. The characterization of the earth as a random medium is complementary to the classical stratified media characterization.

In this thesis, coda waves' recordings from local earthquakes will be analyzed to estimate the three-dimensional spatial distribution of scatterers in the crust. For this

purpose, it is necessary to know how the scattered waves' energy is distributed spatially and as a function of time. Thus, some hypothesis about the media characteristics and how and where the scattering is produced are necessary. The existing models on the scattering process will be reviewed in Chapter 2.

These models for S-coda envelope synthesis are based on the assumption of a homogeneous distribution of isotropic scatterers and they predict results consistent with the observed characteristics of the coda. However, detailed observations show that there may be departures from the observed characteristics of S-coda waves which may be explained by an inhomogeneous distribution of scatterers. This issue will be the subject of Chapter 3.

The problem of estimating deterministically the spatial distribution of scatterers leads to an inversion process of a huge system of equations that can not be solved by traditional methods. They require the use of sophisticated numerical techniques. We are talking about systems of equations with about  $10^5$  unknowns and  $10^5$  equations. These sorts of problems were solved for the first time in medical tomography applications and, since then, the computational methods needed have been applied to other scientific fields. The first approach used to obtain three dimensional reconstructions was an iterative method called ART. Then, other methods based on ART soon appeared. Although these methods are very accurate and the reconstructions are of a high quality, they have an important drawback: they are terribly slow and not appropriate for real time applications. Nowadays, scanners are able to obtain three dimensional images by solving large systems of equations, not by using iterative approaches, but using a remarkably fast non-iterative algorithm: the Filtered Backprojection. The Filtered Backprojection method is based on an important mathematical definition, the Radon Transform, and a theorem that connects the Radon transform and the Fourier Transform of the three-dimensional object to be reconstructed. This theorem is the so-called Fourier Slice Theorem.

In Chapter 4 we will analyze in detail the ART, SIRT and Filtered Backprojection algorithms. ART algorithm has been previously used in other seismological studies (Chen and Long, 2000, [43]). SIRT reconstructions, which are based on ART, are less noisy and better looking than ART reconstructions at the

expense of computation time. We will use this algorithm for the first time in seismological applications. The Filtered Backprojection algorithm had never been used in Seismology because there is no simple way to adapt it to the kind of problem to solve. The Filtered Backprojection algorithm is very sensitive to the geometry defined by the problem to solve. Then, firstly, the Filtered Backprojection algorithm will be derived using a simple approach and, secondly, a generalization by taking into account the special geometry of our problem will be adapted to our case. This is the main mathematical contribution of this thesis.

Chapters 5 and 6 present two applications of the methodology to different geotectonic regions in the earth: a seismically stable region in southern India and an active volcano in south-western Colombia. The three-dimensional spatial distribution of relative scattering coefficients in southern India will be estimated by means of an inversion technique applied to coda wave envelopes recorded by the Gauribidanur Seismic Array (GBA). The inversion analysis will be performed for the first time in this kind of seismological research by means of the Simultaneous Iterative Reconstruction Technique (SIRT) and Filtered Back-Projection method (FBP). Finally, the three-dimensional spatial distribution of relative scattering coefficients will be estimated for the Galeras volcano, Colombia, by means of inversion analysis of coda wave envelopes and using the Filtered Backprojection algorithm. The scientific contribution of these applications is very important, since tomographic results confirm for the first time geological hypothesis on the structure of both regions. On the one hand, the presence of the Closepet granitic batholith to the east of GBA is revealed up to a depth of about 24 km. This granitic intrusion is one of the most important geological features of the region that acts as the major geological boundary in the region. It is believed to be a Precambrian suture zone between the eastern and western Dharwar craton in southern India. On the other hand, the present magmatic plumbing system of Galeras volcano sketched by geological evidences is also confirmed. Two zones of strong scattering are detected: the shallower one is compatible with the presence of a shallow magmatic chamber located at a depth from 4 km to 8 km under the summit. The deeper one is imaged at a depth of ~37 km from the Earth's surface and may be related to a deeper magma reservoir that feeds the system.

Parallel to the theoretical developments in this thesis, an important amount of work

corresponds to programming the numerical algorithms and graphic displays. Numerical algorithms were programmed in C++ using a free version of Borland C++ (BuilderX [3]) and a commercial compiler as Microsoft Visual C++ [4] to assure a high compatibility. The program codes are annexed at the end of the document. Some of the graphic representations were generated with the DISLIN graphic libraries from the Max Planck Institute of Solar Research [5]. Using DISLIN, programs written in C++ were developed to display results. Special look-up tables (pseudocolor) were designed to enhance the significance of the three-dimensional reconstructions. Three-dimensional representations were also developed. The outcome of this effort can be particularly noticed in Chapter 6.

